Over-aging - Are present human populations too old?

R. Stelter

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Robert Stelter†

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Abstract

This paper investigates the problem of an "optimum population" concerning age structures in a 3-period OLG-model with endogenous fertility and longevity. The first-best solution for a number-dampened total social welfare function, including Millian and Benthamite utilitarianism as two extreme cases, identifies the optimal age structure, generally failed in laissez-faire economies. As individuals don't internalize effects of longevity on life-cycle income, they over-invest in health. Additionally, they choose a non-optimal number of offspring. A calibration exercise for 80 countries emphasizes that an over-aging of populations crucially depends on social preferences and observed age structures. Interestingly, we find that in contrast to taxes on health expenditures, taxes or subsidies on children to decentralize the first-best solution are sensitive to social preferences.

Keywords: endogenous fertility; adult mortality; optimal age structure; over-aging; optimal taxation

JEL classification: H20; I10; J18

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†University of Rostock, Institute for Economics, Chair for Business Cycles and Growth, Ulmenstrasse 69, D-18057 Rostock, Germany, E-Mail: robert.stelter@uni-rostock.de, Université Catholique de Louvain, IRES - Place Montesquieu, 3, B-1348 Louvain-la-Neuve, Belgium.

1 Introduction

The demographic transition, most countries underwent during the past 150 years, leads to a variety of age structures. In developed countries, low fertility and mortality rates imply relatively high dependency ratios and mean ages. The high fraction of old individuals arouses fear of an over-aged population. On the contrary, in developing countries, featured by many children and low survival probabilities, populations are seen to be too young. This paper aims at providing an instrument to discuss optimality of age structures and evaluating sub-optimality suggested by observed situations.

The problem of sub-optimal age structures differs between developed and developing countries. The former are presumably too old and the latter too young, implying different social and political challenges, since demographic aging involves strong economic effects, due to varying intergenerational transfers (Weil, 2006). Developed economies, like in Europe, with a high fraction of old individuals are plotted on the upper left of fig. 1a. The high old age dependency ratios have consequences on old age security, health care as well as labor markets and are seen as a risk on social and individual welfare quite often. On the contrary, in developing economies, mean ages are still low, but dependency ratios likewise high, as illustrated in fig. 1b. The large number of births combined with a low life expectancy e.g. requires policies on birth control, nutrition or health care and is as challenging to societies and especially policy makers as presumably over-aged populations. Providing and applying an instrument to evaluate the existence and extent of the sub-optimality of age structures in the short- as well as in the long-run is the main objective of the paper. Each policy implemented to avoid a population that is either over-aged or too young requires such an instrument or point of reference: an optimal age structure.

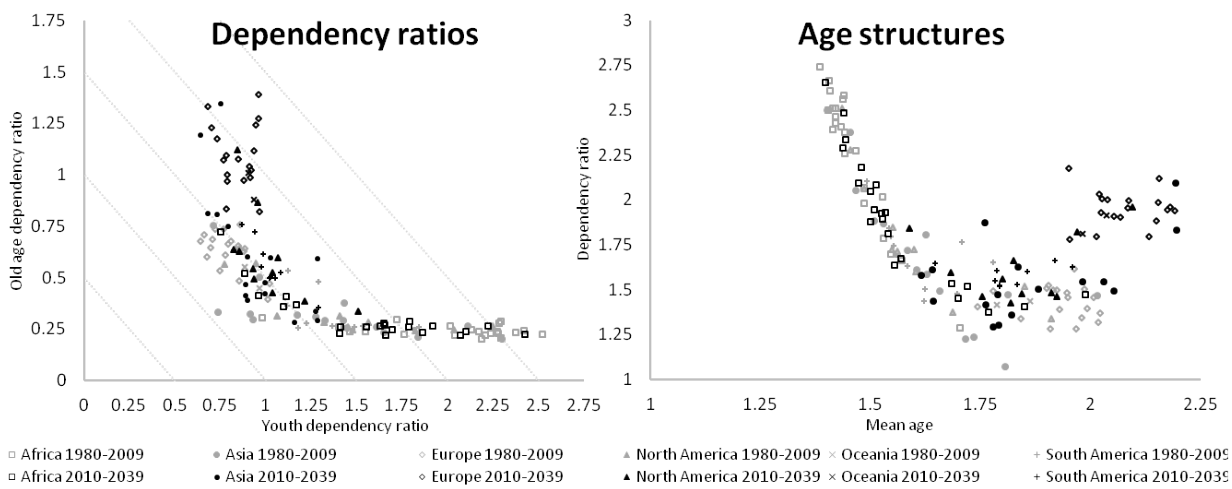

Figure 1: Dependency ratios (1a) and age structures (1b) across countries

Fig. 1 presents age structures in the periods 1980-2009 and 2010-2039 arising from Net Reproduction Rates (NRR) and female life expectancies at age 60, according to section 3.1. Fig. 1a illustrates dependency ratios across countries distinguished by youth and old age dependency ratio. Its overall value rises with distance to the point of origin, see Weil (2006). The U-shaped relation between mean ages and dependency ratios is plotted in fig. 1b. (Data source: United Nations (2013))

An overlapping-generations model (OLG-model) with three co-existing generations, children, adults and retirees, enables to evaluate discrepancies between observed and optimal age structures. Individual decisions in a laissez-faire (LF) economy endogenously determine observed age-structures, illustrated in figure 1. Selfish representative individuals choose savings, the number of offspring and health expenditures to influence longevity.[1] These age structures are faced to the optimal, or social welfare maximizing, solutions of the first-best (FB) economy. Its social welfare function (SWF) follows a flexible approach proposed by Ng (1986) and allows both Millian and Benthamite utilitarianism as extreme as well as intermediate cases (number-dampened total utilitarianism). Comparing LF and FB solutions and decentralizing the latter points out, that observed and optimal age structures differ. Neglecting the general equilibrium effect of longer lives on asset prices, individuals over-invest in health. Supposing an annuity for life, an increasing survival probability reduces returns on savings and consequently life-cycle income. Hence, it is always optimal to tax health expenditures. On the contrary, a dominated or dominating capital dilution effect determines whether children are taxed or subsidized and thus if parents ceteris paribus (c.p.) have a too high or low fertility. Furthermore, lump-sum transfers in working and retirement age correct savings and balance government's budget constraint.

Calibrating the LF economy to reproduce age structures of 80 countries in the period 1980-2009, presented in fig. 1, permits to compute paths of LF and FB age structures into steady states as well as required policy instruments to decentralize FB solutions. Whether observed age structures are over-aged, crucially depends on social preferences, the social discount factor (SDF) and the preference for the population stock. In particular the latter increases FB fertility and alters age structures as well as policy instruments. A sufficient weight of the population size in SWF leads to over-aged populations in the short as well as the long-run. The required preference for the population stock is lower in developed countries, going along with the intuition: their populations are more likely to be over-aged and too young in developing economies. Still, social preferences are fixed arbitrary and discounting future generations utility is problematic from an ethical point of view.

The optimal steady state solution with the highest feasible individual life-cycle utility among all first-best - the "Golden Age" - avoids arbitrary fixed social preferences. "Golden Age" mean ages in all countries are considerably above laissez-faire, whereas findings on the dependency ratio notably depend on observed fertility. In developing countries with high fertility LF exceed optimal dependency ratios and vice versa in populations with low fertility. Even if "Golden Age" implies the highest expected life-cycle utility, results are striking from a demographic point of view. Fertility is substantially below replacement level. Finally, these findings lead to the following dilemma: "Golden Age" is favorable from an ethical and economic point of view but problematic from a demographic perspective. On the contrary, fixing the weight of the population size and the SDF arbitrarily facilitates plausible demographic results as well as investigating dynamics but is questionable from an ethical point of view.

The outline of the paper is as follows. Before introducing the theoretical framework in section 3, the related literature is reviewed. The first subsection (3.1) presents the demographic structure in the OLG-model. Afterwards the solution chosen by individuals in a LF economy (3.2) and the optimal decisions of a social planner (3.3) are evolved. A decentralization of the first-best solution in subsection (3.4) highlights tendencies concerning an over-aged population. Section 4 presents the data set (4.1) and the calibration of the LF model (4.2). Afterwards, we emphasize deviations in observed and optimal age structures in the short (5) as well as long-run (6). Section 7 concludes.

[1]In contrast to Becker and Barro (1988), Soares (2005) and Baudin (2012), we abstract from investigating the role of child mortality and focus on longevity.

2 Literature review

The idea of optimal age structures contributes to the literature on "optimum population", traditionally dealing with population size by applying the "economic principle" on populations (Wolf, 1908). As "an optimum population is the one that achieves a given aim in the most satisfactory way" (Sauvy, 1969), we define maximization of social welfare as objective. The number-dampened total utilitarianism (Ng, 1986) provides a flexible SWF that allows different preferences for the population stock between the two extreme cases of Millian and Benthamite utilitarianism (Blackorby *et al.*, 2005). A Millian or Average SWF optimizes utility of an average or representative individual. As population size is not valued, a very small number of individuals characterized by a high (average) utility presumably is optimal. On the contrary population size is fully internalized in Benthamite, Total or Classical utilitarianism. The sum over welfare or utility of all individuals is maximized. Parfit [1976, 1982, 1984] criticizes that the repugnant conclusion may emerge: it is optimal to increase population size while per capita consumption goes to zero.[2]

Theories on optimal population emphasize different aspects of populations, starting with the population size, see e.g. Dasgupta [1969, 1998, 2005] or Boucekkine *et al.* (2011). Investigating the growth rate adds the dynamic perspective. For an OLG-framework with two overlapping generations the famous "serendipity theorem" (Samuelson, 1975) claims that in the case of a unique and stable steady state the laissez-faire economy converges to the most advantageous golden rule steady state, the "Golden Age", if fertility is fixed optimally.[3] The "serendipity theorem" still holds if mortality, and hence an uncertain life time for the second period, is added (de la Croix *et al.*, 2012). Stressing the idea of an optimal age structure in a related OLG-framework introduces a third aspect to theories of "optimum population", not done yet to the best of our knowledge. Additionally, some major differences arise typically not considered in existing literature: First of all, highlighting age structures adds the dimension "age" to theories of optimal populations taking into account the fact that individuals age over time (Arthur and McNicoll, 1977). Secondly, in line with Boucekkine and Fabbri (2013) applying a number-dampened total utilitarianism offers a more flexible SWF than exclusively focusing on "Golden Age". Thirdly, models on optimal population growth often disregard child rearing costs and/or health expenditures. As fertility only balances the negative capital widening and positive intergenerational transfer effect, optimal population growth is probably too high in those models (Arthur and McNicoll, 1978). Considering costly aspects of fertility and longevity in the LF and FB solution links the paper to frameworks on optimal fertility and longevity, jointly determining population size, growth rate as well as the age structure.

The socially optimal level of fertility is strongly related to the choice of the SWF. In general, fertility and hence population size of a Benthamite SWF exceeds those of a Millian SWF. However, the number of children in the LF economy is either above or below the optimal level in both kinds of SWF (Nerlove *et al.*, 1985) and depends on individual preferences. Utility of offspring is an argument in their parents' utility function if they are altruistic, like in Razin and Ben-Zion (1975), Becker and Barro (1988), Baudin (2011) and Nerlove *et al.* (1986). This kind of perfect altruism leads to dynastic preferences such that the time horizon of parents is infinite.[4] In contrast to altruistic individuals, time horizon is limited by death and therefore finite for selfish parents. In

[2]Boucekkine and Fabbri (2013) figure out that the repugnant conclusion never occurs for realistic parameters in an AK-like growth model. Blackorby and Donaldson (1984) propose a critical-level generalized utilitarianism as an alternative to avoid a population that is too large. Individual utility only contributes positively to the SWF if it exceeds a critical level.

[3]Deardorff (1976) shows that the solution is a global minimum for a Cobb-Douglas utility and production function. Jäger and Kuhle (2009) add the exact general conditions for an interior optimal growth rate of populations.

[4]Optimal family policies to decentralize FB fertility in a framework with quality quantity trade off have to account for interactions between instruments on education and fertility, due to the non-linear budget constraint (Baudin, 2011).

line with Eckstein and Wolpin (1985), Galor and Weil (1996) or van Groezen and Mejidam (2008) we assume that children appear as a kind of consumption good in parents' utility function and go along with costs.[5] Related to the idea of an optimal fertility, missing property rights, causing an inefficiently low number of offspring, are an another possible explanation for the low fertility in developed countries (Schoonbroodt and Tertilt, 2013).

Endogenous health expenditures are one possibility to model an endogenous length of life. Individuals invest in their health capital stock, simultaneously depreciated age specifically in each period. They die, if the health capital stock is zero (Grossman (1972), Ehrlich and Chuma (1990), Dalgaard and Strulik (2014)). Focusing on longevity, we assume that health expenditures determine the probability to survive retirement age. The optimal survival probability might be failed as several effects, like externalities in preventing contagious diseases (Kuhn *et al.*, 2011) or the negative impact of health expenditures on the life-cycle income, are not taken into account (Davies and Kuhn (1992), Philipson and Becker (1998)).

3 The model

3.1 Demographic structure

The model is an extension of the overlapping generations model introduced in the literature by Allais (1947), Diamond (1965) and Samuelson (1958) in discrete time t from 0 to infinity. For simplicity, we assume a single-sex population in a closed economy. The age structure is pooled in three age groups, children, working population and retirees. Hence, at any point in time three generations coexist. The cohort born in period t constitutes children N_t^t.[6] Childhood as first period of life is passive. Simultaneously the cohort N_t^{t-1}, born in $t-1$, is in middle age as working adults. During this active period of life, individuals solve their maximization problem, endowed with one unit of time, allocated on child-rearing and labor.[7] As the old generation N_t^{t-2} is retired and does not participate in labor market, total labor supply is:

$$L_t = (1 - \tau n_t)\, N_t^{t-1}, \tag{1}$$

where τ is the time required to bring up a child. Fertility in t determines size of birth-cohort in t, hence $N_t^t = n_t N_t^{t-1}$. Whereas all individuals survive childhood and middle age, only a fraction q_t becomes old: $N_t^{t-2} = q_t N_{t-1}^{t-2}$.[8] Thus, age structures are as simple as possible and composed of the three at least necessary age groups to describe a human life-cycle: the working period including parenthood surrounded by the two periods in dependency, childhood and retirement (Bommier and Lee, 2003).

The dependency ratio (ξ_t) and the mean age (κ_t) are two indicators to describe age structures. The former links economic and demographic aspects. Individuals participating in the labor market are compared to non-working and thus dependent people. Due to interactions between generations, e.g. intergenerational transfers, this indicator is of particular interest for policy makers. Applied

[5] On the contrary, Neher (1989) or Bental (1989) consider children as an investment good. Further alternative approaches are "warm glove" preferences (or ad hoc altruism) and the exchange model. Parents achieve utility by the number of children and an argument of their children's utility function in the former, see e.g. Benabou (1996). Children doesn't appear at all in parents preferences in the latter, but in the budget constraints. Refunding implicit credits received during childhood serve as old-age provisions (Cigno, 1993).

[6] The subscript indicates the period and the superscript the birth cohort.

[7] Allocating time to work and child rearing links the paper to literature on endogenous labor supply. Nourry (2001) investigates stability in a similar framework focusing on work and leisure.

[8] Likewise $N_t^{t-2} = q_t N_{t-2}^{t-2}$ holds, as the survival probability between childhood and middle age is 1.

to the population of the model, the dependency ratio is:

$$\xi_t = \frac{N_t^{t-2} + N_t^t}{N_t^{t-1}}.$$

Rearranging allows to express the dependency ratio in terms of the survival probability and fertility:

$$\xi_t = \frac{q_t + n_{t-1} n_t}{n_{t-1}}. \tag{2}$$

It has to be mentioned that the term "dependent" is deceptive with respect to the old generation in the LF framework. Indeed, they consume part of output and do not participate in the labor market. Still, retirees are not supported directly by the working generation, as they consume their own savings. Nevertheless, it is not possible to conclude on the age of a population by means of the dependency ratio. Both children and retirees are elements of the numerator. A high value can imply either a very young or old population, as emphasized on the RHS of fig. 1. In other words, an age structure with the shape of a pyramid as well as an inverted pyramid can have the same dependency ratio. Moreover, this demographic indicator is inappropriate to describe the process of aging. Therefore the mean age is additionally considered.[9] Applied to the model the mean age is calculated as:

$$\kappa_t = \frac{\sum\limits_{\iota=1,2,3} \iota N_t^{t-\iota+1}}{\sum\limits_{\iota=1,2,3} N_t^{t-\iota+1}},$$

whereby young individuals are defined to be at age $\iota = 1$. Accordingly, middle age is 2 and old age 3. Rearranging leads to:

$$\kappa_t = \frac{3q_t + 2n_{t-1} + n_{t-1} n_t}{q_t + n_{t-1} + n_{t-1} n_t}.$$

Hence, mean age and dependency ratio are expressed in terms of the survival probability and the individual number of children. As both are endogenous variables, the age structure is endogenous as well.

3.2 The laissez-faire economy

3.2.1 Production and capital market

Production follows the common neoclassical OLG framework (see e.g. de la Croix and Michel (2002)). At each point in time homogeneous firms produce a single commodity Y_t^{LF} with capital K_t^{LF} and labor L_t^{LF} by means of a neoclassical production function with constant returns to scale. Since capital is fully depreciated every period, $Y_t^{\mathrm{LF}} = F\left(K_t^{\mathrm{LF}}, L_t^{\mathrm{LF}}\right)$ is net production. Dividing by labor yields production in intensive terms $f(k_t^{\mathrm{LF}}) = F\left(\frac{K_t^{\mathrm{LF}}}{L_t^{\mathrm{LF}}}, 1\right)$, where k_t^{LF} is the capital-labor ratio. The representative firm maximizes its profits in fully competitive markets. Labor is provided

[9] Alternatively, a decomposition in youth and old age dependency ratio, as illustrated in figure 1, allows to conclude on the age structure (see e.g. Weil (2006)). Nevertheless, the mean age is preferred, as this single indicator measures the full age structure.

by the generation in middle age and remunerated by the wage rate w_t^{LF}. The interest factor R_t^{LF} is paid on capital. The maximization problem of firms in intensive terms is:

$$\max_{k_t^{\mathrm{LF}}} = f\left(k_t^{\mathrm{LF}}\right) - R_t^{\mathrm{LF}} k_t^{\mathrm{LF}} - w_t^{\mathrm{LF}}.$$

Production factors are compensated by their marginal products:

$$R_t^{\mathrm{LF}} = f'\left(k_t^{\mathrm{LF}}\right), \tag{3}$$

$$w_t^{\mathrm{LF}} = f\left(k_t^{\mathrm{LF}}\right) - k_t^{\mathrm{LF}} f'\left(k_t^{\mathrm{LF}}\right). \tag{4}$$

The working generation invests savings, constituting productive capital in the next period $S_t^{\mathrm{LF}} = K_{t+1}^{\mathrm{LF}}$. Therefore, capital of firms is owned by the current old generation. Since labor is determined by equation 1, the capital-labor ratio is:

$$k_t^{\mathrm{LF}} = \frac{K_t^{\mathrm{LF}}}{L_t^{\mathrm{LF}}} = \frac{K_t^{\mathrm{LF}}}{(1-\tau n_t^{\mathrm{LF}})N_t^{t-1}}.$$

Hence, time to bring up children decreases labor and pushes the capital labor ratio. The equilibrium of the capital market in terms of individuals in working age is:

$$s_t^{\mathrm{LF}} = \left(1 - \tau n_{t+1}^{\mathrm{LF}}\right) n_t^{\mathrm{LF}} k_{t+1}^{\mathrm{LF}}. \tag{5}$$

Savings per capita of a middle-aged individual in t balance capital per individual in working age in $t+1$.

3.2.2 Individuals

The representative individual lives at most three periods, childhood, working age, including parenthood, and retirement. She maximizes her expected life-cycle utility at the beginning of middle age and obtains utility from consumption in middle c_t^{LF} and old age d_{t+1}^{LF}. Furthermore, parents receive a "consumption" utility from rearing their children n_t^{LF}, weighted in relation to those of material consumption $u\left(c_t^{\mathrm{LF}}\right)$ by $\gamma > 0$. For simplicity, only the pure number of own children generates utility. Hence, no quality quantity trade off is considered.[10]
With all individuals undergoing child- and parenthood, to experience retirement is uncertain. If she is alive, she consumes d_{t+1}^{LF} and receives a utility discounted by $0 < \beta \leq 1$. Otherwise if she dies, her utility is implicitly assumed to be zero. The survival probability q is a strictly increasing function of health expenditures (h_t^{LF}) during middle age and upper bounded by one:

$$q'(h) > 0,\ q(h \to \infty) = 1.$$

Like in Leung and Wang (2010) individuals themselves choose health expenditures, which are a pure private good. As she is risk neutral with respect to longevity her expected life-cycle utility function is additive and time separable:[11]

$$EU_t = u(c_t^{\mathrm{LF}}) + \beta q(h_t^{\mathrm{LF}}) u(d_{t+1}^{\mathrm{LF}}) + \gamma v(n_t^{\mathrm{LF}}) \tag{6}$$

[10] Abstracting from human capital, parents cannot directly control expected utility of children EU_{t+1}. Considering expected utility of a representative child $EU_t = u(c_t^{\mathrm{LF}}) + \beta q(h_t^{\mathrm{LF}}) u(d_{t+1}^{\mathrm{LF}}) + \gamma v(n_t^{\mathrm{LF}}) + EU_{t+1}$ and hence altruistic parents doesn't change results.

[11] Risk neutral individuals with respect to longevity are a common assumption in literature. Supposing risk neutrality facilitates a time separable and additive life-cycle utility function in contrast to risk averse individuals (Bommier, 2006). In the more general case of risk aversion the shift of certain utilities is concave. Kuhn *et al.* (2010) show that in the case of constant health expenditures the optimal savings are lower in the case of risk aversion. The effect on health expenditures is ambiguous, if savings are assumed to be constant.

and is maximized with respect to the budget constraints in working age and retirement:

$$(1 - \tau n_t^{\mathrm{LF}}) w_t^{\mathrm{LF}} = c_t^{\mathrm{LF}} + a n_t^{\mathrm{LF}} + s_t^{\mathrm{LF}} + h_t^{\mathrm{LF}}, \tag{7}$$

$$d_{t+1}^{\mathrm{LF}} = \tilde{R}_{t+1}^{\mathrm{LF}} s_t^{\mathrm{LF}}. \tag{8}$$

In midlife individuals are endowed with one unit of time allocated to labor and child rearing. To bring up a child causes two kinds of costs, exogenous goods cost a, like in van Groezen and Mejidam (2008), and time cost τ, see for example de la Croix and Doepke (2003). Time cost fixes an upper bound on the individual number of children $n_t^{LF} < \frac{1}{\tau}$. Labor income is spent on consumption in middle age c_t^{LF}, direct child cost in terms of goods $a n_t^{LF}$, health expenditures h_t^{LF} and savings for old age consumption s_t^{LF}. Supposing perfect annuity markets with an annuity for life (Yaari, 1965), the return factor on savings $\tilde{R}_t^{\mathrm{LF}}$ depends on the average survival probability of individuals:

$$\tilde{R}_t^{\mathrm{LF}} = \frac{R_t^{\mathrm{LF}}}{q(h_{t-1}^{\mathrm{LF}})}. \tag{9}$$

Taking prices as given, maximizing expected life-cycle utility with respect to health expenditures, the number of children, consumption in working and retirement age yields to the following set of first-order conditions (FOC):

$$u'(c_t^{\mathrm{LF}}) = \beta \tilde{R}_{t+1}^{\mathrm{LF}} q(h_t^{\mathrm{LF}}) u'(d_{t+1}^{\mathrm{LF}}), \tag{10}$$

$$u'(c_t^{\mathrm{LF}}) = \frac{\gamma}{a + \tau w_t^{\mathrm{LF}}} v'(n_t^{\mathrm{LF}}), \tag{11}$$

$$u'(c_t^{\mathrm{LF}}) = \beta q'(h_t^{\mathrm{LF}}) u(d_{t+1}^{\mathrm{LF}}). \tag{12}$$

The strictly positive set of variables $\left\{c_t^{\mathrm{LF}}, d_t^{\mathrm{LF}}, h_t^{\mathrm{LF}}, k_t^{\mathrm{LF}}, n_t^{\mathrm{LF}}, s_t^{\mathrm{LF}}, w_t^{\mathrm{LF}}, R_t^{\mathrm{LF}}\right\}_{t=0}^{t=+\infty}$, solving the system of FOCs (eq. 10-12) as well as the equations on wages (eq. 4), return factor on savings (eq. 9), capital market equilibrium (eq. 5) and budget constraints (eq. 7, 8), defines the interior solution of the laissez-faire economy.[12] The first FOC (eq. 10) is the familiar Euler equation describing the distribution of consumption between midlife and old age. As the annuity for life changes returns on savings, the assumption of perfect annuity markets alters the decision on savings. On the contrary, neither the second FOC (eq. 11), that illustrates the individual decision on the number of children, nor the FOC (eq. 12) on health expenditures is sensitive to alternative frameworks without perfect annuity markets.[13] The marginal costs of children, goods $\left(au'(c_t^{\mathrm{LF}})\right)$ as well as time cost $\left(\tau w_t^{\mathrm{LF}} u'(c_t^{\mathrm{LF}})\right)$ balance the marginal utility of a child $\left(\gamma v'(n_t^{\mathrm{LF}})\right)$. Deciding on health expenditures (eq. 12), individuals compare the additional utility due to a higher survival probability in old age with the loss of consumption possibilities in middle age. The gain achieved from the last marginal unit of health expenditures $\left(\beta q'(h_t^{\mathrm{LF}}) u(d_{t+1}^{\mathrm{LF}})\right)$ equalizes its opportunity cost $\left(u'(c_t^{\mathrm{LF}})\right)$.[14] Thus, individuals do not take into account the influence of the changing survival probability on life-cycle income. The rising longevity shrinks returns on savings $\tilde{R}_t^{\mathrm{LF}}$. This reduces consumption possibilities during retirement and thus life-cycle income. Hence, individuals do not internalize the so-called Philipson-Becker-effect (Philipson and Becker, 1998).

[12]The Hessian-matrix, required to control for the second-order conditions (SOC), is presented appendix B.

[13]Alternatively, savings of deceased individuals might either be wasted by governments or paid as a lump-sum transfer to the working or retired generation. Since an optimal solution always excludes wasting resources (Heijdra *et al.*, 2014), discussions focus on the latter. A framework with lump-sum transfers to working or retired individuals is detectable in appendix A.

[14]As $u'(c_t) > 0$, a positive level of utility in old age $u(d_{t+1}) > 0$ is required. Intuitively, rational individuals only invest in health, if they enjoy old age.

3.3 The first-best solution

A point of reference is required to evaluate the age structures reflecting individual decisions. This optimal or first-best age structure results from a SWF following the idea of number-dampened total utilitarianism (Ng, 1986). Expected life-cycle utility of the current and all future generations, weighted by the size of the birth cohorts and the social discount factor ρ, is maximized:

$$W = \sum_{t=0}^{\infty} \rho^t \left(N_t^{t-1}\right)^x EU(c_t^{\mathrm{FB}}, d_t^{\mathrm{FB}}, n_t^{\mathrm{FB}}, h_t^{\mathrm{FB}}), \qquad \text{with} \qquad 0 \leq x \leq 1, 0 < \rho \leq 1$$

The characteristic feature of number-dampened total utilitarianism, a concave weight of population size in the welfare function, holds if $0 < x < 1$. Furthermore, the two extreme cases of Average and Total utilitarianism are included. Population size doesn't matter at all in the SWF, if the planner is Millian ($x = 0$). Only the discounted life-cycle utility of a representative individual of each birth cohort is taken into account. On the contrary, a Benthamite Planner ($x = 1$) considers the discounted life-cycle utility of each born individual. Thus number-dampened total utilitarianism has the advantage to imply the two extreme and the whole spectrum of intermediate cases.

Given the infinite time horizon, an upper bound of the planner's dynamic maximization problem requires $\rho < n^{-x}$. This necessitates a discounting of future generation's utility ($\rho < 1$) as soon as fertility is above the replacement level. Weighting present and (unborn) future individuals differently is problematic from an ethical point of view. Blackorby *et al.* (2005) e.g. claim that "for the purpose of social evaluation, the well-being of future generations should not be discounted." Additionally, the choice of the social discount factor (SDF) is arbitrary. One way to avoid any kind of social discounting is to convert the dynamic maximization problem of the Millian planner into a static. We come back to this issue in section 6.[15]

The planner includes preferences of the selfish individuals of each generation, implying that both, individuals and planner, have different time horizons. An individual only achieves utility in periods, she is alive. In contrast, the planner has a dynastic SWF. Due to the varying preferences, solutions differ, even if there is no externality or imperfection in the economy. Considering the preferences of the representative individual and rearranging from a longitudinal in a cross-section point of view results in:

$$W = \sum_{t=0}^{\infty} \rho^t \left[\left(N_t^{t-1}\right)^x \left[u(c_t^{\mathrm{FB}}) + \gamma v(n_t^{\mathrm{FB}})\right] + \left(N_{t-1}^{t-2}\right)^x \frac{\beta}{\rho} q(h_{t-1}^{\mathrm{FB}}) u(d_t^{\mathrm{FB}}) \right],$$

where consumption in working age of the first old generation $\left(c_{-1}^{\mathrm{FB}}\right)$ is not taken into account and the initial health expenditures $\left(h_{-1}^{\mathrm{FB}}\right)$ are preexisting. The social planner's resource constraint at time t is:

$$Y_t = N_t^{t-1}(c_t^{\mathrm{FB}} + a n_t^{\mathrm{FB}} + h_t^{\mathrm{FB}}) + K_{t+1}^{\mathrm{FB}} + N_t^{t-2} d_t^{\mathrm{FB}}.$$

The initial capital stock K_0 and the two cohorts N_0^{-1}, N_0^{-2} are as well historically given. Aggregated production Y_t is allocated on consumption c_t^{FB}, health expenditures h_t^{FB} and exogenous goods cost to bring up children $a n_t^{\mathrm{FB}}$ for all adults N_t^{t-1} as well as for consumption of retirees alive $d_t^{\mathrm{FB}} N_t^{t-2}$.

[15] The "Golden Age" situation additionally enables to relate the results to the literature on optimal population growth, e.g. Samuelson (1975), Deardorff (1976) and de la Croix *et al.* (2012). Different solutions to ensure an upper bound of the problem, required to determine the optimal path, are discussed in the literature. Already Ramsey (1928) highlights this problem in his paper "A Mathematical Theory of Saving". For an overview on social discounting see e.g. Heal (2005).

Additionally, output is used to install capital for production in the next period K_{t+1}^{FB}. Dividing by the size of the cohort N_t^{t-1} leads to the resource constraint in terms of individuals in working age. In doing so, time cost to raise a child has to be taken into account. Hence, output in terms of working individuals is $y_t^{\mathrm{FB}} = \left(1 - \tau n_t^{\mathrm{FB}}\right) f(k_t^{\mathrm{FB}})$ and the resource constraint at t:

$$y_t^{\mathrm{FB}} = c_t^{\mathrm{FB}} + a n_t^{\mathrm{FB}} + \left(1 - \tau n_{t+1}^{\mathrm{FB}}\right) n_t^{\mathrm{FB}} k_{t+1}^{\mathrm{FB}} + q(h_{t-1}^{\mathrm{FB}}) \frac{d_t^{\mathrm{FB}}}{n_{t-1}^{\mathrm{FB}}} + h_t^{\mathrm{FB}}. \tag{13}$$

In order to maximize the social welfare function, the benevolent planner chooses consumption in working and retirement age, health expenditures, the number of children and the capital-labor ratio for the next point in time. The following system of FOCs defines the optimal decisions:

$$u'(c_t^{\mathrm{FB}}) = \frac{\beta}{\rho} \left(n_{t-1}^{\mathrm{FB}}\right)^{1-x} u'(d_t^{\mathrm{FB}}), \tag{14}$$

$$u'(c_t^{\mathrm{FB}}) = \frac{\rho}{\left(n_{t-1}^{\mathrm{FB}}\right)^{1-x}} f'(k_{t+1}^{\mathrm{FB}}) u'(c_{t+1}^{\mathrm{FB}}), \tag{15}$$

$$u'(c_t^{\mathrm{FB}}) = \frac{\gamma v'(n_t^{\mathrm{FB}}) + \rho x \left(n_t^{\mathrm{FB}}\right)^{x-1} \left(u\left(c_{t+1}^{\mathrm{FB}}\right) + \gamma v\left(n_{t+1}^{\mathrm{FB}}\right) + \beta q\left(h_{t+1}^{\mathrm{FB}}\right) u\left(d_{t+2}^{\mathrm{FB}}\right)\right)}{\tau\omega + a + \left(1 - \tau n_{t+1}^{\mathrm{FB}}\right) k_{t+1}^{\mathrm{FB}} - \frac{q(h_t^{\mathrm{FB}}) d_{t+1}^{\mathrm{FB}}}{n_t^{\mathrm{FB}} f'\left(k_{t+1}^{\mathrm{FB}}\right)}}, \tag{16}$$

$$u'(c_t^{\mathrm{FB}}) = \beta q'(h_t^{\mathrm{FB}}) \left(u(d_{t+1}^{\mathrm{FB}}) - u'(d_{t+1}^{\mathrm{FB}}) d_{t+1}^{\mathrm{FB}}\right), \tag{17}$$

with $\omega = f(k_t^{\mathrm{FB}}) - f'\left(k_t^{\mathrm{FB}}\right) k_t^{\mathrm{FB}}$. The set of variables $\left\{c_t^{\mathrm{FB}}, d_t^{\mathrm{FB}}, h_t^{\mathrm{FB}}, k_t^{\mathrm{FB}}, n_t^{\mathrm{FB}}\right\}_{t=0}^{t=+\infty}$ satisfying the 4 FOCs (eq. 14-17) and the resource constraint (eq. 13) defines the interior FB solution.[16] Consumption is allocated optimally between the working and retired generation according to eq. 14. By means of eq. 15 the FOC is converted in the familiar Euler equation, already known from LF economy.

$$u'(c_t^{\mathrm{FB}}) = \beta f'(k_{t+1}^{\mathrm{FB}}) u'(d_{t+1}^{\mathrm{FB}}) \tag{18}$$

The modified golden rule of capital accumulation[17] (eq. 15) describes the optimal investment in the capital stock. In steady state, a capital-labor ratio is installed that balances its marginal product and the inverse of the SDF in the extreme case of total utilitarianism ($x = 1$). Additionally the number of children and thus implicitly population growth is considered if the weight of population size is concave or even absent in the social welfare function ($0 \leq x < 1$).

Considering all effects of a newborn, fertility is optimal if marginal utility is equal to marginal costs. According to FOC (eq. 16) both consist of three elements. First of all, parents enjoy marginal utility from rearing an additional child $\left(\gamma v'(n_t^{\mathrm{FB}})\right)$. Furthermore, the intergenerational or (old age) dependency ratio effect $\left(\frac{q(h_t^{\mathrm{FB}}) d_{t+1}}{n_t^{\mathrm{FB}} f'\left(k_{t+1}^{\mathrm{FB}}\right)} u'\left(c_t^{\mathrm{FB}}\right)\right)$, see e.g. van Groezen and Mejidam (2008), captures the effect of an additional child on the future relation between working and retired generation. Children increase population growth and hence the number of working (and consuming) individuals

[16] The Hessian-matrix to control for the SOC in "Golden Age" as well as in the general FB solution are in appendices C and D.

[17] Phelps (1961) introduced the term "golden rule".

related to the old, only consuming, in $t+1$.[18] A third effect arises if the planner is characterized by a preference for the population stock, see e.g. Baudin (2011). The discounted marginal life-cycle utility of an additional child $\left(\rho x n_t^{\mathrm{FB}^{x-1}} EU_{t+1}^{FB}\right)$ is considered in the SWF and boosts fertility if $EU_{t+1} > 0$. On the contrary, fertility is reduced if output does not ensure a positive expected life-cycle utility. A sufficient output level $y_{t+1} > \bar{y}_{t+1}$ s.t. $EU_{t+1} > 0$ is assumed below. Obviously this effect vanishes if the planner is Millian ($x = 0$). The first two costs in eq. 16 are the marginal goods ($au'(c_t^{\mathrm{FB}})$) and time costs $\tau f(k_t^{\mathrm{FB}})u'\left(c_t^{\mathrm{FB}}\right)$ to bring up children. Due to the time cost for child rearing, the capital widening effect of children, as third costly aspect, is composed of two elements. Current fertility increases the present capital-labor ratio $\tau f'\left(k_t^{\mathrm{FB}}\right) k_t^{\mathrm{FB}} u'\left(c_t^{\mathrm{FB}}\right)$, as time cost lowers labor supply. Nevertheless, the capital-labor ratio in the next period decreases, because labor supply tomorrow is higher $\left(1 - \tau n_{t+1}^{\mathrm{FB}}\right) k_{t+1}^{\mathrm{FB}} u'(c_t^{\mathrm{FB}})$. Consequently, the time cost of children slows down the capital widening effect. Hence, two effects influencing fertility in opposite directions are not internalized by individuals if the optimal solution follows the idea of Millian utilitarianism. Taking into account the dependency ratio effect increases c.p. the number of children, whereas a consideration of the capital dilution effect leads to a reduction. An existing preference for the population stock ($x > 0$) additionally boosts fertility.

Optimally fixed health expenditures follow the last FOC (eq. 17). Compared to de la Croix *et al.* (2012), a survival probability determined by health investments helps to obtain an interior solution. Rearranging eq. 17 emphasizes that

$$\varepsilon_{u\left(d_t^{FB}\right), d_t^{FB}} = \frac{u'\left(d_t^{FB}\right)}{u\left(d_t^{FB}\right)} d_t^{FB} < 1$$

is a necessary condition for an interior solution of the social planner problem. The instantaneous utility function $u\left(d_t^{FB}\right)$ has to be inelastic with respect to changes in old age consumption d_t^{FB}; implying a sufficient consumption level. In other words, the absolute level of consumption of retirees matters. An interior solution is only feasible, if consumption and hence utility during retirement exceeds a critical level. The gain achieved from an additional unit of health expenditures $\left(\beta q'(h_t^{\mathrm{FB}}) u(d_{t+1}^{\mathrm{FB}})\right)$ has to be as high as arising costs. These opportunity costs consist of two elements, less consumption in middle age $\left(u'(c_t^{\mathrm{FB}})\right)$ and decreasing consumption possibilities in old age $\left(\beta q'(h_t^{\mathrm{FB}}) u'(d_{t+1}^{\mathrm{FB}}) d_{t+1}^{\mathrm{FB}}\right)$. The latter is the fully internalized Philipson-Becker effect, not detectable in the FOC (eq. 12) of the individual decision .

All effects are taken into account by the benevolent planner. Choices on fertility and mortality are optimal and hence implicitly the resulting age structure. Thus, first-best age structures are considered as optimal.

3.4 Decentralization of optimal age structures

The interior FB and LF solution imply the optimal as well as the age structure, reflecting individual decisions. As well known, nothing guarantees that the "modified golden rule of capital accumulation" holds in laissez-faire economies. In contrast, in general FB and LF solutions differ, because of the double infinity in goods and agents (Shell, 1971). Fixing fertility optimally is one possibility to achieve the golden rule level in the standard OLG framework, claimed by the "serendipity theorem". Anyway, this intervention is excluded, since individuals decide themselves on the number of offspring.

[18] Opposing working and retired generation, fertility of the previous period influences the current dependency ratio, but has no impact in the future. Current fertility determines the future relation, as present old are going to be deceased. This result would change in an OLG-model with additional age groups.

As individuals don't consider all effects, fertility and mortality deviate from their optimal values everything else equal. However, a conclusion on the relation between LF and FB age structure isn't possible, due to deviating LF and FB capital-labor ratios. Thus, beside the sub-optimal decisions on demographic variables, production levels differ. Only a discussion of c.p. discrepancies is possible either by comparing the FOCs or by decentralizing optimal age structures. The latter is preferred, because of the advantage to display the required policy instruments.

Four instruments implement the optimal solution in a decentralized economy. Two pigouvian taxes ensure, that individuals internalize all effects in fertility as well as mortality. A child allowance or tax η_t corrects the desired number of children and a health subsidy or tax ϕ_t the health expenditures. In addition a lump-sum transfer in working age z_t^m guarantees that the capital-labor ratio in the laissez-faire economy satisfies the modified golden rule level. Finally, excluding debt, a second one in retirement age z_t^o balances the budget constraint of the government in each point in time:

$$(\eta_t n_t + \phi_t h_t + z_t^m) N_t^{t-1} + z_t^o N_t^{t-2} = 0,$$

or, in terms per individual in middle age:

$$\eta_t n_t + \phi_t h_t + z_t^m + \frac{q(h_{t-1}) z_t^o}{n_{t-1}} = 0. \tag{19}$$

Individuals maximize their expected life-cycle utility function (eq. 6) with respect to the adjusted budget constraints, due to the interventions of the government:

$$c_t = (1 - \tau n_t) w_t - (a - \eta_t) n_t - s_t - (1 - \phi_t) h_t + z_t^m, \tag{20}$$

$$d_{t+1} = \tilde{R}_{t+1} s_t + z_{t+1}^o.$$

The corresponding system of FOCs is:

$$u'(c_t) = \tilde{R}_{t+1} \beta q(h_t) u'(d_{t+1}), \tag{21}$$

$$u'(c_t) = \frac{\gamma v'(n_t)}{\tau w_t + a - \eta_t}, \tag{22}$$

$$u'(c_t) = \frac{\beta q'(h_t) u(d_{t+1})}{(1 - \phi_t)}. \tag{23}$$

The FOCs eq. 16, 17, 22 and 23, the budget in midlife and resource constraint (eq. 13 and 20), as well as the constraint of the government (eq. 19) determine the necessary levels for the four policy instruments such that $\left\{c_t^{\mathrm{FB}}, d_t^{\mathrm{FB}}, h_t^{\mathrm{FB}}, k_t^{\mathrm{FB}}, n_t^{\mathrm{FB}}\right\}_{t=0}^{t=+\infty} = \{c_t, d_t, h_t, k_t, n_t\}_{t=0}^{t=+\infty}$. The lump-sum transfer in working age corrects savings or capital accumulation in the LF economy (eq. 24) and those in retirement age (eq. 25) balances the budget constrain:

$$z_t^m = (1 - \tau n_t) f'(k_t) k_t - \eta n_t - \phi_t h_t - \frac{q(h_{t-1}) d_t}{n_{t-1}}, \tag{24}$$

$$z_t^o = -\frac{n_{t-1} (\eta_t n_t + \phi_t h_t + z_t^m)}{q(h_{t-1})}. \tag{25}$$

Proposition 1 ***Taxes on health expenditures***
A tax on individual health expenditures according to

$$\phi_t = \frac{d_{t+1}u'(d_{t+1})}{d_{t+1}u'(d_{t+1}) - u(d_{t+1})} \tag{26}$$

implements the Philipson-Becker-Effect.

Proof Resulting from equalizing 17 to 23 and rearranging, equation 26 describes the optimal policy on health expenditures at each point in time. Due to the non-negativity in consumption and $u'(\cdot) > 0$, the numerator is always positive. On the contrary, only a negative denominator allows a solution of the FOC (eq. 17). The negative result for ϕ_t confirms that health expenditures are taxed at all dates. ∎

The tax on health expenditures indicates that their price is too low and individuals c.p. over-invest in their longevity. Findings change, if externalities in preventing contagious diseases (Kuhn *et al.*, 2011) are considered or health expenditures have an impact on labor productivity (not considered by individuals).[19] In contrast to the unambiguous findings for mortality, results concerning fertility are less obvious. The effects, not considered by individuals, operate in different directions.

Proposition 2 ***Child allowances***
The optimal path on child policies to decentralize first-best fertility follows:

$$\eta_t = \frac{\gamma v'(n_t)\left[\frac{q(h_t)d_{t+1}}{n_t f'(k_t)} - (1-\tau n_{t+1})k_{t+1}\right] + \rho x n_t^{x-1} EU_{t+1}(a + \tau w_t)}{\gamma v'(n_t) + \rho x n_t^{x-1} EU_{t+1}}. \tag{27}$$

1. *Average utilitarianism*
   The child allowance is positive if the dependency ratio effect $\left(\frac{q(h_t)d_{t+1}}{n_t f'(k_t)}\right)$ *dominates the capital dilution effect* $((1-\tau n_{t+1})k_{t+1})$ *and negative in a vice versa situation.*

2. *Number-dampened total and total utilitarianism*
   The child allowance is positive if the dependency ratio effect $\left(\frac{q(h_t)d_{t+1}}{n_t f'(k_t)}\right)$ *and the preference for the population stock, weighted by the ratio of marginal time and good costs to the marginal consumption utility of child rearing* $\left(\rho x n_t^{x-1} EU_{t+1}\frac{a+\tau w_t}{\gamma v'(n_t)}\right)$, *dominate the capital dilution effect* $((1-\tau n_{t+1})k_{t+1})$. *Children are taxed in a vice versa situation.*

Proof Equalizing the FOC on fertility (eq. 16 and 22) and transposing for η_t leads to 27. If $x = 0$, the extreme case of Millian utilitarianism, the optimal policy on fertility simplifies to:

$$\eta_t|_{x=0} = \frac{q(h_t)d_{t+1}}{n_t f'(k_t)} - (1-\tau n_{t+1})k_{t+1} \quad \text{with} \quad \begin{cases} \eta < 0 \text{ if } \; LHS < RHS \\ \eta = 0 \text{ if } \; LHS = RHS \\ \eta > 0 \text{ if } \; LHS > RHS \end{cases}$$

[19] Bloom *et al.* (2004) emphasize the significantly positive effect of good health on output. Another possibility is to ad health expenditures directly as an argument in the utility function. However, as the planner considers individual preferences, no additional externality arises.

In the case of number-dampened total or total utilitarianism $0 < x \leq 1$ the sign of the policy is determined by the numerator of eq. 27. Rearranging leads to:

$$\eta_t|_{0<x\leq 1} \begin{cases} < 0 \text{ if } \frac{q(h_t)d_{t+1}}{n_t f'(k_t)} + \rho x n_t^{x-1} EU_{t+1} \frac{a+\tau w_t}{\gamma v'(n_t)} < (1-\tau n_{t+1})\, k_{t+1} \\ = 0 \text{ if } \frac{q(h_t)d_{t+1}}{n_t f'(k_t)} + \rho x n_t^{x-1} EU_{t+1} \frac{a+\tau w_t}{\gamma v'(n_t)} = (1-\tau n_{t+1})\, k_{t+1} \\ > 0 \text{ if } \frac{q(h_t)d_{t+1}}{n_t f'(k_t)} + \rho x n_t^{x-1} EU_{t+1} \frac{a+\tau w_t}{\gamma v'(n_t)} > (1-\tau n_{t+1})\, k_{t+1} \end{cases}$$

$\blacksquare$

Hence, the dominating effect (or effects) determine(s) the sign of child allowances. A positive value indicates costs of offspring that are too high in the laissez-faire economy. A situation more likely if the planner takes into account the population size in the SWF, as this adds an additional positive effect not considered by individuals. The allowance reduces costs and fertility increases, since children are a normal good. The opposite is true, if the capital dilution effect dominates. The costs of a child are too low to consider the dominating negative influence of additional children on the capital-labor ratio. The tax increases costs and lowers fertility.

Proposition 1 and 2 are robust to the alternative framework with lump-sum transfers. The former isn't altered, as redistribution of deceased individuals' savings is likewise central. Either lump-sum transfers or returns to savings shrink in longevity and hence in health expenditures; not taken into account by individuals. The major difference arises with respect to allocation of consumption to working and retirement age. Given the correct income, individuals would save optimally in the case of perfect annuity markets. On the contrary, the frameworks with lump-sum transfers requires an additional pigouvian tax to correct the individual Euler equation, discussed in appendix A.

Supposed both children and health expenditures are normal goods, taxing health expenditures implies an over-investment in health and a survival probability that exceeds c.p. the optimal level. The lower mortality raises the mean age. This tendency to an over-aged population is strengthened by fertility, if the dependency ratio effect, supported by the preference for the population stock, dominates. The costs of children are too high in the LF economy. Hence, the number of offspring is c.p. below its optimal level and thus upwards biases the mean age, too. On the contrary, a dominating capital dilution effect goes along with a LF fertility that tends to exceed the FB level, reducing the LF mean age.

The case of the dependency ratio is similar and depends positively on the survival probability, too. As the Philipson-Becker effect is not internalized by individuals, the fraction of the non-working population exceeds c.p. the optimal level. The effect of fertility is less obvious, based on different reasons. First of all, according to the relationship between capital dilution and the sum of dependency ratio effect and the preference for the population stock, costs of children are either above or below the LF solution. Additionally, fertility either rises or lowers the dependency ratio. A higher fertility boosts the dependency ratio in t but reduces its value in $t+1$. According to

$$\frac{\partial \xi}{\partial n} = \begin{cases} > 0 \ \text{ if } \ n^2 > q \\ < 0 \ \text{ if } \ n^2 < q \end{cases}$$

the steady state effect is ambiguous. Developing countries, commonly characterized by a relatively high fertility, are featured by a positive steady state relation between the dependency ratio and fertility, as Net Reproduction Rate (NRR)[20] is likely above one. On the contrary, in developed

[20]The NRR is "the average number of daughters a hypothetical cohort of women would have at the end of their reproductive period if they were subject during their whole lives to the fertility and mortality rates of a given period (United Nations, 2013)."

economies with a NRR below and a survival probability closed to one, a negative relation is more likely. Hence, four steady state scenarios are possible. Developing countries with high youth dependency ratio have a ξ that is c.p. too low, if the intergenerational effect and the preference for the population stock dominate. In contrast, a tendency to exceed the optimal level features developed countries. If the capital dilution effect dominates, the situation is vice versa. However, as the effect of the capital-labor ratios is not considered, only a discussion on tendencies is possible.

4 Calibration

Laissez-faire generally deviate from first-best age structures. To highlight the direction and extent of differences as well as required policies to decentralize optimal solutions, the LF economy is calibrated to fit observations of 80 countries. Thereby, we proceed in three steps:

The first step presents in section 4.1 the data set and afterwards the calibration of parameters in 4.2. The latter focuses on the first period 1980-2009, to avoid the crucial assumption that observed age structures are in steady state. More importantly, the LF economy reproduces observations in the first period. Supposing the same initial situation for the first-best economy then highlights the varying evolution of age structures over time. Furthermore, it enables to discuss policies necessary to implement the optimal path - given the initially observed economic and demographic conditions.

The second step focuses on dynamic implications. Assuming social preference that go along with an intermediate first-best fertility, this baseline scenario highlights deviations in age structures across countries in the first (1980-2009) and second period (2010-2039) under investigation. As findings are sensitive to social preferences, the next subsection (5.2) discusses consequences of varying preferences on optimal age structures and related policies for selected countries.

In a last step, section 6 emphasizes age structures across countries in the long-run. Investigating of steady state solutions enables to contrast LF to "Golden Age" age structures across countries. Additionally, different first-best scenarios highlight consequences of varying social preference on age structures and policy instruments.

4.1 Data

Supposing a period of 30 years and implicitly each age, childhood, parenthood and retirement, a calibration of the LF economy for 80 countries ($i = 1, 2, ..., 80$) requires data of three periods ($t = -1, 0, 1$): 1950-1979, 1980-2009 and 2010-2039. These economic and demographic data originate from different sources. The world bank database is source of average health expenditures per capita, GDP and gross fixed capital formation ratio in percentage of GDP. GDP in 1980-2009 approximates Output $Y_{i,0}$ in country i at time $t = 0$. Like all economic variables, GDP is expressed in \$ (PPP constant 2005 international \$). Nehru and Dhareshwar (1993) offer an estimation of gross domestic fixed capital stocks for 93 countries until 1990. Converting their values in PPP \$ and adjusted to the future according to:

$$K_{i,t+1} = (1 - \Delta) K_{i,t} + I_{i,t} \times GDP_{i,t},$$

approximates real capital stocks in 1980 and 2010 ($K_{i,0}$, $K_{i,1}$). The gross fixed capital formation ratio in percentages of GDP ($I_{i,t}$) times GDP involves the yearly investments and capital depreciates with a rate Δ of 5% p.a. Furthermore, health expenditures between age 30 and 59 are required. As age structures themselves fundamentally drive average health expenditures per capita and no adequate data by age group is available, controlling for the effects of age structures approximates $h_{i,1}$. Supposing the distribution of health expenditures over age groups and sexes in Germany allows to calculate health expenditures between age 30 and 59, because the distribution of health expenditures over age groups is generally similar across countries (Dalgaard and Strulik, 2014).[21]

Demographic data emanates from the United Nations (2013) World Population Prospects. Working age populations in 1980 and 2010 determine $N_{i,t}^{t-1}$ in $t = 0, 1$. Both fertility and mortality focus on female populations. The survival probability $q_{i,t}$ relates the average life-expectancy at age 60 in 1980-1985 and 2010-2015 to the length of a period. The average Net Reproduction Rate (NRR)[22] reflects fertility $n_{i,t}$ in $t = -1, 0, 1$ and implicitly captures mortality within childhood and reproductive age. The implied age structures in 1980-2009 ($t = 0$) and 2010-2039 ($t = 1$) are illustrated in figure 1.

4.2 Calibration of laissez-faire economies

The calibration of parameters focuses on period $t = 0$, implying the years 1980-2009, and proceeds in three steps: Fixing functional forms, determining country non-specific and finally identifying country specific parameters.

Step one specifies functional forms for the calibration exercise. Logarithmic instantaneous utility functions for consumption in middle and old age, as well as for utility generated by children, underlie the calibration exercise ($u(\cdot) = v(\cdot) = \ln(\cdot)$). Furthermore, production technology follows a common Cobb-Douglas function with A_i as total factor productivity (TFP):

$$Y_{i,t} = F(K_{i,t}, L_{i,t}) = A_i K_{i,t}^{\alpha} L_{i,t}^{1-\alpha}.$$

The survival probability is a monotonically increasing function of health expenditures $q' > 0$, upper bounded by 1:

$$q(h_{i,t-1}) = \frac{1}{1 + \exp^{\sigma_i - h_{i,t-1}^{\delta_i}}}. \tag{28}$$

Determined by σ_i and δ_i the function is either s-shaped or concave. The country specific parameter σ_i reflects general climate, medical and hygienic environmental situations and is independent from individual health expenditures. The country specific parameter $\delta_i > 0$ captures their influence.[23]

The second step of the calibration fixes the set $\{\alpha, \gamma, \tau\}$ of country non-specific parameters. The partial production elasticity of capital (α) in the Cobb-Douglas function is fixed to 0.3. Time to bring up children is approximated by information on time use offered by OECD (2012). Relating time for care to the sum of time for care and paid work, in the model parents solely allocate their time between child rearing and paid work, leads to an average of 15.3% for both men and women over age 15 in 23 OECD countries. However, this value is sensitive to age structures and the average number of children within countries. Focusing alternatively on the time used for childcare as main or second activity compared to the total time for work and child rearing in all families with at least one child in preschool age results in a much higher average value of 30.5% in 14 OECD countries. As time to bring up a child varies over age and potentially more than one is reared in the household,

[21] Appendix E offers details of the calculation and Table 6 in appendix F presents summary statistics of economic as well as demographic variables.

[22] Projected values, starting in 2010-2014, suppose the medium scenario.

[23] A common assumption in the literature, see e.g. Chakraborty (2004), Leung and Wang (2010) and Leroux *et al.* (2011), is a concave survival probability function with: $q'(h) > 0$, $q''(h) < 0$, $q(h \to \infty) = 1$ and $0 \leq q \leq 1$. The specification $q(h_{i,t-1}) = \frac{\sigma_i + h_{i,t-1}^{\delta_i}}{1+\sigma_i + h_{i,t-1}^{\delta_i}}$ satisfies an upper bound of 1 and concavity for the set of parameters (δ_i, σ_i) fulfilling $h_i^{\delta_i} + \sigma_i > 0$ and $h_i^{\delta_i} + \frac{(1+\sigma_i)(1-\delta_i)}{1+\delta_i} > 0$ for steady state solutions. As this specification inserts additional restrictions and qualitative results are not altered, we prefer the more flexible functional form in equ. 28.

this value is likely to be an overestimation. Following de la Croix and Doepke (2003), assuming that only half of the time is necessary over a period of 30 years, results once again in a value around 15%. Since latter is inline with the literature on OLG-models with endogenous fertility, $\tau = 0.15$ is supposed in the calibration.[24] As part of individual preferences, the weight of utility generated by child rearing is fixed to fulfill the FOC on the number of children (eq. 11) in the US. The required good costs are 207.43 K PPP $ and defined as total expenditures of a child for housing, food, health care etc.[25] With a value of 0.54 utility generated by child rearing is weighted slightly above half of those originated by consumption during working age.

The third step determines the set $\{A_i, a_i, \beta_i, \delta_i, \sigma_i\}$ of country specific parameters. A system of nine equations and unknowns, of which 5 are the required parameters, features each economy and enables to identify the parameter set stepwise. Following the definition of labor (eq. 1) $L_{i,t}$ is calculated in $t = 0, 1$, considering the NRR, population in working age and the time to bring up children in both periods. With $Y_{i,0}$, $L_{i,0}$ and $K_{i,0}$ the TFP afterwards remains as residual in the Cobb-Douglas function and is ascertained. Additionally, the capital market equilibrium (eq. 5) and eq. 4 determine savings and wages in $t = 0$. Eq. 3 in $t = 1$ identifies the interest factor. Then the individual discount factor β_i solves the Euler equation (eq. 10) according to R_{t+1} and savings, required to fit the capital-labor ratio.[26] Good costs to bring up offspring in each country balances the FOC on fertility (eq. 11) in $t = 0$. Finally, the two parameters of the survival probability function (δ_i, σ_i) are jointly determined by the decision on health expenditures (eq. 12) and $q\,(h_{i,t-1})$ itself (eq. 28).

Table 1:
SUMMARY STATISTICS OF PARAMETERS

Parameter	Mean	Std. dev.	Min	Max
A_i (in K)	30.492	17.864	4.284	85.508
a_i (in K)	87.320	114.536	-7.720	470.077
β_i	0.295	0.101	0.162	0.890
δ_i	0.065	0.032	0.021	0.150
σ_i	0.996	0.442	0.281	2.896

Table 1 summarizes country specific parameters. On average the TFP (A_i) is 30,492, with a range between 4,284 in Mozambique and 85,508 in Singapore. The highest good costs per child are observable in Luxembourg with 470.077 K $. In contrast, in many very poor, especially African countries, children still have to contribute to household income, the most in Jordan (-7,720 $). On average the expenditures to bring up a child are 87,320 $. The average individual discount factor (0.295) is very close to the text book value (0.3) for 30 years, see e.g. de la Croix and Michel (2002). Commonly, values are below mean in developed countries with high survival probabilities,

[24] For a period of 25 years Doepke (2004) chooses a time costs of 0.155. Half of the time cost of 0.15 %, estimated by Knowles (1999), is used in the calibration of de la Croix and Doepke (2003). A value of 0.15 % implicitly fixes the average maximal fertility to around 13 children per women. This outcome seems to be rather realistic as Gagnon *et al.* (2008) state: "fertility was particularly high among the settlers of New France ... an average of 9.97 children for each fertile woman who survived married to the age of 50." Thus, even this very high fertility is below the fixed natural limit.

[25] The good costs are the sum over total expenditures at ages 0 to 17 for the younger child in middle-income, husband-wife households with two children. Child care and education expenses are only considered for families with expense (Lino, 2012).

[26] Wang *et al.* (2011) conclude from a survey on 45 regions that time discounting highly varies across countries. For a general discussion on discount rates see e.g. Frederick *et al.* (2002).

e.g. Germany (0.23), UK (0.24) or the US (0.28), whereas developing countries like Guyana (0.89), Mali (0.57) or Mozambique (0.52) have rather high values. Thus, overall discounting of future utility as product of individual discount factor and survival probability tend to converge. The two parameters of the survival probability function have an average of $\delta = 0.065$ and $\sigma = 0.996$. To fit both, mortality and health expenditures, Sierra Leone has the lowest survival probability in absence of health investments, due to $\sigma = 2.9$. A very low longevity features Japan in this theoretical situation as well ($\sigma = 1.96$), even if the observed survival probability is highest. However, health expenditures have c.p. the highest influence on longevity ($\delta = 0.15$). On the contrary, the situation is vice versa in Singapore with the highest survival probability in a hypothetical situation without health investments ($\sigma = 0.281$ and $\delta = 0.078$). The lowest impact of health expenditures on longevity ($\delta = 0.02$) characterizes Algeria.

This set of parameters enables to replicate observations on survival probability in 2010-2015, the capital per individual in working age 2010, NRR and health expenditures between 1980-2009 almost perfectly ($R^2 = 1$). Average projected NRR for 2010-2039 and those predicted by the model vary in an acceptable range ($R^2 = 0.73$).[27]

5 Dynamic implications

5.1 Age structures and policies in 1980-2009 and 2010-2039

Given initial capital stocks per person in working age 1980, NRR 1950-1979 as well as survival probabilities 1980-1985, the set of parameters enables us to calculate paths of LF and FB economies from $t = 0$ (1980-2009) into steady states. In order to compare observed age structures, illustrated in figure 1, with the optimal across countries, the SDF and the weighting of populations in the SWF are initially fixed according to an intermediate scenario (FB_{02}) with $x = 0.02$ and 0.3% p.a. ($\rho = 0.91$). Section 5.2 afterwards discusses consequences of different social preferences.

Table 2:
OBSERVED AND OPTIMAL AGE STRUCTURES IN 1980-2009 AND 2010-2039

Continent	Mean age				Dependency ratio			
	1980-2009		2010-2039		1980-2009		2010-2039	
	Observed	FB_{02}	Observed	FB_{02}	Observed	FB_{02}	Observed	FB_{02}
Africa	1.46	1.45	1.57	1.48	2.26	2.31	1.89	2.31
Asia	1.63	1.59	1.86	1.68	1.69	1.88	1.56	2.03
Europe	1.93	1.89	2.08	2.02	1.43	1.53	1.93	1.87
North America	1.61	1.56	1.84	1.67	1.79	2.01	1.61	2.09
Oceania	1.82	1.77	2.01	1.91	1.42	1.58	1.86	1.85
South Amercia	1.61	1.57	1.83	1.67	1.69	1.89	1.57	1.96
World	1.66	1.62	1.83	1.71	1.79	1.93	1.76	2.07

Figure 2 presents observed age structures, illustrated in the motivation, in grey supplemented by the optimal, according to fixed social preferences, in black. As mortality in 1980-2009 is historically given, differences are solely driven by fertility. On average all continents are featured by a fertility that is too low and thus over-aged situations. The observed mean age exceeds the optimal, see table 2.[28] However, in several developing countries, like Nigeria or Sierra Leone, populations are

[27] Appendix G presents details.

[28] The values are non-weighted averages across continents. Appendix I contains selected numerical results.

too young. In 2010-2039 observed and optimal fertility as well as longevity differ. Still, on average an over-aging is observable; the majority of grey squares is to the right of black ones.

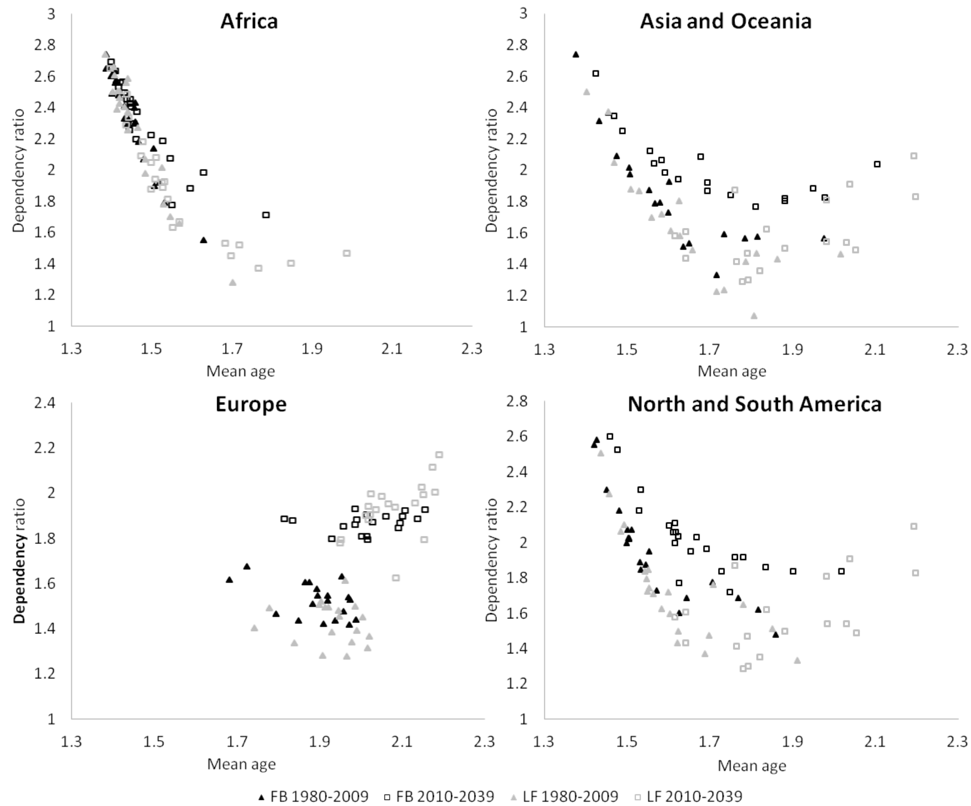

Figure 2: Observed and first-best age structures for FB_{02} in 1980-2009 and 2010-2039

Differences in the dependency ratio 1980-2009, pointed out in fig. 3, are exclusively affected by the number of offspring and hence the youth dependency ratio (YDR). The old age dependency ratio (ODR), determined by longevity and previous fertility, is historically given and hence identical. As the optimal initially exceeds observed fertility in over-aged populations, youth and thus overall dependency ratios are below the first-best level and vice versa in populations that are too young. In 2010-2039 interaction of deviating LF and FB fertility and longevity inserts. In previously over-aged populations the lower observed fertility in 1980-2009 and higher survival probability in 2010-2039 push the observed ODR above the optimal level. However, the effect is ambiguous in those developing countries that were featured by too young populations, as fertility and survival probability operate in opposite direction. Finally, the interaction with the YDR and hence fertility in 2010-2039 determines if the overall share of non-working individuals is above the optimal level. On average optimal dependency ratios are still above observed, e.g. in Asia. On the contrary, in Europe the share of working individuals is now below its optimal level. This switch holds on average across all populations and for the majority of countries. The observed number of dependent people is too low in 1980-2009 but too high in 2010-2039.

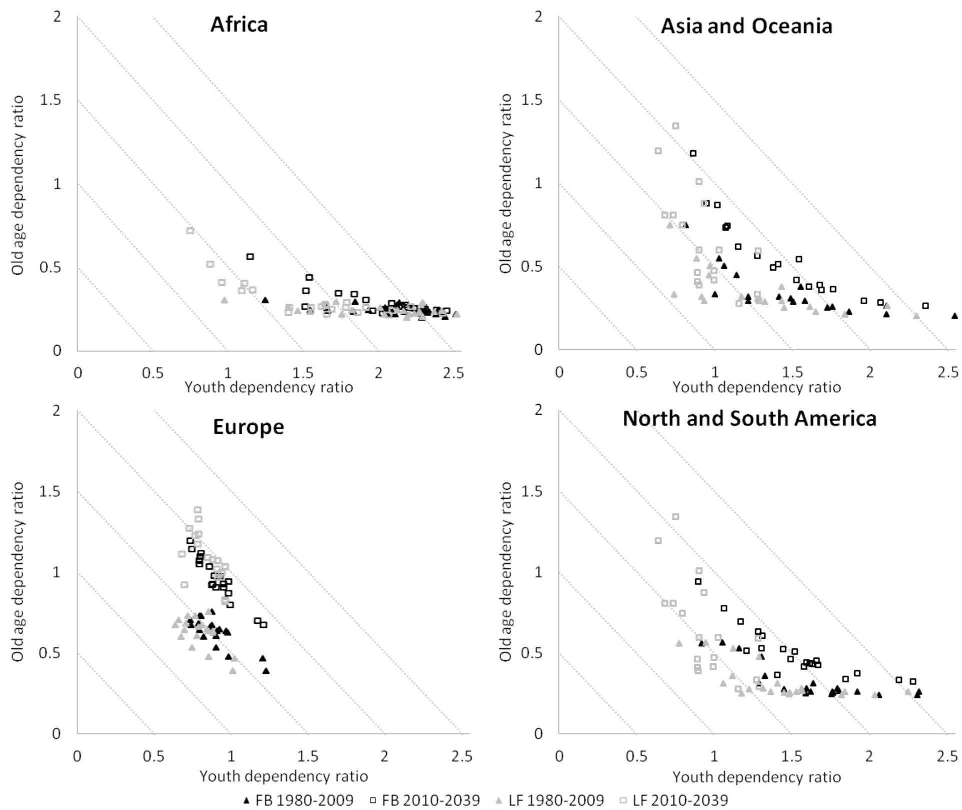

Figure 3: Dependency Ratios in 1980-2009 and 2010-2039

Given the initial situations, figure 4 presents paths of policies to implement optimal population developments. The upper left picture illustrates the non-monotones paths of child allowances related to the total child costs (TCC). With 1.8 % the average value is highest in 1980-2009 ($t = 0$), even if children are already taxed in many countries. Afterwards, the value of $\eta_{t,i}$ drops and taxation of children is observable in most economies. Thus, a higher optimal fertility goes along with a taxation of children, due to the interplay of policy instruments. Parents receive a high lump-sum transfer pushing fertility (lower right figure). As expected, health expenditures are taxed. On average taxation shrinks and converges to steady state values around 9 %. Finally, retiree pay and working individuals receive lump-sum transfers.

In line with the intuition, this first numerical exercise figured out the following major result: Supposing the baseline first-best scenario with $x = 0.02$ and a SDF of 0.3 % p.a., observed age structures in developed economies are indeed over-aged. On the contrary, some developing countries are too young. Additionally, the decentralization illustrates the paths of required policies over time. However, optimal age structures and hence policies are sensitive to social preferences. Taking into account developments into steady states the next subsection discusses the role of social preferences on paths of age structures.

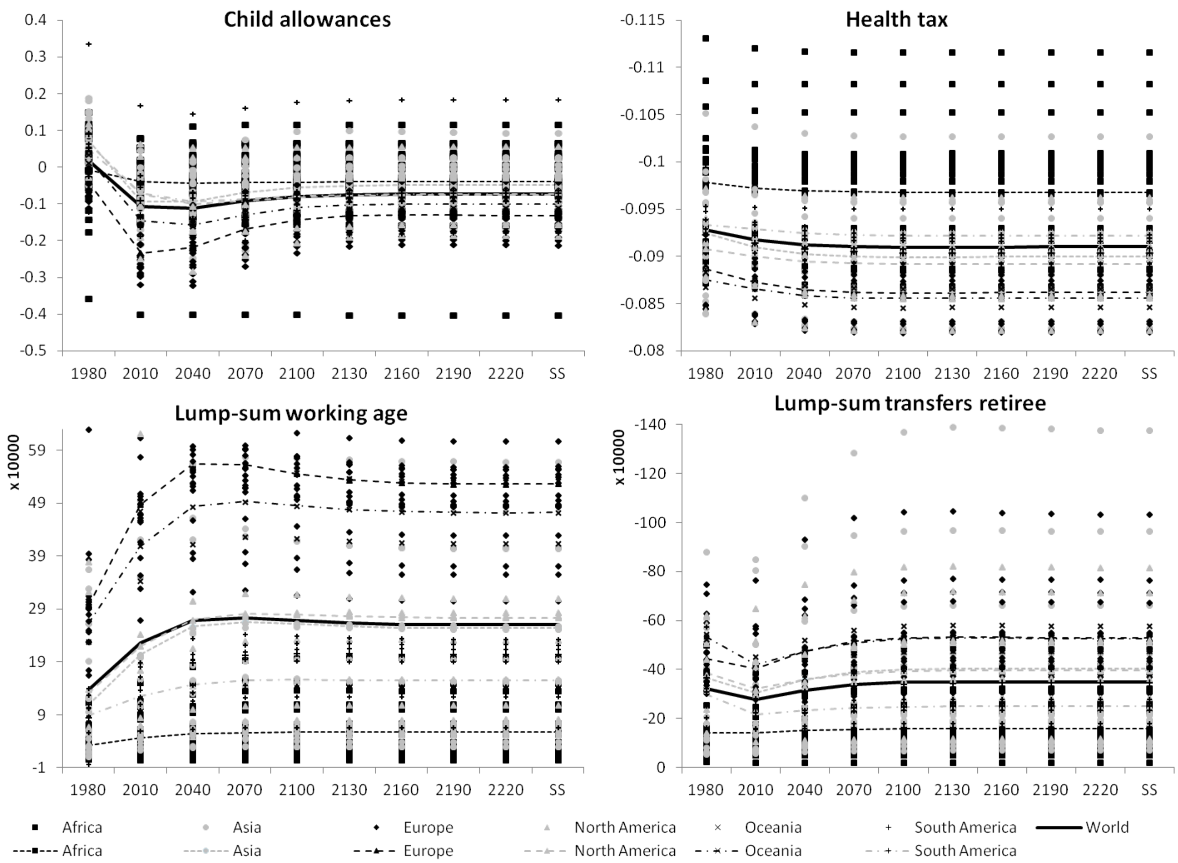

Figure 4: Policy instruments from 1980 to steady state to implement FB_{02} Scenario

5.2 Paths of age structures and policy instruments

To highlight the role of social preferences on paths of optimal age structures and policy instruments from 1980-2009 into steady states, this section focuses on different preferences for the population stock in a reasonable range between $x = 0.005$ and $x = 0.03$, labeled as subscript in FB_x. The former going along with relatively low and the latter with high optimal fertility. Figure 5 and 6 illustrate the impact on age structures and policy instruments for three examples. India serves as an example with relatively high observed fertility. The US represents countries with intermediate and Germany with low NRR.

Starting in 1980-2009 graphs illustrate evolution of laissez-faire (grey) and first-best (black) age structures over time in fig. 5. Moving to the upper right in plots at the top indicates population aging accompanied by a shrinking share of active individuals. A higher LF mean age and dependency ratio in 2010-2039 features the three examples. Afterwards, graphs turn to the lower left, mean age and dependency ratio decline into steady states, marked by circles with surface area. Magnitudes in changes of dependency ratios vary across examples, mainly affected by the interaction of YDR and ODR, shown in the lower part of fig. 5. Whereas YDR permanently grow, ODR only increase between 1980-2009 and 2010-2039. Afterwards shrinking ODR dominate in the examples and drop overall values.

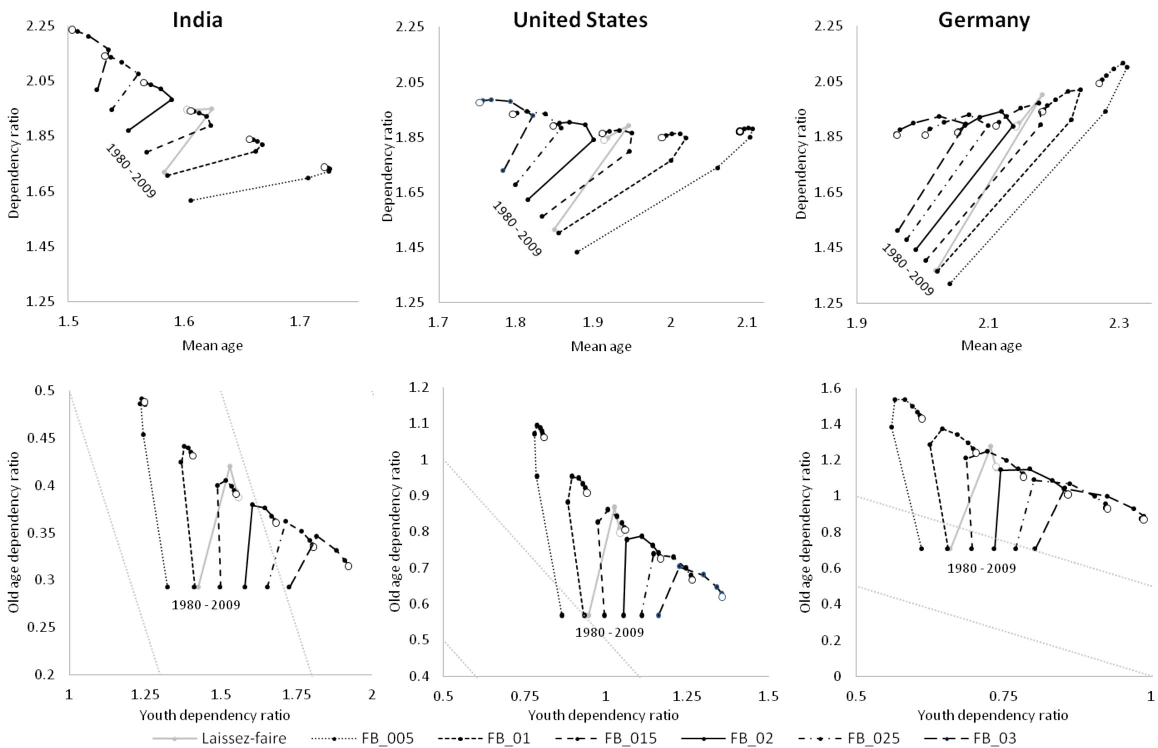

Figure 5: Age structures over time in selected countries

Like LF, FB populations undergo a process of non-monotonous aging. In all FB scenarios, mean ages initially increase. However, sooner or later graphs turn to the right and populations getting younger accompanied by country specific developments of FB dependency ratios. In India, the high fertility example, dependency ratios permanently grow, as the distance to the point of origin in the lower plot of figure 5 increases in all scenarios. On the contrary, in Germany, the low fertility example, developments trace walking sticks. Initially raising dependency ratios go along with aging populations. Once ODR start to shrink, they by and by dominate increasing YDR, overall values drop, accompanied by shrinking mean ages.

The relation between laissez-faire and first-best path of age structures crucially depends on the preference for the population stock, illustrated in fig. 5. A higher preference for the population stock shifts the initial values in 1980-2009 to the upper left and turns the graph counterclockwise. A sufficiently low x, leads to older optimal populations at all dates. In scenario FB_{015}, paths cross. Optimal populations are initially younger than in the laissez-faire economy and older in the future. Due to higher optimal fertility, an even higher preference for the population stock leads to younger FB populations on the whole path. Additionally, the higher fertility increases YDR. On the contrary, it doesn't alter initial ODR, such that starting points are shifted horizontally to the right, shown in the lower pictures of fig. 5. Furthermore, the stronger preference for population stock turns the path clockwise.

The preference for the population stock alters first-best age structures and hence policies to implement optimal paths. Figure 6 presents the non-monotonous paths of optimal health taxes and child allowances.[29] Initially η_i drops from a relatively high level and increases subsequently

[29] Appendix H contains lump-sum transfers in working and retirement age.

into its steady state afterwards, whereby a higher x shifts the graph upwards. Additionally, the level depends on country-specific characteristics. India starts with child subsidies in a range between -2.2 % and 2.1 %. Finally, they converge to -10.5 % respectively 6.9% of TCC. Children are taxed much stronger in Germany, initially with slightly more than 50 % of TCC. The spectrum increases over time. In steady states children additionally cost between 52 % and more than 100 % of TCC. Instruments are even more sensitive to social preferences in US as intermediate observed fertility example: η converges to values between -82 % ($x = 0.005$) and 13 % of TCC ($x = 0.03$).

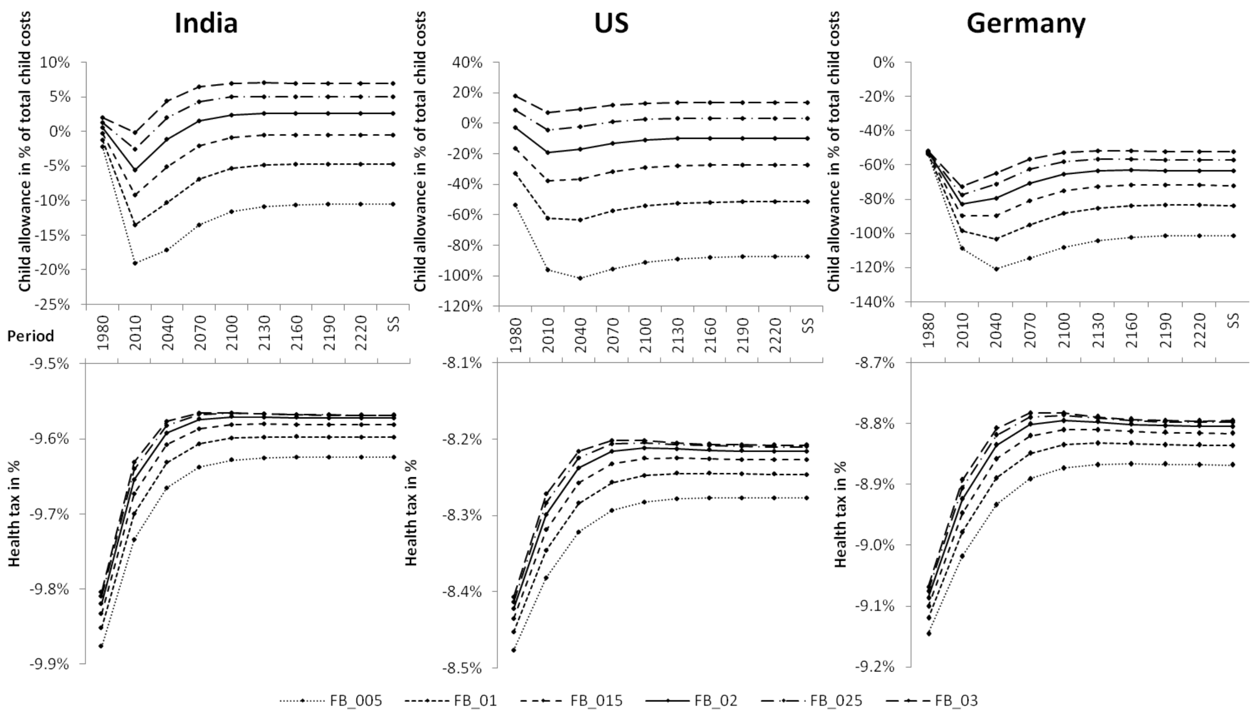

Figure 6: Child allowances (upper fig.) and health taxes (lower fig.) over time in selected counties

The optimal path on health taxes, plotted on the lower part of figure 6, is less sensitive to x. A higher x shifts graphs upwards and thus slightly reduces taxes. Still, health expenditures are always taxed, between 8.2 % and 9.9 % in the given examples, to avoid individual over-investments in health (see proposition 1). Across scenarios taxes initially shrink. After a minimum is reached, they increase and converge into steady state, visible e.g. in Germany and US for $x = 0.03$.

Thus, the second numerical exercise emphasizes that social preferences alter initial optimal age structures, their paths into steady state and required policies. A sufficiently high preference for population size leads to an over-aging as optimal mean ages shrink. The required preference is lower in developed countries. Furthermore, higher preferences for population size reduce taxes on health and children, respectively increases allowances, whereby paths are non-monotonous over time and the latter is more sensitive to social preferences.

6 Steady state implications

Focusing on the long-run and hence steady states additionally enables to confront laissez-faire with "Golden Age" age structures. The "Golden Age" scenario (GA) constitutes the point of reference for an optimum age structure, because of two reasons. First of all, this optimal solution for a Millian SWF with $\rho = 1$ avoids an arbitrary and hardly defensible discounting of future generations. Secondly, the "Golden Age" characterizes the most favorable solution among all optimal from an individual point of view, as it goes along with the highest expected life-cycle utility. However, neglecting population size in the SWF and a dominating capital dilution effect lead to very low "Golden Age" fertility. On average every birth cohort shrinks by one quarter and actually halves in the European area. Africa, the continent with the highest observable number of offspring, coincidences with the highest "Golden Age" fertility. Still, on average the NRR of 0.95 is slightly below replacement level. Thus, adding children to the SWF and considering their goods and time costs clearly avoid an overestimation of fertility, detectable in theories on optimal growth rates (Arthur and McNicoll, 1978).

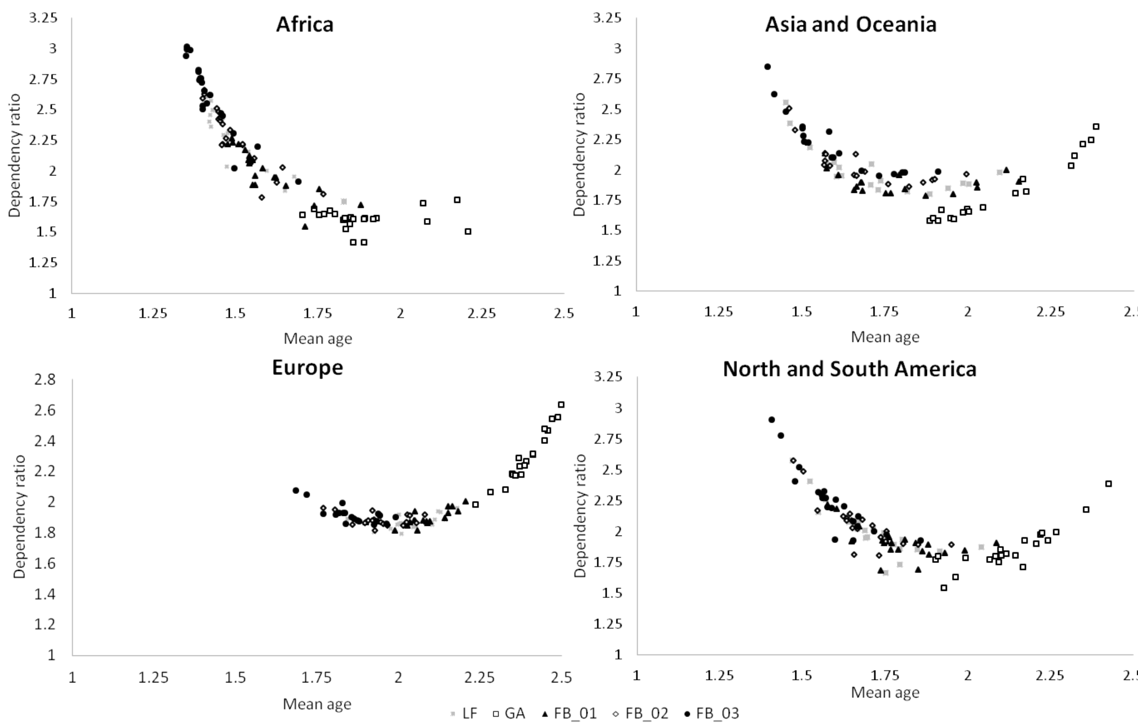

Figure 7: Laissez-faire and first-best age structures across countries

Even if the low "Golden Age" fertility seems to be striking from a demographic perspective, it is rather intuitive. The Millian planner only cares about utility of the representative individual, not about population size. Furthermore, accounting for the "modified golden rule" (eq. 15) a smaller fertility goes along with a lower marginal product of k^{FB} and hence c.p. a higher output per workforce. Finally, this implies more consumption possibilities and thereby utility for individuals. In addition, the higher output enables to invest more in health. Thus, even if taking into account the Philipson-Becker effect would suggest a lower survival probability than in the LF economy, the "Golden Age" exceeds the observed survival probability in all 80 countries.

Table 3:
AVERAGE LAISSEZ-FAIRE AND FIRST-BEST DEMOGRAPHIC VARIABLES IN STEADY STATES

Area	Fertility					Survival probability				
	LF	GA	FB_{01}	FB_{02}	FB_{03}	LF	GA	FB_{01}	FB_{02}	FB_{03}
Africa	2.031	0.953	1.628	2.002	2.327	0.583	0.597	0.591	0.589	0.587
Asia	1.487	0.780	1.298	1.582	1.823	0.721	0.732	0.728	0.726	0.724
Europe	0.890	0.499	0.838	1.020	1.171	0.857	0.882	0.872	0.868	0.864
North America	1.510	0.751	1.297	1.597	1.851	0.793	0.814	0.805	0.802	0.798
Oceania	0.994	0.557	0.929	1.131	1.298	0.874	0.896	0.887	0.883	0.880
South America	1.380	0.730	1.217	1.482	1.706	0.755	0.773	0.766	0.763	0.760
World	1.472	0.745	1.260	1.542	1.783	0.733	0.751	0.744	0.741	0.738

Table 3 and 4 confront "Golden Age" demographic variables and implied age structures with laissez-faire as well as 3 additional first-best scenarios (FB_x). Again, I assume a SDF of 0.3 % p.a. and different preferences for the population stock: a low ($x = 0.01$), intermediate ($x = 0.02$) and high weight for the population size ($x = 0.03$) in SWF. Optimal fertility increases and the cohort size already grows by around 80 % every period in the latter. However, due to lower outputs per capita, survival probabilities decrease compared to the "Golden Age" if x increases. Still, even the high fertility scenario goes along with an optimal longevity above observed in most countries. Fig. 7 additionally plots dependency ratios and mean ages, involved by optimal fertility and mortality, across countries.

Table 4:
AVERAGE OBSERVED AND OPTIMAL AGE STRUCTURES

Area	Mean age					Dependency ratio				
	LF	GA	FB_{01}	FB_{02}	FB_{03}	LF	GA	FB_{01}	FB_{02}	FB_{03}
Africa	1.490	1.886	1.589	1.493	1.431	2.334	1.609	2.005	2.307	2.589
Asia	1.693	2.067	1.761	1.652	1.579	2.023	1.784	1.898	2.074	2.250
Europe	2.031	2.390	2.075	1.946	1.857	1.870	2.293	1.895	1.884	1.920
North America	1.704	2.130	1.789	1.668	1.590	2.077	1.906	1.955	2.129	2.308
Oceania	1.960	2.332	2.009	1.880	1.792	1.875	2.166	1.885	1.912	1.977
South America	1.723	2.122	1.799	1.683	1.605	1.938	1.818	1.857	2.004	2.158
World	1.729	2.116	1.803	1.690	1.615	2.065	1.887	1.929	2.088	2.255

Fig. 8 illustrates the latter distinguished by youth and old age DR. Both, the very low NRR and high survival probability, lead to a "Golden Age" mean age above those in LF steady states, plotted to the right of the LF scenario in fig. 7. Hence, LF populations are far too young compared to the "Golden Age" situation, as point of reference. The increasing optimal fertility combined with a reduced survival probability decreases first-best mean ages, if future generation's utility is discounted and/or the SWF considers population size.

Whether the "Golden Age" exceeds the LF share of non-working individuals or not, in particular depends on LF fertility. All countries go along with a higher old-age and a lower youth first-best than laissez-faire dependency ratio (figure 8). In developed countries, like European ones, the reduced "Golden Age" YDR is dominated by the higher ODR. On the contrary, situation is vice versa in Africa, displayed in the upper left of fig. 8. "Golden Age" age structures are plotted closer to the point of origin and thus have smaller "Golden Age" dependency ratios.

Introducing the SDF and preference for the population stock ($x > 0$) boosts optimal fertility and reduces the survival probability. The optimal mean age and ODR shrink, whereas the YDR increases. First-best age structures move to the lower right, following a convex curve, visualized in figure 8. In developed countries, like the European, the decreasing ODR initially dominates the growing YDR. The distance to the point of origin shrinks and so does the dependency ratio. As soon as they balance each other, the minimal overall dependency ratio is obtained. A further increasing SDF and/or weight of population size in the SWF leads to a growing overall share of non-active individuals. Thus, the optimal DR follows the same U-shaped pattern in the decreasing mean age detectable in observed data of figure 1, if the SDF and/or weight of the population size increases, illustrated by the four first-best scenarios in figure 7. In developing countries, characterized by "Golden Age" dependency ratios closed to the minimum or already to its left in fig. 7, the share of the working population always shrinks if the preference of the population stock increases.

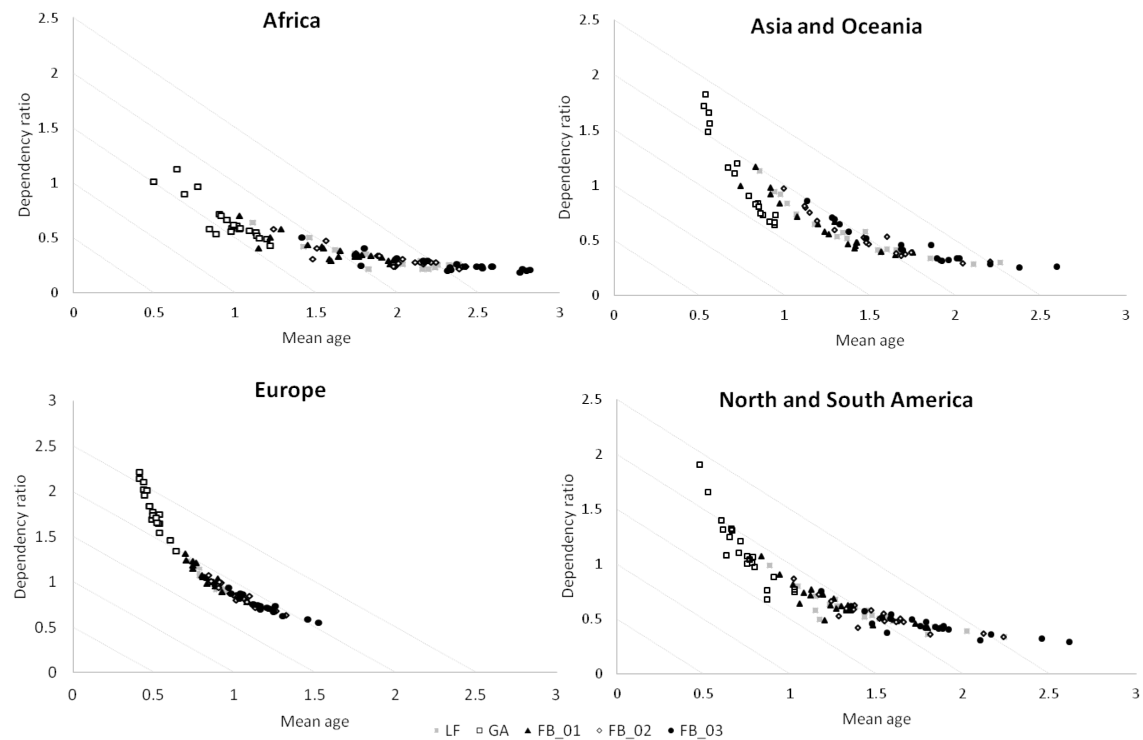

Figure 8: Laissez-faire and first-best dependency ratios across countries

The "Golden Age" goes along with the highest feasible expected life-cycle utility for the represented individuals. To achieve this most favorable optimal age structure from an individual point of view, which is beyond doubt problematic from a demographic perspective, requires extensive policy measures. Their signs are identical across countries, only magnitudes differ. As expected, health expenditures are taxed to account for the Philipson-Becker effect between 8.5 to 12 %. Children are taxed too, due to a capital dilution effect that dominates the intergenerational transfer effect in all economies. Additionally to the good cost a, a child costs 637.9 K \$ (185.1 % of TCC) on average, certainly with a large variation between 9.03 K \$ (102.8%) in Mozambique and 3.32 million \$ (241%) in Singapore.[30] The working generation obtains a lump-sum transfer to increase

[30]The child allowance in % of TCC is highest in Sierra Leone (-388.4 %) and lowest in Guyana (-61.3 %).

the capital-labor ratio and the retired generation pays a transfer to close the budget constraint in all 80 countries. In scenario FB_{01} the extents of instruments decrease. The higher preference for the population stocks in FB_{02} starts to change taxation of children in subsidies in some countries. Additionally to higher child allowances further raising x to FB_{03} shrinks lump-sum transfer in middle age and even alters its sign in Guyana. Hence, assumptions concerning the SWF determine directions and magnitudes of policy instruments, necessary to implement the varying optimal age structures.

Table 5:
Average child allowances and health taxes in steady state

	Health Tax in %				Child allowance in % of TCC			
Area	GA	FB_{01}	FB_{02}	FB_{03}	GA	FB_{01}	FB_{02}	FB_{03}
Africa	-10.0	-9.7	-9.7	-9.7	-167.0	-38.3	-4.0	16.8
Asia	-9.2	-9.0	-9.0	-9.0	-171.0	-40.3	-4.9	16.1
Europe	-8.8	-8.6	-8.6	-8.6	-214.9	-53.7	-13.3	10.2
North America	-9.2	-8.9	-8.9	-8.9	-191.9	-45.3	-7.5	14.7
Oceania	-8.8	-8.6	-8.6	-8.6	-199.7	-49.0	-10.1	12.6
South America	-9.5	-9.2	-9.2	-9.2	-180.2	-43.4	-7.3	14.2
World	-9.3	-9.1	-9.1	-9.1	-185.1	-44.3	-7.4	14.4

The last numerical exercise emphasized the role of social preferences in steady states. Starting from "Golden Age" a higher preference for the population stock reduces first-best mean ages. Fertility is higher but on the costs of a lower individual income and longevity. Combined with arbitrary fixed social preferences, this implies the following dilemma: The "Golden Age" is desirable from an economic and philosophic perspective. But very fast shrinking long-run populations are a questionable demographic aspect. Social discounting and a preference on the population size avoid this issue, but are problematic from an ethical point of view. Additionally, successively increasing x points out that economies with a population, that is old in the presence, are already over-aged for lower preferences on the population stock in steady states, e.g. Germany or Japan.

7 Conclusion

A simple OLG-model with endogenous fertility and longevity offers an instrument to discuss an over-aging of populations. A laissez-faire solution replicates observed age structures in 1980-2009 and enables to compute paths into steady states. Adding a first-best economy, with number-dampened total utilitarianism as objective for optimality, enables us to evaluate a possible over-aging of initial observed age structures as well as those on the path into and in steady state. Several effects not internalizes in individual decisions on demographic variables induce sub-optimal solutions in laissez-faire economies. The calibration exercise points out that scale and direction of the discrepancy in age structures is sensitive to the definition of social preferences.

"Golden Age" populations are always older than LF ones and featured by a shrinking number of very happy individuals in almost every economy. Introducing a social discount factor combined with a successive raise in the preference for the population stock increases optimal fertility and lowers the survival probability as well as expected life-cycle utility. Optimal mean ages and dependency ratios trace the U-shaped pattern detectable across observed age structures in fig. 1. A sufficient weighting of the population size in SWF leads to an over-aging of laissez-faire populations in steady states. Going along with the intuition, such an over-aging is more likely in developed economies with old populations. Introducing SDF and x additionally permits to investigate dynamics. Once again the arbitrary fixed parameters determine whether populations in 1980-2009 as well as on the paths are over-aged. Mean ages of most populations raise first and shrink afterwards into steady state. Still, final exceed initial values in most economies.

Four policies implement optimal age structures. Health expenditures are always taxed, across scenarios and over time. On the contrary, the sign of the child allowances and the lump-sum transfers in working as well as retirement age are sensitive to the SDF (ρ) and the preference for the population stock (x). Furthermore, the non-monotonous path leads to changes in signs over time for some parameter combinations. Generally, a positive child allowance is more likely the higher the latter. However, the discounting of future generation's utility is problematic from an ethical point of view. Furthermore, both parameters, ρ and x, are fixed arbitrary and enable first-best solutions with lower life-cycle utilities than in the laissez-faire economy so that the following dilemma arises:

At least one aspect of the different optimal solutions is problematic. Shrinking, very old populations in "Golden Age" are striking from a demographic point of view but favorable from an ethical and economic perspective. In contrast, first-best solutions with younger age structures are demographically less problematic, but are less advantageous for individuals and imply ethical problems, due to an arbitrary discounting of future generation's utility and preference for the population stock. Furthermore, fertility generally differs from replacement level, questionable in the long run: Populations either extinct or excess. To ensure a constant long run population different options exist, e.g. either to fix parameters in SWF country specific to ensure a constant population or to relate costs of fertility to population size. For the latter fertility endogenously converges to the replacement level. However, in this case optimal and laissez-faire age structures are exclusively driven by longevity in the long-run, whereas fertility differentials are important here. Secondly, the latter requires an arbitrarily fixed "optimal population size" to ensure $n^{FB} = 1$.

Finally, due to assumptions and simplifications, findings should be interpreted carefully. First of all, age structures are aggregated to three generations. An uncertain child survival probability and age-dependent heterogeneity within age groups are not considered, e.g. fast growing health expenditures in old age. Beside demographic aspects, economic assumptions limit results. Estimated parameters might be biased, in consequence of country-specific policies and social security systems, neglected in the model and hence in the calibration exercise, too. Diversity of social security systems across countries would require totally different models to consider them adequately. Such country specific models are left for feature research.

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Appendix

A Alternative framework with Lump-sum transfers

Neglecting any kind of bequest motive, different options to deal with savings of deceased individuals exist. In the baseline framework they are redistributed by an annuity of life, supposing perfect annuity markets. Assuming alternatively lump-sum transfers to working or retired generations enables a discussion on the role of perfect annuity markets. Adjusted budget constraints are

$$(1-\tau n_t^{\mathrm{LF}})w_t^{\mathrm{LF}} = c_t^{\mathrm{LF}} + an_t^{\mathrm{LF}} + s_t^{\mathrm{LF}} + h_t^{\mathrm{LF}} + z_t^M,$$

$$d_{t+1}^{\mathrm{LF}} = R_{t+1}^{\mathrm{LF}} s_t^{\mathrm{LF}} + z_{t+1}^O.$$

Either the working generation $z_t^M > 0$ or the retirees $z_t^O > 0$ receive the lump-sum transfer. The corresponding set of FOC is:

$$u'(c_t^{\mathrm{LF}}) = \beta R_{t+1}^{\mathrm{LF}} q(h_t^{\mathrm{LF}}) u'(d_{t+1}^{\mathrm{LF}}), \tag{29}$$

$$u'(c_t^{\mathrm{LF}}) = \frac{\gamma}{a+\tau w_t^{\mathrm{LF}}} v'(n_t^{\mathrm{LF}}), \tag{30}$$

$$u'(c_t^{\mathrm{LF}}) = \beta q'(h_t^{\mathrm{LF}}) u(d_{t+1}^{\mathrm{LF}}). \tag{31}$$

Changing the assumption on the redistribution of deceased individuals' savings doesn't alter the FOC on fertility (30) and health expenditures (31). Only the FOC on savings (29) changes and now additionally differs from the Euler equation of the Social planner. Hence, all the three FOC deviate from the first-best solution and require three pigouvian taxes. Additionally, two (already existing) lump-sum transfers implement the first-best solution. Again, the governments budget constraint has to be balanced every period:

$$(\eta_t n_t + \phi_t h_t + z_t^m + \psi_t s_t)\, N_t^{t-1} + z_t^o q_t N_{t-1}^{t-2} = (1-q_t)\, N_{t-1}^{t-2} R_t s_{t-1},$$

whereby savings in working age are taxed by ψ.[31] In terms per individual in middle age the government budget writes:

$$\eta_t n_t + \phi_t h_t + \psi_t s_t + z_t^m + \frac{q(h_{t-1}) z_t^o}{n_{t-1}} = \frac{1-q_t}{n_{t-1}} R_t s_{t-1}.$$

Budget constraints of individuals adjust to:

$$c_t = (1-\tau n_t) w_t - (a-\eta_t) n_t - (1-\psi_t) s_t - (1-\phi_t) h_t + z_t^m,$$

$$d_{t+1} = \tilde{R}_{t+1} s_t + z_{t+1}^o.$$

[31] Alternatively, it is possible to implement the pigouvian tax on returns on savings. In this case only those who are "lucky" and survive receive a subsidy or have to pay a tax.

Solving the individual maximization and choosing policy instruments to decentralize the optimal solution s.t. $\left\{c_t^{\mathrm{FB}}, d_t^{\mathrm{FB}}, h_t^{\mathrm{FB}}, k_t^{\mathrm{FB}}, n_t^{\mathrm{FB}}\right\}_{t=0}^{t=+\infty} = \{c_t, d_t, h_t, k_t, n_t\}_{t=0}^{t=+\infty}$ figures out, that propositions 1 and 2 still hold. Additionally, savings are always subsidized according to:

$$\psi_t = 1 - q(h_t). \tag{32}$$

As $q < 1$ a subsidy on savings is always paid. On the contrary, the lump-sum transfer in working age to correct the capital labor ratio is:

$$z_t^m = (1 - \tau n_t)\, f'(k_t)k_t - \psi\,(1 - \tau n_{t+1})\, n_t k_{t+1} - \eta n_t - \phi_t h_t - \frac{q(h_{t-1})d_t}{n_{t-1}},$$

and those to balance the budget constraint adjusts to:

$$z_t^o = \frac{1}{q(h_{t-1})}\left(\left(1 - q(h_{t-1})\right) R_t s_{t-1} - n_{t-1}\left(\eta_t n_t + \phi_t h_t + \psi_t s_t + z_t^m\right)\right).$$

B Hessian-matrix of the laissez-faire economy

The FOCs describe a maximum if the following 3x3 Hessian-matrix is negative definite:

$$H \equiv \begin{pmatrix} \frac{\partial^2 EU_t}{\partial s_t^2} & \frac{\partial^2 EU_t}{\partial s_t \partial n_t} & \frac{\partial^2 EU_t}{\partial s_t \partial h_t} \\ \frac{\partial^2 EU_t}{\partial n_t \partial s_t} & \frac{\partial^2 EU_t}{\partial n_t^2} & \frac{\partial^2 EU_t}{\partial n_t \partial h_t} \\ \frac{\partial^2 EU_t}{\partial h_t \partial s_t} & \frac{\partial^2 EU_t}{\partial h_t \partial n_t} & \frac{\partial^2 EU_t}{\partial h_t^2} \end{pmatrix}$$

Following Young's theorem, the Hessian is symmetric. The 6 necessary partial derivations of the matrix H are:

$$\frac{\partial^2 EU_t}{\partial s_t^2} = u''(c_t) + \beta \tilde{R}_{t+1}^2 q(h_t) u''(d_{t+1})$$

$$\frac{\partial^2 EU_t}{\partial s_t \partial n_t} = (\tau w_t + a)\, u''(c_t)$$

$$\frac{\partial^2 EU_t}{\partial s_t \partial h_t} = u''(c_t) + \beta q'(h_t) \tilde{R}_{t+1} u'(d_{t+1})$$

$$\frac{\partial^2 EU_t}{\partial n_t^2} = \gamma v''(n_t) + (\tau w_t + a)^2\, u''(c_t)$$

$$\frac{\partial^2 EU_t}{\partial n_t \partial h_t} = (a + \tau w_t)\, u''(c_t)$$

$$\frac{\partial^2 EU_t}{\partial h_t^2} = u''(c_t) + \beta q''(h_t) u(d_{t+1})$$

To determine the SOC the Eigenvalues and Sylvester's Criterion are computed by the matrix H in the calibration exercise.

C Hessian-matrix of the "Golden Age" solution

To verify that FOC describe a maximum in the Golden age situation, the following Hessian-matrix is computed for the static case of $\rho = 1$:

$$H \equiv \begin{pmatrix} \frac{\partial^2 W}{\partial d^2} & \frac{\partial^2 W}{\partial d \partial k} & \frac{\partial^2 W}{\partial d \partial n} & \frac{\partial^2 W}{\partial d \partial h} \\ \frac{\partial^2 W}{\partial k \partial d} & \frac{\partial^2 W}{\partial k^2} & \frac{\partial^2 W}{\partial k \partial n} & \frac{\partial^2 W}{\partial k \partial h} \\ \frac{\partial^2 W}{\partial n \partial d} & \frac{\partial^2 W}{\partial n \partial k} & \frac{\partial^2 W}{\partial n^2} & \frac{\partial^2 W}{\partial n \partial h} \\ \frac{\partial^2 W}{\partial h \partial d} & \frac{\partial^2 W}{\partial h \partial k} & \frac{\partial^2 W}{\partial h \partial n} & \frac{\partial^2 W}{\partial h^2} \end{pmatrix}$$

The partial derivations of the matrix H are:

$$\frac{\partial^2 W}{\partial d^2} = \left(\frac{q\left(h\right)}{n}\right)^2 u''(c) + \beta q\left(h\right) u''(d)$$

$$\frac{\partial^2 W}{\partial d \partial k} = -\left(f'\left(k\right) - n\right) \frac{q\left(h\right)}{n} \left(1 - \tau n\right) u''\left(c\right) = 0$$

$$\frac{\partial^2 W}{\partial d \partial n} = \frac{q\left(h\right)}{n} \left(\tau\left(f\left(k\right) - nk\right) + a + \left(1 - \tau n\right) k - \frac{q\left(h\right) d}{n^2}\right) u''(c_t) + \frac{q\left(h\right)}{n^2} u''\left(c\right)$$

$$\frac{\partial^2 W}{\partial d \partial h} = \frac{q\left(h\right)}{n} \left(1 + \frac{q'\left(h\right) d}{n}\right) u''\left(c\right)$$

$$\frac{\partial^2 W}{\partial k^2} = \left(1 - \tau n\right) f''\left(k\right) u'\left(c\right)$$

$$\frac{\partial^2 W}{\partial k \partial n} = -\left(1 - \tau n\right) u'\left(c\right)$$

$$\frac{\partial^2 W}{\partial k \partial h} = \left(1 - \tau n\right) \left(n - f'\left(k\right)\right) \left(1 + \frac{q'\left(h\right)}{n}\right) u''\left(c\right) = 0$$

$$\frac{\partial^2 W}{\partial n^2} = \left(a + \tau\left(f\left(k\right) - nk\right) + \left(1 - \tau n\right) k - \frac{q\left(h\right) d}{n^2}\right)^2 u''\left(c\right) + \gamma u''\left(n\right) + 2\left(\tau k - \frac{q\left(h\right) d}{n^3}\right)^2 u'\left(c\right)$$

$$\frac{\partial^2 W}{\partial n \partial h} = \frac{q'\left(h\right) d}{n^2} u'\left(c\right) + \left(a + \tau\left(f\left(k\right) - nk\right) + \left(1 - \tau n\right) k - \frac{q\left(h\right) d}{n^2}\right) \left(1 + \frac{q'\left(h\right) d}{n}\right) u''(c_t)$$

$$\frac{\partial^2 W}{\partial h^2} = q''\left(h\right) \left(\beta u(d) - \frac{d}{n} u'\left(c\right)\right) + \left(1 + \frac{q'\left(h\right) d}{n}\right)^2 u''\left(c\right)$$

D Hessian-matrix of the first-best solution

The Hessian matrix, to verify that FOC in the first-best solution describe a maximum, is:

$$H \equiv \begin{pmatrix} \frac{\partial^2 W}{\partial d_t^2} & \frac{\partial^2 W}{\partial d_t \partial k_{t+1}} & \frac{\partial^2 W}{\partial d_t \partial n_t} & \frac{\partial^2 W}{\partial d_t \partial h_t} \\ \frac{\partial^2 W}{\partial k_{t+1} \partial d_t} & \frac{\partial^2 W}{\partial k_{t+1}^2} & \frac{\partial^2 W}{\partial k_{t+1} \partial n_t} & \frac{\partial^2 W}{\partial k_{t+1} \partial h_t} \\ \frac{\partial^2 W}{\partial n_t \partial d_t} & \frac{\partial^2 W}{\partial n_t \partial k_t} & \frac{\partial^2 W}{\partial n_t^2} & \frac{\partial^2 W}{\partial n_t \partial h_t} \\ \frac{\partial^2 W}{\partial h_t \partial d_t} & \frac{\partial^2 W}{\partial h_t \partial k_t} & \frac{\partial^2 W}{\partial h_t \partial n_t} & \frac{\partial^2 W}{\partial h_t^2} \end{pmatrix}$$

The partial derivations of the matrix H are:

$$\frac{\partial^2 W}{\partial d_t^2} = \left(\frac{q(h_t)}{n_{t-1}}\right)^2 u''(c_t) + \frac{\beta}{\rho n_{t-1}^x} q(h_t) u''(d_t)$$

$$\frac{\partial^2 W}{\partial d_t \partial k_{t+1}} = \frac{q(h_t)}{n_{t-1}} \left((1 - \tau n_{t+1}) n_t\right) u''(c)$$

$$\frac{\partial^2 W}{\partial d_t \partial n_t} = \frac{q(h_{t-1})}{n_{t-1}} \left(\tau f(k_t) + a + (1 - \tau n_{t+1}) k_{t+1}\right) u''(c_t)$$

$$\frac{\partial^2 W}{\partial d_t \partial h_t} = \frac{q(h_{t-1})}{n_{t-1}} u''(c_t)$$

$$\frac{\partial^2 W}{\partial k_{t+1}^2} = (1 - \tau n_{t+1}) \left(\rho n_t^x f''(k_{t+1}) u'(c_{t+1}) + (1 - \tau n_{t+1}) \left(f'(k_{t+1})^2 u''(c_{t+1}) \rho n_t^x + n_t^2 u''(c_t)\right)\right)$$

$$\begin{aligned}\frac{\partial^2 W}{\partial k_{t+1} \partial n_t} = (1 - \tau n_{t+1}) \Bigg(&\rho n_t^x f'(k_{t+1}) \left(\frac{x}{n_t} u'(c_{t+1}) + \frac{q(h_t) d_{t+1}}{n_t^2} u''(c_{t+1})\right) - u'(c_t) \\ &+ \left(\tau f(k_t) + a + (1 - \tau n_{t+1}) k_{t+1}\right) n_t u''(c_t)\Bigg)\end{aligned}$$

$$\frac{\partial^2 W}{\partial k_{t+1} \partial h_t} = (1 - \tau n_{t+1}) \left(n_t u''(c_t) - \rho \frac{q'(h_t) d_{t+1}}{n_t^{1-x}} f'(k_{t+1}) u''(c_{t+1})\right)$$

$$\begin{aligned}\frac{\partial^2 W}{\partial n_t^2} = &\left(\tau f(k_t) + a + (1 - \tau n_{t+1}) k_{t+1}\right)^2 u''(c_t) + \gamma u''(n_t) + \rho x (x-1) n_t^{x-2} EU_t \\ &+ \rho x n_t^{x-1} \frac{q(h_t) d_{t+1}}{n_t^3} u'(c_{t+1}) - \rho (2 - x) \frac{q(h_t) d_{t+1}}{n_t^{3-x}} u'(c_{t+1}) + \\ &\rho \left(\frac{q(h_t) d_{t+1}}{n_t^3}\right)^2 n_t^x u''(c_{t+1}) + \frac{1}{\rho} \tau^2 k_t^2 n_{t-1}^{2-x} u''(c_{t-1})\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 W}{\partial n_t \partial h_t} = &\rho (1 - x) \frac{q'(h_t) d_{t+1}}{n_t^{2-x}} u'(c_{t+1}) - \rho \frac{q'(h_t) q(h_t) d_{t+1}^2}{n_t^{3-x}} u''(c_{t+1}) \\ &+ \left(\tau \left(f(k_t) + a + (1 - \tau n_{t+1}) k_{t+1}\right)\right) u''(c_t)\end{aligned}$$

$$\frac{\partial^2 W}{\partial h_t^2} = \beta q''(h_t) u(d_{t+1}) - \rho \frac{d_{t+1}}{n_t^{1-x}} \left(q''(h_t) u'(c_{t+1}) - \frac{q'(h_t)^2 d_{t+1}}{n_t} u''(c_{t+1})\right) + u''(c_t)$$

E Calculation of the health expenditures

The total health expenditures per capita in PPP $ are the weighted sum over sexes and age groups:

$$H_{i,\sum} = \sum_{\text{age group}} \sum_{\text{sex}} x_{i,\text{age group, sex}} h_{\text{i,age group, sex}}$$

with $h_{i,\text{age group, sex}}$, the age and sex specific health expenditures and $x_{i,\text{ age group, sex}}$, the share on total population in country i given by the United Nations (2013). Supposed $\frac{h_j^{\text{Germany}}}{h_0^{\text{Germany}}}$, the relation of health expenditures between age groups and sexes in Germany, holds across countries, rearranging allows to express the male health expenditure in the youngest age group for each country:

$$h_{i,\text{0-14, male}} = \frac{H_{\sum}}{\sum_{j=1}^{J=12} x_j \frac{h_j^{\text{Germany}}}{h_1^{\text{Germany}}}}$$

with j as age-groups distinguished by sexes, while $j = 1$ are men 0-14. By means of $h_{i,\text{0-14, male}}$ it is possible to approximate total health expenditures of females between age 30 and 59 for the 80 countries:

$$h_i = 15 * h_{i,\text{ 0-14, male}} * \left(\frac{h_{30-44,\text{ female}}^{Germany}}{h_{0-14,\text{ male}}^{Germany}} + \frac{h_{30-64,\text{ female}}^{Germany}}{h_{0-14,\text{ male}}^{Germany}} \right).$$

F Summary statistics

Table 6:
Summary statistics of economic and demographic variables

Variable	Period	Mean	Std. dev.	Min	Max
GDP in bn $	0	13454	35278	49	279565
Health exp. in K $	0	29.8	32.6	0.9	157.5
Working age population in K	0	26744	79955	143	585725
	1	47251	141807	213	999569
Capital stock in bn $	0	753.0	2022.5	3.1	15924.3
	1	2030.2	5359.3	5.4	38535.8
Survival prob.	0	0.625	0.088	0.401	0.767
	1	0.730	0.124	0.418	0.959
NRR	-1	1.878	0.541	0.941	2.945
	0	1.414	0.562	0.639	2.522
	1	1.115	0.397	0.639	2.427

G Observations and values predicted by the model

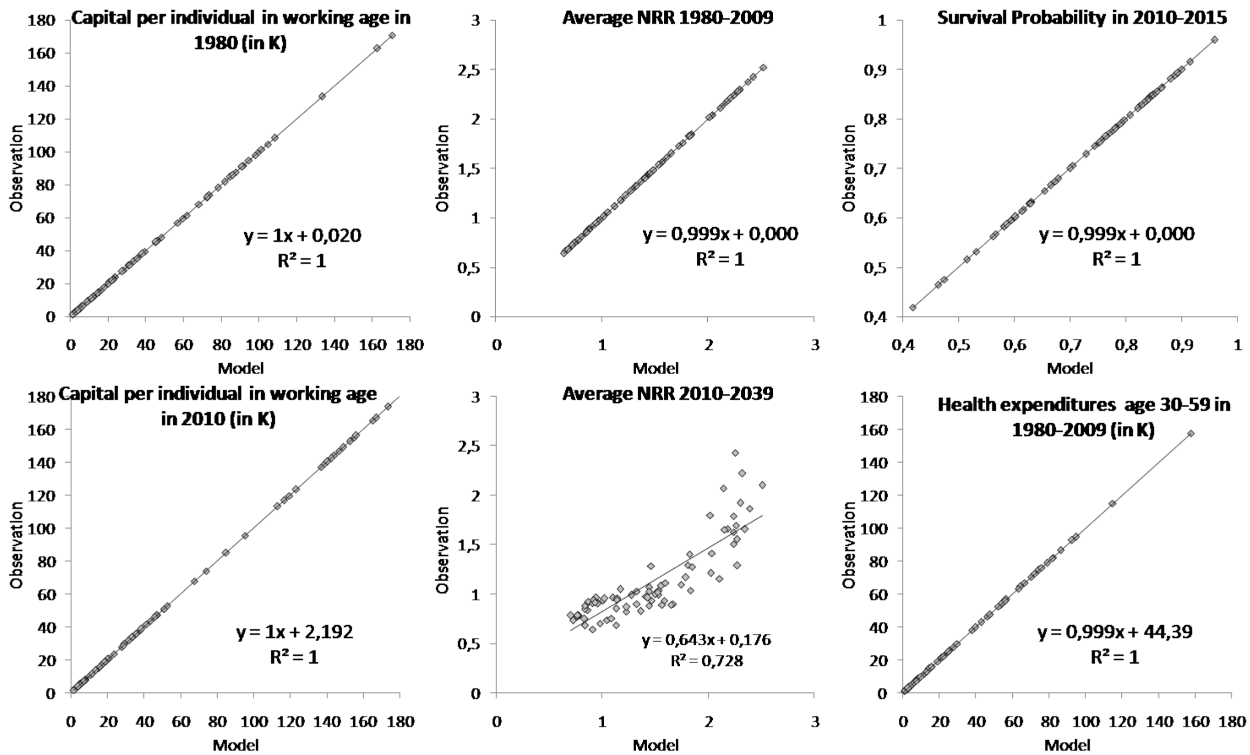

Figure 9: Observed values and those predicted by the model

H Lump-sum transfers in India, US and Germany

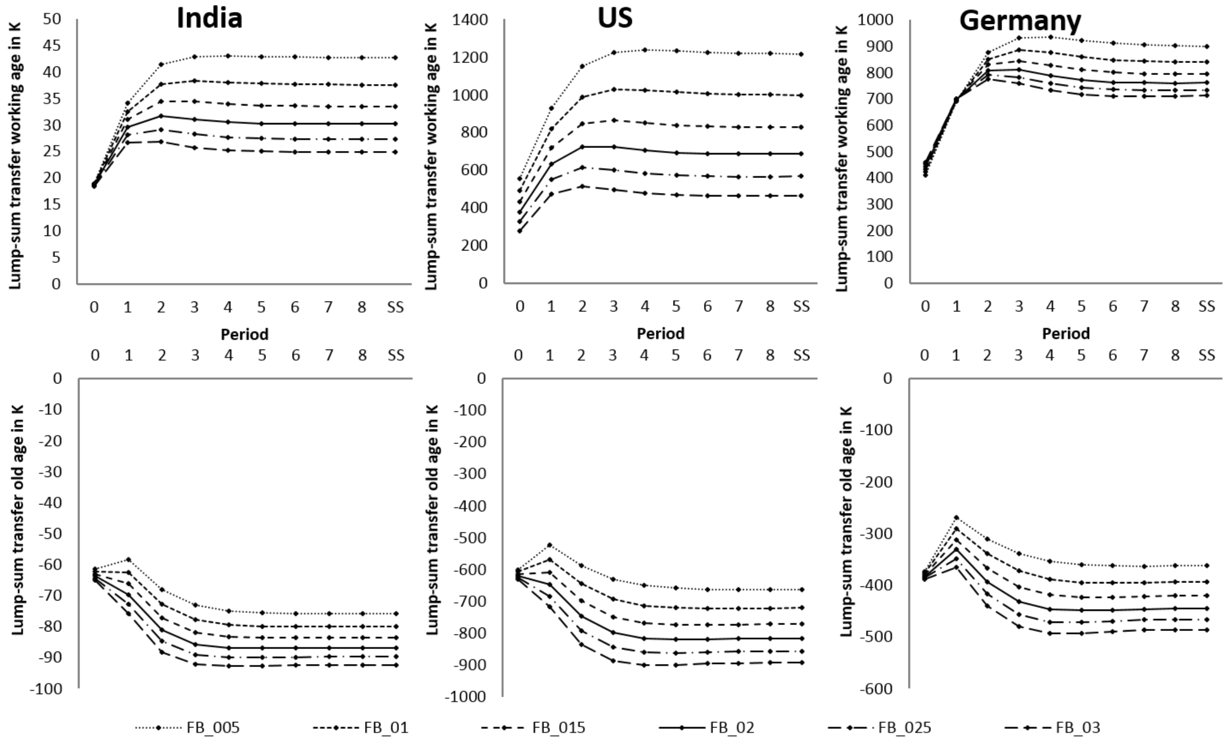

Figure 10: Transition of lump-sum transfers in steady state

I Results for selected countries

Starting with steady state solutions, this section presents numerical results for selected countries. Table 7 presents parameters and age structures. Required policies to decentralize first-best age structures are in table 8. Tab. 9 proofs that solutions are maxima by Sylvester's criterion.[32] Afterwards, tab. 10 documents age structures and policy instruments for the two periods 1980-2009 and 2010-2039. Finally, tab. 11 and 12 illustrate paths of age structures and policy instruments into steady state for the three examples of section 5.2.

[32] H_i indicates the upper left i by i matrix of H.

Table 7:
PARAMETERS AND STEADY STATE AGE STRUCTURES IN SELECTED COUNTRIES

Country	Parameter					Mean age					Dependency ratio				
	A_i	a_i	β_i	δ_i	σ_i	LF	GA	FB_{01}	FB_{02}	FB_{03}	LF	GA	FB_{01}	FB_{02}	FB_{03}
Argentina	27125	40114	0.25	0.08	0.97	1.78	2.21	1.86	1.73	1.65	1.89	1.90	1.84	1.95	2.08
Australia	52985	215737	0.25	0.10	1.11	1.98	2.34	2.03	1.90	1.81	1.90	2.21	1.91	1.93	1.98
Brazil	27109	44721	0.21	0.08	1.02	1.79	2.24	1.88	1.75	1.67	1.86	1.93	1.81	1.92	2.04
Canada	57175	282207	0.20	0.11	1.74	2.04	2.42	2.08	1.95	1.85	1.87	2.38	1.91	1.89	1.93
Chile	32258	52782	0.24	0.09	0.93	1.80	2.22	1.88	1.75	1.67	1.94	1.99	1.90	2.00	2.13
China	16297	17342	0.24	0.09	0.93	1.73	2.00	1.75	1.65	1.59	1.84	1.68	1.81	1.96	2.10
Egypt	25259	6527	0.33	0.04	0.86	1.54	1.93	1.62	1.52	1.46	2.16	1.62	1.95	2.23	2.48
France	47714	186745	0.24	0.13	1.87	2.00	2.37	2.05	1.92	1.83	1.92	2.29	1.94	1.94	1.99
Germany	43751	279839	0.23	0.11	1.70	2.14	2.49	2.18	2.05	1.96	1.89	2.56	1.94	1.87	1.86
Ghana	9963	-223	0.38	0.03	1.18	1.46	1.83	1.56	1.47	1.41	2.30	1.53	1.97	2.27	2.55
Guyana	6665	6053	0.89	0.03	0.99	1.75	1.92	1.73	1.65	1.60	1.67	1.55	1.69	1.81	1.93
India	11434	5325	0.47	0.03	0.88	1.60	1.91	1.66	1.56	1.50	1.95	1.58	1.84	2.04	2.24
Indonesia	17892	12289	0.36	0.02	0.68	1.62	1.96	1.68	1.58	1.52	1.95	1.60	1.83	2.04	2.23
Israel	64032	85011	0.21	0.08	0.73	1.71	2.16	1.79	1.66	1.58	2.05	1.93	1.96	2.14	2.32
Italy	41625	282067	0.20	0.12	1.53	2.18	2.50	2.21	2.08	1.99	1.96	2.63	2.01	1.92	1.90
Japan	45566	242033	0.27	0.15	1.96	2.09	2.39	2.11	1.99	1.91	1.98	2.36	2.00	1.97	1.99
Jordan	33340	-7720	0.25	0.05	0.87	1.45	1.92	1.57	1.46	1.40	2.56	1.68	2.14	2.51	2.85
Luxembourg	74979	470077	0.23	0.09	1.15	2.05	2.41	2.09	1.95	1.86	1.85	2.31	1.87	1.86	1.89
Mali	7619	-1822	0.57	0.03	1.21	1.42	1.70	1.47	1.40	1.35	2.46	1.64	2.22	2.60	2.94
Mozambique	4284	-375	0.52	0.04	0.99	1.48	1.77	1.54	1.46	1.40	2.31	1.65	2.09	2.38	2.66
New Zealand	43483	146622	0.25	0.09	0.96	1.94	2.32	1.99	1.86	1.77	1.85	2.12	1.86	1.90	1.97
Nigeria	10135	-450	0.31	0.04	1.51	1.42	1.86	1.55	1.46	1.40	2.37	1.42	1.89	2.21	2.51
Pakistan	16810	-237	0.26	0.03	0.85	1.46	1.88	1.57	1.48	1.41	2.39	1.58	2.01	2.33	2.63
Sierra Leone	5417	290	0.30	0.11	2.90	1.47	2.21	1.71	1.58	1.49	2.04	1.51	1.55	1.78	2.02
Singapore	85508	460631	0.16	0.08	0.28	2.00	2.37	2.02	1.89	1.80	1.88	2.25	1.90	1.92	1.98
South Africa	28153	29379	0.29	0.05	1.30	1.65	2.08	1.74	1.62	1.55	1.84	1.59	1.72	1.91	2.09
Sweden	46514	196823	0.24	0.10	1.16	2.00	2.37	2.05	1.92	1.83	1.86	2.23	1.88	1.88	1.93
United Kingdom	47026	190080	0.24	0.09	1.08	1.98	2.36	2.03	1.90	1.81	1.83	2.17	1.85	1.86	1.92
United States	63797	207432	0.28	0.11	2.01	1.91	2.36	1.99	1.85	1.75	1.84	2.18	1.85	1.89	1.98
Zambia	7333	-1925	0.42	0.03	0.93	1.41	1.74	1.49	1.40	1.35	2.65	1.69	2.27	2.66	3.02

Table 8:
Steady state policy instruments in selected countries

Country	Child allowances (in K $)				Health tax				Lump-sum middle age (in K $)				Lump-sum old age (in K $)			
	GA	FB_{01}	FB_{02}	FB_{03}	GA	FB_{01}	FB_{02}	FB_{03}	GA	FB_{01}	FB_{02}	FB_{03}	GA	FB_{01}	FB_{02}	FB_{03}
Argentina	-407	-83	-18	17	-0.093	-0.091	-0.091	-0.091	537	277	190	127	-213	-251	-278	-298
Australia	-1306	-268	-48	71	-0.087	-0.085	-0.085	-0.085	1458	767	531	361	-435	-515	-577	-628
Brazil	-489	-102	-29	10	-0.095	-0.093	-0.092	-0.092	604	317	223	155	-228	-275	-304	-327
Canada	-2060	-437	-123	41	-0.088	-0.086	-0.086	-0.085	1976	1062	760	546	-512	-635	-713	-775
Chile	-501	-100	-19	25	-0.092	-0.089	-0.089	-0.089	670	343	233	154	-250	-293	-324	-349
China	-88	-13	7	18	-0.093	-0.092	-0.092	-0.092	151	69	39	17	-89	-92	-103	-112
Egypt	-212	-36	-1	18	-0.090	-0.088	-0.088	-0.088	386	181	110	58	-269	-296	-317	-331
France	-1186	-250	-53	52	-0.088	-0.086	-0.086	-0.086	1292	688	483	335	-366	-435	-488	-531
Germany	-1656	-370	-111	27	-0.090	-0.088	-0.088	-0.088	1380	760	555	409	-318	-393	-444	-486
Ghana	-50	-9	-1	4	-0.101	-0.098	-0.098	-0.098	98	46	29	16	-88	-94	-100	-103
Guyana	-16	-1	4	7	-0.102	-0.100	-0.100	-0.100	28	10	4	-1	-20	-19	-22	-24
India	-56	-10	1	7	-0.098	-0.096	-0.096	-0.096	103	49	30	16	-76	-80	-87	-92
Indonesia	-136	-26	-2	12	-0.094	-0.092	-0.092	-0.092	233	116	74	44	-156	-170	-184	-195
Israel	-1242	-219	-27	71	-0.085	-0.083	-0.082	-0.082	1810	881	569	347	-747	-882	-966	-1026
Italy	-1572	-366	-115	20	-0.092	-0.090	-0.089	-0.089	1316	734	539	399	-294	-359	-403	-439
Japan	-1017	-218	-28	78	-0.088	-0.086	-0.086	-0.086	1095	586	408	277	-297	-340	-383	-419
Jordan	-323	-49	-1	23	-0.089	-0.087	-0.087	-0.087	615	271	156	73	-419	-470	-492	-500
Luxembourg	-2898	-600	-139	105	-0.084	-0.082	-0.082	-0.082	2777	1478	1043	733	-749	-917	-1032	-1125
Mali	-18	-2	1	3	-0.100	-0.098	-0.098	-0.098	45	18	8	0	-52	-50	-51	-50
Mozambique	-9	-1	0	1	-0.111	-0.109	-0.108	-0.108	21	9	5	2	-20	-19	-20	-21
New Zealand	-993	-207	-44	43	-0.089	-0.087	-0.087	-0.086	1124	593	413	284	-357	-425	-475	-515
Nigeria	-70	-13	-3	2	-0.103	-0.100	-0.099	-0.099	124	61	40	24	-116	-132	-140	-145
Pakistan	-125	-23	-3	7	-0.097	-0.094	-0.094	-0.094	236	112	70	40	-184	-204	-215	-221
Sierra Leone	-72	-12	-5	-1	-0.119	-0.113	-0.112	-0.111	73	36	26	19	-35	-52	-59	-65
Singapore	-3321	-692	-175	95	-0.084	-0.082	-0.082	-0.082	3504	1859	1303	910	-1014	-1239	-1375	-1480
South Africa	-415	-81	-18	15	-0.091	-0.089	-0.089	-0.089	577	292	197	130	-318	-378	-415	-443
Sweden	-1242	-263	-61	48	-0.088	-0.087	-0.086	-0.086	1297	692	488	342	-374	-450	-505	-549
United Kingdom	-1278	-270	-65	44	-0.088	-0.086	-0.086	-0.086	1344	716	505	354	-402	-486	-545	-592
United States	-1837	-340	-62	80	-0.084	-0.082	-0.082	-0.082	1962	999	687	465	-583	-721	-817	-893
Zambia	-20	-3	1	3	-0.104	-0.101	-0.101	-0.101	49	20	10	3	-50	-49	-50	-49

Table 9:
SYLVESTER'S CRITERION IN STEADY STATE FOR SELECTED COUNTRIES

Country	Laissez-faire			Golden Age				First-best $x = 0.01$				First-best $x = 0.03$			
	H_1	H_2	H	H_1	H_2	H_3	H	H_1	H_2	H_3	H	H_1	H_2	H_3	H
Argentina	-8E-11	4E-11	-2E-21	-2E-11	1E-23	-9E-24	3E-34	-1E-11	1E-22	-6E-23	1E-33	-1E-11	3E-22	-2E-22	4E-33
Australia	-9E-12	7E-12	-5E-23	-4E-12	3E-25	-3E-25	1E-36	-2E-12	2E-24	-2E-24	5E-36	-2E-12	6E-24	-5E-24	2E-35
Brazil	-1E-10	6E-11	-4E-21	-2E-11	1E-23	-8E-24	2E-34	-1E-11	1E-22	-6E-23	1E-33	-1E-11	3E-22	-1E-22	4E-33
Canada	-1E-11	1E-11	-4E-23	-4E-12	1E-25	-2E-25	3E-37	-2E-12	1E-24	-1E-24	1E-36	-2E-12	4E-24	-3E-24	5E-36
Chile	-5E-11	2E-11	-9E-22	-1E-11	6E-24	-4E-24	9E-35	-8E-12	5E-23	-3E-23	4E-34	-7E-12	1E-22	-7E-23	1E-33
China	-2E-10	7E-11	-2E-20	-3E-11	3E-22	-2E-22	6E-32	-3E-11	2E-21	-1E-21	2E-31	-2E-11	5E-21	-2E-21	8E-31
Egypt	-1E-10	3E-11	-4E-21	-9E-12	2E-23	-7E-24	7E-34	-6E-12	2E-22	-6E-23	3E-33	-5E-12	4E-22	-1E-22	1E-32
France	-1E-11	1E-11	-7E-23	-5E-12	5E-25	-5E-25	2E-36	-4E-12	5E-24	-4E-24	9E-36	-3E-12	1E-23	-1E-23	3E-35
Germany	-2E-11	2E-11	-1E-22	-9E-12	5E-25	-9E-25	2E-36	-6E-12	5E-24	-6E-24	1E-35	-5E-12	1E-23	-1E-23	4E-35
Ghana	-2E-09	3E-10	-7E-19	-8E-11	3E-21	-9E-22	1E-30	-5E-11	3E-20	-7E-21	6E-30	-5E-11	6E-20	-2E-20	2E-29
Guyana	-1E-09	8E-10	-2E-18	-3E-10	4E-20	-3E-20	1E-28	-3E-10	3E-19	-2E-19	3E-28	-2E-10	6E-19	-4E-19	1E-27
India	-7E-10	2E-10	-2E-19	-7E-11	2E-21	-7E-22	8E-31	-5E-11	1E-20	-5E-21	3E-30	-5E-11	3E-20	-1E-20	1E-29
Indonesia	-2E-10	8E-11	-6E-20	-2E-11	1E-22	-5E-23	3E-32	-2E-11	1E-21	-4E-22	1E-31	-2E-11	2E-21	-9E-22	5E-31
Israel	-9E-12	3E-12	-3E-23	-2E-12	1E-25	-7E-26	3E-37	-1E-12	1E-24	-5E-25	2E-36	-9E-13	3E-24	-1E-24	7E-36
Italy	-2E-11	3E-11	-3E-22	-1E-11	7E-25	-1E-24	6E-36	-7E-12	6E-24	-8E-24	2E-35	-6E-12	2E-23	-2E-23	9E-35
Japan	-1E-11	1E-11	-1E-22	-6E-12	7E-25	-9E-25	1E-35	-4E-12	5E-24	-6E-24	3E-35	-4E-12	1E-23	-1E-23	1E-34
Jordan	-8E-11	1E-11	-1E-21	-5E-12	5E-24	-1E-24	6E-35	-3E-12	5E-23	-1E-23	3E-34	-3E-12	1E-22	-3E-23	1E-33
Luxembourg	-4E-12	4E-12	-8E-24	-2E-12	3E-26	-4E-26	4E-38	-1E-12	3E-25	-2E-25	2E-37	-9E-13	7E-25	-6E-25	7E-37
Mali	-2E-09	4E-10	-1E-18	-1E-10	2E-20	-4E-21	2E-29	-8E-11	1E-19	-4E-20	7E-29	-7E-11	3E-19	-8E-20	2E-28
Mozambique	-1E-08	2E-09	-4E-17	-8E-10	6E-19	-2E-19	3E-27	-6E-10	4E-18	-1E-18	1E-26	-5E-10	9E-18	-2E-18	4E-26
New Zealand	-2E-11	1E-11	-2E-22	-6E-12	8E-25	-8E-25	6E-36	-4E-12	7E-24	-5E-24	2E-35	-3E-12	2E-23	-1E-23	1E-34
Nigeria	-3E-09	5E-10	-1E-18	-8E-11	2E-21	-6E-22	6E-31	-5E-11	2E-20	-5E-21	3E-30	-4E-11	5E-20	-1E-20	1E-29
Pakistan	-5E-10	1E-10	-1E-19	-3E-11	2E-22	-5E-23	4E-32	-2E-11	2E-21	-5E-22	2E-31	-2E-11	4E-21	-1E-21	7E-31
Sierra Leone	-3E-08	6E-09	-2E-17	-1E-09	4E-20	-3E-20	2E-29	-5E-10	5E-19	-2E-19	9E-29	-3E-10	1E-18	-4E-19	3E-28
Singapore	-4E-12	3E-12	-3E-23	-1E-12	2E-26	-2E-26	6E-38	-7E-13	2E-25	-1E-25	3E-37	-7E-13	4E-25	-3E-25	1E-36
South Africa	-9E-11	4E-11	-2E-21	-9E-12	7E-24	-4E-24	9E-35	-5E-12	6E-23	-3E-23	4E-34	-5E-12	2E-22	-7E-23	2E-33
Sweden	-1E-11	1E-11	-1E-22	-5E-12	5E-25	-6E-25	3E-36	-4E-12	4E-24	-4E-24	1E-35	-3E-12	1E-23	-1E-23	4E-35
United Kingdom	-1E-11	1E-11	-1E-22	-5E-12	4E-25	-5E-25	2E-36	-3E-12	4E-24	-3E-24	9E-36	-3E-12	1E-23	-8E-24	4E-35
United States	-6E-12	4E-12	-7E-24	-2E-12	8E-26	-9E-26	9E-38	-1E-12	9E-25	-7E-25	4E-37	-1E-12	2E-24	-2E-24	1E-36
Zambia	-3E-09	5E-10	-3E-18	-2E-10	3E-20	-6E-21	4E-29	-1E-10	2E-19	-5E-20	2E-28	-1E-10	5E-19	-1E-19	6E-28

Table 10:
AGE STRUCTURES AND POLICY INSTRUMENTS IN 1980-2009 ($t = 0$) AND 2010-2039 ($t = 1$) SELECTED COUNTRIES

Country	Mean age				Dependency ratio				Child allowance (in K $)		Health tax		Lump-sum transfer			
	Observed		FB_{02}		Observed		FB_{02}						middle age (in K $)		old age (in K $)	
	t=0	t=1	t=0	t=1	t=0	t=1	t=0	t=1	t=0	t=1	t=0	t=1	t=0	t=1	t=0	t=1
Argentina	1.71	1.86	1.63	1.81	1.77	1.60	1.51	1.77	-13.6	-23.1	-0.092	-0.091	140.8	180.7	-252.1	-242.8
Australia	1.86	2.04	1.96	2.02	1.43	1.91	1.63	1.91	4.5	-75.0	-0.087	-0.086	300.2	475.2	-543.8	-451.4
Brazil	1.62	1.92	1.52	1.63	1.50	1.46	1.92	1.98	5.8	-17.2	-0.094	-0.093	110.2	162.4	-417.7	-257.7
Canada	1.91	2.09	1.50	1.61	1.34	1.96	2.03	2.06	14.7	-140.4	-0.088	-0.087	323.2	621.2	-585.4	-515.0
Chile	1.70	1.96	1.58	1.69	1.48	1.62	1.80	1.92	7.9	-22.0	-0.092	-0.090	104.4	189.4	-310.5	-248.1
China	1.73	1.98	1.88	1.99	1.24	1.54	1.61	1.93	6.6	-5.2	-0.097	-0.093	11.4	43.9	-59.0	-64.7
Egypt	1.53	1.68	1.94	2.10	2.02	1.53	1.44	1.87	4.1	-2.9	-0.090	-0.089	42.9	90.9	-169.0	-247.7
France	1.90	2.03	1.43	1.45	1.52	2.00	2.41	2.45	-22.2	-89.5	-0.088	-0.087	298.9	457.8	-422.6	-381.7
Germany	2.02	2.18	1.51	1.58	1.37	2.01	1.98	2.06	-34.3	-161.4	-0.091	-0.089	303.1	507.7	-382.7	-330.3
Ghana	1.44	1.57	1.92	2.03	2.26	1.67	1.53	1.87	-0.1	-0.7	-0.099	-0.098	17.3	24.1	-76.6	-88.5
Guyana	1.62	1.78	1.82	1.90	1.43	1.55	1.62	1.84	7.1	3.3	-0.100	-0.100	-3.3	3.3	-29.4	-20.3
India	1.58	1.76	1.43	1.46	1.72	1.41	2.33	2.30	0.1	-1.7	-0.098	-0.097	18.7	29.5	-63.7	-69.6
Indonesia	1.60	1.79	1.96	2.09	1.61	1.47	1.48	1.85	2.8	-6.0	-0.095	-0.093	28.9	64.6	-113.6	-131.6
Israel	1.63	1.76	1.53	1.63	1.81	1.87	1.85	1.78	5.0	-32.5	-0.084	-0.083	329.7	495.0	-881.8	-804.9
Italy	2.02	2.19	1.50	1.62	1.32	2.17	2.03	2.04	-11.3	-144.3	-0.093	-0.091	267.8	463.0	-375.5	-293.3
Japan	2.02	2.19	1.42	1.48	1.46	2.09	2.56	2.53	-20.1	-99.6	-0.089	-0.087	263.2	421.0	-322.2	-279.7
Jordan	1.40	1.61	1.73	1.84	2.50	1.58	1.68	1.88	8.5	4.0	-0.087	-0.087	62.0	105.3	-407.2	-427.9
Luxembourg	1.99	2.08	1.48	1.55	1.50	1.94	2.07	2.08	-85.9	-272.9	-0.084	-0.083	628.9	1044.5	-748.8	-765.1
Mali	1.44	1.40	1.86	2.02	2.58	2.65	1.48	1.84	-1.2	1.0	-0.099	-0.098	12.9	9.7	-34.9	-49.1
Mozambique	1.47	1.51	1.53	1.65	2.28	2.08	1.89	1.95	-0.3	0.3	-0.109	-0.108	6.8	5.2	-22.8	-19.9
New Zealand	1.79	1.98	1.98	2.11	1.42	1.81	1.53	1.92	9.8	-45.2	-0.088	-0.087	228.1	343.0	-521.9	-388.4
Nigeria	1.44	1.44	1.54	1.69	2.37	2.29	1.88	1.97	-3.4	-3.4	-0.100	-0.099	34.5	38.9	-112.5	-133.4
Pakistan	1.45	1.64	1.57	1.62	2.38	1.44	1.79	1.95	0.4	-2.4	-0.096	-0.095	31.3	55.7	-120.8	-178.5
Sierra Leone	1.48	1.55	1.92	2.00	2.07	1.63	1.55	1.81	-2.6	-3.9	-0.113	-0.112	15.9	22.2	-46.4	-58.4
Singapore	1.80	2.20	1.68	1.82	1.07	1.83	1.62	1.89	128.5	-150.6	-0.086	-0.084	364.7	896.2	-1468.0	-850.2
South Africa	1.55	1.77	1.50	1.62	1.70	1.37	2.00	2.06	4.2	-10.0	-0.089	-0.089	114.1	152.6	-500.4	-373.9
Sweden	1.96	2.02	1.87	1.96	1.61	1.94	1.61	1.86	-50.5	-120.1	-0.089	-0.087	314.5	497.5	-349.3	-387.1
United Kingdom	1.92	2.02	1.55	1.67	1.50	1.88	1.95	2.03	-31.8	-114.3	-0.089	-0.087	291.1	488.8	-400.7	-409.5
United States	1.85	1.97	1.41	1.43	1.52	1.82	2.56	2.50	-11.8	-104.0	-0.084	-0.083	378.6	632.5	-620.7	-648.2
Zambia	1.42	1.44	1.41	1.43	2.43	2.49	2.58	2.56	2.3	1.0	-0.100	-0.101	11.4	10.8	-75.9	-52.9

Table 11:
Age structures in India, Germany and the United States over time

Period	Net reproduction rate				Survival probability				Mean age				Dependency ratio			
	LF	FB_{005}	FB_{015}	FB_{025}	LF	FB_{005}	FB_{015}	FB_{025}	LF	FB_{005}	FB_{015}	FB_{025}	LF	FB_{005}	FB_{015}	FB_{025}
	India															
t=0	1.427	1.324	1.499	1.655	0.518	0.518	0.518	0.518	1.583	1.606	1.568	1.538	1.719	1.617	1.792	1.948
t=1	1.530	1.244	1.490	1.714	0.600	0.600	0.600	0.599	1.624	1.707	1.623	1.560	1.951	1.697	1.890	2.076
t=2	1.552	1.235	1.517	1.768	0.603	0.605	0.604	0.604	1.607	1.725	1.620	1.546	1.947	1.722	1.922	2.120
t=3	1.558	1.240	1.536	1.794	0.604	0.607	0.606	0.605	1.604	1.726	1.613	1.537	1.947	1.732	1.936	2.137
t=4	1.559	1.245	1.546	1.804	0.604	0.608	0.607	0.605	1.603	1.724	1.609	1.533	1.947	1.736	1.941	2.142
t=5	1.559	1.249	1.550	1.807	0.604	0.608	0.607	0.605	1.602	1.722	1.607	1.532	1.947	1.737	1.942	2.142
t=6	1.559	1.250	1.551	1.807	0.604	0.608	0.607	0.605	1.602	1.721	1.606	1.531	1.947	1.737	1.943	2.142
t=7	1.559	1.251	1.552	1.807	0.604	0.608	0.607	0.605	1.602	1.721	1.606	1.531	1.947	1.737	1.943	2.142
t=8	1.559	1.251	1.552	1.807	0.604	0.608	0.607	0.605	1.602	1.721	1.606	1.531	1.947	1.737	1.943	2.142
	Germany															
t=0	0.660	0.611	0.696	0.770	0.685	0.685	0.685	0.685	2.021	2.042	2.005	1.975	1.368	1.320	1.404	1.478
t=1	0.728	0.561	0.686	0.801	0.841	0.845	0.842	0.839	2.182	2.279	2.181	2.100	2.002	1.942	1.896	1.891
t=2	0.737	0.567	0.722	0.861	0.848	0.860	0.858	0.855	2.148	2.312	2.177	2.071	1.901	2.101	1.971	1.929
t=3	0.738	0.583	0.755	0.904	0.849	0.870	0.866	0.862	2.143	2.306	2.150	2.033	1.890	2.118	1.954	1.905
t=4	0.738	0.596	0.775	0.922	0.849	0.874	0.868	0.864	2.143	2.292	2.128	2.012	1.888	2.096	1.924	1.878
t=5	0.738	0.605	0.783	0.926	0.849	0.875	0.869	0.863	2.143	2.281	2.117	2.003	1.888	2.072	1.904	1.863
t=6	0.738	0.609	0.785	0.925	0.849	0.875	0.868	0.863	2.143	2.274	2.112	2.002	1.888	2.056	1.894	1.857
t=7	0.738	0.611	0.784	0.924	0.849	0.874	0.868	0.862	2.143	2.271	2.111	2.003	1.888	2.047	1.890	1.856
t=8	0.738	0.611	0.784	0.923	0.849	0.874	0.868	0.862	2.143	2.269	2.111	2.004	1.888	2.043	1.890	1.856
	United States															
t=0	0.947	0.865	0.995	1.109	0.747	0.747	0.747	0.747	1.850	1.879	1.834	1.799	1.516	1.434	1.564	1.678
t=1	1.026	0.787	0.975	1.146	0.821	0.825	0.821	0.818	1.945	2.061	1.947	1.858	1.893	1.740	1.800	1.884
t=2	1.040	0.779	1.005	1.207	0.830	0.842	0.839	0.836	1.919	2.102	1.950	1.837	1.849	1.850	1.866	1.936
t=3	1.043	0.788	1.033	1.245	0.831	0.853	0.848	0.843	1.914	2.106	1.934	1.814	1.842	1.882	1.877	1.944
t=4	1.043	0.797	1.049	1.261	0.831	0.857	0.850	0.845	1.913	2.101	1.921	1.802	1.841	1.884	1.873	1.940
t=5	1.043	0.803	1.057	1.266	0.831	0.858	0.851	0.845	1.913	2.095	1.914	1.797	1.840	1.880	1.867	1.935
t=6	1.043	0.806	1.059	1.266	0.831	0.858	0.851	0.844	1.913	2.092	1.911	1.796	1.840	1.875	1.864	1.933
t=7	1.043	0.808	1.059	1.265	0.831	0.858	0.850	0.844	1.913	2.089	1.910	1.796	1.840	1.873	1.862	1.932
t=8	1.043	0.808	1.059	1.264	0.831	0.858	0.850	0.844	1.913	2.088	1.910	1.796	1.840	1.871	1.862	1.931

Table 12:
Policy instruments in India, Germany and the United States over time

Period	Child allowances in K $			Health tax in %			Lump-sum transfer working age in K $			Lump-sum transfer retirement in K $		
	FB_{005}	FB_{015}	FB_{025}	FB_{005}	FB_{015}	FB_{025}	FB_{005}	FB_{015}	FB_{025}	FB_{005}	FB_{015}	FB_{025}
	India											
t=0	-5.72	-2.74	0.28	-9.88	-9.83	-9.81	18.85	18.81	18.61	-61.36	-63.03	-64.34
t=1	-6.17	-1.78	-0.77	-9.73	-9.67	-9.64	34.09	30.99	28.08	-58.48	-66.18	-72.82
t=2	-5.20	-0.75	0.68	-9.67	-9.61	-9.58	41.31	34.51	29.11	-68.13	-77.20	-84.77
t=3	-4.57	-0.33	1.51	-9.64	-9.59	-9.57	42.91	34.40	28.23	-72.97	-81.88	-88.99
t=4	-4.31	-0.21	1.74	-9.63	-9.58	-9.57	43.02	33.95	27.66	-74.88	-83.34	-89.91
t=5	-4.21	-0.19	1.78	-9.63	-9.58	-9.57	42.89	33.68	27.43	-75.52	-83.63	-89.91
t=6	-4.19	-0.19	1.77	-9.62	-9.58	-9.57	42.77	33.56	27.35	-75.69	-83.62	-89.78
t=7	-4.18	-0.20	1.76	-9.62	-9.58	-9.57	42.71	33.51	27.34	-75.72	-83.57	-89.70
t=8	-4.18	-0.20	1.75	-9.62	-9.58	-9.57	42.67	33.50	27.34	-75.71	-83.53	-89.66
	Germany											
t=0	-229.65	-226.13	-222.88	-9.15	-9.10	-9.08	411.95	433.40	451.20	-372.72	-379.71	-385.49
t=1	-547.05	-451.91	-388.53	-9.02	-8.95	-8.91	689.11	696.35	699.76	-269.28	-311.27	-348.30
t=2	-684.85	-498.13	-390.36	-8.93	-8.86	-8.82	876.14	829.49	791.96	-309.76	-366.81	-416.85
t=3	-683.44	-463.38	-349.16	-8.89	-8.82	-8.79	932.44	845.18	783.42	-338.96	-402.98	-456.63
t=4	-652.98	-430.86	-323.02	-8.87	-8.81	-8.79	934.38	827.62	758.47	-354.14	-418.97	-470.59
t=5	-629.00	-413.71	-312.64	-8.87	-8.81	-8.79	923.32	810.49	741.92	-360.54	-423.31	-471.71
t=6	-615.10	-407.06	-310.35	-8.87	-8.81	-8.80	913.03	800.52	734.79	-362.56	-422.99	-469.27
t=7	-608.31	-405.51	-310.89	-8.87	-8.82	-8.80	906.40	796.28	733.03	-362.79	-421.68	-467.15
t=8	-605.52	-405.77	-311.82	-8.87	-8.82	-8.80	902.88	795.12	733.28	-362.50	-420.69	-466.10
	United States											
t=0	-230.39	-70.13	36.93	-8.48	-8.44	-8.41	555.30	432.96	327.97	-600.37	-614.62	-626.15
t=1	-519.72	-205.42	-24.88	-8.38	-8.32	-8.28	928.35	721.85	550.54	-524.19	-609.68	-684.31
t=2	-640.34	-223.85	-12.79	-8.32	-8.26	-8.22	1153.80	847.58	613.14	-588.90	-698.43	-792.56
t=3	-647.05	-202.41	7.07	-8.29	-8.23	-8.21	1225.42	863.22	602.04	-630.59	-748.62	-845.19
t=4	-630.28	-185.93	16.51	-8.28	-8.23	-8.20	1236.62	850.93	583.70	-650.90	-768.90	-861.20
t=5	-616.58	-178.13	19.42	-8.28	-8.22	-8.21	1231.84	838.77	572.82	-659.29	-774.37	-862.29
t=6	-608.59	-175.30	19.79	-8.28	-8.23	-8.21	1225.42	831.69	568.36	-662.18	-774.48	-859.93
t=7	-604.56	-174.62	19.52	-8.28	-8.23	-8.21	1220.81	828.54	567.16	-662.87	-773.44	-858.00
t=8	-602.75	-174.65	19.25	-8.28	-8.23	-8.21	1218.15	827.48	567.15	-662.84	-772.59	-857.04

Institut de Recherches Économiques et Sociales
Université catholique de Louvain

Place Montesquieu, 3
1348 Louvain-la-Neuve, Belgique

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