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INTERTEMPORAL FAIRNESS AS EQUALITY OF OPPORTUNITY

by

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There has been considerable discussion in the literature of the criteria for and characteristics of fair income distributions either viewed satistically or viewed intertemporally. The three major stands of thought concentrate on fair or equity restrictions on the Bergson welfare function (e.g. the literature concerning the use of the Gini coefficient as a measure of inequality); on fairness as lack of envy in the context of static commodity allocations and on fairness as distributive justice in the sense of a contract theory. On the other hand a common notion central to ideas of fairness is the equality of opportunity and it is of interest to note that in all but the Rawlsian contract theory this concept enters in a secondary rather than primary way.

For example the first strand of argument basically shows that if society's judgements on equality are expressed by strict concavity and symmetry of the Bergson welfare function together with Pareto optimality then these judgements provide a partial ordering of income distributions with respect to their equity and hence presumably their fairness (2). Fairness is expressed solely in terms of income received or consumption enjoyed with no consideration for the generation of that income. For example if two income distributions are compared one with Lorenz curve everywhere within the other then the former is judged to be fairer than the latter even though it may well be that in the former distribution the ways in which individuals earn their incomes (e.g.earned or unearned income) or the ways in which they dispose of their incomes may be less fair in some sense than in the latter. This approach then is conditional on the judgements that social welfare depends only on individual utilities which depend only on exogenous incomes. It is also true that this approach requires interpersonal comparisons either of utility or income and that when expressed in terms of utility a certain cardinality of individual utility is necessary.

The second strand of argument does not as such require interpersonal comparisons and does not necessarily seek to provide a social ordering for the evaluation of alternative social states although it can be extended to consider this problems (3,7). To avoid the difficulties inherent in the possibility of an individual envying the production possibilities and particularly the ability or technology of another, the emphasis is quite heavily on pure exchange systems in a static context.

The aims is to characterise either those commodity allocations (9) or those net commodity gains over the initial endowments (6) which have the property that each individual, using his own tastes, does not feel that he is worse off with his own allocation than with the allocation of some other individual. Such allocations are said to be fair. In the case of commodity allocations, the results concern rather weak conditions under which fair allocations exist and the remark that an equal income competitive equilibrium is fair while in the net trade case the results concentrate on examining the near equivalence between fair net trades and competitive equilibrium.

The third approach is very general so that there is danger of misleading in attempting any summary. The basic principle concerns the search for tenets of justice to which all individuals will freely agree in a notional original assembly of individuals which meets before individuals know who they will actually be (5). In a static context where all individuals who will ever exist do so simultaneously the perscription for economic justice is that social and economic inequalities are arranged so as to be to the greatest advantage of the least advantaged and that economic and social rewards should be attached to offices that are open to all. In the static context Rawls goes further than this and recommends that just economic states be characterised by maxmin policies i.e. by maximising the utility or income of the individual who is worst off. In the intertemporal context Rawls expresses reservations about this rule. The type of intertemporal situation considered is one in which there are nonoverlapping generations and the sole economic variable relevant to justice is the level of saving by each generation. Since any positive saving always benefits future generations relative to the initial generation then if all generations are assumed identical in terms of preferences and endowment and each generation's utility depends only on its own consumption a direct application of maxmin world Lead to zero saving. Hence "if the economy starts in poverty it is doomed to poverty". It is worth noting that the situation with respect to dissaving is symmetrical so that even if an institutional system permitting dissaving were possible it would not be optimal. Rawls argues that these problems are not relevant since it is universally the case that preferences of a given generation depend in a positive way on the preferences of succeeding generations. This provides each generation with a selfisch motivation to save and then a just savings role is determined in the original assembly by each individual who will ever exist examining how much he would save if he were to belong to any given generation on the assumption that all other generations would also save at his chosen rate. If for all individuals and all potential generations to which they could belong there is a consistent set of savings rates then these define a just set of savings rates. Note that although a just set of savings rates may not be constant across generations, they are nevertheless invariant under changes in the allocation of individuals to generations. While laying down this general prescription, Rwals does not consider the problem of calculating just savings rates in any great detail.

Economists investigating this problem have primarily adopted two approaches. Firstly some interest has been attached to examining the implications of maxmin policies in simple aggregate frameworks(4). In a range of simple growth models varying in their specification of population growth, technical progress and dependence on non-renewable natural resources Solow (8) has examined the existence and nature of maximum constant consumption paths. Secondly closely following Rawl's analysis, Dasgupta (1) has explored the nature of Nash equilibrium savings configurations in a one good model. Each generation has preferences of the form $u_t=u((1-s_t)k_t, u_{t+1}(s_{t+1}))$ where s_t is the savings ratio of generation t and k_t its income. Then each generation maximises its own preferences over s_t assuming that s_{t+1} is constant taking into account the effect its decisions have on future income via $k_{t+1}=s_t k_t$ and hence on its utility. The result is intertemporally consistent as of course will be any Nash equilibrium treating the different generations as the decision makers but because no account is taken of the marginal utility of consumption for different generations may not be Pareto efficient.

There are many extensions of and links between these three views of equity or fairness. If individuals have identical tastes than zero income inequality acrcss time in the Lorenz sense corresponds to maxmin policies over income. Similarly if individuals have identical tastes maxmin constant consumption paths in the solow setting are fair and are the only fair allocations which are efficient. The Dasgupta solution will not be fair in general but in the special case where justice requires a constant savings ratio over time on which Dasgupta focusses some attention his rule is not fair, since with growth in income and constant saving ratios earlier generations will envy later ones.

Some extensions beyond the frameworks used are easy to see. The maxmin criteriou does not depend on the one commodity assumption but only on the existence of well defined preferences. However it does require interpersonal comparability of the welfare levels (at least) of different generations. The Atkinson income inequality approach can similarly be applied to wealth rather than income distributions. The Dasgupta approach could in general be applied where each generation has several decision variables e.g. a multiasset model and each cares directly about the utility levels of soms finite group of descendants. However the approach will always yield zero savings as just if $\frac{\partial u_t}{\partial u_{t+j}} = 0$ for all j since then each generation cares only about its own consumption. This might be thought to be rather serious since altruism does not necessarily appear to be the type of stylised fact that is sufficiently established to be incorporated into the kinds of models used to discuss equity.

Indeed concentration on a multiasset framework avoids the need to incorporate such elements into the model to give a justice oriented motivation for saving since with several assets, saving in one form may be accompanied by dissaving in another form for any given generation. It does not then follow that only a zero saving path will avoid penalising the enjoyment of the first generation. It is easy to give formal definitions of just savings policies in fairly general intertemporal frameworks that preserve the Nash equilibrium and the equality of opportunity aspects of the concept. As a paradigm it is convenient to classify commodities as either consumption goods (commodities which positively or negatively have a direct effect on utility), capital goods (intermediate goods produced withen the system) and natural resources (productive commodities not produced). Labour services are then treated as a consumption good. Denoting vectors of these commodities at time t by c_t, k_t, z_t respectively at each time there is a production set $(c_t, k_{t+1}; k_t, z_t) \varepsilon \Omega_t$ giving producible outputs $(c_t \; k_{t+1})$ obtainable from imputs. Typically these will also be constraint functions of the form $z_t \leqslant \phi(z_{t-1} \ldots z_0)$ which are normally independent of calendar time. For example for nonrenewable mineral resources we may have $z_{it} \leqslant z_{io} - \sum_{s=1}^{t-1} z_{is}$ where z_{io} is the available initial stock, while for resources subject to a simple birth-death process we might have $z_{it} \leqslant z_{it}(1+n)$ where n is the net reproduction rate e.g. fish. Let Ω_t^* be the projection of Ω_t on the output coordinates i.e.

$$\{(c_t k_{t+1})/(c_t k_{t+1} k_t z_t)\} \varepsilon \Omega_t \varepsilon \Omega_t^*(k_t z_t)$$

then we can define vectors $(c_1 \ldots c_t \ldots k_1 \ldots k_t \ldots z_1 \ldots z_t \ldots)$

to be justice feasible if for all t

$$\Omega^*_t(k_{t1} z_t) = \Omega^*_{t+1}(k_{t+1} z_{t+1}) \text{ and } z_{t+1} \leqslant \phi(z_{t-1})$$

and to be just if for all t $(c_t k_{t+1} z_{t+1})$ maximises $u_t(c_t)$ over $(c_t k_{t+1} z_{t+1})$ satisfying $(c_{t1} k_{t+1}) \varepsilon \Omega^*_t$, $\Omega^*_t(k_t z_t) = r^*_{t+1}(k_{t+1} z_{t+1})$ and $z_{t+1} \leqslant \phi(z_t \ldots z_0)$. here u_t is a real valued function reflecting the preferences of generation t. The essentiel ingredient is recursivity of the production system so that each generation requires information only of its inhemted and bequeathed technology. If also $\Omega^*_t = \Omega^*$ then any generation faces the same constraints as any other and so might be said to be treated in a fair fashion. Always however if a just time path exists it will obviously be a Nash equilibrium and the intended interpretation is that each generation should inherit the same output possibilities although the structure of inputs may be very different. In general of course a fair policy in this sense will not be maxmin but it will be fair in the sense that no generation will envy any other generations output possibilities. Also in distinction to Rawl's arguments that the first generation is typically the loser, depending on the nature of the functions ϕ either all generations have a symmetrical vote in determining the possibilities of other generations or possibly through their power to influence the ϕ functions throughout the whole future the initial generation has an above average degree of control over the input composition of the future.

Before considering formal properties of this definition some simple examples will be considered to examine the types of solutions which arise with this concept. As a first step consider a model with two capital goods used to produce output which can be consumed or costlessly and on linear terms transformed into investment in either capital asset so that Ω_t is defined by

$$c_t+\Delta k_{1t}+\Delta k_{2t}=F(k_{1t}k_{2t})$$

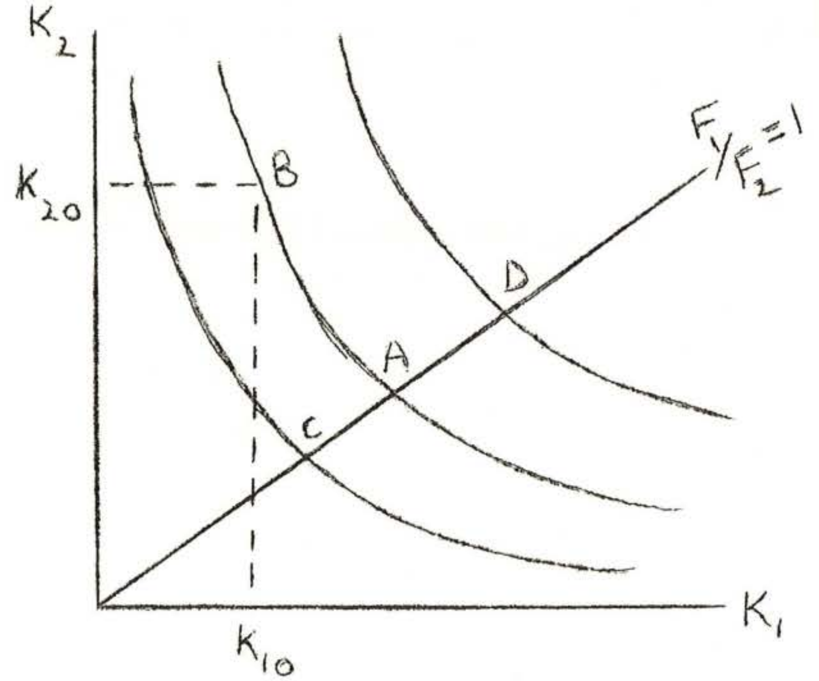
Since there is a single consumption good we may as well take preferences of any generation to be identical without loss of generality so that generation t determines its investments to maximise

$$c_t=F(k_{1t}k_{2t})-\Delta k_{1t}-\Delta k_{2t}$$

subject to $F(K_{1t}K_{2t})=F(k_1t+\Delta k_{1t},k_{2t}+\Delta k_{2t})$ so that the constraint states that each generation should inherit equal potential output. If F is taken to be concave with $k_{i0}>0$ and $F(k_1 0)=F(0_1 k_2)=0$ then necessary and sufficient conditions for a solution of this problem are

$$F(k_{1t}k_{2t})=F(k_{1t}+\Delta k_{1t},k_{2t}+\Delta k_{2t})$$
$$F_1(k_{1t+1},k_{2t+1})=F_2(k_{1t+1},k_{2t+1})$$

Each generation t=0,1... solves this problem so that if we assume F to be homogenous of some degree then the second equation above requises that the ratio of capital stocks be constant for t=1,2... at a level yielding a unit marginal rate of substitution. The first equation states that the particular scale of output with this ratio of capital stocks is set at the level of output corresponding to the initial capital stocks of the initial generation.



So every generation $t=1,2...$ beyond the initial one engages in zero investment in each asset and is at point A in the diagram and so enjoys constant consumption equal to the level of output of the initial capital stocks. However the initial generation is at point B of the deagram and, in the example given, invests in the first asset and disinvests in the second to move all later generations to A. The level of consumption of the first generation will then typically be higher than that of all later generations since by strict quasi concavity of F and the fact that the marginal rate of substitution is unity at A, the disinvestment exceeds the investment.

As an explicit example suppose $F=k_1^a k_2^{1-a}$. The necessary conditions become

$$k_1^a k_2^{1-a}=(k_1+\Delta k_1)^a (k_2+\Delta k_2)^{1-a}$$

$$1=\frac{a}{1-a}\,\frac{k_2+\Delta k_2}{k_1+\Delta k_1}$$

and simple calculation shows that $k_{2t}=\left(\frac{a}{1-a}\right)^{-a} k_{1o}^a k_{2o}^{1-a} \quad t\geq 1$

$$k_{1t}=\left(\frac{a}{1-a}\right)^{1-a} k_{1o}^a k_{2o}^{1-a}$$

so if $a<\frac{1}{2}$ $k_{2t}\Delta k_{2o}$ and $k_{1t}\Delta k_{1o}$ while if a $\frac{1}{2}$ the opposite inequalities hold; the first generation disinvests in the asset with lower product↑v↑ty.

It is of interest to explore some of the other solution concepts in the context of this simple example. Suppose there is some degree of altruism so that preferences for generation t can be written $v_t=u(c_t)+\beta u(c_{t+1})$ in the fashion of Dasgupta. Then his view of just savings would require

$$u'(c_t)=\beta u'(c_{t+1})F_1(t+1)$$

$$u'(c_t)=\beta u'(c_{t+1})F_2(t+1) \qquad \text{for all } t\geq 0$$

so that again $\frac{F_1(t+1)}{F_2(t+1)}=1$ For all $t\geqslant 0$ so that under homogeneity of F the ratio of capital stocks is again constant for all periods beyond the first. However the intertemporal marginal rate of transformation between consumptions is no longer necessarily constant so that consumption may not be constant. If we have constant returns to scale then F_i is constant for all $t\geqslant 1$.

Marginal private utilities u' then grow or fall exponentially at rate βF_i for all generations $t\geqslant 0$. The effect of this on the time pattern of concumption depends on the form of preferences but if we adopt the popular constant elasticity case

$$c_{t+1}=(\beta F_i)^{\frac{1}{\gamma}}c_t \qquad \gamma>0 \qquad t\geqslant 0$$

so that $c_t=(\beta F_i)^{\frac{t}{\gamma}}c_o$ for $t\geqslant 1$. Of course if one has aversion to one's heirs then the model behaves rather perversely. We can then solve for the time path of investments; we have $\frac{k_{1t}}{k_{2t}}$ constant $t\geqslant 1$ imphes $\frac{\Delta k_{2t}}{\Delta k_{1t}}=\frac{k_{2t}}{k_{1t}}$ $t\geqslant 1$ so that

$$c_t+\Delta k_{1t}+\Delta k_{2t}=(\beta F_i)^{\frac{t}{\gamma}}+\left[1+\frac{k_{1t}}{k_{2t}}\right]\Delta k_{2t}=F(k_{1t}k_{2t})=$$
$$k_{2t}f(\frac{k_{1t}}{k_{2t}}) \qquad \text{for } t\geqslant 1$$

where $\frac{k_{1t}}{k_{2t}}$ is constant. Writing $a=\beta F_i$, $b=1+\frac{k_{1t}}{k_{2t}}$, $d=f(\frac{k_{1t}}{k_{2t}})$ the solution is

$$k_{2t}=(1+\frac{d}{b})^{t-1}k_{21}-\sum_{s=1}^{t-1}(1+\frac{d}{b})^{s-1}\frac{a^{t-s}}{b} \qquad \text{for } t=1,2\ldots$$

and $k_{1t}=\frac{k_{1t}}{k_{2t}}\Delta k_{2t}+k_{1t-1}$ for $t=2,3\ldots$ c_o is then arbitrary but given any valve of c_o, Δk_{1o} and Δk_{2o} are determined by

$$c_o+\Delta k_{1o}+\Delta k_{2o}=F(k_{1o}k_{2o})$$

$$c_o^{-\gamma}=\beta c_1^{-\gamma}F_1(k_{11},k_{21})$$

and the fact that future ratio of capital stocks is known. In terms of the diagram, a profligate initial generation may condemn the first generation to the scale given by c or a more thrifty generation to the scale given by b. Again the initial generation is treated asymmetrically relative to all other generations; in a sens it is difficult to see how this could be rationalised in the context of Rawl's contract type theory. However the behaviour of a just economy is richer in this case than in the preceding case in that using or falling living standards are possible.

We can of course apply our formal definition to the case of altruistic preferences as well but then in this simple example the solution is actually independent of any altruism. A just path is defined by each generation t choosing investments to maximise $u(c_t)+\beta u(c_{t+1})$ subject $F(t)=F(t-1)$ which Leads to

$$u'(c_t)=\left[\beta u'(c_{t+1})+\lambda_t\right]F_i(t+1) \qquad i=1,2 \qquad t\geqslant 0$$

$$F(t)=F(t+1)$$

From the first two equations we have $\frac{F_1(t+1)}{F_2(t+1)}=1 \quad t\geqslant 0$

so then the solution is exactly the same as the case of no altruism; the initial generation carries out an investment policy to put the first generation at A and then all later generations remain at A.

With no altruism a maximin policy is also easy to define. As solow states except for trick cases a maxmin policy will require consumption constant over time. The maximum such consumption will be with zero investment in each asset and hence the economy starts and semains at point B in the diagram.

It is also clear that in terms of individual generation preferences all of there policies with the exception of maxmin are Pareto efficient. Necessary condition for Pareto efficiency are of course that each generation should be on the production frontier and that the marginal rate of substitution between assets viewed as outputs should be equal to that between the assets viewed as outputs for the next generation but since both are unity in each solution concept this condition certainly holds. Then given concavity of F. Pareto efficiency follows. One might at first sight expect that the type of consumption inefficiency which arises in a consumption loans model could appear. In other words if each generation disinvested a little more than was just and so raised their own consumption than a Pareto superior consumption sequence to the just sequence could be constructed. On reflection this does not appear to be possible because with only finite amounts $k_{io}>0$ a generation will 'asymptotically arise' which inherits zero capital stocks.

The example could of course be generalised to $c_t+a\Delta k_{1t}+b\Delta k_{2t}=F(k_{1t},k_2t)$ and a similar approach could be applied in the case where investments enter nonlinearly. The propoced concept of justice in the example gives similar results to maxmin in that all generations beyond the first receive identical consumption under the two regimes but the first generation benefits from the proposed criterion. The proposed notion of a fair solution is a Nash equilibrium; for given investments of all generations but one the remaining generation chooses its investments to maximise its utility. The solution concept is also consistent in the sense that if a generation chooses its investments to maximise its utility subject to the constraints that future generations will achoive the same path then the solution coincides with that given by the proposed concept. However the nature of the solutions in the special example are unappealing in some respects; to some extent this is due to the nature of the example so a richer framework involving natural resources is next considered.

Suppose that there is a single all purpose consumption-capital good and a single exhaustible natural resource available to the initial generation in a fixed amount X_o. The production relations become

$$c_t+\Delta k_t=F(k_t,x_t)$$

and the endowment constraints ϕ become $x_{t+1}\leq X_{t+1}=X_t-x_t$ where x_t is the amount of resource used as input by the t th generation and X_t is the remaining stock of the resource. Each generations preferences are assumed to depend only on its own consumption and so a just path would be characterised by each generation t solving the problem $\max\limits_{(\Delta k_t,x_t)}$

$$F(k_t x_t)-\Delta k_t$$

sugect to $F(k_t X_t)=F(k_t+\Delta k_t,X_{t+1})$

$$k_t,x_t>0$$

Again it is natural to suppose that F is concave; $F(0,X)=F(k,o)=0$ and $k_o>0,X_o>0$ so that on any feasible path $k_t>0,X_t>0$. It is easy to see that this requires that for almost all t $x_t<\varepsilon$ for any $\varepsilon>0$ independent of t since if $x_t\geq\varepsilon$ for all but a finite number of t then enventually $\sum\limits_o^T x_t=X_o$ so $X_{T+1}=0$. We also note that disinvestment policies are not feasible unless the marginal product of capital is zero or of the outstanding resource stock is infinite since to satisfy the constraint $k_{t+1}<k_t$ implies $X_{t+1}>X_t$ which is not possible. Given concavity of F this means that disinvestment policies can occur only when F_X has a supremum of ∞ and for most production functions one might be willing to postulate that if this occurs it occurs only at X=0 or when the infinum of F_k is zero which again one might willing to postulate occurs only at $k\to\infty$ if at all. This then means that k_t must be a non-decreasing sequence and suggests that we should rule out of consideration those production functions for which only a finite total output can be produced from a finite initial resource stock no matter how it is allocated over time (8). This suggests that a good case to explore is the Cobb Douglas for which it is possible to show that positive consumption feasible paths exist. Suppose that

$$c_t+\Delta k_t=k_t^a x_t^{1-a}$$

$$k_t^a X_t^{1-a}=(k_t+\Delta k_t)^a(X_t-x_t)^{1-a}$$

$$k_{t+1}=\lambda k_t$$

where λ is a constant. Then $x_t=\left(1-\frac{a}{1-a}\right)X_t=\alpha\left(X_o-\sum_{s=0}^{t-1}x_s\right)$

so that $\Delta x_t=-\alpha x_{t-1}$ and $x_t=(1-\alpha)^t x_o=\alpha(1-\alpha)^t X_o$.

Then $X_o=\sum_{t=0}^{\infty}x_t=\alpha\sum_{t=0}^{\infty}(1-\alpha)^t X_o$ so that we require $0<\alpha<1$ and then the resource constraint holds. But $k_t=\lambda^t k_o s_o$ so

$$c_t=\lambda^{2at}k_o^a\alpha^{1-a}X_o^{1-a}-\lambda^t(\lambda-1)k_o.$$

Use of this technology will also facilitate comparison with Solow's results.

In this case a just growth path is defined by

$$k_t^a x_t^{-a} = \frac{1}{a} k_{t+1} / X_{t+1}$$

$$\frac{k_{t+1}}{k_t} = \left(\frac{X_{t+1}}{X_t}\right)^{\frac{a-1}{a}} \qquad \text{for all } t \geqslant 0.$$

This system appears to be very complicated to handle but some progress can be made by writing it in terms of $z_t = \frac{k_t}{X_t}$ and X_t as follows:

$$X_t^{-1} X_{t+1} = Z_t^a Z_{t+1}^{-a}$$

$$Z_t^{-1} = Z_t^{a-1} Z_{t+1}^{-a} + \left(\frac{1}{a} Z_{t+1}\right)^{-1/a}$$

From the second equation

$$\frac{dz_{t+1}}{dz_t} = \frac{z_{t+1}}{z_t} \frac{\left\{a z_t^{a-1} z_{t+1}^{-a} + \frac{1}{a} z_{t+1}^{-1/a}\right\}}{\left\{a z_t^{a-1} z_{t+1}^{-a} + z_{t+1}^{-1/a}\right\}}$$

Which is everywhere positive and moreover sice $0<a<1$ $\frac{d \log^{Z}{}_{+1}}{d \log^{Z}{}_t} > 1$ so that z_t grows at least geometrically and of course X_t falls. However it appears to be very difficult to deduce anything more precise than this rather general qualitative information. A glance at a system which is, in a sense, approximate to the above shows that even if an analytical solution could be devised its content would not be transparent. So suppose that we define the first order fair criterion to be that to the first order each generation must bequeath what it inherits. In other words the constraint $F(k_t X_t) = F(k_{t+1} X_{t+1})$ is replaced by the constraint $F_k(k_t, X_t)(k_{t+1} - k_t) = -F_X(k_t X_t)(X_{t+1} - X_t)$.

Then in the Cobb Douglas case $F = k_t^a x_t^{1-a}$

the result of each generation maximising its own consumption subject to this constraint Leads to

$$\frac{k_{t+1}-k_t}{k_t} = -\frac{1-a}{a}\left(\frac{X_{t-1}-X_t}{X_t}\right)$$

$$\frac{X_{t+1}-X_t}{X_t} = a^{1/a} k_t^{\frac{a-1}{a}} X_t^{\frac{1-a}{a}}$$

so that defining $z_t = {}^{k}t/X_t$
$$-\left(\frac{X_{t+1}-X_t}{X_t}\right) = a^{1/a} z_t^{\frac{a-1}{a}}$$

$$z_{t+1} = z_t\left(1+\frac{a^{\frac{1-a}{a}}}{z_t^{\frac{1-a}{a}}-a^{1/a}}\right)$$

If we let $\delta(w) = 1+\dfrac{a^{\frac{1-a}{a}}}{w^{\frac{1-a}{a}}-a^{1/a}}$ then the solution of the second

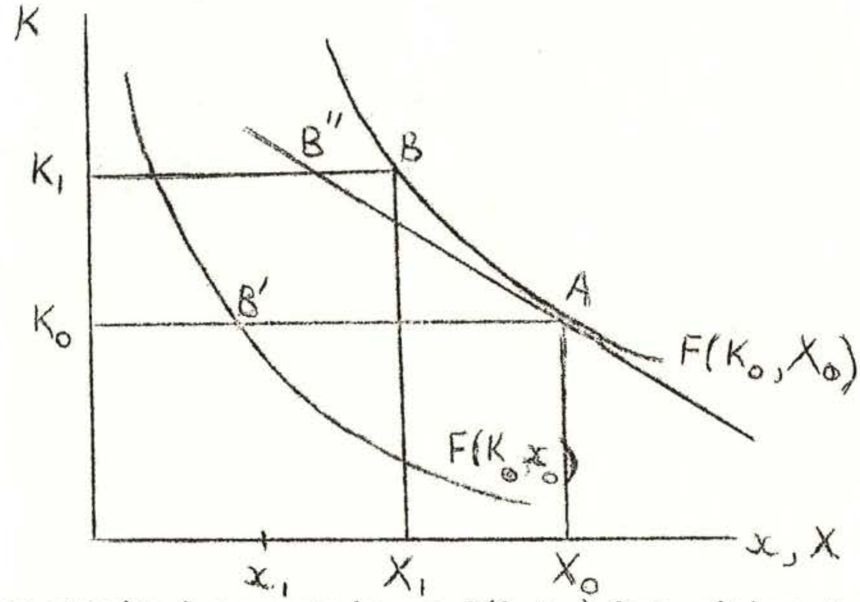
equation is $\quad z_t = z_o \prod_{s=1}^{t} \delta^s(z_o)$ where $\delta^s(z_o)$ means the

repeated composition of z_o under δ i.e.

$$\delta^s(z_o) = \delta(\delta(\ldots\delta(z_o))\ldots)$$

so that $\delta^s(z_o)-1$ represents the growth rate of z at s. However whether $\delta^{s+1}(z_o) \lessgtr \delta^s(z_o)$ depends on the value of a and also on the value of z_o so that it is not possible to be explicit about whether the growth rate is using or falling. Of course z_t is again growing and at least at a geometric rate. We could then proceed to solve for R_t but there is sufficient complexity in the system already to make general argument more rewarding.

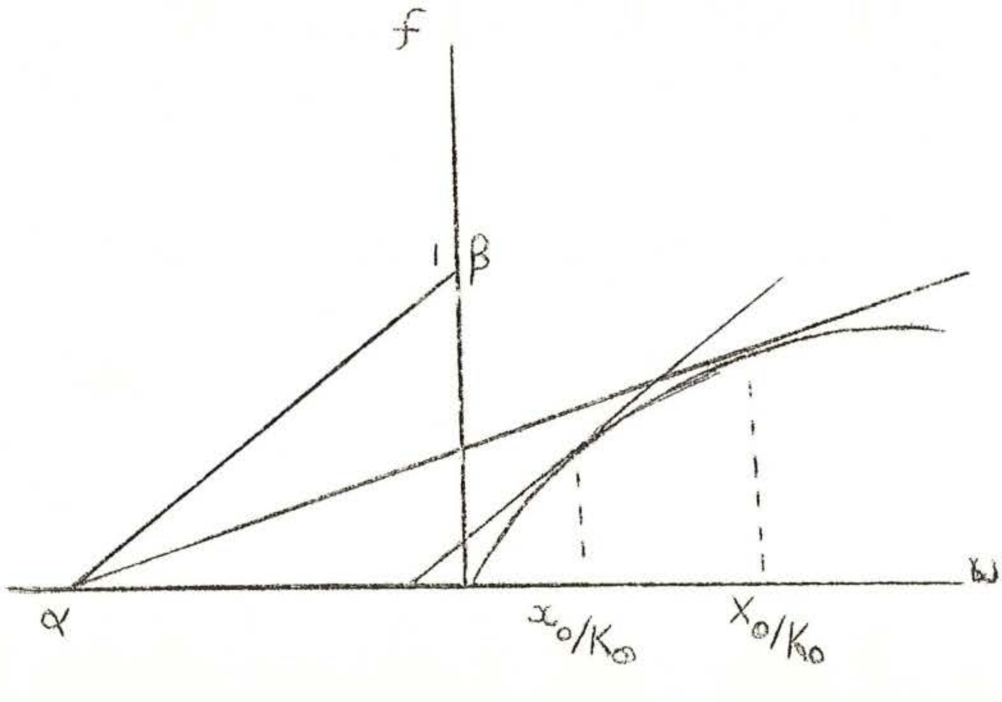
Diagrammatically the situation is as follows. In the fully fair case the initial generation is



constrained to remain on $F(k_o x_o)$ but wishes to choose a point on this curve to which to move satisfying.

$$F_x(k,x)=\frac{F_x(KX)}{F_k(KX)}$$

. If we assume constant returns to scale then such a point can be found by writing $f(w)=F(1,X/k)$, plotting $f(w)$ and then finding $w=x/k_o$ such that the tangent to $f(w)$ at x/k_o has slope equal to $\alpha\beta$.



Translated back to the first diagram this then yields points such as B,B'. The economy then develops by each generation repeating this procedure.

In the first order fair case the procedure is similar except movements are now along tangents such as A B''. Such fair paths inherit some of the desirable properties of the previous example: they are Nash equilibria and are intertemporally consistent. However in general they are not Pareto efficient. For Pareto efficiency we require

$$\frac{\Delta F_x(kx)}{F_x(kx)}=F_k(kx) \text{ and so for a fair}$$

path to be Pareto efficient we require

$$\frac{\Delta\left(\frac{F_x(K,X)}{F_k(K,X)}\right)}{F_{x/F_k}(K,X)}=F_k(K,X). \text{ In general}$$

this will not hold; for example in the first order fair Cobb Douglas case we would require

$$\delta^t(z_o)=\frac{\Delta z_t}{z_t}=ak^{a-1}x^{1-a}=a^z\left(\prod_{s=o}^{t}\delta^s(z_o)\right)^{a-1}X_t\left(\prod_{s=o}^{t}\delta(z_o)\right)^a$$

which clearly does not hold. At a heuristic level, on a fair path as opposed to a Pareto efficient path and hence in particular a maxmin path, the need to keep levels od potential output and wealth constant restrains resource depletion below its Pareto optimal level so that typically capital gains on resources are below the rate of return on capital. To some extent of course this scenario depends on the initial conditions (K_o,X_o) and the technology being such that resources are already relatively scarce. In such cases one would then expect lower investment and hence higher consumption in the early stages than on a maxmin path.

Since analytical argument seems to be unrewarding it is useful to conclude by giving a numerical example indicating the relative growth rates in a Cobb-Douglas framework. With $\alpha = 0.8$ an example of the first seventy five years of a programme in the first order approximation to the proposed criterion is

Time	Capital	Resource Use	Consumption
1	960	960	736
5	1560	13.78	606
10	2130	1.98	527
20	3115	.218	460
50	5613	.008	384
75	7458	.002	355

For purposes of comparison the maxmin criterion path can also be calculated for the same production conditions by simulating

$$z_t = K_t/x_t$$

$$z_{t-1} = z_t(1 + \alpha z_{t-1}^{\alpha-1})^{-1/\alpha}$$

$$K_{t-1} = (K_t + c)(1 + z_{t-1}^{\alpha-1})^{-1} \qquad \text{to yield}$$

Time	Capital	Resource Use	Consumption
1	$.76\ 10^{-8}$	$.3\ 10^{38}$	10.
5	$.89\ 10^{-2}$	$.16\ 10^{14}$	10.
10	.94	$.17\ 10^{6}$	10.
20	$.64\ 10^{-13}$	$.58\ 10^{58}$	10.
50	$.46\ 10^{-1}$	$.23\ 10^{11}$	10.
55	1.7	$.17\ 10^{5}$	10.
65	16	39.6	10.
75	50	.3	10.

From these examples which appear to be numerically representative under the given production conditions several features emerge. Firstly the maxmin criterion appears to inherit some of the cyclic properties attributed to it by Arrow in a single good discrete time model |10| . Secondly there is a clear cost in terms of consumption in requiring that it be constant across time. Thirdly for regions in which both paths are monotonic capital accumulation and resource depletion are faster on the maxmin than on the first order fair path. However the first order fair riterion in yielding falling consumption does permit one generation to envy another and moreover in having the facility to arrange the later structure of inputs to a greater extent than any later generation the first generation has a decided advantage.

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