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Models of Restructured Electricity Systems

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Abstract

The work presented in this thesis is a collection of four different research projects, conducted at the intersection between academic and industrial fields, related to the restructuring of the electricity industry.

Many models have been proposed to organise restructured electricity systems. The first contribution of this thesis proposes a unified mathematical framework for expressing them, using variational inequalities. An advantage of this formalism is that it allows the use of existence and uniqueness theorems. But it also leads to models that are "computable" in the sense that they can be solved by numerical algorithms.

In the second part, we present a model of the European Commission proposals on cross-border trade, in particular access to the network and congestion issues. Access charges reflect the mechanism devised by the European directive, and congestion at the interconnection is managed through a coordinated auction mechanism. The model allows for various domestic regulation of the national non-eligible market, and different forms of competition in the eligible market. We illustrate this flexibility on a stylised example, and identify policy issues to be studied in a more realistic case study.

The third contribution elaborates on the modelling of imperfect competition on electricity markets. More specifically, this chapter illustrates the integration of perfect or imperfect competition behaviours in power models, through conjectural variations. The main contribution of this paper deals with estimation and consistence topics, including a numerical illustration on a stylised model.

The last part is devoted to the modelling of the EU Emission Trading Scheme, which impacts the power industry by limiting the total amount of CO_2 they are allowed to emit. We illustrate the effect of some basic economic assumptions on the investment and generation mix. We also study how these assumptions impact the permit price and affect operators' profitability. The innovative part of this chapter is in the way allocation mechanisms in subsequent commitment periods can potentially distort the behaviour of CO_2 emitting agents.

Contents

Chapter 1

General Introduction

The power sector has been undergoing a profound reorganisation over the past, whether in the European Union or the United states. The objective of this reorganisation is to move from a world where energy prices are regulated to one where the market determines them. The expectation is that moving from cost-based to market prices will provide the adequate signals to improve efficiency.

This overhaul of the electricity sector did not follow a single paradigm; the process was in fact more painful than expected. One quickly realised that the whole system could not be subject to competition. One quickly admitted that one can deregulate the energy market, and submit it to standard markets forces. A different reasoning applied to transmission which, because of its character of natural monopoly, had to remain regulated. The idea then emerged that the overhaul of the electricity system would develop as a mix of market and regulation where the commodity could be submitted to normal competition but some services should remain regulated. This meant that the overhaul would not be a full liberalisation but a *restructuring*: one would only liberalise some parts of the system while trying to regulate the rest in a way that would make the whole thing a coherent economic system. Needless to say the idea of this restructuring was to liberalise as much as possible. Mixing competition and regulation is never an easy task; it quickly turned out to be quite difficult in electricity.

Even competition on the sole electricity market, that is, the liberalised part of the restructuring, did not develop along a single paradigm. The initial idea of freeing the access to the transportation infrastructure was to let the energy market develop on a purely bilateral basis with generators selling directly to customers after paying an access charge to the network. Alternatively other electricity markets developed along a very organised paradigm where all agents were mandated to bid into a *Pool*. Intermediate solutions also developed where the pool was no longer mandatory and one allowed for a mix of bilateral transactions and an organised market. *Power exchanges*, closer to standard organised commodity markets, also developed to replace the pool. In fact many different views emerged as to what the energy market should be. A similar diversity of views appeared for transmission, which itself gave raise to different mixes of market and regulation ideas. Different theories appeared as to how to price congestion in a market-based way; others rejected the whole idea and argued that it suffices to simply socialise congestion costs. One also developed systems where part of congestion is priced on a market base and the other socialised. Because market-based congestion pricing does not cover the infrastructure cost of the grid, other regulated charges for accessing the grid had to be developed. In other words, very much like for energy, quite different ideas appeared going from pure regulation to a complex mix of regulation and markets. There were as many restructuring ventures as systems to restructure.

Not all these restructuring paradigms are equivalent though. The combination of organised and regulated markets is a human construction that is not guaranteed to work. Some major failures, such as the meltdown of the Californian wholesale market in 2000, are still in all memories. They demonstrate that ill-designed markets can fail. Other systems also encountered difficulties in their development, with the result that the "law of unintended consequences" has become the common rule for characterising electricity restructuring. Things do not happen the way one expects them to happen. Proposals are made that look good if one simply looks at first-order effects, but turn out dramatic after taking all interactions and agents incentives into account.

This stresses the need for a sound understanding of such systems before they are implemented. This understanding should consider physical insights into the power sector, so that the physics of the phenomena are properly accounted for. They should also account for economic concepts, so that incentives of agents are adequately represented, and one does not simply assume that the restructured system will behave like the former optimised system.

The complexity of the design of the restructured systems, together with the introduction of new agents in a deregulated energy market, constitute a challenge for power system modelers: in the old regulated world, the integrated nature of the electricity sector would lead to formulate the modelling of power systems as an *optimisation problem*, in which the objective would be to minimise the cost of the integrated industry. This is generally not possible anymore in a deregulated system where several economic agents compete on different markets[1]. However, *equilibrium models* may then be used to formulate a problem in which several agents simultaneously try to optimise their production or consumption, even with part of the market still subject to some regulation. An equilibrium is reached when productions and consumptions balance, such that no agent would be able to depart unilaterally from this equilibrium without incurring some losses.

The following four chapters each propose a collection of models, which deal with specific issues of restructured electricity systems. A brief description of these chapters will follow the general remarks that we present now. The emphasis of this thesis is on the modelling side: we try to represent the main features of different paradigms of restructured electricity system. We do this through models that we call *"computable"* in the sense that we do not require them to have analytical solution, but are satisfied if we can submit them to a numerical algorithm. The enlargement from analytical models to computable models is important in the sense that computable models can in general accommodate not only more realistic description of physical systems, but also more interactions between different phenomena taking place either in regulated or competitive subsystems. Typically the combination of a regulation and a competitive market is obtained by assembling the mathematical relations that describe theses regulated and competitive markets. This can be done without being limited by the number of variables or equations, or by the physical representation of the system.

While we are interested in computable models in the above sense we do not insist on the fact that the models should be numerically tractable. As argued before, restructured electricity markets are human constructions, which can behave in an awkward way. Pricing principles common in regulation, like average cost pricing, destroy all convexity properties of models, leading possibly to the absence of solution or to the multiplicity of solution, and in any case to

[1] In some specific situations, however, competition between different agents may still be captured through an optimisation model, as will be discussed in subsequent chapters.

untractable models. It is unfortunately part of the process of market design, that human being can badly design market. The appearance of non-convexity in a model can make it intractable from a numerical point of view. Our interest in non-tractable models is that they very often signals bad market design.

In the same line, this thesis is not meant to elaborate numerical solutions of equilibrium problems that may suffer from inexistent or multiple solution. We construct models that we hope reproduce important features of the real world and submit them to non-linear solvers such as CONOPT or MINOS to solve our problems. For each type of problem, we argue whether we have good chances to end up with a solution or not. As will be discussed, existence and uniqueness of a solution may be proven for some models, while uniqueness and even guarantee of existence disappear for others.

This thesis is by no means intended to provide an algorithmic contribution. Its novelty resides in the modelling of the institutional changes in electricity markets. As illustrated by the famous example of the Californian crisis, these institutional changes sometimes may end in situations without solutions. The aim is thus not to provide a solution to all situations resulting from market restructuring, but to explore the possible equilibria resulting from the electricity market restructuring, and sometimes to show that a given set of institutional rules, originally intended to help deregulation and increase competition, might eventually result in an absence of equilibrium, and hence a system failure.

In each of the four subsequent chapters, various models are proposed, with increasing complexity. In general, we start with perfect competition models without regulation constraints. These are convex and hence tractable, with a unique solution. But as soon as equilibrium constraints appear, or when regulatory conditions are introduced, the problems become NP-hard. For these equilibrium models, multiple equilibria might appear, which indicates market incompleteness. Conversely, the absence of equilibrium reflects a system which does not function properly. Furthermore, since restructured electricity markets are in constant evolution, they require permanent attention. The models presented in this work may thus help identifying systems that could lead to problematic situations.

To the best of our knowledge, the absence of uniqueness, or even existence, of solutions to these equilibrium problems has been widely ignored by the literature devoted to electricity systems, to date. In this work, we have disregarded numerical and algorithmic issues related to these problems, for the benefit of modelling aspects.

The thesis is organised as a collection of four different papers, all related to specific aspects of the modelling of restructured electricity systems. We finish this general introduction by giving a brief overview of the most prominent features of each of these subsequent chapters.

Variational inequality models

The first of these papers, presented in chapter 2, was written together with Professor Smeers in 1999, and presents a unified mathematical framework providing a common tool to study different paradigms of restructured electricity systems.

Many models have been proposed and extensively discussed in the literature. The objective of this paper is the construction of a unifying mathematical framework for formulating them. Although mathematical formulations overlook a lot of the market microstructure embedded in restructuring models, a common mathematical framework still provides a very convenient description of the different macro structures proposed for designing electricity systems.

The second point of interest is that this formalism allows one to call upon the theory of variational inequalities to analyse restructuring models. Existence and uniqueness theorems can be applied, and duality theorems may prove the equivalence between some of the models of restructured markets.

The third objective of this paper is computational: it is generally impossible to eliminate all inefficiencies, in market design in general, and in electricity restructuring in particular. Trading one inefficiency for another is thus unavoidable. It is therefore useful to be able to numerically assess the magnitude of some of the inefficiencies embedded in certain market designs.

Variational inequalities provide this opportunity, since they lead to implementable models that may be solved with powerful codes. The original contribution of this paper resides in the unifying mathematical framework that is proposed for modelling and comparing a wide range of restructuring paradigms. The numerical case study helps quantifying the inefficiencies inherent to the different restructuring models.

We first study, in a variational inequality framework, bilateral models with or without transmission constraints. We then introduce a market for transmission services. Alternative organisations of the system are then examined: we introduce a first nodal model, followed by the Pool and its link to the bilateral model, and finally introduce regulation in the transmission market in order to recover infrastructure costs.

This paper first appeared as CORE Discussion Paper number 99/66 in December 1999, and was eventually published after going through standard refereeing procedure as chapter 5 (pp. 85-120) in *Complementarity: Applications, Algorithms and Extensions*, book edited by Michael Ferris, Jong-Shi Pang and Olvi Mangasarian, Kluwer Academic Publishers (2001). This chapter is reproduced here with kind permission of Springer Science and Business Media.

This paper was cited in

- *Oligopolistic Competition in Power Networks: A Conjectured Supply Function Approach* (2002), Christopher J. Day, Benjamin F. Hobbs and Jong-Shi Pang,

- *Network-constrained Cournot models of liberalized electricity markets: the devil is in the details* (2005), Karsten Neuhoff, Julian Barquin, Maroeska G. Boots, Andreas Ehrenmann, Benjamin F. Hobbs, Fieke A.M. Rijkers and Miguel Vasquez, Energy Economics, vol 27, issue 3,

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- *A Supply Chain Network Perspective for Electric Power Generation, Supply, Transmission, and Consumption* (2007), Anna Nagurney and Dmytro Matsypura, Advances in Computational Management Science, volume 9,

- *Strategic Analysis of Electricity Markets Under Uncertainty: A Conjectured-Price-Response Approach* (2007), Efraim Centeno, Javier Reneses and Julián Barquin, IEEE Transactions on Power Systems, Vol 22, Issue 1,

- *Application of Benders Decomposition to an Equilibrium Problem* (2005), Jordi Cabero, Álvaro Baíllo, Santiago Cerisola and Mariano Ventosa, presented at the Power Systems Computing Conference, Liège, Belgium,

- *A study of the Transmission Price Conjecture in an oligopolistic power market* (2007), Martin Kurzidem and Göran Andersson, presented at the PowerTech Conference, Lausanne, Switzerland.

Cross-border trade model

Next, Chapter 3 deals with Cross-border trade. One of the important aspects of restructuring electricity markets is related to market integration. In particular, the European Union has been promoting a *real single integrated market*, since 1999, as a major objective of the restructuring. Similar integration objectives were announced by the Federal Energy Regulatory Commission (FERC) in the US, at the same period.

An interesting question is whether it is possible to integrate power systems organised according to different paradigms (such as electric systems in the EU Member States) into one single electricity market. In other words: does this integration require some preliminary harmonisation, or not? While the FERC concludes that this is indeed a prerequisite, Europeans maintain it is not. As a result, the European Commission initiated the *Florence Regulatory Forum*, with the objective of addressing "in particular the tarification of cross-border trade of electricity exchanges and the allocation and management of scarce interconnection capacity". The work of the Forum ended up with the EU Regulation on Cross-Border Exchanges in Electricity, in 2003.

The Regulation provides guidelines for accessing the network. It also establishes rules for congestion management on the interconnections. Finally, the Regulation develops rules that govern cross-border trade of electricity in Europe, through a mechanism that compensates network operators hosting transit. Our contribution stands in the design of a two-stage model that implements these three issues on the transmission market, mixed with various competition intensities or regulatory interventions on the energy market. This two-stage model is briefly described here.

The first stage is concerned with the setting of access charges to the network by local Transmission System Operators (TSO's). In the second stage, an equilibrium is searched both on the energy market (where generators behave more or less competitively on eligible markets, and face regulated prices on captive markets) and on the transmission market (congestion management and cross-border trade compensations).

The first stage game is considered in different versions. In the simplest case, local TSO's exogenously distribute network charges between generators and consumers. This results in a "second stage only" equilibrium. The next version assumes that all TSO's act cooperatively to distribute access charges so as to maximise the global welfare. This provides the "cooperative equilibrium". Finally, the most complex variant assumes that TSO's act separately to maximise their own regional welfare. This "Nash equilibrium" is formulated as an equilibrium problem.

All these three versions are subject to equilibrium constraints in the second stage, which is formulated as a set of complementarity conditions. For given network access charges (output of the first-stage), the existence of the second-stage equilibrium may be proven when all customers are eligible to competition, but neither uniqueness nor existence may be guaranteed anymore when some of the clients are subject to regulation.

The chapter ends with a numerical illustration on a stylised model including four regions. We examine the impact of various market structure assumptions, different regulatory pricing for captive customers as well as competitive assumptions on eligible markets. We conclude by observing that, although standard economic assumptions are well reflected by the model, some mechanisms may indeed not perform as expected. In particular, the compensation mechanism for hosting transit may in fact reward those inducing this transit. This reveals undesired effects, the law of unintended consequences applies in this case.

A first, extensive version of this paper written with Professor Smeers, was featured in 2002 on the Research Library web site of the Harvard Electricity Policy Group. The short version, reproduced in this thesis, was published in the European Journal of Operations Research, volume 181, 2007 under the title *"The EU regulation on cross-border trade of electricity: A two-stage equilibrium model"*. References to the short or long version of this paper appear in

- *Two-settlement systems for Electricity Markets under Network uncertainty and Market Power* (2004), Rajnish Kamat and Shmuel S. Oren, Journal of Regulatory Economics, Vol 25, Issue 1,

- *Organization of electroenergetic sector of Serbian surrounding countries* (article in Serbian, 2004), Zorana Z. Mihajlović-Milanović and Milenko M. Nikolić, Ekonomske Teme, vol 42, issues 1-2,

- *Electricity competition and market splitting: a real-size two countries case* (2003), Julián Barquin et al., Proceedings of the 2nd "International Conference on Liberalization and Modernization of Power Systems: Congestion Management Problems", Irkustsk, Russia,

- *Evaluating Congestion Management Schemes in Liberalized Electricity Markets applying Agent-based Computational Economics* (2007), thesis submitted by Thilko Krause for the degree of Doctor of Technical Sciences at the Swiss Federal Institute of Technology, Zurich.

Conjectural variations models

One important common feature of the restructured electricity markets, in all regions of the world, is the deregulation of a significant part of the energy market. This deregulation intends to improve the market efficiency, replacing regulated average cost-based prices by competitive prices, resulting from an increased pressure of new entrants on eligible markets.

Standard economy assumes that, in a competitive environment, the prices should tend towards marginal costs as the number of economic agents increases. But this is not what seemed to happen on electricity markets in the early 2000s. Prices appeared to remain significantly above marginal costs, at least in high-demand periods. This indicates that economic agents (namely generators) do not behave as price takers: they do exert some market power.

Another standard economic model, namely the Cournot model, does account for this exercise of market power. However it assumes that each firm aims to maximise profits, based on the expectation that its own output decision will not have an effect on the decisions of its rivals. When applying the Cournot paradigm to the electricity sector, the resulting prices are generally higher than those observed on the market.

A model which replicates price observations must therefore be somewhere between perfect competition and Cournot equilibrium. One way to achieve a whole range of different degrees of competition, combining the exercise of market power with assumptions on the reaction of rivals, is through the introduction of *conjectural variations.*

Conjectural Variations define an equilibrium in which agents make a *conjecture* on the *variation* of output by rivals, and take account of this conjecture while maximising their profit.

The advantage of conjectural variations is that it encompasses a whole range of standard economic equilibria and imperfect competition paradigms, ranging from perfect competition or Bertrand equilibrium, through collusion, passing by the Cournot paradigm. One inconvenient is that it requires the estimation of a potentially large number of model parameters in order to fit market data.

Chapter 4 of this thesis first recalls how the various economic paradigms may be rephrased in terms of conjectural variation models. It also shows how a conjectural variation equilibrium may be cast in a single optimisation model – whereas in general, equilibrium models cannot be formulated as such.

The main contribution in this chapter comes from the two proposed procedures for estimating the Conjectural Variation parameters that are introduced in the model.

The first approach assumes that agents iteratively learn from successive observations of equilibrium, and improve their profit by adapting their conjectural variations to the observed output of their rivals. Although the iterative procedure converges quickly, no proof of the uniqueness of the solution to this problem can be advanced. However, when applying this procedure on electricity markets, the resulting equilibrium prices are higher than those of a Cournot equilibrium. The only way to replicate price observations would therefore be to jointly estimate conjectural variation parameters, together with the elasticity of the demand function.

The second approach, based on consistency, looks more promising. A conjectural variation is said to be *consistent* if it is equivalent to the optimal response of the other firms, at the equilibrium resulting of this conjecture. First and second-order equilibrium conditions are derived for such equilibrium, and the estimation problem can then be formulated as an equilibrium problem subject to equilibrium constraint. Analytical solutions are proposed for some stylised cases (linear marginal costs, identical players, . . .).

This work was conducted on my own as a research activity for Electrabel during the years 2002 to 2004, but has not been submitted for publication. Although unpublished, this paper is referenced as mimeo in

- *Organization of electroenergetic sector of Serbian surrounding countries* (article in Serbian, 2004), Zorana Z. Mihajlović-Milanović and Milenko M. Nikolić, Ekonomske Teme, vol 42, issues 1-2.

- *Conjectural variation estimation in electricity markets* (2006), Efraim Centeno, Juan José Sánchez, Ignacio Hoyos and Julián Barquín, presented at the Applied Mathematical Programming and Modelling, Madrid, Spain.

CO_2 emission permit trading models

The last part, in chapter 5, is devoted to a particularly crucial feature of restructured electricity markets. Not only did the European Commission want to move forwards to an integrated electricity market, in order to promote competition and efficiency, it also wanted to push the industry towards cleaner technologies. In the continuation of the Kyoto protocol on the reduction of greenhouse gas emissions, the EU introduced a market mechanism, the Emission Trading Scheme (ETS), which intends to push the industry towards cleaner technologies.

In short, the ETS allocates *emission permits* to carbon-intensive sectors of the industry (part of the permits are granted for free, others are auctioned). The principle is that for each ton of CO_2 that is rejected in the atmosphere, a permit must be obtained by the emitter. Since the total quantity of available permits is strictly limited, so will be the amount of rejected greenhouse gas. Should an emitter fall short of permits, he would then have to buy them to another emitter, owning a surplus of permits. This creates a *market* for permits, thus a price for the latter. This price, in turn, will affect the generation of electricity, because common technologies such as coal-fired or gas-fired generation both reject significant quantities of CO_2 in the atmosphere.

In contrast to the other parts of this work, which focused on short-term equilibrium considerations, this last chapter cares for long-term consequences of this new feature of the restructured system. In particular, the impact of the permit price on the future technological mix at the equilibrium is studied.

Optimal investment, in terms of technological choice, has been widely studied in the literature since this chapter was originally written. This is formulated here as an equilibrium problem between several agents investing in different technologies. Two versions of the equilibrium model are first presented, whether permits are traded exclusively between electricity generators, or may exchange permits with the rest of the industry.

Additional questions treated in this chapter concern (*i*) the potential impact of the ETS on natural gas price. The rationale is the following: the objective of the directive being to reduce the total amount of rejected CO_2, this can only be met if some of the coal-fired generation plants are abandoned to the advantage of cleaner technologies, in particular natural gas. As a result, a considerable increase in gas consumption would definitely impact gas prices. This would induce a leverage effect on power prices; (*ii*) the Directive leaves it to Member States to decide how they will allocate free permits. Free permits obviously create a rent since they may be sold at market price. This creates an incentive to try and obtain as many free permits as possible and hence, depending on the rules retained for the granting of free permits, to alter the behaviour of agents in order to increase profit. This attempt to game the rules is introduced in a manner similar to the conjectural variations presented in chapter 4.

The innovative contribution of this chapter consists in three parts: the introduction of an interaction between the available quantity of permits and the price of natural gas, the modelling of incentives to game the allocation mechanism, and finally the combination of both features plus the trade of permits with the non-electrical carbon intensive industry. All these components, combined in a nonlinear mixed complementarity problem, make the model original.

This work on the Emission Trading Scheme was conducted on my own, in a research project at Electrabel, mostly in 2004. It has however not been submitted for publication yet.

Glossary and Acronyms

CEER The *Council of European Energy Regulators* aims at facilitating the creation of a single competitive, efficient and sustainable internal market for gas and electricity in Europe.

ETSO *European association of Transmission System Operators* (TSO's), founded in response to the emergence of the Internal Electricity Market within the European Union. See TSO below.

EU-ETS The *European Union Emissions Trading Scheme* specifies that large emitters of CO_2 must provide an amount of emission allowances, equivalent to their actual emission. They may get some allowances for free, and are supposed to balance their excess or shortage by trading. The total amount of available allowances is capped to reduce total emissions.

LMP *Locational Marginal Prices* are used to manage the efficient use of the transmission system when congestion occurs. Marginal prices reflect the cost of bringing one additional unit of commodity. LMP recognise that this cost may vary according to location and transmission congestions.

LRMC The *Long-Run Marginal Cost* is the cost of increasing production by one unit, including the cost related to the capacity adjustment that would be optimal for this output increase in the long run.

Pool *Power Pools* arise from public initiative to create competition between generators. They usually take into account many technical details, and are generally mandatory (no bilateral market).

PTDF *Power Transfer Distribution Factors* define the flows on the lines of the network, that would result from a unit injection at some node and withdrawal at some arbitrarily fixed node (the sink node).

TSO *Transmission System Operator*. TSO's provide grid access to the electricity market players (i.e. generating companies, traders, suppliers, distributors and directly connected customers) according to transparent rules. In order to ensure the security of supply, they also guarantee the safe operation and maintenance of the system. In many countries, TSOs are in charge of the development of the grid infrastructure too. TSOs in the European Union internal electricity market are entities operating independently from the other electricity market players.

Chapter 2

Variational inequality models:

Alternative variational inequality representations of restructured electricity systems

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Access to this chapter is restricted to the university's Intranet. It was published as chapter 5 (pp. 85-120) in *Complementarity: Applications, Algorithms and Extensions*, book edited by Michael Ferris, Jong-Shi Pang and Olvi Mangasarian, Kluwer Academic Publishers (2001). This chapter is reproduced in the printed version of this thesis with kind permission of Springer Science and Business Media.

Chapter 3

Cross-border trade model:

A two-stage equilibrium model of the EU regulation on the cross-border trade of electricity

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This chapter has been published as a separate paper in the European Journal of Operational Research (Volume 181, issue 3, September 2007), in a special issue on the Energy Industry.

The Regulation on cross-border exchanges of electricity in the European Union is meant to enhance the trade of electricity between Member States, by facilitating access to the network and improving the management of congestion at the interconnections. This paper presents a computational model that embeds these two features. The problem is cast in the form of a two-stage equilibrium between regional Regulators. In the first stage, they decide on the allocation of their regional network costs between generators and customers. Either they maximise their regional welfare non-cooperatively (Nash equilibrium), or they centralise the decision as a super-regulator (leading to a cooperative equilibrium). In the second-stage equilibrium, the consequences of first-stage's decisions are assessed by modelling the energy market as the result of imperfect competition equilibrium on *competitive market*, coupled with regulated pricing on the domestic *less competitive markets*. The "rules" that come out of the first-stage game largely influence the final equilibrium. We illustrate this on an extensive numerical example, showing that the model behaves properly and identifying policy issues worth of further investigations.

3.1 Introduction

The Directive on *common rules for the internal market in electricity* appeared in the Official Journal, in January 1997 (in [10], Legislation). It constitutes the first serious step of the restructuring of the European electricity sector. The Directive had a major impact on the industry but fell short of creating the internal electricity market claimed by European authorities. Additional legislation (in [10], Legislation) came into force in July 2004. An amending Directive strengthened and repealed the 1996 law. The Regulation on *conditions for access to the network for cross-border exchanges in electricity*, hereafter referred to as the Regulation, accompanying the amending Directive, aims to facilitate the exchanges of electricity among Member States. This paper concentrates on the modelling of the main features of the Regulation.

The Regulation mainly deals with three issues. One is the establishment of a system for accessing the European grid. Congestion management on the interconnections is the second issue. Last but not least, the Regulation creates a Regulatory Committee for developing and implementing the rules that govern cross-border trade of electricity in Europe. This latter governance body is a direct outgrowth of the Florence Regulatory Forum, created by the European Commission to help elaborate the Regulation (in [10], Florence Forum). Access and congestion management are treated separately in the Regulation. This is also the case in the literature that commonly sees congestion management and tariff for accessing the network as separate subjects. Needless to say both the payments due to congestion management and for accessing the network cumulate and it is their sum that influences the decision of agents operating in the electricity system. A first objective of this work is to cast both problems in a single model and to show that they indeed combine to influence agents'decisions. The governance solution selected in the European process also deserves attention. It indeed appears more and more evident that the integration of electricity systems into a single competitive market mandates strong institutions to organise this competition. This is not really surprising in a system as plagued by externalities as electricity. The European and US experiences testify that this need is not always well perceived, let alone accepted. The resistance against strong institutions may result in weak coordination of market designs and operations. This suggests to model the Regulatory Committee as a group of national Regulators engaged in a non-cooperative game. Combining the game of the Regulators with the market simulation game leads to a two-stage equilibrium problem. This inclusion of governance considerations in the market model is a second objective of this paper.

Our model is a stylised view of the European power system but it contains several of its main features. New questions must first be looked at through streamlined models and we believe that we make a first step in that direction. The model is also numerically small. Because it is a two-level equilibrium problem, it raises methodological questions that it is probably also wise to first explore on a prototype. The paper is organised as follows. Section 3.2 summarises the principles of access to the grid and congestion management set by the Regulation. The overall structure of the model is presented in section 3.3. Section 3.4 briefly discusses the modelling implications of the Directives. Section 3.5 elaborates on the three subjects handled in the Regulation. Numerical experiences focusing on the governance problem are presented in section 3.6. Conclusions terminate the paper. Because of space, this paper only gives a brief presentation of the work. The interested reader is referred to [2] for more details. European Regulators recently issued a consultation paper that report different case studies of integration of restructured electricity markets [5].

3.2 The Regulation

The Regulation came into force in July 2004. Its aim is to establish common rules that facilitate the cross-border trade of electricity. These rules are a direct outgrowth of the work of the Florence Regulatory Forum, which continues elaborating them. An extensive discussion of the Regulation is provided in [1]. The European association of electricity Transmission System Operators (site `www.etso-net.org`, hereafter referred to as ETSO) and the Council of European Energy Regulators (site `www.ceer-eu.org`, hereafter referred to as CEER) were instrumental in the work leading to this Regulation. We briefly describe here the main features of the Regulation that we introduce in our model.

3.2.1 Access to the European electricity grid

Articles 3 and 4 of the Regulation present the rules for accessing the grid. Article 3 sets the principles of an inter-TSO compensation mechanism. Article 4 lists the charges that can be levied for accessing the network. These charges serve to cover the cost of the grid and to finance the inter-TSO compensation mechanism. Generators and consumers of electricity pay an access charge respectively referred to as the G and L charges. The G and L charges result from cost allocation mechanisms: total costs are distributed among agents by purely arithmetic rules. Standard economic theory teaches that these accounting-like practices cannot pretend to any efficiency property. An export fee (T) was also levied until 2003 on agents engaged in cross-border trade. This fee has always been modest and disappeared from transactions among participants to the inter-TSO mechanism in 2004 but is still sometimes implemented for other Member States. The Regulation requires that all charges other that G and L disappear when these latter reflect locational signals. We model cost allocations rules and possible mechanisms for computing the G and L charges.

3.2.2 Congestion management

Articles 5 and 6 of the Regulation deal with the allocation of transmission capacities and congestion management. The Regulation concentrates on the interconnections and leaves the handling of domestic congestion to Member states. This separation of roles is common in European law as it is, to some degree, in federal systems. The argument for separating the two congestion problems is here reinforced by the observation that congestion occurs very infrequently on the grid of the Member States but is almost permanent on the interconnections. It is reasoned that European law therefore only needs to deal with the management of congestion at the interconnections. The idea is not new. It led to successes (the Nordic system) and failures (congestion management difficulties in California well before the full meltdown of the system). The Regulation and the Florence Regulatory Forum take it that the intra and inter-zonal separation of the congestion management will work in continental Europe. Discussions therefore mainly concentrated on inter-zonal approaches.

Two methods were proposed. ETSO [9] initially proposed a coordinated auction that introduces a market for transmission capacities across zones. Later the European association of Power Exchanges (EuroPEX) introduced a mechanism for coupling zonal markets [6]. These two proposals have now been merged into what is referred to as "Flow-based Market Coupling" (ETSO-EuroPEX [7]). Both coordinated auction and Flow-based Market Coupling can be traced to congestion management methods extensively discussed before in the US. Ehrenman and Smeers [4] present a comparative analysis of these methods and discuss their relation to Locational Marginal Prices (LMP) and market splitting. We implement an ideal version of the Flow-based Market Coupling model, which can be described as follows.

The relevant electricity system is seen as a set of control areas, each represented by a notional node where all intra-area injections and withdrawals are supposed to take place. These notional nodes are interconnected by transfer capacities. The set of these nodes and transfer capacities form a virtual electrical network in the sense that its elements, even if not physical realities, are endowed with electrical properties: lines have a thermal capacity and there exists a load flow model of the virtual network, built using the DC-approximation of Kirchoff's laws. The Flow-based Market Coupling supposes that markets clear at two levels. Given import/export to a zone, the Power Exchange guarantees intrazonal market clearing. Import and export between zones then clear through a market of coordinated transmission services between zones. Taken together, these two market clearings maximise the value of the transfer capacity of the whole virtual network, taking into account its possibilities as represented by its DC load-flow approximation. Further discussion of these approaches can also be found in Olmos and Perez-Arriaga [12]. There is at this stage no documented evidence that one can construct a virtual network that is both compatible with the institutional segmentation of the European system and a reasonable approximation of the physical network. Feasibility studies are promised that should clarify the issue.

It may be useful to refer the Flow-based Market Coupling to the existing literature and practice of congestion management. Locational Marginal Prices (LMP) and Flowgates are the reference models. They have been extensively discussed in the literature and implemented in practice. They boil down to the same model when applied to a single-period, perfectly reliable system, such as the virtual network used in ETSO and EuroPEX documents. LMP and Flowgates can drastically differ in real-world implementation that deal with multiple settlements, forced outages and real networks. Our model remains within the simplified framework adopted by ETSO and EuroPEX. Flow-based Market Coupling is then equivalent to LMP applied to a zonal system or to a Flowgate model where the flowgates are the interconnections.

3.3 Overall modelling architecture

The first electricity Directive left considerable freedom to Member States to establish their national electricity rules. The amending Directive only slightly restricted this freedom. Most relevant to this paper, the Directives mandate the creation of national Regulators but refrain from introducing a EU Regulator. This has implications both at the national and supra-national level. At the national level, the Directives resulted in a balkanised system that, as the European Commission since recognised, cannot be immediately integrated into a single market. This also creates distorted conditions of competition that should be assessed before they lead to unintended economic losses. At the international level, this leaves the ruling of cross border trade to the willingness of Regulators and TSO to cooperate to coordinate their operations.The aim of our model is to accommodate this diversity of arrangements in a prototype model. The following sketches the main features of the model.

We model the governance of the restructured European electricity system as a two-stage process somehow analogous to a principal/agent economic model. Regulators act as principals and decide essential features of the architecture and structure of the markets under their jurisdiction. These features have to do with the implementation of the national laws inherited from the transpositions of the Directives. They also imply interpretations and implementations of the Regulation. These characteristics determine the operations conditions of national or regional markets. Principal/agent models normally concentrate on asymmetry of information. We do not deal with these questions here but assume instead perfect information. Our aim is more to model the impact of subsidiarity, that is, of the possibility left to national laws and Regulators to differently implement common EU rules. Because of the preliminary nature of this type of investigation by computable models, we limit ourselves to the analysis of a very specific but persisting question, namely the harmonisation of the G and L charges discussed above.

This suggests to the following model architecture. In a first stage, Regulators select the G and L charges in their jurisdiction. This impacts on the competitive conditions of the market inherited from the transposition of the Directives. The second stage of the model therefore assesses the impact of the G and L charges by simulating the market. This results in electricity prices and quantities, which in turn define the consumer welfare and company profits in individual regions or countries. It is assumed that each Regulator selects the G and L charge in its jurisdiction on the basis of the resulting welfare and profit. Needless to say, welfare in a region or country accruing from electricity generation and consumption is influenced by what takes place in the whole market, and hence by the G and L decisions of other Regulators. The key question (even though barely mentioned) is whether a European (federal) Regulator and TSO would benefit the market, compared to a set of Regulators acting together in a more or less coordinated process. Needless to say we do not pretend to answer this questions through an academic prototype model such as presented in this paper. Our goal is instead to suggest a methodology for looking at the problem. In this sense we assume that a federal Regulator is modelled as a set of cooperating Regulators who maximise total welfare. The current situation is modelled as a set of non cooperative Regulators achieving Nash equilibrium. The following first describes the second stage model that simulates the market.

We here gather some fundamental assumptions that will be recalled when relevant in the text. The proposed model assumes that TSOs do not behave strategically, have perfect information and fully cooperate. We realise that this is not a very realistic assumption. It is technically difficult to depart from it, especially since the Forum papers do not provide any information as to how to model imperfect cooperation and transmission of information among TSOs. We assume that transmission capacities are fixed, and hence that the impedances of the lines are also fixed. We also assume that generators operate with fixed capacities, and do not strategically manipulate them. This excludes strategic investment and maintenance policies. Strategic behaviour is thus limited to the operation of the spot market.

The utilisation of the network requires two types of payments: an access charge and a congestion charge. The congestion charge is discussed later in this paragraph. As to the access charge, we assume that the L and G tariffs are structured as energy charges. Therefore, grid charges affect the decisions of market agents on how much energy to produce and consume. We realise that this degrades efficiency. We believe however that this complies with the discussions of the Forum. Looking at alternatives would require a complete overhaul of the model. The proposed model is based on flow-based transmission rights. It is well known that flow-based markets are equivalent to LMP applied to a zonal system, where the flowgates are the interconnections. However, the reality of the continental grid, where loop-flows issues are relevant, suggests that this may be a very poor approximation. We however believe that our representation fits with the proposal of the Forum, and, in the absence of additional information, stick to it. Needless to say, real results could be very different from those obtained with this approximation. The Forum extensively discussed the distinction between implicit and explicit auction for the allocation of interconnection capacities. Because our model simultaneously finds the equilibrium on energy and transmission markets, it corresponds, strictly speaking, to a fully coordinated implicit auction. An alternative interpretation is to assume a fully coordinated explicit auction with an assumption of rational expectation.

Last but not least, it may be worthwhile mentioning that even though we solve Regulator's optimisation Karush-Kuhn-Tucker (KKT) conditions of the first-stage problems (which are non-convex optimisation problems), we did verify numerically that, at the obtained points, any deviation of the strategy of the first-stage agents (the Regulators), keeping the strategies of the others constant, would lead to a degradation of the objective function. In short, we compute KKT points, but verify that they effectively satisfy the conditions of an equilibrium problem.

3.4 Modelling the Directives.

3.4.1 Overall architecture

We denote with $r \in R$ the set of regions, that we here assimilate to Member States. Each region is under the jurisdiction of a Regulator. In this prototype model, we assume a one-to-one correspondence between regions and Regulators. Customers are grouped in customer segments $c \in C$ having the same characteristics in terms of location and regulation. The set of generating firms is denoted with $j \in J$. We here make the assumption that each customer or generating firm is geographically attached to one specific region, and let C^r and J^r respectively denote the subset of customer segments and generating firms located in region r. Because some markets remain captive, or only partially open to competition, we denote with J_c the subset of generators which are effectively able to supply customer c. Conversely, the subset C_j denotes the subset of market segments in which generating firm j is allowed to enter.

3.4.2 Market segmentation

The electricity market of each Member State is decomposed into competitive and non-competitive segments. This partitioning is similar to the distinction between the eligible and non-eligible markets introduced by the first Directive to protect some market segments from competition. The Amending Directive eliminates this distinction and foresees that all consumers be allowed to choose their suppliers in 2007. European experience shows that serious barriers to trade can persist in systems that are, in principle, fully open to competition such as Germany. We take stock of this finding and retain the former segmentation of the market into eligible and non-eligible parts. We simply give it another name (competitive and non-competitive). We also recognise that this distinction between market segments is no longer enshrined in law, and hence leave it to the user to select, on the basis of his/her analysis of barriers to trade, what fraction of each national market is effectively open to competition and which generator can supply which market.

Articles 5 and 6 of the Regulation strongly suggest – and the documents about congestion management of the Regulatory Forum confirm – a zonal organisation of the European electricity system. We therefore assume that the price charged to a particular segment of the competitive market depends on the segment and is unique for a given segment in a zone. Similarly the price charged to a non-competitive segment of the market is unique for a given segment in a zone.

Each market segment, competitive and non-competitive, is modelled by a linear demand system where supplies from the different generators are imperfect substitutes. This permits, at least in principle, us to represent perceived differences of reliability that could be attached to the generator's supplies. We recognise that such a demand system would be difficult to calibrate.

The inverse demand function for each market segment is given by:

$$p_j^c(q^c) = \alpha^c - \sum_{j' \in J_c} \beta_{j,j'}^c q_{j'}^c \tag{3.1}$$

where q^c is the array of variables q_j^c, representing the bilateral supply from producer j to consumer c. The degree of competition to which a given market segment is left exposed to, in the absence of regulation, mainly depends on the number of generators and hence on the cardinality of the set $J_c \subset J$. The resulting prices p_j^c to a given customer c may differ according to the supplier j, which accounts for product differentiation on the basis of criteria such as reliability.

Provided that the matrix of coefficients β is symmetric with respect to indices j and j', the considered demand system can be derived from a utility function for each customer c given by

$$U^c(q^c) = \sum_{j \in J_c} \alpha^c q_j^c - \frac{1}{2} \sum_{j,j' \in J_c} q_j^c \beta_{j,j'}^c q_{j'}^c. \tag{3.2}$$

Defining the customer c's total expense with $E^c(q^c)$ as the product of bilateral supplies q_j^c by the relevant prices p_j^c, the consumer surplus CS^c will be given by equation :

$$CS^c(q^c) = U^c(q^c) - E^c(q^c) = \frac{1}{2} \sum_{j,j' \in J_c} q_j^c \beta_{j,j'}^c q_{j'}^c. \tag{3.3}$$

3.4.3 Generation

The first electricity Directive removed all exclusive rights in generation and supply in the Member States. This step was effective with the result that there now exist several generators in each Member State. The model therefore allows for a set of generators in each control area. Generators may supply the competitive segments of all control areas provided they buy transmission services on existing transfer capacities. We combine the removal of exclusive rights with remaining barriers to trade by assuming that several generators can also supply the non-competitive segment but are subject to a different pricing regime (see sections 3.4.4 and 3.4.5).

A generator is represented by a quadratic generation cost curve. This is clearly a simplification, compared to the common piecewise linear cost function (piecewise constant marginal cost function) obtained by modelling individual plants. This approximation is common in the economic literature, where one prefers to work with smooth functions. It also drastically simplifies our computations. This allows one to represent increasing short run marginal costs. In most cases (co-generation is an exception) it is relatively easy to calibrate this cost curve on more detailed plant data. The curve also contains a fixed cost. This latter does not play any role in the search for an equilibrium but it allows one to check whether market prices allow generators to cover their fixed charges. Electricity market simulation models indeed generally concentrate on short-run equilibrium. It is essential to assess the extent to which long-run (capacity) costs are covered if one wants to assess (not simulate) long-term equilibrium.

Let generation costs be defined with the following quadratic relation

$$CG_j(q_j) = K_j + \sum_{c \in C_j} v_j^c q_j^c + \tfrac{1}{2} \sum_{c,c' \in C_j} q_j^c \omega_j^{c,c'} q_j^{c'}, \tag{3.4}$$

where q_j is the array of bilateral supplies q_j^c to its customers C_j, and K_j is the fixed-cost component which should be recovered at the equilibrium.

The first electricity Directive only required a weak unbundling of transmission from generator. The amending Directive is stronger, even though it still refrains from enforcing a full (ownership) separation of transmission and generation. This may result in uneven possibilities for accessing the network. Unbundling is a subject in itself that is not treated here where we assume a total unbundling of generation and transmission in each control area, and hence non-discriminatory access to the grid.

3.4.4 The competitive market

European power exchanges currently only trade a small fraction of the total electricity produced and consumed in Europe. We neglect the power exchanges in this prototype model and concentrate on the representation of bilateral transactions that constitute the bulk of the market. We accordingly assume that each generator concludes bilateral transactions with customers.

A generator receives the price paid by the consumer on a given market minus the access charge (L). The generator also pays an access to the network (G)[1]. Finally, the generator also pays the potential congestion charge, denoted with X. This is equivalent to assuming that the total cost paid by the generator is equal to its generation cost plus the sum of all G, L and possible congestion charges X implied by its bilateral transaction. These three transmission components are grouped in the transmission cost expression :

$$CT_j(q_j) = \sum_{c \in C_j} (G_{r|j \in J^r} + L_{r'|c \in C^{r'}} + X_j^c) \times q_j^c \tag{3.5}$$

As discussed in section 3.3, depending on the version of the model, the G and L charges are endogenous or exogenous. The congestion charge X is always endogenous.

The total revenue of generator j is given by

$$R_j(q) = \sum_{c \in C_j} p_j^c(q^c) q_j^c = \sum_{c \in C_j} \left(\alpha^c q_j^c - \sum_{j' \in J_c} q_j^c \beta_{j,j'}^c q_{j'}^c \right) \tag{3.6}$$

where prices in various markets now also depend on supplies from other producers. Finally, generator j's profit $\pi_j(q)$ is obtained as its total revenue $R_j(q)$ minus its generation and transmission costs, $CG_j(q_j)$ and $CT_j(q_j)$, respectively.

Competition in the electricity sector can be intense in low demand period and subject to extreme market power when capacity is tight. The sole, convenient, Cournot paradigm cannot accommodate that diversity of situations. But the introduction of conjectural variation together with the Cournot assumption can. Some may contest the theoretical soundness of this fix which

[1] In the initial Cross-Border Trade mechanism implemented by ETSO, an additional export charge (T) used to be explicitly levied on inter-regional transactions to (partially) feed a Compensation Fund for TSOs incurring transit. This charge has disappeared from the new cross-border trade mechanism. The compensation mechanism in the new cross-border trade mechanism is funded directly from the G and L components.

is here retained for its practical advantages. First, there indeed exists a wide variety of mixes of price and quantity competition regimes, that theory alone cannot choose from in market studies. Second, conjectural variation can be used to span this variety of paradigms. There is thus a fundamental indeterminacy in the representation of market power and the user must be called upon to select some assumption. We therefore assume that several generators can supply the competitive segment of the market of each control area. We thus model competition in some market segment through the Cournot assumption, completed with coefficients of conjectural variation. We do not discuss here the calibration of these coefficients of variation, but we use them in the formulation of the (conjectured) marginal revenue :

$$\begin{aligned}
\frac{\partial R_j}{\partial q_j^c}(q) &= p_j^c(q^c) + q_j^c \frac{\partial p_j^c}{\partial q_j^c} \\
&= p_j^c(q^c) + q_j^c \sum_{j' \in J_c} \frac{dp_j^c}{dq_{j'}^c} \frac{\partial q_{j'}^c}{\partial q_j^c} \\
&= \left(\alpha^c - \sum_{j' \in J_c} \beta_{j,j'}^c q_{j'}^c \right) - q_j^c \sum_{j' \in J_c} \xi_{j,j'}^c \beta_{j,j'}^c
\end{aligned} \tag{3.7}$$

The conjectural variation coefficients $\xi_{j,j'}^c$ appear in the last expression, as the conjectured response in output by competitor j' to a unit change in output by operator j. We do here neglect the impact of such a change on the output of any competitor on an another market, that is, we suppose that $\partial q_{j'}^{c'} / \partial q_j^c = 0$ for $c' \neq c$.

Competition on eligible markets implies a profit maximising behaviour. This is equivalent to writing complementarity relations for each pair generator/eligible customer segment. The symbol $\perp$ between two inequalities indicates that at least one of them must hold with equality. In this case, the following notation implies that either q_j^c is strictly positive, forcing the derivative of the profit to zero, or q_j^c is zero, allowing for a negative value of the profit derivative.

$$\frac{\partial \pi_j}{\partial q_j^c}(q) \leq 0 \qquad \perp \qquad q_j^c \geq 0. \tag{3.8}$$

For the sake of notation, in the following, we group the access and congestion charges between generator j and consumer c in a single parameter, denoted with GLX_j^c, and defined by the relation:

$$GLX_j^c = G_{r|j \in J^r} + L_{r'|c \in C^{r'}} + X_j^c. \tag{3.9}$$

GLX_j^c represent the total transmission charges introduced by the Regulation.

The left inequality of complementarity condition (3.8) can be reformulated as a limit on the aggregated access charges for a specific bilateral transaction :

$$\begin{aligned} GLX_j^c \quad &\geq \quad \left(\alpha^c - \sum_{j' \in J_c} \beta_{j,j'}^c q_{j'}^c\right) - q_j^c \sum_{j' \in J_c} \xi_{j,j'}^c \beta_{j,j'}^c - \upsilon_j^c - \sum_{c' \in C_j} \omega_j^{c,c'} q_j^{c'} \\ \perp \quad & \qquad q_j^c \geq 0. \end{aligned} \tag{3.10}$$

One should keep in mind that G and L access charges are uniformly applied at the regional level, as stated in the Regulation, and not transaction-based. The condition can be interpreted as providing the maximum level of access and transmission charges which will make it possible for this bilateral transaction to take place. Strict equality should hold for positive transactions.

3.4.5 The non-competitive market

By definition, barriers to trade limit the supplies to a non-competitive market segment. These parts of the market are thus, more than others, vulnerable to the exercise of market power. The amending Directive mandates an electricity Regulator in each Member State and hence removes the German exception that claimed that competition law is sufficient to monitor market power in electricity. It is reasonable to admit that this Regulator will intervene if the relative lack of competition leads to excessive prices in those market segments. One can also assume that the threat of the intervention of the Regulator may suffice to limit the exercise of market power. France is a case in point. Even though EDF has a strong dominant position in the whole French market, the prices remain comparatively low compared to the rest of Europe without the Regulator having to intervene. EDF has market power but does not exercise it – at least not excessively. This puts an implicit bound on the prices in the non-competitive market. What determines these bounds is a difficult matter. But one can make the assumption that the tradition of the member countries will prevail. France will somehow refer to marginal cost, Belgium to average cost, Germany will leave considerable freedom to its generators, possibly limited to a price cap and the Netherlands will be influenced by England and Wales practice. We thus refer to these standard forms of regulation and assume that each non-competitive part of the market is subject to some form of regulation. The model accommodates the standard regulation modes. Specifically we suppose that the price in a non-competitive part of the market can be based on the marginal costs of some supplier of that market, limited by a price cap or be set at monopoly price (absence of an effective regulation). We also suppose that the price can be equal to some average cost. Even if Regulators will have less investigation power to compute average cost in the future, one may reasonably assume that a supplier, fearing the threat of Regulatory intervention would prepare to show the Regulator or Competition Authorities that it only charges average cost.

The different regulation schemes which can be implemented in the model are briefly described below. We assume here that non-competitive market segments can only be supplied by one single incumbent generator, yet the following results can easily be extended to the case where a small limited number of generators have access to market segments inaccessible to other suppliers.

Absence of Regulation: Going from the simplest to the most complex regulatory framework, we start with the absence of regulation, that is, when non-competitive market segments are left to monopoly pricing. The first-order equilibrium condition on this market segment becomes:

$$GLX_j^c = \alpha^c - (1 + \xi_{j,j}^c)\beta_{j,j}^c q_j^c - v_j^c - \sum_{c' \in C_j} \omega_j^{c,c'} q_j^{c'} \tag{3.11}$$

Price Cap: Introducing a price cap is equivalent to let the incumbent operator maximise its profit, subject to one additional constraint which provides an upper bound $\overline{p}_j^c$ on the equilibrium price (we neglect here more complex versions of average price caps). This can easily be formulated using a Lagrangian multiplier μ_j^c and the first-order equilibrium conditions on the non-competitive market segment become:

$$\begin{aligned} GLX_j^c &= \alpha^c - \beta_{j,j}^c(1 + \xi_{j,j}^c)(q_j^c - \mu_j^c) - v_j^c - \sum_{c' \in C_j} \omega_j^{c,c'} q_j^{c'} \\ p_j^c \leq \overline{p}_j^c \quad &\perp \quad \mu_j^c \geq 0, \end{aligned} \tag{3.12}$$

where we have supposed that a feasible solution exists for a strictly positive supply $q_j^c > 0$. Ill-designed price caps, combined with potentially high access charges, can sometimes lead to marginal cost of supply higher than the cap, ending up in a zero-supply to this non-competitive market.

Long-Run Incremental Cost: In this case, the price is imposed to be equal to the marginal cost of supplying the customer. This therefore equates the market equilibrium price with the derivative of the generator's cost function. Reformulating this condition in terms of access charges, it becomes:

$$GLX_j^c = \alpha^c - \beta_{j,j}^c q_j^c - v_j^c - \sum_{c' \in C_j} \omega_j^{c,c'} q_j^{c'} \tag{3.13}$$

Note that the last relation again assumes that a feasible solution exists for some strictly positive supply. Note also that non-competitive market segment should realistically be supplied by a local generator, which would result in the disappearance of the congestion charge X from the left hand-side of the last relation.

Average Cost: Average cost pricing, for a generator supplying several distinct customers, faces the problem of allocating the costs between the customers, and generally requires to resort to some arbitrary allocation. Different versions of *Fully Distributed Costs* can help solve this issue, yet keeping arbitrary rules: the Regulator tries to identify each source of costs for the generator, and to allocate them between the customers. This is straightforward for access charges, export taxes (when relevant), congestion fees and the linear part of generation variable costs, but is more intricate for fixed costs. It might even be more difficult for quadratic costs.

Reformulating the total generation and transmission costs as:

$$\begin{aligned} CG_j(q_j) + CT_j(q_j) &= \sum_{c \in C_j} \left[(GLX_j^c + v_j^c) q_j^c + \tfrac{1}{2} \omega_j^{c,c} (q_j^c)^2 \right] \\ &+ K_j + \tfrac{1}{2} \sum_{\substack{c,c' \in C_j \\ c \neq c'}} q_j^c \omega_j^{c,c'} q_j^{c'}, \end{aligned} \tag{3.14}$$

it clearly appears that the first terms in brackets account for all variable costs which can be readily allocated to individual customers. In contrast the remaining terms represent costs that cannot be directly assigned to individual customers and hence need to be allocated in some arbitrary way. The regulated price in non-competitive markets therefore becomes :

$$p_j^c = GLX_j^c + v_j^c + \tfrac{1}{2}\omega_j^{c,c} q_j^c + \psi_j^c(q_j), \tag{3.15}$$

where $\psi_j^c(q_j)$ results from some allocation of the common costs.

We briefly describe three possible allocation schemes. *Quantity-based Fully Distributed Costs* result from the allocation of common costs to the different customers, in proportion of the delivered output. The resulting equilibrium condition is given by:

$$\begin{aligned} GLX_j^c &= \alpha^c - v_j^c - (\beta_{j,j}^c + \tfrac{1}{2}\omega_j^{c,c})q_j^c \\ &+ \left(K_j + \tfrac{1}{2}\sum_{c \neq c'} q_j^c \omega_j^{c,c'} q_j^{c'}\right) \times \frac{1}{\sum_{c' \in C_j} q_j^{c'}} \end{aligned} \tag{3.16}$$

In *Revenue-based Fully Distributed Costs*, the regulator allocates common costs in proportion of the revenues accruing from the different customer segments. At the equilibrium, regulated market segments must satisfy:

$$\begin{aligned} GLX_j^c &= \alpha^c - v_j^c - (\beta_{j,j}^c + \tfrac{1}{2}\omega_j^{c,c})q_j^c \\ &+ \left(K_j + \tfrac{1}{2}\sum_{c \neq c'} q_j^c \omega_j^{c,c'} q_j^{c'}\right) \times \frac{p_j^c(q^c)}{\sum_{c' \in C_j} p_j^{c'}(q^{c'}) \times q_j^{c'}} \end{aligned} \tag{3.17}$$

Finally, *Cost-proportional Fully Distributed Costs* allocate common costs with the same share for each market segment as directly identifiable costs. This yields:

$$\begin{aligned} GLX_j^c &= \alpha^c - v_j^c - (\beta_{j,j}^c + \tfrac{1}{2}\omega_j^{c,c})q_j^c \\ &+ \left(K_j + \tfrac{1}{2}\sum_{c \neq c'} q_j^c \omega_j^{c,c'} q_j^{c'}\right) \times \frac{v_j^c + \tfrac{1}{2}\omega_j^{c,c} q_j^c}{\sum_{c' \in C_j} [v_j^{c'} + \tfrac{1}{2}\omega_j^{c',c'} q_j^{c'}] q_j^{c'}} \end{aligned} \tag{3.18}$$

As can be observed, each average-cost pricing scheme results in non-linear equilibrium conditions, which may result in multiple equilibria (or possibly no equilibrium at all)[2]. Solving numerical instances of this model generally requires the computation of good starting points, as briefly discussed in section 3.6.

[2] The lack of existence and uniqueness properties of the equilibrium for such problems has been discussed in section 2.5, for the equivalent problem in its mixed variational inequality formulation.

The possibility to resort to different distributed cost mechanisms raises the question of their relative advantages. The economic response is that one does not know: allocation methods are accounting-like procedures that do not enjoy any economic property. One can certainly devise economically efficient allocation methods (Ramsey pricing, non-linear pricing). But these require demand information that is generally not available. These also lead to discriminatory pricing that are unlikely to be accepted by regulatory and competition authorities. The allocation mechanisms implemented in this paper are retained here because one finds them in practice, not because they are economically justified.

3.5 Modelling implications: The Regulation

The following discussion supposes that TSOs fulfill all ideal economic assumptions. They do not behave strategically, have perfect information, and fully cooperate. These assumptions are probably necessary in a (first) computable model. They also correspond to the goals of the amending directive and of the Regulation: unbundling should ban strategic behaviour, and the Regulation mandates cooperation and exchange of information.

3.5.1 Access to the network

A generator located in some zone will pay the zonal G charge to access the network, whether its trade is intra or inter-zonal. Similarly a consumer pays a zonal L charge in order to access the network. The Regulation also foresees a mechanism to compensate TSOs for the costs that the cross border trade transactions may impose on their system. Although there is still a lot of debate between Regulators and TSOs on the practical implementation, the European Commission has issued guidelines on inter-TSO compensation (in [10], results from the 11th meeting of the Florence Forum), which this paper will stick to.

TSOs divide the transmission network into horizontal and vertical networks. The horizontal network is the "part of the transmission system, which is used to transmit electricity between countries and within the country: it contains the transmission system elements that are influenced significantly by cross-border exchanges". Operators of systems hosting such transit incur network costs. The so-called *Compensation Mechanism* intends to cover the part of the horizontal network costs in a given region, which is deemed due to transit. This compensation "shall be paid by the operators of national transmission systems from which cross-border flows originate, and the systems where those flows end." In [8], ETSO explains that one should aim at measuring the impact of the different transactions on the total cost of the network. The extent to which this can be done is debatable. We here assume that the work has been done and refer to the cost of the transmission network as a function of intra and inter-zonal transactions. We therefore assume a cost function of transmission that consists of a fixed charge and a linear part that depends on these transactions. Failure to find the relation sought by ETSO means that the linear part vanishes and the cost of the network is only represented by a fixed charge.

The G and L charges are all based on an allocation of the cost of the network. We adopt the computation of the compensation mechanism defined in the Commission's guidelines, and consider several standard fixed cost allocation procedures. Because of subsidiarity we suppose that different methods can be implemented in different member States.

The principle of the computation of the various charges is as follows. Each regional network operator (TSO) incurs costs for its horizontal network, which we assume linear with the amount of electricity which flows through the local network[3]. Let $NC_r(q)$ denote the network costs in region r, and let $f^c_{r,j}$ denote the power transmission distribution factor (PTDF, see also section 3.5.2) of a transaction between generator j and customer c, on the horizontal network of region r, e.g. it represents the fraction of a given transaction which effectively passes on the horizontal network of a given region. Network costs can then be expressed by:

$$NC_r(q) = F^r + \sum_{j,c} n^c_{r,j} \times (f^c_{r,j} q^c_j), \tag{3.19}$$

Some of this power flow involves transactions originating from an outer region, and destined to another one. These are grouped under the generic appellation of *transit*. How the actual transit is effectively measured is another issue, which will not be tackled in details here. We retain, as required by the Regulation, that it should be physically measured, and not result from a contract path calculation. The part of network costs which is deemed due to transit will however be compensated for, by the Compensation Mechanism. This part is estimated as a fraction of the annual network costs, given by the ratio of the measured regional transit to the total regional transmitted flow (including transit). The sum over all regions of these compensations gives the total amount to be funded by regions inducing transit. This is implemented as follows.

Let $T_r(q)$ denote the transit flow across region r, and $V_r(q)$ be the maximum between total regional load and generation. The total amount to compensate for all regions, CF, is obtained by

$$CF(q) = \sum_r \frac{T_r(q)}{V_r(q) + T_r(q)} NC_r(q) \tag{3.20}$$

[3]The Regulation recommends using of *forward looking long-run average incremental cost* (LRAIC). For the sake of simplicity and as mentioned above, we will stick here to the sum of a fixed charge and linear function of the flow for modelling the cost of the network. This approximates the LRAIC by a constant value. To the best of our knowledge, there is to date no more sophisticated proposal to compute LRAIC.

The funding of compensations is based on the notion of *cumulated absolute net flow* (denoted $NF_r(q)$), which is defined as the cumulated absolute value of the difference between hourly imports and exports at the borders of a given region. Regions inducing transit contribute to fund the compensation mechanism in proportion of their net flows. More precisely, the contribution of the TSO in region r to the compensation mechanism is obtained by the expression

$$\frac{NF_r(q)}{\sum_{r'} NF_{r'}(q)} \times CF(q) \tag{3.21}$$

Each TSO thus incurs horizontal network costs, $NC_r(q)$, of which a fraction deemed due to transit is compensated for. But the TSO must also contribute to the compensation mechanism for the transit it creates on other regional networks. The resulting net cost should, at the equilibrium, be balanced with the revenue collected through the G and L charges. This balance is expressed by:

$$\sum_{\substack{j \in J^r \\ c \in C_j}} G_r q_j^c + \sum_{\substack{c \in C^r \\ j \in J^c}} L_r q_j^c = \Big(1 - \frac{T_r(q)}{V_r q + T_r(q)}\Big) \times NC_r(q) + \frac{NF_r(q)}{\sum_{r'} NF_{r'}(q)} \times CF(q). \tag{3.22}$$

The left hand side of this expression is the total revenue collected through the G and L charges in region r. The first term of the right hand side represents the financing of the cost of the network attributed to domestic transactions, while the second part is the amount paid to the compensation mechanism. The repartition of access revenues between local generators (through G) or consumers (through L) is left to the discretion of the local Regulator.

The construction of these charges suggests two remarks. Except for technicalities related to the computation of imports and exports, cost allocations rules (more generally average cost pricing) are always expressed as non-linear equalities. These relations are not difficult to specify but they destroy any convexity property of the model. They may generate models that have no solution (when fixed costs are too high) or several solutions. This requires some attention when interpreting the results. Failure to find an equilibrium suggests an error in the fixed charges of the network. It is indeed unlikely that the fixed charges of transmission may be so high as to make the model unfeasible. High G or L values suggest that the model selected to allocate these fixed charges to a small demand, and hence that it chose the wrong equilibrium.

The allocation of the costs of the network also suggests models of different levels of complexity. Suppose first that the cost of the horizontal network reduces to an exogenous fixed charge. This is equivalent to a standard short-term economic problem that simulates the operations of the system within fixed transmission capacities. This suggests two different models. One may suppose that one uses past observed flows as an allocation key. This is the simplest model with exogenous G and L charges. A more complex problem is to allocate the network costs according to the flows computed by the model. The total cost to allocate is then exogenous, but the G and L charges are not.

Last, one may also consider a long-term view where the capacities of the horizontal network are endogenous. Both the cost of the horizontal network and the flows used to allocate this costs are then endogenous. This is the most complex model. We model these three different cases. Note that endogenous network cost should, in principle, be accompanied by endogenous generations capacities. This (significant) extension is left for further research.

3.5.2 Congestion management

Generators and customers also need to pay a congestion charge. Following ETSO/EuroPEX proposal we suppose that access to the interconnection is ruled by a mix of coordinated auction and decentralised market coupling (hereafter referred to as Flow-based Market Coupling) that allocates the transfer capacities to those that attribute the highest value to it.

The management of congestion through the Flow-based Market Coupling method raises relatively little technical difficulty once one supposes that the proposed system is indeed viable. We accordingly make the following assumptions. The European grid is modelled as a virtual network that connects zones (notional zonal nodes) through "transfer capacities". The flows on the transfer capacities are obtained from the injections and withdrawals through power distribution factors (PTDFs). Transfer capacities are known and non-contingent. The Flow-based Market Coupling model then reduces to a set of complementarity relations that essentially state that (i) the flows resulting from the zonal injections and withdrawals remain within the transfer capacities, (ii) zone-to-zone transmission prices are obtained from transfer capacity prices very much like in flowgate models and (iii) the price of a transfer capacity can only be different from zero when it is fully used. We further make the assumption that (a) netting is allowed (this assumption conforms to the Regulation but not to the current EU practice), (*b*) flows that are feasible for the virtual network are also feasible for the real network (this assumption is implicit in ETSO's work) and (*c*) agents take the zone-to-zone prices as given and do not take advantage of the separation of the energy and transmission market to further exert market power (see [3] for a discussion of that assumption).

As a result we obtain the following model. Let $IC_{r,r'}$ denote the transfer capacity from region r towards region r', provided that these regions are directly interconnected. These capacities might be different in one direction or the other. Let also $\phi^c_{j|r,r'}$ denote the fraction of the transaction from j to c flowing through interconnection (r, r') (difference of PTDFs between (r, r') and some hub node). These have symmetric values (that is, $\phi^c_{j|r,r'} = -\phi^c_{j|r',r}$), and are assumed zero for any intra-regional transaction.

The congestion charge for the use of a saturated interconnection capacity is taken equal to the dual variable $\delta_{r,r'}$ of the capacity constraint. This expresses $\forall r \neq r'$ as:

$$\sum_{(j,c)\in CBT} q^c_j \phi^c_{j|r,r'} \leq IC_{r,r'} \qquad \perp \qquad \delta_{r,r'} \geq 0. \tag{3.23}$$

For any interconnection (r, r') used by a particular transaction (j, c), the unit contribution to congestion charges will be $\delta_{r,r'}\phi^c_{j|r,r'}$, and the total congestion charge incurred by this transaction is given by:

$$X^c_j q^c_j = \left(\sum_{r\neq r'} \delta_{r,r'} \phi^c_{j|r,r'} \right) q^c_j. \tag{3.24}$$

This ensures that, at equilibrium, each transaction contributing to the saturation of an interconnection capacity will be penalised proportionally to the actual flow it generates on this interconnection, while a transaction generating a counterflow would be rewarded based on the same rule. One can also see that the above equation defines a point-to-point charge (from j to c) as would result from the application of locational marginal prices to ETSO's virtual network.

3.5.3 The Regulatory Committee

The European law does not foresee any European Regulator. It plans instead for a Regulatory Committee that will decide "on guidelines on compensation on transit flows, on harmonisation of national transmission tariffs and on allocation of cross-border interconnection capacity". It is foreseen that national

Regulators play a crucial role in this Committee. In contrast with Directives, a Regulation applies as such to all Member States in European law. But it can be formulated in vague enough terms that a lot of implementation details remain to be decided by the Committee or the individual Regulators. We mentioned in section 3.2 various points where Member States could take advantage of the freedom allowed by Directives. We take it for granted that the access system derived from the work of the Forum will be enforceable throughout the EU and make the more heroic assumption that the Flow-based Market Coupling proposal of ETSO and EuroPEX will eventually apply. One parameter of the design is currently left to Member States, namely the allocation of the network costs between the G and L components. This should be harmonised but it is not clear that there will be a unique allocation rule. We consider the choice of this parameter as an example of the decentralised governance that the Regulatory Committee could put in place, and model it as a Nash equilibrium where each Regulator selects the fixed cost allocation parameters in a way that maximises the welfare of its own constituency, taking the actions of the other Regulators as given. The actual performance of the market results from generators and customers actions in the second-stage equilibrium. We compare this solution to the one where there would be a single EU Regulator that maximises the overall welfare of the EU. Because those regulatory actions determine a market between producers and consumers, the overall problem is a two-stage game: Regulators decide on the network cost allocation parameters in a first stage taking into account the reaction of the market in the second stage of the game. We thus seek a sub-game perfect equilibrium.

Since network operators are supposed to adjust their access charges G and L in order to balance their costs and revenues, the welfare in each region is the sum of its producers benefits, plus the consumer surplus. Instead of writing the problem with explicit G_r and L_r, we formulate the problem as a decision, by each local Regulator, of the share λ_r of the total regional access charges (which, given the cost of the grid, results in G_r and L_r), which have to be paid by the local generators through access charge G_r.

$$W_r(q, \lambda_r) = \sum_{c \in C^r} CS^c(q^c) + \sum_{j \in J^r} \pi_j(q). \tag{3.25}$$

We successively consider three versions of the model :

- the *Second-Stage Only*, where the parameter defining the repartition of access charge revenues between local generators and customers (λ_r) is fixed *a priori* to some exogenous arbitrary level,

- the *Nash Equilibrium*, where each local regulator decides individually on the value of λ_r so as to maximise its local welfare, taking the choice of other regulators as given, and

- the *Cooperative Equilibrium*, where the total welfare is maximised by a supra-national entity, regardless of its repartition between regions.

For each of these three assumptions, we consider various combinations of captive market regulations, compensation mechanisms, competitive behaviours, as well as the sensitivity of the solution to some model parameters.

In this paper we only focus on two specific results of our study. More detailed numerical experience can be found in [2].

3.6 Sample numerical results

The model has been used first for testing purposes and to explore various issues. We here illustrate two particular questions, both related to the governance of cross-border trade through a Regulatory Committee instead of a EU Regulator.

3.6.1 Model Formulation and Computational Aspects

The model has been written in GAMS. Mathematically, the three versions of the first-stage game lead to model formulations of considerably different complexities.

The *Second-stage only* formulation is a pure equilibrium problem (EP). This was formulated in GAMS as a DNLP problem, with a set of equality constraints $h(q) = 0$ on the one hand, and inequalities which required a formulation using a pair of GAMS constraints of the type:

$$\mu \geq 0, \qquad g(q) \leq 0 \quad \text{and} \quad \mu.g(q) = 0. \tag{3.26}$$

on the other hand. Some of these constraints (in particular cost allocation rules) introduce non-convexities, implying the absence of uniqueness or even

The First-Stage Game

2nd Stage only

$\forall r$:

$\lambda_r = \overline{\lambda}_r$

OR

Nash equilibrium

$\forall r$:

$\max_{\lambda_r} W_r(q)$

OR

Cooperative equilibrium

$\max_{\lambda_1 \dots \lambda_R} \sum_r W_r(q)$

Second-Stage Equilibrium

Equilibrium Conditions on the Eligible Markets :
equations (3.8-3.10)

Equilibrium Conditions on the Captive Markets :
equations (3.11-3.13) and (3.16-3.18)

Auction on Interconnection Capacities :
equations (3.23-3.24)

Compensation Mechanism for Transit Costs :
equation (3.20)

Access Charges to cover Network Costs :
equations (3.19) and (3.22)

Figure 3.1: The global structure of the model

existence of the solution. This model was solved using CONOPT2 with default options.

Second in complexity, the *Cooperative equilibrium* results in a mathematical program subject to equilibrium constraints (MPEC). The first stage is formulated as an optimisation problem while the second stage is the same as in the EP. This can still be written in a single optimisation model, subject to a set of equilibrium constraints, possibly non-linear, which again destroys uniqueness and existence guarantees for the solution. We keep the GAMS formulation of the equilibrium constraints as in the *second-stage only* formulation, but this time we maximise an objective function which is the sum of the regional welfares, depending on additional variables λ_r (the repartition of access charges between local generators and consumers). It was solved using CONOPT2.

Finally, the most complex model is the *Nash equilibrium* since this one can be cast in the category of Equilibrium Problem with Equilibrium Constraints (EPEC). This was impossible to formulate as a single-instance model. It was solved by a succession of MPECs, similar to those in the *cooperative equilibrium*, in which one single regulator optimises its local welfare by tuning its repartition λ_r, leaving other regulators' decision unchanged ($\lambda_{r' \neq r}$ fixed). The full second-stage equilibrium constraint set is still included in each instance. Empirical convergence of the iterative process turned out to be very good in this case, though again neither uniqueness nor existence of the equilibrium can be guaranteed *a priori*. CONOPT2 with default options turned out to be very efficient on our problem.

Since several solutions may exist for each of these three problems, we tried to provide the algorithm with a reasonable starting point. This starting point was constructed, based on the content of discussions that were held by the Florence Forum committees at the time, namely that network charges should mainly be covered by access charges to consumers, rather than generators. This, coupled with initial guesses on bilateral transactions would lead to the computation beforehand of network flows, tariffs, prices, ... Multiplicity of equilibria has been observed in some cases, with distinct solutions in terms of access charges repartition, but yielding almost no difference in value for the total welfare.

Under some regulatory assumptions, however, it turns out that the equilibrium point ends opposite to starting point, in terms of repartition of access charges between generators and consumers. This indicates that charging con-

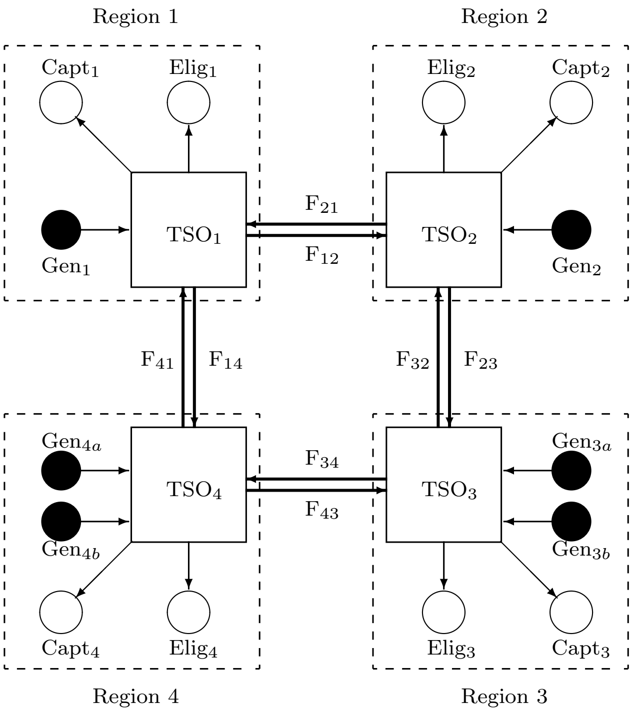

Figure 3.2: Stylised model

sumers only to cover network costs may be inefficient in terms of global welfare.

3.6.2 Data description

We consider a stylised model comprising four regions. Regions 1 and 2 are comparatively small with respect to regions 3 and 4. In each region r, there is one Transmission System Operator (TSO_r), one or two generation companies (Gen_r), and two customer segments ($Elig_r$ is competitive and accessible to each of the four producers, while $Capt_r$ is partially removed from competition by barriers to trade, and supplied by the local generator(s) only). The four regions are linked together by interconnection capacities. Note that we here assume two generators serving the less competitive markets in regions 3 and 4. The situation is sketched in figure 3.2. An extensive description of the data, as well as tables and figures illustrating numerical results can be found in [2].

3.6.3 Impact of market structure and regulation on the global welfare

Figure 3.3 illustrates the variations in total welfare, for all envisaged combinations of market structure and regulation. Each dot represents the combination of an assumption about the first-stage game, chosen between :

- **Fixd** for fixed allocation of network charges, or second-stage only,
- **Nash** for the Nash non-cooperative equilibrium, and
- **Coop** for the cooperative equilibrium between network regulators,

and the regulation in the second-stage game, one of :

- **PROF** for profit maximisation without restriction,
- **PCAP** for price cap
- **R-FDC**, **C-FDC** and **Q-FDC** for fully distributed costs, in proportion of revenues, costs and quantities respectively, and
- **LRIC** for long-run incremental costs.

The variations on the vertical axis are expressed in terms of improvement with respect to the worst case, which corresponds to the fixed (arbitrary) allocation of network costs, leaving less-competitive customers exposed to profit maximisation in the second stage.

The most significant numerical values are displayed in table 3.1. Columns distinguish between the first-stage game assumptions. A first part in the table gives the total welfare for the retained second-stage regulatory rules. In the second part, the range of equilibrium prices in captive markets across the four regions are provided. Similarly, prices on competitive eligible markets are displayed in the third part of the table.

"Second-stage only" equilibrium. We first consider a model version where the allocation of network costs between generators and customers is fixed to arbitrary shares, without any concern of welfare optimisation. This first version of the model assumes that competition in the competitive markets is represented by a Cournot equilibrium, where each agent is aware of its own impact

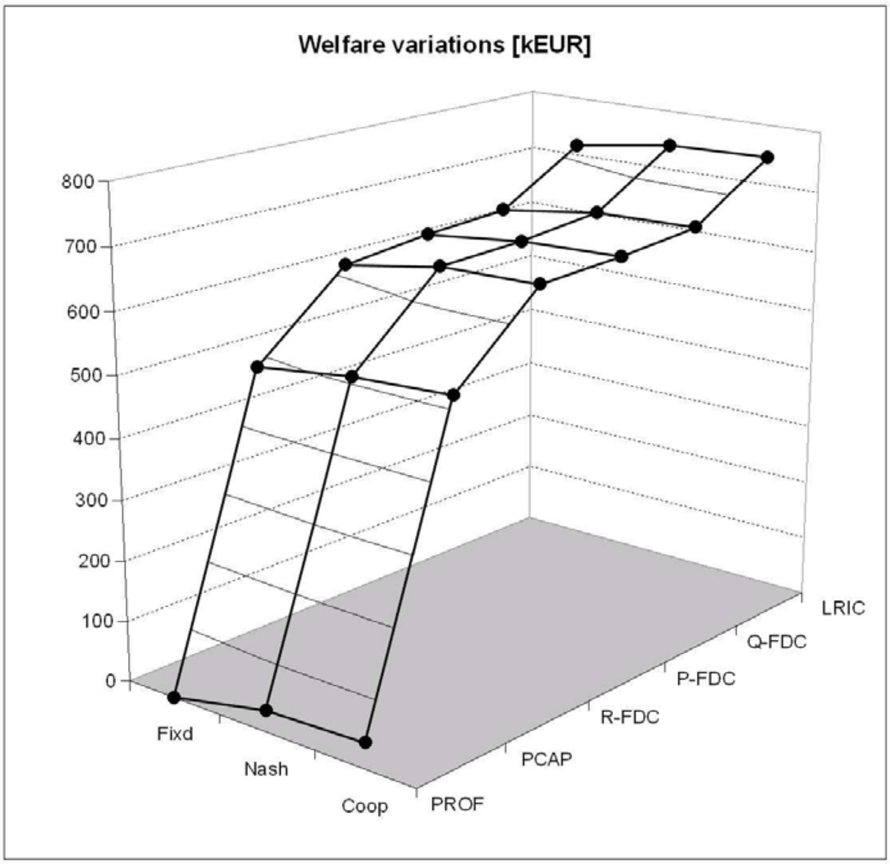

Figure 3.3: Welfare evolution according to structure and regulation

	2nd Stage Equilibrium	**Nash Equilibrium**	**Cooperative Equilibrium**
Total Welfare [million EUR]			
PROF	3.807	3.848	3.849
PCAP	4.305	4.331	4.335
R-FDC	4.423	4.420	4.456
C-FDC	4.431	4.463	4.469
Q-FDC	4.443	4.475	4.481
LRIC	4.532	4.561	4.566
Captive Mkt Price [EUR/MWh]			
PROF	64 ... 83	64 ... 84	64 ... 83
PCAP	50	50	50
R-FDC	38 ... 45	38 ... 45	38 ... 45
C-FDC	38 ... 44	39 ... 44	39 ... 44
Q-FDC	38 ... 43	38 ... 42	38 ... 43
LRIC	26 ... 30	26 ... 30	27 ... 30
Eligible Mkt Price [EUR/MWh]			
PROF	30 ... 35	31 ... 35	29 ... 35
PCAP	31 ... 35	32 ... 35	30 ... 35
R-FDC	31 ... 36	32 ... 35	31 ... 35
C-FDC	31 ... 36	32 ... 36	31 ... 35
Q-FDC	31 ... 36	32 ... 35	31 ... 35
LRIC	32 ... 36	33 ... 36	31 ... 36

Table 3.1: Welfare, Equilibrium Prices and Compensation Fund

on prices. Conjectural variation parameters have been introduced in order to provide a more general representation of imperfect competition equilibria. Several pricing rules are envisaged for the less competitive markets: no regulation, price caps, fully distributed costs and long run incremental cost.

Welfare evolution according to the regulation of the less competitive market. Quite expectedly, the welfare is most degraded when generators are allowed to practise unrestricted profit maximisation; it is largest when prices reflect long-run incremental costs. In between, for the different cost allocation methods, welfare increases from revenue-proportional, then cost-proportional, to quantity-proportional allocation. Price cap welfare depends on the level of the cap and might encompass the whole range from the lowest (when caps are so high that they boil down to pure profit maximisation prices) to the highest (when caps are set equal to long run incremental cost).

Moving to a Nash non-cooperative equilibrium. Allowing the Regulators to optimise their local welfare non-cooperatively leads to a pure strategy Nash equilibrium (if it exists). Equilibrium prices are barely affected by this move to a two-stage problem. The total welfare is significantly increased by this move, but this results from the fact that we come from some arbitrarily fixed repartition of access charges (between G and L), to a repartition which maximises (in a Nash sense) the regional welfare. The potential improvement therefore depends on the values of the initial arbitrary repartition. In any case, this implies that the allocation of the costs between G and L matters.

Moving from Nash to Co-operative equilibrium. Moving one step further towards welfare optimisation, we choose now to look at the total welfare, over all four regions, instead of independently optimising local welfare. We should therefore, in each scenario of regulation of the less competitive market, obtain a total welfare which is greater than (or equal to) the value obtained in the corresponding Nash equilibrium. This may be verified in the numerical results in [2].

Looking at equilibrium prices, however, we see that they are not dramatically modified by this move to a cooperative equilibrium. The main change (with respect to the Nash equilibrium) resides in the repartition of access charges between customers and generators. While regions 2, 3 and 4 are not affected (or very little) by the cooperation of Regulators, region 1 (the "smallest" one in terms of volume) is largely perturbed by this cooperation: in region 1, the access charge passes from 100% on customers to 100% on generators in Cooperative equilibrium. The result is that the generator in region 1 loses a considerable amount of profit (depending on the less competitive market rule) when switching from the Nash to the Cooperative equilibrium. This mainly benefits the generator in region 2. Profits in regions 3 and 4 do not change significantly.

3.6.4 Impact of Competitive Assumptions on the Equilibrium

We get interesting variants of the base case by tuning the market power parameters. We can modify the assumed degree of competition between agents by changing the coefficient of conjectural variation. The latter, denoted with $\xi^c_{j,j'}$, $(j \neq j')$, expresses the belief of generator j about the reaction of competitor j' to a unit change in output sales from producer j to customer c.

Figure 3.4 illustrates the influence of the assumptions made on the competitive behaviour. In the standard Cournot paradigm, all of these parameters are set to 0 (i.e. all agents believe that competitors will not react to a change in their output). This correspond to the "base case" which served as reference situation in this study.

Setting the conjectural variation parameters to a value of $+1$, for each pair of agents competing on an eligible market, leads to a *collusive equilibrium* between agents. In the latter case, all equilibrium volumes drop around 50% of the base case level. The end result is also disastrous in terms of welfare which decreases by around 25% with respect to the *base case* and prices on eligible markets are soaring up to 55 EUR/MWh.

On the contrary, when set to a negative value, the equilibrium tends towards perfect competition. Note, for the interpretation of the results, that we are here talking of a short-term perfect competition equilibrium, that is, one where prices do not embed the capital cost of the equipment. Probably because of the difficulty of covering the fixed charges of the network, the model does not find any feasible solution for the perfectly *competitive equilibrium*. But solutions obtained with intermediate values (getting very close to this com-

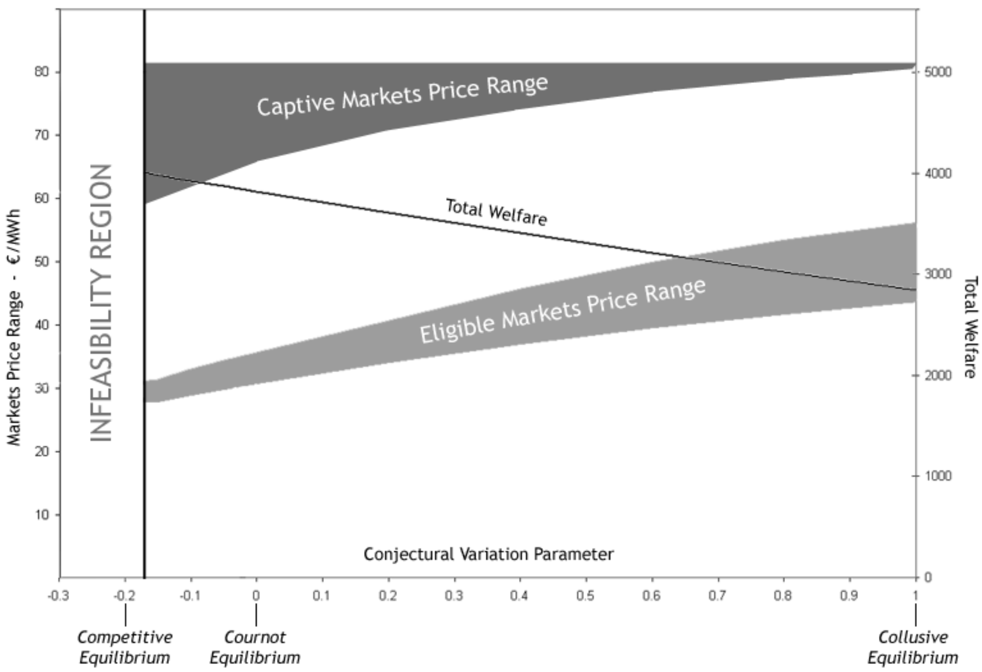

Figure 3.4: Impact on Competitive Assumptions

petitive equilibrium limit) show a significant increase in total welfare (above 5%), while prices charged to the competitive customer segments quickly drop towards marginal prices although less competitive customers prices remain at their level.

These situations are interesting and deserve more investigation; they signal conditions where the allocation mechanism hampers the development of competition, in this case by preventing a competitive equilibrium.

We can also play with the generator's awareness of its own impact on market price, by modifying the value of parameter $\xi^c_{g,g}$. When this parameter is set to +1, the agent is aware of the impact of its own decisions on market price. When set to 0, it is not aware at all and behaves like a price taker. Again this latter extreme case may lead to infeasible constraints because of the difficulty of covering the fixed costs of the network. In this particular case, a feasible solution can only be achieved with a relaxed version of the problem, where the compensation fund is exogenously set to zero, that is, when no compensation for alleged transit cost is enforced. The increase of welfare is however considerable, since each customer (including captive ones) is now priced at marginal cost. Keeping compensation mechanisms as in the *base case*, we get a feasible solution with values of $\xi^c_{g,g}$ down to 0.15, but cannot go below that value. The welfare increases by almost 25% with respect to the *base case*, and prices on all customer segments drop significantly, also for the captive customer.

3.7 Conclusions

We developed a computational framework, destined to encompass the two important issues in Cross-Border Trade: congestion of interconnection capacities, and access to interconnected networks.

The model manages the *congestion* of interconnection capacities through a mix of coordinated auction and decentralised market coupling, where each agent pays the opportunity cost for congested interconnection capacities. Using simplified power distribution factors, the auction price of the interconnection is taken as the dual value of using congested interconnection capacities.

Pricing the *access* to interconnected networks does not only involve the coverage of local network costs in areas of injection and delivery, but also transit costs induced on other regional networks through loop flows. We have implemented several former combinations of ETSO proposals for compensation mechanisms ([2] and this paper). This suggests that it will also be possible to implement the final mechanism.

The problem is cast in the form of a game between regional Regulators, and is modelled by a two-stage equilibrium problem. In the first stage, Regulators decide on the allocation of their regional network costs between generators and customers, taking the consequences of their decisions on the energy market into account. The first-stage game has different versions : either they maximise their regional welfare non-cooperatively (Nash equilibrium), or they centralise the decision as a super-regulator (leading to a cooperative equilibrium).

In the second-stage equilibrium, the consequences of first-stage's decisions are assessed by modelling the energy market as the result of imperfect competition equilibrium on *competitive market* (where competition intensity and market power exercise are modelled through conjectural variation parameters, ranging from perfect competition to collusion paradigms), coupled with regulated pricing on the domestic *less competitive markets* (ranging from oligopoly pricing to long-run incremental cost pricing, passing by price caps and costs allocation rules). The "rules" that come out of the first-stage game largely influence the final equilibrium.

The total welfare is mainly dependent on the regulation of the less competitive market. However, the repartition of the welfare between the agents (generators and customers) differs considerably, according to the network cost allocation rules selected in the first stage. The distortion of competition that results from this allocation shows up as expected, and welfare increases when one moves from a non-cooperative to a cooperative equilibrium. But the model also reveals questions that deserve more analysis in a realistic case study: some allocation methods foreseen by the compensation mechanism do not necessarily perform as expected. They may reward those deemed to cause transit cost. They may also become inapplicable in extreme conditions. This signals that they may induce undesired effects in non-extreme conditions. In some cases, it appears impossible to find an equilibrium when competition increases too much.

Even though both the compensation methods and allocations of network costs may not influence total welfare much, they drastically influence the allocation of that welfare. This is true whether across control areas or inside each area. Last but not least, the model reveals a close interaction between the access charge and the value of the interconnection (see [2]). This indicates that these two problems which have been considered separately by the Florence Forum and are commonly seen as distinct issues in the literature may be more interdependent than expected.

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Chapter 4

Conjectural variations models:

Integrating market power and conjectural variations in power models

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This chapter tries to illustrate the integration of perfect or imperfect competition behaviours in power models. Two major concerns are the implementation of market power mechanisms, as well as the representation of conjectural variation. We will try to demonstrate how this relates to the *conjectured supply curves* introduced by Day et al. in [1]. The last part of this paper deals with consistence and estimation topics, including a numerical illustration on a stylised model.

4.1 Introduction and General Notation

Several power models have been developed to represent competitive behaviours of power generators, supplying their customers through a transmission network. *Perfect Competition* models assume that generators supply their power at their marginal cost, which is rather optimistic but quite easy to implement. Generators in the real world exert (or at least try to exert) their market power to influence prices. On the other hand, the *Cournot* game, which is often referred to in competition models and captures market power behaviours, generally results in too high prices. Further, one of its basic assumptions is that when a generator makes a strategic move to increase its profits, he/she believes that competitors will not react. In general, market players about to make a strategic move do not expect their competitors to wait statically: they try to anticipate competition's reaction through some conjecture. To overcome these weaknesses, we explore an approach for modelling both market power and conjectural variations in power models.

We introduce the following notation. Let $f \in F$ be the set of power generators. We denote by $i \in I$ the set of network nodes, and the power consumption at node i by $q_i(p_i)$, at some price p_i. The consumption at one particular node might be supplied by different generators, and their respective shares are denoted by s_{fi}, such that

$$q_i = \sum_f s_{fi}. \tag{4.1}$$

On the generation side, the cost function $C_{fj}(g_{fj})$ gives the cost for generator f to produce and inject g_{fj} MW at node j. All generated power is supposed to be injected in the network at its generation node.

Let y_{ij} be the power flowing on a network line (i, j). The vectors of network flows y, injections g and consumptions q must satisfy a set of network constraints (which we will not elaborate upon in this document). We simply denote the set of network constraints by the equation:

$$(g, q, y) \in NC, \tag{4.2}$$

where the set NC of network constraints should always include the Kirchoff's first law, but it might also include Kirchoff's second law, or even nomograms. See [4] for details on network constraints formulation.

We make the standard assumption that one particular network node is the *hub*, and denote by w_i the transportation charge from the hub to node i. Smeers [4] showed how the optimal values of these transportation charges may be obtained from dual values of the optimisation problem.

4.2 Demand and Utility Functions

We assume that consumers react to price along a linear demand function. The (inverse) linear relation is the following:

$$p_i = p_{i0} - \frac{p_{i0}}{Q_{i0}}\ q_i, \tag{4.3}$$

where $q_i = \sum_f s_{if}$ is the total demand of customer i, and the price p_i does not directly depend on the generator f. For each customer, this allows for the definition of a *Utility Function.* The latter may be formulated by :

$$U_i(q_i) = p_{i0}q_i - \tfrac{1}{2}\frac{p_{i0}}{Q_{i0}}q_i^2. \tag{4.4}$$

It is important to mention that this formulation is likely to raise problems if we wish to represent price-inelastic customers: in this case, p_{i0} should be set to $+\infty$, and the following sections would not apply. Section 4.8, at the end of this paper, mentions a method to overcome this problem.

4.3 The Generator's Problem

Each generator tries to maximise its own profit. Therefore, it tries to find the sales quantities s_{fi} which maximise the following problem:

$$\begin{aligned}
\max \quad & \sum_i (p_i - w_i^*)s_{fi} - \sum_j \left[C_{fj}(g_{fj}) - w_j^* g_{fj}\right] \\
& (s_f, g_f) \in X_f \qquad s_{fi} + s_{-fi} = q_i \qquad \forall i \in I \\
& p_i = p_{i0}\left(1 - \frac{q_i}{Q_{i0}}\right)
\end{aligned} \tag{4.5}$$

As we will show, the key element is in the agent's perception of the very first term, $p_i s_{fi}$, and more specifically in its belief about how he could influence this term. This will give give the generator an incentive to exert its (potential) market power or to speculate on conjectural variations.

Although the price of transmission services w_i from the hub to node i is a variable from the market model's point of view, it is assumed to be viewed as fixed (exogenous) by producers, and therefore denoted with w_i^* (see [1, 2]): generators do not believe that they are able to influence w_i^* or w_j^*.

4.4 The Transmission Network Equilibrium Problem

The transmission network we consider here is very simplified. We consider a DC-approximation without losses. It is similar to the *flowgate* concept. Consider that customers and generators are located in geographical regions, and that neighbour regions are connected with interconnection capacities (that is, flowgates). We denote with K the set of bidirectional interconnections. Let IC_k^+ and IC_k^- define the interconnection capacities in the forward and backward directions. Let κ_{ki} denote the *power distribution factor*, giving the fraction of a transaction going from node i to the hub node, through interconnection k. These power distribution factors are used to take into account Kirchoff's second law.

Let I^0 denote the set of network nodes without the hub. We need to compute the *net injections* at each node, using variables $z_i, i \in I^0$, defined by:

$$\forall i \in I^0 : \qquad z_i = \sum_f (g_{fi} - s_{fi}). \tag{4.6}$$

The interconnection capacity constraints may be represented using the following relations:

$$\forall k : \qquad -IC_k^- \leq \sum_i \kappa_{ik} z_i \leq IC_k^+ \tag{4.7}$$

Efficient rationing of scarce interconnection capacity can be achieved using opportunity cost pricing, which, in a complementarity form, may be expressed as computing the transmission price from node $i \in I^0$ to the hub, summing the dual values of used interconnections, weighted by the power distribution factors:

$$\begin{aligned}
&\sum_{i \in I^0} \kappa_{ik} z_i - IC_k^+ \leq 0, \qquad \delta_k^+ \geq 0, \qquad \delta_k^+ (\sum_{i \in I^0} \kappa_{ik} z_i - IC_k^+) = 0 \\
&-\sum_{i \in I^0} \kappa_{ik} z_i - IC_k^- \leq 0, \qquad \delta_k^- \leq 0, \qquad \delta_k^- (-\sum_{i \in I^0} \kappa_{ik} z_i - IC_k^-) = 0 \\
&-w_i^* = \sum_k \kappa_{ik} (\delta_k^+ + \delta_k^-).
\end{aligned} \tag{4.8}$$

4.5 Market Power and Conjectural Variation

In this section we focus on the interpretation of the first term in the generator's equilibrium conditions. We introduce the symbol ð to denote a *conjectured* partial derivative, which expresses generator f's own *belief* about its influence on price as a result of a change in its output – this may thus differ from its actual influence:

$$\frac{\partial}{\partial s_{fi}} p_i s_{fi} = p_i + s_{fi} \frac{ðp_i}{ðs_{fi}}. \tag{4.9}$$

Let us first introduce the *market power* parameter $\theta_{fi} \in [0, 1]$, which accounts for generator f's awareness of the impact of its own moves on market price. We assume that if valued zero, then the generator behaves as a price taker (i.e. generator f does not consider that prices will move if it changes its supply s_{fi}). If valued one, on the contrary, then generator f is perfectly conscious of the change in price induced by a change in supply. Any value in between could be used to tune various behaviours. This leads us to the following relation :

$$\frac{\eth p_i}{\eth s_{fi}} = \theta_{fi} \frac{\partial p_i}{\partial s_{fi}}, \qquad \theta_{fi} \in [0,1]. \tag{4.10}$$

Conjectural variations operate in the last partial derivative, and have to be considered carefully, in order to reconciliate them with Day's conjectured supply curves. Remembering the mathematical expression of the inverse demand function:

$$p_i = p_{i0} - \frac{p_{i0}}{Q_{i0}} q_i = p_{i0} - \frac{p_{i0}}{Q_{i0}} \sum_{f'} s_{f'i}, \tag{4.11}$$

we may state the partial derivative of this expression, with respect to the sales variable from generator f to customer i, with:

$$\frac{\partial p_i}{\partial s_{fi}} = -\frac{p_{i0}}{Q_{i0}} \sum_{f'} \frac{\partial s_{f'i}}{\partial s_{fi}} \tag{4.12}$$

Now we define the *conjectural variation* parameters $\phi^i_{f'f}$, representing the belief of generator f about the competitor's reaction if it increases its sales by one unit. Hence, we have the relations :

$$\begin{aligned} \phi^i_{f'f} &= \frac{\partial s_{f'i}}{\partial s_{fi}}, \quad \forall f' \neq f, \\ \phi^i_{ff} &= 1. \end{aligned} \tag{4.13}$$

Sometimes conjectural variation might not be estimated for each competitor separately, but a global conjectural variation could be used instead to represent the reaction of all competitors together. We define Φ^i_{-f} as the aggregate conjectural variation from generator f's perspective:

$$\Phi^i_{-f} = \sum_{f' \neq f} \phi^i_{f'f}. \tag{4.14}$$

Using these notation, generator f's subjective influence on price becomes:

$$\frac{\eth p_i}{\eth s_{fi}} = -\theta_{fi} \frac{p_{i0}}{Q_{i0}} \sum_{f'} \phi^i_{f'f} = -\theta_{fi} \frac{p_{i0}}{Q_{i0}} \left(1 + \Phi^i_{-f}\right). \tag{4.15}$$

This now leads us to the full formulation of generator f's partial derivative of revenues, with respect to sales:

$$\frac{\partial}{\partial s_{fi}} p_i s_{fi} = p_i - s_{fi} \theta_{fi} \frac{p_{i0}}{Q_{i0}} \sum_{f'} \phi^i_{f'f}. \tag{4.16}$$

The equilibrium condition for generator f can now be restated as in the following equation, where p_i has been replaced by its full linear expression:

$$p_{i0} - \frac{p_{i0}}{Q_{i0}} \left(\sum_{f'} s_{f'i} + s_{fi}\theta_{fi} \sum_{f'} \phi_{f'f} \right) = w_i^* + \frac{\partial}{\partial s_{fi}} \sum_j \left[C_{fj}(g_{fj}) - w_j^* g_{fj} \right]. \tag{4.17}$$

In the latter expression, the right-hand side of the equation defines the marginal cost of supplying customer i with one additional unit, given the transmission charges which prevail on the network.

The following subsections illustrate some special cases for values of the market power and conjectural variations.

4.5.1 Special Case: Perfect competition

Under the perfect competition paradigm, no generator is able to influence the price. They believe that equilibrium prices do not depend on the quantity they supply on this market, hence they behave as price takers. This is easily obtained by setting the *market power* parameters θ_{fi} to zero. Equilibrium condition (4.17) then reduces to :

$$p_{i0} - \frac{p_{i0}}{Q_{i0}} \sum_{f'} s_{f'i} \equiv p_i^* = w_i^* + \frac{\partial}{\partial s_{fi}} \sum_j \left[C_{fj}(g_{fj}) - w_j^* g_{fj} \right]. \tag{4.18}$$

In other terms, every generator equates his marginal cost to the equilibrium price in order to maximise its profit. This is exactly the perfect competition paradigm.

4.5.2 Special Case: Cournot competition

In order to obtain a pure *Cournot* equilibrium, the following conditions have to be fulfilled:

$$\forall f: \quad \theta_{fi} = 1, \qquad \forall f' \neq f: \quad \phi^i_{f'f} = 0, \tag{4.19}$$

Proof: Remember that, under Cournot assumptions, every generator perfectly anticipates the customer's price reaction to a change in supply, but considers that competitors will not move. This exactly corresponds to the numerical condition above: $\theta_{fi} = 1$ implies that the generator is fully aware of its influence on the price, but $\phi^i_{f'f} = 0$ for $f' \neq f$ assumes that the generator considers that competitors will not react to its own decisions. Note that the equilibrium condition (4.17) becomes:

$$p_{i0} - \frac{p_{i0}}{Q_{i0}} \left(s_{fi} + \sum_{f'} s_{f'i} \right) = w_i^* + \frac{\partial}{\partial s_{fi}} \sum_j \left[C_{fj}(g_{fj}) - w_j^* g_{fj} \right]. \quad (4.20)$$

Re-arranging this equation yields:

$$p_{i0} - \frac{p_{i0}}{Q_{i0}} \sum_{f'} s_{f'i} \equiv p_i^* = w_i^* + \frac{\partial}{\partial s_{fi}} \sum_j \left[C_{fj}(g_{fj}) - w_j^* g_{fj} \right] + \frac{p_{i0}}{Q_{i0}} s_{fi}. \quad (4.21)$$

The interpretation of this equilibrium condition is that for each generator, the equilibrium price is equal to the marginal cost (generation and transportation), plus a mark-up which is proportional to the market share. Looking at the particular case where all of the $|F|$ generators would have the same constant marginal cost mc, this equilibrium condition becomes:

$$p_{i0} - (1 + \frac{1}{|F|}) \frac{p_{i0}}{Q_{i0}} q_i = mc, \quad \Longrightarrow \quad p_i^* = \frac{p_{i0} + |F| \times mc}{1 + |F|}, \quad (4.22)$$

■

4.5.3 Special Case: Collusion

It is interesting to mention here that some limits have to be put on the *conjectural variation* parameters. *Collusion* will occur if equilibrium conditions of all competitors boil down to monopolistic equilibrium. This will be the case if, for every competitor, the following conditions are satisfied:

$$\forall f: \quad \theta_{fi} = 1, \qquad \forall f' \neq f: \quad \phi^i_{f'f} = 1, \quad (4.23)$$

that is, for any change in the supply from generator f, the reaction of other producers together is exactly equivalent.

Proof: Let us first simplify the equilibrium condition (4.17) by assuming that for each generator f the marginal generation and transportation cost is equal to the same constant mc. The equilibrium conditions for each agent now write:

$$\forall f: \qquad p_{i0} - \frac{p_{i0}}{Q_{i0}} \left(\sum_{f'} s_{f'i} + s_{fi}\theta_{fi} \sum_{f'} \phi_{f'f} \right) = mc. \quad (4.24)$$

Summing the $|F|$ identical equilibrium conditions, and substituting q_i where possible, gives:

$$|F| \times p_{i0} - \frac{p_{i0}}{Q_{i0}} \left(|F| \times q_i + \sum_f \left[s_{fi} \theta_{fi} \sum_{f'} \phi^i_{f'f} \right] \right) = |F| \times mc. \quad (4.25)$$

Provided that all market power and conjectural variation parameters are set to one, this results in:

$$|F| \times p_{i0} - \frac{p_{i0}}{Q_{i0}} \left(|F| \times q_i + \sum_f \left[s_{fi} \times |F| \right] \right) = |F| \times mc. \quad (4.26)$$

Re-arranging and dividing by $|F|$ results in the following equilibrium condition:

$$p_{i0} - 2\frac{p_{i0}}{Q_{i0}} q_i = mc, \quad \Longrightarrow \quad p_i^* = \frac{p_{i0} + mc}{2}, \quad (4.27)$$

which is exactly the definition of the monopolistic price under a linear demand function. ■

4.5.4 Special Case: Price War

On the other hand, *Price war*, or equivalently Bertrand competition, will result of the following set of numerical values for conjectural variation:

$$\forall f: \quad \sum_{f' \neq f} \phi^i_{f'f} = -1. \quad (4.28)$$

Proof: Let us restart from the individual equilibrium conditions for each generator:

$$\forall f: \qquad p_{i0} - \frac{p_{i0}}{Q_{i0}} \left(\sum_{f'} s_{f'i} + s_{fi} \theta_{fi} \sum_{f'} \phi_{f'f} \right) = mc_f. \quad (4.29)$$

The sum of conjectural variation parameters now boils down to zero, according to the numerical hypothesis mentioned above. Hence, the equilibrium condition for each generator will become:

$$\forall f: \qquad p_{i0} - \frac{p_{i0}}{Q_{i0}} q_i = mc_f \quad \Longrightarrow \quad p_i^* = mc_f. \quad (4.30)$$

This is equivalent to price war. ■

4.6 General Formulation: Optimisation Problem

We may now reformulate the generators' problem as an optimisation problem. The equilibrium conditions exposed for each generator are equivalent to those derived from the following maximisation problem:

$$\begin{aligned}
\max \quad & \sum_f \left[\sum_i \left(p_{i0} s_{fi} + \tfrac{1}{2} \sum_{f'} s_{f'i} K^i_{f'f} s_{fi} \right) - \sum_j C_{fj}(g_{fj}) \right] \\
& \forall f : \sum_i s_{fi} = \sum_j g_{fj}, \\
& \forall i : \sum_f s_{fi} = q_i, \\
& (g, q, y) \in NC, \text{ for some vector } y_i, \qquad (4.31)
\end{aligned}$$

where the coefficients of the quadratic form $K^i_{f'f}$ are given by:

$$\begin{aligned}
K^i_{f'f} &= -\frac{p_{i0}}{Q_{i0}} \qquad \forall f' \neq f, \\
K^i_{ff} &= -\frac{p_{i0}}{Q_{i0}} \times (1 + \theta_{fi} \sum_{f'} \phi^i_{f'f}). \qquad (4.32)
\end{aligned}$$

Bounding the conjectural variation parameters to their extreme cases of *price war* or *collusion*, the expression between braces in the last equation may range between 1 and $(1 + |F|)$. Individual bounds on the parameters are as follows, for each generator f :

$$\begin{aligned}
& 0 \leq \theta_{fi} \leq 1, \\
& \forall f' \neq f : \quad \begin{array}{l} -1 \leq \phi^i_{f'f} \leq +1, \\ \sum_{f' \neq f} \phi^i_{f'f} \geq -1. \end{array} \qquad (4.33)
\end{aligned}$$

4.7 Comparison with Day and Hobbs' approach

The quadratic form in (4.31) is equivalent to the sum of a uniform square matrix containing $-p_{i0}/Q_{i0}$ in every element, plus a diagonal matrix accounting for market power and conjectural variation parameters. We will now try to link the last formulation with Day et al. formulation as mentioned in [1]. The optimisation problem, using conjectured supply curves, is described by exactly the same optimisation model, only the definition of the quadratic form differs.

$$
\begin{array}{lcl}
K^i_{f'f} & = & -\frac{p_{i0}}{Q_{i0}} \qquad \forall f' \neq f, \\
K^i_{ff} & = & -\frac{p_{i0}}{Q_{i0}} - diag(B''_{fi}) \\
B''_{-fi} & = & \frac{B'_{-fi}}{1+B'_{-fi}\frac{Q_{i0}}{p_{i0}}} \\
p_{fi} - p^*_i & = & B'_{-fi}\left[s_{-fi}(p_{fi}) - s^*_{-fi}\right]
\end{array} \tag{4.34}
$$

The purpose of this section is to try to reconcile formulations (4.32) and (4.34). That is, we would like to show that the following equation holds:

$$
\theta_{fi}\frac{p_{i0}}{Q_{i0}}\sum_{f'}\phi^i_{f'f} = \frac{\frac{p_{fi}-p^*_i}{s_{-fi}(p_{fi})-s^*_{-fi}}}{1+\frac{p_{fi}-p^*_i}{s_{-fi}(p_{fi})-s^*_{-fi}}\frac{Q_{i0}}{p_{i0}}} \tag{4.35}
$$

This equation may be reformulated as :

$$
B'_{-fi} = \frac{p_{fi}-p^*_i}{s_{-fi}(p_{fi})-s^*_{-fi}} = \frac{p_{i0}}{Q_{i0}}\frac{\theta_{fi}\sum_{f'}\phi^i_{f'f}}{1-\theta_{fi}\sum_{f'}\phi^i_{f'f}} \tag{4.36}
$$

Considering the approach developed in section 4.5, we have stated in equation (4.15) that generator f's conjecture about its own influence on price is given by:

$$
\frac{\eth p_i}{\eth s_{fi}} = -\theta_{fi}\frac{p_{i0}}{Q_{i0}}\sum_{f'}\phi^i_{f'f}. \tag{4.37}
$$

This might be interpreted as follows: in order for generator f to move the price from its equilibrium value p^*_i to a value p_{fi}, it has to change its supply from s^*_{fi} to s_{fi}, such that:

$$
(p_{fi}-p^*_i) = \frac{\eth p_i}{\eth s_{fi}} \times \left(s_{fi}-s^*_{fi}\right) = -\theta_{fi}\frac{p_{i0}}{Q_{i0}}\sum_{f'}\phi^i_{f'f} \times \left(s_{fi}-s^*_{fi}\right). \tag{4.38}
$$

On the other hand, moving its supply from s^*_{fi} to s_{fi} implies a conjectural move from its competitors by:

$$
\left(s_{-fi}(p_f i) - s^*_{-fi}\right) = \sum_{f'\neq f}\phi^i_{f'f} \times \left(s_{fi}-s^*_{fi}\right). \tag{4.39}
$$

Dividing equation (4.38) by equation (4.39), we get:

$$
\frac{p_{fi}-p^*_i}{s_{-fi}(p_f i)-s^*_{-fi}} = \frac{-\theta_{fi}\frac{p_{i0}}{Q_{i0}}\sum_{f'}\phi^i_{f'f}}{\sum_{f'\neq f}\phi^i_{f'f}}.
$$

Remembering that $\phi^i_{ff} = 1$, this becomes:

$$\frac{p_{fi} - p_i^*}{s_{-fi}(p_f i) - s^*_{-fi}} = \frac{p_{i0}}{Q_{i0}} \frac{\theta_{fi} \sum_{f'} \phi^i_{f'f}}{1 - \sum_{f'} \phi^i_{f'f}}. \tag{4.40}$$

Comparing equations (4.36) and (4.40), we may observe that the only difference between the two relations resides in the presence of the factor θ_{fi}, before the sum in the denominator, in the expression derived from the approach in section 4.5. Actually, the approach followed by Day et al. assumes that θ_{fi} equals one for every generator. According to this assumption, both methods are equivalent. But the methodology developed here offers the opportunity to model some (or all) of the generators as pure *price takers*. This is not the case in the conjectured supply curve method: even for generators ignoring competitors' reaction to their own moves (that is, setting $\phi^i_{f' \neq f}$ or respectively B'_{-fi} to zero), the approach presented here allows for some generators to behave as *price takers* or anything in between, while the conjectured supply function method would then result in a pure Cournot.

On the other hand, this approach is based on a linear demand function which is most often very difficult to estimate on real markets. The conjectured supply function approach does not suffer from this drawback. We will however elaborate on this topic in the next section.

4.8 Extension to Inelastic Demand Functions

As mentioned in the previous section, this approach cannot handle inelastic demands, although these often represent the only way to model power markets operationally. Inelastic demands would result in infinite coefficients in the quadratic form K^i. In this section we examine the required transformations to allow for such demands. First, the equation (4.3) does not hold anymore. Demand at every node is assumed to be fixed at its nominal level $\bar{q}_i$. Similarly to Smeers (see [4] at page 18), we define a *conjectural* linear relation between price and individual supply from a generator around the equilibrium:

$$p_{fi} = p_i^* - \xi_{fi} \left(s_{fi} - s^*_{fi} \right) \tag{4.41}$$

where parameter ξ_{fi} could be compared to the *subjective* partial derivative $\eth p_i / \eth s_{fi}$ as defined in equation (4.9). On the other hand, conjectural variations introduced in section 4.5 must satisfy the additional constraint:

$$\forall f: \quad \sum_{f' \neq f} \phi^i_{f'f} = \Phi^i_{-f} = -1 \tag{4.42}$$

in order to be compatible with the inelastic demand.

The conjectured supply relations may be derived using:

$$\begin{aligned} s_{f'i} - s^*_{f'i} &= \phi^i_{f'f} \times \left(s_{fi} - s^*_{fi}\right) \\ \Rightarrow \quad s_{f'i} &= s^*_{f'i} + \frac{\phi^i_{f'f}}{\xi_{fi}} \left(p_{fi} - p^*_i\right). \end{aligned} \tag{4.43}$$

This is actually just rephrasing the Conjectured supply curves using other parameters, coherent with definitions and notations introduced in previous sections. Using these relations, we follow Smeers [4] to get the equilibrium conditions:

$$p_i + \xi_{fi} s_{fi} - w^*_i + \frac{\partial}{\partial s_{fi}} \left[\sum_j C_{fj}(g_{fj}) - w^*_j\right] = 0 \tag{4.44}$$

and show that the optimisation model:

$$\begin{aligned} \max \quad & \tfrac{1}{2} \sum_{fi} \xi_{fi} s^2_{fi} - \sum_{fj} C_{fj}(g_{fj}) \\ \text{s.t.} \quad & \sum_i s_{fi} = \sum_j g_{fj} \\ & \sum_f s_{fi} = \bar{q}_i \\ & (g, q, y) \in NC \end{aligned} \tag{4.45}$$

provides has the same first-order equilibrium conditions as above, and hence provides the solution to the generators' problem.

One problem here is that the equilibrium price p^*_i is not defined explicitly anymore in this context, but has to be extracted from dual information. In fact, the dual variable of the supply constraint satisfying $\bar{q}_i$ provides the price.

4.9 Parameters Estimation: A First (Non-Consistent) Approach

4.9.1 General Principle

In this section we neglect the network constraints affecting the equilibrium computation. We consider a single node and hence no transmission price matters.

The problem for generator f is to find the sales quantities s_{fi} which maximise the following problem:

$$\begin{array}{ll} \max & \sum_i p_i s_{fi} - C_f\left(\sum_i s_{fi}\right) \\ \text{s.t.} & s_{fi} + s_{-fi} = q_i \\ & p_i = p_{i0} - \dfrac{p_{i0}}{Q_{i0}} q_i . \end{array} \tag{4.46}$$

Conjectural variations and market power express through the following first-order condition (assuming that supplies are all strictly positive) with respect to consumer i :

$$p_{i0} - \frac{p_{i0}}{Q_{i0}}\left(\sum_{f'} s_{f'i} + s_{fi}\theta_{fi}\sum_{f'}\phi^i_{f'f}\right) = \frac{\partial C_f}{\partial s_{fi}}\left(\sum_j s_{fj}\right) \tag{4.47}$$

Considering market segment i, and writing the system of equilibrium equations for each generator f, we may rewrite for each generator:

$$Q_{i0} \times \left(1 - \frac{1}{p_{i0}}\frac{\partial C_f}{\partial s_{fi}}\right) = \sum_{f'} s_{f'i} + s_{fi}\theta_{fi}\sum_{f'}\phi^i_{f'f} \tag{4.48}$$

Let us arbitrarily fix one of the supply variables to a given value: we set $s_{1i} = \overline{s}_{1i}$. We write the equilibrium conditions for all other generators $f = 2, \ldots, |F|$. It comes:

$$\begin{array}{rcl} Q_{i0} \times \left(1 - \dfrac{1}{p_{i0}}\dfrac{\partial C_2}{\partial s_{2i}}\right) & = & \sum_{f'} s_{f'i} + s_{2i}\theta_{2i}\sum_{f'}\phi^i_{f'2} \\ \ldots & = & \ldots \\ Q_{i0} \times \left(1 - \dfrac{1}{p_{i0}}\dfrac{\partial C_{|F|}}{\partial s_{|F|i}}\right) & = & \sum_{f'} s_{f'i} + s_{|F|i}\theta_{|F|i}\sum_{f'}\phi^i_{f'|F|}, \end{array} \tag{4.49}$$

or, defining by

$$\lambda_{fi} = 1 + \theta_{fi} \sum_{f'} \phi^i_{f'f}, \tag{4.50}$$

we express the same conditions in a matrix formulation:

$$\begin{pmatrix} Q_{i0} - \overline{s}_{1i} & - & \frac{Q_{i0}}{p_{i0}} \frac{\partial C_2}{\partial s_{2i}} \\ Q_{i0} - \overline{s}_{1i} & - & \frac{Q_{i0}}{p_{i0}} \frac{\partial C_2}{\partial s_{3i}} \\ & \vdots & \\ Q_{i0} - \overline{s}_{1i} & - & \frac{Q_{i0}}{p_{i0}} \frac{\partial C_{|F|}}{\partial s_{|F|i}} \end{pmatrix} = \begin{pmatrix} \lambda_{2i} & 1 & \dots & 1 \\ 1 & \lambda_{3i} & \dots & 1 \\ \vdots & & \ddots & \vdots \\ 1 & 1 & \dots & \lambda_{|F|i} \end{pmatrix} \begin{pmatrix} s_{2i} \\ s_{3i} \\ \vdots \\ s_{|F|i} \end{pmatrix} \tag{4.51}$$

We denote by $\Lambda_i(1)$ the square matrix in the right-hand side of the last equation. This matrix is invertible, provided that at most one agent behaves as a purely competitive price taker (hence $\lambda_{fi} = 1$). Note that if there were more than one price taker, those agents might then be aggregated in one single competitive agent without loss of generality. Denoting by $\Gamma_i(1)$ the inverse matrix, we get:

$$\begin{cases} \gamma_{iff'}(1) &= \dfrac{1}{(1-\lambda_{fi})} \dfrac{1}{(1-\lambda_{f'i})} \dfrac{1}{\sum_{k>1} \frac{1}{1-\lambda_{ki}} - 1}, \qquad f \neq f', \\ \gamma_{iff}(1) &= \dfrac{1}{(1-\lambda_{fi})^2} \left[\dfrac{1}{\sum_{k>1} \frac{1}{1-\lambda_{ki}} - 1} - (1-\lambda_{fi}) \right] \end{cases}$$

provided that all values of λ_{fi} differ from 1. Should agent $|F|$ alone behave as a price taker, $\lambda_{|F|i} = 1$, then the solution would be:

$$\begin{cases} \gamma_{iff}(1) &= \dfrac{-1}{(1-\lambda_{fi})}, \qquad f < |F|, \\ \gamma_{if|F|}(1) = \gamma_{i|F|f}(1) &= \dfrac{1}{1-\lambda_{fi}}, \qquad f < |F|, \\ \gamma_{i|F||F|}(1) &= 1 - \displaystyle\sum_{1<f<|F|} \dfrac{1}{1-\lambda_{fi}}, \end{cases}$$

and all other elements in matrix $\Gamma_i(1)$ would be zero.

This gives us the following relation defining the output at equilibrium of all agents s_{fi}, $f \neq 1$, as a function of the output $\overline{s}_{1i}$:

$$\begin{pmatrix} s_{2i} \\ s_{3i} \\ \vdots \\ s_{|F|i} \end{pmatrix} = \Gamma_i(1) \times \begin{pmatrix} Q_{i0} - \overline{s}_{1i} & - & \frac{Q_{i0}}{p_{i0}} \frac{\partial C_2}{\partial s_{2i}} \\ Q_{i0} - \overline{s}_{1i} & - & \frac{Q_{i0}}{p_{i0}} \frac{\partial C_2}{\partial s_{3i}} \\ & \vdots & \\ Q_{i0} - \overline{s}_{1i} & - & \frac{Q_{i0}}{p_{i0}} \frac{\partial C_{|F|}}{\partial s_{|F|i}} \end{pmatrix} \tag{4.52}$$

This system is quite easy to solve if one considers constant marginal costs for the producers, but needless to say this is rather simplistic. It might be impossible to solve if marginal costs are non-continuous functions, which is often the case. We will however assume that marginal costs are continuous and that a solution exists.

Consider this system at equilibrium. We then would have $s_{fi} = s^*_{fi}$ and the derivative of the previous expression would yield:

$$\frac{\partial s^*_{fi}}{\partial s^*_{1i}} \equiv \phi^i_{f1} = -\sum_{f' \neq 1} \gamma_{iff'}(1) \tag{4.53}$$

This, according to the conjectural variations that were made on all producers $f = 2, \ldots, |F|$, defines the conjectural variation parameters for generator 1. We might then proceed iteratively, as in a fixed-point or Newton's method: given some initial set of conjectural variation parameters $\{\phi^{i,0}_{f'f}\}$, we invert the matrix $\Lambda_i(f)$ for each generator and obtain a new set of conjectural variation parameters $\{\phi^{i,1}_{f'f}\}$ through equation (4.53). Iterating on this system, we hope to converge to stable values of the conjectural variation parameters.

4.9.2 Extension to non-constant Marginal Costs

In this section we try to account for a particular case of variable marginal generation costs. We make the assumption of linear marginal costs (i.e. quadratic cost functions). Even though this is definitely not true in general, it might give a good approximation in order to reach a stable solution. Let us assume that marginal costs are given by:

$$\frac{\partial C_f}{\partial s_{fi}}(s_{fj}, \ldots, s_{fI}) = a_{fi} + \sum_j b_{fi,j} s_{fj}. \tag{4.54}$$

Provided that the coefficients $b_{fi,j}$ are symmetric in (i, j), this would give a quadratic cost function expressed by:

$$C_f(s_{fj}, \ldots, s_{fI}) = c_f + \sum_i a_{fi} s_{fi} + \tfrac{1}{2} \sum_{i,j} s_{fi} b_{fi,j} s_{fj} \tag{4.55}$$

The system of equilibrium conditions (4.49) may hence be rewritten as:

$$\begin{array}{rcl} Q_{i0} \times \left(1 - \frac{a_{2i}}{p_{i0}} - \sum_j \frac{b_{2i,j}}{p_{i0}} s_{2j}\right) & = & \sum_{f'} s_{f'i} + s_{2i}\theta_{2i} \sum_{f'} \phi^i_{f'2} \\ \dots & = & \dots \\ Q_{i0} \times \left(1 - \frac{a_{2i}}{p_{i0}} - \sum_j \frac{b_{|F|i,j}}{p_{i0}} s_{|F|j}\right) & = & \sum_{f'} s_{f'i} + s_{|F|i}\theta_{|F|i} \sum_{f'} \phi^i_{f'|F|}, \end{array} \tag{4.56}$$

Now we adapt the definition (4.50) to cope with the new linear terms in supply variables, appearing in the left-hand side of the system:

$$\lambda'_{fi} = 1 + \theta_{fi} \sum_{f'} \phi^i_{f'f} + \frac{b_{fi,i} Q_{i0}}{p_{i0}}, \tag{4.57}$$

and we build matrix $\Lambda'_i(1)$ and its inverse $\Gamma'_i(1)$ as in the previous step, using the new values λ'_{fi}. The new equilibrium is now obtained by the system of equations:

$$\begin{pmatrix} s_{2i} \\ s_{3i} \\ \vdots \\ s_{|F|i} \end{pmatrix} = \Gamma'_i(1) \times \begin{pmatrix} Q_{i0} - \bar{s}_{1i} & - & \frac{a_{2i}Q_{i0}}{p_{i0}} - \sum_{j \neq i} \frac{b_{2i,j}Q_{i0}}{p_{i0}} s_{2j} \\ Q_{i0} - \bar{s}_{1i} & - & \frac{a_{3i}Q_{i0}}{p_{i0}} - \sum_{j \neq i} \frac{b_{3i,j}Q_{i0}}{p_{i0}} s_{3j} \\ & \vdots & \\ Q_{i0} - \bar{s}_{1i} & - & \frac{a_{|F|i}Q_{i0}}{p_{i0}} - \sum_{j \neq i} \frac{b_{|F|i,j}Q_{i0}}{p_{i0}} s_{|F|j} \end{pmatrix} \tag{4.58}$$

Again, the derivative of these conditions at equilibrium provides estimates for conjectural variation parameters since:

$$\phi^i_{f1} \equiv \frac{\partial s^*_{fi}}{\partial s^*_{1i}} = \sum_{f'} \gamma'_{iff'}(1). \tag{4.59}$$

The main step in this process therefore resides in the inversion of matrices $\Lambda'_i(f)$ for each customer and producer, and this inversion should be performed on an iterative basis.

4.9.3 Computation of the Optimal Parameters

The above algorithm has been implemented on a small example with one single customer i and four competitors on the generation side.

Conjectural variation parameters $\phi^{i,0}_{f'f}, f \neq f'$, have initially been set to zero. Figure 4.1 shows their evolution when iteratively computed using (4.53).

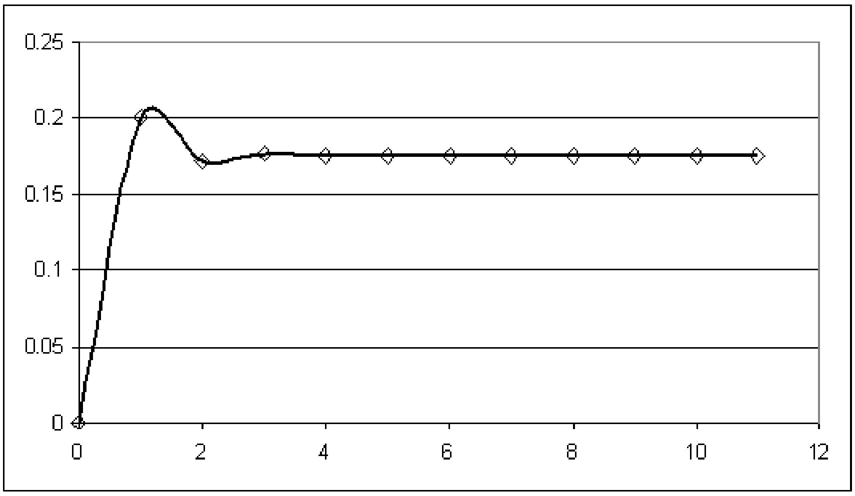

Figure 4.1: Convergence without variable marginal costs.

It can be seen that convergence is pretty quick, and that the solution illustrates a behaviour which lies somewhere between Cournot and collusive paradigms. Note also that the procedure converges towards the same solution whatever the starting point (i.e. setting $\phi^i_{f'f} = 1$ instead of 0 does not change the final solution).

Considering non-constant marginal costs (i.e. setting $b_f i, i > 0$) leads to a slightly inferior value: at each iteration step, we get the values of $\phi^i_{f'f}$ inverting matrix Γ'_i, then we construct the new λ'_i adding the second derivative of the cost function.

Consider a single customer segment, having a *reference demand* q^{ref} of 100 MWh, for a *reference price* p^{ref} of 100 €/MWh. We assume an elasticity ϵ^{ref} of -0.5 at this reference price. Following Smeers' notation for the demand function, we have to map this on parameters p_0 and Q_0 such that

$$p(q) = p_0 - \frac{p_0}{Q_0} q. \tag{4.60}$$

The last relation is compatible with the reference price p^{ref} at the reference load q^{ref}, with the reference elasticity ϵ^{ref}, under the following conditions:

$$p_0 = p^{ref} \times \left(1 - \frac{1}{\epsilon^{ref}}\right), \qquad Q_0 = q^{ref} \times \left(1 - \epsilon^{ref}\right). \tag{4.61}$$

Assume there are four competing generators supplying this market. We first assume that all generators have identical cost functions:

$$C_f(s_f) = c_f + a_f s_f + \tfrac{1}{2} b_f s_f^2, \tag{4.62}$$

where we arbitrarily set a_f at 25 €/MWh, we neglect the fixed costs ($c_f = 0$) and consider a constant marginal cost (that is, $b_f = 0$) in a first step. Since all generators have identical cost functions (and we do not model consumer preferences for specific suppliers – we assume homogeneous supply), the equilibrium volumes should be the same for each generator. We know from the traditional Cournot assumptions that with $|F|$ competing identical suppliers and a linear demand function, then the equilibrium is obtained at:

$$s_f^* = \frac{1}{|F|+1} Q_0 \left(1 - \frac{a_f}{p_0}\right) \tag{4.63}$$

$$p^* = p_0 - \frac{|F|}{|F|+1} (p_0 - a_f) = a_f + \frac{p_0 - a_f}{|F|+1} \tag{4.64}$$

The latter equation is interpreted as follows: in an oligopolistic game where all agents behave *à la* Cournot, the equilibrium price is equal to the marginal generation cost, plus a mark-up which is monotonously decreasing with the number of agents. The asymptotic case being an infinite number of generators, where the price settles at its perfect competition level, i.e. the marginal cost.

From relation (4.64) we know that in a Cournot game with 4 players, the price will settle at 80 €/MWh, for a total supply reaching 110 MWh (hence 27.5 from each generator). The profit of all generators together in this case would be equal to their revenue, minus generation costs, that is 6050.0 €.

For the sake of comparison, the monopolistic situation would reach an equilibrium price of 162.5 €/MWh, yielding a total profit of 9453.1 €.

Using conjectural variations in this framework, and under the assumptions that all players behave similarly and have identical linear cost functions, the equilibrium is obtained at:

$$s_f^{cv} = \frac{1}{|F| + \theta_f \sum_{f'} \phi_{f'f}} Q_0 \left(1 - \frac{a_f}{p_0}\right) \tag{4.65}$$

$$p^{cv} = p_0 - \frac{|F|}{|F| + \theta_f \sum_{f'} \phi_{f'f}} (p_0 - a_f) = a_f + \frac{(p_0 - a_f)\theta_f \sum_{f'} \phi_{f'f}}{|F| + \theta_f \sum_{f'} \phi_{f'f}} \tag{4.66}$$

Introducing conjectural variations in this game, we obtain a value of 0.215 for the conjectured variation when four players are involved (hence $\phi_{ff} = 1$ and

$\phi_{f' \neq f} = 0.215$). Each of them of them supplies 24.355 MWh (for a total of 97.418 MWh), which is significantly less than in the pure Cournot game. The equilibrium price settles at 105.16 €/MWh, the profit of each generator culminates at 7809.5 €, which is far better than the Cournot case. Figure 4.2 illustrates the variation of the total generation profit when the number of competitors increases from the monopolistic situation, up to 8 competing generators.

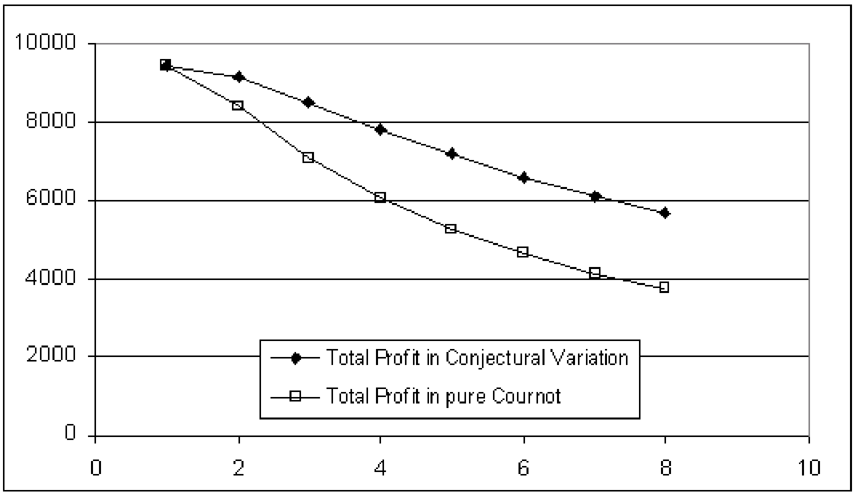

Figure 4.2: Evolution of the total profit as a function of the number of competitors.

One common criticism towards the Cournot equilibrium is that it generally results in too high prices. Needless to say, in the above described method, positive values of parameters $\phi_{f'f}$ would yield yet higher prices. This is embarrassing, and the methodology can hardly be applied to power markets, where prices are acknowledged to be lower than Cournot prices. The next section however provides an method for estimating a set of values for elasticity and conjectural variation parameters, which replicates market results observations as closely as possible.

4.9.4 Market-based Estimation Procedure

Generators in the real world do necessarily behave optimally. In sections 4.9.1 to 4.9.3, we have shown how to obtain the *optimal* conjectural variation parameters for each competitor, given some hypothesis on marginal costs and customer elasticity. This might not always reflect a real-world situation. If we want to estimate the set of parameters (elasticity and conjectural variation) for a model which intends to simulate as accurately as possible the actual competition mechanisms, we have to find market-based estimation procedures. Further, we have assumed that the elasticity at some reference point was known for every customer. Again, this is far from realistic. We might however try to overcome these problems, provided that we have the following information (or make solid assumptions about it).

Let us consider a single market segment, and a single time segment. We observe the equilibrium price p^*, and the respective supplies from each generator s_f (this might be left to hypothesis in cases where only the aggregate consumption q^* is known). We also need to have a reasonable idea of the *marginal cost of supply* $mc_f(s_f)$ – and if possible information on its derivative around the equilibrium point. Assume we can approximate the marginal cost at equilibrium, for each generator, as in expression (4.54).

We want to estimate both the market elasticity, as well as conjectural variation parameters, that are coherent with elasticity and marginal costs. Assume some arbitrary initial elasticity ϵ_0 at the observed equilibrium. Following the method described in section 4.9.2, isolating each generator's supply successively, we obtain the values $\phi_{ff'}$ for each pair or generators, unless we have specific information about some player's conjectural variation (in the latter case, we fix the value of conjectural variation coefficients to the alleged ones). We then denote the aggregate market power parameter with

$$\vartheta_f = \theta_f \sum_{f'} \phi_{f'f}. \tag{4.67}$$

Suppose that competitors actually behaved according to the alleged values of the conjectural variation parameters. We rewrite the equilibrium conditions and search for the value of elasticity which would fit the best (that is, for which the squared error would be minimal). This is obtained by solving the following problem:

$$\begin{aligned} &\min_{\epsilon} && \sum_f \eta_f^2 \\ &\text{s.t.}\, \forall f && p^* + \frac{p^*}{\epsilon q^*} s_f \vartheta_f (1+\eta_f) = a_f + b_f s_f \end{aligned} \tag{4.68}$$

Having solved this problem, we obtain a new value (ϵ_1) for the elasticity. We then recompute the new conjectural variation parameters, coherent with this elasticity, and so on. We hope to converge towards a stable solution both in terms of elasticity and conjectural variations. In the end, we attribute to each generator the *amended* aggregated conjectural variation parameter, $\vartheta_f \times (1 + \eta_f)$, where the deviation η_f is hopefully small.

4.9.5 Summary

Table 4.1 summarises the whole estimation procedure. Note that step 3e of the procedure does not need to be performed for players for which we have *a priori* information about their behaviour. For such players, we would keep the alleged values instead of updating the conjectural variation parameters which come out of the algorithm.

Theoretical convergence for this procedure has not been studied. Since this is a two-stage equilibrium, there is no evidence of the existence of a solution for this problem. However, empirical tests showed that the procedure converges, when conjectural variation parameters are estimated on a single market i.

4.9.6 Generalisation to the Multiple Customer Model with Transmission Constraints

Consider the more general case of a multi-area problem, where several customer segments are located at different nodes of a transmission network. We have stated in section 4.5 that the equilibrium conditions should then account for transmission costs. Assume, in a first step, that each node represents a country. We further make the simplification that competitors have their generation assets located in one single region for each company. We assume, as in section 4.9.4, that we have linear approximation of marginal generation costs, as in equation (4.54).

1. **Market Information:** $\forall\, f$ on market i, get supply s_{fi} and approximate marginal cost (a_{fi} and b_{fi}). Record equilibrium price p_i^* and volume q_i^*.

2. **Initialise:** Set $j = 0$ and give ϵ_0 some arbitrary negative value.

3. **Compute Conjectural Variation Parameters:**

   (a) **Initialise:** Set $k = 0$, set $\theta_{fi} = 1$ and set $\phi^i_{ff'} = 0$ and $\phi^i_{ff} = 1$.

   (b) **Select** the first generator, $\tilde{f} = 1$.

   (c) **Compute matrix $\Lambda_i'(\tilde{f})$:** Set non-diagonals to 1, and diagonals to

   $$\lambda'_{fi} = 1 + \theta_{fi} \sum_{f'} \phi^i_{ff'} - \frac{\epsilon_j q_i^* b_{fi}}{p_i^*}$$

   (d) **Invert the matrix $\Lambda_i'(\tilde{f})$:** Compute elements of $\Gamma_i'(\tilde{f})$.

   (e) **Retrieve parameters:** Update conjectural variation parameters:

   $$\phi^i_{f\tilde{f}} = \sum_{f' \neq \tilde{f}} \gamma'_{iff'}(\tilde{f})$$

   (f) **Next generator:** if $\tilde{f} < |F|$, switch to $\tilde{f} \leftarrow \tilde{f}+1$ and go to step (3c).

   (g) **Check Convergence:** If the values $\phi^i_{f\tilde{f}}$ have converged, then stop. Otherwise, iterate $k \leftarrow k+1$ and go to step (3b).

4. **Compute Elasticity:** Solve the optimisation problem:

   $$\begin{aligned} &\min_{\epsilon_{j+1}} \quad \sum_f \eta^2_{f,j} \\ &\text{s.t.}\,\forall f, \quad p_i^* + \frac{p_i^*}{q_i^* \epsilon_{j+1}} s_{fi}\theta_{fi} \sum_{f'} \phi^i_{ff'}(1+\eta_{f,j}) = a_{fi} + b_{fi}s_{fi} \end{aligned}$$

   and retrieve the elasticity ϵ_{j+1}.

5. **Check Convergence:** If the values ϵ_j have converged, then stop. Otherwise set $j \leftarrow j+1$ and go to step 3.

6. **Values of Aggregated Conjectural Variation Parameters:**

   $$\vartheta_f = (1+\eta_{f,j})\theta_f \sum_{f'} \phi^i_{ff'}.$$

Table 4.1: Elasticity and Conjectural Variation: Estimation Procedure

Complementarity constraints describe the equilibrium conditions for each pair generator-customer (f, i). In particular, we may extend relation (4.17) to its complementarity form, where j_f denotes the node at which generator f is located. Let π_f denote the profit of generator f. Its first-order derivative with respect to the supply s_{fi} is given by:

$$\frac{\partial \pi_f}{\partial s_{fi}} = p_{i0} - \frac{p_{i0}}{Q_{i0}} \left(\sum_{f'} s_{f'i} + s_{fi} \vartheta_{fi} \right) - a_{fi} - \sum_{i'} b_{fi,i'} s_{fi'} - w_i^* + w_{j_f}^*. \quad (4.69)$$

The equilibrium condition now becomes, for each pair (f, i) :

$$\frac{\partial \pi_f}{\partial s_{fi}} \leq 0, \qquad s_{fi} \geq 0, \qquad s_{fi} \times \frac{\partial \pi_f}{\partial s_{fi}} = 0. \quad (4.70)$$

The transmission prices $(w_i^* - w_{j_f}^*)$ are derived using equilibrium constraints, as described in section 4.4.

Considering the estimation problem described for the single-customer case, and extending the best-fit problem (4.68), we are now facing a mathematical problem subject to equilibrium constraints: we minimise the sum of squared estimation errors η_{fi} for each pair (f, i), subject to two sets of equilibrium constraints, (4.70) and (4.8). The full model is the following. Given the observed equilibrium prices and volumes p_i^* and q_i^*, as well as observations of the transmission prices w_i^*, one tries to find the elasticities ϵ_i which minimise the errors η_{fi} on equilibrium conditions:

$$\begin{aligned} \min_{\epsilon} \quad & \sum_{fi} \eta_{fi}^2 & (4.71) \\ \text{s.t.} \quad \forall f, i \quad & \frac{\partial \pi_f}{\partial s_{fi}} \leq 0, \qquad s_{fi} \geq 0, \qquad s_{fi} \times \frac{\partial \pi_f}{\partial s_{fi}} = 0. \end{aligned}$$

where $\partial \pi_f / \partial s_{fi}$ is adapted from equation (4.69) to incorporate the error term:

$$\frac{\partial \pi_f}{\partial s_{fi}} = p_i^* + \frac{p_i^*}{\epsilon_i q_i^*} s_{fi} \vartheta_{fi} (1 + \eta_{fi}) - a_{fi} - \sum_{i' \in I} b_{fi,i'} s_{fi'} - w_i^* + w_{j_f}^*. \quad (4.72)$$

The impact of using equilibrium constraints here is that, in the case of a customer segment i which should not be supplied by a particular generator f, that is, $s_{fi} > 0$, then the estimation error has no impact anymore. This might cause problems if generator f chooses not to supply customer i although its profit would actually increase if it did. The latter situation would occur when $s_{fi} = 0$ and $(\partial \pi_f / \partial s_{fi}) > 0$.

4.10 Parameters Estimation: A Consistent Approach

4.10.1 The Principle of Consistency

One major problem we are facing lies in the numerical estimation of the conjectural parameters value. In section 4.9, we have shown one possible way of building these parameters, based on the notion of continuously repeated game. This (iterative) method for estimating parameters leads to conjectural variations which tend to some kind of collusive behaviour. Actually, the results obtained using this approach reveal equilibrium prices which are higher than those obtained with a plain Cournot model. Whether this is realistic or not is another matter.

In the following, we try an alternative approach to obtain consistency. Assume in a first step that we know the value of the elasticity parameter. We concentrate on the base case of a single market, hence we drop the market index i for the sake of readability. Using the general notation $mc_f(s_f)$ for firm f's marginal cost, we write the equilibrium conditions for each agent :

$$p_0 - \frac{p_0}{Q_0}\left(\sum_{f'} s_{f'} + s_f \theta_f \sum_{f'} \phi_{f'f}\right) = mc_f(s_f) \tag{4.73}$$

We assume, at this stage at least, that all players exert some market power (that is, none of them is price taker and we set $\theta_f = 1$ for each f), and we trivially set parameters $\phi_{ff} = 1$ (which means that a player increasing its supply by one unit knows that he does so).

Let us give the following intuitive interpretation to the conjectural variation parameters. Consider player f at the equilibrium. He/she does not necessarily know that this is the equilibrium. Assume however that each player is right about his conjectural variation assumption, that is, they do correctly anticipate the reaction of their competitors if they individually change their supply by a deterministic small amount δ_f. This is exactly the definition of *consistency*: Perry [6] writes that *"a conjectural variation is consistent if it is equivalent to the optimal response of the other firms at the equilibrium defined by that conjecture"*.

If each of the player does anticipate this reaction correctly, then the equilibrium shall be such (a) that equation (4.73) holds for each player, and (b) we may also write that if one of the players, say k, were to depart from equilibrium by an amount δ_k, then the new equilibrium conditions for other agents would result for them in a move by an amount $\phi_{fk} \times \delta_k$. This yields the following system of conditions for each $j \neq k$:

$$\begin{aligned} p_0 \quad &- \quad \frac{p_0}{Q_0}\left(\sum_f (s_f + \phi_{fk}\delta_k) + (s_j + \phi_{jk}\delta_k)\theta_j \sum_f \phi_{fj}\right) \\ &= \quad mc_j(s_j + \phi_{jk}\delta_k), \qquad\qquad \forall j \neq k. \end{aligned} \tag{4.74}$$

In a more compact notation, if we denote with $\tilde{s}_f$ the new (correctly) anticipated move of agent f :

$$\begin{aligned} \tilde{s}_k &= s_k + \delta_k, \\ \tilde{s}_f &= s_f + \phi_{fk}\delta_k, \end{aligned} \tag{4.75}$$

then the equilibrium conditions after the move become:

$$p_0 - \frac{p_0}{Q_0}\left(\sum_f \tilde{s}_f + \tilde{s}_j\theta_j \sum_f \phi_{fj}\right) = mc_j(\tilde{s}_j), \quad \forall j \neq k. \tag{4.76}$$

Writing the systems (4.73) and (4.74) for each of the players k simultaneously, leads to a full problem with $|F|^2$ non-linear equations, $|F|$ unknown supplies at equilibrium s_f, and $|F| \times (|F| - 1)$ unknown conjectural variation parameters $\phi_{f' \neq f}$, totalling $|F|^2$ variables. All other parameters are set exogenously (p_0 and Q_0 depend on the reference price, quantity and elasticity; marginal costs mc_f are given as functions of s_f and $\theta_f = 1$ and $\phi_{ff} = 1$ for each player).

Let us restate the underlying hypothesis, aiming at a better understanding of the effect and the estimation of conjectural variation parameters. Assuming each of the players does correctly anticipate the reaction of each of its competitors if he/she were to depart from its equilibrium by a small amount δ_f, then the system built from expressions (4.73) to (4.74) should provide us with the correct conjectural variation parameters.

Assume we are able to solve this problem for a given value of elasticity, the resulting equilibrium provides us with a set of equilibrium supplies s_f^*. If the total supply q^* (and hence the equilibrium price p^*) does not match the observed reference value, we might consider tuning the elasticity parameter in order to try and converge towards these target values.

Equations (4.73) are generally referred to as the *first-order conditions*. In a sense, the systems of equations (4.74) for each pair (k, j) may be interpreted as the *second-order conditions* at the equilibrium.

These equilibrium conditions may be further investigated. Consider the first-order condition (4.73) for agent k, as well as the second-order condition (4.74) for the same supplier, when agent j departs unilaterally from equilibrium by a small amount δ_j. These two equations are explicitly rewritten here:

$$\begin{aligned} p_0 \quad &- \quad \frac{p_0}{Q_0}\left(\sum_f s_f + s_k\theta_k \sum_f \phi_{fk}\right) = mc_k(s_k) \\ p_0 \quad &- \quad \frac{p_0}{Q_0}\left(\sum_f (s_f + \phi_{fj}\delta_j) + (s_k + \phi_{kj}\delta_j)\theta_k \sum_f \phi_{fk}\right) \\ &= \quad mc_k(s_k + \phi_{kj}\delta_j), \qquad j \neq k. \end{aligned} \tag{4.77}$$

Subtracting these two equations, one gets :

$$\begin{aligned} -\frac{p_0}{Q_0}\left(\sum_f (\phi_{fj}\delta_j) + \phi_{kj}\theta_k\delta_j \sum_f \phi_{fk}\right) &= mc_k(s_k + \phi_{kj}\delta_j) - mc_k(s_k) \\ &= \phi_{kj}\delta_j \frac{\partial mc_k}{\partial s_k} \end{aligned} \tag{4.78}$$

In the last equation we introduced the derivative of the marginal cost (hence the second derivative of the cost function, its curvature). Rearranging this equation, we obtain the system of relations :

$$\sum_f \phi_{fj} + \phi_{kj}\theta_k \sum_f \phi_{fk} = -\phi_{kj}\frac{Q_0}{p_0}\frac{\partial mc_k}{\partial s_k} \tag{4.79}$$

for each ordered pair (j, k) of suppliers. Coupled with the first-order equilibrium conditions (4.73), we have a fully-determined system.

4.10.2 Illustration on Particular Cases

4.10.2.1 Constant Marginal Costs

If all suppliers have constant marginal cost, then the right-hand sides in system (4.79) boil down to zero. Remembering that we have made the assumption that each supplier is aware of its impact on price ($\theta_k = 1$) and that $\phi_{kk} = 1$, it becomes :

$$(1 + \sum_{f \neq j} \phi_{fj}) + \phi_{kj}(1 + \sum_{f \neq k} \phi_{fk}) = 0. \tag{4.80}$$

This system has a trivial solution for $\sum_{f \neq k} \phi_{fk} = -1$ for each supplier k. We have shown in section 4.5 that this is equivalent to a price war equilibrium when marginal costs are all equal. If marginal costs of suppliers differ, the equilibrium is obtained at a price equal to the highest marginal cost. This is the result of the underlying assumption that all producers supply a strictly positive quantity: we wrote the first-order equilibrium conditions as equalities, not as complementarity relations. As a next step in this study, we shall try to incorporate complementarity conditions in this problem (note: preliminary tests tend to show that the equilibrium remains at the highest marginal cost).

4.10.2.2 Linear Marginal Costs with two identical Suppliers

Assume that we have two competitors, both having identical marginal cost functions, which we assume to be linear.

$$mc_f(s_f) = a + bs_f \tag{4.81}$$

The system of equations (4.79) may hence be rewritten (using subscripts j and k for competitors) :

$$\begin{aligned} (1 + \phi_{kj}) + \phi_{kj}(1 + \phi_{jk}) &= -\phi_{kj}\frac{Q_0}{p_0}b \\ (1 + \phi_{jk}) + \phi_{jk}(1 + \phi_{kj}) &= -\phi_{jk}\frac{Q_0}{p_0}b \end{aligned} \tag{4.82}$$

With this perfectly symmetric system, and since marginal cost functions are identical, there is no reason that values of ϕ_{jk} and ϕ_{kj} differ. We therefore replace them with a single variable $\phi_{jk} = \phi_{kj} = \phi$. Rearranging the last equation gives the second degree polynomial:

$$1 + (2 + b\frac{Q_0}{p_0})\phi + \phi^2 = 0. \tag{4.83}$$

This equation may be solved, depending on the relative values of b and Q_0/p_0.

Increasing Marginal Costs

For positive values of parameter b, and since the parameters of the demand function p_0 and Q_0 are strictly positive, equation (4.83) can always be solved for ϕ.

$$b > 0 \qquad \rightarrow \qquad \phi = -1 - b\frac{Q_0}{2p_0} + \sqrt{b^2\frac{Q_0^2}{4p_0^2} + b\frac{Q_0}{p_0}} \tag{4.84}$$

This result may be interpreted as follows. When marginal costs increase with the level of production (which is typically the case in the power generation problem), one always ends up with values of ϕ in the range $[-1, 0[$, between Bertrand and Cournot equilibria. Further, when the ratio of the marginal cost increase b to the slope of the demand function (p_0/Q_0) is small, the value of ϕ is close to -1, that is, close to the price war equilibrium. The more this ratio increases, the closer we get to a Cournot equilibrium ($\phi \rightarrow 0$).

At the equilibrium, we might also want to look at the markup above marginal cost. This markup is obtained by rewriting the first-order equilibrium condition. We denote with p^* the equilibrium price, and with s_j^* and s_k^* the supplies at equilibrium. Rearranging for markup, one obtains for supplier j:

$$p_0 - \frac{p_0}{Q_0}(s_j^* + s_k^*) - a - bs_j^* = p^* - mc_j(s_j^*) = \frac{p_0}{Q_0}(1+\phi)s_j^*, \tag{4.85}$$

Substituting for ϕ in the last equation leaves:

$$p^* - mc_j(s_j^*) = \left(\sqrt{\frac{b^2}{4} + b\frac{p_0}{Q_0}} - \frac{b}{2}\right)s_j^* \tag{4.86}$$

This relation defines the link between the equilibrium price and the quantity which will be supplied by each agent at equilibrium. Graphically, we may illustrate the equilibrium as follows : Individual *supply curves* are obtained

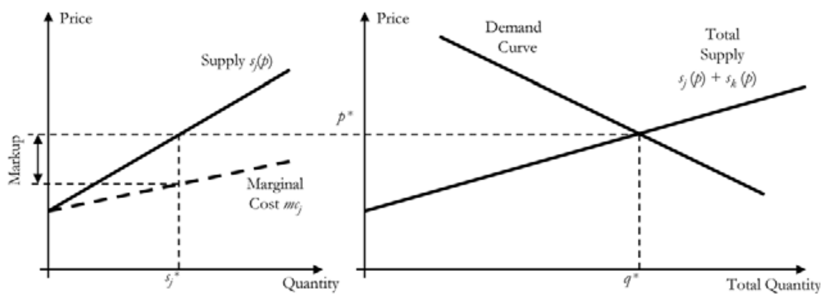

Figure 4.3: Supply Curve and Equilibrium.

linking supplied quantities to the market price:

$$s_j^* = \frac{p^* - a}{\frac{b}{2} + \sqrt{\frac{b^2}{4} + b\frac{p_0}{Q_0}}} \tag{4.87}$$

With two suppliers, one may then equate demand and supply to obtain the equilibrium :

$$s_j^* + s_k^* = 2\frac{p^* - a}{\frac{b}{2} + \sqrt{\frac{b^2}{4} + b\frac{p_0}{Q_0}}} \equiv q^* = \frac{Q_0}{p_0}(p_0 - p^*) \tag{4.88}$$

Solving the last equation for p^*, one gets:

$$p^* = p_0 \frac{b + \sqrt{b^2 + 4b\frac{p_0}{Q_0}} + 4\frac{a}{Q_0}}{b + \sqrt{b^2 + 4b\frac{p_0}{Q_0}} + 4\frac{p_0}{Q_0}} \tag{4.89}$$

and correspondingly, the quantities at equilibrium are given by:

$$q^* = s_j^* + s_k^* = 4\frac{p_0 - a}{b + \sqrt{b^2 + 4b\frac{p_0}{Q_0}} + 4\frac{p_0}{Q_0}} \tag{4.90}$$

Decreasing Marginal Costs

For negative values of b, that is, when marginal costs decrease with production level (which is definitely not the case in the power industry), there are no solutions for "small" values of b. Consistent values for ϕ can only be reached if the absolute value of b is larger than four times the slope of the demand function.

$$b < -4\frac{p_0}{Q_0} \qquad \rightarrow \qquad \phi = -1 - b\frac{Q_0}{2p_0} - \sqrt{b^2\frac{Q_0^2}{4p_0^2} + b\frac{Q_0}{p_0}} \tag{4.91}$$

This, in principle, results in positive values of ϕ, starting at a pure collusive equilibrium ($\phi = +1$) for $b = -4p_0/Q_0$, and progressively moving towards a Cournot equilibrium when b reaches larger negative values. However, when we try to compute the equilibrium supplies, we obtain an inconsistent result (with negative values). Indeed, the equilibrium condition states that

$$\begin{aligned} p_0 &- \frac{p_0}{Q_0}(s_j + s_k) - (1+\phi)\frac{p_0}{Q_0}s_j = a + bs_j \\ p_0 &- \frac{p_0}{Q_0}(s_j + s_k) - (1+\phi)\frac{p_0}{Q_0}s_k = a + bs_k \\ \phi &= -1 - b\frac{Q_0}{2p_0} - \sqrt{b^2\frac{Q_0^2}{4p_0^2} + b\frac{Q_0}{p_0}} \end{aligned} \tag{4.92}$$

Summing the two first relations and substituting ϕ, one gets the following relation defining the total supply :

$$\left(\frac{b}{2}+\frac{2p_0}{Q_0}-\sqrt{b^2\frac{Q_0^2}{4p_0^2}+b\frac{Q_0}{p_0}}\right)(s_j+s_k)=2(p_0-a) \tag{4.93}$$

In the left parentheses, the first two terms add up to a negative value, by existence condition of equation (4.91). The following square root has a negative sign, hence the total value within these parentheses cannot be positive. Since the price intercept p_0 should be larger than the initial marginal cost a, the right-hand side is positive and this equation can therefore only be solved for a negative total supply $(s_j + s_k)$. This inconsistent result prevents us from using this approach for conjectural variation parameters estimation, in the case of decreasing marginal costs.

4.10.2.3 Linear Marginal Costs with several identical Suppliers

When dealing with more than two (identical) suppliers, the problem may be extended as follows. We still assume that, for strictly identical suppliers, there is no reason to distinguish the conjectural variation parameters ϕ_{fj} between one agent and another. We therefore adopt the notation $\phi_{f\neq j} = \phi$ and $\phi_{ff} = 1$. Let F denote the total number of suppliers. Rearranging equation (4.79) with these assumptions, we obtain:

$$1+(F-1)\phi+\phi(1+(F-1)\phi)+\phi b\frac{Q_0}{p_0}=0. \tag{4.94}$$

Rearranging this expression, we have a second-degree polynomial in ϕ :

$$\phi^2+\frac{F+bQ_0/p_0}{F-1}\phi+\frac{1}{F-1}=0 \tag{4.95}$$

which solves for:

$$\phi=\frac{1}{F-1}\left[-\frac{F}{2}-b\frac{Q_0}{2p_0}\pm\sqrt{\frac{F^2}{4}+(\frac{bQ_0}{2p_0}-1)F+(\frac{b^2Q_0^2}{4p_0^2}+1)}\ \right] \tag{4.96}$$

We may easily verify that, with $F = 2$, we boil down to equation (4.84). For the same reason as in the previous section, the only valid solution can be obtained with the “plus” sign, and increasing marginal costs ($b \geq 0$).

Using a similar method as in the 2-suppliers case, we get *supply curves* for each player:

$$s_j^* = \frac{p^* - a}{\frac{b}{2} + \frac{p_0}{Q_0}(1 - \frac{F}{2}) + \sqrt{\frac{p_0^2(F-2)^2}{4Q_0^2} + \frac{b^2}{4} + b\frac{Fp_0}{2Q_0}}} \tag{4.97}$$

Summing up all supply curves and equating them with the demand function, one obtains the equilibrium price and total quantities.

$$p^* = p_0 \frac{b + \sqrt{\frac{p_0^2(F-2)^2}{Q_0^2} + b^2 + 2Fb\frac{p_0}{Q_0}} + (2-F)\frac{p_0}{Q_0} + 2F\frac{a}{Q_0}}{b + \sqrt{\frac{p_0^2(F-2)^2}{Q_0^2} + b^2 + 2Fb\frac{p_0}{Q_0}} + (2+F)\frac{p_0}{Q_0}} \tag{4.98}$$

$$q^* = 2F \frac{p_0 - a}{b + \sqrt{\frac{p_0^2(F-2)^2}{Q_0^2} + b^2 + 2Fb\frac{p_0}{Q_0}} + (2+F)\frac{p_0}{Q_0}} \tag{4.99}$$

4.10.2.4 Suppliers with different linear Marginal Costs

Consider the case of two suppliers which do not have the same marginal cost function. The analytic solution may be extended to more than two players but, for the sake of comprehension, we do here restrict our attention to this simple case. The consistence conditions (4.79) may be rewritten:

$$\begin{aligned}(1+\phi_{kj}) + \phi_{kj}(1+\phi_{jk}) &= -\phi_{kj}\frac{Q_0}{p_0}b_k \\ (1+\phi_{jk}) + \phi_{jk}(1+\phi_{kj}) &= -\phi_{jk}\frac{Q_0}{p_0}b_j\end{aligned} \tag{4.100}$$

This system solves for :

$$\phi_{kj} = -1 - b_j\frac{Q_0}{2p_0} + \sqrt{1 - \frac{2 + b_jQ_0/p_0}{2 + b_kQ_0/p_0} + b_j^2\frac{Q_0^2}{4p_0^2} + b_j\frac{Q_0}{p_0}}, \tag{4.101}$$

and the symmetric solution holds for ϕ_{jk}. We easily verify that this equation boils down to equation (4.84) when $b_j = b_k$. At the equilibrium, the following relations hold:

$$\begin{aligned}
\sigma_j &\equiv \frac{1}{\frac{b_j}{2} + \frac{p_0}{Q_0}\sqrt{1 - \frac{2+b_j Q_0/p_0}{2+b_k Q_0/p_0} + b_j^2 \frac{Q_0^2}{4p_0^2} + b_j \frac{Q_0}{p_0}}}, \\
\sigma_k &\equiv \frac{1}{\frac{b_k}{2} + \frac{p_0}{Q_0}\sqrt{1 - \frac{2+b_k Q_0/p_0}{2+b_j Q_0/p_0} + b_k^2 \frac{Q_0^2}{4p_0^2} + b_k \frac{Q_0}{p_0}}}, \\
s_j^* &= \sigma_j \left[p_0 - a_j - \frac{p_0(\sigma_j + \sigma_k) - \sigma_j a_j - \sigma_k a_k}{\sigma_j + \sigma_k + Q_0/p_0} \right] \\
s_k^* &= \sigma_k \left[p_0 - a_k - \frac{p_0(\sigma_j + \sigma_k) - \sigma_j a_j - \sigma_k a_k}{\sigma_j + \sigma_k + Q_0/p_0} \right]
\end{aligned} \tag{4.102}$$

where we have introduced parameters σ_k and σ_f for the sake of readability. Note here that, if one of the two players has a constant marginal cost (take for instance $b_j = 0$ and $b_k > 0$), he will however exert some market power: the value of the conjectural parameter will be strictly larger than -1, hence differing from the price taker behaviour.

In the more general case of multiple users with linear costs, the equilibrium supplies may be rewritten as:

$$\begin{aligned}
\sigma_j &\equiv \frac{1}{b_j + \frac{p_0}{Q_0}(1 + \sum_{f \neq j} \phi_{fj})} \\
s_j^* &= \sigma_j \left[p_0 - a_j - \frac{\sum_f \sigma_f (p_0 - a_f)}{Q_0/p_0 + \sum_f \sigma_f} \right]
\end{aligned} \tag{4.103}$$

Unfortunately we have not been able to solve analytically for ϕ_{jk} in the last case.

4.10.2.5 Generalisation to non-linear Marginal Costs

In this subsection we will limit ourselves to the case of two suppliers. Considering conditions (4.79) for two players, one may express their general solution as:

$$
\begin{aligned}
\phi_{jk}(s_k, s_j) &= -1 - \frac{\partial mc_k}{\partial s_k}\frac{Q_0}{2p_0} + \sqrt{\left(1 + \frac{\partial mc_k}{\partial s_k}\frac{Q_0}{2p_0}\right)^2 - \frac{2 + \frac{\partial mc_k}{\partial s_k}\frac{Q_0}{p_0}}{2 + \frac{\partial mc_j}{\partial s_j}\frac{Q_0}{p_0}}} \qquad (4.104)\\
\phi_{kj}(s_j, s_k) &= -1 - \frac{\partial mc_j}{\partial s_j}\frac{Q_0}{2p_0} + \sqrt{\left(1 + \frac{\partial mc_j}{\partial s_j}\frac{Q_0}{2p_0}\right)^2 - \frac{2 + \frac{\partial mc_j}{\partial s_j}\frac{Q_0}{p_0}}{2 + \frac{\partial mc_k}{\partial s_k}\frac{Q_0}{p_0}}}
\end{aligned}
$$

where the derivatives of marginal costs may depend on the level of supply, hence the possibility that ϕ depends on the value of variables s_j and s_k.

In the following we try to find some analytical solution for the case of non-linear marginal costs. Let us consider a two-player game, assuming quadratic marginal costs (which allows for a quite good representation of supply costs in the electricity sector).

$$mc_f(s_f) = a_f + b_f s_f + \tfrac{1}{2}c_f s_f^2 \qquad (4.105)$$

Defining the functions $\sigma_j(s_j, s_k)$ and $\sigma_k(s_k, s_j)$ as in system (4.102), one gets the following system of equations at equilibrium:

$$
\begin{aligned}
\sigma_j(s_j, s_k) &\equiv \frac{1}{\frac{b_j}{2} + \frac{p_0}{Q_0}\sqrt{\left(1 + (b_j + c_j s_j)\frac{Q_0}{2p_0}\right)^2 - \frac{2+(b_j+c_j s_j)Q_0/p_0}{2+(b_k+c_k s_k)Q_0/p_0}}},\\
\sigma_k(s_k, s_j) &\equiv \frac{1}{\frac{b_k}{2} + \frac{p_0}{Q_0}\sqrt{\left(1 + (b_k + c_k s_k)\frac{Q_0}{2p_0}\right)^2 - \frac{2+(b_k+c_k s_k)Q_0/p_0}{2+(b_j+c_j s_j)Q_0/p_0}}}, \qquad (4.106)\\
s_j^* &= \sigma_j(s)\left[p_0 - a_j - \frac{p_0(\sigma_j(s) + \sigma_k(s)) - \sigma_j(s)a_j - \sigma_k(s)a_k}{\sigma_j(s) + \sigma_k(s) + Q_0/p_0}\right]\\
s_k^* &= \sigma_k(s)\left[p_0 - a_k - \frac{p_0(\sigma_j(s) + \sigma_k(s)) - \sigma_j(s)a_j - \sigma_k(s)a_k}{\sigma_j(s) + \sigma_k(s) + Q_0/p_0}\right]
\end{aligned}
$$

Such non-linear systems of equations might be hard to solve with brute force. However, one may try to inject tentative values of s_j and s_k to obtain estimators of σ_j and σ_k, which in turn provide new tentative values for supplies at the equilibrium.

4.10.3 Complementarity Problem for Incomplete Games

Throughout section 4.10.2 we have considered the case of a single market (customer segment) where each of the suppliers provides a strictly positive quantity $s_f > 0$ at the equilibrium. This is too restrictive to cope with our general objective.

First, players interact on multiple markets (e.g. the same players are in competition to some extent, in Belgium and the Netherlands). Hence the necessity to cope with a more general definition of marginal costs : firm f's marginal cost to supply one more unit to market i may now depends on the its current level of supplies to each market, $mc_{fi} = mc_{fi}(s_{f1}, \ldots, s_{fI})$. Further, players might face different transmission charges to access the different markets. This is related to the network equilibrium problem described in section 4.4. Assume here that we denote with τ_{fi} the access charge when firm f supplies market i. The equilibrium conditions (4.73,4.79) have to be adapted to take these features into account. Equilibrium conditions for each pair market-agent become :

$$p_{i0} - \frac{p_{i0}}{Q_{i0}} \left(\sum_{f'} s_{f'i} + s_{fi}\theta_f \sum_{f'} \phi^i_{f'f} \right) = mc_{fi}(s_{f1}, \ldots, s_{fI}) + \tau_{fi}, \tag{4.107}$$

while consistency conditions, for each market, and each pair of firms supplying this market, may be rewritten as :

$$\sum_{f'} \phi^i_{f'k} + \phi^i_{fk}\theta_k \left(\sum_{f'} \phi^i_{f'f} + \frac{Q_{i0}}{p_{i0}} \left(\frac{\partial mc_{fi}}{\partial s_{fi}} + \frac{\partial \tau_{fi}}{\partial s_{fi}} \right) \right) = 0. \tag{4.108}$$

Second, firms may abstain from supplying some markets. This can be expressed using complementarity formulations. When a firm actually supplies a market, the first-order condition (4.107) should hold strictly. But if a profit-maximising firm refuses to supply a specific market, it then implies that its marginal revenue is smaller than its marginal cost. The first-order equilibrium conditions can therefore be rewritten using a (non-negative) slack variable σ_{fi}, such that :

$$p_{i0} - \frac{p_{i0}}{Q_{i0}} \left(\sum_{f'} s_{f'i} + s_{fi}\theta_f \sum_{f'} \phi^i_{f'f} \right) - mc_{fi} - \tau_{fi} + \sigma_{fi} = 0,$$

$$\forall\, i, f : \qquad s_{fi} \geq 0 \quad \perp \quad \sigma_{fi} \geq 0. \tag{4.109}$$

where $\perp$ denotes the complementarity operator, meaning that one of the two adjacent inequalities must be satisfied as an equality. Similarly, the consistency conditions should be rewritten as :

$$\sum_{f'} \phi^i_{f'k} + \phi^i_{fk}\theta_k \left(\sum_{f'} \phi^i_{f'f} + \frac{Q_{i0}}{p_{i0}} \left(\frac{\partial mc_{fi}}{\partial s_{fi}} + \frac{\partial \tau_{fi}}{\partial s_{fi}} \right) \right) - \frac{Q_{i0}}{p_{i0}} \frac{\partial \sigma_{fi}}{\partial s_{ki}} = 0,$$

$$\forall\ i, f \neq k : \qquad s_{fi} \geq 0 \quad \perp \quad \frac{\partial \sigma_{fi}}{\partial s_{ki}} \geq 0. \tag{4.110}$$

The intuitive interpretation of the last complementarity relation is the following: as long as firm f supplies market i with strictly positive values, its profit-maximising condition should hold strictly. Its slack variable should therefore always remain at zero, and *a fortiori* the derivative of this slack will still be nil. If we consider the case where firm f abstains from supplying market i, then we know that σ_{fi} will be strictly positive (except for one degenerate point of measure zero), but we also consider the following interpretation for its sensitivity to other players' moves. Assume firm k increases its output on market i while firm f abstains because $\sigma_{fi} > 0$. The increase by firm k will imply a global increase of the total volume (because of conditions (4.33) on the possible values for $\phi^i_{f'k}$). This will result in a price depletion, hence firm f's gap between marginal revenue and marginal cost (at $s_{fi} = 0$) can only increase, i.e. the slack variable σ_{fi} can only increase if firm k increases its output.

An additional complementarity condition should be introduced, that may have an impact on the solution. Consistency implies that firms anticipate correctly their rivals' reaction to a unilateral move. Assume that firm f stays out of market i (that is, $\sigma_{fi} \geq 0$). If its slack is *strictly* positive, then a small move by any other player cannot bring firm f back into the market. This would only be possible if firm f were on the break-even point (where both s_{fi} and σ_{fi} are zero). Therefore, we may also infer that as long as σ_{fi} is strictly positive, firm f will not enter the market, hence the consistent conjectural variation should be ϕ^i_{fk} for any other player k. Under complementary form, this is obtained by :

$$\forall\ i, f \neq k : \qquad \sigma_{fi} \geq 0 \qquad \perp \qquad \phi^i_{fk} \leq 0 \tag{4.111}$$

The full problem for multiple markets, where some firms might refuse to supply some markets, is then described by equations (4.109), (4.110) and (4.111). This system of non-linear complementarity constraints is not easy to solve.

Numerical difficulties arise on non-trivial problems, and the quest for a good starting point to feed the algorithm is of crucial importance. Since this type of equilibrium problem does not have the required convexity properties, we cannot guarantee uniqueness or even existence of an equilibrium. However, as mentioned in the general introduction, this thesis is not addressed at numerical methods, but well at the formulation of equilibrium models. We leave numerical issues for future work.

4.10.4 Numerical Experiments : Variations on a Theme

Consider the following simple example. In order to avoid unit conversion trouble, we consider a single hour problem. The demand function has an intercept of 350 €/MWh on the price axis and of 116.7 MWh on the volume axis. This demand function passes through a *reference point* for 100 MWh at a price of 50 €/MWh, where the elasticity is equal to -1/6.

Identical Constant Marginal Costs :
Assume two producers with constant identical marginal costs, $a_1 = a_2 = 30.0$ and $b_1 = b_2 = 0$. As expected from section 4.10.2, for identical producers with constant marginal costs, the equilibrium price is set equal to the marginal cost. At this price, the total supplied volume is equal to 106.7 MWh, but the solution is degenerate in that one does not know how market shares are distributed between the two firms. No market power can be exerted in this case, both firms act as if they were price takers.

Identical Increasing Marginal Costs :
Assume we now set $b_1 = b_2 = 0.1$. Both firms now have identically increasing linear marginal costs. Formula (4.84) tells us that there should be some exercise of market power in this situation, with $\phi_{12} = \phi_{21} = -0.8333$. This indicates us that the solution will be closer to the perfect competition equilibrium than to Cournot. Yet some market power is exerted: at equilibrium, each firm supplies 48.5 MWh, resulting in a price of 59.1 €/MWh while at this level marginal costs barely reach 34.85 €/MWh, hence a mark-up reaching almost 25 €/MWh.

Increasing the Number of Players :
Table 4.2 summarises the situation when the number of (identical) firms increase. We progressively depart from a situation where some market power is exerted (although not as much as in a standard Cournot equilibrium) to get closer and closer to the perfect competition situation, both in terms of markups above marginal costs, and in the value of the conjectural variation parameters.

Number Players F	Equil. Volume Q^*	Equil. Price p^*	Marginal Cost $mc_f(s_f^*)$	Market Power (markup)	Conj.Var. Variation $\sum_{f' \neq f} \phi_{f'f}$
2	96.97	59.09	34.85	24.24	-0.833
3	104.42	36.75	33.48	3.37	-0.968
4	105.36	33.92	32.63	1.29	-0.984
5	105.73	32.81	32.11	0.70	-0.988
6	105.93	32.20	31.77	0.43	-0.990

Table 4.2: Equilibrium when the number of firms increases

Two Firms with Different Constant Marginal Costs :
Let us consider a variant of the first example. Here we set $a_1 = 20$ and $a_2 = 40$, leaving $b_f = 0$. The intuition here is that firm 1, having a smaller marginal cost, should try to keep firm 2 out of the market by setting the price at some level which would deter entry. Indeed, this is exactly what the model's results provide. Firm 1 set its output quantity at $S_1 = 103.33$ so that the price drops to 40.0 €/MWh. At this point, any additional quantity provided by firm 2 would decrease the equilibrium price, below firm 2's marginal cost. In case we would add new firms, with higher marginal costs, the equilibrium would remain the same: firm 1 would decide on its output s_1 at such a level that it would prevent competitors to enter.

Two Firms with Different Increasing Marginal Costs :
Let $a_1 = 20$ and $a_2 = 40$, but we now set $b_1 = b_2 = 0.1$. Let us apply the rationale from the last paragraph. If firm 1 tries to prevent its rival from entering the market, it has to produce a quantity $s_1 = 103.33$ such that the equilibrium price remains below firm 2's marginal cost. However, by doing so, firm 1 would dramatically reduce its markup (the marginal cost of producing

this quantity would be equal to 30.33 €/MWh). Indeed, firm 1's profit in this case would be around 1530 €. By allowing the competitor to enter the market, and although the degree of competition is rather high when using consistent conjectural variations, the equilibrium would be attained at 59.09 €/MWh, with supplies by firm 1 reduced to 65.151 MWh. The resulting profit (2330 €) being higher than in the entry-deterrence case, firm 1 has an incentive to let firm 2 enter the market.

Introducing more firms :
Assume we introduce a third player such that $a_3 = 50$ and $b_3 = 0.1$. In principle, the equilibrium with only two agents was resulting in a price such that it would attract a player of the type of firm 3. However, the entry of player 3 would be deterred by firms 1 and 2, which would collectively set the price at 50 €/MWh. Their respective profits would be reduced, but not as much as if they had allowed firm 3 to enter the game.

Extending to Multiple Markets :
We next consider the competitive mechanisms when several markets are subject to competition between a set of firms. We split the demand function across two identical markets, so that the total aggregated demand remains the same as for the single market case[1]. Consider the case of two identical suppliers with increasing marginal costs ($a = 30.0$ and $b = 0.1$). Intuitively, one would assume that the sum of the outcomes on both markets should be equivalent to the case of a single market. But this is not what happens. In fact, splitting customers across several distinct markets results in lower equilibrium prices and higher outputs: both players adopt a more aggressive behaviour on each market. The equilibrium price drops from 59.1 to 52.9 €/MWh, and the total volume increases by 2 MWh.

[1]The reference point for each demand curve now becomes 50 MWh at 50 €/MWh, with an elasticity of -1/6 on each market. The price intercept does not change and remains at 350 €/MWh, but the volume intercept is divided by two on each market.

4.10.5 Consistency and the Elasticity

The methodology presented in this section allows to compute the numerical value of conjectural variation parameters, based on polynomial approximations of competitors' marginal costs, and assumptions on the demand function.

The latter has been built on some observation of *actual* equilibrium on the market, through measured values of p^{obs} and q^{obs}, and on some hypothesis on the market elasticity. We showed in section 4.9.3 that price and quantities intercepts of the demand function were linked to market observations through the assumption on elasticity ϵ. In the case of a single market, these relations are :

$$p_0 = p^{obs} \times \left(1 - \frac{1}{\epsilon}\right), \qquad Q_0 = q^{obs} \times (1 - \epsilon)\,. \tag{4.112}$$

As mentioned in section 4.10.1, the equilibrium which would result from the consistent model will not necessarily replicate the market observations. We therefore may adjust the value of the elasticity parameter, in order to replicate the actual price and quantities observations. In general, this could be achieved using linesearch algorithms. This approach is convenient for single-market problems but might be tricky for multiple-market problems. We wish however to give an analytical insight on special cases.

Players with identical linear marginal costs :
We start from the equilibrium equation (4.98), where we substitute p_0 and Q_0 using (4.112). We want to find the value of ϵ such that the model equilibrium price p^* is equal to the observed value p^{obs}. For the sake of readability we let the mathematic developments aside, and jump to the result which states :

$$\epsilon = \frac{\frac{p^*-a}{bQ^*}\frac{F(F-2)}{F-1} - 1}{\frac{F^2}{F-1}\frac{(p^*-a)^2}{bp^*Q^*} - \frac{F}{F-1}\frac{p^*-a}{p^*}} \tag{4.113}$$

This result gives the value of the elasticity parameter which, if used in the imperfect competition model, will result in an equilibrium corresponding to market observations. Although this result is rather theoretical, it has a very interesting interpretation. Given our set of assumptions on consistency, demand curves and marginal costs, for the model to replicate the market observations, we need to use a value of elasticity as defined in the last equation. We do however impose this elasticity to be negative. This introduces constraints on the signs of the numerator (negative) and denominator (positive). Rearranging these sign constraints, one gets:

$$\begin{aligned} \frac{p^* - a}{bQ^*} &< \frac{F-1}{F(F-2)}, \\ \frac{p^* - a}{bQ^*} &> \frac{1}{F}. \end{aligned}$$

These existence conditions may be re-expressed, as a *consistency condition* on the observed equilibrium price:

$$a + \frac{b}{F}Q^* < p^* < a + \frac{F-1}{F(F-2)}bQ^*, \tag{4.114}$$

which has a very nice interpretation: for a given volume exchanged at equilibrium, Q^*, the price in a consistent conjectural variation equilibrium must be (i) larger than the aggregated system marginal cost, but is (ii) capped upwards by a maximum mark-up which decreases as the number of supplier increases.

While the first part of this proposition looks obvious (and should hold even for any equilibrium, not only consistent conjectural variations), the second is less trivial and has a deep impact on the possible price range in a consistent world. The largest allowed mark-up (that is, the upper bound on the difference between p^* and the system marginal cost) is equal to $bQ^*/((F-2)F)$. For two players, there is hence no upper bound on the equilibrium price: any value above the marginal cost will be replicable by a model having the appropriate value of the elasticity. This is not the case anymore as soon as at least three players are in the game. Indeed, the upper bound on the price of a consistent conjectural variation equilibrium converges quite quickly to the marginal cost, as F increases.

Consider a market with three identical players, where an equilibrium has been observed for $Q^{obs} = 100$. Assume each supplier has a marginal cost $20+s_f$. The bounds on possible values for the price at a consistent equilibrium are the system marginal cost (53.333) on one side and an upper limit to market power (86.666) on the other side.

Assume the equilibrium is obtained at a price $p^{obs} = 70$. Applying equation (4.113) provides a value of -0.466. The higher the observed price, the smallest the absolute value of the elasticity (e.g. if the observed price had been of 80, the adequate value of the elasticity should be taken as -0.111).

One may also note from equation (4.114) that for constant marginal costs (that is, for $b = 0$), the interval reduces to a single point: a consistent conjectural variation can only be obtained when the price equals the marginal cost, whatever the number of agents (at least two).

Distinct linear marginal costs :
We have not been able to find an analytical solution for the elasticity when each supplier has a distinct linear marginal cost. However, it is possible to get a reasonable guess of this parameter using the following method.

Assume we can approximate the aggregated system marginal cost around the observed equilibrium Q^{obs} by a linear expression

$$\tilde{mc}(Q) = \tilde{a} + \frac{\tilde{b}}{F}Q. \tag{4.115}$$

This is equivalent to compute the "average" marginal cost. Simplifying the problem by stating that, on average, each player has the same linear marginal cost function is equivalent to substitute $\tilde{a}$ and $\tilde{b}$ in equation (4.113). We show that this approach is useful to provide a good starting point by using the same example as in previous paragraph. Assume three players with respective values for a_f of 25, 20 and 5, and respective values for b_f of $\frac{2}{3}$, 1 and 2. If all players are actively supplying at equilibrium, then one can show that :

$$\frac{1}{\tilde{b}} = \frac{1}{F}\sum_f \frac{1}{b_f} \qquad \text{and} \qquad \tilde{a} = \frac{\sum_f \frac{a_f}{b_f}}{\sum_f \frac{1}{b_f}}. \tag{4.116}$$

From these relations we may easily verify that $\tilde{a} = 20$ and $\tilde{b} = 1$, which corresponds to our previous example. Using the tentative value of the elasticity as suggested, the resulting equilibrium price reaches 71.98 €/MWh, that is, only 2 € above the observed price. Tuning the elasticity for replicating the observation however requires the elasticity to reach -0.638. This illustrates the influence on equilibrium that one player may be able to exert, if he is significantly larger than its competitors. Starting with the approached value is nevertheless a better starting point than anything else up to date.

4.11 Apply the Method on a “Real World” case

The method described in section 4.10 has in turn been tested on a more realistic data set. The set involves three suppliers (generators), two captive (regulated) and one eligible customer segments.

Marginal cost for suppliers are represented by their stacking curve (merit order). These are step functions which must be approximated by continuous differentiable functions in order to fit in the mathematical framework described in section 4.10. In order to do so, we first need to implement a method for approximating step functions by (for instance) quadratic functions.

4.11.1 Quadratic Approximation

In order to obtain an acceptable quadratic approximation of the supply stack in a first step, we have chosen the following method. Let G_f denote the set of generating units owned (or controlled) by agent f. Let us denote with β_g the capacity of each unit, and with ψ_g its variable operating cost. We further assume that agent f's generation fleet is ordered from the most efficient (smallest ψ) to the most expensive (largest ψ). On the other hand, assume we want to approximate the agent's marginal cost by

$$mc_f(s_f) = a_f + b_f s_f + c_f s_f^2. \tag{4.117}$$

We then try to find the value of coefficients a_f, b_f and c_f such that the surface comprised between the stacking curve and the quadratic approximation be minimal. Actually, rather than computing the real surface, we both avoid sign problems and put more penalty weight on large discrepancies by integrating the square of the distance between the stack curve and the quadratic approximation.

Denoting with q_{g-1} and $q_g = q_{g-1} + \beta_g$ the interval of the step function corresponding to generating unit g with variable cost ψ_g, the coefficients of the quadratic approximation are obtained by solving :

$$\begin{aligned}\min_{a_f,b_f,c_f} Z &= \textstyle\sum_{g\in G_f}(a_f-\psi_g)\times\Big\{(a_f-\psi_g)\times(q_g-q_{g-1})\\ &\qquad +b_f(q_g^2-q_{g-1}^2)+\tfrac{2}{3}c_f(q_g^3-q_{g-1}^3)\Big\} \\ &\quad +\tfrac{1}{3}b_f^2Q_{max}^3+\tfrac{1}{2}b_fc_fQ_{max}^4+\tfrac{1}{5}c_f^2Q_{max}^5\end{aligned} \tag{4.118}$$

4.11.2 Model Enhancement : Capacity Limitation

Once parameters a_f, b_f and c_f have been estimated for each generator, they can be fed into the equilibrium game using consistent conjectural variation parameters. The full model which was derived in section 4.10.3 can be summarised as:

$\forall\ i, f$:

$$p_{i0}-\tfrac{p_{i0}}{Q_{i0}}\left(\textstyle\sum_{f'} s_{f'i}+s_{fi}\theta_f\sum_{f'}\phi^i_{f'f}\right)-mc_{fi}-\tau_{fi}+\sigma_{fi}=0,$$

$$s_{fi}\geq 0 \quad \perp \quad \sigma_{fi}\geq 0,$$

$\forall\ i, f\neq k$:

$$\textstyle\sum_{f'}\phi^i_{f'k}+\phi^i_{fk}\theta_k\left(\sum_{f'}\phi^i_{f'f}+\frac{Q_{i0}}{p_{i0}}\left(\frac{\partial mc_{fi}}{\partial s_{fi}}+\frac{\partial \tau_{fi}}{\partial s_{fi}}\right)\right)-\frac{Q_{i0}}{p_{i0}}\frac{\partial\sigma_{fi}}{\partial s_{ki}}=0,$$

$$s_{fi}\geq 0 \quad \perp \quad \tfrac{\partial\sigma_{fi}}{\partial s_{ki}}\geq 0,$$

$$\sigma_{fi}\geq 0 \quad \perp \quad \phi^i_{fk}\leq 0.$$

However, it turns out when implementing this model on test cases, that one sometimes obtain equilibria in which the output of a particular firm, characterised by a quadratic marginal cost function, turns out to exceed the total installed capacity of this agent.

In much the same way as we have constrained the supply of firm f to be non-negative on each market i by inserting a slack variable σ_{fi} and a complementary condition linking variables s_{fi} and σ_{fi}, we also introduce a set of non-negative slacks ς_{fi} to prevent firm f to increase its sales on market i whenever its total production level reaches its available capacity. This affects first-order conditions as well as consistency conditions.

Further, one should also consider that whenever firm f produces at full capacity, and that its complementarity slackness variable ς_{fi} is *strictly* positive, then any small deviation by competitor k should not impact firm f's supply to market i. No reaction from firm f to a unilateral move by firm k, on a given market i, implies a zero consistent conjectural variation. Therefore a strictly positive value $\varsigma_{fi} > 0$ implies $\phi^i_{fk} = 0$ for all $k \neq f$.

Taking this enhancement into consideration, one obtains the full model with capacity limitation, where we have denoted with $\overline{S}_f = \sum_{g \in G_f} \beta_g$ the total generation capacity, available to firm f :

$\forall\ i, f$:

$$p_{i0} - \tfrac{p_{i0}}{Q_{i0}} \left(\textstyle\sum_{f'} s_{f'i} + s_{fi}\theta_f \sum_{f'} \phi^i_{f'f} \right) - mc_{fi} - \tau_{fi} + \sigma_{fi} - \varsigma_{fi} = 0,$$

$$s_{fi} \geq 0 \quad \perp \quad \sigma_{fi} \geq 0,$$

$$\textstyle\sum_i s_{fi} \leq \overline{S}_f \quad \perp \quad \varsigma_{fi} \geq 0,$$

$\forall\ i, f \neq k$:

$$\textstyle\sum_{f'} \phi^i_{f'k} + \phi^i_{fk}\theta_k \left(\sum_{f'} \phi^i_{f'f} + \frac{Q_{i0}}{p_{i0}} \left(\frac{\partial mc_{fi}}{\partial s_{fi}} + \frac{\partial \tau_{fi}}{\partial s_{fi}} \right) \right) - \frac{Q_{i0}}{p_{i0}} \left(\frac{\partial \sigma_{fi}}{\partial s_{ki}} - \frac{\partial \varsigma_{fi}}{\partial s_{ki}} \right) = 0,$$

$$\textstyle\sum_i s_{fi} \leq \overline{S}_f \quad \perp \quad \frac{\partial \varsigma_{fi}}{\partial s_{ki}} \leq 0,$$

$$\sigma_{fi} \geq 0 \quad \perp \quad \phi^i_{fk} \leq 0,$$

$$\varsigma_{fi} \geq 0 \quad \perp \quad \phi^i_{fk} \leq 0.$$

This model does provide a consistent estimation of conjectural variation parameters, but does not yet ensure that at the equilibrium, one will be able to replicate observed prices and volumes since we have made an assumption on the demand function, by fixing its elasticity and deriving p_{i0} and Q_{i0} using relations (4.112). In order to be able to replicate observed prices and quantities, one should introduce elasticities as explicit variables and constrain equilibrium prices and volumes to match p_i^{obs} and Q_i^{obs}.

Unfortunately, when implemented on a "real world" case, it turns out that this *EPEC* (equilibrium problem, subject to equilibrium constraints) is very difficult to solve, unless provided with quite efficient starting point values.

4.12 Conclusions

In the early sections of the paper, we have developed a mathematical framework which encompasses market power and conjectural variations, in various imperfect competition paradigms. We have shown that this framework could, according to the numerical values associated to certain parameters, resolve to perfect competition, plain Cournot, Collusion or Price War paradigms. We have then established the link between our approach and the one developed by Day and Hobbs. We have also shown how this framework could be adapted to inelastic demand.

One major problem which arises in most imperfect competition models, is the estimation of parameters which drive market power or conjectural variations. In section 4.9, we have developed a market-based approach for estimating these parameters. This approach provides a iterative way to estimate both customer elasticities and players' conjectural variations. We have shown that for constant marginal costs, the computation of conjectural variation parameters has very good convergence properties. The convergence is slower (although still good) when we consider linear approximations for marginal costs. In all the cases which were tested during this study, convergence has always been achieved within ten iterations.

For constant marginal costs, the values obtained for conjectural variation parameters only depend on the number of players: neither the relative values of marginal costs, nor the customer's elasticity influence the final value. When we consider increasing marginal costs, however, the final value for each player depends on cost functions' second derivatives, as well as on market characteristics (equilibrium prices and quantities, and elasticity).

We have shown that, using the recursive estimation, one gets the generators' optimal conjectural variation parameters, according to the hypothesis that (a) the initial phase is a standard Cournot game, and (b) at each step, generators learn from the equilibrium how to improve their position. Of course, collusion would definitely lead to a yet more profitable position for generators, but this contradicts the initial assumption that we start from a Cournot game. Note that if one has a good reason to believe that some generator would *not* behave in this manner, but that we know its conjectural variation parameters by another means, then we only need to skip step (3e) for this generator in the estimation procedure of section 4.9.5.

The elasticity that comes out of this procedure is, in a way, the value which best fits the generators supply, given their conjectural variation parameters. In a market where players would behave exactly according to their optimal conjectural variation parameters, then the solution of model (4.68) would provide us with the actual value of the market elasticity at the equilibrium point. In cases where players deviate from their optimal strategy, however, the resulting value of the elasticity would only provide us with a good guess.

This first method, however, seems to converge towards conjectural variations that reflect an equilibrium between Cournot and collusion. This is probably not realistic in power markets. Therefore, an alternative estimation method has been proposed, in which *consistency* plays a key role. Consistency implies that conjectures do correspond to the actual reactions of competitors.

The last section of this paper therefore focuses on consistency for conjectural variations, and proposes a set of analytical results on stylised examples. Furthermore, this consistent behaviour has an interpretation in terms of supply curves equilibrium. It also show that the exercise of market power quickly vanishes as the number of players increase.

This consistency approach, when tested on prototype models, provides equilibrium conditions which are much more realistic with respect to power markets (prices are greater than marginal costs, but below Cournot price). Furthermore, tuning the elasticity parameter also allows to replicate market observations of prices and volumes at equilibrium.

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Chapter 5

CO_2 emission permit trading models:

Equilibrium models of emission permits trading in the electricity generation industry

Contents

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This chapter summarises the key findings resulting from the study of CO_2 emission permits, allowances and trading. We illustrate the effect of some basic economic assumptions on the equilibrium, more specifically on the investment and generation mix. We also study how these assumptions impact the permits price, and how this affects operators' profitability.

5.1 Introduction

Ever since the Kyoto protocol in 1992, the common wisdom in the European electricity industry seems to disregard coal-fired power plants in favour of natural gas, the latter being cleaner than the former in terms of greenhouse gas emission. Before the protocol was eventually ratified, as a result of its acceptance by Russia, the European parliament and the Council have adopted a directive [1], which aims at a reduction of greenhouse gas emissions in the different Member States. This directive defines the *Emission Trading Scheme* of the European Union (hereafter denoted with EU-ETS).

This directive defines two commitment periods during which some maximal CO_2 emission levels have to be satisfied, the first of these periods beginning in January 2005. The emission targets, or *allowances*, are defined on a national basis for each Member State, and progressively decrease in order to fulfil the Kyoto requirements at the European level[1]. Loyal to the principle of subsidiarity, the Council has left to national (or regional) governments the freedom of deciding how allowances should be redistributed between greenhouse gas emitters, restricting itself to a few guidelines but keeping the right to accept or reject National Allocation Plans. One important rule is that at least 90% of the allowances have to be distributed for free (most Member States are likely to allocate all their allowances free of charge), and these allowances are tradeable between different sectors of the economy across the European Union[2]. Each (major) installation which produces greenhouse gas should be allocated a certain initial endowment for each commitment period, according to rules which are left to the national governments. The operator of the installation would

[1] The fact that Kyoto objectives do not need to be met individually for each Member State, but collectively at the EU level is often referred to as the *burden sharing agreement* and is destined to facilitate the development of some of the least performing Member States.

[2] Only a subset of the industry – consisting in energy activities, metal and mineral industries, pulp and paper – are eligible for emission permits trading. Other sectors such as chemistry will be incorporated in later stages.

then have the right to buy additional permits on the market, or sell its possible excess allowances.

The Emission Trading Scheme as designed by the European Union is supposed to let market players establish the "right price" for emission permits. This price is deemed to be of vital importance to the electricity sector, because it could entail a reshuffling of the stacking of power plants, more specifically by inverting coal-fired and gas-fired power plants in the merit order. Whether or not it actually is so important will be discussed in this chapter. Additionally, the National Allocation Plan published by the Flemish government clearly promotes the renewal of the electrical generation fleet with gas-fired units, and encourages the decommissioning of coal plants.

There seems to be some consensus in the European electricity sector, that all future installations will be based on natural gas technology. Today, for a baseload unit, the comparison of coal versus natural gas does not seem to bring a clear-cut decision in favour of one or the other, but strictly positive permit prices would immediately tilt the balance in favour of natural gas. A similar phenomenon occurred in the US in the past decade: the comparison of coal and gas forward curves in the mid-nineties, coupled with the promise to decommission a number of coal-fired units, has lead to the building of a large number of new gas plants. This, however, turned out to be a complete failure because of a sudden rise of the natural gas price, consecutive to the large increase in gas demand.

The question we would therefore like to address is to sketch the possible evolution(s) of coal *vs* natural gas prices in the coming years, coupled with the future of the European cap-and-trade system on greenhouse gas emissions.

First, coal prices these days are all-time high. This is for a great deal explained by the "China Boom", together with the shortage of freight capacity. However, both these factors are not likely to persist. China cannot sustain a 10% annual growth for years, and some countries like Korea have started to build a number of new tankers for coal transportation. Coal prices are therefore expected to come back to more reasonable levels, since its extraction cost is fundamentally not changing and transportation tariffs should rapidly drop back to competitive levels. The question which remains is to know whether or not price cycles will subsist in the future.

Second, gas prices could continue to be driven by oil prices, which have skyrocketed during the last years, reaching record levels above 100 US dollars per barrel. In 2007, OPEC has revised its long-term oil price forecast and does not expect it to drop back below 50-60 US dollars per barrel [6]. Some economists even predict future oil prices to go out of control. This would pull gas price upwards. Alternatively, one could observe a worldwide globalisation of gas-to-gas competition, conditionally to the building of new LNG-regasification facilities for import in the US. This, since natural gas prices in the US are largely higher than they are in Europe, would result in a smoothing of the price differential, that is, a price decrease in the US but an increase in Europe. The question is to know how long it would take for these regasification installations to be available. In both cases, natural gas prices would then significantly rise in the coming years.

This, although we definitely do not pretend to hold the truth, tends to indicate that apart from greenhouse gas problems, one should not totally discredit investments using the coal technology. The question is then to assess the long-term impact on electricity costs of the Emission Trading System: what is likely to happen *after* the second commitment period in 2012? One possibility is known as the *Kyoto forever* scenario, which is self-explanatory. Alternatively, one should not forget possible legal recourse to the European Court of Justice, aiming to revoke the Council directive. This is not unlikely at all. In this case, nothing would probably be implemented before the US develops their own CO_2 emission reduction policy, which apparently is in preparation but could still take some years to be adopted.

Finally, one should not forget the possibility to turn on their heel and head back to the nuclear technology. This eliminates the greenhouse gas problem.

These questions are approached in the following. We propose a methodological framework and a simple model, designed to help us provide answers to the questions above. This paper is organised as follows: in section 5.2 we describe a simple problem, present a starting point for the study (disregarding the CO_2 emission question), and then introduce the EU-ETS mechanism. Section 5.3 briefly reviews the impact of fixed-price permits in a CO_2 constrained market. This, however, has already been conducted in numerous studies and will not be further elaborated upon here. In section 5.4, we introduce a *market* for permits trading, making the permit price a model output, rather than input hypothesis. We also consider several possible extensions of the simple model, ranging from the interaction with non-electrical industrial CO_2 emitters, to the potential impact on natural gas prices of a sudden increase in consumption. We also introduce gaming possibilities while looking at endowment allocation mechanisms.

5.2 Model Description: A Simple Example

To tackle the CO_2 issue we envisage to approach the problem using a long-run stationary assumption. We therefore assume that electrical demand, spread over a set of time segments, should be satisfied with the output of newly invested power plants.

We assume, in this base case, that two distinct technologies are available. Each technology is characterised by its *investment cost* (proportional to the amount of new generation capacity which is built in this technology), by its *variable generation cost* (proportional to the total amount of energy produced with this technology) and, for later usage, by its *emission rate* (that is, the amount of rejected CO_2 per generated unit of energy).

More precisely, we state the problem in terms of long-term investment choice between coal-fired and gas-fired units. While the former has a lower long-run marginal cost when used as a baseload unit (at least with the data we will describe later on for the base case), the latter is more suitable for use during peak periods. This is basically due to the relatively low investment cost for a Combined-Cycle Gas Turbine (CCGT) unit compared to that of a coal-fired unit, but generation is cheaper with coal, making it more profitable for a time-intensive usage. On top of these considerations, we will add the problem of CO_2 emissions and the cost increase it induces on both technologies' variable cost. Coal is definitely more carbon-intensive than natural gas is, and this could handicap coal-based generation with respect to its competitor for baseload units.

5.2.1 The Prototype Model and Data

Let us build a (simplistic yet realistic) model with two demand periods $t = t_1, t_2$, where demands $D(t)$ are fixed and known. For the sake of generality, let $\delta(t)$ denote the duration of time segment t, expressed in hours. Assume further in a first step that the question of CO_2 emission permits is irrelevant to construct the base case, we simply drop this problem.

We assume there are two available technologies, denoted with the subscript $i = \{c, g\}$, where c stands for coal and g for natural gas. Let K_i correspond to the hourly-equivalent investment cost of technology i (that is, as a first approximation, the annual cost of investing 1 MW, divided by the number of hours in a year). We here assume that investment costs include all fixed costs such as maintenance and personnel. Let also c_i stand for variable operating costs, which include fuel costs, variable operation and logistic costs, ... in €/MWh.

Let the variable X_i denote the amount of capacity (in MW) which is to be invested into technology i. Let also $q_i(t)$ correspond to the amount of power which is generated using this technology during time period t, expressed in MW.

One fundamental hypothesis, in the construction of this starting point for our study, is that at this stage each technology is controlled by an independent operator. These operators behave competitively, in the sense that they act as price-takers on the electrical market. The simplest problem can be stated in its optimisation form as:

$$\begin{array}{rll} \min\limits_{q,X} & \sum\limits_{t,i} \delta(t) \times [K_i X_i + c_i q_i(t)] & \\ \text{s.t.} \quad \forall t & \sum\limits_{i} q_i(t) = D(t) & \quad (5.1) \\ \forall i, t & 0 \leq q_i(t) \leq X_i. & \end{array}$$

where one decides on the amount of capacity X_i to be invested in each technology i, such that the demand $D(t)$ be satisfied in every time period t by the sum of supplies $q_i(t)$.

More specifically, when one reduces the problem to a two-technology (namely c for coal and g for natural gas), the extensive problem becomes:

$$\begin{array}{rll} \min\limits_{q,X} & \sum\limits_{t} \delta(t) \times [K_g X_g + K_c X_c + c_g q_g(t) + c_c q_c(t)] & \\ \text{s.t.} \quad \forall \text{t} & q_g(t) + q_c(t) = D(t) & \quad (5.2) \\ \forall t & 0 \leq q_g(t) \leq X_g, \quad 0 \leq q_c(t) \leq X_c. & \end{array}$$

The objective of this problem is to minimise total costs (investment and generation) subject to demand satisfaction and capacity constraints. Alternatively, this problem can also be stated in its *equilibrium formulation*. To obtain the latter, one writes the first-order equilibrium complementarity conditions for both operators, and one adds the balance constraints which ensures that the total demand level is met. The equivalent equilibrium formulation can be stated in its complementarity form, after introduction of the dual variables

	High Demand Period	Low Demand Period	
Demand Level	100.0	75.0	[MW]
Duration	1	1	[h]
	Coal Technology	**Nat.Gas Technology**	
Investment Cost	15.0	10.0	[€/MW.h]
Variable Cost	15.0	22.0	[€/MW.h]
Efficiency	44	55	[%]
Emission Rate	0.75	0.35	[tCO_2/MW.h]

Table 5.1: Sample Problem Data for Load and Generation

$\rho_i(t)$ and $\pi(t)$, weighted by the appropriate time segment durations $\delta(t)$. The symbol $\perp$ is understood to mean that at least one of the inequalities must hold with equality. This yields :

$$\begin{aligned} \pi(t) \leq c_i + \rho_i(t) \quad &\perp \quad q_i(t) \geq 0, \\ \sum_t \delta(t)(K_i - \rho_i(t)) \geq 0 \quad &\perp \quad X_i \geq 0, \\ q_i(t) \leq X_i \quad &\perp \quad \delta(t)\rho_i(t) \geq 0 \\ \sum_i q_i(t) = D(t) \quad &\perp \quad \delta(t)\pi(t) \end{aligned} \tag{5.3}$$

Table 5.1 provides sample data for our prototype model. The first part of the table describes the demand system, with two equal-length periods, one corresponding to the "peak" period and the other to the baseload. The second part of the table deals with generation characteristics. The *investment costs* are hourly equivalent values, i.e. annuities divided by 8760, derived from costs analysis that can be found in various studies within Electrabel, where values of 130 and 85 €/kW.yr were mentioned for coal and natural gas, respectively. Investment costs with a similar order of magnitude can be found in [5]. These values represent the annual fixed charge consecutive to an investment in these types of units, and include capacity, maintenance, personnel, logistics... Variable costs have been computed as follows [3]. For coal, we have considered an average level of 45 US$ per ton, coupled with an efficiency of 44 %. This yields a pure fuel variable cost of about 12.5 €/MWh, on top of which we have added nearly 2.50 additional variable logistic and maintenance costs, to reach the final value of 15 €/MWh. For natural gas, we started from gas quotes at 22 p/therm. After proper conversion and considering an efficiency of 55 %, one slightly exceeds 20 €/MWh. Adding another 1.50 for variable logistic and maintenance costs, we obtain the value of 22 €/MWh as stated in table 5.1.

The long-run marginal costs, for each technology, will depend on their respective load factor θ_i, that is, on the ratio between their actual production and the potential generation at full capacity during the whole relevant period. We use the following expression to obtain the hourly long-run marginal cost:

$$lrmc_i = \frac{1}{\theta_i} K_i + c_i \tag{5.4}$$

Obviously, the decision to invest in one or the other technology will only be sustainable if the average revenue over the relevant time segments at least equals this long-run marginal cost.

Interpreting this data set, a gas unit is cheaper to invest in, but its variable costs are higher than those of a coal-fired units. This kind of tradeoff usually carries as expected consequence that one should invest in coal-fired units for baseload, whereas peak units should be gas ones – at least as long as one neglects emission costs.

If one solves the optimisation problem (5.1), it indeed yields the expected solution that a 75 MW investment should be conducted with the coal technology to cover the baseload, while the remaining 25 MW for the peak demand should be covered by a gas-fired unit.

At the equilibrium, power prices settle at 42 and 18 /MWh in the on-peak and off-peak periods, respectively. Since the average price perceived by each

[3] Variable costs mentioned in this study rely on orders of magnitude of fuel prices and exchange rates in early 2005. Fuel prices have significantly increased since then while exchange rates somewhat alleviated this increase.

operator is exactly equal to their long run marginal cost, the resulting profit is zero. Indeed, the coal-fired unit gets and average baseload revenue of 30 /MWh (which exactly covers its long-run marginal cost for a baseload coal unit), while the gas operator only runs during the peak period (load factor $\theta_g = 0.5$), hence incurring a long-run marginal cost of 42 which is exactly the price at peak.

5.2.2 Introducing Emissions

Before we start looking at the two main approaches which can be implemented to study the impact of emission allowances, we look at the situation as it is before emission permits are introduced. At the current solution, 150 electrical MWh are generated from coal and only 25 from natural gas.

Let us denote with e_i the *emission rate* of each technology. This corresponds to the amount of CO_2 which is injected into the atmosphere, for each generated electrical MWh. Assume that the emission rate of the coal technology is of 0.75 ton per MWh, whereas natural gas only emits 0.35 ton per produced MWh.

Starting from our reference case, the current economic solution to the investment problem would result in a total emission of 121.25 tons of CO_2 (112.5 of which, due to coal-fired generation). This figure will be critical when compared to results in the following sections, when emission permits are introduced, whether at fixed price or in limited quantity.

5.3 Fixed-Price Models

5.3.1 Emission Permits at Exogenous Price

Assume now that emissions rights are introduced in the model. Let E_i denote the initial permit endowment for technology i, and the variable z_i is the shortage (or surplus) which has to be bought (or sold) on the emission market at some fixed price η. The emission rates have been taken as 0.75 and 0.35 tons of CO_2 per MWh, produced with coal and gas, respectively. This difference in emission rates (gas being cleaner than coal) should intuitively pull the equilibrium further towards natural gas investments.

We consider in a first step that endowments are purely exogenous: operators take them as given exogenous values. However, they have to balance their total CO_2 emissions with their initial endowment, through the buying (or selling) of additional (excess) permits. Exogenous-price permits can be easily modelled using the following formulation:

$$\begin{aligned}
\min_{q,z,X} \quad & \sum_i \left[\eta z_i + \sum_t \delta(t) \times (K_i X_i + c_i q_i(t)) \right] \\
\text{s.t.} \quad \forall t \quad & \sum_i q_i(t) = D(t) \\
\forall i \quad & 0 \leq q_i(t) \leq X_i, \\
\forall i \quad & \sum_t \delta(t) \times [e_i q_i(t)] = E_i + z_i.
\end{aligned}$$

The objective function contains a supplementary term, ηz_i, which accounts for the number of permits each operator has to buy ($z_i > 0$) or sell ($z_i < 0$) at price η. One also includes the additional constraint which balances the total emissions with the initial endowment E_i and the traded permits z_i.

It is often mentioned that the dispatch of generating units, between coal and gas-fired plants, will be determined by the permit price and not by the initial endowment. We do however wonder whether, in an investment perspective, the initial endowment can impact the first-stage investment decision. It can be shown, in this deterministic approach, that when the permit price is known, the initial endowment does not change the investment decision (it is in fact equivalent to the substraction of a constant term in the objective function, hence not changing anything in the optimal decision). When the permit price is known, the only critical factor is the fuel price spread.

This permit price affects the long-run marginal cost of any CO_2-emitting technology, proportionally to its emission rate e_i. Even if some permits are allocated for free, they have an opportunity value (they could be sold instead of used) which is equal to their market price. Therefore the long-run marginal cost of technology i becomes:

$$lrmc_i(\theta) = \frac{K_i}{\theta_i} + c_i + e_i\eta \tag{5.5}$$

Given two specific technologies such as coal and natural gas, for example, it is hence easy to derive the permit price which would *arbitrate* between them at a given load factor. It suffices to equate both long-run marginal costs for a given load factor θ:

$$\frac{K_c}{\theta} + c_c + e_c\eta = \frac{K_g}{\theta} + c_g + e_g\eta, \tag{5.6}$$

and to derive the permit price :

$$\eta^*_{c,g}(\theta) = \frac{(K_g - K_c)\frac{1}{\theta} + (c_g - c_c)}{e_c - e_g}. \tag{5.7}$$

Assuming that investment costs are not supposed to change drastically for these technologies (at least in the medium-term), one can see that the arbitrage price between coal and gas basically depends on the fuel price spread $(c_g - c_c)$ and on the difference in emission rates $(e_c - e_g)$.

If the permit price exceeds a given threshold, only natural gas technology should be retained even for baseload. It might be interesting, however, to look at the switching level in terms of fuel spread and permit prices. This is illustrated on figure 5.1: permit prices above the frontier line should encourage investments using natural gas technology, whereas permit prices below would be in favour of coal.

Note that if the fuel spread were to reach about 20 €/MWh (that is, if natural gas prices were doubled), it would require a permit price of 40 €/ton to discourage a baseload investment in coal. Even though the former does not look absolutely impossible, it is clear that the latter is way above any currently mentioned possible price.

Note also that the slope of the switch line does not depend on the load factor for the envisaged unit. It only depends on the difference in emission rates between the two technologies. Further, the switch lines for load factors of 100 % and 75 % are reasonably close to each other. However, further reducing

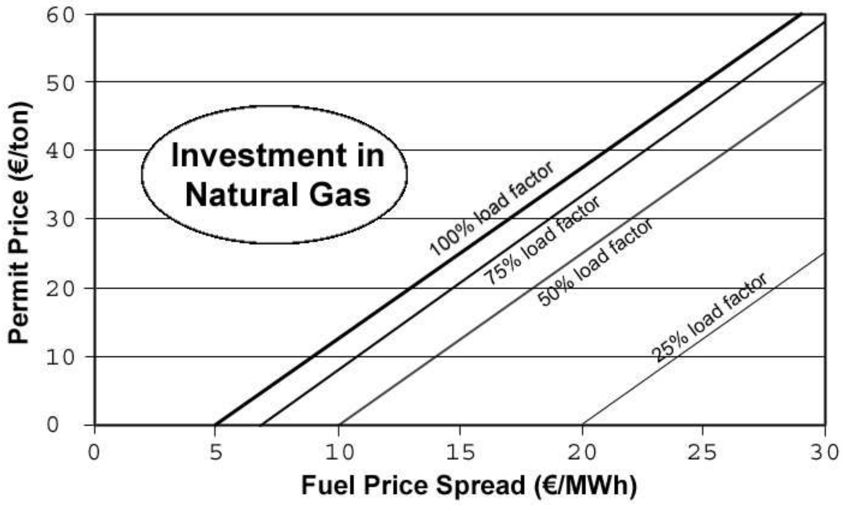

Figure 5.1: Switching Gas for Coal depending on Fuel Spread and Permit Price

the load factor rapidly moves the switch line far away on towards the right (higher fuel spreads are required).

5.3.2 What about nuclear units?

One may question the relevance of limiting investment opportunities to only two technologies, namely pulverised coal and natural gas in a combined-cycle. Although these are definitely commonly referred to, one could also view new investments in nuclear units as a viable alternative, most certainly in the perspective of high emission permit prices.

Figure 5.2 shows the best technological choice (using a pure least-cost criterion) as a function of the number of running hours and of the permit price.

Adding a third technology to the problem, namely nuclear energy, try to find the optimal technology mix. This depends on the load factor and permit price. Denoting with K_n and c_n the investment and variable costs of the nuclear technology, and noting that its emission rate is nil ($e_n = 0$), then for given permit price and load factor, one chooses the technology which minimises:

$$\min\left\{\frac{1}{\theta}K_n + c_n\ ;\ \frac{1}{\theta}K_c + c_c + \eta e_c\ ;\ \frac{1}{\theta}K_g + c_g + \eta e_g\right\} \tag{5.8}$$

In a way similar to the computation of the *arbitrage* value of the permit price between coal and gas, one may also derive frontier lines between nuke and coal, or nuke and gas. This yields:

$$\begin{aligned} \eta^*_{n,c}(\theta) &= \frac{(K_n - K_c)\frac{1}{\theta} + (c_n - c_c)}{e_c}, \\ \eta^*_{n,g}(\theta) &= \frac{(K_n - K_g)\frac{1}{\theta} + (c_n - c_g)}{e_g}. \end{aligned} \tag{5.9}$$

Nuclear technology injects no CO_2 and has a relatively low variable generation cost (we have chosen a typical value of 10 €/MWh). But its investment cost is very important. Based on some typical values found in the literature, we have taken a value of 210 €/kW.yr, which translates into a hourly investment cost of nearly 25 €/MWh.

As can be seen on figure 5.2, once nuclear investments are re-introduced and if gas prices remain reasonably low, very few room is left for coal. The latter is only interesting for very low permit prices and high load factors.

According to the horizontal line, which corresponds to the current level of permit prices, as observed on the market, one sees that nuclear opportunities might be considered but only for very high load factors (above 8000 hours). All of the remaining investments should be aimed towards natural gas. Note that if the permit price were to increase significantly, up to for instance 20 €/MWh, then the load factor threshold for nuclear units would be importantly reduced, as nuclear units might become interesting as of 6500 hours per year. This, however, must probably be considered as unrealistic, given the lack of flexibility of such generation plants.

The situation would however drastically change, if natural gas prices increase. Indeed, considering long-term generation costs of 30 €/MWh (instead of 22) for natural gas, leaving nuclear and coal unchanged, would lead to the situation depicted on figure 5.3.

We see that, although the increase in variable cost is limited to nearly 35%, natural gas investments almost completely disappear from this region of the

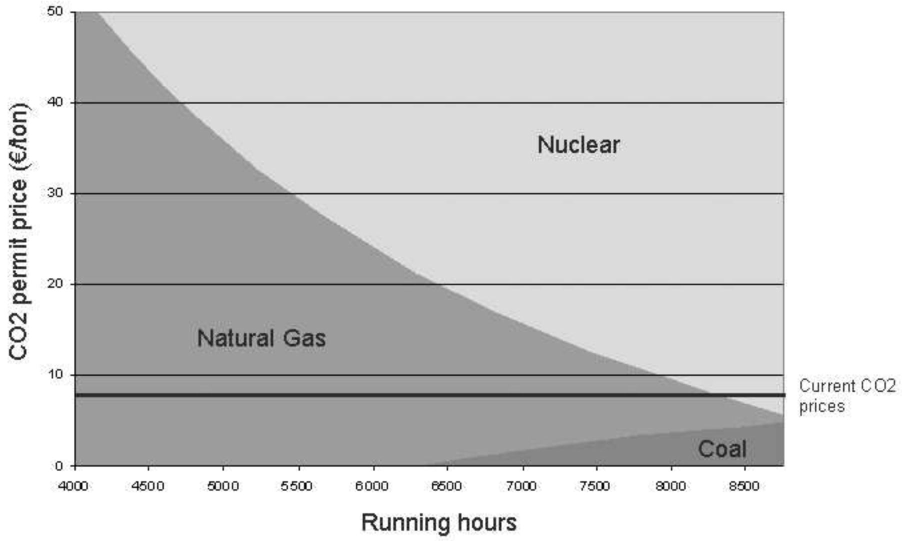

Figure 5.2: Natural Gas, Coal and Nuclear opportunities at low Gas prices

merit-order (between 4000 and 8000 hours per year). Using today's price for the emission permits, investment in the natural gas technology would only be viable for peak units.

It is however not realistic to assume that nuclear units would be chosen for not more than 6000 hours. But even in this area of load factors, the coal technology would still be preferred to natural gas, even for permit prices as high as 15 €/ton.

Therefore, it is important to keep in mind that a not-so-important change in natural gas prices could more than significantly alter the optimal technology mix. The load factor at which all three technologies are equivalent (from the long-run marginal cost point of view) is obtained by solving the system which equates :

$$lrmc_n(\theta) = lrmc_c(\theta) = lrmc_g(\theta).$$

Solving the last part of the equation gives η as a function of θ. Substituting this in the first equation gives :

$$\frac{K_n}{\theta} + c_n = \frac{K_c}{\theta} + c_c + e_c \frac{(K_g - K_c)\frac{1}{\theta} + (c_g - c_c)}{e_c - e_g},$$

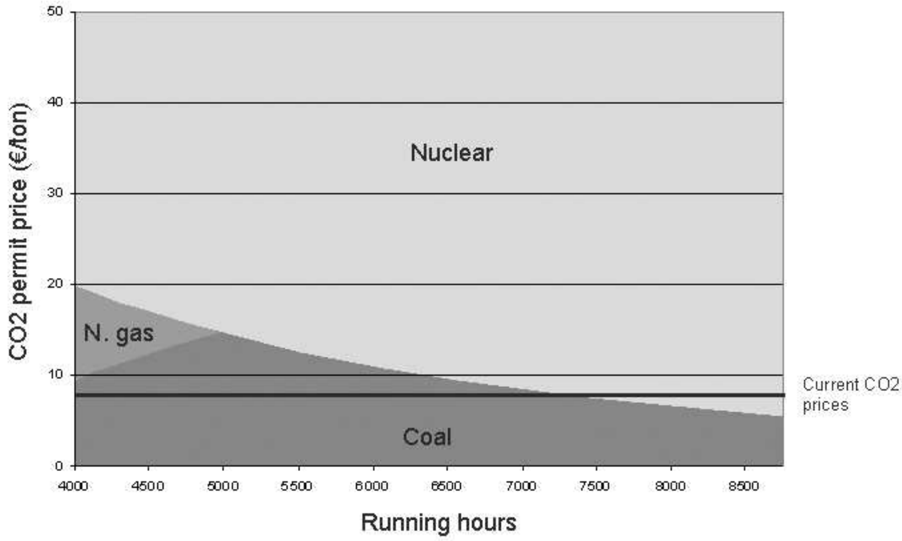

Figure 5.3: Natural Gas, Coal and Nuclear opportunities at high Gas prices

which, once solved for θ, gives the expression:

$$\theta = -\frac{K_n(e_c - e_g) + e_g K_c - e_c K_g}{c_n(e_c - e_g) + e_g c_c - e_c c_g} \tag{5.10}$$

as the load factor value for which all three technologies have the same long-run marginal cost, including emission cost.

We do however believe that there might be some implicit link between the prices of natural gas and emission permits, since an increase in emission permit prices would result in an increased natural gas consumption, which in turn should end up in a gas price increase. This link is not looked at in the current section, but will however be addressed in section 5.4.3.

5.4 Endogenous Price Models

5.4.1 Base Case

In section 5.3, we have essentially been looking at the impact on the investment mix of the relative positioning of the permit price with respect to the spread in fuel generation costs.

In this section, we focus on the interaction between the emitters' need for CO_2 permits and the permit price. This implies the modelling of permit exchanges in the context of an equilibrium model. The right way to price a rare resource is to model explicitly the resource limitation. In other words, we introduce a new constraint which limits the total amount of available permits. The fact that emissions and permits must be balanced for each agent then *de facto* imposes a physical limitation on the total emissions. This *emission constraint* will provide, as dual variable, the opportunity value of one additional permit, that is, economically speaking, the permit price.

We first consider the problem in a "closed-bubble" economy: the CO_2 allocation and trading occurs only between the two operators, and there is no trade with the rest of the industry. In this context, the permit price should settle at a level which provides the right incentive to reduce emissions, that is, adjusting the coal capacity such that the emission constraint can be satisfied. Suppose that, given a current permit price, the emission level would be too high to satisfy the emission limit. The trading dynamics would then adjust the price upwards, until its level prevents producers to invest too much in coal. Conversely, if the price is too high, then natural gas investments will be conducted in excess. The resulting emissions would then remain strictly below the limit, leaving spare permits whose value would then be worthless, driving the permit price back to zero (or, at least, downwards).

Following this reasoning, the conclusion is that in a CO_2 constrained situation, the permit price will settle *exactly* at a level which would satisfy the emission constraint, resulting in a decrease of the investment in the dirtiest technology.

Formulating an equilibrium problem consists of putting together several players' problems, each with its own distinct objective, and trying to find a point from which no one would be incited to depart individually. This is why we start by describing each operator's individual problem and then look at a way to integrate them into a single instance.

The Operator's problem: Each operator behaves so as to maximise its profit at some market prices $\pi(t)$, incurring investment and generation costs as well as costs or revenues accruing from the trading activity on the emission permit market, at price η. On both markets, we assume that there is no market power, or equivalently that both operators are supposed to be price-takers. The fact that the total demand has to be satisfied does not intervene in the individual agent's problem. It implicitly will through the power price signal, $\pi(t)$. The problem faced by agent i can be stated as follows:

$$\begin{array}{llll} \min\limits_{q,z,X} & \sum\limits_t \delta(t) \times [K_i X_i + (c_i - \pi(t)) q_i(t)] & +\eta z_i & \\ \text{s.t.} & q_i(t) \leq X_i & < -\delta(t)\rho_i(t) \leq 0 > & \\ & \sum\limits_t \delta(t) e_i q_i(t) - E_i - z_i \leq 0 & < -\epsilon_i \leq 0 > & \end{array} \tag{5.11}$$

The dual expressions ($-\delta(t)\rho_i(t)$ and $-\epsilon_i$) have been formulated in a way to obtain positive, time-independent values at the equilibrium, which is more straightforward when interpreting results.

In order to be able to express the full equilibrium problem, we need to state this one as first-order complementarity conditions. To do so, we first transform the agent's optimisation problem into its Lagrangian formulation. The latter can be obtained by subtracting from the initial objective function, the cross-product of constraints and their dual expressions. This yields :

$$\begin{aligned} \min\ L(X_i, q_i, z_i, \rho_i, \epsilon_i) &= \sum_t \delta(t) \times [K_i X_i + (c_i - \pi(t)) q_i(t)] + \eta z_i \\ &+ \sum_t \delta(t) \rho_i(t) (q_i(t) - X_i) \\ &+ \epsilon_i \times \left(\sum_t \delta(t) e_i q_i(t) - E_i - z_i \right). \end{aligned} \tag{5.12}$$

The different complementarity conditions can be extracted from the last expression as in the following of this section. The complementarity condition relative to a positive variable can generally be expressed as :

$$\min_x L(x \mid x \geq 0) \quad \Leftrightarrow \quad \frac{\partial L}{\partial x} \geq 0 \quad \perp \quad x \geq 0.$$

Computing this first-order complementarity conditions for our problem yields the following interpretation. The most easily interpretable first-order complementarity conditions are those relative to the supply $q_i(t)$ and investment X_i variables. After a little rearrangement, these express as :

$$\pi(t) \leq c_i + \rho_i(t) + \eta e_i \quad \perp \quad q_i(t) \geq 0, \tag{5.13}$$

$$q_i(t) \leq X_i \quad \perp \quad \rho_i(t) \geq 0, \tag{5.14}$$

$$\sum_t \delta(t)(K_i - \rho_i(t)) \geq 0 \quad \perp \quad X_i \geq 0. \tag{5.15}$$

The first complementarity condition states that generation will only take place for a given technology, if the power price covers the variable and investment costs (more precisely the dual value of the capacity limitation during the relevant time period), plus the CO_2 emission's opportunity cost. The second and third conditions express that if investment takes place, then the whole investment charge has to be reported and covered during time periods where the unit is ran at full capacity.

The complementarity condition on ϵ_i just results in the agent's permit balance. Either the balance holds strictly and permits have for this agent a positive opportunity value, or the balance is loose, and permits are worth nothing. This is expressed by :

$$\sum_t \delta(t) e_i q_i(t) \leq E_i + z_i \quad \perp \quad \epsilon_i \geq 0. \tag{5.16}$$

Finally, the condition relative to the actual emission permits trade z_i simply reduces to $\eta = \epsilon_i$. Note that the latter relation could have been avoided by substituting the last constraint of problem (5.11) in the objective function. This would have suppressed variable ϵ_i.

We now have a full set of four complementarity conditions ((5.13) to (5.16)) and one equality ($\epsilon_i = \eta$) which fully describe each agent's problem.

The Full Problem: In order to obtain the full equilibrium problem, one still needs to add the coupling constraints: the total supply must equal the demand, and the total amount of traded permits must be balanced (each bought permit must have a selling counterpart).

Writing the full system, for all agents, and adding the coupling constraints (total supply equates demand, and total trade equals zero) as well as the allocation mechanism, one gets:

$$\begin{array}{rcl} \pi(t) \leq c_i + \rho_i(t) + \eta e_i & \perp & q_i(t) \geq 0, \\ \sum_t \delta(t)(K_i - \rho_i(t)) \geq 0 & \perp & X_i \geq 0, \\ \eta = \epsilon_i & & < z_i > \\ \sum_t \delta(t) e_i q_i(t) - E_i - z_i \leq 0 & \perp & -\epsilon_i \leq 0, \\ q_i(t) \leq X_i & \perp & -\delta(t)\rho_i(t) \leq 0 \\ \sum_i q_i(t) = D(t) & & < \delta(t)\pi(t) > \\ \sum_i z_i = 0 & & < \eta > \end{array} \tag{5.17}$$

Testing this model on the data described in 5.2 yields the following results. Without any CO_2 consideration, we have shown in section 5.2 that the optimal solution would be to cover 75 MW in baseload using coal, and the 25 MW on peak with natural gas. We have also pointed out that this solution would result in an emission level of 121.25 tons of CO_2. The introduction of endogenous permits on this simple example can result in two quite different situations, depending on the available amount of permits with respect to the base-case emission level.

5.4.1.1 Emission rights are in excess

As long as the available amount of permits remains high enough, the permit price will stick to zero. Changing the level of the fuel price spread will simply result in a change of investment decision: a spread strictly smaller than 5 will result in an all-gas investment decision. Above 10, only coal will be retained, even for the peak load coverage. Between 5 and 10, coal will be preferred for the 75 MW of baseload and natural gas for peak. In all of these cases, however, the permit price remains at zero since the overage of emission permits make the problem irrelevant. Again, profits of both operators remain at zero, whatever the relative prices of coal and natural gas.

5.4.1.2 Shortage of emission rights

Let us now consider the case where scarcity of emission permits yields an opportunity cost, and check the impact on equilibrium. The important thing to remember is that this opportunity cost will be higher for a coal-fired plant than for a natural gas one. In particular, in our example, a permit price of 10 €/ton would impact variable generation costs of gas and coal with an increase of 3.50 and 7.50 €/MWh, respectively.

We start from our base-case assumptions: without CO_2 considerations, coal would be preferred for baseload but gas should cover peak. However, the available amount of permits will not allow for a full baseload generation on coal. Therefore coal investments will be reduced and some natural gas will come to baseload. How does this affect the power and permit prices?

We conduct the detailed computation here for our prototype example, but the same reasoning can be extended to more complex problems.

Since both technologies are present in baseload (that is, both in the on-peak and the off-peak periods), the output variables $q_i(t)$ are all *strictly* positive. This implies that the first-order condition relating power price and costs holds with a strict equality. On the other hand, the coal unit will always run at full capacity, while the gas unit will reach the limit only during the peak period. This has important implications: it implies a value $\rho_g(t_{OFF}) = 0$, which imposes via the second complementarity condition that $\rho_g(t_{ON}) = 2K_g$ since $\delta(t_{OFF}) = \delta(t_{ON}) = 1$. As a result, the first complementarity conditions can be rewritten as equality constraints :

$$\begin{aligned} \pi(t_{OFF}) = \quad & c_c + \eta e_c + \rho_c(t_{OFF}) \quad && = c_g + \eta e_g \\ \pi(t_{ON}) = \quad & c_c + \eta e_c + \rho_c(t_{ON}) \quad && = c_g + \eta e_g + 2K_g. \end{aligned} \tag{5.18}$$

We do however know that capacity dual value for coal, $\rho_c(t)$, must on average also equal the coal investment cost. Therefore, reformulating the previous equations, it comes :

$$K_c = \frac{\rho_c(t_{OFF}) + \rho_c(t_{ON})}{2} = (c_g - c_c) + K_g - \eta(e_c - e_g).$$

In our two-period example, the permit price (provided that coal is economically a viable baseload solution regardless of the EU-ETS problem) will be given by:

$$K_c + c_c \leq K_g + c_g \qquad \Rightarrow \qquad \eta^* = \frac{(K_g + c_g) - (K_c + c_c)}{e_c - e_g}. \tag{5.19}$$

This expression can be rephrased as follows: the permit price, provided that coal would turn out to be a cheaper baseload alternative, will settle at a level such that both technologies are economically equally attractive. The larger the economic advantage of coal, the higher the permit price. The larger the pollutant differential between coal and gas, the lower the permit price. Note that this expression exactly corresponds to equation (5.7) with a load factor of one.

Power prices in both time segments are given by :

$$\begin{aligned} \pi(t_{OFF}) &= c_g + e_g \times \frac{(K_g + c_g) - (K_c + c_c)}{e_c - e_g} \\ \pi(t_{ON}) &= c_g + e_g \times \frac{(K_g + c_g) - (K_c + c_c)}{e_c - e_g} + 2K_g. \end{aligned} \tag{5.20}$$

If, however, the gas solution is cheaper than coal for baseload, then all investments will take place in natural gas and the permits will be valued zero (in this specific example of closed economy).

5.4.1.3 Capacity Limitation

The permit price is thus determined by the relative values of costs for both technologies and by the difference in their emission levels. In this subsection, we will show how the total available amount of emission rights will define the capacity which will be invested in each technology. This can be easily illustrated on our simple example.

Assume that gas is a cheaper solution for baseload. As we mentioned above, the problem greatly simplifies in this case, since all 100 MW are invested in natural gas and the permit price is zero[4]. Neither operator realises profits.

Assume, on the contrary, that coal is a more interesting solution. Then the scarcity of the emission permits will limit the potential capacity which will be invested in coal. We know that the amount of permits which would allow the full baseload to be satisfied with coal, leaving only the peak to natural gas, is equivalent to 121.25 tons. Therefore, as soon as the total quantity A of available permits drops below this level, there is a physical impossibility to ensure the whole baseload production on coal. The investment in coal technology will then be such that we hit the boundary on emission permits, though generating as much as possible using the cheapest (but dirty) technology.

In our numerical example, if A were limited to 100 units, then it would cap the potential investment in coal to 48.4 MW, even though coal might be more profitable economically. Permit prices are thus adjusted to discourage investments in the coal technology.

The scarcity of emission permits creates a rent which is profitable for operators. It becomes a profit which is allocated between both operators, depending on their respective endowment. The total rent is equal to the permit price, times the total number of allocated permits. The profit allocation follows directly from the allocation mechanism (each emission permit obtained for free is directly converted onto profits).

Table 5.2 provides detailed numerical results for the base case. We may in particular verify that baseload long-run marginal costs are equalised by the level of the permit price. We also note that, as a consequence of the price-taking hypotheses we have assumed about the producers' behaviour, they would make no profit if no CO_2 constraint were imposed, but the latter, once activated, creates a *rent* which is in direct proportion of the amount of permits given for free. We also observe that the average power price covers the long-run marginal cost for both technologies in baseload, but the peak price is set to compensate the gas investments for a peak unit only.

[4]This solution is always feasible, provided that *at least* the minimal amount of permits, required to satisfied demand with the cleanest technology, are available.

	Coal	Gas	Total	
Investment	48.4	56.1	100.0	[MW]
Generation	96.9	78.1	175.0	[MWh]
Emissions	72.7	27.3	100.0	[ton]
Endowment	50.0	50.0	100.0	[ton]
Trading	22.7	-22.7	0.0	[ton]
Investment Cost	15.00	10.00		[€/MWh]
Variable Fuel Cost	15.00	22.00		[€/MWh]
Emission Cost	3.75	1.75		[€/MWh]
Baseload LRMC	**33.75**	**33.75**		[€/MWh]
Profit	250.00	250.00	500.00	[€]
Power Prices				
Off-peak		23.75		[€/MWh]
On-peak		43.75		[€/MWh]
Average		**33.75**		[€/MWh]
EU-ETS Price				
Permit		5.00		[€/ton]

Table 5.2: Numerical Solution for the Sample Problem

5.4.1.4 Graphical Representation

In order to give a graphical intuition of the equilibrium solution, both in terms of the impact of the limited amount of permits and the spread between fuel costs, one may have a look at figure 5.4 which illustrates the equilibrium properties on the prototype model.

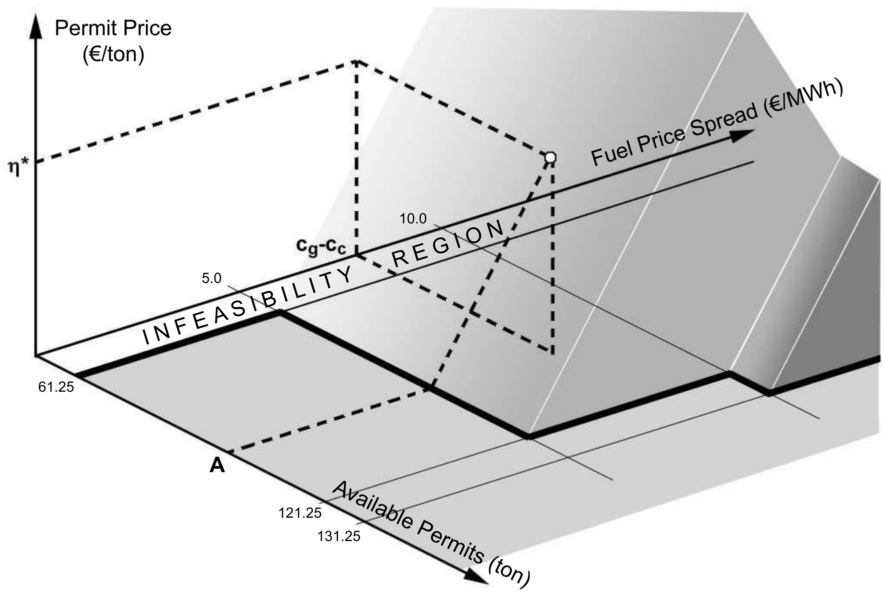

Figure 5.4: Equilibrium Price for the Prototype Model

The fuel price spread should justify whether or not coal should be preferred to gas or vice-versa. But the total amount of available permits will potentially limit the capacity investment in coal. The combination of these two factors, together with the load profile, defines the thick black line in figure 5.4. There is however an *infeasibility region* in which no solution can be found, because the available amount of permits would not be sufficient even while working with the cleanest technology only. On the one side of this line, there are too many available permits with respect to the least-cost technological mix, hence permits are worth nothing (this corresponds to the flat region in the graph). On the other side of the line, permit prices will increase linearly with the fuel spread (except in the infeasible region, where prices should be considered as infinite). This part of the surface, however, is spread over several faces. In the leftmost one, investment in coal are targeted for baseload only (no coal at all on the 61.25 ton border, increasing linearly up to 75 MW coal at the 121.25 ton border). Then there is a switching face: before one invests for coal in the peak period, the fuel price spread should increase by another 5 €/MWh. Only then coal capacity would start to replace gas in the peak period, which corresponds to the third face, in which coal investment progressively passes from 75 to 100 MW.

Therefore, starting from a given total endowment A, and a given fuel price spread $(c_g - c_c)$, one immediately derives the permit price as well as the type of technological mix.

5.4.1.5 Generalisation

The previous reasoning can easily be extended to any kind of load profile, and a graphical solution can still be derived as long as only two technologies are involved.

Consider in this subsection that we have to satisfy a demand, varying over a time horizon T. Disregarding technical constraints such as ramping rates and startup costs, one may neglect the chronological load profile, and simplify the problem by aggregating demand from the highest to the lowest level: the resulting load can be characterised by a *load-duration curve*, as illustrated on figure 5.5, where $\tau(D)$ denotes the amount of time, across the whole time horizon, during which the demand exceeds D.

Let us make the basic assumption that coal's fuel cost is always lower than gas' fuel cost, but on the contrary that investment costs are lower for gas than for coal. The fuel price spread, together with the investment cost differential, defines the load factor above which the coal technology should be preferred to natural gas, as long as we disregard the emission problem.

Using the load-duration curve, one can infer the amount of capacity which should be invested in coal or gas. All demand levels below the level corresponding to this "critical" load factor will be supplied with coal-fired generation, while all demand exceeding this level will be supplied from natural gas. It is also easy to compute the corresponding quantity of CO_2 emissions, and hence the amount of allowances A^* which would be required to satisfy this load

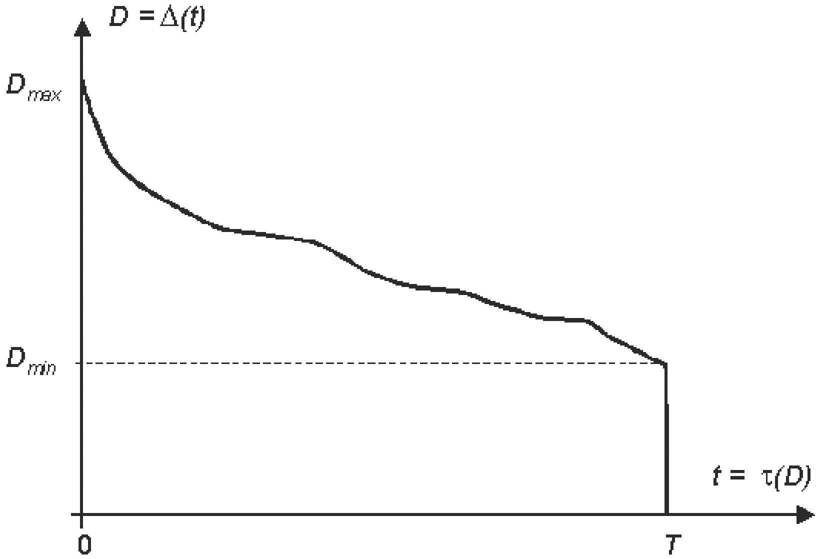

Figure 5.5: Load Duration Curve

profile with the least-cost technology mix (disregarding the permit price).

Let $D \in [D_{min}, D_{max}]$ denote the values of demand during this time horizon. We define the load-duration function $\tau(D)$ as the measure of time during which the demand has been greater than or equal to D. Conversely, we denote with $\Delta(t)$ the demand level which is attained or exceeded during t hours.

Considering the fuel price spread $(c_g - c_c)$, we can easily determine the load factor $\theta^*_{c,g}$ which delimits both technologies, CO_2 put aside. This load factor, conditionally to both the numerator and denominator being strictly positive and that the inequality $c_g - c_c < K_c - K_g$ holds[5], is given by:

$$\theta^*_{c,g} = \frac{c_g - c_c}{K_c - K_g} \tag{5.21}$$

This means that every capacity which is due to function at least a fraction $\theta^*_{c,g}$ of the time should be based on coal. Under this level of load factor, gas is less expensive. Using the load-duration curve notation, this is equivalent to chose

[5]This condition defines the only relevant situation, since a value of the load factor $\theta^*_{c,g}$ outside of the range]0,1[would result in one technology being systematically preferred to the other, from baseload to peak units.

capacities according to :

$$X_c = \Delta(\theta^*_{c,g} T), \qquad X_g = D_{max} - \Delta(\theta *_{c,g} T). \tag{5.22}$$

Assume one has some capacities X_c and X_g such that $X_c + X_g = D_{max}$. If the variable cost c_c of the coal is lower than the variable cost c_g of natural gas (and if one neglects the emission limit) then one would expect to produce only with coal as long as the demand level would not exceed X_c, and to supply the excess demand with natural gas. As a result one would generate Q_c and Q_g MWh using coal and gas, respectively, where :

$$\begin{aligned} Q_c &= \textstyle\int_0^{X_c} \tau(x)dx, \\ Q_g &= \textstyle\int_{X_c}^{D_{max}} \tau(x)dx. \end{aligned} \tag{5.23}$$

The resulting production levels would require an amount of permits, as a function of the fuel price spread, which is given by:

$$A^* = e_c Q_c + e_g Q_g = \int_0^{\Delta(\frac{K_c - K_g}{c_g - c_c} T)} \tau(x) e_c dx + \int_{\Delta(\frac{K_c - K_g}{c_g - c_c} T)}^{D_{max}} \tau(x) e_g dx \tag{5.24}$$

The consequence is straightforward. If the available amount of permits is greater than or equal to A^*, then the plain least-cost solution can be applied, and the permit value is zero.

However, this required amount of allowances will probably not be available, which will *de facto* put a limitation on the amount of capacity which can be invested in coal. This capacity should however be used as much as possible (under the assumption that the fuel price spread is in favour of coal), which will in turn define a marginal load factor for the coal unit. Let A be the actual amount of available permits. Then the problem is then to find the value of $\overline{X}_c$ which solves the equation

$$\int_0^{\overline{X}_c} \tau(x) e_c dx + \int_{\overline{X}_c}^{D_{max}} \tau(x) e_g dx = A < A^*. \tag{5.25}$$

This coal capacity will be used in baseload, and defines the marginal load factor at the equilibrium $\overline{\theta}_{g,c} = \tau(\overline{X}_c)/T$. The latter, in turn, determines the CO_2 permit price at the solution, since :

$$\eta = \frac{(c_g - c_c) - (K_g - K_c)\frac{1}{\overline{\theta}_{g,c}}}{e_c - e_g}. \tag{5.26}$$

The proof of this last statement can be extended from the results developed in section 5.3. To establish this result, we will consider three distinct time sets : t_1 groups time segments in which demand is strictly lower than the coal capacity $\overline{X}_c$, t_2 is when the coal plant is operated at full capacity and the gas plant provides the remainder (but not at full capacity), and finally let t_3 denote time segments when both plants are at full capacity.

Applying the first complementarity condition of system (5.17) for the three time segments yields, after proper withdrawal of the dual variables ρ (which must be set to zero when the unit is not operated at full capacity) :

$$\begin{array}{lclcl} \pi(t_1) & = & c_c + \eta e_c & < & c_g + \eta e_g \\ \pi(t_2) & = & c_c + \eta e_c + \rho_c(t_2) & = & c_g + \eta e_g \\ \pi(t_3) & = & c_c + \eta e_c + \rho_c(t_3) & = & c_g + \eta e_g + \rho_g(t_3), \end{array} \tag{5.27}$$

and the condition on the dual investment value for coal is such that :

$$\begin{array}{lcr} T \times K_c & = & \delta(t_2)\rho_c(t_2) + \delta(t_3)\rho_c(t_3), \\ T \times K_g & = & \delta(t_3)\rho_g(t_3). \end{array} \tag{5.28}$$

Solving this system provides a value of the permit price at the equilibrium given by the expression :

$$\eta^* = \frac{(c_g - c_c) - (K_g - K_c)\frac{T}{\delta(t_2)+\delta(t_3)}}{e_c - e_g} \tag{5.29}$$

We may easily verify that the fraction $(\delta(t_2) + \delta(t_3))/T$ exactly corresponds to the marginal load factor $\overline{\theta}_{g,c}$, since it corresponds to the ratio of full-capacity usage of the coal plant and total time. This allows us to write as a general result:

$$\eta^* = \frac{(c_g - c_c) - (K_g - K_c)/\overline{\theta}_{g,c}}{e_c - e_g} \tag{5.30}$$

where $\overline{\theta}_{g,c}$ is the marginal load factor of the coal technology.

Figure 5.6 illustrates the combined effect of limited emission permits, together with the fuel price spread, on the permit price. The thick black line provides the theoretical amount of permits which would be required at a given fuel price spread, if no CO_2 constraint existed. Permits in excess would drive their price down to zero, while on the other side of the limit the permit price continuously and linearly increases. The shape of this limit depends mainly on the load-duration curve. The infeasibility region remains since a minimum

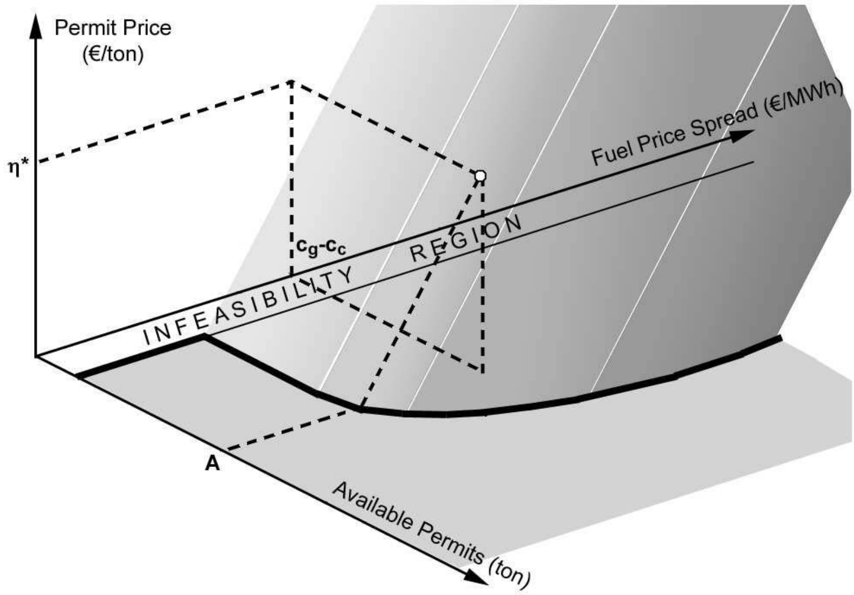

Figure 5.6: Equilibrium Price for a Generalised Model

of emission permits is required to fulfill the whole demand with the cleanest available technology.

5.4.2 Open Economy

The assumption that the power industry remains in its own "bubble" totally isolated from the rest of the world is, of course, unrealistic. In the EU-ETS scheme, various industrial sector will interact on the market of permits, resulting in a permit price which depends on different components of the industry. Many claim that electricity generation will be the main driver for permit prices, but even if this is true, this does not mean that the rest of the market will not influence this price.

The "closed-bubble" approach provides some intrinsic value of the emission permit for the restricted market we are looking at. This intrinsic value is probably not equal to that of the rest of the permits market. The size and magnitude of the difference should determine the flow of permits exchanged with the outside world.

In order to account for this, we should introduce some representation of the rest of the carbon-emitting industry in our stylised model. This can be achieved in a more or less sophisticated way. However, given the stylised character of this prototype study, we should limit ourselves to a very simple representation of this component of the market.

For this reason we have chosen to retain a simple linear relationship between the market price of emission permits, and the net balance of permits exchange between the power industry and the rest of the industry. Let ΔZ denote this net balance, positive if the power industry buys permits from other agents, negative if net seller. When the price signal from the power industry is rather low, this should incite other actors to buy permits from power producers (and hence probably pull prices upwards). On the contrary, if the price signal arising from the power industry alone is high, other agents on the emission market would be interested to sell their permits at high price. In between lies some indifference price (denoted with η^o, and corresponding to the intrinsic value of the permit in the rest of the carbon-emitting industry) at which nothing occurs. In the following, we refer to this price as the *switch price*. The slope of the linear relation gives, in a sense, a measure of the "liquidity" of the permits market: the larger the difference in intrinsic prices, the bigger the flow of permits. Setting this liquidity at zero would completely cut the power industry from the rest of the world.

We provide here the necessary mathematical framework to include the trade of permits with the non-electrical carbon-emitting industry. We denote with ΔZ the net balance of permits between electric and non-electric industries. Positive values account for a net flow from the outside to the power industry, negative values for the contrary.

This net flow basically depends on the relative values of *intrinsic prices* of the permit in each part of the market if they were totally separated. If this intrinsic value is higher in the electricity sector than in the rest, then the natural flow would be from the outside towards the power industry. We do not write an explicit equilibrium model for the non-electrical industry, but we assume that its intrinsic price is equal to some value η^o.

Therefore, we model the net balance as:

$$\Delta Z = \phi \times (\eta - \eta^o), \qquad \phi \geq 0. \tag{5.31}$$

Leaving $\phi = 0$ results in the same model as in the previous section. But the higher the value of this parameter, the more intense the interaction between the power industry and the rest of the industry. For this reason, ϕ can be interpreted as some measure of the liquidity of the permits market. At the limit, if the rest of the industry were able to provide an infinitely liquid market for permits, and were insensible to coal and gas prices, then the permit's market price should always be η^o.

The corresponding amount of permits must, of course, be accounted for in the balance constraint. We therefore replace the last equilibrium condition of system (5.17) by

$$\sum_i z_i - \Delta Z = 0 \qquad < \eta >, \tag{5.32}$$

where the interesting feature is that the value of ΔZ itself depends on the dual value of the balance constraint in which it intervenes.

This interaction on the equilibrium has two possible sorts of impacts. In the first, the net flow will be such that the total amount of available permits is affected but no other constraint hits a bound. In this case, the permit price will not be affected. The change will mainly concern the investment variables because a modification of the available quantity of permits alters the potential coal capacity. This first effect is the one which is illustrated in the numerical example described at the end of this section. As a consequence, the equilibrium permit price of the whole market adjusts to the electricity industry's intrinsic price. This situation is the one in which the power industry drives the permit market price.

In the second possible situation, the net flow alters the amount of available permits enough to hit a technical bound (the thick black line of figure 5.4). In this case the price will be altered to adjust the discrepancy between the two markets.

Let us illustrate this phenomenon on our prototype two-period example. Assume the initial endowment dedicated to the power industry is not sufficient to ensure the full baseload coverage by the coal technology. This implies:

$$A < \big[\delta(t_{OFF}) + \delta(t_{ON})\big] e_c D(t_{OFF}) + \delta(t_{ON}) e_g \big[D(t_{ON}) - D(t_{OFF})\big] \tag{5.33}$$

The marginal load factor for the coal technology will therefore be equal to one. Hence the intrinsic permit price for the electricity industry should still be given by

$$\eta^e = \frac{(c_g - c_c) - (K_c - K_g)}{e_c - e_g}, \tag{5.34}$$

where η^e denotes the intrinsic value of the permit in the electricity industry.

Assume that at this price, however, the flow of permits would be positive. If the price gap between electrical and non-electrical industries is large enough, then the flow of permits will be such that the amount of available permits, $A + \Delta Z$, will make it possible to cover the whole baseload with coal. The permit price will then decrease as $(\eta^e - \eta^o)$ increases, until it reaches the level where coal technology becomes profitable even for the peak period. Simplifying with $\delta(t_{OFF}) = \delta(t_{ON}) = 1$ for the sake of readability, the flow of permits will then stabilise at:

$$\Delta Z = \phi \times (\eta - \eta^o) = 2e_c D(t_{OFF}) + e_g\big[D(t_{ON}) - D(t_{OFF})\big] - A \tag{5.35}$$

while the permit price smoothly drops in the range:

$$\frac{(c_g - c_c) - (K_c - K_g)}{e_c - e_g} \geq \eta \geq \frac{(c_g - c_c) - 2(K_c - K_g)}{e_c - e_g}. \tag{5.36}$$

Once the permit price reaches this level, it would stabilise again and the net flow of permits would restart to increase, allowing for more and more coal capacity in the peak period.

Assume, on the contrary, that the intrinsic price differential is in the other direction: the net flow of permits would then be negative. The final equilibrium price should however not be affected, as long as the net permit flow does not deprive the electricity industry of an amount of its initial permits such that it reaches the minimum required to satisfy all demand with the cleanest technology. From this level, the equilibrium permit price would start to increase with η^o. This would happen from

$$\eta^o \geq \eta^e + \frac{1}{\phi}\big[A - e_g D(t_{ON}) - e_g D(t_{OFF})\big]. \tag{5.37}$$

and the resulting equilibrium permit price would then raise infinitely with η^o.

For the sake of illustration we consider a switch price of 8 €/MWh, which roughly corresponds to the current market price. If the power industry's intrinsic price is below 8, then the non-electric operators would want to buy permits from power producers. If, on the contrary, it is above 8, then the rest of the industry would want to sell some of their permits to power producers. We further assume in our example that they would provide an additional 2.5 permits, for each euro above the switch price.

In this case, keeping everything else unchanged (namely a total of 100 available emission permits and a fuel price spread of 7 €/MWh), the introduction of this interaction does not directly affect the equilibrium price. The intrinsic price of the power industry remains at 5 €/ton, but this has as a consequence that the amount of permits, available to power producers, is reduced from 100 to 92.5. This puts more pressure on coal and hence further reduces the capacity investment in this technology (ending up with 39 MW instead of 48).

A possible way to improve this model could be to make the switch price η^o depend on market prices of coal and natural gas, where we could think of $\eta^o(c_c, c_g)$ as an increasing function of c_g, because of the expected positive correlation between permit prices and gas demand, and hence gas prices. The dependence on c_c is likely to be less tight although dramatically high permit prices coupled with reasonable gas prices would result in a severe reduction of demand for coal (in favour of gas), which in turn would put the pressure downwards on the coal price.

Assume this relation to be linear:

$$\eta^o(c_c, c_g) = \overline{\eta}^o + \gamma_g(c_g - \overline{c}_g) + \gamma_c(c_c - \overline{c}_c) \tag{5.38}$$

where $\gamma_g > 0$ and $\gamma_c \leq 0$, and $\overline{c}_g$ and $\overline{c}_c$ are some reference levels for the prices of gas and coal, respectively. The permits balance constraint would then become:

$$\sum_i z_i - \phi\eta + \phi\big[\overline{\eta}^o + \gamma_g(c_g - \overline{c}_g) + \gamma_c(c_c - \overline{c}_c)\big] = 0 \qquad < \eta > . \tag{5.39}$$

Further investigation on the impact on the equilibrium of such relations has not been conducted yet.

5.4.3 Gas Price Reaction

Natural Gas prices will probably be affected by the EU-ETS scheme. In this section we shall try to provide the economic tools which could help assessing the impact of such a relationship. We start from the base case, without emission permits, and see how the latter will affect gas consumption, and potentially gas prices. Remember that the base case would see all baseload demand supplied by a coal-fired unit, leaving only the peak 25 MWh for gas generation.

Once endogenous permits are introduced, while leaving the cost of gas generation at the same level of 22 €/MWh, gas-fired production is pushed up to 78 MWh, that is, roughly three times the base case. Does it seem reasonable to stick to the idea that gas prices will remain unaffected? Probably not. It is however hard to predict exactly how gas prices should evolve, but we will provide the modelling framework to at least conduct some sensitivity testing.

We introduce a simple modelling of the relation between gas price and gas consumption in the electricity industry: suppose that gas price evolve with a constant elasticity ϵ. This implies that an increase by 1 % in gas consumption results in an increase by ϵ % in gas price. To fully define the relationship, we need some *reference point* defined by a gas price $\overline{c}_g$ and a gas consumption level $\overline{q}_g$. The latter is expressed in its equivalent electricity production, summed over all time periods.

A reference point definition is always somewhat arbitrary. In this study we have chosen as reference, the gas consumption level corresponding to the solution of the problem without permit constraints (in which only the peak is covered by a gas-fired unit), and to take for the reference price the same value as in the base case, that is, the equivalent of 25 MWh at 22 €/MWh. Let us assume an elasticity value of $\epsilon = 0.5$.

The relation which should be added to the whole model is defined by the non-linear constant elasticity function, that is:

$$\frac{dc_g}{c_g} = \epsilon \frac{dq_g}{q_g} \qquad \Rightarrow \qquad c_g = \overline{c}_g \left(\frac{\sum_t \delta(t) q_g(t)}{\overline{q}_g} \right)^{\epsilon}, \quad \epsilon > 0. \tag{5.40}$$

This relation does however imply an additional non-linearity in the model, which makes it a bit more complex to solve. We will denote with λ the dual variable of this relation, yet this is only for the sake of the rewriting of the full model in its complementary shape, see section 5.4.5.

Note that at the equilibrium, and under the additional condition that the whole baseload (that amounts to D_{min}) cannot be satisfied with coal-fired electricity, using the prototype model in a closed-bubble, the permit price can still be computed explicitly :

$$\eta^* = \frac{\overline{c}_g \left[\frac{\sum_t \delta(t) \left(D(t) - \frac{A - e_g \sum_\tau D(\tau)}{2(e_c - e_g)} \right)}{\sum_t \delta(t)(D(t) - D_{min})} \right]^\epsilon - c_c - (K_c - K_g)}{e_c - e_g}. \tag{5.41}$$

This expression indeed yields a value of the permit price which reaches 47.21 € per ton, using the usual data.

Allowing the gas price to raise when consumption increases has dramatic effects on our prototype model. Since the available quantity of allowances does not change, the constraint on coal capacity is maintained. Therefore, the total electricity produced with gas subsists, whether gas price raise or not. But since the gas consumption is multiplied by three, one should expect a rather severe increase of the gas price. This is illustrated on figure 5.7, for various values of the total permit endowment (on the horizontal axis). The grey line shows the evolution of the gas price (in €/MWh) as the total available quantity of permits changes. The tighter the emission constraint, the more gas is required to replace coal, hence the higher the gas price. At our reference point (100 available permits), the gas price reaches 38.9 €/MWh.

This gas price increase has an unexpected and rather perverse side-effect. Since the permit price at the equilibrium settles at a level such that it arbitrates between coal and gas technologies at the coal's marginal load factor. If the gas price increases significantly, then so should the permit price. And it does: the dotted black line in figure 5.7 gives the evolution of the CO_2 emission permit price. One can see that at our reference point, the permit price reaches a value of 47.2 €/ton, which is almost ten times its value in the reference case! As a result, the baseload long-run marginal cost climbs up to a little over 65 €/MWh. This is almost twice as high as the baseload price which was obtained in the base case.

If one does however integrate the results of the previous section, by allowing the interaction between the electricity industry and the rest of the CO_2-emitting industry, and examines its combination with the gas price dynamics, then we see that the impact on final prices of the price responsiveness is largely softened. The explanation is quite simple: if the permit price increases because of the gas price dynamics, then this would incite external CO_2 traders to bring more permits into the electricity generation market, which in turn would partially relax the emission constraint, hence the need for additional gas would be reduced.

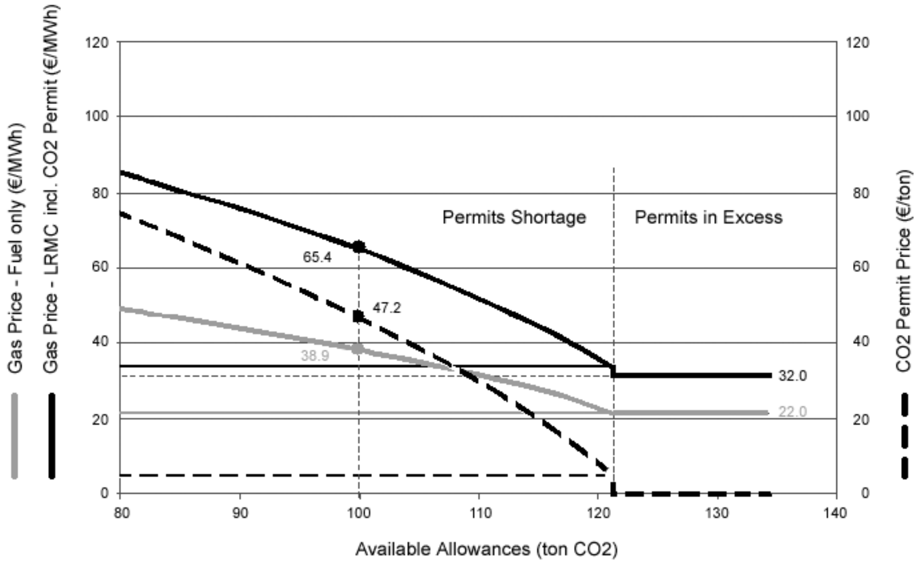

Figure 5.7: Impact of Varying Gas Prices

Numerically, starting with 100 permits but allowing the rest of the industry to interfere (with the same characteristics as in section 5.4.2, one would end up with a final gas price of 26.6 €/MWh, which is significantly lower than in the closed-economy case. The resulting permit price would settle at 16.6 €/ton, which brings a little over 20 additional permits to the electricity industry. The resulting baseload long-run marginal cost reaches 42.4 €/MWh.

As a conclusion to this section, the modelling of an endogenous gas price function results in increasing gas prices, as the total emission constraint becomes tighter. This gas price increase impacts final power prices, but is even amplified by the fact that permit prices raise accordingly with the fuel price spread.

It should be noted that, around the equilibrium, a reduction by 10 % of the available allowances would result in an increase by 33 % of the total gas consumption, which would then impact both gas and permit prices, ending up in a 17 % increase of the baseload power price ! However, this effect is mitigated by the interaction with the rest of the industry, since in this case the final impact on power price would be limited to 6%, which is however not negligible.

5.4.4 Gaming the Allocation Mechanism

Within the EU-ETS framework, each Member State has been assigned emission targets for different commitment periods. Allocation mechanisms between industry operators within the different countries are however left to subsidiarity. Hence, different countries might in the long run adopt different allocation schemes (Grandfathering, best available technology, ...).

The question one wants to assess in this section is whether the retained mechanisms provide the right incentives to reduce emissions, without distorting competition. More generally, we want to assess the impact of potentially differentiated allocation schemes on the competition between generators, and analyse the incentives sent to carbon-emitting installations.

Consider the generic problem faced by a power plant operator i, who decides to invest some new capacity X_i. We simplify the problem of emission permits as follows. In a stationary regime, we assume that emission permits E_i are allocated between installations in proportion of their energy output, whatever their respective emission rate. However, the total available amount of permits to be allocated between agents is limited to some value A. In the following, we describe the way such allocation mechanisms and total emission limitation can be introduced. It must be noted, however, that these allocation mechanisms generally imply non-linearities in the model, which are not necessarily easy to solve.

Several types of allocation mechanisms can be envisaged. In the following, we will more specifically focus on three types of permits allocation, namely

- fixed endowments,
- allocation in proportion of the generated energy,
- allocation in proportion of emissions.

The first allocation scheme is the one that has been used throughout this document, until now. It consists of allocating fixed amounts of permits to each carbon-emitting installation, whatever the use they make of these permits (even if they totally stop emitting, they would still get the permits). This could be a variant of the "best available technology" scheme. Obviously, there are few chances to game this mechanism.

In the second, permits are granted to each operator in proportion of their *energy output*. This would maybe make sense in a global view, where each operator would control a set of more or less polluting assets. In this example where one operator controls a single dirty asset, and the other a single clean asset, this allocation is biased.

In the last one, the national authority grants permits in proportion of actual emissions. This corresponds in some way to grand-fathering, assuming that allocation based on past history may be part of a stationary phenomenon.

We will here focus on allocations proportionally to generated energy. The extension of the results in this section to other mechanisms are straightforward.

If the total amount of permits, available to the electricity industry, is limited to A, and that each operator obtains a fraction of this amount in proportion of its own generation, then the allocation mechanism can be very simply stated by writing

$$E_i = \frac{\sum_t \delta(t) q_i(t)}{\sum_j \sum_t \delta(t) q_j(t)} \times A \tag{5.42}$$

As long as operators believe this endowment cannot be altered by their actions (i.e. as long as they behave competitively with respect to the allocation mechanism), then this will not impact the final solution in terms of prices and outputs. Each operator receives a fixed amount E_i of permits, but the permit price will settle independently at its equilibrium level, given by equation (5.30).

Therefore the endowment is viewed as an externality, its quantity cannot be changed and its value being set by relative costs of gas and coal, it ends up as a fixed term in the objective function of each operator and hence can simply be dropped from the problem. This is the reason why allocation mechanisms do not influence the permit price (as long as they do not change the total available quantity of permits).

Hence, whatever the retained allocation mechanism, the ultimate equilibrium in terms of prices, emission and generation, is the same – at least as long as there is no attempt to game the system. The only effect of the allocation mechanism is to change the initial endowment, and hence the final profit of each operator. Since generators have been modelled as price-takers both on the power market and the permits market, they make no profit except, for the permits they get for free and value at their opportunity cost. But things could be quite different if operators know how their endowment responds to their own decisions.

Impact of the different Allocation Mechanisms: Table 5.3 provides the detailed results for the tests conducted on the three allocation mechanisms, both in a closed-bubble and in an open economy. The reader will verify that in each endowment mechanism, the profits are distributed differently between the two operators, but the physical solution in terms of investment, generation and emission is common. The only fundamental difference is the initial allocation of permits, which of course requires different traded volumes and, *in fine*, different profits.

One may also note that the opening of the permit market to the rest of the carbon-emitting industry results in a supplementary reduction of the coal capacity, as mentioned in section 5.4.2, but the profit distribution benefits largely to the "clean" gas operator. This can be explained by the fact that the system sells permits to the rest of the industry, and that these excess permits can only be obtained by reducing the coal capacity, hence increasing both gas-based generation and emissions. The second and third allocation mechanisms therefore attribute more allowances to the gas operator, therefore increasing its profit. Note that the profit increase for this operator is exactly equal to the rent accruing from selling excess permits to the rest of the world.

Note also that profit increases for the coal generator when one switches from the second (proportional to generation) to the third (proportional to emissions) allocation mechanism. This is also quite straightforward since, for the same production level, the relative emissions from coal-based generation are way higher those based on gas. Whether the "fixed endowment" mechanism ends up above or below the others, in terms of profits, is purely arbitrary and depends on the relative values of initial endowments.

Introducing Potential Gaming: The question we want to address in this section is: do these three allocation mechanisms send the right economic signals? In other words, would one or several operators be tempted to modify its behaviour (investment, generation, emissions) in order to try to distort the

	Coal	**Gas**	**Total**	
CLOSED-BUBBLE – PERMIT PRICE 5.0 €/ton				
Investment	48.4	56.1	100.0	[MW]
Generation	96.9	78.1	175.0	[MWh]
Emissions	72.7	27.3	100.0	[ton]
Fixed Endowment				
Endowment	50.0	50.0	100.0	[ton]
Trading	22.7	-22.7	0.0	[ton]
Profit	*250.0*	*250.0*	*500.0*	[€]
Prop. to Generation				
Endowment	55.4	44.6	100.0	[ton]
Trading	17.3	-17.3	0.0	[ton]
Profit	*277.0*	*223.0*	*500.0*	[€]
Prop. to Emissions				
Endowment	72.7	27.3	100.0	[ton]
Trading	0.0	0.0	0.0	[ton]
Profit	*363.5*	*136.5*	*500.0*	[€]
OPEN ECONOMY – PERMIT PRICE 5.0 €/ton				
Investment	39.1	60.9	100.0	[MW]
Generation	78.1	96.9	175.0	[MWh]
Emissions	58.6	33.9	92.5	[ton]
Fixed Endowment				
Endowment	50.0	50.0	100.0	[ton]
Trading	8.6	-16.1	-7.5	[ton]
Profit	*250.0*	*250.0*	*500.0*	[€]
Prop. to Generation				
Endowment	44.6	55.4	100.0	[ton]
Trading	14.0	-21.5	-7.5	[ton]
Profit	*223.0*	*277.0*	*500.0*	[€]
Prop. to Emissions				
Endowment	63.3	36.7	100.0	[ton]
Trading	-4.8	-2.7	-7.5	[ton]
Profit	*316.5*	*183.5*	*500.0*	[€]

Table 5.3: Endowments without Gaming

allocation mechanism? Do operators have some incentive to modify their generation plan in order to obtain more permits?

A model is proposed in the following, which tackles this problem. The modelling is based on the notion of *reasonable assumption* and implements the following concept: one (or several) operator believes that, according to the allocation mechanism and in a stationary situation[6], he would obtain a given amount of additional permits for any additional generated MWh, above the equilibrium situation. Conversely he also anticipates a reduction in endowments if he reduces his output.

We wonder whether one operator would be induced to change its behaviour in terms of production level, in order to bias the allocation mechanism with the view of increasing its profit. We try here to answer this question in a stationary regime, that is, where operators immediately obtain emission permits as a function of their decisions (investments and generation levels).

In order to assess this question, we introduce an *anticipation* of the possible future endowment as follows. We will assume that operators have information on how permits are allocated, and that they have an intuition of how their emissions or production levels may influence their endowment in the long run. We do however let the competitors act as price-takers, they only try to twist the allocation mechanism by the way of their actual emission profile.

The price-taking behaviour of each operator consists of solving the following problem (see equation (5.11)) :

$$\begin{aligned} \min \; L(X_i, q_i, z_i, \rho_i, \epsilon_i) \;&=\; \sum_t \delta(t) \times [K_i X_i + (c_i - \pi(t))q_i(t)] + \eta z_i \\ &+\; \sum_t \delta(t)\rho_i(t)(q_i(t) - X_i) \qquad (5.43) \\ &+\; \epsilon_i \times \left(\sum_t \delta(t) e_i q_i(t) - E_i - z_i \right). \end{aligned}$$

In case of scarcity of emission permits, which is the only relevant situation for

[6]This assumption mostly implies that endowments would immediately follow changes in output levels.

us, the latter problem resumes to its simplified expression:

$$\begin{aligned} \min\ L(X_i, q_i, z_i, \rho_i, \epsilon_i) &= \sum_t \delta(t) \times [K_i X_i + (c_i - \pi(t)) q_i(t)] \\ &+ \sum_t \delta(t) \rho_i(t)(q_i(t) - X_i) \\ &+ \eta \times \left(\sum_t \delta(t) e_i q_i(t) - E_i \right). \end{aligned} \tag{5.44}$$

We do not change the assumption that operators behave as price-takers, but we elaborate on their expectation of a change of permit endowments if they strategically move their production. In other words, we introduce an *anticipated* relation $\hat{E}_i \leftarrow \hat{E}_i(q_i)$.

This, when writing down the complementarity condition with respect to $q_i(t)$ has the an important implication. Assume the coal operator believes that he can influence the allocation, one gets:

$$\pi(t) \leq c_i + \rho_i(t) + \eta \left(e_i - \frac{1}{\delta(t)} \frac{\partial \hat{E}_i}{\partial q_i(t)} \right) \quad \perp \quad q_i(t) \geq 0.$$

Note that the player is still a price taker, it does not believe that the price η will change. Simply he anticipates that the allocation mechanism will give him a different amount of permits.

In other words, this condition is equivalent to say that the operator subjectively underestimates its own emission rate. As a result, he will probably try to produce more than what he would do under normal circumstances and try to obtain more emission rights.

At the equilibrium, however, the total available amount of emission rights is capped and maintains the former limit on the coal-fired unit production. For this reason, neither the actual investment in coal technology, nor the production level of the coal unit will be affected. But the increased pressure exerted by the coal operator will make the permit price rise.

Numerical Evidence Let the gas variable cost c_g be 22.0 €/MWh, while we leave coal generation price c_c down at 15.0/ €MWh. The initial total endowment is equal to 100 permits. Equation (5.19) tells us that the permit price should establish at 5.0 €/ton. This is indeed the model result if no gaming takes place.

Assuming that the coal operator hopes to increase its own endowment by 0.25 ton, for each additional MWh he produces. This is equivalent to define a conjectured parameter:

$$\alpha_c(t) = \frac{1}{\delta(t)} \frac{\partial E_i}{\partial q_i(t)} = 0.25. \tag{5.45}$$

Assuming the coal operator is active on the market, his equilibrium conditions with respect to its production level become

$$\pi(t) = c_c + \rho_c(t) + \eta \left(e_c - \alpha_c(t)\right). \tag{5.46}$$

Since the equilibrium condition is biased but that the output cannot change, due to the global cap constraint, the only solution is to change the permit price (and, subsequently, the power price at equilibrium). The new permit price becomes 13.33 €/ton, and impacts power price by an additional 3.41 €/MWh in both time periods.

One can show that, generalising this behaviour to both operators, the new equilibrium permit price becomes :

$$\hat{\eta}^*(\alpha_c, \alpha_g) = \frac{(K_g + c_g) - (K_c + c_c)}{(e_c - \alpha_c) - (e_g - \alpha_g)}. \tag{5.47}$$

We consider, as a case study, that only the coal operator behaves this way. More precisely, coal-based generation emits 0.75 ton of CO_2 for each produced MWh, but we model the fact that the coal operator believes he could get 0.25 permit for each supplementary generated MWh. We measure the impact of this behavioural change in table 5.4. Two very distinct phenomena occur, which have to be looked at separately.

Gaming in a closed-bubble : If both power producers are only allowed to trade permits between them, then the resulting endowment taking gaming into account would not be affected. Indeed, the fixed bound on emissions still sets an implicit upper bound on the coal-based generation. Therefore the total coal production remains unchanged, and endowments are unaffected with respect to their levels in the absence of gaming.

But the permit price is affected! The most significant change in results arises because of the pressure on permit trading by the coal operator, yielding a higher price for emission permits. In our example, permit prices rise from

5.0 to 13.33 €/ton. Profits for the gas operator are modified accordingly (passing, for the fixed endowment, from 250 to 666.7 €). But if the profit of the coal operator also increases, it does not do so in the same way. The change in behaviour which was introduced biases the "price-taking" equilibrium conditions. As a result, profits realised by the coal operator increase with respect to the "no-gaming" situation, but not as much as those of the gas operator, which eventually benefits the most from the imperfectly competitive behaviour of his rival: gas operator's profits increase by 166%, whereas coal operator only increase by 77%.

Gaming in an open economy : The results of the gaming strategy in an open economy are quite different. Here, if one operator, by changing his behaviour, is able to put some pressure on the permit price, then he is indirectly able to influence the total amount of permits available to the power industry, since higher prices would result in more permits flowing towards the electrical sector.

In this example, inserting the interaction with the non-electrical carbon-emitting industry as in section 5.4.2 results in a significant increase of the amount of permits available to the power producers (from 100 to 108.3 tons) and hence a subsequent increase in total invested coal capacity. In this case, the opening of the interaction does not affect the permit price, but this depends on the liquidity of the permit market[7].

Here the coal operator is really able to manipulate both prices and quantities. By raising the permit price, he attracts additional permits from the rest of the industry, and captures these permits in order to increase his output (generation and emissions). This is true only if the gaming has an actual impact on allocations. In the case of fixed endowment, the coal operator increases his production hoping for additional permits, but these obviously will not come. In the two other allocation mechanisms, profits increase significantly for the coal-based generator, to the prejudice of the gas-based generator.

[7]Higher values of the amount of permits per € would smoothen the differential in the permit price.

	Coal	Gas	Total	
CLOSED-BUBBLE – PERMIT PRICE 13.33 €/ton				
Investment	48.4	56.1	100.0	[MW]
Generation	96.9	78.1	175.0	[MWh]
Emissions	72.7	27.3	100.0	[ton]
Fixed Endowment				
Endowment	50.0	50.0	100.0	[ton]
Trading	22.7	-22.7	0.0	[ton]
Profit	*343.7*	*666.7*	*1010.4*	[€]
Prop. to Generation				
Endowment	55.4	44.6	100.0	[ton]
Trading	17.3	-17.3	0.0	[ton]
Profit	*415.2*	*595.2*	*1010.4*	[€]
Prop. to Emissions				
Endowment	72.7	27.3	100.0	[ton]
Trading	0.0	0.0	0.0	[ton]
Profit	*645.8*	*364.6*	*1010.4*	[€]
OPEN ECONOMY – PERMIT PRICE 13.33 €/ton				
Investment	65.1	34.9	100.0	[MW]
Generation	130.2	44.8	175.0	[MWh]
Emissions	97.7	15.7	113.3	[ton]
Fixed Endowment				
Endowment	50.0	50.0	100.0	[ton]
Trading	47.7	-34.3	13.3	[ton]
Profit	*232.6*	*666.7*	*899.3*	[€]
Prop. to Generation				
Endowment	74.4	25.6	100.0	[ton]
Trading	23.3	-9.9	13.3	[ton]
Profit	*558.0*	*341.3*	*899.3*	[€]
Prop. to Emissions				
Endowment	86.2	13.8	100.0	[ton]
Trading	11.5	1.8	13.3	[ton]
Profit	*714.9*	*184.4*	*899.3*	[€]

Table 5.4: Endowments with Gaming from Coal operator

5.4.5 The Complete Model

The full model, incorporating all features introduced from section 5.4.2 through 5.4.4, can now be expressed as:

$$
\begin{aligned}
\pi(t) \leq c_i + \rho_i(t) + \eta\left(e_i - \frac{1}{\delta(t)}\frac{\partial \hat{E}_i}{\partial q_i(t)}\right) &\quad \perp \quad q_i(t) \geq 0, \\
\sum_t \delta(t)(K_i - \rho_i(t)) \geq 0 &\quad \perp \quad X_i \geq 0, \\
\eta = \epsilon_i &\quad\quad\;\; < z_i > \\
\sum_t \delta(t) e_i q_i(t) - E_i - z_i \leq 0 &\quad \perp \quad -\epsilon_i \leq 0, \\
q_i(t) \leq X_i &\quad \perp \quad -\delta(t)\rho_i(t) \leq 0 \\
\sum_i q_i(t) = D(t) &\quad\quad\;\; < \delta(t)\pi(t) > \\
E_i \times \sum_j \sum_t \delta(t) q_j(t) - A \times \sum_t \delta(t) q_i(t) = 0 &\quad\quad\;\; < \chi_i > \\
c_g = \overline{c}_g \left(\frac{\sum_t \delta(t) q_g(t)}{\overline{q}_g}\right)^{\epsilon}, &\quad\quad\;\; < \lambda > \\
\textstyle\sum_i z_i - \phi\eta + \phi\big[\overline{\eta}^o + \gamma_g(c_g - \overline{c}_g) + \gamma_c(c_c - \overline{c}_c)\big] = 0 &\quad\quad\;\; < \eta > .
\end{aligned}
\tag{5.48}
$$

5.4.6 Models Complexity

The base case model described in (5.17) is very easy to solve as it may be reformulated as a simple optimisation problem with quadratic objective function and linear constraints. The same holds for the extension to an open economy, as the quantity of permits exchanged with the outside industry may be introduced in the objective function by the means of a utility function. They were written in GAMS and solved with standard quadratic (CPLEX) or non-linear (CONOPT) solvers.

All subsequent models (namely including gas price reaction and gaming of the allocation mechanisms) require complementarity formulations. Introduction of the gas price reaction requires non-linear expressions. The resulting model was formulated as non-linear EP, with explicit complementarity relations, and solved as a non-linear problem, with CONOPT, MINOS or KNITRO. Modelling of the gaming mechanism alone (without the non-linear gas reaction) is an equilibrium problem subject to equilibrium constraints (EPEC) and was formulated as a linear complementarity program, solved with PATH. The modelling of both features together give rise to a non-linear EPEC, which was formulated as a non-linear problem, with explicit complementarity constraints, and solved with the above mentioned NLP solvers.

5.5 Conclusion and Further Work

The first objective of this paper was to provide a modelling framework of the CO_2 trading scheme, in its yet poorly defined shape. We have proposed an extensive modelling formalisation of this trading scheme and built a prototype model in accordance with this formalisation.

As second step, we have analysed the impact of some economic assumptions on the future equilibrium. This analysis was meant to be qualitative, not quantitative. In particular, we have shown how interaction with the non-electrical industry, price-responsiveness of natural gas, and intentional gaming of the allocation mechanisms, may interfere with a standard base-case equilibrium.

Among the notable results of this important step, we have established that in a two-technology world the permit price would systematically settle at a level which arbitrates between both technologies at the marginal load factor of the dirtiest one. Another important result is that envisaging price-responsiveness for natural gas seems quite wise, but the analysis has revealed a secondary (and perverse) effect: if gas prices increase because of a raising gas consumption, then the permit price will also increase because the arbitrage condition is still true. At last we have also pinpointed the fact that some operators might be able to manipulate the allocation mechanisms in order to increase their own endowment as well as their own profit.

Eventually, this study should pave the way towards the implementation of EU-ETS trading mechanisms in large-scale optimisation and simulation models of the electricity system. As far as this last point is concerned, some data problems may be encountered.

A realistic implementation of the National Allocation Plans (NAP) in a model of the electricity industry should not raise much technical problems but requires these NAPs to be known. This is not the case for Member States at this stage.

Concerning the price-responsiveness of natural gas, econometric estimations of the elasticity (or whatever other parameter depending on the functional shape of this price-responsiveness) shall be required. Gas experts in the department could probably help us finding some answers. With regard to the interaction between electric and non-electric industries on the permit market, data mining should eventually provide a supply-demand curve.

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