

ON THE DYNAMICS OF SUPPLY-CONSTRAINED EQUILIBRIA

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ABSTRACT

The paper defines a simple tâtonnement process of adjustments in prices and quantities, where excess demand results in nominal price increases and excess supply results in quantity rationing of supply at unchanged prices. Under reasonable assumptions, the process converges to a supply-constrained equilibrium. The result contributes to our understanding of how supply-constrained equilibria come about.

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1. Introduction

In recent years, I have been intrigued by the pervasiveness, in standard general equilibrium models, of underutilisation of resources reflecting pure coordination failures rather than price distortions. As explained in section 1.1 of Drèze (1997), my interest in that topic grew out of empirical as well as theoretical preoccupations. That paper is concerned with the existence of supply-constrained equilibria, i.e. equilibria where some markets clear through quantity constraints on supply, with no constraints on demand. It contains the following result, based on standard assumptions: if some relative prices are downward rigid, unlimited underutilisation of the associated resources is possible, i.e. may persist once established, even when the downward rigid prices are compatible with competitive equilibria. Stronger results are proved in Herings and Drèze (1998), where in particular sufficient conditions are given under which there exists a connected set of supply-constrained equilibria, including a competitive equilibrium and a no-trade equilibrium.

The pervasive existence of supply-constrained equilibria at arbitrary prices (possibly but not necessarily competitive) raises an important question: how do they come about? The discussion in Drèze (1997) emphasizes the role of expectations, in economies with incomplete markets. “In particular, the coordination failure could take the form of supply constraints expected in the future and spilling over to current markets. Or it could take the form of supply constraints experienced today, as a legacy of the past or as a consequence of a current surprise (like an oil shock or the Gulfwar), and accompanied by expectations of future similar constraints, resulting together in current and anticipated underutilisation of resources”.¹ That possibility is illustrated in Drèze (1997, section 4) on a streamlined “Real Business Cycle” model and on the Barro-Grossman-Malinvaud three-good model.

The discussion in Drèze (1997) also makes a passing reference to dynamics. “The movement from one supply-constrained equilibrium to another is a topic in dynamics, inviting the study of adjustment processes defined over prices (...),

¹Drèze (1997, p. 1746).

quantities, price expectations and plans”.² The present paper is a modest step in that direction. It deals with an exchange economy where resources are supplied inelastically to the market (they cannot be consumed directly).³ Two kinds of commodities are distinguished: F -commodities, whose prices are flexible and whose supply is never constrained; and G -commodities, whose prices are downward rigid but whose supply is subject to quantity constraints.

A tâtonnement process in continuous time is defined, in terms of prices and supply constraints. It starts from arbitrary vectors of prices and supply constraints. Prices of F -commodities adjust up or down, proportionately to excess demands, as under Walrasian tâtonnement. Prices of G -commodities adjust upward only, when aggregate effective demand exceeds unconstrained supply. Supply constraints for G -commodities adjust up or down, proportionately to the discrepancy between effective demand and effective (i.e. constrained) supply. Under “natural” assumptions, the process converges to a supply-constrained equilibrium (theorem 1). That result may be seen as a first extension of standard stability analysis to sticky prices.

The “natural” assumptions are strict gross substitutability, as could be expected if only to clear the markets for F -commodities through price adjustments, and absence of inferior goods. Non-inferiority plays for quantity-adjustments a role comparable to that of substitutability for price-adjustments.⁴ It is the microeconomic translation of a positive “marginal propensity to consume”.

This paper extends earlier work. In Drèze (1991), I analyse the stability of an adjustment process in downward-rigid prices and supply constraints, for a production economy with a linear technology. The adjustment process concerns the input markets. An always desired numeraire in fixed supply limits (artificially, I think today) price level increases. Under a non-inferiority assumption, I obtain finite convergence of a discrete non-tâtonnement process to an approximate supply constrained equilibrium. The same model is studied by Herings *et al.* (1998), who dispense with the non-inferiority assumption and prove convergence to equilibrium (supply-constrained under downward price rigidities, competitive under price flexibility) of an adjustment rule which however uses more information and does not

²Drèze (1997, p. 1753).

³As explained below, I have in mind an application to input markets.

⁴In Herings and Drèze (1998), the two assumptions are combined in the single assumption A.7 viewed as the natural extension of substitutability to a framework allowing for supply constraints.

seem amenable to economic interpretation. Still, their methodology should prove relevant to a number of problems, including one mentioned in section 6 below.

In Drèze (1992), I give a summary description of a tâtonnement process in continuous time which starts from a supply-constrained equilibrium and converges to a competitive equilibrium thanks to price increases for commodities in equilibrium and to exogenous upward adjustments of supply constraints when all commodities are supply-constrained. Unfortunately, an inconsistency is present in the definition of the process.⁵ Still, some ideas used in the present paper originate there, and a consistently reformulated version of that process might be worth studying.

The technical side of this paper is based on earlier work with Paul Champsaur and Claude Henry; see Champsaur et al. (1977). Using that work, the proof of the theorem below boils down to elementary calculus.

The present paper has one merit: it studies a natural, transparent process, requiring only market-by-market information and mimicking some aspects of the functioning of decentralised market economies; the equilibrium concept is sensible and empirically relevant; the conclusion of the theorem is clear and simple.

As against this merit, there are a number of drawbacks. First, I do not prove existence of solutions, referring only to section 5 of Champsaur et al. (1977) for a description of a suitable method for proving existence. Of course, I had in mind the requirements for existence while designing the process. I do not expect problems in verifying existence, but had to postpone the task due to time constraints.

Second, the model below does not include production. I started with the intention of using the same specification as in Drèze (1991), where the linear technology sidesteps the issue of equilibrium on product markets: supply is unbounded at prices computed from input prices (possibly with a mark-up); all the action is

⁵A full description of that process is given in the mimeographed paper “Stability of Monotone Price Adjustment” which I circulated and presented at the KNAW Conference on “Theory of markets and their functioning” held in Amsterdam. July 1998. A perceptive remark by Roald Ramer led me to uncover the inconsistency. For the benefit of Conference participants, the inconsistency consists in specifying that some markets are in equilibrium (equality of effective supply and demand) while others are in excess demand. This violates Walras law, which is used in the proof of the stability theorem. A neat beginners’s mistake! I thank Roald Ramer and present my apologies to participants, asking them to discard the paper circulated in Amsterdam.

concentrated on input markets. I now realise that the approach used here could, and should, be extended to a standard production economy, as in Drèze (1997) or Herings and Drèze (1998). Still, I invite readers to follow me in interpreting the exchange economy below as describing trade in inputs. Downward price rigidities are more prevalent on input markets than on product markets, for reasons that deserve further analysis.

Third, the model does not include money - in spite of the fact that downward price rigidities are more often nominal than real. The overall price level in the model below is exogenous at the initiation and evolves according to the adjustment rule in virtual time. Of course, the speed of adjustment is arbitrary, and no significance should be attached to the specific rates of adjustment introduced below on grounds of simplicity. An explicit treatment of money is essential to study inflation and the role of monetary policy. In the model below, one could distinguish between present and future commodities, and recognise that manipulations of nominal interest rates affect the relative prices of present versus future commodities, thereby sidestepping some (not all) nominal rigidities. Introducing money supplied by banks at set nominal interest rates to accommodate a transactions demands, as done in Drèze and Polemarchakis (1998 a, b), should not raise any difficulty.⁶

Fourth, interesting dynamics concern non-tâtonnement, with trading out of equilibrium. A step in that direction is taken in Drèze (1991), but rests on a naïve specification of expectations. That territory is insufficiently explored at present and defines a natural priority for further research - not a new one, of course, witness the concluding sentence in Drèze (1991): "...the major challenge to all students of price dynamics remains that of modelling non-deterministic expectations and stochastic sequential decision-making by consumers and firms." One merit of the process studied here is that its simplicity (and relevance) makes it a natural candidate for a non-tâtonnement extension.

The paper is organised as follows. The model is defined in section 2 and the process in section 3. Section 4 is devoted to the stability theorem. Section 5 is devoted to two special cases, where the simplicity of the process reveals some interesting qualitative properties. Section 6 offers a concluding remark.

⁶With set interest rates and an accommodating supply of loans, the commodity "loans" would belong to the G -commodities, with a demand-determined constraint on effective lending.

2. The Model

I consider an exchange economy defined by:

- $F + G$ commodities, indexed respectively $i, j = 1 \dots F$ and $k, \ell = 1 \dots G$; commodities in the first group have flexible prices $q \in R_+^F$ and are referred to as F -commodities; commodities in the second group have downward-rigid prices $p \in R_+^G$ and are referred to as G -commodities;
- H agents, indexed $h = 1 \dots H$, with initial endowments $(\psi^h, \omega^h) \in R_+^F \times R_+^G$, consuming $(z^h, x^h) \in R_+^F \times R_+^G$, with preferences represented by $u^h(z^h, x^h)$.⁷

All endowments are supplied to an exchange, and all consumptions are bought from the exchange. Thus, direct consumption of endowment is ruled out (as is typical for inputs, once leisure is distinguished from specific labour services).

I study a virtual tâtonnement process in prices and quantities, defined in continuous time $t \geq 0$, by:

- supplies (ψ^h, ω^h) and demands (z^h, x^h) addressed by the agents to the exchange;
- price and quantity signals $(q(t), p(t), \alpha(t)) \in R_+^F \times R_+^G \times [0, 1]^G$ addressed by the exchange to the agents.

Thus, at all $t \geq 0$, the exchange announces:

- for each F -commodity i a price $q_i(t) \geq 0$;
- for each G -commodity k , a price $p_k(t) > 0$ and a rationing coefficient $\alpha_k(t) \in [0, 1]$.

This entails the stipulation that the exchange buys from each agent h the quantity ψ_i^h of F -commodity i and the quantity $\alpha_k(t)\omega_k^h$ of G -commodity k , $i = 1 \dots F$, $k = 1 \dots G$. Rationing is thus assumed proportional (as a shortcut for the more natural random scheme used e.g. in Drèze, 1991).

⁷The non-standard notation, whereby different symbols (prices and quantities) are used for F -commodities and G -commodities, reflects a preference for clarity over elegance.

The income of agent h at time t is

$$y^h(t) = \sum_i q_i(t) \psi_i^h + \sum_k p_k(t) \alpha_k(t) \omega_k^h. \quad (1)$$

The demand of agent h at time t is⁸

$$(z^h(t), x^h(t)) \in \operatorname{argmax} \{u^h(z^h, x^h) | q^T z^h + p^T x^h \leq y^h(t)\}.$$

I formulate my assumptions directly on individual and aggregate demand functions.

Assumption 1

For each $h = 1 \cdots H$, demand is defined by the continuously differentiable demand functions $(z^h, x^h)(q, p, y^h)$, homogeneous of degree zero in (q, p, y^h) , which satisfy, for all $(q, p, y^h) \in R_+^F \times R_+^G \times R_+$, $q^T z^h + p^T x^h = y^h$ (local non-satiation of preferences).

Remark

The differentiability assumption can be weakened to Lipschitz continuity, at some technical complication which did not seem to pass a cost-benefit test.

Assumption 2

For all $i = 1 \cdots F$, for all $k, \ell = 1 \cdots G$, for all $h = 1 \cdots H$ and for all $(q, p, y^1 \cdots y^H) \in R_+^F \times R_+^G \times R_+^H$:

$$\frac{\partial z_i^h}{\partial y^h} \geq 0, \quad \frac{\partial x_k^h}{\partial y^h} \geq 0 \quad \text{and} \quad \sum_h \omega_\ell^h \frac{\partial z_i^h}{\partial y^h} > 0, \quad \sum_h \omega_\ell^h \frac{\partial x_k^h}{\partial y^h} > 0 \quad (\text{non-inferiority}).$$

Note that strict inequality is only required for weighted sums across individuals.

Assumption 3

For all $i, j = 1 \cdots F, j \neq i$, for all $k, \ell = 1 \cdots G, \ell \neq k$, and for all $(q, p, y^1 \cdots y^H) \in R_+^F \times R_+^G \times R_+^H$:

$$\sum_h \left(\frac{\partial z_i^h}{\partial q_j} + \psi_j^h \frac{\partial z_i^h}{\partial y^h} \right) > 0, \quad \sum_h \left(\frac{\partial z_i^h}{\partial p_k} + \omega_k^h \frac{\partial z_i^h}{\partial y^h} \right) > 0,$$

⁸ q^T , a row vector, is the transpose of q , a column vector.

$$\sum_h \left(\frac{\partial x_k^h}{\partial q_i} + \psi_i^h \frac{\partial x_k^h}{\partial y^h} \right) > 0, \quad \sum_h \left(\frac{\partial x_k^h}{\partial p_\ell} + \omega_\ell^h \frac{\partial z_k^h}{\partial y^h} \right) > 0 \text{ (substitutability).}$$

Notation and definitions

Aggregate supply, exogenous and fixed, is normalised to unity for each commodity (choice of physical units). That is:

$$\sum_h \psi_i^h = 1, \sum_h \omega_k^h = 1, \quad i = 1 \cdots F, \quad k = 1 \cdots G.$$

Aggregate demand at time t is

$$\begin{aligned} Z_i(t) &= \sum_h z_i^h(q(t), p(t), y^h(t)) \\ &:= \sum_h z_i^h(q(t), p(t), \alpha(t)) = Z_i(q(t), p(t), \alpha(t)); \\ X_k(t) &= \sum_h x_k^h(q(t), p(t), y^h(t)) \\ &:= \sum_h x_k^h(q(t), p(t), \alpha(t)) = X_k(q(t), p(t), \alpha(t)). \end{aligned}$$

In the definition of my tâtonnement process, and in the proof of the theorem to follow, I use extensively indicator functions, denoted $\mathcal{J}(\cdot)$. Thus,

$$\begin{aligned} \mathcal{J}(\alpha_k = 1) &= 1 \quad \text{if} \quad \alpha_k = 1 \\ &= 0 \quad \text{if} \quad \alpha_k < 1 \\ &= 1 - \mathcal{J}(\alpha_k < 1), \quad \text{a.s.o..} \end{aligned}$$

Compact symbols for complex indicators are introduced as needed.

The following almost standard definition is used in the statement of the theorem.⁹

⁹The term “supply-constrained equilibrium” was introduced by Van der Laan (1980) and borrowed by Dehez and Drèze (1984) then Drèze (1997). Herings (1996) refers to “underemployment equilibrium”, and that term is retained by Herings and Drèze (1998); it avoids the possible misinterpretation of “supply-constrained” as “constrained by insufficient supply”.

Definition

A supply-constrained equilibrium is a tuple $(q, p, \alpha, Z, X) \in R_+^F \times R_+^G \times [0, 1]^G \times R_+^F \times R_+^G$ such that :

- for all $i = 1 \cdots F$, $Z_i = Z_i(q, p, \alpha) = 1$;
- for all $k = 1 \cdots G$, $X_k = X_k(q, p, \alpha) = \alpha_k \leq 1$.

I am interested here in economies with $F \geq 0$ and $G \geq 1$.¹⁰

3. The Process

My main concern is the stability of the following Tâtonnement Process TP, presented in the form of adjustment rules for the prices $q(t), p(t)$ and the quantity signals $\alpha(t)$ announced by the exchange. The rules mimick market adjustments with flexible prices for the F -commodities, downward rigid prices and quantity constraints on supply for the G -commodities. The rules are simple and specific, but amenable to standard generalisations (to strictly-positive monotone Lipschitz-continuous functions of the arguments below).

Process TP

- *Initiation:*
 $(q(0), p(0)) \in R_{++}^F \times R_{++}^G$ are given (arbitrary);
 $\alpha(0) \in (0, 1]^G$ is given (arbitrary);
- *Adjustment:* for all $t \geq 0$, for all $i = 1 \cdots F, k = 1 \cdots G$:
 $\frac{dq_i}{dt} := \dot{q}_i(t) = q_i(t) \cdot (Z_i(t) - 1)$;
 $\frac{dp_k}{dt} := \dot{p}_k(t) = p_k(t) \cdot (X_k(t) - 1) \cdot \mathcal{J}(X_k(t) > 1 = \alpha_k(t))$;
 $\frac{d\alpha_k}{dt} := \dot{\alpha}_k(t) = (X_k(t) - \alpha_k(t)) \cdot [1 - \mathcal{J}(X_k(t) > 1 = \alpha_k(t))]$.

It should be understood that $Z(t) = Z(q(t), p(t), \alpha(t))$, $X(t) = X(q(t), p(t), \alpha(t))$, where $(q(t), p(t), \alpha(t))$ are defined by process TP.

Thus, prices of F -commodities adjust upward or downward in proportion to excess demand. (Reminder: aggregate supply is normalised to unity.)

¹⁰For $G = 1$, price rigidities reduce to a constraint on price normalisation, but equilibria with $\alpha < 1$ still exist. Although I discuss a special case with $F = 0$ in section 5.1, I have always regarded $F > 1$ as the relevant case.

Prices of G -commodities are downward rigid. They adjust upward in proportion to excess demand when, and only when, supply of the relevant commodity is unconstrained ($\alpha_k = 1$). That feature reflects the desirable property that “quantities adjust faster than prices” (for those commodities whose prices display stickiness); it is encapsulated in the indicator function attached to the adjustment rule for $p_k : \mathcal{J}(X_k > 1 = \alpha_k)$ is zero when $X_k \leq 1, \alpha_k < 1$ or both.

The quantity signals (supply constraints) adjust upward or downward in response to effective excess demand, subject to remaining in $[0, 1]$.¹¹ When $X_k > \alpha_k$ and $\alpha_k < 1$, α_k is raised: idle resources which are demanded are released; conversely, resources for which there is no demand go out of use.

This is a simple, straightforward and well-behaved process. The existence theory in section 5 of Champsaur *et al* (1977)¹² should apply routinely – an assertion that remains to be verified systematically.

4. A Stability Theorem

At any point along the tâtonnement process, it is natural to define (virtual) nominal “national income” $Y(t)$ as the aggregate value of resources used, which is also the sum of individual incomes $y^h(t)$:

$$Y(t) = \sum_i q_i(t) + \sum_k p_k(t)\alpha_k(t).$$

Aggregation of the budget equations (1) yields

$$\sum_i q_i(t) (Z_i(t) - 1) + \sum_k p_k(t) (X_k(t) - \alpha_k(t)) = 0, \quad (2)$$

$$\begin{aligned} \sum_i \dot{q}_i(t) (Z_i(t) - 1) + \sum_i q_i(t) \dot{Z}_i(t) + \sum_k \dot{p}_k(t) (X_k(t) - 1) \\ + \sum_k p_k(t) (\dot{X}_k(t) - \dot{\alpha}_k(t)) = 0. \end{aligned} \quad (3)$$

¹¹Thus, $\dot{\alpha}_k = 0$ when $X_k > \alpha_k = 1$; the lower bound, $\alpha_k \geq 0$, is never violated because $X_k \geq 0$.

¹²Definitions, details and references can be found there.

Using (2),

$$\begin{aligned}
\frac{dY}{dt} &= \sum_i \dot{q}_i(t) + \sum_k \dot{p}_k(t) + \sum_k p_k(t) (X_k(t) - \alpha_k(t)) \\
&\quad [1 - \mathcal{J}(X_k(t) > 1 = \alpha_k(t))] \\
&= \sum_i \dot{q}_i(t) + \sum_k \dot{p}_k(t) + \sum_k p_k(t) (X_k(t) - \alpha_k(t)) - \sum_k \dot{p}_k(t) \quad (4) \\
&= \sum_i q_i(t) (Z_i(t) - 1) + \sum_k p_k(t) (X_k(t) - \alpha_k(t)) = 0.
\end{aligned}$$

Thus, nominal national income is constant along the process. The evolution of real income is of unknown sign, because the adjustments in the prices of F -commodities may be positive or negative.

Theorem 1

Let there exist continuous solutions $(q(t), p(t), \alpha(t), (z^h, x^h)(t)_{h=1 \dots H})$ to the system defined by process TP .

Under assumptions 1, 2 and 3, process TP is quasi-stable and every limit point of a trajectory is a supply-constrained equilibrium.¹³

Proof

I use the Lyapunow function

$$\Lambda(t) = \sum_i q_i(t) |Z_i(t) - 1| + \sum_k p_k(t) |X_k(t) - \alpha_k(t)|$$

and prove that $\frac{d\Lambda}{dt} := \dot{\Lambda}(t) \leq 0$ with $\dot{\Lambda}(t) = 0$ iff $\Lambda(t) = 0$, $Z_i(t) = 1$ for all $i = 1 \dots F$, $X_k(t) = \alpha_k(t) \leq 1$ for all $k = 1 \dots G$.

I drop the explicit time reference, since all variables and derivatives are eval-

¹³A dynamic process P is “quasi-stable” iff any limit point of a trajectory is an equilibrium (i.e. rest point) of the process.

uated at t , and calculate:

$$\begin{aligned}
\dot{\Lambda}(t) = & \sum_i [\dot{q}_i (Z_i - 1) + q_i \dot{Z}_i] [\mathcal{J}(Z_i > 1) + \mathcal{J}(Z_i = 1, \dot{Z}_i > 0)] \\
& - \sum_i [\dot{q}_i (Z_i - 1) + q_i \dot{Z}_i] [\mathcal{J}(Z_i < 1) + \mathcal{J}(Z_i = 1, \dot{Z}_i < 0)] \\
& + \sum_k \dot{p}_k (X_k - 1) \\
& + \sum_k p_k (\dot{X}_k - \dot{\alpha}_k) [\mathcal{J}(X_k > \alpha_k) + \mathcal{J}(X_k = \alpha_k, \dot{X}_k > 0)] \\
& - \sum_k p_k (\dot{X}_k - \dot{\alpha}_k) [\mathcal{J}(X_k < \alpha_k) + \mathcal{J}(X_k = \alpha_k, \dot{X}_k < 0)].
\end{aligned} \tag{5}$$

Write $\mathcal{J}_i^>(Z)$ for $[\mathcal{J}(Z_i > 1) + \mathcal{J}(Z_i = 1, \dot{Z}_i > 0)]$, $\mathcal{J}_i^<(Z)$ for $1 - \mathcal{J}_i^>(Z)$, $\mathcal{J}_k^>(X)$ for $[\mathcal{J}(X_k > \alpha_k) + \mathcal{J}(X_k = \alpha_k, \dot{X}_k > 0)]$ and $\mathcal{J}_k^<(X)$ for $1 - \mathcal{J}_k^>(X)$. Using (3), equation (5) simplifies to:

$$\dot{\Lambda}(t) = 2 \sum_i \dot{q}_i (1 - Z_i) \mathcal{J}_i^<(Z) - 2 \sum_i q_i \dot{Z}_i \mathcal{J}_i^<(Z) - 2 \sum_k p_k (\dot{X}_k - \dot{\alpha}_k) \mathcal{J}_k^<(X), \tag{6}$$

where

$$\begin{aligned}
\dot{Z}_i = & \sum_j \dot{q}_j \sum_h \frac{\partial z_i^h}{\partial q_j} + \sum_k \dot{p}_k \frac{\partial z_i^h}{\partial p_k} + \sum_h \frac{\partial z_i^h}{\partial y^h} \left[\sum_j \dot{q}_j \psi_j^h + \sum_k (\dot{p}_k + p_k \dot{\alpha}_k) \omega_k^h \right] \\
= & \sum_j \dot{q}_j \sum_h \left(\frac{\partial z_i^h}{\partial q_j} + \psi_j^h \frac{\partial z_i^h}{\partial y_j^h} \right) + \sum_k \dot{p}_k \sum_h \left(\frac{\partial z_i^h}{\partial p_k} + \omega_k^h \frac{\partial z_i^h}{\partial y^h} \right) \\
& + \sum_h \frac{\partial z_i^h}{\partial y^h} \sum_k p_k \dot{\alpha}_k \omega_k^h, \\
\dot{X}_k = & \sum_j \dot{q}_j \sum_h \left(\frac{\partial x_k^h}{\partial q_j} + \psi_j^h \frac{\partial x_k^h}{\partial y_j^h} \right) + \sum_\ell \dot{p}_\ell \sum_h \left(\frac{\partial x_k^h}{\partial p_\ell} + \omega_\ell^h \frac{\partial x_k^h}{\partial y^h} \right) \\
& + \sum_h \frac{\partial x_k^h}{\partial y^h} \sum_\ell p_\ell \dot{\alpha}_\ell \omega_\ell^h.
\end{aligned} \tag{7}$$

Write $\frac{\partial Z_i}{\partial q_j}$ for $\sum_h \left(\frac{\partial z_i^h}{\partial q_j} + \psi_j^h \frac{\partial z_i^h}{\partial y_j^h} \right)$ and define similarly $\frac{\partial Z_i}{\partial p_k}$, $\frac{\partial X_k}{\partial q_j}$ and $\frac{\partial X_k}{\partial p_\ell}$.

Substituting from (7) into (6),

$$\begin{aligned}
\dot{\Lambda}(t) = & 2 \sum_i \dot{q}_i (1 - Z_i) \mathcal{J}_i^<(Z) + 2 \sum_k p_k \dot{\alpha}_k \mathcal{J}_k^<(X) \\
& - 2 \sum_j \dot{q}_j \left[\sum_i q_i \frac{\partial Z_i}{\partial q_j} \mathcal{J}_i^<(Z) + \sum_k p_k \frac{\partial X_k}{\partial q_j} \mathcal{J}_k^<(X) \right] \\
& - 2 \sum_\ell \dot{p}_\ell \left[\sum_i q_i \frac{\partial Z_i}{\partial p_\ell} \mathcal{J}_i^<(Z) + \sum_k p_k \frac{\partial X_k}{\partial p_\ell} \mathcal{J}_k^<(X) \right] \\
& - 2 \sum_\ell p_\ell \dot{\alpha}_\ell \left[\sum_i q_i \mathcal{J}_i^<(Z) \sum_h \frac{\partial z_i^h}{\partial y^h} \omega_\ell^h \right. \\
& \quad \left. + \sum_k p_k \mathcal{J}_k^<(X) \sum_h \frac{\partial x_k^h}{\partial y^h} \omega_\ell^h \right] \leq 0.
\end{aligned} \tag{8}$$

In order to establish the non-positivity of $\dot{\Lambda}(t)$ in (8), some regrouping and validation is needed. The first term will be combined with the third, and the second with the fifth. The reasoning thus proceeds in three steps.

Step 1

In the third term, $\dot{q}_j \geq 0$ when $\mathcal{J}_j^>(Z) = 1$ and $\dot{q}_j \leq 0$ when $\mathcal{J}_j^<(Z) = 1$. When $\dot{q}_j > 0$, $\frac{\partial Z_i}{\partial q_j} \mathcal{J}_i^<(Z) > 0$ by assumption 3 because $i \neq j$, and similarly $\frac{\partial X_k}{\partial q_j} > 0$. Accordingly, all components of the sum over j are positive when $\dot{q}_j > 0$ with $\sum_i \mathcal{J}_i^<(Z) + \sum_k \mathcal{J}_k^<(X) > 0$, and they contribute negatively to $\dot{\Lambda}(t)$. When $\dot{q}_j \leq 0$, i.e. when $\mathcal{J}_j^<(Z) = 1$, F -commodity j is included in $\sum_i q_i \frac{\partial Z_i}{\partial q_j} \mathcal{J}_i^<(Z)$. In that case, it is necessary to use the property that $\sum_i q_i \frac{\partial Z_i}{\partial q_j} + \sum_k p_k \frac{\partial X_k}{\partial q_j} = 1 - Z_j$.¹⁴ From this follows that

$$\begin{aligned}
\sum_i q_i \frac{\partial Z_i}{\partial q_j} \mathcal{J}_i^<(Z) + \sum_k p_k \frac{\partial X_k}{\partial q_j} \mathcal{J}_k^<(X) &= 1 - Z_j - \sum_i q_i \frac{\partial Z_i}{\partial q_j} \mathcal{J}_i^>(Z) \\
&\quad - \sum_k p_k \frac{\partial X_k}{\partial q_j} \mathcal{J}_k^>(X) \leq 1 - Z_j,
\end{aligned}$$

¹⁴At the risk of being pedantic, it is a property of the Slutsky matrices that

$$\begin{aligned}
\sum_h \left[\sum_i q_i \left(\frac{\partial z_i^h}{\partial q_j} + z_j^h \frac{\partial z_i^h}{\partial y^h} \right) + \sum_k p_k \left(\frac{\partial x_k^h}{\partial q_j} + z_j^h \frac{\partial x_k^h}{\partial y^h} \right) \right] &= 0 \\
= \sum_i q_i \frac{\partial Z_i}{\partial q_j} + \sum_k p_k \frac{\partial X_k}{\partial q_j} + \sum_h \left[\sum_i q_i \frac{\partial z_i^h}{\partial y^h} + \sum_k p_k \frac{\partial x_k^h}{\partial y^h} \right] (z_j^h - \psi_j^h).
\end{aligned}$$

For each h , the terms in the square bracket add up to one by weak monotonicity, and $\sum_h (z_j^h - \psi_j^h) = Z_j - 1$.

where the inequality follows from the fact $\frac{\partial Z_i}{\partial q_j} \mathcal{J}_i^>(Z) > 0$ when $\dot{q}_j < 0$ because $i \neq j$, and $\frac{\partial X_k}{\partial q_j} > 0$. Accordingly, combining the first term in (8) with the components of the sum over j in the third term for which $\dot{q}_j \leq 0$, $\mathcal{J}_j^<(Z) = 1$, one obtains

$$2 \sum_j \dot{q}_j \mathcal{J}_j^<(Z) \left[1 - Z_j - \sum_i q_i \frac{\partial Z_i}{\partial q_j} \mathcal{J}_i^<(Z) - \sum_k p_k \frac{\partial X_k}{\partial q_j} \mathcal{J}_k^<(X) \right] \leq 0,$$

where the inequality follows from $\dot{q}_j < 0$ whereas the terms in square bracket are positive, as just shown. Consequently, the first and third terms in (8) taken together contribute non-positively to $\dot{\Lambda}(t)$, and negatively when $\dot{q}_j \neq 0$ for some j , with $\sum_i \mathcal{J}_i^<(Z) + \sum_k \mathcal{J}_k^<(X) > 0$.

Step 2

The fourth term in (8) is non positive, by assumption 3, because $\frac{\partial Z_i}{\partial p_\ell} > 0$ and $\mathcal{J}_k^<(X) = 1$ implies $\dot{p}_k = 0$, so that $k \neq \ell$ and $\frac{\partial X_k}{\partial p_\ell} > 0$; whereas $\dot{p}_\ell \geq 0$. This term is strictly negative when $\dot{p}_\ell > 0$ for some ℓ , with $\sum_i \mathcal{J}_i^<(Z) + \sum_k \mathcal{J}_k^<(X) > 0$.

Step 3

The last term in (8) is non-positive when $\dot{\alpha}_\ell \geq 0$, $\mathcal{J}_\ell^>(X) = 1$. The components for which $\dot{\alpha}_\ell \leq 0$, $\mathcal{J}_\ell^<(X) = 1$, may be combined with the second term in (8) to yield

$$\sum_\ell \dot{p}_\ell \dot{\alpha}_\ell \mathcal{J}_\ell^<(X) \left[1 - \sum_h \omega_\ell^h \left(\sum_i q_i \frac{\partial z_i^h}{\partial y^h} \mathcal{J}_i^<(Z) + \sum_k p_k \frac{\partial x_k^h}{\partial y^h} \mathcal{J}_k^<(X) \right) \right] \leq 0.$$

The sums in parentheses are for each h less than or equal to one, so their weighted sum over h is less than or equal to $\sum_h \omega_\ell^h = 1$. Non-positivity then follows from $\dot{\alpha}_\ell \mathcal{J}_\ell^< \leq 0$. Thus the second and last term in (8) taken together contribute non-positively to $\dot{\Lambda}(t)$, and negatively when $\dot{\alpha}_\ell \neq 0$ for some ℓ , with either $\sum_i \mathcal{J}_i^<(Z) + \sum_k \mathcal{J}_k^<(X) > 0$ if $\dot{\alpha}_\ell > 0$ or $\sum_i \mathcal{J}_i^>(Z) + \sum_k \mathcal{J}_k^>(X) > 0$ if $\dot{\alpha}_\ell < 0$.

We may thus conclude that $\dot{\Lambda}(t) \leq 0$. To verify that $\dot{\Lambda}(t) < 0$ unless $\Lambda(t) = 0$, note first that $\sum_i \mathcal{J}_i^<(Z) + \sum_i \mathcal{J}_i^<(X) > 0$ unless $Z_i \geq 1$ for all $i = 1 \dots F$ and $X_k \geq \alpha_k$ for all $k = 1 \dots G$. Similarly, $\sum_i \mathcal{J}_i^>(Z) + \sum_k \mathcal{J}_k^>(X) > 0$ unless $Z_i \leq 1$ for all i and $X_k \leq \alpha_k$ for all k . In either case, (2) would imply $Z_i = 1$ for all i and $X_k = \alpha_k$ for all k , i.e. $\Lambda(t) = 0$. It thus follows from the conclusions of the three steps that $\dot{\Lambda}(t) < 0$ whenever $\dot{q}_j \neq 0$ for some j or $\dot{p}_\ell > 0$ for some ℓ or $\dot{\alpha}_\ell \neq 0$ for

some ℓ , i.e. whenever $\Lambda(t) \neq 0$. Using theorem 6.1 in Champsaur *et al.* (1977), the proof is complete. ■

5. Two Special Cases

5.1 Fixed Prices

This special case, of purely pedagogical interest, is obtained by setting $F = 0$ and $\dot{p}_k(t) = 0$ for all k and t . Thus, I define a quantity-adjustment process (QP) for an economy with fixed prices.

Process QP

$F = 0$; $p(t) = p > 0$ is given and constant.

- *Initiation*:

$\alpha(0) \in (0, 1]^G$ is given (arbitrary).

- *Adjustment*:

$$\frac{d\alpha_k}{dt} := \dot{\alpha}_k(t) = (X_k(t) - \alpha_k(t)) [1 - \mathcal{J}(X_k(t) > 1 = \alpha_k(t))].$$

The quasi-stability of process QP follows as a direct corollary of theorem 1. Equation (8) simplifies to

$$\dot{\Lambda}(t) = 2 \sum_k p_k \dot{\alpha}_k \mathcal{J}_k^<(X) - 2 \sum_\ell p_\ell \dot{\alpha}_\ell \sum_k p_k \mathcal{J}_k^<(X) \sum_h \frac{\partial x_k^h}{\partial y^h} \omega_\ell^h \leq 0,$$

where the inequality still follows as per step 3 in the proof of the theorem.

With constant prices, $Y(t) = \sum_k p_k \alpha_k(t)$ now defines real (as well as nominal) national income. Its evolution is given by

$$\frac{dY}{dt} := \dot{Y}(t) = \sum_k p_k \dot{\alpha}_k(t) = - \sum_k p_k (X_k(t) - 1) \mathcal{J}(X_k(t) > 1 = \alpha_k) \leq 0.$$

Thus, the aggregate use of resources is monotone non-increasing over time; it is constant over time intervals when there is no excess demand for a fully used resource, and strictly decreasing over time when such excess demand arises – a situation often described as one of “inflationary pressure” (and characterised by price inflation under process TP). In a sense, process QP has a “contractionary bias”: the quantity adjustments carried out to bring effective supply in agreement

with effective demand trace out a contractionary spiral (lower incomes from sales of endowment result in lower demand overall); when prices are fixed, aggregate income contraction is the only avenue open to eliminate excess demand for some fully used commodity, i.e. one for which $X_k > 1 = \alpha_k$.

It is interesting to note that a “contractionary bias” is not inherent to quantity adjustments per se; rather, it is a feature associated with supply rationing, as opposed to demand rationing. For the model of section 2, one can define demand-constrained equilibria, and a dual process whereby constraints on demand are adjusted upward or downward in response to effective excess demands. Under such a process, Say’s Law operates: (unconstrained) supply creates its own demand; the role of quantity constraints is to choke off demand on markets where it exceeds potential supply, thereby creating spillovers which raise demand on other markets. In contrast, Say’s Law is inoperative in presence of constraints on supply.¹⁵ It is unfortunate, but undeniable, that quantity adjustments in decentralised market economies take the form of supply constraints, resulting in the contractionary bias illustrated above. That unfortunate feature results both from market forces (the competition among buyers to overcome demand rationing, a competition rationally anticipated by sellers), and from market imperfections (small numbers of sellers, organised collusion, price rigidities aimed at correcting the inefficiencies resulting from incomplete markets ...¹⁶). This is not the place to expand on that fruitful theme.

A further insight into the operation of process QP is obtained by assuming, as usually done in macroeconomics, that aggregate demand for any commodity k is a function of prices p and aggregate income Y . In the present special case, a simple assumption entails a similar property for income derivatives, namely:

Assumption 4.

For all $k, \ell = 1 \cdots G$ and for all $(p; y^1 \cdots y^H) \in R_+^G \times R_+^H$, the covariances, across agents, of marginal propensities to consume $\frac{\partial x_k^h}{\partial y^h}$ with endowments ω_ℓ^h are

¹⁵Morishima (1976, Chap. 7) contains an interesting discussion of Say’s Law under price rigidities. Here, one could remark that “demand creates its own supply” - within the limits of resource availability.

¹⁶That last imperfection is spelled out as a rationale for (second-best) wage rigidities in Drèze and Gollier (1993).

equal to zero:

$$\sum_h \frac{\partial x_k^h}{\partial y^h} \left(\omega_\ell^h - \frac{1}{H} \right) = 0.$$

It follows from assumption 4 that, in our special case,

$$\dot{X}_k = \sum_h \frac{\partial x_k^h}{\partial y^h} \sum_\ell p_\ell \dot{\alpha}_\ell \omega_\ell^h = \frac{1}{H} \sum_h \frac{\partial x_k^h}{\partial y^h} \sum_\ell p_\ell \dot{\alpha}_\ell := \frac{\partial X_k}{\partial Y} \dot{Y} \leq 0,$$

where the inequality follows from assumption 2 and $\dot{Y} \leq 0$. This also implies $\dot{X}_k = 0$ for all k when $\dot{Y} = 0$, i.e. when $\sum_k \mathcal{J}(X_k > 1 = \alpha_k) = 0$.

Thus, if process QP starts from $\alpha_k(0) < 1$ for all k , then $\dot{Y}(0) = 0$, $\dot{X}_k(0) = 0$ for all k , and all the discrepancies between effective demands and effective supplies are corrected by bringing the quantity constraints $\alpha_k(t)$ progressively closer to the initial effective demands $X_k(0)$.

Two cases are usefully distinguished. If $X_k(0) < 1$ for all k , then process QP will converge to an equilibrium where the supply of every commodity is constrained below one: the process simply validates $X(0)$ as a supply-constrained equilibrium, at the given (arbitrary) prices p .¹⁷ The process is then exempt from contractionary bias, since Y is constant. It eliminates excess demands by raising supply constraints where needed, and brings effective supply down to the level of effective demand elsewhere. In an application to input markets, firms simply hire the inputs (or use the capacities) needed to satisfy demands, and fire inputs (or keep capacities idle) when not needed to satisfy demand.

When $X_k(0) > 1$ for some k , the corresponding α_k will rise from $\alpha_k(0) < 1$ until it reaches unity.¹⁸ From that point on, the process becomes contractionary: $\dot{Y} < 0$, entailing $\dot{X}_\ell < 0$ for all ℓ ; in order to absorb the excess demand for commodity k , income must fall, and will fall as a consequence of decentralised quantity adjustments, entailing falling demand for all commodities; the contractionary spiral is at work. This is of course the point where the special case of fixed prices

¹⁷Of course, $X(0) = X(p, \alpha(0)) = X(p, Y(0))$ is not an arbitrary vector. For given p , assumption 1 (continuously differentiable demands) implies that $X(0)$ varies continuously with $Y(0)$, defining a one-dimensional set of supply-constrained equilibria, in agreement with the general theory in Herings (1996).

¹⁸So far process QP parallels the achievements of the short-term process in Herings *et al.* (1998).

becomes uninteresting. More generally, the price p_k of the G -commodity in excess demand will rise, as in process TP , speeding up the overall adjustment and limiting the impact of the contractionary bias.

One final remark is in order. If process QP converges to an equilibrium with binding supply constraints for all commodities, the associated coordination failure could be remedied by raising some or all supply constraints (the α 's) exogenously. This would lift Y and consequently all X_k 's. In this case, supply would create its own demand – leaving it to process QP to bring about the further adjustments in individual α_k 's needed to match the increments in the corresponding X_k 's. This remedy to the coordination failure could be used until some α (s) reach unity. At that stage, price adjustments for the fully used commodities offer the only avenue to raise Y . My second special case expands on this remark.

5.2 Fixed Shares

Another instructive special case is obtained when the demand functions have the special property of “fixed shares”, embodied in the following assumption.

Assumption 5

There exist a vector $\gamma \in \Delta^F := \{\gamma \in R_+^F | \sum_i \gamma_i = 1\}$ and a vector $\delta \in \Delta^G$ such that, for all $i = 1 \cdots F$, for all $k = 1 \cdots G$ and for all $(q, p, \alpha) \in R_{++}^F \times R_{++}^G \times (0, 1]^G$, $q_i Z_i(q, p, \alpha) = \gamma_i Y(q, p, \alpha)$ and $p_k X_k(q, p, \alpha) = \delta_k Y(q, p, \alpha)$.

Applied to inputs, this assumption imposes fixed factor shares, as under a Cobb-Douglas production function. Applied to consumption goods, this assumption imposes fixed budget shares, as under a Cobb-Douglas utility function. This is of course a very special case.¹⁹ It leads to a simple property, for which less restrictive sufficient conditions are worth investigating.

Corollary

Let $(\bar{q}, \bar{p}, \bar{\alpha}, \bar{Z}, \bar{X})$ define a supply-constrained equilibrium and let $(\bar{\bar{q}}, \bar{\bar{p}}, \bar{\bar{\alpha}}, \bar{\bar{Z}}, \bar{\bar{X}})$ be a limit point of Process TP with initiation $q(0) = \bar{q}$, $p(0) = \bar{p}$, $\alpha(0) \in (0, 1]^G$. Under assumption 5:

- (i) If $\sum_k \bar{p}_k \alpha_k(0) > \sum_k \bar{p}_k \bar{\alpha}_k$,

¹⁹It is also the special case privileged in the “Real Business Cycles” literature.

then $\bar{q} > \bar{q}$, $\bar{p} \geq \bar{p}$, $\bar{\alpha} \geq \bar{\alpha}$ with $\bar{\alpha}_k > \bar{\alpha}_k$ whenever $\bar{\alpha}_k < 1$;²⁰

- (ii) If $\sum_k \bar{p}_k \alpha_k(0) < \sum_k \bar{p}_k \bar{\alpha}_k$,
then $\bar{q} < \bar{q}$, $\bar{p} = \bar{p}$, $\bar{\alpha} < \bar{\alpha}$.

Proof of (i)

It follows from (4) that, along the process, $\dot{Y}(t) = 0$, so that $Y(t) = Y(0) = \sum_i \bar{q}_i + \sum_k \bar{p}_k \alpha_k(0) > \sum_i \bar{q}_i + \sum_k \bar{p}_k \bar{\alpha}_k$.

Consequently, for all $i = 1 \cdots F$, $Z_i(0) = \frac{\gamma_i Y(0)}{\bar{q}_i} > \bar{Z}_i = 1$ and for all $k = 1 \cdots G$, $X_k(0) = \frac{\delta_k Y(0)}{\bar{p}_k} > \bar{X}_k = \bar{\alpha}_k$.

This in turn implies $\dot{q}_i(0) = Z_i(0) - 1 > 0$. But $\dot{q}_i(0) Z_i(0) + q_i(0) \dot{Z}_i(0) = \gamma_i \dot{Y}(0) = 0$, so that $\dot{Z}_i(0) = [-\dot{q}_i(0) Z_i(0) / q_i(0)] < 0$.

These relations continue to hold for $t > 0$, so long as $Z_i(t) > 1$. As $Z_i(t)$ approaches 1, $\dot{q}_i(t)$ approaches 0. Limit points accordingly verify $\bar{Z}_i = 1$, $\bar{q}_i > q_i(0) = \bar{q}_i$.

Turning to G -commodities, two cases should be distinguished, according as $X_k(0) > 1$ or $X_k(0) \leq 1$. For all k such that $X_k(0) > 1$, if $\alpha_k(0) = 1$, then the reasoning just adduced for F -commodities applies unchanged and $\bar{X}_k = 1 = \bar{\alpha}_k \geq \bar{\alpha}_k$ and $\bar{p}_k > p_k(0) = \bar{p}_k$. If $\alpha_k(0) < 1$, then $\dot{\alpha}_k(0) = X_k(0) - \alpha_k(0) > 0$ with $\dot{p}_k(0) = 0$, $\dot{X}_k(0) = 0$. Thus, for all t such that $\alpha_k(t) < 1$, $\dot{\alpha}_k(t) \geq X_k(0) - 1 > 0$ and $\alpha_k(t)$ rises monotonically until $\alpha_k(\bar{t}_k) = 1$ for some value $\bar{t}_k \leq [1 - \alpha_k(0)] / [X_k(0) - 1]$. From that point onwards, $\dot{\alpha}_k(t) = 0$, $\dot{p}_k(t) = X_k(t) - 1 \geq 0$, and limit points will again satisfy $\bar{X}_k = 1$, $\bar{p}_k > p_k(0) = \bar{p}_k$, $\bar{\alpha}_k = 1 \geq \bar{\alpha}_k$, with $\bar{\alpha}_k > \bar{\alpha}_k$ whenever $\bar{\alpha}_k < 1$.

For all k such that $X_k(0) \leq 1$, the adjustment proceeds on quantities alone, with $\dot{\alpha}_k(0) = X_k(0) - \alpha_k(0)$, $\dot{X}_k(0) = 0$, $\dot{p}_k(0) = 0$. These relations continue to hold for $t > 0$ so long as $X_k(t) = X_k(0) \neq \alpha_k(t)$, so that limit points verify $\bar{\alpha}_k = \bar{X}_k = X_k(0)$. But $Y(0) > \sum_i \bar{q}_i + \sum_k \bar{p}_k \bar{\alpha}_k$ implies $X_k(0) > X_k(\bar{q}, \bar{p}, \bar{\alpha}) = \bar{\alpha}_k$, so that $\bar{\alpha}_k > \bar{\alpha}_k$.

Combining the different cases, the conclusion of the corollary follows, with

²⁰Vector inequalities obey $x \geq y$ iff $x_i \geq y_i$ for all i ; $x \geq y$ iff $x \geq y$ and $x \neq y$; $x > y$ iff $x_i > y_i$ for all i .

$\bar{\alpha} \geq \bar{\alpha}$, i.e. $\bar{\alpha}_k > \bar{\alpha}_k$ for some k . Indeed, $\sum_k \bar{p}_k \alpha_k(0) > \sum_k \bar{p}_k \bar{\alpha}_k$ rules out $\bar{\alpha}_k = 1$ for all k ; and $\bar{\alpha}_k < 1$ implies $\bar{\alpha}_k > \bar{\alpha}_k$ both when $X_k(0) > 1$ and when $X_k(0) \leq 1$.

Proof of (ii)

In this case, $Z_i(0) < 1$ for all $i = 1 \cdots F$ and $X_k(0) < \bar{X}_k = \bar{\alpha}_k \leq 1$ for all $k = 1 \cdots G$. For F -commodities, the argument in the proof of (i) holds with inequality signs reversed, so that $\bar{q} < \bar{q}$. For G -commodities, the argument in the next-to-last paragraph in the proof of (i) holds with inequality signs reversed, so that $\bar{p} = \bar{p}$, $\bar{\alpha} < \bar{\alpha}$. ■

This corollary has two important implications, one normative and one positive. Both concern the consequences of exogenous shocks *interpretable* as shocks affecting individual incomes, here introduced as exogenous modifications of the vector α . A natural, but not exclusive, interpretation would be that expectations about sales of the endowment (e.g. employment) enter the definition of (permanent) income, and are subject to exogenous shocks (revisions of expectations). Another interpretation would be that macroeconomic policies affect incomes and income expectations.

The normative implication is the following. Consider a sequence of supply-constrained equilibria $\bar{e}^\nu = (\bar{q}^\nu, \bar{p}^\nu, \bar{\alpha}^\nu, \bar{Z}^\nu, \bar{X}^\nu)$ where $\bar{e}^{\nu+1}$ is a limit point of process TP with initiation $(\bar{q}^\nu, \bar{p}^\nu, \alpha^{\nu+1}(0))$, where $\sum_k \bar{p}_k^\nu \alpha_k^{\nu+1}(0) > \sum_k \bar{p}_k^\nu \bar{\alpha}_k^\nu$. It follows from the corollary that $\bar{\alpha}^{\nu+1} \geq \bar{\alpha}^\nu$. Accordingly, the sequence $\bar{\alpha}^\nu$, $\nu = 1, 2, \dots$, will converge to $\iota^G = (1 \cdots 1) \in R^G$. That is, the sequence $\bar{e}^\nu$ will converge, $\nu \rightarrow \infty$, towards a *competitive equilibrium*. In other words, fiscal policy is potentially effective in this model. It can overcome the coordination failure sustained by downward price rigidities, and bring the economy to full use of its resources.

The positive implication is the following. The operation of process TP leads the economy to a supply-constrained equilibrium. An exogenous shock will upset the equilibrium and restart the process. For shocks interpretable as income shocks (leading to a combination of price and quantity adjustments), the corollary defines comparative-statics properties which constrain the evolution of the economy. The predictions allowed by the corollary are thus usefully sharper than those implied by the static theory in Drèze (1997) or Herings and Drèze (1998), which they complement.

It might also prove possible to characterise further the inflation-employment trade-off implied by assumption 5 and its robustness to alternative assumptions.

Thanks to its simplificatory power, the “fixed shares” assumptions permits other applications, that will not be developed here for lack of space, but two of which will be described succinctly. One application consists in introducing constraints on relative prices, i.e. real rigidities. A price index is a linear function of (q, p) , say $a^T q + b^T p$. A real lower bound on the price p_k then takes the form $p_k \geq \lambda_k(a^T q + b^T p)$. Consistency requires $b^T \lambda < 1$, under which condition one can solve for the reduced conditions $p \geq (I - \lambda b^T)^{-1} \lambda a^T q := Cq$. Consider then a sequence of supply-constrained equilibria $\bar{e}^\nu$, where $\bar{e}^{\nu+1}$ is a limit point of process TP with initiation $(\bar{q}^\nu, p^{\nu+1}(0), \bar{\alpha}^\nu)$ and $p_k^{\nu+1}(0) = \mu \max(\bar{p}_k^\nu, (C\bar{q}^\nu)_k) + (1-\mu)\bar{p}_k^\nu$. Under assumption 5, there exists $\mu > 0$ such that the sequence of supply-constrained equilibria $\bar{e}^\nu$ converges to a supply-constrained equilibrium satisfying $p_k \geq \lambda_k(a^T q + b^T p)$ for all $k = 1 \cdots G$. In this way, both nominal and real rigidities can be taken into account.

Another application brings out explicitly the time dimension. In an elementary two period framework, a single nominal interest rate relates the present (first period) values of future resources to their spot prices in the second period. Using assumption 5, it is easy to spell out some consequences of exogenous changes in that interest rate. Starting from a supply-constrained equilibrium, an increase in the nominal rate of interest defines new initial conditions for process TP . The process then converges to a new supply-constrained equilibrium, that compares as follows with the starting one: (i) the price level is lower in period one, but higher in period two; (ii) real national income is lower in period one, but higher in period two. The model is thus of potential usefulness for discussing monetary policy.

6. Concluding Remark

Theorem 1 contributes a partial answer to the question raised in paragraph 2 of the introduction: how do supply-constrained equilibria come about? The partial answer is: through a natural process of price and quantity adjustments, with arbitrary starting point. Elementary dynamics thus confirm the robustness of that equilibrium concept, previously recognised in static analysis.

A complementary program, illustrated in section 5.2, would study how the inefficiency inherent in supply constraints can be remedied through a combination

of trend inflation and demand stimulation, in economies where removing downward nominal price rigidities is either undesirable (the rigidities are second best) or impossible (e.g. the market power of some agents cannot be curbed, on “political economy” grounds). The work of Herings *et al.* (1998) carries out that program, for a sophisticated adjustment rule. It was remarked in my interpretation of process QP that my simple adjustment process parallels the achievements of their short-run phase. It should be clear that process TP parallels the achievements of their first and second phases combined. The normative implication of my corollary outlines an alternative to their third phase. As of now, all avenues remain worth exploring.

Section 5 above is also of some methodological interest. It illustrates how additional macroeconomic implications follow from additional assumptions, of the kind routinely used by macroeconomists. To microeconomists, such assumptions look unappealing. But so do standard assumptions like convexity of production sets. The integration of microeconomics and macroeconomics, to which many of us aspire, cannot ignore that particular path.

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