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Advances in Space Research 60 (2017) 2314–2318

ADVANCES IN SPACE RESEARCH (a COSPAR publication)

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Non-randomness of exponential distance relation in the planetary system: An answer to Lecar

Vladimir Pletser *

Institut d'Astronomie et de Géophysique G. Lemaitre, Université de Louvain, 1348 Louvain-la-Neuve, Belgium

Received 29 June 2017; received in revised form 7 August 2017; accepted 17 August 2017
Available online 30 August 2017

Abstract

One of the usual main objections against attempts in finding a physical cause for the planet distance distribution is based on the assumption that similar distance distribution could be obtained by sequences of random numbers. This assumption was stated by Lecar in a short article of 1973 that is still referred to nowadays. We show here how this assumption is incorrect.

Keywords: Titius-Bode's law; Planetary distances; Secondary exponential distance relations; Randomly distributed numbers; Dole's planetary systems' computer simulation

1. Introduction

For centuries, scientists were limited to only one planetary system and three satellite systems within the Solar System to test various theories and models on the cosmogony of these systems. Within these models, the distribution of distances of planets and satellites has always been a major point of discussion. A large body of literature exists on the Titius-Bode law of planetary distances and its various modifications (see e.g. Nieto, 1972). The origin of the distance relations in Solar System's planetary and satellite systems has also been in dispute for long and is not settled yet: either it is the result of some physical process or it is due to chance alone.

However, since about 25 years, several thousands of exoplanets and thousands of multiple planetary systems have been discovered, one recently announced has seven Earth-like planets arranged in a regular configuration (Gillon et al., 2017). Some of these exo-multiplanetary systems have been fitted with Titus-Bode-like relations (Lazio et al., 2004; Christodoulou and Kazanas, 2008; Poveda and Lara, 2008; Qian et al., 2011; Cuntz, 2012; Altaie et al., 2016; Aschwanden and McFadden, 2017; Pletser and Basano, 2017). Generalized Titius-Bode relations have been applied to exoplanetary systems with at least four planets, to predict undiscovered planet positions (Bovaird and Lineweaver, 2013). This approach has been questioned (Huang and Bakos, 2014) as not all predicted planets have been detected.

The question whether the observed exponential planetary distance relations is due to some random process alone is still debated and open. Several authors tested random planet distances based on statistics (Dworak and Kopacz, 1997), or stochastic processes (Cresson, 2011), or some other approaches. Hayes and Tremaine (1998) concluded that the significance of Bode's law is simply that stable planetary systems tend to be regularly spaced, based on comparison of the actual Solar System with simple random systems generated with distances distributed uniformly random in log of distances and with some distance constraints

* Address: Technology and Engineering Centre for Space Utilization, Chinese Academy of Sciences, PITC-325-1, No. 9 Dengzhuang South Road, Haidian District, Beijing 100094, China.
E-mail address: Vladimir.Pletser@csu.ac.cn

http://dx.doi.org/10.1016/j.asr.2017.08.021
0273-1177/

between successive planets. Lynch (2003) found that the estimated probability of chance occurrence depends on the restrictions imposed on the population of orbits obtained in Monte Carlo simulations, concluding that the possibility that the observed regularity in the patterns of the planetary and satellite systems has some physical explanation is still open. Continuing with Lynch proposed method and with different constraints, Neslusan (2004) showed that corresponding sequence of distances matches power laws by chance with a probability of only 0.3 or 3 per cent depending on the type of constraints, respectively without physical limitations and exclusion of relatively close planetary orbits.

One of the usual main objections against attempts in finding a physical or dynamical explanation of the planet distance distribution is based on the simplest assumption that similar distance distributions could be obtained by sequences of random numbers sorted in increasing order. Lecar (1973) stated this assumption, based on a limited set of results of Dole's computer simulations (Dole, 1970), in a short letter to Nature, sometime still referred to nowadays.

Dole (1970) generated planetary-like systems by injecting small nuclei into a Laplace-type nebula of gas and dust. The semi-major axis and the eccentricity of the initial nuclei injection orbit were chosen at random. The nuclei would grow into proto-planets by accreting dust and gas, if their mass were large enough and their temperature low enough. Coalescence took place if the orbits crossed or came within a critical interacting distance d, function of the planetesimal semi-major axis a and mass m expressed in primary mass units:

$$d = a\left(\frac{m}{1+m}\right)^{1/4} \tag{1}$$

Some of Dole's planetary systems showed features similar to those of the actual Solar System, namely the spacing of orbits and size of individual planets. From this, Lecar (1973) argued that the spacing ratio expressed in Bode's planetary distance law could be generated by sequences of random numbers subject to the constraint that adjacent planets cannot be "too close to each other". The physical reason of the constraint is simply, if two planets were too close to each other during the accretion process, they would coalesce or cease to grow because they were competing for the same material. This idea was previously suggested and discussed by Dermott (1972).

Further, Lecar displayed in the Fig. 1 of his article the distances of seven Dole's computer generated systems in logarithmic plots against increasing integers, next to the plot of the actual planetary system, which the reader was invited to recognize. By simple visual comparison, it was difficult to differentiate the planetary system from Dole's systems. From this, Lecar concluded "... *that this offers an equally satisfactory rationalization of Bode's mnemonic.*"

If the physical reason for the "closeness not too close" condition is perfectly correct and obvious, Lecar's choice of computer generated systems reflects an arbitrary selection effect of best fits, his visual comparison method is limited and not scientific, his conclusion based on Dole's results is incorrect and his arguments are inconsistent.

2. Comparing the incomparable

2.1. Careful choice of Dole's random systems

Dole (1970) conducted 200 simulation runs resulting in computer generated planetary-like systems. He conducted additionally another series of simulations resulting in multiple star like systems that are not considered here. Out of the 200 planetary-like systems, Dole displayed in four figures schematic plots of planet distances and sizes for 20 systems. Among these 20 systems, two systems had eight planets, five had nine, twelve had ten, and one had eleven.

Among these 20 systems, seven systems were chosen carefully similar to the actual planetary system and with the most regular distance distribution (see Fig. 1), i.e. with ten planets and an average spacing ratio close to the one of the planetary system.

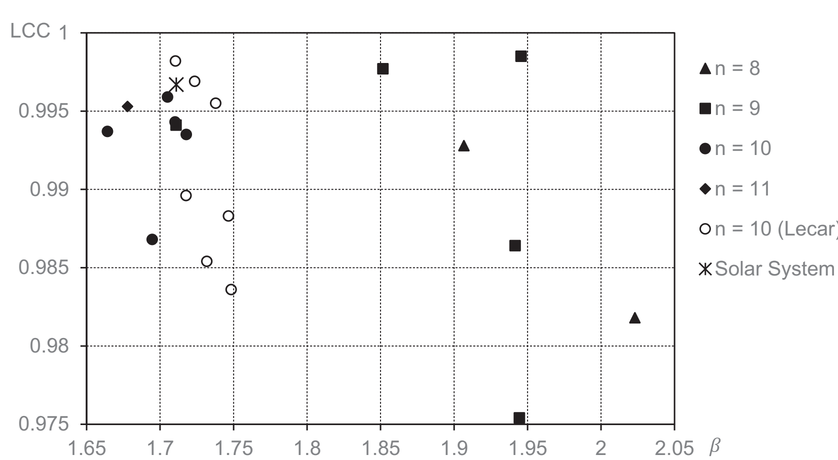

Fig. 1. Distribution of the averaged spacing ratios $\bar{\beta}$ between planets and the Linear Correlation Coefficient (LCC) values of linearized exponential regressions versus increasing integers for Dole's displayed 20 systems having 8 (triangle), 9 (square), 10 (circle) and 11 (diamond) planets (open circles for the seven systems of 10 planets chosen by Lecar). Values of β and LCC for the actual Solar System are indicated by an asterisk.

Fig. 1 of Lecar's paper displayed these seven systems next to the actual planetary system, which the reader was invited to recognize. A simple visual comparison was obviously not sufficient to differentiate between the eight displayed plots of ten dots in approximate straight lines, mainly because all eight systems had Linear Correlation Coefficients (LCC) close to or greater than 0.99. Quantitative goodness-of-fit tests would have been needed.

One can ask why any of the other 13 systems showed by Dole were not chosen or why the remaining 193 systems were not considered. Is it because they were not regular enough in terms of more or less constant spacing ratios to be exhibited next to the plot of the Solar System? Wouldn't it have been better to consider all 200 Dole's systems and either choose at random seven or more among these 200 systems or to make an accurate statistic on these 200 planetary-like systems and then make a comparison with the actual Solar System?

2.2. Linear fit

It is obvious that a more or less accurate linear fit will always be found through ten numbers generated at random and sorted in ascending order, in function of increasing integers. It will be easily found, after several trials, that most of the LCC's range between 0.9 and 0.999. Can we deduce from this that the present planetary spacing ratio can be similarly expressed by this simple process? Furthermore, imposing in the random generation process the constraint that two successive numbers cannot be "too close to each other" will increase artificially the fit accuracy, as the allowed region of existence of a next random value is reduced.

3. A more quantitative answer

Instead of Bode's artificial relation, pure exponential relations of the form

$$a_n = \alpha\beta^n \tag{2}$$

represent more accurately, for increasing integers n, the distances a_n of not only the planets but also of the satellites of Jupiter, Saturn and Uranus, with α and β different for each system (Pletser, 1986, 1990). Starting from Mercury ($n = 1$) up to Pluto[1] ($n = 10$) and considering a mean distance of 2.78 AU for the asteroid belt and no other "hole" in the distribution of planetary bodies, one has $\alpha = 0.214$ AU and $\beta = 1.711$ for the actual planetary system with a LCC of 0.9967 (Pletser, 1990).

[1] Although Pluto was moved from the family of planets to the group of planetesimals in 2006 by the IAU, it remains here in this discussion as Lecar's arguments were made in the seventies when the planetary system counted nine planets and the asteroid belt. There would be no great change in this answer if we inserted $n = 9$ instead of $n = 10$.

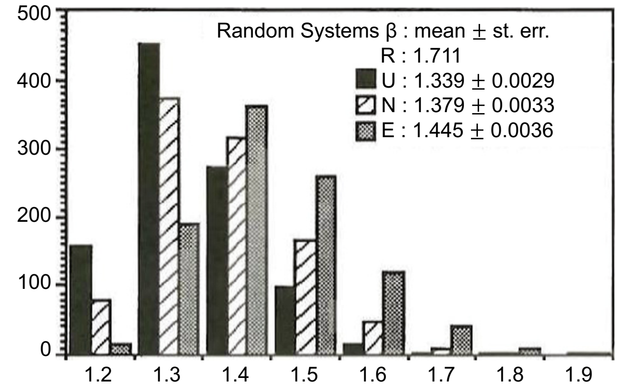

Fig. 2. Frequency histograms of the random systems 's with class size of 0.1 centred around 1.2, 1.3, etc. for the uniform (U), normal (N) and exponential (E) random generators, with means and standard errors (R: real planetary system β; a value of $\beta = 4.091$ found with the uniform generator is not represented).

3.1. Random distances with the "closeness not too close" condition

Lecar's argument is only based on spacing ratios and other orbital elements or planetary characteristics are not considered. Therefore, if Lecar's conclusion is correct, we should expect the real planetary system β_r and LCC to be close to mean values of β's and LCC's of random planetary-like systems of ten bodies "not too close to each other" with distances a_i generated at random. This is performed with the "closeness not too close" condition in an upper limit case, where we associate to the ten random values the present planets mass. We consider the Earth-Moon and Pluto-Charon systems masses and, instead of the asteroid belt, a hypothetical body having an equivalent total asteroidal mass. These masses replace the planetesimal masses m in Dole's critical distance d in Eq. (1). Three random generators are used: uniform linear mixed-congruential U (Knuth, 1969), normal N (Kinderman and Ramage, 1976), and exponential E (Ahrens and Dieter, 1972). Sets of ten random numbers are generated and sorted. An entire set is rejected if one of the difference $(a_i - a_{i-1})$ is found smaller than d_i^*, the greatest of the two local critical distances d_i and d_{i-1} (1). This is repeated until $N = 1000$ "random systems" are found compliant with the "closeness not too close" condition. Linearized exponential regressions of the random distances a_i on successive integers i give the exponential distance relations.

3.2. Statistical tests on the values of β 's

The β frequency histograms are given in Fig. 2, with means $\bar{\beta}$ and standard errors.

As the three frequency distributions have positive skewness, the modes and medians are smaller than the means. It is seen that the real planetary system β_r is greater than the random systems' means for the three generators. If we accept Lecar's conclusion, the β_r should be representative of the β's of the population of all systems of ten bodies with the present planets mass and generated at random by the same process, i.e. the β_r should be close to this population mean β. Two non-parametric distribution-free tests (Bailey, 1983) are used to test this hypothesis.

For the sign test, the differences $(\beta - \beta_r)$ are computed. If the above hypothesis is correct, we should expect a similar number of positive and negative differences, i.e. the distribution should be binomial with a probability $p = 0.5$. Using the large-sample normal approximation to the binomial, the normal test variable z is

$$z = \frac{x - Np}{\sqrt{Np(1-p)}} \tag{3}$$

where x is the number of positive or negative differences and N is the sample size (here $N = 1000$). Four positive and 996 negative differences $(\beta - \beta_r)$ are found, giving $z = \pm 31.37$, leading to the rejection of the above hypothesis with a nil significance level.

For the Wilcoxon's signed rank sum test, the sums T of the ranks of the positive and negative differences are 1017 and 499 483. The normal test variable z is

$$z = \frac{\left| T - \frac{N(N+1)}{4} \right| - \frac{1}{2}}{\sqrt{\frac{N(N+1)(2N+1)}{24}}} \tag{4}$$

where N is the sample size (here $N = 1000$), the numerator is the absolute value of the difference between T and its theoretical average, reduced by a 'continuity' correction of one-half, and the denominator is the variance of the numerator. It yields then $z_+ = 0.037$ and $z_- = 27.28$, leading again to the rejection of the above hypothesis with a nil significance level.

3.3. Statistical tests on the values of LCC's

Even so, if we accept Lecar's conclusion, the fit accuracy of sequences of ten random numbers with the "closeness not too close" condition should be at least similar to the fit accuracy of the real planetary distances, i.e. the planetary system's LCC should be close, for example, to the modes of the LCC frequency histograms, shown in Fig. 3. The main modes are all smaller than the real planetary system values. For all β classes, only 5.4% (U), 4.4% (N) and 3.4% (E) of the systems have LCC's greater than 0.995.

Dermott (1972) used another method in attempting to assess the poor statistical significance of Bode's law. Comparing the observed distances a_{obs} and the ones calculated by the exponential relation, the means $\langle|\Delta_n|\rangle$ and maxima $|\Delta_n|_{max}$ of

$$|\Delta_n| = \left| \frac{\log a_{obs} - \log \alpha}{\beta} - n \right| \tag{5}$$

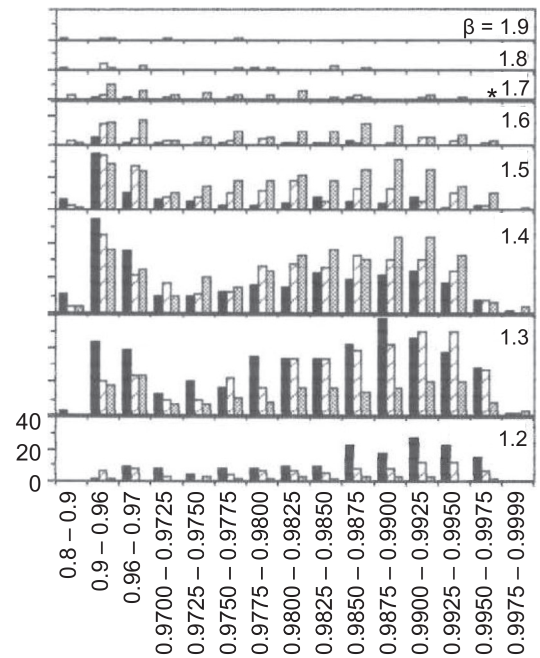

Fig. 3. Frequency histograms of the random systems LCC's with different class sizes for the three random generators (same notation as in Fig. 2) and for the different β classes. The main modes correspond to the pairs of classes [β; LCC] = [1.3; 0.9875–0.9900] (U), [1.3; 0.9900–0.9925] (N) and [1.4; 0.9875–0.9900] (E). The position of the (β; LCC) = (1.711; 0.9967) for the actual planetary system is marked by an asterisk *.

are computed. Let us call N' and N'' the number of random systems having their mean values $\langle|\Delta_n|\rangle$ and their maximum values $|\Delta_n|_{max}$ less than the actual planetary system ones. The probabilities $P'(= N'/N)$ and $P''(= N''/N)$ to find random systems similar to the real system can be estimated. For the actual planetary system, $\langle|\Delta_n|\rangle = 0.207$ and $|\Delta_n|_{max} = 0.375$. For the 1000 generated random systems, P' and P'' are respectively $2.5 \cdot 10^{-2}$ and $8 \cdot 10^{-3}$ (U), $2 \cdot 10^{-2}$ and $3 \cdot 10^{-3}$ (N), $1.9 \cdot 10^{-2}$ and $9 \cdot 10^{-3}$ (E) for the three generators. These values are ten times less than the probabilities found by Dermott and are certainly significant. However, Dermott made this calculation only for the Uranus system of five satellites.

3.4. With proto-planet masses

One can argue that, instead of the present planets mass reflecting the present planetary system state, the protoplanets higher mass at the end of the accretion phase should be used in the calculation of Dole's critical distances d in Eq. (1). This was done (Pletser, 1987) by considering the present planets mass increased up to the Solar abundance and it showed similar results.

Random systems with the mass of the satellites of Jupiter, Saturn and Uranus were also investigated in this way (Pletser, 1988) and similar results were found. i.e. random systems having on average mean values of their β's and LCC's smaller than the actual systems.

4. Conclusion

We therefore conclude that the distance relation of the present planetary system, particularly the mean spacing ratio, cannot be expressed similarly by sequences of random numbers subject to the constraint of "closeness not too close" with the planets mass. This condition is certainly necessary but not sufficient to explain the exponential spacing observed in the present planetary system. Random processes are believed to be important for planetesimal accretion but these would act only locally and not for the overall distribution of distances.

Acknowledgements

The suggestions of two anonymous Reviewers are kindly acknowledged. The Author is supported by the Chinese Academy of Sciences Visiting Professorship for Senior International Scientists (Grant No. 2016VMA042).

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