

Study of eddy current losses in printed windings for the optimization of slotless permanent magnet machines

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*"It does not matter how slowly you go
as long as you do not stop."*

— supposedly said by
Confucius

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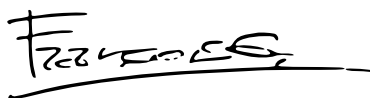
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Résumé

La technologie des PCB flexibles permet de développer de nouveaux types de bobinages qui améliorent les performances des moteurs électriques. On peut en effet remplacer le bobinage filaire habituel par un circuit imprimé, sur lequel il est possible de dessiner des pistes de cuivre aux formes très variées, ce qui amène de nouveaux degrés de liberté exploitables pour l'optimisation des moteurs.

Ce travail se concentre plus précisément sur le cas des petits moteurs synchrones à aimants permanents avec entrefer lisse, pour lesquels les PCB flexibles sont particulièrement bien adaptés. Le choix d'utiliser des aimants se justifie par le développement des terres rares qui permettent maintenant d'obtenir une grande densité d'énergie. Le choix d'une topologie avec un entrefer lisse procure de nombreux avantages et est guidé par le fait que, d'une part elle laisse la place à un arrangement des conducteurs complètement libre, et que d'autre part elle permet la réduction des pertes fer, du couple de dentures, des vibrations ou encore du bruit, grâce à l'absence de dents ferromagnétiques.

Bien que les bobinages sur PCB aient démontré leur efficacité à faible vitesse, on y observe rapidement l'apparition de courants induits lorsque la vitesse du moteur augmente. Car à même section de conducteur, les pistes de cuivre imprimées sont plus larges que le fil que l'on retrouve dans des bobinages standards et par conséquent de plus grandes surfaces conductrices sont exposées perpendiculairement au champ magnétique des aimants en rotation.

L'objectif principal de la thèse est donc de modéliser les pertes par courants induits pour les estimer et réduire leur impact lors de la conception et de l'optimisation des moteurs.

La première partie de la thèse concerne la modélisation de ces pertes. Un modèle de départ utilisant des éléments finis 3D et, s'appuyant sur une formulation de la magnétodynamique en potentiel vecteur, est présenté. Partant de ce modèle de référence, dès lors que l'on connaît l'expression du champ magnétique induit par les aimants permanents, on montre qu'il est possible d'estimer les pertes par courants induits à l'aide d'un modèle 2D FEM de façon beaucoup plus rapide et moins coûteuse en temps de calcul. Cette réduction du coût de calcul permet entre autres

de modéliser un bobinage de moteur au complet et d'y observer l'évolution des courants induits. Ce modèle, en plus d'être rapide et précis, a la particularité intéressante de prendre aussi en compte la forme et la topologie des conducteurs. Par la suite un modèle analytique est également proposé afin de disposer d'un outil capable d'estimer les pertes par courants induits en un temps encore plus réduit. Ce modèle analytique nous donne la possibilité de prendre en compte ces pertes dans le cadre d'une procédure d'optimisation qui demanderait un très grand nombre d'évaluations. Afin de valider les modèles proposés, un banc d'essai expérimental a été mis en place.

La deuxième partie de la thèse met en oeuvre les différents modèles créés afin d'étudier, d'une part l'impact de ces pertes sur le design optimal d'un moteur selon son point de fonctionnement donné et, d'autre part la pertinence d'utiliser des modèles éléments finis pour l'optimisation d'un moteur. En plus de confirmer que les pertes par courants induits sont essentielles à prendre en compte dans le cas de bobinages PCB, un premier résultat montre que la principale variable d'ajustement lorsque l'on souhaite changer le point de fonctionnement du moteur, est la largeur des pistes conductrices. Une seconde observation indique qu'il est possible de tendre vers une conception presque optimale uniquement à l'aide de modèles analytiques.

La troisième partie de la thèse propose une stratégie permettant de limiter les pertes par courants induits. Une simple diminution de la largeur des conducteurs n'est pas suffisant car elle va de pair avec une augmentation du nombre de spires du bobinage. Or dans ce cas, la tension d'alimentation étant limitée par l'électronique de puissance, cela impliquerait de mettre des spires en parallèle au détriment d'éventuels courants de recirculation. La solution suggérée pour résoudre ce problème est de dessiner des coupures isolantes dans les pistes de cuivre de la façon la plus adéquate possible. Cela revient à réaliser des parallélisations à une échelle plus locale, sans impacter la tension d'alimentation du moteur. Ce travail a mis en évidence les configurations optimales pour ces coupures et a montré qu'elles l'étaient pour toute une gamme de points de fonctionnement.

Abstract

Flexible PCB technology allows to develop new types of windings to improve the performance of electrical motors. By replacing the usual wire winding with a flexible printed circuit board, it becomes possible to design copper tracks with complex shapes which was not the case in the past. Therefore, these additional degrees of freedom can be exploited for motor optimization.

Specifically, this work focuses on the case of slotless micromotors with permanent magnets, for which flexible PCB are particularly well suited. The choice to use magnets is motivated by the developments of rare-earth materials which allow to obtain a high power density. The choice of the slotless topology is driven by the fact that, on the one hand, this topology leaves room for a completely free arrangement of the conductors in the airgap, and on the other hand, the absence of ferromagnetic teeth presents numerous advantages, for example, reduction of iron losses, cogging torque, vibrations or even of the noise.

Although the PCB winding outperforms the classical wire winding at low speeds, it is more prone to eddy currents as the motor speed increases. Indeed, for the same conductor cross-section, the printed copper tracks are wider than classical wires and therefore larger conductive areas are exposed to a perpendicular magnetic flux varying over time.

The objective of the thesis is therefore to model the eddy current losses in order to be able to estimate and reduce their impact during motor design and optimization process.

The first part of this thesis deals with the modeling of the eddy currents. A reference model using 3D finite elements and based on a vector potential formulation of magnetodynamics is primary explained. Starting from this model, it is shown that, knowing the airgap permanent magnets magnetic field, it is possible to estimate the eddy current losses using a 2D FEM model which is much faster and requires much less computational efforts. This model, in addition to being fast and accurate, has the interesting feature of also taking into account the shape and skewing of the conductors because it runs in the surface of printed circuit board. Subsequently, an analytical model is also proposed in order to estimate the eddy current losses in

a fraction of the time. This analytical model allows us to take into account these losses in the context of an optimization process that would require a large number of evaluations. In order to validate the proposed models, an experimental test bench was set up.

The second part of this thesis implements the models in order to study, first the impact of these losses on the optimal design of a motor for a given operating point, second the relevance of using finite element methods for optimization. It has been shown that eddy current losses have a significant impact on the winding design when using PCB. In addition to the above, a first important result is that the main adjustment variable is the width of the conductor tracks, when one wishes to change the operating point of a motor. A second observation indicates that it is possible to approach an almost optimal motor design with only the help of analytical models.

The third part of this thesis proposes a strategy to reduce the eddy current losses. A simple reduction of the width of the conductors is not sufficient because it goes hand in hand with an increase in the number of tracks composing the winding. But in this case, the supply voltage being limited by the power electronics, it would be necessary to put copper tracks in parallel at the expense of possible recirculation currents. The main idea adopted to tackle this problem consists in drawing insulating slits along large copper tracks. This amounts to carry out local parallelizations without impacting the motor supply voltage. This work highlighted the optimal slit configurations and showed that they were optimal for a range of operating points.

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Introduction

1

1.1 Context & Motivations

1.1.1 The need

A *Micromotor* refers to a small electric motor whose main dimensions range from a few millimeters to a few centimeters. In general, people largely underestimate the number of micromotors used and active every day around us, even though they are present in a wide variety of applications and have a strong impact on our living comfort and our environment. Whether it is a hand drill, a surgical tool, a rear-view mirror of a car, a fan, a kitchen blender or yet a drone, all of these are equipped with electric micromotors.

The demand for these small motors is constantly increasing [1] [2] and there is a permanent desire to improve their performances. As the following examples show, better performances can be sought in various ways depending on the application in which the motor will be used.

In industry, some workers often use power tools. Reducing the *weight* and *size* of such tools would obviously improve their working comfort. A dental drill has to go at a high *speed* to be effective, but at the same time its *temperature* must remain cool when in the dentist's hand. A bionic hand requires motors with the *torque* capabilities of real fingers. The earth's resources are increasingly scarce [3], climate and energy consumption are a major concern today. Thus, improving the *efficiency* of micromotors also becomes one of the main objectives in their development, especially, when they are integrated to devices sold by millions, such as a car.

In any cases, no matter what the targeted features of the motor are sought after, the need for improvement is always relevant and driven by the desire to enhance quality, comfort and durability.

Although it exists various types of micromotors, this thesis focuses on permanent magnet motors. Indeed, the development of rare-earth based permanent magnets has led to a significant increase in power density. In addition, technological advances in electronics have allowed the emergence of high quality AC and BLDC controllers.

Motors equipped with these types of controllers have a drastically increased lifetime and higher efficiency than their classical brushed DC counterparts. Furthermore, as they no longer use commutators, they are able to operate in harsh environments. In this thesis, we consider both AC synchronous motors and brushless DC motors.

1.1.2 The motors

Permanent magnet (PM) micromotors can be classified into two main categories depending on whether their armature winding is inserted into the slots of a magnetic circuit or directly placed in the air gap as shown in Fig. 1.1.

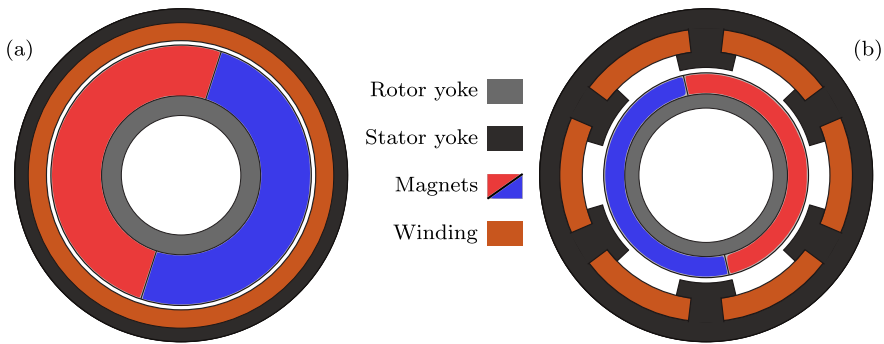


Figure 1.1: Representation of (a) a slotless motor and (b) a slotted motor. Figure adapted from [4].

To produce high torques, the slotted topology is the traditional one, as the ferromagnetic teeth allow for reaching high magnetic flux densities in the airgap; therefore a high torque constant. However, even if the airgap magnetic field is necessarily reduced in slotless topologies due to an apparent airgap thickness increase, the impact on the torque capability is moderate for this kind of motor because the absence of teeth saves more place for the winding. Indeed, the torque capabilities of a motor are directly related to the flux linked with the winding and also to the inverse of its electrical resistance. At least, this is the case at low to moderate speeds and at given external dimensions. Therefore, this additional space can be used either to reduce the electrical resistance of the winding, by increasing the cross section of the conductors, or to increase the flux caught by the winding. This latter can be done by increasing the number of loops constituting the winding or increasing the rotor radius and thus the poles surface in the motor. Furthermore, an increase of the rotor radius could also reduce the impact of an increase of the apparent airgap, because, on the one hand, there is more space for the rotor, and thus more room for magnets which increases the magnetic field amplitude, on the other hand, a greater radius implies a longer leverage arm to produce torque. Finally, the availability of high-energy neodymium-magnets tends to compensate the large airgap of slotless topologies. Studies have also shown that high-energy magnets, whose operating point are less sensitive to the airgap thickness, sometimes allow even better torque capabilities to be obtained with a slotless topology than with a

slotted one, especially when outer radii are small [5]. Although the slotted topology is commonly the best for achieving high torque capabilities, we have just seen that the slotless topology has the potential to reach similar performance with proper use of the space left by the ferromagnetic teeth.

To produce high speeds, the slotless topology has a clear advantage over the slotted one. Indeed, removing the ferromagnetic teeth allows to drastically reduce the extra iron losses which are strongly related to the speed. In addition, they usually have a negligible armature reaction field which also helps to reduce the iron losses. Unfortunately, the winding of slotless motors is also more exposed to the PM magnetic field and hence there are extra eddy current losses in it, which balance the gain on these iron losses.

Globally, in terms of power capabilities, which is simply the product of the speed and torque capabilities, both topologies are known to be similar with a slight advantage for the slotless as the speed increases [6]. Today, the choice of one category or the other depends mainly on the application for which the motor is designed, and the constraints associated with it.

Besides that, the slotless configuration offers some other benefits. For example, the cogging torque created by the attraction between magnets on the rotor and the iron teeth of the stator is obviously removed. This also reduces the noise and vibrations and allows a smooth motion. Then, the weak reaction field of the armature leads to a good linearity between the torque and the supply current which helps for the controllability [7, 8]. Third, these motors are generally well suited to miniaturization, thanks to their simpler design easier to scale down [9], their manufacturing and assembly costs are usually reduced [10].

In this thesis, we focus on the slotless topology.

1.1.3 The windings

A key element of the motor is the winding because, with the magnets, they represent the two sources of magnetic flux which interact together to produce torque. The windings are distinguished by their fabrication technique as well as by their topology and shape.

First, the windings made of copper wires have been the traditional choice among micromotors, for decades. The most common shapes when using wound wires for the slotless configurations are the diamond, rhombic and zigzag types [9, 11–13], as shown in Fig. 1.2. All three are distributed windings. A great benefit of these is that their end-windings are reduced to their simplest form and can be entirely located in the motor airgap [14]. It can also be noticed that, unlike in some purely axial windings (or rectangular shapes), these end-windings actually do participate in the generation of the motor torque and not only in the generation of Joule losses.

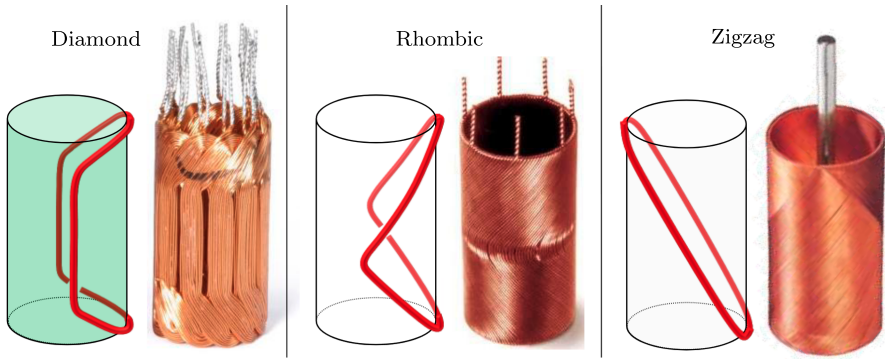


Figure 1.2: Common topologies of wire windings for slotless motors. From [15].

In slotless machines, the windings do not have to follow the direction of the slots, but they can be freely arranged in the airgap. This brings a lot of opportunities for optimization. Yet, the industry got stuck on these few shapes of winding for years due to the complexity of the fabrication process when using copper wire. Another drawback of the copper wire is its constant rounded cross section which limits the possibility of improving the fill factor of the winding.

However, recent developments of new technologies have been performed these past years to fabricate windings differently. Numerous feasible shapes and topologies, which were impossible to develop previously with simple wires, are now offering a substantial potential for improvement. Therefore, these technologies have the potential of redefining the optimal use case of the slotless motors and how they compare with the slotted ones. Indeed, it has been demonstrated theoretically that the use of non-constant conductor sections and curved optimized paths for the current can significantly increase the performances of the motor [16].

In practice, Bräuer et al. realized windings using 3D screen printing as shown in Fig. 1.3. Nevertheless, they did not optimize the shape [17] and their winding remains similar to the diamond topology. The big step in their work is that printing the winding allowed them to obtain non-constant sections which decreases the electrical resistance. Like most of slotless windings their prototype is also self-supporting [18] which is a very interesting property, especially when the objective is to miniaturize the motor. Indeed, self-supporting windings do not require additional structural elements to maintain them.

Tsai et al. suggested an improved winding for an axial flux machine built using micro-electroforming [19], as shown in Fig. 1.4. Micro-electroforming is a process that forms products by the electrodeposition of metal onto a cathode substrate [20]. Although this process is clearly not the most economical method, it can be used for a wide range of product scales. Furthermore, unlike etching techniques micro-electroforming process reduces the waste because it allows better use of materials.

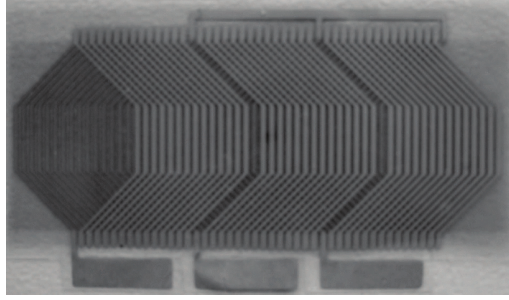


Figure 1.3: Sample of a screen printed airgap winding of a three phase synchronous machine. From [17].

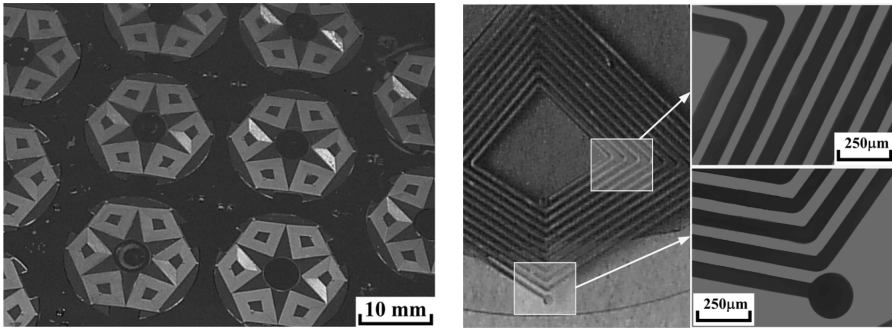


Figure 1.4: Electroformed winding structure after the UV exposure and microscopy photograph of a coil. Images from [19].

Burnand et al. proposed a winding built by using laser cutting which offered the possibility to design the winding with an optimized complex topology [21], as shown in Fig. 1.5. They cut the coils of the winding from a 1-mm sheet of copper, then curved them on a dedicated tool before welding them together. After that process, the conductors forming the winding were isolated using resin.

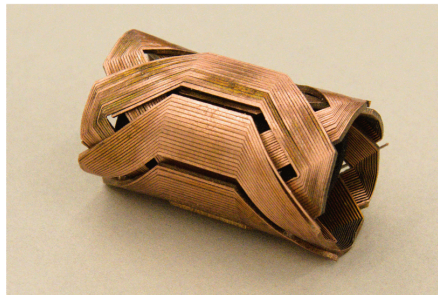


Figure 1.5: A laser-cut winding prototype made of six coils. From [21].

Among the alternatives to the copper wires, the use of flexible printed circuit boards (flex-PCB) is another which has been growing in recent decades. The technique consists first, in printing the copper conductors on flat PCBs which are made of flexible materials and second, in rolling them to finally obtain a cylindrical winding. Fig. 1.6 shows the structure of a double-layered PCB and Fig. 1.7 shows the final result after it has been rolled and heated to form an armature winding which is self-standing. Polyamide insulating layers can be really thin, reach small bending radii, while having mechanical properties that allow them to support thick copper layers leading to a very high fill factor [22]. Although the etching process involved leads to a waste of material, this technique remains economical and has the advantage of being extremely repeatable and accurate [23]. The technique was originally patented in 1987 [24] by Herbert Malsky (USA). Since then, multiple designs have applied this technique, from sensors and inductors [25] [26] to motors [27–29]. Others have used the PCB technique to build the windings and the sensors simultaneously [30]. Quite a thorough literature review on PCB windings is available in [31].

Dehez et al. showed that the printed circuit board technology can be used to obtain optimized complex shapes with non-constant cross sections [32]. Those improved windings are designed to increase the flux while maintaining a relatively low electrical resistance. To reduce the electrical phase resistance, horizontal shortenings are achieved in the end-windings as proposed in [33]. Because the conductor width can vary along its path, the clearance between the tracks is also kept minimal to get a high fill factor. An example of such an optimized winding shape is shown in Fig. 1.7.

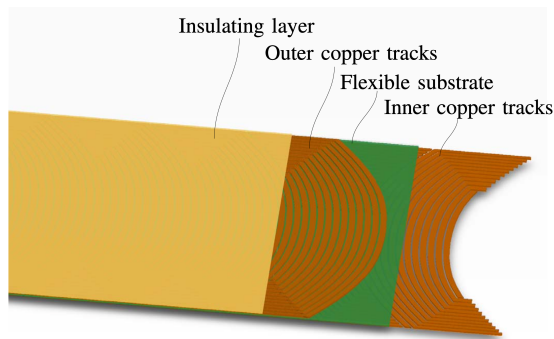


Figure 1.6: Scattered view of the windings on the flexible PCB from [22].

At the time of this writing, the flexible PCB technology is a competitive technology which is already commercially available at reasonable costs and outperforms traditional motors by up to 40% when compared on the basis of an optimized motor constant [22].

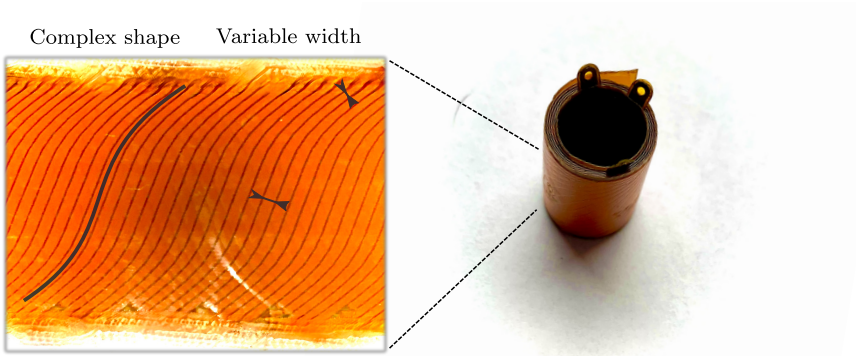


Figure 1.7: Example of an optimized winding shape that illustrates some of the features achieved through the use of printed technology.

Searchers have also shown a growing interest on additive manufacturing (AM) [34]. Indeed, in [35–41], they have all demonstrated that there is a lot of potential in the new means of designing and manufacturing electrical components. Although most of them are far from being used in commercial motors, all these innovations, which are constantly improving, are promising in the development and enhancement of motor windings for the future. In addition to having better fill factors and shapes which improve the torque constant, AM can also be used to enhance the thermal management of the motors, allowing integrated heat pipes for cooling the windings [42] for example.

This thesis focuses on the printed circuit board technology.

1.1.4 The losses

In any motor, energy conversion is made at the expense of some losses dissipated as heat. First, it means that efficiency is not 100% and that some electrical energy is not used for mechanical purposes. Second, the heat tends to increase the motor temperature which can be a limiting factor [43]. Indeed, if the temperature rises too high, magnets lose their remanence, the insulators are damaged, and the properties of the materials change. Therefore, running the motor in hot conditions can be damaging and thus it is important to keep it at reasonably cold temperatures. In this context, it becomes a necessity to limit the losses, especially if there is no room for a coolant system. Having a good knowledge of the losses occurring in a motor allows to run it safely and sustainably.

A list of these losses is proposed in Fig. 1.8, in which they have been classified by the motor regions. These losses are the fruit of two distinct physical phenomena.

First, there is the mechanical friction which leads to the windage losses and the friction due to the bearings. Those losses are mostly linked to the speed of the motor and they become generally important only at very high speed.

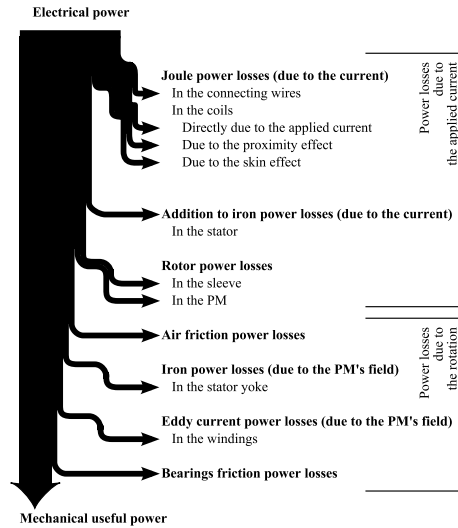


Figure 1.8: Power balance in electrical motors. Adapted from [44].

Second, there is the Joule effect leading to all the remaining losses. According to the motor topology and its working point, some losses are more or less important than others in the power balance. Indeed, rotor losses and iron losses in the stator, due to the supply current, are much less important in the case of a slotless motor. This is because their armature reaction field is weaker than in slotted motors. Neglecting these two, it remains the iron losses in the stator yoke and the eddy current losses in the winding due to the rotation of the magnets, as well as the Joule losses in the winding due to the supply current.

Sometimes, proximity and skin effects are also considered in the literature. They are not additional losses per se but can be seen as two phenomena which modify the behavior of the current. The proximity effect consists of eddy currents induced by the supply current flowing in the neighboring conductors. The skin effect tends to repel the current on the conductor sides when the frequency becomes high, increasing the effective electrical resistance. Skin effect can be computed as in [45, 46] for example. The two phenomena could be integrated into the supply losses value as it will be the case throughout this thesis. Note that, those two physical phenomena also apply to eddy current losses even though it is not generally detailed. It can also be noticed that eddy current losses are also composed, on the one hand, of local eddy currents and, on the other hand, of recirculating currents which occur between two connected regions seeing different back emf as shown in Fig. 1.9.

All those losses have been studied by many researchers over the last decade to find the best way of designing a motor. Concerning the iron losses, although they remain difficult to estimate, it exists plenty of models in the literature as listed in [47] for example. The use of PCB windings instead of classical wires does not modify the validity of these iron losses models. Most of them can thus be directly applied without any changes.

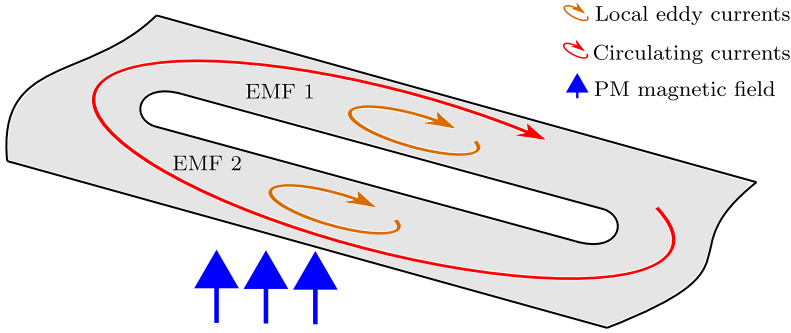


Figure 1.9: Distinction between the circulating currents arising between two connected conductors and the local eddy currents.

Concerning the winding, some authors evaluated the impact of the conductors position [48, 49], others studied the impact of the conductors connections and twists of conductors [50]. Yamazaki et al. [51], for example, showed that there is an optimum for the number of conductors to be put in parallel in the winding. This optimum is a trade-off between the amount of eddy currents induced by the PM and the supply losses. Indeed, eddy current losses and supply losses have antagonist behaviors as it will be shown later. In [52–54] they investigated the effects of circulating currents between parallel strands. Some authors developed methods to evaluate these losses trying to reduce the computational efforts, either by using time-harmonic FEM as in [55] or analytical methods as in references [56–59].

Overall, they all show that eddy current losses are as important as the supply losses to consider, in order to obtain pertinent motor designs. However, all these works analyzed eddy current losses in the context of slotted motors. Only a few research papers consider slotless windings. In addition, when they do so, most of them give information about axial flux machines [60] or linear structures [60, 61]. In [62], Wang et al. dealt with eddy current losses in the case of an AFPM and suggested various ways of connecting multi-conductors in order to reduce them. Burnand et al. proposed an analytical model of eddy current losses for small RFPM machines when the conductors are straight along the motor axis [63]. Similarly, Min S.G. proposed a 2D analytical model in square conductors in the case of a sandwiched structure [64].

To summarize, only a few discuss about eddy current losses in slotless motors. Most of those who deal with the matter only consider axial conductors and ignore the impact of their shape and skewing. However, eddy losses can be a huge drawback in slotless machines since their winding is totally immersed in the airgap magnetic field. Furthermore, in the case of PCB windings, for the same cross-section area, the copper tracks are wider than the classical wires (printed conductive tracks are wide and thin). Therefore, larger areas of conducting material are exposed to a perpendicular time varying magnetic flux, which favors the appearance of eddy currents [65]. At high speed, the Joule losses associated with these eddy currents

can deteriorate the performances PCB windings reach at low to medium speed. Because any motor has a limited capability to dissipate the heat, an excess in eddy currents also means that less supply current is allowed to be injected to produce torque.

1.2 Objectives

As related in section 1.1.1 of this introduction, there is a real need for high-performance micromotors that slotless PM motors can fulfill. Section 1.1.2 motivated the choice to focus on the slotless topology of motors because it has many benefits over the slotted one while having similar power capabilities.

Despite the fact that the winding is a major part of the motor that strongly impacts its performance, it has not varied much since the 1950's [22]. Therefore, as highlighted in section 1.1.3, the use of flexible PCB windings is now a breakthrough technology that allows optimizing the shape and width of the conductors to obtain much better performances by far. At low speed, meaning when the Joule losses due to the supply current flowing in the winding is what matters the most, this technology has already demonstrated its great capabilities, improving the nominal torque of existing motors by up to 40%. However, as the speed increases, more and more physical phenomena and other types of losses progressively start to impact on performance and must be considered when designing a motor. The next step to bring PCB windings into higher speed motors is to be able to deal with eddy current losses as they are likely to experience it. However, the literature addressing these losses in slotless machines is quite poor and almost inexistent when with regard to PCB windings. Solving the problem of eddy current losses in PCB windings will extend and bring the benefits of this breakthrough technology to a wider range of applications. Furthermore, the results will not only apply to PCB windings, but also to any type of winding in which the conductors would have a similar topology and shape.

In this context, the objective of this thesis is threefold:

- To propose models allowing to predict eddy current losses in PCB windings, both quickly and accurately;
- To exploit these models to study the impact of these losses on the performance and the optimal design of a slotless motor with PCB windings;
- To use these models to compare different strategies, at the conductor level, and find an adequate trade-off between the eddy current losses and the supply current losses, maintaining the supply voltage of the machine within the limit of the power electronics.

1.3 Content & Organization

This manuscript is organized as follows.

Chapter 2 introduces a full 3D finite element model which implements the quasi-static Maxwell's equations to assess the airgap magnetic field and the eddy current losses. From that accurate but expensive model, successive observations are made to reduce the eddy losses computation part into a simpler 2D planar model which has to be computed only in the conductors. Though, this simpler model for the losses needs to be provided with the airgap magnetic field as a source term. The last part of the chapter is thus dedicated to proposing an adequate analytical representation of this field which can be directly used in the model.

Chapter 3 presents the results of an experimental validation of the model proposed in chapter 2. This validation is carried out on a test bench which consists in deceleration tests on a rotor made of a cylindrical permanent magnet. Four types of windings were designed and built for this purpose, each being associated with the validation of a specific hypothesis. In addition, a study of the sensitivity to the various parameters which could affect the measures and/or the estimations of the losses is proposed.

Chapter 4 derives some analytical expressions to quickly evaluate the eddy currents in the windings. Two methods to derive these expressions are proposed. A first one results from the application of the Π -theorem and a second one results from the resolution of a global equivalent circuit modeling the eddy current loops in a conductor segment. The empirical coefficients appearing in these expressions are determined from 2D and 3D finite element models presented in chapter 2. The last part of the chapter is dedicated to the application of this analytical model to curved conductors and to the estimation of its accuracy.

Chapter 5 implements the loss models proposed in chapter 1 and 3 within full motor optimizations. First, the importance of considering the eddy current losses is highlighted. Second, optimal configurations obtained using either analytical or FEM models are compared. Third, the evolution of the optimal motor configuration is studied according to the nominal speed. Finally, the optimization process is discussed in order to make it as fast and efficient as possible. In this chapter, all conductors of a winding phase are assumed in series, so that there is no need to consider recirculation currents. It also means that the problem of the supply voltage is not yet addressed.

Chapter 6 focuses on the winding only and studies the possibility of making slits along the large conductors to reduce the amount of eddy currents. It investigates the trade-off between supply losses and eddy current losses in order to maximize the power density of the motor. Considering the problematic of a limited supply voltage and thus the necessity to put tracks in parallel to keep a moderate torque constant, this chapter also deals with the topic of circulating currents that appear when wiring conductors in parallel.

Finite element model

2

The goal of this chapter is to derive a light but precise model of the eddy currents in the PCB winding, using the finite element method (FEM). The reason for choosing finite elements is that they allow accurate consideration of the shape of the conductors even though it may be complex. At first, a standard 3D FEM formulation is implemented and will serve as a reference, since it is commonly accepted that it provides the best possible approximation for a given mesh decomposition. Although, a 3D simulation may be accurate, it requires a huge amount of computational effort and a lot of time. Therefore, successive simplifications of the problem are made and justified in order to obtain an equivalent 2D problem which is simpler to solve.

2.1 Reference motor

The present work does not target a specific application and therefore has no real motor to optimize. However, a reference motor is defined to support the developments throughout this thesis.

Except if mentioned otherwise, the motor will be considered with an axial length of 22 *mm* and an external diameter of 16 *mm* as shown in Fig. 2.1. The rotor has a radius of 4 *mm*, the total winding thickness is 2 *mm* and the iron yoke thickness is 1.5 *mm*. The airgap is fixed to 0.5 *mm*. For the sake of simplicity, the rotor is assumed to be a plain magnet at first. The amplitude of the remanent magnetic field B_r is 1.3 *T*. The rotor has a one pole pair with parallel magnetization, thus only a fundamental harmonic of the PM magnetic field exists.

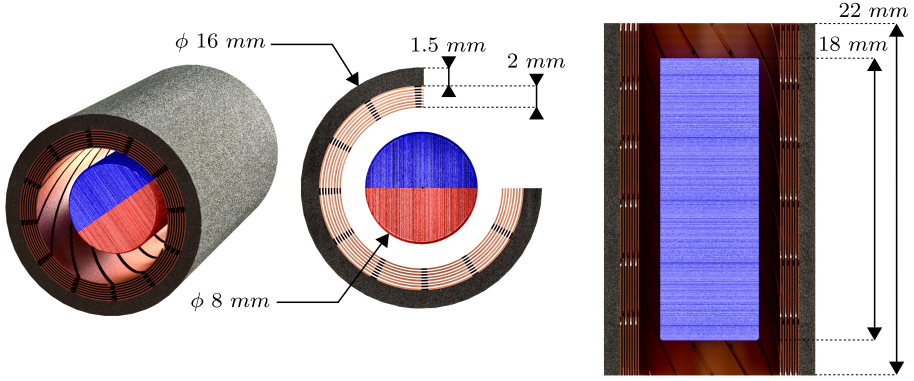


Figure 2.1: Perspective view and dimensions of the reference motor.

2.2 Coupled 3D formulation

2.2.1 Governing equations

Many electromagnetic problems can be represented by the set of following equations:

$$\nabla \cdot \mathbf{b} = 0 \quad (\text{magnetic Gauss law}) \quad (2.1)$$

$$\nabla \cdot \mathbf{d} = q \quad (\text{electric Gauss law}) \quad (2.2)$$

$$\nabla \times \mathbf{h} = \mathbf{j} + \partial_t \mathbf{d} \quad (\text{Ampere law}) \quad (2.3)$$

$$\nabla \times \mathbf{e} = -\partial_t \mathbf{b} \quad (\text{Faraday law}) \quad (2.4)$$

$$\mathbf{b} = \mu_0 \mu_r \mathbf{h} + \mathbf{b}_r \quad (2.5)$$

$$\mathbf{d} = \epsilon_0 \epsilon_r \mathbf{e} + \mathbf{p}_e \quad (2.6)$$

$$\mathbf{j} = \sigma \mathbf{e} + \mathbf{j}_s \quad (\text{local Ohm's law}) \quad (2.7)$$

The first four equations are the well-known generalized Maxwell-Heaviside laws [66, 67] whereas the last three are constitutive equations linking quantities of different nature. The vector fields $\mathbf{b}$, $\mathbf{d}$, $\mathbf{h}$, $\mathbf{e}$ and $\mathbf{j}$ are respectively the magnetic flux density, the electric flux density, the magnetic field, the electric field and the current density. The scalar field q , which is the electric charge, the remanent magnetic flux density $\mathbf{b}_r$, the electric polarization $\mathbf{p}_e$ and the current density $\mathbf{j}_s$ are source terms in these equations.

All the quantities can be separated in two groups $\{\mathbf{b}, \mathbf{b}_r, \mathbf{e}, q, \mathbf{p}_e\}$ and $\{\mathbf{h}, \mathbf{d}, \mathbf{j}, \mathbf{j}_s\}$. Within each group the quantities are only linked by geometrical differential operators with no use of any material properties or physical constants. But linking quantities from one group to another requires at least one physical constant, this is the purpose of constitutive equations. In the constitutive equations, the electrical conductivity σ , the relative magnetic permeability μ_r and the relative electric permittivity ϵ_r can be either scalars or tensors to consider the anisotropic properties of the matter. Note that (2.7) is only valid for non-moving conductors; otherwise, it becomes $\mathbf{j} = \sigma(\mathbf{e} + \mathbf{v} \times \mathbf{b}) + \mathbf{j}_s$ with $\mathbf{v}$ the speed of the conductor.

The previous equations have never been invalidated since their completion by Maxwell in the late 19th century [68]. Together, they form a mathematical representation of the same physical phenomenon, namely the electromagnetic field [69–71]. Note that the charge conservation can be deduced by taking the divergence of (2.3) and applying (2.2):

$$\nabla \cdot \mathbf{j} + \partial_t q = 0 \quad (2.8)$$

When there is no interaction of the electromagnetic field with the external elements, the system is fully described with two constants μ_0 and ϵ_0 . In the internal system unit, the electric permittivity in vacuum ϵ_0 is equal to $1/36\pi * 10^{-9} \text{ Fm}^{-1}$ and the magnetic permeability in vacuum μ_0 is equal to $4\pi * 10^{-7} \text{ Hm}^{-1}$. Both constants are linked through the speed of light c in a vacuum as $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$.

In the context of electrical machines, the quasi-static approximation of these equations is generally considered. The frequency at which the electric field varies in the motor is of the same order of magnitude as its rotational speed ω . Assuming $\sigma = 5.87 * 10^7 \text{ Sm}^{-1}$ (copper conductivity), $\omega \approx 1 * 10^4 \text{ rads}^{-1}$, and $\epsilon_r = 1$ it can be observed that

$$\partial_t \mathbf{d} \sim \omega \epsilon_0 \epsilon_r \mathbf{e} \lll \sigma \mathbf{e} = \mathbf{j} \quad (2.9)$$

which means that the electric flux density term can be neglected in equation (2.3). It also leads to an uncoupling of the equations (2.2) and (2.6) so that we don't have to be concerned about them anymore.

In order to analyze the eddy currents in the windings, only the time-varying remanent magnetic flux density of the magnets $\mathbf{b}_r$ needs to be read as a source term. The three-phase winding is also studied in the open loop, so that $\mathbf{j}_s = 0$. Furthermore, no static charges are considered in the motor $q = 0$. Note that it also means that the conservation equation (2.8) simply becomes:

$$\nabla \cdot \mathbf{j} = 0 \quad (2.10)$$

Finally, the system to solve in the motor is a magnetodynamics problem:

$$\nabla \cdot \mathbf{b} = 0 \quad (\text{magnetic Gauss law}) \quad (2.11)$$

$$\nabla \times \mathbf{h} = \mathbf{j} \quad (\text{Ampere's law}) \quad (2.12)$$

$$\nabla \times \mathbf{e} = -\partial_t \mathbf{b} \quad (\text{Faraday law}) \quad (2.13)$$

$$\mathbf{b} = \mu_0 \mu_r \mathbf{h} + \mathbf{b}_r \quad (2.14)$$

$$\mathbf{j} = \sigma \mathbf{e} \quad (\text{local Ohm's law}) \quad (2.15)$$

In the reference motor, shown in Fig. 2.2, four distinct subsets of the domain Ω can be distinguished. At first, the air, the insulator and all the elements which are neither conductive nor magnetic form the subset Ω_a . Second, the winding in which the eddy currents are induced is assumed non magnetic and is denoted Ω_c . Then the permanent magnets, for which we only consider their magnetic aspect, but we ignore their electric conductivity, are denoted Ω_m . Finally, the iron parts, which can have non-linear magnetic properties, are denoted Ω_i . The subsets' properties are summarized in Table 2.1. As an example, Fig. 2.2 shows the subsets in the case of the reference motor.

Table 2.1: Particularities of each domain.

Domain	$\mathbf{b}_r$	σ	μ_r
Ω_a	0	0	1
Ω_c	0	$\neq 0$	1
Ω_i	0	0	$\neq 1$
Ω_m	$\neq 0$	0	1

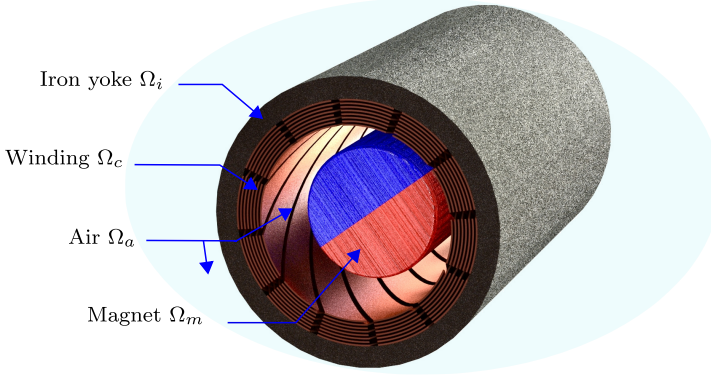


Figure 2.2: Definition of the domains of the reference motor.

2.2.2 Magneto-dynamic $\mathbf{a} - \mathbf{v}$ formulation

In order to obtain the eddy current $\mathbf{j}$ in the windings, equations (2.10) to (2.15) are usually not solved directly. The system is first reduced to fewer unknown quantities. There are several ways to combine the previous equations, each of them bringing out different quantities into light. Any combination can be made but the preferred formulation chosen in this work is the common $\mathbf{a} - \mathbf{v}$ formulation explained below. The main advantage of this formulation is that it deals very well with multiply-connected domains, without the need of cutting mesh techniques or equivalent [72], property that will be used in chapter 6. In addition, boundary conditions are very simple to apply in this formulation.

As the vector field $\mathbf{b}$ is divergence free, by definition it derives from a magnetic vector potential $\mathbf{a}$ so that:

$$\mathbf{b} = \nabla \times \mathbf{a} \quad (2.16)$$

with $\mathbf{a}$ defined up to a gradient. It means that $\mathbf{b}$ also equals to $\nabla \times \mathbf{a}'$ with $\mathbf{a}' = \mathbf{a} + \nabla f$ for any scalar function f .

Injecting (2.16) into the Faraday law (2.13), one obtains:

$$\nabla \times (\mathbf{e} + \partial_t \mathbf{a}) = 0 \quad (2.17)$$

If the curl of a quantity is null, it means that this quantity derives from a scalar potential. Hence, defining v the electric scalar potential, it can be written that:

$$\mathbf{e} + \partial_t \mathbf{a} = -\nabla v \quad (2.18)$$

The conductivity σ is assumed to be constant in the conductors. The magnetic permeability $\mu = \mu_0 \mu_r$ is also assumed to be constant, except in the iron parts where it can be non-linear. There is no loss of generality in this assumption because if they are varying in space, the domain can be decomposed into subdomains in which they would be constant [72]. Therefore, from the previous equations, Ampere's law (2.12) can be successively rewritten using the scalar and vector potentials:

$$\nabla \times (\nabla \times \mathbf{a}/\mu) - \nabla \times (\mathbf{b}_r/\mu) = \mathbf{j} \quad (2.19)$$

$$\nabla \times (\nabla \times \mathbf{a}/\mu) = \sigma \mathbf{e} + \nabla \times (\mathbf{b}_r/\mu) \quad (2.20)$$

$$\nabla \times (\nabla \times \mathbf{a}/\mu) = -\sigma \partial_t \mathbf{a} - \sigma \nabla v + \nabla \times (\mathbf{b}_r/\mu) \quad (2.21)$$

In conductive domains ($\sigma \neq 0$), the conservation equation has to be solved to also determine the scalar electric potential:

$$\nabla \cdot \mathbf{j} = 0 \quad (2.22)$$

$$\sigma \nabla \cdot \mathbf{e} = 0 \quad (2.23)$$

$$\nabla \cdot (\partial_t \mathbf{a} + \nabla v) = 0 \quad (2.24)$$

Equations (2.21) and (2.24) could be directly solved in the temporal domain, given initial conditions for the quantities $\mathbf{a}$ and v . However, this approach leads to two difficulties. First, the initial conditions, especially the magnetic field distribution in the machine, are generally unknown and therefore zero values are classically implemented. It means that a transition needs to be simulated between this initial state and the steady state we are looking for at the nominal speed. But this transition has no physical reality. Second, the time derivative is solved step by step in an iterative way. This means that all steps, starting from the initial state, need to be computed even though they are unnecessary for the analysis of the eddy current losses. To bypass these difficulties and reduce the computational efforts, the problem can be rewritten and computed in the time-harmonic domain.

Considering that the rotor is spinning at a constant speed ω , i denoting the imaginary number, the equations to solve the problem in steady state become:

$$\nabla \times (\nabla \times \mathbf{a}/\mu) = -\sigma i \omega \mathbf{a} - \sigma \nabla v \quad \text{in } \Omega_c \quad (2.25)$$

$$\nabla \cdot (i \omega \mathbf{a} + \nabla v) = 0 \quad \text{in } \Omega_c \quad (2.26)$$

$$\nabla \times (\nabla \times \mathbf{a}/\mu) = \nabla \times (\mathbf{b}_r/\mu) \quad \text{in } \Omega_a, \Omega_i, \Omega_m \quad (2.27)$$

To close the system and make it unique, it is also mandatory to add boundary conditions as well as a gauge.

At the conductor boundary, the current density can't go through the frontier and therefore $\mathbf{j} \cdot \mathbf{n} = 0$ with $\mathbf{n}$ the normal vector to the boundary surface must

be imposed. In terms of unknowns it means that $\mathbf{n} \cdot (i\omega\mathbf{a} + \nabla v) = 0$ must be respected. At infinity, the vector potential must vanish [73], so that $\mathbf{a}(\infty) = 0$. Either the volume of air surrounding the machine is large enough so that directly setting $\mathbf{a}|_{\partial\Omega} = 0$ won't modify the results, or shell transformations can be used to apply boundary conditions at infinity without truncation approximation [74].

Finally, to make the field $\mathbf{a}$ unique, a co-tree gauge is applied [75, 76]. Note that this gauge is not mandatory if the solver is an iterative one because in that case a singular matrix is not problematic. The co-tree gauge consists in creating a tree κ which goes through all the vertices of the mesh without forming any loops. Then the problem is solved in the dual space κ' composed of all the edges which are not part of the tree. An example is given in Fig. 2.3.

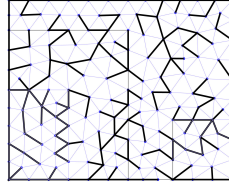


Figure 2.3: Tree κ (in bold) and associated co-tree κ' (other edges) in 2-D. From [76].

These equations have been thoroughly studied and are nowadays easily solved using common finite element solvers [77, 78]. In this work, all the finite element models are computed using the open source Onelab package [79] that combines the mesh generator Gmsh [80] and the solver GetDP [81–84]. The problems are solved using first order mixed finite elements [85, 86].

The eddy current losses are simply obtained by integration of the power loss density $\rho||\mathbf{j}||^2$ over the conductive tracks Ω_c . Using (2.18), these losses can be directly written according to the unknowns $\mathbf{a}$ and v :

$$P_{eddy} = \frac{\sigma}{2} \int_{\Omega_c} ||-\partial_t \mathbf{a} - \nabla v||^2 d\Omega \quad (2.28)$$

The factor $1/2$ appears because the mean losses in time are considered here.

2.3 Model reduction

Section 2.2.2 provided the equations to solve. However, the domain spaces $\Omega_a, \Omega_c, \Omega_m, \Omega_i$ still need to be discretized and meshed properly. The magnet, the iron and the air surrounding are simple hollowed cylinders quite easy to mesh but the winding is way more complex. Not only does the complexity come from the shape of the conductive tracks but also much more from the huge ratio between their length ($\sim cm$) and their thickness (tens of μm), and from the scale difference between the motor as a whole and the details of the copper tracks. Furthermore,

the gaps between the tracks are also very small (hundreds of μm). In consequence, either the elements composing the mesh are completely distorted and their quality is insufficient or the number of elements is tremendously huge, and the problem becomes impossible to solve with a standard computer. As a simple visual example, Fig. 2.4 shows a potential mesh for one PCB layer of windings. To correctly mesh only the external 2D surface of this domain, about one million elements are already required.

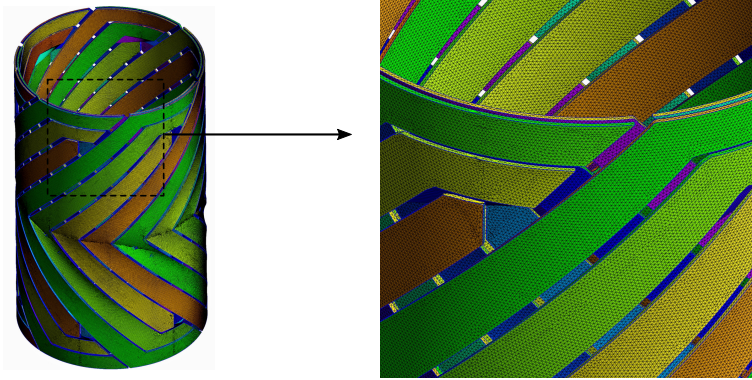


Figure 2.4: One PCB layer of conductive tracks which has been meshed in 2D with about one million unstructured elements using Gmsh.

Consequently the difficulty is to draw a meaningful geometry which can be meshed with a reasonable number of elements, and of good quality to produce an exploitable solution. At the time of this writing, when using a standard computer, it is not feasible to solve (2.25) in 3D for a complete motor with the full details of its winding. In this context, the purpose of this section is to simplify the 3D FEM model. Therefore, the objective is to obtain a model which can be effectively run on a machine to study eddy currents, in PCB windings of any complex shape.

2.3.1 Simplified 2D model

To simplify the model, we make the following hypothesis a priori:

- H1** The reaction field due to the eddy currents is weak and negligible compared to the PM magnetic field in the airgap;
- H2** The eddy currents are mainly 2D so that their normal component (in the radial direction) is zero;
- H3** Variations of the PM magnetic field along the thickness of the conductors are negligible;
- H4** Conductors can be segmented into smaller sections which can be simulated aside from each other.

Thanks to the hypothesis H1, the magnetic flux density in the airgap can be decomposed into two components. One due to the magnets $\mathbf{b}_0$ and another part due to the reaction of the conductor $\mathbf{b}_i$, so that $\mathbf{b} = \mathbf{b}_0 + \mathbf{b}_i$ in the conductors. This is possible because governing equations remain linear as long as there is no material saturation. And the only parts where such saturation could occur are far enough from the conductors not to be impacted by this weak reaction field.

Therefore, the two fields also derive from vector potentials such that $\mathbf{b}_0 = \nabla \times \mathbf{a}_0$ and $\mathbf{b}_i = \nabla \times \mathbf{a}_i$ with $\mathbf{a} = \mathbf{a}_0 + \mathbf{a}_i$.

As $\mathbf{b}_i \ll \mathbf{b}_0$, a first 3D FEM simulation (or any other model) can be made by replacing all conductors with air, $\Omega'_a = \{\Omega_a, \Omega_c\}$, to obtain the source term $\mathbf{a}_0$.

Then, a second simulation can be made by focusing only on the conductors' domain Ω_c , for which the equations of the magnetodynamics lead to:

$$\nabla \times (\nabla \times \mathbf{a}_i / \mu) = -\sigma i \omega \mathbf{a}_i - \sigma i \omega \mathbf{a}_0 - \sigma \nabla v \quad (2.29)$$

$$\nabla \cdot (i \omega \mathbf{a}_i + i \omega \mathbf{a}_0 + \nabla v) = 0 \quad (2.30)$$

Note that $\mathbf{b}_r$ does not appear anymore, but its influence is included in the source term $\mathbf{a}_0$. Thanks to H1, $\mathbf{a}_i$ can be removed from equation (2.30) because it can be considered as a negligible term compared to $\mathbf{a}_0$.

$$\nabla \cdot (i \omega \mathbf{a}_0 + \nabla v) = 0 \quad (2.31)$$

The above Poisson's equation now has a purely resistive behavior. This kind of simplification is generally known in the literature as resistance limited eddy currents [58, 64].

In cylindrical coordinates, developing the vector fields as $\mathbf{u} = (u_r, u_\theta, u_z)$, equation (2.31) can be re-written:

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} + \frac{\partial^2 v}{\partial z^2} = -i \omega \left(\frac{1}{r} \frac{\partial(r a_{0r})}{\partial r} + \frac{1}{r} \frac{\partial a_{0\theta}}{\partial \theta} + \frac{\partial a_{0z}}{\partial z} \right) \quad (2.32)$$

It is worth noticing that this hypothesis also implies that the reaction fields of neighboring conductors are too weak to affect the conductor itself. Consequently, all unconnected conductors can be simulated independently from each other.

Hypotheses H2 and H3, together, allow us to study the conductor within surfaces. Assuming the conductor is located at a mean radius r_w the following transformation yields:

$$\begin{pmatrix} r = r_w \\ \theta \\ z \end{pmatrix} \rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \theta r_w \\ z \end{pmatrix} \quad (2.33)$$

This means that the winding can be virtually unrolled so that the simulation can be achieved on a flat plane xy . Note that this transformation makes the conductor thickness virtually infinite because there is no more variation in r .

Applying the transformation to (2.32) it becomes:

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = -i\omega \left(\frac{\partial a_{0x}}{\partial x} + \frac{\partial a_{0y}}{\partial y} \right) \quad (2.34)$$

This equation has to be solved in the 2D domain of the conductor $\Omega_c(\mathbb{R}^2)$ which can be seen as a projection of $\Omega_c(\mathbb{R}^3)$ onto the cylinder of radius r_w .

Only the x and y components of the vector potential are useful in the equation. Then, this indicates that only the radial component of the PM magnetic field is necessary. Indeed, if one writes the relation between $\mathbf{b}_0$ and $\mathbf{a}_0$, one notes that, as the radial variation of the problem being removed, the tangential and radial components of the PM magnetic field are completely independent. The tangential part can be adjusted using only a_{0z} which has no impact on the resolution of (2.34):

$$\mathbf{b}_0(r, \theta, z) = \left(\frac{1}{r} \frac{\partial a_{0z}}{\partial \theta} - \frac{\partial a_{0\theta}}{\partial z} \right) \hat{\mathbf{e}}_r + \left(\frac{a_{0r}}{\partial z} \hat{\mathbf{e}}_\theta - \frac{1}{r} \frac{\partial a_{0r}}{\partial \theta} \hat{\mathbf{e}}_z \right) \quad (2.35)$$

$$\mathbf{b}_0(x, y, z) = \underbrace{\left(\frac{\partial a_{0y}}{\partial x} - \frac{\partial a_{0x}}{\partial y} \right) \hat{\mathbf{e}}_z}_{\text{Radial part}} + \underbrace{\left(\frac{a_{0z}}{\partial y} \hat{\mathbf{e}}_x - \frac{\partial a_{0z}}{\partial x} \hat{\mathbf{e}}_y \right)}_{\text{Tangential part}} \quad (2.36)$$

The current density can be expressed as:

$$\mathbf{j} = -\sigma i\omega \mathbf{a}_0 - \sigma \nabla v \quad (2.37)$$

$$= -\sigma i\omega \left(a_{0x} + \frac{\partial v}{\partial x} \right) \hat{\mathbf{e}}_x - \sigma i\omega \left(a_{0y} + \frac{\partial v}{\partial y} \right) \hat{\mathbf{e}}_y \quad (2.38)$$

The hypothesis H4 states that a long conductor can generally be segmented into smaller sections which can be simulated independently from each other. Indeed, by choosing an adequate decomposition of the domain, more specifically on the basis of naturally existing narrowings within the topology, the sum of the losses in each section is almost equal to the losses estimated simulating the whole conductor. Consequently, this hypothesis allows us to study and simulate the eddy currents on reduced domains.

Finally, the expression of the eddy current losses is given by:

$$P_{eddy} = \frac{\sigma g}{2} \int_{\Omega_c} || -i\omega \mathbf{a} - \nabla v ||^2 \quad (2.39)$$

with g the thickness of the conductors which is assumed to be constant.

A consequence of the proposed hypothesis is that the modeling of the eddy currents can be decoupled from the computation of the magnetic flux density due to PM. That offers the possibility to perform the simulation in two steps: a first one at the motor scale without any conductor, to provide a source vector potential $\mathbf{a}_0$,

and a second one at the scale of the conductive tracks. This latter is done on 2D domains only, which allows to model all the details of the conductor shape. Finally, the equation to solve is (2.34) with $\mathbf{a}_0$ either as a solution of (2.27) or from any other magnetostatics model.

Ferromagnetic materials may have non-linear properties such that the magnetic permeability μ sometimes depends on the magnetic field $\mathbf{h}$ which is the solution itself. These non-linearities are taken into consideration at the scale of the motor when computing $\mathbf{a}_0$. In case the model presented above is used, (2.27) must be solved iteratively, adapting at each step the value of μ according to the last value of $\mathbf{h}$ obtained, up to convergence. At the scale of the winding, when solving (2.34), the magnetic permeability μ no longer appears in the equation and the influence of the stator iron around is assumed to be negligible thanks to the hypothesis H1. Indeed, considering the stator as a semi-infinite block of iron with infinite permeability $\mu = \infty$, in this case one could apply the method of images [87] and one would find that at worst the amplitude of the magnetic field induced by the eddy currents is only doubled because of the iron. But twice the value of $\mathbf{b}_i$ remains a negligible quantity compared to the amplitude of the PM magnetic field $2\mathbf{b}_i \ll \mathbf{b}_0$.

2.3.2 Hypothesis 1: Weak reaction field

In order to validate the first hypothesis, we computed the amplitude of the magnetic field in front of moving magnets with and without the presence of conductors. More precisely, we observed the magnetic flux embraced by the regions where the conductors are located. If no difference is observed, it means that the impact of the conductor's reaction field is negligible.

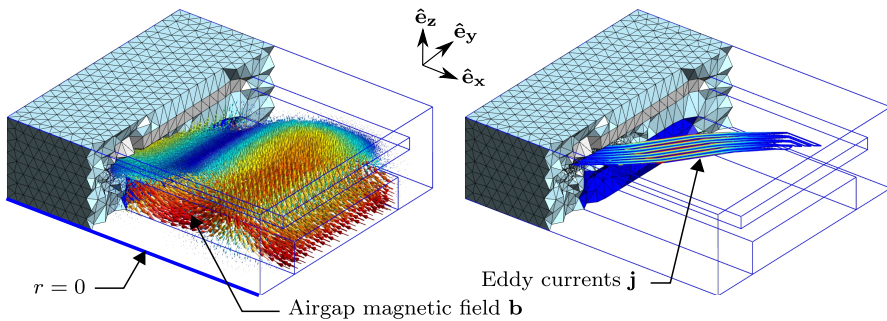


Figure 2.5: Simulation of the eddy currents in 3D. The motor is first unrolled so that the conductive segments are modeled as simple extruded polygons.

Since hypothesis H1 is completely independent of the curvature of the motor, for the sake of simplicity, the simulations are made using only planar elements in cartesian coordinates. Different shapes of conductors are tested.

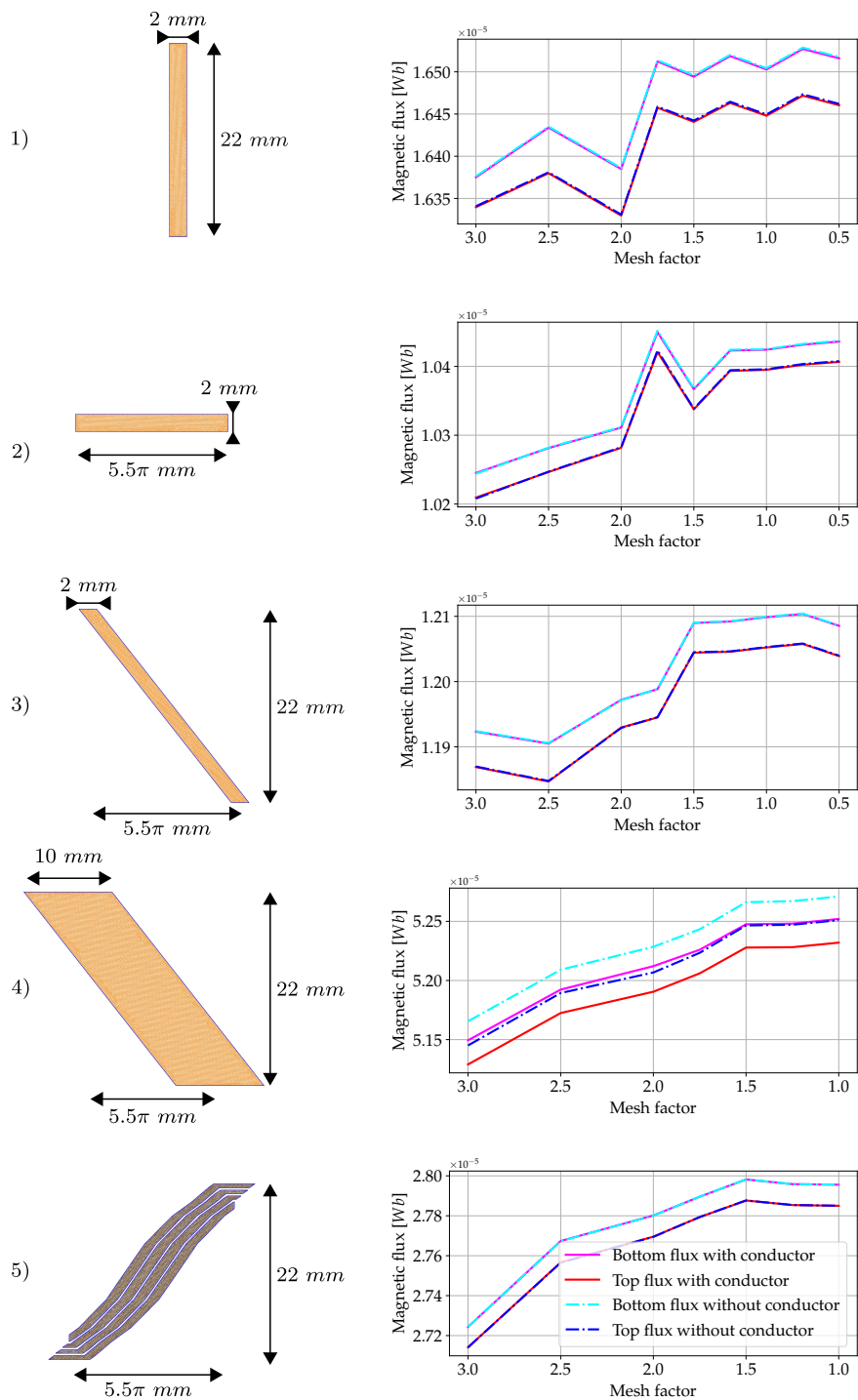


Figure 2.6: Comparison of the magnetic flux embraced whether or not their reaction field of the conductors is considered. The values are plotted against the mesh size used. A mesh factor of 1.0 corresponds to a characteristic length of 0.15 mm.

Vertical, horizontal and skewed segments are considered. They have a width of 2 mm , that is actually already large compared to real windings, in order to study situations with important eddy currents. Then, a yet larger segment of 10 mm which is more inductive is considered. Finally, a group of curved segments from a realistic PCB winding is evaluated. The conductors have a thickness of $80\text{ }\mu\text{m}$. The speed of the motor is set to 100 krpm . This way, being in some worst case conditions, the conclusions of the study will apply to most situations in real motors.

A simulation example is shown in Fig. 2.5 with the group of curved segments. Once the magnetic field has been computed, it is integrated to obtain the magnetic flux ϕ embraced by the top S_{top} and bottom S_{bottom} surfaces of the conductor:

$$\phi = \int_S \mathbf{b} \cdot \mathbf{n} dS \quad (2.40)$$

It is observed that the amplitude of the magnetic field at a single point in space is sensitive to numerical noise and errors. We therefore prefer to use the flux value ϕ instead. In addition, it allows to compare the different results on the basis of simple scalar values. In the case we do not consider the conductor, its domain is still drawn so that the mesh remains exactly the same but is regarded as a simple volume of air. Simulations were performed with various mesh size characteristics and the results are visible in Fig. 2.6 for the five shapes considered. The mesh factor corresponds to a relative characteristic length of the finite elements.

Fig. 2.6, shows that the presence of the conductor has clearly no impact on the flux embraced. At worst, for the most inductive conductor (the large skewed segment of 10 mm) there is a variation of 0.4% which is assumed to be negligible.

Hence, no variation in the flux embraced by the conductor means that the magnetic reaction field of the conductor can be assumed to be very small in comparison of the PM magnetic field in the airgap. This supports the first hypothesis of the model reduction.

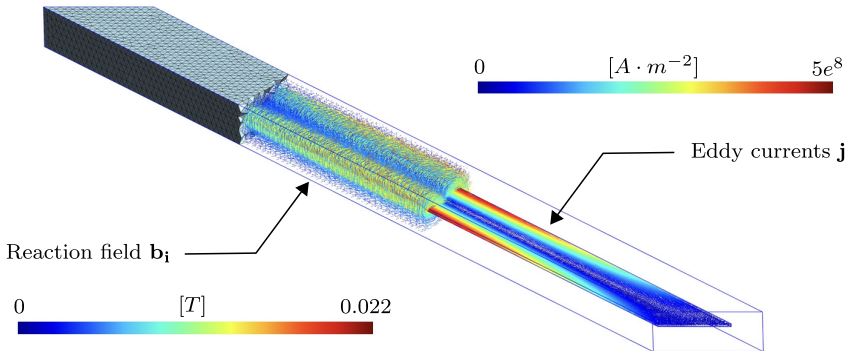


Figure 2.7: Model of a single segment in a box of air with a uniform field $\mathbf{b}_0$.

In addition, Fig. 2.7 illustrates the simple skewed conductor segment of 2 mm width, placed in the smallest air box that does not affect the reaction field by more than one percent, for which a uniform source $\mathbf{b}_0 = 1\hat{\mathbf{e}}_z\text{ T}$ is imposed. As expected, it can be observed that the reaction field is very low even at a speed of 100 krpm . Indeed, the highest local value of the reaction field $\mathbf{b}_i$ is 22 mT , thus two orders of magnitude lower than the source field.

2.3.3 Hypothesis 2: Tangential currents

In order to validate the second hypothesis, a 3D simulation of the motor has been performed for which the winding has been modeled by a simple hollowed cylinder of copper as shown in Fig. 2.8. Decomposing this cylinder into different layers electrically isolated from each other, a second simulation has been performed, as shown in Fig. 2.9, to observe whether or not it generates the same amount of losses.

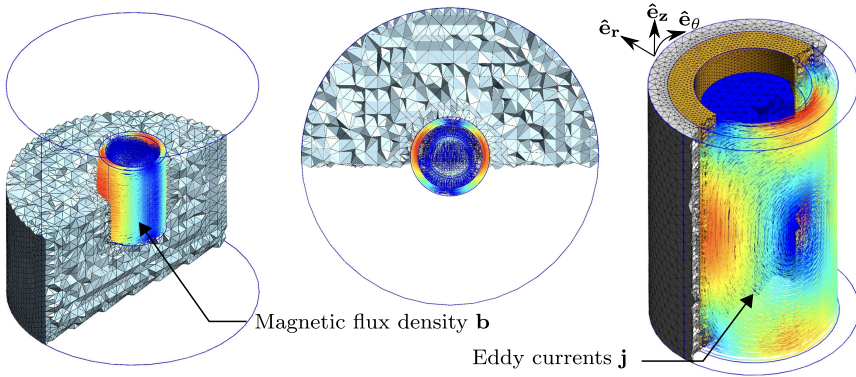


Figure 2.8: Simulation in 3D where the winding is considered as a plain conductor.

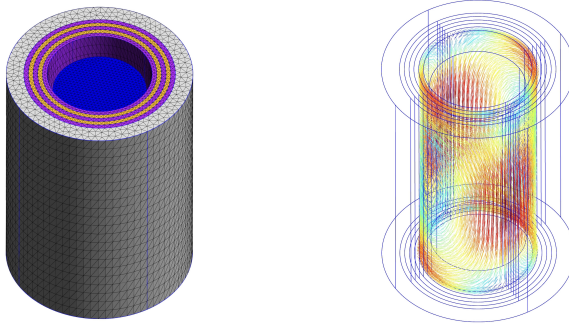


Figure 2.9: Decomposition of the conductor cylinder in multiple layers.

In both cases, the total eddy current losses P_{eddy} are computed with (2.28). In order to quantify the amount of currents that go radially between the layers instead of being tangential, the following value is also computed:

$$P_{eddy\ rad} = \frac{1}{2} \int_{\Omega_c} \rho ||(\mathbf{j} \cdot \mathbf{n}_r) \mathbf{n}_r||^2 \quad (2.41)$$

where $\mathbf{n}_r$ is the unit vector in the radial direction.

At a speed of 100 *krpm*, the simulation gives $P_{eddy} = 643.58\ W$ for the whole bloc of copper when the characteristic mesh length is 0.15 *mm* (~ 1000000 degrees of freedom). The precision of the mesh is such that there is less than one percent of error when the characteristic length is halved. The size of the air box surrounding the motor is such that its impact on the losses is less than 0.1%. The results of slicing the copper cylinder are shown in Tab. 2.2. with the first layer being the one with the smallest radius. Notice that the values are extremely large because we are considering a plain bloc of copper instead of a real winding. Therefore, these values are of no comparison with the losses within a real motor.

Table 2.2: Losses computation in each layer of the conductor cylinder.

	Single bloc conductor		Isolated layers	
	$P_{eddy}\ [W]$	$P_{eddy\ rad}\ [W]$	$P_{eddy}\ [W]$	$P_{eddy\ rad}\ [W]$
Layer 1	205.15	1.83	205.17	1.83
Layer 2	138.05	1.03	138.04	1.02
Layer 3	107.73	0.52	107.71	0.5
Layer 4	97.20	0.21	97.22	0.21
Layer 5	95.41	0.05	95.44	0.05
Total	643.54	3.64	643.58	3.61

First of all, we observe a decrease in losses as the radius increases, which can be explained by a reduction in the magnetic field amplitude when moving away from the magnet. Second, $P_{eddy\ rad}$ is two orders of magnitude lower than P_{eddy} which means that the amount of currents flowing radially is negligible. And third, there are negligible differences in the results when considering the electrically isolated or non-electrically isolated layers. That confirms the intuition that eddy currents are mostly tangential to the cylinder.

2.3.4 Hypothesis 3: Invariance in the radial direction

An observation which can be made in Fig. 2.6 (at section 2.3.2) is that the variation of the magnetic flux ϕ , and thus of the magnetic flux density $\mathbf{b}$, between the top and bottom surfaces of the conductors is quite low. This means that the whole conductor can be assumed to see a uniform field along its thickness. In order to support this idea and hence validate the third hypothesis, we can turn to analytical models of the airgap magnetic field in slotless machines, such as those proposed in [88] or in [89].

Generally, it states that b is somehow proportional to $1 + (r_s/r)^2$ with r_s the radius of the stator yoke, which here is equal to 6.5 mm . It means that over the thickness of the conductor $g = 80 \text{ }\mu\text{m}$, the relative variation of the field is equal to:

$$\frac{\Delta b_{0r}(r)}{b_{0r}(r)} \approx \frac{\left(\frac{r_s}{r - g/2}\right)^2 - \left(\frac{r_s}{r + g/2}\right)^2}{1 + \left(\frac{r_s}{r}\right)^2} \quad (2.42)$$

which gives roughly a 1.7% variation when computed at the mean radius of the winding $r_w = 5.5 \text{ mm}$. A similar reasoning can be done with the tangential component $b_{0\theta}$ which varies in $1 - (r_s/r)^2$.

Although the variation of 1.7% in the magnetic field is only a few percent and is already in favor of considering this latter constant over the conductor thickness, the matter is to be aware on the variation of the losses. The losses being proportional to the square of the magnetic field we can directly compare the result of P_{eddy} whether the radius is fixed or not along the conductor:

$$\frac{P_{eddy \text{ constant } b}}{P_{eddy \text{ variable } b}} \approx \frac{g \left(1 + \left(\frac{r_s}{r_w}\right)^2\right)^2}{\int_{r_w - g/2}^{r_w + g/2} \left(1 + \left(\frac{r_s}{x}\right)^2\right)^2 dx} \quad (2.43)$$

which gives a value of 0.9999 showing that hypothesis H3 is valid.

2.3.5 Hypothesis 4: Segmentation of the conductor

In Fig. 2.10, three conductors composed of straight segments are disposed parallel to each other and span multiple pole pitches. The mean losses in one segment are 2.317 W when they are simulated alone (with no influence from neighboring segments) and 2.317 W when they are simulated with neighboring but non-connected. In addition, it can be observed that the reaction field, is not only weak compared to the source field, but is also concentrated inside the conductor and does not significantly affect the neighboring segments. Indeed, along both sides of adjacent conductors the currents flow in opposite directions and therefore the reaction fields tend to cancel out each other between the conductors. These results are in total accordance with hypothesis H1.

When considering the full winding, connecting the successive segments to each other, the mean losses per segment are 2.324 W . This means that modeling a segment independently between two vias generates an error of only 0.3%, which is negligible. Indeed, the vias act as narrowings in the conductor which locally increase the resistance and behave as walls breaking eddy current loops. The loop vanishes at the walls to reappear behind. This observation validates hypothesis H4.

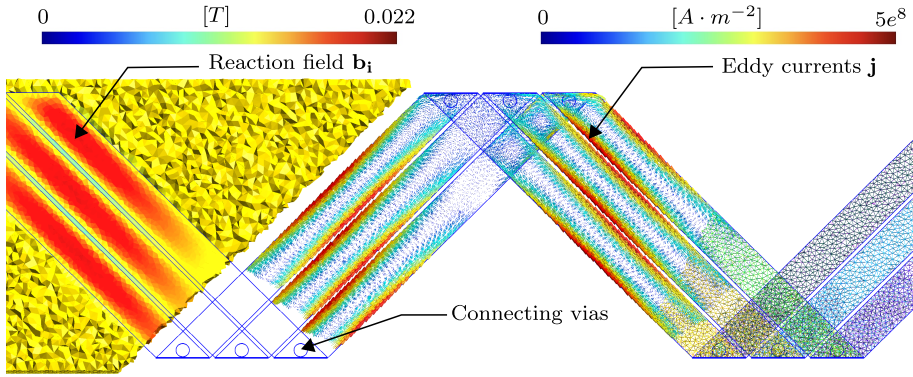


Figure 2.10: 3D simulation of some connected segments disposed side by side and for which the source term $\mathbf{b}_0 = 1 \text{ T}$ is directly imposed in the small box of air around.

The conductive parts which are located between two narrowings can thus be considered as independent for the evaluation of the eddy current losses, while keeping a very accurate estimation.

2.4 Source term representation

The problem of estimating the eddy current losses of the winding has been reduced to a simple 2D Poisson equation (2.34). However, to do so, the source term $\mathbf{a}_0$ is required. To obtain this source term, a magnetostatics problem is previously solved in the motor without any conductor:

$$\nabla \times (\nabla \times \mathbf{a}_0 / \mu) = \nabla \times (\mathbf{b}_r / \mu) \quad (2.44)$$

Then the solution of this magnetostatics problem is injected in the eddy current problem. From a practical point of view, it is required to be able to represent the quantity $\mathbf{a}_0$ in order to transfer it from one simulation to another.

The magnetostatics solution could be stored as it is, elements by elements, and directly re-injected in the Poisson equation. Nevertheless, this would use a lot of physical memory to store these data compared to the actual information about the field it should contain. Indeed, the number of finite elements used in the simulations is often quite large in order to be accurate. In addition, the meshes of both problems have different characteristic lengths because one is at the motor scale while the other is at the conductor scale, which poses compatibility issues. Furthermore, the model used to solve the magnetostatics problem might be anything else, different from a finite element method. Therefore, to facilitate the task of simulation softwares and have something more practical to use, share and analyze, it has been preferred to approach the magnetic field distribution with an analytical function.

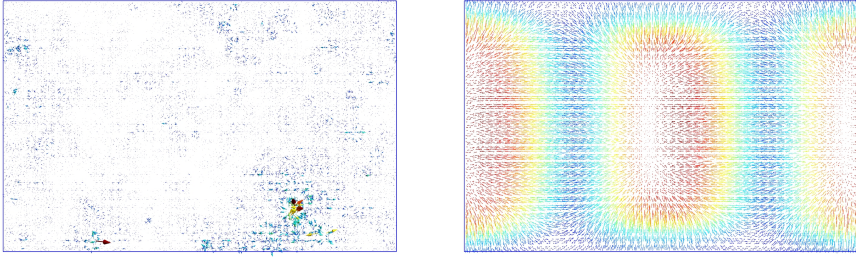


Figure 2.11: Solution of a magnetostatics problem $\mathbf{a}_0$ (left) and its associate magnetic flux density $\mathbf{b}_0$ (right).

As shown in Fig. 2.11, the vector field $\mathbf{a}_0$ usually obtained after a FEM magnetostatics simulation is totally discontinued and has no physical meaning. Because of that, interpolations and approximations of such a kind of field are thus very difficult and often lead to huge errors. To overcome this problem, it has been decided to manipulate the magnetic flux density $\mathbf{b}_0$ instead. The magnetic flux density being more progressive and smoother in nature, a small error of approximation of this one won't have as much consequence on the estimation of the losses as an error made on the potential vector.

The data transformation scheme is thus as follows. Using any existing model, $\mathbf{b}_0$ is obtained first. Then, its radial component is converted into an analytical expression which can be very easily manipulated, transferred and/or stored. Finally, from this analytical expression of b_{z0} , a smooth possible solution of $\mathbf{a}_0$ is constructed to be the source term of our eddy current problem. The objective of this section is therefore to propose an adequate analytical representation of the airgap magnetic field which can easily be transformed into an appropriate vector potential.

2.4.1 Analytical representation of the magnetic field

Finite elements are known to propose the best possible approximation on a certain mesh. However, this interesting property is only true as long as the quantities are evaluated at the exact element positions. Though, to obtain the evolution of a quantity along a single axis or simply to be able to use standard interpolation toolboxes it is easier to work with structured grids. Therefore, in order to obtain a regular grid containing the values of $\mathbf{b}_0$ evenly spaced, it is interesting to directly build the mesh using transfinite elements (at least in the conductors) during the magnetostatics simulation such that, the interpolation produces the minimal error. The objective is to find a 2D fit of the normal component of the magnetic field b_{z0} for each conductor that we want to simulate. Therefore, as shown in Fig. 2.12, a 2D scalar map of the field is extracted at various radii r_i . Each map is plotted in the coordinates xy with $x = \theta r_i$ and $y = z$ applying the transformation (2.33).

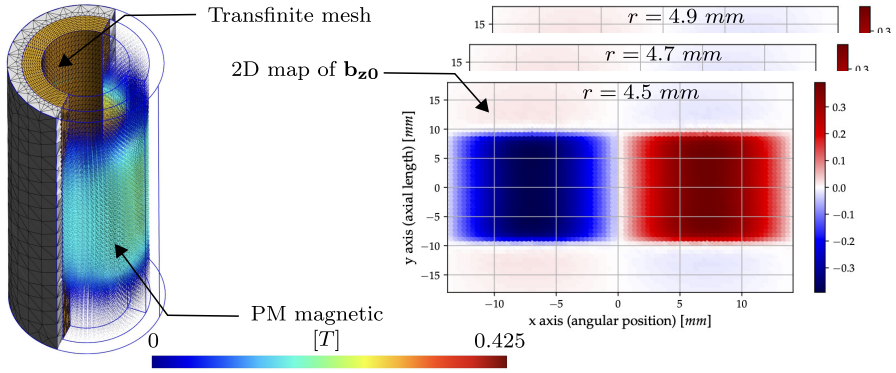


Figure 2.12: Extraction of the PM magnet magnetic field $\mathbf{b}_0$ and mapping of the normal component of the field into successive 2D grids at each radii of the layers.

Separation of variables is possible and b_{z0} can be seen as the product of two functions $b_{z0}(x, y) = f(x) \cdot g(y)$. Indeed, for any y_0 we have that $b_{z0}(x, y)/b_{z0}(x, y_0) = g(y)$ which is independent of x . In this factorization, $f(x)$ represents the evolution along the angular position while $g(y)$ represents the axial evolution and allows to model the decrease of the field at the edges.

Naturally, to fit the PM airgap magnetic field, a Fourier transformation can be applied to obtain the evolution along the angular position $f(x)$. The Fourier transform could be extended to two dimensions to obtain $g(y)$. And yet, the main drawback is that the closer the studied layer is to the magnets, the more the field shape will be square and thereby will contain a lot of spatial harmonics. This means that many coefficients will be necessary to correctly represent the field.

Quite commonly, the evolution of the magnetic field along the y axis looks like a simple smoothed step at the rotor ends. Hence, a good choice for $g(y)$ is the sigmoid function. This function requires only three coefficients which are a position y_0 , a certain amplitude A and finally a smoothness k :

$$g(y) = \frac{A}{1 + e^{k(|y| - y_0)}} \quad (2.45)$$

When the motor is not symmetric, a more general expression with five coefficients can be used instead, with the subscripts r and l referring to the left and right edges respectively:

$$g(y) = A \left(-1 + \frac{1}{1 + e^{-k_l(y - y_{0l})}} + \frac{1}{1 + e^{k_r(y - y_{0r})}} \right) \quad (2.46)$$

As a result, the method consists, first, to identify the peak value of the field $b_{z0}(x_{max}, y_{max})$. Second, the Fourier transform is applied along $b_{z0}(x, y = y_{max})$ to find the function $f(x)$. Finally, the sigmoid coefficients describing $g(y)$ are found

by minimizing the root mean square error:

$$\min \sqrt{\sum_{k=1}^n \frac{(f(x_k)g(y_k) - b_{z0}(x_k, y_k))^2}{n}} \quad (2.47)$$

The points (x_k, y_k) are the discrete positions at which the magnetic field is known.

Minimizing the mean square error is very suitable because it gives more importance to high values of the magnetic field, which have a significant impact on the eddy current losses. The interpolation can be performed on a single line, along $x = x_{max}$ for example, or by using all available data points. Both possibilities are illustrated in Fig. 2.13 and it can be considered that they give the same resulting sigmoids.

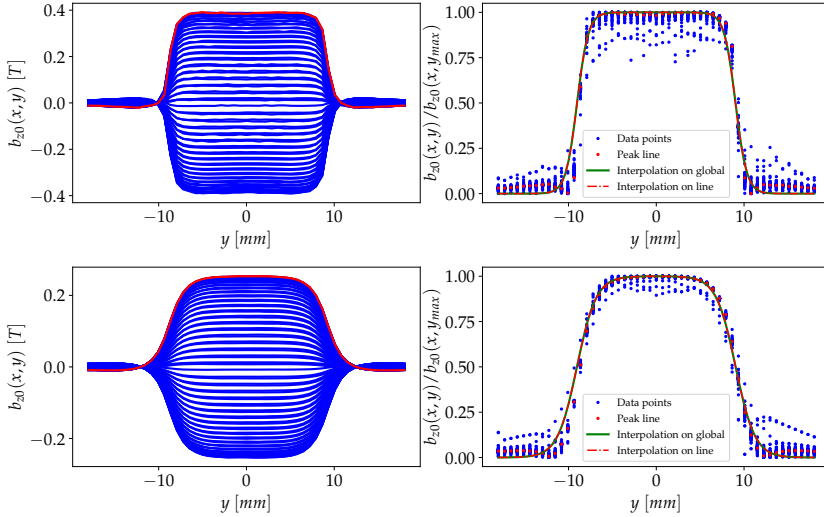


Figure 2.13: Extraction of the sigmoid function $g(y)$ for two different radii. Red lines and points are ignored and treated as outliers.

Table 2.3: Mean of the interpolation error $\hat{e}$ and its standard deviation $\sigma(e)$ for different layers. Layers 1, 5 and 10 are at radii of 4.5 mm, 5.4 mm and 6.5 mm respectively.

	Π (1D Fourier x Sigmoid)	
	$\hat{e}$	$\sigma(e)$
Layer 1	0.55%	2.13%
Layer 5	0.52%	1.66%
Layer 10	0.72%	1.64%

The results, when applying this method to approach the magnetic field at three different radii, are available in Tab. 2.3.

It is observed that the error is very small. As a reminder, in our case study, since the rotor has a single pole pair parallel magnetized, only the fundamental exists for $f(x)$. Thus, for each radius, only three parameters are needed to represent the field:

$$b_{z0}(x, y) = \frac{A}{1 + e^{k(|y| - y_0)}} \sin(x/\pi r) \quad (2.48)$$

On each radius, b_{z0} is expressed as the product of two independent functions. The question arises as to whether there is a third function exists to model the evolution along the z axis (radial direction). Unfortunately, as it can be seen in Fig. 2.14, the patterns on various layers do not superimpose even after normalization. Consequently, the parameters of the sigmoid are different for each of the layers and a simple piecewise interpolation is used to obtain the value of the field at intermediate radii.

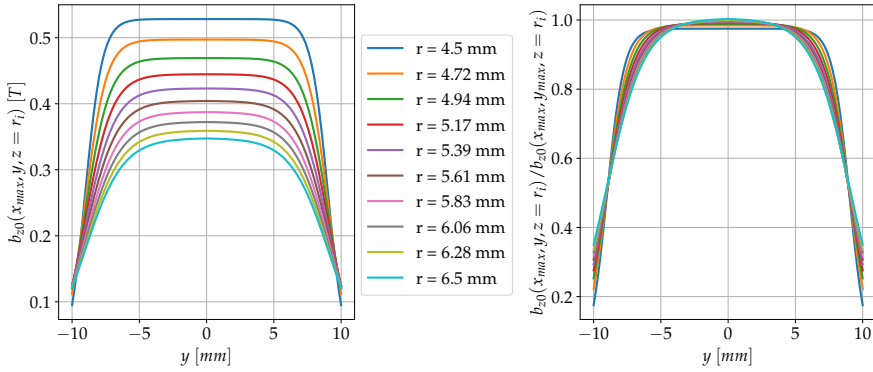


Figure 2.14: PM airgap magnetic field at various radii.

2.4.2 Potential vector

The magnetic flux density and the vector potential are linked by the curl operator such that $\mathbf{b}_0 = \nabla \times \mathbf{a}_0$. The vector potential can be chosen arbitrarily as long as it respects this constraint. As demonstrated in appendix B, a possible solution for $\mathbf{a}_0$ is to take:

$$\mathbf{a}_0 = \begin{pmatrix} -f(x, z) \int g(y, z) dy \\ 0 \\ 0 \end{pmatrix} \quad (2.49)$$

In the specific case of the sigmoid function the integral is given by:

$$\int g(y, z) dy = A \left(y + \frac{\ln(1 + e^{-k_l(y - y_{0l})})}{k_l} - \frac{\ln(1 + e^{-k_r(y - y_{0r})})}{k_r} \right) \quad (2.50)$$

2.5 Comparison between 2D and 3D FEM

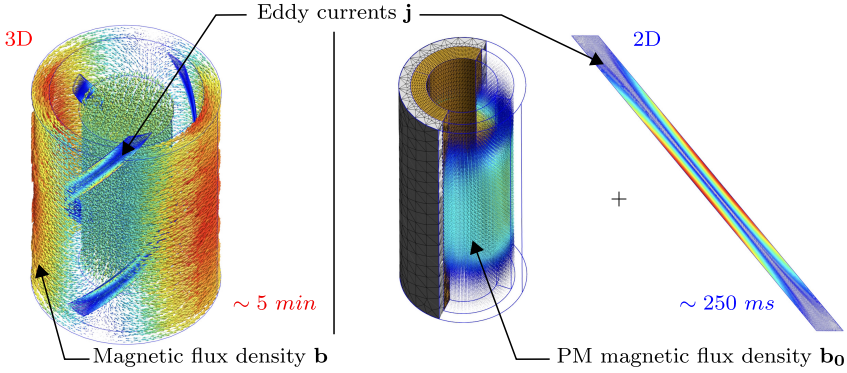


Figure 2.15: Case study to compare the estimations of the eddy current losses using either the 3D model or the 2D model with a source term previously extracted.

The purpose of this section is to validate that the 2D model, considering all the simplifications and hypotheses which have been made, is coherent with the full 3D model which serves as a reference. Since it is difficult to model the whole winding in 3D, in the context of this comparison, only three conductor segments have been considered. They have a 2 mm width, an $80\text{ }\mu\text{m}$ thickness and span a pole pitch. These three segments are each at a different radius. The eddy current losses in these conductors are first computed in 3D using the coupled equations (2.25), (2.26) and (2.27) with the result $P_{\text{eddy}} = 1.130\text{ W}$. Then, three simulations in 2D, solving equation (2.34) are performed on a planar segment. These latter are using a previously established source term b_{z0} . The total losses obtained are 1.134 W . It means there is a difference in the final solution of 0.35% . Although, it is not a demonstration per se, it shows that the reduced 2D model has a lot of potential. Simulations of the test case are shown in Fig. 2.15. Using a laptop with an Intel i9-9980HK (8 cores) CPU, the 3D simulation took about 5 min while the 2D simulations took about 250 ms each.

Because the PM magnetic field b_{z0} needs to be obtained first to be able to run the 2D simulations, a fair comparison requires to consider the time of a 3D magnetostatics simulation which also takes about 5 min . The two methods therefore have a similar total computation time. However, there is a significant gain in computation effort when multiple winding shapes are simulated at the expense of a single source term extraction. If N winding shapes are simulated, all other things being equal, using the 3D magnetodynamics model would take $N * 5\text{ min}$, while extracting once the source term and then use the 2D model developed above would take $5\text{ min} + N * 250\text{ ms}$. In addition, as only the conductor has to be meshed, a lot of conductors can be placed next to each other with very small spaces between them. Finally, thanks to the fact that the simulation is run on a planar 2D domain, it allows to draw many complex geometries without any difficulties. Also, because 2D elements bring less degrees of freedom in the system, the characteristic length of the mesh can be drastically reduced while keeping the computational efforts very low.

2.6 Complex topologies

In reality, windings are not simply composed of few straight segments, but they are composed of many tracks of complex geometries. In PCB windings, the board is usually double sided and the current path alternates between the top and the bottom of the PCB, going through vias. Also, in cylindrical motors, the flexible circuit is rolled on itself to form the armature winding. It means that there are no layers at different radii but a continuous variation of the conductor radius. The evolution of the radius can be given by a spiraled function:

$$r(x) = \sqrt{r_{in}^2 + x \frac{t_{pcb}}{\pi}} \quad (2.51)$$

with t_{pcb} the thickness of the PCB and r_{in} the internal radius of the winding.

Even though the hypothesis H4 of the model allows us to decompose the winding into smaller pieces. Sometimes, it may be interesting to simulate a larger set of conductors all together by considering the overlap between the top and the bottom tracks. The extreme case is that of a whole winding, with the purpose of evaluating the impact of the recirculating currents due to parallel connected tracks.

In any case, the top and bottom layers of the PCB have to be drawn at separate places (different z values for example) inside the simulation domain to avoid that their tracks cross each other. Therefore, to connect both layers and simulate the connections even so, pairs of boundaries can be defined in the geometry, along which periodical conditions are applied. It means that the scalar potential v is imposed to be the same at both boundaries. An example is shown in Fig. 2.16.

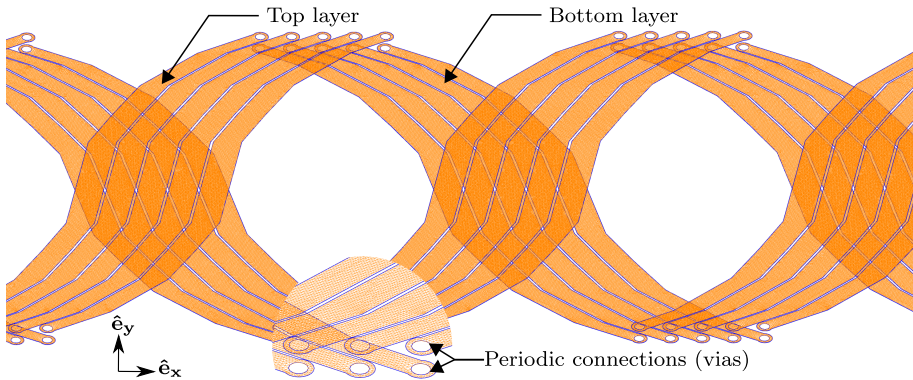


Figure 2.16: Real winding meshed, with the top and bottom layers separated along the z axis, but virtually linked through periodic boundary conditions on circles representing the vias.

2.7 Conclusion

This chapter started from the Maxwell's equations to derive a 3D finite element model of the eddy current losses. This model being very demanding to simulate a complete motor in 3D, because of the complexity to mesh the winding and in particular because of the many different scales it implies, an alternative which consists in decomposing the problem in two steps is proposed. The first step is to find the value of the PM magnetic field, ignoring the winding. This first step is achieved at the motor scale. Along this chapter, it was done using a 3D FEM magnetostatics model, although any other model can be used. The second step is to obtain the eddy currents with a simplified 2D FEM model. To derive this simplified model, four main hypotheses are made.

- H1** The reaction field due to the eddy currents is weak and negligible compared to the PM magnetic field in the airgap;
- H2** The eddy currents are mainly 2D so that their normal component (in the radial direction) is zero;
- H3** The variations of the PM magnetic field along the thickness of the conductors are negligible;
- H4** The conductors can be segmented into smaller sections which can be simulated independently from each other.

Each hypothesis has been justified in a section of this chapter, and using them has been shown to build a model a hundred times faster while remaining very accurate.

The suggested 2D model derives from a magnetic vector potential formulation $\mathbf{a} - v$. It solves the electric scalar potential v in the plane of the unrolled winding. Working in a 2D plane allows to handle more complex geometries with tiny elements because the mesh can be refined more easily, at a lower computational cost. Working in the plane of the unrolled winding allows to take into consideration the impact of the track shapes and skewing. The model has to be provided with a source vector potential $\mathbf{a}_0$ representing the magnetic field due to the permanent magnets. In this context, this chapter also proposes an analytical representation of this magnetic field. It has the purpose to store it efficiently using only few coefficients.

Publication related to this chapter:

- G. François, F. Baudart, F. Henrotte and B. Dehez, *Numerical Investigation of Eddy Current Losses in Airgap PCB Windings of Slotless BLDC Motors*, 2018 21st International Conference on Electrical Machines and Systems (ICEMS), 2018, pp. 2702-2708.

Experimental validation & Sensitivity

3

The 2D model of chapter 2 and the underlying hypotheses have been previously compared and tested against 3D finite element models. In this chapter, the objective is to validate the model experimentally.

3.1 Case study

Four types of stators have been realized. These are not functional windings but simple straight segments of conductors whose dimensions are precisely known. They are composed of seven layers ranging from 5.5 mm to 7.4 mm of radius. Their active height h is 18 mm . The particularities of the windings are illustrated in Fig. 3.1 and explained hereafter. They are shown flat and then rolled in their final cylindrical shape as shown in Fig. 3.2.

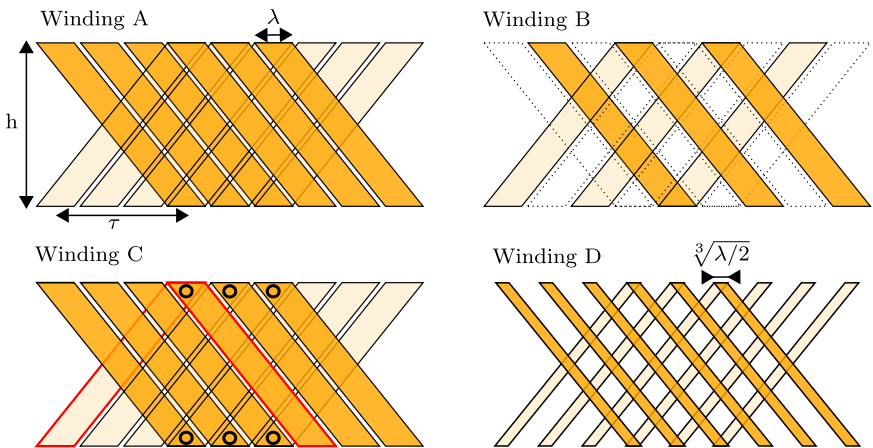


Figure 3.1: Winding A is the reference with isolated segments. Winding B is like A with only one of two segments. Winding C connects the segments between the top and bottom layers. Winding D is composed of segments with a smaller width.

Winding A is composed of seven layers, each of them having 12 segments on the top layer of the PCB and 12 segments on the bottom layer of the PCB, which are skewed along a full pitch. Per layer, all the segments have the same width given by $\lambda = 0.47r_l$, where r_l is the mean radius of the layer. This means that, on average, the angular width of the segments is 0.47 rad . The spiral evolution of the winding, which requires adapting the widths as the radius varies, is overcome by progressively increasing the space between two consecutive segments.

Winding B corresponds to the reference with only one of two segments. With this second winding it is expected to observe half of the losses compared to those of the reference winding. This result would also help to validate hypothesis H1 of section 2.3.1. The reaction field is also expected to be weak enough not to impact the eddy currents induced in the neighboring conductors. Indeed, if there are interactions between neighboring segments, separating them from each other as proposed in this second winding would reduce these interactions and hence the losses should not be strictly halved.

Winding C corresponds to the reference for which top and bottom conductors have been connected through vias, leading to 6 independent paths for the current instead of 164 isolated segments. The amount of losses is estimated to be almost equivalent to that of the reference losses. If that is the case, the result would partially validate hypothesis H4 of section 2.3.1 stating that winding segments which are connected with vias can be considered independently.

Winding D is similar to the reference A, except that segment width has been shrunk by a factor $1/\sqrt[3]{2}$. In this case, one would expect to obtain half of the losses compared to the reference winding, confirming eddy current losses depend on the cube of the width of the conductive tracks.

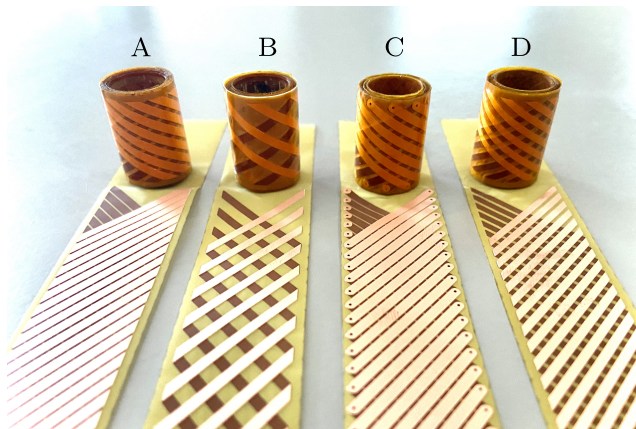


Figure 3.2: The four windings used for the validation of the eddy current losses model.

The source field is provided by a spinning NdFeB/N48 magnet of 8 mm in diameter, 70 mm in length and which has a remanent magnetic field of 1.38 T. The magnet being very long compared to the stator, it allows to consider that the magnetic field is invariant along the z axis, at least along the active part of the winding.

3.2 Test bench

3.2.1 Principle

In order to measure the eddy current losses, deceleration tests are performed. The magnet is first put in rotation and then left in freewheel with the stator around. The rotor speed ω is measured in time and knowing its inertia I_{rotor} , the breaking torque $T_{breaking}$ can be computed. The breaking torque is composed, on the one hand, of the interaction of the eddy currents with the airgap magnetic field, denoted T_{eddy} and, on the other hand, of all the other phenomena that we are not interested in for the measurement. Those other torques are mainly due to the friction in the bearings. To a lesser extent they also come from the interaction of the magnets with metallic elements around and/or caused by the friction between the air and the rotor. The sum of these other torques will be denoted T_{bench} .

First, by running the test bench with a fake winding of paper, the Euler's second law can be written as:

$$T_{breaking} = T_{bench} = I_{rotor} \frac{\partial \omega_1}{\partial t} \quad (3.1)$$

Indeed, paper being non-conductive, there are no eddy currents to consider.

Second, by running the test bench with a real winding, the value of T_{eddy} is non-zero and the equation becomes:

$$T_{breaking} = T_{bench} + T_{eddy} = I_{rotor} \frac{\partial \omega_2}{\partial t} \quad (3.2)$$

The fake paper stator and the real stators have exactly the same dimensions. The friction between the rotor and the air flowing in the airgap is thus assumed to be equivalent for both windings. The only change between both runs is the presence of the conductive segments. As their presence does not impact the mechanical conditions of the test bench, it is assumed that T_{bench} is perfectly identical in (3.1) and (3.2).

The two equations (3.1) and (3.2) can be used to estimate the eddy current losses:

$$P_{eddy}(\omega) = \omega T_{eddy}(\omega) = \omega I_{rotor} \left(\frac{\partial \omega_2(\omega)}{\partial t} - \frac{\partial \omega_1(\omega)}{\partial t} \right) \quad (3.3)$$

Equation (3.3) shows that the accuracy of the measurements depends on the accuracy of the speed measure and of the accurate knowledge of the inertia.

3.2.2 Setup

The test bench is visible in Fig. 3.3. The rotor part, in addition to the magnet, is composed of a stainless steel cylinder and has inertia I_{rotor} equals to $25.2 \text{ kg} \cdot \text{mm}^2$. To bring this rotor at full speed initially, a spindle is brought closer to the top part of the rotor permanent magnet. As this head is conductive, it is subject to eddy currents and therefore acts as an inductive coupling. Once the rotor speed reaches its steady state value, the spindle is lifted up sufficiently to eliminate the magnetic coupling. The rotor spins and is axially supported inside an air bearing in order to reduce the friction to its minimum. To measure its speed a photodiode is used. The analog output voltage is then sampled at 100 kHz and the mean speed for each rotation is computed. The curve is fitted numerically with piecewise linear interpolations and the derivative is computed. Finally, to support the winding, a 3D printed socket has been designed. The part is compliant in order to be able to easily insert and maintain the winding.

In this configuration, thanks to the high sampling frequency, there is nearly no uncertainty on the speed of the rotor. Inertia of the rotor has been computed using a CAD software. The properties of the materials implemented in the software were calibrated by measuring the masses of the rotor parts. A scale accurate to one gram was used. Since both the magnet and the steel cylinder have a uniform mass density, in addition to the fact that the cylinder has been machined with very good tolerances so that the geometry is known precisely to a few microns, the inertia is confidently known to $\pm 0.5\%$.

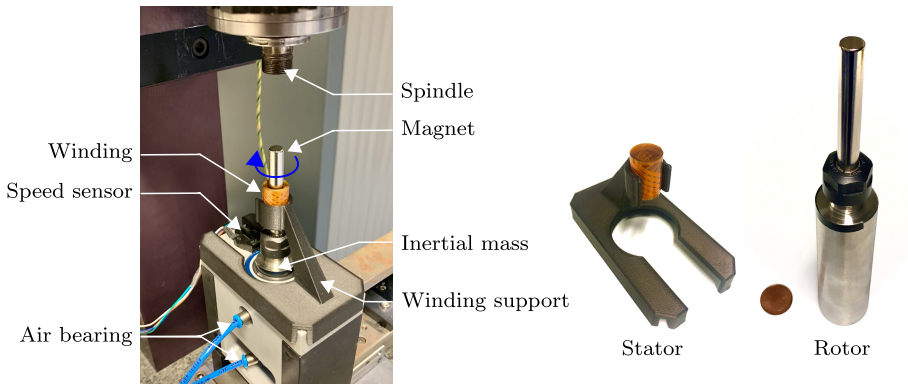


Figure 3.3: Experimental test bench to measure the eddy current losses observing the speed decrease rate.

3.2.3 Benefits

The main advantage of this method is that it can estimate the losses at any speed. Indeed, the accuracy remains the same whatever the measurement range because it is directly proportional to the inertia, which remains constant. In comparison, a test bench which directly measures the torque with a torque sensor would be limited in range. For example, a torque sensor with a capacity of 1 Nm , a resolution of 0.1 Nm and assumed to be perfectly selected for a given speed, would have an uncertainty of 100% if the speed decreases and the torque measurement drops to 0.1 Nm .

Therefore, measurements can be done at low speeds to limit the risks in terms of safety and, to be able to neglect the temperature variations during the experiment. As the temperature impacts the material properties and thus influences the eddy current losses, the less it varies during the experiment the more stable and reliable is the measure.

In this work, during the tests, the rotor speed was pushed to a maximum value of 1500 rpm producing about 10 mW of thermal heating in the winding. Each test lasted about one minute considering the acceleration and deceleration phases of the rotor. Therefore, in a conservative approach, the total energy released during an experiment was limited to 0.6 J . A prototype winding contains roughly $1e^{-6}\text{ m}^3$ of copper and considering a thermal capacity of $3.43e^6\text{ JK}^{-1}\text{m}^{-3}$ it leads, at worse, to a temperature increase of 0.18 K during the experiment, which can be assumed negligible.

3.2.4 Measurements

Fig. 3.4 shows the speed decrease for the deceleration tests. Although, the curves might be difficult to distinguish, each experiment has been run three times. In the case of the windings, the experiment was run with three different windings of the same type. One can observe that there is a great repeatability in the experiment and that there are no noticeable differences between two windings of the same type. In addition, the experiment gives very similar results when using a dummy winding made of paper and when running without winding, meaning that most of the friction losses come from the air bearing and not from the air flow in the airgap.

In Fig. 3.5, the instantaneous breaking torque applied to the rotor is shown according to the speed. The eddy current losses are then obtained using (3.3) and shown in Fig. 3.6.

3.3 Sensitivities

In reality the parameters of the problem cannot be exactly known with absolute accuracy and there are always some uncertainties from the fabrication of the parts to the assembly of the experiment. Some parameters have more impact on the losses than others and their deviation from the expected nominal value can affect the com-

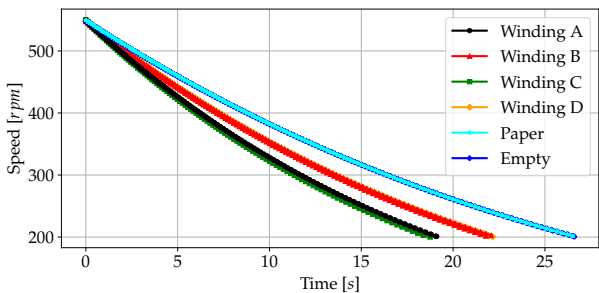


Figure 3.4: Speed decrease of different windings.

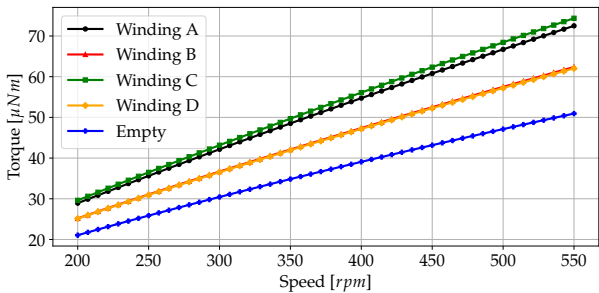


Figure 3.5: Breaking torque estimation according to the speed.

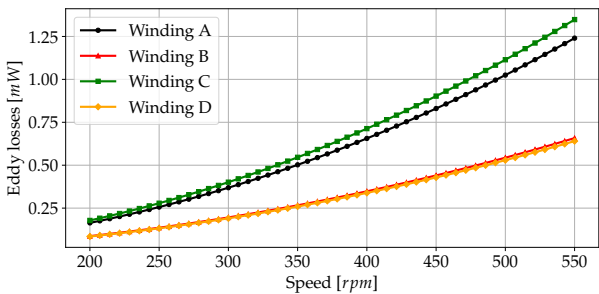


Figure 3.6: Eddy current losses for the four windings.

parison between the numerical model and the experimental results. In this context, the purpose of this section is to evaluate the sensitivity of the problem to these parameters and to build a confidence range for the eddy current losses estimation. All the numerical estimations are performed using the 2D FEM of chapter 2.

3.3.1 Airgap magnetic field

According to the Ampere's law (2.12), eddy currents are directly proportional to the intensity of the magnetic field. Therefore, eddy current losses are proportional to the square of this intensity. This means that the source magnetic field is a very important parameter. To obtain the magnetic field distribution due to the PM rotor, two methods are considered in this section. First, the information can be directly extracted from the data given by the magnet provider. Second, a magnetostatics problem can be solved.

For the test bench, the problem can be solved in 2D with a uniform disk of the magnet and only air around it. The Poisson equation to solve is thus the following:

$$\nabla^2 \mathbf{a} = \nabla \times \mathbf{b}_r \quad (3.4)$$

with $\mathbf{b}_r$ equals to $B_r (\hat{\mathbf{e}}_r \cos \theta + \hat{\mathbf{e}}_\theta \sin \theta)$ inside the magnet ($r \leq r_{\text{magnet}}$) and zero outside. The remanent field of the magnet B_r equals 1.38 T. The solution for the normal component, which is the one we are interested in, is given by:

$$b_{z0}(r, \theta) = \frac{B_r}{2} \left(\frac{r_{\text{magnet}}}{r} \right)^2 \quad (3.5)$$

Both results are plotted in Fig. 3.7. It can be observed that approximately 10% difference exists between the two of them. This difference can be explained by a dispersion in the remanence of the magnets effectively built, or by a non-homogeneity of the materials, or even by the impact that the coating could have on the actual dimensions of the permanent magnet. Finally, it could also be simply a measurement error. Considering the reference stator, which is the winding A, eddy current losses can be computed for both source terms, resulting in two different estimations of 1.38 mW and 1.14 mW, at 500 rpm.

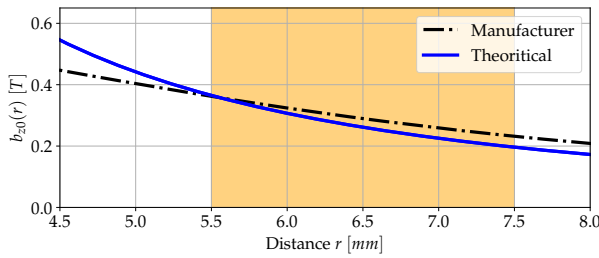


Figure 3.7: Comparison between the magnetic flux density obtained by solving a simple analytical model or by using the data of the magnet provider. The colored zone corresponds to the position of the winding.

It is assumed that the uncertainty about B_r can be represented using a normal probability distribution:

$$p(B_r = x) = \mathcal{N}(\mu, \sigma_{99\%}^2) \quad (3.6)$$

with the mean value $\mu = 1.38 \text{ T}$ and the standard deviation $\sigma_{99\%}$. The magnet provider claims a maximum remanent field variation of $\pm 10\%$. Considering there are 99% chances for B_r to remain in this $\pm 10\%$ interval, the value of $\sigma_{99\%}$ is chosen equal to $3/0.138 \text{ T}$. A parametric sweep is performed on B_r , using both source terms, and is plotted against the probability of an event in Fig. 3.8.

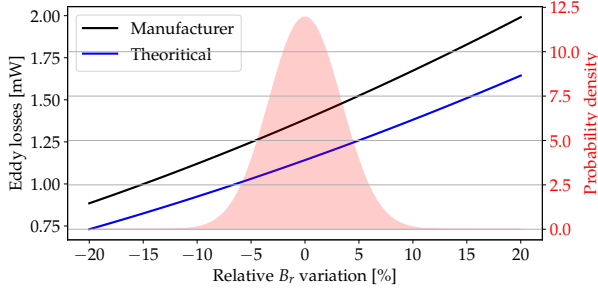


Figure 3.8: Eddy current losses according to the remanent field variation.

Fig. 3.9 shows the expected probability density of measuring eddy current losses according to the data we choose to trust. The magnetic field in the airgap is directly

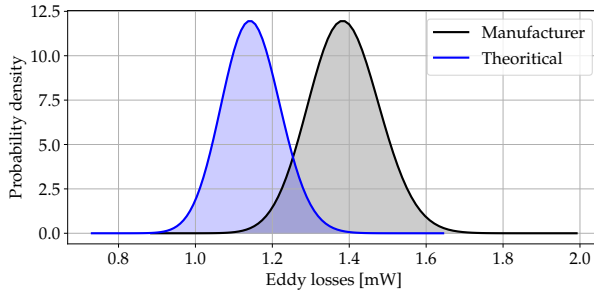


Figure 3.9: Expected probability density of measuring eddy current losses.

proportional to the strength of the magnet: $b_{z0} \propto B_r$. Consequently, as $P_{eddy} \propto b_{z0}^2$, it can also be written that:

$$P_{eddy}(B_r) = P_{eddy}(B_r = B_r^*) \left(\frac{B_r}{B_r^*} \right)^2 \quad (3.7)$$

At the nominal value of $B_r = 1.38 \text{ T}$, the relative sensitivity of the losses to the remanent magnetic field of the magnet is given by:

$$\frac{\partial P_{eddy} / \partial B_r}{P_{eddy}} = \frac{2}{1.38} = 145\% T^{-1} \quad (3.8)$$

3.3.2 Temperature

Another parameter to consider is the temperature. It impacts the eddy current losses by modifying the material properties. On the one hand, it modifies the remanent field of the magnet, which can be modeled using:

$$B_r(T) = B_{r\ 20^\circ C}(1 - \beta_T(T - 20)) \quad (3.9)$$

with $B_{r\ 20^\circ C}$ the remanent field at $20^\circ C$ and β_T the thermal coefficient of the magnet. On the other hand, it also modifies the copper conductivity:

$$\sigma(T) = \frac{1}{\rho_{20^\circ C}(1 + \alpha_T(T - 20))} \quad (3.10)$$

with $\rho_{20^\circ C}$ the electric resistivity at $20^\circ C$ and α_T the thermal coefficient of the copper.

Common values in the literature are $\alpha_T = 3.93e^{-3}K^{-1}$ and $\beta_T = 1e^{-3}K^{-1}$. Therefore, because $P_{eddy} \propto \sigma(T)B_r^2(T)$, the sensitivity to the temperature can be expressed as:

$$\frac{\partial P_{eddy}/\partial T}{P_{eddy}} = -\frac{\alpha_T}{1 + \alpha_T(T - 20)} - \frac{2\beta_T}{1 - \beta_T(T - 20)} \approx -(\alpha_T + 2\beta_T) \approx -0.6\%K^{-1} \quad (3.11)$$

As discussed previously, the deceleration tests do not require to go at high speeds, which has the benefit of negligible heating of the bench during the experiment. The temperature of the test bench is thus at the ambient temperature of the room which can easily be measured with the accuracy of $\pm 1^\circ C$.

As with the magnet, uncertainty about the temperature can be modeled using a normal probability distribution. It is considered that there is a 99% probability that temperature was in the range of $21 \pm 1^\circ C$ in the lab. Fig. 3.10 shows the impact of temperature and the expected probability density of measuring eddy current losses. This ambient temperature corresponds to the situation where a winding is being tested and the rotor is rotating slowly to accurately measure eddy currents. Obviously, this temperature would be drastically higher and its variations much greater than $\pm 1^\circ C$ inside a motor under real operating conditions.

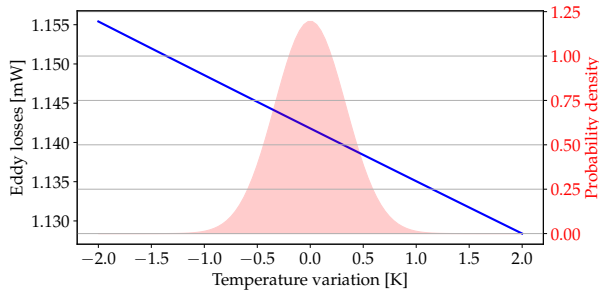


Figure 3.10: Eddy current losses according to the temperature variation.

3.3.3 Segment dimensions

Uncertainty on the shape of the conductors after production can occur in all three directions of space, so that it can be the thickness, the width and the length of the segments, as defined in Fig. 3.11. Considering only one of the stator segments, sensitivities have been computed for those three parameters.

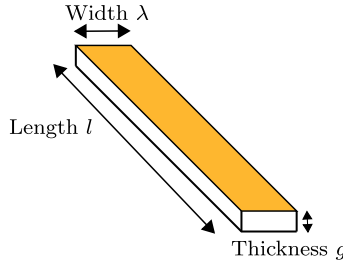


Figure 3.11: Dimensions of a straight segment.

The estimation of the losses is obviously directly proportional to the thickness g , by construction of the FEM model itself. The manufacturer provides choices for the copper thicknesses at intervals of $30 \mu m$. Therefore, it can be assumed that the tolerance on the pcb thickness is at least of $\pm 15 \mu m$ around the nominal value which is $80 \mu m$. Fig. 3.12 shows the impact of the copper thickness and the probability density of deviation from the nominal value. The sensitivity of the losses to the parameter g is $1.25\%\mu m^{-1}$.

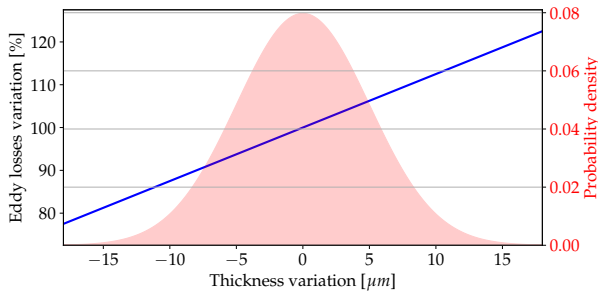


Figure 3.12: Sensitivity of the eddy current losses estimation to the segment thickness. Probability density of deviation from the nominal thickness.

In the plane of the flat pcb, to avoid short circuits between the tracks, manufacturers usually recommend leaving a clearance of at least $150 \mu m$. This threshold value will also be used to limit the uncertainty and it will be considered that there

is 99% chance that the length and/or the width of a segment is within the range of $\pm 150 \mu m$ around its nominal values. Fig. 3.13 and Fig. 3.14 respectively show the impact of the length and the impact of the width, as well as the probability density of deviation from their nominal value. A sensibility to the length of only $3.96 \%mm^{-1}$ is observed, which can be considered weak. On the contrary there is a very important sensitivity to the width, which is $148 \%mm^{-1}$. Note that the nominal width of the segments differs at each layer, the previous value is given for $\lambda = 2 mm$.

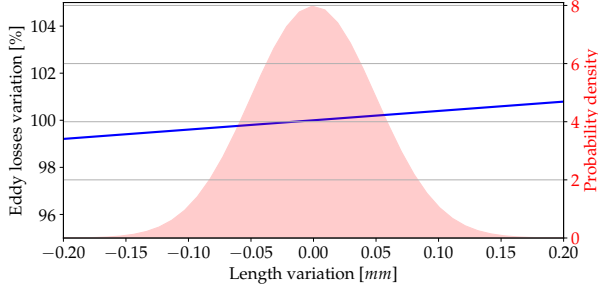


Figure 3.13: Sensitivity of the eddy current losses estimation to the segment length. Probability density of deviation from the nominal length.

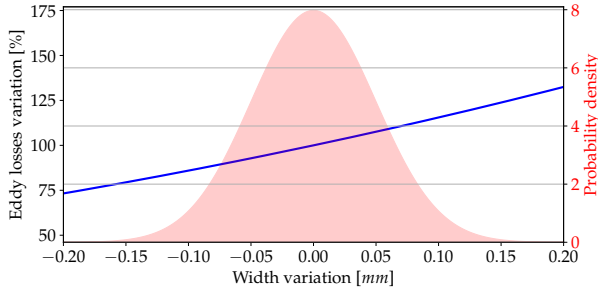


Figure 3.14: Sensitivity of the eddy current losses estimation to the segment width. Probability density of deviation from the nominal width.

3.3.4 Winding position

It has been observed previously that the sensitivity to the source field is quite important with a value of $145 \%T^{-1}$. The amplitude of the source field is impacted by the magnet but also by the position of the conductor itself in the airgap. Indeed, if a layer of the winding is positioned at a smaller radius than expected, it sees a magnetic field with a higher amplitude, and conversely.

Three ways to misplace the different layers of the winding are considered in this section. First, it may be possible that the winding is wound starting from a wrong inner radius r_{in} , what shifts all the layers. Second, the thickness of the pcb t_{pcb} may differ from the expected value and the external layers will be impacted even though the initial radius r_{in} is correct. And third, assuming a perfect wound winding, its center and the magnet center can still be out of alignment by a distance dr when placed in the test bench.

An analysis of the losses variability, for the reference winding A, according to the initial radius of the spiraloid r_{in} is given in Fig.3.15. The reference is $r_{in} = 5.5 \text{ mm}$ and it has been considered that at worst this radius can vary by $\pm 100 \text{ }\mu\text{m}$ with a 99% confidence. This choice is motivated by the fact that the winding is wrapped around a cylinder of reference during the fabrication process. The uncertainty on the inner radius of the winding is thus linked to the accuracy of the machining process to obtain this reference cylinder. The relative sensitivity to r_{in} is $-54 \text{ \%mm}^{-1}$. The minus sign indicates that the larger the winding, the less the eddy currents there are.

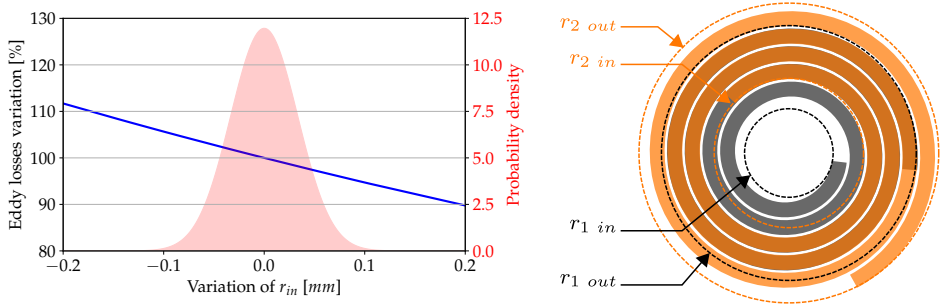


Figure 3.15: Sensitivity of the eddy current losses to a variation of the inner radius r_{in} of the winding at which the spiral shape starts.

The next parameter studied is the pcb thickness t_{pcb} . Most of the time its variability occurs during the fabrication process. The support material can be slightly deformed because of the heating up or because of the pressure applied during the rolling process. In addition, the exact amount of glue used to maintain the layers together is difficult to master and it directly influences the pcb thickness. As the thickness varies, the number of turns of the spiraloid varies and the layers are finally not positioned where they should be. Fig. 3.16 shows the impact of the pcb thickness as well as the probability density of deviation. A sensibility to the pcb thickness of $-0.18 \text{ \%}\mu\text{m}^{-1}$ is observed. It has been considered that there is 99% chance that the thickness is in the interval $\pm 25 \text{ }\mu\text{m}$ around its nominal value. This corresponds to a tolerance of about 10% as mentioned by the manufacturers.

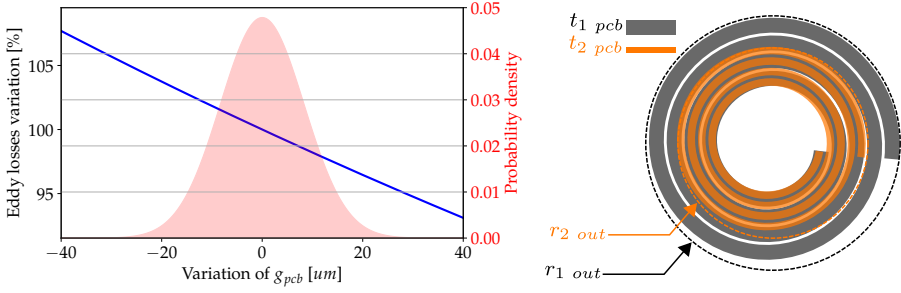


Figure 3.16: Sensitivity of the eddy current losses to a variation of the pcb thickness.

Unlike the previous two, eccentricity does not appear at the time of the building but during the measurement phase, when the winding is placed on the bench. To minimize the positioning error, the 3D printed part maintaining the winding is fixed without any possibility of manual adjustment. Except the mechanic tolerances of the bench (which can be assumed negligible), the possible error in radial position is hence due to the compliance of the 3D black printed part shown in Fig.3.3.

From the winding point of view, the eccentricity of the magnet generates a magnetic field which is no longer composed of a single sine, but of multiple spatial harmonics. Fig. 3.17 shows the amplitude of the magnetic flux density, either at a fixed position in the winding and varying over time, or equivalently at a given time but throughout the angular positions.

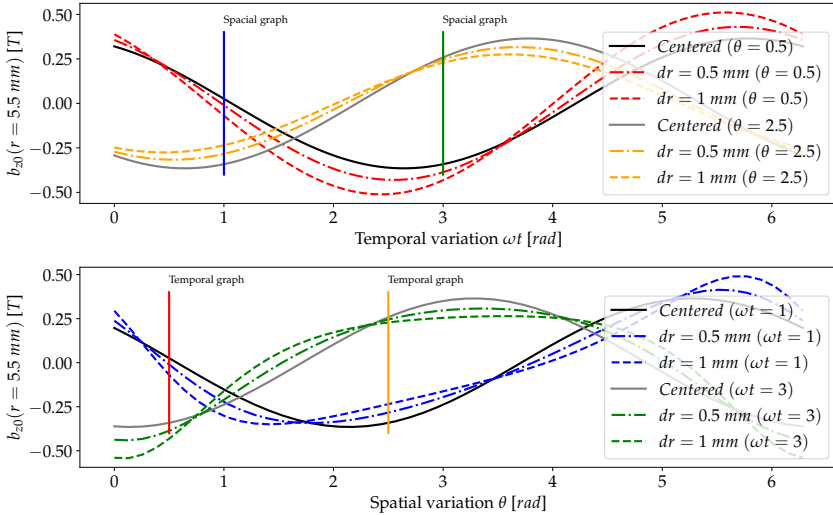


Figure 3.17: Evolution of the magnetic flux density when the magnet is off-centered.

Let us point out that, at a given position, whatever the eccentricity, it always evolves like a sine although the amplitude and phase may vary. The field is thus shifted in time but it has no impact when considering the mean of the losses.

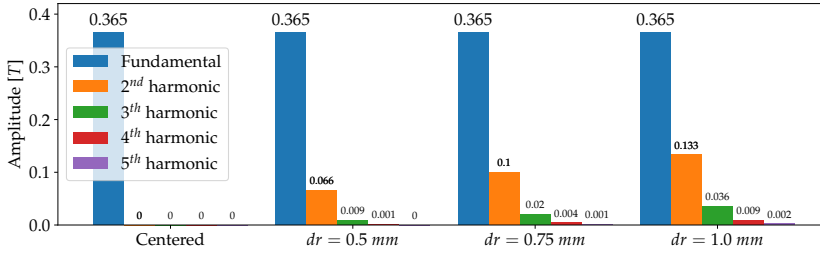


Figure 3.18: Spatial harmonics decomposition of b_{z0} for off-centered magnets.

However, in space, the field is now composed of multiple harmonics. The Fourier decomposition for different values of dr is shown in Fig. 3.18. All the harmonics are in phase with the fundamental. Also, the fundamental remains the same regardless of the eccentricity. In other words, adding eccentricity, is simply considering additional spatial harmonics in the source term b_{z0} . A sweep analysis on dr is proposed for the reference winding A in Fig. 3.19. The observed sensitivity is $40 \text{ \%mm}^{-1}$. An uncertainty of at worst 0.5 mm has been considered on the misalignment.

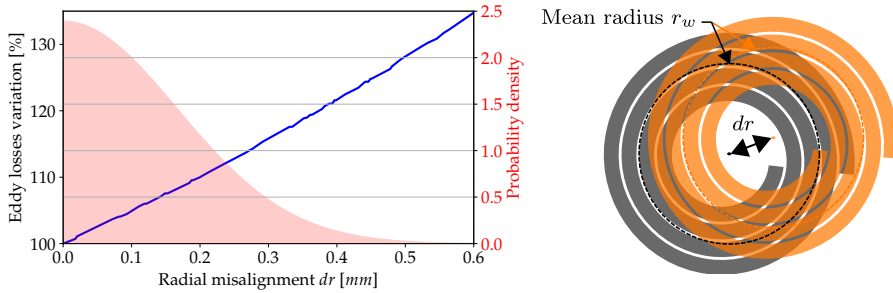


Figure 3.19: Sensitivity of the eddy current losses to an eccentricity of the magnet.

3.3.5 Summary

A summary of the sensitivities and the probability for each parameter to impact the eddy current losses is proposed in Tab. 3.1. With the exception of temperature, all numerical values of the confidence intervals are apriori computed values based on information from the manufacturer.

Table 3.1: Summary of the sensitivities studied and the apriori uncertainties.

Parameter	Symbol	Sensitivity	Interval (99%)	Losses delta
Source field	B_r	$145 \%T^{-1}$	$\pm .138 T$	$\pm 20.0 \%$
Temperature	T	$-0.6 \%K^{-1}$	$\pm 1^{\circ}C$	$\mp 0.6 \%$
Segment thickness	g	$1.25 \% \mu m^{-1}$	$\pm 15 \mu m$	$\pm 18.75 \%$
Segment length	l	$3.96 \% mm^{-1}$	$\pm 0.15 mm$	$\pm 0.6 \%$
Segment width	λ	$148 \% mm^{-1}$	$\pm 0.15 mm$	$\pm 22.2 \%$
Spiraloid radius	r_{in}	$-54 \% mm^{-1}$	$\pm 0.1 mm$	$\mp 5.4 \%$
Pcb thickness	t_{pcb}	$-0.18 \% \mu m^{-1}$	$\pm 25 \mu m$	$\mp 4.5 \%$
Eccentricity	dr	$40 \% mm^{-1}$	$\pm 0.5 mm$	$\pm 20 \%$

3.4 Experimental validation

3.4.1 Hindsight uncertainty

In the context of a validation of the numerical model, the objective is to reduce the uncertainty of the system to its minimum. This is why the final windings have been measured in order to perform simulations as close as possible to real conditions.

Based on images of the winding section on a microscopic scale, such as that in Fig. 3.20, it has been estimated that most of the segment thicknesses are included in a $\pm 5 \mu m$ interval around the nominal value which is $80 \mu m$.

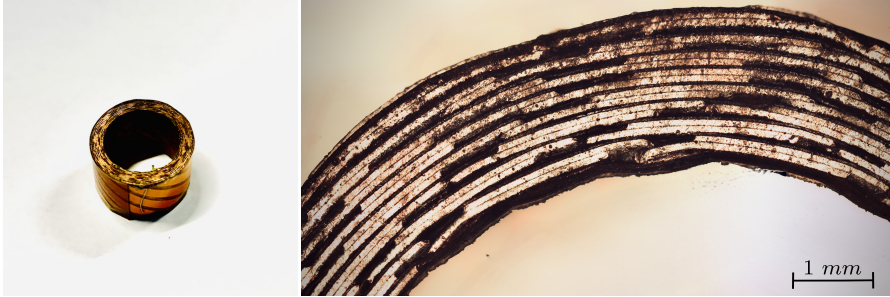


Figure 3.20: Section of winding A, after having been cut with the micro-trunking machine and then polished

The heights of the copper segments were measured at their nominal value of only $18 mm$ with accuracy of $\pm 20 \mu m$. However, the widths of these segments were on average only 95% of their expected value. The measurements were made with an accuracy of about $\pm 50 \mu m$.

All internal radii of the checked windings were measured at their nominal value of $5.5 mm$ with accuracy of $\pm 2.5 \mu m$. Their external radii were also measured at their nominal value of $7.4 mm$ on average for windings A, C and D, but with a

lower accuracy of $\pm 100 \mu m$. It is assumed that this inaccuracy can be uniformly spread on all the internal layers, which means that the thickness of a pcb layer is assumed to be known with an uncertainty of about $\pm 7 \mu m$. In the specific case of winding B, where there is a lot of free space between the tracks, the adhesive fits into the gaps reducing the pcb thickness of about $35 \mu m$.

As it can be seen in Tab. 3.1, besides the uncertainty on the winding, the two most important sources of poor estimation of the losses are the uncertainty of the PM magnetic field and of the eccentricity. No tools were available to accurately measure the intensity of the magnetic field around the magnet. Therefore, the uncertainty about the exact value of B_r , as well as the question of which is the best model to use between the manufacturer's data and the 2D analytical model remain. The eccentricity is also quite difficult to measure without specific sensors. But it can be reduced by running the deceleration test multiple times. As a matter of fact, the losses are minimal when the rotor and the stator are perfectly aligned. Choosing the lowest result among these multiple runs of the same experiment allows to reduce the chances of being impacted by eccentricity.

3.4.2 Comparison & discussion

The next step is to compare the measurements presented in section 3.2.4 with an estimation of the losses. Because of the intrinsic uncertainties of the experiment, the estimation cannot be a single value but has to be a continuous set of possible estimations, weighted by their probability of occurrence. It is assumed that the event probability corresponds to the combined probability of deviation of the parameters discussed above. Therefore, the link between the estimated losses in the simulation and the actual losses can be expressed as follows:

$$P_{eddy} = P_{simulation} X_{B_r} X_T X_g X_\lambda X_{t_{pcb}} \quad (3.12)$$

$$X_{B_r} \sim \mathcal{N}(\mu = 1.0, \sigma = 6.67\%) \quad (3.13)$$

$$X_T \sim \mathcal{N}(\mu = 1.0, \sigma = 0.20\%) \quad (3.14)$$

$$X_g \sim \mathcal{N}(\mu = 1.0, \sigma = 2.47\%) \quad (3.15)$$

$$X_\lambda \sim \mathcal{N}(\mu = 1.0, \sigma = 2.08\%) \quad (3.16)$$

$$X_{t_{pcb}} \sim \mathcal{N}(\mu = 1.0, \sigma = 0.42\%) \quad (3.17)$$

where X_i is a random variable that allows to take into account the uncertainty of each specific parameter and to model its impact on the losses. All these random variables are centered at the unit so that on average the simulation is expected to represent the actual situation.

While the probability density function of each variable X_i , is expressed by a standard gaussian function that is fairly easy to work with, the calculation of the combined probability density function of the eddy current losses is less obvious. It has been shown that the product of normal distributions can be obtained exactly by using modified Bessel functions [90, 91]. However, for the sake of simplicity, we approximated the combined probability density function of eddy current losses by

applying the Monte-carlo method. For each of the parameters, numerical values are randomly chosen using the normal distributions. Then the total variation of the losses is computed using the linearized sensitivities of Tab. 3.1. This is repeated millions of times and a discrete distribution of the losses can be obtained. Although it may appear very similar, this discrete distribution is no longer exactly gaussian. Finally, to obtain a continuous and smooth curve, a gaussian kernel density estimator has been used [92].

This whole process is only valid mathematically if all the parameters are independent from each other. In our case, to respect this condition as much as possible, certain parameters are neglected. The segment length l , the internal radius r_{in} and the eccentricity dr are thus assumed perfectly known in what follows. It has been chosen to ignore these three parameters because they are mastered and/or their impact is negligible, and they were directly correlated with other parameters.

The standard deviation σ_{global} of this global discrete distribution, obtained by combining the uncertainties, is 7.45%. As the curve looks pretty similar to a true gaussian, a 99%-confidence interval can still be approximated with $[100\% - 3\sigma_{global}, 100\% + 3\sigma_{global}] = [77.65\%, 122.35\%]$.

The measure of the eddy current losses at 500 *rpm*, which is visible in Fig. 3.6, is compared with the results of 2D FEM simulations in Fig. 3.21. For sake of readability, the probability densities have been scaled so that their maximum are arbitrarily at the same unitary value. The uncertainty of $\pm 0.5\%$ of the measurements is represented by the vertical rectangles. Table 3.2 shows the nominal values and the relative error between estimations and measurements.

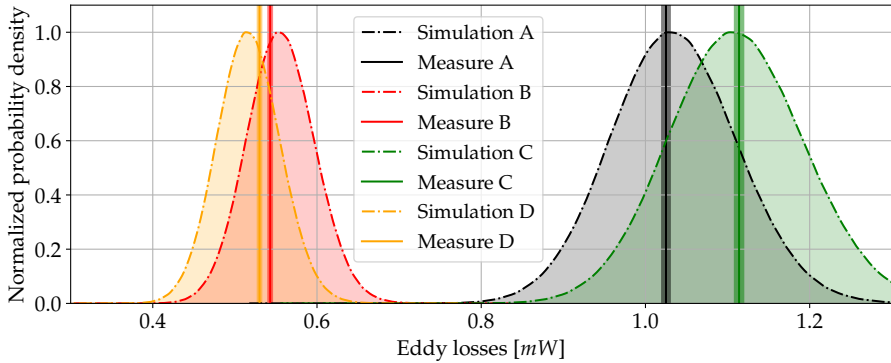


Figure 3.21: Comparison between measurements and simulations at 500 *rpm*, considering uncertainties and inaccuracies.

The simulations take into consideration the fact that segments widths were smaller than expected and also considers the smaller external radius of winding B. The simulations use the theoretical model to compute the source field instead of using the data provided by the magnet manufacturer. The characteristic length of the mesh is 100 μm and convergence has been reached up to less than 1.5% variation when this length is halved.

Table 3.2: Relative error between the measurements and simulations at 500 *rpm*.

	A	B	C	D
Measured <i>mW</i>	1.025	0.543	1.114	0.530
Simulated <i>mW</i>	1.035	0.556	1.111	0.517
Relative error %	0.97	2.39	0.27	2.45

There was a conception mistake and adding vias to the extremities of the segments in winding C increases the volume of copper available for eddy currents. Actually, the copper volume ratio between winding A and winding C is approximately 93.5%. Therefore, it cannot be considered that these two windings differ only in the connectivity of the segments. This difference in copper volume explains the 93.2% ratio between 1.035 *mW* and 1.111 *mW*.

The relative error obtained for each of the windings is quite small. The measures and the simulations are in good agreement which means that it seems reasonable to consider that the numerical model is valid. Each of these windings was used for a specific purpose, the result of which is discussed in the paragraphs below.

Winding B had the objective to validate the hypothesis that the reaction field of a segment is too weak to influence its neighbors. The measured losses are not strictly halved in comparison of winding A. However, the deviation is not significant and is most probably explained by the shrinkage of the winding as the adhesive has filled the large spaces between the tracks. The results obtained are thus in favor of the validation of the hypothesis.

Winding C had the objective to validate that the segments separated by a via can be studied independently from each other. Since they actually have different fill factors, a direct comparison between windings A and C is less useful than expected. That said, if the losses are compared relatively to their copper volume, then there is no significant difference, and the hypothesis is largely validated.

Winding D had the objective to verify that $P_{eddy} \propto \lambda^3$. The relation is not strictly respected considering the values of 1.025 *mW* and 0.530 *mW* for windings A and D respectively. But, once again, these values remain largely acceptable regarding the tolerances and uncertainties which have been considered. As the measurements are well within the gaussians drawn in Fig. 3.21, it can be considered that the relation is verified.

3.5 Conclusions

The objective of this chapter was to validate by experiment the 2D FEM model developed in chapter 2. Four specific windings were also built to test some of its underlying assumptions as well. Eddy current losses were measured using deceleration tests which have the advantage of being very accurate even at low speeds.

The chapter started with the description of these windings and discussed the working principle of the test bench.

Because the actual system parameters may deviate from the expected design, the second part of this chapter was dedicated to compute the sensitivity of the losses to those parameters. Combined with uncertainty on the latter they allowed to build the probability density for measuring eddy current losses. It appeared that the most impacting parameters are the amplitude of the source magnetic field, the relative rotor eccentricity and the width of the copper segments.

Finally, the uncertainty in the dimensions was reduced a posteriori by measuring the built windings and, the simulations were compared to the experimental measurements. Both were in good agreement within the range of $\pm 2.5\%$, which validates the 2D FEM model.

Analytical model

4

The purpose of a model is to estimate quantities inherent to a motor without effectively building it. Different models can be developed to assess the same quantities, but the choice of a model will depend on the intended use. Usually, to understand the physics of motors, the models implemented are accurate but computationally expensive. As part of an optimization which requires a large number of evaluations, there are a large number of calls to the model. Therefore, a model whose execution time is low would be preferred, even if this latter offers less accurate results.

In chapter 2, a simplified FEM model has been developed so that it can accurately predict the eddy current losses of an entire winding. This model can be successfully used in an optimization routine aimed at maximizing the performance of an electrical machine. However, it remains computationally expensive. For instance, to compute the losses in a $20 \times 2.5 \text{ mm}^2$ segment with 20000 elements it takes about 1 s. This may not sound like much, but on the same computer, evaluating an analytical expression with a dozen of operations to perform only takes about $100 \mu\text{s}$. In this context this chapter is devoted to the development of an analytical model that would be sufficiently accurate to evaluate the eddy current losses in a winding and with a computation time potentially reduced by four orders of magnitude.

Two approaches will be considered. The first is to identify an analytical expression that fits the results of numerous FEM simulations. The second one is based on the construction of equivalent circuits which are then solved to obtain the expression for the eddy current losses.

The chapter is structured as follows. The two approaches are first developed considering a simple straight segment conductor. Then, they are extended to more complex shapes. Finally, the analytical expressions obtained are compared to the 2D FEM model in order to quantify their accuracy.

4.1 Problem definition

As for the development of the 2D FEM model, the focus is at first on a single straight segment isolated from the others. This choice is motivated by the results of the two previous chapters. The reaction field being weak, interactions between neighboring tracks can be neglected (H1 of section 2.3.1), and the vias acting like walls for the loops of eddy currents, the sections of the conductor between two vias can be modeled alone (H4 of section 2.3.1).

Given a source magnetic field of amplitude b_{z0} which is assumed to be normal to the conductor and invariant along its thickness and height, the objective is to determine P_{eddy} in a segment such as the one shown in Fig. 4.1. The source field considered results from the rotation of a one pole pair rotor with parallel magnetization. Therefore, its evolution in time is given by $b_{z0}(t) = b_{z0} \cos(\pi x/\tau - \omega t)$ with ω the speed of the motor. Material properties as the conductivity σ and the magnetic permeability μ are supposed to be constant in this problem.

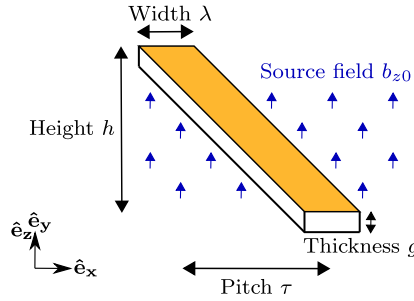


Figure 4.1: Geometric parameters defining a straight segment of the conductor.

4.2 Identification & factorization

This section develops a formal methodology to establish a heuristic formula for eddy current losses out of numerical simulations, using the Vaschy-Buckingham theorem or Π theorem [93].

4.2.1 Π theorem

As stated in [94]:

The Π theorem is used to determine the number of dimensional groups required to describe a phenomenon. According to this theorem the number of dimensionless groups to define a problem equals the total number of variables, n , minus the fundamental dimensions, m . If we call these dimensionless groups π_1, π_2, π_3 , etc., then the equation expressing the relationship between the variables has a solution of the form :

$$\pi_1 = f(\pi_2, \pi_3, \dots, \pi_{n-m}) \quad (4.1)$$

The problem of eddy currents losses as described in the previous section has 9 quantities that can vary. There are the dimensions of the the segment h, λ, g, τ and its electromagnetic properties σ and μ . There are also the amplitude and the frequency of the source magnetic field b_{z0} and ω . Finally, there is the mean value of the eddy current losses P_{eddy} which one seeks to obtain. These 9 quantities depend on 4 independent fundamental units (m, kg, s, A). In order to apply the theorem, five dimensionless variables are suggested in Tab. 4.1. Note that the dimensionless groups can be chosen totally arbitrarily, regardless of the underlying physical reality, without influencing the results.

Table 4.1: Dimensionless groups of the problem.

π_1	π_2	π_3	π_4	π_5
$\frac{\mu P_v}{\omega b_{z0}^2}$	$\sigma \mu \omega g^2$	$\frac{g}{\tau}$	$\frac{h}{\tau}$	$\frac{\lambda}{\tau}$

According to the theorem, there exists a function f_1 such that:

$$\frac{\mu P_v}{\omega b_{z0}^2} = f_1\left(\sigma \mu \omega g^2, \frac{g}{\tau}, \frac{h}{\tau}, \frac{\lambda}{\tau}\right) \quad (4.2)$$

with $P_v = P_{eddy}/g\lambda h$, namely the loss density.

4.2.2 Separation of variables

The next step in the reasoning is to show that the function f_1 is separable. For a function F of two variables x, y , if the quotient $F(x, y = y_i)/F(x = x_0, y = y_i)$ is independent of y_i , then F is said separable and can be rewritten as the product of two functions of one variable $F(x, y) = X(x) * Y(y)$. The notation $x \perp y$ will be used when this is the case.

In the context of this study, 3D FEM simulations of a single conductive segment (similar to the one in Fig. 2.7) have been performed, varying the dimensions and the speed. A sweep of the values has been carried out for the following ranges:

- λ from 0.5 mm to 3 mm
- h from 0.5 cm to 2 cm
- τ from 0.5 cm to 3 cm
- ω from 10 krpm to 100 krpm

The variables σ, μ and g have been kept constant, and equal to $6.56 \cdot 10^7$ S/m, $4\pi \cdot 10^{-7}$ Tm/A and 60 μ m, respectively.

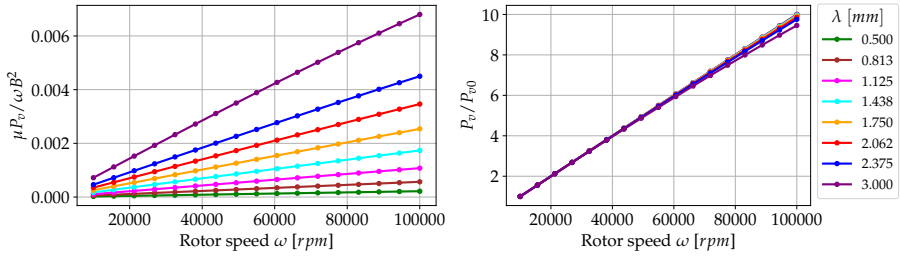


Figure 4.2: Left: Evolution of the variable π_1 for various λ . Right: Evolution of $P_v(\omega)/P_v(10000 \text{ rpm})$ for various λ .

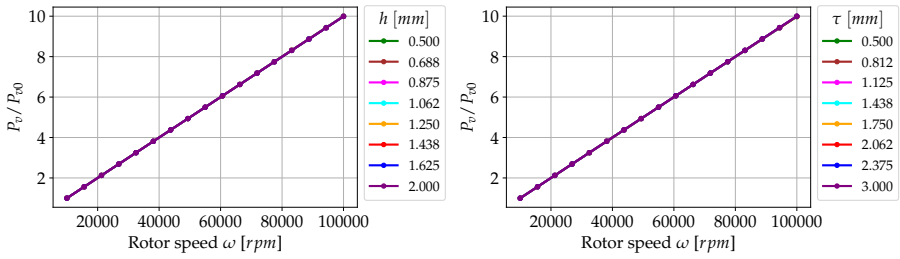


Figure 4.3: Evolution of $P_v(\omega)/P_v(10000 \text{ rpm})$ for various h (left) and τ (right).

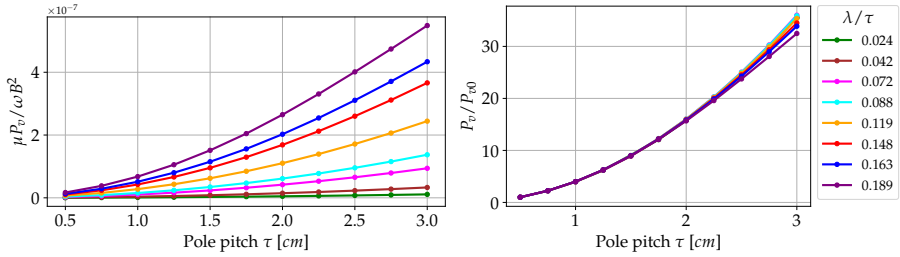


Figure 4.4: Highlight of the independence of variable π_3 from variable π_5 .

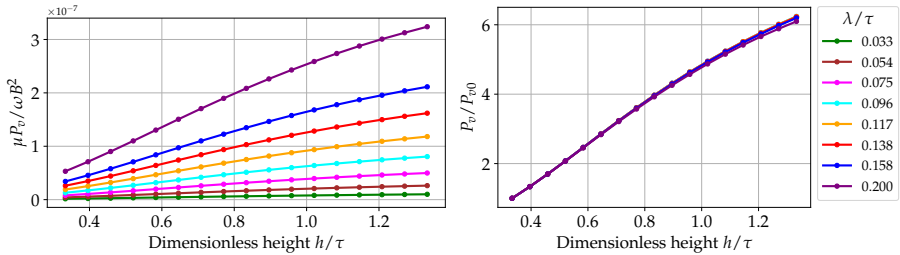


Figure 4.5: Highlight of the independence of variable π_4 from variable π_5 .

Fig. 4.2 and 4.3 indicate that $\lambda \perp \omega$, $h \perp \omega$ and $\tau \perp \omega$, showing that the effect on the losses of the frequency and the winding geometry are decoupled. At 100 *krpm*, the difference in $\frac{P_v}{P_{v0}}$ between $h = 0.5 \text{ cm}$ and $h = 2 \text{ cm}$ is only 0.14%. The variation between $\tau = 0.5 \text{ cm}$ and $\tau = 3 \text{ cm}$ is also negligible, being 0.25%. However, at this speed, the difference between $\lambda = 0.5 \text{ mm}$ and $\lambda = 3 \text{ mm}$ is 5.44% implying that the variable separation is somewhat less accurate. Indeed, as the width of the conductor increases, this latter becomes more inductive and the reaction field is no longer as weak and counteracts a little the source field. It means that in addition to the active power, the reactive power becomes non-negligible and the curve expressing the losses deviates slightly from the purely resistive behavior.

Fig. 4.4, keeping h/τ and λ/τ constant, suggests that $g/\tau \perp (h/\tau, \lambda/\tau)$ despite that for large values of λ/τ the variable separation becomes less accurate due to a more inductive behavior again. Finally, Fig. 4.5 shows that h/τ and λ/τ can be studied independently, again with an inductive phenomenon for large λ . Based on these observations, the density of eddy current losses can be written as:

$$P_v = \mu^{-1} b_{z0}^2 \omega f_2(\sigma \mu \omega g^2) f_3\left(\frac{g}{\tau}\right) f_4\left(\frac{\lambda}{\tau}\right) f_5\left(\frac{h}{\tau}\right) \quad (4.3)$$

4.2.3 Curve fitting

It is observed that f_2 , f_3 and f_4 go through the origin and constantly increase/decrease toward $+\infty$ or $-\infty$. Consequently, it has been arbitrarily proposed to fit them with monomials. However, when h/τ goes to infinity the loss density should tend towards a constant value. Therefore, in the case of f_5 , since this function is the only one to depend on h , it needs to tend asymptotically to a constant value. For this reason it was chosen to fit f_5 with an arctangent function limited to the unit, so that it ends up with the following parametric function:

$$P_v = \alpha_0 \mu^{-1} b_{z0}^2 \omega (\sigma \mu \omega g^2)^{\alpha_1} \left(\frac{g}{\tau}\right)^{\alpha_2} \left(\frac{\lambda}{\tau}\right)^{\alpha_3} \left(\frac{2}{\pi} \operatorname{atan}\left(\alpha_4 \frac{h}{\tau}\right)\right)^{\alpha_5} \quad (4.4)$$

A learning set of about 1500 points uniformly distributed in the space of variables $\{\lambda, h, \tau, \omega\}$, evaluated with the 3D FEM model of chapter 2, has been used to identify the free α_i parameters and the following algebraic expression has been obtained:

$$P_v = 5.00 e^{-4} \mu^{-1} b_{z0}^2 \omega (\sigma \mu \omega g^2)^1 \left(\frac{\tau}{g}\right)^2 \left(\frac{\lambda}{\tau}\right)^2 \left(\frac{2}{\pi} \operatorname{atan}\left(\frac{2\pi h}{\pi 2\tau}\right)\right)^2 \quad (4.5)$$

After simplifying the terms that cancel each other out, the sought heuristic formula for eddy current losses is:

$$P_{eddy} = 5.00 e^{-4} \sigma b_{z0}^2 \omega^2 g h \lambda^3 \left(\frac{2}{\pi} \operatorname{atan}\left(\frac{\pi h}{2\tau}\right)\right)^2 \quad (4.6)$$

The final expression has then been validated on another set of about 1000 points, using our 2D FEM model of chapter 2. The distribution of the relative error, when compared to the 2D numerical solutions, is shown in Fig. 4.6. The mean of the relative errors is 1.12 % and the standard deviation is 6.41 %.

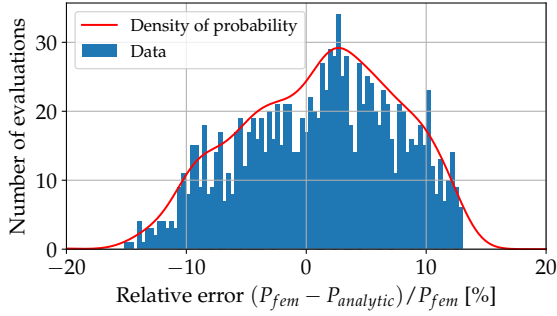


Figure 4.6: Distribution of the relative error.

4.3 Equivalent circuit

Instead of directly fitting curves based on numerical simulations to obtain heuristic formulae, a second approach consists in representing the problem by an equivalent circuit and to find the expression of the power dissipated in the equivalent resistance.

4.3.1 Model

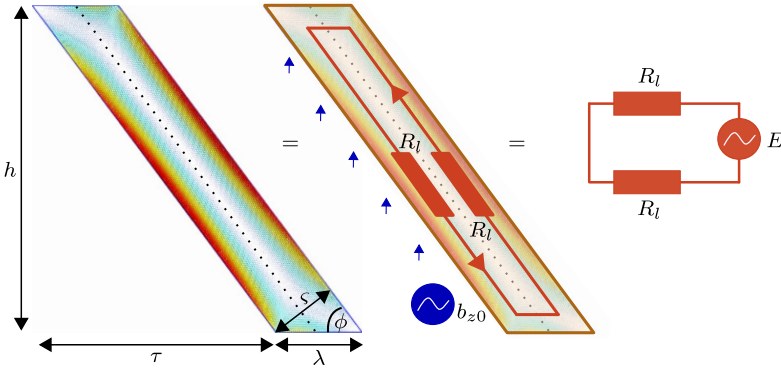


Figure 4.7: Circuit representation of the eddy current problem.

As shown in Fig. 4.7, eddy currents always take the shape of closed loops. It is assumed that these currents can be represented by a unique current flow looping back on itself. All along the segment, two currents of the same amplitude but in opposite directions run on their sides and see an electrical resistance R_l . It is also assumed that the distance where both current lines close on themselves is negligible compared to the length of the current loop itself. Therefore, the horizontal resistance of the segment is neglected. The equivalent circuit is thus composed of two resistances R_l and an electromotive force E that represents the interaction with the source magnetic field which goes through the conductor.

Eddy current losses in the equivalent circuit of Fig. 4.7 are computed as:

$$P_{eddy} = \frac{E^2}{2R_l} \quad (4.7)$$

where E is the rms value of the electromotive force.

The resistance R_l can be approximated using the Pouillet's law:

$$R_l = \frac{\sqrt{h^2 + \tau^2}}{\sigma(\varsigma/2)g} = 2 \frac{\sqrt{h^2 + \tau^2}}{\sigma g \lambda \sin(\phi)} = 2 \frac{\sqrt{h^2 + \tau^2}}{\sigma g \lambda} \frac{\sqrt{(h/\tau)^2 + 1}}{h/\tau} = 2 \frac{h^2 + \tau^2}{\sigma g \lambda h} \quad (4.8)$$

Faraday's law states that the emf $E(t)$ is given by the rate of change of the magnetic flux $\Psi(t)$:

$$E(t) = -\frac{d\Psi(t)}{dt} \quad (4.9)$$

And the magnetic flux captured by the segment can be approximated by:

$$\Psi(t) = b_{z0} h \lambda \cos(\omega t) \quad (4.10)$$

where λh is the surface of the segment. Consequently, the rms value of the electromotive force is given by:

$$E = \frac{\omega b_{z0} h \lambda}{\sqrt{2}} \quad (4.11)$$

The mean value of the eddy current losses is thus given by:

$$P_{eddy} = \frac{\sigma g h \omega^2 b_{z0}^2 \lambda^3}{8} \frac{(h/\tau)^2}{1 + (h/\tau)^2} \quad (4.12)$$

4.3.2 Adjustment variable

The above expression (4.12) is obtained considering that:

- the current density was uniformly distributed;
- the whole segment sees the same magnetic field amplitude ignoring its sinusoidal distribution.

This is clearly not exact and in order to consider these non uniform distributions, the values of R_l and Ψ can be modified by applying a distribution factor κ . These factors have to be determined and allow a degree of freedom in the expression of the losses:

$$P_{eddy} = \frac{\kappa_\Psi^2 E^2}{2\kappa_R R_l} = \alpha_0 \sigma g h \omega^2 b_{z0}^2 \lambda^3 \frac{(h/\tau)^2}{1 + (h/\tau)^2} \quad (4.13)$$

with α_0 , the adjustment variable.

To obtain a value for α_0 , about 1000 2D FEM simulations have been performed, varying h , λ , τ and ω in the same ranges as for the first approach. The value $\alpha_0 = 4.37e^{-4}$ has been chosen to obtain a zero average error.

The relative error has been computed on the same test set as for the previous approach (section 4.2.3) and the results are shown in Fig. 4.8. The error is 0.13% on average and its standard deviation is 4.61 %.

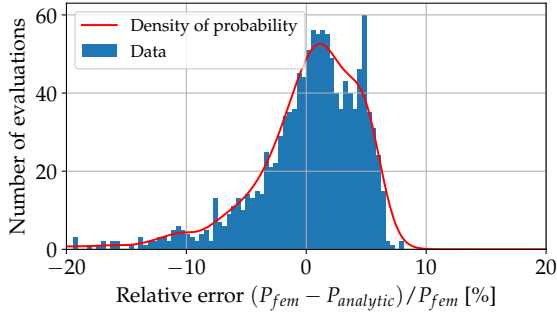


Figure 4.8: Distribution of the relative error.

Because there is only one degree of freedom with α_0 , theoretically only one simulation of reference is needed to complete the analytical model. The question arises to know how good an estimation based on a single simulation would be compared to the use of $\alpha_0 = 4.37e^{-4}$ obtained with 1000^+ points. The answer is shown in Fig. 4.9. Quite intuitively, the standard deviation remains centered around the value of 4.61 %. However, the chances of getting a non-zero error on average are very important if one uses only a single random evaluation of the losses to set α_0 .

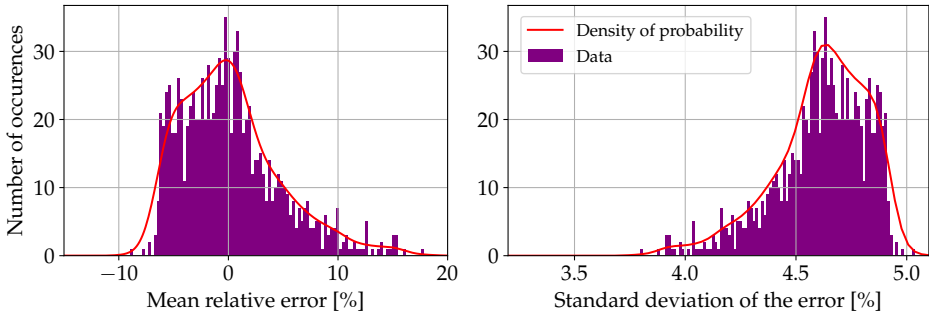


Figure 4.9: Distribution of the mean relative error (left) and the associated standard deviation (right) when α_0 is determined to use a single point in the learning dataset.

4.4 Expressions comparison

Actually, the expressions (4.6) and (4.13) obtained by using both approaches are quite similar. Their only difference is in the way the skewing of the segment is taken into consideration through the variable h/τ .

$$P_{eddy} = \sigma g h \omega^2 b_{z0}^2 \lambda^3 g_i(h/\tau) \quad \text{with} \quad g_1 = 5.00e^{-4} \left(\frac{2}{\pi} \operatorname{atan} \left(\frac{\pi h}{2\tau} \right) \right)^2$$

$$\text{or} \quad g_2 = 4.37e^{-4} \frac{(h/\tau)^2}{1 + (h/\tau)^2}$$

By plotting both functions g_1 and g_2 , as in Fig. 4.10, it can be observed that they intersect each other twice and their difference is bounded. Their maximum relative difference is 14.2% when they reach their asymptotic values of $5e^{-4}$ and $4.37e^{-4}$.

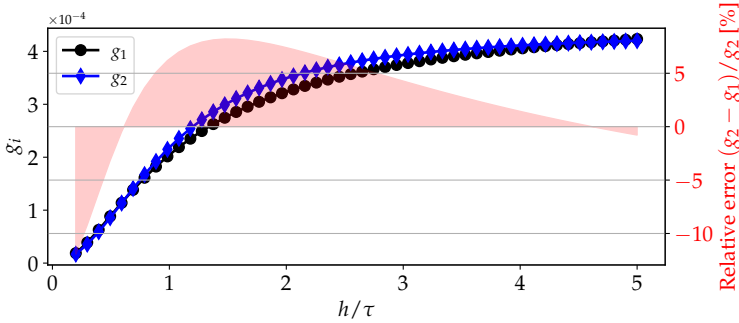


Figure 4.10: Comparison of the two approaches.

As g_1 is arbitrarily chosen whereas g_2 is the result of analytical developments, it can be assumed that this latter is more reliable, and it is g_2 that will be used in the following of this work.

4.5 Complex shapes

Practically, during optimization, the segments constituting the winding are not straight but curved. In this case, as shown in Fig. 4.11, two approaches have been considered to apply the analytical formula. Either the segments are approximated by global straight segments of total height h , width λ and pitch τ or they are decomposed into straight sections.

With the first approach, equation (4.13) is simply applied as it is using the approximated dimensions.

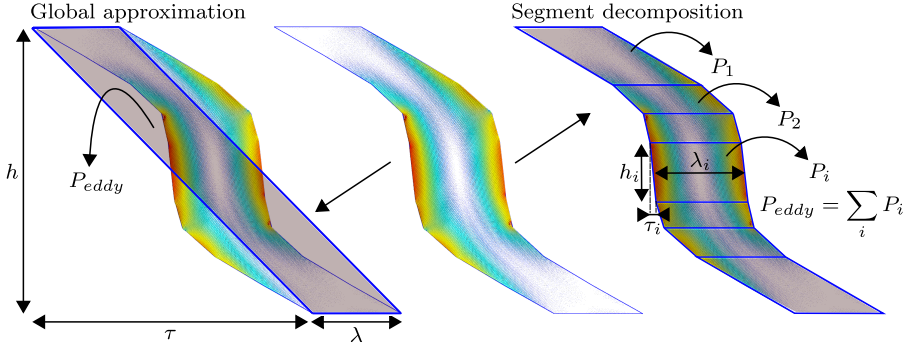


Figure 4.11: The analytical formula for a straight segment can be applied to more complex shapes either by considering a global equivalent trapezoid or by decomposing the shape into smaller independent straight sections.

With the second approach, mean losses are computed on each of the sections independently:

$$P_i = 4.37e^{-4} \sigma g h_i \omega^2 b_{z0}^2 \lambda_i^3 \frac{(h_i/\tau_i)^2}{1 + (h_i/\tau_i)^2} \quad (4.14)$$

with h_i the height of the i^{th} section, λ_i its mean width and τ_i its pitch, and the total losses on the segment are estimated as the sum of the losses on each section:

$$P_{eddy} = \sum_i P_i \quad (4.15)$$

The above summation is possible because the sections are spatially independent of each other and thus the currents flowing in them are also considered independent of each other.

To demonstrate the relevance of such a decomposition, a straight segment was successively divided in several sections, as shown in Fig. 4.12, and 2D numerical simulations were performed to estimate the losses. In this case, the boundaries of the sections do not correspond to any physical boundary. Therefore, there is no longer any pertinence to impose $\mathbf{j} \cdot \mathbf{n} = 0$ everywhere. On the contrary, in this specific case, it has been observed that it is preferable to simulate opened sections where the loop of current is not forced to vanish at both ends. Therefore, the condition $\mathbf{j} - (\mathbf{j} \cdot \mathbf{n})\mathbf{n} = 0$ is applied instead, imposing a normal direction to the current and allowing to minimize the error. The difference in the behavior of the electrical current between those two types of boundaries is shown in Fig. 4.13. The results of the successive divisions are listed in Tab. 4.2 where it can be seen that there is almost no difference in loss when using opened sections whereas there is a significant difference when using closed sections. In this numerical experiment, the cuts were drawn horizontally, as presented in the second approach, to approximate a curved segment. These horizontal cuts slightly distort the path of the eddy currents, but this has very little impact on the losses. Since the analytical model developed in section 4.3 assumes only two opposite flows of the current along the length of the segment and neglects transversal resistances, it models this case of scenarios perfectly.

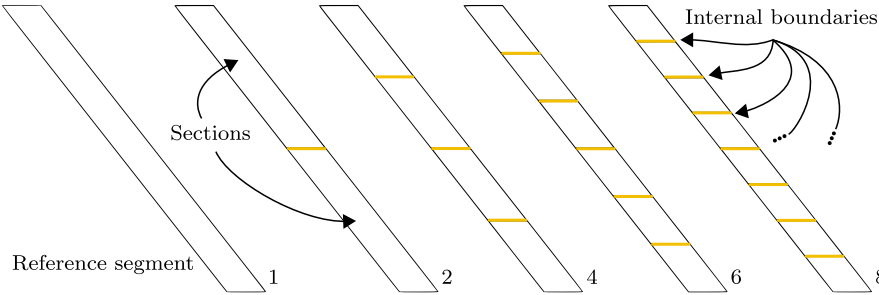


Figure 4.12: Splitting of a segment into sections in which the losses are computed independently from each other.

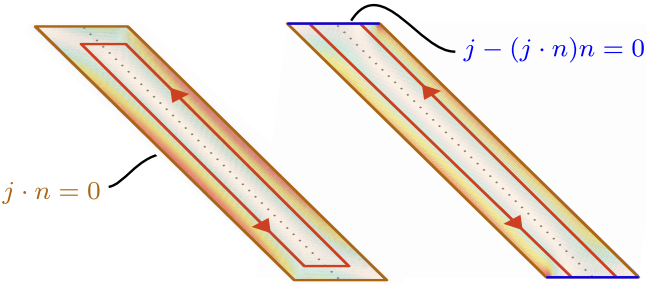


Figure 4.13: Both types of boundary condition for the segment ends.

Table 4.2: Relative estimation of eddy current losses when splitting a segment in sub-sections.

	Closed sections $\mathbf{j} \cdot \mathbf{n} = 0$	Opened sections $\mathbf{j} - (\mathbf{j} \cdot \mathbf{n})\mathbf{n} = 0$
1 section	100%	
2 sections	95.6%	100.8%
4 sections	86.8%	102.5%
6 sections	78.0%	104.1%
8 sections	69.3%	105.7%

4.6 Relative error

This section aims at observing the relative error of the analytical model according to some parameters of the problem, especially the dimensions of the segments and the frequency of the source magnetic field. The comparison was carried out on a new data set that covers a wider range of values for the parameters.

In order to quantify the inductive phenomena for each problem, the skin depth has been also evaluated and compared to the perpendicular width ς of the conductor. The skin depth is defined as it follows.

$$\delta = \sqrt{\frac{2\rho}{\omega\mu_0}} \quad (4.16)$$

with ρ the copper electrical resistivity, ω the angular frequency of the source magnetic field and μ_0 the magnetic permeability of vacuum.

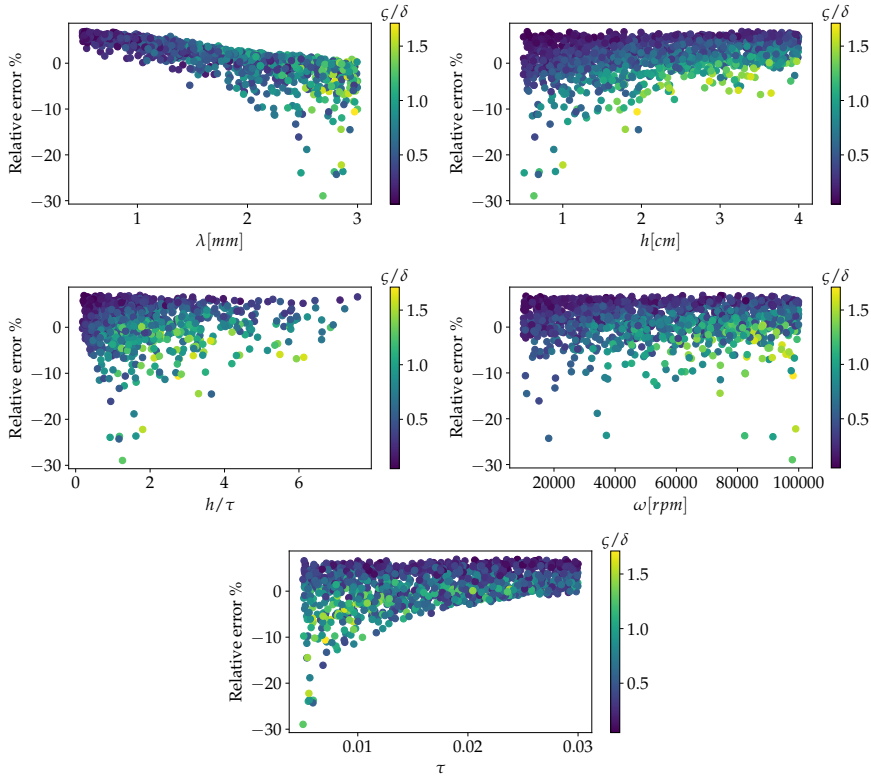


Figure 4.14: Relative error between the finite element model and the analytical model in the case of simple straight segments and no current can cross the boundaries.

Fig. 4.14 shows the relative error between the finite element model and the analytical model in the case of simple straight segments for which no current can cross the boundaries. On average the relative error is 1.22% with a standard deviation of 4.15%. This low average is explained by the fact that this case is very close to the case for which the value α_0 was actually determined.

It can be observed that the error is not correlated with the speed. On the contrary, it is linearly correlated with the width of the segment. The larger are the

segments, the more inductive they are and therefore a purely resistive model tends to overestimate the eddy current losses.

A further observation is that the model is more accurate for higher values of h/τ . In other words, the less skewed the segment is, the more accurate the loss estimation will be. A possible explanation comes from the fact that the less the segment is skewed, the more the assumption of a uniform source magnetic field is respected.

For smaller values of h , the model can suggest erroneous estimations up to 20 – 30% error. This can be explained by the fact that the smaller the segment, the less valid is the assumption that the transverse electrical resistance can be neglected compared to the longitudinal resistance. This phenomenon is even more present for large segments.

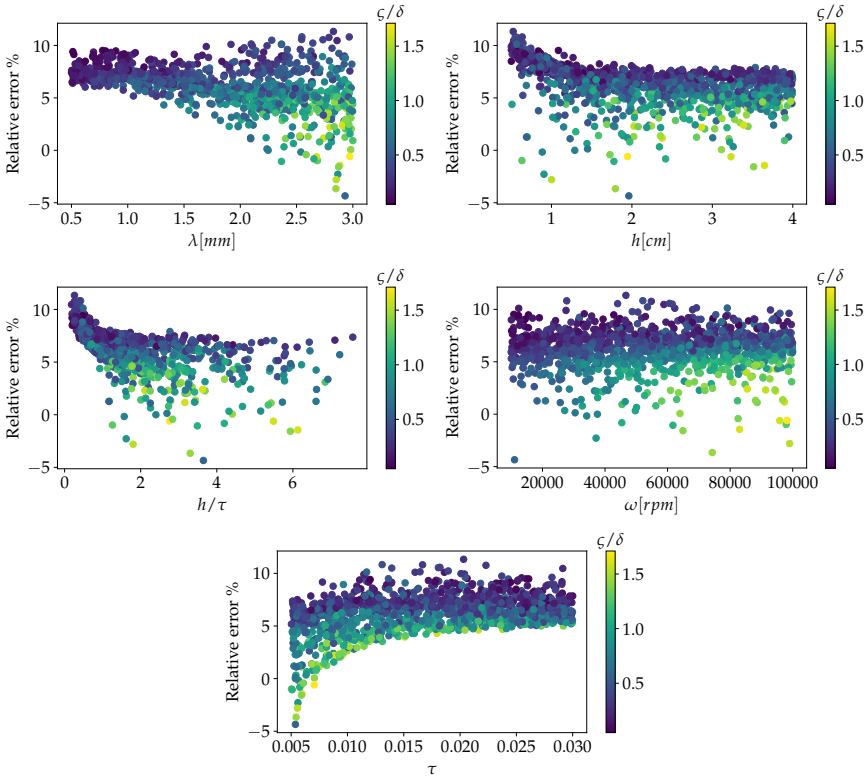


Figure 4.15: Relative error between the finite element model and the analytical model in the case where the boundaries are open at both ends so that the current can cross them.

Fig. 4.15 also shows the relative error in the case of simple straight segments. However, in the latter cases, the condition $\mathbf{j} - \mathbf{j} \cdot \mathbf{n} = 0$ has been applied at both

ends of the segment so that the currents do not have to loop back on themselves. On average, the relative error is now 6.19% but with a standard deviation of only 1.90%. This higher value for the relative error is explained by the fact that α_0 has not been optimized to model this case of scenario, where the current can leave out the segment. However, the lower standard deviation shows that it is actually more representative of the developed analytical model as there is no transverse current. By the way, it can be observed that the correlation of the error with the segment width has been reduced.

As for the segments for which no current can cross the boundaries, it is observed that the error is independent of the speed.

As expected, we also observe that the error tends to increase as the skin depth to width ratio increases, regardless of the boundary conditions applied.

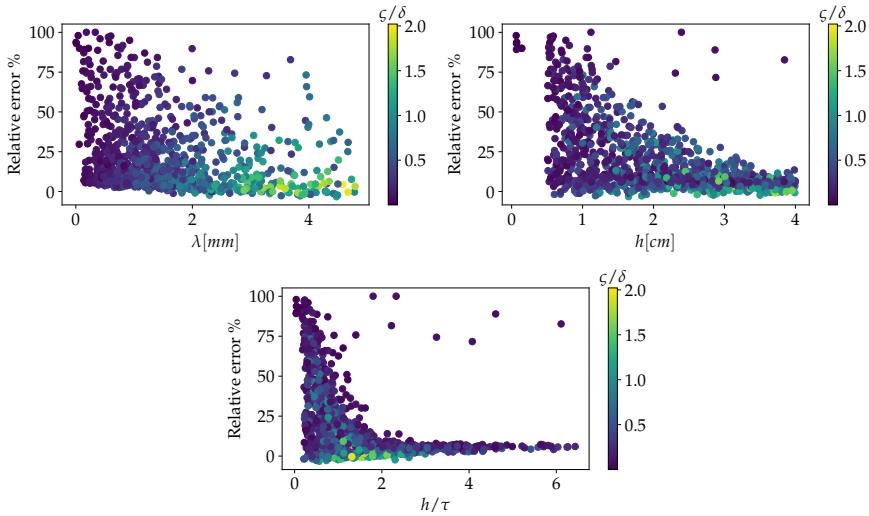


Figure 4.16: Relative error between the finite element model and the analytical model when the segments are curved and no current can cross the boundaries.

Fig. 4.16 shows the relative error in the case of a curved segment composed of height sections, as illustrated in Fig. 4.11. When the curved segment is approximated by one equivalent straight segment, the relative error is 17.12% with a standard deviation of only 20.25%. This large error shows that approaching a curved segment with a single straight segment is not adapted to estimate the losses.

Applying the model to the eight sections separately, the relative error for the whole segment falls to 1.74% on average with a standard deviation of only 4.73%. It means that decomposing the curved segments in smaller straight sections, the accuracy of the model is nearly the same as for straight segments.

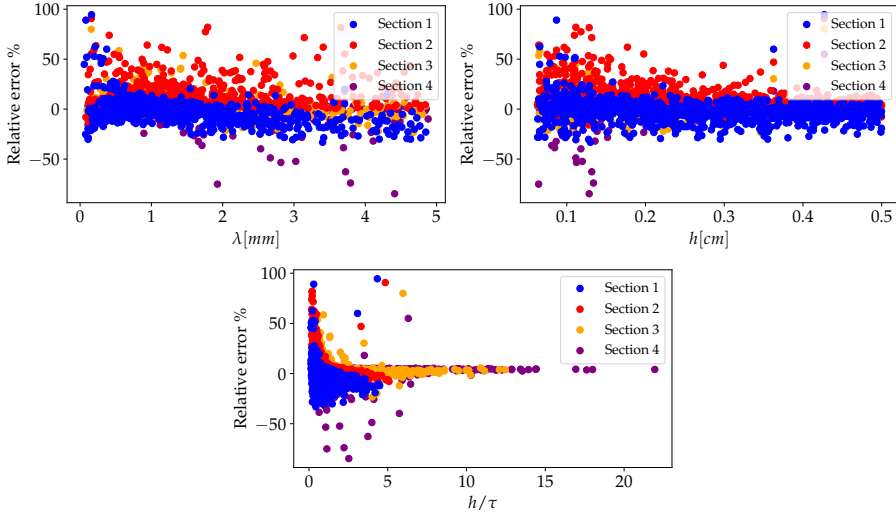


Figure 4.17: Relative error between the finite element model and the analytical model for each of the first four sections.

Fig. 4.17 shows the relative error for each of the first four sections. The first section is the one at the start of the segment and the fourth is in the center. In general, the first section tends to be more horizontal and the fourth more vertical, so that, overall, the eight sections put together in series form a rounded S shape.

From section one to four, the average error is -2.03% , 10.33% , 2.85% and 1.50% respectively, while their standard deviation is 11.54% , 12.52% , 7.40% and 8.03% respectively.

Results are quite similar to those obtained previously when the whole segment was straight. Indeed, we find the same results but at the scale each section.

4.7 Validity range

The previous section evaluated the error as a function of the problem parameters. In this section, the reverse exercise is proposed to determine in which situation the analytical model can be used when a maximum error margin is a priori set.

Based on the error evaluated in Fig. 4.14 for straight segments, Tab. 4.3 lists the intervals for which an error of 10% maximum was detected. For example, for all segments with λ between 0.5 mm and 2.24 mm , the analytical model gave a correct estimate of the losses, with an accuracy of 10%. In Tab. 4.3, the percentages in parentheses represent the proportion of the 10%-validated range compared to the complete initial range studied.

We can observe that the width of the conductors is a good discriminant to eliminate outliers and to be able to use the analytical model in most cases. This means that regardless of the other parameters, as long as they remain within the ranges for which the model was studied, it is only required to verify that λ is below 2.24 mm to ensure an accuracy of 10%. The 0% range for speed means that no significant interval was found for which all estimates are less than 10% error.

This result concerns the case of straight segments but it is reasonable to assume that it also works for curved segments since we have previously shown that the evolution of the error for the latter is the same when studied at the scale of each section composing the curve.

Table 4.3: Parameter range for which the analytical model and the 2D FEM model give a loss estimate with a maximum difference of 10%.

	Range evaluated	10% error max
λ [mm]	0.5 - 3	0.5 - 2.24 (70%)
h [cm]	0.5 - 4	1.96 - 4 (58%)
τ [cm]	0.5 - 3	1.04 - 3 (78%)
ω [krpm]	10 - 100	- (0%)
h/τ	0.17 - 8	3.65 - 8 (56%)
ς/δ	0.048 - 1.711	0.048 - 0.405 (22%)

In practice, it is very rare to see tracks as wide as $2 - 3 \text{ mm}$, especially at high speed. If we restrict the analysis to the $0.5 - 1 \text{ mm}$ interval, we can observe an error of 5.38% with a standard deviation of 0.98%. Therefore, by adjusting the coefficient α_0 to obtain a zero error on average, we obtain a maximum error of 4.59%.

From a development point of view, the question is then to determine the most relevant interval to fit the analytical expression. A large interval leads to a large uncertainty, but choosing too small intervals with different coefficients for each might be a trap because of the risk of overfitting.

4.8 Conclusions

In this chapter, different analytical models able to predict the eddy current losses have been developed.

The first approach was to vary all the parameters of the problem and find an expression that fits the results of 3D finite element simulations. The II theorem was used to determine dimensionless variables of the problem. The use of this theorem has the advantage of not having to know the physics of the problem a priori. The drawback of this approach is that it requires making an arbitrary choice about which functions to fit.

The second approach was to express the problem as an equivalent circuit to be solved. In order to take into account a non-uniform distribution of the currents and a non-uniform distribution of the source field, a degree of freedom was added to the model through the parameter α_0 . This parameter was determined by minimizing the error on a learning dataset consisting of uniformly distributed segments.

It appears that both approaches lead to an almost identical expression. The only small difference consists in the consideration of the skewing.

Both approaches were presented on straight segments. In order to extend the analytical model to curved segments, it was proposed either to approximate the curve by a straight segment or to decompose it into smaller trapezoidal sections on which the formula can be applied. The first method is not recommended as it leads to relative errors of up to 20 – 30%. However, the decomposition of the problem into sections showed that an average error of less than 2% could be obtained with a standard deviation of about 5%.

Finally, it was pointed out that the width of the conductor is a strong indicator of whether the estimate of the analytical model will be potentially wrong or not. Confidence intervals were also proposed for the use of the model with 10% accuracy.

Publication related to this chapter:

- G. François, F. Baudart, F. Henrotte and B. Dehez, *Numerical Investigation of Eddy Current Losses in Airgap PCB Windings of Slotless BLDC Motors*, 2018 21st International Conference on Electrical Machines and Systems (ICEMS), 2018, pp. 2702-2708.

Impact of eddy currents on the optimal motor

5

This chapter has two objectives. The first one is to observe the impact of eddy currents on the optimal design of a motor. In order to evaluate the importance of considering them in the optimization process. The second one is to compare the motor designs obtained using either analytical or finite element models during optimization. Since analytical models require much less computational effort, the question is whether or not they are sufficient to lead to the optimal solution when used in the optimization routines.

This chapter is structured in three main parts. First, the case study is described and the formulation of the optimization problem is presented. Second, the models implemented in the optimization routines and the optimization strategy are developed. Finally, the results of the multiple optimizations are analyzed with respect to the two objectives of this chapter.

5.1 Optimization problem

When designing a motor, a framework is always set at first, either by experience, by external specifications, or simply by arbitrary choices. Then, within this framework, one strives to optimize the machine to best meet a given objective. This section sets the parameters and defines the meaning of what a good motor is through the definition of an objective function.

5.1.1 Motor description

The motor under study is cylindrical with a total length l_{motor} and an external diameter d_{ext} . It is composed of four concentric parts which are the shaft, the one pole pair permanent magnet, the winding and the stator yoke. Their thicknesses are respectively denoted t_{shaft} , $t_{magnets}$, $t_{winding}$ and t_{yoke} . Similarly, their lengths can be denoted l_{shaft} , $l_{magnets}$, $l_{winding}$ and l_{yoke} . The airgap between the magnets and the winding has a thickness t_{airgap} . All of these parameters are illustrated in Fig. 5.1.

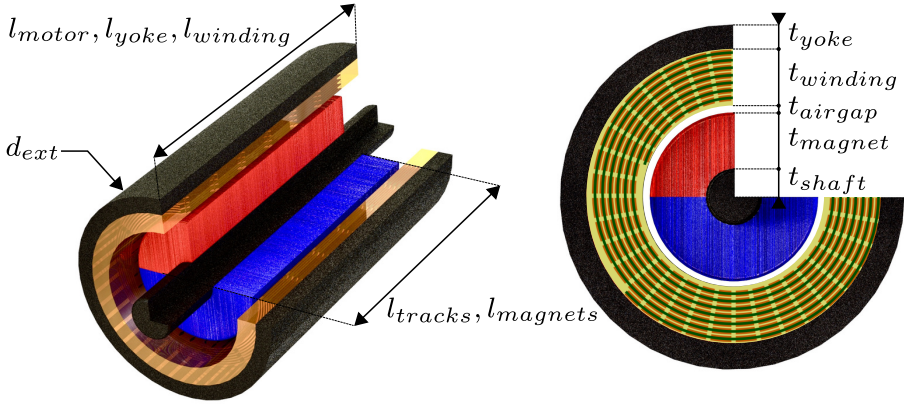


Figure 5.1: Perspective view and section of the parameterized motor.

Material properties are assumed uniformly distributed in each part. The magnets are parallel magnetized with a remanent magnetic field B_r . The copper tracks have a constant conductivity σ . The stator yoke is composed of iron with a known anhysteretic curve $\mu(\mathbf{h})$ and all other elements have a magnetic permeability close to μ_0 .

The winding consists of several layers forming a spiraloïd. Each layer of PCB is itself composed of two layers of copper with a thickness t_{copper} , one layer of substrate $t_{substrate}$ and one insulation layer $t_{insulation}$ so that the total thickness of a winding layer is:

$$t_{layer} = 2t_{copper} + t_{substrate} + t_{insulation} \quad (5.1)$$

In order to be optimized the winding shape is also parameterized. Using the symmetries, only a quarter of the loop has to be defined and it is approximated with n rectilinear sections s_i defined by their end points (φ_{i-1}, z_{i-1}) and (φ_i, z_i) as shown in Fig. 5.2. Each loop spans over a pole pitch and has a height of l_{tracks} , which is the active length of the winding. The full winding is achieved using symmetries and periodic repetitions along the rolling direction. A three phase winding is considered with $3N_t N_l$ loops, where N_l is the number of layers and N_t is the number of loops per layer and per phase. Finally, a maximum azimuthal width λ is imposed for the copper tracks and a minimum space between them t_{gap} .

The lengths of the sections s_i are defined as follows:

$$l_i = \sqrt{\Delta z_i^2 + r_i^2 \Delta \varphi_i^2} \quad (5.2)$$

where Δz_i and $\Delta \varphi_i$ are respectively the vertical and angular distances between the two extremities of the section s_i and r_i is the mean radius of the section.

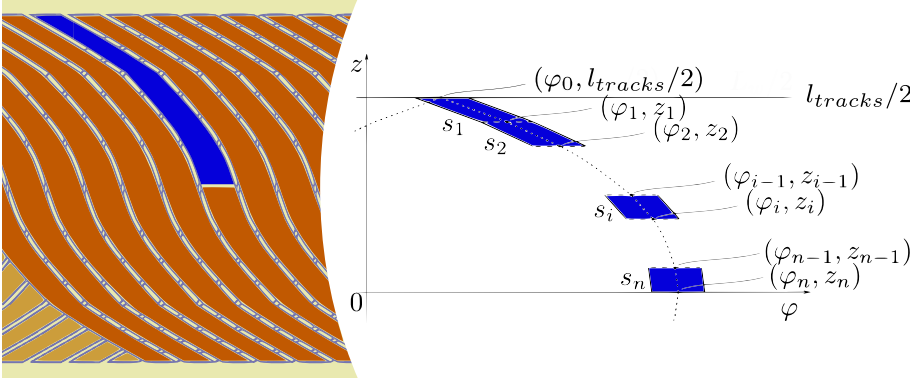


Figure 5.2: Parameterization of the winding shape.

The width ς_i of the section i is determined by the most constraining rule imposed by the two parameters N_t and λ .

With $3N_t$ loops on a PCB layer, the maximum width for a section s_i is thus given by:

$$\varsigma_i N_t = \frac{2\pi r_i}{3N_t} \frac{\Delta z_i}{l_i} - t_{gap} \quad (5.3)$$

where the term $\frac{\Delta z_i}{l_i}$ is present to consider the skewing of the track.

The constraint imposed by the parameter λ converts to a constraint on the normal width as:

$$\varsigma_i \lambda = \lambda \frac{\Delta z_i}{l_i} \quad (5.4)$$

Consequently, the width of a section is actually computed as:

$$\varsigma_i = \min(\varsigma_i N_t, \varsigma_i \lambda) \quad (5.5)$$

5.1.2 Fixed parameters

Not all of the above parameters describing the motor are optimized. To begin with, the materials are fixed in advance. Namely, an NdFeB/N42 magnet with $B_r = 1.3 T$ is used and *Supra 50* laminated steel is chosen for the stator yoke. The thickness of the PCB layers is of $245 \mu m$ with a copper thickness of $90 \mu m$.

For mechanical reasons, the thickness of the airgap is also fixed in advance. The smaller is the distance between the magnets and the winding, the greater is the amplitude of the magnetic field at the conductors and therefore the better is the torque capacity of the motor. Consequently, most of the optimizations should aim to minimize t_{airgap} . Only a large increase of the friction losses would balance this phenomenon. However, this occurs at very high speeds or for more viscous fluids

such as water for example, yet this is not the case that we consider in this work. Therefore, the t_{airgap} is set at 0.5 mm which is the smallest dimension that can be produced at a reasonable cost of machining and assembly.

For the sake of simplicity and in order to reduce the number of variables to be optimized, the various lengths of the motor parts are left invariant. The stator and the shaft extend over the entire length of the motor. Their lengths have been set to:

$$l_{yoke} = l_{winding} = l_{shaft} = l_{motor} = 22 \text{ mm}$$

The external diameter of the motor is fixed at:

$$d_{ext} = 16 \text{ mm}$$

The active length of the winding l_{tracks} and the length of the magnets $l_{magnets}$ are both equal to 18 mm . They are slightly shorter what eventually let space for bearings if we were to build the motor practically. Also, the thickness of the shaft is set to $t_{shaft} = 1.5 \text{ mm}$. Reducing the number of variables allows to reduce the computational efforts of the optimizations and thus their time and ease of convergence. It is assumed that these simplifications do not fundamentally change the results of this chapter. At low speeds, setting the lengths to their maximum is anyway the correct choice since the more magnets and copper there are, the higher the torque capability. Moreover, the longer the stator yoke, the easier it is for the magnetic field to loop. It is at higher speeds that the optimization is forced to adapt thicknesses rather than the lengths when losses due to the spin of the rotor become important. However, playing with the lengths only affects the edge effects and these are relatively weak for long motors.

For the description of the winding shape, it has been observed that in practice a number of sections $n = 6$ is a good compromise between the smoothness of the conductor tracks obtained and the number of variables to be optimized. In a quarter of loop, the heights z_i , visible in Fig. 5.2, are equally distributed between zero and half of the height l_{tracks} of the tracks. In addition, φ_0 and φ_6 are equal to 0 and $\pi/2$, respectively. The minimal space t_{gap} between two conductors is imposed at $200 \mu\text{m}$.

5.1.3 Optimization variables

The remaining nine parameters can vary independently of each other. These are the optimization variables and are summarized in Tab.5.1.

Note that t_{yoke} also varies but is directly dependent on the other thicknesses because of the constraint on the external diameter which is:

$$t_{shaft} + t_{magnets} + t_{airgap} + t_{winding} + t_{yoke} = 0.5d_{ext} \quad (5.6)$$

Similarly, the number of layers is also linked to the total thickness available for the winding:

$$N_l = t_{winding}/t_{layer} \quad (5.7)$$

where in practice, N_l is rounded down to the nearest unit. The remaining space due to the rounding is allocated to the stator yoke.

Table 5.1: Optimization variables

Variable	Signification	Range
Winding shape		
λ	Maximum azimuthal track width	0.2-2 mm
N_t	Number of loops per phase and per layer	2-10
$\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5$	Angles of the s_i end points	$0 - \pi/2$ rad
Internal thicknesses		
$t_{magnets}$	Thickness of the magnets	0.5-5 mm
$t_{winding}$	Thickness of the winding	0.5-5 mm

The number of loops N_t and their maximum width λ might seem redundant but they are not. Indeed, λ is a maximum limit and not the actual value for the conductor azimuthal width in this chapter. The limit is only activated if the value of N_t would have led to a space available for the track larger than λ .

In practice, the optimization deals with coefficients ranging between 0 and 1. Zero corresponds to the angle φ_{i-1} and the unit corresponds to $\pi/2$. This way, the constraints on the minimum angles are naturally respected regardless of the value of the coefficient in the interval $[0, 1]$.

Using the set of variables in Tab. 5.1, the constraint on the outer shell can be easily respected. The lengths are fixed, and so does l_{motor} , and the diameter d_{ext} is set by adjusting the variable t_{yoke} to respect (5.6).

5.1.4 Objective

There are multiple definitions of what a good motor is depending on the use case of the machine. The objective can be either to reach the highest efficiency if it is in the context of energy conversion, or to be the lightest if it is in the context of embedded mechanisms, for example.

Multiple objectives may be pursued, although combining or prioritizing them always results in the definition of a single objective function. Pareto fronts can sometimes be used to visualize solutions that are said to be equivalent at an intermediate stage, but at the end of the design process, other criteria are added so that only one solution is preferred.

In this chapter, the motor that will be considered the best will be the one with the highest power density. A nominal speed and some constraints on the outer shell are assumed. Furthermore, as for any motor, thermal constraints need to be respected as well. The optimization problem can be expressed as:

At a given speed ω and for a given motor envelope (l_{motor}, d_{ext}) , the objective is to achieve the highest possible torque T_{nom} , keeping the motor under a limit temperature.

The electromagnetic torque developed by a PM motor is given by:

$$T_{nom} = k_t I_{nom} \quad (5.8)$$

with k_t the torque constant, and I_{nom} the amplitude of the currents flowing through the winding.

The torque constant k_t depends on the motor parameters through the field generated by the rotor in the winding. It is also influenced by the shape and the number of turns of the winding. The torque constant also depends on the type of power supply, DC or AC. The developments to obtain the expressions of k_t for both types are in appendix C. It is an intrinsic characteristic of the motor. This means that regardless of the load, k_t will remain the same for a given motor.

The current is limited by the Joule losses it causes in the winding and by the resulting temperature increase within the motor. The rise of temperature is not only caused by the Joule losses but also by all the other losses. Knowing that, the temperature must be limited within the motor, there is a maximum amplitude of current I_{nom} which can flow in the winding steadily. The temperature must be restricted in the winding in order to avoid accelerated aging of the insulation. In addition, the temperature must also be limited in the magnets in order to prevent their demagnetization. If the temperature is too high, the PM can even become permanently demagnetized. Among others, those two phenomena are the main factors that imply to manage and limit the temperature in the motor.

In light of the above, the optimization of the torque capability can be expressed as:

$$\max T_{nom} = k_t I_{nom} \quad \text{s.t.} \quad T_{winding} \leq T_{winding}^* \quad (5.9)$$

$$T_{magnets} \leq T_{magnets}^* \quad (5.10)$$

with $T_{winding}$ and $T_{magnets}$ the temperatures in the winding and in the magnets, and $T_{winding}^*$ and $T_{magnets}^*$ their respective limits before the risk of a motor deterioration.

Considering that the thermal capabilities of a motor are mostly characterized by the heat transfer between its external frame and the ambient, it can be written that:

$$T_{frame} - T_{ambient} = R_{th} P_{diss} \quad (5.11)$$

with T_{frame} the temperature of the motor at its external frame, $T_{ambient}$ the ambient temperature, R_{th} the thermal resistance between the motor frame and the outside, and P_{diss} the losses dissipated as heat to the outside of the motor.

Assuming that the constraints on the temperatures inside the motor can be gathered into a unique constraint on its maximal temperature at the external frame T_{frame}^* , the optimization can be expressed equivalently as:

$$\max T_{nom} = k_t I_{nom} \quad \text{s.t.} \quad P_{losses} \leq \frac{T_{motor}^* - T_{ambient}}{R_{th}} = P_{diss} \quad (5.12)$$

It means that for the motor to be sustainable in time, all the losses generated inside the motor P_{losses} must be lower or equal to the maximum power it can dissipate P_{diss} .

Losses can be distinguished between two categories. On the one hand, there are the Joule losses due only to the supply current and independent of the speed, and on the other hand, there are all the other losses which are functions of the speed but assumed independent of the supply current. Hence, it can be written that:

$$P_{losses} = P_{supply}(I_{nom}) + P_{speed}(\omega) \quad (5.13)$$

As detailed in appendix C, Joule losses due to a supply AC current are simply given by:

$$P_{supply} = \frac{3}{2} R_{ph} I_{nom}^2 \quad (5.14)$$

with R_{ph} the electrical resistance of one phase.

It can be considered that the maximal torque is achieved when the constraint is activated $P_{losses} = P_{diss}$. Indeed, if this were not the case, it would mean that more current could be injected to produce an additional torque. It would also mean that the maximal torque has not been reached. Therefore, by inverting the relation, the nominal current is given by:

$$I_{nom} = \sqrt{\frac{2}{3}} \sqrt{\frac{P_{diss} - P_{speed}(\omega)}{R_{ph}}} \quad (5.15)$$

As the torque is directly proportional to the supply current through the torque constant, the optimization problem can be reduced to:

$$\max T_{nom} = k_t \sqrt{\frac{2}{3}} \sqrt{\frac{P_{diss} - P_{speed}(\omega)}{R_{ph}}} \quad (5.16)$$

without any constrain.

For a DC power supply, the Joule losses are given by:

$$P_{supply} = 2R_{ph} I_{nom}^2 \quad (5.17)$$

and similarly, the optimization problem becomes:

$$\max T_{nom} = k_t \sqrt{\frac{P_{diss} - P_{speed}(\omega)}{2R_{ph}}} \quad (5.18)$$

In equations (5.18) and (5.16), the torque constant k_t and the phase electrical resistance R_{ph} are both intrinsic parameters of the motor. Furthermore, without any losses of generality, P_{diss} varies with the thermal environment of the motor through the temperature $T_{ambient}$ but also through the thermal resistance R_{th} .

Assuming that two different motors with the same external dimensions (l_{motor}, d_{ext}) have similar thermal capabilities, then P_{diss} is identical for the two motors if they are at the same ambient temperature. Consequently, by fixing the ambient temperature to compare the motors fairly, the term P_{diss} becomes a constant since the optimization problem is stated to be at a given outer shell.

Eventually, it can be observed that the objective function of Eq. (5.16) reduces to:

$$\max T_{nom} = k_t \sqrt{\frac{2}{3}} \sqrt{\frac{P_{diss}}{R_{ph}}} \quad (5.19)$$

when the nominal speed goes to zero as $P_{speed}(0) = 0$. In these circumstances, P_{diss} being a constant, the objective function can be simplified as:

$$\max \sqrt{\frac{2}{3}} \frac{k_t}{\sqrt{R_{ph}}} = k_p \quad (5.20)$$

where k_p is known as the motor constant. This latter criterion is often used to compare motors based on intrinsic values regardless of the operating point or the environment. The same conclusion applies to a DC power supply. Therefore, the objective functions (5.18) and (5.16) proposed in this chapter can also be seen as an extension of a classical criterion in order to be able to consider extra losses, still without any thermal model.

In this chapter, three types of losses depending on the speed are considered. The eddy current losses in the winding due to the PM, P_{eddy} , the iron losses in the stator yoke due to the PM, P_{iron} , and the friction losses that occur in the airgap between the air and the motor $P_{friction}$. Hence, we have that:

$$P_{speed} = P_{eddy} + P_{iron} + P_{friction} \quad (5.21)$$

The power dissipated by the motor is set using the following reasoning. A maximum temperature of $180^\circ C$ is admitted in the motor. It is assumed an ambient temperature of 20° and a heat transfer coefficient of $h = 25 \text{ W m}^{-2} K^{-1}$. For the motor described in section 5.1.1, it leads to:

$$P_{diss} = \pi d_{ext} l_{motor} h \Delta T \approx 6 \text{ W} \quad (5.22)$$

Note that this approach suggests that the hottest temperature in the motor occurs at the external frontier. This is true only if the heat flux is transferred radially.

5.2 Models

In order to evaluate the objective function, it is needed to estimate k_t , R , P_{eddy} , P_{iron} and $P_{friction}$, according to the motor parameters and the nominal speed. This section aims at describing the models used to estimate each of them, except P_{eddy} which was the main purpose of the previous chapters.

The finite element model applied to obtain P_{eddy} is the 2D model described in section 2.3.1 and its analytical counterpart is Eq. (4.13) used on equivalent straight segments of the copper tracks. For k_t and R , as for P_{eddy} , both FEM and analytical models are proposed. P_{iron} and $P_{friction}$ are obtained in both cases using analytical expressions.

5.2.1 Electrical resistance R_{ph}

5.2.1.a Analytical

In order to determine the electrical phase resistance R_{ph} of the winding, we use a simple approach based on the Pouillet's law, as explained in [32] and [22]. This latter consists in approximating the parallelogrammatic conductive track of each winding section by rectangles. The contribution of the section s_i to the global resistance is therefore given by:

$$R_i = \rho \frac{l_i}{s_i g} \quad (5.23)$$

where ρ is the copper resistivity, g the track thickness, s_i the perpendicular width of the section s_i and l_i its length.

The phase resistance is obtained through:

$$R_{ph} = 4N_t N_l \sum_i^{N_s} R_i \quad (5.24)$$

with the sum applied on the N_s sections defining a quarter of loop.

5.2.1.b FEM

To obtain the electrical resistance numerically, a voltage difference V is applied at both ends of the quarter of loop and the resulting current is measured. The latter is obtained by solving an *Electrokinetics* problem:

$$\begin{cases} \nabla \times \mathbf{e} = 0 & \mathbf{n} \times \mathbf{e}|_{\Gamma_{input}, \Gamma_{output}} = 0 \\ \nabla \cdot \mathbf{j} = 0 & \mathbf{n} \cdot \mathbf{j}|_{\Gamma_{lateral}} = 0 \\ \mathbf{j} = \sigma \mathbf{e} \end{cases} \quad (5.25)$$

with $\mathbf{e}$ and $\mathbf{j}$ respectively the electrical field and the current density. Γ is the track frontier and $\mathbf{n}$ its normal. In order to solve this problem, it can be re-written in its scalar electric potential v -formulation inside the conductor:

$$-\nabla \cdot (\sigma \nabla v) = 0, \quad \mathbf{e} = -\nabla v \quad (5.26)$$

Then, the total current density can be integrated such that the electrical resistance of the quarter of loop is given by:

$$R_{quarter} = \frac{V}{\int_{\Gamma_{input}} \mathbf{j} \cdot \mathbf{n} d\Gamma_{input}} \quad (5.27)$$

And the phase resistance is obtained through:

$$R_{ph} = 4N_t N_l R_{quarter} \quad (5.28)$$

Note that only a quarter of loop positioned at the mean radius of the winding is modeled in order to reduce the domain and thus the computation time. However, the whole winding could be simulated to yield a more accurate value of the phase resistance.

Fig. 5.3 illustrates the two methods to estimate the electrical resistance of a quarter of loop.

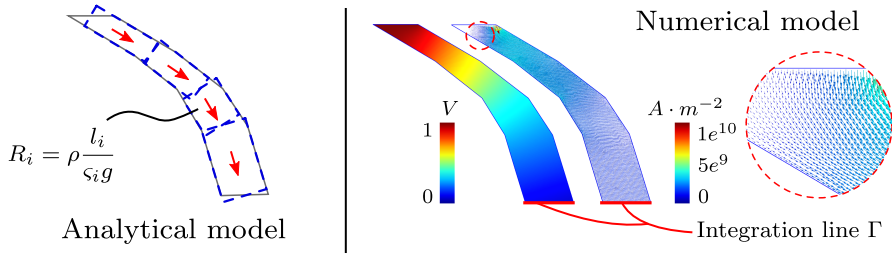


Figure 5.3: Analytical and numerical modeling of the electrical resistance.

5.2.2 Torque constant k_t

5.2.2.a Analytical

The magnetic field in the airgap can be obtained solving the Maxwell's equations with the following assumptions:

- The end effects are negligible, with the consequence that the model becomes 2D.
- The iron yokes of the stator and rotor have finite but constant permeability μ .
- There are no conducting domains, such that the magnetic field is only due to the permanent magnets.

In these conditions, it arises that the magnetic vector potential a has only a z component along the axial direction of the machine (see appendix A). As there is only one pair of poles, it can be written that:

$$a_z(r, \varphi, \theta) = \hat{A}_z(r) \sin(\varphi - \theta) \quad (5.29)$$

where $\hat{A}_z(r)$ is found by applying the method of separation of variables as described in [95], and θ being the relative position between the magnetic axis of the winding and the magnetic axis of the rotor.

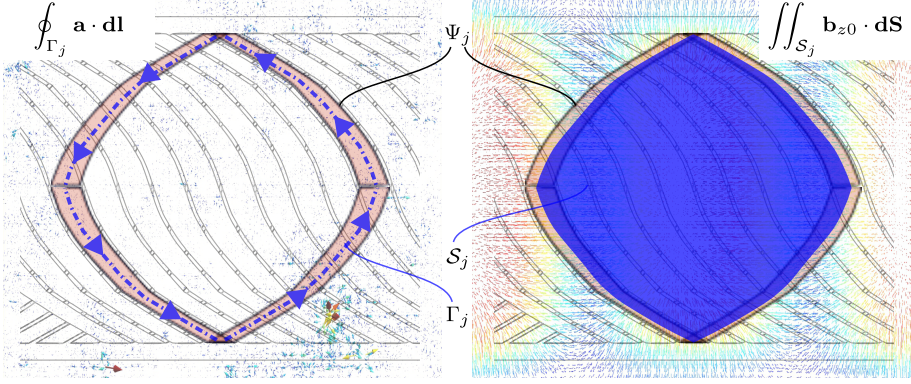


Figure 5.4: Two methods of integration to compute the magnetic flux seen by a loop.

From the vector potential, the flux intercepted by a loop j of the winding is deduced through:

$$\Psi_j(r, \theta) = \oint_{\Gamma_j} \mathbf{a} \cdot d\mathbf{l} \quad (5.30)$$

where Γ_j is the neutral fiber of the loop j as shown in Fig. 5.4.

As the winding segments are formed of linear sections, the flux contribution of a specific section s_i of loop j is practically given by [32]:

$$\Psi_{ji}(r, \theta) = \hat{A}_z(r) \frac{\Delta z_i}{\Delta \varphi_i} [\cos(\varphi_i - \theta) - \cos(\varphi_{i-1} - \theta)] \quad (5.31)$$

and finally, the maximum amplitude of the torque constant, evaluating the field at the mean radius of the winding and assuming an AC supply, is obtained with:

$$k_t = \frac{3}{2} N_l \sum_j^{N_t} \sum_i^{4N_s} \Psi_{ji}(\bar{r}_w, 0) \quad (5.32)$$

the i index sum being applied on the $4N_s$ sections defining the entire loop j . The relation between the torque constant and the flux is detailed in appendix C.

In order to consider the nonlinear behavior of the magnetic materials, their magnetic permeability μ is evaluated using their anhysteretic curves. As explained in [96], these anhysteretic curves are modeled using a logistic regression function. The tail of the function is modified in order to make sure the $\mathbf{b}(\mathbf{h})$ curve tends to a slope of μ_0 for high values of $\mathbf{h}$. The magnetic field is computed using an iterative process until successive re-evaluations of μ converge towards a stable value. As μ is a constant in each domain, only global values can be considered here, so this process is applied based on the average magnetic field in each part.

5.2.2.b FEM

The flux embraced by a winding loop is determined by:

$$\Psi_j(\theta) = \iint_{\mathcal{S}_j} \mathbf{b}_{z0} \cdot d\mathbf{S} \quad (5.33)$$

with $\mathcal{S}_j$ the surface whose contour is defined by the loop j , as shown in Fig. 5.4.

The source magnetic field $\mathbf{b}_{z0}$ is found, as discussed in section 2.4 and previous ones, using 3D FEM simulations. The torque constant is finally given by:

$$k_t = \frac{3}{2} N_l \sum_j^{N_t} \Psi_j \quad (5.34)$$

5.2.3 Iron losses P_{iron}

Iron losses come from eddy currents occurring at different scales. At the macroscopic scale, eddy current losses are due to a change in the magnetic field. At the microscopic scale, the movements of the walls of the magnetic domains, which are discontinuous, induce local currents and generate what is called hysteresis losses. Finally, at the mesoscopic scale, the smooth movement of the domain walls inside a varying magnetic field also generates losses, namely the excess losses.

Three types of models can be found in the literature to model these losses [97,98].

Steinmetz-based models

The Steinmetz formula is one of the most common fits to model iron losses [99].

$$P_{iron} = m \alpha f_e^\beta B_{max}^\gamma \quad (5.35)$$

α, β, γ being the Steinmetz coefficients, m the mass of the stator yoke, B_{max} the maximum amplitude of the magnetic field in the yoke and f_e its frequency.

Many variations and improvements of (5.35) exist in the literature as in [100–102], for example. Steinmetz-based approaches assume sinusoidal excitation which makes them very appropriate when applied to transformers [103,104]. However, because the parameters depend on the frequency, they are not very suitable for waveforms with high harmonic content. Moreover, being only identified in alternative cases the Steinmetz parameters are not very suitable for rotating excitation.

Loss-separation models

They consist in separating iron losses into static hysteresis losses and dynamic eddy current losses [105] and, more generally, in modeling separately the distinct physical mechanisms mentioned above, which leads to the widely used Bertotti's formulation [106], as follows:

$$P_{iron} = \underbrace{p_{hyst}}_{\text{static losses}} + \underbrace{p_{ec} + p_{exc}}_{\text{dynamic losses}} = k_h f_e B_{max}^2 + k_{ec} (f_e B_{max})^2 + k_{exc} (f_e B_{max})^{1.5} \quad (5.36)$$

where p_{hyst} , p_{ec} and p_{exc} are the hysteresis losses, the eddy current losses and the excess losses respectively. The parameters k_h , k_{ec} and k_{exc} are usually identified from measurements. Alternatively, p_{ec} can also be found by solving Maxwell's equations in the iron parts as in [107].

In electrical machines, the magnetic field is not alternating but rotating. Even though the separation still holds, it is important to take this rotation movement into account and to adapt p_h , p_{ec} and p_{exc} accordingly as proposed in [108–110].

Hysteresis models

The idea is to describe the curve $\mathbf{b}(\mathbf{h}, \text{history})$ as accurately as possible and to include it in a FEM model. The iron losses are then given by:

$$P_{iron} = \int_0^{1/f_e} \left(\mathbf{h} \cdot \frac{\partial \mathbf{b}}{\partial t} \right) dt \quad (5.37)$$

These are the most accurate models but require considerable computation time. The hysteresis curve can either be represented mathematically with simple functions or described empirically with raw measured data, or formulated mathematically as a function of the origin of the losses, which leads to a model closely related to the physics. Implementations of these three classes of hysteresis models are given in [111–115], as examples.

A rough estimate of losses based on a Steinmetz-type model was deliberately chosen for its simplicity.

5.2.4 Friction losses $P_{friction}$

The friction losses due to the air flow in the airgap can be estimated by the equation of rotating cylinders in enclosure developed in the works of Polkowski [116], [117]:

$$P_{friction} = k C_f \rho_a \pi \omega^3 r_m^4 l_{magnets}$$

with k the roughness coefficient, ρ_a the mass density of the air flow, r_m the external rotor radius and C_f the torque coefficient. The rotor surface and the inside of the PCB winding are smooth surfaces and thus the roughness coefficient can be approximated by $k \approx 1$.

Bilgen and Boulos [118] measured friction torque of enclosed smooth cylinders and formulated the following expressions for the torque coefficient:

$$\begin{aligned} C_f &= 0.515 \frac{(\delta/r_m)^{0.3}}{Re_\delta^{0.5}} & 500 < Re_\delta < 10^4 \\ C_f &= 0.0325 \frac{(\delta/r_m)^{0.3}}{Re_\delta^{0.2}} & 10^4 < Re_\delta < 10^6 \end{aligned} \quad (5.38)$$

with $Re_\delta = \frac{\omega r_m \delta}{\nu_a}$ the Couette Reynolds number, ν_a the kinematic viscosity and $\delta = r_w - r_m$ the airgap thickness.

5.2.5 Additivity of the losses

Models have been proposed to determine the eddy currents $\mathbf{j}_{eddy}$ as well as the supply current $\mathbf{j}_{supply}$. The objectif of this section is to determine under which conditions both can be superimposed to obtain the total Joules losses dissipated in the winding.

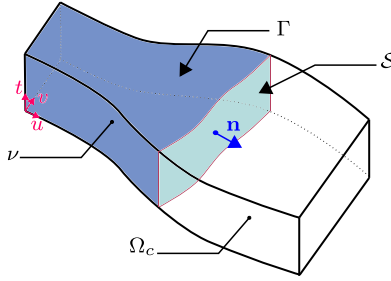


Figure 5.5: A general volume of conductor parameterized in all three dimensions.

Let's assume without any loss of generality that a conductive track of volume Ω_c is parameterized in all three dimensions of the space, as shown in Fig. 5.5, with:

$$[0, 1] \times [0, 1] \times [0, 1] \rightarrow \mathbb{R}^3 : (u, v, t) \rightarrow \gamma(u, v, t) = (x(u, v, t), y(u, v, t), z(u, v, t)) \quad (5.39)$$

where γ is a regular linear application. The total Joule losses are given by:

$$P_{Joule} = \int_{\Omega_c} \rho \|\mathbf{j}_{eddy} + \mathbf{j}_{supply}\|^2 \quad (5.40)$$

$$= \int_{\Omega_c} \rho \|\mathbf{j}_{eddy}\|^2 + \int_{\Omega_c} \rho \|\mathbf{j}_{supply}\|^2 + 2\rho \int_{\Omega_c} \mathbf{j}_{eddy} \cdot \mathbf{j}_{supply} \quad (5.41)$$

$$= P_{eddy} + P_{supply} + 2\rho \int_{\Omega_c} \mathbf{j}_{eddy} \cdot \mathbf{j}_{supply} \quad (5.42)$$

with P_{eddy} and P_{supply} the eddy and supply losses when computed using independent models. Therefore, in order to show that these losses can be simply added together after being studied independently, it must be shown that the crossed term $\int_{\Omega_c} \mathbf{j}_{eddy} \cdot \mathbf{j}_{supply}$ is equal to zero.

Making the assumption that the supply current *follows* the conductor shape and is uniform across any section of it, it can be written that:

$$\mathbf{j}_{supply}(\gamma(u, v, t)) = j_{supply}(u) \mathbf{n}(u, v, t) \quad (5.43)$$

with $\mathbf{n}(u, v, t) = \frac{\partial \gamma}{\partial u} \times \frac{\partial \gamma}{\partial v}$ being the normal to the section $\mathcal{S}$. Therefore, the crossed

term can be developed as:

$$\int_{\Omega_c} \mathbf{j}_{eddy} \cdot \mathbf{j}_{supply} = \int_0^1 \left(\iint_{\mathcal{S}(u)} (\mathbf{j}_{eddy} \cdot \mathbf{j}_{supply}) d\mathcal{S} \right) \left\| \frac{\partial \gamma}{\partial u} \right\| du \quad (5.44)$$

$$= \int_0^1 \left(\iint_{\mathcal{S}(u)} (\mathbf{j}_{eddy} \cdot \mathbf{j}_{supply}(u) \mathbf{n}) d\mathcal{S} \right) \left\| \frac{\partial \gamma}{\partial u} \right\| du \quad (5.45)$$

$$= \int_0^1 j_{supply}(u) \left(\iint_{\mathcal{S}(u)} \mathbf{j}_{eddy} \cdot \mathbf{n} d\mathcal{S} \right) \left\| \frac{\partial \gamma}{\partial u} \right\| du \quad (5.46)$$

By using the charge conservation, one obtains $\nabla \cdot \mathbf{j}_{eddy} = 0$ and using the divergence theorem it becomes:

$$0 = \int_{\nu} \nabla \cdot \mathbf{j}_{eddy} = \oint_{\Gamma} \mathbf{j}_{eddy} \cdot d\mathbf{\Gamma} \quad (5.47)$$

$$= \int_{\Gamma/\mathcal{S}(u)} \mathbf{0} \cdot d\mathbf{\Gamma} + \int_{\mathcal{S}(u)} \mathbf{j}_{eddy} \cdot \mathbf{n} d\mathcal{S} \quad (5.48)$$

$$= \int_{\mathcal{S}(u)} \mathbf{j}_{eddy} \cdot \mathbf{n} d\mathcal{S} \quad (5.49)$$

with ν any volume such that Γ , its frontier is composed of $\mathcal{S}(u)$ a section of the conductor at a given t and $\Gamma/\mathcal{S}$ a closing surface which is never crossed by the eddy currents ($\mathbf{j}_{eddy} \cdot \mathbf{n}(\Gamma/\mathcal{S}) = 0$). As such, a volume ν exists for any conductor and any section $\mathcal{S}$. Using (5.46), it results that:

$$\int_{\Omega_c} \mathbf{j}_{eddy} \cdot \mathbf{j}_{supply} = 0 \quad (5.50)$$

Finally, it can be said that if the supply current is such that assumption (5.43) is respected, then the power supply and the eddy current losses can be studied independently and the final value of their mean losses can be added.

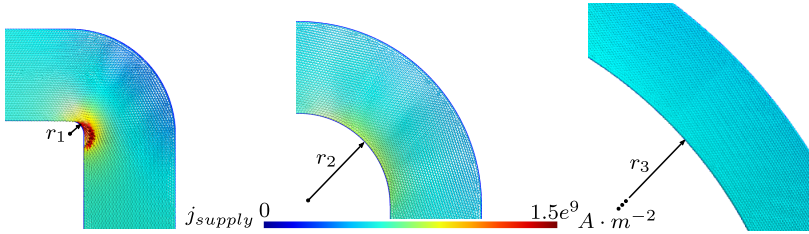


Figure 5.6: Map of the supply current density for various curvatures of the track.

In most cases, the assumption of a smooth conductor shape with a uniform supply current density is obtained. In practice, the copper tracks have large curvature radii compared to their width which is the main factor influencing the current distribution. The exceptions are for the vias and/or the transitions between horizontal shortenings and the main part of the winding, but they represent only a very small portion of the total current path. The impact of the curvature radius is visible in Fig. 5.6.

5.2.6 Optimization algorithm

Models and optimization routines were implemented in python. The optimizations were performed using the NSGA-II algorithm available in the Platypus python framework [119] [120].

Basically, the algorithm starts with a population of N randomly chosen individuals. At each generation, the algorithm applies on these N individuals, named parents, a crossover with another individual, with a probability p_{cross} , and then it applies a mutation on each of these with a probability p_{mut} to create N new individuals named children. Finally, it selects the N best individuals among the parents and the children.

This section investigates the impact that some of the genetic algorithm parameters could have on the optimization convergence.

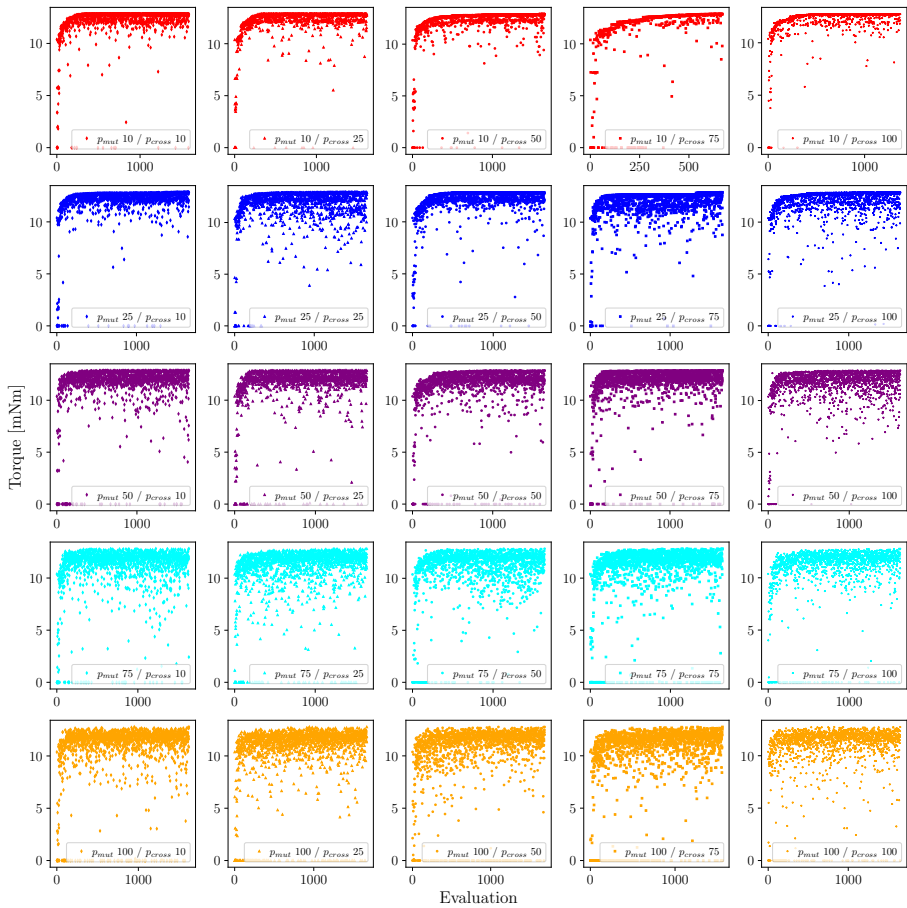


Figure 5.7: Impact of mutation and crossover rates on the optimization convergence.

Fig. 5.7 shows multiple optimizations of the same problem but varying the parameters p_{cross} and p_{mut} . For each optimization, the best motor obtained and the number of iterations to find it, are listed in Tab. 5.2. The best performances have been highlighted in blue.

Table 5.2: Analysis of the meta parameters. In each cell $x/y[mNm]$ indicates that the best torque y was found at iteration x .

$p_{mut} \backslash p_{cross}$	10%	25%	50%	75%	100%
10%	1161 / 12.82	1504 / 12.85	1648 / 12.91	659 / 12.87	1320 / 12.74
25%	1639 / 12.85	1620 / 12.93	1601 / 12.84	1628 / 12.88	1570 / 12.87
50%	1104 / 12.86	1369 / 12.86	1281 / 12.86	1391 / 12.86	1516 / 12.90
75%	1222 / 12.81	1562 / 12.83	1132 / 12.84	1507 / 12.82	1615 / 12.78
100%	950 / 12.77	944 / 12.77	1099 / 12.77	1266 / 12.75	826 / 12.75

In order to observe the influence of the parameters on the results versus the randomness inherent to a genetic algorithm, five simulations using all the same default parameters were computed. The five optimizations visible in Fig. 5.8 found their best result at evaluation 1448, 1662, 758, 934 and 1094 respectively and the optimal torques found were 12.71, 12.72, 12.69, 12.73 and 12.74 mNm . The mean number of evaluations required is thus of 1180 with a standard deviation of 332. The standard deviation of the resulting torque with these five optimizations is 0.16%.

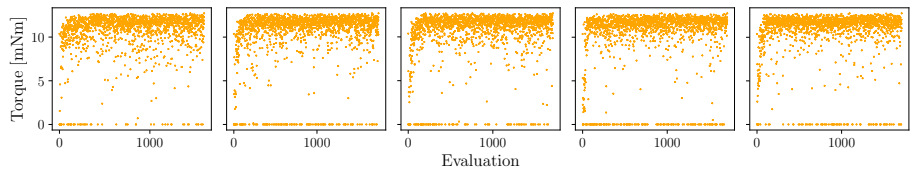


Figure 5.8: Five independent simulations using the default parameters of the Platypus library ($p_{mut} : 100\%$ / $p_{cross} : 100\%$).

Regarding the above observations, no major difference in results is identified in terms of convergence. This means that the results and conclusions that will be obtained from the optimizations are not sensitive to these meta-parameters. In this context, the Platypus library has been used with its default values $p_{crossover} = 100\%$ and $p_{mutation} = 100\%$. Note that those two percentages are very uncommon in the theoretical literature on genetic algorithms, but seem to be a standard in the python coders community. It has been assumed that the difference lies in the implementation details of the algorithm.

5.2.7 Optimization strategy

The simulations were performed on a laptop with an Intel i9-9980HK (8 cores) CPU. Each FEM evaluation of a motor required about 2 min while the analytical evaluations lasted less than 1s. An initial random population of 15 individuals was used for each optimization. The FEM optimizations ran ten hours each, while the optimizations based on the analytical models were limited to 7 minutes. These time limitations allowed to obtain few hundreds of individuals evaluated for both types of optimization and to have comparable results.

In order to help for the convergence of the FEM optimizations, the best solution of their analytical counterpart was part of their initial population. As an example, Fig. 5.9 shows the convergence of both optimizations, ran successively at a speed of 40 *krpm*. The first 15 first individuals, in purple, are randomly chosen. Then, the analytical optimization is performed in blue. To start the FEM optimization, 14 individuals are randomly chosen, and the best solution previously obtained completes the initial population, which is represented by the second group of purple points. The torque estimation of this best motor is not the same in both optimizations because the models give different estimations of the motor quantities. The FEM optimization is finally performed in red.

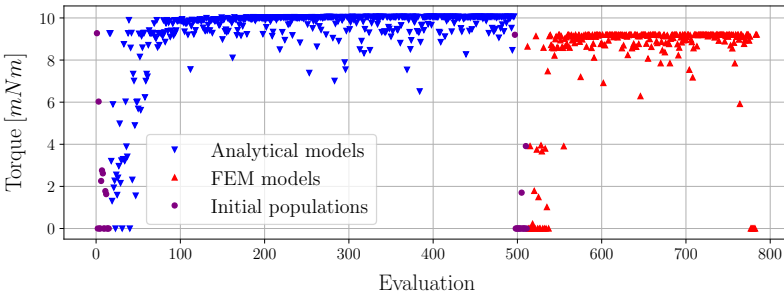


Figure 5.9: Example of two successive optimizations. The first one is using the analytical models and the second one is using the FEM models to estimate the torque.

5.3 Results analysis

The optimization was performed for different speeds ranging from 1 *krpm* to 100 *krpm*, using both types of models.

In order to facilitate the convergence of these optimizations, they were performed one after another and the optimal configuration obtained for a given speed was included in the initial population of the next optimization to be performed. This allowed the results to be smoothed when comparing the optimal motor configurations against their nominal speed, especially for variables which have only a small influence on the objective function. In other words, the best individual not only transited between the analytical and FEM optimizations as explained in the previous section, yet it was also re-used between two successive problems solved at different speeds.

The benefit of doing this study at various speeds instead of a single one is that even if some convergence issues appear all the conclusions will be made based on the trend that emerges.

The analysis is divided into three parts. The first part compares estimations of the motor characteristics given by the analytical models and the FEM models. The second part compares the optimal motor parameters obtained using both types of models. Finally, the third part analyzes the evolution of these optimal parameters with respect to the nominal speed.

5.3.1 Motor characteristics' estimation

FEM and analytical predictions were compared for machine designs resulting from optimizations based on analytical models performed at various nominal speeds.

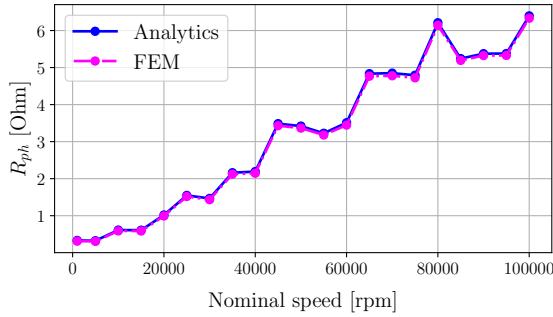


Figure 5.10: Phase resistance of motor windings, each resulting from an analytical model based optimization, at various operating speeds. Each optimized motor has been re-evaluated using FEM and analytical models.

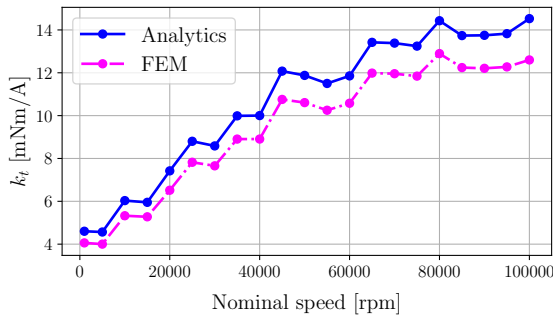


Figure 5.11: Torque constant of the motors, each resulting from an analytical model based optimization, at various operating speeds. Each optimized motor has been re-evaluated using FEM and analytical models.

Fig. 5.10 shows that the analytical estimation of the electrical resistance matches its FEM counterpart perfectly with less than a percent of difference. On the contrary, even though the saturation has been taken into account in the analytical model, the estimation of the torque constant is up to 15% higher than with the FEM model, as illustrated in Fig. 5.11.

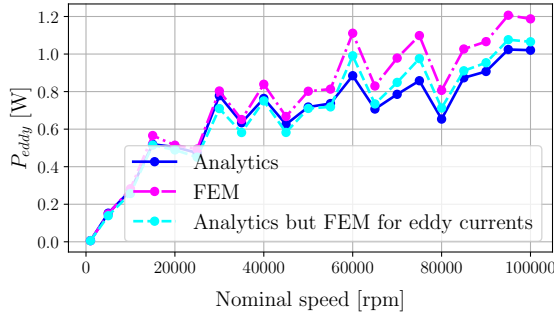


Figure 5.12: Eddy losses within motors, each resulting from optimization based on analytical models, at various nominal speeds. Each optimized motor has been re-evaluated using FEM and analytical models.

Fig. 5.12 shows the predictions of the eddy current losses. It is observed that the numerical and analytical models also give results that are far apart from each other. To understand the origin of this gap, eddy current losses were computed using the FEM model and the circuit analogy model, both of which being provided with an identical source term, i.e. the analytical computation of the magnetic field in the airgap. This comparison is observed in Fig. 5.12 with the blue curves. Since the two predictions of the losses are closer for most of the speeds, this seems to mean that the gap between the complete FEM model and the analytical model is to a great extent due to the bad estimation of the source term, which is also consistent with the results of Fig. 5.11 showing the torque constant.

The results of Fig. 5.12, showing the eddy current losses, highlight that the average value of the airgap magnetic field is lower in the case of the analytical model. Consequently, the 15% increase of k_t in Fig. 5.11 could be mainly due to the non-consideration of the edge effect.

5.3.2 Best motor parameters

Figure 5.13 shows the optimized torque obtained using either the analytical model or the FEM model but where the results have been evaluated with both the analytical model and the FEM model. It clearly appears that the results of both optimizations finally evaluated with the FEM model provide the same optimized torque and vice versa. This result suggests that both models can be used indifferently for optimization and will produce the same optimal motor configuration.

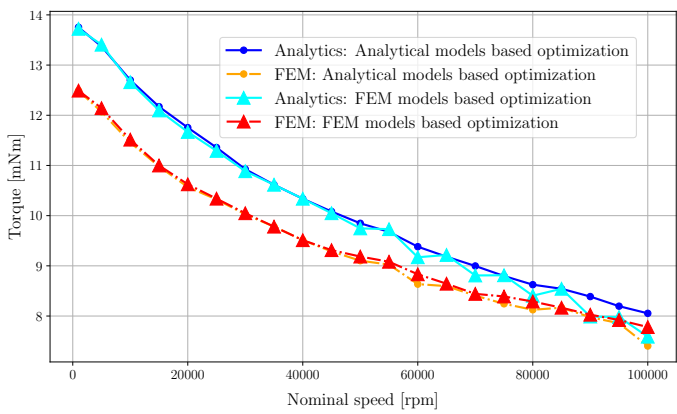


Figure 5.13: Comparison between the analytical model and the FEM model based optimizations, at various nominal speeds. Each resulting motor configuration has been evaluated using analytical and numerical models.

For high speeds it can be observed that some curves are less smooth, probably due to convergence issues. Indeed, the main limitation in this study is the time required to optimize a motor using 3D finite element models. It has an impact on the quality of the meshes used for the 3D simulations.

We have limited the number of elements to a maximum of 1 million per magnetostatics simulation for the estimation of the magnetic field. But the higher the speed, the smaller the optimal track widths, and consequently the uncertainty on the source magnetic field is relatively increased. In other words, when a large conductor spans a dozen finite elements for the discretization of the magnetic field along its width, a half-smaller conductor might only have five elements. In addition, at high speeds, the skin depth of the conductors is reduced but the characteristic length of the mesh remains the same and thus becomes relatively less suitable.

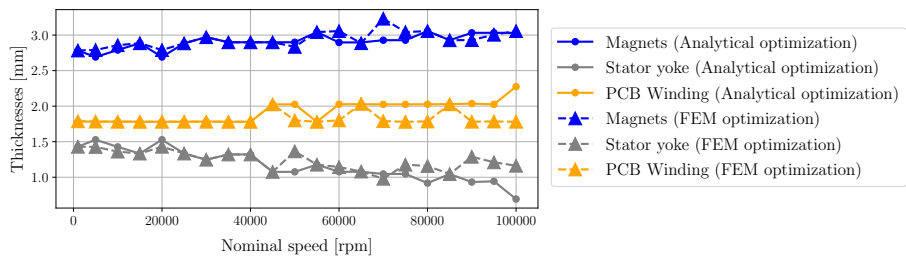


Figure 5.14: Comparison between the optimal thicknesses using analytical and FEM model based optimizations.

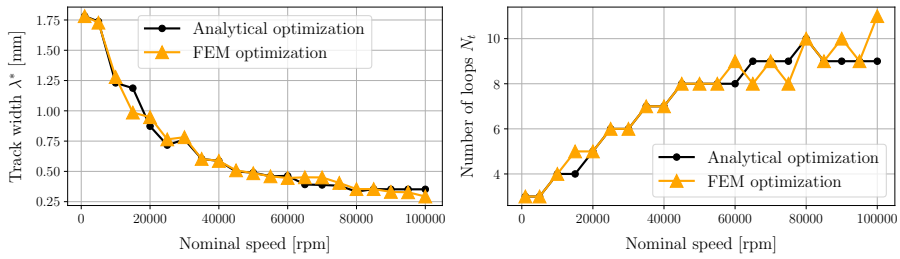


Figure 5.15: Comparison of optimal widths and number of loops between analytical and FEM model based optimizations.

Fig. 5.14 and 5.15 show the optimal internal thicknesses, the largest azimuthal width among the conductor sections λ^* and the number of loops N_t respectively. Both graphs strengthen the suggestion that FEM and analytical models can be used interchangeably.

As illustrated in Fig. 5.15, another problem that makes convergence more difficult is the fact that the number of loops is a discrete parameter. Moreover, since the number of layers is also discrete, it also induces discontinuities in the evolution of the optimal winding thickness.

In view of the tremendous time required for the 3D FEM simulations, the fact that analytical optimizations yield similar results is encouraging news. Although, FEM models are still important to correctly estimate the motor characteristics, the use of analytical models to find the best motor parameters represents a pure gain of 98.5% in computational efforts. Hence, the third part of this section will only focus on the analytical model based optimizations.

5.3.3 Impact of operating speed

Considering the motors optimized with analytical models only, their torque-speed characteristic curves have been computed. These torque-speed curves have been obtained by re-evaluating the motor configuration at many different speeds (using the analytical models), with no further change.

The resulting curves shown in Fig. 5.16 highlight the importance of taking into account the nominal speed when sizing the motor. Indeed, a motor optimized considering a low nominal speed could deliver a continuous torque up to 14 mNm but barely reaches the 30 $krpm$ continuous speed, while a motor optimized at higher speed, even though it is curbed to 10 mNm might be able to run continuously at more than 120 $krpm$. In other words, the standstill torque, on the $\omega = 0$ rpm axis, decreases when the motor is optimized at increasing speed, while the maximum reachable speed, on the $T = 0$ Nm axis, increases.

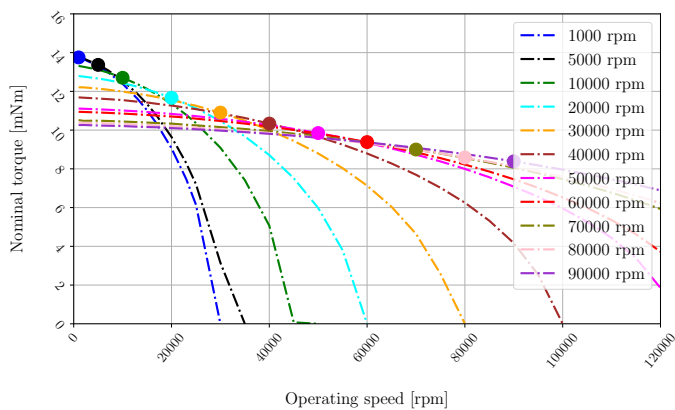


Figure 5.16: Torque-speed characteristic curves (dashed lines) of height motors optimized for a different nominal point (dots).

In Fig. 5.16, the first blue curve at low speed also corresponds to the result that we would obtain if one did not take into account the eddy current losses. This clearly illustrates that optimizing a motor disregarding the speed-dependent losses quickly leads to inadequate solutions in the case of high speed applications.

It can be observed that the characteristic curves tend to drop directly into a rounded shape whereas standard equivalent motors generally have a flatter curve at low speed. This is specific to the use of PCB windings for which eddy current losses appear quickly compared to more traditional windings where Joule losses due to the supply current remain predominant for a wide range of speeds.

Fig. 5.17 shows the distribution of the four types of losses considered in this optimization problem. It shows that the eddy current losses increase with the nominal speed while the supply losses decrease to maintain the total of the dissipated losses constant. Next to that, the friction and iron losses also increase with the speed but they remain limited and almost negligible in comparison of the eddy current losses.

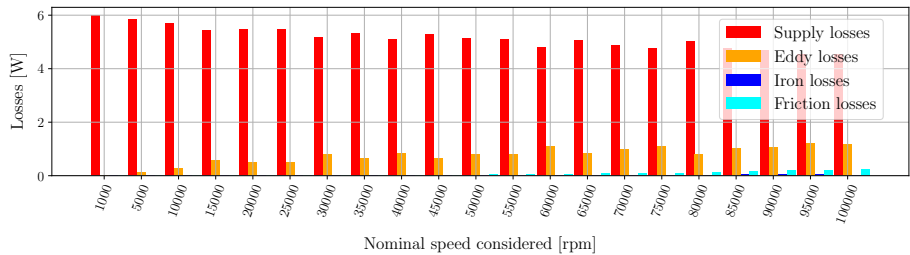


Figure 5.17: Different loss sources within motors, each resulting from an analytical model based optimization, at various nominal speeds.

Fig. 5.18 shows the optimal thicknesses distributions depending on the nominal speed. It highlights that the optimal thickness of the stator yoke progressively decreases with the speed in favor of thicker magnets and windings. The stator yoke is slightly reduced in order to minimize the iron losses. However, when running the optimizations multiple times, it has been observed that there are significant variations in the optimal thicknesses results. This means that the optimal torque is not very sensitive to these thicknesses. Therefore, the strategy consisting in adding the best motor configuration obtained for a given speed, into the initial population of the next optimization problem to be performed, at another speed, proved to be worthwhile.

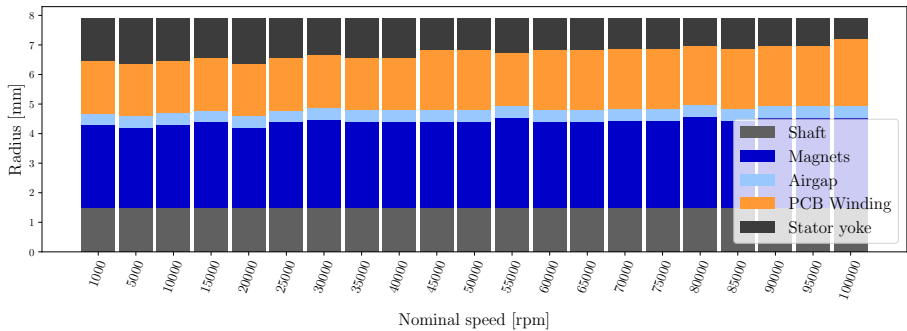


Figure 5.18: Optimal space distributions resulting from analytical optimizations.

It should be noted that iron losses are generally quite difficult to master and can strongly impact the optimal sizing of the motor. As far as the stator yoke is concerned, it has been considered for the sake of this chapter that laminated sheets of *Supra 50* were used which leads to almost no iron losses. However, in some other circumstances with less efficient materials, these losses could have been drastically higher and could have impacted more the stator thickness.

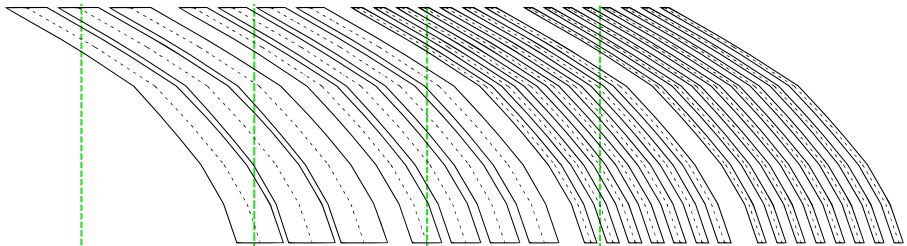


Figure 5.19: From left to right, best configuration of the winding shape at 5 *krpm*, 20 *krpm*, 50 *krpm* and 90 *krpm*.

Fig. 5.19 shows some of the winding shapes resulting from the optimizations. The optimizations tend to generate smooth and round curves. It can be visually observed that the shape of the segments is also quite independent of the speed. This visual observation is also supported by Fig. 5.20 showing the coefficients describing the shape. They all seem to be approximately constant on average.

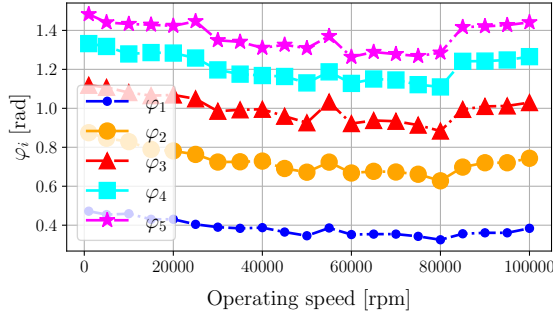


Figure 5.20: Optimal values of the coefficients describing the winding shape.

Fig. 5.19 also highlights that the number of loops increases with speed in order to reduce the conductor widths. Actually, the motor optimized at the smallest nominal speed has the largest track. Indeed, at low speed there are no eddy current losses, and the optimization simply tends to minimize the resistance. On the contrary, at high speed the eddy current losses become critical and it is worth increasing the electrical resistance by decreasing the track width as it allows to strongly reduce the eddy current losses. The evolution of the track width and of the number of loops according to the speed, visible in Fig. 5.15, confirm this observation.

The two variables defining the width of the tracks are λ and N_t . They impose the two constraints ς_λ and ς_{N_t} on the width of the conductor, respectively. Only the smallest is activated to define the actual width of a section. These constraints are applied section per section on a quarter of loop and their values are shown in Fig. 5.21. Section s_1 is the smallest one because it is the most horizontal in the loop.

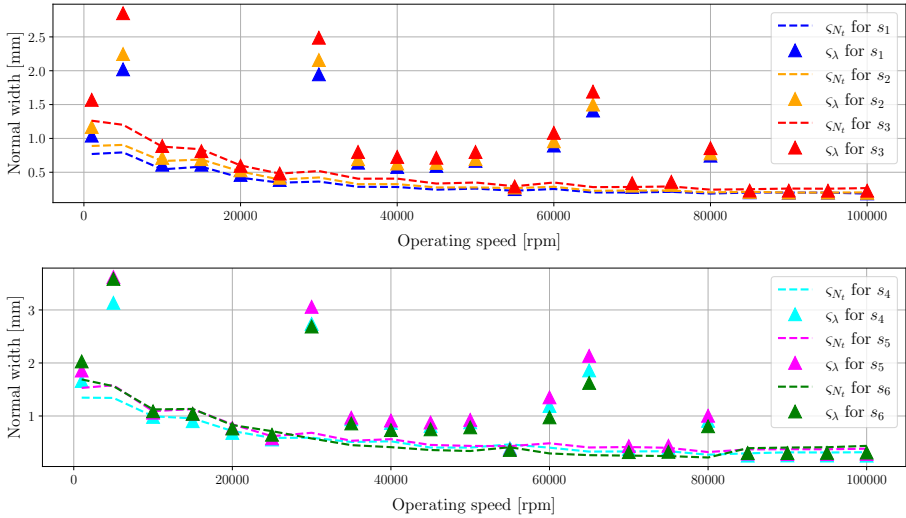


Figure 5.21: Constraints for the width of the sections s_i in a quarter of loop.

It can be observed that the most constraining value alternates between ς_λ and ς_{N_t} , as the speed varies. Actually, ς_λ is only useful when increasing the number of loops would increase the electrical resistance too much and would not be optimal, but a limitation of the width would still be necessary to avoid too much eddy current. When ς_λ is not activated its value has no physical meaning. It can be seen that for high speeds, ς_λ is the only relevant quantity that matters because there is no room anymore for more loops since a minimum space between the tracks has to be kept.

Overall, it seems that the objective function is not very sensitive to the parameters, exception made for the width of the conductor and thus also for the number of loops. To strengthen this idea, the thicknesses and the shape of the winding have been fixed at the values of the optimal configuration obtained at 1 *krpm* and only the track width and the number of loops have been adapted with the speed. Fig. 5.22 compares the torques obtained from these re-evaluations with the analytical models to the optimal torques previously obtained from the optimizations. A drop of only 7% of the torque capability is observed at 100 *krpm*, which seems to confirm the idea that the width of the winding is the most important parameter to consider through λ and N_t , despite the visible convergence issues.

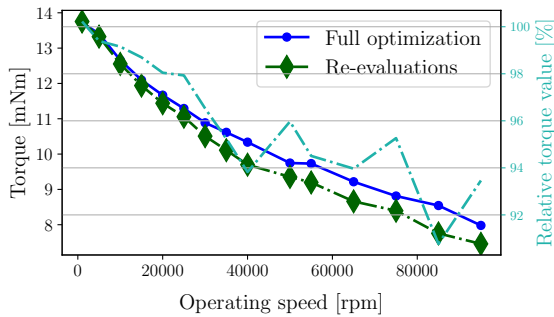


Figure 5.22: Re-evaluation of the torque, considering the internal thicknesses and the winding shape fixed at the values of the best configuration for a 1 *krpm* speed.

Going further the optimization problem has been performed again at all speeds, with only N_t and λ as degrees of freedom. The other parameters have been fixed according to the previously obtained optimal configuration at 1 *krpm*. The optimizations have been carried out using analytical models and the results are shown in Fig. 5.23. It can be observed that up to 60 *krpm*, the resulting motor has a performance at least 99% as good as what would be obtained using a complete optimization.

Fig. 5.24 shows the largest azimuthal width among the optimal conductor sections λ^* and the best number of loops N_t . It proposes a comparison between the results of the full optimizations and the results of the track width optimizations, with only N_t and λ as variables. It appears that the results are pretty close, the main difference being on the number of loops.

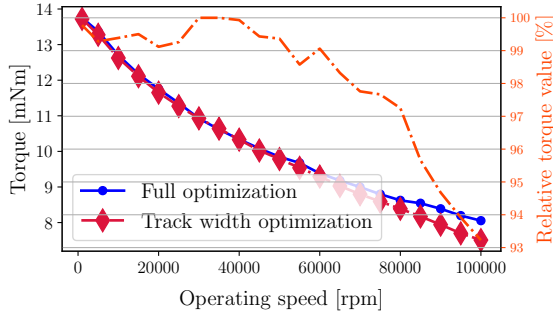


Figure 5.23: Re-optimization of the motors, considering the internal thicknesses and the winding shape fixed at the values of the best configuration for a 1000 rpm speed.

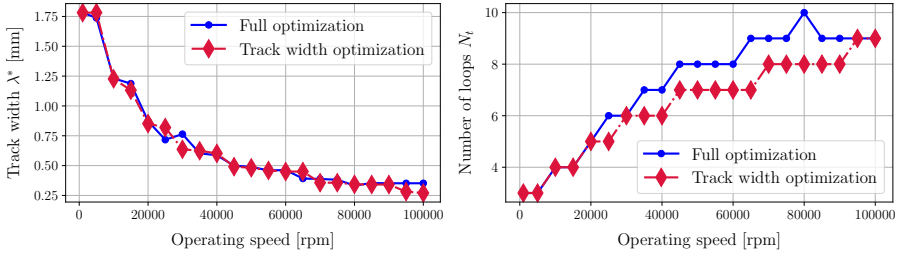


Figure 5.24: Comparison between a full optimization and a track width optimization. The optimal widths (on the left) and the optimal number of loops (on the right).

All these results support the idea that the sizing of the internal motor thicknesses is not as important as a well-chosen copper track width in the context of PCB windings. From an industrial point of view, this idea is quite interesting since it means that a manufacturer can propose only one motor structure and simply adapt the winding according to the desired operating point, while remaining competitive. In practice, most of them do so and we can therefore conclude that these results confirm that this is indeed a good practice and especially that it can also be applied to the flex-PCB technology.

5.4 Conclusions

This chapter optimized a motor for various nominal speeds, and in different ways using either FEM models or analytical models.

The problem was made independent from the thermal conditions by assuming that all motors of the same size would dissipate the same constant amount of power. To run these optimizations, models from chapters 2 and 4 were used to estimate the eddy current losses. Models for assessing the torque constant and the electrical resistance were also presented. For each, an analytical and a FEM version of the models were proposed.

These optimizations resulted in three main outcomes. The first is that analytical models are quite sufficient to optimize a slotless PM machine from an electromagnetic point of view. The second is the importance of taking eddy current losses into consideration when designing such a motor, otherwise, they will prevent the motor from going at high speeds. The third is that one of the most important parameters to adjust when the targeted speed changes is the width of the conductive tracks. This adaptation goes hand in hand with a suitable number of winding turns. Consequently, it appears that a motor fully optimized at low speed is nearly optimal at high speed by simply changing the winding. This result elegantly re-demonstrates what is already standard practice in the industry.

In this chapter, the focus has been made on reducing the losses by considering that all the conductors in a winding phase are connected in series. This implies that the resulting torque constant and the electrical phase resistance are the highest possible, for a given motor. In addition, we observed that when optimizing the motor at high nominal speed, the number of loops N_t increases to reduce the width of the conductors, which leads to even larger torque constants. Consequently, to drive a motor at high speed when the latter already has a large torque constant k_t , a power supply with a very high voltage U is required. However, it is not always possible to have this kind of power supply and the motor should be optimized while having the constraints of the external electronics in mind. This topic will be addressed in the next chapter in order to propose a solution.

Publications related to this chapter:

- G. François, F. Baudart and B. Dehez, *Analytical Estimation of Eddy Current Losses in PCB Winding for the Optimal Sizing of PM Slotless Motor*, 2019 IEEE International Electric Machines & Drives Conference (IEMDC), 2019, pp. 862-869.
- G. François, F. Baudart and B. Dehez, *Influence of eddy current losses in the optimization of linear coreless BLDC motors with PCB windings*, 2019 12th International Symposium on Linear Drives for Industry Applications (LDIA), 2019, pp. 1-6.

Strategies for reducing eddy current losses

6

The previous chapter brought forward a complete motor optimization to evaluate the impact of the eddy current losses and showed that winding design is a major aspect of loss reduction. It was determined that the width of the copper tracks and their number of loops were almost exclusively the elements that varied with speed when optimizing a motor. Consequently, this chapter will only focus on the winding. As the speed increases, it makes sense to replace the large tracks by multiple thinner ones because the eddy current losses are proportional to the square of the track width, considering the conductor volume constant [121]. These smaller tracks were connected in series in the previous chapter and lead to very high nominal voltages which might not be handled by the power electronics.

Another option consists in connecting these smaller tracks in parallel in order to reduce the torque constant and thus the required supply voltage. However, this option can lead to recirculating currents due to a non-balanced back emf in the conductors [122]. Some authors already studied the impact of parallel connections in the case of traditional wiring [123]. Others already dealt with this problem in the case of PCBs, but only in planar transformers [124] or in axial flux motors when the superimposed layers see a field's amplitude variation [125]. In [126] they studied multi-conductors connected in parallel, either horizontally on the same layer of the PCB or vertically between the top and bottom layers.

This chapter aims to extend the kind of work done in [126] to the optimized windings proposed in chapter 5 to get rid of the need for high voltage supply. In this context, starting from a large conductor path of any shape, it is proposed to handle the eddy current losses in an original way that consists in cutting slits along this conductor to reduce its effective width. This cut creates thinner and locally parallelized conductors, which is shown to be really efficient to tackle the problem. Therefore, the traditional parallelization of thinner conductors is considered as a particular case in this approach, where the slits become so long that they separate the track into distinct sub-tracks connected in parallel to the terminals of the winding.

The addition of slits in the conductor also comes with an increase of the electrical resistance, and therefore the supply losses. The impact of slitting possibilities is studied focusing on the trade-off between the supply losses and the eddy current losses, since they are the two most important inside the winding.

This chapter starts by defining the performance criteria used to compare windings and how the modeling of the losses is achieved. Then, parametric analyses are performed to evaluate the impact of different slitting configurations. An experimental validation is proposed to strengthen some of the conclusions in the case of the single slit study. Finally, an example of how to optimize a winding with slitting is given.

6.1 Performance criteria

In order to get the most out of the motor thermal capabilities, a tradeoff is sought between the supply losses and the eddy current losses, so that the sum of the two is minimal. To remove the dependence on nominal torque and speed from the analysis, normalized values were established:

$$\alpha = P_{supply}/T^2 \quad (6.1)$$

$$\beta = P_{eddy}/\omega^2 \quad (6.2)$$

with T the torque and ω the speed of the motor. Both α and β are now unique for a single motor design, regardless of the working point.

On the one hand, if we develop α , we have:

$$\alpha = \frac{3R_{ph}I^2}{2T^2} = \frac{3R_{ph}I^2}{2k_t^2I^2} = \frac{3R_{ph}}{2k_t^2} \quad (6.3)$$

which only depends on the electrical phase resistance R_{ph} and the torque constant k_t , both being intrinsic to the motor.

On the other hand, chapter 4, showed that:

$$P_{eddy} = \sigma gh\omega^2 b_{z0}^2 \lambda^3 g_i(h/\tau) = \beta\omega^2 \quad (6.4)$$

with β assumed to be independent from the speed as long as inductive phenomena are negligible and clearly independent from the supply current and thus from T .

Together, the coefficients α and β make the link between a working point (T, ω) and the losses generated in the winding $P_{winding}$.

$$P_{winding} = \alpha T^2 + \beta \omega^2 \quad (6.5)$$

When comparing two motors, the one with the lowest values for both α and β would be the better one, as the sum of supply and eddy current losses would be the lowest, regardless of the working point. If one of these two motors has a lower α but

a higher β , or conversely, the better one would depend on the working point. Under these circumstances, a threshold is defined:

$$\gamma = \frac{\beta_B - \beta_A}{\alpha_B - \alpha_A} \quad (6.6)$$

The motor with the lowest α has the lowest sum of supply and eddy current losses for all working points such that $T^2/\omega^2 > -\gamma$ and conversely. Such a comparison is shown in Fig. 6.1 with two motors arbitrarily chosen for the example. One is more sensitive to the speed while the other is more sensitive to the torque.

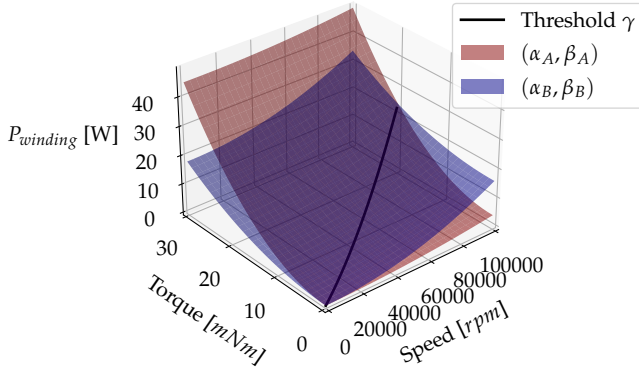


Figure 6.1: Winding losses of two different motors as an example, where none dominates the other throughout all the working points.

6.2 Study case & Modeling

To contextualize the study, a reference winding shown in Fig. 6.2 is considered. From an actual winding, one phase is unrolled, and the focus is on the total space available for that phase to re-arrange the copper tracks and connections into it. For the sake of simplicity, and thanks to symmetries and repetitions, we only focus on four segments of this winding. Additionally, the spiraled effect is neglected, and a constant radius of 6 mm is assumed. The width of the segments is 2 mm and their height is 40 mm.

The interested is only in a relative comparison of the loss reduction. Therefore, the source magnetic field is not computed from a motor configuration but imposed directly and arbitrarily with an amplitude of 1 T and without variation along the winding height. Practically, the source term for the eddy current losses can be written

$$\mathbf{a}_0(x, y) = -yB_0 \cos(x\pi/\tau - \omega t) \hat{e}_x \quad (6.7)$$

with $\tau = 3\pi$ and $B_0 = 1$ T, without any loss of generality.

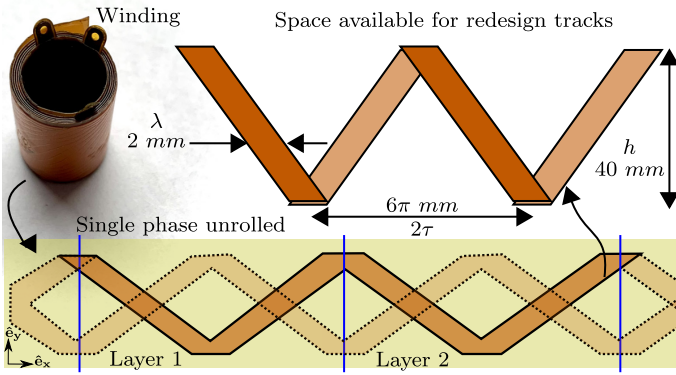


Figure 6.2: From an actual complete winding to a set of four reference segments.

To obtain the coefficients α and β , the three intermediate values R_{ph} , P_{eddy} and k_t must be computed. R_{ph} and P_{eddy} are obtained using the 2D FEM models described in 5.2.1.b and 2.3.1 respectively. The torque constant k_t is obtained by integration of (6.7) as explained in 5.2.2.

The domain Ω_C of the studied conductor is shown in Fig. 6.3. This elementary track is composed of two segments on the top layer and two segments on the bottom layer. There is an input Γ_{input} and an output Γ_{output} for the supply current. At the places where connections are to be made between the top and bottom layers to link the segments together, periodic conditions are applied which recopy the electric scalar potential from one layer to another. Finally, the shape of these segments can be straight, or curved as shown in Fig. 6.4.

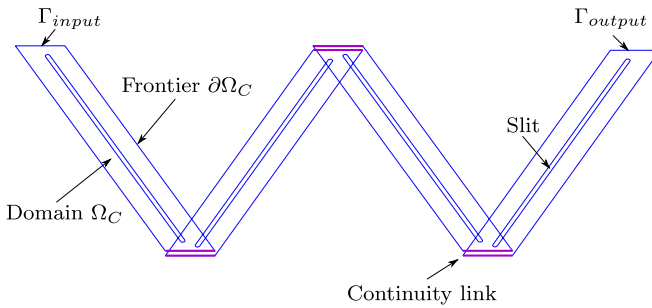


Figure 6.3: The 2D domain of a conductor with α and β as unknowns.

The Ω_C domain is now a multi-connected domain and therefore different from that of previous chapters, due to the holes generated by the slits. Hence, as proposed in [127], the magnetic vector potential $A - v$ formulation is used to solve eddy currents in this thesis. As explained in [128], although the use of $A - v$ is not the

best solution for reducing the number of degrees of freedom, it is the most straightforward to implement with multiply-connected regions. Indeed, if one uses a scalar potential formulation with multiply-connected regions, one usually has to deal with cuts in the mesh as explained in [129] for example. Though, today, new approaches allow to obtain the best of both worlds with the use of scalar potentials without the need for cuts [130]. However, this best of both worlds is not yet widely available and the cohomology concepts required to implement it are beyond the scope of this work.

However, it is not yet very democratized and the concepts in cohomology necessary to the good understanding of the topic are still out of the scope of the author's knowledge at the time of writing.

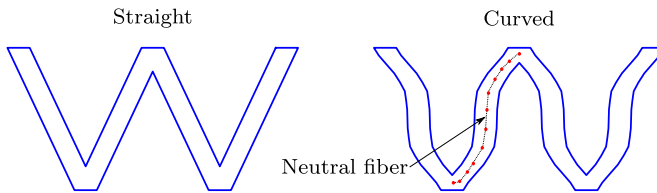


Figure 6.4: Examples of shapes for the elementary track without slits.

Note that the simulations can be performed with the top and bottom layers considered either as two different planes connected through the periodic links as in Fig. 6.3 or merged into a single conducting plane as in Fig. 6.4. The first case allows to get closer to reality and to model the superimposed surfaces correctly. The second case eliminates the need for connections between layers, which are arbitrarily chosen, leading to numerical discrepancies, and allows to focus more on the impact of the slit pattern only.

6.3 Single slit

6.3.1 Slit description

In this case slits are cut along the neutral fiber of the segments. The four slit patterns shown in Fig. 6.5 have been studied. The *simple slit* pattern is characterized by one slit per segment whose center is at mid height. The *double slit* pattern is made of two slits per segment, whose centers are at 1/4 and 3/4 of the height, respectively. The *long slit* pattern consists of a single slit covering the entire length of the winding, and which splits it in two tracks connected at the terminals. The *V slit* pattern consists of a slit spanning over two consecutive segments in a symmetric way.

The normal width of the slit, denoted ς_s , is a parameter but with a minimum of $200\text{ }\mu\text{m}$, which corresponds to the limits recommended by most PCB manufacturers. The slit length is measured by the coverage ξ , defined as the ratio between the total length of all the slits and the total length of the neutral fiber of the winding.

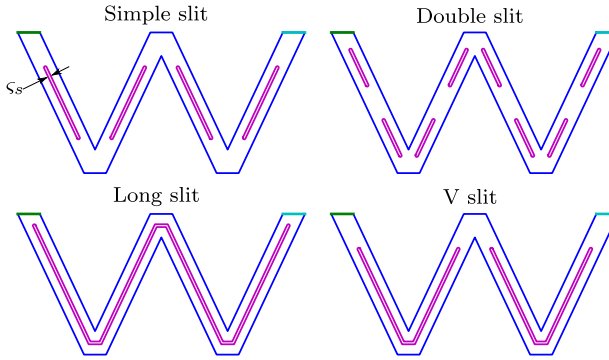


Figure 6.5: Description of the four patterns of slits studied in this paper.

For the *V slit* and the *long slit* patterns, since the slits extend over consecutive segments, two different types of interconnection between the upstream and downstream slitted segments are considered. They are illustrated in Fig. 6.6. In the *parallel* connection, the upstream and downstream slitted segments reverse their position, while in the *crossed* connection they keep their relative position.



Figure 6.6: Illustration of the *parallel* and *crossed* connections. They only apply to the *long* and the *V slits*.

6.3.2 Parametric analysis

Fig. 6.7 and 6.8 show the evolution of α and β coefficients for a straight and for a curved segment, respectively. As can be observed on the graphs, the classification between the slit configurations remains quite the same regardless of the segment shape. The same observation can be made with many different shapes even though only two of them are plotted here. Therefore, it is assumed that impact of the slit configuration can be discussed for the specific case of straight segments, without loss of generality.

Regardless of the winding shape, the *long slit* pattern with *parallel* connections appears to be the best option to reduce the β factor, related to eddy current losses, while keeping the α factor, linked to supply losses, as low as possible. However, this *long slit* pattern is not efficient if it is used with *crossed* connections. In this case, there is a small increase of α because, intrinsically, this type of connection has a longer current path. Most importantly, the reduction of β is nearly half-canceled due to the appearance of significant recirculating currents. These occur in case of *crossed*

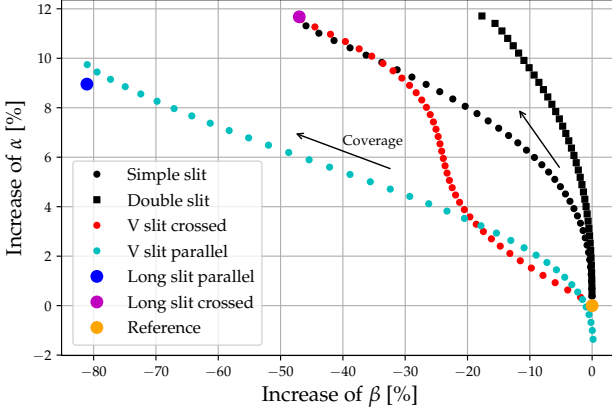


Figure 6.7: Evolution of the α , β coefficients compared to the reference conductor without slits in the case of straight segments.

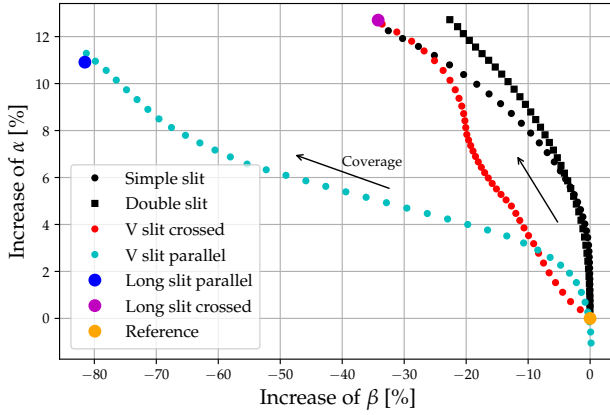


Figure 6.8: Evolution of the α , β coefficients compared to the reference conductor without slits in the case of curved segments. The shape used is the one in Fig. 6.4.

connections because the slitted segments connected in series along the winding are systematically on the same side relative to the direction of movement. As a consequence, the two sub-tracks connected in parallel at the winding terminals are subject to the same back emf amplitude, but with a phase shift given by:

$$\frac{(\lambda + \zeta'_s)/2}{r_{mean}} \quad (6.8)$$

with $\zeta'_s = \zeta_s \frac{h/\tau}{\sqrt{1 + (h/\tau)^2}}$ the azimuthal width of the slit. For the parallel connection,

the slitted segments (connected in series all along the winding) are alternatively on one side and the other relative to the direction of movement (of the PM). As a consequence, both sub-tracks that are connected in parallel at the winding terminals are subject to a back emf having the same phase.

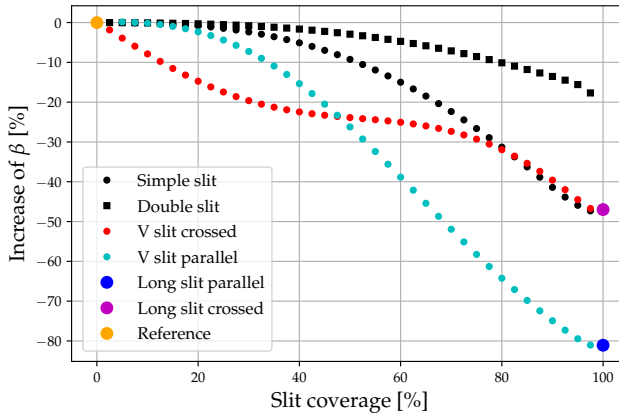


Figure 6.9: Evolution of β and consequently of eddy current losses, compared to the reference track without slits according to the total coverage of the slits.

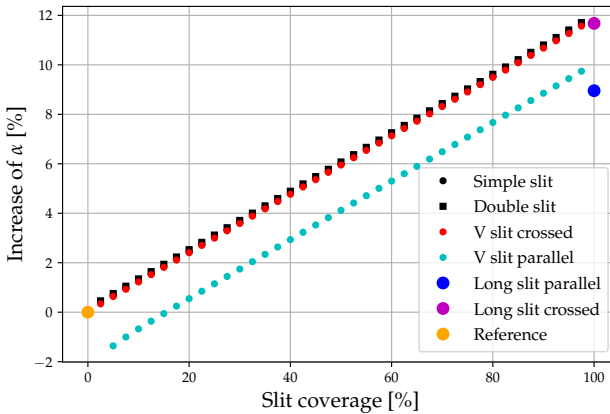


Figure 6.10: Evolution of α and consequently of the phase resistance, compared to the reference track without slits according to the total coverage ξ of the slits.

Compared to the *double slit* pattern, the *simple slit* pattern gives better results. Fig. 6.9 and 6.10 reveal that, for the same slit coverage, α is actually unchanged while the β is worse using the *double slit* pattern. Actually, recirculating currents are more important in the case of the *double slit* pattern.

For the same reasons as for the *long slit* pattern, the *V slit* pattern gives quite different results depending on the type of connection, the best being obtained with a

parallel connection. For this specific configuration, Fig. 6.7, 6.8 and 6.10 reveal that a slight decrease of the factor α is obtained for a slit coverage close to 0%. However, this gain is purely a numerical artifact due to the way the interconnections between the upper and lower segments of the PCB are accounted for in the model, forcing the current lines to follow a slightly longer path for the reference winding.

Fig. 6.7, 6.8 and 6.9 also show that, for a slit coverage close to 100%, the *V slit* pattern with parallel connection reach the same decrease of the β factor than the *long slit* pattern, but at the expense of a larger increase of the α factor. This difference in the α factor is also due to a numerical artifact.

To get convinced, simulations on the *V slit* pattern with *parallel* connection and the *long slit* pattern have been done considering the top and bottom layers merged into a single conducting plane. It is also considered that the slit would follow exactly the same path as for the *long slit* pattern, as represented in Fig. 6.11.

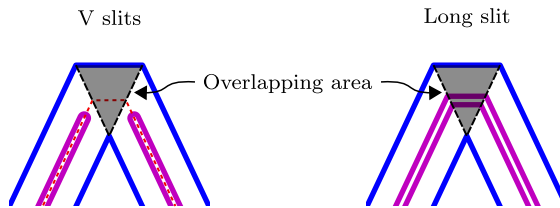


Figure 6.11: The overlapping area where the *long slit* turns back serves as a limit for the *V slits*. If the slit enters in this triangle, the place for vias reduces.

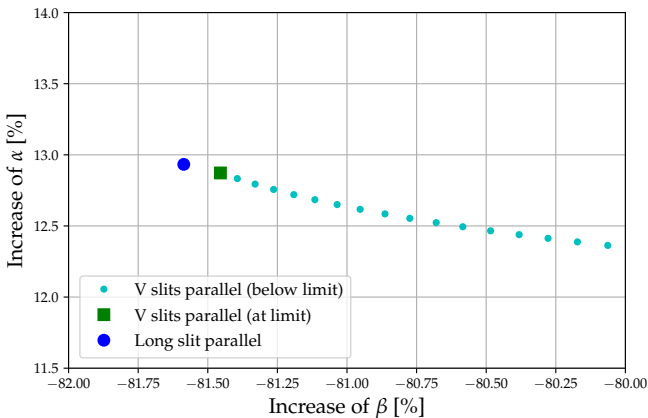


Figure 6.12: Comparison between a *long slit* pattern and *V slit* patterns with a coverage that varies up to the limit of the overlapping area.

The results given in Fig. 6.12 show that the performance of the *V slit* pattern tends to that of the *long slit* pattern, what could have been expected.

Both patterns, the *long slit* and the *V slit* show similar performance when the coverage is close to 100 %. So, if the slits of the *V slit* pattern are terminated at the limit of the overlapping area between the upper and lower layer conductors, β is only 0.13% higher than for the long slit pattern. But this small increase in β comes with a significant practical advantage over the *long slit* pattern. This is because the technological constraints on the minimum size of vias used to connect the two layers of a PCB limit the minimum width of the segments that must be connected by means of these vias. In the case of the *long slit* pattern, this minimum width applies independently to each of the two conductors, upstream and downstream, whereas in the case of the *V slit* pattern, it applies to both conductors together. In the end, it will therefore be possible to design windings with a greater number of conductors of narrower width, and therefore more suitable for very high speeds with this type of slit pattern.

Actually, this theoretical value of 0.13% observed when the two planes are merged turns into a difference of 1.5% when the top and the bottom layers are considered as two separated planes with more realistic connections. Fortunately, this is still a little difference compared to the practical benefit of the *V slit* pattern.

The larger the overlapping area, the greater the difference between the *V slit* and *long slit* patterns. Yet, as the main benefit of using a *V slit* pattern is the ability to place larger vias, this is not really a problem if it performs less well with large overlapping areas. Indeed, if the area is so large that the difference in β values is high, then it can be assumed that there is enough place to put vias and still implement a *long slit* pattern with a greater number of conductors.

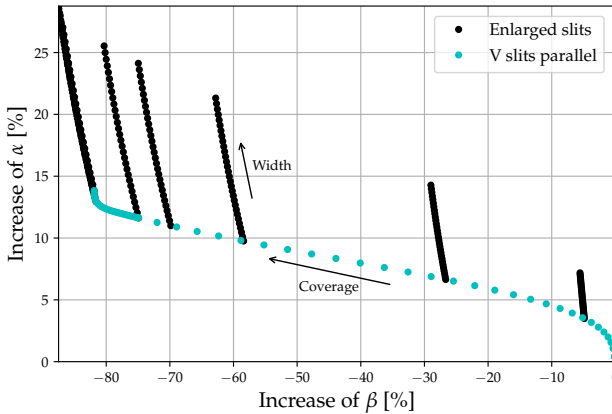


Figure 6.13: Evolution of α and β when the slit width is enlarged (black points).

Finally, Fig. 6.13 shows the impact of increasing the slit thickness ζ_s , for various values of slit coverage. It always seems preferable to increase the length of a cut while keeping it the thinnest as possible to avoid increasing α too much, and thus the supply losses.

6.3.3 Experimental

In order to validate the observations made in the previous section, ten windings with the various patterns were designed. For practical considerations they have a slightly different sizing. Their useful height is 18 mm , their inner radius is 5 mm and their outer radius is 8.65 mm . The copper thickness of the tracks is $80\text{ }\mu\text{m}$ and there are 5 layers in total. Except otherwise mentioned the slits have a width of $300\text{ }\mu\text{m}$. The configurations of these windings are listed below.

Winding A is the reference winding. It is composed of straight segments of constant width and connected through vias. Their width is set to 4 mm in order to obtain a maximum of eddy currents and to enlarge the range of measurements. Two groups of segments were considered for each of the three phases. It means that there are 6 independent tracks on the winding, each of them being composed of one segment on the top and one segment on the bottom, per layer. It leads to a total of 60 segments. The passage of the current from the top layer to the bottom layer is done through three vias arranged to suit all the slit patterns studied, as shown in Fig. 6.14.

Winding B is the reference winding with a *simple slit* pattern drawn on each of the segments. The slits have a coverage of 80% of the height.

Winding C is the reference winding with a *double slit* pattern drawn on each of the segments. The total coverage of the slits is also 80% of the height, so the total cut out area is exactly the same as for the winding B.

Winding D is the reference winding with a *V slit* pattern implementing *parallel* connections. The slits stop just at the limit of the overlapping areas.

Winding E is the reference winding with a *V slit* pattern implementing *parallel* connections. The slits have a lower coverage than in the D winding because they only cover 70% of the height.

Winding F is the reference winding with a *V slit* pattern implementing *crossed* connections. The slits stop just at the limit of the overlapping areas.

Windings G, H and I are similar to the reference winding with *long slit* patterns implementing *parallel* connections. The widths of these long slits are $300\text{ }\mu\text{m}$, $345\text{ }\mu\text{m}$ and $390\text{ }\mu\text{m}$, respectively.

Windings J is the reference winding with a *long slit* pattern implementing *crossed* connections.

All of these winding types are visible in Fig. 6.14. Three windings of each type have been fabricated and placed in the speed decay test bench described in section 3.2 to obtain the eddy current losses. The electrical resistance has been measured with an RLC-meter on the remaining unrolled PCBs. Three measurements of the resistance have been achieved for each of the stator types. The measurements

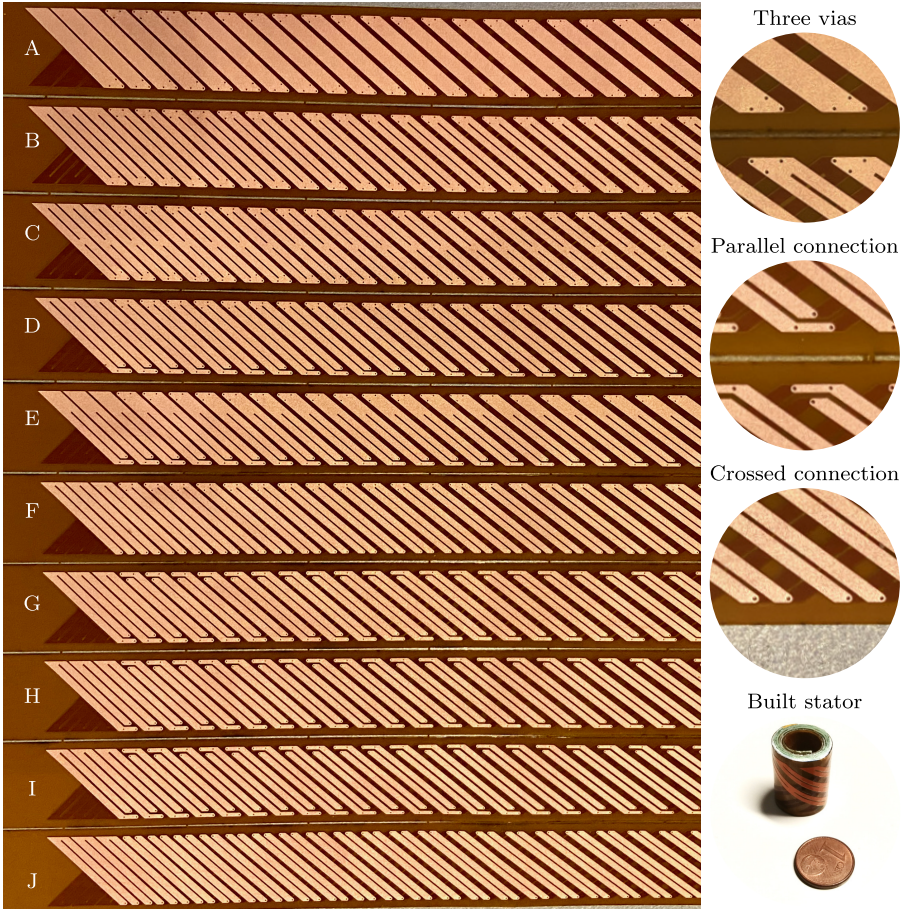


Figure 6.14: Experimental windings to observe the impact of slits patterns.

are compared to the simulations in Fig. 6.15 where each color is associated with the same type of winding. Each measurement of the eddy current losses corresponds to a different rolled physical winding and electrical resistance measurements were made on a fourth unrolled winding. The relative evolution of the electrical resistances and of the eddy current losses corresponds to the relative evolution of α and β respectively and so they are plotted along these two axes. The comparison is made between the simulations points and the envelopes defined by the nine pairs $\alpha - \beta$ when reporting the measures of each stator type.

Despite a significant uncertainty, especially on the resistance, it can be observed that the measurements match the simulations quite well. This again supports the validity of the 2D FEM model of eddy currents presented in the section 2.3.1. Let us point out that this second experimental validation shows that the model also works for multiply-connected domains. More importantly, the relative classification of the various slit patterns is verified.

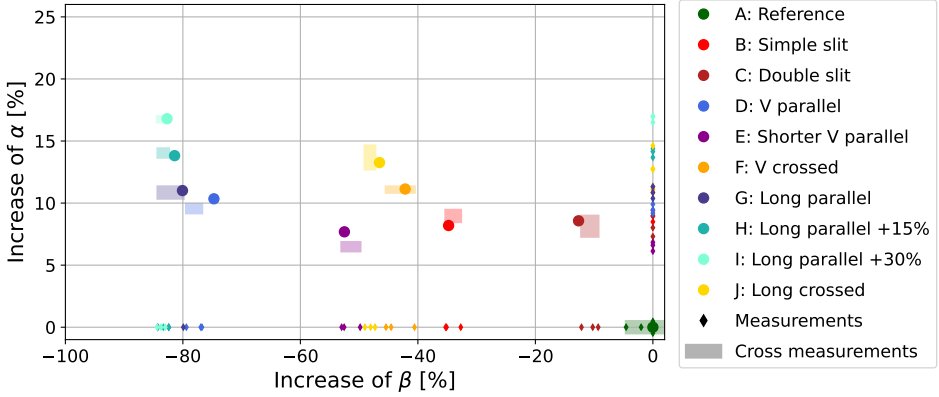


Figure 6.15: Comparison between the expected α and β obtained with FEM simulations and the measurements of the various prototypes.

In Fig. 6.15, it can be observed that the reduction of β and thus of the eddy current losses is the most important for *long slit* and *V slit* patterns with *parallel* connections (D,G,H,I). As mentioned previously, this is explained by the fact that *parallel* connections prevent recirculating currents more than the *crossed* connections (F,J). Fig. 6.15 also shows the superiority of the *simple slit* (B) pattern over the *double slit* pattern (C) which can be explained by a geometry less favorable to recirculating currents.

6.4 Multi-slits

6.4.1 Slit description

In this section, several slits are made in parallel and equally spaced across the width of the segment, as illustrated in Fig. 6.16.

Only the *long slit* and the *V slit* patterns with *parallel* connections are considered in what follows as they proved to be the most promising configurations. In order to avoid the numerical artifacts observed in the previous section for slit coverage close to 100%, the winding has been modeled as a single domain, as explained in the section 6.2.

As in the case of a single slit the *long slit* pattern can be seen as a particular *V slit* pattern with 100% coverage. In addition, the interest of using *V slit* patterns is again to be able to save space at the ends of the segment and not to be limited by the minimum size of the vias. In the study, all *V slit* patterns were considered to be arranged side by side in the same direction, but the direction of these V shapes could have been alternated to distribute the space gained between the upper and lower ends of the segments.

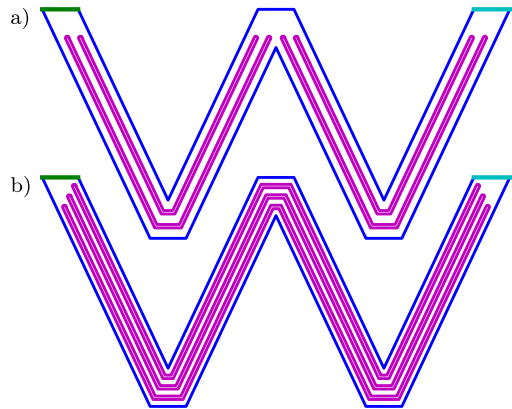


Figure 6.16: Multiple slits on a track: a) Two *V slit* patterns; b) Three *long slit* patterns.

6.4.2 Parametric analysis

Fig. 6.17 shows the evolution of α and β when cutting one, two or three slits along the tracks. The cyan points represent *V slit* patterns with a minimal slit width of $200\ \mu\text{m}$ and progressively increasing coverage up to 100%. The blue points represent the cases with one, two or three slits cut according to a *long slit* pattern and also with a minimal slit width of $200\ \mu\text{m}$. Finally, starting from the blue points, the red points are the same windings with *long slit* patterns but with enlarged slits.

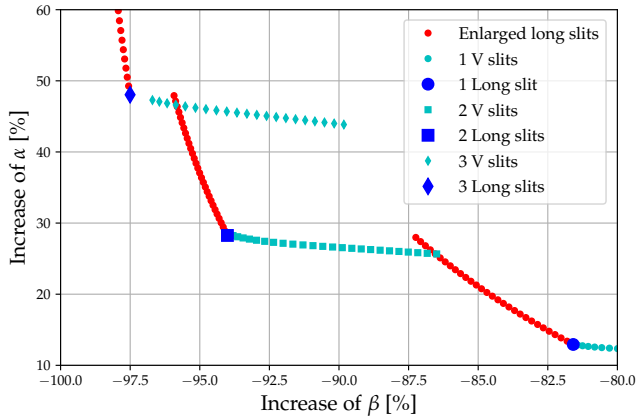


Figure 6.17: Evolution of α , β when using multiple slits and varying their width.

It can be seen in Fig. 6.17 that by increasing the number of side-by-side slits on a track, and by operating on their width, a larger range of α and β factors can be covered compared to a single slit. It is also observed that there is a step in both α and β factors when there is a change in the number of slits in parallel. Finally, Fig. 6.17 illustrates that there are several combinations of slit coverage and slit width giving the same α and β factors.

6.5 Optimal slit configuration

As highlighted in the previous section, varying the pattern, number, thickness and length of the slits results in different values of α and β . However, there is no slit configuration that minimizes both factors simultaneously. Otherwise, there would have been a single winding configuration minimizing both supply and eddy current losses whatever the operating conditions. As a consequence, the question is to determine which slit configuration to choose, according to the working point for which the motor should be optimized. To answer this question, we must first note that, for the same working point (T, ω) , two slit configurations A and B will have the same losses if:

$$\left(\frac{T}{\omega}\right)^2 = -\frac{\beta_B - \beta_A}{\alpha_B - \alpha_A} \quad (6.9)$$

By extension, all the slit configurations whose α and β factors lie on a straight line with an angular coefficient of $1/\gamma$ will result in the same losses for a motor whose working point is such:

$$\left(\frac{T}{\omega}\right)^2 = -\gamma \quad (6.10)$$

In the end, the slit configuration that minimizes the losses for a given working point is the one that will position a line of angular coefficient $-\left(\frac{\omega}{T}\right)^2$ as close as possible to the origin of the $\alpha - \beta$ plane.

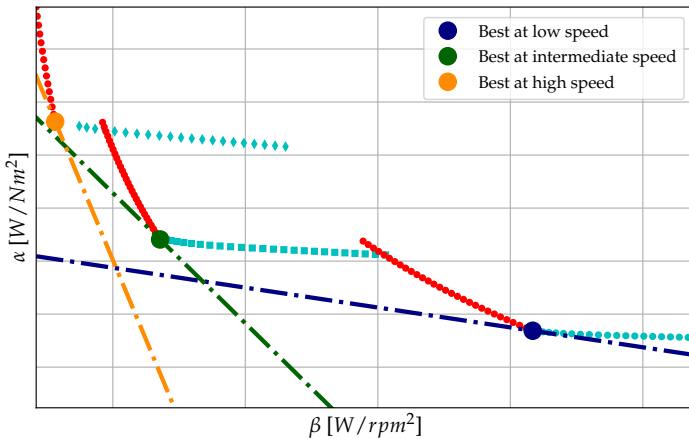


Figure 6.18: Search for the optimal slit configuration.

On this basis, as illustrated in Fig. 6.18, for motors with different working points, it can be observed that it is always better to choose for the *long slit* patterns or *V slit* patterns, but with a coverage approaching 100%, and to add a new slit with the smallest possible width when it is technically feasible, rather than trying to enlarge

an existing slit. As a matter of fact, the thinner the slit, the smaller the impact on resistance. However, considering it would technically not be possible to properly cut more than four parallel slits along the conductor, enlarging the three existing slits would be a good solution for very high speed motors to further reduce the β , but at the expense of a significant increase of α .

Beyond that, it is also important to point out that a configuration characterized by a number of slits i , a minimal thickness and coverage close to 100% in the case of the *V slit* pattern, is optimal for a range of working points such as:

$$-\frac{1}{\gamma_{i,1-i}} \leq \left(\frac{\omega}{T}\right)^2 \leq -\frac{1}{\gamma_{i,i+1}} \quad (6.11)$$

where $\gamma_{i,j}$ is the angular coefficient of the line passing through the points (α, β) of the configuration i and j .

Considering the example of Fig. 6.18, the absolute values of $\gamma_{i,j}$ are of no real interest since we have modeled only four segments of a winding. But their ratios indicate the relative range of operating points we could expect by implementing these patterns in motor windings. It has been computed that:

$$\sqrt{\gamma_{1,0}/\gamma_{1,2}} = 2.83 \quad \sqrt{\gamma_{2,1}/\gamma_{2,3}} = 2.14 \quad (6.12)$$

This means that for a given speed, the selected *low speed* configuration is ideal to produce a torque ranging from simple to almost triple and the selected *intermediate speed* configuration is ideal to produce a torque ranging from simple to double.

6.6 Conclusions

This chapter studied the impact of cutting slits along the winding conductive tracks to reduce the eddy current losses in the PCB winding. This approach is original because it brings the parallelization of conductors at the local scale.

In order to evaluate the impact of slitting, the motors were compared using the intrinsic parameters α and β . They correspond to kind of normalized values of supply losses and eddy current losses, respectively. These parameters directly link between a working point (T, ω) and the total losses that will be generated in the winding.

Among the several slit patterns considered, two stood out, the *V slit* and the *long slit*. The *V slit* pattern consists of a slit spanning over two consecutive segments in a symmetric way. When the slit coverage reaches 100%, it splits the tracks into two sub-tracks connected to the terminals and this becomes what is called a *long slit* pattern, which is equivalent to a traditional parallelization of the winding. Both patterns show almost similar performances when the coverage is close to 100%. But the *V slit* pattern comes with the practical benefit of being less constrained by the minimum size of the vias used to connect the two layers of the PCB.

Two different types of interconnection between upstream and downstream slitted segments were considered. The *crossed* connection where the upstream and downstream segments cross at the ends to maintain their relative position along the track, and the *parallel* connection where the two segments are always parallel so that they alternate their relative position. It was observed that the *parallel* connection was always better because it allows to equilibrate the back-emf in the segments and reduces recirculating currents.

It was shown that the slit configuration that minimizes the losses for a given working point lies on the line with angular coefficient $-(\omega/T)^2$ and as close as possible to the origin of the $\alpha - \beta$ plane. The study carried out on the impact of the number of parallel slits and their width has shown that it has always appeared preferable, when technically feasible, to increase the number of slits rather than enlarge the existing ones.

Finally, it was observed that the optimal configurations do not only apply to single working points but to ranges of values, making them stable optima.

Publication related to this chapter:

- G. François and B. Dehez, *Impact of slit configurations on eddy current and Joule losses in PCB windings of PM machines*, 2021 IEEE International Electric Machines & Drives Conference (IEMDC), 2021, pp. 1-7.

Conclusion

Content Overview

This thesis implemented models to predict eddy current losses in slotless permanent magnet machines with PCB windings. It showed that these losses are important to consider in the design phase, and suggested strategies to limit them. Two models were developed, one focusing on accuracy and the ability to take into account conductor shapes and topologies, the other focusing on computational speed.

Chapter 2 started from the Maxwell's equations to study the eddy current problem with 3D FEM simulations. The formulation derived in this thesis is based on the classical Av formulation which has the advantage of remaining straightforward when used with multiple-connected domains. From this standard formulation, simplifications were made to obtain, on the one hand, a decoupling between the problem of eddy current losses and the knowledge of the magnetic field in the airgap, and on the other hand, a simpler 2D FEM formulation for eddy currents. A large part of chapter 2 was therefore allocated to validate the relevance of these simplifications. It came down to the conclusion that once the component of the airgap magnetic field which is normal to the PCB surface is known, the eddy current problem can be solved in a few seconds using the suggested 2D model. A particularity of this model is also that unlike most papers in the literature, in which the section of the motor is modeled, the plane considered for the simulation is the surface of the PCB. Hence, this model allows to evaluate the impact of shapes, variations of the track width and the skewing of the conductors. Finally, chapter 2 proposes a way to transform the results of a 3D FEM magnetostatics simulation into an analytical expression which can be used as a source term in the 2D model of eddy currents.

Chapter 3 described the experimental tests realized to validate the FEM model proposed in chapter 2. Eddy current losses have been obtained for different prototypes of windings, by measuring the speed decay of a PM rotor. The main advantage of this kind of measurement is that the accuracy of the results is directly related to a good knowledge of the rotor inertia, which is relatively easy to have. Furthermore, it does not require to go at high speed to get accurate results, which makes things even easier. The measurements were mostly in agreement with

theoretical expectations. Next to that, the chapter was also devoted to evaluating the sensitivity of the results to the problem parameters. It was shown that variations in the shape of the winding and in the radial position of the winding with respect to the rotor can lead to significant variations in the losses. However, they are not such a concern because they can usually be set and measured accurately. It was also shown that the temperature can have a significant impact on the results. Though in this thesis, we got rid of its influence by performing the experiments at low speed and thus with low heat dissipation. Finally, the conclusion was that the most critical issue to properly estimate the eddy current losses was to have a thorough knowledge of the airgap magnetic field.

Chapter 4 aimed to derive an analytical expression to estimate the eddy current losses as quickly as possible. The purpose of this expression is either to be used on the fly without any specific FEM code or especially to be used in an optimization routine. Indeed, in the context of an optimization, numerous motors have to be evaluated and the computational efforts needed for each evaluation become a critical aspect to consider. Two ways to obtain this analytical expression were proposed. The first suggestion was to use the Π -theorem, on the data obtained by running a sweep analysis with the initial 3D FEM model. The second was to solve equivalent circuits of the copper tracks. Both led to very similar results. This chapter also explained how the analytical expression can be used in the case of complex conductor shapes.

Chapter 5 used the models presented in the previous chapters to estimate the eddy current losses in an optimization process. The latter intended to search for the highest possible torque considering a given speed, a given outer shell and the constraint of keeping the motor under a limit temperature. Four main outcomes were obtained in this chapter. First, eddy currents are important to consider in the optimization process, otherwise the motor may never reach the speed for which it was intended for, because eddy currents slow it down too much. Second, it was found that there is no real need for 3D FEM models (in the context of optimization), which are highly demanding in terms of computational resources, since almost the same results can be achieved with simpler analytical expressions. Third, it was observed that the dimensions of the motor and the shape of the winding were not very sensitive to the working point. Fourth, it was highlighted that the most important variable to consider was the width of the conductors. A limitation of the analysis was that optimizations were made assuming that all tracks of a winding were connected in series. Indeed, this means that recirculating currents were neglected if some tracks needed to be put in parallel to reduce the supply voltage. In addition, a section was dedicated to demonstrating that, to a large extent, it can be assumed that eddy current losses and supply losses can simply be added together without worrying too much about linearity problems.

Chapter 6 challenged the problematic of parallelizing the tracks by adopting a novel approach which consists to consider the large initial track and to cut slits in it to break the loops of eddy currents. This approach is original because it brings the parallelization of the conductors to the local scale. Several patterns of slits were

considered. Among them, the *long slit*, which is equivalent to a traditional full parallelization of the winding, was found to be the best when the connections between the top and the bottom layers kept the tracks *parallel* to each other. However, the *V slit* pattern, consisting of a slit spanning over two consecutive segments in a symmetric way, remained the preferred choice because its performance was almost the same when the slit coverage was close to 100%, but with the significant benefit of having much more space at the end-windings to make the connections. It was shown that all the conclusions can be drawn for various shapes of the conductors. The intrinsic parameters α and β , which are sort of normalized values of supply losses and eddy current losses respectively, were used to find the optimal slit configurations at given working points. The results showed that, when the nominal speed increases relatively to the nominal torque, it is always preferable, when technically feasible, to increase the number of slits than enlarge the existing ones. This allows to keep the best trade-off between the reduction of eddy current losses and supply current losses. Finally, these conclusions drawn from simulations were confirmed through experiments on ten windings with different slit configurations.

Original contributions

The original contributions of the thesis can be articulated on three parts, which are the development of new models to estimate eddy current losses, the application of these models for motor optimizations and the insights gained to improve the design process of a slotless BLDC motor with PCB windings.

Models

- It was highlighted that the airgap magnetic field and eddy currents in the windings can be computed successively without considering the influence of their reaction field.

Therefore, assuming that the magnetic field is known as a source term :

- A 2D finite element model has been implemented to obtain an accurate estimation of the losses in a whole winding within a reasonable time. Its particularity is that it is in the plane of the PCB and therefore allows to take into account the shape of the conductors, their width and their skewing.
- An analytical model has been developed which allows to evaluate eddy current losses very quickly.

Although developed for radial flux motors, these models can also be applied to linear or axial flux motors.

Application

In order to optimize a motor, one can be interested in reducing the computation efforts to its minimum and simplifying the problem into simpler steps. In this context:

- It has been shown that the use of the analytical models leads to almost the same optimal motor as with the use of FEM models.

Focusing on the winding:

- A new approach has been developed to tackle eddy current losses considering local parallelization of the tracks by using slits, instead of increasing the number of smaller tracks. This approach allows to keep a reasonable supply voltage and separates the torque constant determination from the conductor width optimization.
- In order to compare the various slit patterns, the parameters α and β have been introduced. They allow to pre-compute a map of the optimal slit configurations which is independent of the nominal working point.

Insights gained

In the context of a torque optimization, for a given speed and for a given envelope, it was observed that:

- The internal motor thicknesses and the winding shape are not very sensitive to the operating point.
- The width of the conductors is the key element to adapt to the operating point.
- The *long slit* pattern with *parallel* connections between the top and bottom layers is the best option if technically feasible, otherwise the *V slit* pattern with coverage close to 100% remains a very good compromise to break eddy current loops while keeping space for large vias.
- Optimal slit configurations are valid for significant ranges of operating points.

Using all these outcomes should allow to save on computational resources when optimizing a motor.

Perspectives

Although there are many assumptions on the models and thus limitations on the results throughout this work, that would be interesting to discuss further, we decided to highlight only three main areas of improvement.

Optimization methodology

Based on the observations made during the various optimizations of slotless motors with PCB windings in chapter 5 and 6, we are able to come up with a methodology to design a motor using a rational amount of computational resources as follows. For a given torque and speed, a first step would be to optimize the whole motor using only analytical models, such as the one developed in chapter 4. The shape of the winding would be variable, but no slitting is considered at first glance and the voltage supply does not matter. Then, a final adjustment could be made by optimizing the winding with more precision using the 2D FEM model developed in chapter 2, which can take into account the complex shapes as well

as the slitting configuration. The initial track width would be determined by the voltage constraints and the desired torque constant. In cases where a motor should be re-optimized at another speed and/or torque than its initial purpose, our recommendation would be to start from the existing basis and adjust the winding. Ideally, it would simply consist in picking the right slit configuration, as explained in section 6.5, on a set of possibilities previously computed in the $\alpha - \beta$ plane.

An interesting perspective would be to evaluate the extent to which the above suggestion is valid for optimizing a motor, addressing the limit of each of the observations that lead to propose this methodology. For example, in this work, for the reference motor considered, it has been observed that, between 0 rpm and 100 krpm , the internal thicknesses of the motor are not very sensitive to the operating point. Note that the same observation has been made for a motor of different dimensions in [131]. However, this could be related to the fact that there was little iron loss. So, the question is, if there were more iron losses, how would the speed range evolve in which the motor structure can be considered not to change significantly to the point of requiring further optimization?

Analytical model

Even though harmonics of the source magnetic field can be taken into account in the 2D FEM model, this is not the case in the analytical model. This is an important limitation because as it stands now, the analytical model cannot be used to evaluate the losses when the rotor has a higher number of pole pairs, when it is composed of a permanent magnet with non-parallel magnetization or simply when the rotor is misaligned relatively to the winding. A simple transformation $\omega \rightarrow n\omega$ has been tried but does not give any great results to consider the n^{th} harmonic. Therefore, a modification of the structure of the formula should be made in order to correctly take into account the harmonics.

In chapter 6, when studying the slits configurations, the FEM formulation considered the recirculating currents (currents around a slit due to a phase or an amplitude difference in the back emf). However, they have not been studied separately from local eddy currents. A more in-depth study should be conducted, first to determine the importance of each type of eddy currents in the total losses for different slit configurations, and second to be able to derive an analytical expression to quickly estimate these circulating currents, similar to what we have for local eddy currents.

Operating cycles

The optimizations were all performed assuming that the motor runs at a constant nominal speed. However, in most real-world applications, the motors are cyclically loaded and have a speed profile that is not at all constant. Therefore, it would be relevant to observe how the results of this work change if we now consider these operating cycles. Instead of maximizing the torque, the objective function could be the energy delivered by the motor during a cycle, for example. This would correspond even better with reality, but it also requires a dynamic model of the heat dissipation.

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Appendix

Vector potential of a 2D field



This appendix has the objective to demonstrate that any magnetic field in the form $\mathbf{b} = (b_x, b_y, 0)$ with a null component along the z axis, can be derived from a vector potential $\mathbf{a}$ having only a z component: $\mathbf{a} = (0, 0, a_z)$.

As a reminder, thanks to the Gauss law ($\nabla \cdot \mathbf{b} = 0$) the magnetic field derives from a vector potential:

$$\mathbf{b} = \nabla \times \mathbf{a}$$

In the maxtrix form and without loss of generality we thus obtain:

$$\begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = \begin{pmatrix} \frac{\partial a_y}{\partial z} - \frac{\partial a_z}{\partial y} \\ \frac{\partial a_z}{\partial x} - \frac{\partial a_x}{\partial z} \\ \frac{\partial a_x}{\partial y} - \frac{\partial a_y}{\partial x} \end{pmatrix} \quad (\text{A.1})$$

Imposing that the third component of the magnetic field b_z is equal to zero means that we must impose the following condition on the vector potential:

$$\frac{\partial a_x}{\partial y} = \frac{\partial a_y}{\partial x} \quad (\text{A.2})$$

Since $\nabla \times (\nabla f) = 0$ is true for any function f , the vector potential is defined up to a gradient and one can choose arbitrarily any f such that $\mathbf{a}' = \mathbf{a} + \nabla f$ is also an adequate vector potential for $\mathbf{b}$.

Let's define the following function:

$$f^x = - \int a_x(x, y, z) dx \quad (\text{A.3})$$

Consequently we can write:

$$\begin{pmatrix} a'_x(y, z) \\ a'_y(y, z) \\ a'_z \end{pmatrix} = \begin{pmatrix} a_x(x, y, z) + \frac{\partial f^x}{\partial x} \\ a_y(x, y, z) + \frac{\partial f^x}{\partial y} \\ a'_z \end{pmatrix} = \begin{pmatrix} a_x - \int \frac{\partial a_x}{\partial x} dx \\ a_y - \int \frac{\partial a_x}{\partial y} dy \\ a'_z \end{pmatrix} \quad (\text{A.4})$$

$$\stackrel{(\text{A.2})}{=} \begin{pmatrix} a_x - \int \frac{\partial a_x}{\partial x} dx \\ a_y - \int \frac{\partial a_y}{\partial x} dx \\ a'_z \end{pmatrix} = \begin{pmatrix} a_x - a_x + K_x(y, z) \\ a_y - a_y + K_y(y, z) \\ a'_z \end{pmatrix} \quad (\text{A.5})$$

Where $K_x(y, z)$ and $K_y(y, z)$ are independent of the x component.

Because $\mathbf{a}'$ must also respect the condition A.2, we have:

$$\frac{\partial a'_x(y, z)}{\partial y} = \frac{\partial a'_y(y, z)}{\partial x} = 0 \Rightarrow a'_x(z) \quad (\text{A.6})$$

Let's repeat the same reasoning on $\mathbf{a}'$ choosing this time:

$$f^y = - \int a'_y(y, z) dy \quad (\text{A.7})$$

We obtain:

$$\begin{pmatrix} a''_x(z) \\ a''_y(z) \\ a''_z \end{pmatrix} = \begin{pmatrix} a'_x(z) + \frac{\partial f^y}{\partial x} \\ a'_y(y, z) + \frac{\partial f^y}{\partial y} \\ a''_z \end{pmatrix} = \begin{pmatrix} a'_x - \int \frac{\partial a'_y}{\partial x} dy \\ a'_y - \int \frac{\partial a'_y}{\partial y} dy \\ a''_z \end{pmatrix} = \begin{pmatrix} a'_x(z) - 0 \\ a'_y - a'_y + K'_y(z) \\ a''_z \end{pmatrix} \quad (\text{A.8})$$

Again let's choose an appropriate function to find $\mathbf{a}'''$:

$$f^z = -a''_x(z)x - a''_y(z)y \quad (\text{A.9})$$

$$\begin{pmatrix} a'''_x \\ a'''_y \\ a'''_z \end{pmatrix} = \begin{pmatrix} a''_x(z) + \frac{\partial f^z}{\partial x} \\ a''_y(z) + \frac{\partial f^z}{\partial y} \\ a'''_z \end{pmatrix} = \begin{pmatrix} a''_x(z) - a''_x(z) \\ a''_y(z) - a''_y(z) \\ a'''_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ a'''_z \end{pmatrix} \quad (\text{A.10})$$

Since $a'''_x = a'''_y = 0$ we also have $\frac{\partial a'''_x}{\partial z} = \frac{\partial a'''_y}{\partial z} = 0$ and consequently:

$$\begin{pmatrix} b_x \\ b_y \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{\partial a'''_z}{\partial y} \\ \frac{\partial a'''_z}{\partial x} \\ 0 \end{pmatrix} \quad (\text{A.11})$$

Building the vector potential

B

The objective of this appendix is to show how to build an adequate vector potential to use as a source term in equation (2.34), to solve the eddy currents problem. This equation is reminded below:

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = -i\omega \left(\frac{\partial a_{0x}}{\partial x} + \frac{\partial a_{0y}}{\partial y} \right) \quad (\text{B.1})$$

It is assumed that the normal component of the magnetic field is expressed as $b_{z0} = f(x, z = r_i)g(y, z = r_i)$ with r_i a fixed radius. Using the second hypothesis of the 2D FEM model of section 2.3.1, there is no variation of the z component along the conductor thickness.

Any vector potential can be chosen as long as it respects $\mathbf{b}_0 = \nabla \times \mathbf{a}_0$. Using the linearity property of the curl operator, $\mathbf{a}_0$ can be split in two vector potentials which are associated to the normal and tangential components of the magnetic field respectively, such that $\mathbf{a}_0 = \mathbf{a}_{tan} + \mathbf{a}_{norm}$:

$$\mathbf{b}_0 = \begin{pmatrix} 0 \\ 0 \\ b_{z0} \end{pmatrix} + \begin{pmatrix} b_{x0} \\ b_{y0} \\ 0 \end{pmatrix} = \nabla \times \mathbf{a}_{norm} + \nabla \times \mathbf{a}_{tan} = \nabla \times (\mathbf{a}_{tan} + \mathbf{a}_{norm}) \quad (\text{B.2})$$

Hence, the source term can be constructed as the sum of $\mathbf{a}_{tan}$ and $\mathbf{a}_{norm}$.

Let's focus on the tangential component of the field first. As demonstrated in appendix A, any magnetic field in the form $\mathbf{b} = (b_x, b_y, 0)$ with a null component along the z axis, can be derived from a vector potential having only a z component: $\mathbf{a}_{tan} = (0, 0, a_{tan z})$. However, Eq. (B.1) which is the main equation to solve the eddy current problem is independent of this third component of the potential. It means that no matter the value of $a_{tan z}$, the resulting losses will be the same. Therefore, zero value can be chosen for $a_{tan z}$ without loss of generality.

Focusing on the normal component of the field, we have that the following equations must be respected:

$$\begin{pmatrix} 0 \\ 0 \\ b_{norm} \ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ f(x)g(y) \end{pmatrix} = \begin{pmatrix} \frac{\partial a_{normz}}{\partial y} - \frac{\partial a_{normy}}{\partial z} \\ \frac{\partial a_{normx}}{\partial z} - \frac{\partial a_{normz}}{\partial x} \\ \frac{\partial a_{normy}}{\partial x} - \frac{\partial a_{normx}}{\partial y} \end{pmatrix} \quad (\text{B.3})$$

As any arbitrary solution which respects the constraints is acceptable, a possible solution for $\mathbf{a}_0$ is to take:

$$\mathbf{a}_0 = \begin{pmatrix} -f(x) \int g(y) \, dy \\ 0 \\ 0 \end{pmatrix} \quad (\text{B.4})$$

Indeed, it can be verified that:

$$\begin{pmatrix} 0 - 0 \\ \frac{\partial (-f(x) \int g(y) \, dy)}{\partial z} - 0 \\ 0 - \frac{\partial (-f(x) \int g(y) \, dy)}{\partial y} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ f(x)g(y) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ b_{norm} \ 0 \end{pmatrix} \quad (\text{B.5})$$

Torque constant and supply losses formulae

C

C.1 Torque

The general expression of the torque T is given as [132]:

$$T = \underbrace{\frac{\partial W_{cmag0}(\theta_m)}{\partial \theta_m}}_{\text{Cogging torque}} + \underbrace{\sum_{k=1}^n \frac{\partial \phi_{k0}(\theta_m)}{\partial \theta_m} i_k}_{\text{Electrodynamic torque}} + \underbrace{\frac{1}{2} \sum_{k=1}^n \sum_{j=1}^n \frac{\partial L_{kj}(\theta_m)}{\partial \theta_m} i_k i_j}_{\substack{\text{Reluctant torque} \\ (+ \text{ electrodynamic torque})}}$$

In the case of slotless motors, the cogging torque is null and the magnets see the same magnetic circuit for every mechanical position θ_m of the rotor, so there is no variation of magnetic co-energy (due to the PM) W_{cmag0} with respect to θ_m . In the case of surface mounted permanent magnets, the reluctant torque is also null and the windings see the same magnetic circuit for every mechanical position θ_m of the rotor. Such that there is no variation of the inductances L_{kj} when the rotor is turned. Therefore previous equation reduces to:

$$T = \sum_{k=1}^n \frac{\partial \phi_{k0}(\theta_m)}{\partial \theta_m} i_k$$

The magnetic flux generated by the PM and intercepted by the windings ϕ_{k0} is supposed to be purely sinusoidal with respect to the motor angle θ_m (which is nearly the case with distributed windings) and becomes for a three-phase system:

$$\begin{bmatrix} \phi_{10} \\ \phi_{20} \\ \phi_{30} \end{bmatrix} = \phi_0 \begin{bmatrix} \sin(p\theta_m) \\ \sin(p\theta_m + 2\pi/3) \\ \sin(p\theta_m + 4\pi/3) \end{bmatrix}$$

With ϕ_0 the maximum flux intercepted by a phase.

The machine can be supplied either by using sinusoidal currents, as it is common in the case of large synchronous machines, or by using chopped DC currents as in small BLDC machines (note that they are also synchronous machines by nature).

C.1.1 Sinusoïdal currents

In this case the electrical current i in each phase is given by:

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = I_{peak} \begin{bmatrix} \sin(\omega_e t + \theta_{e0}) \\ \sin(\omega_e t + 2\pi/3 + \theta_{e0}) \\ \sin(\omega_e t + 4\pi/3 + \theta_{e0}) \end{bmatrix}$$

With I_{peak} the peak current, ω_e the electrical frequency, t the time, and θ_{e0} an initial phase shift in the alternating current. The torque equation then becomes:

$$\begin{aligned} T = p\phi_0 I_{peak} & \left(\cos(p\theta_m) \sin(\omega_e t + \theta_{e0}) \right. \\ & + \cos(p\theta_m + 2\pi/3) \sin(\omega_e t + 2\pi/3 + \theta_{e0}) \\ & \left. + \cos(p\theta_m + 4\pi/3) \sin(\omega_e t + 4\pi/3 + \theta_{e0}) \right) \end{aligned}$$

And by recalling that $2 \sin \alpha \cos \beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$:

$$\begin{aligned} T &= \frac{1}{2} p\phi_0 I_{peak} \left(\sin(\omega_e t + \theta_{e0} + p\theta_m) + \sin(\omega_e t + \theta_{e0} - p\theta_m) \right. \\ &\quad + \sin(\omega_e t + 2\pi/3 + \theta_{e0} + p\theta_m + 2\pi/3) \\ &\quad + \sin(\omega_e t + \theta_{e0} + 2\pi/3 - p\theta_m - 2\pi/3) \\ &\quad + \sin(\omega_e t + 4\pi/3 + \theta_{e0} + p\theta_m + 4\pi/3) \\ &\quad \left. + \sin(\omega_e t + \theta_{e0} + 4\pi/3 - p\theta_m - 4\pi/3) \right) \\ &= \frac{1}{2} p\phi_0 I_{peak} \left(\sin(\omega_e t + \theta_{e0} + p\theta_m) + \sin(\omega_e t + \theta_{e0} - p\theta_m) \right. \\ &\quad + \sin(\omega_e t + 4\pi/3 + \theta_{e0} + p\theta_m) + \sin(\omega_e t + \theta_{e0} - p\theta_m) \\ &\quad \left. + \sin(\omega_e t + 8\pi/3 + \theta_{e0} + p\theta_m) + \sin(\omega_e t + \theta_{e0} - p\theta_m) \right) \\ &= \frac{3}{2} p\phi_0 I_{peak} \sin(\omega_e t + \theta_{e0} - p\theta_m) \end{aligned}$$

The torque is maximal and constant when $\omega_e t + \theta_{e0} - p\theta_m = \pi/2$. Under this condition we obtain:

$$T = \frac{3}{2} p\phi_0 I_{peak} = k_t I_{peak}$$

With k_t being the motor torque constant when using sinusoïdal currents.

C.1.2 DC currents

In this case the electrical current i in each phase is constant by time intervals. During each interval, only two phases are supplied with opposite currents such that for example:

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} I_{dc} \\ -I_{dc} \\ 0 \end{bmatrix}$$

With a DC current supply I_{dc} , the torque equation then becomes:

$$T(\theta_m) = p\phi_0 I_{dc} \left(\cos(p\theta_m) - \cos(p\theta_m + 2\pi/3) \right)$$

The torque is not constant anymore and its mean value has to be considered instead. In the case of a classic three-phase winding supply, the period for which the currents remain unchanged is generally $\tau_{chopped} = \pi/3\omega_e$. Therefore the mean torque on this period can be computed as:

$$\begin{aligned} T &= p\phi_0 I_{dc} \frac{1}{\tau_{chopped}} \int_0^{\tau_{chopped}} \left(\cos(\omega_e t) - \cos(\omega_e t + 2\pi/3) \right) dt \\ &= p\phi_0 I_{dc} \frac{3\omega_e \sqrt{3}}{\pi \omega_e} \\ &= p \frac{3\sqrt{3}}{\pi} \phi_0 I_{dc} \end{aligned}$$

The torque constant when using DC currents is thus given by:

$$k_t = p \frac{3\sqrt{3}}{\pi} \phi_0$$

C.2 Supply losses

C.2.1 Sinusoïdal currents

Considering sinusoïdal currents with an amplitude I_{peak} , the losses in the winding can be written as:

$$\begin{aligned} P_{supply} &= R_{ph} (i_1^2 + i_2^2 + i_3^2) \\ &= R_{ph} I_{peak}^2 \left(\sin^2(\omega_e t + \theta_{e0}) + \sin^2(\omega_e t + 2\pi/3 + \theta_{e0}) + \sin^2(\omega_e t + 4\pi/3 + \theta_{e0}) \right) \\ &= R_{ph} I_{peak}^2 \left(\frac{1 - \cos(2(\omega_e t + \theta_{e0}))}{2} + \frac{1 - \cos(2(\omega_e t + 2\pi/3 + \theta_{e0}))}{2} + \dots \right. \\ &\quad \left. \dots \frac{1 - \cos(2(\omega_e t + 4\pi/3 + \theta_{e0}))}{2} \right) \end{aligned}$$

$$P_{supply} = \frac{3}{2} R_{ph} I_{peak}^2$$

with R the electrical resistance of one phase.

C.2.2 DC currents

If we now consider that only two phases are supplied simultaneously by a current I_{dc} and $-I_{dc}$ respectively, as it is the case for BLDC machines, the losses can be written:

$$P_{supply} = R_{ph}(I_{dc})^2 + R(-I_{dc})^2 = 2R_{ph}I_{dc}^2$$

with R_{ph} being the same electrical phase resistance.