

UNIVERSITE CATHOLIQUE DE LOUVAIN
Ecole Polytechnique de Louvain (EPL)
Département de Mécanique
Unité de Thermodynamique et Turbomachines (TERM)



**Simulation of turbulent aircraft wake vortex flows
and their impact on the signals returned by a coherent
Doppler LIDAR system**

Laurent Bricteux

Composition du jury:

Pr. M. Papalexandris (président)
Pr. G. Winckelmans (promoteur)
Pr. L. Dufresne
Pr. H. Jeanmart
Pr. J.F. Remacle
Pr. P. Sobieski

Thèse présentée en vue
de l'obtention du grade de
Docteur en Sciences Appliquées

To Colette, Léon and Mike

Contents

Acknowledgements	1
List of acronyms	3
1 Introduction: aircraft wake vortices and LIDAR measurements	5
1.1 Scope of the thesis	5
1.2 Contents and achievements of the thesis	6
1.3 The aircraft wake vortex phenomenon	8
1.4 Turbulence physics	12
1.4.1 Incompressible Navier Stokes equations	12
1.4.2 The energy cascade of Kolmogorov-Richardson	13
1.4.3 Homogeneous isotropic turbulence (HIT).	13
1.5 Introduction to LIDAR Measurement	17
2 Presentation of the numerical codes used for the simulation of turbulent flows	21
2.1 Fourth order finite difference Navier Stokes solver	22
2.1.1 The need for a high order code	22
2.1.2 Description of the code	24
2.1.3 Validation case 1: DNS of the Taylor-Green vortex flow at $Re = 1600$	32
2.1.4 Validation case 2: DNS of a four vortex system at $Re =$ 5000	34
2.1.5 Validation case 3: Channel flow at $Re_\tau = 180$	39
2.1.6 Conclusions	48
2.2 Pseudo-spectral (PS) Navier Stokes solver	49
2.2.1 Description of the code	49
2.2.2 De-aliasing strategy and energy conservation	51
2.2.3 Validation case: DNS of the Taylor-Green vortex at $Re =$ 1600	55
2.2.4 DNS of homogeneous isotropic turbulence at $Re_\lambda = 100$	55
2.2.5 Conclusion	57

3	Contributions to the large eddy simulation of turbulent flows	61
3.1	The large eddy simulation of turbulent flows	61
3.2	The Smagorinsky model (SMAG)	63
3.3	The WALE SGS model	65
3.3.1	Definition of the model	65
3.3.2	LES of the channel flow at $Re_\tau = 395$	68
3.3.3	SGS viscosity of vortical flows	73
3.4	The sampling based dynamic procedure	76
3.4.1	Description of the procedure	76
3.4.2	Decaying incompressible isotropic turbulence, using 4th order FD	78
3.4.3	Incompressible channel flow, using 4th order FD	81
3.5	High order hyper-viscosity model	84
3.6	The Chollet-Lesieur spectral eddy viscosity model	85
3.7	Multiscale models	86
3.7.1	<i>Type I</i> models	87
3.7.2	<i>Type II</i> models	88
3.8	Asymptotic behaviour of SGS models.	90
3.8.1	Introduction	90
3.8.2	Results	90
3.8.3	Conclusions	93
3.9	A new multiscale WALE type model	102
3.9.1	Definition of the model	102
3.9.2	Results for the channel flow	102
3.10	Conclusions	107
4	LES of aircraft wake vortices in weak atmospheric turbulence: reaching the “equilibrium turbulent vortex system”	109
4.1	Introduction	109
4.2	Computation of the background turbulent flow	110
4.3	Description of the wake vortex flow	116
4.4	Results	120
4.4.1	Flow visualisation	120
4.4.2	Evolution of the kinetic energy and of the dissipation	127
4.4.3	Longitudinal modal energy	130
4.4.4	Energy spectra at equilibrium	132
4.4.5	Circulation decay and trajectories	132
4.4.6	Circulation and tangential velocity profiles	136
4.4.7	Turbulent stresses	138
4.5	Conclusions	146
5	LES of aircraft wake vortices in ground effect, without and with wind.	147
5.1	Introduction	147
5.2	Description of the flow and numerical setup	149

5.3	Case without wind	151
5.3.1	Flow visualisations	151
5.3.2	Evolution of the vortex strength	161
5.3.3	Modal analysis	163
5.3.4	Analysis of the vortex trajectory	163
5.4	Case with cross wind	166
5.4.1	Simulation of a turbulent boundary layer at $Re_\tau = 590$	166
5.4.2	Flow visualisation	173
5.4.3	Quantitative results	179
5.5	Conclusions	182
6	Modelling of a coherent Doppler LIDAR using a fiber LASER.	185
6.1	Coherent Doppler pulsed LIDAR measurement	185
6.1.1	Doppler effect	187
6.1.2	Coherent detection principle	187
6.1.3	Pulsed vs continuous mode	191
6.2	Backscattering particles	191
6.2.1	Molecules	192
6.2.2	Atmospheric aerosols	193
6.3	Optical scintillation	199
6.3.1	Description	199
6.3.2	Relation between atmospheric turbulence and optical scintillation	199
6.4	Development of a LIDAR measurement simulator	203
6.4.1	LIDAR signal	203
6.4.2	The Feuilleté model	208
6.5	Performance parameters of a LIDAR	210
6.5.1	Signal to noise ratio	210
6.5.2	Spatial resolution	211
6.5.3	Velocity accuracy	211
6.5.4	LASER beam coherence	211
6.6	Fluid flow interpolation technique	212
6.6.1	LIDAR scanning patterns	215
6.7	Radial velocity profiles processing for vortex systems	216
6.8	LIDAR measurement simulation results	219
6.8.1	Airborne remote sensing of a pair of aircraft wake vortices in atmospheric turbulence	219
6.8.2	Remote sensing of a turbulent boundary layer	222
6.8.3	Remote sensing of aircraft wake vortices in ground effect	227
6.9	Conclusions	232
7	Conclusions	233

A	2D finite differences code used for simulation of 2D ground effects	247
A.1	Description of the code	247
A.2	Description of the flow	248
B	The $Q - R$ classification of Perry and Chong	251
C	Generation of a random “turbulence” field	253
D	Vortex-in-cell code	255

Acknowledgements

Ce travail a été financé dans le cadre du projet LASEF de la Région Wallonne à laquelle je sais le plus vif gré.

Je voudrais d’abord exprimer ma profonde gratitude envers mon promoteur, Grégoire Winckelmans pour la confiance qu’il m’a témoignée en m’intégrant dans son groupe de recherche, en m’acceptant comme doctorant et en me donnant les moyens de mener à bien ce travail. Sa maîtrise de la mécanique des fluides, des mathématiques et du calcul numérique ainsi que son goût du travail bien fait ont été pour moi une source d’inspiration de grande valeur.

Ce travail doit aussi beaucoup à Hervé Jeanmart : son dynamisme et son efficacité m’ont aussi servi d’exemple. Grâce à son intérêt soutenu pour ma recherche, beaucoup de questions posées lors de ce travail ont trouvé une réponse.

Je remercie aussi Jean-François Remacle d’avoir accepté de faire partie de mon comité d’encadrement et d’avoir ainsi porté un regard extérieur avisé sur mes travaux.

Louis Dufresne m’a fait le grand honneur de participer à ce jury. Je lui en suis très reconnaissant. Il m’a de plus initié aux méthodes spectrales et s’est toujours intéressé de près à mes recherches. Frédérique et moi n’oublions pas non plus les agréables journées passées ensemble à Montréal.

Les compétences en télédétection de Piotr Sobieski ont été très utiles pour le projet LASEF et l’élaboration de ce texte. Je le remercie aussi pour sa participation à l’évaluation de ce travail.

J’ai aussi beaucoup apprécié l’enthousiasme et la confiance témoignés par Benoit Macq, coordinateur du projet LASEF.

Ma gratitude va également vers Miltiadis Papalexandris qui a accepté de présider le jury de cette thèse.

J’ai aussi pu compter sur l’aide de mes collègues. Je pense en particulier à Goéric Daeninck, toujours prêt à rendre service et à prodiguer de bons conseils. Ses multiples compétences tant en mécanique des fluides qu’en programmation ont été appréciables. Je n’oublie pas non plus Matthieu Duponcheel qui m’a aidé de ses précieux conseils, m’a soutenu lors de mémorables sessions de “débogage” grâce à ses fameuses routines MATLAB et a relu de nombreux passages du présent texte.

Ce fut aussi un réel plaisir de travailler avec Roger Cocle avec qui j’ai

partagé la rédaction de deux articles. Je salue ici sa rigueur et son efficacité. Grâce à Laurent Georges, la partie “simulation numérique” de la thèse n’est pas née d’une feuille blanche. Je lui dois beaucoup car son travail a permis de conduire aux bons choix technologiques du code de différences finies ainsi qu’à la première version de ce code.

Aux autres chercheurs du groupe “Turbulence & écoulements tourbillonnaires” : Timothée, Cédric, Ivan et Stavros, je souhaite beaucoup de plaisir et un vif succès dans leurs travaux. J’aimerais aussi souligner qu’il m’a été agréable de partager le bureau des “anciens” : Raphaël, Thomas, Vincent et Olivier.

J’ai eu beaucoup de plaisir à découvrir les fondements théoriques de la mesure LIDAR avec Sébastien. Brousmiche de l’unité TELE, les projets LASEF et FIDELIO lui doivent beaucoup. J’ai toujours apprécié les discussions fructueuses que nous avons eues. Je lui souhaite le meilleur pour son travail doctoral. Je remercie aussi Sébastien. Lugan pour le traitement des signaux LIDAR.

Que tous les membres de TERM soient aussi remerciés pour leur chaleureux accueil et l’ambiance de travail conviviale qui règne dans l’unité. Ma gratitude va également vers l’équipe de gestion du calcul intensif, en particulier Bernard et Jean-Paul, pour leur aide sur le cluster Lemaître.

Ma famille ainsi que ma belle famille m’ont beaucoup soutenu sur le plan logistique et les relectures, je leur en suis très reconnaissant.

Je suis redevable à ma chère épouse Frédérique de beaucoup de sacrifices, notamment les longs dimanches sans balade. Je la remercie aussi pour sa confiance inébranlable, sa compréhension et le réconfort qu’elle m’a apporté dans les moments plus difficiles.

Et enfin, merci à ma petite Adeline pour le bonheur qu’elle me procure.

List of acronyms

BPLO	Back-Propagated Local Oscillator.
CFD	Computational Fluid Dynamics.
CPU	Central Processing Unit of a computer.
DNS	Direct Numerical Simulation.
EDR	Eddy Dissipation Rate.
DSP	Digital Signal Processing.
FAR-Wake	Fundamental Research on Aircraft Wake phenomena. EU-FP6 Research project, http://www.far-wake.org/
FD	Finite Differences.
FFT	Fast Fourier Transform.
FFTW	“Fast Fourier Transform in the West” FFT library.
FIDELIO	FIber laser DEvelopment for next generation LIidar Onboard detection system. EU-FP6 Research project http://www.fidelio-fp6.org/
FSF	Filtered Structure Function.
HIT	Homogeneous Isotropic Turbulence.
HV	Hyper-Viscosity.
ICAO	International Civil Aviation Organization.
IF	Intermediate Frequency.
IGE	In Ground Effect.
IR	InfraRed.
LASEF	“Laser A Source d’Emission Fibrée” (fiber LASER). Belgian “Région Wallonne” Research project.
LASER	Light Amplification by Stimulated Emission of Radiation.
LES	Large Eddy Simulation.
LIDAR	Light Detection And Ranging.
LO	Local Oscillator.
LOS	Line Of Sight.
MG	MultiGrid.
PRF	Pulse Repetition Frequency.
PS	Pseudo Spectral.

RADAR	Radio Detection And Ranging.
RANS	Reynolds Averaged Navier Stokes.
RF	Radio Frequency.
RMS	Root Mean Square.
RVM	Regularized Variational Multiscale.
SGS	Subgrid Scale.
SODAR	Sound Detection And Ranging.
SNR	Signal to Noise Ratio.
SMAG	Smagorinsky model.
TKE	Turbulent Kinetic Energy.
UCL/TELE	Communications and remote sensing Laboratory. Université catholique de Louvain, http://www.tele.ucl.ac.be/
VIC	Vortex In Cell.
WALE	Wall Adapting Local Eddy Viscosity.

Chapter 1

Introduction: aircraft wake vortices and LIDAR measurements

1.1 Scope of the thesis

This thesis concerns the numerical simulation and the remote sensing of aircraft wake vortex flows. The investigation is twofold: it concerns, on the one hand, the physics and the numerical simulation of aircraft wake vortices and, on the other hand, the technique to detect and measure properties of those vortices. This work is performed in the framework of the research project LASEF¹ funded by “the Région Wallonne” of Belgium. The aim of the LASEF project is the design and the simulation of a ground-based coherent Doppler LIDAR² based on a fibered LASER source. One of the original ideas emanating from this project is to couple state of the art fluid dynamics simulations with a LIDAR signal simulator. This work is thus truly multi-disciplinary, as it involves many fields, such as fluid dynamics, numerical simulation and remote sensing. The LASEF project was done in close collaboration with the remote sensing and signal processing laboratory of the Université catholique de Louvain and the optical laboratory of the Multitel Research Center. The achievements realized in the scope of this study have also some valuable impact on other research project of the sixth framework program of the European commission: FIDELIO³ and FAR-Wake⁴.

¹ “LASER à source d’émission fibrée.”

² The LIDAR measurement technique will be introduced in the section 1.5

³ Fiber laser DEvelopment for next generation LIdar Onboard detection system

⁴ Fundamental Research on Aircraft Wake Phenomena

1.2 Contents and achievements of the thesis

- Chapter 1

The first chapter is dedicated to a general introduction of the concepts used throughout this thesis. We first present the physical phenomenon of aircraft wake vortices. The theoretical basis relative to turbulence and its numerical simulation are given in the second part. A qualitative introduction to the LIDAR measurement technique is provided in the last part.

- Chapter 2

This chapter sets out the numerical techniques used to perform the fluid dynamics simulations. In the first part, we present a state of the art parallel fourth order finite difference code. This code was initially designed and developed by L. Georges during his doctoral work [46]. The performances and the reliability of this code were further improved in the present work and with the help of T. Lonfils and M. Duponcheel in the framework of their graduation thesis and during the Ph.D. thesis of M. Duponcheel. The implementation of several subgrid scale (SGS) models was also performed. The direct numerical simulation (hereafter referred as DNS) of several flows of interest was performed to validate the methodology and assess the quality of the code. The second part of this chapter is devoted to a pseudo-spectral code. This code was initially developed by the research group of D. Carati and it was improved in the framework of this thesis to use an energy conserving dealiasing strategy and incorporate several subgrid scale models. Several DNS of flows of interest are also presented.

- Chapter 3

All the subgrid scales models used and developed to perform large eddy simulation of aircraft wake vortices flows are presented in this part of the text. The advantages and drawbacks of each model are presented, with a quantitative support which consists in theoretical considerations or numerical investigations. A sampling-based dynamic procedure is also presented with promising results for homogeneous turbulence and mitigated results for turbulent channel flow. In order to perform LES of vortical flows in the presence of ground, a new model was implemented and tested successfully on channel flow. Finally, a study concerning the asymptotic behaviour of subgrid scale models performed in collaboration with R.Cocle in the framework of his Ph.D. thesis [22] is presented.

- Chapter 4

The first aircraft wake vortex application we investigate is the behaviour of a pair of aircraft wake vortices put in very low level atmospheric turbulence. The LES of this flow is performed, at a very high Reynolds number. This study shows some new aspects of the evolution and the decay of aircraft wake vortices. This work highlights the turbulent equilibrium reached by such a flow and allows to predict the dissipation rate of a turbulent vortex pair. Relevant diagnostics are presented and analysed, and fluid flow visualizations permit to get a better understanding of the flow topology. The fluid flow databases resulting from this study serve also to simulate LIDAR signals as described in chapter 6. As this study also demonstrates, the axial velocities fluctuations in such a vortex system are high enough to be measured, these databases also helped to simulate airborne wake vortex detection as in Lugan et al. [75].

- Chapter 5

The second aircraft wake vortex application is a wall-resolved LES of wake vortices in ground effect, as it is a particularly relevant flow for an airport ground based LIDAR. The LES is performed at a fairly high Reynolds number ($Re = 20000$), which requires large grids and appropriate SGS models. We also study the influence of a cross-wind. For that purpose, we performed a preliminary simulation of a turbulent boundary layer. Relevant diagnostics are presented and fluid flow simulations provide some new insights on the topological aspects of these particular flows.

- Chapter 6

In this last chapter, the theoretical foundations of coherent Doppler LIDAR remote sensing are explained. In particular all the important aspects of atmospheric transmission of a LIDAR signal are studied, such as the LASER particle interaction and the performances of such a device. An analysis of the physical phenomenon of optical scintillation and its relation with fluid mechanics has also been carried. This study was further followed by S. Brousmiche to investigate the effects of the turbulence on a LASER beam propagation by means of the phase screens technique [13]. Then, the LIDAR simulator developed in close collaboration with S. Brousmiche is presented. Results of the simulation of LIDAR measurement of turbulent wind and several flows related to the aircraft wake vortex problematics are provided. Also some techniques to retrieve relevant informations from the sensed LIDAR velocity profiles are presented.

1.3 The aircraft wake vortex phenomenon

The lift force acting on an aircraft is due to the pressure difference between the lower side (pressure side) and the upper side (suction side) of its wing. As a result of this pressure difference, there is a spanwise flow at the edges of the wing from the pressure side to the suction side. This pressure difference forces the suction side streamlines to converge toward the center of the wing and the pressure side streamlines to diverge from it. This spanwise flow combined with the free-stream velocity produces a swirling motion of the air trailing downstream of the wing. A picture of the streamlines of this flow is displayed in Fig. 1.1. Just behind the trailing edge, a vortex sheet is shed which rolls-up rapidly within a few span lengths to form a pair of counter rotating vortices of equal strength. The circulation, Γ , of the velocity field, $\mathbf{u}$, for each vortex is a quantity of primary importance: it is an image of the strength of this vortex. For a closed contour $\mathcal{C} = \partial\Omega$ enclosing a patch of vorticity defined by the field $\boldsymbol{\omega} = \nabla \times \mathbf{u}$,

$$\Gamma = \int_{\Omega} \boldsymbol{\omega} \cdot \mathbf{n} dS = \oint_{\mathcal{C}} \mathbf{u} \cdot d\mathbf{l}. \quad (1.1)$$

In the particular case of a vortex alone, it is natural to choose a circular integration contour $\mathcal{C}$ and since the velocity field has the form $\mathbf{u} = u_{\theta}(r)\hat{\mathbf{e}}_{\theta}$ it follows that:

$$\Gamma(r) = 2\pi r u_{\theta}(r) \quad (1.2)$$

The velocity profile is thus closely related to the circulation profile. A typical vortex circulation profile is illustrated in Fig. 1.2, this is a “low order algebraic” circulation profile which will be the vortex model used further in this work. The vorticity, the circulation and the tangential distribution are given by the following relations:

$$\omega(r) = \frac{\Gamma_0}{\pi} \frac{r_c^2}{(r^2 + r_c^2)^2}, \quad (1.3)$$

$$\Gamma(r) = \Gamma_0 \frac{r^2}{(r^2 + r_c^2)}, \quad (1.4)$$

$$u_{\theta}(r) = \frac{\Gamma(r)}{2\pi r}. \quad (1.5)$$

The parameters are Γ_0 , the total circulation of the vortex and r_c , the radius of maximum induced tangential velocity. It is an indicator of the core size of a vortex; typically it amounts, after the initial roll-up phase, to 3 – 5% of the wing span b .

A typical velocity profile for a pair of vortices is illustrated in Fig. 1.3. Examining Fig. 1.3 tells us that a downwash velocity will alter the trajectory of an airplane passing between the two vortices. If it passes on one of the sides of the vortex pair, it will feel an upwash velocity. If the airplane enters in a vortex zone, a strong rolling moment will be applied to it. The momentum

conservation allows to bind the initial circulation Γ_0 of an aircraft vortex pair to the lift force itself, equal to the weight of the aircraft:

$$L = Mg = \rho U b_0 \Gamma_0, \quad (1.6)$$

where U is the flight velocity and ρ is the fluid density. The circulation is thus proportional to the mass M of the aircraft and inversely proportional to its flight speed U . Typical values for the circulation are $\Gamma_0 \approx 430 \text{ m}^2/\text{s}$ for an Airbus A340 and $\Gamma_0 \approx 630 \text{ m}^2/\text{s}$ for the new A380. The length b_0 is the distance between the vortex cores; its value is approximately $b_0 = \frac{\pi}{4}b$ (assuming elliptical loading) with b the wingspan. The vortex spacing b_0 can be deduced following the lifting line theory [95]. Wake vortices generated at take-off are very strong since the airplane evolves at low speed and is fully loaded. Aircraft wake vortices are also the source of the drag induced by the lift force acting on the aircraft: the so-called “induced drag”. For further informations, one can read [95].

Numerical example

We consider the case of a generic heavy aircraft in cruise conditions, for which we estimate some of the important parameters to illustrate the concepts introduced here. The wing span is $b \approx 60 \text{ m}$. Assuming an elliptically loaded wing, one has $b_0 \approx \frac{\pi}{4}b = 47 \text{ m}$. The vortex core size is approximated by $r_c \approx 0.05 b_0 = 2.35 \text{ m}$. The mass of the aircraft is $M \approx 210 \text{ tons}$. The speed of the aircraft in cruise condition is $U \approx 250 \text{ m/s}$. Given this, Eq. (1.6) allows to compute the circulation $\Gamma_0 \approx 490 \text{ m}^2/\text{s}$. Assuming that the lift to drag ratio also called finesse, or glide ratio, has the following typical value for such an aircraft: $f = \frac{L}{D} \approx 15$. The total drag force value which is equal to the total thrust of the engines in cruise condition is $D = 135 \text{ kN}$ (this value is close to the thrust of one CFM-56 engine). In cruise conditions, the induced drag amounts to roughly 50% of the total drag: $D_i = 67.5 \text{ kN}$. The required cruise power is: $P = DU = 34 \text{ MW}$. Making a power budget on the gas turbine yields a fuel consumption given by the following relation:

$$\dot{m} = \frac{P}{\eta_{tot} LHV}.$$

The approximative value of the global engine efficiency is chosen as: $\eta_{tot} = 0.3$. The lower heating value (LHV) of the kerosene is $LHV = 43000 \text{ kJ/kg}$. This yields a fuel consumption $\dot{m} = 2.6 \text{ kg/s}$. The specific fuel consumption value is $m_s = 11 \text{ kg/km}$.

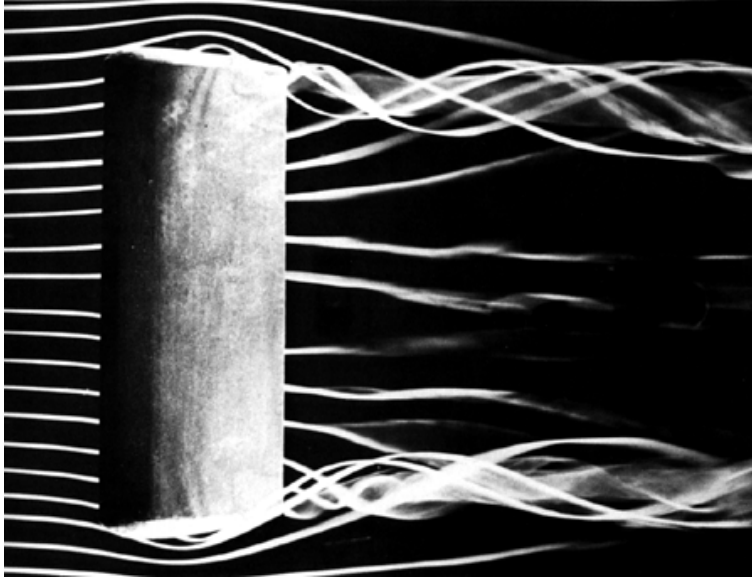


Figure 1.1: Topology of the fluid flow in the vicinity of a wing: the vortex sheet emanates from its trailing edge and this sheet rolls-up in two coherent counter-rotating vortices. Photo from H. Werlé, ONERA.

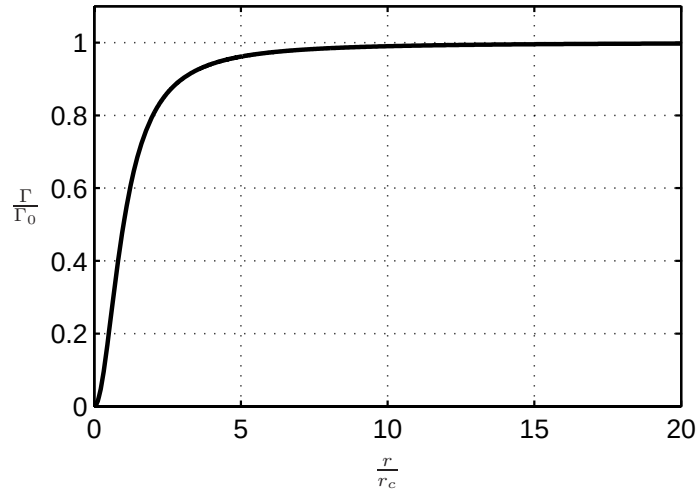


Figure 1.2: Typical model circulation profile $\Gamma(r)$ for a wake vortex.

Table 1.1: International Civil Aviation Organization (ICAO) aircraft separation distances to avoid wake vortex encounter, in nautical miles (1 NM = 1.852 km). For all other combinations, the separation is 3 NM. Table reproduced from [51].

Leader aircraft max take-off weight	Follower aircraft (metric tons)	Separation Nautical miles	Time delay [s] Approach speed: 70m/s
Heavy	Heavy ($\geq 136 T$)	4	106
Heavy	Medium ($< 136 T$)	5	132
Heavy	Light ($\leq 7 T$)	6	159
Medium	Light	5	132

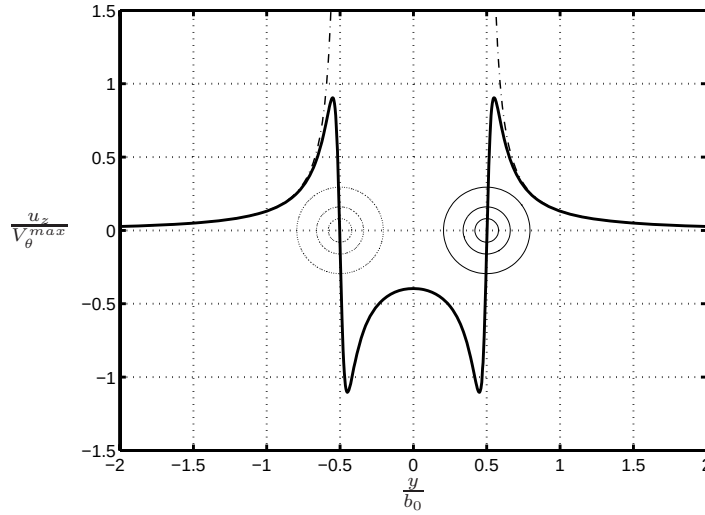


Figure 1.3: Vertical velocity profile in the far wake of an aircraft. The three contour levels are plotted for the values $\frac{\omega b_0^2}{\Gamma_0} = 0.1, 1, 10$. The dotted circles represents the negative vorticity for a clockwise rotating vortex. The dashed-dotted lines represent the potential velocity profile $V_\theta = \frac{\Gamma_0}{2\pi r}$. The curve is normalized by $V_\theta^{max} = \frac{\Gamma_0}{4\pi r_c}$ in the case of a low order algebraic model.

1.4 Turbulence physics

A fluid can experience two very different regimes. In the laminar regime, the velocity field is a regular quantity, eventually unsteady (laminar unsteady flow). Several equivalent experimental devices will reproduce almost exactly the same flow, with the same velocity structures. In the turbulent regime, the velocity field becomes an apparently chaotic quantity which cannot be exactly reproduced even when using exactly the same experimental device, as mentioned in Carati et al. [17]. The value of the Reynolds number defined by Eq. (1.11) determines if a flow is turbulent or not. Most of the flows in nature are in the turbulent regime. In this section we first recall the equations of motion for a Newtonian fluid and we then introduce the phenomenology of turbulence.

1.4.1 Incompressible Navier Stokes equations

Newtonian fluids, like air or water, obeys to the Navier-Stokes equations. The Navier-Stokes equations are a direct translation of the principles of classical mechanics into the language of fluid dynamics.

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (u_j u_i) = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \tau_{ij} + F_i \quad (1.7)$$

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (1.8)$$

The equation (1.7) expresses the conservation of momentum for a parcel of fluid. $P = p/\rho$ is the reduced pressure, ρ is the fluid density, $\nu = \mu/\rho$ is the kinematic viscosity. The vector $\mathbf{F}$ represents the forcing. It has the dimensions of an acceleration. For instance it can be a streamwise pressure gradient in the case of a channel flow or a Coriolis force in the case of turbomachinery or atmospheric flow. The mass conservation for an incompressible flow requires the velocity field to be solenoidal as stated by the constraint (1.8). The viscous stress tensor is written as (Newtonian fluid):

$$\tau_{ij} = 2\nu S_{ij}. \quad (1.9)$$

S_{ij} being the strain rate or deformation tensor.

$$S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (1.10)$$

Considering U and L , the typical velocity and length scales of the flow, the Reynolds number is defined as:

$$Re = \frac{U L}{\nu} \quad (1.11)$$

Re is the ratio of the inertia forces to the viscous forces in a fluid flow. For a given geometry, this number is the unique parameter determining the topology

and the regime of the flow. Above a critical value the laminar regime is unstable and the flow is unsteady and, for higher Reynolds, turbulence is observed. For aircraft wake vortex systems, the Reynolds number is defined as $Re = \frac{U_0}{\nu}$. Considering for instance the previous example, assuming that $\nu = 4 \times 10^{-5} m^2/s$ one obtains $Re = 1.2 \times 10^7$, which is illustrative of the fact that this flow is at very high Reynolds number and thus dominated by convection.

1.4.2 The energy cascade of Kolmogorov-Richardson

One way to characterize a turbulent flow is to refer to the hierarchy of the different scales associated to it. A turbulent flow can be seen as a set of “eddies” of different sizes. At the top, one can find the integral length scale, at which the energy is injected into the flow. At the end of this hierarchy lie the dissipative scales; they dissipate the energy through viscous stresses. Between these two extremes, there is what is called the inviscid energy cascade. The big scales transfer their energy to smaller scales, those smaller scales pass their energy to even smaller scales and so on down to the dissipative scales where the viscous diffusion process becomes dominant. The generation of small scales from larger ones is mostly an inviscid process; which is due to the non linear nature of the convective term present in the Navier-Stokes equations.

1.4.3 Homogeneous isotropic turbulence (HIT).

Homogeneous isotropic turbulence is a particular turbulent flow which has the main characteristic to have orientation independent statistics in the entire domain. An experimental realization of this flow has been performed by Comte-Bellot et al. [25]. It consists in a flow developing behind a grid. A typical experimental energy spectrum of homogeneous isotropic turbulence is displayed in the figure 1.4. By use of dimensional analysis, Kolmogorov found that the shape of the energy spectrum in the inertial range has the following form:

$$E(k) = C_K \epsilon^{2/3} k^{-5/3}, \quad (1.12)$$

where k is the wavenumber and C_K the Kolmogorov constant for which a modern value (computed on the 4096³ DNS of homogeneous isotropic turbulence [126]) is $C_K = 1.6$. A typical energy spectrum is displayed at Fig. 1.4. The turbulent kinetic energy per mass unit is defined as⁵:

$$E = \frac{1}{V} \int_V \frac{u_i u_i}{2} dV = \left\langle \frac{u_i u_i}{2} \right\rangle = \int_0^\infty E(k) dk. \quad (1.13)$$

The kinetic energy dissipation [m^2/s^3] per unit mass is:

$$\epsilon = -\frac{dE}{dt} = \langle \tau_{ij} S_{ij} \rangle = \nu \langle 2S_{ij} S_{ij} \rangle. \quad (1.14)$$

⁵The notation $\langle f \rangle$ denotes the volume average of a quantity f : $\langle f \rangle = \frac{1}{V} \int_V f dV$.

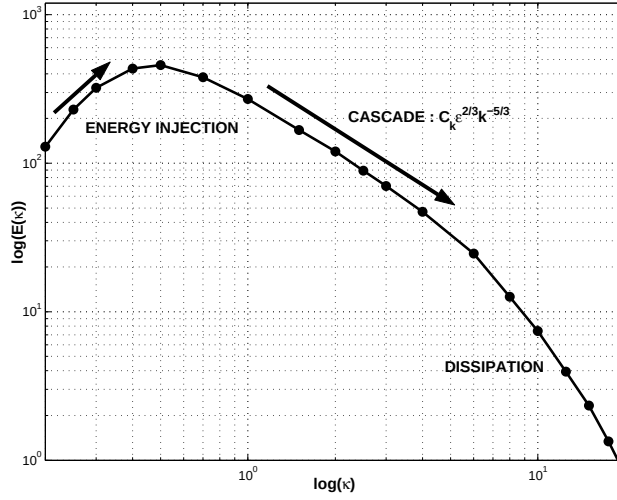


Figure 1.4: Measured spectrum of the Comte-Bellot grid turbulence experiment [25]. Measurement points are represented by bullets.

It is also linked to the enstrophy $\mathcal{E}$ in the following way:

$$\epsilon = 2\nu\mathcal{E} = 2\nu \left\langle \frac{\omega_i \omega_i}{2} \right\rangle. \quad (1.15)$$

The dissipation is determined by the large scales of the flow. The integral lengthscale $\mathcal{L}$ is the largest scale of a turbulent flow, it exists however other definitions. This length is determined approximatively by the domain geometry or the size of the obstacle generating the turbulence. Using the RMS velocity u_{RMS} one has:

$$\epsilon \propto \frac{u_{RMS}^3}{\mathcal{L}}. \quad (1.16)$$

The Kolmogorov scale defines the order of magnitude of the size of the eddies at which the dissipation takes place:

$$\eta = \left(\frac{\nu^3}{\epsilon} \right)^{\frac{1}{4}}. \quad (1.17)$$

This helps to estimate the number of grid points needed to perform a direct numerical simulation (DNS), for which all the scales of the flow are properly discretized on the grid. The number of grid points needed in one direction is $N \propto \frac{\mathcal{L}}{\eta} = Re^{\frac{3}{4}}$. As a consequence, the number of grid points is $N^3 \propto Re^{\frac{9}{4}}$. On dimensional analysis grounds, it can also be showed that the total number of time steps needed to capture all the temporal frequencies, is also dependent on the Reynolds number: $N_t = Re^{\frac{3}{4}}$. Hence, the total “cost” of the simulation scales as Re^3 , which limits drastically the Re number for which DNS can be

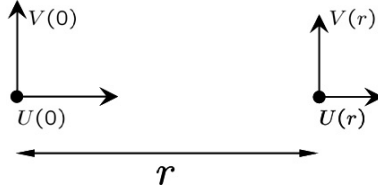


Figure 1.5: Two points correlation function axis convention.

performed. The criterion to choose the grid size h to have obtain well resolved DNS is:

$$k_c \eta \geq 1.5 \iff h \leq 2\eta. \quad (1.18)$$

The cutoff wavenumber, of the highest mode that can be represented on a grid of size h is that of wavelength $\lambda_c = 2h$, and thus:

$$k_c = \frac{\pi}{h}. \quad (1.19)$$

Since $h \approx 2\eta$ for a DNS, we conclude that the smallest viscous eddies are of size $\lambda_c = 2h \approx 4\eta$. The turbulent kinetic energy is the energy of the turbulent fluctuations as there is no mean flow. This allows to compute the RMS velocity as:

$$u_{RMS} = \sqrt{\frac{2}{3}E}. \quad (1.20)$$

As there is no geometrical length available in HIT flows, the Taylor micro-scale is defined. It is a length based on the RMS velocity definition and the dissipation:

$$\lambda = \sqrt{\frac{15\nu u_{RMS}^2}{\epsilon}}. \quad (1.21)$$

This scale does not correspond to a real eddy scale. It is however related to the two points correlation functions (longitudinal and transverse, see Fig.1.5);

$$R_L(r) = \langle U(r)U(0) \rangle, \quad R_T(r) = \langle V(r)V(0) \rangle \quad (1.22)$$

$$\frac{R_T(r)}{R_T(0)} = g(r) = 1 - \frac{r^2}{\lambda^2} + \dots \quad (1.23)$$

$$\frac{R_L(r)}{R_L(0)} = f(r) = 1 - \frac{r^2}{2\lambda^2} + \dots \quad (1.24)$$

This scale allows to define a convenient Reynolds number for HIT flows:

$$Re_\lambda = \frac{\lambda u_{RMS}}{\nu}. \quad (1.25)$$

The HIT theory allows to predict the temporal evolution of the turbulent kinetic energy, the dissipation rate and the integral length scale:

$$E(t) = E_0 \left(\frac{t}{t_0} \right)^{-p} \quad (1.26)$$

$$\epsilon(t) = \frac{pE_0}{t_0} \left(\frac{t}{t_0} \right)^{-(p+1)} \quad (1.27)$$

$$L(t) = \frac{E^{3/2}}{\epsilon} = \frac{t_0}{p} E_0^{\frac{1}{2}} \left(\frac{t}{t_0} \right)^{1-p/2}. \quad (1.28)$$

Theoretical values for the slope exponent are $p = \frac{6}{5} = 1.20$ to $p = \frac{10}{7} = 1.43$ [3]. Values between 1.15 and 1.45 are found in the literature [80], both experimental and numerical. The presentation of the turbulence theory made in this introductory section is not exhaustive. For further informations, one can refer to [3] and [30].

1.5 Introduction to LIDAR Measurement

Light Detection And Ranging (LIDAR) is a technique in which a beam of light, usually emitted from a LASER, is used to make remote measurements. This beam of light will interact with the medium or the target of interest. Some of the light emitted is scattered back to the LIDAR. This backscattered light is then transformed into a current from which useful information on the target can be retrieved by means of an appropriate treatment. The time for the light to go back and forth to the target at the speed of the light, c , is used to determine the distance (range) to the target. The physical principle of the LIDAR is thus the same as the RADAR, the difference lies in the fact that a RADAR uses radio waves while a LIDAR uses optical waves. The first LIDAR measurement was performed in 1962 to measure the distance from the earth to the moon by Smullin and Fiocco [103]. More informations on the subject can be found in the book of Fujii [108] or Weitkamp [116].

There are three basic generic types of LIDAR:

- Range finder LIDARs are used to measure the distance from the LIDAR instrument to a solid or hard target. A typical illustration of this is the LASER range meter often used in estate survey. The device illuminates the target by means of a LASER beam and measures the time t for the light to travel back and forth. This time is used to compute the distance d from the source at which the target is located using:

$$d = \frac{ct}{2}, \quad (1.29)$$

where c is the speed of the light ($c = 3 \times 10^8 m/s$).

- Differential absorption LIDARS are used to measure chemical concentrations (such as ozone, water vapor, pollutants) in the atmosphere
- Doppler LIDARs are used to measure the velocity of a target. When the light transmitted from a LIDAR hits a target moving toward or away from the lidar, the wavelength⁶ of the light scattered from the target is changed. This is known as a Doppler shift. The same principle is used for police radar: an electromagnetic wave is sent toward a moving vehicle. The vehicle scatters back this radiation with a Doppler shift proportional to its speed. Appropriate signal processing of the returned signal allows to measure the velocity of the moving target under consideration. If the target is moving away from the LIDAR, the returned wave will have a longer wavelength (referred as red shift); if the target is moving toward the LIDAR, the return wave will be at a shorter wavelength (blue shifted). The atmosphere contains many microscopic dust and aerosol particles which are carried by the wind. These are the targets of interest as they are small and light enough to move at the fluid flow velocity and thus enable a remote measurement of that velocity to be made.

⁶or the frequency since $\lambda = c \times f$

The LIDAR measurement chain considered in the framework of the LASEF project is sketched in Fig. 1.6. The main components illustrated are a fiber-based LASER, a telescope which sends and receives the optical waves, an acquisition system to record the electrical signals and appropriate signal processing algorithms to recover quantitative information on the flow field of interest.

There exists other measurement techniques that can be applied to wake vortex detection. We mention the two principal ones:

- SODARS (Sonic Detection and Ranging) SODAR remote sensing works with acoustic waves. Doppler effect can also be exploited to measure wind velocities or detect wake vortex. Their use in airport can however be criticized considering the frequency range at which they operate. Indeed, their working frequency is centered around 2 kHz . The measurements can be altered by severe interferences due to the sound released by aircraft engines for which typical values of the three harmonics of the front fan blade passage frequencies are $750 - 1500 - 2250\text{ Hz}$.
- RADARS A new trend in aircraft wake vortex sensing is to use X-band radars $8 - 12\text{ GHz}$. Usually, radars work in the Rayleigh range, for which the size of the objects they sense is much smaller than the radiation wavelength λ . The efficiency of such a detection varies as λ^{-4} . These radars are able to perform remote sensing efficiently only if there are large enough particles, e.g. rain or snow. Radar echoes in the X-Band are observed in clear air due to refractive index gradients themselves induced by temperature and moisture inhomogeneities.

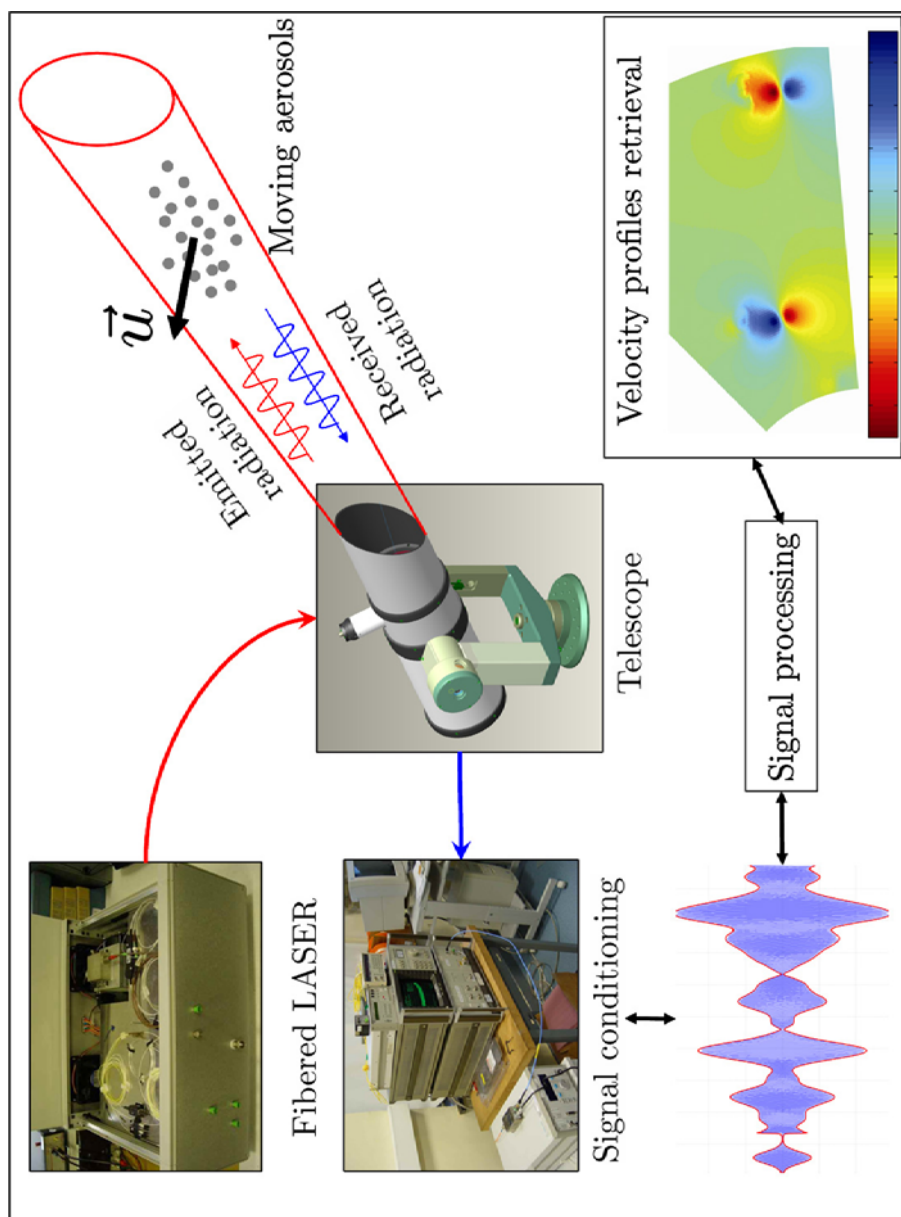


Figure 1.6: Overview of the LASEF LIDAR measurement chain.

Chapter 2

Presentation of the numerical codes used for the simulation of turbulent flows

In order to simulate aircraft wake vortex flows and winds in the most realistic way, high quality numerical codes, as well as good subgrid scale models are required. The SGS modelling theory will be presented in Chap. 3. High quality methods are methods that conserve mass, momentum and kinetic energy (in the absence of viscosity), as do the Navier-Stokes equations. The kinetic energy conservation is mandatory for several reasons. At first, one should avoid a code which injects energy, because it might lead to a blow up of the simulations. Secondly, some methods dissipate energy; this renders the control of the energy dissipated by the SGS model very difficult. Moreover, in a wake vortex flow simulation, the fact that the numerical method dissipates too much energy leads to a modification of the dynamics of the vortices. Another criterion of quality is the ability of the code to convect turbulent structures properly, which is linked to the dispersion errors.

The aim of this chapter is to present the numerical codes developed to simulate the flows relevant in the framework of LIDAR measurement simulations. The first code is based on a fourth order finite difference (FD) method while the second is based on a pseudo-spectral method. Both codes are validated on canonical flows of interest for the simulation of aircraft wake vortices. The benchmark flows used to assess the quality of the fourth order FD code are the DNS of the Taylor-Green flow at $Re = 1600$, the DNS of a four vortex system at $Re = 5000$, and the DNS of the channel flow at $Re_\tau = 180$. The pseudo-spectral code is tested on the DNS of the Taylor-Green flow at $Re = 1600$ and homogeneous turbulence at moderate Re_λ .

2.1 Fourth order finite difference Navier Stokes solver

We present a state-of-the-art parallel fourth order finite difference (FD) code. This code was initially designed and developed by L. Georges [46] during his doctoral work. The performances and the reliability of this code were further improved in the present work and with the help of T. Lonfils and M. Duponcheel in the framework of their graduation thesis and during the Ph.D. thesis of M. Duponcheel. The Direct Numerical Simulation (DNS) of several flows of interest are presented to validate the methodology and assess the quality of the code.

2.1.1 The need for a high order code

Approximating derivatives by finite differences introduce truncation errors. A method of order p has a truncation error which has an order of magnitude $\mathcal{O}(\Delta^p)$, Δ being the mesh spacing. The order is thus the power of Δ with which the truncation error tends to zero. The order of the error due to this truncation should be as high as possible to have an accurate approximation of the derivatives in the PDE's. Having high order methods is also interesting if one considers dispersion errors. Dispersion errors are related to the ability of convecting flow structures properly. The dispersion errors can lead to spurious oscillations for simulation that are marginally resolved. Considering a 1D first order derivative approximated using a second order centered finite difference leads to write¹

$$\frac{\delta}{\delta x} e^{ikx} = \frac{e^{ik(x+\Delta x)} - e^{ik(x-\Delta x)}}{2\Delta x} = i \frac{\sin(k\Delta x)}{\Delta x} e^{ikx} = ik' e^{ikx}, \quad (2.1)$$

k' being the effective (or modified) wavenumber due to the use of FD approximation. In Fig. 2.1, the effective wavenumber vs. the real one is plotted for different schemes and orders. Increasing the order makes the modified wavenumber curve go closer to the exact one. Examining these curves indicates that the order of the discretization p measures only the accuracy at low wavenumbers. As suggested by Ferziger and Perić [40], we examine an error criteria which measures the accuracy of a method taking into account the whole wavenumber range. This criterion is

$$e_p^1 = \frac{\int (k - k_p')^2 E(k) dk}{\int k^2 E(k) dk}. \quad (2.2)$$

Another interesting error measurement would be based on the enstrophy deficit:

$$e_p^2 = \frac{\int (k^2 - k_p'^2) E(k) dk}{\int k^2 E(k) dk} \quad (2.3)$$

¹The notation $\frac{\delta}{\delta x}$ is used for the numerical approximation of the derivatives.

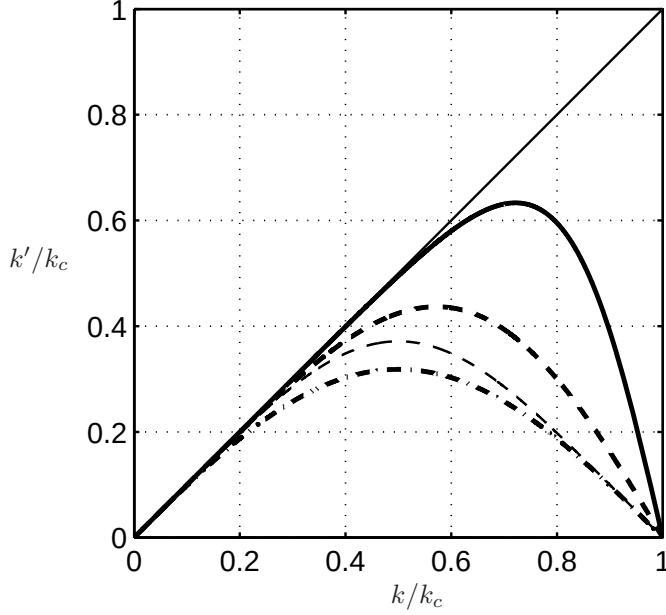


Figure 2.1: Modified wavenumber: spectral (solid thin); classical (C) second order FD (dash dot); classical fourth order FD (dash); Vasilyev (V) fourth order FD (dash thin); sixth order Padé (P) scheme (solid).

Table 2.1 contains values of these two errors evaluated using a HIT flow detailed in Section 2.2.4. The important fact to note is that these two error definitions decrease drastically with the order. For a classical FD scheme (C), going from $p = 2$ to $p = 4$ reduces e_p^1 and e_p^2 by 60%. The error reduction is evolving in the same way for the two definitions.

It is also interesting to consider the Padé schemes. These schemes are well known to offer a high order with a compact stencil. Moreover, it was shown by Lele [77] that the dispersion errors generated by these schemes are quite low. The sixth order Padé scheme (P) approximating the derivative f' of a function f can be written in the following way:

$$\beta f'_{i-2} + \alpha f'_{i-1} + f'_i + \alpha f'_{i+1} + \beta f'_{i+2} = c \frac{f_{i+3} - f_{i-3}}{6\Delta x} + b \frac{f_{i+2} - f_{i-2}}{4\Delta x} + a \frac{f_{i+1} - f_{i-1}}{2\Delta x}$$

The coefficients a, b, c, α, β are obtained using Taylor series. For the sixth order scheme, one has $\beta = 0$, $\alpha = 1/3$, $a = 14/9$, $b = 1/9$ and $c = 0$. This type of scheme is considered as implicit in space as it require to solve a linear system (tridiagonal if $\beta = 0$) to provide the approximated derivatives (for more details see e.g. [71]). The modified wavenumber relation for this scheme can be found

Table 2.1: Variation of the errors vs the order of the discretisation p . The evaluation of ϵ_p is based on a spectrum computed from a field at $Re_\lambda = 90$ of the 256^3 DNS detailed at section 2.2.4.

Order	e_p^1	e_p^2	$\frac{e_p^1}{e_\lambda^1}$	$\frac{e_p^2}{e_\lambda^2}$
$p = 2$ (C)	0.056	0.25	1.0	1.0
$p = 4$ (C)	0.023	0.10	0.4	0.4
$p = 4$ (V)	0.038	0.15	0.7	0.6
$p = 6$ (P)	0.006	0.024	0.1	0.1

in [70]. Using this scheme reduces the errors e_p^1 and e_p^2 by a factor 10. As a conclusion, the gain obtained on e_p^1 and e_p^2 by using method with $p > 2$ is significant.

The errors values evaluated for the scheme of Vasiliyev [114] (V) are discussed in the next section, where it is presented.

2.1.2 Description of the code

Staggered mesh (Marker And Cell, MAC)

A staggered grid is used in order to avoid the odd even decoupling phenomenon. If all the variables are located at the same grid points, and if centered schemes are used for the pressure gradient and the velocity divergence, the pressure values at neighboring points are uncoupled. This leads to spurious oscillations on the pressure field. One solution to avoid this problem is to use a grid where the pressure unknowns are not located at the same place as the velocity unknowns. The velocity unknowns are located around the pressure ones, shifted by a distance which is half of the cell edge length, as depicted in Fig. 2.2. With such a discretization, special care has to be taken for the boundary conditions, as detailed by Georges [46].

Energy conserving finite difference discretisation

We define the notation for the n points centered finite difference stencil for the first derivative in the direction ξ_1 of the computational space:

$$\frac{\delta_n \phi}{\delta_n \xi_1} = \frac{\phi\left(\xi_1 + \frac{n\Delta_1}{2}, \xi_2, \xi_3\right) - \phi\left(\xi_1 - \frac{n\Delta_1}{2}, \xi_2, \xi_3\right)}{n\Delta_1}. \quad (2.4)$$

The interpolation operator using the same stencil is defined as

$$\overline{\phi}^{n\xi_1} = \frac{1}{2} \left(\phi\left(\xi_1 + \frac{n\Delta_1}{2}, \xi_2, \xi_3\right) + \phi\left(\xi_1 - \frac{n\Delta_1}{2}, \xi_2, \xi_3\right) \right). \quad (2.5)$$

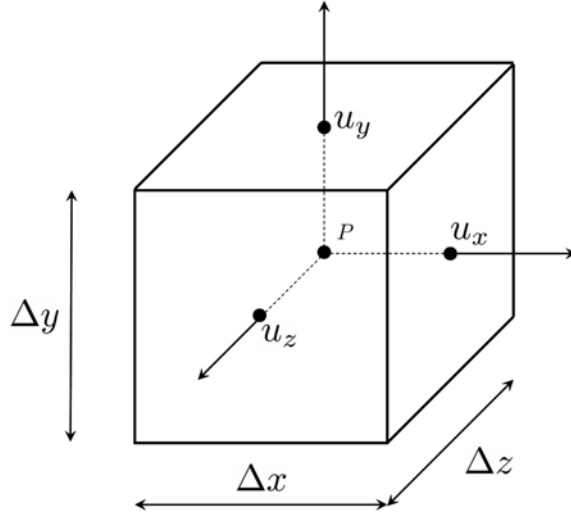


Figure 2.2: Sketch of a grid cell for a staggered arrangement of the variables. The pressure point is located at P ; the velocity points are shifted by a half cell width.

The Jacobian of the mapping between the physical and the computational space is written

$$J_n(\xi_i) = \frac{\delta_n x_i}{\delta_n \xi_i}, \quad (2.6)$$

so that derivatives in the physical space can be computed from the ones in the computational space:

$$\frac{\delta_n \phi}{\delta_n x_i} = \frac{1}{J_n(\xi_i)} \frac{\delta_n \phi}{\delta_n \xi_i}. \quad (2.7)$$

The gradient can be computed at fourth order as described by Vasilyev [114]:

$$G_i(\phi) = \frac{9}{8} \frac{\delta_1 \phi}{\delta_1 x_i} - \frac{1}{8} \frac{\delta_3 \phi}{\delta_3 x_i}. \quad (2.8)$$

The fourth order is obtained by an appropriate linear combination of the derivative stencils. The fourth order divergence operator of a field u_i can be computed as:

$$D(u_i) = \frac{9}{8} \frac{\delta_1 u_i}{\delta_1 x_i} - \frac{1}{8} \frac{\delta_3 u_i}{\delta_3 x_i} \quad (2.9)$$

Examining Fig. 2.1 indicates that this scheme does not perform as well as the classical fourth order one. However, this scheme is able to conserve kinetic energy on a staggered grid. Moreover, looking at Table 2.1 shows that the gain obtained on the error e_p^1 and e_p^2 compared to a second order scheme are still significant.

The convective term of the Navier stokes equations can be written in several equivalent forms, the divergence form (*DIV*), the advective form (*ADV*), and the skew symmetric form (*SKEW*):

$$H_i^{DIV} = \frac{\partial}{\partial x_j} (u_j u_i) \quad (2.10)$$

$$H_i^{ADV} = u_j \frac{\partial u_i}{\partial x_j} \quad (2.11)$$

$$H_i^{SKEW} = \frac{1}{2} H_i^{DIV} + \frac{1}{2} H_i^{ADV} \quad (2.12)$$

The fourth order finite difference discretization of these forms are:

$$\begin{aligned} H_i^{DIV} &= \frac{9}{8} \frac{\delta_1}{\delta_1 x_j} \left(\left(\frac{9}{8} \overline{u_j^{1x_i}} - \frac{1}{8} \overline{u_j^{3x_i}} \right) \overline{u_i^{1x_j}} \right) \\ &\quad - \frac{1}{8} \frac{\delta_3}{\delta_3 x_j} \left(\left(\frac{9}{8} \overline{u_j^{1x_i}} - \frac{1}{8} \overline{u_j^{3x_i}} \right) \overline{u_i^{3x_j}} \right) \end{aligned} \quad (2.13)$$

$$\begin{aligned} H_i^{ADV} &= \frac{9}{8} \frac{1}{J_1(\xi_j)} \overline{\left(\frac{9}{8} \overline{u_j^{1\xi_i}} - \frac{1}{8} \overline{u_j^{3\xi_i}} \right) \frac{\delta_1 u_i}{\delta_1 \xi_j}}^{1\xi_j} \\ &\quad - \frac{1}{8} \frac{1}{J_3(\xi_j)} \overline{\left(\frac{9}{8} \overline{u_j^{1\xi_i}} - \frac{1}{8} \overline{u_j^{3\xi_i}} \right) \frac{\delta_3 u_i}{\delta_3 \xi_j}}^{3\xi_j} \end{aligned} \quad (2.14)$$

$$H_i^{SKEW} = \frac{1}{2} H_i^{DIV} + \frac{1}{2} H_i^{ADV} \quad (2.15)$$

The skew symmetric form was retained as it exactly conserves the directional discrete kinetic energy and the discrete momentum is conserved up to the fourth order on a non uniform mesh. The numerical approximation of the Laplacian of a scalar field ϕ is

$$L(\phi) = \left(\frac{\delta^2 \phi}{\delta x_1^2} + \frac{\delta^2 \phi}{\delta x_2^2} + \frac{\delta^2 \phi}{\delta x_3^2} \right). \quad (2.16)$$

The scheme used to compute a second derivative is

$$\frac{\delta^2 \phi}{\delta x_i^2} = \left(\left(\frac{9}{8} \frac{\delta_1}{\delta_1 x_i} - \frac{1}{8} \frac{\delta_3}{\delta_3 x_i} \right) \left(\frac{9}{8} \frac{\delta_1}{\delta_1 x_i} - \frac{1}{8} \frac{\delta_3}{\delta_3 x_i} \right) \phi \right). \quad (2.17)$$

The numerical operator L is symmetric and positive definite; thus it has a truly dissipative behaviour.

Time stepping

The major difficulty in obtaining a time-accurate numerical solution for an incompressible flow arises from the fact that the time derivative does not appear explicitly in the continuity constrain, Eq. (1.8). The numerical integration

of the convective term is done by means of an explicit multistep second order scheme. This scheme is a blending between the leapfrog and the classical Adams-Bashforth scheme. When this scheme integrates the following differential equation:

$$\frac{d\phi}{dt} = F(\phi, t) ,$$

the discrete form is:

$$\phi^{n+1} = \alpha\phi^n + (1 - \alpha)\phi^{n-1} + \frac{1}{2}\Delta t ((4 - \alpha)F^n - \alpha F^{n-1}) .$$

If $\alpha = 0$ one recovers the classical leapfrog scheme, while the case $\alpha = 1$ corresponds to the second order classical Adams-Bashforth scheme. This scheme has the advantage that the stability curve (obtained for the linear model problem $\frac{d\phi}{dt} = \lambda\phi$) comes closer to the imaginary axis as α decreases. It allows to use bigger time steps for convection dominated problems as observed in Fig. 2.3. The diffusion term can be integrated either with the same explicit scheme or with a Crank-Nicholson implicit scheme. In the case where the molecular or the subgrid scale dissipation are high, the viscous constraint leads to use very small time steps; in this case one prefers an implicit time stepping. A fractional-step method is used to integrate the momentum conservation equation and satisfying the incompressibility constraint. This scheme is equivalent to the one described by Lee et al. [72]. For clarity, this fractional step algorithm is here written for $\alpha = 1$; but all other cases are implemented. Knowing the velocity field at step n , the momentum equation is integrated first:

$$\frac{u_i^* - u_i^n}{\Delta t} = -G_i(\phi^n) + \frac{1}{2}(3H_i^n - H_i^{n-1}) + \frac{1}{2}\nu \left(\frac{\delta^2}{\delta x_1^2} + \frac{\delta^2}{\delta x_2^2} + \frac{\delta^2}{\delta x_3^2} \right) (u_i^* + u_i^n) . \quad (2.18)$$

This yields a velocity field u_i^* which is not divergence free. The second step of this algorithm consists in solving a Poisson equation for the pressure difference $\Phi^{n+1} = \phi^{n+1} - \phi^n$ at the next time step:

$$D(G(\Phi^{n+1})) = \frac{1}{\Delta t} D(u_i^*) . \quad (2.19)$$

The velocity field is then projected in a divergence-free space:

$$\frac{u_i^{n+1} - u_i^*}{\Delta t} = -G_i(\Phi^{n+1}) . \quad (2.20)$$

The boundary conditions are:

$$G_n(\Phi^{n+1}) = 0 \quad (2.21)$$

$$u_i^* = u_i^{n+1} \quad (2.22)$$

So far, the subgrid scale model terms used in LES are integrated explicitly in every case. There is ongoing work to be able to also make it partially implicit.

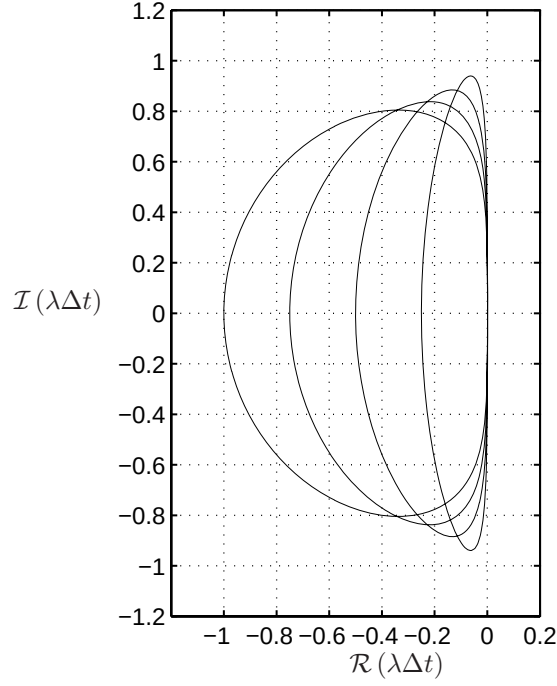


Figure 2.3: Multistep stability region. From left to right: $\alpha = 1$, $\alpha = \frac{3}{4}$, $\alpha = \frac{1}{2}$, $\alpha = \frac{1}{4}$.

Multigrid (MG) Poisson Solver

The equation (2.19) is a Poisson equation, which has to be solved at each time step. We here summarize the way this equation is solved. For further details, refer to Georges [46] or Briggs [11]. Consider the generic Poisson equation defined on a domain Ω with a given set of boundary conditions defined on $\partial\Omega$:

$$\nabla^2 \psi = R. \quad (2.23)$$

ψ and R being scalar fields. In its fourth order finite difference discretized form, this equation can be re-written as a linear system of equations:

$$A^h \psi^h = f^h, \quad (2.24)$$

where the superscript h stands for the discretized version of the fields and A^h is the matrix representation of the operator $D(G(\psi))$. Solving this equation also amounts to reach the stationary state of a diffusion equation:

$$\frac{\partial \psi}{\partial t} = \alpha (\nabla^2 \psi - R) \quad (2.25)$$

where α is a diffusion coefficient. If one uses the explicit Euler time stepping to solve this evolution problem, it reads:

$$\psi^{n+1} = \psi^n + \alpha \Delta t \left(\frac{\delta^2 \psi^n}{\delta x_1^2} + \frac{\delta^2 \psi^n}{\delta x_2^2} + \frac{\delta^2 \psi^n}{\delta x_3^2} - R^h \right) \quad (2.26)$$

This strategy is equivalent to the iterative point Jacobi algorithm to solve the Poisson equation. This scheme is subject to a diffusion stability constraint on the pseudo time step Δt :

$$\alpha \Delta t < \beta \frac{1}{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}, \quad (2.27)$$

the local grid spacing being Δx , Δy and Δz . This limit was computed by Georges [46] who found $\beta = 0.367$. On a given mesh, the smallest grid spacing limits the pseudo time step and thus the convergence speed. If the discretization of the physical problem requires non uniform grids, i.e with thinner cells in the regions of strong gradients in the solutions, the stability constraint of the scheme can become very severe, leading to the use of very small steps. If the discrete Laplacian is computed with the numerical values of ψ as soon as they are updated, this time stepping is equivalent to the point Gauss-Seidel algorithm. Another improvement is to over relax the scheme, meaning that the solution at the next time step is obtained as a linear combination of the old value and the value ψ^* predicted by the point Gauss-Seidel or Jacobi algorithm:

$$\psi^{n+1} = (1 - \omega)\psi^n + \omega\psi^* \quad (2.28)$$

The value of the over relaxation factor is such that $1 < \omega < 2$. There exists an optimum value for ω which leads to the fastest convergence rate with this method. This value is quite easy to find in 2D on an uniform mesh but is more complex to compute on 3D non uniform grids. These improvements are however subject to the same stringent stability constraints. To solve this problem, the code allows to perform implicit integration in the directions where the grid is non-uniform. The following scheme is then used:

$$\begin{aligned} \psi^{n+1} = & \psi^n + \alpha \Delta t \left(\Theta_x \frac{\delta^2 \psi^{n+1}}{\delta x_1^2} + (1 - \Theta_x) \frac{\delta^2 \psi^n}{\delta x_1^2} \right. \\ & + \Theta_y \frac{\delta^2 \psi^{n+1}}{\delta x_2^2} + (1 - \Theta_y) \frac{\delta^2 \psi^n}{\delta x_2^2} \\ & \left. + \Theta_z \frac{\delta^2 \psi^{n+1}}{\delta x_3^2} + (1 - \Theta_z) \frac{\delta^2 \psi^n}{\delta x_3^2} - R^h \right) \end{aligned} \quad (2.29)$$

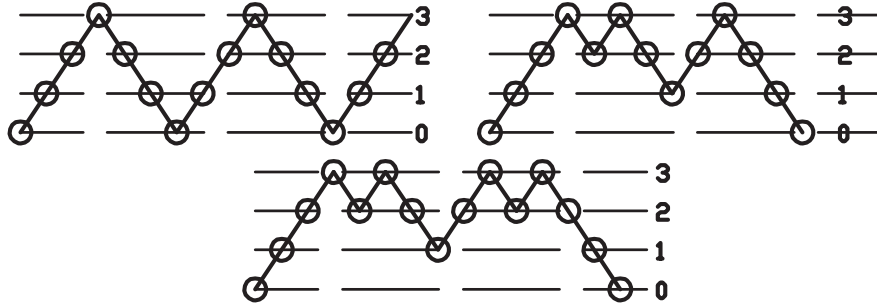
This set of integration schemes corresponds to the line Jacobi or line Gauss-Seidel methods where the parameters Θ_x , Θ_y and Θ_z determine the type of pseudo-time integration. These parameters are ranging from 0 to 1, with $\Theta_i = 0$

for an explicit Euler, $\Theta_i = 1/2$ for a Crank-Nicolson and $\Theta_i = 1$ for an implicit Euler. If $\Theta_i \geq 1/2$ in one direction, the integration is unconditionally stable for this direction. The convergence rate thus remains unaffected by an important grid stretching or anisotropy.

These algorithms are easy to implement but they are inefficient to solve large scale problems. There exist many variants of these iterative algorithms but they are all limited by the fact that they are only efficient to damp the high frequency components of the error, which limits considerably the convergence rate. Multigrid algorithms eliminate this drawback. One of the key concept of this technique is that a signal with a low frequency is “seen” as a signal at a higher frequency on a coarser grid: this introduces the coarse grid correction concept. The MG algorithm implemented in the present code is listed in Table 2.2. The coarse grid correction concept requires operators to transfer fields from coarse grid to fine grid, this is called prolongation and further noted I_h^{2h} , and from fine grids to coarse ones which is noted I_{2h}^h . The prolongation operator is a simple trilinear interpolation while the restriction operator requires that the discrete integral of the source term must be zero as explained in [46] which defines a volume weighted scheme to enforce this condition. One of the essential components of the MG algorithm is the “smoother”, which relaxes the solution on the different grid levels. Usually, the number of relaxations is chosen as $2 \leq n_1, n_2 \leq 5$. The standard MG algorithm is formulated in a recursive way, the different grid levels are visited successively from the finest mesh (level 0) to the coarsest one (level M) and from the coarsest level to the finest. This description is valid for a **V** cycle ($\mu = 1$). μ is a parameter which determines the pattern of the cycle, see Table 2.2 and Fig. 2.4. There exist also **W** ($\mu = 2$) cycles, and **F** cycles. An **F** cycle is formed by a **W** cycle followed by a **V** cycle. In the **W** and **F** cases, a more important work is done on the coarse grids, expecting an increase of the convergence rate.

Table 2.2: Multigrid μ cycle algorithm as defined by Briggs [11].

μ cycle algorithm: $v^h \leftarrow M\mu^h(v^h, f^h)$
1. Relax n_1 times $A^h u^h = f^h$ with v^h as initial guess.
2. If coarsest grid level goto 4. Else $f^{2h} \leftarrow I_h^{2h}(f^h - Av^h)$. $v^{2h} \leftarrow 0$ $v^{2h} \leftarrow M\mu^{2h}(v^{2h}, f^{2h})$ μ times
3. Correction step: $v^h \leftarrow v^h + I_{2h}^h v^{2h}$
4. Relax n_2 times $A^h u^h = f^h$ with v^h as initial guess.

**Figure 2.4:** Examples of V , F , W cycles for a four level multigrid. Drawing from Georges [46].

2.1.3 Validation case 1: DNS of the Taylor-Green vortex flow at $Re = 1600$

The Taylor-Green vortex flow allows to study the generation of small scales and the transition to turbulence of an analytical base flow. The paper of Brachet et al. [10] is a reference on the subject. The initial condition is defined as:

$$\begin{cases} u = U \sin\left(\frac{2\pi x}{L}\right) \cos\left(\frac{2\pi y}{L}\right) \cos\left(\frac{2\pi z}{L}\right) \\ v = -U \cos\left(\frac{2\pi x}{L}\right) \sin\left(\frac{2\pi y}{L}\right) \cos\left(\frac{2\pi z}{L}\right) \\ w = 0 \end{cases}$$

The flow evolves naturally from this simple initial condition because of its initial pressure gradient along the z axis. The computational box is periodic in the three directions of space and the periodicity lengths are usually chosen as $L_x = L_y = L_z = L$. Moreover, since the initial condition is analytic and simple, this flow can be used to validate accurately the quantities computed in a flow solver, particularly, the average kinetic energy, the average enstrophy, the initial dissipation and the norm of the strain rate tensor, or other more complex quantities. The Reynolds number of this flow is here defined as $Re = \frac{UL}{2\nu}$. We here choose to simulate this flow at $Re = 1600$. This requires a large grid to ensure that the DNS criteria $k_c\eta \geq 1.5$ is satisfied. In this case, one needs a number of $N = 256$ grid points in each direction. The dissipation evolution is displayed in Fig. 2.5. To assess the validity of our result, we compare the finite difference solution with the spectral one detailed in Section 2.2.3. Examining the curves in Fig. 2.5 shows that the two solutions yield the same results in the initial transitional phase. However, after a certain time, some discrepancies can be observed. These differences come from the fact that the finite difference resolution was the same as the spectral one. Moreover, the time stepping scheme is more accurate for the spectral code. These differences are minor and lead to the conclusion that the finite difference code is valid to perform simulations of transitional flows. The figure 2.6 provides a visualization of the flow at a late time of the flow ($t = 10$). It can be observed that the flow has become very complex; yet it is not representative of an homogeneous isotropic turbulence state since the flow is still quite organized.

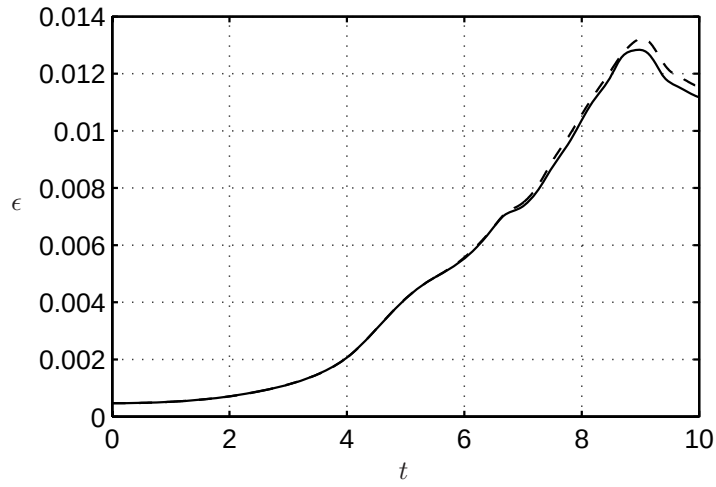


Figure 2.5: Taylor-Green vortex at $Re = 1600$: evolution of the dissipation for a 256^3 computation. Fourth order finite differences (dash), spectral result (solid).

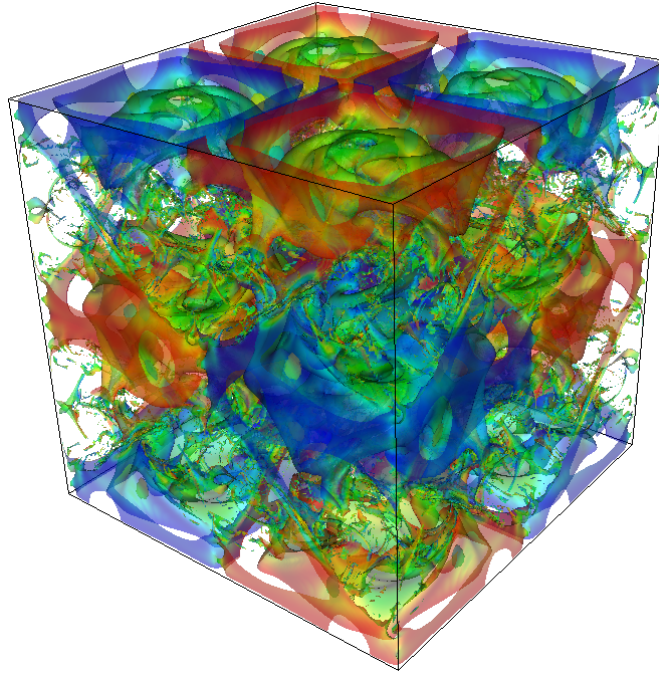


Figure 2.6: Visualization of the solution at $t = 10$. The vorticity norm is displayed and is colored by the vertical vorticity component value ω_z .

2.1.4 Validation case 2: DNS of a four vortex system at $Re = 5000$

This validation case was proposed in the framework of the “AWIATOR” project of the European commission. This benchmark allows to assess the quality of a code regarding its ability to capture properly the dynamics of a complex rotational flow, see Dufresne et al. [35]. The base flow condition is made of a four vortex system with two alternate pairs of counter-rotating vortices, as sketched in Fig. 2.7. The outer or principal vortices, identified by the subscript 1, are characterized by their circulation Γ_1 , their core size radius a_1 , and their spacing b_1 . The inner or secondary pair is in turn characterized by Γ_2 , a_2 , and b_2 respectively. Normalization of the dimensional quantities can be made using the natural reference length and time scales $L_{\text{ref}} = b_1$ and $T_{\text{ref}} = b_1^2/\Gamma_1$. The non dimensional time is defined here as $t^* = tV_1/b_1$. The Reynolds number is here defined as $Re = \frac{\Gamma_1}{\nu} = 5000$. For the present comparison, we considered the single particular case where

$$\begin{aligned}\frac{\Gamma_2}{\Gamma_1} &= -0.3, \quad \frac{b_2}{b_1} = 0.3, \\ \frac{a_1}{b_1} &= 0.075, \quad \frac{a_2}{a_1} = 2/3.\end{aligned}$$

The core structure of each of the four vortices is defined such that, for a given vortex with core radius $a = r_c$ and circulation Γ , the longitudinal vorticity (ω_x) distribution is given by the “low-order algebraic” profile

$$\omega_x = \frac{\Gamma}{\pi} \frac{r_c^2}{(r^2 + r_c^2)^2}. \quad (2.30)$$

Where $r = \sqrt{(y - y_0)^2 + (z - z_0)^2}$ is the distance to the vortex centerline of coordinates $\{y_0, z_0\}$.

The simulation results were obtained on a domain of periodicity lengths

$$\frac{L_x}{b_1} = 0.983, \quad \frac{L_y}{b_1} = 5.20, \quad \frac{L_z}{b_1} = 3.12,$$

using a grid with

$$N_x = 64, \quad N_y = 320, \quad N_z = 192.$$

One can also define the “equivalent” two vortex sytem:

$$\Gamma_0 = \Gamma_1 + \Gamma_2 \quad (2.31)$$

and

$$b_0 = \frac{\Gamma_1 b_1 + \Gamma_2 b_2}{\Gamma_0}. \quad (2.32)$$

It is a two vortex system with the same lift:

$$L = \rho U \Gamma_0 b_0 = \rho U (\Gamma_1 b_1 + \Gamma_2 b_2). \quad (2.33)$$

The secondary vortices are perturbed using a sinusoidal displacement of wavelength L_x and amplitude $\epsilon = 10^{-6}b_1$. The visualizations, Fig. 2.9, picture the system evolution and are in good agreement with the simulation of Dufresne et al. [34] using the pseudo spectral code. At the beginning, Fig. 2.9, the flow is essentially twodimensional. The secondary vortices are orbiting around the primary vortices. At $t^* \approx 1.5$, the secondary vortices are subject to medium wavelength instabilities and the flow becomes then three-dimensional. These instabilities lead to the so-called “omega loops”, representative of this flow, as illustrated in Fig. 2.9. Finally, the flow becomes turbulent through the partial reconnection of the secondary vortices with the primary ones, which generates much small scales and some vortex “bursting” events. The evolution of the kinetic energy, reported in Fig. 2.8, is compared successfully with the DNS of Dufresne et al. [34]. Up to $\tau \approx 1.5$, the energy decay comes from the simple viscous diffusion of the vortex cores. The fast decay phase that follows is the result of the generated turbulence.

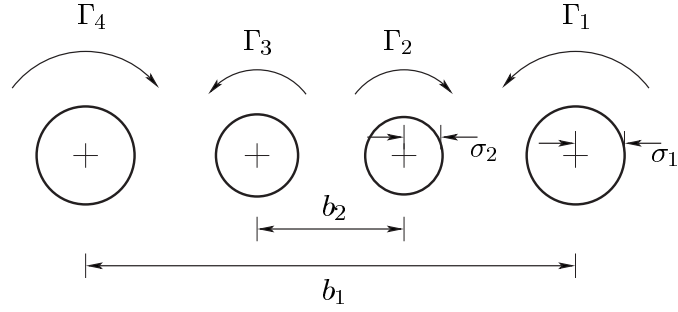


Figure 2.7: Schematic of the four vortex system configuration (figure from [35]).

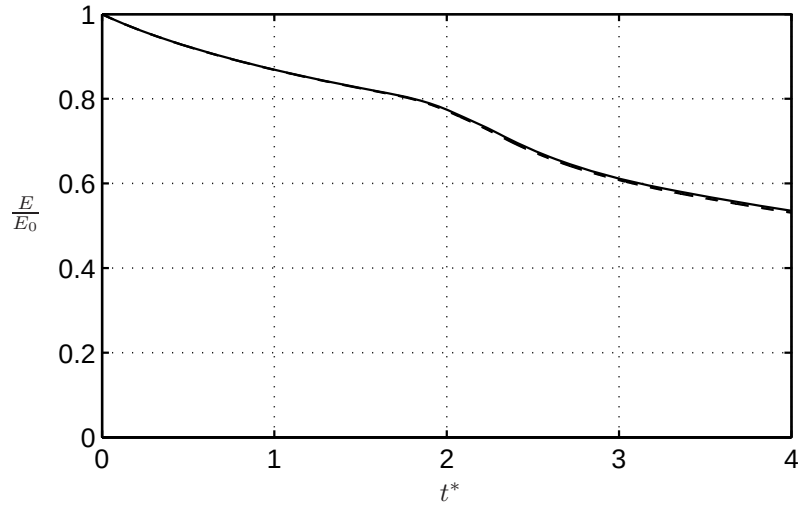


Figure 2.8: Kinetic energy evolution for the four vortex system at $Re = 5000$: fourth order FD (dash), pseudo spectral method reference computation (solid).

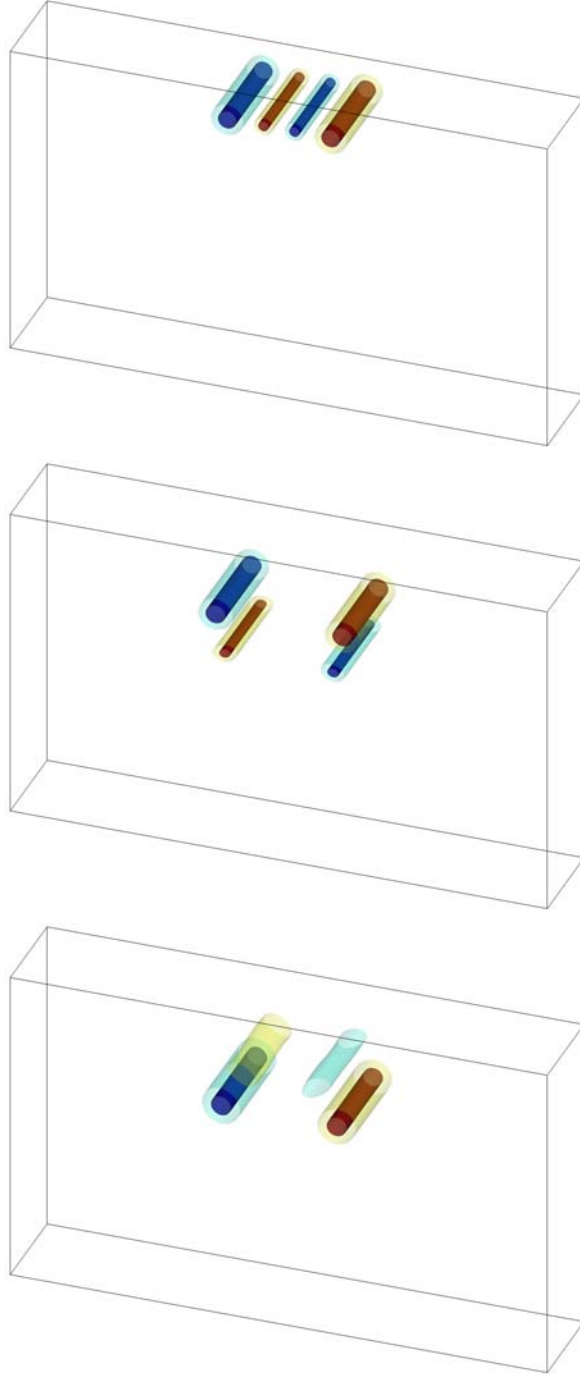


Figure 2.9: Visualization of the flow field using isosurfaces of $\|\omega\|b_1^2/\Gamma_1 = 2$ and 10. From top to bottom: $t^* = 0$, $t^* = 0.32$, $t^* = 1.3$

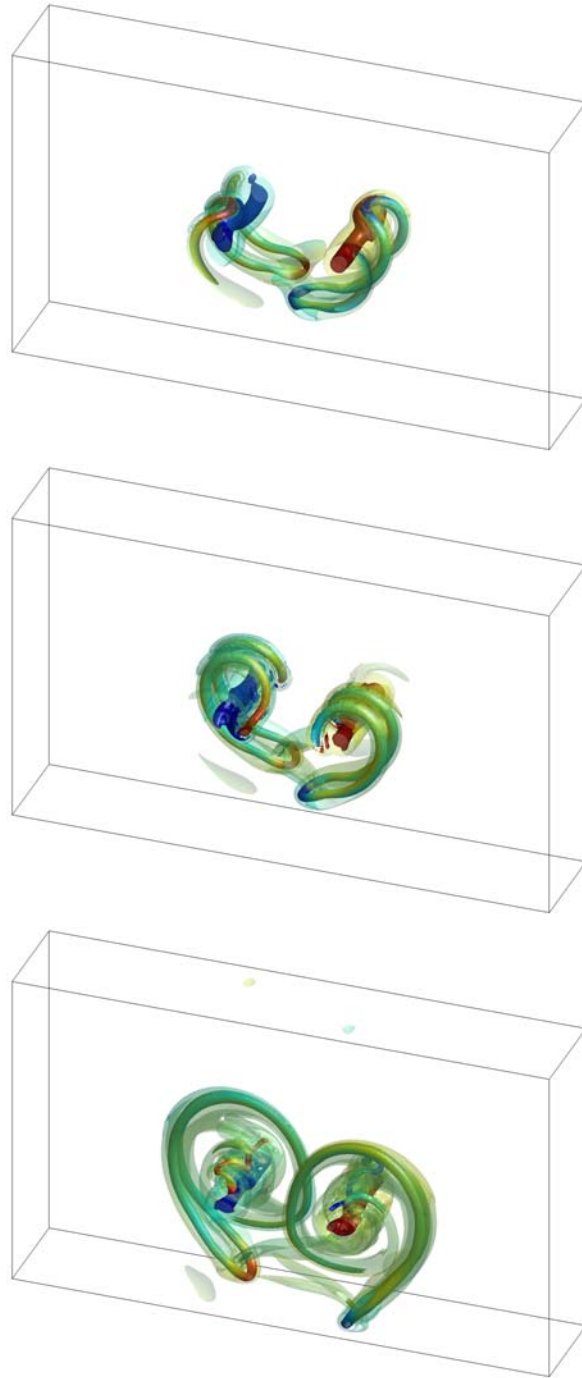


Figure 2.9: (continued) From top to bottom: $t^* = 1.9$, $t^* = 2.0$, $t^* = 2.5$.

2.1.5 Validation case 3: Channel flow at $Re_\tau = 180$

We propose to validate the code on the turbulent channel flow at $Re_\tau = 180$. It is a well known benchmark case and many reference results are available. The validation will compare our results with the spectral ones obtained by Moser, Kim and Mansour [84]. We will also introduce, in this section, all the concepts which are related to this particular flow, as it will be invoked further in this work. A sketch illustrating the conventions used for this flow is displayed in Fig. 2.10.

Usually, the velocity reference for this problem is the friction velocity u_τ which is linked to the average wall shear stress τ_w in the following way:

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}}. \quad (2.34)$$

u_τ and ν are used to render quantities dimensionless (these are written with the superscript $+$). For instance, the dimensionless distance to the wall is written:

$$y^+ = \frac{u_\tau y}{\nu}. \quad (2.35)$$

The Reynolds number based on the channel half width δ and the friction velocity u_τ is

$$Re_\tau = \frac{u_\tau \delta}{\nu} = 180. \quad (2.36)$$

At the center of the channel, $y = \delta$, one has $y^+ = Re_\tau$. The forcing pressure gradient is computed from the targeted Re_τ :

$$\frac{1}{\rho} \frac{dp_f}{dx} = -\frac{u_\tau^2}{\delta}. \quad (2.37)$$

The reference time is $T_0 = \delta/u_\tau$. A dimensionless time is defined: $t^+ = t/T_0$. The bulk mean velocity:

$$U_b = \frac{1}{L_y} \int_0^{L_y} U(y) dy, \quad (2.38)$$

with $U(y)$, the time-averaged and (x, z) space averaged velocity, can also be used as a reference velocity scale. It is also convenient to have a relation between Re_τ and the Reynolds number based on the bulk velocity as reported in Table 2.3:

$$Re = \frac{U_b 2\delta}{\nu}. \quad (2.39)$$

Table 2.3: Relation between Re_τ and Re for turbulent channel flows (from various DNS in the literature).

Re_τ	Re
180	5600
395	13800
590	22000
950	37500
2000	87200

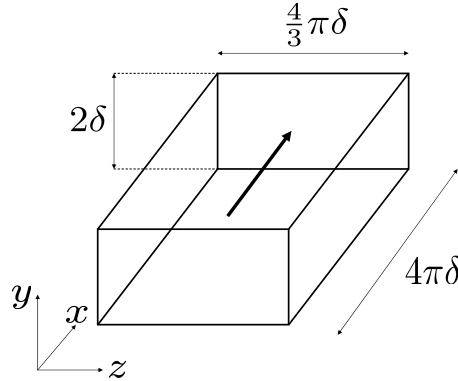


Figure 2.10: Channel flow setup and reference axis system. The flow is periodic in x and z directions. No-slip boundary conditions are applied at $y = 0$ and $y = 2\delta$.

The grid contains 128 points in each direction. The points in the spanwise (z) and the streamwise (x) direction are equally spaced. In the direction normal to the walls (y), the grid is stretched in order to capture correctly the strong gradients appearing close to the walls. The grid has the same number of nodes as that of Moser et al. [84]; but they used a spectral code, and thus fully resolved the highest wavenumbers. The stretching in the y direction is defined by the following 1D mapping :

$$\frac{y}{\delta} = 1 + \frac{\tanh(\alpha(\xi - 1))}{\tanh(\alpha)}. \quad (2.40)$$

The coordinate in the computational space is noted ξ . This mapping was already used by Jeanmart [61] and Gullbrand [54]. The “*tanh*” mapping is compared for different values of α to that used by Moser et al. [84] in Fig. 2.11. The factor $\alpha = 1.6$ guarantees that the first grid point is located at $y^+ \approx 0.5$ and that the resolution at the center of the channel is still acceptable ($\Delta y_c^+ = 4.9$). Preliminary studies on a grid with $\alpha = 2.75$ (which is the correct value for the LES at $Re_\tau = 395$) did not allow to obtain results in agreement with the

reference solutions in the outer region of the flow while the viscous sublayer was well resolved. The grid resolution was too fine at the wall and too coarse in the center of the channel : $\Delta y_c^+ \approx 8$. This grid is however better adapted for “wall resolved” LES investigations. We noticed also that the mapping with $\alpha = 1.6$ is very close to the cosine mapping used by Moser et al. [84] in their spectral code. The resulting grid is however less refined close to the wall than that used in the reference solution (cosine stretching). The time step is set to $\Delta t^+ = 2.5 \cdot 10^{-4}$. It is chosen small enough to avoid flow relaminarization occuring when the time step is too big compared to the Kolmogorov time scale:

$$\tau_k^+ = \left(\frac{u_\tau^4}{\epsilon \nu} \right)^{1/2}. \quad (2.41)$$

According to Choi and Moin [19], for a channel flow at $Re_\tau = 180$, one should verify that $\Delta t^+ < 0.009$. The base flow was initialized with a velocity profile of the type :

$$\frac{u(y)}{u_{max}} = 1 - \left(1 - \frac{y}{\delta} \right)^{2p}, \quad (2.42)$$

with $p = 3$. This base flow is perturbed by adding harmonic modes on the analytical velocity profile. The first stage of the flow is the transition from the initial velocity field to an equilibrium turbulent flow.

Once a turbulent regime is reached, the statistics of interest are collected. The overall process is quite lengthy as one needs a period of about $10 T_0$ to obtain a statistically converged flow and the same time is then also required to gather the statistics. In a channel flow problem, the averages denoted by $\langle \bullet \rangle$ are carried out by averaging in time and in the homogeneous directions (x and z).

The first result presented to assess the quality of the simulation is the mean streamwise velocity profile, Fig. 2.12. The first zone of interest on this graph is the viscous sublayer ($y^+ < 5$) where the viscous stress dominates the turbulent stress. In this region the mean profile has the form $u^+ = y^+$. For $5 < y^+ < 70$, there is a buffer layer which allows the transition from the region where the viscous stress dominates, to the region where the turbulent stress dominates. For $y^+ > 70$, the turbulent stress dominates over the viscous stress. The curve in Fig. 2.12 is built using the real u_τ , as measured, and not the aimed one as based on the Re_τ definition. As observed in Fig. 2.12 and Fig. 2.13, there is a good agreement with the reference spectral results.

The mean quantities $\langle u \rangle \langle v \rangle \langle w \rangle \langle uv \rangle \langle uw \rangle \langle vw \rangle$ are gathered during the computation while the fluctuating quantities are derived from the mean ones using the following rule:

$$\langle \phi' \psi' \rangle = \langle \phi \psi \rangle - \langle \phi \rangle \langle \psi \rangle. \quad (2.43)$$

The RMS of a fluctuating value is defined as : $\phi'_{RMS} = \left\langle \phi'^2 \right\rangle^{1/2}$. The turbulent

shear stress for a fully developed channel flow is given by the following relation:

$$-\frac{\langle u'v' \rangle}{u_\tau^2} + \frac{du^+}{dy^+} = 1 - \frac{y}{\delta}. \quad (2.44)$$

When the flow is fully established, the sum of the turbulent shear stress and the viscous one gives a linear profile. This fact offers a qualitative criterion to check that the channel flow simulation has reached a statistical equilibrium. As observed in Fig. 2.14, this is indeed the case. Again, there is a good agreement with the spectral results. The RMS velocity fluctuations are reported in Fig. 2.16. The agreement is very good for vertical v'_{RMS} and spanwise w'_{RMS} velocity fluctuations. The underprediction of the streamwise velocity profile and velocity fluctuation u'_{RMS} can be explained by the drop in the modified wavenumber due to the finite order of the code, as observed by Kravchenko and Moin [70]. The reason that the mean velocity profile is under-predicted can be explained by the fact that, with a 4th order finite difference code, due to the modified wavenumber effect, one has a deficit of dissipation, as stated by Eq. (2.3). This deficit could explain the fact that the velocity profile is under-predicted. The profile would be even lower when using a second order code. This is indeed the case when one compares to the results of Daeninck [29]. The turbulent kinetic energy reported in Fig. 2.15 is also under-predicted. Even if the velocity fluctuations v'_{RMS} and w'_{RMS} agree very well with the reference results, the u'_{RMS} underestimation affects this result. The ratio of the turbulent shear stress over the turbulent kinetic energy is provided in Fig. 2.17. This ratio is close to 0.3, the value used for setting the RANS $k - \epsilon$ model constant: $C_\mu = (0.3)^2 = 0.09$ used in the following closure relation:

$$\nu_t = C_\mu \frac{k^2}{\epsilon} \quad (2.45)$$

Figs. 2.18 and 2.19 display the flow kinetic energy, (more important in the center of the channel) and the spanwise vorticity (concentrated near the walls).

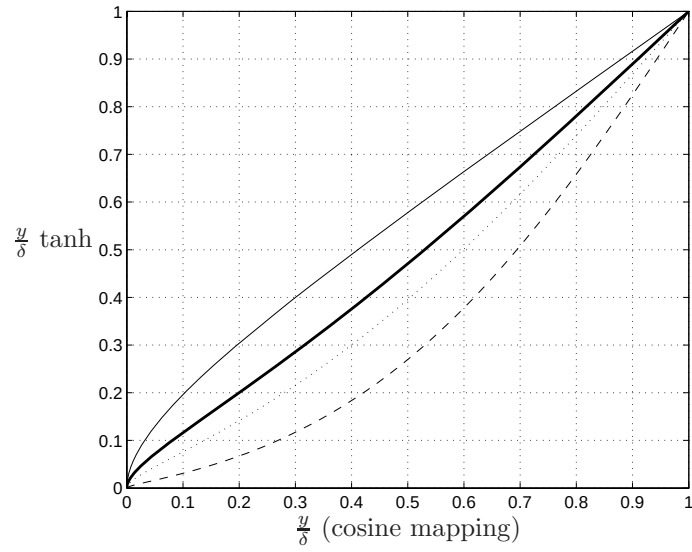


Figure 2.11: Stretching function for different values of the parameter α . $\alpha = 1.0$ (solid-thin); $\alpha = 1.6$ (solid-thick); $\alpha = 2.0$ (dot); $\alpha = 2.75$ (dash-thick)

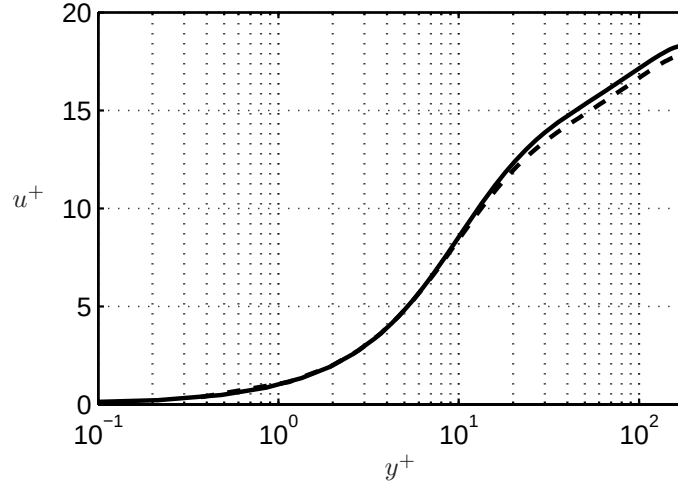


Figure 2.12: Mean streamwise velocity profile, inner (wall) coordinates are used. Reference results of [84] (solid) and present work (dash).

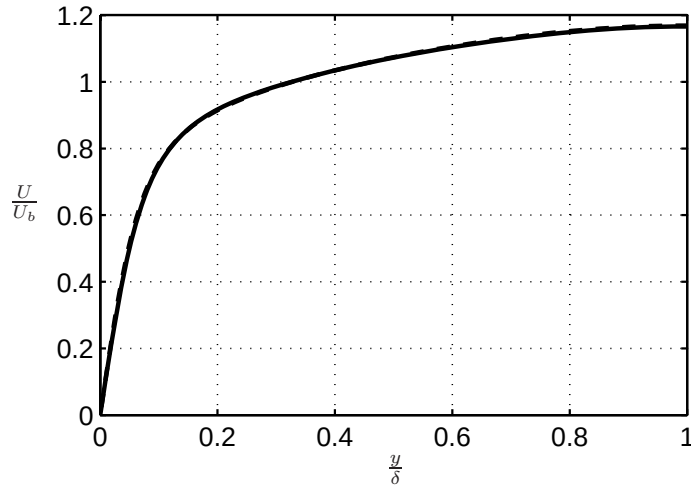


Figure 2.13: Mean streamwise velocity profile, outer coordinates are used. Reference results of [84] (solid) and present work (dash).

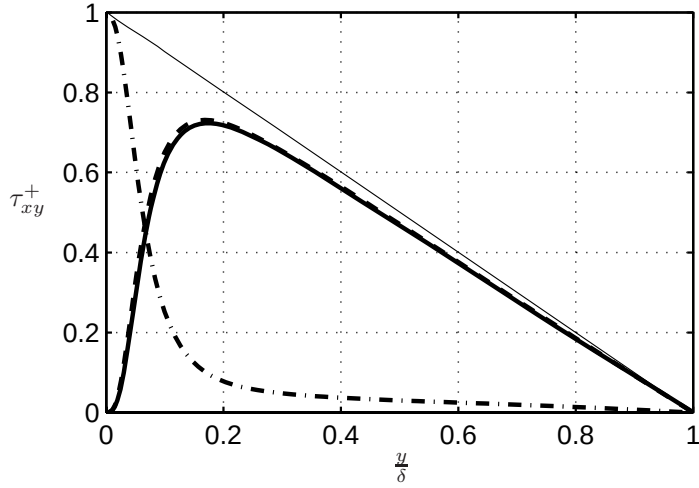


Figure 2.14: Turbulent mean shear stresses: viscous stress (dash-dot); turbulent stress, present work (dash) and reference of [84] (solid); total stress (thin solid).

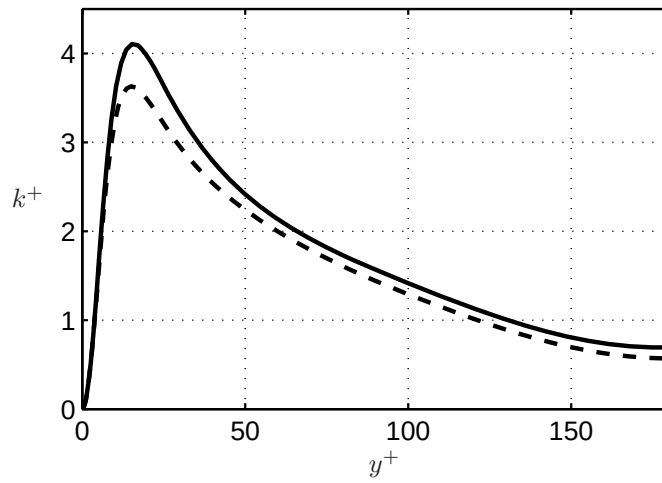


Figure 2.15: Turbulent kinetic energy profile. Reference results of [84] (solid) and present work (dash).

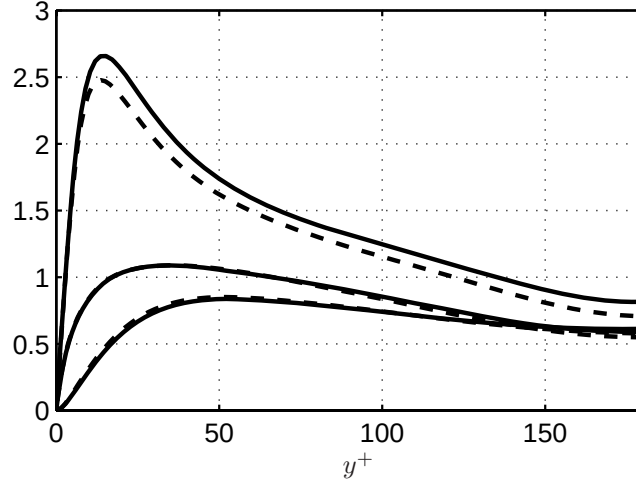


Figure 2.16: RMS velocity profiles: u'_{RMS} (top), w'_{RMS} (middle), v'_{RMS} (bottom). Reference results of [84] (solid) and present work (dash).

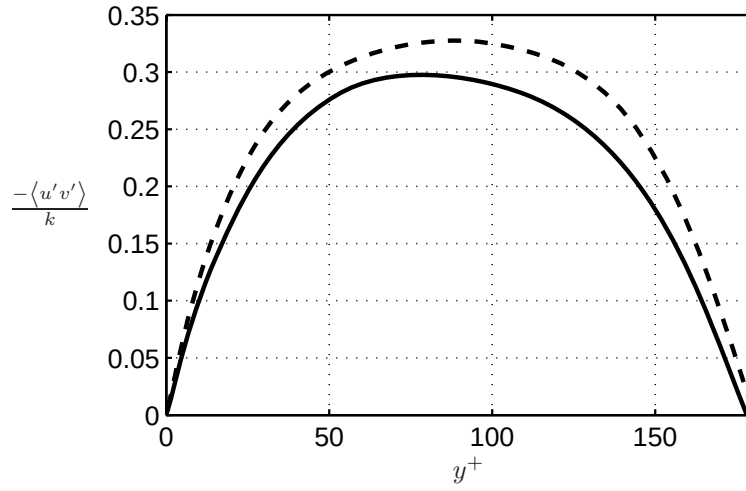


Figure 2.17: Turbulent shear stress over kinetic energy profile. Reference results of [84] (solid) and present work (dash).

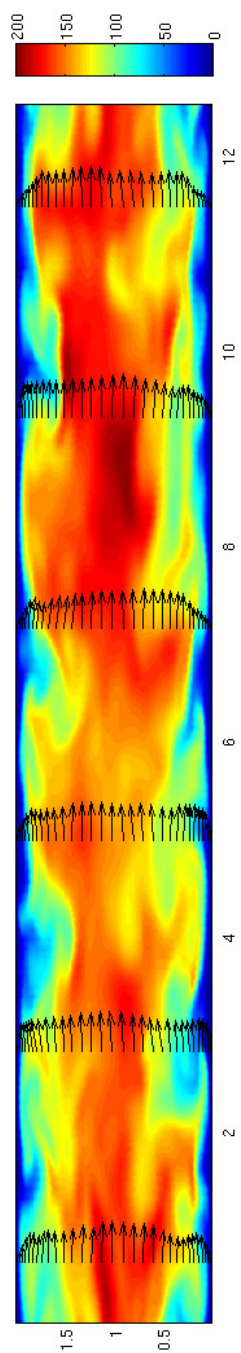


Figure 2.18: Snapshot of the normalized total kinetic energy (base flow+turbulence), with instantaneous velocity profiles taken at some stations.

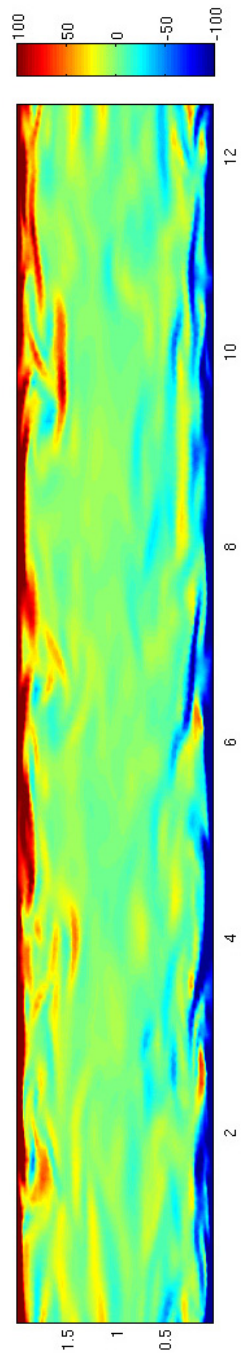


Figure 2.19: Snapshot of the normalized spanwise vorticity ω_z^+ .

2.1.6 Conclusions

Using the fourth order finite difference code, we performed a direct numerical simulation of three important flows. The first one is the Taylor-Green vortex which is a transitional flow: going from a very simple structured initial condition, it evolves quickly to a very complex “turbulent” flow. We compared successfully the dissipation history with that produced by a spectral reference code. As the code is energy conserving, it dissipates the proper amount of discrete energy, and diagnostics such as kinetic energy or dissipation history are of quality. For this reason, the second flow which was simulated, a four vortex system, also produced results that agree well with the reference solution. The code is thus able to predict properly the dynamics of vortical flows. The third flow we tested is a turbulent channel flow. We found results in good agreement with the reference ones of Moser et al. [84]. However there are still little discrepancies, i.e the mean velocity profile is under-predicted as well as the stream-wise fluctuations. This is due to the fact that the grid used has the same resolution as the spectral one (128^3). The same DNS was performed by Georges [46] with the same code but with a twice finer resolution (256^3). This improves the agreement with the spectral results. To conclude, the fourth order finite difference code is well suited for turbulent flows, also involving vortical structures, and is able to simulate wall bounded flows.

2.2 Pseudo-spectral (PS) Navier Stokes solver

Spectral methods make use of series expansions of the fields of interest with orthogonal basis functions. The main asset of spectral methods is that any derivative of a function is evaluated exactly and the error on the approximation decreases faster than any power of the mesh spacing h (this property is referred as spectral convergence). As a pseudo spectral solver is used in the present work, we introduce briefly the basis of spectral methods, the books of Canuto et al. [15] and Boyd [9] give exhaustive informations on spectral methods. The main orthogonal basis functions are the Chebyshev and Legendre polynomials and the trigonometric functions (Chebyshev and Legendre polynomials can be viewed as special cases of Jacobi polynomials [15]). The choice of the basis functions depends on the boundary conditions of the problem. For instance Chebyshev polynomials are well suited for wall bounded flow problems as they are able to handle Dirichlet boundary conditions. Fourier expansions require problems which are periodic to maintain spectral convergence. In this work we consider a Fourier-based pseudo-spectral method. The evaluations of terms which involve non linear products, such as the convective term of the Navier Stokes equations or the different effective viscosity based subgrid scale models, requires the evaluation of convolutions. Since these convolution require a cost which is $\mathcal{O}(N^2)$, the non linear products are evaluated in the physical space and transformed back in the Fourier space. This requires a cost which is reduced to $\mathcal{O}(N \log(N))$ thanks to the efficiency of the Fast Fourier Transform (FFT) algorithm. The fact that the non linear products are evaluated in physical space instead of Fourier space explains the name “pseudo-spectral” [91]. Computing the non linear products in the physical space may introduce aliasing errors; the origin of these errors and the way to eliminate them are presented hereafter.

2.2.1 Description of the code

This code was initially developed by the team of D. Carati of the Université Libre de Bruxelles. The initial version of this code was modified in order to be energy conserving and to be able to perform LES with eddy-viscosity SGS models.

Pseudo spectral discretisation

The spectral approximation of the velocity vector $\mathbf{u}$ is symbolically written as

$$\mathbf{u}(\mathbf{x}, t) = \sum_{\mathbf{k}} \hat{\mathbf{u}}(\mathbf{k}, t) \exp(i \mathbf{k} \cdot \mathbf{x}), \quad (2.46)$$

The spatial wavenumber vector is $\mathbf{k} = \{k_x, k_y, k_z\}$ and the spatial coordinates are $\mathbf{x} = \{x, y, z\}$. The “ $\hat{\cdot}$ ” notation is used to identify Fourier transforms. Taking advantage of the following property of the Fourier transform of a func-

tion $g(x)$:

$$\frac{d^n}{dx^n} g(x) \longleftrightarrow (ik)^n \hat{g}(k), \quad (2.47)$$

the Navier Stokes equations can be rewritten using the spectral approximation:

$$\left(\frac{d}{dt} + \nu |\mathbf{k}|^2 \right) \hat{\mathbf{u}}(\mathbf{k}) = \hat{\mathbf{f}}(\mathbf{k}). \quad (2.48)$$

This gives the temporal evolution of the Fourier coefficients $\hat{\mathbf{u}}(\mathbf{k})$ of the velocity field. The non linear term takes the following form:

$$\hat{\mathbf{f}}(\mathbf{k}) = -[\widehat{\nabla \cdot (\mathbf{u}\mathbf{u})}](\mathbf{k}). \quad (2.49)$$

The products are evaluated in physical space and then transformed back in the Fourier space to assemble the non linear term in Eq. 2.49. This avoids to compute expensive convolutions in Fourier space. The fast Fourier transforms are computed using the parallel FFTW routines developed by Frigo et al. [45].

Time stepping

Considering the system of partial differential equations symbolically written as:

$$\frac{du}{dt} = H(u, t), \quad (2.50)$$

this system has to be integrated in time. We chose to implement the classical Runge Kutta 3 variant of Williamson [117] as it requires only two levels of storage and three assemblies of $H(u, t)$. This is a third order scheme. If the velocity at time t^n is known the algorithm is:

$$\begin{aligned} U &= u^n \\ G &= H(U, t^n) \\ U &= U + \frac{1}{3} \Delta t G \\ G &= -\frac{5}{9} G + H(U, t^n + \frac{1}{3} \Delta t) \\ U &= U + \frac{15}{16} \Delta t G \\ G &= -\frac{153}{128} G + H(U, t^n + \frac{3}{4} \Delta t) \\ u^{n+1} &= U + \frac{8}{15} \Delta t G \end{aligned}$$

Incompressibility constraint

At the end of each sub-step of the Runge Kutta scheme, the velocity field is forced to be divergence free. This basically accounts for the effect of $-\nabla P$ in

the Navier-Stokes equations. A divergence free (or solenoidal) velocity field $\mathbf{u}^s$ can be obtained from any velocity field $\mathbf{u}$ by letting:

$$\hat{\mathbf{u}}^s(\mathbf{k}) = \hat{\mathbf{u}}(\mathbf{k}) - \mathbf{k} \frac{(\hat{\mathbf{u}} \cdot \mathbf{k})}{|\mathbf{k}|^2} \quad (2.51)$$

This avoids to store and compute a pressure field as it is required with a FD solver.

2.2.2 De-aliasing strategy and energy conservation

Aliasing errors

A brief introduction to aliasing in pseudo spectral methods is provided here. As it is not an exhaustive description, one can refer to the book of Canuto et al. [15] for more informations.

According to Boyd [9], on a grid of N points, Fourier components $\exp(ikx)$ with $k > N/2$ appear as low wavenumbers; the high wavenumber is said to be aliased to the low. We illustrate this phenomenon in 1D. Considering the Fourier transform of length M of an aliased product:

$$c_k^{aliased} = \sum_{n+m=k} \tilde{a}_n \tilde{b}_m + \sum_{n+m=k \pm M} \tilde{a}_n \tilde{b}_m, \quad (2.52)$$

the first sum is the alias-free result and the second sum is the alias error at wavenumber k . The principle of the dealiasing method implemented in the PS code is described hereafter. If one shifts the grid in physical space by a quantity Δ , in the Fourier space this operation amounts to multiply each Fourier mode $\tilde{a}_n$ by the factor $\exp(in\Delta)$. Considering the evaluation of the product on a mesh shifted by a spacing Δ and shifting the result back to the original mesh, one obtains:

$$c_k^{shifted} = \sum_{n+m=k} \tilde{a}_n \tilde{b}_m + \exp(\pm iM\Delta) \sum_{n+m=k \pm M} \tilde{a}_n \tilde{b}_m. \quad (2.53)$$

The alias-free sum remains unchanged by the shift while the alias error is multiplied by $\exp(\pm iM\Delta)$. If the shift is chosen so that $\Delta = \pi/M$, corresponding to half of the grid spacing, this gives: $\exp(\pm iM\Delta) = -1$. The dealiased product is then obtained by computing:

$$c_k = \frac{1}{2} \left(c_k^{aliased} + c_k^{shifted} \right). \quad (2.54)$$

This method allows to retain all the modes but requires two evaluations of each product. In 1D, the aliasing errors are thus exactly annihilated. The extension to 3D of the phase shift technique requires to evaluate eight times each product. As it is not affordable in terms of computational cost, the products are evaluated twice on two shifted grids with an ellipsoidal truncation of the high

order modes, as done in Jeanmart [61]. The ellipsoidal truncation retains only the modes (k_x, k_y, k_z) such that:

$$\frac{k_x^2}{(n_x/2)^2} + \frac{k_y^2}{(n_y/2)^2} + \frac{k_z^2}{(n_z/2)^2} \leq 1. \quad (2.55)$$

This attenuates considerably the aliasing errors. Other techniques also exists as the 2/3 rule or the random phase shift technique. They will not be used in the scope of the present work but are explained with great detail in Jeanmart [61], Rogallo [99] and Canuto [15]. It is also worth noting that using a skew symmetric form (see Eq. (2.12)) of the convective term of the Navier-Stokes equations reduces the aliasing error, as shown in Blaisdell et al. [5]. In this case the computational cost is the same as it amounts to evaluate twice the non linear products: one for the advective part and one for the conservative part of the skew symmetric operator.

Energy conservation and time reversibility.

To stress the fact that the de-aliasing strategy is important with regard to energy conservation, we propose to present results on the following benchmark developed by Duponcheel et al. [39]. This test is based on the time reversibility of the incompressible Euler equations:

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (u_j u_i) = - \frac{\partial P}{\partial x_i} \quad (2.56)$$

$$\frac{\partial u_i}{\partial x_i} = 0. \quad (2.57)$$

$P = p/\rho$ is the reduced pressure. If $(u(\mathbf{x}, t), P(\mathbf{x}, t))$ is a solution of the system, then $(u(\mathbf{x}, -t), P(\mathbf{x}, -t))$ is also a solution. It is legitimate to expect a numerical discretization of the Euler equations to preserve this time-reversibility property. The test case is the inviscid Taylor-Green vortex, as presented in Section 2.1.3, with ν deliberately set to zero. This flow becomes very complex before the time is reversed at $t = 10$ to try to recover the initial condition. As the requirement to pass the time-reversibility test are to be energy conserving and to be highly accurate in time, it constitutes a severe test that solver should pass. We will not provide all the details of this benchmark as it is well explained in [39]. The diagnostics analyzed are the energy evolution, the enstrophy evolution and the 3D Skewness evolution:

$$\mathcal{S}^{3D} = \left\langle \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k} S_{ij} \right\rangle. \quad (2.58)$$

The time reversibility results are displayed in Fig. 2.20. The time evolution exhibits clearly that the PS code performs quite well to recover the initial state of the three diagnostics. Only small differences can be observed on the recovered energy level, which is due to the time stepping errors. The presence of

spurious oscillations on the energy curve is due to the partial dealiasing; these oscillations are however time reversible. If we consider the pseudo-spectral solver without dealiasing procedure, the forward simulation blows up because of spurious injection of energy. Another interesting fact is that the random phase shift technique described in Rogallo [99] also yields to a blow up of the simulation, this one being due to the fact that the aliasing errors are not reduced in the same way during the forward and backward phase. It is thus reasonable to conclude that the pseudo-spectral code with the phase shift technique and ellipsoidal truncation passes the time reversibility test and is therefore suited to perform high quality DNS and LES of turbulent flows.

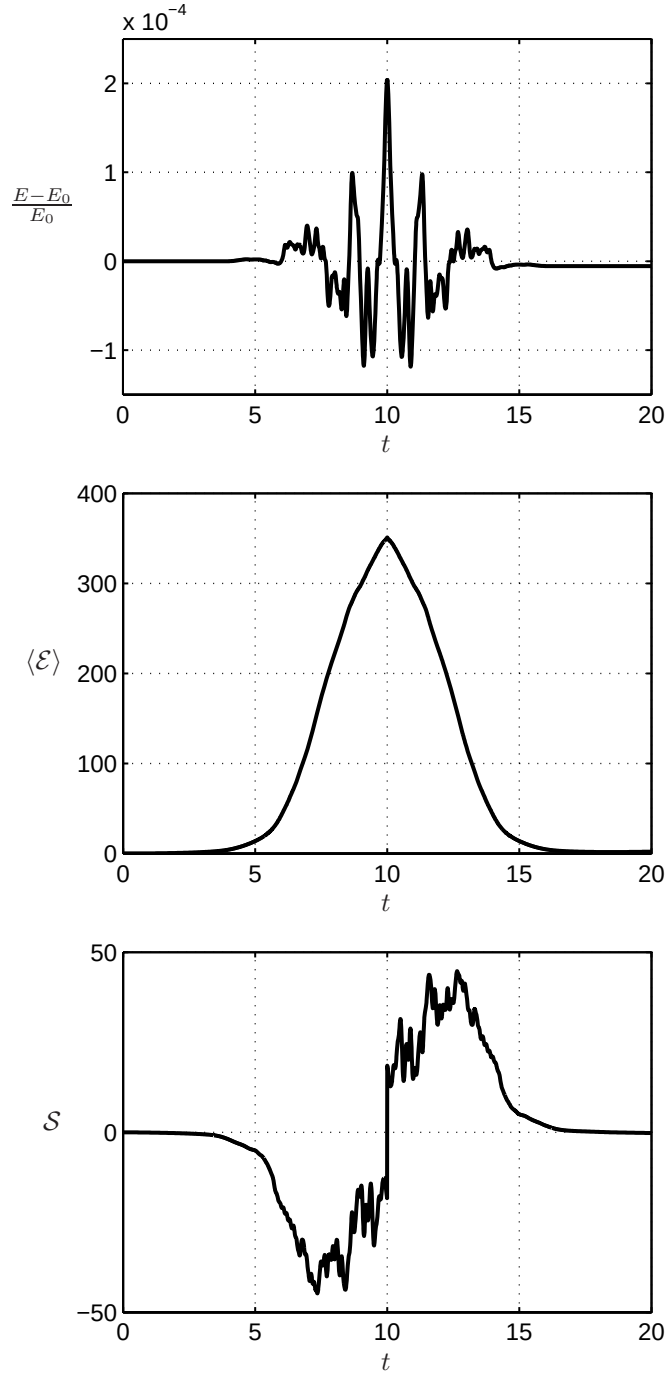


Figure 2.20: Temporal evolution of the energy (top), enstrophy $\langle \mathcal{E} \rangle$ (mid) and skewness $\mathcal{S} = \langle \partial_k u_i \partial_k u_j S_{ij} \rangle$ (bottom). The time reversal is performed at $t = 10$.

2.2.3 Validation case: DNS of the Taylor-Green vortex at $Re = 1600$

As a validation, we test the quality of the results provided by the PS code on the Taylor-Green vortex flow. The reference results are documented in Jeanmart [61]. The flow is simulated in a first step at $Re = 400$ and the results match perfectly with those of the reference (same temporal evolution of the dissipation). They are however not illustrated here. We prefer to illustrate a more challenging case fully resolved at $Re = 1600$ and to compare our results with Jeanmart [61] at the same Re but for different grid resolutions: one modest at $N = 96^3$ and one fairly resolved at $N = 192^3$. Since the case with $N = 192^3$ provides a resolution criteria $k_c\eta$ larger than unity, it is expected to obtain results which are in good agreement with those of the fully resolved DNS. Examining Fig. 2.21 shows that the 192^3 curves fit closely the 256^3 reference over a broad range of time. However after the peak, the 192^3 curve yields slightly different results since this computation is less resolved at this peak and the results are thus polluted for the rest of the time, see Fig. 2.22. The 96^3 leads to results that are not in agreement with the two other curves except in the first phase of the flow when the dissipation is weak. The energy spectra are displayed in Fig. 2.23. The compensated spectrum at a later time ($t = 10$) exhibits a $k^{-5/3}$ behaviour over the range of wavenumbers ($4 \leq k \leq 20$); however, even at this late time, the flow cannot be considered as homogeneous isotropic turbulence, as the large and medium scales still exhibit significant anisotropy, as displayed in Fig. 2.6.

Table 2.4: Resolution criteria for the Taylor-Green vortex with different grid resolutions. The Kolmogorov scale, η was computed at the time where the maximum of dissipation occurs, $t = 9$.

N	k_c	L/η	$k_c\eta$
96	48	537	0.56
192	96	537	1.12
256	128	537	1.5

2.2.4 DNS of homogeneous isotropic turbulence at $Re_\lambda = 100$

In order to calibrate the coefficient in SGS models for different codes, it is convenient to have a DNS of homogeneous isotropic turbulence. This particular flow is presented in Section 1.4.3. The method to obtain a random homogeneous isotropic initial velocity field is described in Appendix C. To initiate the turbulence flow we use the iterated short decays initialization method described in [61]. There are 256 Fourier modes in each direction: the maximum wavenumber resolved is thus $k_c = \frac{\pi}{h} = 128$. The initial field is such that

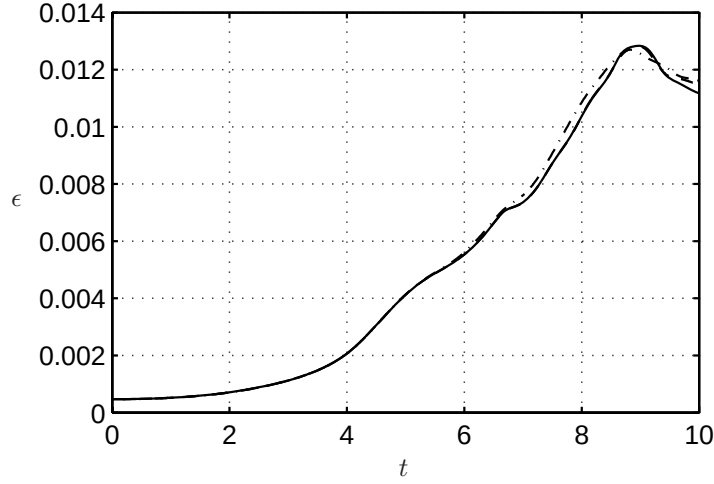


Figure 2.21: Evolution of the dissipation of the Taylor-Green vortex flow for different grid resolutions. 256^3 (thin solid), 192^3 (dash), 96^3 (dash-dot).

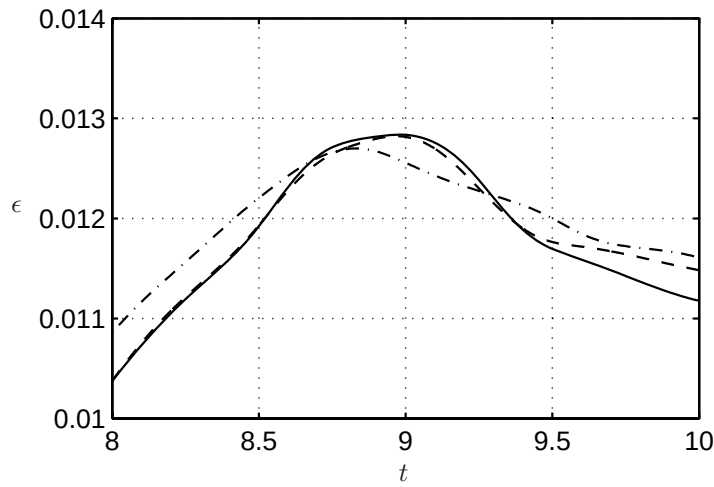


Figure 2.22: Zoom on the dissipation curves 2.21 for different grid resolutions. 256^3 (thin solid), 192^3 (dash), 96^3 (dash-dot).

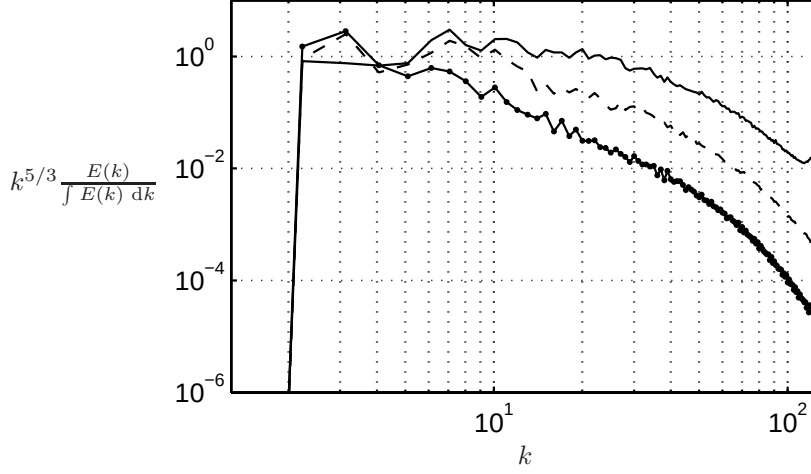


Figure 2.23: Compensated and normalized energy spectra at different times for the 256^3 pseudo-spectral computation. Spectra at time $t = 4$ (solid-bullets), $t = 6$ (dash), $t = 10$ (solid).

$k_c \eta = 1.1$ which means that the DNS is properly resolved. The evolution of this criterion is displayed in Fig. 2.27. As the energy and dissipation decay, the Kolmogorov length scale increases, leading to a computation which becomes better resolved as the flow evolves. The evolution of the kinetic energy is displayed in Fig. 2.24. As the $Re_\lambda \approx 105$ number is still fairly low, it is not possible to compare with the theoretical behaviour provided in section 1.4.3 which is valid at very high Reynolds. The Reynolds number based on the Taylor microscale λ is also displayed going from $Re_\lambda \approx 105$ to $Re_\lambda \approx 55$. Compensated energy spectra are provided at $t = 5$ and $t = 10$. It can be observed that the simulation at this time is well resolved as $k_c \eta > 1.5$. One can observe an inertial range, located in the region where the compensated spectrum exhibits a plateau ($4 \leq k \leq 20$).

2.2.5 Conclusion

We validated with success the pseudo-spectral code on the Taylor-Green benchmark flow. The code is energy conserving and yielded good results on the time reversibility benchmark. We have thus a pseudo-spectral code which works in parallel (as it use a parallel version of FFTW) and is able to accurately simulate turbulent and transitional flows.

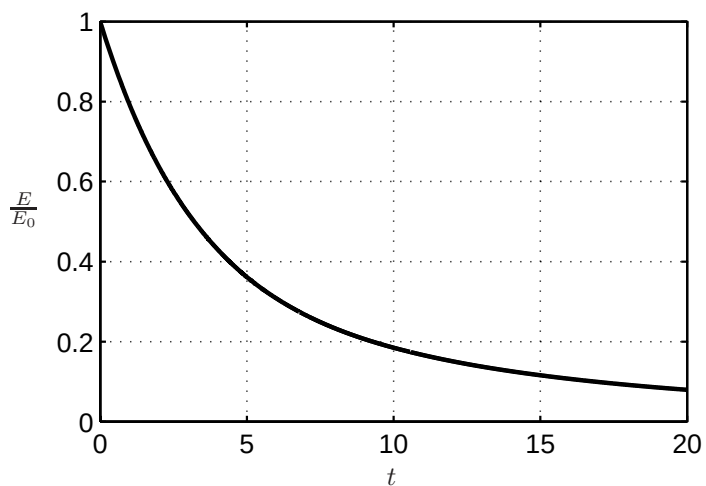


Figure 2.24: Kinetic energy evolution for the homogeneous isotropic turbulence.

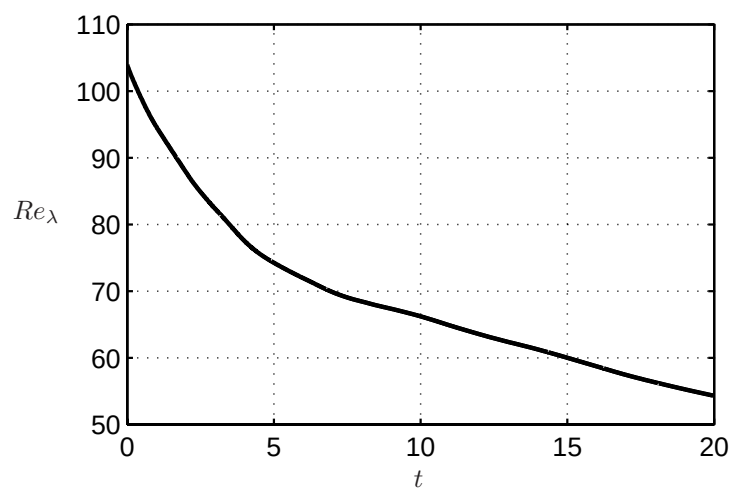


Figure 2.25: Evolution of the Reynolds number based on the Taylor microscale: Re_λ

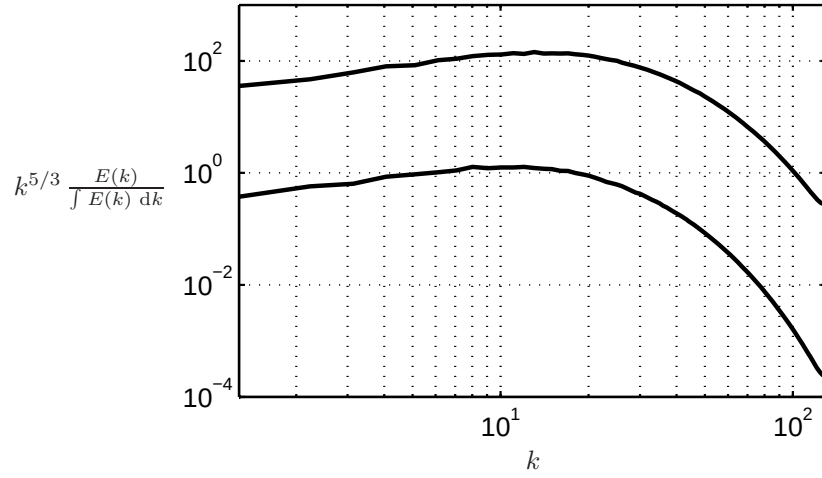


Figure 2.26: Compensated normalized energy spectrum at time $t = 5$ and $t = 10$. The spectrum at $t = 5$ is shifted up for clarity.

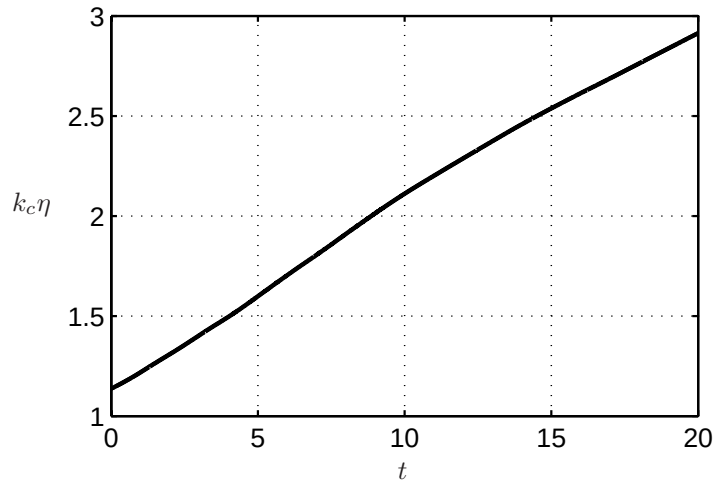


Figure 2.27: Evolution of the resolution factor $k_c \eta$.

Chapter 3

Contributions to the large eddy simulation of turbulent flows

3.1 The large eddy simulation of turbulent flows

Different families of approaches have been considered to numerically study turbulent flows. LES ¹ lies in a complexity level (and thus physical realism) between RANS ² calculations in which all scales of the turbulence are modeled and DNS in which all the scales of the turbulent flow are resolved. LES are simulations where the grid is not fine enough to capture all the scales of the flow field. In LES, only the large scales of the flow are resolved. There are thus small (subgrid) scales which are not captured by the grid. The effect of the unresolved scales on the large ones is modeled by means of a SGS ³ model. Recalling that the resolution criterion for the grid spacing h in order to perform a DNS is:

$$k_c \eta \geq 1.5 \iff h \leq 2\eta.$$

“Challenging” LES correspond to cases where the cutting is well within the inertial range, thus

$$k_c \eta < 0.15 \iff h > 20\eta.$$

A factor of ten (or even much more) between LES and DNS grid size is often encountered in industrial applications. In more academic cases, one would rather meet factors of four to six.

The truncation operator which consists in projecting a field on a LES grid is written $\widetilde{\bullet}$. Using this notation, the evolution equations for the LES velocity

¹Large Eddy Simulation

²Reynolds Averaged Navier Stokes

³SubGrid Scale

field are:

$$\frac{\partial \widetilde{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\widetilde{u_j u_i}) = -\frac{\partial \widetilde{P}}{\partial x_i} + \frac{\partial}{\partial x_j} (2\nu \widetilde{S_{ij}}) \quad (3.1)$$

$$\frac{\partial \widetilde{u}_i}{\partial x_i} = 0 \quad (3.2)$$

One can transform the momentum equation so as to let the SGS stress tensor $\widetilde{\tau}_{ij}^{sgs}$ appearing.

$$\frac{\partial \widetilde{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\widetilde{u_i u_j}) = -\frac{\partial \widetilde{P}}{\partial x_i} + \frac{\partial}{\partial x_j} 2\nu \widetilde{S_{ij}} - \frac{\partial}{\partial x_j} (\widetilde{u_i u_j} - \widetilde{u_i} \widetilde{u_j}) \quad (3.3)$$

The SGS tensor appearing explicitly in this equation is defined as:

$$\widetilde{\tau}_{ij}^{sgs} = \widetilde{u_i u_j} - \widetilde{u_i} \widetilde{u_j}. \quad (3.4)$$

This stress tensor has to be modeled since it is cannot be expressed in terms of the resolved field only. The eddy viscosity model hypothesis assumes that the effect of the small scales on the resolved scales is dissipative and acts like a viscosity term. The model for the sgs tensor, τ_{ij}^M is written:

$$\widetilde{\tau}_{ij}^M - \frac{1}{3} \widetilde{\tau}_{kk}^M \delta_{ij} = 2\nu_{sgs} \widetilde{S_{ij}} \quad (3.5)$$

The trace of the SGS stress tensor corresponds to twice the opposite of the SGS energy:

$$\widetilde{\tau}_{kk}^{sgs} = -(\widetilde{u_k u_k} - \widetilde{u_k} \widetilde{u_k}) = -2\widetilde{K} \quad (3.6)$$

Typically, it is not modelled; it is incorporated in the pressure:

$$\widetilde{P} - \frac{1}{3} \widetilde{\tau}_{kk}^{sgs} = \widetilde{P} + \frac{2}{3} \widetilde{K} = \widetilde{\mathcal{P}} \quad (3.7)$$

The effective model equations are then:

$$\frac{\partial \widetilde{u}_i}{\partial t} + \frac{\partial \widetilde{u_i u_j}}{\partial x_j} + \frac{\partial \widetilde{\mathcal{P}}}{\partial x_i} = \frac{\partial}{\partial x_j} (2\nu \widetilde{S_{ij}}) + \frac{\partial}{\partial x_j} (2\nu_{sgs} \widetilde{S_{ij}}). \quad (3.8)$$

The average dissipation due to a given model can be computed as:

$$\epsilon_{sgs} = \left\langle \widetilde{\tau}_{ij}^M \widetilde{S_{ij}} \right\rangle, \quad (3.9)$$

where the operator $\langle \bullet \rangle$ means that we average in the homogeneous directions. The fundamentals of the LES being exposed, we further deliberately omit the $\widetilde{\bullet}$ notation for the sake of clarity. The different SGS models that were used or investigated along this work are presented and analysed in this chapter. We illustrate the use of each of these models on canonical flows like homogeneous

isotropic turbulence and channel flow. Some of the models were implemented in the pseudo-spectral code, some in the finite difference code, some in both codes.

The Chapter is organized in the following way: the different models are presented from the most classical models to the most advanced ones. We present first the simplest models like the Smagorinsky, the WALE and the hyperviscosity models. A new sampling based dynamic procedure is also presented and some results are discussed. A classification of the eddy-viscosity based model is presented as well as the multiscale models. The asymptotic behaviour of several SGS models is also discussed. Finally the new RVM_s -WALE model is presented with results on the channel flow.

3.2 The Smagorinsky model (SMAG)

This is the first and most commonly used SGS model [102]. This model serves as a “reference” in many LES investigations. It is easy to implement and obtain at reduced computational costs. However it suffers from several drawbacks and fails to reproduce physical behaviour for many canonical flows, as explained further. We choose to present it for the sake of completeness and also because it can serve as a basis to develop other models; for instance the multiscale models presented later in this work. This effective viscosity based model is defined as:

$$\tau_{ij}^M = 2 \nu_{sgs} S_{ij} . \quad (3.10)$$

The SGS viscosity is defined by choosing the norm of the strain rate tensor for the local inverse time scale:

$$\nu_{sgs} = C_S \Delta^2 (2 S_{ij} S_{ij})^{\frac{1}{2}} . \quad (3.11)$$

The length scale is usually taken as $\Delta = h = (h_x h_y h_z)^{\frac{1}{3}}$ where the h_x, h_y, h_z are the size of the grid cells. Although other definitions can be used, we keep this one throughout this work. The SGS viscosity is thus dependent of a tensorial invariant of S_{ij} . As a consequence its value is the same in any coordinate system. The theoretical value for C_S is $(0.3)^3 = 0.027$. The relation established by Lilly [73], [74], links the Smagorinsky model constant to the Kolmogorov constant $C_K = 1.6$:

$$C_S = \frac{1}{\pi^2} \left(\frac{2}{3 C_K} \right)^{\frac{3}{2}} . \quad (3.12)$$

As it is valid for flows at very high Re only, the Smagorinsky constant is not universal. Meneveau et al. [86] have shown that, when the cutting falls outside of the inertial range (i.e. inside part of the viscous dissipation range), the coefficient becomes dependent on the ratio $\frac{\Delta}{\eta}$. They found, using the von Karman and Pao spectrum, the theoretical behaviour of the model constant as $\frac{\Delta}{\eta}$ varies:

$$C = e^{-\gamma} \left(\frac{\eta}{\Delta} \right)^2 (1 - e^{-\gamma})^{-3/2} , \quad (3.13)$$

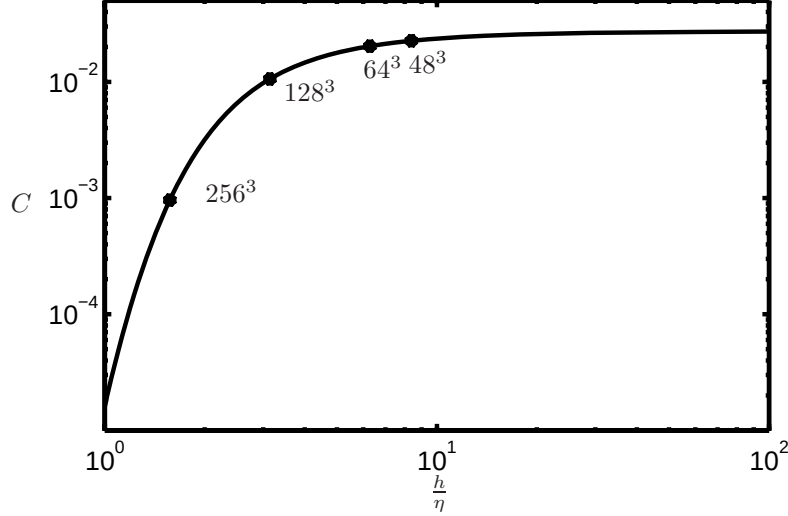


Figure 3.1: Variation of the Smagorinsky coefficient for different grid resolutions as computed by [86].

where,

$$\gamma = \frac{3}{2} C_K \left(\pi \frac{\eta}{\Delta} \right)^{4/3}. \quad (3.14)$$

The curve displayed in Fig. 3.1 showing the predicted variation of C allows to observe that when $h \rightarrow \eta$, the C value decreases sharply. The asymptotic value of C as $\frac{h}{\eta} \rightarrow \infty$ is C_S . One of the major drawbacks of the Smagorinsky model is its behaviour for wall bounded flows. Near a wall, the turbulent fluctuations vanishes and the SGS viscosity should also vanish, which is not the case for this model since it gives non-zero values of ν_{sgs} as soon as a velocity gradient appears. Due to this reason, this model dissipates too much close to the wall and can lead to a relaminarization of the flow in a channel. One solution for this problem is to use damping functions for the near wall regions. The SGS viscosity is modulated by a function $f_w(y)$,

$$\nu_{sgs} = C_S \Delta^2 f_w(y) (2 S_{ij} S_{ij})^{\frac{1}{2}}. \quad (3.15)$$

The Van Driest damping function is defined as:

$$f_w(y) = \left(1 - \exp \left(-\frac{y u_\tau}{25\nu} \right) \right)^2. \quad (3.16)$$

This function does not provide the correct near wall behaviour (cfr. Section 3.3.1). Another damping function is the one of Piomelli, which is an alternate form of the Van Driest one with the correct y^{+3} behaviour:

$$f_w(y) = 1 - \exp \left(-\left(\frac{y u_\tau}{25\nu} \right)^3 \right). \quad (3.17)$$

Damping functions requires to evaluate u_τ but enforces the SGS viscosity to vanish close to the walls. However, this is an *ad hoc* modification difficult to implement in case of complex geometries.

The Smagorinsky model can also be used with a dynamic procedure as described by Germano et al. [49]. This avoids to use damping functions and to adapt the constant as the flow evolves. The spectral behaviour of this model is however not adequate, as will be illustrated in the Section 3.8.

The implementation of this model is quite easy using a finite difference approach. It is however much more costly in a pseudo-spectral approach. One has to compute the components of the strain rate tensor S_{ij} in the Fourier space, evaluate the products in the physical space and go back to Fourier space. This implies that the evaluation of the SGS model has approximately the same cost as the evaluation of the convective term (in term of number of FFT's performed). Some results obtained with this model on isotropic homogeneous turbulence are presented in Section 3.8.

3.3 The WALE SGS model

3.3.1 Definition of the model

The WALE model is an effective eddy viscosity model developed by Nicoud and Ducros [88]. This acronym stands for “Wall Adapting Local Eddy viscosity”. The model is based on the square of the velocity gradient tensor and the local eddy-viscosity depends on both the strain rate tensor S_{ij} and the rotation rate tensor Ω_{ij} . If the velocity field of a wall bounded flow is expanded in Taylor series close to the wall, one obtains:

$$\begin{aligned} u &= \alpha_1 y + \alpha_2 y^2 + \mathcal{O}(y^3) \\ v &= \beta_1 y + \beta_2 y^2 + \mathcal{O}(y^3) \\ w &= \gamma_1 y + \gamma_2 y^2 + \mathcal{O}(y^3) \end{aligned}$$

The incompressibility constraint implies that: $\beta_1 = 0$. Letting $y \rightarrow 0$ and introducing wall units, we have

$$u \propto y^+ \quad v \propto (y^+)^2 \quad w \propto y^+$$

Considering the component τ_{12} of the SGS stress tensor:

$$\tau_{12}^{sgs} = \widetilde{\widetilde{uv}} - \widetilde{uv} \quad (3.18)$$

Close to the wall, it comes that:

$$\tau_{12}^{sgs} \propto (y^+)^3 \quad (3.19)$$

This implies that the SGS stress should vary as $\tau_{12}^M \propto (y^+)^3$ at the wall. S_{12} behaving as a constant close to the wall, it follows that ν_{sgs} must vary as

$(y^+)^3$. A good SGS model for wall bounded flow should have this behaviour. The WALE model is designed to have the correct y^{+3} behaviour of ν_{sgs} at the wall. The way chosen to build such a SGS viscosity is to consider the traceless symmetric part of the square of the velocity gradient tensor:

$$\mathcal{S}_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_k} \frac{\partial u_k}{\partial x_j} + \frac{\partial u_j}{\partial x_k} \frac{\partial u_k}{\partial x_i} \right). \quad (3.20)$$

The deviatoric part of this tensor is:

$$\mathcal{S}_{ij}^d = \mathcal{S}_{ij} - \frac{1}{3} \delta_{ij} \mathcal{S}_{kk}. \quad (3.21)$$

The invariant $\mathcal{S}_{ij}^d \mathcal{S}_{ij}^d$ of this tensor behaves like y^{+2} for $y \rightarrow 0$, a simple way to build a spatial operator which behaves like y^{+3} is to take $OP1 = (\mathcal{S}_{ij}^d \mathcal{S}_{ij}^d)^{3/2}$. The way to obtain a quantity with the dimension of a viscosity is to build a second operator $OP2$ and build the SGS viscosity in the following way:

$$\nu_{sgs} = C_w \Delta^2 \frac{OP1}{OP2} = C_w \Delta^2 \frac{(\mathcal{S}_{ij}^d \mathcal{S}_{ij}^d)^{3/2}}{(S_{ij} S_{ij})^{5/2} + \alpha (\mathcal{S}_{ij}^d \mathcal{S}_{ij}^d)^{5/4}}. \quad (3.22)$$

The first term in $OP2$ is $\mathcal{O}(1)$ near a wall and its exponent allows to build a viscosity. The second term in $OP2$ is negligible near the wall and avoids to have a zero denominator when the first term vanishes. The value $C_w = 0.25$ was calibrated based on homogeneous isotropic turbulence at high Reynolds. The same value was also applied successfully in pipe and channel flows. Laporte [71] used it for vortex merging investigations at high Re , he took in this case $C_w = 0.0025$, thus a hundred times smaller than the calibrated value. The parameter α is left as a degree of freedom of the model, it is set to $\alpha = 1$ in the model by Nicoud and Ducros [88] (its influence was not investigated). The scalar quantity $\mathcal{S}_{ij}^d \mathcal{S}_{ij}^d$ can also be computed by means of the following identity:

$$\mathcal{S}_{ij}^d \mathcal{S}_{ij}^d = \frac{1}{6} (S^2 S^2 + \Omega^2 \Omega^2) + \frac{2}{3} S^2 \Omega^2 + 2 IV_{S\Omega}. \quad (3.23)$$

Here we use the following notations:

$$S^2 = S_{ij} S_{ij}, \quad \Omega^2 = \Omega_{ij} \Omega_{ij}, \quad IV_{S\Omega} = S_{ik} S_{kj} \Omega_{jl} \Omega_{li}. \quad (3.24)$$

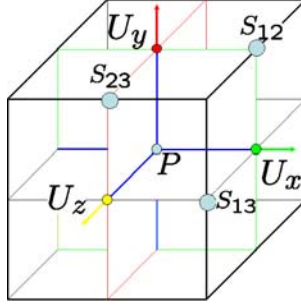


Figure 3.2: Location of the components of S_{ij} on a MAC cell, the diagonal components are located at pressure points.

which can be further expanded (since $S = S^T$, $\Omega = -\Omega^T$) in terms of individual components of S_{ij} and Ω_{ij} :

$$\begin{aligned}
 \mathcal{S}_{ij}^d \mathcal{S}_{ij}^d &= \frac{1}{6}T_1^2 + \frac{1}{6}T_2^2 + \frac{2}{3}T_3 + \sum_{p=4}^9 T_p \\
 T_1 &= (S_{11}^2 + S_{22}^2 + S_{33}^2 + 2S_{12}^2 + 2S_{13}^2 + 2S_{23}^2) \\
 T_2 &= (2\Omega_{12}^2 + 2\Omega_{13}^2 + 2\Omega_{23}^2) \\
 T_3 &= T_1 \cdot T_2 \\
 T_4 &= 2(S_{11}^2 + S_{12}^2 + S_{13}^2)(-\Omega_{12}^2 - \Omega_{13}^2) \\
 T_5 &= -4(S_{11}S_{12} + S_{12}S_{22} + S_{13}S_{23})\Omega_{13}\Omega_{23} \\
 T_6 &= 4(S_{11}S_{13} + S_{12}S_{23} + S_{13}S_{33})\Omega_{12}\Omega_{23} \\
 T_7 &= 2(S_{12}^2 + S_{22}^2 + S_{23}^2)(-\Omega_{12}^2 - \Omega_{23}^2) \\
 T_8 &= -4(S_{12}S_{13} + S_{22}S_{23} + S_{23}S_{33})\Omega_{12}\Omega_{13} \\
 T_9 &= 2(S_{13}^2 + S_{23}^2 + S_{33}^2)(-\Omega_{13}^2 - \Omega_{23}^2)
 \end{aligned}$$

This model was implemented in the fourth order finite difference code. Using a staggered grid, as depicted in Fig.3.2, requires some care to compute ν_{sgs} . The components of S_{ij} should be first computed at the pressure points. This allows to evaluate ν_{sgs} at those points and then interpolate it to the desired location.

In the case of a pure shear flow : $u = \beta y, v = 0, w = 0$, the WALE model produces $\nu_{sgs} = 0$, while the Smagorinsky model provides a constant: $\nu_{sgs} = \frac{1}{2}\beta^2$. The amount of SGS viscosity is thus negligible in the case of laminar wall bounded flows. This avoids the use of wall functions or a dynamic procedure for the simulation of wall bounded flows. An a-priori examination of the term $\mathcal{S}_{ij}^d \mathcal{S}_{ij}^d$ however shows that, in case of a pure solid body rotation ($S = 0$ but $\Omega \neq 0$) the WALE model produces a non zero ν_{sgs} . This is investigated more in Section 3.3.3.

3.3.2 LES of the channel flow at $Re_\tau = 395$

We propose here to assess the WALE model on the channel flow at $Re_\tau = 395$. The computational box is the same as Moser et al. [84], $L_x = 2\pi\delta$, $L_y = 2\delta$, $L_z = \pi\delta$. Usually the grid chosen for the LES is quite coarse: $N_x = 64$, $N_y = 48$, $N_z = 48$, yet it is still wall resolved, i.e the first grid point is located at $y^+ < 1$. The stretching used is the same as in Eq. (2.40) with $\gamma = 2.75$ a value appropriate for the LES, as shown by Jeanmart [61]. The time step is set to $\Delta t^+ = 0.001$

Results and discussion

In Fig. 3.3, a very good agreement between the mean velocity profile of the LES and that of the DNS [84] is observed. The value of the coefficient C_w was calibrated on this test case. Choosing a too high C_w value leads to an over-estimation of the velocity profile while a too low value leads to an under-estimation. The proper value for the fourth order FD code in such moderate Reynolds number flow and still fairly well-resolved is $C_w = 0.2$. The decomposition of the mean shear stress is shown in Fig. 3.4. It can be observed that the model contribution is quite weak and peaks close to the wall. Recall that the sum of the stresses obeys to the following relation:

$$-\frac{\langle u'v' \rangle}{u_\tau^2} + \frac{d\langle u^+ \rangle}{dy^+} + \langle \tau_{xy}^{sgs} \rangle = \langle \tau_{xy}^{tot} \rangle = \left(1 - \frac{y}{\delta}\right), \quad (3.25)$$

recalling that $\langle \cdot \rangle$ means an averaging in time and homogeneous (x, z) directions. It is important to recall that LES or DNS channel flow statistics must be gathered over a sufficiently long time to be of good quality. As displayed in Fig. 3.6, the value of mean shear stress at the wall is monitored over a sufficiently long time ($t^+ = 100$) to obtain a good time average of this signal, thus the averages of the other fluctuating quantities should also be properly converged. The fact that the quantities provided by the simulations verify the relation (3.25) is an indicator that the statistics are well converged. The SGS viscosity is also displayed in Fig. 3.5. It can be observed that this curve has the proper $(y^+)^3$ behaviour in the near wall region. The value of $\frac{\nu_{sgs}}{\nu} \approx 1$ at the center of the channel as also observed by Daeninck [29]. This confirms that the LES is still resolving much the flow and that the flow itself is at moderate Reynolds number. The comparison of the RMS velocities from the DNS with those calculated from the LES (including the SGS model contribution) is not entirely valid. Indeed since the SGS model, τ_{ij}^M have a zero trace, one can only reconstruct, and thus fairly compare with DNS, the deviatoric part of the Reynolds stress tensor R_{ij} as detailed in Winckelmans et al. [120]. The trace of the Reynolds stress tensor is the DNS turbulent kinetic energy, which is different from the resolved LES turbulent kinetic energy. The fair comparison, on the deviatoric components, consists in comparing the reduced traceless part of the tensor R_{ij}^{DNS*} to $R_{ij}^{LES*} - \langle \tau_{ij}^M \rangle$. The comparison of the reduced turbulent

intensities is presented in Figs. 3.7, 3.8, 3.9. The reduced intensities produce an overshoot of the maximum near wall value which is more pronounced for the component $\langle u'u' \rangle$. These overshoots are also observed with other models and/or in other studies, as in Hughes et al. [57], Georges [46] and Jeanmart [62].

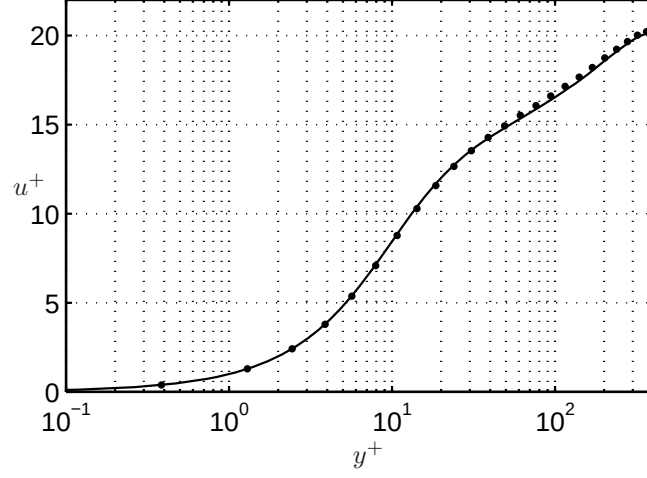


Figure 3.3: Mean velocity profile for the channel flow at $Re_\tau = 395$. Reference DNS (solid), LES using WALE model (bullets).

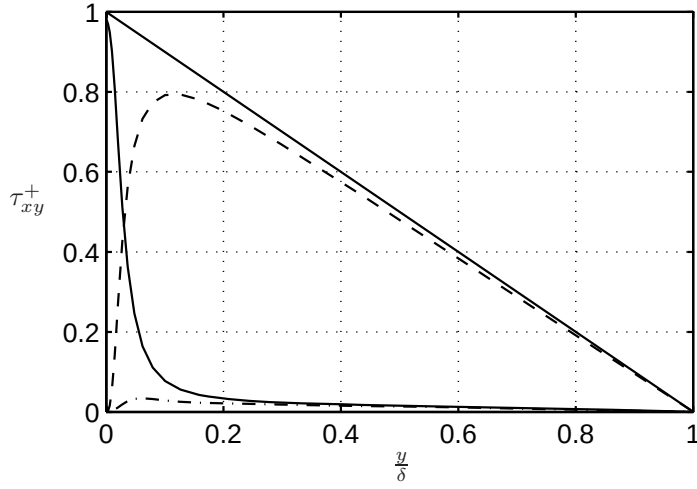


Figure 3.4: Decomposition of the mean shear stress profile: τ_{xy}^{tot} (solid thick), τ_{xy}^{visc} (solid thin), $-\langle u'v' \rangle$ (dash), τ_{xy}^{sgs} (dash-dot)

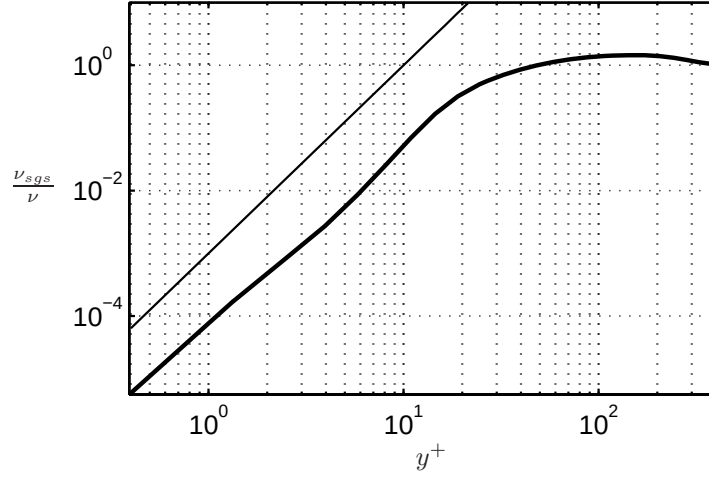


Figure 3.5: SGS viscosity profile for the channel flow at $Re_\tau = 395$ using the WALE model. The thin straight line corresponds to a y^{+3} behaviour.

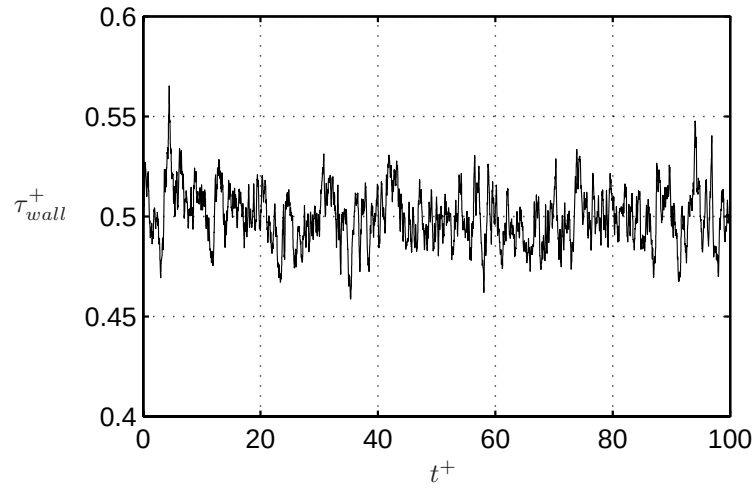


Figure 3.6: Time history of wall shear stress for the period during which statistics were gathered.

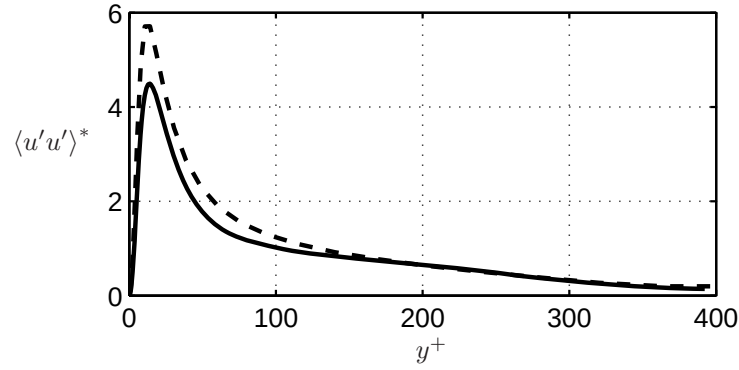


Figure 3.7: Reduced turbulence intensity : $\langle u'u' \rangle^*$. Reference DNS (solid), LES using WALE model(dash).

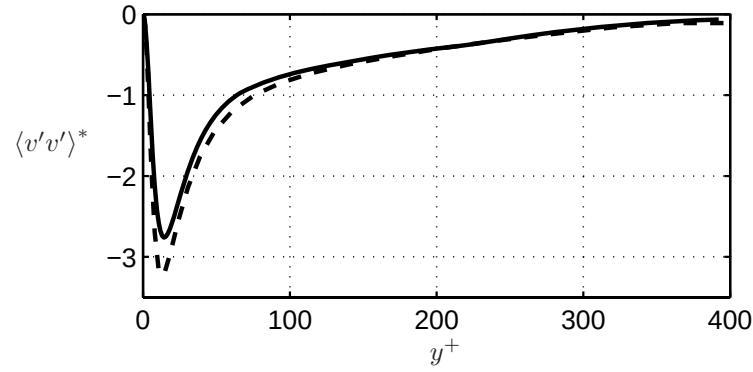


Figure 3.8: Reduced turbulence intensity : $\langle v'v' \rangle^*$.

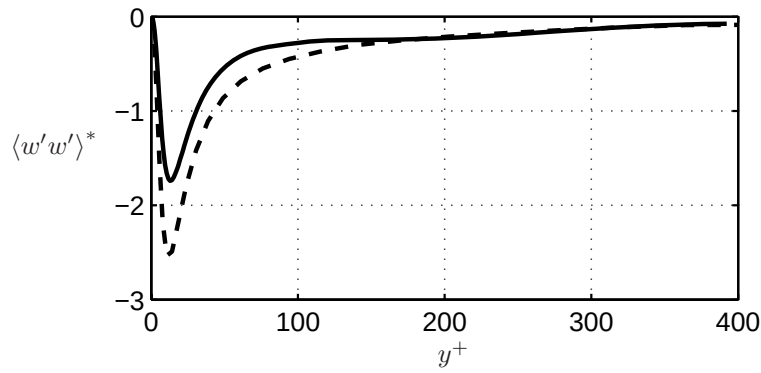


Figure 3.9: Reduced turbulence intensity : $\langle w'w' \rangle^*$.

3.3.3 SGS viscosity of vortical flows

We are interested in studying the capacity of the WALE SGS model to correctly perform LES of vortical flows. An isolated vortex with a low order algebraic profile is considered here. The spatial distribution of the circulation and velocity profile for such a vortex are defined in Section 1.3. We recall here the vorticity profile:

$$\omega = \frac{\Gamma_0}{\pi} \frac{r_c^2}{(r^2 + r_c^2)^2}, \quad (3.26)$$

leading to a maximum value

$$\omega_{max} = \frac{\Gamma_0}{4\pi r_c^2}. \quad (3.27)$$

The value of the SGS viscosity provided by the WALE model is compared with that provided by the Smagorinsky model. These values were obtained analytically using a symbolic computation software. If one evaluates the value of ν_{sgs} in the case of the WALE model at the center of the vortex, one obtains that:

$$\nu_{sgs} = \frac{\Gamma_0}{2\pi} C_w \left(\frac{2}{3}\right)^{1/4} \left(\frac{\Delta}{r_c}\right)^2 = \left(\frac{32}{3}\right)^{1/4} C_w \Delta^2 \omega_{max} \quad (3.28)$$

The curves displayed in Fig. 3.10 clearly show that the WALE model yields a value of ν_{sgs} which is too high in the region at the center of the vortex, where solid body rotation is dominant. The value of ν_{sgs} is even bigger than that provided by the Smagorinsky model, which already dissipates too much vortical flows. This behaviour is due to the presence of the term Ω^2 in the expression of ν_{sgs} , cfr. Eq. (3.23). The Smagorinsky model dissipates only in the regions where the norm of S_{ij} is important and exhibits a low peak around $r = r_c$. The WALE model produces a ν_{sgs} at the center of the vortex which is approximatively 15 times more important than the peak value obtained with the Smagorinsky model. The two models can also be compared on the “zero circulation vortex”. It was given and studied by Kloosterziel and Carnevale [66] and by Dufresne and Dumas [33]. Laboratory experiments, which have applications in quasi-geostrophic fluid dynamics, were also done by Kloosterziel and Van Heijst [67]. This flow is essentially an axisymmetric shear layer. This flow is defined by the following velocity field, we took here $p = 7$:

$$\frac{u_\theta(r)}{\omega_0 a} = -\frac{1}{2} \left(\frac{r}{a}\right) \exp\left(\left(-\frac{r}{a}\right)^p\right). \quad (3.29)$$

The vorticity field is written:

$$\frac{\omega(r)}{\omega_0} = \left(\frac{1}{2}p \left(\frac{r}{a}\right)^p - 1\right) \exp\left(\left(-\frac{r}{a}\right)^p\right), \quad (3.30)$$

where ω_0 is the vorticity at the center and a is the vortex core size. The results are displayed in Fig. 3.11. This flow is chosen to illustrate the fact that the

WALE model produces an uniform ν_{sgs} in the case of a solid body rotation,

$$\nu_{sgs} = \left(\frac{1}{24} \right)^{1/4} C_w \Delta^2 \omega_0. \quad (3.31)$$

As the Smagorinsky model acts only in the region where S_{ij} is important, the value of ν_{sgs} is important only at the location of the axi-symmetric shear layer. The WALE model will produce lower values of ν_{sgs} in this region due to the fact that it is designed to be inactive in the region of pure shear flow. To conclude, even if the WALE model provides good results in the case of wall bounded flows like channel flows, this model dissipates too much in the case of vortical flows and is even worst than Smagorinsky. As a conclusion, this model cannot be used for investigations of aircraft wake vortices flows without modifications.

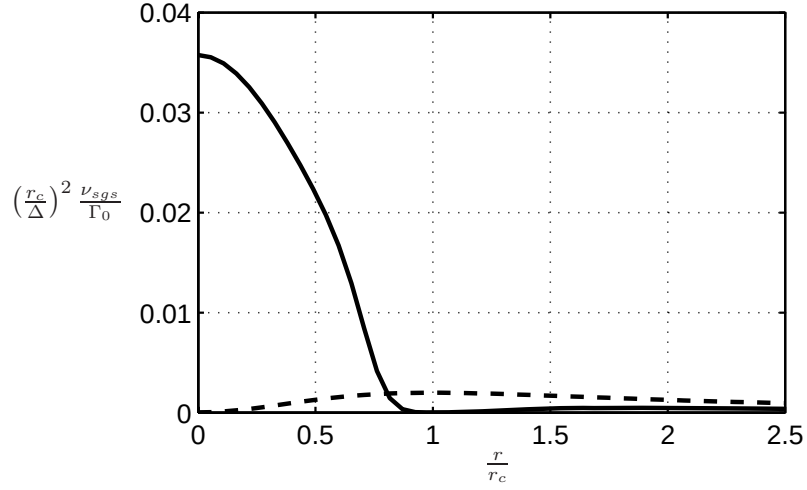


Figure 3.10: ν_{sgs} profile for the low order algebraic vortex, dash: Smagorinsky; solid: WALE

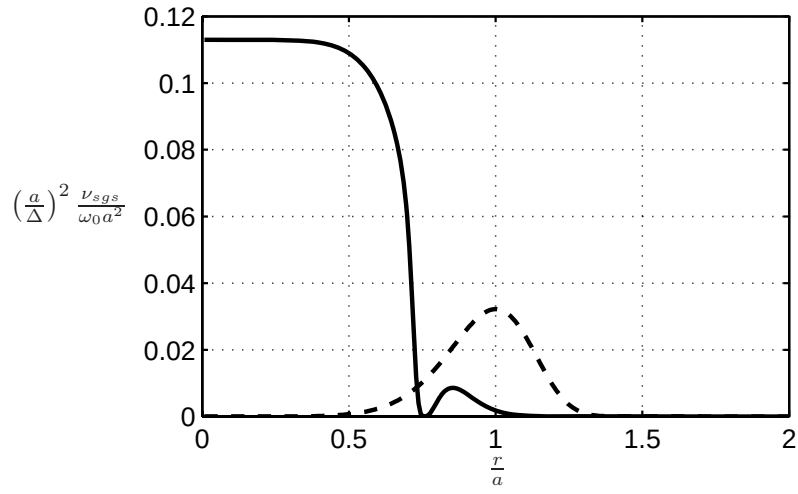


Figure 3.11: ν_{sgs} profile for the zero circulation vortex, dash: Smagorinsky ; solid: WALE

3.4 The sampling based dynamic procedure

This section is mainly reproduced from the DLES6 conference proceedings. The part concerning the investigations on 1D Burgers turbulence performed by L.Georges was removed. For further information, the reader can consult Winckelmans et al. [123].

3.4.1 Description of the procedure

The dynamic procedure for LES performed solely in physical space (i.e., no Fourier transform) is considered. It amounts to a procedure working at the force (vector) level that is natural and quite general: it only requires a numerical tool for restriction of the discrete LES field and forces to a coarser level. It is here investigated using finite differences and with restriction done by sampling. It gives good results on flows with homogeneous directions: Burger's turbulence as presented in [123] and homogeneous isotropic turbulence. Results on the turbulent channel flow are also presented: they are encouraging but not yet satisfactory (velocity profile underpredicted). The obtained profile of $C\Delta^2$ is found to have the proper near and far wall behaviours, but with too low amplitude. Further improvements are required: they might include some filtering (using tensor-product stencil-3 discrete filters, also iterated) prior to the sampling, to mitigate the aliasing effects due to the sampling; they might also require to modify the procedure itself, following what was done by others when using the classical procedure expressed at the force level. Practical LES mainly consists of a "restriction": a truncation to much less information than that required to numerically capture the complete field, $\vec{u}(\vec{x})$, as in DNS. We here use the word "restriction" to clearly distinguish it from the word "filtering". Indeed, the concept of filtering usually means "regular filtering", thus a regular convolution written as

$$\bar{\vec{u}}(\vec{x}) = \int G(\vec{x} - \vec{x}') \vec{u}(\vec{x}') d\vec{x}'$$

where the filter G has a regular Fourier transform (e.g., the Gaussian filter). Such filtering does not remove scales; it damps those at high wavenumbers. It works also such that applying it twice is not the same as applying it once: $\bar{\bar{\vec{u}}} \neq \bar{\vec{u}}$. Contrary to regular filtering, restriction corresponds to truncation. Considering a grid of size h , the discrete LES-restricted field is here written $\vec{u}_h = R_h(\vec{u}(\vec{x}))$. Notice that, contrary to regular filtering, restriction is such that applying it twice is the same as applying it once (idempotent): $R_h(\vec{u}_h) = \vec{u}_h$. The time variation of the discrete field is evaluated using the discrete "forces" (convection term, SGS term, viscous term). This is here written as a discrete numerical operator acting on the discrete field:

$$\partial_t \vec{u}_h + \nabla_h P_h = -\nabla_h \cdot (\vec{u}_h \vec{u}_h) + \nabla_h \cdot (2\nu \vec{S}_h) + \nabla_h \cdot \tau_h^{sgs} \stackrel{\text{def}}{=} Q_h(\vec{u}_h) . \quad (3.32)$$

Notice that the pressure correction step (the projection to ensure incompressibility) is here seen as part of the proper time integration. In the same way, the equations for LES on a grid $2h$ would be written as

$$\partial_t \vec{u}_{2h} = Q_{2h}(\vec{u}_{2h}) . \quad (3.33)$$

We here present and investigate a procedure for dynamic LES performed solely in physical space (i.e., no recourse to spectral space): finite differences (FD), etc. For simplicity, we only consider LES without explicit filtering; the approach being easily extended to LES with added explicit filtering (e.g., as those in [119]), when using discrete filters. We here investigate the case with FD (staggered approach), and with sampling used as the basic restriction operator: either “pure sampling” (thus with some aliasing) or “regularized sampling” (using discrete filtering to mitigate the aliasing effects). Results were already presented in [122]. Similar work for such “LES without filtering”, and the related dynamic procedure, was proposed in Knaepen et al. [68]: they investigated it using a pseudo-spectral code (collocated approach) and mimicking FD behaviour. The procedure is really quite simple, and is designed to be self-consistent with the numerics (whatever those are). In the case of FD, it means:

1. Further restrict the LES field, to obtain the discrete field on the coarser grid:

$$\vec{u}_{2h} = R_{2h}(\vec{u}_h) \quad (3.34)$$

2. Evaluate the discrete forces at both levels and using the same numerics:

$$Q_h(\vec{u}_h) \quad \text{and} \quad Q_{2h}(\vec{u}_{2h}) \quad (3.35)$$

3. The equivalent of a “Germano identity” involving the forces is then obtained; it corresponds to the following statement: *the restriction of the discrete forces (evaluated using the discrete field) should be equivalent to the discrete forces evaluated using the restricted field:*

$$R_{2h}(Q_h(\vec{u}_h)) = Q_{2h}(\vec{u}_{2h}) . \quad (3.36)$$

4. To obtain the best coefficient(s) in the SGS model, the square of the error on this identity is averaged over the homogeneous direction(s) and is minimized.

Consider, for instance, using the Smagorinsky SGS model:

$$\nabla_h \cdot \tau_h^{sgs} = \nabla_h \cdot \left(2 C h^2 |\vec{S}_h| \vec{S}_h \right) . \quad (3.37)$$

The error vector on the identity is then:

$$\begin{aligned}
\vec{e} &= [R_{2h}(\nabla_h \cdot (\vec{u}_h \vec{u}_h)) - \nabla_{2h} \cdot (\vec{u}_{2h} \vec{u}_{2h})] \\
&\quad - \left[R_{2h} \left(\nabla_h \cdot \left(2\nu \vec{S}_h \right) \right) - \nabla_{2h} \cdot \left(2\nu \vec{S}_{2h}^* \right) \right] \\
&\quad - (C h^2) \left[R_{2h} \left(\nabla_h \cdot \left(2|\vec{S}_h| \vec{S}_h \right) \right) - \nabla_{2h} \cdot \left(8|\vec{S}_{2h}^*| \vec{S}_{2h}^* \right) \right] \\
&\stackrel{\text{def}}{=} \vec{a} - (C h^2) \vec{b}.
\end{aligned} \tag{3.38}$$

Averaging the error squared over the homogeneous direction(s)

$$\langle \vec{e} \cdot \vec{e} \rangle = \langle \vec{a} \cdot \vec{a} \rangle - 2(C h^2) \langle \vec{a} \cdot \vec{b} \rangle + (C h^2)^2 \langle \vec{b} \cdot \vec{b} \rangle, \tag{3.39}$$

and minimizing gives $(C h^2) = \frac{\langle \vec{a} \cdot \vec{b} \rangle}{\langle \vec{b} \cdot \vec{b} \rangle}$. Notice that, since (to make it simpler and cheaper) we do not enforce that $\vec{u}_{2h}$ be divergence free (i.e., we avoid solving the Poisson equation for P_{2h} required to project it), we have to use the deviatoric part of the tensor: $\vec{S}_{2h}^*$. In most LES, one can also neglect the molecular viscous term contribution in the error minimization, as it is small compared to the SGS contribution. So far, this is the approach followed here. Recall also that this term is linear and thus has no contribution in the classical spectral-based dynamic procedure. When using the staggered approach for incompressible flows (as here, with FD), an intermediate interpolation is required as part of the “global restriction step”, see Section 3.4.2. For the present validations, the procedure is investigated using FD and the Smagorinsky SGS model. Two cases are investigated: LES of decaying incompressible isotropic turbulence, using the staggered approach and fourth order FD and LES of channel flow, using the same code. Reference solutions are those obtained using a pseudo-spectral method (DNS and LES with the spectral-based dynamic procedure: with restriction to grid $2h$ done using a sharp Fourier cutoff).

3.4.2 Decaying incompressible isotropic turbulence, using 4th order FD

We consider next LES of decaying incompressible isotropic turbulence. Our code simulates unsteady incompressible flows, using the staggered approach (MAC), 4th order FD, the convection scheme of Vasilyev [114] (which conserves energy), the fractional step method of Choi and Moin, explicit convection and SGS terms (Adams-Bashforth), and the choice between explicit or implicit molecular diffusion (in case of implicit, Crank-Nicholson with ADI). It is multi-block and parallel, and it uses a Multigrid Poisson solver. Due to the staggered approach, the restriction operation from grid h to grid $2h$ requires an intermediate interpolation step, see Fig. 3.12. For instance, to obtain the u_{2h} , the interpolation only uses the u_h values that lie in the same plane as that of grid $2h$; the values on the planes at $-h$ and h are not used. This ensures that the global step (interpolation + sampling) is indeed a good restriction.

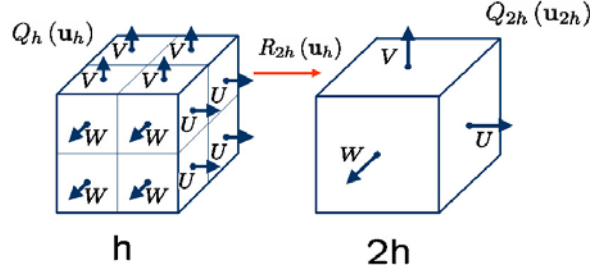


Figure 3.12: Schematic of the restriction operation on a 3D MAC cell.

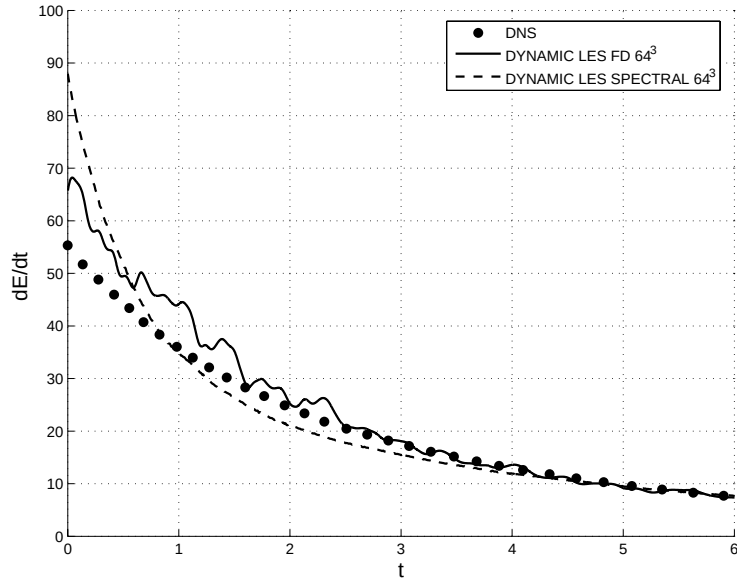


Figure 3.13: Evolution of total dissipation for 64^3 LES of decaying isotropic turbulence (reference DNS is spectral 256^3).

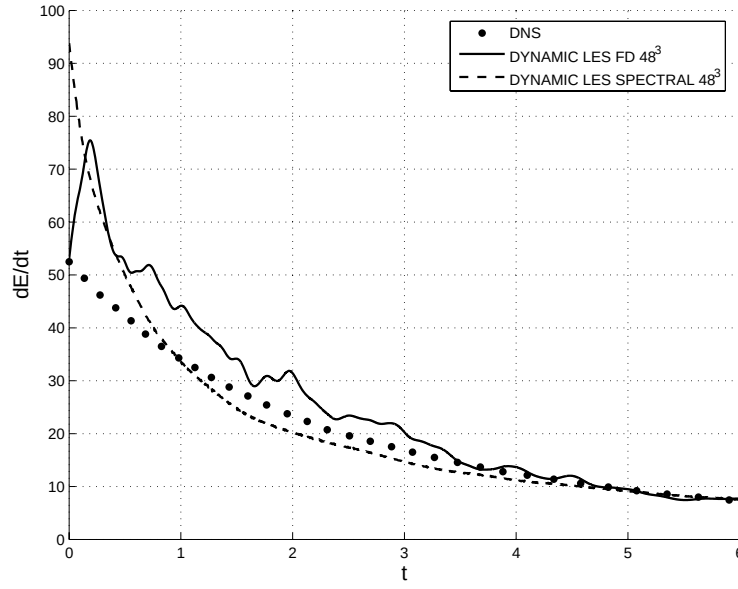


Figure 3.14: Evolution of total dissipation for 48^3 LES of decaying isotropic turbulence (reference DNS is spectral 256^3).

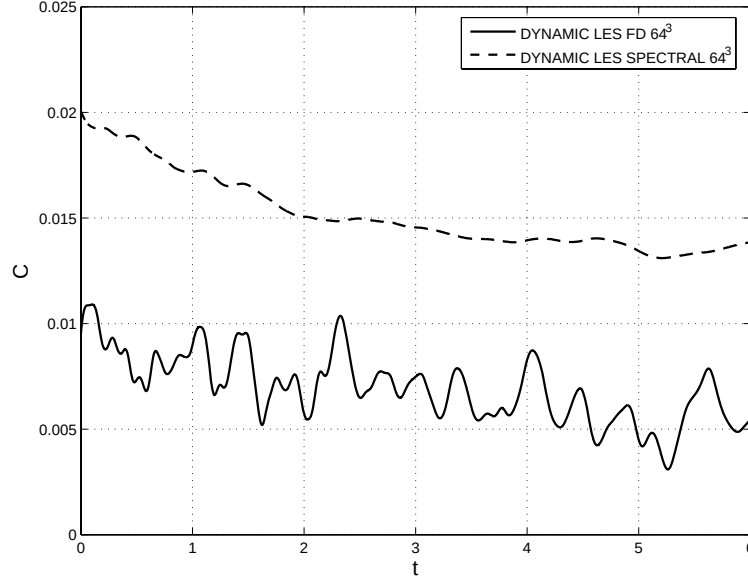


Figure 3.15: Evolution of the dynamic coefficient for 64^3 LES of decaying isotropic turbulence (reference DNS is spectral 256^3).

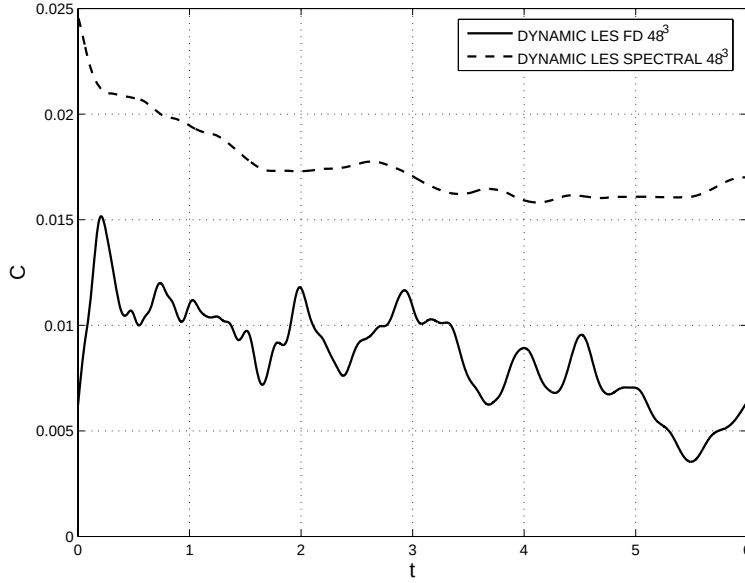


Figure 3.16: Evolution of the dynamic coefficient for 48^3 LES of decaying isotropic turbulence (reference DNS is spectral 256^3).

We consider a 256^3 de-aliased pseudo-spectral DNS, projected to 64^3 (easy LES) and 48^3 (more challenging), and then run further in time using the Smagorinsky model (same cases as those considered in [119]). Comparisons are made with dynamic LES using the de-aliased pseudo-spectral method and with the dynamic procedure based on sharp spectral cutoff. It is seen, on Figs. 3.13 and 3.14, that both dynamic LES have too much dissipation initially: a long recognized weakness of the dynamic procedure when LES is started from restricted DNS data. Apart from that, the dissipation histories of the two LES compare well with each other. The sampling based dynamic procedure also performs well. The only noticeable difference in the results are the more abrupt variations observed in the histories (dissipation $\frac{dE}{dt}$ and dynamic C) in the case of FD: this was expected as the present procedure is theoretically not as pure as the spectral-based procedure. Of course, the history of the resolved energy ($E(t)$, not shown) is smooth in both cases.

3.4.3 Incompressible channel flow, using 4th order FD

There remains to test the proposed approach on a flow with walls. LES of the channel flow at $Re_\tau = \frac{u_\tau H}{\nu} = 395$ is thus considered next, the reference being the DNS of Moser et al. [84]. The computational domain is $2\pi H \times 2H \times \pi H$ in respectively streamwise, normal, and spanwise directions. The FD grid used for LES is quite coarse: $64 \times 48 \times 48$, thus $\Delta x^+ = 38.8$ and $\Delta z^+ = 25.8$. The

same stretching law as the DNS detailed in Chap. 2, Eq. (2.40) is used in y :

$$\frac{y}{H} = 1 + \frac{\tanh(\alpha(\zeta - 1))}{\tanh(\alpha)} \quad (3.40)$$

with $\zeta = y = 0$ at the lower wall and $\zeta = y/H = 2$ at the upper wall, and with the stretching parameter set to $\alpha = 2.75$ (which gives $\Delta y_w^+ = 0.8$ at the wall and $\Delta y_c^+ = 45.4$ at the center). LES results are presented in Fig. 3.18. So far, the velocity profile is underpredicted compared to the DNS. Examining the dynamic $C\Delta^2$ curve, we also see that it is zero up to $y^+ \approx 40$: too far. Notice that clipping was also used (whenever the value produced by the dynamic procedure is negative, it is set to zero). Away from the wall, the order of magnitude of $C\Delta^2$ is found to be in agreement with that obtained for dynamic and spectral LES using the same grid, see [61]. Notice that Morinishi and Vasilyev [83] tested the dynamic procedure for filtering-based LES and with the Germano identity expressed at the force (vector) level. They also observed the same excessive near-wall clipping behaviour. They were able to cure this problem, and thus improve the results, by taking into account the y variation of $C\Delta^2$ in the minimization procedure. As it is not clear whether this procedure can be used safely for wall bounded flows, and considering the fact that the numerous interpolations and the evaluation of the different terms are costly, it is difficult to apply it to perform LES on large grids to solve problems as the one presented in the chapters 4 and 5. Indeed, in these cases dealing, with all the terms on two large grids would be unaffordable. This is the reason why this procedure was not investigated further in the scope of the present work.

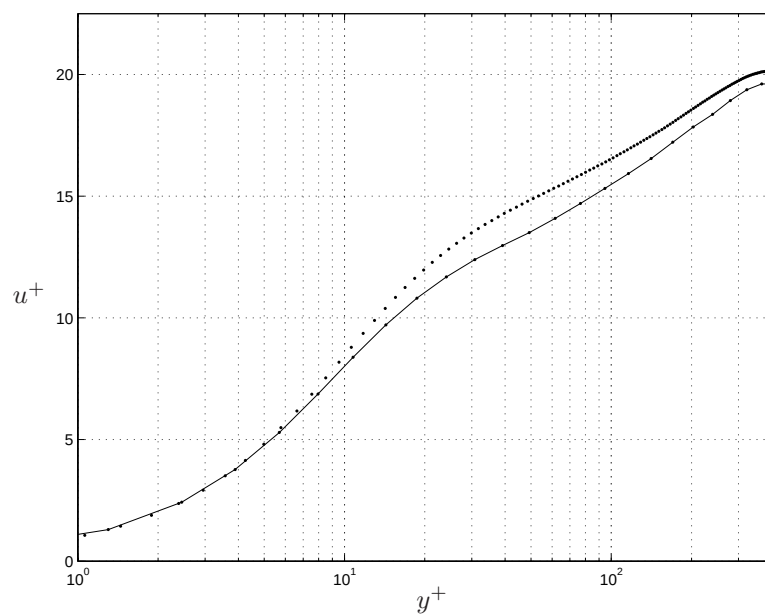


Figure 3.17: LES of channel flow at $Re_\tau = 395$: averaged velocity profile. DNS profile shown in bullets

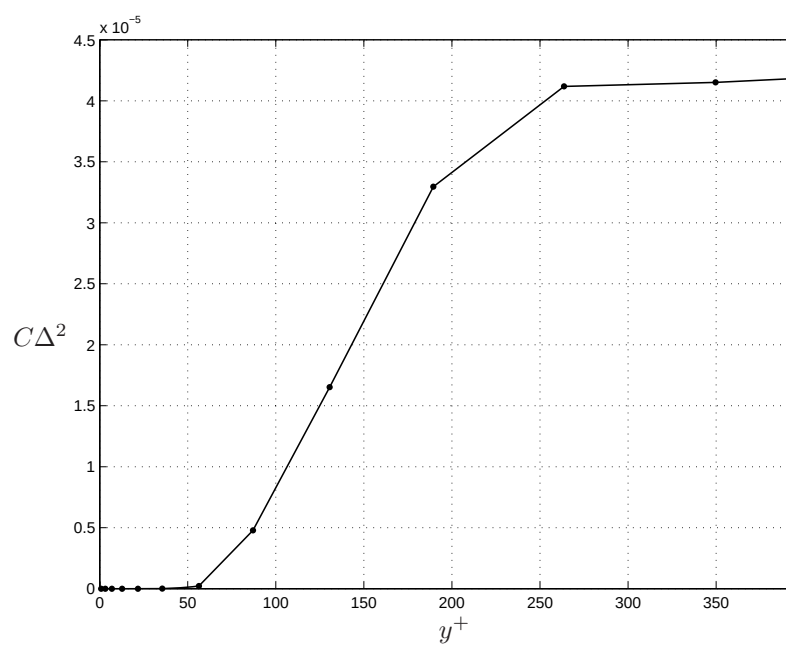


Figure 3.18: LES of channel flow at $Re_\tau = 395$: $C \Delta^2$ (right).

3.5 High order hyper-viscosity model

Hyper-viscosity (HV) based models are usually used in spectral simulations. They are easy to implement and cost almost the same as the DNS in terms of memory and CPU. In the literature, hyper-viscosity based modelling is also seen as a DNS of hyper-viscous Navier Stokes equations, as mentioned in Lamorgese et al. [76]. Usually, when hyper-viscosity is used, the molecular viscosity is set deliberately to $\nu = 0$, it is used as a model for “quasi-Euler” simulations [6]. The hyper-viscosity dissipation affects only the very high wavenumbers. It is the reason why the energy spectra fall in a very sharp way at the highest wavenumbers. In general, the hyper-viscosity model is defined in physical space as:

$$\tau_{ij}^M = (-1)^p \nabla^{2p} (2\nu_h S_{ij}) . \quad (3.41)$$

If one uses a constant ν_h (i.e., a linear model) where

$$\nu_h = \frac{1}{T_0} C^{(p)} h^{(2p+1)} , \quad (3.42)$$

T_0 being a global time scale, one obtains in spectral space:

$$\widehat{\nabla \cdot \boldsymbol{\tau}^M}(\mathbf{k}) = -C^{(p)} \frac{1}{T_0} (|\mathbf{k}|h)^{2(p+1)} \widehat{\mathbf{u}}(\mathbf{k}) . \quad (3.43)$$

When $p = 0$, this amounts to a classical eddy viscosity model with constant scaling instead of using $(2 S_{ij} S_{ij})^{1/2}$. Usually, higher values are chosen for the parameter p . Lamorgese et al. [76] performed studies with $p = 0, 1$ and 7 . In the remaining of this work we use $p = 7$. The energy dissipation due to the SGS model can be evaluated from the energy spectra:

$$\epsilon_h(t) = 2\nu_h \int_0^\infty k^{2(p+1)} E(k, t) dk . \quad (3.44)$$

Thus, for high p , the biggest contribution to the dissipation only comes from the very high wavenumbers. The way to define the effective dissipation scale is the following:

$$\eta_h = \left(\frac{\nu_h^3}{\epsilon_h} \right)^{\frac{1}{6p+4}} . \quad (3.45)$$

Results obtained with this model on homogeneous turbulence are presented in Sections 3.8 and 4.2.

3.6 The Chollet-Lesieur spectral eddy viscosity model

Chollet and Lesieur proposed an eddy viscosity model defined in Fourier space [20], [69]. The Chollet-Lesieur model produces negligible eddy-viscosity as long as there is no energy near the cut-off. This model requires to know the value of $E(k_c)$ at each time step. By choosing $\sqrt{k_c E(k_c)}$ as velocity scale and k_c^{-1} as length scale, one can write the spectral eddy-viscosity:

$$\nu_e = \nu_e^+ \sqrt{\frac{E(k_c, t)}{k_c}}$$

This requires the definition of a dimensionless function of k/k_c :

$$\nu_e^+ = 0.441 C_K^{-\frac{3}{2}} \left[1 + 34.5 \exp \left(-3.03 \frac{k_c}{k} \right) \right]$$

where $C_K \approx 1.6$ is the Kolmogorov constant. As illustrated in Fig. 3.19, the spectral eddy-viscosity function starts with a plateau for low wavenumbers and rises to a finite “cusp” close to the cutoff wavenumber. This model is designed as to have a spectral $k^{-5/3}$ behaviour over the whole range. The SGS

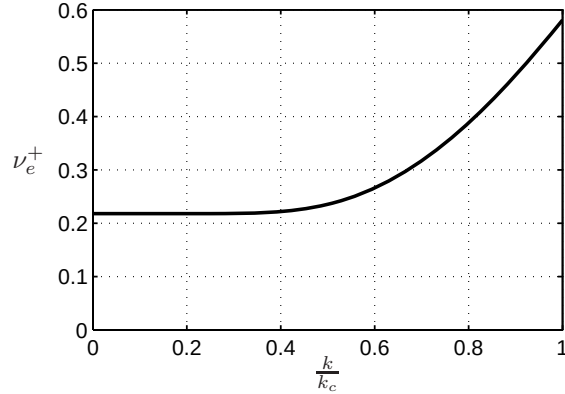


Figure 3.19: Dimensionless spectral eddy viscosity function

dissipation is obtained as:

$$\epsilon^{sgs}(k) = 2 \left[\nu_e^+ \sqrt{\frac{E(k_c, t)}{k_c}} \right] k^2 E(k). \quad (3.46)$$

3.7 Multiscale models

Eddy-viscosity models can be classified in two sets, according to the spectral content of the field on which they are acting:

- *Type I* models (“x-complete” models): models acting on the complete LES field. The SGS stress is then modelled using $\tau_{ij}^M = 2\nu_{\text{sgs}}S_{ij}$ with $S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ the strain rate tensor. For the vorticity formulation presented in appendix D, Eq. (D.1) and used in section 3.8, we use $T_{ij}^M = 2\nu_{\text{sgs}}Q_{ij}$ with $Q_{ij} = \frac{1}{2}\left(\frac{\partial \omega_i}{\partial x_j} + \frac{\partial \omega_j}{\partial x_i}\right)$.
- *Type II* models (“x-small” models): models acting on a “small-scale” LES field. The SGS stress is then modelled using $\tau_{ij} = 2\nu_{\text{sgs}}S_{ij}^s$ with S_{ij}^s the strain rate tensor of the small scale field. For the vorticity formulation, we use Q_{ij}^s .

As to the effective viscosity, ν_{sgs} , it is itself either evaluated using the complete LES field (“x” is then replaced by “complete”) or it is evaluated using the small-scale LES field (“x” is then replaced by “small”). The evaluation also requires the effective LES cut-off length: we here use, for all models, the grid size Δ (also uniform as we consider HIT). This choice is appropriate since any ratio between cut-off length and grid size can be put into the C coefficient.

Because of homogeneity and isotropy, the “x” choice for the effective viscosity scaling really does have much influence in HIT: the “complete-complete” and the “small-complete” models (of course each with its appropriate C coefficient) are indeed verified to lead to similar results (global and spectrum). The same applies when comparing the “complete-small” and the “small-small” multiscale models in HIT.

Of course, this is not true in more complex flows, such as transitional flows: then the “small-complete” models clearly outperform the “complete-complete” models. However, we stress that this is not the scope of this paper: we here investigate the “spectral response” of SGS models in fully developed turbulent flows.

As to the small-scale LES field, it is here obtained using the compact (stencil 3) tensor-product second order discrete filter, and that can also be iterated n times to produce a higher order (order $2n$) filter [118, 60, 62]:

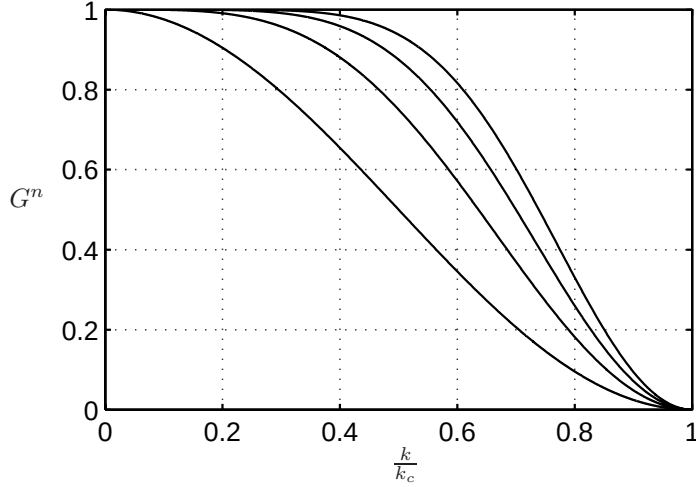
$$\mathbf{u}^{s(n)} = \mathbf{u} - \bar{\mathbf{u}}^{(n)} \quad (3.47)$$

with

$$\bar{\mathbf{u}}^{(n)} = \begin{bmatrix} \left(I - (-\delta_x^2/4)^n\right) \\ \left(I - (-\delta_y^2/4)^n\right) \\ \left(I - (-\delta_z^2/4)^n\right) \end{bmatrix} \mathbf{u}$$

Table 3.1: Classification scheme for multiscale models

	ν_{sgs} evaluated on	S_{ij} evaluated on	SGS model
<i>Type I</i>	Complete field	Complete field	$\tau_{ij}^M = 2 \nu_{\text{sgs}}(\mathbf{u}) S_{ij}$
	Small field	Complete field	$\tau_{ij}^M = 2 \nu_{\text{sgs}}(\mathbf{u}^s) S_{ij}$
<i>Type II</i>	Complete field	Small field	$\tau_{ij}^M = 2 \nu_{\text{sgs}}(\mathbf{u}) S_{ij}^s$
	Small field	Small field	$\tau_{ij}^M = 2 \nu_{\text{sgs}}(\mathbf{u}^s) S_{ij}^s$

**Figure 3.20:** Filter transfer function G^n , from left to right $n = 1, 2, 3, 4$.

where $\delta_x^2 f_{i,j,k} = f_{i+1,j,k} - 2f_{i,j,k} + f_{i-1,j,k}$, etc. In Fourier space, the obtained filtered field is then

$$\begin{aligned}
 \bar{\mathbf{u}}^{(n)}(\mathbf{k}) &= G^{(n)}(k_x h) G^{(n)}(k_y h) G^{(n)}(k_z h) \mathbf{u}(\mathbf{k}) \\
 &= (1 - \sin^{2n}(k_x \Delta/2)) \\
 &\quad (1 - \sin^{2n}(k_y \Delta/2)) \\
 &\quad (1 - \sin^{2n}(k_z \Delta/2)) \mathbf{u}(\mathbf{k})
 \end{aligned} \tag{3.48}$$

The filter transfer function $G^{(n)}$ is plotted for different values of n in Fig. 3.20.

3.7.1 *Type I* models

We begin with the “x-complete” models, thus where the SGS viscosity is applied to the complete field ($\mathbf{u}$ or $\boldsymbol{\omega}$ depending on the formulation chosen for the

Navier-Stokes equations). The classical Smagorinsky (SMAG) model is the most common model. We also use it as baseline for comparisons. It uses

$$\nu_{\text{sgs}} = C_S \Delta^2 (2 S_{ij} S_{ij})^{1/2} , \quad (3.49)$$

and is thus a “complete-complete” model. The discussion about the C_S value (and also about each coefficient associated to the SGS models presented after) can be found in [22].

We also recall that the coefficient C_S can be obtained dynamically, using the “dynamic procedure” [49, 50]: this is then the “dynamic” version of the Smagorinsky model. We stress that any SGS model can be made dynamic (see, e.g., a dynamic “mixed model” in [119]). In HIT, the dynamically determined model coefficient does not vary in space, hence the dynamic procedure does not change the spectral behavior of the SGS model. Since the present investigation focuses on calibration and on studying the spectral behavior of SGS models, we here do not use the dynamic procedure on purpose.

A second model considered here is the Filtered Smagorinsky (SMAG_s) model: the “small-complete” version of the Smagorinsky model, thus using

$$\nu_{\text{sgs}} = C_{S_s}^{(n)} \Delta^2 (2 S_{ij}^s S_{ij}^s)^{1/2} , \quad (3.50)$$

where S_{ij}^s is the strain rate based on the small-scale field.

A third model considered here (also a “small-complete” model) is the FSF model [32]. Here, the effective viscosity is obtained from the “filtered structure function” $F_2^{s(n)}$:

$$\nu_{\text{sgs}} = C_F^{(n)} \Delta \sqrt{F_2^{s(n)}} , \quad (3.51)$$

where

$$F_2^{s(n)} = \left\langle \|\mathbf{u}^{s(n)}(\mathbf{x} + \mathbf{x}') - \mathbf{u}^{s(n)}(\mathbf{x})\|^2 \right\rangle_{|\mathbf{x}'| = \Delta} . \quad (3.52)$$

Numerically, this is evaluated using the nearest neighbors ($3^3 = 27$) values. Furthermore, and to follow [32] exactly, the small-scale field $\mathbf{u}^{s(n)}$ is here obtained (and also for $F_2^{s(n)}$ in the FSF_s model, see later) by recursive application (n times) of the 2nd order FD Laplacian:

$$\mathbf{u}^{s(n)} = ((-\Delta^2/4) \nabla^2)^n \mathbf{u} . \quad (3.53)$$

Such “small-complete” models have indeed been shown to capture well the transition to turbulent flow (e.g., the turbulent flow that results from the growth of instabilities in wake vortex simulations [71, 23]). Of course, in HIT, they perform no better than the Smagorinsky model.

3.7.2 Type II models

We now consider the “x-small” models, thus the true multiscale models, where the effective viscosity is applied to the small-scale LES field ($\mathbf{u}^s$ or ω^s depending on the formulation chosen).

The first model considered is a “complete-small” model: the “regularized” version (here using the tensor product discrete filter) of the “variational multiscale” model (VM) of Hughes et al. [56, 57]. It was first proposed and tested, in HIT, by Winckelmans and Jeanmart [118] who called it the “regularized variational multiscale” (RVM) model. It was then further tested in [60, 62] in HIT, and also in channel flow using the dynamic procedure (i.e., a “dynamic RVM model”). The RVM model was also further proposed and tested by [115] and also by Stolz et al. [105, 104]. In the present work, we use

$$\nu_{\text{sgs}} = C_R^{(n)} \Delta^2 (2 S_{ij} S_{ij})^{1/2} . \quad (3.54)$$

The second model considered is the regularized version of the “small-small” variational multiscale model of [56, 57]. It is here noted RVM_s. Such regularized version was proposed and evaluated by [115] and by [105, 104]. We here use

$$\nu_{\text{sgs}} = C_{R_s}^{(n)} \Delta^2 (2 S_{ij}^s S_{ij}^s)^{1/2} . \quad (3.55)$$

The coefficient values are $C_{R_s}^{(1)} = 0.066$ and $C_{R_s}^{(3)} = 0.11$.

Another model is also considered: the “small-small” version of the filtered structure function model, as proposed by [104]. It is here noted FSF_s model, and we use

$$\nu_{\text{sgs}} = C_{F_s}^{(n)} \Delta \sqrt{F_2^{s(n)}} . \quad (3.56)$$

3.8 Asymptotic behaviour of SGS models.

The content of this section is the result of a common work performed with R. Cocle and G. Winckelmans. This work was already partially presented at the TSFP5⁴ 2007, see [12], and will appear in the proceedings of the conference QLES 2007 that will be published by Springer as part of the ERCOFTAC Series, in a book entitled “Quality and Reliability of Large-Eddy Simulations.”

3.8.1 Introduction

We investigate LES of decaying HIT in the limit of very high Reynolds number and for LES cutting well into the inertial range (i.e., LES where $\Delta/\eta > 20$). The molecular viscosity dissipation being negligible compared to the SGS dissipation, the LES are run with ν set to zero. The LES grid is also taken large enough so as to capture the true asymptotic spectral behavior of each model (see later).

The results presented were computed using the VIC (see Cocle [22] and Appendix D) and the PS codes.

The initial condition is build following the algorithm described by Rogallo in [99]. This initial field is isotropic, divergence free and has a prescribed spectrum. We describe in Appendix C the way to build it. This initial field then evolved using LES. For the investigations using the VIC code, the following SGS models are investigated: SMAG, FSF (with $n = 1$) and RVM (with $n = 1$ and $n = 3$); RVM_s and FSF_s. For the sake of comparison, simulations were also carried out using a dealiased PS method. The models investigated are the SMAG model, the RVM_s (with $n = 3$) model and the high order HV model. The asymptotic value of each model’s coefficient was obtained by Cocle [22].

3.8.2 Results

From grid turbulence experiments (e.g., Comte-Bellot and Corrsin [25]) and from theory, we know that, after the transient required to obtain a “statistically converged turbulence”, the kinetic energy evolves as a power law:

$$\frac{E(t)}{E_0} = \left(\frac{t}{t_0} \right)^{-p} \quad (3.57)$$

where the value of the decay exponent p is found between 1.15 and 1.45 in the literature. Mohamed and LaRue [80] suggest that a good value is $p = 1.30$. Fig. 3.21 shows the energy evolution, using a 128^3 grid (both with the VIC code and the PS code). For the RVM model ($n = 1$ and $n = 3$), the obtained exponent is $p \approx 1.38$: well into the range of admissible values. The other *Type II* models (i.e. RVM_s and FSF_s) give the same energy decay exponent as the

⁴Turbulence and Shear Flow Phenomena

RVM model. The same results hold when using the spectral code with the RVM_s model, and even with the high order HV model (the exponent is then found as $p \approx 1.37$).

A significant difference is however noticed with the *Type I* models (here the SMAG, FSF and SMAG_s models) that act on the whole field. As they produce spectra that are not as good as the one produced by the *Type II* models, this also influences the obtained p value: we obtain $p = 1.49$ for the SMAG model (equivalently for both codes) and $p = 1.45$ for the FSF model (also for the SMAG_s model, not shown). Clearly, the 1.49 value obtained with the “complete-complete” model (SMAG) is out of acceptable range. That obtained with the “small-complete” models (FSF and SMAG_s) is at the border of the acceptable range: already better but still not as good as the *Type II* models that act on the small-scale field (RVM, RVM_s, FSF_s).

Fig. 3.22 shows the obtained energy spectra for a 128^3 grid (also time-averaged between $t = 10$ and 40 , when the turbulence is statistically converged). To better highlight the inertial range in the spectra, we also show the compensated energy spectra on Fig. 3.23. We observe that the *Type II* models with $n = 3$ provide the broader inertial range (i.e., with $k^{-5/3}$ behavior). The width of this captured inertial range decreases with the order of the filter used to obtain the small-scale field (compare RVM with $n = 1$ to RVM with $n = 3$). In opposition, the SMAG and the FSF models do not exhibit any inertial range on such a 128^3 grid. The same conclusion holds for the SMAG_s model, even though not shown here. Nevertheless, all spectra are clearly altered by a “bump-like” behavior at the medium to high wavenumbers, though to a lesser extent for the *Type II* models using high order ($n = 3$) filters. This non-physical energy accumulation is present because the shape of the dissipation spectra of the SGS models is never exactly the required one. Indeed, since any SGS model must dissipate, it can never lead to an inertial range obtained over all medium to high wavenumbers of the LES (recall that an inertial range corresponds to an energy cascade without dissipation). Hence, one always obtains the “bottleneck” effect with energy accumulation (bump) followed by dissipation. However, the behavior is much improved when the *Type II* models are used. As they act only at the small scales, their influence on the large scales is reduced and allows to capture a $k^{-5/3}$ inertial range on part of the LES.

This conclusion is also valid for the spectrum obtained using the high order HV model: it provides an inertial range as broad as the *Type II* models with $n = 3$ (even a bit broader), yet the “bump” is more pronounced. Our result compares well with [76] who also shows the presence of the “bottleneck” effect in the energy spectra when a high order HV model is used.

An argument that can partially explain the presence of this bottleneck is that a LES is always locally under-resolved with respect to its own SGS viscosity! We can illustrate this concept using the SMAG model. Consider first the cell Reynolds number based on the SGS viscosity:

$$Re_{\Delta} = \frac{|S| \Delta^2}{\nu_{\text{sgs}}} = \frac{1}{C_S} = \frac{1}{0.027} \approx 37.$$

This value is too high to meet the quality criterion of a locally well-resolved simulation. We can also evaluate the $k_c \eta_{\text{sgs}}$ criterion based on a pseudo Kolmogorov length scale η_{sgs} . The SGS dissipation is then:

$$\epsilon_{\text{sgs}} = \langle 2 \nu_{\text{sgs}} S_{ij} S_{ij} \rangle = C_S \Delta^2 \langle (2 S_{ij} S_{ij})^{3/2} \rangle$$

leading to

$$\begin{aligned} \eta_{\text{sgs}} &= \left(\frac{\nu_{\text{sgs}}^3}{\epsilon_{\text{sgs}}} \right)^{1/4} = \left(\frac{(C_S \Delta^2 (2 S_{ij} S_{ij})^{1/2})^3}{C_S \Delta^2 \langle (2 S_{ij} S_{ij})^{3/2} \rangle} \right)^{1/4} \\ &\approx (C_S^2 \Delta^4)^{1/4} \approx 0.16 \Delta \end{aligned}$$

and thus

$$k_c \eta_{\text{sgs}} = \frac{\pi}{\Delta} 0.16 \Delta \approx 0.50.$$

We confirm that the obtained value for this criterion is too small to consider a well-resolved computation (which requires 1.50 or more). It thus leads to a local energy accumulation due to the under-resolution, as observed in the previous spectra. If we impose a classical well-resolved criterion of $k_c \eta_{\text{sgs}} = 1.50$, we have to take a SMAG coefficient greater than $9 C_{S,\infty}$. It is shown in [22] that the spectrum obtained when using $10 C_{S,\infty} = 0.27$ is indeed similar to that of a “DNS with a viscosity such that it is well-resolved on that grid”. In particular, no inertial range was observed.

We also investigate the asymptotic behavior of the models when performing LES on larger and larger grids, see Fig. 3.24. We consider the SMAG model, the RVM_s model ($n = 3$) and the high order HV model, and we compare the shape of the spectra obtained using the PS code and higher resolutions (128^3 , 256^3 , and even 512^3 for HV). As in the previous results, we show the unequivocal presence of the “bump-like” behavior. As expected, the $k^{-5/3}$ inertial range becomes broader as the grid is taken larger. Such LES has indeed reached its “asymptotic behavior”, with self-similar obtained spectra: going to an even finer grid will produce an even larger $k^{-5/3}$ inertial range followed by the same bump part; this is verified by running the high order HV model on a 512^3 grid. On Fig. 3.24, we also illustrate that each model produces its own energy spectrum in the dissipation range and that the bump part conserves the same shape when enlarging the grid. Indeed, if we decompose the energy spectrum in the following way,

$$E(k, t) \approx E_0(t) k^{-5/3} \tilde{E}(k/k_c) \quad (3.58)$$

we observe that $\tilde{E}(k/k_c)$ preserves its shape whatever the time or the LES resolution. We also observe that, for the RVM_s model and the high order HV model, the width of the captured inertial range is, in each case, appreciably smaller than the number of modes computed. For the 128^3 , the inertial range extends on $k \approx [2 - 8]$; for the 256^3 , it grows to $k \approx [2 - 20]$ and, for the

512^3 , to $k \approx [2 - 40]$. In opposition, the SMAG model, even with the 256^3 resolution, is not able to capture any significant inertial range.

Finally, we also investigate the behavior of the Chollet-Lesieur spectral eddy viscosity model on 128^3 and 256^3 grids using the PS code. The results are compared with the RVM_s model. Considering the kinetic energy evolution in Fig. 3.27, one can observe that the spectral eddy viscosity approach underpredicts the energy decay for the modest resolution (128^3 grid) while the temporal behavior seems to be better predicted with the finest grid. Examining the compensated spectra in Fig. 3.28 for the 128^3 grid leads to the conclusion that this model is not able to produce the expected $k^{-5/3}$ behavior of the energy spectrum, while the RVM_s model produces it even if its inertial range is narrow. On the finer grid, both models give the $k^{-5/3}$ behavior. The inertial range is broader in the case of the RVM_s model. The spectral eddy viscosity model yields spectra which are also altered by the presence of the bottleneck effect, even if less pronounced than in the case of the RVM_s model. The aimed $k^{-5/3}$ behavior on the whole range of wavenumbers for which the model is designed is not obtained. As this model is defined in Fourier space only, it is not suitable for finite differences, VIC method or finite volumes schemes and complex geometries.

3.8.3 Conclusions

The results obtained by performing LES of HIT in the limit of high Reynolds number allow to highlight the good spectral behavior of the multiscale subgrid models (those acting on a “small” field) even if they clearly suffer from the presence of a “bump-like behavior” in their spectra. This behavior is also observed with the high order HV model; in this case, the bump is even more pronounced while also keeping a significant inertial range. To validate the quality of our conclusions, this analysis was done using two different numerical methods: the VIC method and a pseudo-spectral method. One of the original aspect of this work is that all LES were performed on large grids and at very large Reynolds number, allowing to obtain the asymptotic behavior of each model. The comparisons show that the RVM and RVM_s models perform significantly better than the Smagorinsky model: a much wider inertial range is obtained. It is also found that, with the same SGS model, the VIC code produces spectra that are very similar to those obtained by the reference PS code (see, e.g., the results with the SMAG model in Fig. 3.23). Recall that the VIC code also has very good qualities: essentially no dispersion and very little numerical dissipation (the PS code has none, of course). The issue of the code numerics interacting with the SGS model is thus of lesser importance in VIC than in other numerical methods.

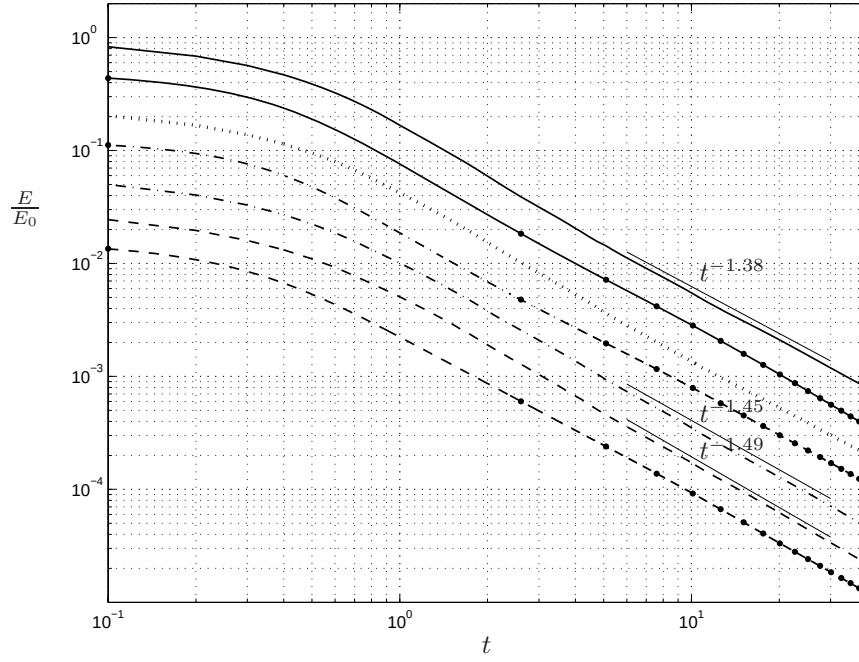


Figure 3.21: 128^3 LES at very high Reynolds number: Evolution of the energy (the curves are shifted vertically by 0.5 to better distinguish them): 1) using the VIC method: SMAG (dash), FSF with $n = 1$ (dash-dot), RVM with $n = 1$ (dot) and RVM with $n = 3$ (solid); 2) using the pseudo-spectral method: SMAG (dash with bullets), HV (dash-dot with bullets) and RVM_s with $n = 3$ (solid with bullets).

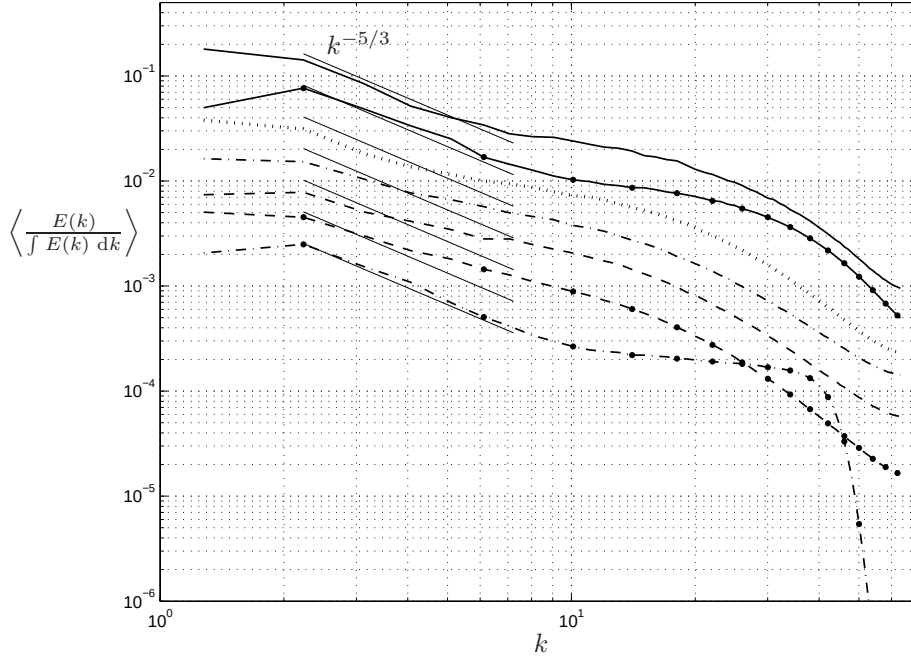


Figure 3.22: 128^3 LES at very high Reynolds number: normalized energy spectra (also time-averaged between $t = 10$ and 40 ; the curves are shifted vertically by 0.5 to better distinguish them): 1) using the VIC method: SMAG (dash), FSF (dash-dot), RVM with $n = 1$ (dot) and RVM with $n = 3$ (solid); 2) using the pseudo-spectral method: SMAG (dash with bullets), HV (dash-dot with bullets) and RVM_s with $n = 3$ (solid with bullets).

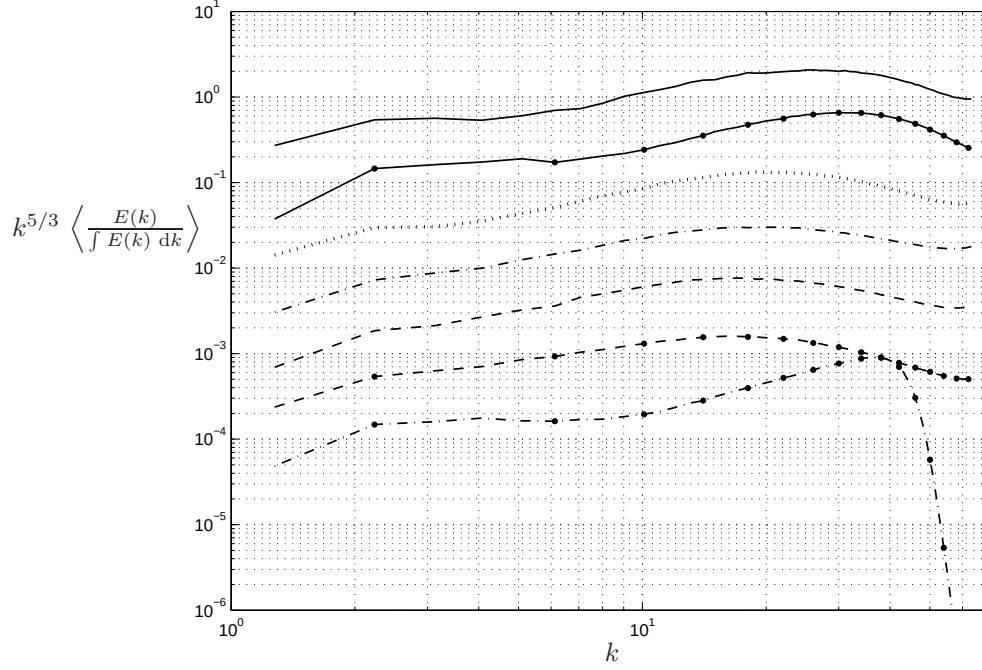


Figure 3.23: 128^3 LES at very high Reynolds number: dimensionless compensated energy spectra (also time-averaged between $t = 10$ and 40 ; the curves are shifted vertically by 0.5 to better distinguish them): 1) using the VIC method: SMAG (dash), FSF (dash-dot), RVM with $n = 1$ (dot) and RVM with $n = 3$ (solid); 2) using the pseudo-spectral method: SMAG (dash with bullets), HV (dash-dot with bullets) and RVM_s with $n = 3$ (solid with bullets).

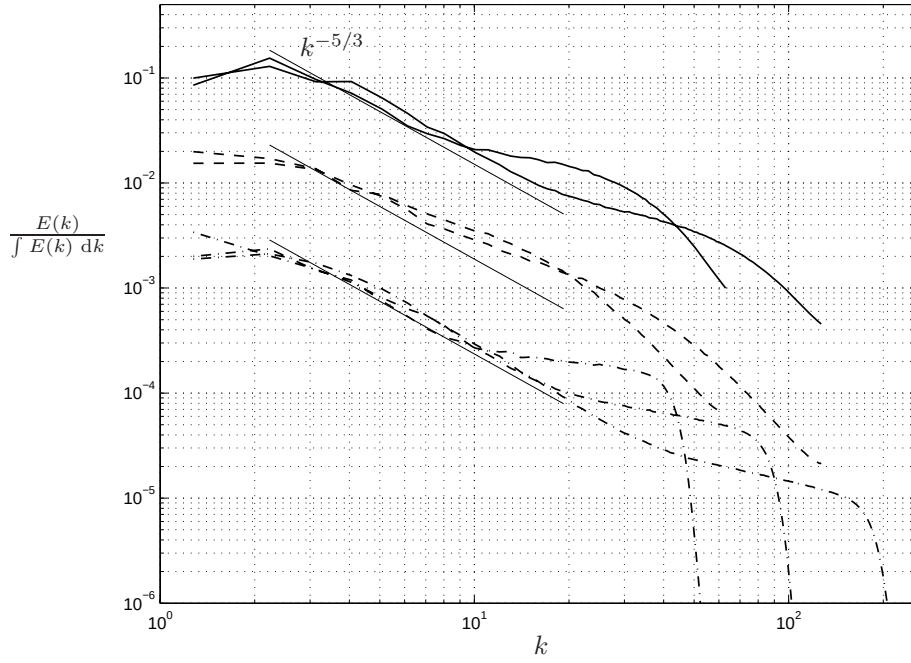


Figure 3.24: LES at very high Reynolds number: Asymptotic behaviour of the energy spectra when using larger grids (pseudo-spectral method at $t = 20$; the curves are shifted vertically by 0.125 to better distinguish them): RVM_s with $n = 3$ on 128^3 and 256^3 grids (solid), SMAG on 128^3 and 256^3 grids (dash), HV on 128^3 , 256^3 and 512^3 grids (dash-dot).

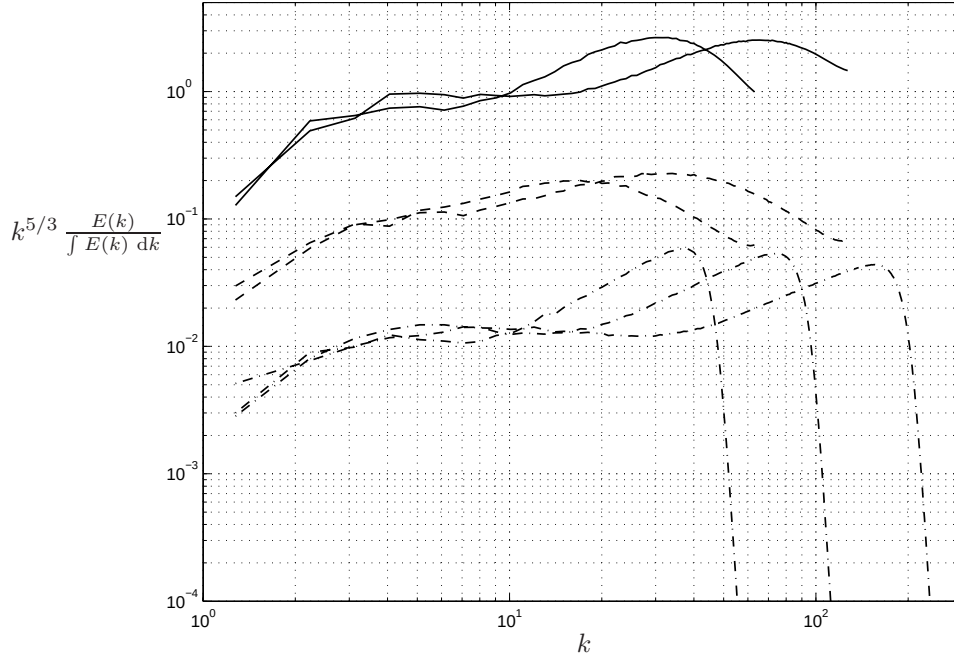


Figure 3.25: LES at very high Reynolds number: Asymptotic behaviour of the compensated energy spectra when using larger grids (pseudo-spectral method at $t = 20$; the curves are shifted vertically by 0.125 to better distinguish them): RVM_s with $n = 3$ on 128^3 and 256^3 grids (solid), SMAG on 128^3 and 256^3 grids (dash), HV on 128^3 , 256^3 and 512^3 grids (dash-dot).

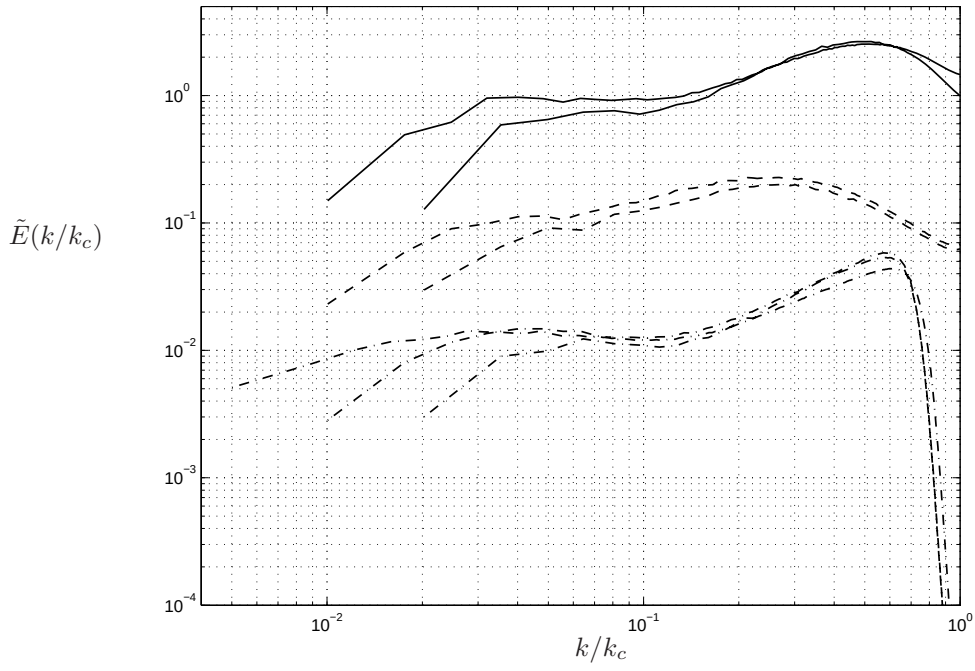


Figure 3.26: LES at very high Reynolds number: Illustration of the shape similarity in the dissipation range (pseudo-spectral method at $t = 20$; the curves are shifted vertically by 0.125 to better distinguish them): RVM_s with $n = 3$ on 128^3 and 256^3 grids (solid), SMAG on 128^3 and 256^3 grids (dash), HV on 128^3 , 256^3 and 512^3 grids (dash-dot).

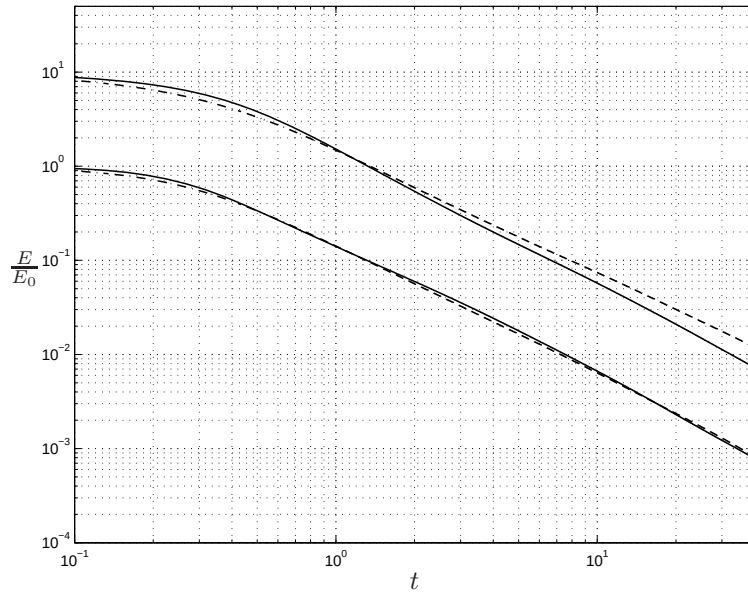


Figure 3.27: LES at very high Reynolds number: Comparison of the energy decay for the SEV model (dash-dot) and the RVM_s model (solid). Curves obtained on the 128^3 grid are shifted up for clarity.

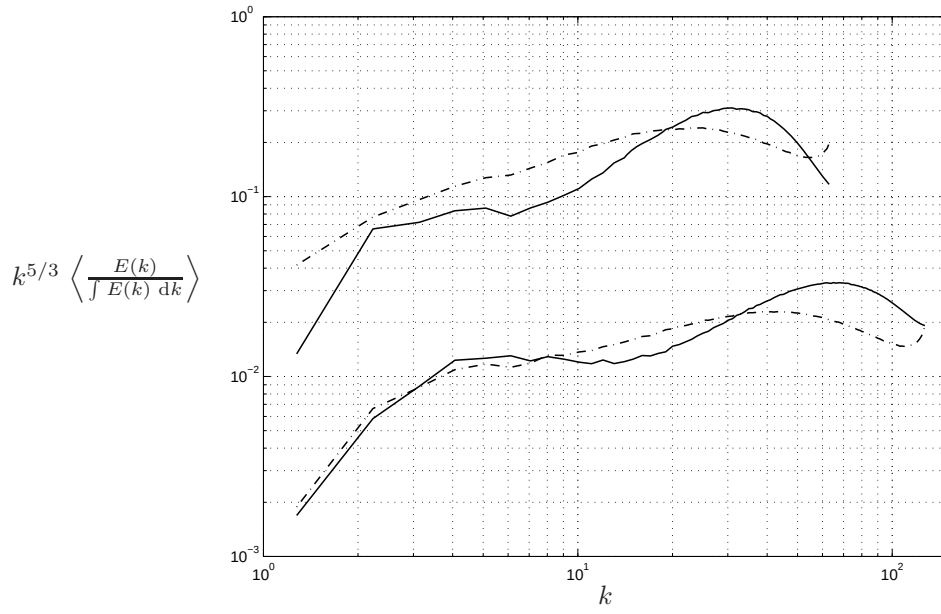


Figure 3.28: LES at very high Reynolds number: Comparison of the normalized and time averaged energy spectra for the SEV model (dash-dot) and the RVM_s model (solid). Grid resolutions are 128^3 and 256^3 . Spectra obtained on the 128^3 grid are shifted up for clarity.

3.9 A new multiscale WALE type model

As was shown previously, the WALE model works well for wall bounded flows, i.e, it provides the correct $y^+{}^3$ behaviour of ν_{sgs} close to the walls. However, this model fails in the presence of vortical structures. On the other hand, it was shown that the multi-scale models (small-small) are suitable for wake vortices simulations [23]. The drawback of the multiscale models is that they produce too high values of the SGS viscosity close to the walls as demonstrated formally and by numerical experiment in [62]. This is because the LES flow is unsteady and anisotropic in the near wall region; hence, there is a significant spatial non-uniformity in the fields at any given time leading to $\mathbf{u}^s(x, 0, z) \neq 0$, implying a nonzero dissipation of the multiscale models in the laminar sublayer. For that purpose, Georges [46], [47] tested the multiscale model with a Piomelli wall damping function. This model provided good results on channel flow and wake vortex systems in ground effect. However, using a damping function requires the evaluation of y^+ , which is not obvious to define in particular for boundary layers which are laminar close to the walls as it is the case for wake vortices in ground effect, see Chapter 5. In this work we propose to develop a model for which ν_{sgs} is computed using the WALE model evaluated on a small scale field and acting on the small scale field. This is thus a “small-small” model with a natural damping, we call it the RVM_s-WALE model.

3.9.1 Definition of the model

The SGS viscosity is computed on the small scale field using the definition of the WALE model:

$$\nu_{sgs} = C \Delta^2 \frac{(\mathcal{S}_{ij}^s \mathcal{S}_{ij}^d)^{3/2}}{(S_{ij}^s S_{ij}^s)^{5/2} + \alpha (\mathcal{S}_{ij}^s \mathcal{S}_{ij}^d)^{5/4}}$$

The SGS stress tensor is written:

$$\tau_{ij}^M = 2 \nu_{sgs} S_{ij}^s.$$

3.9.2 Results for the channel flow

The LES of the turbulent channel flow at $Re_\tau = 395$ was performed in order to validate and calibrate the model for wall-bounded flows. The wall behaviour of ν_{sgs} is displayed in Fig. 3.29. This one conforms a bit less with the theory than the one obtained with the pure WALE model displayed in Fig. 3.5. This is due to the fact that ν_{sgs} is evaluated on the small scale field, to avoid having to store both S_{ij} and S_{ij}^s . However this model offers however a natural damping of the SGS viscosity at the wall with a behaviour close to the theoretical one. Fig. 3.30 illustrates that the coefficient which yields the better mean velocity profile and, thus, the correct amount of dissipation is $C_W^{(1)} = 0.56$. The optimal velocity profile is displayed in Fig. 3.31 and the shear stress decomposition is

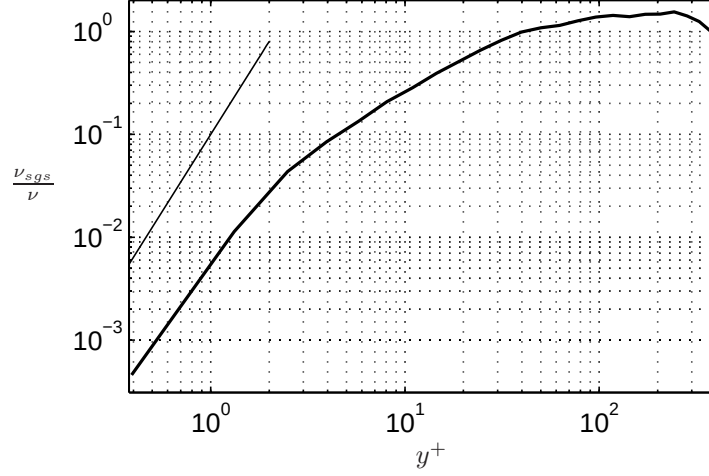


Figure 3.29: Normalized SGS viscosity for the channel flow at $Re_\tau = 395$, LES with the RVM_s - WALE model. The thin straight line corresponds to a y^{+3} behaviour.

shown in Fig. 3.32. It is important to note that in this case, the contribution of the SGS model to the mean total shear stress is very weak. This is due to the fact that the model is acting on small scales with a SGS viscosity evaluated on the small-scale field:

$$\tau_{12}^M = 2\nu_{sgs}(\mathbf{u}^s)S_{12}^s. \quad (3.59)$$

There is no reason to have an important time averaged profile on this quantity which is a major difference with the models acting on the complete field, for which S_{12} has a non negligible averaged profile. It can also be observed that the peak value of the resolved $\langle u'v' \rangle$ is higher than what is usually found with classical models (e.g., WALE model). This is a good feature of this model, which is able to recover the shear stress balance with only a minor average contribution of the model. The reduced turbulence intensities are also displayed in Figs. 3.33, 3.34, 3.35. The conclusion drawn for the WALE model stays valid here.

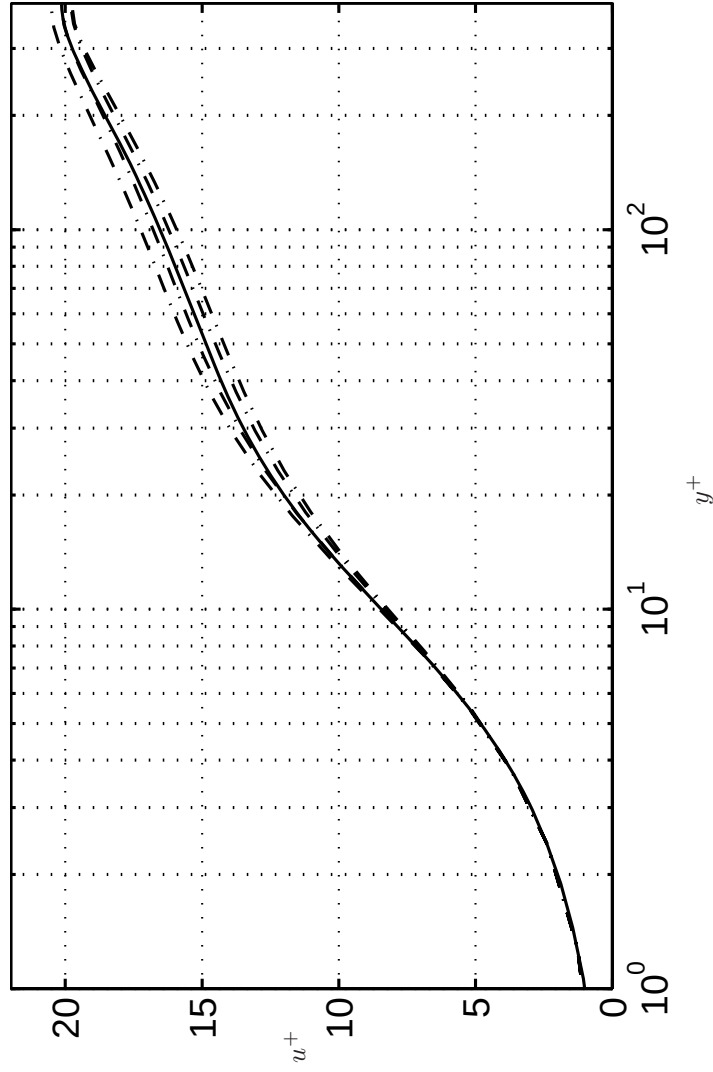


Figure 3.30: Comparison of the mean velocity profiles. LES using RVM_s -WALE model (dash-dot) for the 4 coefficients investigated from top to bottom: $C = 0.8$, $C = 0.56$, $C = 0.3$, $C = 0.2$. Reference DNS (solid).

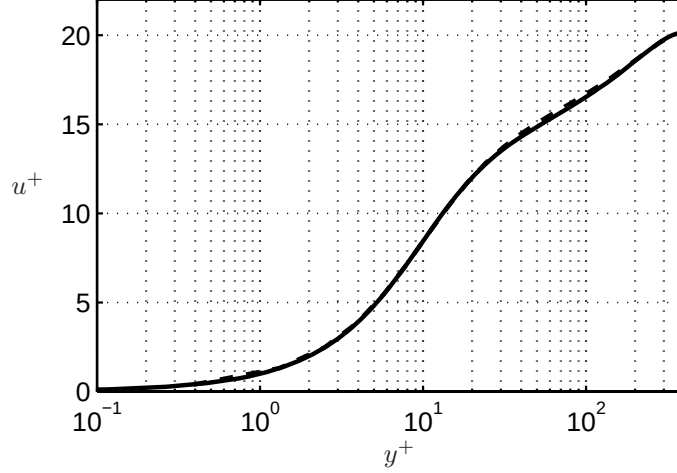


Figure 3.31: Mean velocity profile obtained with the optimal coefficient $C = 0.56$. Reference DNS (solid), LES using RVM_s -WALE model(dash).

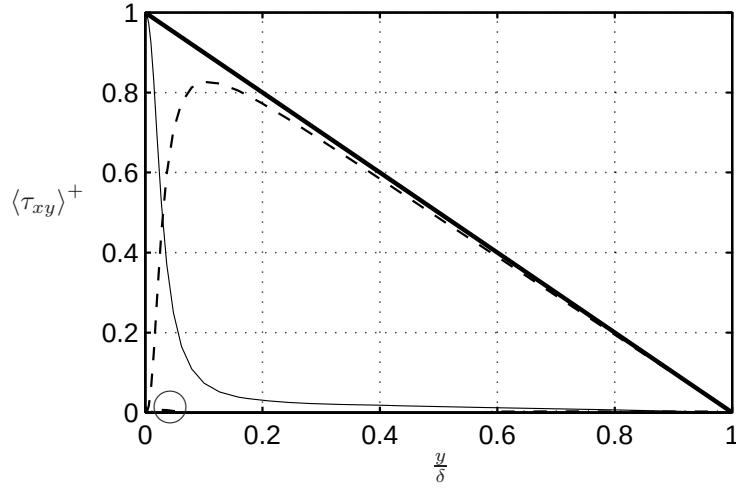


Figure 3.32: Decomposition of the mean shear stress profile: $\langle \tau_{xy}^{tot} \rangle$ (solid thick), $\langle \tau_{xy}^{visc} \rangle$ (solid thin), $-\langle u'v' \rangle$ (dash), $\langle \tau_{xy}^M \rangle$ (dash-dot) (highlighted by the circle as its contribution is very small).

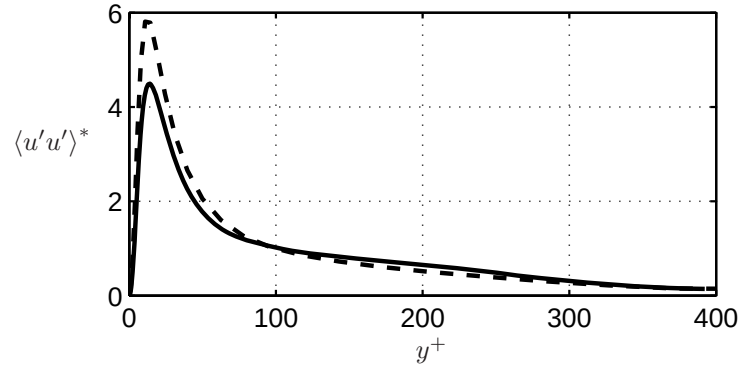


Figure 3.33: Reduced turbulence intensity : $\langle u'u' \rangle^*$. Reference DNS (solid), LES using RVM_s -WALE model(dash).

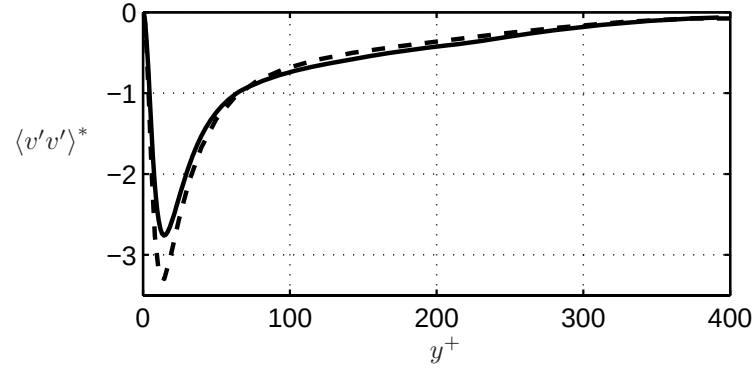


Figure 3.34: Reduced turbulence intensity : $\langle v'v' \rangle^*$.

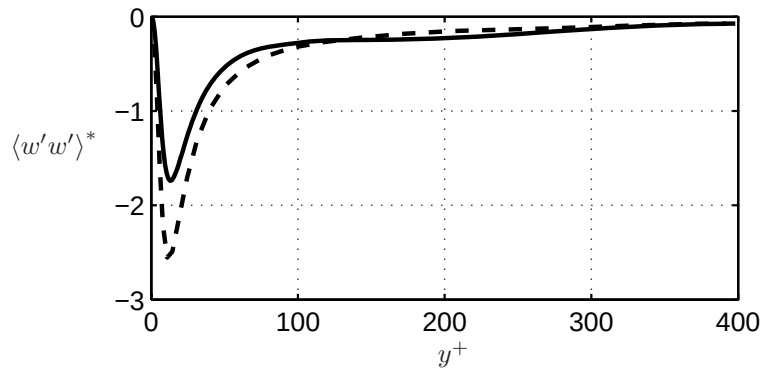


Figure 3.35: Reduced turbulence intensity : $\langle w'w' \rangle^*$.

3.10 Conclusions

In this chapter, several SGS models were presented and investigated. Starting from simple models with modest performance and going to more advanced ones. The WALE model was carefully investigated for wall bounded flows, it was clearly shown that, in the case of a channel flow, it yielded very good results. However, we showed that this model is too much dissipative in vortex cores. The sampling-based dynamic procedure is conceptually appealing. Even if this procedure allows to obtain good results in the case of homogeneous turbulence, it does not allow to properly simulate channel flows due to the large amount of negative model coefficient values yielded by the procedure. Performing LES of homogeneous turbulence on very large grids allowed to analyse the spectral behaviour of several models investigated in the present work. A major trend resulting from this analysis is that the multiscale models acting on the small scale fields exhibit the better spectral behaviour, even if they suffer from the presence of a bottleneck effect. This study also highlighted the asymptotic behaviour of the SGS models investigated. The hyper-viscosity results also provide good results on homogeneous turbulence regarding to the obtained inertial range and the energy decay. However it suffers from the bottleneck effect which is more pronounced than in the case of the multiscale models. For this reason, this model will be used as a basis of comparison for the study concerning the aircraft wake vortex system in turbulent atmosphere detailed in Chapter 4. Moreover, the good agreement between the spectral results and the VIC ones, is an indicator of the high quality of this method, also used for aircraft wake vortex simulations as done by Cocle [22]. The Chollet-Lesieur spectral-eddy viscosity model produces fair results in the case of homogeneous turbulence but is clearly of less quality than the multiscale models, in particular the RVM_s one.

One of the main outcome of this chapter is that the new multiscale WALE type model appears to be the best model to perform LES of vortical flows in presence of walls. This model offers a natural damping at the wall with a behaviour close to the theoretical one, and it avoids excessive diffusion in the vortex cores. Moreover the contribution of this model is negligible in the shear stress budget. This model was calibrated and tested on the channel flow at $Re_\tau = 395$. It will be further used in Chapter 5 on more complex flows.

Chapter 4

LES of aircraft wake vortices in weak atmospheric turbulence: reaching the “equilibrium turbulent vortex system”

4.1 Introduction

The far wake of an aircraft, after completion of the initial roll-up phase, is essentially composed of a pair of counter rotating vortices. This is a consequence of the lift and the finite aspect ratio of the wing. The initial circulation Γ_0 of those vortices is linked to the lift L of the aircraft flying at a speed U by

$$L = \rho U \Gamma_0 b_0 ,$$

where b_0 is the distance between the cores of the vortices at the end of the roll-up and ρ is the air density. The strong rolling moment induced by such vortices on a following aircraft entering the wake can be hazardous. This potential hazard is at the origin of the separation standards between aircraft in landing and take-off phases. There is thus a real need to understand the physical behaviour of such a flow. Numerical simulation of this flow can help better understand the life and the decay of aircraft wake vortices.

The Reynolds number of such a flow is very high: $Re = \Gamma_0/\nu = O(10^7)$, the molecular viscosity thus plays a minor role on the large scales of this flow. Nevertheless the viscosity dissipates all the energy at the small scales. This fact limits the feasibility to perform direct numerical simulations and requires appropriate modelling. In this chapter, we propose to simulate using the pseudo-

spectral code described in Chapter 2, the interaction between an homogeneous isotropic turbulence at a given very low level of dissipation and a pair of counter rotating vortices. Two SGS models will be investigated: an hyper-viscosity model and the RVM_s model. Both models were defined in Chapter 3. The homogeneous isotropic turbulence computed to serve as background flow is described and some quantitative results are analyzed to assess the quality of this simulation. The question we address in the present work is “what is the state of equilibrium of a turbulent vortex system?” The wake vortex flow is then described and a visualization of the flow is provided to enlight this description. Quantitative results of interest are also analyzed: energy decay, circulation decay, evolution of the dissipation, evolution of the enstrophy, spectra and temporal evolution of the modal energy. The analysis of axial velocity fields shows that they are significant. This fact legitimates the co-axial LIDAR wake vortex detection. Finally, some conclusions concerning the existence of an equilibrium state of a turbulent vortex system are drawn.

It is also worth mentioning that similar studies had been performed and can be found in the literature. Jeanmart [61] performed LES using the same approach, i.e. inserting a pair of aircraft wake vortices in homogeneous turbulence. The available grid resolution at this time was however quite modest, highlighting the need to perform the study on larger grids. Takahashi et al. [109] investigated the interaction between a single columnar vortex immersed in homogeneous turbulence. They observed that the turbulent eddies are wrapped around the columnar vortex and become spirals. A similar behaviour will be observed here in the case of a pair of vortices. However the conclusions of their work is not directly applicable to aircraft wake vortices as they studied the evolution of a single vortex. More recently, Gerz et al. [48] performed LES (using a “corrected” Smagorinsky model) of two and four vortex systems varying the atmospheric turbulence level. They allowed for the Crow instability to develop, which is not the purpose of the work presented here, as the turbulent vortex system is the principal aim of the investigation. This work was already presented at the American Physical Society, Division of Fluid Dynamics 58th Annual Meeting [124], and partially appears in the proceedings of [125] and [12].

4.2 Computation of the background turbulent flow

A turbulent atmosphere is here modeled by means of a homogeneous isotropic turbulent flow (HIT). The intensity of the turbulence in the atmosphere is quantified by means of the dissipation level ϵ also called the eddy dissipation rate (*EDR*). As the aim of the investigation is the final turbulent state of a wake vortex, the HIT dissipation level is set to a very low value: $\epsilon = 10^{-5} m^2/s^3$. The initial velocity field is build following the algorithm described by Rogallo in [99]. This algorithm is described in Appendix C. The generated initial field is isotropic, divergence free and has a given spectrum but does not present the

characteristic of real turbulence. In order to obtain a field which represents homogeneous isotropic turbulence, this field evolves without any forcing until it reaches approximatively the prescribed dissipation level. The theory of homogeneous isotropic turbulence provides the temporal evolution laws for the turbulent kinetic energy, the dissipation and the integral length as introduced in Section 1.4. These equations are recalled here:

$$E(t) = E_0 \left(\frac{t}{t_0} \right)^{-p}, \quad (4.1)$$

$$\epsilon(t) = \frac{pE_0}{t_0} \left(\frac{t}{t_0} \right)^{-(p+1)}, \quad (4.2)$$

$$L(t) = \frac{E^{\frac{3}{2}}}{\epsilon}. \quad (4.3)$$

The time evolution of these quantities are displayed in Fig. 4.1 and Fig. 4.2. For the hyperviscous model, the decay exponent was found to be $p = 1.38$. This value is in agreement with the literature (Mohamed and Iarue [80]) which suggests $1.15 \leq p \leq 1.45$. The energy spectra are also displayed in Fig. 4.8 and Fig. 4.9. They both exhibit the same broad inertial range, though the shape of the spectra are different in the dissipative range. The hyper-viscosity model has a very sharp decay in the dissipative range preceded by a “bump”, which is typical of hyper-viscous models. The RVM_s models also exhibit a “bump” just before the dissipative scales. However this bump contains less energy than the case with the hyper-viscosity model. This spectral behavior was already detailed in Section 3.8 of the present work. Note that it is important to consider a turbulent field instead of a database of random numbers. To illustrate this fact, we compare two initial fields: a random field and a field which has evolved with the LES equations. The two fields have the same spectrum, this implies that they have the same dissipation and turbulent kinetic energy. Both fields are also divergence free. Thus they have the same global characteristics. However, the skewness of the flow, defined here as:

$$\mathcal{S} = \frac{\left\langle \left(\frac{\partial u_x}{\partial x} \right)^3 \right\rangle}{\left\langle \left(\frac{\partial u_x}{\partial x} \right)^2 \right\rangle^{\frac{3}{2}}}, \quad (4.4)$$

is different: $\mathcal{S} = 0$ in the case of the random numbers database and $\mathcal{S} = -0.31$ in the case of the turbulence database, which is in line with grid turbulence which provides $-0.4 \leq \mathcal{S} \leq -0.3$ as stated by Davidson [30]. To highlight the topological difference of the two fields, we refer to the Perry and Chong [92] classification described in the appendix B. $R-Q$ diagrams are displayed for the random number field in Fig. 4.3 and, in Fig. 4.4, for the case of the simulated turbulence. In the case of the random number field, the $R = 0$ axis is a symmetry axis of the scatter plot. This means that the vortex compression or stretching states of a turbulent flow can be found with an equal probability.

The symmetry is lost along the $Q = 0$ axis. There is however only slightly more points with $Q < 0$. It is thus reasonable to deduce that each of the four states of the flow has the same probability to occur with a random number database. For the HIT case, 60% of the values are such that there exists a strong positive correlation between Q and R such that $\langle QR \rangle > 0$. This is equivalent saying that the two most common states are vortex stretching and bi-axial strain (as shown in Table. B.1): as it is the case in most turbulent flows as explained in [30]. This fact enforces the assumption that the vortices will interact with turbulence due to the vortex stretching and strain mechanism. Moreover, coherent vortical structures can be observed using the λ_2 criterion displayed in Fig. 4.5, which is not the case for a random numbers database even with the same spectrum for which no coherent structure can be educted with the same criterion. These arguments allow to highlight the importance of inserting the vortices in a HIT field instead of a random numbers field.

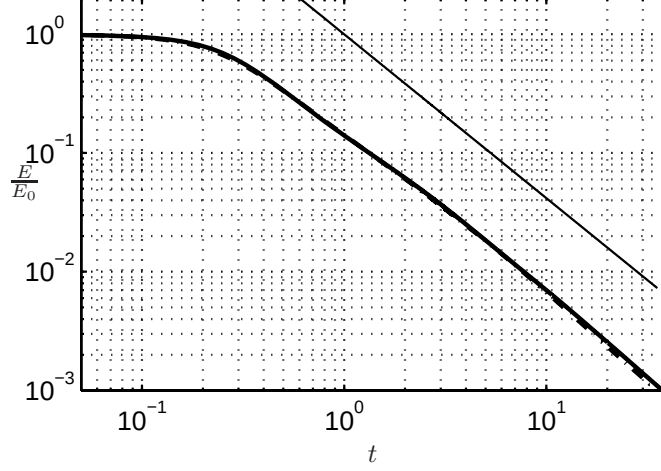


Figure 4.1: Kinetic energy evolution. RVM_s model (dash-dot), hyper-viscosity model (solid). The straight line shows the t^{-p} slope, with $p = 1.38$.

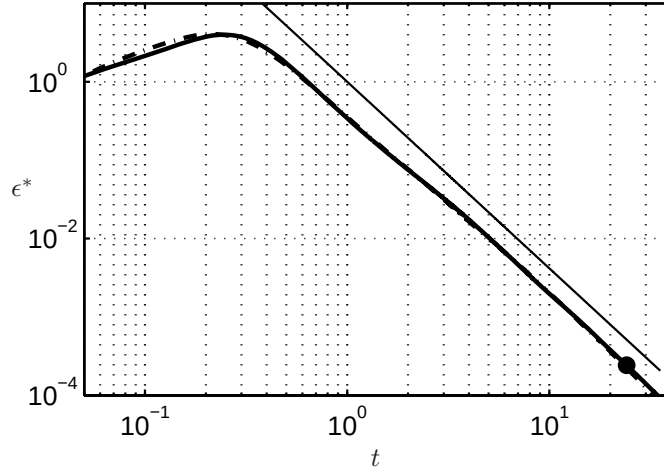


Figure 4.2: Enstrophy evolution normalized by the physical quantities of the vortex pair as defined at 4.3. The straight line shows the $t^{-(p+1)}$ slope, with $p = 1.38$. The bullet shows the time corresponding to the required dissipation level $\epsilon^* = 2.4 \times 10^{-4}$ mentioned in Section 4.3.

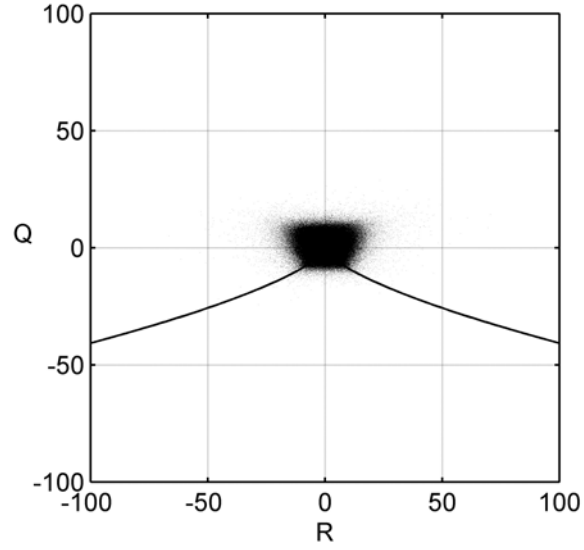


Figure 4.3: *Q-R plot for the random numbers database. The thin solid line shows the locus where $Q = -3(R^2/4)^{1/3}$*

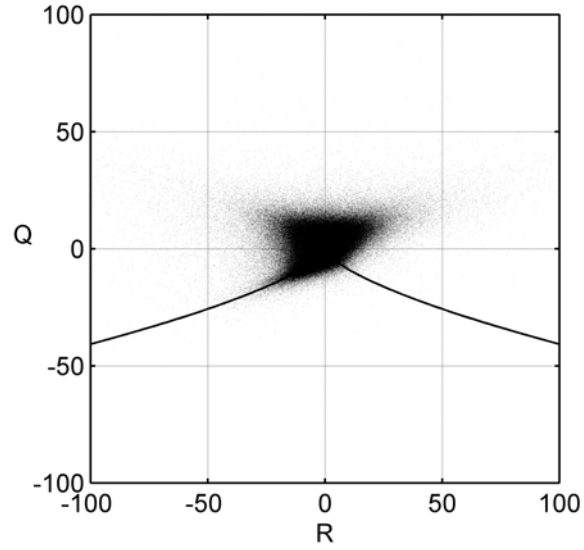


Figure 4.4: *Q-R plot for the turbulence database. The thin solid line shows the locus where $Q = -3(R^2/4)^{1/3}$*

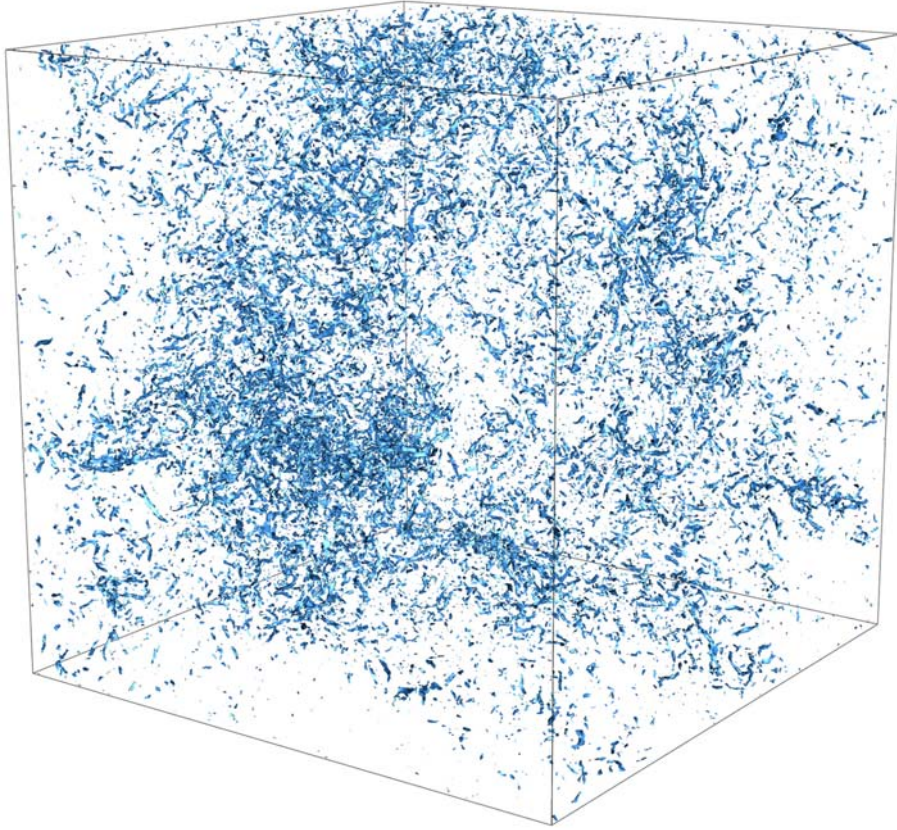


Figure 4.5: Iso surfaces of the λ_2 criterion (as defined by Jeong and Hussain [59]) for the homogeneous turbulence. The same plot for the random numbers results in an empty box.

4.3 Description of the wake vortex flow

The fluid flow considered here is related to the passage of a heavy aircraft in a turbulent atmosphere. This is adequately simulated by superposition of the homogeneous isotropic turbulent flow (obtained as a pre-simulation) with a pair of analytical vortices. The simulation of a wake vortex system (after completion of the initial roll-up) thus requires to choose a distribution function to initialize the wake flow. In the present work, a Hallock-Burnham (low order algebraic) velocity profile is used. This distribution was shown to provide a good fit for real aircraft wake vortices velocity profiles (as measured by LIDARS or as a result of wind tunnel experiments) see Gerz et al. [51]. This vortex model is defined by Eqs. (1.3), (1.4) and (1.5), the velocity profile is recalled here:

$$u_\theta(r) = \frac{\Gamma_0}{2\pi r} \frac{r^2}{(r^2 + r_c^2)}.$$

The parameter r_c is set here at $r_c = 0.05 b_0$ which is a somewhat realistic value for aircraft wake vortices after the initial roll-up phase. There are more than six grid points in the core of the vortices ($2r_c/h \geq 6$). The vortex cores are thus fairly well discretized and their dynamics should be captured correctly. The streamlines of this flow are displayed at Fig. 4.6. Typical dimensional parameters of this flow are listed in Table 4.1. The ratio $\alpha = \frac{L}{b_0}$ is chosen so

Table 4.1: Parameters for the wake vortex flow

Vortex circulation	Γ_0	$400 \text{ m}^2/\text{s}$
Vortex spacing	b_0	50 m
atmosphere EDR	ϵ	$10^{-5} \text{ m}^2/\text{s}^3$
Periodicity length	L_x, L_y, L_z	$4 b_0$
Number of Fourier modes	N_x, N_y, N_z	256

as to minimize the influence of the vortex periodic images. Since enforcing the periodicity in the x direction is less critical than in the y and z directions, we performed this study in two dimensions. As a measurement of this influence, we chose the ratio between the theoretical descent speed $V_0 = \frac{\Gamma_0}{2\pi b_0}$ and the one obtained by taking the influence of all the other vortex pairs (For that purpose, the velocity is evaluated by means of the Biot-Savart formula). In Fig. 4.7 we can observe that for $\alpha = 4$, one has $V_{mod} = 0.98V_0$; this is the reason why we set $\alpha = 4$. If one considers the mean energy of the base flow vortex system in the periodic domain:

$$E_{2D} = \frac{1}{L^2} \int_S \frac{u^2 + v^2}{2} dS, \quad (4.5)$$

the results obtained are $E_{2D} = 0.964V_0^2$ using the RVM_s model and $E_{2D} = 0.976V_0^2$ using the high order hyper viscosity. The same vortex system in an

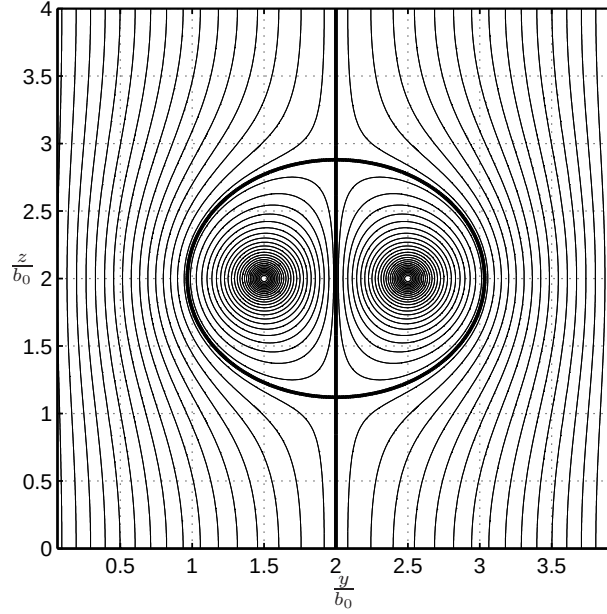


Figure 4.6: Streamlines and Rankine oval for the two vortex system flow. The streamlines are shown in the relative reference frame: that moving at the vortex system descent velocity $V_0 = \frac{\Gamma_0}{2\pi b_0}$.

unbounded domain has a total energy of

$$\tilde{E}_{2D} = \frac{1}{L^2} \int_{S_\infty} \frac{u^2 + v^2}{2} dS = \frac{\Gamma_0^2}{2\pi} \left(\log \left(\frac{b_0}{r_c} \right) - \frac{1}{2} \right) = 0.980 L^2 V_0^2. \quad (4.6)$$

The difference in energy is thus small, indicating again that the size of the computational domain is indeed acceptable. As the obtention of a vortex system in a turbulent state is the aim of this investigation, we do not allow for the long wavelength Crow instability to develop ($\lambda_x \approx 8b_0$). This explains the limited extension of the domain along the vortex direction. The V_0 velocity is used as the velocity scale of our problem. From there we can also define the dimensionless time:

$$\tau = t \frac{V_0}{b_0}. \quad (4.7)$$

The dissipation rate is normalized in the following way:

$$\epsilon^* = \frac{\epsilon b_0}{V_0^3}. \quad (4.8)$$

In the case of a low level turbulence with a dissipation rate $\epsilon \approx 10^{-5} m^2/s^3$, one has with the parameters defined at Table. 4.1 $\epsilon^* = 2.4 \times 10^{-4}$. Another dimensionless number used to represent the strength of the turbulence is $\frac{(\epsilon b_0)^{1/3}}{V_0}$:

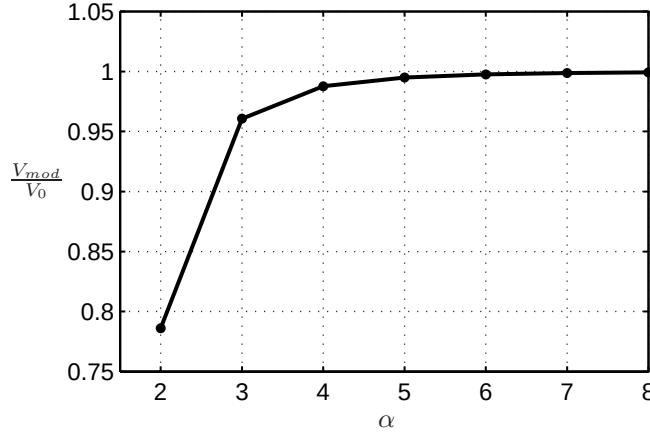


Figure 4.7: Influence of α on the descent velocity of the vortex pair. One observes that, for $\alpha = 4$, only a minor influence of the periodic image vortex pairs is felt (2% on the descent velocity).

in our case it is 0.062 which corresponds effectively to very low level turbulence. A typical order of magnitude for the Reynolds number of this flow, is $Re = \frac{U}{\nu} \approx 3 \times 10^7$. This motivates the LES with $\nu = 0$ approach used in this study. The initial spectra decomposition is displayed in Figs. 4.8 and 4.9. This decomposition illustrates the fact that adding the analytical vortex pair to the homogeneous turbulence field results in an increase of the energy of the large scales of the analytical spectrum. The large scales part of the flow is essentially dominated by the two vortices which are much more energized than the large scales of the turbulence. The bottleneck SGS effect can also be clearly observed, and is more pronounced for the hyper-viscosity model. As both turbulence spectra exhibit a broad inertial range, the initial turbulence field represents quite well a low level atmospheric turbulence.

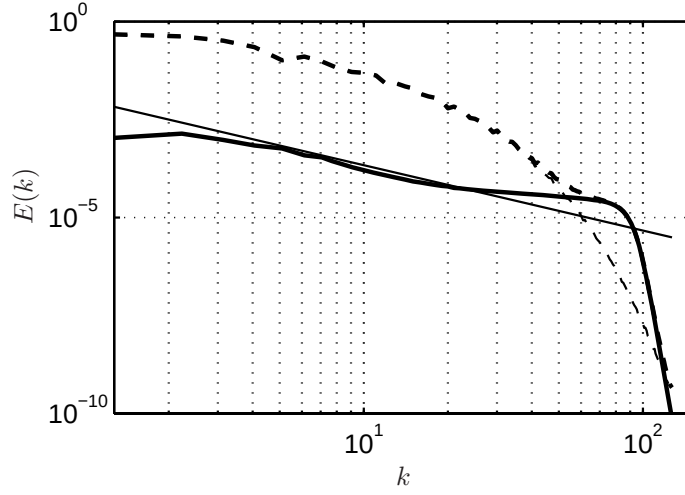


Figure 4.8: Initial spectrum decomposition for the hyper-viscosity model. Background turbulence (solid-thick); vortex system alone (dash-thin); total field (dash-thick). The straight line shows the $k^{-5/3}$ slope.

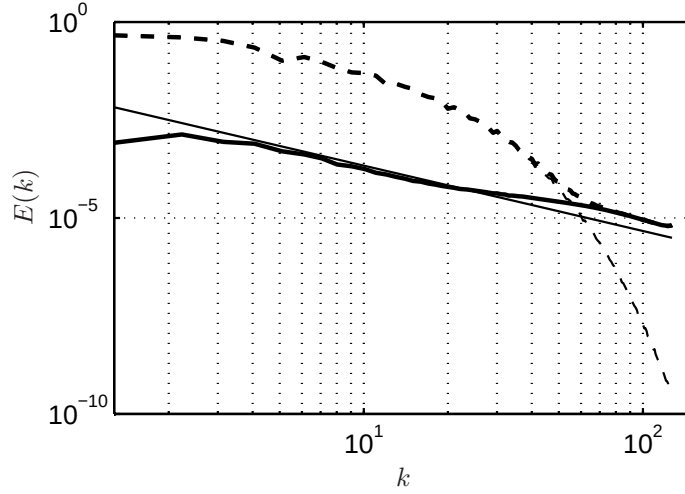


Figure 4.9: Initial spectrum decomposition for the multiscale model. Background turbulence (solid-thick); vortex system alone (dash-thin); total field (dash-thick). The straight line shows the $k^{-5/3}$ slope.

4.4 Results

4.4.1 Flow visualisation

The flow field is visualized by means of the vorticity field in Fig. 4.10. We display two isosurfaces of $\|\boldsymbol{\omega}\|$. These isosurfaces are colored by the intensity of ω_x . The initial condition frame displays the vortex pair surrounded by turbulence. As the turbulence level is very weak, only few structures can be observed. At a first stage, the aircraft wake vortices stretch the vortical structures of the ambient turbulence. This leads to the generation of significant transverse structures around the cores of the vortices as observed at $\tau = 0.5, 1.0$. These can be viewed as medium scales in the energy spectra. They appear in the regions of intense stretching close to the flow stagnation points, see Fig. 4.6, and they are aligned with the stretching directions. As observed in Fig. 4.6, the oval limiting streamlines emanating from the upper and lower stagnation points corresponds to the stretching directions.

The surrounding turbulence is continuously amplified by the non linear interactions and the system progressively becomes more and more complex. It is however worth noting that the turbulence stays confined in a “Rankine-like” oval. Eventually, the system becomes completely turbulent.

To better illustrate the topology of this flow, cross plane snapshots of the axial velocity are displayed in Fig. 4.11. First, it can be observed that there is significant axial velocity present in the flow and this velocity component increases as the vortex system becomes turbulent. These axial velocity fluctuations are due to the transverse structures. The axial velocity component value amounts to approximately 10% of the maximal velocity, which is roughly equivalent to V_0 ($V_\theta^{max} = 10 V_0$ in the case of a low order algebraic kernel with $r_c = 0.05 b_0$). This validates the fact that a LIDAR system can sense velocities using an axial scanning pattern as described in Fig. 6.14. This fact is not a priori obvious, as the velocity field of a vortex is mainly tangential. Again, one can observe that the axial velocity component fluctuations are enclosed in a “Rankine-like” oval.

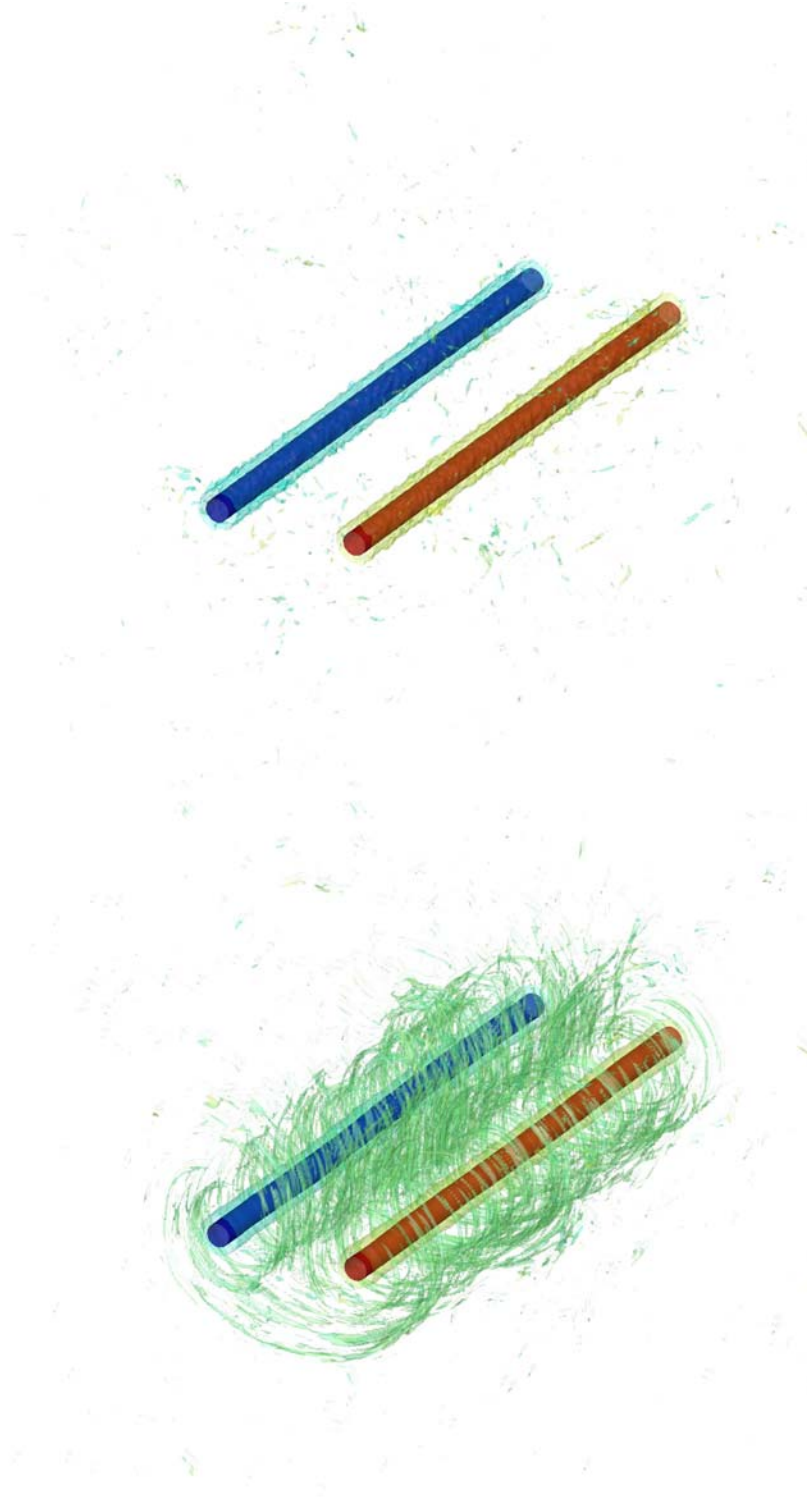


Figure 4.10: Visualization of the flow field using isosurfaces of $\|\vec{\omega}\|b_0^2/\Gamma_0 = 2$ and 10; $\tau = 0$ and $\tau = 0.5$.

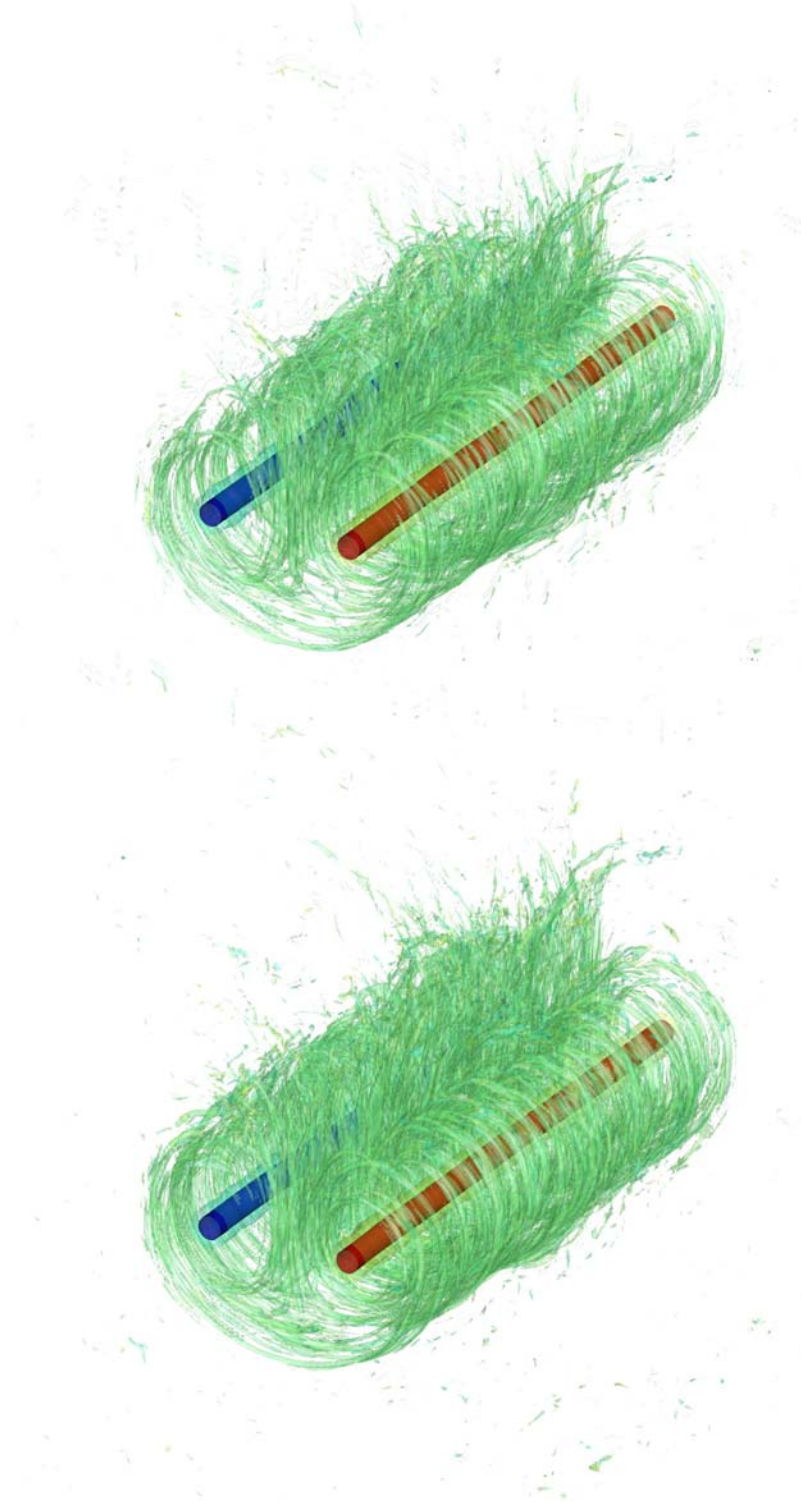


Figure 4.10: (Continued) $\tau = 1.0$ and $\tau = 1.4$.

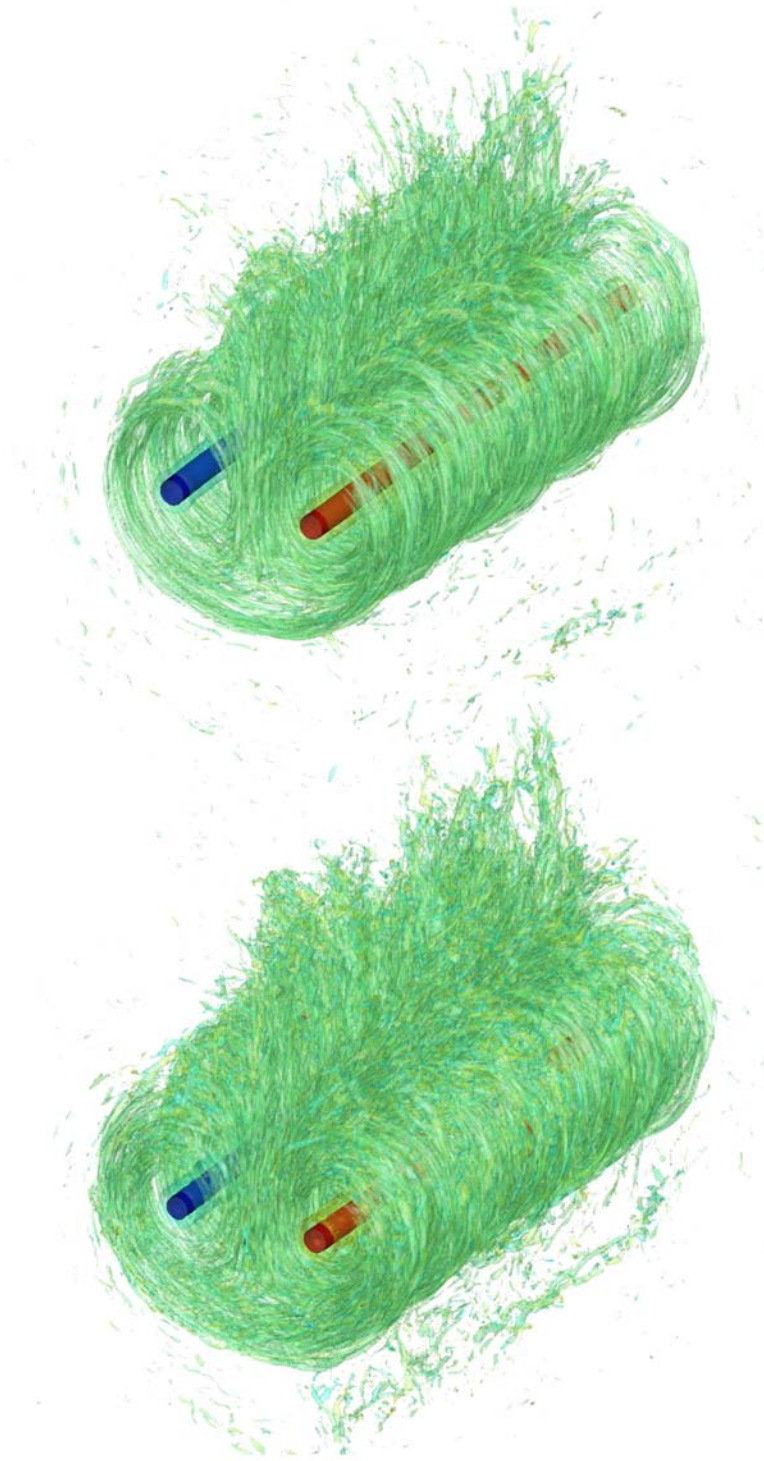


Figure 4.10: (Continued) $\tau = 2.0$ and $\tau = 3.0$.

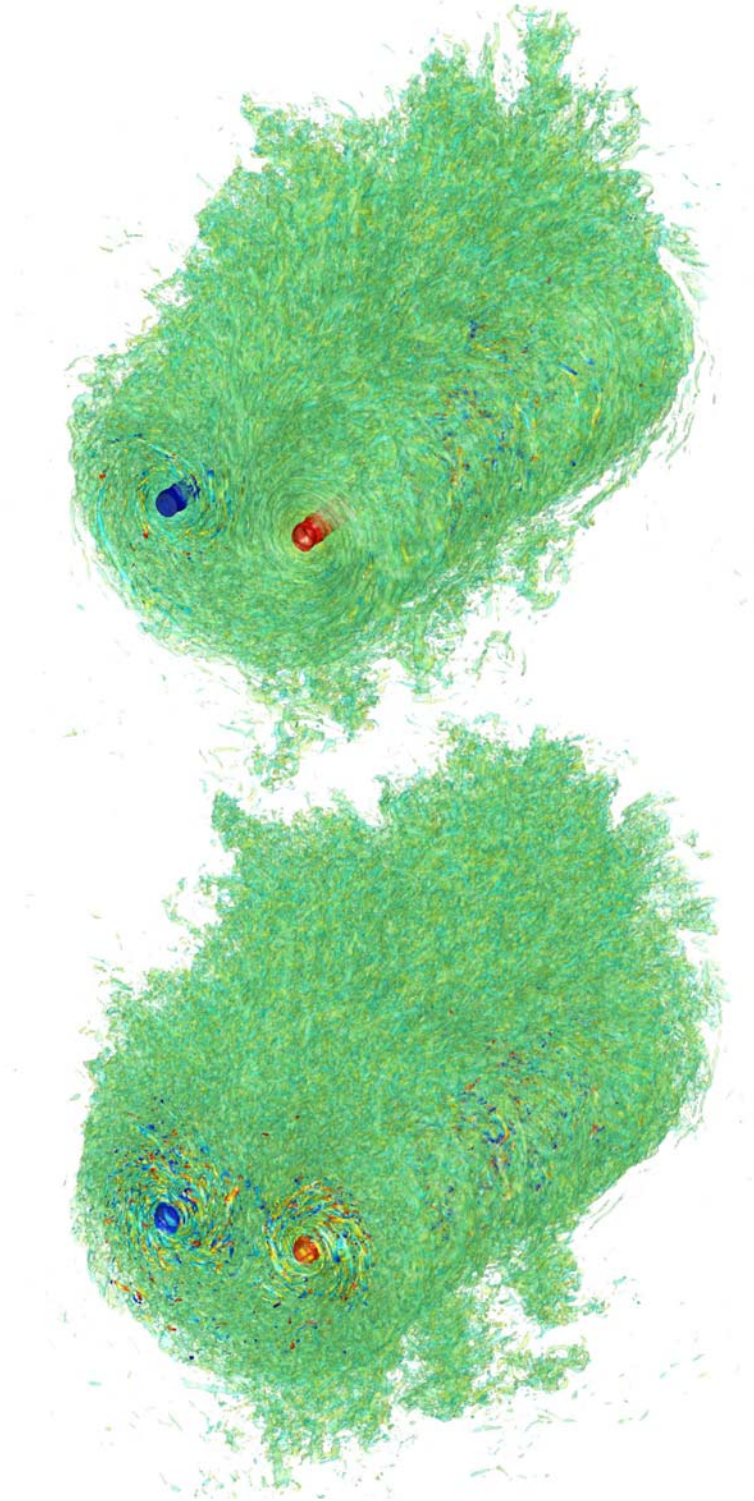


Figure 4.10: (Continued) $\tau = 4.0$ and $\tau = 5.0$.

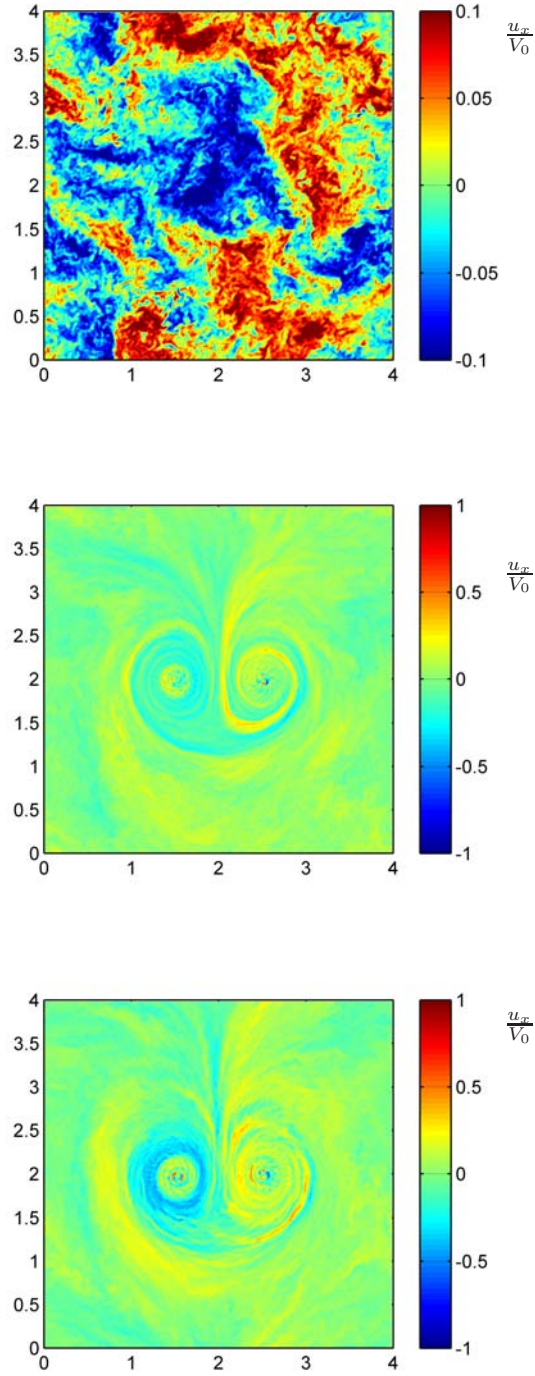


Figure 4.11: Visualization of the flow field using RVM_s , colormap of u_x in an arbitrary cross plane. $\tau = 0$, $\tau = 1.0$ and $\tau = 2.0$. The color scale at time $\tau = 0$ is different to visualize the turbulent scales.

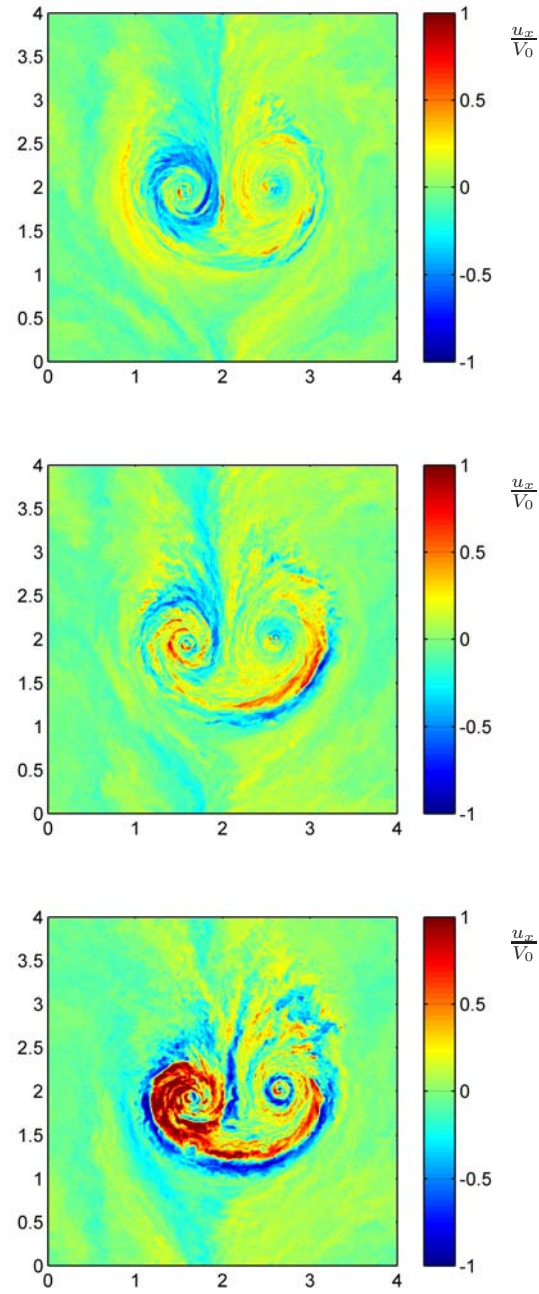


Figure 4.11: (Continued) $\tau = 3.0$, $\tau = 4.0$ and $\tau = 5.0$.

4.4.2 Evolution of the kinetic energy and of the dissipation

The results produced by the hyper-viscosity model and the RVM_s model are compared. The temporal evolution of the total kinetic energy is displayed in Fig. 4.12. The curves exhibit a similar global behaviour: a first period where the global kinetic energy is essentially conserved, followed by a decay phase when the system is turbulent (i.e., when the instabilities saturate). One can remark that the energy curve when using the RVM_s model decays sooner ($\tau \approx 0.2$) than when using the HV model ($\tau \approx 2$). This is due to the fact that the RVM_s model is acting as soon as the stretching of the turbulent structures by the strong vortices creates medium scales. Later, the two curves decay at approximately the same rate. The dissipation evolution displayed in Fig. 4.13 is more relevant to illustrate the physical aspects of the problem. The important result is that both curves reach a level of normalized dissipation of $\epsilon^* = 0.05 - 0.06$ which is much higher than the initial level $\epsilon^* = 2.4 \times 10^{-4}$ of the background turbulence. The stretching of the surrounding turbulent structures by the strong vortices leads to a sharp intensification of the dissipation—or enstrophy—of the flow. This clearly shows that the vortex pair is generating its “own turbulence” and is reaching an “equilibrium state” before decaying for $\tau \geq 6$. These curves allow to deduce the order of magnitude of dissipation for a turbulent vortex system at equilibrium. Examining the evolution of the 3D and 2D kinetic energy defined by Eq. (4.5) and reported in Fig. 4.14 leads to observe that when the system is fully turbulent ($\tau \geq 4.5$), the energy of the 2D averaged field (base flow) decays at almost the same rate as the energy of the 3D field. This implies that the turbulent kinetic energy which corresponds to the difference between those curves remains constant. The same conclusion holds for the hyper-viscosity model. This is also indicative of equilibrium: base flow energy is constantly transformed into turbulent kinetic energy. A base flow vorticity plot is also displayed in Fig. 4.15. It can be observed that there is some vorticity exchange through the mid plane due to the turbulence: this is what leads to the decay of circulation of each half plane as detailed in section 4.4.5 and Fig. 4.19. The obtained dissipation of the equilibrium turbulent vortex system, $\epsilon^* = 0.05 - 0.06$, is of great value: it provides the minimum (i.e. lower bound) dissipation of turbulent aircraft wake vortices, apart of course of the decay following the results of long wavelength Crow-type instabilities.

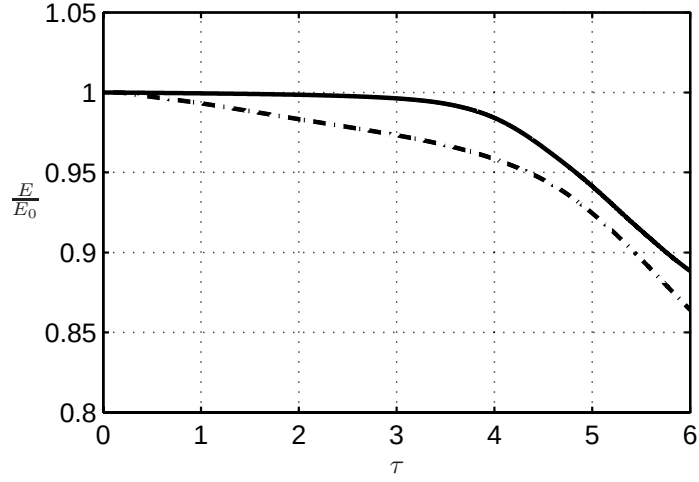


Figure 4.12: Time evolution of the total energy. High order hyper-viscosity (solid), RVM_s (dash).

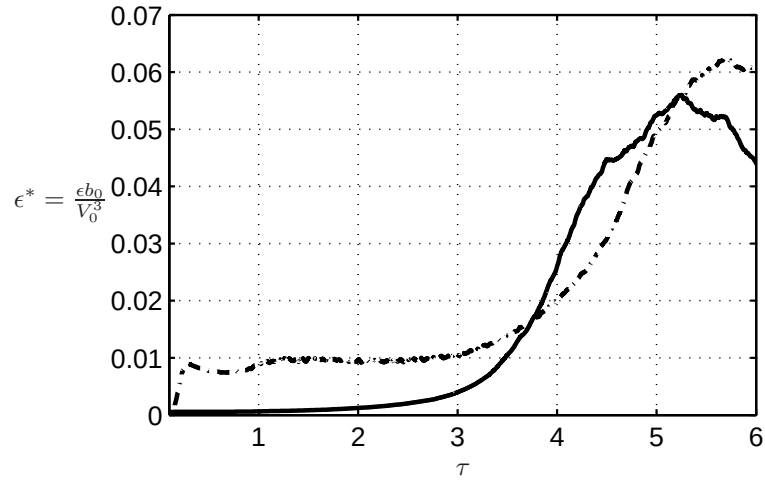


Figure 4.13: Time evolution of the dissipation. High order hyper-viscosity (solid), RVM_s (dash).

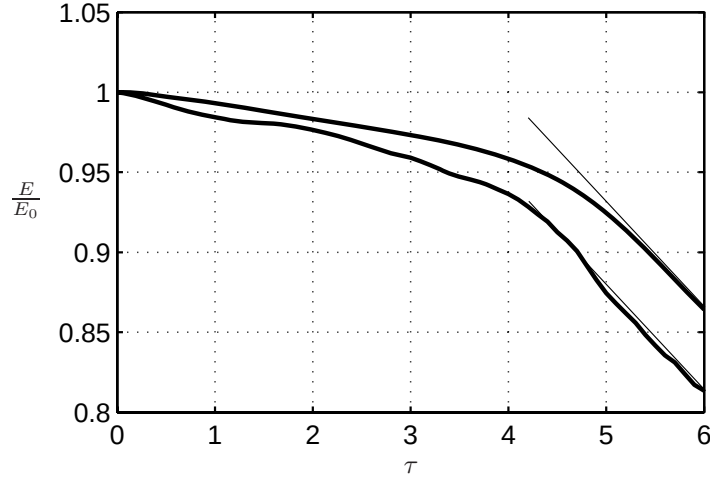


Figure 4.14: 3D (top) and 2D (bottom) kinetic energy evolution using the RVM_s model. The straight thin lines show the equivalence of the finally obtained decay rates.

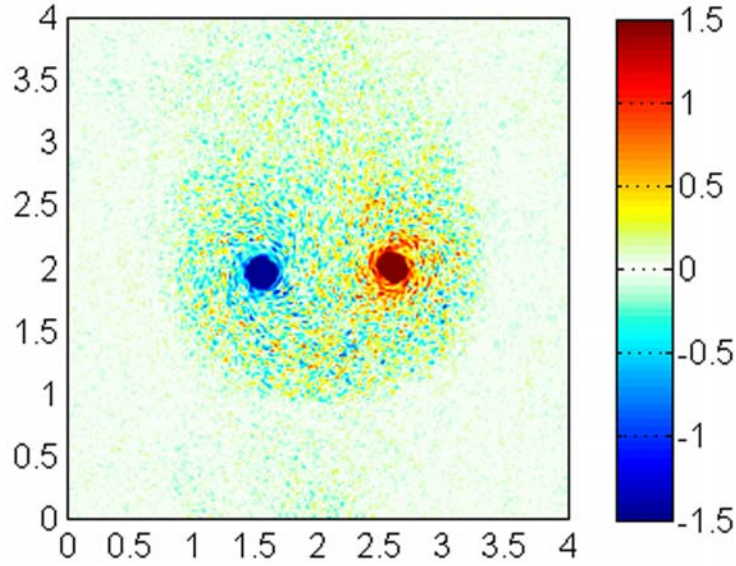


Figure 4.15: Axial vorticity of the base flow $\omega_x b_0^2 / \Gamma_0$. (figure highly saturated to show the vorticity outside the cores)

4.4.3 Longitudinal modal energy

The evolution of the longitudinal modal energy is another important diagnostic: it tells which axial modes are becoming energetic and when they saturate. Taking into account the periodicity of the system in the longitudinal direction, we write for the velocity field :

$$\mathbf{u}(\mathbf{x}, t) = \sum_{m=-N_x/2}^{N_x/2} \hat{\mathbf{u}}(k_x, y, z, t) \exp(i k_x x), \quad (4.9)$$

$k_x = \frac{2\pi m}{L_x}$ being the longitudinal wavenumber. The longitudinal Fourier transform of $\mathbf{u}$ is defined as :

$$\hat{\mathbf{u}}(k_x, y, z, t) = \frac{1}{L_x} \int_0^{L_x} \mathbf{u}(x, y, z, t) \exp(-i k_x x) dx \quad (4.10)$$

The longitudinal modal energy for the mode indexed by the integer m , and using the symmetry property $E_m = E_{-m}$, is defined as :

$$E_m = L_x \int \int \hat{\mathbf{u}} \cdot \hat{\mathbf{u}}^* dy dz \quad (4.11)$$

The temporal evolutions of the modal energy are displayed in Fig. 4.16 for the case using hyper-viscosity modelling, and in Fig. 4.17 for the case using RVM_s modelling. All the modes start with given energy amplitude which is due to the background turbulence. These modal energies grow slowly in time and eventually saturate. This is indicative of the fact that the system has reached a turbulent equilibrium. It is also worth noting that the level reached by the first modes attains very similar values, which means that the models used are consistent with each other.

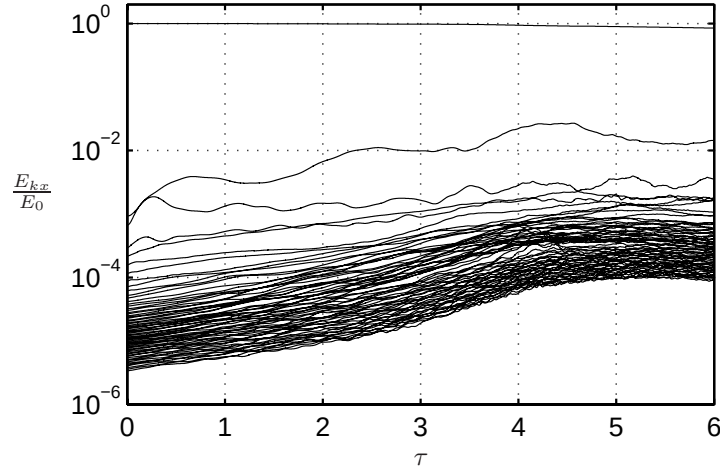


Figure 4.16: Longitudinal modal energy evolution for the turbulent two vortex system (hyper-viscosity model). Only the 64 first modes are represented for the sake of clarity.

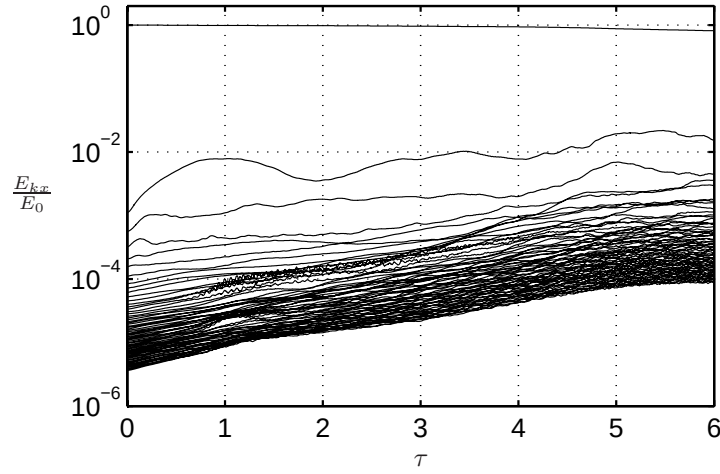


Figure 4.17: Longitudinal modal energy evolution for the turbulent two vortex system (RVM_s model). Only the 64 first modes are represented for the sake of clarity.

4.4.4 Energy spectra at equilibrium

To characterize the spectral behaviour of the flow at equilibrium, we chose to use the 3D spectrum usually defined for homogeneous isotropic turbulence. This spectrum is displayed in Fig. 4.18. The flow including the vortices is far from an isotropic or homogeneous state. However, it exhibits an inertial range. One can also compare the behaviour of the two models investigated: the results are consistent with the conclusions drawn in Section 3.8.

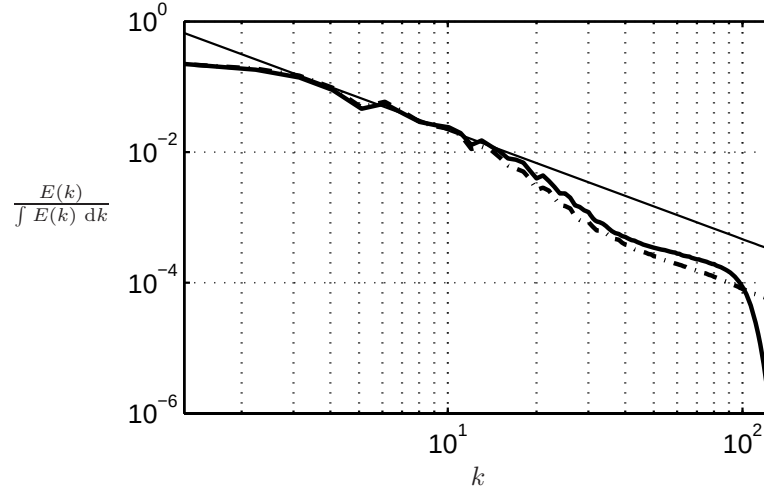


Figure 4.18: Comparison of the energy spectra at $\tau = 5$. The straight thin line shows the $k^{-5/3}$ slope, RVM_s model (dash-dot), hyper-viscosity model (solid).

4.4.5 Circulation decay and trajectories

A quantity of particular importance in wake vortex problems is the vortex circulation Γ . For an airplane, the higher the Γ , the higher the induced rolling moment on a following aircraft. It can be computed from the velocity field or the axial vorticity component:

$$\Gamma = \oint_{\partial\Omega} \mathbf{u} \cdot d\mathbf{l} = \iint_{\Omega} \omega_x dy dz$$

It provides also a good image of the life and the decay of a vortex. The temporal evolution of this quantity is displayed in Fig. 4.19. One observes that the predicted results are in good agreement for the two SGS models investigated. The final circulation attained when the vortex system is completely turbulent is still very high. Another usual measurement of the vortex intensity is the

Γ_{5-15} , which is an average of the circulation distribution. It was originally defined as

$$\Gamma_{5-15} = \frac{1}{10} \int_5^{15} \Gamma(r) dr, \quad (4.12)$$

for aircraft with a wingspan $b = 60 [m]$, where $\Gamma(r)$ is the circulation distribution in a disk of radius r centered on the vortex. In fact, Γ_{5-15} gives a good information about the strength of the rolling moment induced by a wake vortex on another aircraft. Moreover, as will be explained in Chap. 6, this quantity can be measured with a Doppler LIDAR more easily than Γ as the values of the velocity away from the vortex center (thus weaker and less distinguishable from the ambient turbulence) are not accounted in this definition. One defines the Γ_{5-15} for any type of aircraft by

$$\Gamma_{5-15} = \frac{1}{\frac{b}{6}} \int_{\frac{b}{12}}^{\frac{b}{4}} \Gamma(r) dr. \quad (4.13)$$

The span loading factor s is the ratio b_0/b (a typical value being $\pi/4$ for an elliptic loading). The evaluation of $\Gamma(r)$ is here obtained by integrating the mean axial vorticity field $\overline{\omega}_x$ in a disk of radius r

$$\Gamma(r) = \int_0^{2\pi} \int_0^r \overline{\omega}_x r' dr' d\theta'. \quad (4.14)$$

The global circulation Γ temporal evolution is reported in Fig. 4.19. At first sight, one observes that the circulation has not decayed much: at $\tau = 6$, only 5% of the total circulation is lost. The two SGS model used provides results which agree with each other. The Γ_{5-15} evolution is displayed in Fig. 4.20. This quantity has decayed more than Γ : about 20% of its initial value is lost. The two models yield results which are slightly different (3% on the final value at $\tau = 6$). Indeed this criterion is more sensitive to the shape of the circulation profile, which is different for the two models used, see Fig. 4.21. The trajectories of the starboard vortex using the two different models are reported in Fig. 4.22. The two models yields trajectories which are very close to each other. The evolution of the trajectories follow the theoretical behaviour of a vortex pair evolving in the absence of turbulence and viscosity up to $\tau \geq 3.5$. Then the descent velocity decreases as does Γ .

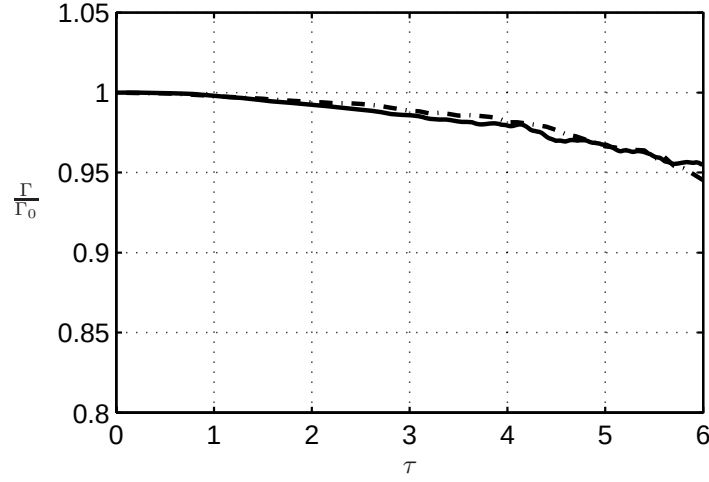


Figure 4.19: Temporal evolution of the circulation Γ : high order hyper-viscosity (solid) and RVM_s (dash).

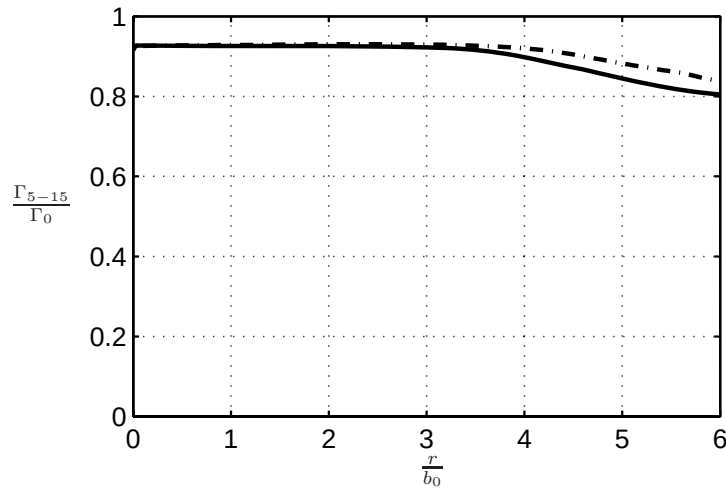


Figure 4.20: Evolution of the Γ_{5-15} circulation: hyper-viscosity (solid) and RVM_s (dash-dot).

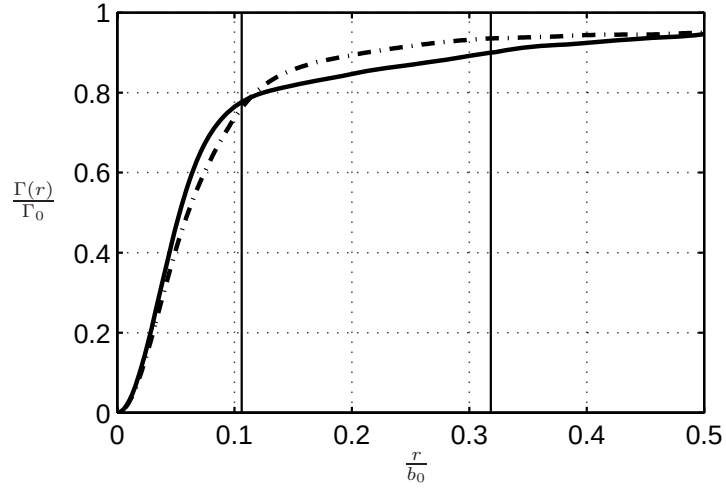


Figure 4.21: Circulation profiles at $\tau = 5.0$: hyper-viscosity (solid) and RVM_s (dash-dot). The solid vertical lines represents the Γ_{5-15} integration limits.

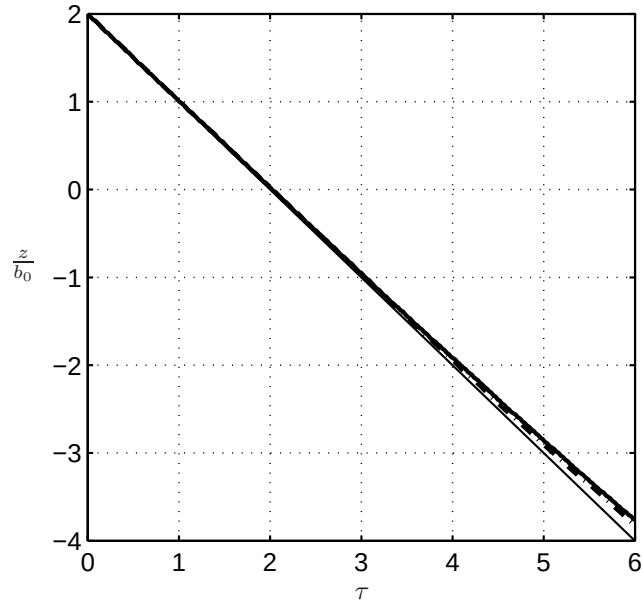


Figure 4.22: Measured trajectory of the starboard vortex: RVM_s (dash-dot) and hyper-viscosity (solid-thick). Theoretical trajectory using initial V_0 (solid thin).

4.4.6 Circulation and tangential velocity profiles

The tangential velocity profiles can be inferred from the circulation profiles $\Gamma(r)$ using the relation:

$$u_\theta(r) = \frac{\Gamma(r)}{2\pi r}. \quad (4.15)$$

The $\Gamma(r)$ profiles used to compute the tangential velocity profiles are reported in Fig. 4.23. The figures are displayed for the RVM_s SGS model but the conclusion stays valid if one examine the curves obtained using the hyper-viscosity model. The circulation profile in the vortex core behaves as $\Gamma(r) \propto r^2$, this is illustrated in Fig. 4.24 using logarithmic scales. The tangential velocity profiles shown in Fig. 4.25 are normalized with the theoretical maximal tangential velocity which is that at $r = r_c$ and is:

$$u_\theta^{max} = \frac{\Gamma_0}{4\pi r_c}, \quad (4.16)$$

in the case of a low order algebraic profile. The peak velocity value has decayed by approximately 15% at $\tau = 5$. In the zone $r/b_0 \geq 0.15$ the velocity profiles collapse, this is due to the fact that this zone is more influenced by the global circulation, which decays very little as observed in Fig. 4.19.

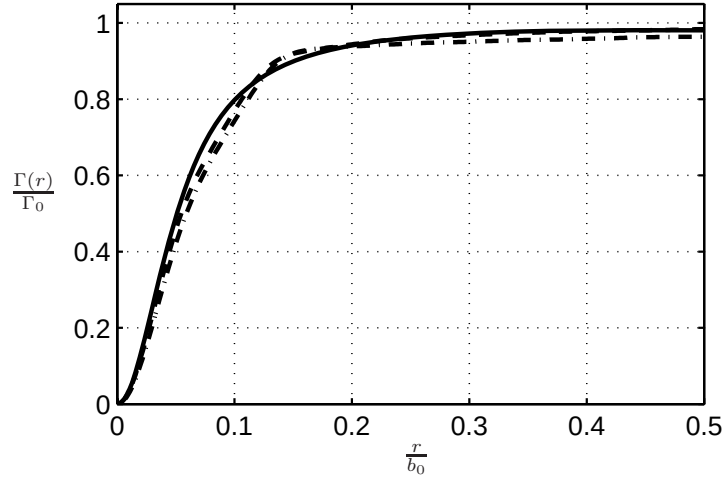


Figure 4.23: Circulation profiles at $\tau = 0$ (solid), $\tau = 3.0$ (dash), $\tau = 5.0$ (dash-dot), using the RVM_s model.

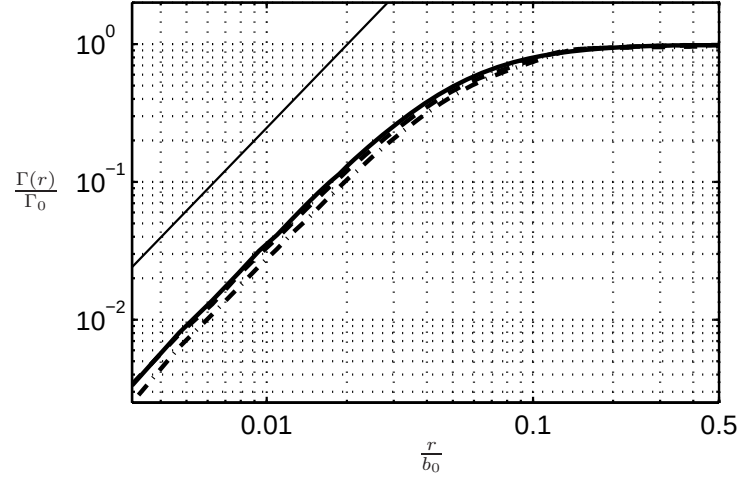


Figure 4.24: Circulation profiles in logarithmic axis system at $\tau = 0$ (solid), $\tau = 3.0$ (dash), $\tau = 5.0$ (dash-dot). The thin line shows the r^2 slope. Results obtained using the RVM_s model.

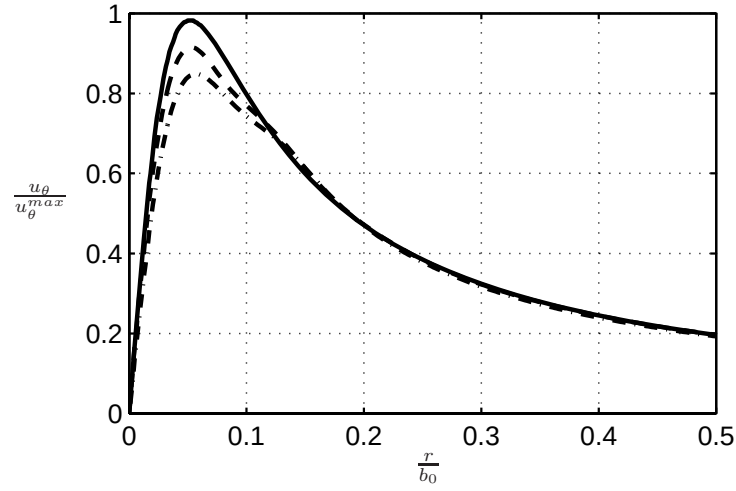


Figure 4.25: Tangential velocity profiles at $\tau = 0$ (solid), $\tau = 3.0$ (dash), $\tau = 5.0$ (dash-dot), using the RVM_s model.

4.4.7 Turbulent stresses

Other interesting results that can be extracted from the fields are the turbulent stresses. However, as the problem does not reach a statistically converged state (as it would be in a channel flow for instance), the averages cannot be carried out in time. Instead, longitudinal averages are computed at a given time, when the vortex system has reached its equilibrium at ($\tau = 5$). The data are gathered along the vortices, meaning that there is only $N_x = 256$ samples in the x direction. Moreover, as illustrated in Fig. 4.26 the cores of the vortices are deformed by short wave instabilities of Widnall type. This can lead to velocity fluctuations which are artificially too high. The $\overline{v'w'}$ component of the Reynolds stress tensor is displayed in Fig. 4.27. Several observations can be made on this plot. First it is interesting to note that the maxima lobes are not aligned with the coordinates axis. The analytical strain rate tensor component for a vortex pair S_{23} is also displayed in Fig. 4.28. The 45° phase shift between the strain and turbulent stress should serve as a warning to modelers who use $k - \epsilon$ models type. In this case, the constitutive relation $\overline{v'w'} = -\nu_T S_{23}$ is invalid. There is also a qualitative agreement between the computed $\overline{v'w'}$ and the experimental ones of Chow et al. [21]. Considering the transformation of a tensor from polar to Cartesian coordinates:

$$\begin{aligned}\overline{v'w'} &= \left(\overline{u_r'^2} - \overline{u_\theta'^2}\right) \sin \theta \cos \theta + \overline{u_r' u_\theta'} (\cos^2 \theta - \sin^2 \theta) \\ &= \frac{1}{2} \left(\overline{u_r'^2} - \overline{u_\theta'^2}\right) \sin(2\theta) + \overline{u_r' u_\theta'} \cos(2\theta).\end{aligned}$$

In Fig. 4.28, the maxima and minima of $\overline{v'w'}$ are located for $\theta = (\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4})$ which are also the extrema of the function $\sin(2\theta)$ and the zeros of the function $\cos(2\theta)$. From this, it is relevant to conclude that the major contribution to the turbulent shear stress comes from the Townsend term $(\overline{u_r'^2} - \overline{u_\theta'^2})$ and as a consequence one should have:

$$\overline{u_r'^2} > \overline{u_\theta'^2}. \quad (4.17)$$

This conclusion can be reinforced by examining the topology of the vortex filaments in Fig. 4.26. It is clear that these filaments will mainly induce u_r' and u_x' fluctuations and negligible u_θ' . The $\overline{u_x' u_x'}$ turbulent stress is displayed in Fig. 4.29. The turbulent fluctuations in polar coordinates are displayed in Figs. 4.30 to 4.31. One observes that the $\overline{u_r' u_r'}$ component is very intense in the core of the vortex. This is due to the short wave instabilities as displayed in Fig. 4.26. The values of the $\overline{u_\theta' u_\theta'}$ component have the same order of magnitude as those of $\overline{u_r' u_r'}$, they are however less spread. This means that outside the core of the vortices, one satisfies (4.17). The $\overline{u_r' u_\theta'}$ component is displayed in Fig. 4.32. This plot is helpful if we want to compute a RANS turbulent eddy viscosity ν_t :

$$\tau_{r\theta}^t = -\overline{u_r' u_\theta'} \equiv \nu_t \frac{\partial \overline{U}_\theta}{\partial r}. \quad (4.18)$$

As observed in Fig. 4.32, the structure of $\overline{u'_r u'_\theta}$ is not axisymmetric in the vortex cores. This is due to the influence of the straining field of the other vortex which produces the typical 45° oriented lobes. The Townsend term is also displayed in Fig. 4.33. The turbulent kinetic energy (TKE) is reported in Fig. 4.34. One observes that the values exhibits an axisymmetric structure. The tangentially averaged turbulent stresses components are displayed in Fig. 4.35. The three components $\overline{u'_x u'_x}$, $\overline{u'_r u'_r}$, $\overline{u'_\theta u'_\theta}$ allow to recover the tangentially averaged TKE profile. We also computed the tangentially averaged profile of $-\overline{u'_r u'_\theta}$ which is reported in Fig. 4.36. As the quantity $\overline{u'_r u'_\theta}$ is not axisymmetric, this tangential average is not fair but one can observe that its value is very low and changes its sign. This may lead to negative values of ν_t .

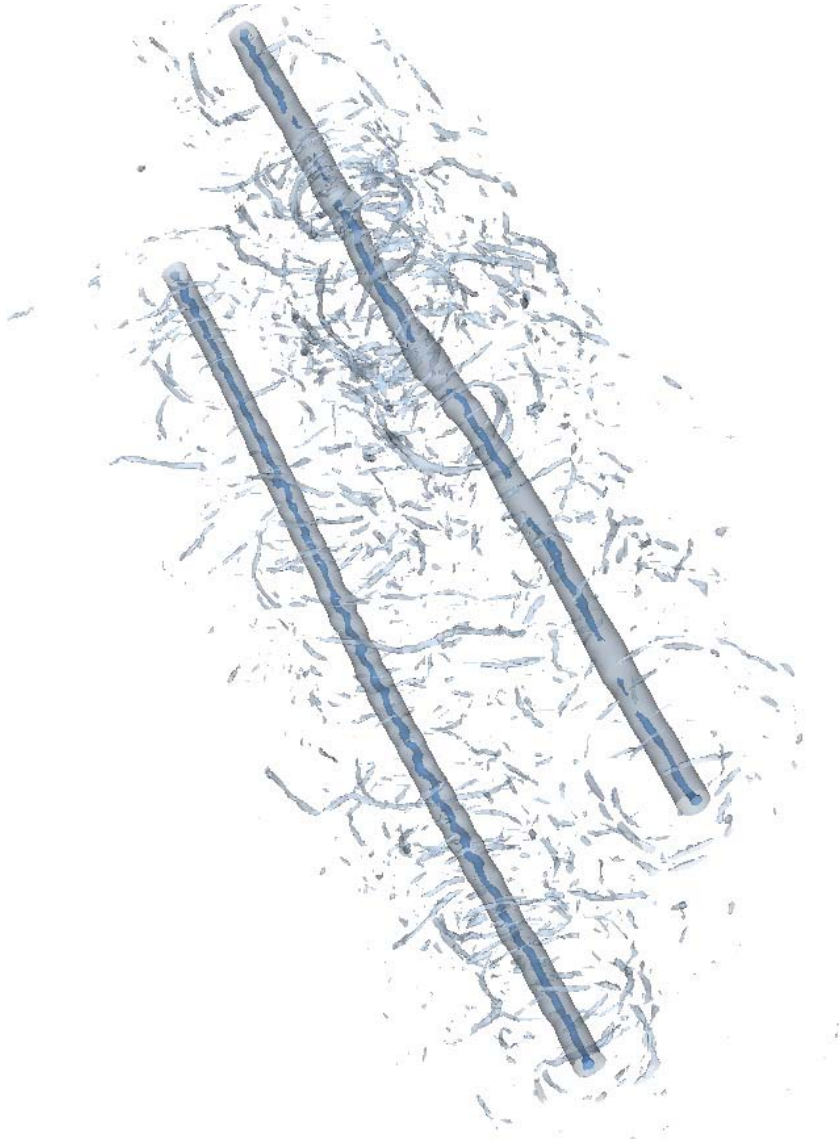


Figure 4.26: Iso surfaces of the λ_2 criterion (as defined by Jeong and Hussain [59]) at $\tau = 5.0$. Short wave instabilities can be observed in the vortex cores.

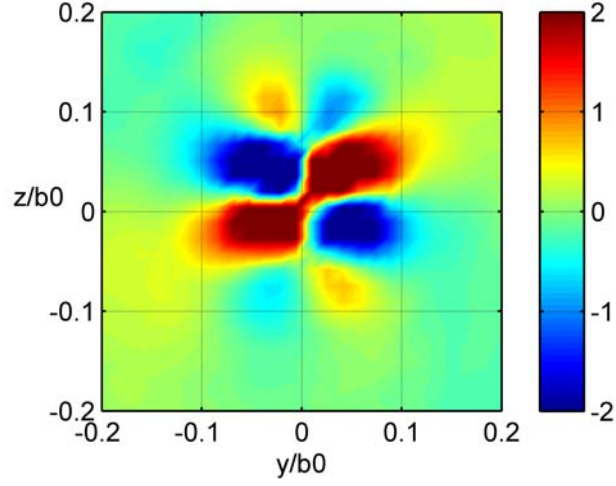


Figure 4.27: Axially averaged normalized turbulent shear stress $\frac{\overline{v'w'}}{V_0^2}$ inside one vortex at $\tau = 5.0$.

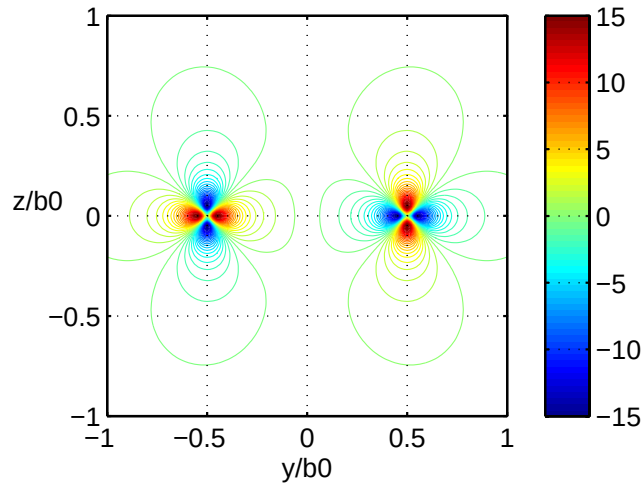


Figure 4.28: Analytical normalized strain rate tensor component $S_{23}^* = \frac{S_{23}b_0^2}{\Gamma_0}$ for a vortex pair.

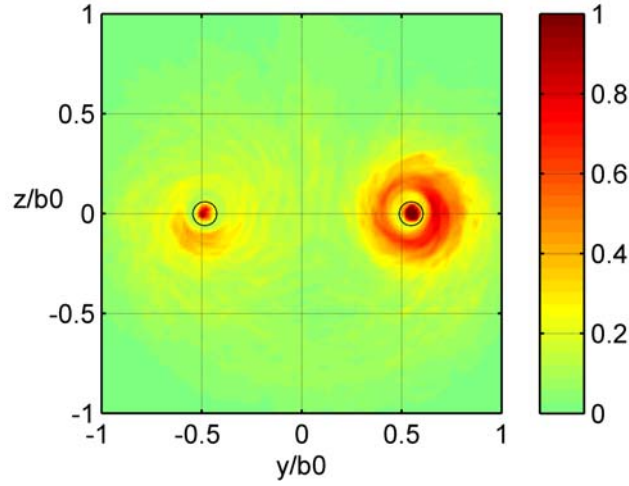


Figure 4.29: Axially averaged normalized turbulent shear stress $\frac{\overline{u'_x u'_r}}{V_0^2}$ for the vortex pair at $\tau = 5.0$. The black circle radius is $r = r_c$.

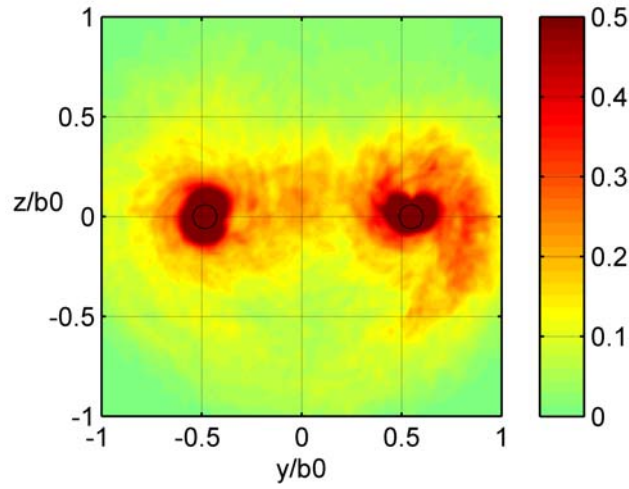


Figure 4.30: Axially averaged normalized turbulent shear stress $\frac{\overline{u'_r u'_r}}{V_0^2}$ for the vortex pair at $\tau = 5.0$.

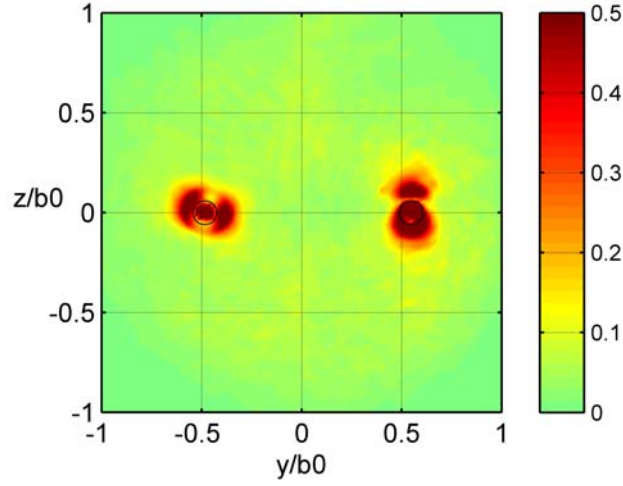


Figure 4.31: Axially averaged normalized turbulent shear stress $\overline{\frac{u'_\theta u'_\theta}{V_0^2}}$ for the vortex pair at $\tau = 5.0$.

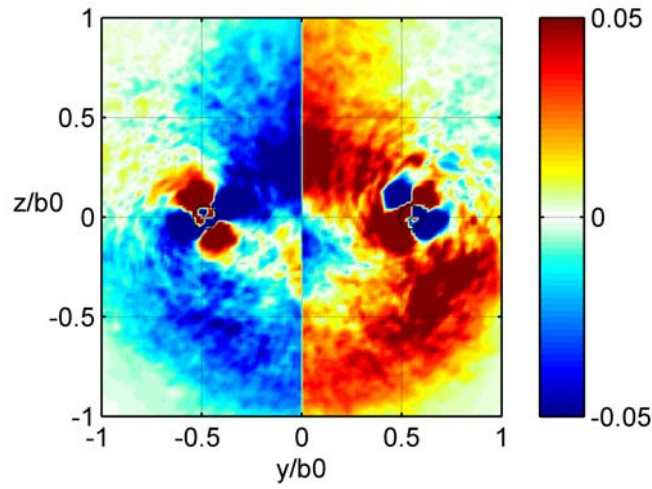


Figure 4.32: Axially averaged normalized turbulent shear stress $\overline{\frac{u'_r u'_\theta}{V_0^2}}$ for the vortex pair at $\tau = 5.0$.

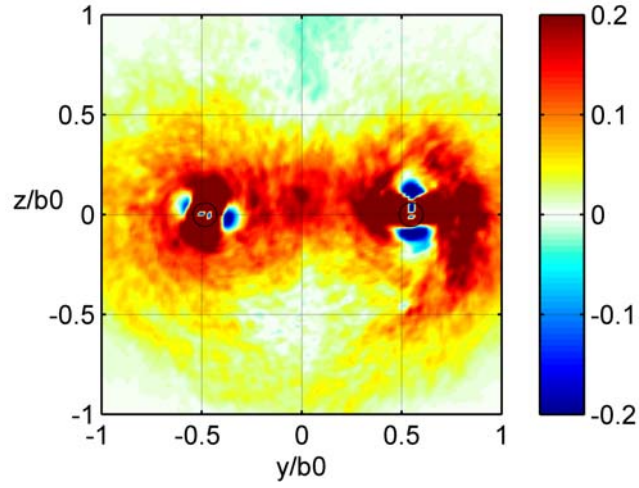


Figure 4.33: Axially averaged Townsend term: $\frac{\overline{u'_x u'_x} - \overline{u'_\theta u'_\theta}}{V_0^2}$ for the vortex pair at $\tau = 5.0$.

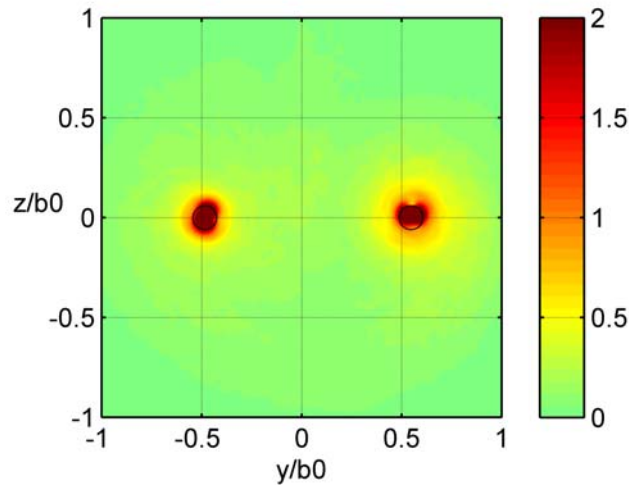


Figure 4.34: Axially averaged normalized TKE $\left(\frac{1}{2} \frac{\overline{u'u'} + \overline{v'v'} + \overline{w'w'}}{V_0^2} \right)$ for the vortex pair at $\tau = 5.0$.

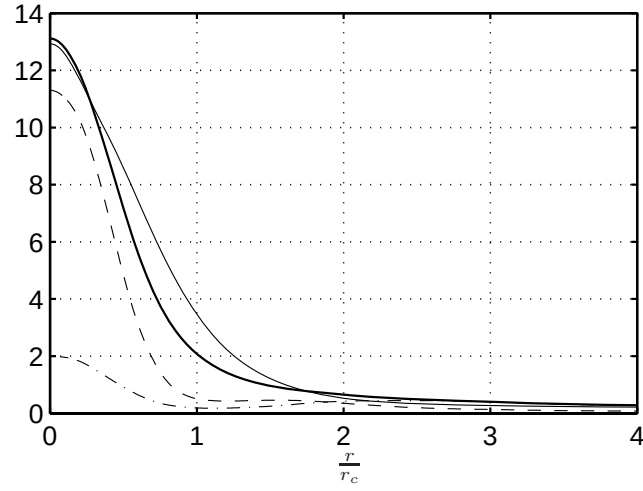


Figure 4.35: Normalized turbulent shear stresses averaged in the (x, θ) directions : $\frac{\overline{u'_r u'_r}}{V_0^2}$ (solid-thin), $\frac{\overline{u'_\theta u'_\theta}}{V_0^2}$ (dash), $\frac{\overline{u'_r u'_\theta}}{V_0^2}$ (dash-dot) and TKE (solid-thick).

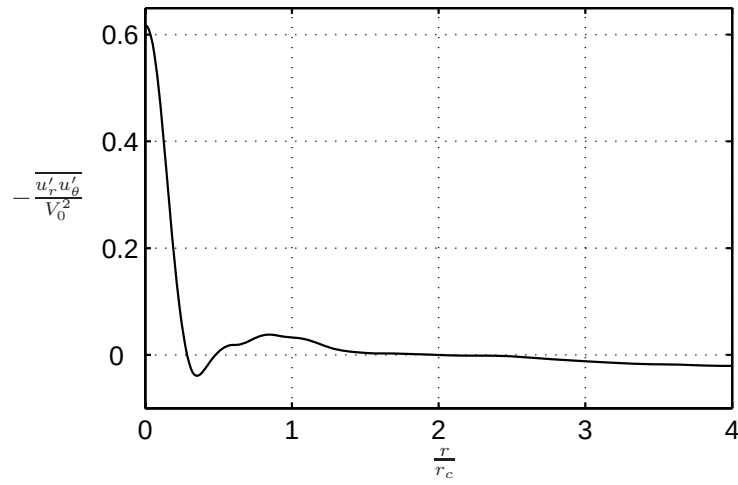


Figure 4.36: Normalized turbulent shear stresses averaged in the (x, θ) directions : $\frac{-\overline{u'_r u'_\theta}}{V_0^2}$.

4.5 Conclusions

In this study, we have performed the LES of a pair of aircraft wake vortices with realistic parameters in very low level atmospheric turbulence. The two SGS models investigated gave results which were consistent with the theory (energy decay slope and inertial range) in the case of a homogeneous isotropic turbulence background.

Both models provide results which are in good agreement with each other regarding the time evolution of global diagnostics of the vortex system put in low level turbulence: energy, dissipation and circulation. The dissipation evolution curve allowed to highlight an important fact: the vortex system creates its own turbulence up to reaching an equilibrium. The level of dissipation of a vortex system at equilibrium can thus be estimated. Moreover the two SGS models yield the same final level of turbulence. This fact can also be deduced observing other diagnostics: the energy of each mode reaches a plateau, the 3D visualization exhibits a turbulence which is much higher than the initial prescribed level. The two SGS models (RVM_s and hyper-viscosity) used to compute this flow seem to perform very well for this class of problems. They provided quantitative results which agree fairly well with each other.

The visualization of axial velocity fluctuations also allows to conclude that their value is significant enough to perform axial Doppler LIDAR measurements, as is done in airborne aircraft wake vortex detection.

The equilibrium energy spectra also exhibits a broad inertial range which is indicative that the flow is fully turbulent. Converged statistics are not easy to obtain as this flow evolves in time and the vortex cores encounter short wave instabilities. Turbulent shear stresses as evaluated when the vortex pair is at equilibrium are consistent with experiments. They can also be used for further turbulence modeling.

Finally, we stress that, for a real turbulent vortex system, the Crow instability (on purpose not permitted in our simulation) will eventually be excited, and the turbulent vortex system will then decay much faster, through vortex reconnection, formation of turbulent vortex rings, etc. This should be considered as future work but would require a computational box with $L_x = L_y = L_z = 8b_0$ leading to a 512^3 grid. Another interesting case would be to perform the same computation using a box which is twice larger in x direction, thus repeating the HIT flow twice along x axis. In this case, the size of the box would be $L_x \times L_y \times L_z = 8b_0 \times 4b_0 \times 4b_0$. This would allow the Crow instability to appear, but in this case, the large scales lose their isotropic character.

Chapter 5

LES of aircraft wake vortices in ground effect, without and with wind.

We propose to investigate the time-developing large eddy simulation (LES) of a longitudinally uniform two-vortex system in ground effect (IGE), at a Reynolds number of 20000. This flow is particularly relevant since it is when the aircraft takes off or lands that the wake vortex hazard is the most important. Furthermore, in the framework of the LASEF project, the case of a ground based LIDAR is under investigation. It is thus natural to simulate this kind of flow in order to use it with the LIDAR simulator described in Chapter 6. These studies were performed together with CENAERO and were also used in the framework of the FP6 european project FAR-Wake. Furthermore, the analysis is in direct continuation with the DNS performed by Duponcheel et al. [38], at a Reynolds number of 5000 without wind: the initial condition, the finite difference code described in Chapter 2 as well as the post-processing tools are indeed the same. An exhaustive review concerning past work and present knowledge of the subject had been carried by Dufresne and Winckelmans [36].

5.1 Introduction

In the study of Duponcheel et al. [38], a direct three-dimensional simulation (DNS) of a pair of vortices interacting with a smooth ground was performed. As the simulations of Luton & Ragab [78] at a low Reynolds show only a weak interaction between the secondary and primary vortices, Duponcheel et al. decided to carry out their simulations at a higher Reynolds number, yet still low enough to avoid the need for a subgrid scale model, i.e., $Re = \Gamma_0/\nu = 5000$. Moreover, most of the previous researches investigated the interaction with the ground of vortices presenting core sizes that are larger than those

of real aircraft vortices. Therefore, a realistic core size for the vortices was chosen. The simulations showed that the secondary vortices are unstable, but the structure of the instability is somewhat different from the results of Luton & Ragab [78], due to the higher Reynolds number and the different core size. The short-wavelength elliptic instability that appears on the secondary vortex (produced by the separation of the boundary layer generated at the ground) grows in amplitude, saturates and eventually causes a strong interaction of the secondary and primary vortices. This finally leads to a turbulent flow field and an enhanced decay rate compared to two-dimensional simulations. The mechanism of the main instability was found to be similar to that observed by Moet [81] in LES at higher Reynolds numbers (yet not *wall-resolved* LES as here). However, this fast decay phase occurs later than observed in the simulations of Moet [81], Proctor et al [97] or measured for real aircraft by Holzäpfel & Steen [55]. This can be partially explained by the *lower growth rate* of the instability due to the lower Reynolds number.

In direct continuation of the DNS by Duponcheel et al. [38], the first objectives of the present work is to increase the Reynolds number. This is done using a *wall-resolved* LES. As the simulations have to be *wall-resolved*, the increased Re is limited to 20000. In fact, the LES is carried out using the same code, a grid refined close to the viscous wall, and the same initial condition. This enables us to investigate the influence of the Reynolds number. As mentioned in Duponcheel et al. [38] and Winckelmans et al. [125], a critical point is to choose a proper subgrid scale (SGS) model. The required SGS models are expected to be inactive during the gentle, well-resolved, phase of the flow, as well as in the laminar vortex cores, and to be active only on the high wavenumbers, during the turbulent phase. Moreover as we are simulating wall bounded flows, the model used should also have the proper wall behaviour. To meet all these constraints, the advanced RVM_s -WALE model defined in Chap. 3 was selected. The performance of this model will be compared with the filtered Smagorinsky model tested by Georges [47] on the same flow.

The comparison of the LES results with the DNS ones at $Re = 5000$ is reported in [47]. For the sake of completeness, we also report the results of this DNS, however the aim here is more to compare the two advanced SGS models: filtered Smagorinsky and RVM_s -WALE.

In order to investigate the influence of the atmospheric conditions, a three-dimensional unsteady turbulent wind is supplemented to the two-vortex system at $Re = 20000$. It means that a complex time-dependent wind presenting physical velocity fluctuations is applied. The present approach is therefore different than simply imposing a time-averaged wind velocity profile. The LES using unsteady turbulent wind is computationally expensive, but is expected to significantly improve the understanding of the physics and instability mechanisms encountered in real atmospheric conditions. A cross-wind (also inducing the so-called tilting phenomenon) is considered. The time-averaged streamwise velocity of the wind at the initial vortices height is taken equivalent to their initial descent velocity out of ground effect, $V_0 = \Gamma_0/2\pi b_0$. This value was se-

lected as such cross-wind is expected to induce the most hazardous trajectory of the upwind vortex. The LES at $Re = 20000$ with wind shows flow properties presenting strong similarities with the LES without wind. However, the detached boundary layers forming the secondary vortex system are unstable faster, through the perturbing effects of the unsteady wind. Therefore, the strong interaction with the primary vortices occurs earlier. Furthermore, the obtained decay rate is significantly higher.

5.2 Description of the flow and numerical setup

The flow consists in a pair of counter rotating vortices evolving under their mutual influence and interacting with the ground in a viscous way. This simulates quite realistically the wake vortex system in the far field of an aircraft flying close to the ground (after completion of the initial roll-up). A careful choice of the computational and physical parameters for such a flow is mandatory and was investigated in [38]. Defining the initial distance between the vortex centers as b_0 , the lengths of the computational domain are $L_x = 4b_0$, $L_y = 8b_0$ and $L_z = 3b_0$. The initial height of the vortex pair is set to $h_0 = b_0$, as illustrated in Fig. 5.1. The domain is periodic in the x and y directions. A no-slip condition is set at the ground ($z = 0$) and a slip condition at the top of the computational domain ($z = 3b_0$). The number of grid points are $N_x = 256$, $N_y = 512$ and $N_z = 256$, leading to a total of 33.5 millions nodes. The grid needed to capture properly the boundary layer is different than that used for the DNS case. The boundary layer developing at the ground is twice thinner for the $Re = 20000$ case than for the case at $Re = 5000$. A clustering of the grid points close to the wall is thus needed to properly capture it. The stretching law of this grid is defined in the following way:

$$\frac{z}{L_z} = \frac{(\beta + 1) - (\beta - 1) \left(\frac{\beta + 1}{\beta - 1} \right)^{(1-\eta)}}{1 + \left(\frac{\beta + 1}{\beta - 1} \right)^{(1-\eta)}}. \quad (5.1)$$

The computational coordinate is η . The parameter β defines the clustering of the points close to $z = 0$. When $\beta \rightarrow 1$, more grid points are clustered near the wall; we set here $\beta = 1.1$. As depicted in Fig. 5.2, there are enough grid points inside the vortex cores to capture their dynamics correctly ($r_c/h \approx 3$). Given the number of nodes, the simulations were performed in parallel, using 32 AMD Opteron 2.6 GHz processors. The simulation of a wake vortex system requires to choose a distribution function to initialize the vortices. As in the DNS at $Re = 5000$, a *low-order algebraic* profile defined in was used. This vortex model is defined by Eqs. (1.3), (1.4) and (1.5), the velocity profile is recalled here:

$$u_\theta(r) = \frac{\Gamma_0}{2\pi r} \frac{r^2}{(r^2 + r_c^2)},$$

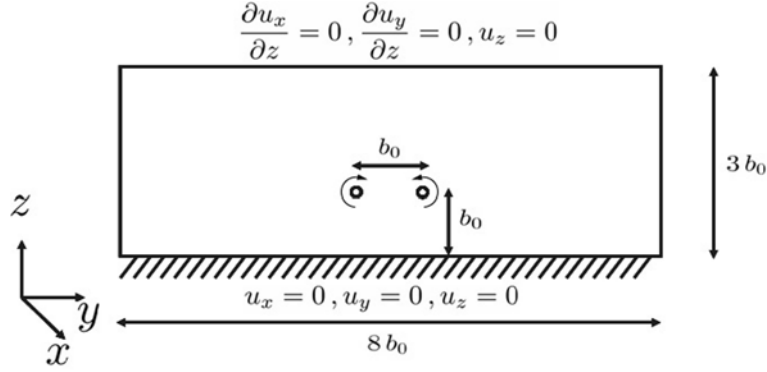


Figure 5.1: Flow setup for the aircraft wake vortices in ground effect. The length of the domain in the x direction is $L_x = 4 \times b_0$

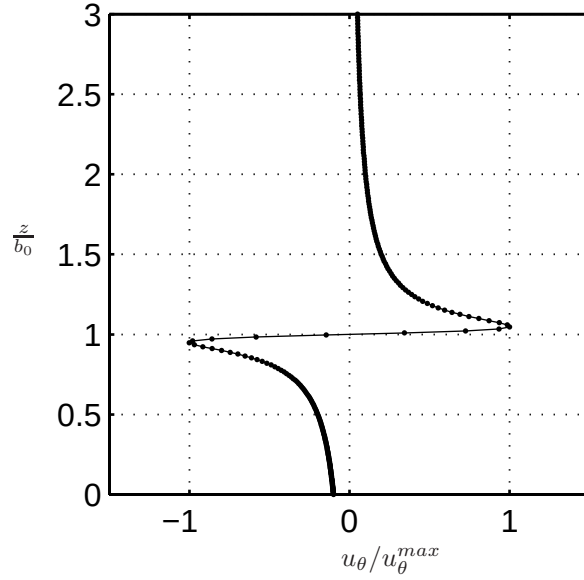


Figure 5.2: Velocity profile with the grid points inside the core

where Γ_0 is the initial circulation of the vortex and r_c is the radius of maximum induced tangential velocity. It is set here to $r_c = 0.05 b_0$ which is a realistic value for aircraft wake vortices. The Reynolds number of this flow is set to $Re \triangleq \Gamma_0 / \nu = 20000$. The velocity scale for this problem is based on the initial

descent velocity of the vortex pair out of ground effect,

$$V_0 = \frac{\Gamma_0}{2\pi b_0}. \quad (5.2)$$

This also defines the non-dimensional time $\tau \triangleq t \frac{V_0}{b_0}$. The initial velocity, $\vec{u}^{BS}$, at each point of the Oyz plane is computed according to the Biot-Savart law, taking into account a finite number (typically 50) of image vortex pairs, due to the periodicity in the spanwise direction (y) and the mirror vortex pairs in the direction perpendicular to the ground (z). The 2-D field is then extruded in the x direction. Finally, in the simulations without wind, a 3D random perturbation is added to this velocity field

$$\vec{u}^v = \vec{u}^{BS} + u_\theta(r_c) \vec{\lambda}. \quad (5.3)$$

where $\vec{u}^v$ is the resulting velocity field. The amplitude of the perturbation λ_i is a random variable with uniform distribution in $[-0.001, 0.001]$. This perturbation level is 10 times lower than the value reported in the paper of Duponcheel et al. [38].

The SGS model used in the present work is the RVM_s -WALE model presented in Section. 3.9.

5.3 Case without wind

5.3.1 Flow visualisations

The temporal 3D evolution of the normalized vorticity field is displayed in Fig. 5.3. An axially averaged normalized vorticity field visualization is also provided in Fig. 5.4. The flow is evolving in the following way: due to the mutually induced velocity, the vortex pair moves down. The impermeability condition at the wall can be modeled by a mirror vortex pair, resulting in an hyperbolic trajectory. However, as the induced tangential velocity at the wall is nonzero, the no-slip condition is at the origin of the development of boundary layers beneath the vortices. Those are clearly visible in Fig. 5.3. Starting from the stagnation point on the symmetry plane between the vortices, the pressure gradient forcing the boundary layers is first favorable and then adverse. This leads to their separation, when the primary vortices are close enough to the wall. The detached vortex-sheet then rolls-up and forms two coherent secondary vortices (see Fig. 5.3). They induce an upward velocity on the primary vortices and force the latter to rise; this is the rebound phenomenon. At this time, the flow is still essentially two-dimensional. But the secondary vortices are unstable with respect to short-wavelength modes. These perturbations are growing while the secondary vortex is orbiting around the primary vortex, as can be seen in Figs. 5.3 at times $\tau = 2$ and $\tau = 3$. The instability mechanism can be related to the elliptic instability of Widnall. At $\tau = 3$, the secondary vortex has lost its coherence, and its vorticity is distributed around the primary

vortex which is itself very disturbed (see Fig. 5.4). The flow eventually evolves to a fully turbulent vortex system.

Fig. 5.5 compares axially averaged subgrid scale viscosity with the rms of the vorticity fluctuations. The two colormaps exhibit the same topology meaning that the model yields subgrid scale viscosity where vorticity fluctuations are present. The model does not act in the boundary layer as this one is still laminar at this early time $\tau = 3.0$ see also Fig. 5.6. This observation remains valid for other times.

Fig. 5.6 shows the longitudinally averaged axial vorticity profiles at the wall $\langle \omega_x^w \rangle^*$. As this quantity is an image of the wall shear stress, the zeros of these profiles give the locations at which the boundary layers separates and re-attaches. As the flow is becoming turbulent, the profiles loose their symmetry and coherence.

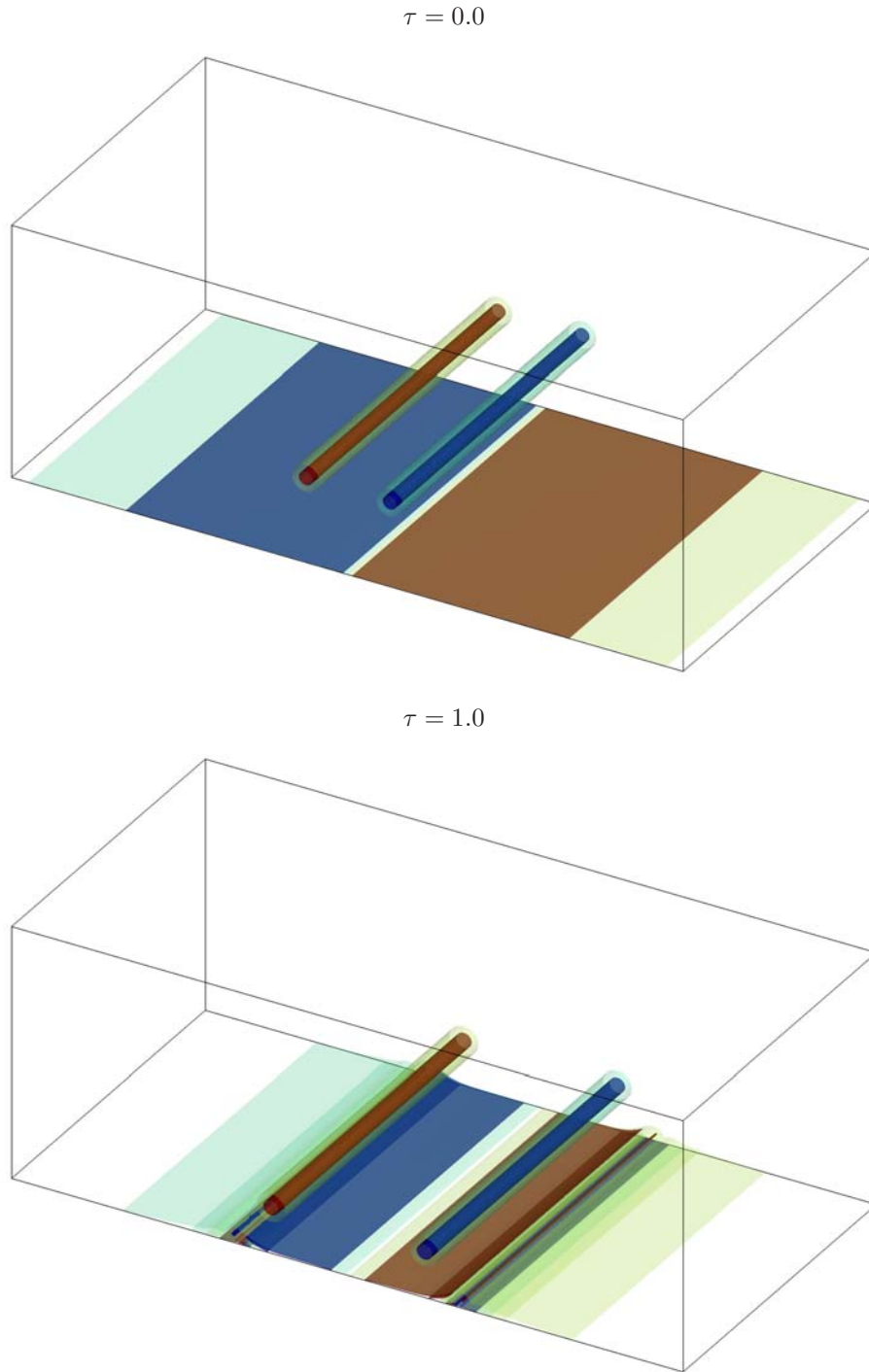
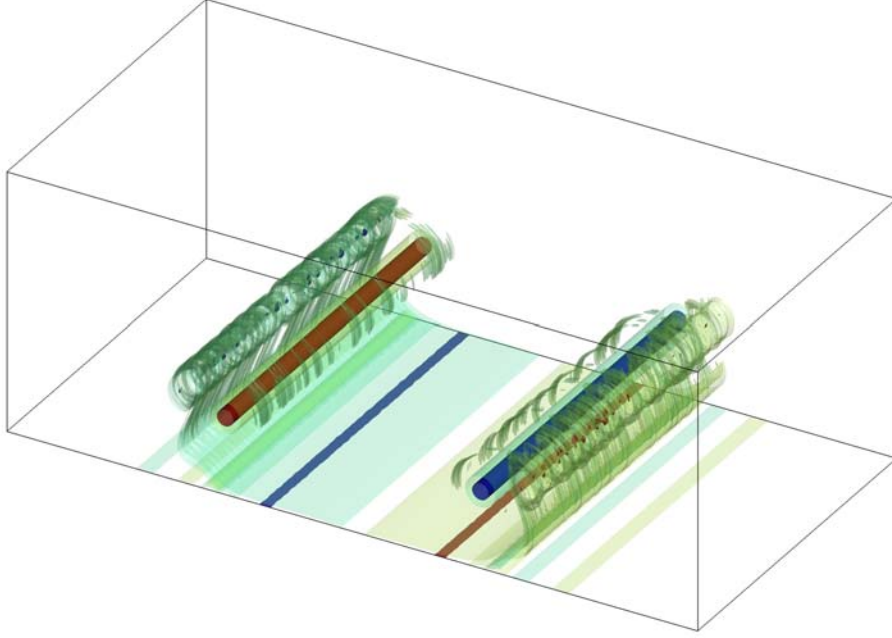


Figure 5.3: Visualization of the flow field using isosurfaces of $\|\vec{\omega}\|b_0^2/\Gamma_0 = 1$ and 10.

$$\tau = 2.0$$



$$\tau = 3.0$$

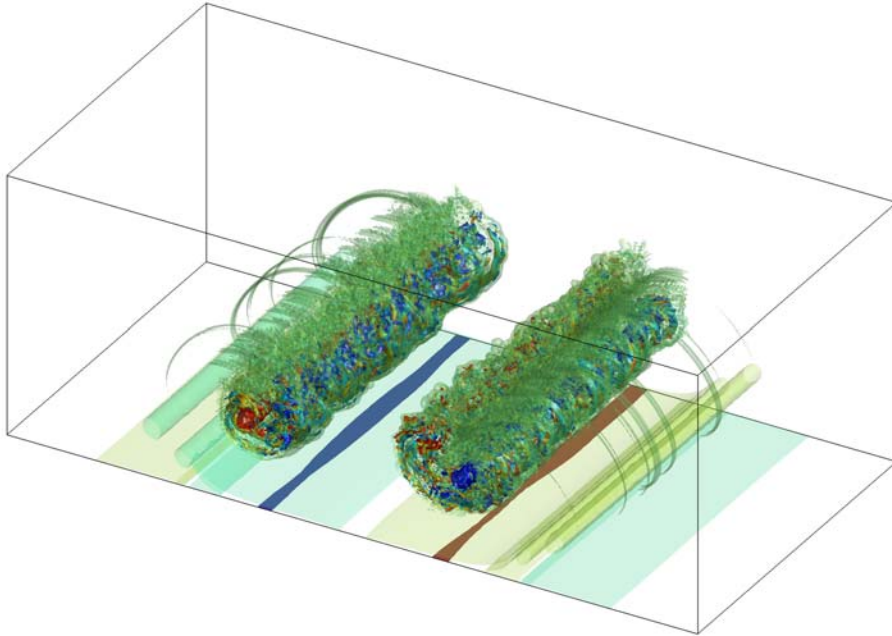


Figure 5.3: (Continued) $\|\vec{\omega}\|b_0^2/\Gamma_0 = 1$ and 10.

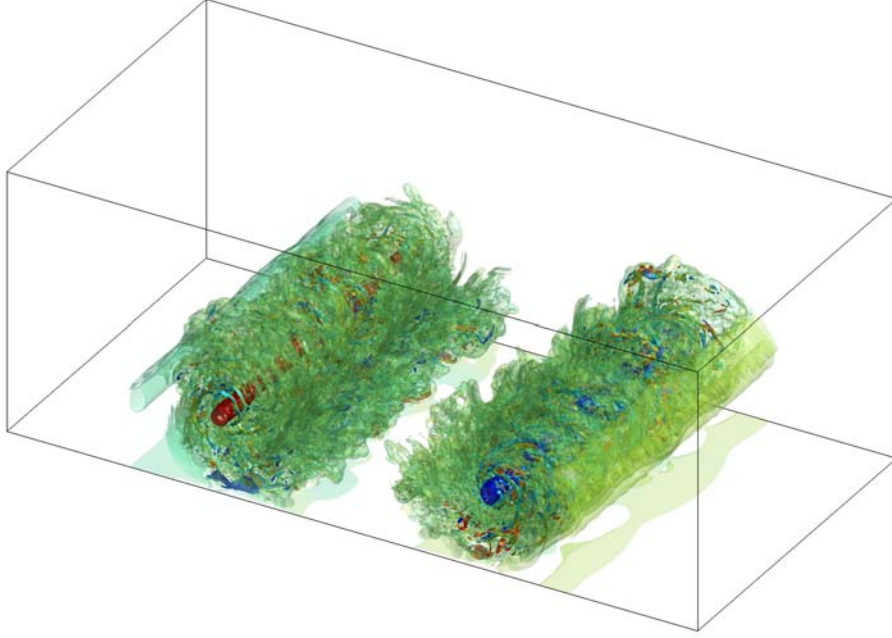
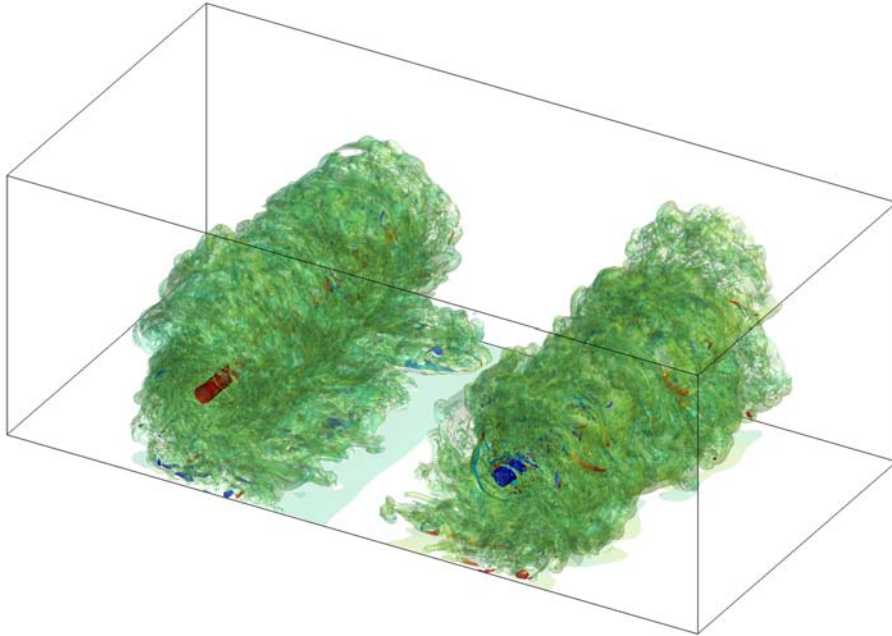
$\tau = 4.0$  $\tau = 5.0$ 

Figure 5.3: (Continued) $\|\vec{\omega}\|b_0^2/\Gamma_0 = 1$ and 10.

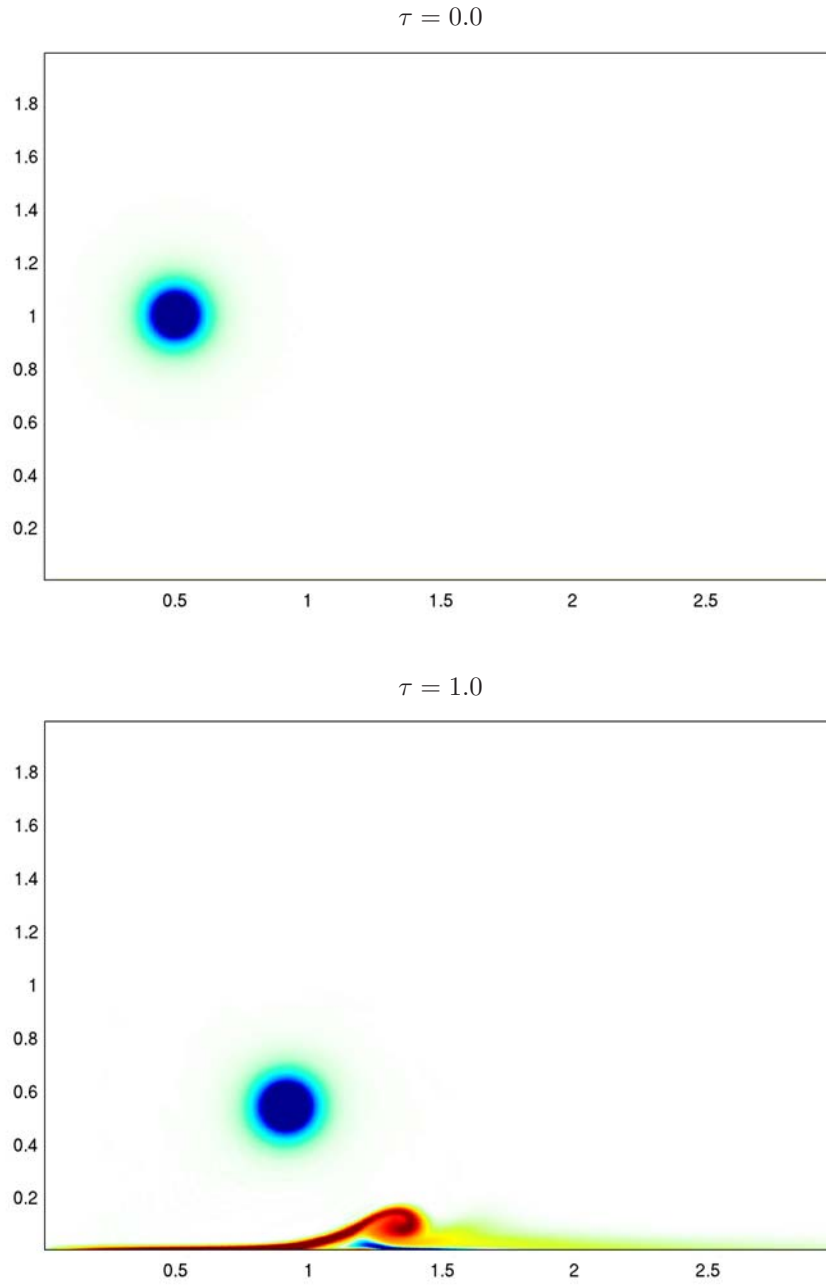


Figure 5.4: Visualization of the longitudinally averaged axial vorticity.

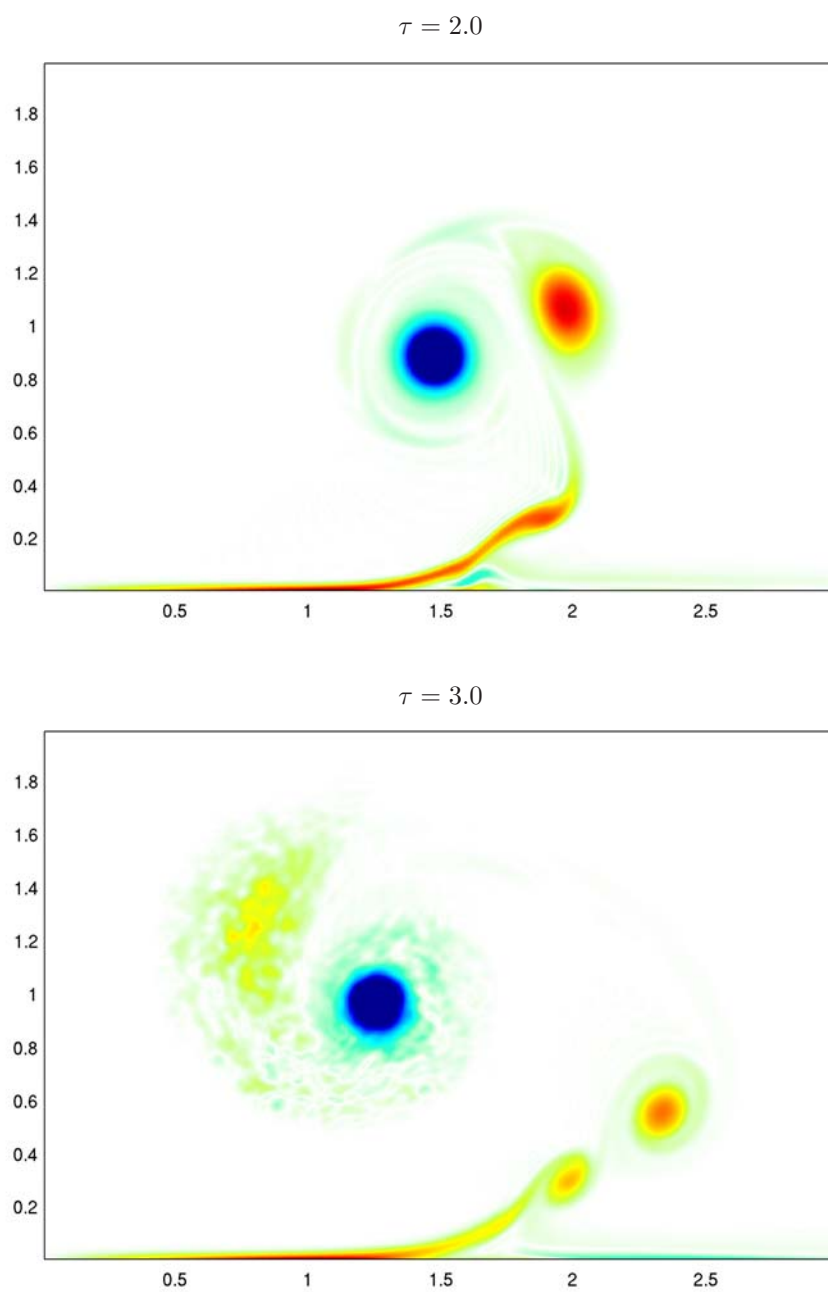
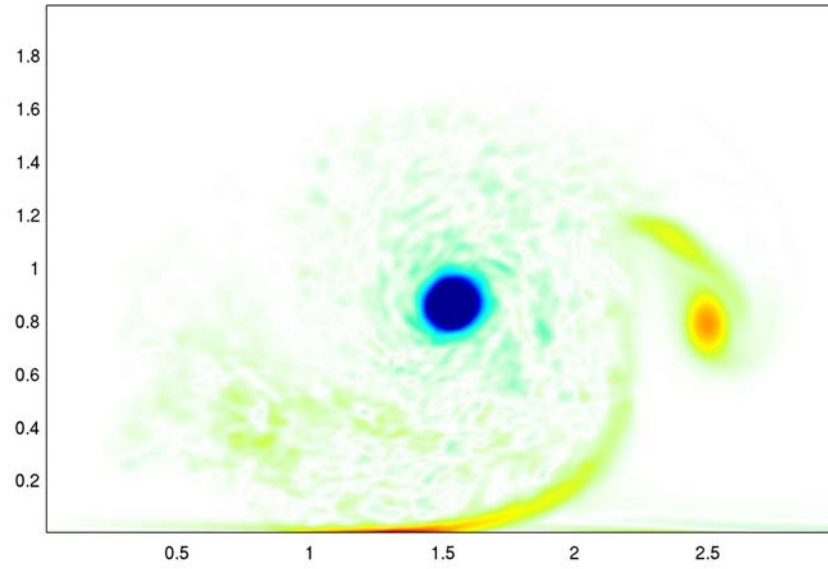


Figure 5.4: (Continued).

$$\tau = 4.0$$



$$\tau = 5.0$$

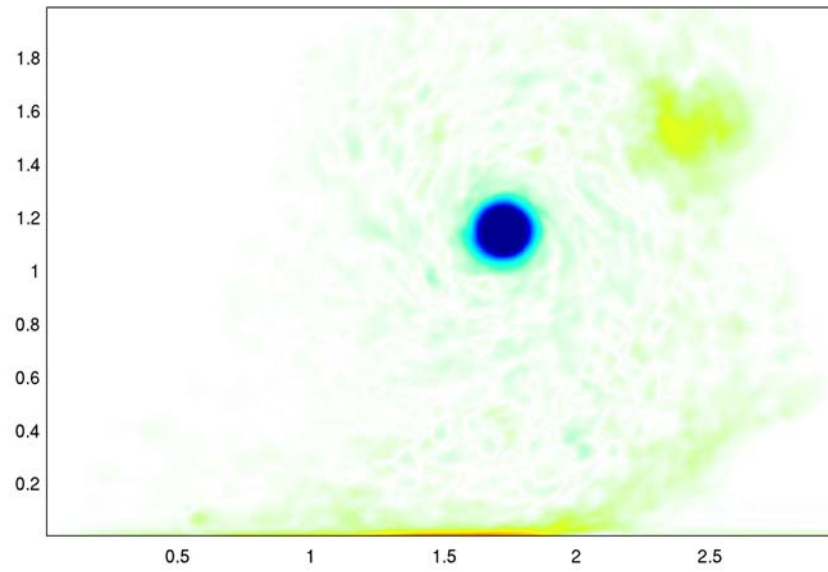


Figure 5.4: (Continued) .

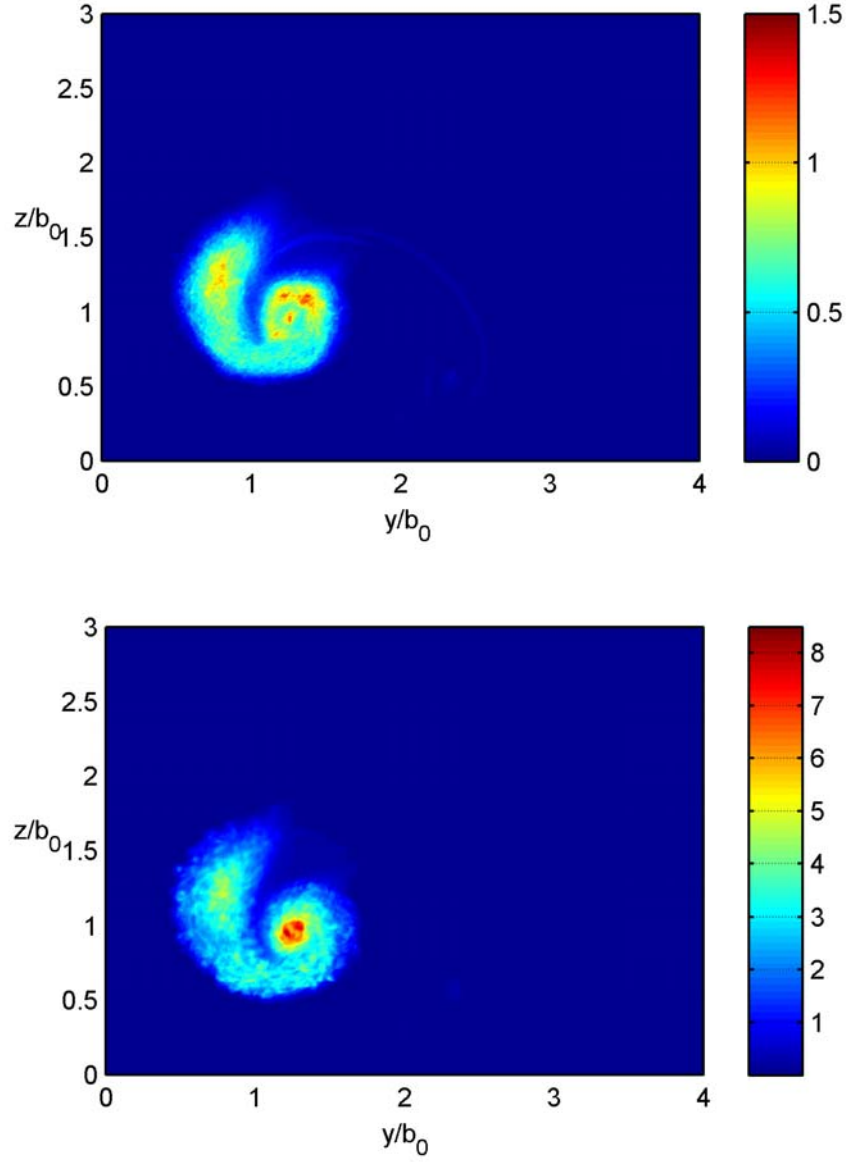


Figure 5.5: Comparison, at $\tau = 3.0$, of the longitudinally averaged SGS viscosity ν_{sgs}/ν (top) and of the rms axial vorticity $\sqrt{\omega_x'^2} b_0^2 / \Gamma_0$ (bottom).

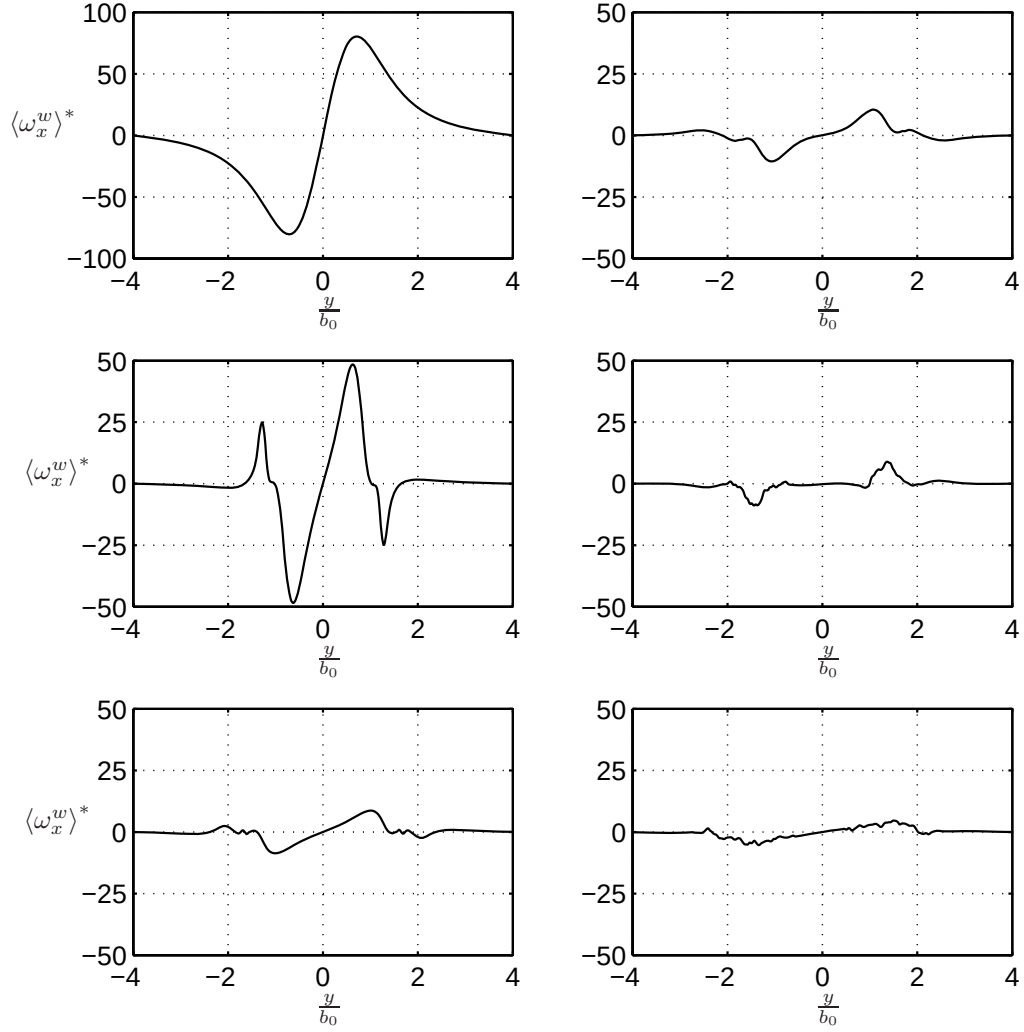


Figure 5.6: Longitudinally averaged axial wall vorticity profiles $\langle \omega_x^w \rangle^* = \langle \omega_x^w \rangle \frac{b_0^2}{\Gamma_0}$. From top to bottom and from left to right: $\tau = 0, 1, 2, 3, 4, 5$. The scale at $\tau = 0$ is larger for clarity.

5.3.2 Evolution of the vortex strength

Fig. 5.7 provides the evolution of the kinetic energy. We compare the 2D DNS, as described in Appendix A with the results obtained using the new RVM_s - WALE model presented in this work, the filtered Smagorinsky model with a Piomelli damping and the classical Smagorinsky model with a Piomelli damping used by Georges [46]. The RVM_s - WALE and the filtered Smagorinsky models are inactive during the first laminar part of the simulation: the obtained curves are indeed very close to the 2D DNS. On the contrary, the classical Smagorinsky model removes a relevant part of the resolved kinetic energy. This is the major motivation that lead us to the implementation and use of advanced SGS models. From $\tau \approx 2.8$, the energy evolution departs from the 2D case. In fact, the dissipation rate of the 3D case increases dramatically because the interaction between the secondary and primary vortices generates small three-dimensional structures. Although the model choice plays an important role on the energy level, there is no significant influence on the fast decay starting time. Furthermore, when high wavenumbers become saturated, the dissipation rate is found to be equivalent between both models. Moreover, the decay curves obtained with RVM_s - WALE model and the filtered Smagorinsky one are very close to each other.

The evolution of Γ_{5-15} , defined in Section 4.4.5 and reported in Fig. 5.8, is quite different. The initial 2-D decay phase is governed by the molecular viscosity and is due to the slow spreading of the core size. The first decay phase is negligible (it was not the case for $\text{Re} = 5000$ as showed by Duponcheel et al. [38]). Let's mention that the curves of the three SGS models are seen to be quite close during this initial phase. As for the energy evolution, the fast decay phase starts at $\tau \approx 2.8$, and independently of the SGS model. This starting time happens significantly earlier than in the DNS at $\text{Re} = 5000$ [38]. This was also observed by Moet [81], Proctor et al. [96] and also by Holzäpfel and Steen. [55]. The fast decay rate of the classical Smagorinsky model is at first more important than the multiscale ones, and finishes at a similar level. The Γ_{5-15} being an important hazard criterion, the previous behaviour again emphasizes the importance of proper SGS modeling.

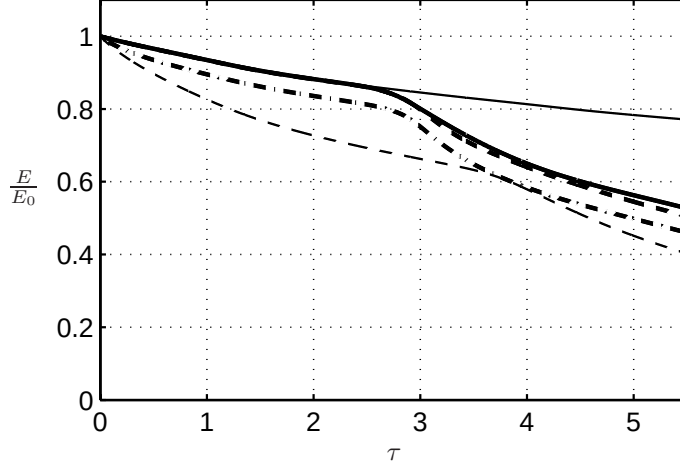


Figure 5.7: Temporal evolution of the kinetic energy: DNS-2D (solid-thin); RVM_s - WALE (solid-thick); Filtered Smagorinsky with Piommelli damping (dash-thick); Smagorinsky with Piommelli damping (dash-dot-thick); DNS at $Re = 5000$ (dash-thin).

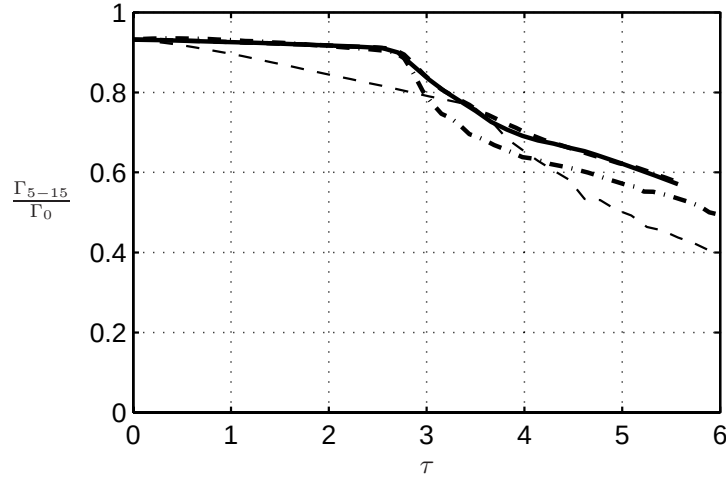


Figure 5.8: Temporal evolution of Γ_{5-15} : RVM_s - WALE (solid-thick); Filtered Smagorinsky with Piommelli damping (dash-thick); Smagorinsky with Piommelli damping (dash-dot-thick); DNS at $Re = 5000$ (dash-thin).

5.3.3 Modal analysis

The modal analysis is particularly important in order to understand the difference between the two-dimensional evolution and the three-dimensional one. Three-dimensional instabilities described above come from the deformation of the primary and the secondary vortices. These instabilities grow and evolve in time. This can be properly investigated by a modal analysis. In the modal space, the discrete Fourier transform of the velocity field in the axial direction is:

$$\widehat{\mathbf{u}}(k_m, y_j, z_l) = \frac{1}{N_x} \sum_{i=1}^{N_x} \mathbf{u}(x_i, y_j, z_l) e^{-I i k_m L_x / N_x} \quad (5.4)$$

where $I^2 = -1$ and k_m is the wavenumber (with $m = 1, 2, \dots, N_x/2$). The discrete kinetic energy is thus, in the Fourier domain,

$$\widehat{E}(k_m, y_j, z_l) = \frac{1}{2} \left(|\widehat{u}_x(k_m, y_j, z_l)|^2 + |\widehat{u}_y(k_m, y_j, z_l)|^2 + |\widehat{u}_z(k_m, y_j, z_l)|^2 \right). \quad (5.5)$$

The average energy of each mode in the axial direction is then

$$E(k_m) = \frac{1}{L_y L_z} \sum_{j=1}^{N_y} \sum_{l=1}^{N_z} \left(\widehat{E}(k_m, y_j, z_l) \Delta y_j \Delta z_l \right). \quad (5.6)$$

The wavelength of each mode is $\lambda_m = \frac{L_x}{m} = \frac{2\pi}{k_m}$. Following Laporte [71], one also defines the growth rate σ of the instabilities by:

$$\sigma^* = \sigma \frac{b_0}{V_0} = \frac{1}{2} \frac{d}{d\tau} \log \left(\frac{E(k_m)}{E_0(0)} \right) \quad (5.7)$$

where $E_0(0)$ is the modal energy of the base mode at time $t^* = 0$. Fig. 5.9 describes the evolution of the modal energy for some characteristic wavenumbers. During the descent, all the modes are stable and their energy decreases. When the separation of the boundary layer occurs ($\tau \approx 1.0$), some wavenumbers become unstable. In the DNS of Duponcheel et al. [38], the growth rate σ^* of the instabilities is approximatively equal to $\sigma^* \approx 3$. In the LES at $Re = 20000$, Fig. 5.9 shows a growth rate of $\sigma^* \approx 4.5$. This is in good agreement with the results of Georges [46] using the filtered Smagorinsky model.

5.3.4 Analysis of the vortex trajectory

Fig. 5.10 compares the trajectories for the center of the right-hand side primary vortex, as obtained by the DNS and two LES simulations. The center is defined as the maximum of the longitudinally averaged axial vorticity. During the first part of the descent, all the trajectories are equivalent, as they correspond to the inviscid theory. As soon as the boundary layer starts to separate, the rebound occurs and the trajectories are significantly different between the DNS at $Re = 5000$ and the three LES at $Re = 20000$. The loop trajectory induced

by the rebound presents a smaller radius at $Re = 20000$. From Duponcheel et al. [38], the $Re = 5000$ trajectory is hitherto equivalent to results from two-dimensional simulations. Nevertheless, after $\tau \approx 3$, corresponding to the strong 3D interaction between the secondary and primary vortices, the velocity induced by the secondary vorticity is much modified in the 3D case, because the secondary vortex has lost its coherence. The computed trajectory is irregular as the primary vortex is turbulent so that the position of the maximum of vorticity fluctuates around a mean position. Moreover, the initial perturbation being asymmetric, asymmetric trajectories are also observed. At $Re = 20000$, the trajectory is less irregular and is nearly independent of the SGS model used. Again, for this diagnostic the RVM_s -WALE model and the filtered Smagorinsky yields results which are very close to each other. Numerical two-dimensional and three-dimensional investigations performed in [37] have also shown that the trajectories are far from being independent of the domain size, because the boundary conditions affect significantly the flow and especially the boundary layers. They allowed to show that the confinement effect for the domain we use here is sufficiently low and that it affects only very weakly the instability mechanism.

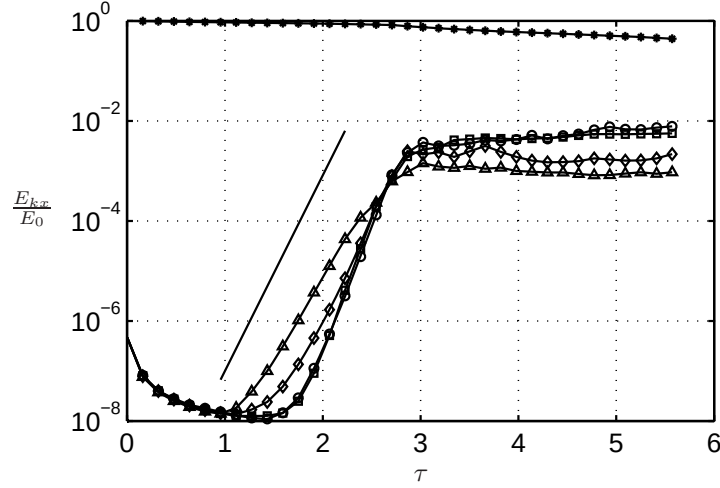


Figure 5.9: Time history of the modal energy $E(k)/E(k=0)$ for the modes $\frac{k b_0}{2\pi} = 0$ ($\bullet$), 0.5 ($\circ$), 1.5 ($\square$), 2.75 ($\diamond$) and 4 ($\triangle$). The thin line indicates the slope $\sigma^* = 4.5$.

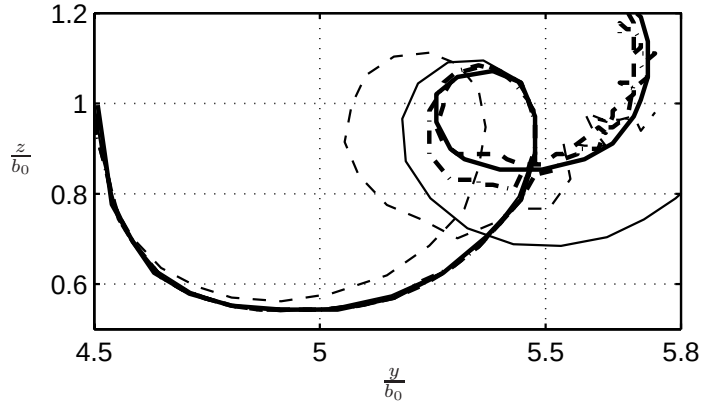


Figure 5.10: Trajectories of the primary vortex based on the maximum of vorticity: DNS-2D (solid-thin); RVM_s - WALE (solid-thick) ; Filtered Smagorinsky with Piomelli damping (dash-thick); Smagorinsky with Piomelli damping (dash-dot-thick); DNS at $Re = 5000$ (dash-thin).

5.4 Case with cross wind

The influence of a cross wind is investigated. Another study of this flow was performed by [47]. However the computational box used was quite narrow in the spanwise direction, $L_y = 8b_0$, leading to unwanted periodicity effects. We decided to double this length using $L_y = 16b_0$. The simulation is quite big to achieve as the computational box includes $N_x \times N_y \times N_z = 256 \times 1024 \times 256 = 67 \times 10^6$ nodes. The computation ran on 64 AMD opteron 2.6 GHz processors during two weeks. In the first part of this section, we present the DNS of a turbulent wind simulated by a half channel flow. In the second part of this section, we provide visualizations of this flow and analyze some quantitative results such as the energy and Γ_{5-15} evolutions. The RVM_s-WALE model was used and compared with the filtered Smagorinsky model of [47].

5.4.1 Simulation of a turbulent boundary layer at $Re_\tau = 590$

There are several ways to take into account the impact of a cross wind on a pair of aircraft wake vortices in ground effect. One can choose to superimpose an analytical velocity profile to the wake vortex flow but it is quite unrealistic. We choose, in the present work, to superpose the base flow with a turbulent boundary layer flow simulated on the same grid. The turbulent boundary layer is obtained as a pre-simulation and the aircraft wake vortices are inserted in this flow. One elegant way to obtain a realistic turbulent boundary layer on a flat plate is to compute a “half channel flow” (this leads however to a flow which is confined in the wall normal direction in opposition to a true boundary layer which grows in the streamwise direction). The boundary conditions are the same as these defined in the case without wind. As the Reynolds number of the vortex flow is defined $Re = \frac{\Gamma_0}{\nu} = 20000$, the viscosity is fixed. We have to impose a forcing on the flow to obtain a realistic turbulent boundary layer which mimics an atmospheric cross wind. For that purpose, we define the ratio between the initial descent speed of the vortex pair $V_0 = \frac{\Gamma_0}{2\pi b_0}$ and the wind speed U_w at the initial height of the vortex pair. We choose to fix this ratio to one.

$$\alpha_V = \frac{V_0}{U_w(z = b_0)} = 1 \quad (5.8)$$

A preliminary look at the order of magnitudes allows to choose the parameters of the channel flow simulation. The symbol convention is the same as the one defined in Chap. 2. The channel half width is thus $\delta = L_z = 3b_0$. If we assume that the locus $z = b_0$ lies in the logarithmic zone of the boundary layer, we can write:

$$\frac{V_0}{u_\tau} = \left(\frac{1}{\kappa} \log \left(\frac{b_0 u_\tau Re}{\Gamma_0} \right) + 5.5 \right). \quad (5.9)$$

By solving this equation, one finds a friction velocity u_τ which leads to $Re_\tau = 530$. This allows to check that $z^+(b_0) \approx 175$ so that the initial z location of

the vortices lies indeed in the logarithmic region of the turbulent boundary layer. The magnitude of the pressure gradient is thus known. In fact, the Re_τ is slightly increased, to 590, in order to compare our flow statistics with the DNS of Moser et al. [84], the standard channel flow reference. The consistency of the wind turbulent fluctuations can thus be properly validated. The mesh selected is the same as that described in Section 5.2. In fact, this mesh is sufficiently fine to perform a DNS of the half-turbulent channel flow at $Re_\tau = 590$. First, considering the uniform grid spacing in x and y directions, as well as the minimum and maximum spacings in the wall-normal direction, respectively,

$$\Delta x, \quad \Delta y, \quad \Delta z_{min}, \quad \Delta z_{max}, \quad (5.10)$$

the present mesh leads to the following resolution in term of wall units

$$\Delta x^+ = \Delta y^+ = 3.1, \quad \Delta z_{min}^+ = 0.67 \quad \text{and} \quad \Delta z_{max}^+ = 3.8. \quad (5.11)$$

This grid is almost twice finer than that of the DNS by Moser et al. [84], performed using spectral methods. This increase of mesh quality, yet important, is mandatory to reach an equivalent resolution using fourth-order accurate finite differences, as done here. The computational domain has the following lengths in wall units,

$$L_x^+ = 787, \quad L_y^+ = 1573 \quad \text{and} \quad L_z^+ = 590. \quad (5.12)$$

Jimenez and Moin [63] recommend the following minimal periodicity lengths that would sustain turbulence:

$$\lambda_x^+ = 100 \quad (5.13)$$

$$\lambda_y^+ = 250 - 350 \quad (5.14)$$

In the present case investigated, we use:

$$\lambda_x^+ = 704 (L_x = 4 b_0) \quad (5.15)$$

$$\lambda_y^+ = 1400 (L_y = 8 b_0) \quad (5.16)$$

For LES of wake vortices IGE at $Re > 20000$, near wall modeling will be required as the number of grid points would increase out of reach of the present computational resources. The obtained streamwise mean velocity profile is displayed in Fig. 5.12. The agreement with the results of Moser et al. [84] is very good even with a symmetry boundary condition which implies that the flow is different at the top boundary compared to the center of a full channel flow. The turbulent stress is also very close to the reference results, see Fig. 5.13. Examining turbulence intensities in Fig. 5.14, shows discrepancies with the reference results. As the vertical component of the velocity is set to $w = 0$ at the top boundary, its fluctuations are also zero. The turbulent energy is however redistributed among u' and v' so that the turbulent kinetic energy profile stays roughly the same as illustrated in Fig. 5.15. The turbulence production profile,

$$G^+ = -\frac{\langle u'v' \rangle}{u_\tau^2} \frac{d\langle U^+ \rangle}{dy^+}, \quad (5.17)$$

is displayed in Fig. 5.16. The turbulence production reaches its maximum in the buffer region for $z^+ \approx 15$. This curve rises steadily from $G^+ = 0$ at the wall to the peak value and then declines with a $1/y^+$ behaviour in the log layer. The results are also fully consistent with those of Moser et al. [84].

A snapshot of the wind flow structures is presented in Fig. 5.17. This illustrates the complexity of the flow in which the vortices are inserted.

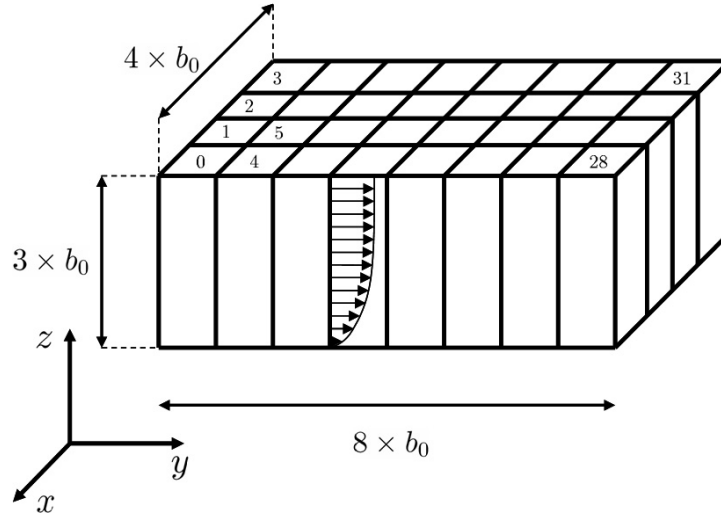


Figure 5.11: Decomposition of the computational domain for the DNS of a turbulent boundary layer at $Re_\tau = 590$. The decomposition of the domains assigned to each processors is also shown.

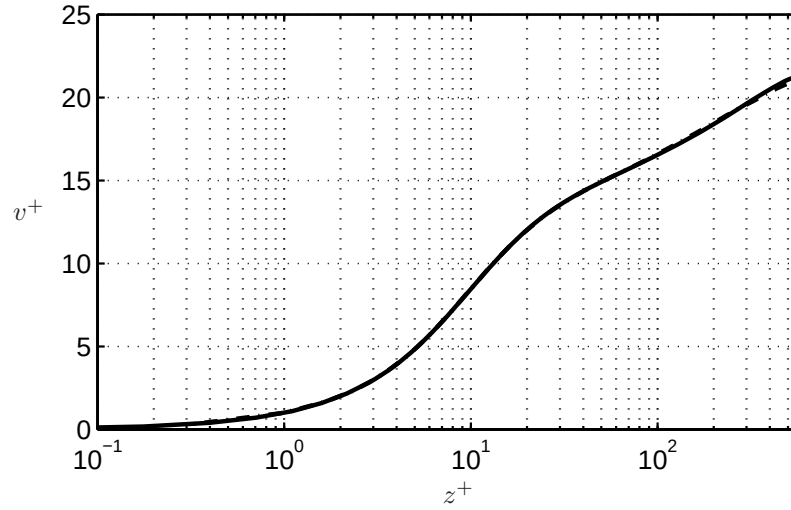


Figure 5.12: Mean streamwise velocity profile. Present work (dash-dot); reference solution of Moser et al. for the full channel flow (solid).

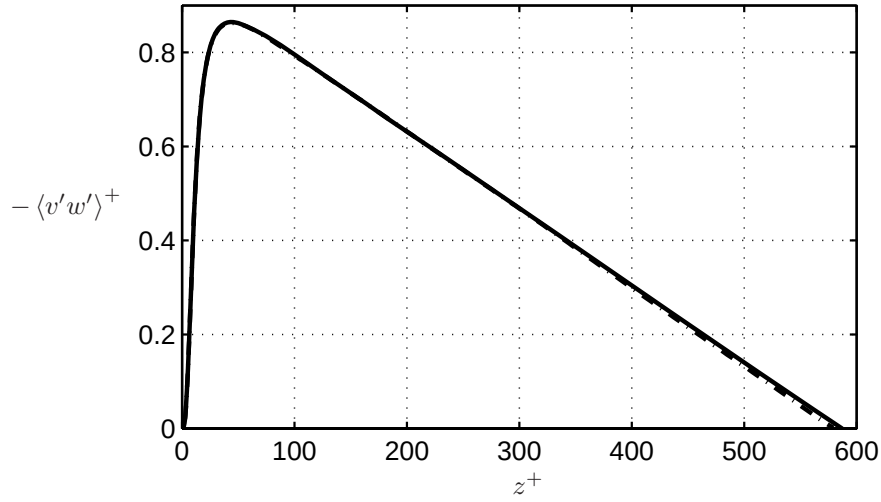


Figure 5.13: Turbulent shear stress. Present work (dash-dot); reference solution of Moser et al. for the full channel flow (solid).

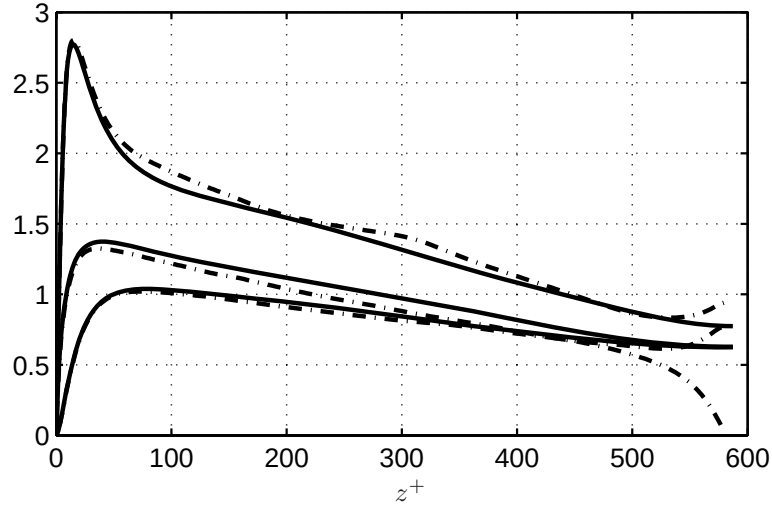


Figure 5.14: Velocity fluctuation profiles. Present work (dash-dot); reference solution of Moser et al. for the full channel flow (solid). top: $\langle u'u' \rangle^+$, middle: $\langle w'w' \rangle^+$, bottom: $\langle v'v' \rangle^+$.

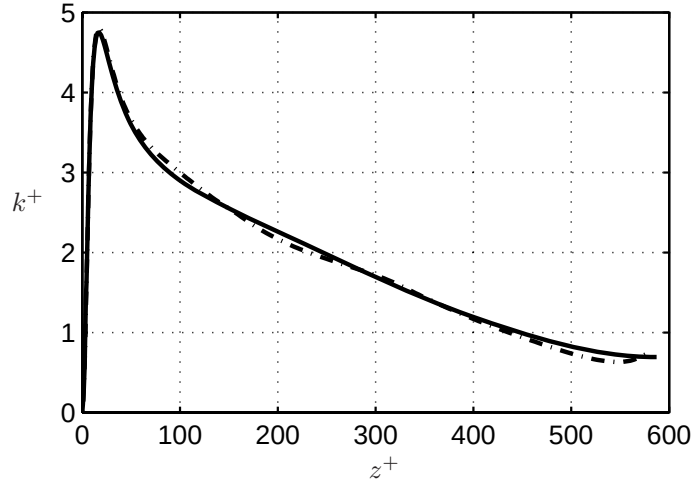


Figure 5.15: Turbulent kinetic energy profile. Present work (dash-dot); reference solution of Moser et al. for the full channel flow (solid).

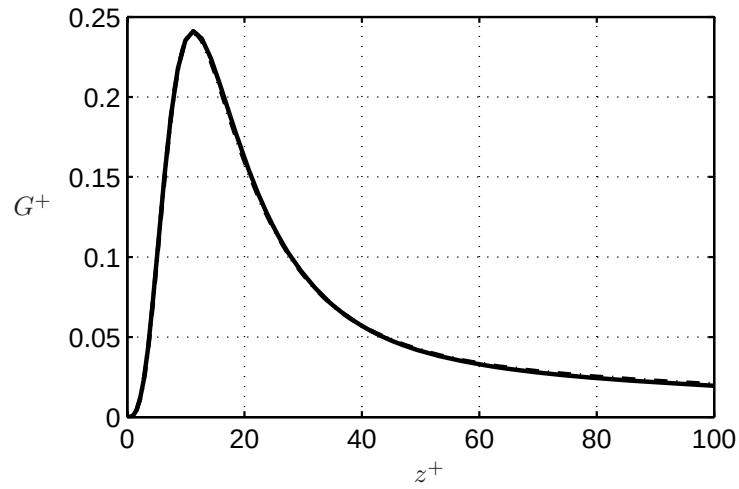


Figure 5.16: Turbulence production profile. Present work (dash-dot); reference solution of Moser et al. for the full channel flow (solid).

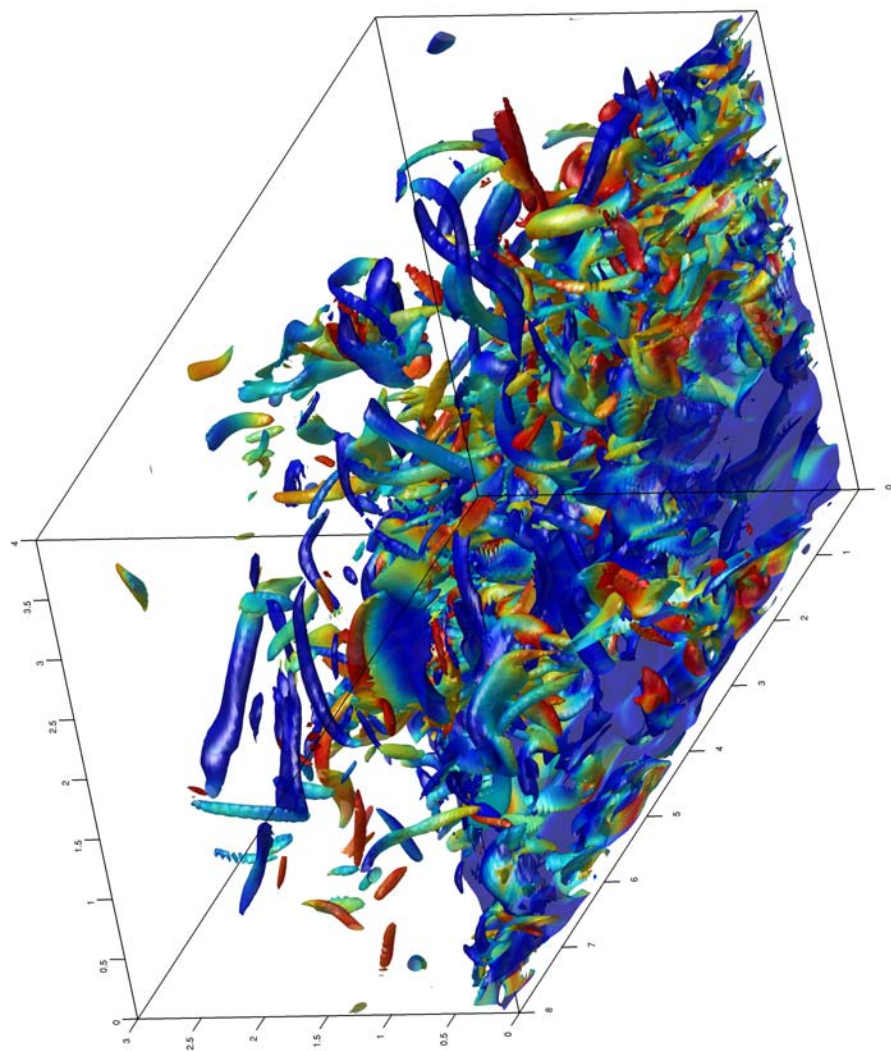


Figure 5.17: Snapshot of the wind vorticity field. One isosurface of the normalized vorticity norm $\|\omega\|u_\tau/H = 100$ is displayed, the colors vary according to the spanwise vorticity component ω_z . The flow is going toward the page.

5.4.2 Flow visualisation

The time evolution of the two-vortex system in ground effect with a cross-wind is now investigated. The temporal development of the flow is displayed in Fig. 5.18. The symmetry is here lost and the upwind and downwind vortices present different behaviours. The initial condition, at $\tau = 0.0$, is obtained by superimposing the cross-wind obtained in Section 5.4.1 to the initial vortex pair. In order to avoid periodicity effects in the streamwise y direction, the turbulent wind is duplicated in the y direction. This leads to a new computational domain with $L_y = 16b_0$. The same forcing as the one used in the half channel simulation is applied in the y direction. Turbulent vortical structures of the wind are clearly visible near the ground and are aligned with the y periodic direction. Again, the no-slip condition at the ground leads to the formation of boundary layers. The near-wall vorticity of the cross-wind has an opposite sign compared to the boundary layer generated by the upwind vortex so that the magnitude of this vortex sheet is significantly lowered. At the opposite, the contribution is positive for the downwind vortex and the magnitude is increased. A longitudinal vortex meandering is also induced by the cross-wind: the primary vortices are deformed by a mode with a wavelength $\lambda = L_x$ as observed in Fig. 5.18, $\tau = 1.3$. Let's remember that this phenomenon is here limited by the size of the computational box. Again, a roll-up process of the wind turbulent structures occurs, as well as a “cleaning” of the structures between the upwind and the downwind vortices, see Fig. 5.18 and Fig. 5.19 at time $\tau = 1.3, 2.0$. Again, the boundary layers generated by the vortex pair and the wind are subject to an adverse pressure gradient that finally leads to separation and the generation of a counter-rotating secondary vortex pair, as observed in Fig. 5.18 and Fig. 5.19 at time $\tau = 1.3, 2.0$. The positive contribution of the cross-wind to the downwind boundary layer favors its separation. On the contrary, the separation is delayed for the upwind secondary vortex, because of the opposite sign of vorticity due to the cross-wind. The evolution of the secondary vortices are thus different in time and in magnitude leading to an asymmetric rebound phenomenon, named tilting. Furthermore, this tilting of the primary vortex pair can also be enhanced by the wind shear itself. As for the case with head-wind, the secondary vortices are directly unstable after separation, because of the perturbing effects of the unsteady cross-wind. The secondary vortices being unstable, the interaction mechanism with the primary vortex pair is then similar to the LES without wind. During this interaction process, the trajectory of the primary vortices can already be commented using the longitudinally averaged streamwise vorticity pictures, Fig. 5.19. The tilting is indeed better observe in Fig. 5.19. Analyzing the vortex core locations, it is clear that the upwind primary vortex remains located, for a long period ($\tau \leq 3.0$), between the initial location of the vortex pair. From an operational point of view, this is a “hazardous situation” where the upwind primary vortex stays on the runway and may cause trouble to a following aircraft. This was in fact the main motivation to choose the present cross-wind, with $U_w(h_0) = V_0$,

for investigation. The rebound of the downwind vortex is more pronounced than that of the upwind vortex, and this vortex is efficiently transported away from the runway. The downwind vortex evolution is also important as it may interfere with a neighboring runway. It is therefore important to investigate whether the intensity of this vortex is decreased significantly. In opposition with the case without wind, new secondary vortices are not created from successive boundary layer separations. Nevertheless, a certain amount of vorticity is again shed from the ground but is not coherent. At the end of the evolution, the turbulence is also located near the ground, Fig. 5.18 and Fig. 5.19 at $\tau = 4.0, 5.0$. As for the previous LES, each vortex can be considered as isolated by its own surrounding turbulence and no further structures will be emitted from the wall. The turbulence “cloud” surrounding the vortices grows in time and the height of the computational box eventually has a blocking effect so that the vortices can no longer evolve freely. This can be observed in Fig. 5.19. As a consequence, the size of the computational box possibly has a relevant influence on the evolution of the two-vortex system during the last phase of the simulation. Nevertheless, this effect can be clearly neglected for $\tau < 4$, leaving the aforementioned discussion unaffected.

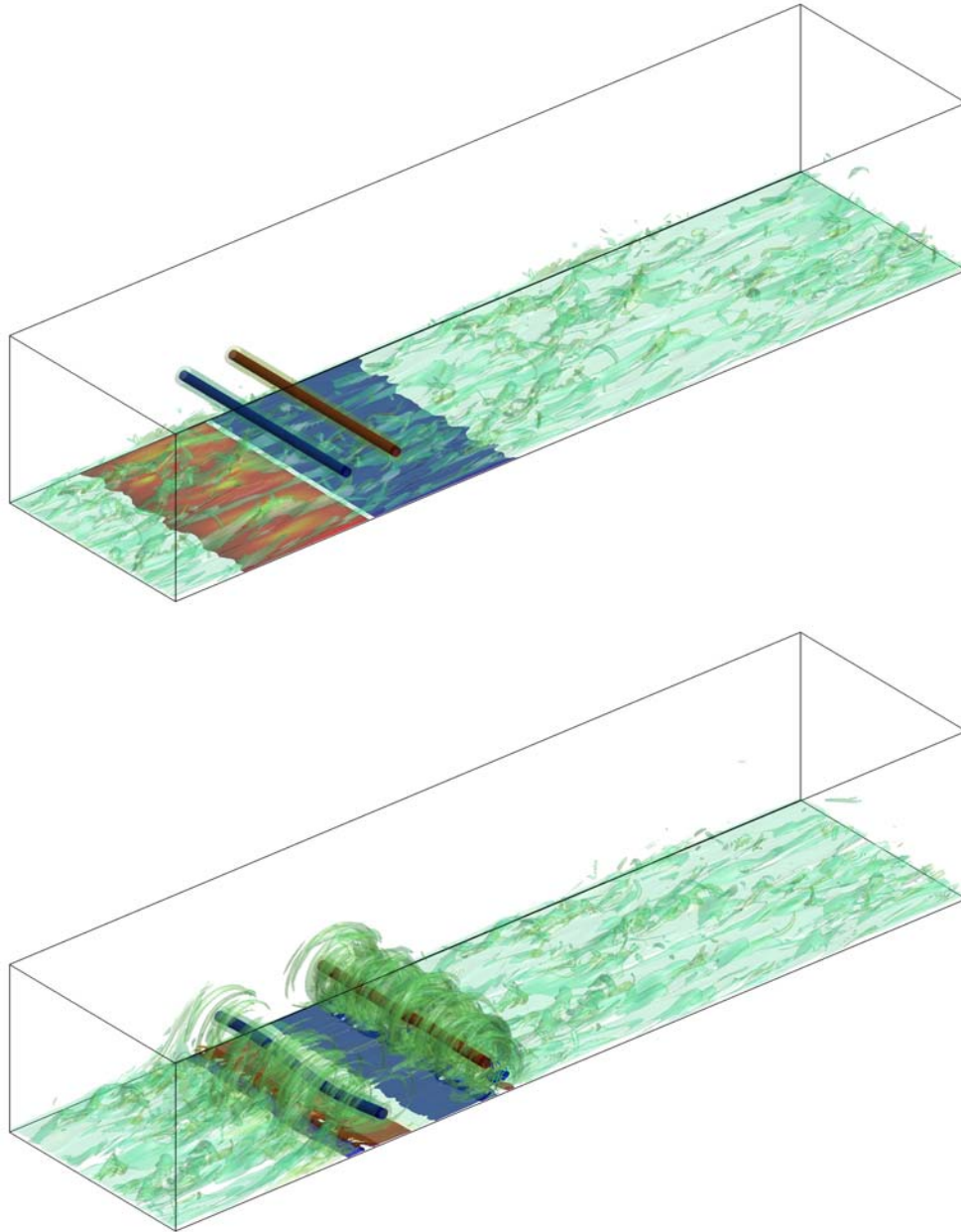


Figure 5.18: Visualization of the flow field using isosurfaces of $\|\vec{\omega}\| b_0^2 / \Gamma_0 = 1$ and 10 for the LES with cross-wind: $\tau = 0.0$ and $\tau = 1.3$.

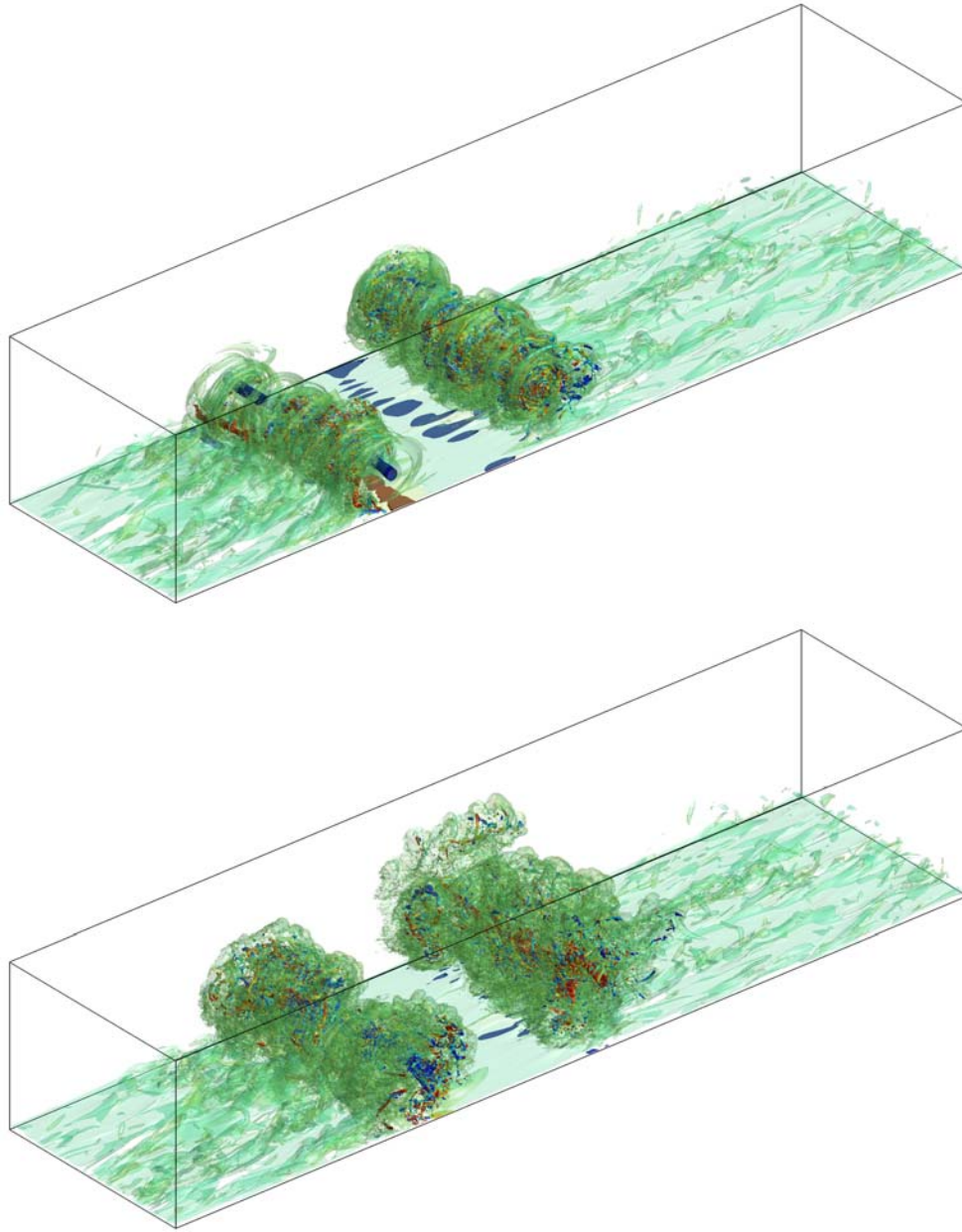


Figure 5.18: (Continued) $\tau = 2.0$ and $\tau = 3.0$.

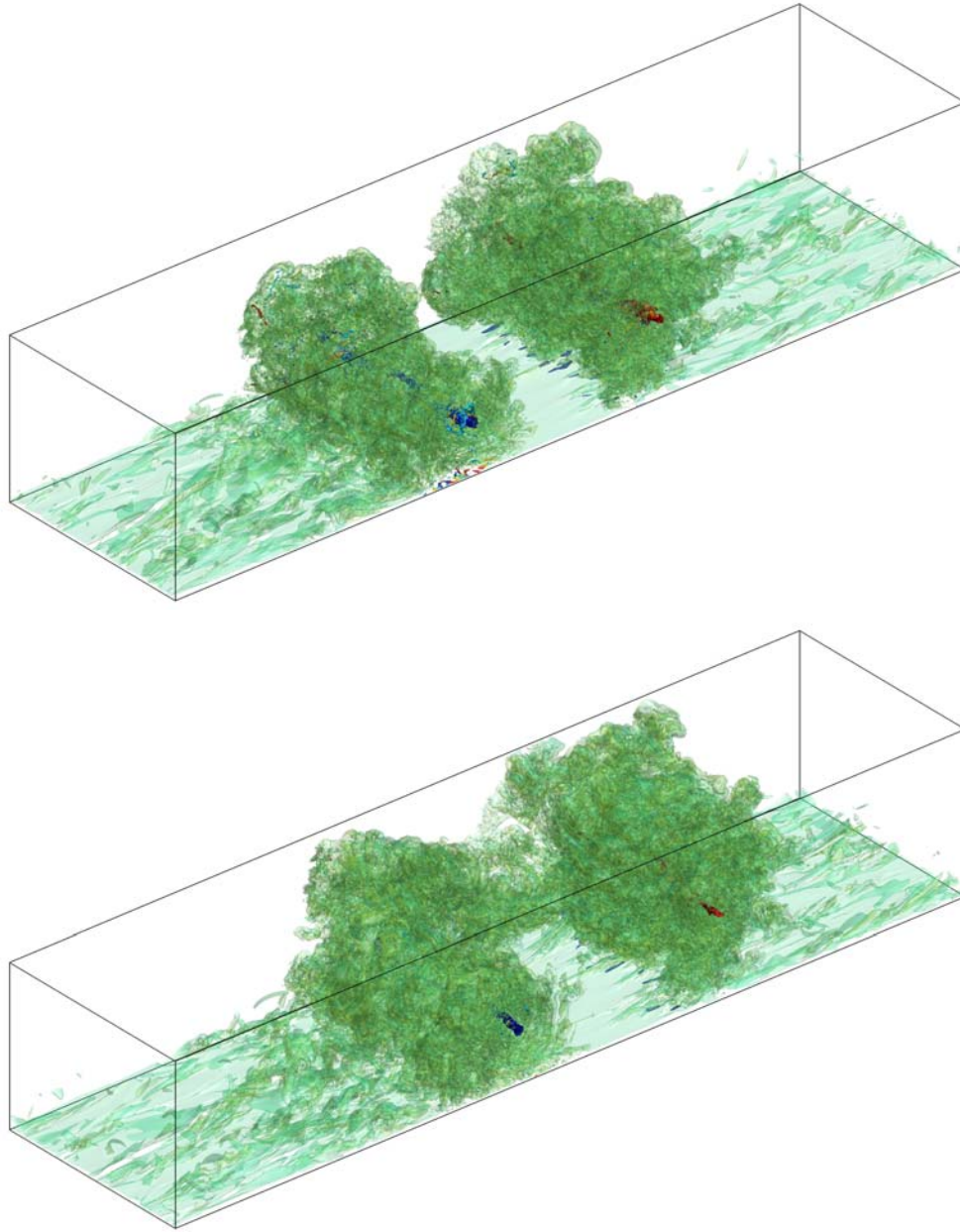


Figure 5.18: (Continued) $\tau = 4.0$ and $\tau = 5.0$.

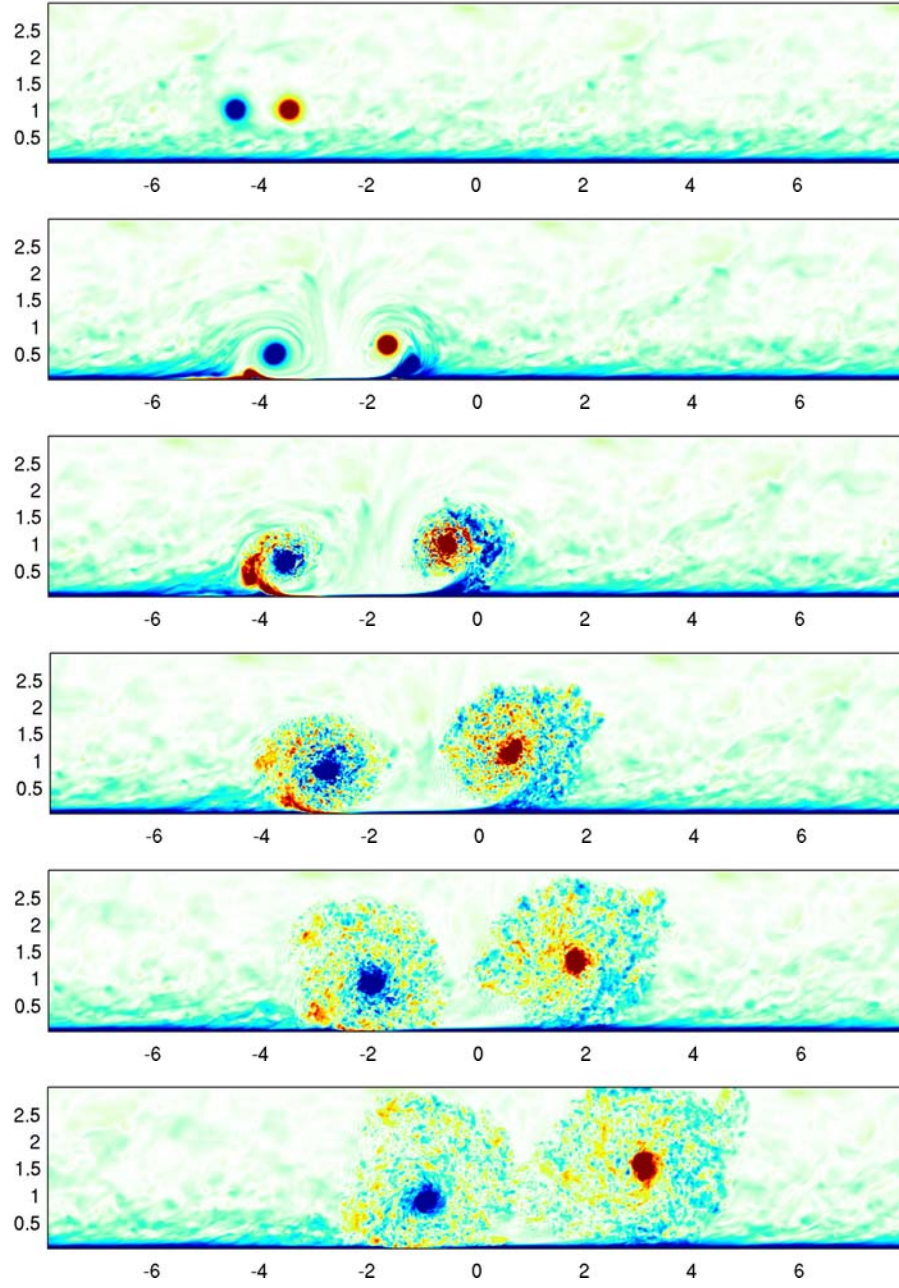


Figure 5.19: Longitudinally averaged axial vorticity field, $\bar{\omega}_x$ for the LES with cross-wind. From top to bottom: $\tau = 0.0$, $\tau = 1.3$, $\tau = 2.0$, $\tau = 3.0$, $\tau = 4.0$, $\tau = 5.0$.

5.4.3 Quantitative results

Fig. 5.20 reports the evolution of the kinetic energy of the system, $E^v(\tau)$. The kinetic energy ,

$$E(\tau) = \frac{1}{2} \int_{\Omega} \mathbf{u} \cdot \mathbf{u} d\Omega \quad (5.18)$$

has to be considered with care as highlighted by Georges et al. [47]. For sufficiently large computational domains Ω and without wind, it does not depend on the domain size. In the presence of a turbulent wind, the situation is quite different. The kinetic energy contribution of the wind can indeed not be neglected. In order to analyze the kinetic energy evolution relative to the primary vortex pair, termed $E^v(\tau)$, this last value is estimated by computing the difference between the kinetic energy of the system $E(\tau)$, as defined by Eq. (5.18), and the kinetic energy of the initial wind only, $E^w(0)$:

$$E^v(\tau) \approx E(\tau) - E^w(0). \quad (5.19)$$

It can be observed in Fig. 5.20 that the Filtered Smagorinsky model and the RVM_s -WALE model give very similar results. The Smagorinsky model leads to a faster decay of the energy during the first phase but eventually reaches a similar level of energy as the other two. The presence of a cross wind lead to a significant decrease of the starting time for the fast decay phase : $\Delta\tau = 1.2$. It is not surprising as the strong interaction between the primary and secondary vortex pairs occurs earlier, the secondary vortices being directly unstable after separation. The evolution of Γ_{5-15} is displayed in Fig. 5.21. A distinction is made between the upwind and the downwind vortices. The comparison is also made with the case without wind. The first slow decay phase is the same for the case without wind and the upwind and downwind vortices. The starting time of the fast decay phase happens earlier for the downwind vortex. It happens also much more earlier than in the case without wind. Anyway, the decrease of Γ_{5-15} is more important in presence of wind. Let's also mention that, for the LES with cross wind, the downwind vortex is more efficiently dissipated than the upwind one. Axial energy spectrum for different times are provided in Fig. 5.22. An inertial $k^{-5/3}$ range can be observed at the late times as the system has become fully turbulent. The longitudinal modal energy plot in Fig. 5.23 shows the growth of the instabilities which saturate when the system is fully turbulent. Note also that, at $\tau = 0$, in opposition to Fig 5.9 there is already an energy spectrum with a given structure. This spectrum is due to the superposition of the wind and the vortices. The trajectories of the vortex pair are displayed in Fig. 5.24. The transport of the two vortices is clearly observed as well as the rebound and tilting phenomena.

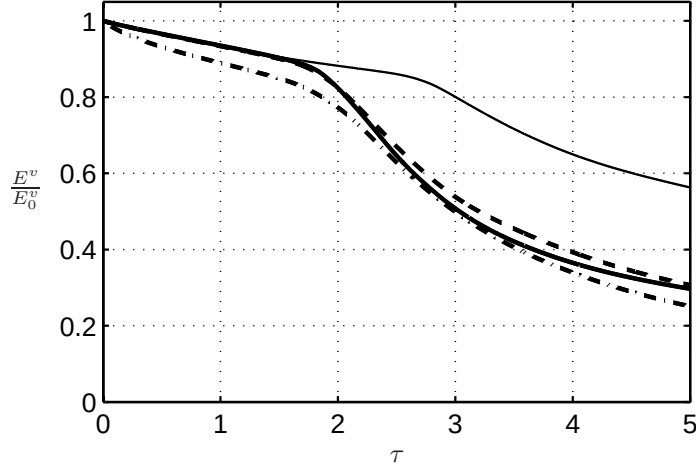


Figure 5.20: Temporal evolution of the kinetic energy for the LES of the vortex system in ground effect without wind using RVM_s -WALE (solid-thin); vortex system with cross-wind using RVM_s -WALE (solid); using filtered Smagorinsky with Piommelli damping (dash); using Smagorinsky with Piommelli damping (dash-dot).

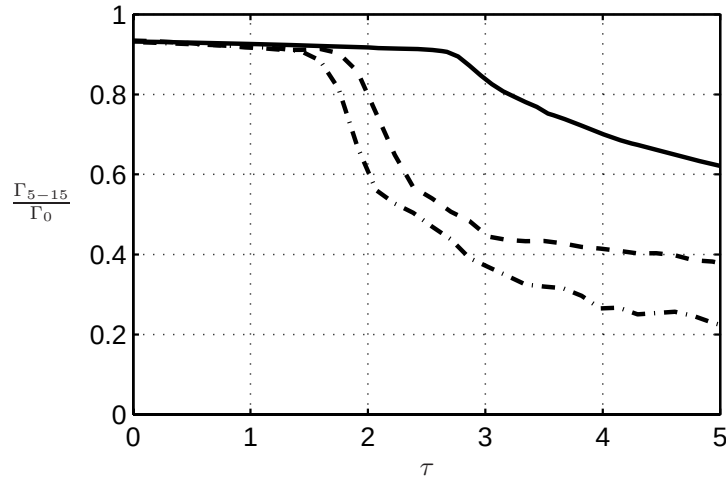


Figure 5.21: Evolution of Γ_{5-15} for the LES of the vortex system in ground effect using the RVM_s -WALE model: LES without wind (solid line); LES with cross-wind considering the upwind (dash) and downwind (dash-dot) vortex.

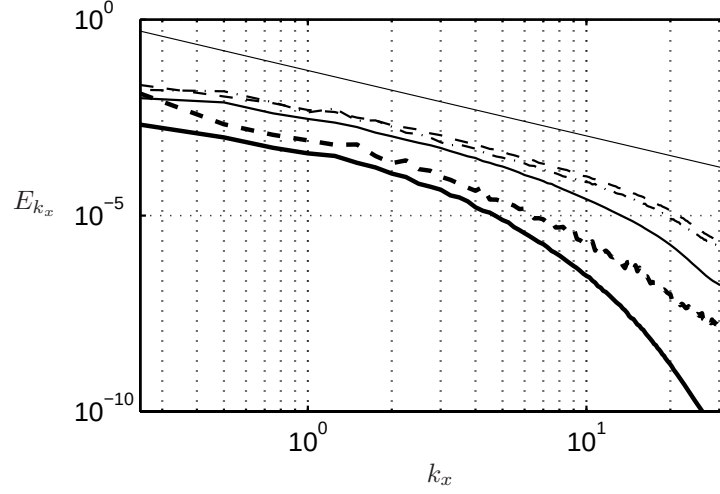


Figure 5.22: Axial energy spectrum at times $\tau = 0.0$ (solid thick), 1.0 (dash-thick), 2.0 (dash-dot), 3.0 (dash), 5.0 (solid thin). The thin line shows the slope $k^{-5/3}$.

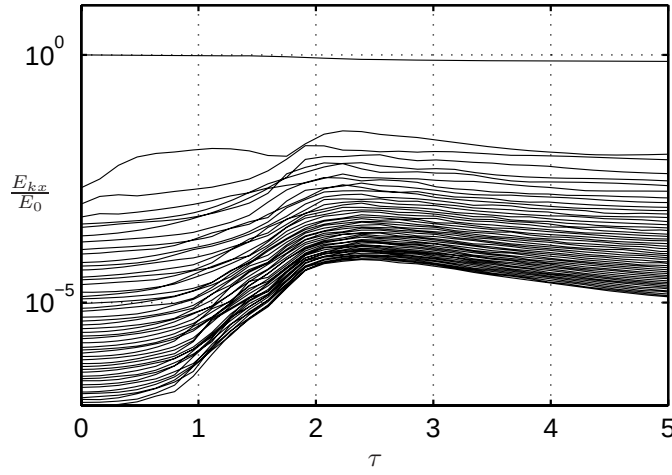


Figure 5.23: Longitudinal modal energy evolution for wake vortices in ground effect with cross-wind (RVM_s - WALE model).

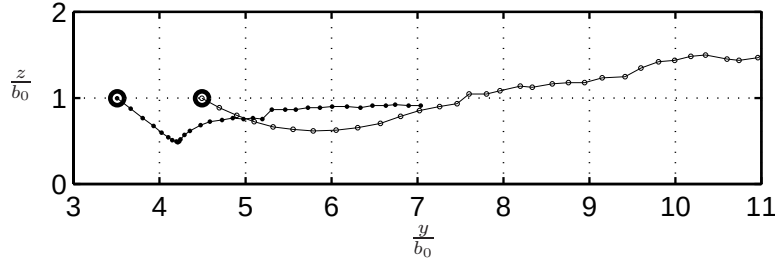


Figure 5.24: Trajectory of the primary vortex cores : LES with cross-wind considering the downwind (hollow circles) and the upwind (plain circles) vortices (RVM_s -WALE model).

5.5 Conclusions

In this study, large eddy simulations (LES) at a Reynolds number of 20000 of a two-vortex system in ground effect have been performed. This is in the direct continuation of the direct numerical simulations (DNS) of Duponcheel et al. [38]. The simulation parameters being almost equivalent (the same initial condition, the same code and almost the same mesh, yet with different wall-normal stretching). The present work intends to investigate the influence of the SGS model and the Reynolds number. In order to investigate the influence of the atmospheric conditions, LES at $Re = 20000$ are also performed with additional turbulent cross-wind. The wind is simulated here using a half-channel flow where a pressure gradient forcing is applied in the wind streamwise direction. Using the present approach, the unsteady wind correspond to DNS of half-channel flows at $Re_\tau = 590$ where the mean streamwise velocity of the wind at the initial height $h_0 = b_0$ is equal to the initial descent velocity V_0 of the vortex pair (formed of two vortices separated by b_0). This wind regime was chosen because the resulting cross-wind leads to a potentially hazardous upwind vortex trajectory.

The results showed that the subgrid-scale (SGS) modeling strategy plays an important role. The impact was indeed significant on the kinetic energy, Γ_{5-15} evolution. In view of the diagnostic based on energy, the Filtered Smagorinsky, and the RVM_s -WALE models based on a flow scales discrimination, clearly outperforms the Smagorinsky model. Indeed, these multiscale models are inactive during the gentle, laminar phases of the simulation and in the vortex cores. The RVM_s -WALE and the Filtered Smagorinsky model performs similarly for this kind of flow.

Concerning the computational cost, the filtered Smagorinsky model is a bit more cost effective than the RVM_s -WALE model. However, the RVM_s -WALE model is more universal as it offers a natural wall damping behaviour without requiring the evaluation of y^+ (as it is the case with the Piomelli damping) which may be complex using unstructured grids.

The DNS at $Re = 5000$ [38] and the present case present strong similarities, also in terms of instability mechanisms, even though the secondary and primary vortices interaction at $Re = 20000$ generates more rapidly complex three-dimensional vortical structures and turbulence. As expected, the increase of the Reynolds number leads to a fast decay phase that happens earlier, and that presents a higher growth rate. This was also confirmed using a modal analysis reported in [47]. It is also expected that the present LES results (energy and circulation decay rate, vortex trajectories), are already at sufficiently high Reynolds number that they are representative of what should be expected at much higher values, relevant to aircraft wakes. This could be further confirmed by comparisons to quality LIDAR data of wakes measured in ground effects.

The presence of wind also reduces the starting time for the fast turbulent decay phase. In fact, the instability of the secondary vortices is triggered almost immediately after separation by the wind turbulence. The strong interaction with the primary vortices then happens earlier. Furthermore, the intensity of the primary vortex pair is reduced significantly. This can be monitored using the energy and Γ_{5-15} evolutions. For the downwind vortex, the cross-wind has a positive contribution to the boundary layer generated by the downwind vortex so that its separation is favored and the resulting secondary vortex more intense. In opposition, the contribution to the upwind vortex boundary layer is negative and the separation is delayed with a lower magnitude of the resulting secondary vortex.

Chapter 6

Modelling of a coherent Doppler LIDAR using a fiber LASER.

The work described in this chapter is the result of the collaboration with the Telecommunication and Remote Sensing Laboratory of Université catholique de Louvain. It is focused on the physical understanding and modeling of a ground based LIDAR using a fiber LASER. A LIDAR simulator was designed and implemented to demonstrate the feasibility of ground based aircraft wake vortices and wind measurements.

6.1 Coherent Doppler pulsed LIDAR measurement

As explained in Section 1.5 the LIDAR system which is here investigated is dedicated to the measurement of air flows (wind, vortices) using the Doppler effect on optical waves. The basic operation was described in chapter 1. Before describing in more details how this device works, we mention several characteristics which identified our LIDAR system.

- Configuration of the transmission / reception:

Two configurations exist: bistatic LIDARs and monostatic ones. In the case of a monostatic LIDAR, the received optical wave presents an important overlap with the emitted wave. The transmitter and the receiver can share the same optical devices or be close to each other. In the bistatic case, the overlap between the two fields has a smaller spatial extent. In the present work, only the monostatic case is considered. A monostatic LIDAR is less sensitive to the large scale turbulence effects, as the back

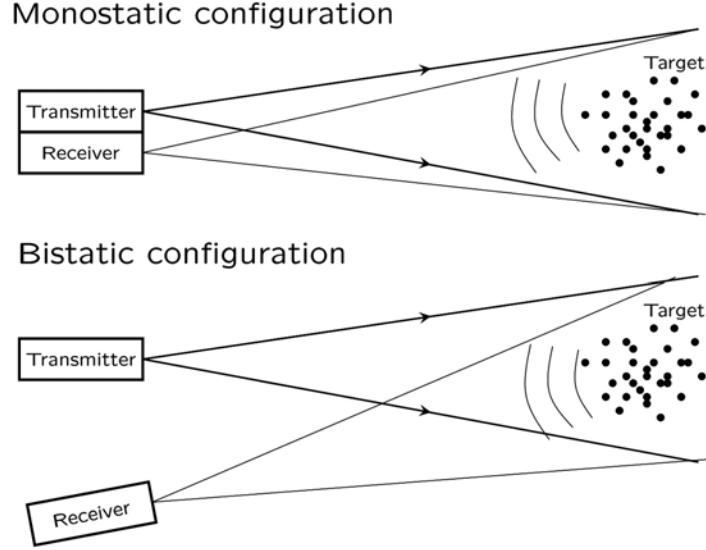


Figure 6.1: LIDAR field of view configuration.

propagated optical wave follows the same path as the emitted one. This distinction is illustrated in Fig. 6.1.

- Detection principle:

There are two ways to extract useful information from the back propagated optical wave. The first one is called the direct (or incoherent) detection. In this case the electrical signal obtained at the photodetector comes directly from the back-propagated optical wave. It is a signal lying in the range of optical frequencies. In the case of a coherent detection, the back-propagated optical wave is first mixed with a reference wave called the local oscillator (referred hereafter as LO). This reference wave corresponds to a small fraction of the flux transmitted from the master LASER itself. The useful information is then shifted at a frequency which is the difference between the frequency of the local oscillator and the back-propagated wave. Usually, this frequency lies in the radio frequency range (RF) in which signal processing techniques can be applied.

- The LASER source:

Two types of LASER are used in LIDAR applications. The solid state LASERs and the fiber ones. A fiber LASER is a LASER in which the active gain medium is an optical fiber. In the present work, we consider only fiber LASERs operating at a wavelength of $\lambda = 1.55 \mu m$. The main advantage is that fiber LASERs can support beam energies up to $100 \mu J$

with a pulse duration of several hundreds nanoseconds and a pulse repetition frequency up to 20 kHz , thus higher than solid state LASERS. Other advantages of fiber LASERS are their compactness, the quality of the LASER beam, the availability of efficient and low cost photodiodes and a very narrow spectral bandwidth. Moreover, at this wavelength the cornea of the human eye is opaque: this means that a LASER beam directed toward the eye will heat its surface and trigger the person to close it. It is the reason why this kind of LASER is referred to as “eye safe”. Moreover, it has been shown in Vallat [111] that the atmospheric power transmittance for this particular wavelength reaches 96%. This means that energy of a LASER beam at this wavelength is not absorbed by the atmospheric constituents.

6.1.1 Doppler effect

The Doppler frequency shift Δf resulting from the interaction of an electromagnetic wave of wavelength λ interacting with backscattering particles moving with a velocity V_R towards the system projected along the axis of propagation of the wave is given by the relation:

$$\Delta f = -\frac{2V_R}{\lambda} \quad (6.1)$$

In the case of an infrared LIDAR at $\lambda = 1.55\mu\text{m}$, a velocity of 1 m/s results in a Doppler shift of $\Delta f \approx 1.3\text{ MHz}$.

6.1.2 Coherent detection principle

The general architecture of a coherent Doppler LIDAR is depicted in Fig. 6.2. The main LASER is used to generate the master optical wave at frequency, f_{LO} . This master wave is split into two waves. First, the reference wave, also called the local oscillator, which contains only a fraction of the power. Second, the wave containing the main part of the power that will be sent to the atmosphere and serves to the sensing. The acousto-optical modulator (referred as AOM in Fig. 6.2) is used to modulate in frequency the master signal at frequency f_{LO} so that the signal which is sent to the atmosphere is at a frequency $f_{LO} + f_{IF}$. Doppler frequency shift might be positive or negative. This is the reason why the frequency of the beam sent in the atmosphere is shifted by an amount f_{IF} using the acousto-optic modulator. The order of magnitude for this frequency is $f_{IF} \approx 80\text{ MHz}$. The frequency shift also allows to work at frequencies that are more adequate for electronic and signal processing. The sensing continuous optical wave is then transformed into a pulsed wave by means of a “Q switching” device.

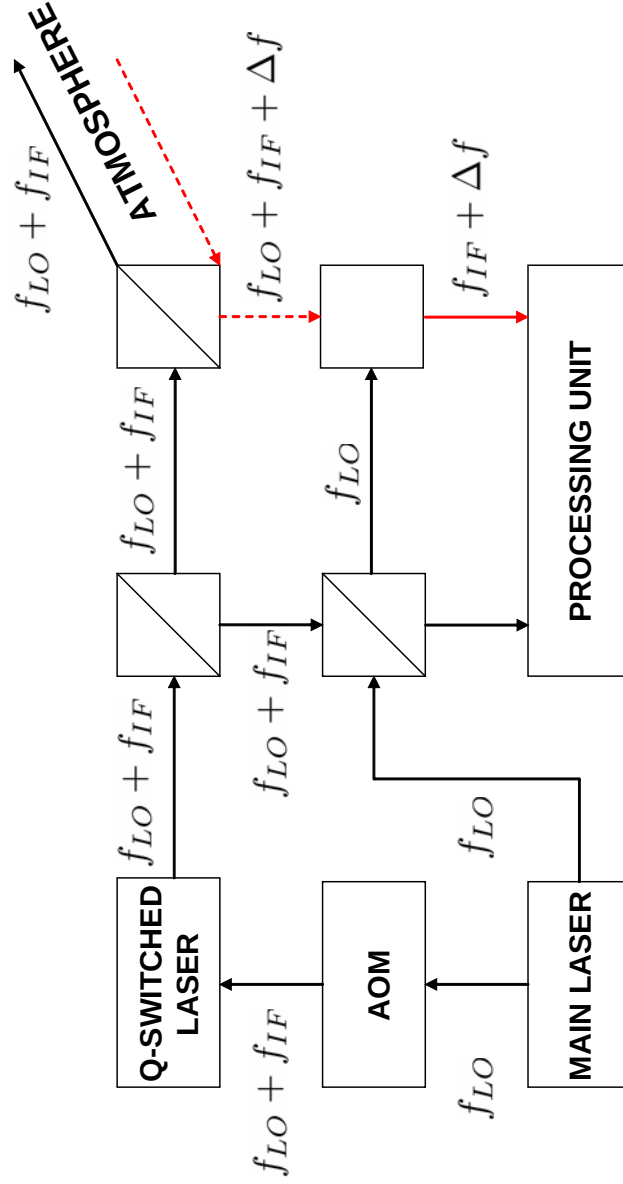


Figure 6.2: Flow sheet of heterodyne measurement. f_{LO} is the local oscillator frequency. f_{IF} is the intermediate frequency. Δf is the Doppler shift.

The pulsed wave is emitted toward the atmosphere. The backscattered wave is then collected on the photodetector at a frequency $f_{LO} + f_{IF} + \Delta f$ and mixed with the reference wave at frequency $f_{LO} \approx 2 \times 10^8 \text{ MHz}$ for a $1.55 \mu\text{m}$ LASER. This results in a signal which is at a frequency $f_{IF} + \Delta f$ lying in the RF range and allowing for either negative or positive Doppler shifts. To illustrate the coherent detection in a more formal way, consider the back-propagated wave at the pulsation $\omega_S = \omega_{LO} + \omega_{IF} + \omega_D$ with $\omega_D = 2\pi\Delta f$. The optical signal (here an electric field, in $[Vm^{-1}]$) coming back from the atmosphere can be written as:

$$E_S = E_{S,0} \exp(j(\omega_S t + \phi_S)) \quad (6.2)$$

ϕ_S being a random phase. The local oscillator signal has the form:

$$E_{LO} = E_{LO,0} \exp(j\omega_{LO} t). \quad (6.3)$$

The photodiode, which is a quadratic detector is sensitive to the square of the amplitude of the optical wave illuminating its surface. Adding the local oscillator wave and the backscattered optical wave yields the resulting wave E :

$$E = E_S + E_{LO}. \quad (6.4)$$

The power of a planar electromagnetic wave impinging on a photodiode of area A is proportional to the square of the electric field:

$$P = \epsilon_0 c A \langle |E|^2 \rangle, \quad (6.5)$$

where ϵ_0 is the free space permittivity and c is the speed of the light. The operator $\langle \rangle$ refers to an averaging over the response time of the photodiode. An order of magnitude for the bandwidth of the photodiode is $B = 150 \text{ MHz}$. The current depends on the photodiode quantum efficiency η and the optical frequency of the LASER f_{LO}

$$I = \frac{\eta q}{h f_{LO}} P = S \epsilon_0 c A \langle |E|^2 \rangle. \quad (6.6)$$

The photodiode sensitivity appearing in Eq. (6.6) is defined as:

$$S = \frac{\eta q}{h f_{LO}} \quad (6.7)$$

where q is the charge of the electron and h is the Planck constant¹. Taking the temporal average of the square of the field magnitude leads to:

$$\begin{aligned} \langle |E|^2 \rangle &= \langle E_{LO,0}^2 \rangle + \langle E_{S,0}^2 \rangle + \\ &+ \langle 2E_{LO,0}E_{S,0} \cos(\omega_S t + \phi_S) \cos(\omega_{LO} t) \rangle \\ &+ \langle 2E_{LO,0}E_{S,0} \sin(\omega_S t + \phi_S) \sin(\omega_{LO} t) \rangle. \end{aligned} \quad (6.8)$$

¹ $h = 6.6262 \times 10^{-34} \text{ Js}$

Using the following trigonometric identity:

$$\cos(\omega_S t - \omega_{LO} t + \phi_S) = \cos(\omega_S t + \phi_S) \cos(\omega_{LO} t) + \sin(\omega_S t + \phi_S) \sin(\omega_{LO} t),$$

Eq. (6.8) transforms into:

$$\langle |E|^2 \rangle = \langle E_{LO,0}^2 + E_{S,0}^2 + 2E_{LO,0}E_{S,0} \cos(\omega_S t - \omega_{LO} t + \phi_S) \rangle. \quad (6.9)$$

The equation for the optical power on the photodiode is then:

$$\begin{aligned} P &= \epsilon_0 c A \langle |E|^2 \rangle \\ &= \epsilon_0 c A (E_{LO,0}^2 + E_{S,0}^2 + 2E_{LO,0}E_{S,0} \cos((\omega_S - \omega_{LO})t + \phi_S)) \end{aligned} \quad (6.10)$$

Taking the definition of the power into account, this can be simplified:

$$P = P_{LO} + P_S + 2\sqrt{P_{LO}P_S} \cos((\omega_S - \omega_{LO})t + \phi_S). \quad (6.11)$$

This equation leads to a similar expression for the current at the photodiode:

$$I = S (P_{LO} + P_S + 2\sqrt{P_{LO}P_S} \cos((\omega_S - \omega_{LO})t + \phi_S)). \quad (6.12)$$

This current can be decomposed into three terms:

- $I_{LO} = S P_{LO}$: a direct current due to the local oscillator,
- $I_S = S P_S$: a direct current due to the signal,
- $I_{het} = 2S \sqrt{P_{LO}P_S} \cos((\omega_S - \omega_{LO})t + \phi_S)$: the heterodyne current.

As can be observed, this technique has two main properties: it translates an optical signal into an RF signal with a pulsation $\omega_S - \omega_{LO} = \omega_{IF} + \omega_D$, and the low power optical signal coming back from the atmosphere is amplified by the term $\sqrt{P_{LO}P_S}$. For instance, choosing an intermediate frequency $f_{IF} \approx 80 MHz$ and fixing Δf to a realistic value of $5 MHz$, results in a RF signal with a frequency of $85 MHz$.

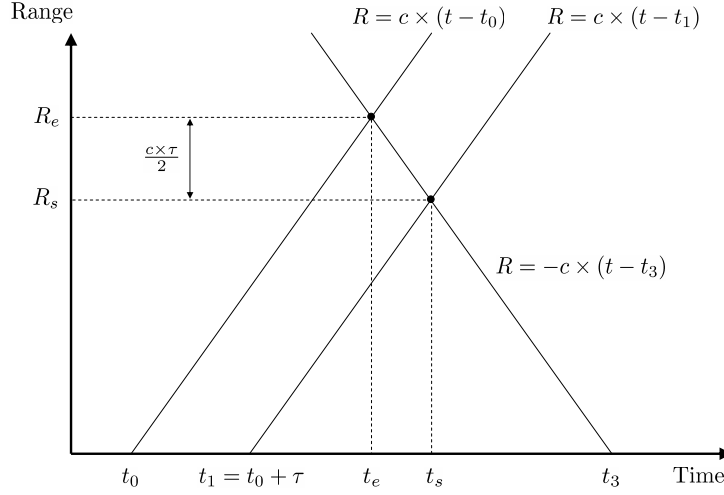


Figure 6.3: Range time diagram

6.1.3 Pulsed vs continuous mode

The LIDARs (or more commonly, any RADAR) can be classified according to the temporal behaviour of the transmitted beam. A pulsed LIDAR allows for ranging by timing the scattered signal while a continuous wave LIDAR (CW) collects data by adjusting the focus range of the transmitted LASER BEAM and scanning across the region of interest. In this work, we are only interested in pulsed LIDARs in order to obtain a quantitative information in strength and position for wake vortices or retrieving wind velocity profiles. The time between the pulse emission and the measured return Δt provides the useful range R information on the sensed objects as $R = \frac{c\Delta t}{2}$. A pulsed LIDAR can be collimated allowing to have a more uniform SNR along the LOS². The space-time diagram displayed in Fig. 6.3 illustrates how a pulsed LIDAR works. A LASER pulse of duration τ is transmitted at time t_0 , this wave propagates at the speed of light c . At a time t , the pulse is located at a range $R = ct$ from the LIDAR. The spatial extension of the pulse is $\Delta R = c\tau$. At a time t_3 , the receiver gets the waves backscattered from the particles which have coordinates in the (t, R) plane obeying to the relationship $R = -c(t - t_3)$.

6.2 Backscattering particles

This section concerns the source of the LIDAR signal. Molecules and aerosols present in the atmosphere contribute to the backscattering, absorption and diffusion of the power they received. These particles can be anthropogenic or

²Line Of Sight

natural. Both have an impact on the LIDAR signal. In scattering problems, an important parameter is

$$x = \frac{2\pi D}{\lambda}$$

which scales the typical diameter size of the particles D to the wavelength λ of the radiating field. The power received by an optical device of area Ω backscattered by particles distributed in a volume V can be expressed as:

$$P_B = \int_{\Omega} \int_V \beta U d\Omega dV$$

where the cloud of particles in a volume V scatter an incident radiation U [Wm^{-2}] in a solid angle Ω [sr]. P_B is the power collected at the reception device. The parameter β [$m^{-1}sr^{-1}$] is the backscattering coefficient of the particles. As it is directly linked to the power budget of LASER based remote sensing devices, this quantity is of primary importance. This coefficient is computed in different ways, for molecules and for aerosols, as will be explained further.

6.2.1 Molecules

The atmosphere is essentially composed of the following molecules: O_2 , N_2 , CO_2 , H_2O , CH_4 , O_3 , N_2O . All these molecules are uniformly distributed in the atmosphere except O_3 and H_2O . These molecules can potentially serve to backscatter LASER beams.

Brownian motion

Molecules which are present in the atmosphere are subject to Brownian motion. The velocity distribution function of the molecules in thermal equilibrium is given by the Maxwell distribution. The isotropic Maxwell-Boltzmann probability density function is given by:

$$f(v) = 4\pi \left(\frac{m}{2\pi k_B T} \right)^{\frac{3}{2}} v^2 \exp \left(-\frac{mv^2}{2k_B T} \right),$$

where m is the molecular mass of the gas, T is the thermodynamic temperature and k_B is the Boltzmann constant. The probability that the velocity norm of a molecule lies between v and $v + dv$ is given by $f(v)dv$. This probability distribution is displayed in Fig. 6.4.

Rayleigh Scattering

Rayleigh theory describes the scattering of light by particles that are small compared to the wavelength of the incident radiation. This theory predicts that the intensity of the light scattered by molecules varies as λ^{-4} . Its backscattering

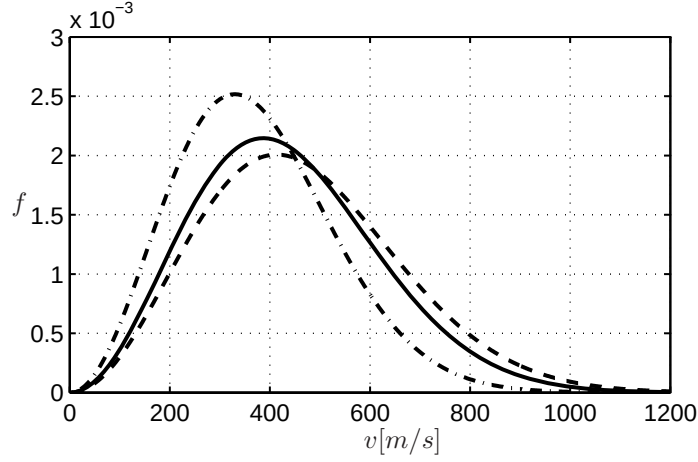


Figure 6.4: Maxwell-Boltzmann distribution at $T = 288K$ for N_2 (dash), O_2 (solid), CO_2 (dash-dot). As the molecular masses of O_2 and N_2 are very close, the two corresponding curves can be approximated by a single distribution using the mean molecular mass of air.

coefficient is calculated in Vallat [111] for standard pressure and temperature conditions:

$$\beta_{mol} \approx \frac{1.272}{\lambda^4} \times 10^{-29} m^{-1} sr^{-1}. \quad (6.13)$$

For the wavelength of interest here, ($\lambda = 1.55\mu m$), one obtains

$$\beta_{mol} = 2.2 \times 10^{-8} m^{-1} sr^{-1}.$$

This value is compared with the one of the aerosols in a following section.

6.2.2 Atmospheric aerosols

Aerosol is the short name for “aerial solid”. These are tiny solid or liquid particles suspended in a gas. They can be natural or anthropogenic and can also be classified into primary and secondary aerosols. Primary aerosols are mineral dust, salt or soots while secondary aerosols result from a chemical reaction. Their size ranges from less than $10nm$ to more than $100\mu m$. This range of sizes represents scales from a gathering of a few molecules to the size where the particles no longer can be carried by the gas. The atmosphere contains significant concentrations of aerosol particles. The highest concentrations of aerosols are usually found in urban areas. They can be as high as $10^7 - 10^8 cm^{-3}$. Typical aerosols which can be found in an urban area are displayed in Fig. 6.5, as the size of these aerosols is close to the wavelength considered, the scattering will be in the Mie regime which is more efficient as explained further. Aerosols have a wide range of sizes and shapes, and may be described

by size distributions. Considering $n_N(D_p)dD_p$ [$cm^{-3}\mu m^{-1}$], the number of particles per cm^3 having diameters in the range $[D_p, D_p + dD_p]$, one can write:

$$n_N(D_p) = \frac{dN}{dD_p}. \quad (6.14)$$

The total number of particles per cm^3 is then:

$$N = \int_0^\infty \frac{dN}{dD_p} dD_p \quad (6.15)$$

For convenience, logarithmic variables are often used:

$$n_N^0(\log_{10}(D_p)) = \ln(10) D_p n_N(D_p). \quad (6.16)$$

The most commonly used aerosols distribution is a sum of n log-normal distributions:

$$n_N^0(\log_{10}(D_p)) = \sum_{i=1}^n \frac{N_i}{\sqrt{2\pi} \log(\sigma_i)} \exp \left(-\frac{(\log_{10} D_p - \log_{10} \overline{D_{pi}})^2}{2 \log_{10}^2 \sigma_i} \right). \quad (6.17)$$

This can also be written:

$$n_N(D_p) = \frac{1}{\ln(10)} \sum_{i=1}^n \frac{N_i}{\sqrt{2\pi} \log(\sigma_i) D_p} \exp \left(-\frac{(\log_{10} D_p - \log_{10} \overline{D_{pi}})^2}{2 \log_{10}^2 \sigma_i} \right).$$

N_i being the concentration number, $\overline{D_{pi}}$ the mean diameter and σ_i the standard deviation of the mode i . The parameters for urban and rural environment are tabulated, in Jaenicke [58]. A typical size distribution chart is displayed in Fig. 6.6 for an urban environment according to the data of Jaenicke [58].

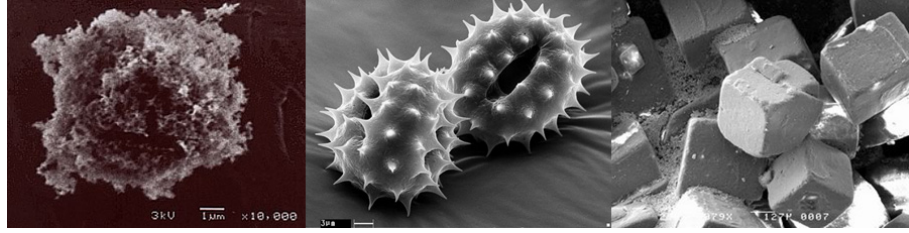


Figure 6.5: Microscope photography of aerosols. Soot particle (left), pollen (middle), salt crystals (right).

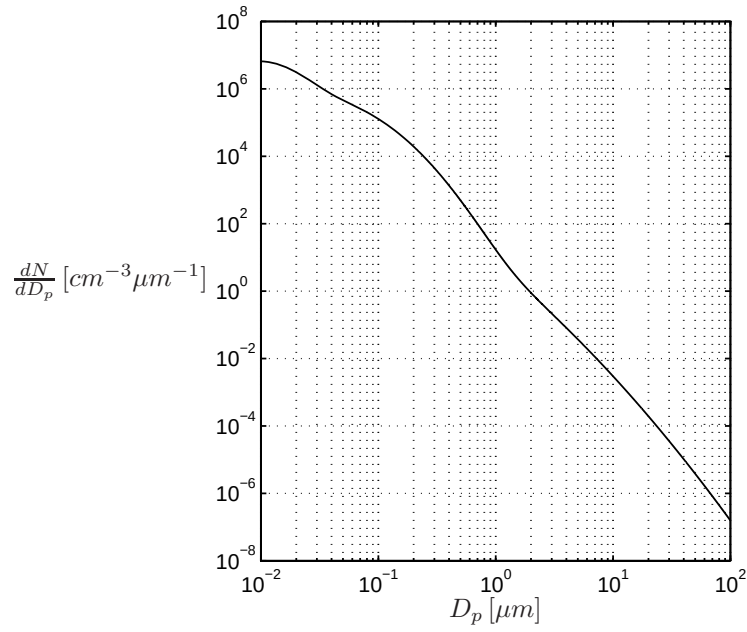


Figure 6.6: Aerosol number size distribution for an urban environment model.

Mie Scattering

As previously mentioned, Rayleigh scattering is valid for particles much smaller than the radiation wavelength ($x \ll 1$). On the other end, geometrical optics laws are only valid for objects much bigger than the radiation wavelength ($x \gg 1$). Mie theory allows to compute the scattering of electromagnetic waves by spheres characterized by a single refractive index n and embedded in an homogeneous medium; this theory is valid whatever the value of the parameter x . As the cases $x \ll 1$ and $x \gg 1$ are covered by simpler theories, Mie theory is more general and is used to cover the cases where $x \approx 1$. The backscattering cross section, σ_b , is defined by the portion of the incident wave scattered in all directions. In the same way, σ_a is the portion of the incident wave absorbed inside the particle. By conservation of energy, the extinction cross section can be obtained by energy conservation:

$$\sigma_e = \sigma_b + \sigma_a. \quad (6.18)$$

These cross-sections can be made dimensionless by dividing them by the geometrical cross-section of the spherical particle having a radius r_p :

$$Q_e = \frac{\sigma_e}{\pi r_p^2}, \quad Q_b = \frac{\sigma_b}{\pi r_p^2}, \quad Q_a = \frac{\sigma_a}{\pi r_p^2}.$$

The Mie theory allows to compute the “efficiency” factors Q_e, Q_b, Q_a . We will not provide the details of the theory here; for further informations, see, e.g., the book of Van de Hulst [112]. A typical scattering efficiency curve computed using Mie theory is provided in Fig. 6.7.

The quantity of importance for Doppler LIDARs is the volume aerosol backscattering coefficient β_a [$m^{-1}sr^{-1}$]. It depends on the aerosol size distribution:

$$\beta_a = \int_0^\infty \pi r^2 Q_b(\nu, r, n) \frac{dN}{dr} dr. \quad (6.19)$$

For the infrared optical wavelength considered here, and for an altitude up to 1000 m, a convenient range of values of this coefficient is: $\beta_a \approx 10^{-7} - 10^{-6} m^{-1}sr^{-1}$. Cariou et al. [18] mention that β_a varies from $10^{-6} m^{-1}sr^{-1}$ at ground level to $10^{-10} m^{-1}sr^{-1}$ at 10 km altitude.

Aerosol motion

We are interested to know how the aerosol particles follow the fluid flow to be measured. The quantity which describes the time required for a particle to adjust, or “relax”, its velocity to a new condition of forces is called the relaxation time τ_R of the particle and is given by

$$\tau_R = \frac{\rho_p d_p^2}{18\mu} \quad (6.20)$$

where ρ_p and d_p are the density and the diameter of the spherical particle, and μ is the dynamic viscosity of the fluid. This is valid if the particle motion is in Stokes regime, ie

$$Re_p = \frac{\rho U_p d_p}{\mu} < 1 \quad (6.21)$$

where ρ is the fluid density ($\nu = \mu/\rho$ is the kinematic viscosity). The relaxation time is a convenient measure of the trend of particles to attain velocity equilibrium with the fluid (here air). Considering an aerosol particle with $\rho_p = 1 \text{ g/cm}^3$, and taking $d_p = 1 \text{ }\mu\text{m}$, leads to a relaxation time $\tau_R = 3 \times 10^{-8} \text{ s}$. These values, combined with a typical order of magnitude of particle speed in atmospheric wind or vortex flows, $U_p \approx 10 \text{ m/s}$, leads to $Re_p = 0.66$. With typical diameter and density values for atmospheric aerosols, the relaxation time τ_R is small, meaning that the aerosols are good tracers for the flow of interest.

Spectral broadening

The value $\beta_{mol} = 2.2 \cdot 10^{-8} \text{ m}^{-1} \text{ Sr}^{-1}$ is not negligible compared to that of the aerosols $\beta_a \approx 10^{-7} \text{ m}^{-1} \text{ Sr}^{-1}$. However the intensity spectrum of the light backscattered by the molecules is very broad compared to that backscattered by the aerosols, see Fig. 6.8. As a consequence, the Rayleigh diffusion cannot provide any information as its signature is similar to the detection noise (see e.g. [111]).

This spectral broadening can be explained considering particle dynamics. Molecules present in the atmosphere are subjected to high-velocity random thermal motion (Brownian motion). A LIDAR backscattered echo from air molecules is thus Doppler broadened due to the large range of molecule velocities. As aerosol particles are much more massive than gas molecules, the Brownian motion of these particles presents significantly lower velocities than the thermal motion of air molecules, hence the resulting Doppler broadening of aerosols return is negligible. As a consequence, the spectral width of the backscatter from aerosols is nearly identical to the laser spectral width, and is therefore much smaller than that of the Rayleigh return.

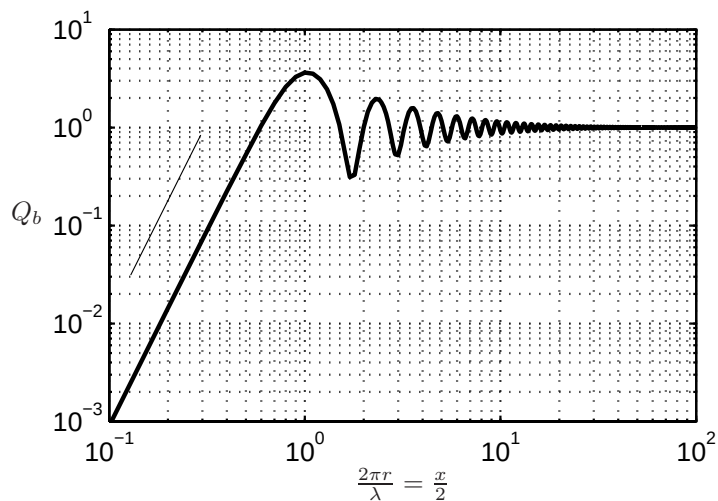


Figure 6.7: Backscattering efficiency for conductive spheres. The thin straight line illustrates the λ^{-4} slope in the Rayleigh range.

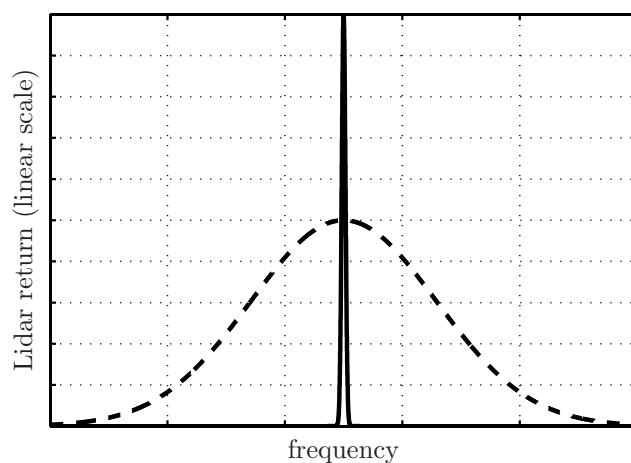


Figure 6.8: Diagram illustrating the spectral broadening effect. Typical signature of the aerosols (solid) and typical signature of the molecules (dash).

6.3 Optical scintillation

The optical turbulence phenomena, also called optical scintillation, is of primary importance for LASER beam propagation in the atmosphere and particularly for LIDAR remote sensing. Indeed, this phenomenon can alter the performances of a remote sensing system such as a LIDAR.

6.3.1 Description

Air flows relevant for LIDAR measurements in the atmosphere are in turbulent regimes. The Reynolds number associated with these flows are usually very large. In turbulent flows, the pressure, the temperature, moisture and refractive index are subjected to temporal and spatial fluctuations and affect the propagation of electromagnetic waves. In the case of a LASER beam propagation, only the temperature, density and pressure fluctuations have a significant impact on the refractive index. The effect of the moisture is negligible in this case but has to be considered for propagation of radio waves, as explained in Vasseur [113]. It can be showed, from Maxwell's equations (see e.g. [1]) that the magnitude of an electromagnetic wave $\mathbf{E}$ propagating in a turbulent medium with a refractive index n obeys to the following relation:

$$\nabla \mathbf{E} + k^2 n^2(r) \mathbf{E} + 2\nabla (\mathbf{E} \cdot \nabla \log(n(r))) = 0. \quad (6.22)$$

The Gladstone-Dale law binds the spatial fluctuations of the refractive index with the pressure P [mbar] and temperature T [K] fluctuations:

$$n_1(\mathbf{r}) = n(\mathbf{r}) - 1 = \alpha_1 \left(1 + \frac{\alpha_2}{\lambda^2}\right) \frac{P(\mathbf{r})}{T(\mathbf{r})}. \quad (6.23)$$

The constants value are $\alpha_1 = 77.6 \times 10^{-6} \text{ K/mbar}$ and $\alpha_2 = 7.52 \times 10^{-3} \text{ m}^2$. The value of the constant part of the refractive index is unity. The order of magnitude of the refractive index fluctuations is $n_1 = \mathcal{O}(10^{-6})$.

6.3.2 Relation between atmospheric turbulence and optical scintillation

Structure function

One way to characterize a turbulent flow is to use structure functions. According to Kolmogorov's turbulence theory, the temporal variations of the velocity increment between two space locations, bound by the displacement vector $\mathbf{r}$, can be described in an universal way. We define the structure tensor of a velocity field:

$$D_{ij}(\mathbf{r}) = \langle [v_i(\mathbf{r}_1 + \mathbf{r}) - v_i(\mathbf{r}_1)] [v_j(\mathbf{r}_1 + \mathbf{r}) - v_j(\mathbf{r}_1)] \rangle \quad (6.24)$$

In homogeneous isotropic turbulence, due to the homogeneity of the flow, the velocity increment statistics depend only on the displacement vector and, furthermore, due to the isotropic character of the statistics, only the norm $r = |\mathbf{r}|$

of this vector is important. Recall that these two hypotheses are only valid locally. In shear or vortical flows this is not valid anymore. As the aim of this work is to illustrate the effect of the turbulence on the propagation of optical waves, we will keep these hypotheses. In this case, one may rewrite the structure tensor function:

$$D_{ij}(r) = [D_{rr}(r) - D_{tt}(r)] n_i n_j + D_{tt}(r) \delta_{ij} \quad (6.25)$$

where

$$D_{rr}(r) = \langle [v_r(\mathbf{r}_1 + \mathbf{r}) - v_r(\mathbf{r}_1)]^2 \rangle \quad (6.26)$$

is the longitudinal structure function and,

$$D_{tt}(r) = \langle [v_t(\mathbf{r}_1 + \mathbf{r}) - v_t(\mathbf{r}_1)]^2 \rangle, \quad (6.27)$$

is the transverse structure function. The two structure functions D_{rr} and D_{tt} are related to the velocity component which is respectively parallel or transverse to the displacement vector $\mathbf{r}$, see Fig. 6.9. Assuming an incompressible flow, it

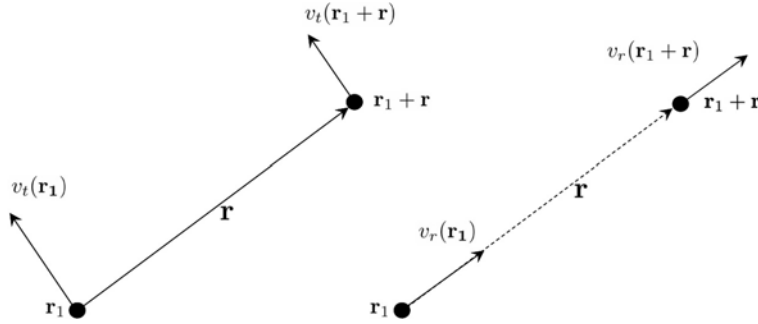


Figure 6.9: Transverse and longitudinal structure functions.

also comes that:

$$D_{tt} = \frac{1}{2r} \frac{d}{dr} (r^2 D_{rr}). \quad (6.28)$$

The incompressible homogeneous isotropic turbulence can thus be characterized by using only the longitudinal structure function. Kolmogorov showed that if r is a length scale lying in the “inertial range” of turbulence, then one has:

$$D_{rr} = C_V^2 \epsilon^{2/3} r^{2/3}, \quad D_{tt} = \frac{4}{3} D_{rr} \quad (6.29)$$

C_V^2 being the velocity structure constant $C_V^2 \approx 2.1$. This particular form of D_{rr} is valid only for $l_0 < r < L_0$, where L_0 is the external scale of the turbulence (below this scale, eddies are supposed to be isotropic) and l_0 the internal scale. The order of magnitude of the internal scale is $l_0 = \mathcal{O}(10 \eta)$, recalling that η is the Kolmogorov length scale.

Table 6.1: *Scintillation intensity vs C_n^2 .*

C_n^2	Scintillation
$10^{-15}m^{-2/3}$	weak
$10^{-14}m^{-2/3}$	medium
$10^{-13}m^{-2/3}$	strong

Structure functions and refractive index fluctuations

The structure functions being defined, we relate these to the fluctuations of the atmospheric refractive index. This implies the introduction of a new concept: the passive scalar. A passive scalar is a globally conserved quantity in a fluid flow which does not influence the flow dynamics. The temperature of the atmosphere is not a conserved quantity: if a parcel of air is moved vertically, its temperature will change adiabatically. This leads to define the potential temperature:

$$\Theta = T - \gamma_a z. \quad (6.30)$$

It is the difference between the absolute temperature and the temperature variation with altitude for an adiabatic atmosphere. The adiabatic lapse, $\gamma_a = 9.8 \text{ K/km}$, is the temperature decrease rate without heat exchange with altitude z . The moist adiabatic lapse rate is less than the dry adiabatic lapse rate and depends on the moisture content of the air mass. The potential temperature is a conserved passive quantity (it does not affect the dynamics of the flow and it is conserved). The structure function for the potential temperature can be written in the same way as the simplified expression obtained for the velocity structure function:

$$D_\Theta(r) = C_\Theta^2 r^{2/3}, \quad (6.31)$$

where C_Θ^2 is the temperature structure constant. For a given altitude z , the refractive index variation can be linked with the potential temperature. If we neglect pressure and moisture variations, we can write:

$$dn = -\frac{P}{T^2} \alpha_1 \left(1 + \frac{\alpha_2}{\lambda^2}\right) d\Theta \quad (6.32)$$

As $d\Theta$ is a passive scalar, dn is also a passive scalar and the structure function for the refractive index can be written:

$$D_n(r) = C_n^2 r^{2/3} \quad (6.33)$$

The refractive index fluctuations have the same statistical properties than the turbulence and they are mainly due to temperature fluctuations; $C_n^2 [m^{-2/3}]$ is a physical parameter which describes the intensity of the refractive index fluctuations.

Optical turbulence scales

As the refractive index fluctuations vary with the turbulent temperature fluctuations, a refractive index field also contains different scales. The most common model used for the refractive index spectrum is the modified von Karman spectrum:

$$\Phi_n(k) = 0.033 C_n^2 (k^2 + k_o^2)^{-11/6} \exp\left(-\frac{k^2}{k_i^2}\right). \quad (6.34)$$

The parameter $k_i = \frac{5.92}{\eta}$ is the wavenumber associated to the inner scale and k_o is that associated to the outer scale L_0 , the typical size of the largest scales. As shown by Brousmiche [13], the effect of the large scales is to deviate the

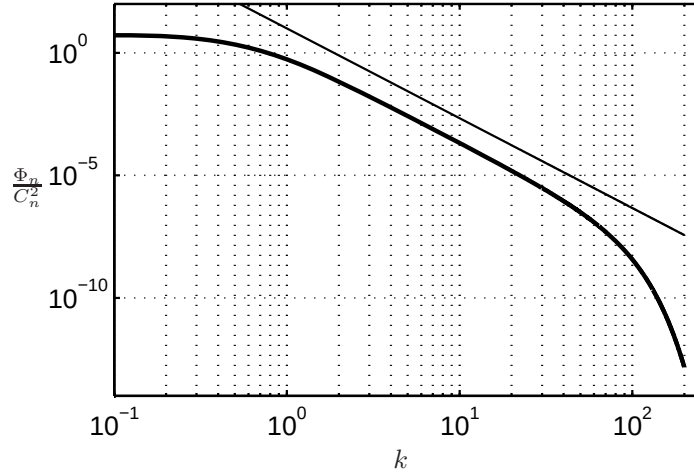


Figure 6.10: Modified von Karman refractive index spectrum. The thin straight line shows a $k^{-11/3}$ slope. Here, we used $l_0 = 1\text{cm}$ and $L_0 = 10\text{m}$.

LASER beam (refraction), while the effect of the small scales is diffraction. In other words, the large scales can be seen as lenses of various sizes and the small scales induce a loss of coherence of the LASER beam.

The propagation time required for the pulse to go back and forth between the LIDAR, and the target is much smaller than the time scale of the turbulence. For this reason, with a monostatic LIDAR the deviations of the beam encountered in the forward propagation are identical, thus compensated by those encountered along the backward propagation.

6.4 Development of a LIDAR measurement simulator

This Section is the result of a common work with S.Brousmiche (Telecommunication and Remote Sensing Laboratory of Université catholique de Louvain). We here summarize the physics of the LIDAR measurement principle and the methodology used in the LIDAR simulator. For more detailed information, the reader can consult the work of Brousmiche [13].

6.4.1 LIDAR signal

Before entering in the details of the LIDAR simulator, it is relevant to define some physical concepts related to the LIDAR theory.

LASER waves are electromagnetic waves which obey to the Maxwell equations:

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \quad (6.35)$$

$$\nabla \times \mathbf{H} = \epsilon \frac{\partial \mathbf{E}}{\partial t} \quad (6.36)$$

$$\nabla \cdot \epsilon \mathbf{E} = 0 \quad (6.37)$$

$$\nabla \cdot \mu \mathbf{H} = 0. \quad (6.38)$$

Here $\mathbf{E}$ is the electric field and $\mathbf{H}$ is the magnetic field, μ and ϵ are the permeability and the permittivity of the medium in which the wave is propagating. We assume that the medium is dielectric, linear, isotropic, homogeneous, non dispersive and non magnetic (magnetic permeability equal vacuum permeability $\mu = \mu_0$). Taking the curl of Eq. 6.35 and using Eq. 6.36 leads to:

$$\nabla^2 \mathbf{E} = \frac{n^2}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}, \quad (6.39)$$

n being the refractive index of the medium. The magnetic field satisfies an identical equation:

$$\nabla^2 \mathbf{H} = \frac{n^2}{c^2} \frac{\partial^2 \mathbf{H}}{\partial t^2}. \quad (6.40)$$

Since the vector wave equation is satisfied for each component of $\mathbf{E}$ and $\mathbf{H}$, it is possible to summarize the behaviour of all components of $\mathbf{E}$ and $\mathbf{H}$ through a single scalar wave equation. It is thus convenient to represent optical waves by a scalar field, written $\psi(\mathbf{u}, t)$. The vector $\mathbf{u}$ indicates the transverse coordinate in a plane perpendicular to the propagation direction, the notation convention for the fields and the vectors is illustrated in Fig. 6.11. The optical field obeys the wave propagation equation:

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{c^2}{n^2} \nabla^2 \psi. \quad (6.41)$$

The units of the optical field are $\left[(Wm^{-2})^{1/2}\right]$. For instance, the lowest order Gaussian mode, TEM_{00} , has the form:

$$\psi(\mathbf{u}, 0) = \psi_0 \exp\left(-\frac{\mathbf{u}^2}{\sigma_0^2} - jk\frac{\mathbf{u}^2}{2F_0}\right), \quad (6.42)$$

the beam diameter being $2\sigma_0$, and F_0 being the focal length. If $F_0 > 0$, the beam is said convergent or focused; if $F_0 < 0$, the beam is divergent the case $F_0 = \infty$ corresponds to a collimated beam. This optical field is usually normalized so that, in the transmitter plane,

$$P\left(t - \frac{z}{c}\right) = \int_{-\infty}^{\infty} |\psi(\mathbf{u}, z, t)|^2 d\mathbf{u}. \quad (6.43)$$

Since a pulsed LASER is considered in the present work, the power of the emitted beam varies with time.

$$P(t) = \frac{U_L}{\sqrt{\pi}\tau} \exp\left(-\frac{t^2}{\tau^2}\right) \quad (6.44)$$

The LASER pulse energy is $U_L [J]$; for the fiber LASER considered in the LASEF project, its value is approximatively $150 \mu J$. The very challenging limit (which is not yet attained at this time) with a fiber LASER is $1 mJ$. Increasing the energy is however quite problematic, as it induces non linear effects in the fiber which lead to severe loss of beam quality. The pulse duration is τ and this value is usually a few hundred nanoseconds.

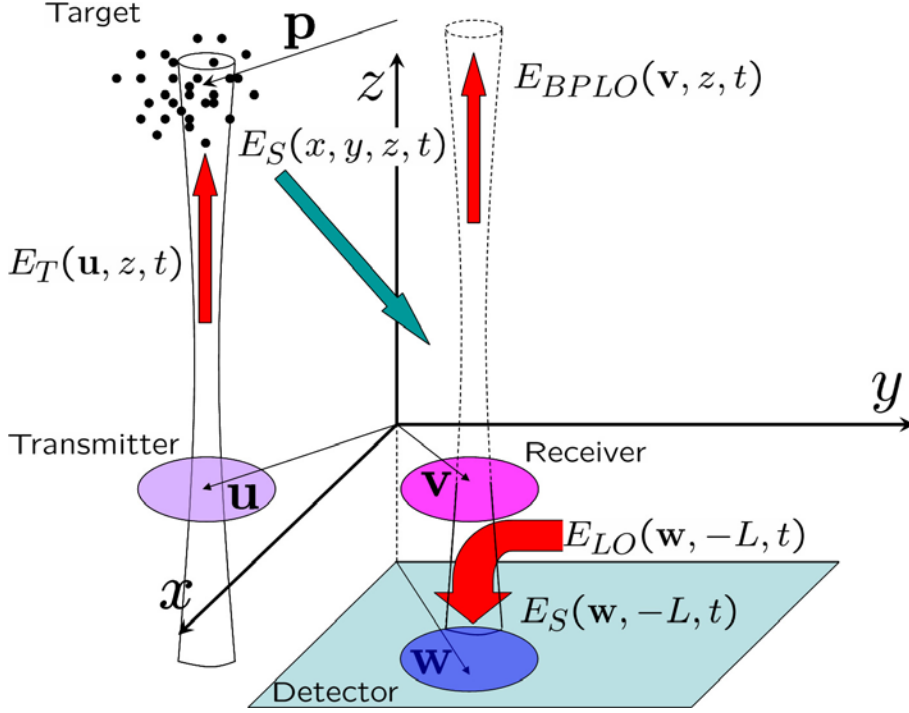


Figure 6.11: Monostatic coherent LIDAR optical fields convention. The z axis is oriented with one LOS.

For instance, a pulse with a duration of 200 ns covers a portion of space of 60 m . Ideally, a Gaussian pulse form should be chosen, because its representation in the Fourier space is itself a Gaussian. The real pulse generated by a Q-switching device is less smooth and presents a shape which is closer to a square pulse as illustrated in Fig. 6.12. Recall that a pulse which has a form close to a square has a spectrum which is broader than a Gaussian pulse; this leads to a decrease on the velocity accuracy. The optical field is split using the slowly varying envelope approximation:

$$\psi(\mathbf{u}, z, t) = E(\mathbf{u}, z, t) \exp(-j\omega t). \quad (6.45)$$

This approximation is legitimate due to the fact that the envelope $E(\mathbf{u}, z, t)$ is varying slowly compared to the frequency ω . The scalar field $E(\mathbf{u}, z, t)$ can also be reduced in a transverse varying part and a line of sight varying part:

$$E(\mathbf{u}, z, t) = E_0(\mathbf{u}, t) \exp(-\gamma z). \quad (6.46)$$

The propagation exponent $\gamma = \alpha + jk$ contains information on the atmospheric attenuation $\alpha\text{ [m}^{-1}\text{]}$ and on the spatial frequency of the wave $k = \frac{2\pi}{\lambda}\text{ [rad m}^{-1}\text{]}$. In the present work, the optical wavelength is fixed at $\lambda = 1.55\mu\text{m}$ (infrared).

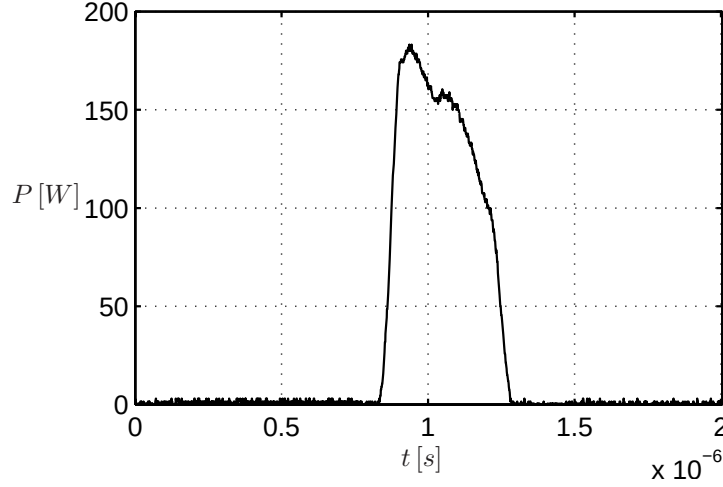


Figure 6.12: Real pulse generated by a Q-switching device. Energy value is $U_L = \int_{-\infty}^{\infty} P(t) dt \approx 56 \mu J$ and pulse duration is $\tau \approx 440 ns$.

The global linear extinction coefficient is $\alpha [m^{-1}]$; an order of magnitude is $\alpha = 0.1 [km^{-1}]$. This extinction coefficient includes the absorption and the diffusion of all the atmospheric constituents that the beam illuminates. In this description, we use the back-propagated local oscillator (BPLO) formalism. As explained in Belmonte [4], this formalism allows to reduce the problem of computing LIDAR returns to the propagation of the transmitted and back-propagated fields along the LOS and perform the heterodyne mixing in the target plane instead of the detector plane. This eliminates the complexity of propagating random optical field from the scatterers via the receiver optics to the detector plane (as it should be done if the mixing was made in the detector plane). The optical field transmitted in the atmosphere is written $\psi_T(t)$ while the back-propagated local oscillator is $\psi_{BPLO}(t)$. They both have units of $[W/m^2]$. On the emitting plane, both fields are written:

$$\psi_T(\mathbf{u}, 0_+, t) = \sqrt{T_T P_T(t)} e_T(\mathbf{u}, 0_+) \exp(j2\pi f_T t) \exp(j\phi_T(t)) \quad (6.47)$$

$$\psi_{BPLO}(\mathbf{v}, 0_+, t) = \sqrt{T_{BPLO} P_{LO}} e_{BPLO}(\mathbf{v}, 0_+) \exp(j2\pi f_{LO} t). \quad (6.48)$$

P_{LO} is the power of the local oscillator beam and $P_T(t)$ the power of the beam which is transmitted in the atmosphere. The phase lag between the emitted field and the backscattered one is ϕ_T . The truncation factor, which is the fraction of the power of a beam having a radius σ_0 transmitted through an aperture of radius R_0 , is defined as:

$$T_T = 1 - \exp\left(-2 \frac{R_0^2}{\sigma_0^2}\right). \quad (6.49)$$

In this work, we let $T = T_T = T_{BPLO}$. The normalized optical fields e_T and e_{BPLO} are defined in the following way:

$$e_T(\mathbf{u}, 0_+) = \frac{E_T(\mathbf{u}, 0_+, t)}{\sqrt{P_T(t)}}, \quad e_{BPLO}(\mathbf{v}, 0_+) = \frac{E_{BPLO}(\mathbf{v}, 0_+, t)}{\sqrt{P_{LO}}}. \quad (6.50)$$

The locus $z = 0_+$ is the emission plane with its normal vector directed toward the atmosphere. The transverse coordinate in the emission plane is $\mathbf{u}$ while the transverse coordinate in the reception plane is $\mathbf{v}$. The general expression for the transmitted optical field computed in the target plane (at a distance z of the emitter and at transverse location $\mathbf{p}$) is written:

$$\begin{aligned} \psi_T(\mathbf{p}, z, t) = & \sqrt{T_T P_T \left(t - \frac{z}{c}\right)} K(z) \times \\ & \exp\left(j2\pi f_T \left(t - \frac{z}{c}\right)\right) \exp\left(j\phi_T \left(t - \frac{z}{c}\right)\right) \times \\ & \int_{-\infty}^{\infty} e_T(\mathbf{u}, 0_+) G(\mathbf{p}; \mathbf{u}, z) d\mathbf{u} \end{aligned} \quad (6.51)$$

The function $K(R)$ is the dimensionless one-way atmospheric transmission factor at a range R :

$$K(R) = \exp\left(-\int_0^R \alpha(R) dR\right). \quad (6.52)$$

The general Green function appearing in the Huygens-Fresnel integral of the last term in Eq. (6.51) is defined as the product of the free space Green function:

$$G^f(\mathbf{p}; \mathbf{u}, z) = \frac{k}{2\pi iz} \exp\left(\frac{ik}{2z} (\mathbf{p} - \mathbf{u})^2\right) \quad (6.53)$$

and a wavefront distortion term which takes turbulence into account, leading to:

$$G(\mathbf{p}; \mathbf{u}, z) = G^f(\mathbf{p}; \mathbf{u}, z) \exp\left(ik \int_0^z n_1(p, z') dz'\right). \quad (6.54)$$

The back propagated optical field computed in the receiver plane contains the influence of each wavelets issued from a particle i located at $(\mathbf{p}_i, z_i)$.

$$\psi_S(\mathbf{v}, 0_+, t) = \sum_i \psi_T\left(\mathbf{p}_i, z_i, t - \frac{z_i}{c}\right) G(\mathbf{v}; \mathbf{p}_i, z_i). \quad (6.55)$$

The Green function $G(\mathbf{v}; \mathbf{p}_i, z_i)$ allows to compute the lighting of the particle i alone located at $(\mathbf{p}_i, z_i)$ on the receiver plane at location $(\mathbf{v}, 0)$. Each particle backscatters a fraction of the emitted energy with a different phase θ_i . For particles having a scattering cross section $\sigma [m^2 sr^{-1}]$,

$$r_i = |r_i| \exp(j\theta_i) = \lambda \sigma^{1/2} \exp(j\theta_i). \quad (6.56)$$

The heterodyne current at the photodetector output can be computed in the following way

$$i_S(t) = 2S \Re \left\{ \int_{-\infty}^{\infty} \psi_S(\mathbf{v}, 0_+, t) \psi_{BPLO}^*(\mathbf{v}, 0_+, t) d\mathbf{v} \right\}. \quad (6.57)$$

The back-propagated local oscillator can be computed in the target plane using the Huygens Fresnel principle:

$$e_{BPLO}^*(\mathbf{p}, z) = \int_{-\infty}^{\infty} e_{BPLO}^*(\mathbf{v}, 0_+) G^*(\mathbf{p}; \mathbf{v}, z) d\mathbf{v}. \quad (6.58)$$

The heterodyne current can also be evaluated in the target plane as:

$$i_S(t) = 4ST \sqrt{P_{LO}} \times \quad (6.59)$$

$$\Re \left\{ \exp(j2\pi f_{IF} t) \sum_i r_i K(z_i) \sqrt{P_T \left(t - \frac{z_i}{c}\right)} \exp\left(j4\pi f_{IF} \frac{z_i}{c}\right) e_S(\mathbf{p}_i, z_i) e_{BPLO}^*(\mathbf{p}_i, z_i) \right\}$$

The sum involved in the right hand side is carried over a very high number of particles. Consider a low particle density, such as 10^6 aerosols per m^3 . The scattering volume value should be $10 m^3$ for a beam with a diameter of $0.3 m$ and a propagation along $150 m$. The total number of particles contributing to the signal is thus 10^7 . The sum appearing in Eq. (6.59) which accounts for the contribution of each particle is not computable. This highlights the need of an appropriate modelling.

6.4.2 The Feuilleté model

This model was proposed by Salameitou et al. [106]. It allows to compute the LIDAR signal by considering a slicing of the atmosphere in layers with homogeneous properties. The region of interest is divided into slices labeled c_m of depth Δz , each slice having its center located at the range z_m along the considered LOS. At a given time t , the number of slices that contribute to the LIDAR signal is M , which obeys to following relation:

$$M \Delta z = c\tau, \quad (6.60)$$

where τ is the pulse duration. The depth of the slices is such that the atmospheric properties can be considered as homogeneous inside each slice. The depth should also satisfy the following constraints:

$$\lambda \ll \Delta z \ll \frac{c}{\delta f}. \quad (6.61)$$

The first constraint comes from the use of the Huygens-Fresnel integrals on which the formulation considered here is based. The second constraint depends on the spectral sharpness of the LASER and translates the fact that the depth

of the slice should be small compared to the coherence length. As the upper and lower bounds differ by several orders of magnitude, usually the slice thickness is taken as their geometric mean:

$$\Delta z = \sqrt{\frac{c\lambda}{\delta f}}. \quad (6.62)$$

Typically, for a fiber LASER as considered here, one could reach $\delta f = 8kHz$ with $\lambda = 1.55\mu m$. For a LASER pulse with $\tau = 200ns$, this provides $\Delta z = 0.25m$ and $M = 120$. Another constraint should be linked with the physical aspect of the flow under investigation. For instance, a vortex with a core size of $r_c = 2.5m$ is well discretized if $\Delta z \leq r_c/3$. This leads to $\Delta z = 0.8m$ which is satisfied in the present case. Note that Eq. (6.62) is quite severe in terms of computational cost: if the maximum range is $1000m$, this leads to a total number of $N = 4000$ slices. Practically, we use in this case the constraint based on the vortex, which provides $\Delta z = 1m$ and $N = 1000$ and satisfy the model constraints. The spatial discretization scheme being defined, Eq. (6.59) giving the heterodyne current can be written for each layer c_m :

$$\begin{aligned} i_{S,m}(t) = & 2S\sqrt{T_T T_{BPLO} P_{LO}} \\ & \Re \left\{ \exp(j2\pi f_{IF} t) \sum_{i \in c_m} r_m K(z_m) \left(P_T \left(t - \frac{z_m}{c} \right) \right)^{1/2} \right. \\ & \left. \exp \left(-j4\pi f_{IF} \frac{z_i}{c} \right) e_S(\mathbf{p}_i, z_i) e_{BPLO}^*(\mathbf{p}_i, z_i) \right\}. \end{aligned} \quad (6.63)$$

For each slice, a new dimensionless function ξ is defined:

$$\xi_m(t) = \sqrt{T_{BPLO} T_T} K(z_m) \sum_{i \in c_m} r_m \exp \left(-j4\pi f_{IF} \frac{z_i}{c} \right) e_S(\mathbf{p}_i, z_i) e_{BPLO}^*(\mathbf{p}_i, z_i). \quad (6.64)$$

The time dependency of this function is due to the motion of the particles. It is assumed that the particle motion is uniform and rectilinear during the transmitter pulse duration:

$$z_i = z_{i,0} + v_{z,m} t. \quad (6.65)$$

Where $z_{i,0}$ is the position of the particle when the pulse reaches the slice m . By assuming that the irradiances e_T and e_{BPLO} does not depend on the range in each layer, and using the rectilinear uniform motion assumption, one can write:

$$\begin{aligned} \xi_m(t) = & \sqrt{T_{BPLO} T_T} K(z_m) \exp \left(-j4\pi f_{IF} \frac{v_{z,m} t}{c} \right) \\ & \sum_{i \in c_m} r_m \exp \left(-j4\pi f_{IF} \frac{z_{i,0}}{c} \right) e_{S,m}(\mathbf{p}_i) \cdot e_{BPLO,m}^*(\mathbf{p}_i) \end{aligned} \quad (6.66)$$

The fact that all the particles inside one slice are assumed to have the same velocity during the pulse duration allows to take the velocity dependent Doppler

term out of the sum. $\xi_m(t)$ is a zero-mean Gaussian random variable determining the stochastic character of the signal current $i_s(t)$ which includes the speckle effect due to the individual aerosol particles movement and the turbulent variations of the refractive index of air. Its variance is given by

$$\langle |\xi_m|^2 \rangle = K(z_m) \beta_m \frac{\pi R_p^2}{z_m^2} \gamma_m \Delta z. \quad (6.67)$$

where $K(z_m) = \exp[-2 \int_0^{z_m} \alpha(z) dz]$ is the dimensionless two-way extinction with $\alpha(z) [m^{-1}]$ the linear extinction coefficient, $\beta_m [m^{-1} sr^{-1}]$ the backscattering coefficient, $R_p [m]$ the telescope radius and γ_m the system efficiency defining the fraction of the backscattered laser beam converted to a coherent signal. This system efficiency includes the turbulence optical effects taken into account by means of the phase screen technique as detailed in Brousmiche et al. [14]. With this definition of ξ_m , the heterodyne current can be written:

$$i_{S,m}(t) = 2S \sqrt{P_T \left(t - \frac{z_m}{c}\right)} P_{LO} \cos \left(2\pi \left(f_{IF} - \frac{2v_m \left(t - \frac{z_m}{c}\right)}{\lambda_T} \right) t \right) \Re \{ \xi_m(t) \}. \quad (6.68)$$

The current due to the contribution of M the slices contributing to the signal at time t is written:

$$\begin{aligned} i_S(t) &= \sum_M i_{S,m}(t) \\ &= 2S \sqrt{P_{LO}} \exp(j2\pi f_{IF} t) \\ &\quad \sum_M \sqrt{P_T \left(t - \frac{z_m}{c}\right)} \xi_m(t) \exp(-j2kv_m t). \end{aligned} \quad (6.69)$$

The LIDAR signal is discretized in time with a sampling period $T_s = 1/F_s$. At each instant, the pulse covers M slices which have uniform physical properties. Finally,

$$\begin{aligned} i_S(nT_s) &= \sum_M i_{S,m}(nT_s) \\ &= 2S \sqrt{P_{LO}} \exp(j2\pi f_{IF} nT_s) \\ &\quad \sum_M \sqrt{P_T \left(nT_s - \frac{z_m}{c}\right)} \xi_m(nT_s) \exp(-j2kv_m nT_s). \end{aligned} \quad (6.70)$$

6.5 Performance parameters of a LIDAR

6.5.1 Signal to noise ratio

The signal to noise ratio compares the useful current to the noise current. It informs on the ability of the LIDAR to detect low level signals within the noise.

it is evaluated at a range z using:

$$\text{SNR}(z) = \frac{\langle |i_S(t)|^2 \rangle}{\langle |i_n(t)|^2 \rangle} = \frac{T_T^2 U_l \beta \lambda K^2(z) A}{\hbar B} \frac{1}{z^2} \gamma(z). \quad (6.71)$$

This parameter depends on the energy of the LASER U_l , the truncation factor of the telescope T_T , which is taken into account twice in the monostatic case, the telescope aperture area A_R , the backscattering coefficient β and the system efficiency γ . Since a larger bandwidth B allows for more noise, this factor appears at the denominator. The noise is usually dominated by the quantum noise due to interaction of the local oscillator field with the photodetector.

6.5.2 Spatial resolution

The spatial resolution δR of a pulsed LIDAR is given by the following relation:

$$\delta R = \frac{c\tau}{2}. \quad (6.72)$$

If one consider a short pulse duration of $\tau = 200 \text{ ns}$, one has a spatial resolution of $\delta R = 30 \text{ m}$. This resolution is quite coarse compared to the scale of a wake vortex core which has a radius of $r_c \approx 3 \text{ m}$. One could reduce the pulse duration; however, in this case, the fiber has to support more peak power.

6.5.3 Velocity accuracy

The precision of the velocity measurement is estimated as the standard deviation of the velocity. This one depends on the pulse duration, the signal to noise ratio SNR and the LASER wavelength. A lower bound can be found for the accuracy on the velocity measurement; we here provide the one used in [13].

$$\sigma_V > \frac{\lambda}{2\tau} \frac{1}{\sqrt{2SNR}}. \quad (6.73)$$

If the pulse duration is decreased to enhance the spatial resolution, the velocity measurement become less accurate accordingly. This calls for appropriate LIDAR signal processing algorithms which are able to obtain a better space resolution than that imposed by Eq. (6.72).

6.5.4 LASER beam coherence

The real intensity profile of a LASER beam is not truly Gaussian. The convention adopted to characterize the quality of a beam is the M^2 factor. This factor is the ratio between the energy in all the modes and the energy in the first mode. One has thus $M^2 = 1$ for a truly Gaussian beam. The evolution of the SNR with M^2 is giving by the following law, as given in Cariou et al. [18]:

$$\frac{SNR(M^2)}{SNR(1)} = \frac{2}{1 + M^4} \quad (6.74)$$

If a loss of 3 dB is considered, the threshold M^2 value is $M^2 = 1.73$. This is thus a quantity of primary importance however, in the case of the LIDAR simulator, so far the quality factor is set to $M^2 = 1$. Usually a LASER with a good coherence has $M^2 \leq 2$ which is the target specification of the LASEF project. Low power fiber lasers, using single mode fibers, deliver a perfect Gaussian beam, with a quality factor $M^2 < 1.1$. However, higher power LASER and amplifiers need larger fiber cores, to store enough energy and to avoid non-linear effects. This indeed limits the quality of the beam. Note also that the small scales turbulence fluctuations of the refractive index deteriorate the beam quality.

6.6 Fluid flow interpolation technique

In order to compute the Doppler term $\exp(-j2kv_mt)$ appearing in the equation giving the heterodyne current (Eq. 6.69), the radial velocity in each layer m and for each LOS is mandatory. As the fluid flow velocity field $(v(y, z), w(y, z))$ issued from the Navier-Stokes simulation is computed on cartesian grid, it is necessary to interpolate the data onto the LIDAR line of sight and perform a projection of the velocity vector on the LOS in order to have the radial velocity. An illustration of this operation is provided in Fig. 6.14.

The interpolation scheme selected in this work to interpolate information from a CFD grid to the LIDAR measurement points is the M'_4 scheme. This technique is widely used in vortex particles methods to redistribute vorticity from a distorted set of particles onto a regular cartesian grid. In this case we perform the inverse operation as we transfer information from a cartesian rectangular grid onto a polar one. This scheme is defined as:

$$\phi^I(\vec{x}) = \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} W(\vec{x} - \vec{x}_{ij}^S) \phi_{ij}^S,$$

where $\vec{x}$ is the lidar sensing location while $\vec{x}^S$ is the location at which the information is computed. The kernel for the M'_4 scheme is defined as:

$$W(x) = \begin{cases} 1 - \frac{5}{2} \left| \frac{x}{h} \right|^2 + \frac{3}{2} \left| \frac{x}{h} \right|^3 & \text{if } \left| \frac{x}{h} \right| \leq 1 \\ \frac{1}{2} \left(2 - \left| \frac{x}{h} \right| \right)^2 \left(1 - \left| \frac{x}{h} \right| \right) & \text{if } 1 < \left| \frac{x}{h} \right| \leq 2 \\ 0 & \text{if } \left| \frac{x}{h} \right| > 2 \end{cases}$$

In Fig. 6.13, one observes the number of points on the cartesian grid which influence the target point on a LOS. The quality of this interpolation scheme is illustrated in Fig. 6.15 in the case of a single vortex velocity field mapped from a cartesian grid to a polar one aligned with the LIDAR lines of sight.

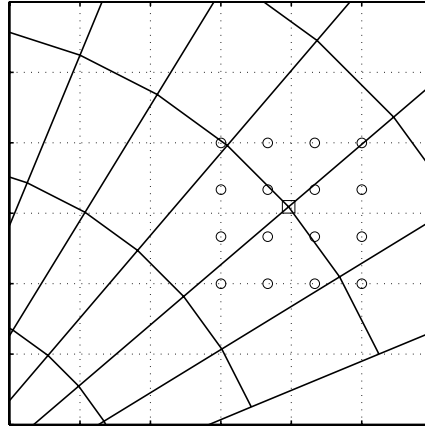


Figure 6.13: Interpolation Stencil: the value at a target point (square) located on a given LOS at a given range is obtained by interpolation using the 16 neighbouring points (circles) of the cartesian CFD grid with a spacing h .

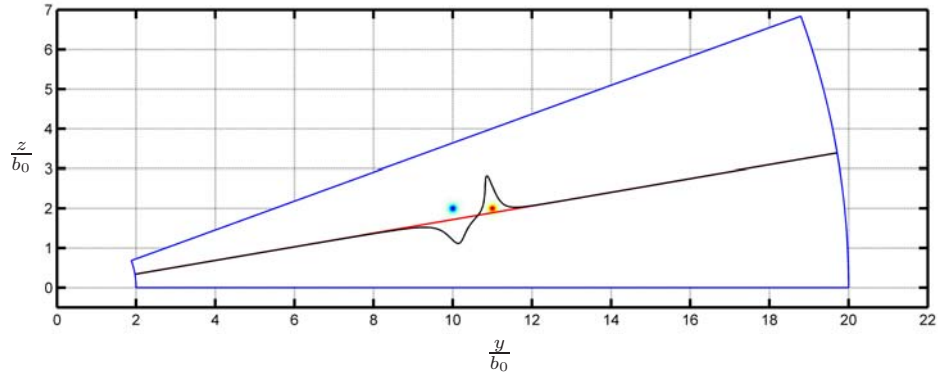


Figure 6.14: Radial velocity profile sensed along one LOS for a pair of analytical vortices separated by a distance b_0 .

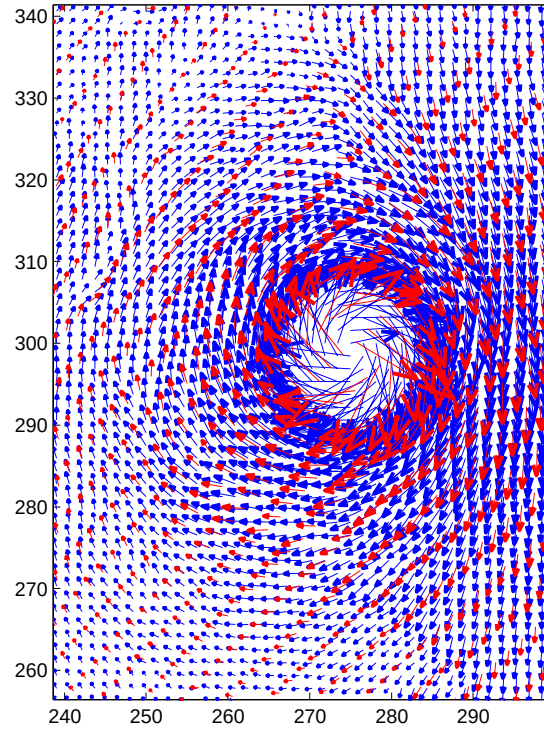


Figure 6.15: Interpolation of a vortical velocity field from a cartesian grid (blue) onto a LIDAR LOS based polar grid (red).

6.6.1 LIDAR scanning patterns

There are two ways to scan the wake of an aircraft with a LIDAR. The first one, for which the LASER beam is aligned with the vortex axis is mainly used in the case of aircraft based detection. In this case, the LIDAR only senses the axial velocity component of the wake which is weaker than the tangential ones. However this instantaneous axial component is not negligible, as highlighted by the numerical simulations performed in Chap. 4. Several investigations of axial vortex detection were carried out in the framework of the project FIDELIO. In particular the Simulation of a LIDAR-based aircraft wake vortex detection using a bi-Gaussian spectral model was performed using the wake vortex flows databases documented in Chap.4, see Lugan et al. [75]. In the second case, the scanning plane is perpendicular to the vortex axis; this configuration is mainly used for ground based measurements. This scanning configuration allows to sense the tangential velocities of a vortex. This allows to better discriminate the velocity signal due to the vortices from that due to the background turbulence, which is weaker. When using a ground-based scan-

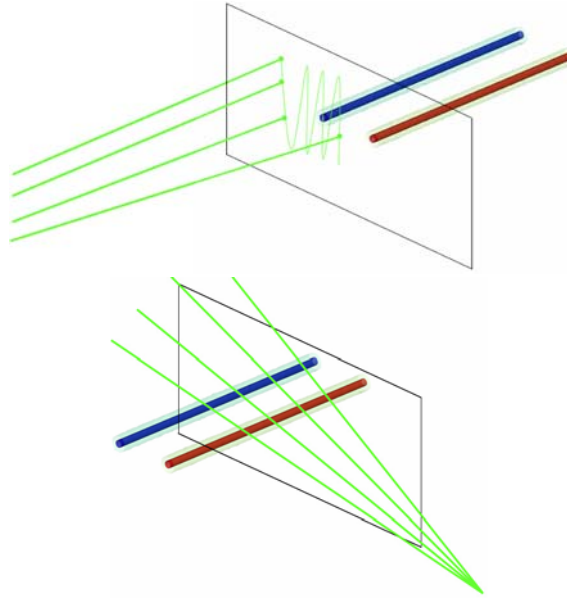


Figure 6.16: Two different scanning strategies for aircraft wake vortex detection

ning pattern, the angular speed $\dot{\theta}$ of the scanner is directly linked to the pulse repetition frequency (PRF): the frequency at which the pulses are released from the Q-switching device and the angular resolution $\Delta\theta$ by the following relation: $\dot{\theta} = PRF \times \Delta\theta$. For each line of sight, one LASER pulse is sent, sweeping the whole range along it.

6.7 Radial velocity profiles processing for vortex systems

The final measurement results provided by the LIDAR after appropriate signal processing of the heterodyne current at the photodetector are radial velocity profiles as a function of the range R and the LOS elevation angle α . Since the information on the two velocity component in the scanning plane cannot be recovered, it is useful to exploit the radial velocity profiles as much as possible. The first information that can be extracted is the presence of the vortices. Plotting the colormap of the radial velocity profile allows, by appropriate image processing to detect the presence of vortices. This strategy does not allow to get quantitative information on the intensity (and thus the hazard) of the vortices. We define the LIDAR polar coordinates system (R, α) , where R is the range

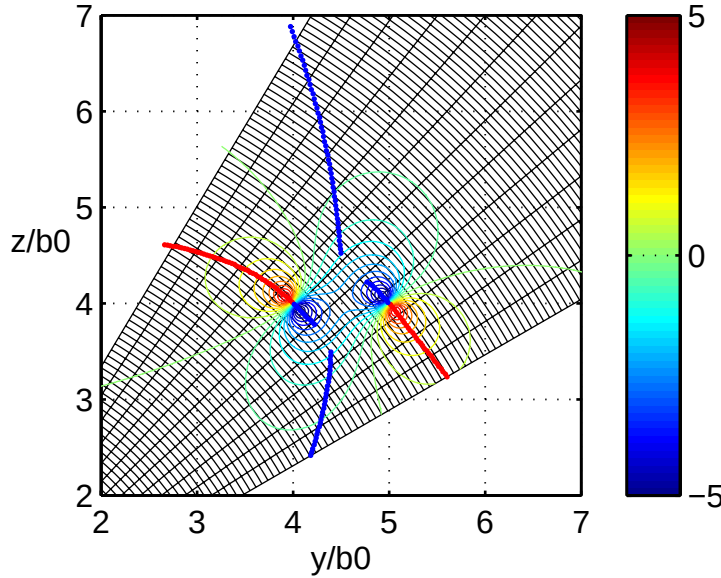


Figure 6.17: Radial velocity contour $\frac{V_R}{V_0}$ for a pair of analytical vortices. The red and blue lines show the loci of maximal and minimal values of V_R along each LOS.

and α the LOS elevation angle. The system (r, θ) is the vortex local coordinate system. The maximum of V_R along one LOS having an angle of elevation α^* is a good estimator of the vortex tangential velocity u_θ at the location (R^*, α^*) of this maximum. Indeed, the velocity field induced by one vortex and projected along a line of sight is written:

$$V_R = u_\theta(r) (\sin(\theta) \cos(\alpha) - \cos(\theta) \sin(\alpha)) = u_\theta(r) \sin(\theta - \alpha) \quad (6.75)$$

The maximum value of V_R will be attained at a point where the LOS is tangent to the streamline of the vortex. In this case, $\theta = \frac{\pi}{2} + \alpha$, and thus $V_R = u_\theta(r)$. If one assumes that a vortex is an axisymmetric object, the tangential velocity found at one point on a streamline is the same everywhere on the same line. If the location of the vortex center (x_c, y_c) is known, the radial distance r is the distance from the center to the point (R^*, α^*) . From the tangential velocity estimator, it can be deduced that :

$$\Gamma(r) = 2\pi r u_\theta(r) \quad (6.76)$$

As an example, if we consider a low order algebraic vortex model centered in (x_c, y_c) with $\tan(\beta) = (y_c/x_c)$ and $R_c = \sqrt{x_c^2 + y_c^2}$, one finds:

$$V_R = \frac{\Gamma_0}{2\pi} \frac{x_c \sin(\alpha) - y_c \cos(\alpha)}{R^2 + R_c^2 - 2RR_c \cos(\beta - \alpha) + r_c^2} \quad (6.77)$$

The maximal radial velocity is retrieved at $R^* = R_c \cos(\beta - \alpha)$ by solving

$$\frac{\partial V_R}{\partial R} = 0. \quad (6.78)$$

With Eq. (6.76), the circulation and velocity profiles of a single vortex can thus be deduced from $V_R^*(R^*, \alpha^*)$. However, there are always two vortices, and there is indeed a non negligible influence of the second vortex on the measured value of V_R at the maximum. As shown in Fig. 6.18, for the starboard vortex, there is an excess approximation if the circulation is evaluated from the positive maxima locus (red curve Fig. 6.17), or a default approximation if the negative one is used (blue curve Fig. 6.17). The idea to be able to evaluate correctly the

$$\Gamma_{5-15} = \frac{1}{\frac{b}{6}} \int_{\frac{b}{12}}^{\frac{b}{4}} \Gamma(r) dr \quad (6.79)$$

vortex circulation indicator³ is to average the excess and default approximations. This leads to the circulation profiles in Fig. 6.19 from which the integral circulation measurement Γ_{5-15} should be recovered properly. The fact that the axisymmetry is lost for a pair of vortices is of less importance as we consider only velocity values in the range $r \in [b/12, b/4]$ which is still close to the center of the vortex.

³ b is the aircraft wingspan. For an elliptical span loading, $b_0 = \frac{\pi}{4}b$ where b_0 is the vortex spacing.

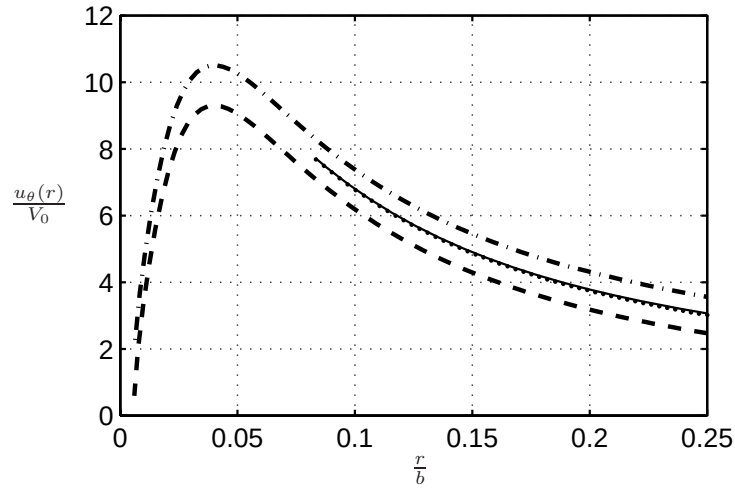


Figure 6.18: Estimation of $u_\theta(r)$ using the Maximum of V_R on each LOS in the case of a two vortex system (starboard vortex). Excess approximation, positive V_R values (dash-dot); Default approximation, negative V_R values (dash). Average between the excess and default predictions (bullets) for $r \in [b/12, b/4]$. Theoretical $u_\theta(r)$ (solid) $r \in [b/12, b/4]$.

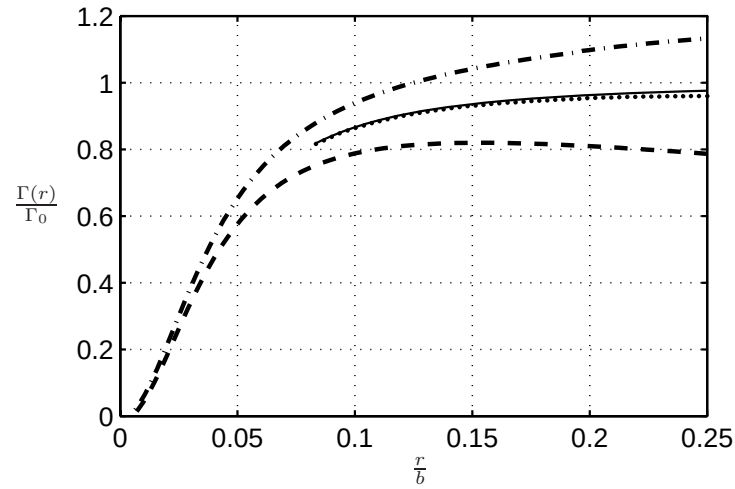


Figure 6.19: Retrieved circulation profiles, same conventions.

6.8 LIDAR measurement simulation results

This section aims to illustrate the results obtained by simulating a virtual LIDAR using the fluid flows data detailed in the course of the preceding chapters. First, an airborne LIDAR simulation is presented. This one does not take into account atmospheric effects. It demonstrates that an axial shooting is appropriate for airborne vortex detection. Second, we present the simulation of the remote sensing of a turbulent wind using the simulator described in Section 6.4. Finally we illustrate the remote sensing of aircraft wake vortices in ground effect, using the same simulator.

6.8.1 Airborne remote sensing of a pair of aircraft wake vortices in atmospheric turbulence

The databases obtained by performing LES of aircraft wake vortices put in a weak atmospheric turbulence allowed to perform the simulation of an axial airborne based wake vortex detection. This work was carried out by S. Lugan (Telecommunication and Remote Sensing Laboratory of Université catholique de Louvain) in the framework of the FP6 european project FIDELIO. We illustrate here the methodology and the results obtained using the fluid flow databases generated in the present work and detailed at Chap. 4. In order to have a sufficient spatial extension of the fluid databases, the database including the wake vortices and the turbulence have to be concatenated with databases containing turbulence only. This requires to carefully choose the turbulence database in order to avoid significant discontinuities, as illustrated in Fig. 6.20. Fig. 6.21, shows the results of an axial scanning of aircraft wake vortices as the one depicted in Fig. 6.16. The principle of this simulation is to send a Gaussian power pulse with a given duration, say 200 ns , and thus a fixed spatial extension 60 m along the line of sight. The velocity values which are collected along the beam are weighted by the value of this Gaussian pulse. This allows to generate averages and variances of the projected velocity.

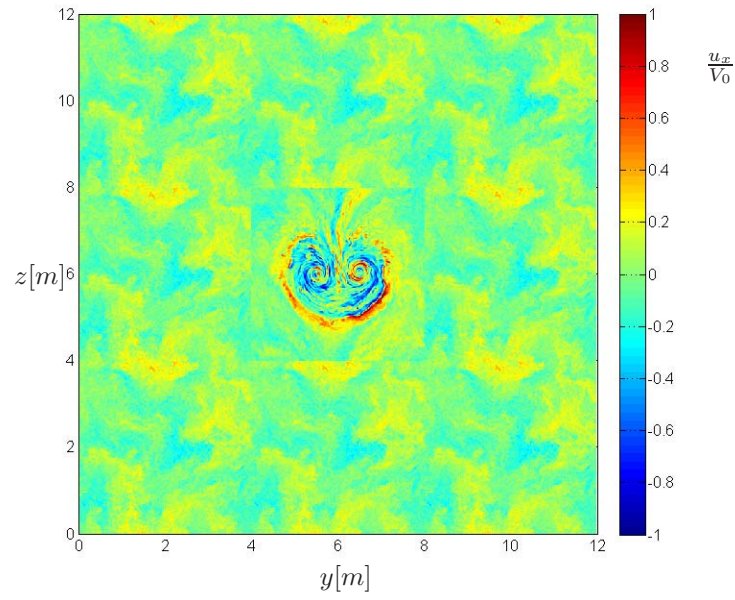


Figure 6.20: Contours of axial velocity component completed with surrounding turbulence of same intensity.

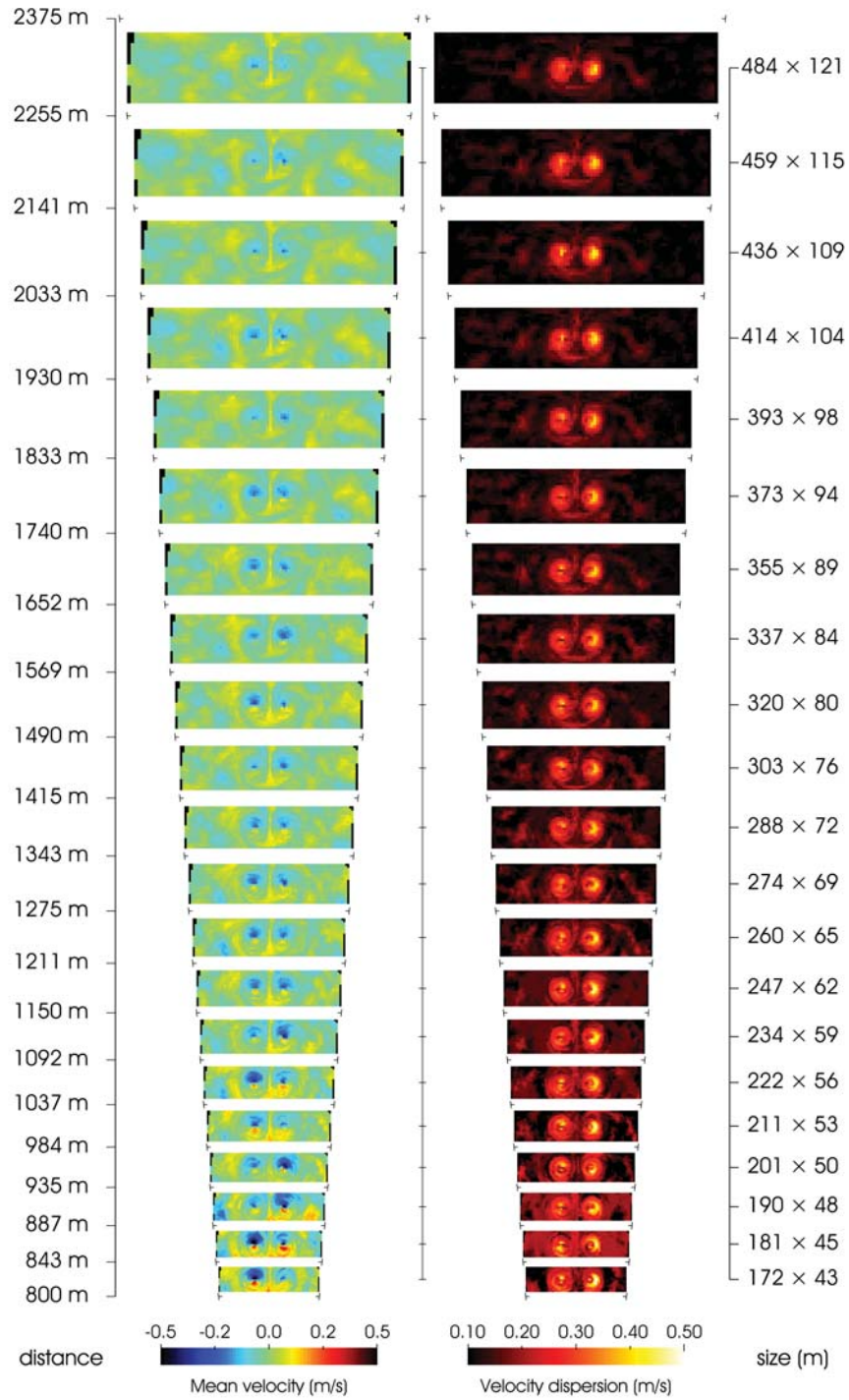


Figure 6.21: Simulation of a synthetic airborne Doppler LIDAR scanning. Image courtesy of S.Lugan.

6.8.2 Remote sensing of a turbulent boundary layer

The content of this section is a partial reproduction of the proceedings [14] (International Geoscience And Remote Sensing Symposium (IGARSS 2007)). The authors are S.Brousmiche and L.Bricteux.

Doppler LIDAR can help in detecting the presence of aircraft wake vortices on airport runways. Usually the detection of aircraft wake vortices is accompanied by wind speed measurements. We propose here to assess the feasibility of wind speed anemometry for airport applications with a ground based LIDAR which usually serves for aircraft wake vortices detection. For that purpose, we simulate the signal returned by a LIDAR with a given set of characteristics and interacting with a simulated turbulent wind. The LIDAR simulation algorithm takes into account LASER beam propagation in a turbulent atmosphere. The parameters of the simulated LIDAR are detailed in Table 6.2. The minimal and maximal LOS elevation angles θ_{min} and θ_{max} and the maximum range are chosen so that the scanning pattern is included in the computational domain used for the fluid flow simulation. The velocity components of the simulated wind used for the LIDAR simulation are displayed in Fig. 6.22.

Table 6.2: Parameters for the simulation of the LIDAR remote sensing of a turbulent boundary layer.

Parameter	Value
Laser wavelength λ	$1.55 \mu m$
Intermediate frequency	$100 MHz$
Beam radius	$0.03 m$
Focal length of the telescope	∞
Pulse energy	$100 \mu J$
Pulse duration	$200 ns$
Maximum range	$600 m$
Sampling frequency	$512 MHz$
Lens radius	$0.05 m$
θ_{min}	3°
θ_{max}	20°
Pulse repetition frequency	$250 Hz$
Scanning angular speed	$2^\circ/s$

Considering the particular scanning pattern illustrated in Fig. 6.23, and due to the fact that, with a Doppler LIDAR, only the radial velocity

$$V_R = u_x \cos \theta + u_y \sin \theta \quad (6.80)$$

is available, we define another quantity to compare the results obtained after

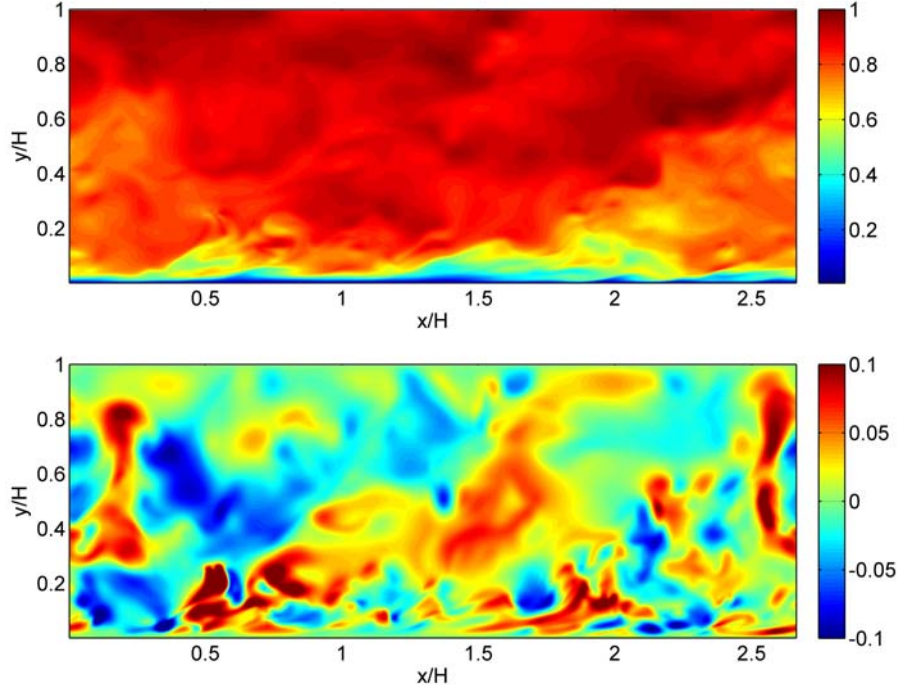


Figure 6.22: Snapshot of velocity: $\frac{u_x}{U}$ (top) and $\frac{u_y}{U}$ (bottom) in a x - y plane. These velocity components are used to compute the Doppler shift.

the signal processing with those issued from the fluid flow simulation:

$$V_R^\theta = \frac{V_R}{\cos(\theta)}. \quad (6.81)$$

This quantity is close to u_x in the regions where u_y is weak. We compute an average profile of V_R^θ as a function of the altitude, as illustrated in Fig. 6.23. To perform this average, one defines bins of vertical thickness h and accumulate V_R^θ retrieved on each LOS intersecting the bin. This leads to a profile of $\langle V_R^\theta \rangle$ as a function of the altitude. Figure 6.24 shows the comparison between a simulated speed profile obtained for low elevation angle and an aggregated speed profile obtained with $N_{agg} = 30$. The first observation is that only the large structures of the wind are conserved. This is a consequence of the windowing process in the DSP algorithm developed by S.Brousmiche [14]. Second, the quality of the results after aggregation tends to be reduced as the range increases. This is due to the fact that the correlation between simulated wind speed profiles decreases with the range, as a consequence of the scanning method. Fig.6.25 shows the averaged profile $\langle V_R^\theta \rangle$ obtained with simulated and retrieved signals. We observe very good results above 40 m. Below this altitude, the performances are

a bit limited by the higher wind velocity fluctuations and the fixed processing window length, yet are still quite good. Figure 6.26 shows the time averaged SNR for a strong optical turbulence ($C_n^2=10^{-13} \text{ m}^{-2/3}$). The results are given for different emitted laser beam radii σ_L and a telescope radius of $1.67 \sigma_L$.

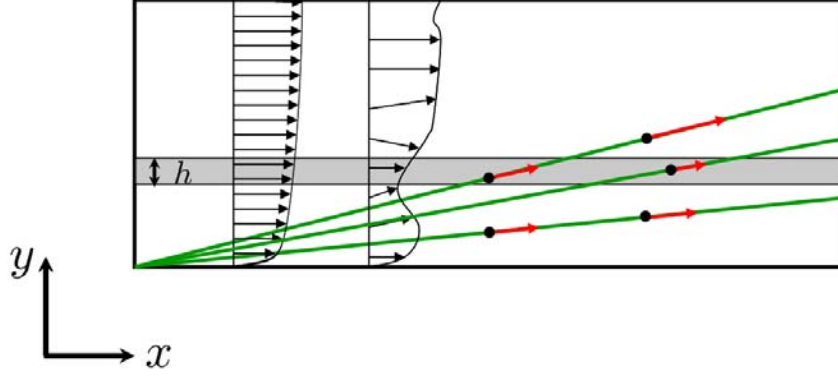


Figure 6.23: Lidar scanning strategy in a slice of the fluid volume. The green lines represent lines of sight, the averaged $\bar{u}_x$ velocity profile is displayed on the left. A snapshot of an instantaneous velocity profile taken at one location in the $x - y$ plane is displayed on the right. The red vectors represent V_R , the wind velocity vector projected on the LOS. The grey band of thickness h represents a bin in which the V_R^θ profiles are averaged.

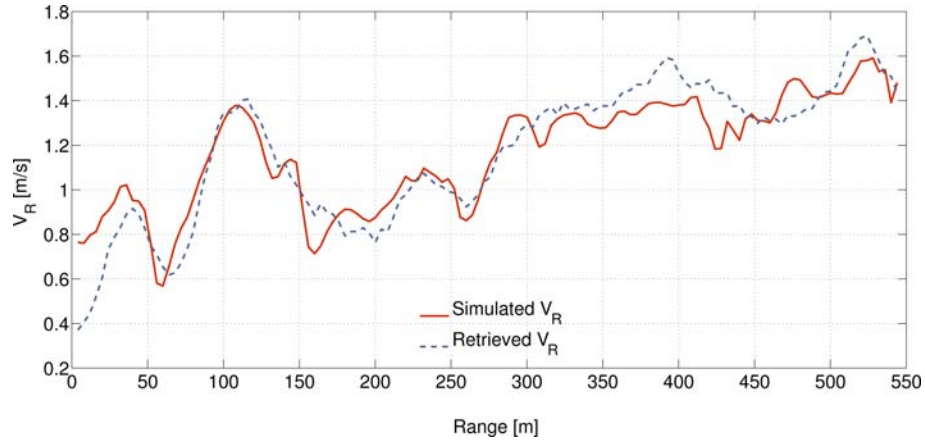


Figure 6.24: Example of radial speed profile as a function of range, obtained with $N_w=200$, $p=4$ and $N_{agg}=30$. The lidar parameters are: $\tau = 200 \text{ ns}$, $\sigma_L = 30 \text{ mm}$, $R_p=50 \text{ mm}$ and $C_n^2 = 10^{-13} \text{ m}^{-2/3}$.

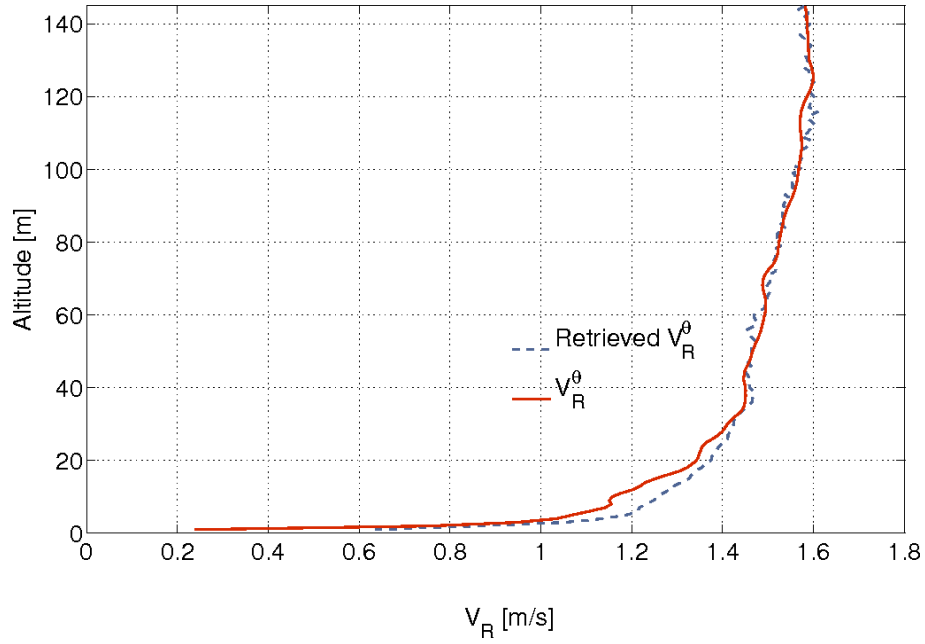


Figure 6.25: Comparison of simulated (x -averaged) and calculated mean velocity profile with altitude. The main parameters are: $\tau = 200$ ns, $\sigma_L = 10$ mm, $C_n^2 = 30^{-13} \text{ m}^{-2/3}$, $N_w = 200$.

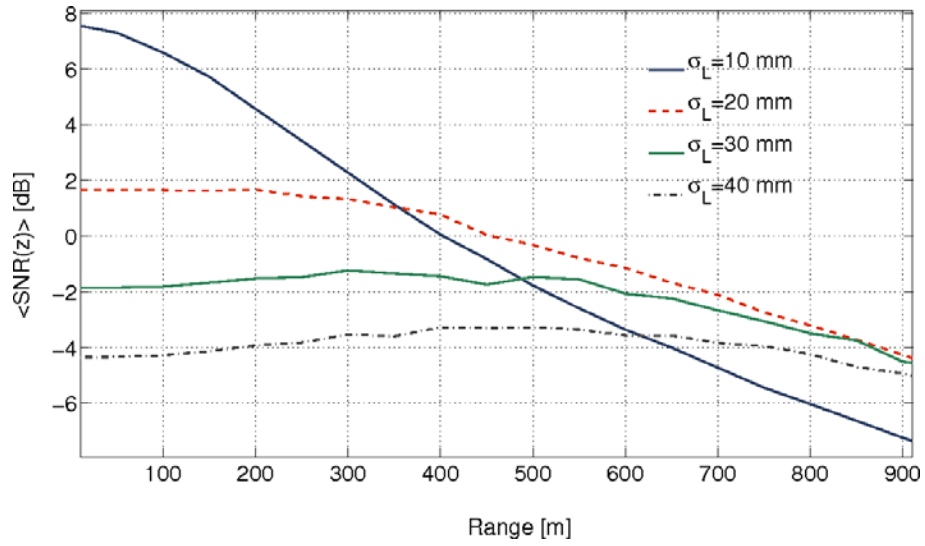


Figure 6.26: Time averaged SNR for variable emitted laser beam radius σ_L in the case of a strong optical turbulence ($C_n^2 = 10^{-13} \text{ m}^{-2/3}$).

6.8.3 Remote sensing of aircraft wake vortices in ground effect

We propose to perform a simulation of LIDAR detection of aircraft wake vortices in ground effect, a case which is particularly relevant for an airport ground based LIDAR. For that purpose, we use the computation performed at Section 5.3. Although the Reynolds number is not realistic (lower than reality), the database already provides velocity fields with the correct velocities for the large to medium scales. Only the very high wavenumbers spectral content of this database is not physical (the very fine scales of the real flow wont be present). As a LIDAR is anyway unable to see those, this fact is of less importance. The scanning pattern used for an airport configuration is displayed in Fig. 6.27. The parameters of the “virtual” LIDAR we simulate are enumerated in Table 6.3. Again, the geometrical parameters are chosen so that the scanning pattern is included in the computational domain used for the fluid flow simulation. The velocity components of the simulated wind used for the LIDAR simulation are displayed in Fig. 6.28. These parameters have the typical values as for the

Table 6.3: Parameters for the simulation of the LIDAR remote sensing of aircraft wake vortices in ground effect.

Parameter	Value
Laser wavelength λ	$1.55 \mu m$
Intermediate frequency	$50 MHz$
Beam radius	$0.03 m$
Focal length of the telescope	∞
Pulse energy	$100 \mu J$
Pulse duration	$200 ns$
Maximum range	$300 m$
Sampling frequency	$512 MHz$
Lens radius	$0.05 m$
θ_{min}	5°
θ_{max}	45°
Pulse repetition frequency	$1 kHz$
Scanning angular speed	$2^\circ/s$

LIDAR designed in the LASEF project. The current retrieved from one arbitrary LOS, as provided by the LIDAR signal simulator is plotted in Fig. 6.29 without the noise and in Fig. 6.30 with noise. There are about 19000 LOS which cover the whole scanning pattern. The first current samples exhibit a Gaussian shape, this is the “pre-trigger” pulse which is used to know exactly what is the shape of the pulse that was sent on each LOS. This is useful as the pulse shape can vary from pulse to pulse. Fig. 6.31 displays the radial

velocity V_R mapped on the LIDAR scanning pattern at an early time. This mapping was performed by means of the interpolation technique described in Section 6.6. The processing of the signals retrieved on each LOS was performed by S.Lugan. It allowed to recover a map of the radial velocity as displayed in Fig. 6.32. The quality of the recovered results is indeed very good, as it allows to detect the presence of the vortices and also give quantitative information on the value of V_R which can help to measure the intensity of the vortices, as explained at Section 6.7.

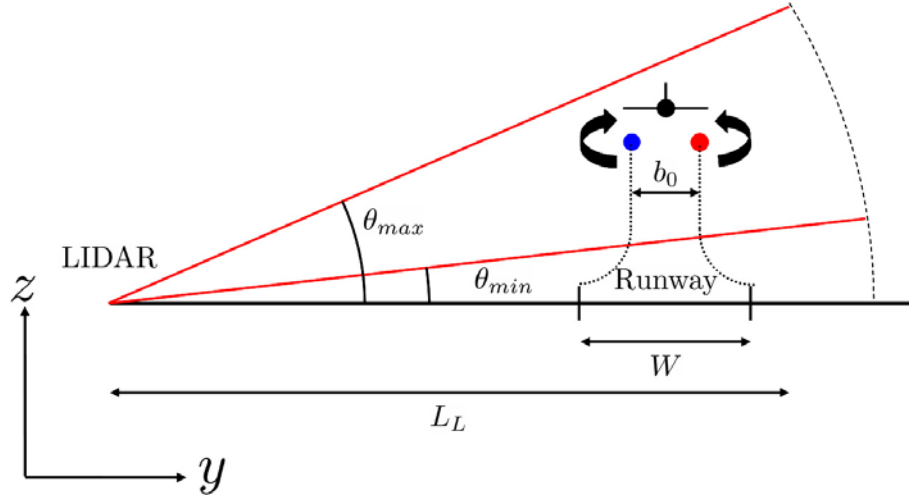


Figure 6.27: Lidar scanning pattern on a runway for aircraft wake vortices released close to the ground.

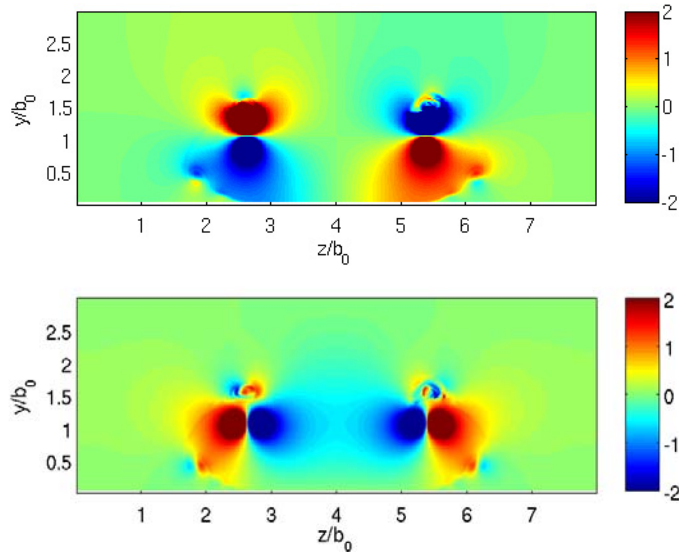


Figure 6.28: Snapshot of velocity: $\frac{u_x}{V_0}$ (top) and $\frac{u_y}{V_0}$ (bottom) in a x - y plane. These velocity components are used to compute the Doppler shift. The dimensionless time corresponds to an early stage of the flow.

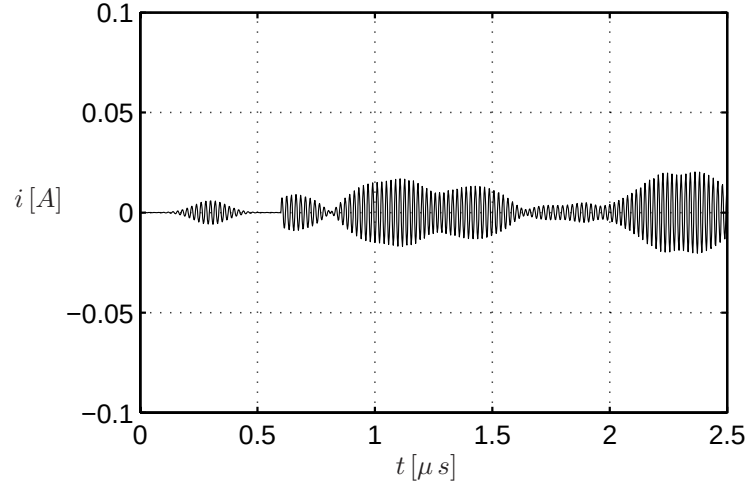


Figure 6.29: Simulated current signal, without noise, along one LOS. The “pre-trigger” pulse is included at the beginning of the signal $t \leq 0.5\mu s$.

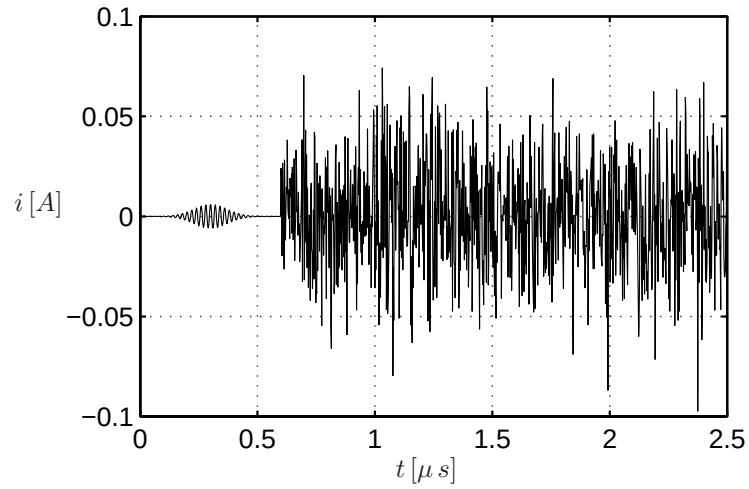


Figure 6.30: Simulated current signal, with noise, along one LOS. The “pre-trigger” pulse is included at the beginning of the signal $t \leq 0.5\mu s$.

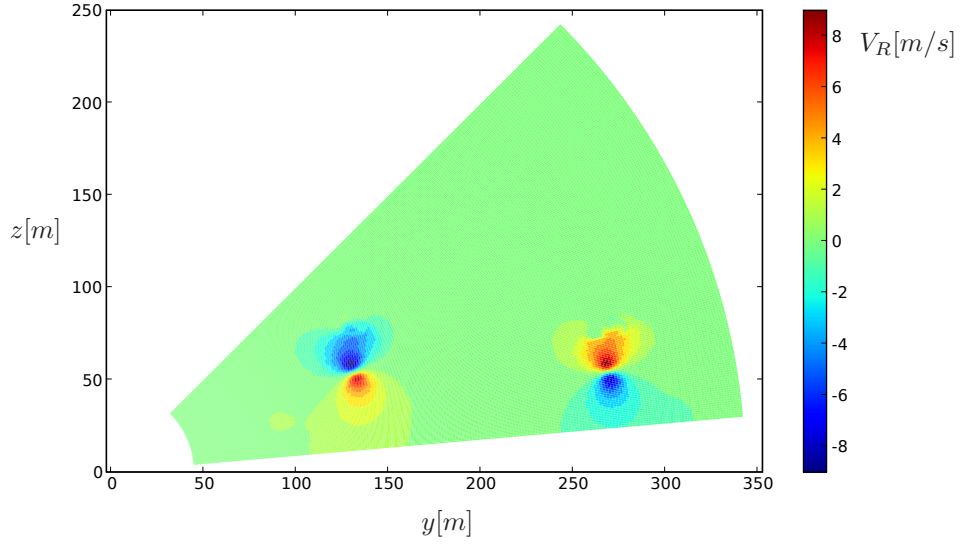


Figure 6.31: Snapshot of LOS projected velocity V_R mapped from the CFD database of Fig. 6.22 (at $\tau = 2.5$) on the LIDAR scanning pattern.

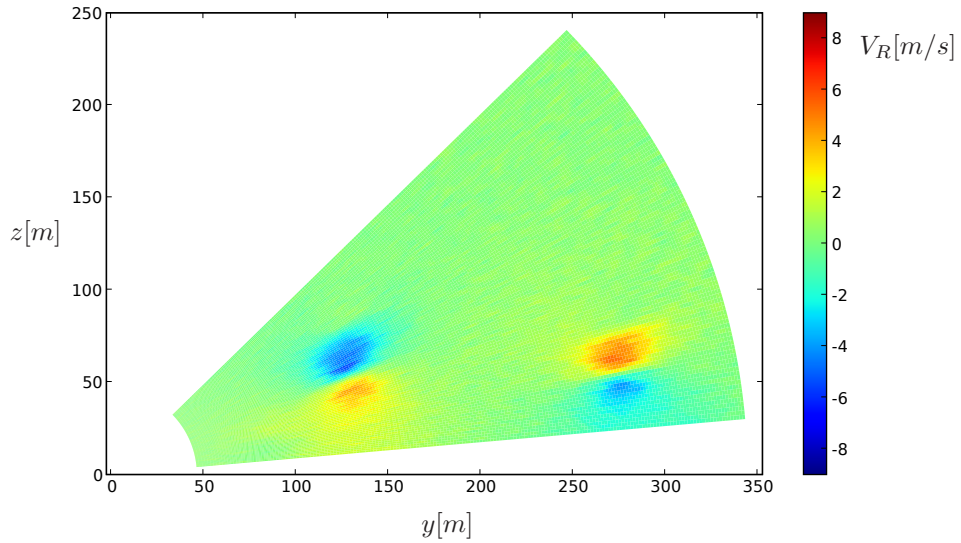


Figure 6.32: Retrieved LOS projected velocity V_R (at $\tau = 2.5$) after processing of the signals, provided by the LIDAR simulator using the data of Fig. 6.31.

6.9 Conclusions

The physics of the LIDAR remote sensing technique has been presented in a concise way for the case of a coherent Doppler LIDAR. Starting from these theoretical considerations, a LIDAR signal simulator was designed and implemented. The originality of the work is to couple this LIDAR signal simulator with high quality CFD databases. We thus follow a multidisciplinary approach, including remote sensing, fluid dynamics and signal processing methods. This simulator has been tested successfully on two flows of interest for an airport based LIDAR: a pair of aircraft wake vortices in ground effect at an early time and a turbulent wind. The velocity results obtained with the LIDAR simulator are consistent with the velocity fields coming from the fluid flows simulations, which serve as input data to the LIDAR simulator. This validate both the simulator and the signal processing codes. Moreover, using the fluid simulations performed on the turbulent vortex at equilibrium, an airborne axial vortex detection was performed and shows that this sensing technique is able to perform detection of aircraft wake vortices.

A technique to obtain the circulation of the vortices based only on the LOS projected velocities has been presented, with promising results on analytical vortex pairs. Future work would be to assess this technique on the data provided by the signal processing.

Other fluids databases were generated within the scope of this work: wake vortices in ground effect without and with wind, at early and late stages of the flow temporal evolution. LIDAR signals were simulated with these databases. Future work should be to exploit these results to deeper asses the quality of the signal processing algorithms.

The LIDAR simulator could also be improved by managing time evolution of the flow field databases. Instead of scanning snapshots of the flow field, it would be interesting to take into account the movement of the vortices during the scanning period. An order of magnitude for the velocity descent being $V_0 \approx 1.3 \text{ m/s}$, this means that if the scan duration is more than 5 s , the effect of the vortices movement will become significant.

Chapter 7

Conclusions

Many targets have been met in the course of this work. First of all, two high quality numerical codes have been improved in a significant way, validated and adapted to better perform LES. The first code is a fourth order difference code for which a preliminary version was implemented during the Ph.D. thesis of L. Georges [46]. This code allowed to perform simulation on very large grids as the one presented in Chapter 5. Recently, this code has been completely rewritten by M. Duponcheel, and its modularity has been improved in a significant way. Further work should include the ability to perform turbulent heat transfer and RANS-LES coupling. The second code is a pseudo-spectral solver, for which the first version was developed by the team of D. Carati at Université Libre de Bruxelles (ULB). This code was also carefully validated and improved in order to be able to perform LES using a wide class of eddy viscosity type models. The dealiasing strategy was also improved for the code to be energy conserving and pass the time reversibility benchmark. This code also allowed to perform simulation on very large grids, as those presented in Chapters 3 and 4. Further work should also include the ability to solve a scalar transport equation. This was already implemented but not yet tested in an exhaustive way.

Several advances have been made in the field of the large eddy simulation of complex turbulent flows. SGS models were investigated, from the simpler ones (such as the Smagorinsky model) to most advanced models such as the multiscale ones. The WALE model seemed to be promising regarding the very good results obtained for wall bounded flows. However this model has an over-dissipative behaviour in vortices. A new sampling-based dynamic procedure formulated solely in physical space was also investigated, providing good results for HIT flows. However, this procedure yields mitigated results in channel flow, due to an excessive clipping of the model coefficient. Multiscale models were also investigated and the assessment of their spectral behaviour leads to conclude that the models acting on the small scale part of the LES field are better than those acting on the complete part. A study also allowed to highlight the asymptotic behaviour of SGS models at very high Reynolds number. A new

model (the RVM_s -WALE) was designed and successfully tested on channel flow. It takes advantage of the WALE model, which has the correct near wall behaviour and of the multiscale models which acts only when there are small scales in the flow. This model is thus a good candidate to perform simulations of aircraft wake vortices in ground effect. However simulating problems at high Reynolds number in the presence of walls is still not yet tractable using, as here, wall-resolved LES. Regarding this, further work should be to investigate appropriate near wall modelling.

The first wake vortex application investigated was the LES of a aircraft wake vortex system. This simulation allowed to highlight the equilibrium state of such a flow and to obtain the dissipation of the equilibrium turbulent vortex system. It provides the minimum dissipation of turbulent aircraft wake vortices. The two SGS models (RVM_s and hyper-viscosity) used to compute this flow appear to perform quite well for this class of problems. They provided quantitative results which are consistent with each other. The circulation profiles and the turbulence intensities constitute valuable information for modelling. The visualization of axial velocity fluctuations also allows to conclude that their value is significant enough to perform axial Doppler LIDAR measurements, as is done in airborne aircraft wake vortex detection. These results were already intensively used in the case of the FP6 project FIDELIO. Further work on the equilibrium turbulent vortex could be to simulate this flow using a computational box long enough to trigger and obtain the Crow instability. This would require a computational box with $L = 8 b_0$ leading to a 512^3 grid. These results should be of great value in the framework of the “Green-Wake” project. The vortex core size influence could also be investigated (it would however require even larger grids). It could also be interesting to study the influence of the atmospheric turbulence level on the equilibrium dissipation value. Further work concerning the influence of atmospheric parameters such as the stratification effects will be carried out in the framework of the Ph.D thesis by I. De visscher. This will also help to further calibrate operational wake vortex models.

Wall-resolved LES of aircraft wake vortex flows in ground effect at $Re = 20000$ with and without wind were also performed. First, these simulations constitute a challenge from a computational point of view. Indeed, only parallel high quality codes are able to perform these tasks. Moreover, the computational grid should be carefully designed in order to capture the relevant physical phenomena. Second, accurate SGS models with correct near the ground behaviour and in the vortices should be used. As we used the new RVM_s -WALE model, it was indeed the case. The results obtained with this model agree very well with those obtained by Georges. One original aspect of the present work was to model the wind using a half channel flow simulation. The presence of wind reduces the starting time of the fast turbulent decay phase of the wake vortices. Moreover the tilting and rebound phenomena were highlighted by flow visualizations. The fluid flows simulated are also of great value for the LASEF project. As the Reynolds number of the simulations is still modest, it should be interesting to increase it, this however requires near-wall modelling.

Finally, in the continuity of the work related to aircraft wake vortex modelling, a ground based coherent Doppler LIDAR simulator was implemented in collaboration with the remote sensing laboratory. The original aspect of this simulator is to couple the LES results obtained in the present work with a virtual LIDAR. The signals obtained by LIDAR simulation can help to develop signal processing algorithms for a real LIDAR. The theory of the coherent Doppler LIDAR measurement technique was briefly presented, starting from electromagnetic wave propagation considerations to the “feuilleté model” of Salamiou et al. [106] which is the model implemented in the simulator. This simulator was tested successfully on two flows of interest for an airport based LIDAR: a pair of aircraft wake vortices in ground effect at an early time and a turbulent wind. The velocity results obtained with the LIDAR simulator are consistent with the velocity fields coming from the fluid flows simulations, which serve as input data to the LIDAR simulator. An airborne axial vortex detection was also performed on the LES databases of the turbulent vortex at equilibrium. This confirms that this sensing technique is able to perform airborne detection of aircraft wake vortices. A technique of computing the circulation of the vortices based only on the LOS projected velocities was presented with promising results on analytical vortex pairs. Further work would be to assess this technique using the data provided by the signal processing or real data. The LIDAR simulator itself could also be improved by managing time evolution of the flow field databases and also by taking the aerosol concentration into account.

Bibliography

- [1] Andrews L.C. and Phillips R.L., *Laser beam propagation through random media*, SPIE press, 1998.
- [2] Adams J., Swarztrauber P. and Sweet R., Efficient FORTRAN Subprograms for the Solution of Elliptic Partial Differential Equations (FISH-PACK ver.4.0), *NCAR Technical Note-TN/IA-109* 1975.
- [3] Batchelor, G. K. *The theory of homogeneous turbulence*, Cambridge University Press 1953.
- [4] Belmonte A., Feasability study for the simulation of beam propagation: consideration of coherent LIDAR performance. *applied optics*, vol 39, No 30, 2000.
- [5] Blaisdell G.A., Spyropoulos E.T., Qin J.H., The effect of the formulation of non linear terms on aliasing in spectral methods. *Applied Numerical Mathematics*, **21**, 207–219, 1996.
- [6] Borue V. and Orszag S. A., Self-similar decay of three-dimensional homogeneous turbulence with hyperviscosity, *Phys. Rev. E*, **51**, 856–859, 1995.
- [7] Borue V., and Orszag S. A., Kolmogorov’s refined similarity hypothesis for hyperviscous turbulence, *Phys. Rev. E*, **53**, 21–24, 1996.
- [8] Borue V., and Orszag S. A., Local energy flux and subgrid-scale statistics in three-dimensional turbulence, *J. Fluid Mech.*, **366**, 1–31, 1998.
- [9] Boyd J. P., *Chebyshev and Fourier spectral method*, Dover publications, Inc 1999.
- [10] Brachet, M. E., Meiron, D. I., Orszag, S. A., Nickel, B. G., Morf, R. H. & Frisch U. Small-scale structure of the Taylor-Green vortex, *J. Fluid Mech.* **130**, 411–452, 1983.
- [11] Briggs W.L., A multigrid tutorial, *SIAM* 1987.

- [12] Bricteux L., Cocle R., Duponcheel M., Georges L., Winckelmans G., Assessment of multiscale models for LES: spectral behaviour in very high Reynolds number turbulence and cases with aircraft wake vortices. *International conference on Turbulence and Shear Flow Phenomena, TSFP5, Munich Germany, 2007*.
- [13] Brousmiche S., Modélisation de l'interaction LASER-turbulence en vue du dimensionnement d'un LIDAR hétérodyne. *DEA thesis*, UCL, Louvain-la-Neuve 2006.
- [14] Brousmiche S., Bricteux L., Sobieski P., Macq B. and Winckelmans G., Numerical simulation of a heterodyne Doppler LIDAR for wind measurement in a turbulent atmospheric boundary layer. *Proceedings of the International Geoscience And Remote Sensing Symposium (IGARSS 2007)*, Barcelona Spain.
- [15] Canuto C., Hussaini M. Y., Quarteroni A. and Zang T. A., *Spectral Methods in Fluid Dynamics*, Springer-Verlag, New-York, 1988.
- [16] Carati, D., Winckelmans, G. S. & Jeanmart, H. (2001). On the modelling of the subgrid-scale and filtered-scale stress tensors in large-eddy simulation, *J. Fluid Mech.* **441**, 119–138.
- [17] Carati D., Benoci, C., Lacor C., Merci B., Winckelmans G., Advanced turbulence modelling, lesson 1: introduction to simulation methods for turbulent flows. Belgian graduate school in mechanics (GRASMECH) course (2004-2005).
- [18] Cariou, J.-P., Augère B., Valla M. Laser source requirements for coherent LIDARS based on fiber technology. *C.R. Phys.* **7** 213-223, ed. Elsevier 2006.
- [19] Choi H., Moin P., Effects of the computational time step on the numerical simulation of turbulent flow. *J. Comp. Phys.*, **113**,1-4 1994.
- [20] Chollet, J.-P. & Lesieur, M. Parameterization of small scales of three-dimensional isotropic turbulence utilizing spectral closures, *J. Atmos. Sci.* **38**, 2747–2757, 1981.
- [21] Chow J., Zilliac G. and Bradshaw P. Turbulence Measurements in the Near Field of a Wingtip Vortex. Technical Memorandum 110418, NASA Ames Research Center. Moffett Field, CA.
- [22] R. Cocle, Combining the vortex-in-cell and parallel fast multipole methods for efficient domain decomposition simulations : DNS and LES approaches, Ph.D. Thesis, Université catholique de Louvain, 154/2006, 2007.
- [23] Cocle R., Dufresne L. and Winckelmans G., Investigation of multiscale subgrid models for LES of instabilities and turbulence in wake vortex systems, *Complex Effects in LES: Springer*, **56**, 141-159, 2007.

- [24] Cocle R., Winckelmans G. S. and Daeninck G., Combining the vortex-in-cell and parallel fast multipole methods for efficient domain decomposition simulations, *J. Comp. Phys.*, in press november 2007.
- [25] Comte-Bellot G. and Corrsin S., The use of a contraction to improve the isotropy of grid-generated turbulence, *J. Fluid Mech.*, **25** 657–682, 1966.
- [26] Corrsin S., On the spectrum of isotropic temperature fluctuations in an isotropic turbulence, *Journal of applied physics*, **22**(4), 1951.
- [27] Cottet G.-H. and Koumoutsakos P. D., *Vortex Methods: Theory and Practice*, Cambridge Univ. Press, 2000.
- [28] Crow S.C., Stability theory for a pair of trailing vortices. *AIAA journal*, vol. **8**, No 12, 1970.
- [29] Daeninck G., Developments in hybrid approaches: Vortex method with known separation location, Vortex method with near-wall Eulerian solver, RANS-LES coupling, Ph.D. Thesis, Université catholique de Louvain, 127/2006 (2006).
- [30] Davidson P.A., *Turbulence*, Oxford University Press (2004).
- [31] Douxchamps D., Multidimensional photogrammetry of short-lived events. Ph.D. Thesis, Université catholique de Louvain, 46/2004 (2004).
- [32] Ducros F., Comte P. and Lesieur M., Large-eddy simulation of transition to turbulence in a boundary layer spatially developing over a flat plate, *J. Fluid Mech.*, **326**, 1–36 (1996).
- [33] Dufresne L. and Dumas G., A spectral/B-spline method for the Navier–Stokes equations in unbounded domains, *J. Comput. Phys.*, **185**, 532 (2003).
- [34] Dufresne L., Winckelmans G., and Capart R. Numerical investigation of long-wavelength instabilities in aircraft wake vortex system. In *Proceedings of the 6th National Congress on Theoretical and Applied Mechanics*, Gent, May 2003.
- [35] Dufresne L., Capart R. and Winckelmans G., Report of benchmark study, AWIATOR final report D1.1.4-16, 2005. (Also to be submitted as paper: L. Dufresne, R. Bau- mann, T. Gerz, G. Winckelmans, H. Moet, R. Capart and L. Nybelen, Large-eddy simulation of wake vortex flows at very high Reynolds numbers: a comparison of different methodologies).
- [36] Dufresne L. and Winckelmans G., Previous work and present knowledge on wake vortices near the ground. European project FAR-Wake (AST4-CT-2005-012238) WorkPackage 3, Subtask 3.1.1, Deliverable 3.0.

- [37] Duponcheel, M. and Lonfils, T., Investigations numériques de tourbillons de sillage en effet de sol : analyse des effets tridimensionnels, Engineering graduation thesis, Univ. cath. Louvain, Louvain-la-Neuve, 2005.
- [38] Duponcheel M., Lonfils T., Bricteux L. and Winckelmans G., Simulation of three-dimensional wake vortices in ground effect using fourth order incompressible code, *7th National Congress on Theoretical and Applied Mechanics*, Belgium, 2006.
- [39] Duponcheel M., Orlandi P., Winckelmans G., Time-Reversibility of the Euler Equations as a Benchmark for Energy Conserving Schemes. Paper Submitted to J. Comp. Phys. (2008).
- [40] Ferziger J.H. and Peric M., *Computational methods for fluid dynamics*, Springer, 3rd ed., 2002.
- [41] Frehlich R. G. and Kavaya J. , Coherent laser radar performance for general atmospheric refractive turbulence, *Applied Optics*, **30**(36), 1991.
- [42] Frehlich R. G., Effects of refractive turbulence on coherent laser radar, *Applied Optics*, **32**(12), 1993.
- [43] Frehlich R. G., Simulation of laser propagation in a turbulent atmosphere, *Applied Optics*, **39**(3), 2000.
- [44] Frehlich R. G., Optimal local oscillator field for a monostatic coherent laser radar with circular aperture, *Applied Optics*, **32**(24), 1993.
- [45] Frigo M. and Johnson S., The Design and Implementation of FFTW3, Proceedings of the IEEE, special issue on “Program Generation, Optimization, and Platform Adaptation”. **93**(2), 2005.
- [46] Georges L., Development and validation of a LES methodology for complex wall bounded flows : application to high-order structured and industrial unstructured solver. Ph.D. Thesis, Université catholique de Louvain, 148/2007, 2007.
- [47] Georges L., Geuzaine P., Duponcheel M., Bricteux L., Lonfils T. and Winckelmans G., European project FAR-Wake (AST4-CT-2005-012238), WorkPackage 3, Subtask 3.1.1, Technical Report 3.1.1-3: LES of two-vortex systems in ground effect with and without wind.
- [48] Gerz T., Baumann R., Decay characteristics of single and double wake vortex pairs in different atmospheric flow realisations. *Proceedings of the 25th International Congress of the Aeronautical Science, ICAS 2006*.
- [49] Germano M., Piomelli U., Moin P. and Cabot W., A dynamic subgrid-scale eddy-viscosity model, *Phys. Fluids A*, **3**, 1991.

- [50] Ghosal S., Lund T. S., Moin P. and Akselvoll K. , A dynamic localization model for large-eddy simulation of turbulent flows, *J. Fluid Mech.*, **286**(229), 1995.
- [51] Gerz T., Holzapfel F. and Darracq D. , Aircraft wake vortices —A position paper—, Wakenet, the european thematic network on wake vortex position paper, 2001.
- [52] Goodman J. W. *Introduction to Fourier optics*, Mc Graw-Hill electrical and computer engineering sciences, 1968.
- [53] Goodman J. W. *Statistical Optics*, Wiley, 1985.
- [54] Gullbrand J., Katopodes-Chow F., The effect of numerical errors and turbulence models in large eddy simulation of turbulent flows, with and without filtering. *J. Fluid Mech.*, **495**, 323-341.
- [55] Holzapfel F. and Steen M. Aircraft wake-vortex evolution in ground effect : Analysis and parameterization. In 44th AIAA Aerosp. Sci. Meet. Exhib., pages AIAA paper No.2006-1077, Reno, NV, Jan. 9-12 2006.
- [56] Hughes T.J.R., Mazzei L., Oberai A.A. and Wray A.A., The multiscale formulation of large eddy simulation: decay of homogeneous isotropic turbulence, *Phys. Fluids*, **13**, 505-512, 2001.
- [57] Hughes T.J.R., Oberai A.A. and Mazzei L., Large eddy simulation of turbulent channel flow by the variational multiscale method, *Phys. Fluids*, **13**, 1784-1799, 2001.
- [58] Jaenicke R., Tropospheric aerosol, in aerosol-cloud-climate interactions, edited by P.V.Hobbs. Academic Press, San Diego,CA; pp. 1-31, 1986.
- [59] Jeong J. & Hussain F., On the identification of a vortex, *J. Fluid Mech.* **285**, 69-94, 1995.
- [60] Jeanmart H. and Winckelmans G., Comparison of recent dynamic subgrid-scale models in turbulent channel flow, *Proc. Summer Program 2002*, Center for Turbulence Research, Stanford University and NASA Ames, 105-116, 2002.
- [61] Jeanmart H., Investigation of novel approaches and models for large-eddy simulation of turbulent flows, Ph.D. Thesis, Université catholique de Louvain, 39/2004, 2002.
- [62] Jeanmart H. and Winckelmans G., Investigation of eddy-viscosity models modified using discrete filters: a simplified “regularized variational multi-scale model” and an “enhanced field model”, *Phys. Fluids*, **19**(5), 055110, 2007.

- [63] Jimenez J. and Moin P., The minimal flow unit in near-wall turbulence. *J. Fluid Mech.*, **225**, 213, 1991.
- [64] Johnston R. A., Woolder N.J. , Reavell F.C. , Bernhardt M. and Dainty C., Horizontal scintillation detection and ranging $Cn^2(z)$ estimation, *Applied Optics*, vol. 20, no. 19, 1981.
- [65] Kim, J., Moin, P. & Moser, R., Turbulence statistics in fully developed channel flow at low Reynolds number, *J. Fluid Mech.* **177**, 133–166, 1987.
- [66] R. C. Kloosterziel and G. F. Carnevale, On the evolution and saturation of instabilities of two-dimensional isolated circular vortices, *J. Fluid Mech.*, **338**, 217, 1999.
- [67] R. C. Kloosterziel and G. J. F. Van Heijst, An experimental study of unstable barotropic vortices in a rotating fluid, *J. Fluid Mech.*, **223**, 1, 1991.
- [68] Knaepen B, Debliquy O, Carati D, Large-eddy simulation without filter. *J. Comp. Phys.*, **205**, 98-107, 2005.
- [69] Kraichnan, R. H. (1976). Eddy viscosity in two and three dimensions, *J. Atmos. Sci.* **33**, 1521–1536.
- [70] Kravchenko A.G. and Moin P., On the effect of numerical errors in large eddy simulations of turbulent flows. *J. Comp. Phys.*, **131**, 310-322, 1997.
- [71] Laporte F., Simulation numérique appliquée à la caractérisation et aux instabilités des tourbillons de sillage d'avions de transport, *PhD thesis*, INPT, Toulouse, 2002.
- [72] Lee, M.J. and Oh, B.D. and Kim, Y.B., Canonical Fractional-Step Methods and Consistent Boundary Conditions for the Incompressible Navier-Stokes Equations, *J. Comp. Phys.*, 168, 73–100, 2001.
- [73] Lilly D. K., On the application of the eddy viscosity concept in the inertial subrange of turbulence, NCAR Manuscript 123, Boulder, Colorado, 1966.
- [74] Lilly D. K., The representation of small-scale turbulence in numerical simulation experiments, *Proc. of the IBM Scientific Computing Symposium on Environmental Sciences*, 195, 1967.
- [75] Lugan S., Bricteux L., Douchamps D., Macq B., Sobieski P., Winckelmans G. Simulation of LIDAR-based aircraft wake vortex detection using a bi-Gaussian spectral model. *Proceedings of the I.E.E.E international geoscience and remote sensing symposium "sensing and understanding our planet"*, Barcelona 2007.
- [76] Lamorgese A. G., Caughey D. A., Pope S. B., Direct numerical simulation of homogeneous turbulence with hyperviscosity, *Phys. Fluids*, **17**, 015106, 2005.

- [77] Lele S.K., Compact finite differences schemes with spectral-like resolution, *J. Comp. Phys*, **103**(16), 1992.
- [78] Luton J.A. and Ragab S.A. , Three-dimensional interaction of a vortex pair with a wall, *Phys. Fluids*, **9**(10), 2967–2980, 1997.
- [79] Menzies R. T. and Hardesty R. M, Coherent Doppler LIDAR for Measurements of Wind Fields, Proceedings of the IEEE, vol. 77, No3, March 1989.
- [80] Mohamed M.S. and LaRue J.C., The decay power law in grid-generated turbulence, *J. Fluid Mech.*, **219** 195–214, 1990.
- [81] Moet H., Simulation numérique du comportement des tourbillons de sillage dans l’atmosphère. PhD thesis, Inst. Nat. Poly. Toulouse, Toulouse, 2003.
- [82] Moet H., Darracq D., Laporte F., Corjon A., Investigations of ambient turbulence effects on vortex evolution using LES. *Proceedings of the 38th aerospace sciences Meeting conference and exhibit AIAA 2000*.
- [83] Morinishi Y, Vasilyev O, Vector level identity for dynamic subgrid scale modeling in large eddy simulation. *Phys. Fluids*, **14**, 2002.
- [84] Moser R., Kim J., Mansour N., Direct numerical simulation of turbulent channel flow up to $Re_\tau = 590$, *Phys. Fluids*, **11**,S1070-6631(99)02204-7, 1999.
- [85] Modeling and simulation of Beam Control Systems: Introduction and Overview, MZA presentation, Directed Energy Profesional Society
- [86] Meneveau C. and Lund T. S., The dynamic Smagorinsky model and scale-dependent coefficients in the viscous range of turbulence, *Phys. Fluids*, **9**(12), 1997.
- [87] Meyzonette J.L., Radars LASER, Ecole d’été des systèmes optiques. Institut d’optique théorique et appliquée ESO, Orsay cedex France;
- [88] Nicoud F. and Ducros F., Subgrid-scale stress modelling based on the square of the velocity gradient tensor, *Flow, turbulence and combustion*, **62** 183–200, 1999.
- [89] Orlandi P., Carnevale G.F., Lele S.K., Shariff K. DNS study of stability of trailing vortices, *Center for turbulence research, proceedings of the summer program* , 187–208, 1998.
- [90] Orlandi P., Vortex dipole rebound from a wall, *Phys. Fluid A*, **2**, 1429–1436, 1990.
- [91] Orszag S.A., Comparison of pseudo spectral and spectral approximations, *Stud. Appl. Math.* **51**, 1972.

- [92] Perry, A.E. and Chong, M.S. Topology of flow patterns in vortex motions and turbulence. Appl. Scientific Research Vol. 54 (3/4) , pp.357-374, 1994.
- [93] Pope S.B., Turbulent flows, *Cambridge University Press*, 2000.
- [94] Pearson G.N., Roberts P.J., Eacock J.R. et Harris M., Analysis of the performance of a coherent pulsed fiber LIDAR for aerosol backscatter applications, *Applied Optics*, **41**(30), 2002.
- [95] Prandtl L. and Tietjens O.G, Fundamentals of hydro- and aero-mechanics, Dover publ., NY, 1957.
- [96] Proctor F.H., Han J., Numerical study of wake vortex interaction with the ground using the Terminal Area Simulation System. In *37th Aerospace Sciences Meeting and Exhibit*, Reno, NV. AIAA paper No.97-0754, Jan. 11–14 1999.
- [97] Proctor F.H., Han J., Hamilton D.W. Wake vortex transport and decay in ground effect: Vortex linking with the ground. In 38th AIAA Aerosp. Sci. Meet. Exhib., pages AIAA 2000–0757, Reno, NV, Jan. 10–13, 2000.
- [98] Faddy J. M., Pullin D.I., Flow structure in a model of aircraft trailing vortices. *Phys. Fluids*, **17**/8,S1070-6631, 2005.
- [99] Rogallo R., Numerical experiments in homogeneous turbulence, Technical Report No. NASA Tech. Memo. 81315, NASA Ames Research Center, 1981.
- [100] Rye B. J., Frehlich R. G., Optimal truncation et optical efficiency of an apertured coherent LIDAR focused on an incoherent backscatter target, *Applied Optics*, **31**(15), 1992.
- [101] Schlichting H., Gersten K., Boundary layer theory, 8th edition. *Springer*, 2000.
- [102] Smagorinsky, J., General circulation experiments with the primitive equations, *Mon. Weath. Rev.* **93**, 99–165, 1963.
- [103] Smullin L.D. and Fiocco G.: Optical echoes from the Moon, *Nature*, 194, 1267, 1962.
- [104] Stolz S., Schlatter P. and Kleiser L., High-pass filtered eddy-viscosity models for large-eddy simulations of transitional flow, *Phys. Fluids*, **17**, 065103, 2005.
- [105] Stolz S., Schlatter P., Meyer D. and Kleiser L., High-pass filtered eddy-viscosity models for LES, *Direct and Large-Eddy Simulation V*, edited by Friedrich R., Geurts B.J. and Métais O., Kluwer, 81–88, 2004.
- [106] Salamitou P., Dabas A., Flamant P. H., Simulation in the time domain for heterodyne coherent laser radar, *Applied physics*, **34**(3), 1995.

- [107] Spalart P. R., Airplane trailing vortices, *Ann. Rev. Fluid Mech.*, **30**, 107–138, 1998.
- [108] Fujii T. and Fukuchi T., *LASER remote sensing* CRC Press, Florida, 2005.
- [109] Takahashi N., Ishii H., and Miyazaki T., The influence of turbulence on a columnar vortex *Phys. Fluids*, **17**, 035105, 2005.
- [110] R. Targ, M. J. Kavaya, R. M. Huffaker et R. L. Bowles, Coherent LIDAR airborne windshear sensor: performance evaluation, *Applied Optics*, **30**(15), 1991.
- [111] Vallat M. , Etude d'un LIDAR doppler impulsional à laser Erbium fibré pour des mesures de champ de vent dans la couche limite de l'atmosphère, ph.D. Thesis june 2005, ENST, Paris.
- [112] van de Hulst H.C., *Light scattering by small particles*, New York, Dover, 1981.
- [113] Vasseur H., Bandwith Limitations on millimetre wave communications due to tropospheric clear air phenomena. Ph.D. Thesis, Université catholique de Louvain, 1995.
- [114] Vasilyev O, High order finite differences schemes on non-uniform meshes with good conservation properties. *J. Comp. Phys*, **157**(2), 746-761, 2000.
- [115] Vreman A.W., The filtering analog of the variational multiscale method in large-eddy simulation, *Phys. Fluids*, **15**, L61, 2003.
- [116] Weitkamp C., *Lidar: Range Resolved Optical Remote Sensing of the Atmosphere*, Springer Science+Business Media, Inc., New York, 2005.
- [117] Williamson, J. H. Low-storage Runge-Kutta schemes, *J. Comp. Phys.* **35**, 45–56, 1980.
- [118] Winckelmans G. S. and Jeanmart H., Assessment of some models for LES without and with explicit filtering, *Direct and Large-Eddy Simulation IV*, edited by Geurts B.J., Friedrich R. and Métais O., ERCOFTAC Series **8**, Kluwer, 55–66, 2001.
- [119] Winckelmans G. S., Wray A. A., Vasilyev O. V. and Jeanmart H., Explicit-filtering large-eddy simulation using the tensor-diffusivity model supplemented by a dynamic Smagorinsky term, *Phys. Fluids*, **13**, 1385-1403, 2001.
- [120] Winckelmans G., Jeanmart H., Carati D., On the comparison of turbulence intensities from large-eddy simulation with those from experiment or direct numerical simulation, *Phys. Fluids*, **14**, 5, 2002, 1809-1811.

- [121] Winckelmans G. S., Chapter 5: "Vortex Methods," in Volume 3 (Fluids) of the *Encyclopedia of Computational Mechanics*, Erwin Stein, René de Borst, Thomas J.R. Hughes (Eds), John Wiley & Sons, 2004.
- [122] Winckelmans G., Georges L., Bricteux L., Jeanmart H. Sampling-based dynamic procedure for LES in physical space. 57th Annual Meeting of the American Physical Society Division of Fluid Dynamics, Seattle, WA, Nov. 21-23, 2004, Bulletin of the American Physical Society, 2004, 49, 9.
- [123] Winckelmans G., Bricteux L., Georges L., The Sampling-based dynamic procedure for LES without filtering: validation using finite differences. *Proceedings of the Ercoftac workshop Direct and Large Eddy Simulation 6 (DLES6)*. Poitiers France, Springer, Sept. 2005.
- [124] Winckelmans G, Dufresne L, Bricteux L, LES investigation of aircraft wake two-vortex system in low level atmospheric turbulence (abstract), 58th Annual Meeting of the American Physical Society Division of Fluid Dynamics, Chicago, IL, Nov.20-22, 2005, Bulletin of the American Physical Society, 2005, **50**, 9.
- [125] Winckelmans G., Cocle R., Dufresne L., Capart R., Bricteux L., Daeninck G., Lonfils T., Duponcheel M., Desenfans O., Georges L. Direct numerical simulation and large-eddy simulation of wake vortices: going from laboratory conditions to flight conditions. *Proceedings of the European conference on computational fluid dynamics, ECCOMAS CFD 2006*. Eds. P.Wesseling, E.Oñate and J.Periaux. TU Delft, The Netherlands.
- [126] Yokokawa M., Itakura K., Uno A. , Ishihara T. and Kaneda Y., 16.4-Tflops Direct Numerical Simulation of Turbulence by a Fourier Spectral Method on the Earth Simulator. Conference on High Performance Networking and Computing archive. *Proceedings of the 2002 ACM/IEEE conference on Supercomputing*. Baltimore, Maryland, 2002.
- [127] Zeman O., The persistence of trailing vortices: A modeling study. *Phys. Fluids*, **7**(1), 135-143, 1995.

Appendix A

2D finite differences code used for simulation of 2D ground effects

A.1 Description of the code

The code solves the 2D incompressible Navier stokes equation in $\psi - \omega$ formulation:

$$\frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = \nu \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) \quad (\text{A.1})$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \quad (\text{A.2})$$

The kinematic viscosity of the fluid is ν , u and v are the fluid velocity components aligned with Ox and Oy . The velocity field can be deduced from the stream function ψ :

$$u = \frac{\partial \psi}{\partial y} \quad (\text{A.3})$$

$$v = -\frac{\partial \psi}{\partial x} \quad (\text{A.4})$$

The discretization is done by means of second order collocated finite differences. The Poisson equation is solved efficiently with the Fishpack [2] Fortran library. The temporal integration is performed by means of a multistep scheme which is described in chap.2.

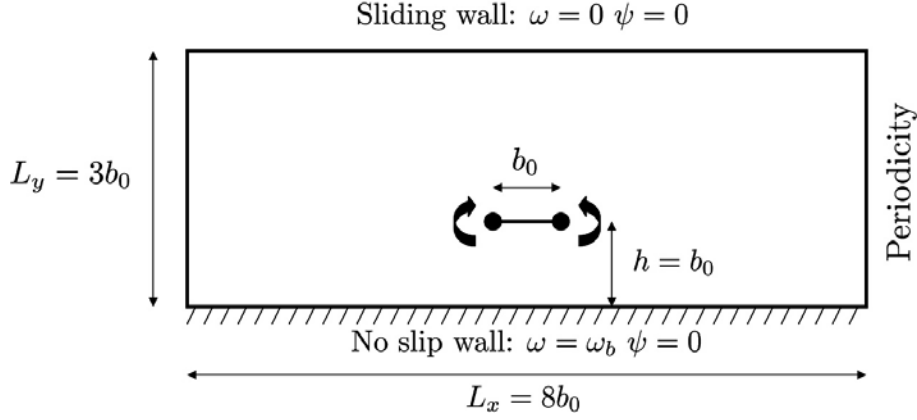


Figure A.1: Flow setup

A.2 Description of the flow

As depicted at Fig. A.1, the boundary conditions are periodic in the spanwise direction. The ground is modeled by a no slip wall where the two components of the velocity vanishes. The generation of vorticity at the wall is ensured by the Woods condition on the ground points indexed $(i, 0)$:

$$\omega_b(i, 0) = \frac{-3}{\Delta y^2} \psi(i, 1) - \frac{1}{2} \omega(i, 1)$$

The upper boundary of the domain should not exist, it is modeled by a symmetry condition, thus the viscous stresses vanishes at the upper boundary. The vortex are initialized with low order algebraic profile as defined in Chapter 5 with b_0 the initial spacing and Γ_0 the initial circulation. The velocity peak radius is set to $r_c = 0.05 b_0$. The reference time is $t_0 = \frac{\Gamma_0}{2\pi b_0^2}$. The number of grid points is $N_x \times N_y = 2048 \times 768$. The time step chosen is:

$$\Delta\tau = \frac{\Delta t}{t_0} = \frac{1}{2\pi} 10^{-3}. \quad (\text{A.5})$$

A visualisation of this flow is provided in Figs. A.2 and A.3.

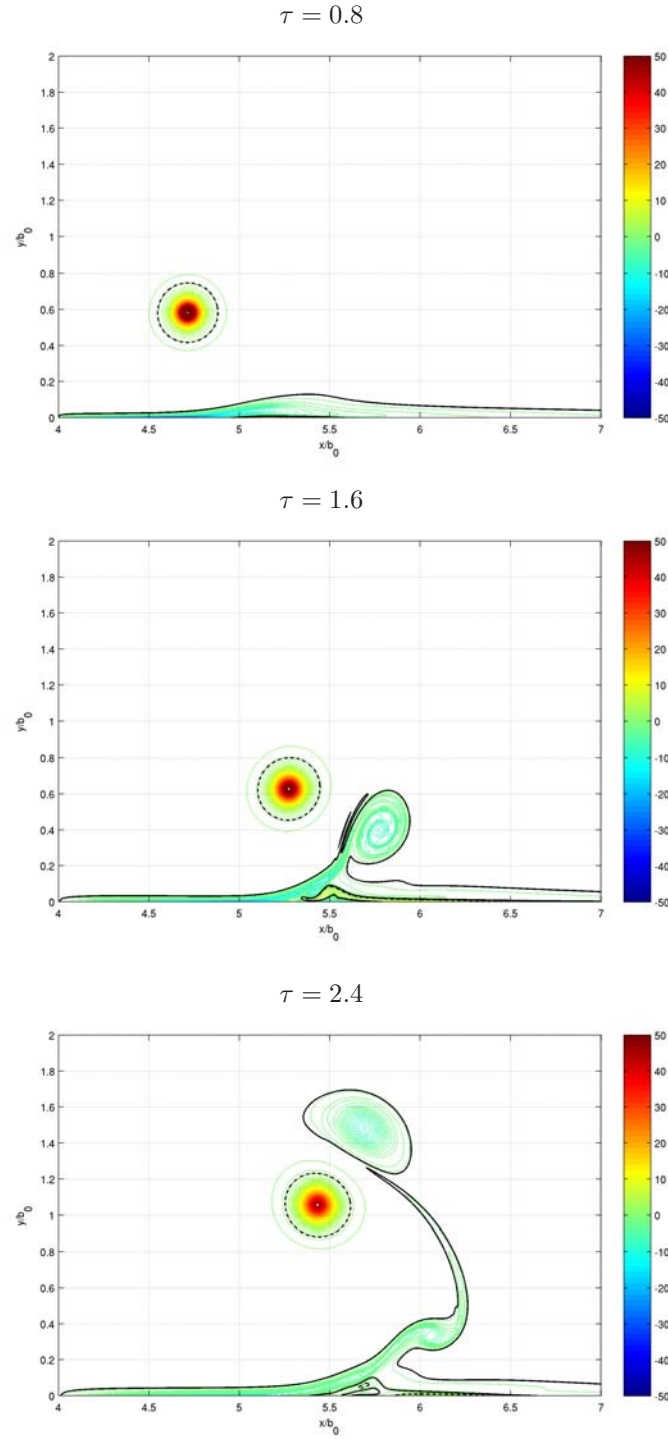


Figure A.2: Visualization of the flow field using isocontours of $\|\vec{\omega}\|b_0^2/\Gamma_0$. Solid line is $\|\vec{\omega}\|b_0^2/\Gamma_0 = -0.1$ and dash line is $\|\vec{\omega}\|b_0^2/\Gamma_0 = 1.0$

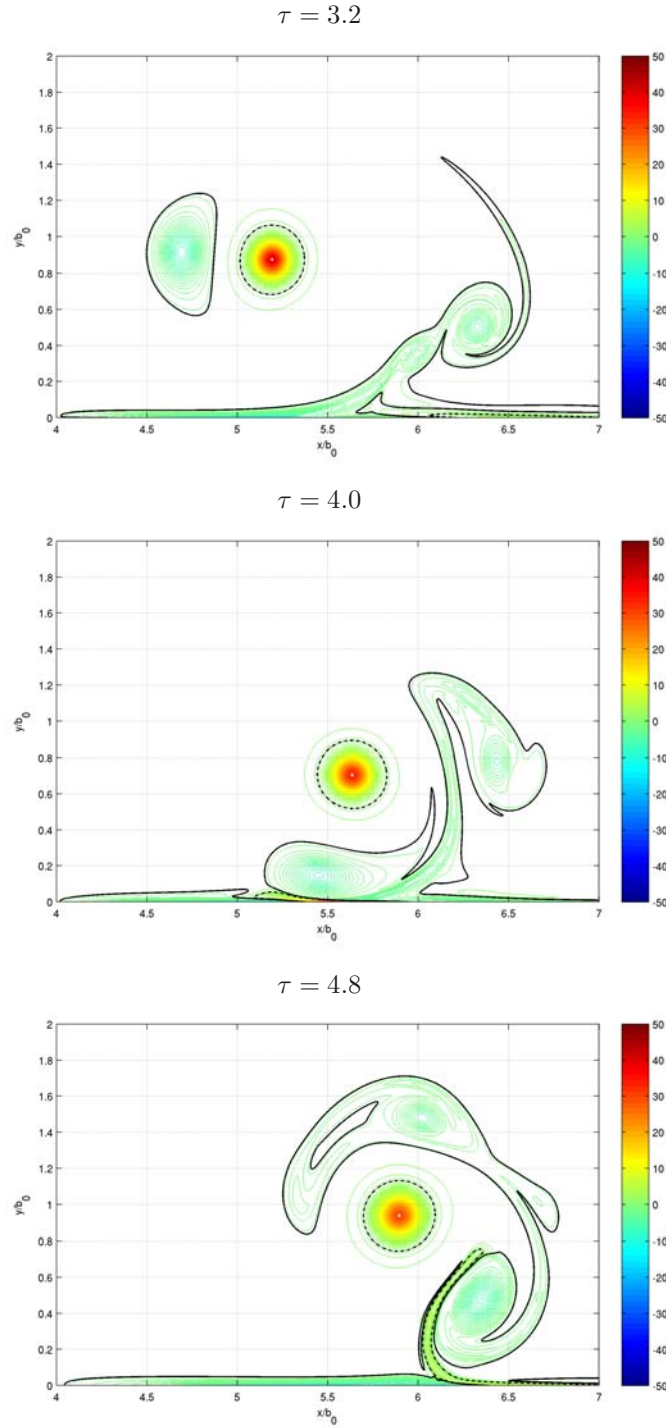


Figure A.3: Visualization of the flow field using isocontours of $\|\vec{\omega}\|b_0^2/\Gamma_0$. Solid line is $\|\vec{\omega}\|b_0^2/\Gamma_0 = -0.1$ and dash line is $\|\vec{\omega}\|b_0^2/\Gamma_0 = 1.0$

Appendix B

The $Q - R$ classification of Perry and Chong

Let $\mathbf{u}$ be the velocity at one point of position vector $\mathbf{x}$ in a fluid flow and let $\mathbf{u} + d\mathbf{u}$ be the velocity at a neighbouring point of position vector $\mathbf{x} + d\mathbf{x}$. At the first order, the relative velocity in the neighborhood of a point is :

$$du_i = \frac{\partial u_i}{\partial x_j} dx_j$$

Where we defined the velocity gradient tensor :

$$A_{ij} = \frac{\partial u_i}{\partial x_j} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) = S_{ij} + \Omega_{ij}$$

At any point in the flow, one can always find principal axes for the strain rate tensor S_{ij} . If the coordinate system is aligned with the principal axes, then S_{ij} becomes diagonal. This second order tensor has three invariants which are directly linked to its eigenvalues (λ_i). The characteristic polynomial for this tensor is written:

$$P_C(\lambda) = \det(A - \lambda I)$$

The associated equation to find the eigenvalues takes the form :

$$\lambda^3 - P\lambda^2 + Q\lambda - R = 0$$

The eigenvalues are independant of the choice of a coordinate system, so are the coefficient P, Q, R .

$$P = A_{ii} = \lambda_1 + \lambda_2 + \lambda_3 \quad (\text{B.1})$$

$$Q = \frac{1}{2}P^2 - \frac{1}{2}A_{ij}A_{ji} = \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 \quad (\text{B.2})$$

$$R = \det(A_{ij}) = \lambda_1\lambda_2\lambda_3 \quad (\text{B.3})$$

For incompressible flows $P = 0$. The aim of the classification is to use the invariants R and Q to classify each point in a flowfield. In particular, one can differentiate between regions of intense vorticity and regions of strong strain. It can be showed that when Q is large and negative, there is a lot of strain and very little vorticity. Conversely, a large positive Q indicates the dominance of vorticity. A positive R indicates vortex stretching and a negative R indicates vortex compression. Instead of considering the sign of Q , one can also use the discriminant

$$Q^* = -3 \left(\frac{R^2}{4} \right)^{1/3}, \quad (\text{B.4})$$

which is such that the eigenvalues of the gradient tensor are all real for $Q < Q^*$ while there is a pair of complex conjugates for $Q > Q^*$.

Table B.1: Perry and Chong classification table.

Q	$R < 0$	$R > 0$
$Q > Q^*$	vortex compression	vortex stretching
$Q < Q^*$	bi-axial strain	axial strain

Appendix C

Generation of a random “turbulence” field

The velocity field in the spectral domain, $\widehat{\mathbf{u}} = \widehat{u}_1 \mathbf{e}_1 + \widehat{u}_2 \mathbf{e}_2 + \widehat{u}_3 \mathbf{e}_3$, is built in the following way :

$$\begin{aligned}\widehat{u}_1 &= \left(\frac{\alpha k k_2 + \beta k k_3}{k (k_1^2 + k_2^2)^{1/2}} \right) , \\ \widehat{u}_2 &= \left(\frac{\beta k_2 k_3 - \alpha k k_1}{k (k_1^2 + k_2^2)^{1/2}} \right) , \\ \widehat{u}_3 &= \left(\frac{-\beta (k_1^2 + k_2^2)^{1/2}}{k} \right)\end{aligned}\tag{C.1}$$

where

$$\begin{aligned}k &= (k_1^2 + k_2^2 + k_3^2)^{1/2} , \\ \alpha &= \left(\frac{E(k)}{4\pi k^2} \right)^{1/2} \exp(i\theta_1) \cos(\phi) , \\ \beta &= \left(\frac{E(k)}{4\pi k^2} \right)^{1/2} \exp(i\theta_2) \sin(\phi)\end{aligned}$$

θ_1, θ_2 and ϕ are uniformly distributed random numbers on the interval $[0, 2\pi]$. The initial spectrum is defined by:

$$E(k) = \frac{a k^4 \exp(-b k^{1.1})}{(q^4 p^4)^p}.\tag{C.2}$$

Here we used $a = 2.74$, $b = 0.0278$, $q = 3.33$ and $p = 1.23$. The shape of this spectrum (Eq. C.2) does not correspond to HIT as it has no well-defined $k^{-5/3}$

inertial range. However, it grows like k^4 at small scales which is consistent with turbulence without net momentum, and it decays exponentially at high wavenumbers to mimic a dissipation range. In any case, the precise definition of this initial spectrum is not important here, as it quickly evolves to a statistically converged turbulence spectrum.

Appendix D

Vortex-in-cell code

The details of the vortex in cell method are presented in [22]. We summarize here the main features of the code as it was used in Chapter 3.

The Navier-Stokes equations are solved in their vorticity-velocity formulation:

$$\frac{D\boldsymbol{\omega}}{Dt} - \nabla \cdot (\mathbf{u} \boldsymbol{\omega}) = \nu \nabla^2 \boldsymbol{\omega} + \nabla \cdot \mathbf{T}^M \quad (\text{D.1})$$

with $\boldsymbol{\omega} = \nabla \times \mathbf{u}$ the LES vorticity field and $\mathbf{T}^M$ the SGS model. The numerical solution of (D.1) is obtained using the vortex-in-cell (VIC) approach. The vorticity field is represented by regularized vortex particles:

$$\omega_\sigma(\mathbf{x}, t) = \sum_p \alpha_p(t) \frac{1}{(\sqrt{\pi} \sigma)^3} \exp\left(-\frac{|\mathbf{x} - \mathbf{x}_p(t)|^2}{\sigma^2}\right), \quad (\text{D.2})$$

with $\alpha_p = \int \boldsymbol{\omega} d\mathbf{x} = \boldsymbol{\omega}_p h^3$ the strength of particle p , $\mathbf{x}_p$ its position, h the discretization size (grid size, also used for particle redistribution), and σ the regularization parameter. Interpolations between particles and grid, as well as particle redistribution, are all done using the M'_4 scheme. The vector stream-function $\boldsymbol{\psi}$ is obtained by solving the Poisson equation $\nabla^2 \boldsymbol{\psi} = -\boldsymbol{\omega}$ on the grid using Fishpack [2]. The velocity field (needed for convection and stretching) is then obtained from $\mathbf{u} = \nabla \times \boldsymbol{\psi}$, using 4th order finite differences (FD). The convective part is done using the Lagrangian approach: $d\mathbf{x}_p/dt = \mathbf{u}(\mathbf{x}_p)$; this ensures good convection (i.e., negligible dissipation and dispersion errors). The time variation of the particle strengths (i.e., both the vorticity stretching and the dissipation terms) is evaluated on the grid, using FD. The global time marching procedure is carried out using the Leap Frog scheme for the convection and the Adams-Bashforth scheme for the diffusion. Finally, the divergence-free character of the vorticity field is ensured by reprojection of the discrete vorticity field, which also requires solving a Poisson equation.