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An assessment methodology for timber beams in a restoration context, based on a dimensionless formulation of EC5 design criteria

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ABSTRACT

For existing timber structures that require renovation or modification, the fast identification of critical zones that need special attention, further inspection, or reinforcement is of major importance. This article proposes a simple and efficient methodology that allows, crossed with other assessment tools, to identify these critical zones. The methodology is based on a dimensionless reformulation of the eurocode 5 design ULS and SLS criteria, allowing to draw “Zd curves” that avoid long calculations, quickly highlight the structural capacity of the beam, and point out which criterion prevails (bending, shear, short term and long term deflection). This approach is thus likely to provide an efficient and simple tool in order to facilitate the discussion between the stakeholders of a renovation project, whatever their global assessment methodology.

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Assessment; efficient design; Eurocode 5; restoration; timber beams; Zd curves.

1. Introduction

During tens of centuries, wood was the most commonly used material to cover human basic needs and to provide shelters against wind, rain, and cold. In the whole world and particularly in Asia and Europe, timber structures part of ancient religious or housing buildings, some of them dating from the 7th and 8th centuries, are still standing today and attest the skills of builders through the centuries. The promotion of the extension of the service life of such structures is important for the sustainable development of cities and the conservation of the historical heritage.

Despite the fact that there are few aging issues on the intrinsic mechanical properties of timber when it is properly used (Cavalli et al. 2016), many reasons can justify interventions on old timber structures: lack of maintenance and effects of mushrooms and insects larvae, bad initial design, or simply because the renovation modifies the load cases that must respect new design codes specifications (Cruz et al. 2015, Harte et al. 2015).

The structural assessment is an important part of a conservation/renovation project that must be considered in parallel with several other relevant fields (architecture, history, archaeology, etc.) (Kouroussis, Ben Fekih, and Descamps 2017). The assessment itself is divided into different phases, depending on the detailedness of the investigations on the structure or on some elements of the structure, the remaining level of doubt, the historical

value of the building, the feasibility and simplicity of the repair/strengthening/demolition, and of course the economic considerations (Dietsch and Kreuzinger 2011, Diamantidis 2001). In all cases, the skills of an interdisciplinary team are necessary to reach realistic and efficient judgement. This is the reality for all existing structures irrespective of the material but however particularly true for structures made of timber (D'Ayala et al. 2015).

For structural engineers, dealing with the design of a new structure or with the structural assessment of an existing one requires different approaches. However, in both cases they need to deal with the same basic requirements such as the mechanical resistance, the long term stability and behavior and the safety. For a new structure, the required performance is commonly checked by the use of harmonized technical rules described in design codes, and the use of standard construction products associated to available technical sheets and data (Harte et al. 2015). However, in this article we must consider that existing buildings might be patrimonial buildings (building historically significant) for which the specifications of the design codes (in our case, the eurocode 5) must be used with a critical eye. Indeed, even if the same design rules can be used, the survey, diagnosis and assessment techniques may slightly differ between a new and an old structure. In other words, the values of the different coefficients must be intelligently chosen: for the assessment of existing structures, the effects

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of the construction process and subsequent life of the structure, during which it may have undergone alteration, deterioration, misuse and other changes to its as-built state must be taken into account. In particular, the modification factors k_{mod} , the deformation factor k_{def} and the partial factors for example, should be adjusted to fit the project requirements (Vrouwenvelder 2002, Sousa et al. 2013). However these aspects are not directly addressed in this article, mainly for two reasons. The first one is that no harmonized codes exist for the structural assessment of existing/ancient structures, which obliges to consider the codes related to new structures. The second one is that the validity of the proposed methodology is not affected by the values of k_{mod} , k_{def} and other design coefficients.

Interventions on existing buildings require the identification of the critical zones, that means zones which need special attention and potential further inspection or study (Cruz et al. 75 2015). According to its background and field of expertise, each stakeholder might propose its own definition of the critical zone. In this article, critical zones are related to sections where the ULS design criteria is overtaken (bending and shear), but also to the respect of SLS design criteria, more related to a global behavior and not a specific zone.

This article is inspired from the theory of dimensionless morphological indicators (Latteur 2000, Latteur et al. 2017, Samyn 2000). It proposes a structural assessment methodology for timber beams based on a dimensionless formulation of the eurocode 5 design criteria, which makes faster and easier the assessment process. The methodology proposes the use of what the authors called “ Z_d curves” that avoid long calculations, quickly highlight the structural capacity, and point out which criterion prevails (bending, shear, short term or long term deflection). The proposed methodology allows for the input of just a couple of parameters (essentially the geometrical slenderness l/h of the beam) while all the others are compiled with very few assumptions in a single dimensionless factor called Z_d , reducing at its minimum the gathering of information and the calculation process, which makes the assessment faster and greatly simplified. The theoretical background of the proposed methodology is developed in Section 2 where the ULS (ultimate limit state) and SLS (serviceability limit state) design criterion of timber beams in bending, according to the Eurocode 5 (EN 1995-1-1:2004) are compiled. Finally, Section 3 presents the results and a discussion based on a practical example.

2. Dimensionless formulation of ULS and SLS design criteria for uniformly distributed loaded beams

Timber floors are often composed of structural elements that can be modeled as simply supported beams subjected

to uniformly distributed loads. Although in this article the methodology is developed for this specific configuration, it stays valid and can be further extended to any other types of timber beams subjected to more particular load combinations with any kind of support at any place. All the ULS and SLS design criteria refer to the specifications of the Eurocode 5 and assume that beams are made out of solid timber or GLT without any notches at the extremities. All beams are supposed to be maintained transversely in such way that the lateral buckling is avoided ($k_{crit} = 1$) and their geometrical slenderness l/h (span over height) is considered between 5 and 30.

2.1 Bending criterion

For a beam with a cross-rectangular section $b.h$, assuming that q_{ULS} is the uniformly distributed load coming from the ULS combination, the criterion for the beam check in bending is:

$$\sigma_{m,d} = \frac{M_d}{S} = \frac{q_{ULS} \cdot l^2}{\frac{8}{b \cdot h^2}} \leq f_{m,d}, \quad (1)$$

where S is the flexion modulus, l the span of the beam, M_d the maximum bending moment, and $\sigma_{m,d}$ and $f_{m,d}$ the design bending stress and the design bending strength, respectively. According to Eurocode 5, partial safety factor γ_M , modification factor k_{mod} , depth factor k_h , and system factor k_{sys} can be introduced in the bending criterion as follows:

$$\frac{q_{ULS} \cdot l^2}{\frac{8}{b \cdot h^2}} \leq \frac{k_h \cdot k_{sys} \cdot k_{mod} \cdot f_{m,k}}{\gamma_M}, \quad (2)$$

where $f_{m,k}$ is the characteristic bending strength. One may notice that $\frac{b \cdot k_h \cdot k_{sys} \cdot k_{mod} \cdot f_{m,k}}{0,75 \cdot \gamma_M \cdot q_{ULS}} = Z_d$ is a dimensionless factor that allows writing a new expression of the bending criterion

$$\left(\frac{l}{h}\right)^2 \leq Z_d. \quad (3)$$

As previously discussed, parameters (k_{mod} and γ_M) are defined according to European standards for the design of new structures. When working on existing structures, keeping the values given in Eurocode 5 is an assumption that should be discussed and improved, even if it does not affect the methodology.

2.2 Shear criterion

For a beam with a cross-rectangular section $b.h$, assuming that q_{ULS} is the uniformly distributed load, the shear criterion can be written as follows:

$$\tau_{v,d} = \frac{V_d}{\frac{2}{3}b.h} = \frac{q_{ULS} \cdot l}{\frac{2}{3}b.h} \leq f_{v,d}, \quad (4)$$

where l is the span of the beam, V_d the design value of shear in the beam, and $\tau_{v,d}$ and $f_{v,d}$ the design shear stress and the design shear strength, respectively. According to common assumption made in Eurocode 5, partial safety factor γ_M , modification factor k_{mod} , system factor k_{sys} and crack factor k_{cr} can be introduced in the shear criterion as follows:

$$\frac{0,75 \cdot q_{ULS} \cdot l}{b.h} \leq \frac{k_{sys} \cdot k_{mod} \cdot k_{cr} \cdot f_{v,k}}{\gamma_M} \quad (5)$$

$$\frac{l}{h} \leq k_{cr} \cdot f_{v,k} \cdot \frac{b \cdot k_{sys} \cdot k_{mod}}{0,75 \cdot q_{ULS} \cdot \gamma_M}, \quad (6)$$

where $f_{v,k}$ is the characteristic shear strength. The crack factor k_{cr} for the effective width (or the shear resistance) must be taken equal to 0.67 for solid timber and glued laminated timber. If the following dimensionless parameter $k_{M-V} = \frac{k_h \cdot f_{m,k}}{k_{cr} \cdot f_{v,k}}$ (k_{M-V} is the “transition factor”; see Section 3) is introduced, one may write Equation (6) in a new dimensionless formulation:

$$k_{M-V} \cdot \left(\frac{l}{h}\right) \leq \frac{b \cdot k_{sys} \cdot k_{mod} \cdot k_h \cdot f_{m,k}}{0,75 \cdot q_{ULS} \cdot \gamma_M} \quad (7)$$

$$k_{M-V} \cdot \left(\frac{l}{h}\right) \leq Z_d. \quad (8)$$

2.3 SLS instantaneous deflection criterion

Assuming that $q_{SLS,inst}$ is the uniformly distributed load coming from the SLS combinations $\sum_j G_j + \left[Q_1 + \sum_{i \geq 2} \psi_{0,i} Q_i\right]$ where G_j are permanent actions (dead loads for example) and Q_i variable actions (imposed loads, snow loads, thermal loads, or wind loads, for example), the criteria of instantaneous deflection at mid-span (considering the shear deflection) is given by:

$$w_{inst} \leq \frac{5 \cdot q_{SLS,inst} \cdot l^4}{384 \cdot E \cdot I} + \frac{q_{SLS,inst} \cdot l^2}{8 \cdot G \cdot A'} \leq \frac{l}{\eta_{inst}}, \quad (9)$$

where η_{inst} is the criterion followed for the SLS checks, E the mean young modulus, G the mean shear modulus, I the moment of inertia, and A' the shear area. For a rectangular cross section $b.h$, the inertia I equals to $b.h^3/12$ and A' equals to $5/6.b.h$, so equation (9) becomes:

$$\frac{5}{32} \left(\frac{l}{h}\right)^3 + \frac{3}{20} \cdot \frac{E}{G} \left(\frac{l}{h}\right) \leq \frac{b.E}{\eta_{inst} \cdot q_{SLS,inst}}. \quad (10)$$

A new dimensionless parameter for SLS instantaneous deflection k_{inst} is now introduced in an attempt to standardize the formulation of SLS check:

$$k_{inst} = \frac{q_{SLS,inst}}{b.E} \cdot Z_d. \quad (11)$$

Equation (10) then becomes:

$$\eta_{inst} \cdot k_{inst} \cdot \left(\frac{5}{32} \left(\frac{l}{h}\right)^3 + \frac{3}{20} \cdot \frac{E}{G} \left(\frac{l}{h}\right) \right) \leq Z_d. \quad (12)$$

For solid timber, ratio E/G is always equal to 16, whatever the strength class.

One may notice that:

$$\begin{aligned} k_{inst} &= \frac{q_{SLS,inst}}{b.E} \cdot Z_d \\ &= k_{mod} \cdot \left(\frac{4}{3} k_{sys} \cdot k_h \cdot \frac{q_{SLS,inst}}{q_{ULS}}\right) \cdot \left(\frac{f_{m,k}}{\gamma_M \cdot E}\right). \end{aligned} \quad (13)$$

In this expression:

- k_{mod} varies from 0.5 to 1.1;
- $\frac{f_{m,k}}{\gamma_M \cdot E}$ varies from $154 \cdot 10^{-5}$ to $240 \cdot 10^{-5}$ for softwoods, from $146 \cdot 10^{-5}$ to $269 \cdot 10^{-5}$ for hardwoods and from $167 \cdot 10^{-5}$ to $190 \cdot 10^{-5}$ for glued-laminated timber according to EN 338 (NBN - EN338: 2009);
- if k_{sys} and k_h are equal to 1 (conservative), it is possible to find a conservative estimation for $\left(\frac{4}{3} k_{sys} \cdot k_h \cdot \frac{q_{SLS,inst}}{q_{ULS}}\right)$. Indeed: for a very light structure $G_j \ll Q_i$ or $\sum_j G_j \rightarrow 0$ and so:

$$\begin{aligned} \lim_{\left(\sum_j G_j\right) \rightarrow 0} \left(\frac{\sum_j G_j + \left[Q_1 + \sum_{i \geq 2} \psi_{0,i} Q_i\right]}{1,35 \cdot \sum_j G_j + 1,5 \cdot \left[Q_1 + \sum_{i \geq 2} \psi_{0,i} Q_i\right]} \right) \\ = \frac{1}{1,5} = 0.666; \end{aligned}$$

for a very heavy structure (or when variable loads are negligible compared to dead loads) $\sum_j G_j \rightarrow \infty$ or $G_j \gg Q_i$

$$\begin{aligned} \lim_{\left(Q_1 + \sum_{i \geq 2} \psi_{0,i} Q_i\right) \rightarrow 0} \left(\frac{\sum_j G_j + \left[Q_1 + \sum_{i \geq 2} \psi_{0,i} Q_i\right]}{1,35 \cdot \sum_j G_j + 1,5 \cdot \left[Q_1 + \sum_{i \geq 2} \psi_{0,i} Q_i\right]} \right) \\ = \frac{1}{1,35} = 0.741 \end{aligned}$$

So, $\left(\frac{q_{SLS,inst}}{q_{ULS}}\right)$ varies between 0.666 and 0.741. Let's assume an average value of 0.703. This assumption

only generates a maximum error of 5.3% in all cases as $(1.5-1.421)/1.5 = 0.053$ and $(1.421-1.35)/1.5 = 0.053$ too. Equation (13) then becomes:

$$k_{inst} \cong k_{mod} \cdot (0,9373) \cdot \left(\frac{f_{m,k}}{\gamma_M \cdot E} \right).$$

2.4 SLS long term deflection

Assuming that $q_{SLS,fin}$ is the uniformly distributed load coming from the worse SLS combination among $(1 + k_{def}) \sum_j G_j + Q_1 + \sum_{i \geq 2} \psi_{0,i} Q_i + k_{def} \sum_{i \geq 1} \psi_{2,i} Q_i$, the long term deflection criterion is given by:

$$w_{fin} = k_{fin} \cdot w_{inst} \leq \frac{l}{\eta_{fin}} \quad (14)$$

with

$$k_{fin} = \frac{q_{SLS,fin}}{q_{SLS,inst}} \quad (15)$$

The same reasoning as the one for the SLS instantaneous criterion leads to the criteria of final deflection at mid span:

$$k_{fin} \cdot \eta_{fin} \cdot k_{inst} \cdot \left(\frac{5}{32} \left(\frac{l}{h} \right)^3 + \frac{3}{20} \cdot \frac{E}{G} \left(\frac{l}{h} \right) \right) \leq Z_d \quad (16)$$

k_{fin} always varies between 1 and $1+k_{def}$. Indeed:

- For a very light structure, $G_j \ll Q_i$ or $\sum_j G_j \rightarrow 0$. The loads that cause creep are permanent loads and the equivalent long-term part of variable loads. Indeed, a variable load Q_i may be divided into a short-term and a long-term part. $\psi_{2,i} \cdot Q_i$ is the long-term part of the variable action Q_i that causes creep, where $\psi_{2,i}$ is the combination factor for variable action Q_i defined in Eurocode 0 (EN 1990 2002). A small value for ψ_2 factor means a small contribution to the creep deflection. Finally, for a light structure only subjected to loads that do not cause important deflections ($\sum_j G_j \rightarrow 0$ and $\psi_2 \rightarrow 0$) the following relation tends to 1:

$$\begin{aligned} \sum_j G_j, \psi_2 \rightarrow 0 & \left(\frac{(1 + k_{def}) \sum_j G_j + Q_1 + \left(\sum_{i \geq 2} \psi_{0,i} Q_i + k_{def} \sum_{i \geq 1} \psi_{2,i} Q_i \right)}{\sum_j G_j + \left[Q_1 + \sum_{i \geq 2} \psi_{0,i} Q_i \right]} \right) \\ & = \lim_{\psi_2 \rightarrow 0} \left(1 + k_{def} \cdot \left(\frac{\sum_{i \geq 1} \psi_{2,i} Q_i}{Q_1 + \sum_{i \geq 2} \psi_{0,i} Q_i} \right) \right) = 1 \end{aligned}$$

This limit shows that the SLS of such a structure is actually governed by its instantaneous deflections and not the creep deflections. For example, this

situation is relevant for light timber roofs, under an altitude of 1,000 m. Indeed, under this altitude, climatic variable loads do not cause creep deflection ($\psi_2 = 0$ under 1,000 m according to EN 1990 2002).

- For a very heavy structure (or when variable loads are negligible compared to dead loads), $G_j \gg Q_i$ or $\sum_j G_j \rightarrow \infty$ and:

$$\begin{aligned} \sum_j G_j \rightarrow \infty & \left(\frac{(1 + k_{def}) \sum_j G_j + Q_1 + \left(\sum_{i \geq 2} \psi_{0,i} Q_i + k_{def} \sum_{i \geq 1} \psi_{2,i} Q_i \right)}{\sum_j G_j + Q_1 + \sum_{i \geq 2} \psi_{0,i} Q_i} \right) \\ & = 1 + k_{def}. \end{aligned}$$

This situation is of course the worst situation in terms of creep deflection as permanent loads are the worst contributors to the long term deflection (their ψ_2 value is equal to the maximum value of 1).

The variation ranges for factor k_{fin} are thus always between 1 and $1+k_{def}$ and are shown in Table 1 for each serviceability class.

3. Discussion and results

The ULS criterion (bending and shear) of a timber beam with a uniformly distributed load is:

$$\text{Max} \left[\left(\frac{l}{h} \right)^2, k_{M-V} \cdot \left(\frac{l}{h} \right) \right] \leq Z_d \quad (17)$$

Indeed, Equation (3) and Equation (8) show that the value of l/h over which the bending criteria prevails over the shear criteria is precisely equal to factor k_{M-V} . This is the reason why it is called the “*M-V transition indicator*”. Indeed, this happens when equations (3) and (8) correspond to the same value of Z_d , thus for $\left(\frac{l}{h} \right)^2 = k_{M-V} \left(\frac{l}{h} \right)$ and thus for $\frac{l}{h} = k_{M-V}$. Figure 1 compares the bending criterion (equation (3)) with the shear criterion (equation (8)) of all strength classes of solid wood and glulam from the lower strength class C14 until the highest D70 (with $k_{cr} = 0.67$ and $k_h = 1$). They can help to evaluate which one of both criteria prevails in any situation whatever the value of Z_d . Figure 1 shows that:

- The shear criterion (function of the power 1 of ratio l/h) prevails over the bending criterion (function of the power 2 of ratio l/h) for short spans.

Table 1. Variation ranges for k_{fin} for each serviceability class.

	k_{fin}
Service class 1: $k_{def} = 0.6$	[1–1.6]
Service class 2: $k_{def} = 0.8$	[1–1.8]
Service class 3: $k_{def} = 2$	[1–3]

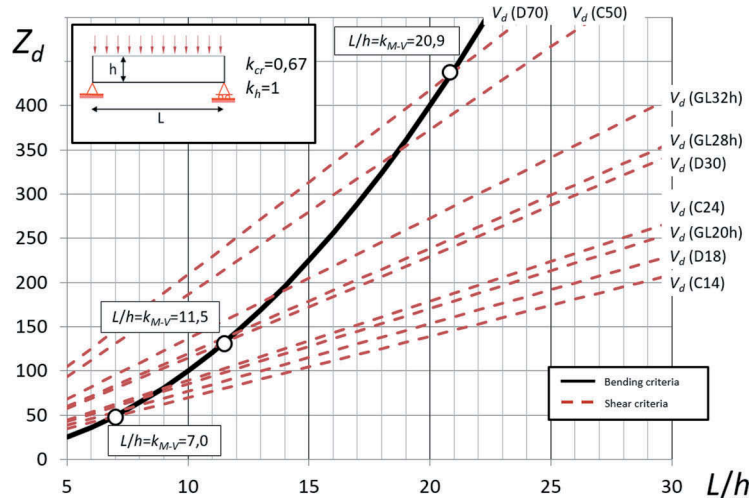


Figure 1. Z_d curves: bending criterion (the same for all strength classes) compared to shear criteria for softwoods (C), hardwoods (D), and glued-laminated timber (GL).

- The higher the strength class, the greater is the value of l/h for which bending begins to prevail over shear ($l/h = 7$ for C14 grade and $l/h = 20.9$ for D70 grade).
- For an existing structure, knowing that the shear criterion prevails over the bending criterion can help to guide restorers and experts in charge of the structural assessment. Indeed, in that case one may suggest to focus on the beam ends and special attention should be drawn to notice shear cracks and all other pathologies in this area (reduced section, knots, etc.). If ever the bending criterion prevails over the shear criterion, one may suggest to focus on mid-span cross sections to search for cracks and knots in the upper or lower fibers that may affect the bending strength.

A numerical example illustrates hereafter how the developed approach and its graphical representation could help practitioners in their task of doing the structural assessment. Let's consider a restoration project that suggests the reuse of an existing timber floor (service class 1, $k_{mod} = 0.8$). The distance between beams is 330 mm, the span 3,000 mm and the cross section is $b = 50$ mm and $h = 220$ mm.

If the architectural project suggests new functions that impose a characteristic dead load of 3 kN/m^2 and a characteristic live load of 5 kN/m^2 , under the assumption that timber joists are D30 grade, one may easily and quickly check that:

$$\begin{aligned} Z_d &= \frac{b \cdot k_h \cdot k_{sys} \cdot k_{mod} \cdot f_{m,k}}{0,75 \cdot \gamma_M \cdot q_{ULS}} \\ &= \frac{50 \cdot 1 \cdot 1,0 \cdot 8,30}{0,75 \cdot 1,3 \cdot (1,35 \cdot (3/3) + 1,5 \cdot (5/3))} = \frac{1200}{3,77} \\ &= 318. \end{aligned}$$

Then, [figure 2](#) shows that the ideal value for l/h is 17.5. So, the minimum height required for a width $b = 50$ mm is $3,000/17.5 = 172$ mm. As the beams are 220 mm high, the ULS criteria are satisfied. Under the same loading conditions ($G_k = 3 \text{ kN/m}^2$ and $Q_k = 5 \text{ kN/m}^2$) if we consider now a D18 and D70 grade, $Z_d = \frac{720}{3,77} = 191$ and $Z_d = \frac{2800}{3,77} = 742$, respectively. According to [Figure 2](#), the ideal corresponding values of l/h for D18 and D70 are, respectively, 13.6 and 30,0. The criteria are thus satisfied whatever the strength grade, since the l/h value for the existing floor beams is $3000/220 = 13.6$. This value is higher than the minimum value of l/h for the lowest strength grade D18. For this case of study, knowing easily, from the early stage of the project, that efforts and money can be affected to other points than the assessment of the timber grade is of course very interesting.

Of course, both ULS and SLS criteria have to be fulfilled, what means that the global design criterion of a timber beam with a uniformly distributed load is:

$$\begin{aligned} &Max \left[\left(\frac{l}{h} \right)^2, k_{M-V} \cdot \left(\frac{l}{h} \right), Max(\eta_{inst}, k_{fin} \cdot \eta_{fin}) \right] \\ &k_{inst} \cdot \left(\frac{5}{32} \left(\frac{l}{h} \right)^3 + \frac{3}{20} \cdot \frac{E}{G} \left(\frac{l}{h} \right) \right) \leq Z_d \end{aligned} \quad (18)$$

The two first terms of Equation (18) are the ULS criteria (bending and shear) and the third one, which is a maximum function too, is the SLS criteria (maximum between instantaneous and long term deflection). Analyzing the third term, for a given value of η_{inst} the instantaneous deflection criterion is reached

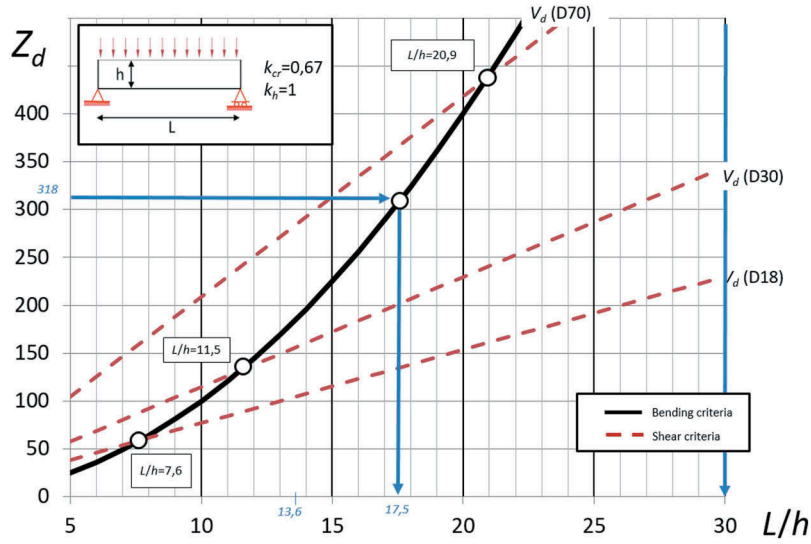


Figure 2. Bending criterion compared to shear criteria for D18, D30, and D70 grades.

when $w_{inst} = l/\eta_{inst}$. The long term deflection is then equal to:

$$w_{fin} = k_{fin} w_{inst} = k_{fin} \frac{l}{\eta_{inst}}. \quad (19)$$

The associated value of η_{fin} is thus equal to:

$$\eta_{fin} = \frac{l}{w_{fin}} = \frac{\eta_{inst}}{k_{fin}}. \quad (20)$$

As k_{fin} varies between 1 and $(1+k_{def})$, the worse value of η_{fin} that can occur is equal to $\eta_{inst}/(1+k_{def})$, with:

$$\left(\eta_{fin, \min} = \frac{\eta_{inst}}{(1 + k_{def})} \right) < \left(\eta_{fin} = \frac{\eta_{inst}}{k_{fin}} \right) < \eta_{inst}. \quad (21)$$

According to table 7.2 of Eurocode 5 (depicted in Table 3), the requirement for ratio η_{fin}/η_{inst} for beams on 2 supports varies from $150/300=0.5$ to $300/500=0.6$.

This means that factor $k_{fin} \cdot \frac{\eta_{fin}}{\eta_{inst}}$ is: (i) for service class 1, limited to $(1 + 0.6) \times 0.6 = 0.96$, which means that the long term criterion is never relevant; and (ii) for service class 2, limited to $(1 + 0.8) \times 0.6 = 1.08$, which means that the long term criterion is only relevant in a few rare cases related to a great value of factor k_{fin} .

However, it is not rare that construction specifications or designers impose a factor η_{fin}/η_{inst} greater than 0.6 and even sometimes equal to 1. This is the reason why all figures presented below with all criteria (ULS and SLS) are labelled over the value of $\eta_{fin, \min} = \eta_{inst}/(1+k_{def})$ shown in Table 2. Figures 3 and 4 show the final “ Z_d curves” with both ULS and SLS criteria for grade D30, and, respectively, for $\eta_{inst} = 300$ or $\eta_{inst} = 500$.

Table 2. Minimum values of η_{fin} for each service class.

Minimum values of η_{fin} ($=\eta_{inst}/(1+k_{def})$)	η_{inst}	
	300	500
Service class 1 ($k_{def} = 0,6$)	188	313
Service class ($k_{def} = 0,8$)	167	278
Service Class 3 ($k_{def} = 2,0$)	100	167

Table 3. Examples of limiting values for deflections of beams according to Eurocode 5, (EN 1995-1-1 2004) (Table 7.2).

	W_{inst}	$W_{net, fin}$	W_{fin}
Beam on two supports	$l/300-l/500$	$l/250-l/350$	$l/150-l/300$
Cantilevering beams	$l-l/250$	$l/125-l/175$	$l/75-l/150$

For the example discussed above ($Z_d = 318$ and $k_{mod} = 0.8$), the SLS criteria prevails over the ULS criteria for both criteria of $\eta_{inst} = 300$ or $\eta_{inst} = 500$. For $\eta_{inst} = 300$ the joists satisfy the ULS and SLS but the minimum height of the beam is now $3000/15.8 = 190$ mm (bigger than the 172 mm calculated above with the ULS criteria). For $\eta_{inst} = 500$, the minimum height of the beam is now $3000/13.3 = 225.5$ mm. As the height of the beam is only 220 mm, the SLS are not satisfied.

Finally, Figure 4 shows that for $\eta_{inst} = 500$ the SLS criteria prevail over bending criterion whatever k_{mod} values (the bending criterion is never relevant for D30). The shear criterion prevails over SLS criteria only for extremely low value of l/h (<7.5).

4. Conclusions

This article sets out the calculation and design of timber beams using a new dimensionless formulation of the EC5 ULS and SLS design criteria, and developing the concept of “ Z_d curves”. These curves avoid

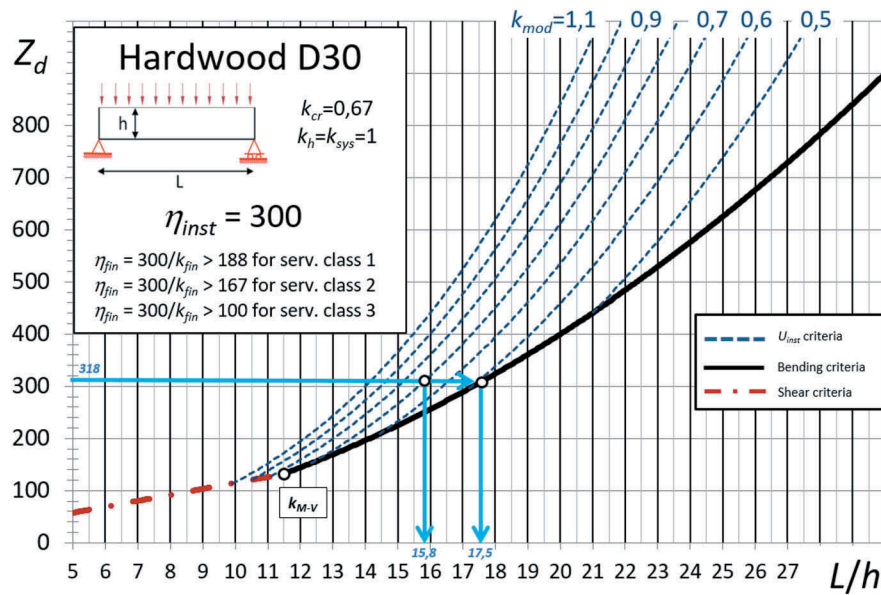


Figure 3. Z_d curve for D30 and $\eta_{inst} = 300$.

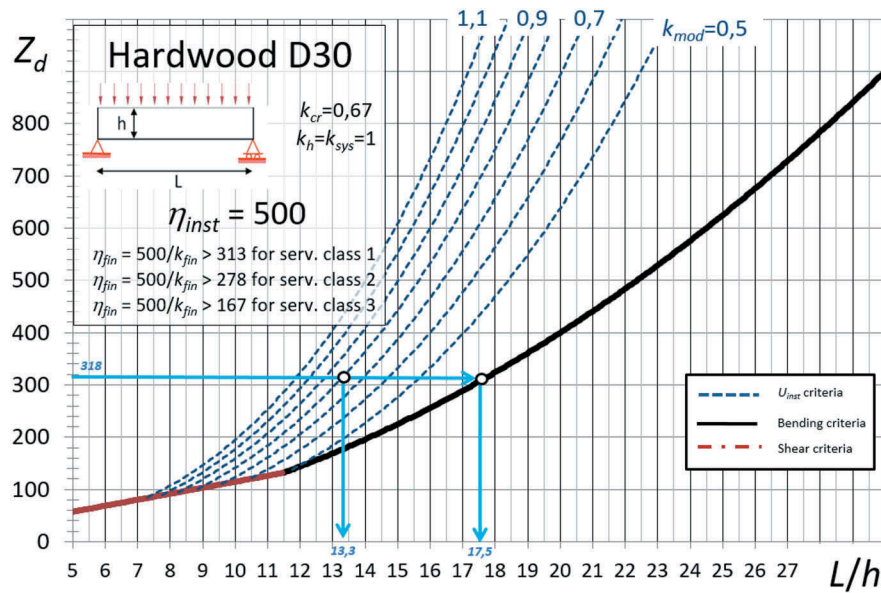


Figure 4. Z_d curve for D30 and $\eta_{inst} = 500$.

long calculations, quickly highlight the structural capacity, and point out which criterion prevails (bending, shear, short term or long term deflection). The proposed methodology allows for the input of just a couple of parameters (essentially the geometrical slenderness l/h of the beam) while all the others are compiled with very few assumptions in a single factor called Z_d , reducing at its minimum the gathering of information and the calculation process, which makes the assessment process faster and greatly simplified. The interest of this methodology for a

practical restauration context is illustrated by an example. The present research is part of a larger ongoing research that aims developing a graphical tool for practitioners during the structural assessment of any existing timber structure more complex than a simply supported beam with a uniformly distributed load. Furthermore, new technologies are nowadays available for on-site work too, and the methodology described in this article could easily lead to the development of a “ Z_d App” that could be used by practitioners directly on their smartphones.

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