ICRAT 2018

A Probabilistic Model for Precedence Rules and Reactionary Delay

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UCL and EUROCONTROL

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European airspace

30 000 daily flights

Close to capacity limit

Managed at the country level

Traffic likely to double in the next years

➔ Big challenge

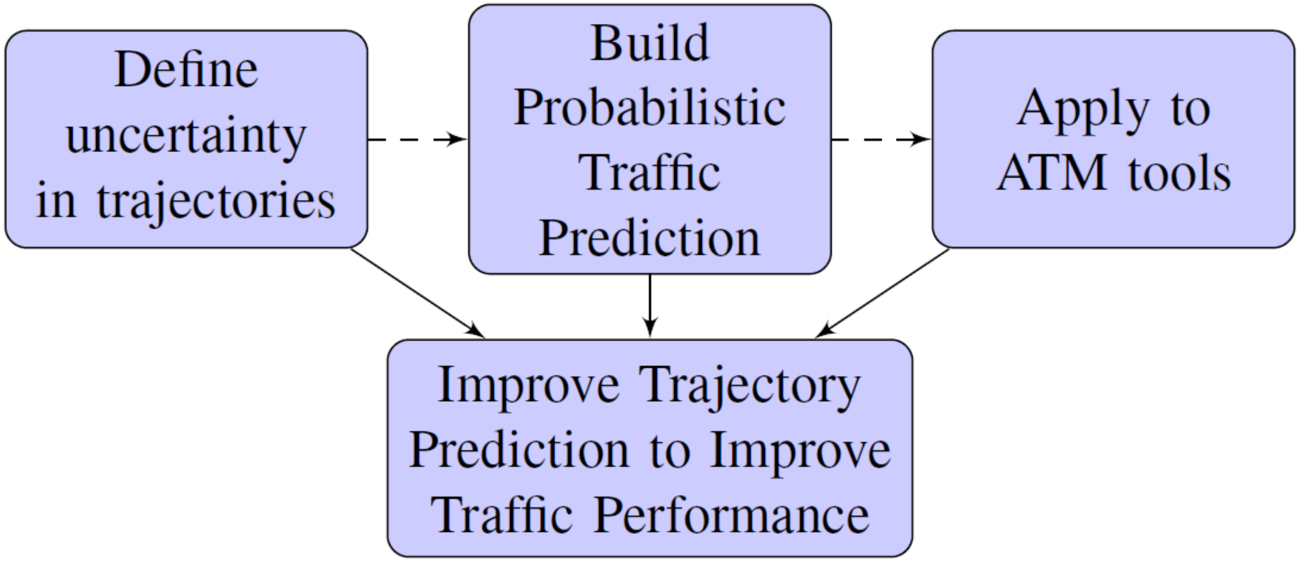
Research program

- Single European Sky ATM Research Programme (SESAR)
- Launched in 2007
- Jointly managed by the European Commission and EUROCONTROL (European Organisation for the Safety of Air Navigation)
- Fundamental, applied and industrial research
- Target number: reduce by 10% the environmental impact and flight time

COPTRA

Combining probable trajectories

COPTRA aims to propose an efficient method to build probabilistic traffic forecasts on the basis of flight trajectory predictions



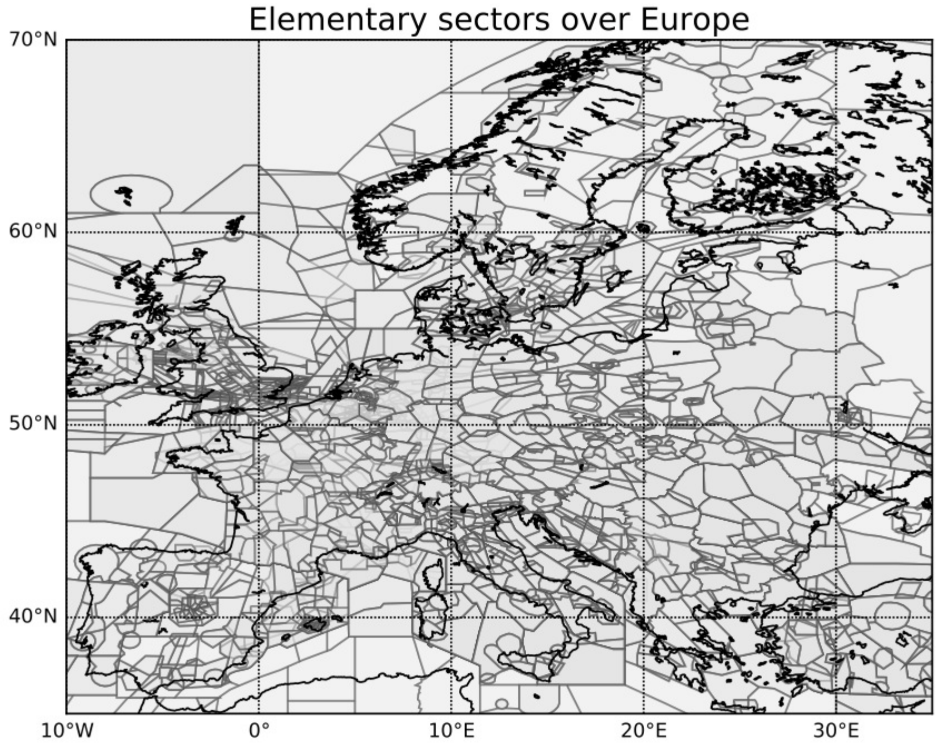
Partners: EUROCONTROL, CRIDA, BR&TE, ITU and UCL

Outlook

- COPTRA aims and available data
- Occupancy count algorithm
- Propagation of reactionnary delay algorithms

Sector Occupancy Counts

The prediction of sector occupancy is critical for ATC operations



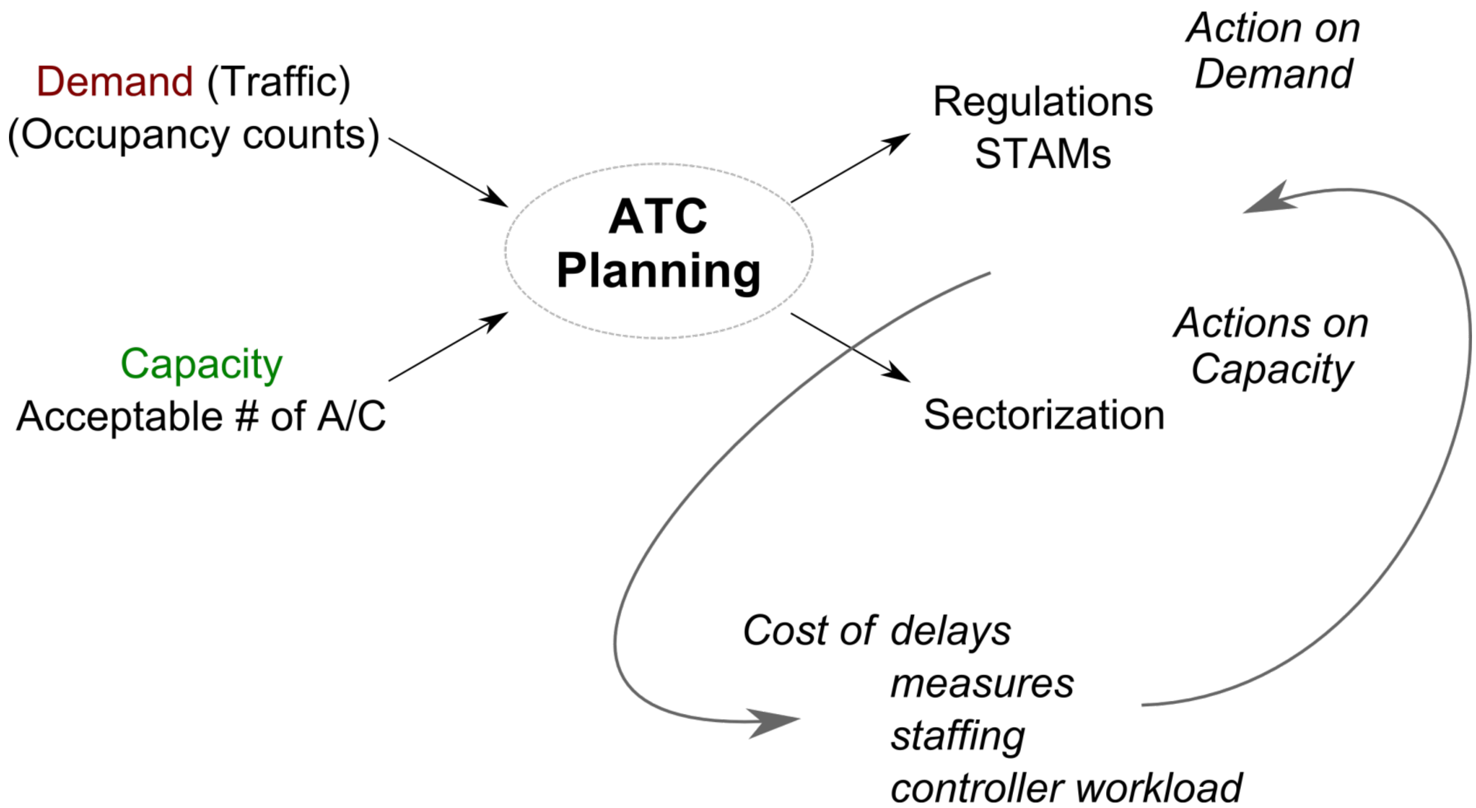
Grouping and managing sector.
Assigning controllers.
…

Compute probabilistic sector occupancy from probabilistic trajectories.

Air Traffic Control

Data available: expected number of fligths

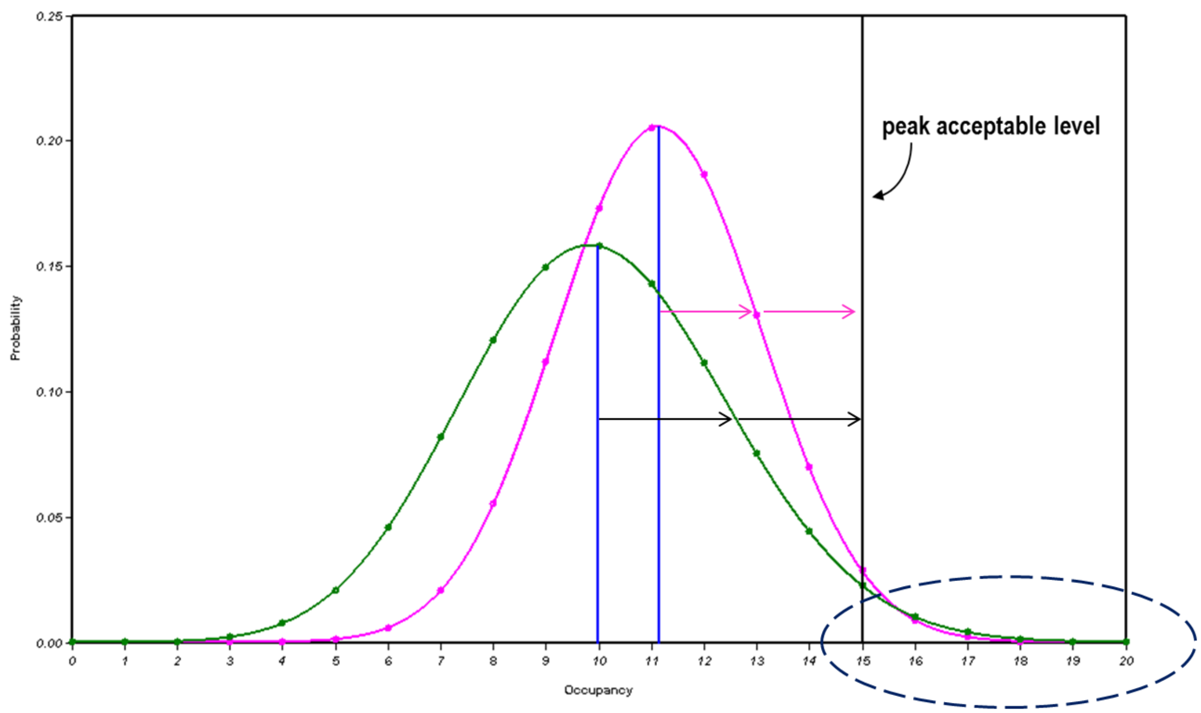
Actions taken based on experience of the ATC



Goal

Increase the average occupancy

Knowledge of the full distribution would allow to increase the average occupancy or to reduce the sector congestion



Input

Flight data provided by Eurocontrol (DDR2 semipublic database and Network manager data)

12 may 2016, 33.219 flights in 1.991 elementary sectors

DDR2

Hello francoisgz, D: 5 ; G: 5 .

Demand Data Repository - Historical Page

Home | Historical Traffic | Filtered Traffic |DCT Mapping| Forecast Traffic | Dataset Files | Tools Download | Events | Reports

Quick Reference Guide
User Manual: PDF

EUROCONTROL

2016-05

Year | Month | Airac | Nest Official | SO6 M1 | SO6 M3 | EXP2 | ALL_FT+ | Events | Airac | Reset | ?

	Sun 1	Mon 2	Tue 3	Wed 4	Thu 5	Fri 6	Sat 7	Sun 8	Mon 9	Tue 10	Wed 11	Thu 12	Fri 13	Sat 14	Sun 15	Mon 16
Events	32	33	33	32	32	33	33	33	35	38	38	38	38	35	34	36
MONTH	MAY-2016															
AIRAC	1605															
NESTO.	1605															
EXP2																
SO6 m1																
SO6 m3																
ALL_FT+																
Ranking	27	17	20	14	25	24	31	23	12	16	15	9	3	30	28	21
Nb Flights	27709	30464	30161	30845	28691	28880	25380	29217	31239	30493	30739	31610	32432	26330	27698	29979

	Tue 17	Wed 18	Thu 19	Fri 20	Sat 21	Sun 22	Mon 23	Tue 24	Wed 25	Thu 26	Fri 27	Sat 28	Sun 29	Mon 30	Tue 31
Events	36	37	39	38	35	36	41	46	48	76	47	42	41	41	41
MONTH															
AIRAC										1606					
NESTO.										1606					
EXP2															
SO6 m1															
SO6 m3															
ALL_FT+															
Ranking	18	11	10	2	29	22	7	13	4	8	1	26	19	5	6
Nb Flights	30443	31367	31492	32722	26740	29230	31661	31231	32149	31622	33342	27938	30245	32124	31755

Download Queue | Legend | Download | Clear Selection | ?

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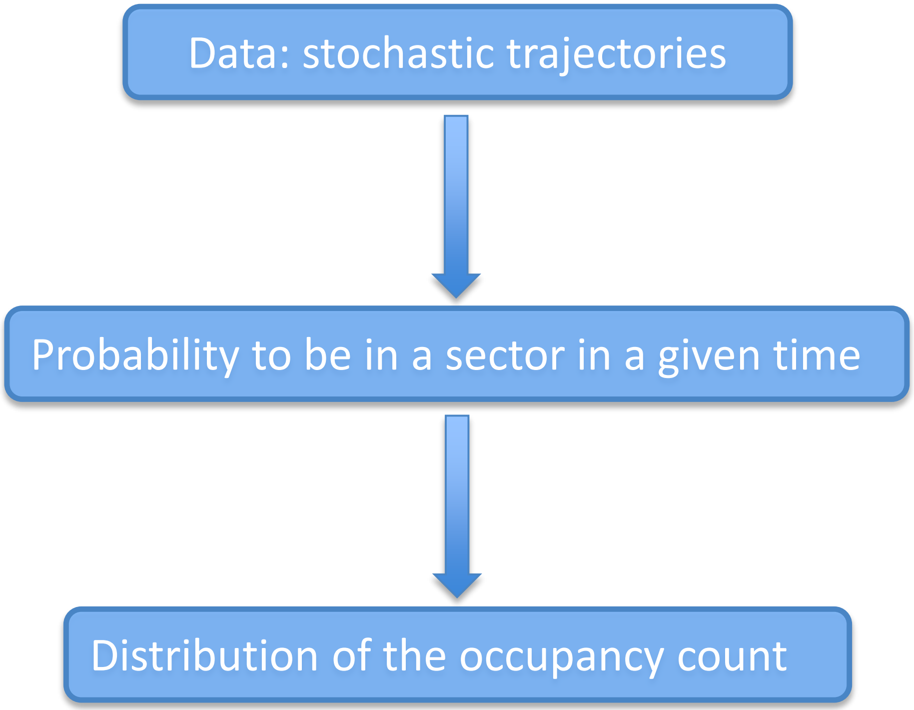
How to compute the probabilistic occupancy count?

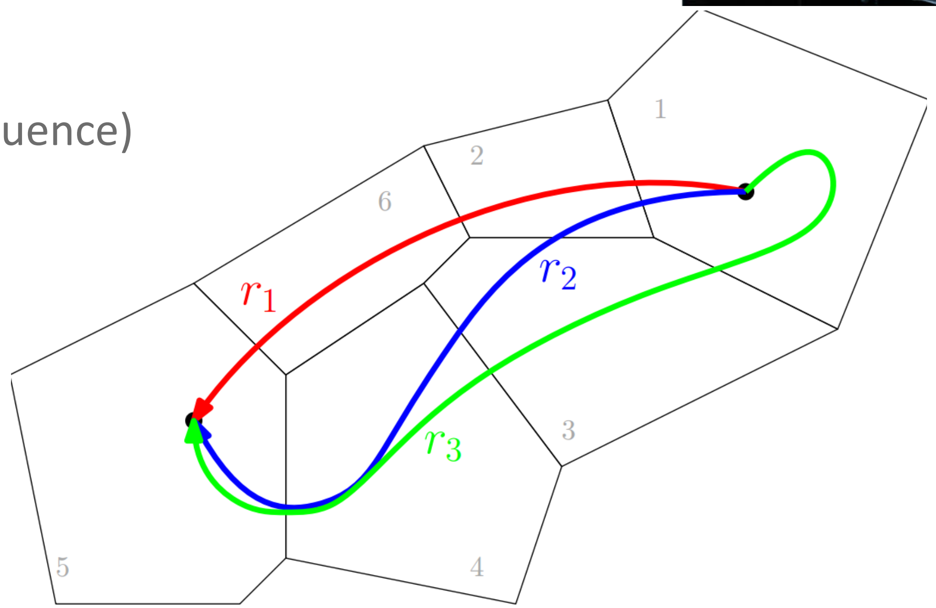


Basic trajectory model

Starting and arrival airport

Many possible trajectories (sectors sequence)

Possible delay on each sequence



For flight f, we consider the set $\mathcal{R}_f = \{r_{f,1}, \ldots, r_{f,n}\}$ of probable trajectories (or scenarios). Each of the trajectories $r_{f,i} \in \mathcal{R}_f$ is associated with the probability $w_{f,i}$ that the flight will fly it.

Delay distribution of entry and leaving time:

$$\tau_{e,f,s,i} \sim \mathcal{Q}_{\tau_{e,f,s,i}}$$

$$\tau_{l,f,s,i} \sim \mathcal{Q}_{\tau_{l,f,s,i}}$$

Probability to be in a sector

SESAR JOINT UNDERTAKING

COPTRA

Probability that flight f is in sector s at time t, if it follows trajectory i:

$$p_{f,s,t,r_{f,i}} := \Pr[\tau_{e,f,s,i} \leq t < \tau_{l,f,s,i}] = \Pr[\tau_{e,f,s,i} \leq t] - \Pr[\tau_{l,f,s,i} \leq t]$$

Entry time — Leaving time — Entry time — Leaving time

Probability that flight f is in sector s at time t:

$$p_{f,s,t} = \sum_{i=1}^{n} w_{f,i} p_{f,s,t,r_{f,i}}$$

Probability to follow trajectory i

One value for each combination of flight, time and sector: 95 billions values

Probabilistic occupancy count of a sector

Dynamic programming scheme

If the probability that each flight is in sector s at time t is known, how can we compute the probabilistic occupancy count of this sector efficiently?

Naive technique: count all the possibilities to have one flight, two flights, etc.

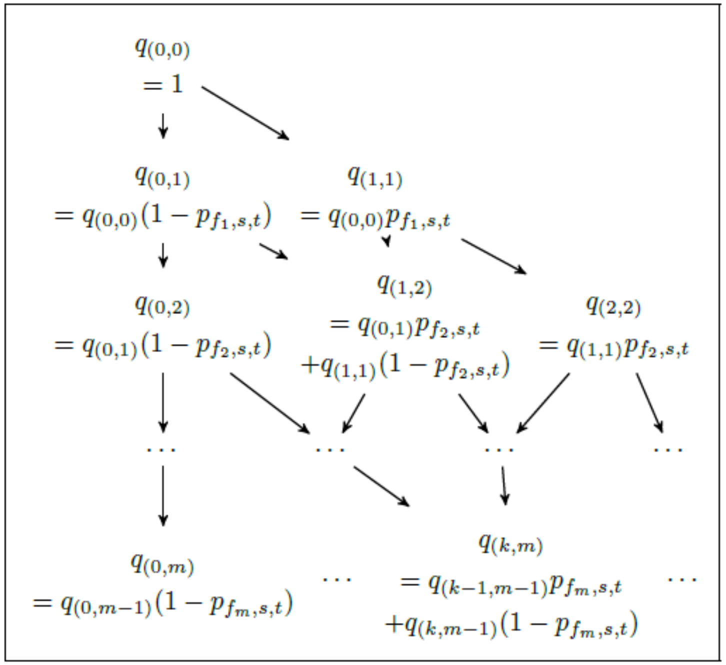
Dynamic programming technique: list the flights and add them recursively

q(i,j): probability that i flights among the j first ones are in the sector

We need to compute the value q(k,m) for all k

We have that:

$$q_{(i,j)} = q_{(i,j-1)} \left(1 - p_{f_j,s,t}\right) + q_{(i-1,j-1)}\, p_{f_j,s,t}$$

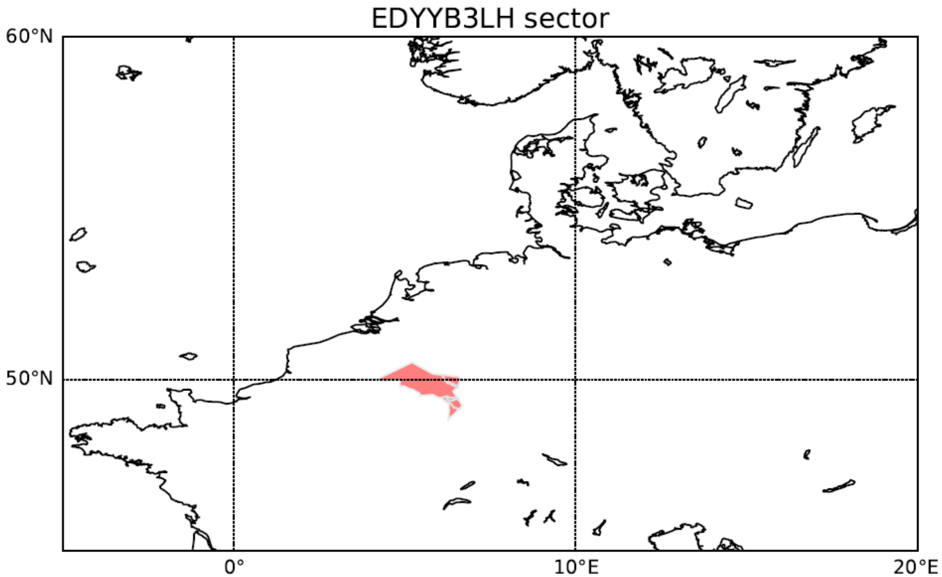


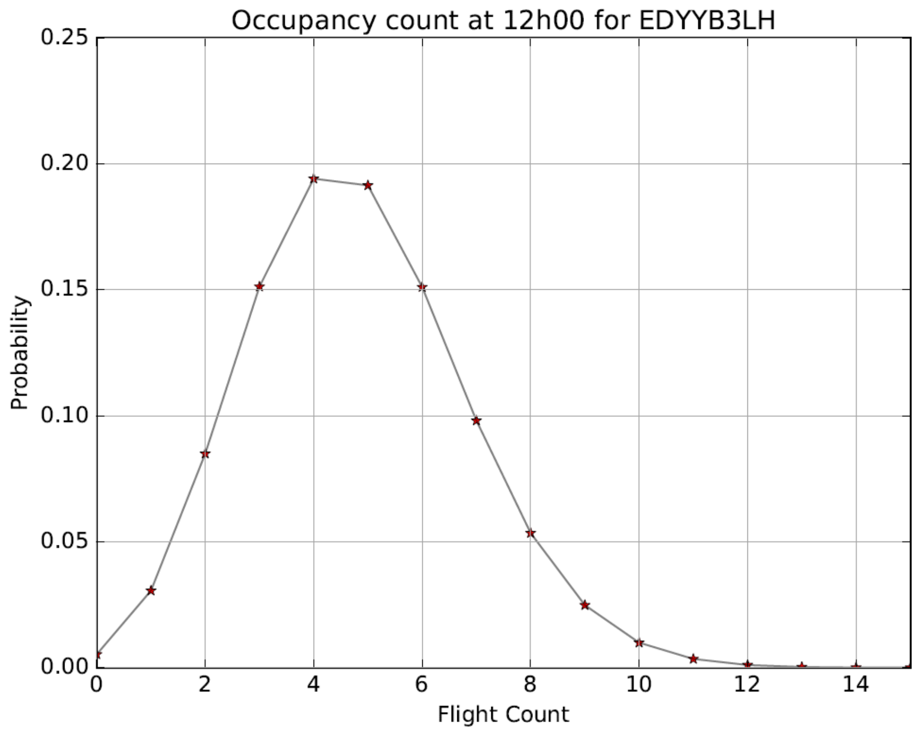
Occupancy pdf

With this method, we are able to compute the occupancy count with a quadratic computational time

We computed the full probabilistic distribution for the 1991 sectors at every minute (3 millions distributions)

Programs running in 3 hours





Expected occupancy

Evolution over the day

Outlook

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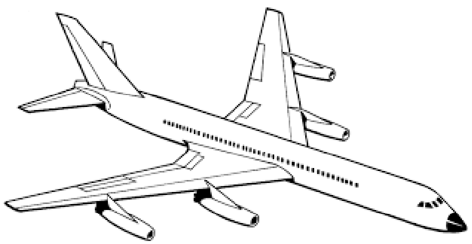
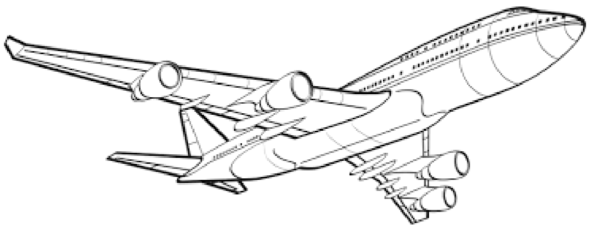
Precedence relations (reactionnary delay)

- What is precedence ?





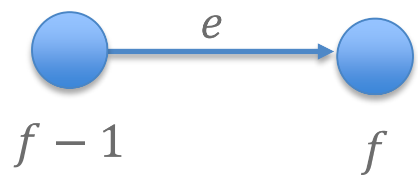
  - Most aircrafts perform several consecutive flights
  - Next flight must wait for the end of previous flight and turnover time
  - Precedences also come from correspondence of crews and passengers

Up to 40% of all delay times is due to propagation of initial delays (CODA report 2016)

Precedence relations

Model for precedence delays

- Each flight is assumed to have an initial a priori delay x_f^0



Travel time : t_{f-1}

Turnover time : t_e

- The delay after precedence x_f must satisfy

$$x_f \geq x_{f-1} + t_{f-1} + t_e$$

and

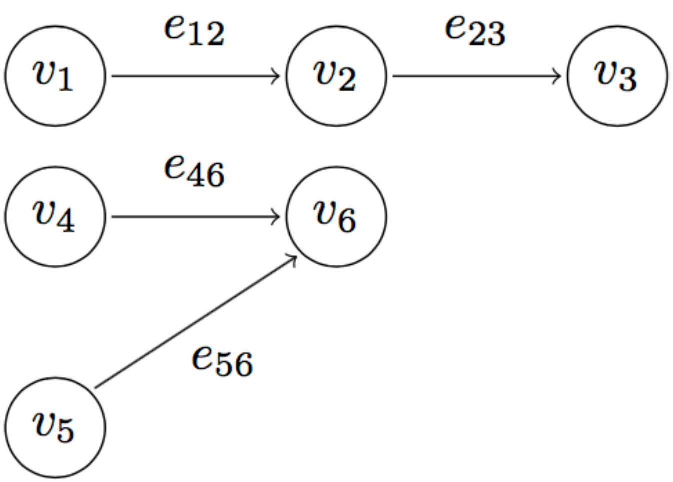
$$x_f \geq x_f^0.$$

Hence

$$x_f = \max(x_f^0, x_{f-1} + t_{f-1} + t_e)$$

Model for precedence delays

An example of precedence network of 6 flights and the precedence equations



$$
\begin{aligned}
x_{f_1} &= \max\{x^0_{f_1}\} \\
x_{f_2} &= \max\{x^0_{f_2}, x_{f_1} + t_{f_1} + t_{e_{12}}\} \\
x_{f_3} &= \max\{x^0_{f_3}, x_{f_2} + t_{f_2} + t_{e_{23}}\} \\
x_{f_4} &= \max\{x^0_{f_4}\} \\
x_{f_5} &= \max\{x^0_{f_5}\} \\
x_{f_6} &= \max\{x^0_{f_6}, x_{f_4} + t_{f_4} + t_{e_{46}}, x_{f_5} + t_{f_5} + t_{e_{56}}\}
\end{aligned}
$$

How can we apply this to delay distributions?

Monte Carlo algorithm

Draws M scenarios according to the Distributions of the flights delay

For each scenario, apply the precedence relation rules

For each flight, return the delays obtained in each scenario

INPUT: *an acyclic graph* G *as in Definition 1 associated with the initial distributions* X_i^0*, the travel time distributions* T_{f_i} *and turnaround time distributions* $T_{e_j i}$ *for each flight* f_i*. A large integer* M*: the sample size to be used.*

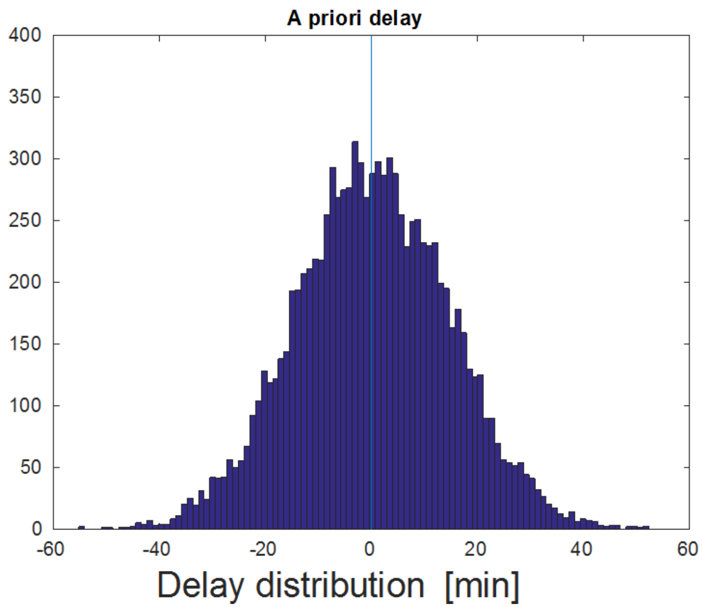
OUTPUT: *The distributions* X_i

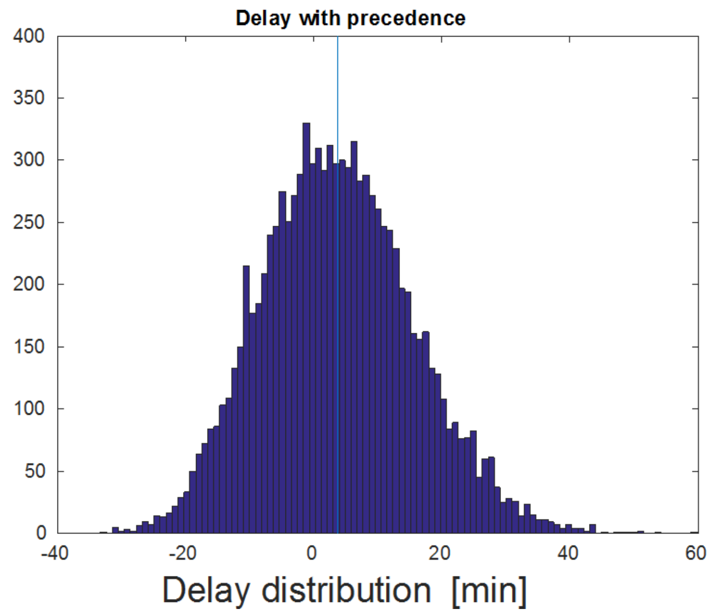
Compute a topological ordering of the graph G and sort the flights $f_1 f_2 \dots f_N$ accordingly.
for $s = 1, \dots, M$ **do**
  Generate one sample $p^s x_i^0$ for each distribution X_i^0, and $t_{f_i}^s \sim T_{f_i}$ and $t_{e_i}^s \sim T_{e_i}$.
  for $i = 1, \dots, N$ **do**
    if Flight f_i has no precedence **then**
      $X_i = X_i^0$
    else if f_i is preceded by flights with indexes in P_i **then**

$$p^s x_i = \max\{p^s x_i^0, \max_{j \in P_i}\{p^s x_j + t_{f_j}^s + t_{e_{ji}}^s\}\}$$

    end if
  end for
end for
return The statistical distributions as sample sets $X_i = \{p^s x_i\}_{s=1:M}$

Monte Carlo algorithm





- Input: a priori delay distribution for each flight
- Output: delay distributions accounting precedence

Pro: Can be used with any a priori delay distribution

Con: High computing time depending on the number of simulation chosen

Analytical Algorithm

Considers Gaussian à priori delay distribution

INPUT: *an acyclic graph G as in Definition 1 associated with the initial Gaussian distributions X_i^0, the travel time distributions T_{f_i} and turnaround time distributions $T_{e_{ji}}$ for each flight f_i.*

OUTPUT: *The distributions X_i*

Compute a topological ordering of the graph G and sort the flights $f_1 f_2 \dots f_N$ accordingly.
for $i = 1, \dots, N$ **do**
  if Flight f_i has no precedence **then**
    $X_i = X_i^0$
  else if f_i is preceded by flights with indexes in P_i **then**
    Set $\mu_{X_i} = \mu_{X_i^0}$ and $\sigma^2_{X_i} = \sigma^2_{X_i^0}$
    for $j \in P_i$ **do**
      Define the mean of $\overline{X}_j = X_j + T_{f_j} + T_{e_{ji}}$, as $\mu_{\overline{X}_j} = \mu_{X_j} + t_{f_j} + t_{e_{ji}}$.
      Set $\theta = \sqrt{\sigma^2_{X_i} + \sigma^2_{\overline{X}_j}}$
      Compute the new mean and variance using Equations (6) and (7).

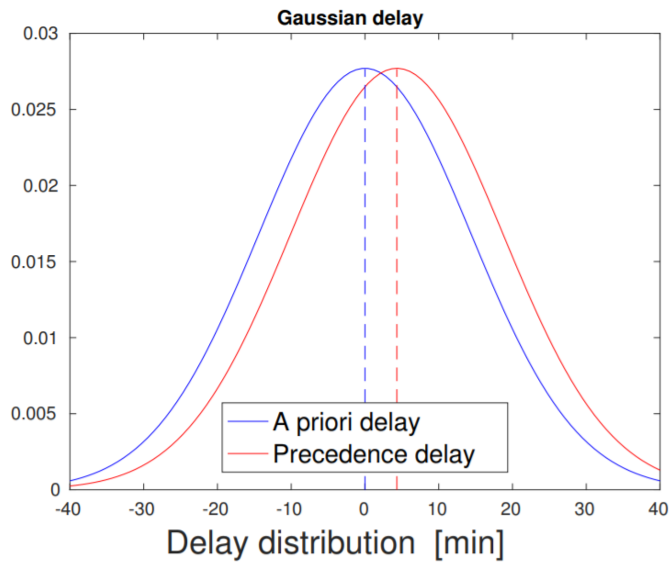
$$\mu_{X_i} = \mu_{\overline{X}_j} \Phi\left(\frac{\mu_{\overline{X}_j} - \mu_{X_i}}{\theta}\right) + \mu_{X_i} \Phi\left(\frac{\mu_{X_i} - \mu_{\overline{X}_j}}{\theta}\right) + \theta \phi\left(\frac{\mu_{\overline{X}_j} - \mu_{X_i}}{\theta}\right) \tag{8}$$

$$\sigma^2_{X_i} = (\sigma^2_{\overline{X}_j} - \mu^2_{\overline{X}_j}) \Phi\left(\frac{\mu_{\overline{X}_j} - \mu_{X_i}}{\theta}\right) + (\sigma^2_{X_i} - \mu_{X_i}) \Phi\left(\frac{\mu_{X_i} - \mu_{\overline{X}_j}}{\theta}\right) + (\mu_{\overline{X}_j} + \mu_{X_i}) \theta \phi\left(\frac{\mu_{\overline{X}_j} - \mu_{X_i}}{\theta}\right) - \mu^2_{X_i} \tag{9}$$

    end for
  end if
end for

For each precedence relation, compute
the new delay distribution
based on
Maximum of Gaussian analytic expression

Analytical algorithm



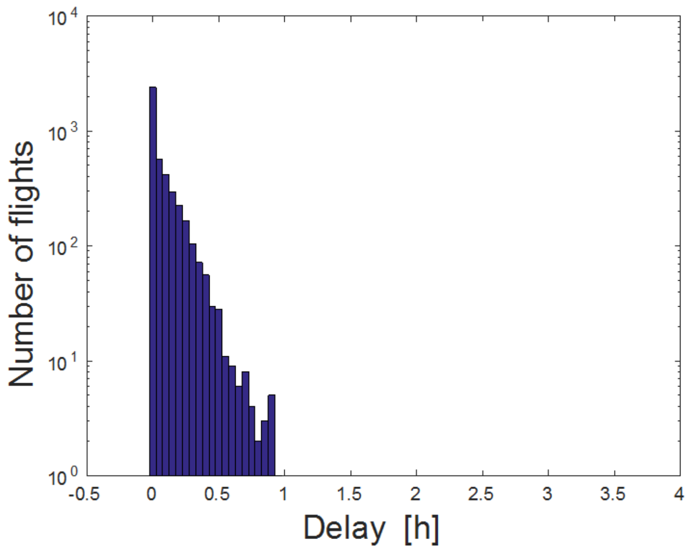
- Input: a priori delay distribution for each flight, expressed as Gaussian
- Output: delay distributions accounting precedence

Pro: Fast computing time

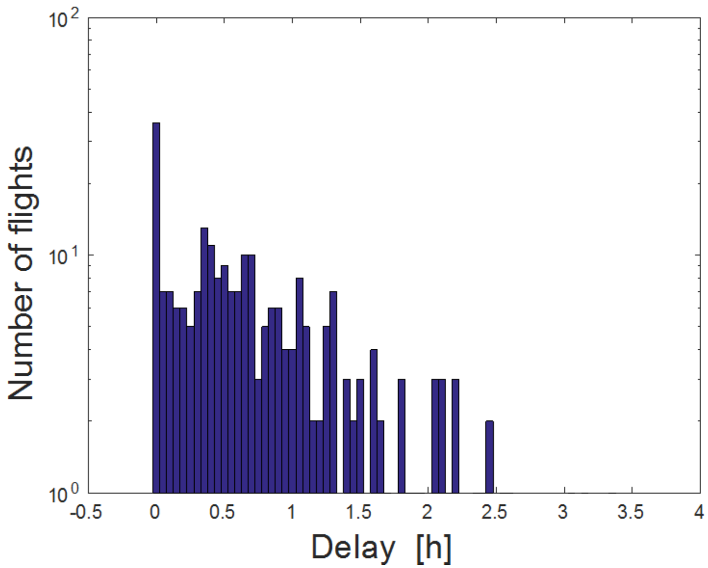
Cons: Needs to have the a priori delay expressed as a Gaussian distribution

Impact of precedence on the network

- Flights later in the schedule of an aircraft tend to be more affected by precedence delays: repartition of mean delay of flights



Flights second in daily schedule of an aircraft



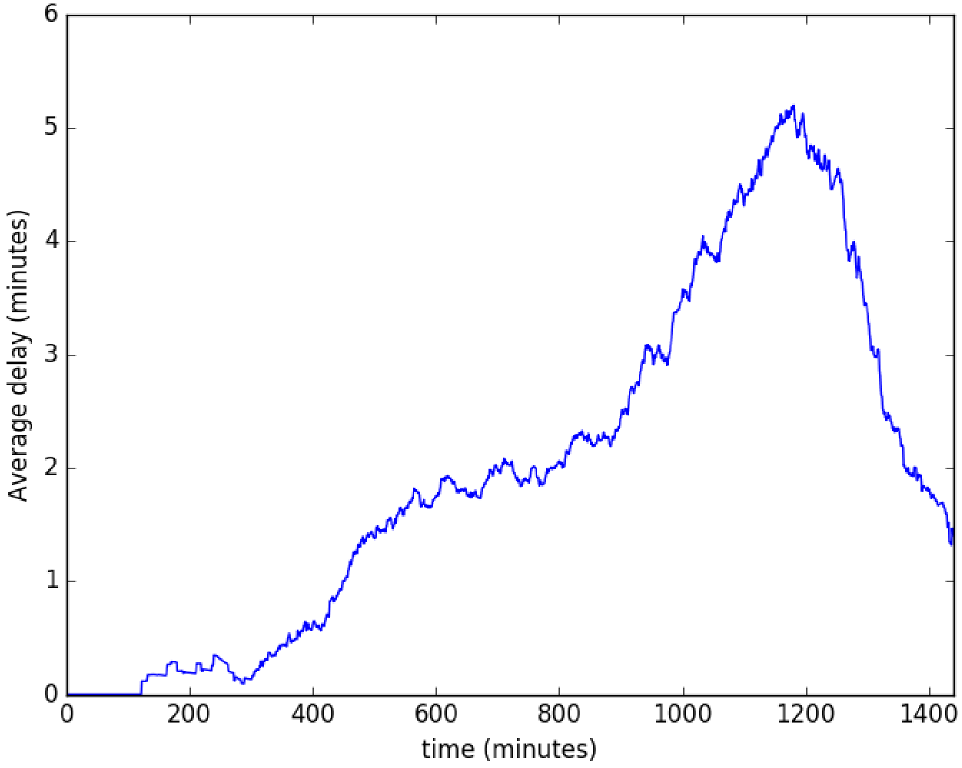
Flights at the end of the daily schedule of an aircraft

Impact of precedence on the network

- Later legs of an aircraft tend to be more affected by precedence delays

Impact of precedence on the network

The average delay of flights delayed, throughout the day



Conclusion

Contribution to the COPTRA project:

- An algorithm computing the probabilistic occupancy count for each sector at any time of the day based on the trajectories and delay distribution of each flight.
- Given a priori delay distributions on each flight, we update the delay distribution to include the impact of precedence between flights with a Monte Carlo algorithm or an analytic algorithm.

Possible future work:

- Consider acyclic relations of precedence. This promises algorithmic challenges.

Questions?