

**CORRUPTION:
A CONTROL THEORY APPROACH**

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ABSTRACT

This paper attempts an analysis of the problem of corruption, in a dynamic optimization framework. A simple ecological balance model is developed to study the evolution of the corrupt and non-corrupt populations. Government policies, used to reduce corruption, are oriented towards the maximization of the utility functional of the representative non-corrupt individual, and it is shown that for some values of the structural parameters it is not optimal to eliminate corruption, even in the long run. Furthermore, restricting our attention to the case where the two populations do not vanish, allowing for the participation of corrupt agents in the decision process not only increases the long run level of corruption, but may also give rise to the non-existence of an optimal policy leading to this steady state.

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1. INTRODUCTION

In a few instances where the economic literature has dealt with the problem of corruption, the marginal revenue/marginal cost approach of standard microeconomic analysis is used to explain why agents engage in those illegal practices. As variations on this theme we find different market behaviour assumptions, ranging from the monopoly of either the 'buyer' or the 'seller', to the 'competitive market' hypothesis. Examples are Becker (1968) and Rose-Ackerman (1975,1978). The static framework is a common characteristic to all these¹.

The recent steps taken by the Portuguese authorities to intensify the struggle against corruption², regarded as a rising problem in our economy and society³, have inspired us to a somewhat different approach: Departing from a social objective function, we consider a dynamic maximization framework, and discuss the evolution over time of the corrupt and non-corrupt populations, that is, the number of corrupt and non-corrupt individuals in the economy. We hope this will help to understand corrupt/non-corrupt interactions left unexplained by other studies.

Throughout this paper corruption means any deviation of resources from the non-corrupt to the corrupt populations, all individuals engaged in this transfer being (or becoming) members of the later. A typical example is the case of a bribe: an individual with discretionary power in the attribution of a public contract does not assign it to the most efficient individual (or firm); instead, he chooses a different agent, and receives either money or an in kind payment (the bribe) from the later. Both the briber and the bribee are corrupt agents, according to the definition given above. Note that the value of the transfer involved is, in general, larger than that of the bribe.

We assume that corruption can be reduced by the action of the economic authority; in our model, however, criteria to decide upon the opportunity or level of policy actions are not based on ethical considerations, but only on economic grounds. Using a simple ecological balance model⁴, we show that an optimal policy,

¹ Yaniv (1975) can be seen as an exception.

² Namely, Government Law n. 369 of the 6/10/83.

³ In its first year in office, the High Authority Against Corruption is already investigating 380 processes ("Alta Autoridade abre 150 novos processos...", in EXPRESSO, 14 July 1984, page 2, col. 1).

⁴ Hirsch and Smale (1974) provides an introduction to ecological balance models. For an (amusing) application to macroeconomics

in the sense that discounted utility of the representative non-corrupt individual is maximized, exists. For particular values of the structural parameters we show that it is not optimal to eliminate corruption, even in the long run. Also, if corrupt agents are allowed to participate in the decision process, the long run level of corruption will be increased, for long run solutions where both populations coexist; again for some values of the parameters of the model, an optimal policy leading to this steady state will not exist, and either the economy will engage in cyclical behaviour, or one of the populations will disappear.

2. THE MODEL

Consider an economy with a stationary population, formed by corrupt and non-corrupt individuals in number of C and NC , respectively. Therefore,

$$P = C + NC, \quad \dot{P} = dP/dt = 0 \quad (1)$$

As previously indicated, corruption is reduced by the action of the economic authority, who decides on the amount of resources that are to be allocated to that end, this quantity being denoted by L . Policies act by directly reducing the number of corrupt individuals, which is expressed by the law of motion of that population:

$$\dot{C} = \gamma vC - \theta L, \quad \dot{C} = 0 \text{ if } C = P \text{ or } C = 0 \quad (2)$$

where v represents the resources deviated by each element of the corrupt population, vC being total resources (or production) deviated from one population to the other, and γ is the rate at which corrupt activities generate an increase in the corrupt population. θ measures the efficiency of resources allocated by the economic authority to the reduction of corruption, and depends on the institutional environment, namely legislation on the prevention and punishment of corrupt activities, moral integrity of the people exerting political and economic power, existing practices in the public sector, etc. γ , v and θ are assumed to be positive constants⁵. Note that any decrease in the corrupt

see Snower (1982).

⁵ Optimizing behaviour of corrupt agents, that is, choice of v such that discounted utility of these agents is maximized, is a possible extension of this paper.

population, due to a policy action, by (1) also implies an increase in the non-corrupt population.

Any policy has an opportunity cost⁶, equal to the corresponding reduction in W , the production of consumption goods available to the non-corrupt population. Therefore, W is given by:

$$W = \max\{\alpha \cdot NC - L - (1 + \xi)\nu C, 0\} \quad (3)$$

In this equation α is the average productivity of non-corrupt individuals, and ξ represents the efficiency cost involved in corrupt activities (see Johnson (1975) and Rose-Ackerman (1978)). These coefficients are also assumed to be constant. The max condition comes from the non-negativity of W ; for C big enough, resources deviated by each corrupt agent will be less than ν , because these agents are already absorbing all the production of the non-corrupt sector. A similar condition holds for L :

$$0 \leq L \leq \max\{\alpha \cdot NC - (1 + \xi)\nu C, 0\}$$

Defining :

$$\psi = (1 + \xi)\nu$$

$$\eta = \gamma\nu$$

$$l = L/NC$$

$$c = C/NC$$

$$w = W/NC$$

we rewrite equations (2) and (3) as:

$$\dot{c} = (\eta c - \theta l)(1 + c) \quad (4)$$

$$w = \max\{\alpha - l - \psi c, 0\} \quad (5)$$

⁶ Which is used to measure L .

and figure 1 can be drawn. In this figure points A, B and C have coordinates $(0, \alpha)$, $(\alpha/\psi, 0)$ and $(\alpha\theta/(\eta+\psi\theta), \alpha\eta/(\eta+\psi\theta))$, respectively. The feasible points, that is, that verify $c, l, u \geq 0$, are the points on and inside the triangle OAB, or on the positive half of the horizontal axis. This area is divided by the $\dot{c}=0$ or $l=\eta c/\theta$ line⁷; above this line c is decreasing, and conversely. Notice that for $c > c_m$ the economy will move inevitably to $c=0$.

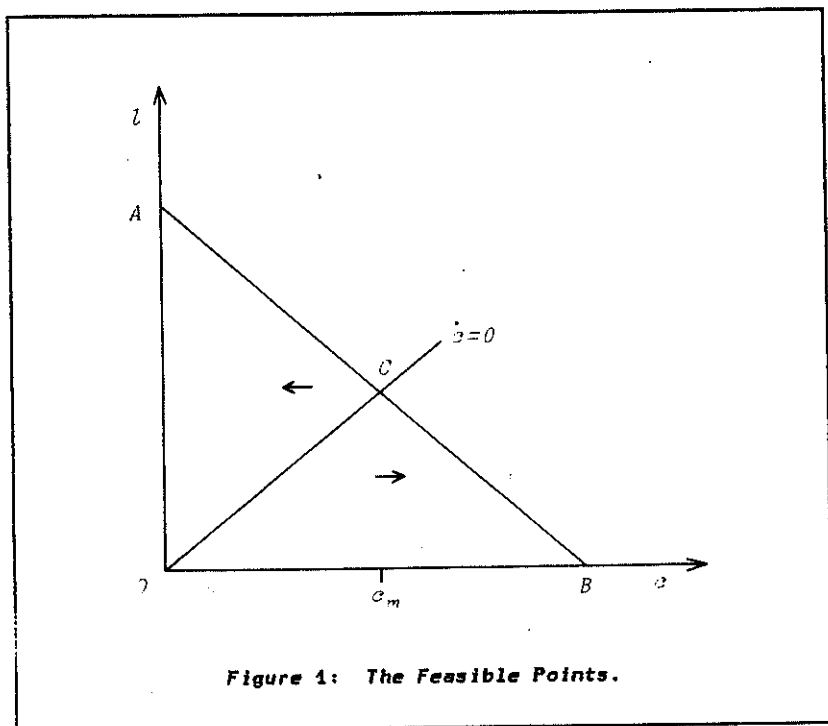


Figure 1: The Feasible Points.

The representative non-corrupt individual maximizes his discounted utility; accordingly, his objective function is:

⁷ It could be argued that on the non-negative half of the vertical axis $\dot{c}=0$ also holds. However, notice that point O is strictly preferred to any other point on this line where $l > 0$, since by moving to O consumption can be increased at no cost (corruption is already eliminated). Point O is on the $l=\eta c/\theta$ line.

$$\max \int_0^{\infty} U(W/NC) \cdot \exp(-\beta t) \cdot dt \quad (6)$$

which is also the objective function of the economic authority. β is the social rate of discount, a fixed constant, and $U(\cdot)$ is a twice differentiable monotonically increasing function with negative second derivative, for all positive values of W/NC . We also assume that:

$$\lim_{x \rightarrow 0^+} U(x) = -\infty \quad (7)$$

Hence, having in mind the previous remark about a state $c > c_m$, by (6) and (7) we know that any such state will be avoided; also $c = c_m$ will be avoided, because $\dot{c} = 0$ can only be obtained if $w = 0$. In what follows we will assume that the initial state, that is, the ratio C/NC at $t=0$, which is denoted by c_1 , verifies $c_m > c_1 + \delta$, where δ is a positive but very small number⁸. Therefore, as the representative individual will maintain $w > 0$, the max condition in (5) can be ignored, and this equations reads:

$$w = \alpha - 1 - \psi c$$

Accordingly, the policy problem (the choice of L) can be rewritten as:

$$\max \int_0^{\infty} U(\alpha - 1 - \psi c) \cdot \exp(-\beta t) \cdot dt$$

$$\text{s. t.} \quad \dot{c} = (\eta c - \theta 1)(1 + c)$$

$$\alpha - 1 - \psi c \geq 0$$

$$c(0) = c_1$$

$$c, 1 \geq 0$$

⁸ The case $c_1 \geq c_m$ is, in our opinion, of little practical interest.

3. THE OPTIMAL POLICY

The optimal time path for l (and the associated time path for c) can be obtained by solving the above control problem. The current value Hamiltonian associated with this problem⁹ is:

$$H = U(\alpha - 1 - \psi c) + \lambda(\eta c - \theta l)(1 + c)$$

where λ is the costate variable. Necessary conditions¹⁰ are:

$$\partial H / \partial l = -U' - \theta \lambda (1 + c) = 0 \quad (8 a)$$

$$\partial H / \partial c = -\dot{\lambda} + \beta \lambda = -\psi U' + \lambda[\eta(1 + c) + \eta c - \theta l] \quad (8 b)$$

$$\partial H / \partial \lambda = \dot{c} = (\eta c - \theta l)(1 + c) \quad (8 c)$$

where (8 c) merely reproduces (4). Solving (8 a) for λ , we obtain:

$$\lambda = -U' / [\theta(1 + c)]$$

that implies, by differentiation:

$$-\dot{\lambda} = \lambda[(\dot{l} + \psi \dot{c})U''/U' + \dot{c}/(1 + c)]$$

Using these two equations, (8 b) can be written as :

$$\dot{l} = [(\eta + \psi \theta)(1 + c) - \beta]U'/U'' - \psi(\eta c - \theta l)(1 + c) \quad (9)$$

Hence, optimal time paths for l and c can be obtained by solving together the differential equations (8 c) and (9).

⁹ Here we assume that constraints $c, l, w \geq 0$ are not binding, i. e., interiority of the solution. The general case is discussed in appendix A.

¹⁰ Due the assumptions made on the form of $U(\cdot)$ these conditions are also sufficient.

4. LONG RUN SOLUTIONS AND ADJUSTMENT PATHS

Depending on the values of the parameters, we can have different long run solutions¹¹. First consider that $\beta > \eta + \gamma\theta$. In this case the long run solution is:

$$\bar{c} = \beta / (\eta + \gamma\theta) - 1 \quad (10)$$

$$\bar{l} = \eta \bar{c} / \theta \quad (11)$$

A first implication to be noted is that elimination of corruption is not optimal, even in the long run. The cost incurred with such a policy outweighs the value of the additional consumption that can thus be obtained. But other important issue is whether an optimal adjustment path leading to the long run solution exists or not. Assume that the initial state of the economy c_1 is near the above long run equilibrium state $\bar{c}$, given by (10); by linearizing (8 c) and (9) around that equilibrium point, that is, looking for optimal time paths for c and l in the neighbourhood of $\bar{c}$, we have:

$$\begin{bmatrix} \dot{l} \\ \dot{c} \end{bmatrix} = (1 + \bar{c}) \begin{bmatrix} \theta\psi & \frac{\eta + \gamma\theta}{1 + \bar{c}} \frac{U'}{U''} - \gamma\eta \\ -\theta & \eta \end{bmatrix} \begin{bmatrix} l - \bar{l} \\ c - \bar{c} \end{bmatrix}$$

where U' and U'' are evaluated at the point $(\bar{c}, \bar{l})$. The behaviour of this system is characterized by the two eigenvalues of the matrix on the right-hand side. The trace of this matrix is positive, since $\theta\psi + \eta > 0$, and its determinant is:

$$\Delta = \theta \frac{\eta + \gamma\theta}{1 + \bar{c}} \frac{U'}{U''} < 0$$

¹¹ By long run solution we mean a pair (c, l) verifying feasibility $(c, l, w \geq 0)$, stationarity $(\dot{c} = \dot{l} = 0)$ and optimality $(\dot{c}$ and $\dot{l}$ given by (8 c) and (9)); this pair is denoted by $(\bar{c}, \bar{l})$. We are disregarding the trivial case $c_1 = 0$, where $\dot{c} = 0$ for all values of l , $c_1 = \bar{c} = \bar{l} = 0$ and there is no motion at all.

Thus, we conclude that one of the eigenvalues is positive, since their sum is positive, and the other negative, because their product is negative. Therefore, we have a saddle point, which means that there is only one optimal path taking the economy from the initial state c_1 to the long run solution $(\bar{c}, \bar{l})$. Not only an optimal time path for the control variable and, hence, for the state variable, exists (in the neighbourhood of the equilibrium state), but also it is unique. The corresponding phase diagram is presented in figure 2.

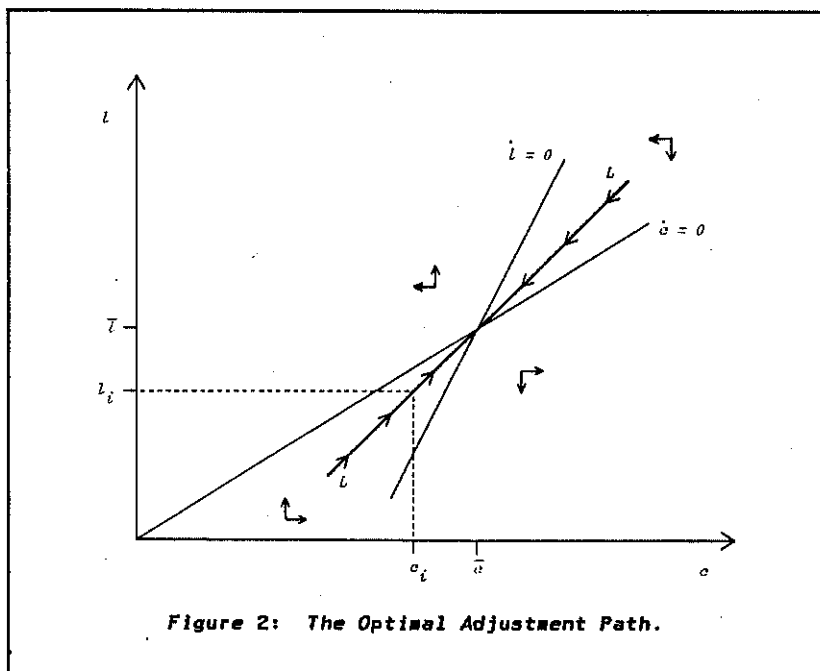


Figure 2: The Optimal Adjustment Path.

In the vicinity of the long run equilibrium $(\bar{c}, \bar{l})$ the slope of the $\dot{c}=0$ locus is:

$$dl/dc|_{\dot{c}=0} = \eta/\theta$$

and that of the locus $\dot{l}=0$ is:

$$\frac{dl}{dc}\bigg|_{\dot{l}=0} = \frac{\eta}{0} - \frac{\eta + \psi\theta \cdot U'}{\psi\theta(1+\bar{c}) U''} > \frac{\eta}{0}$$

In the linearized model the equation of the separatrix LL, whose points are the only verifying optimality in the motion to the equilibrium point $(\bar{c}, \bar{l})$, is (see appendix B):

$$l = \bar{l} + \frac{1}{2} \left[\frac{\eta}{\theta} - \psi + \sqrt{\left(\frac{\psi + \frac{\eta}{\theta}}{\theta} \right)^2 - \frac{4 \cdot \frac{\eta}{\theta} + \theta \psi \cdot U'}{\theta(1+\bar{c}) U''}} \right] (c - \bar{c})$$

Notice that at $t=0$ l can be assigned any value on the $c=c_1$ line, since its initial value is not given; only its dynamics are constrained (by condition (9)) which means that adjustment to the equilibrium point along the stable arm of the saddle point can always take place. Hence, starting from an initial state $c_1 < \bar{c}$, the economic authority sets $l=l_1$, and the economy will move along LL to the equilibrium point $(\bar{c}, \bar{l})$; along the adjustment path l and c will be increasing. If $c_1 > \bar{c}$ the opposite behaviour should be observed.

Now consider the particular case $\beta = \eta + \psi\theta$. This will imply $\bar{c} = \bar{l} = 0$, meaning that the origin is now the equilibrium point¹². Moreover, since neither l nor c can be negative, we can only have trajectories in the first quadrant, i. e., only points (and paths) above $\bar{l}$ and to the right of $\bar{c}$ can be considered. Optimal motion to the equilibrium point may still be verified, along the right half of the LL line¹³. In this case corruption is eliminated in the long run.

Thirdly, let us consider the case where $\beta < \eta + \psi\theta$. The long run solution is no longer given by the above formulae, since c and l must be non-negative. From the control problem associated with this particular case, (see appendix A) we get the same system of differential equations. However, unlike in the first case, either of the constraints $c > 0$ and $l > 0$ will become active at some date; moreover, the long run solution can only be $\bar{c} = \bar{l} = 0$. This point can be made clear by an heuristic argument: assume that the dynamics are such that at a given moment the line $c=0$ is attained. Then, since $\dot{c}=0$ ¹⁴, it is optimal to set $l=0$ (l has no beneficial effect, once corruption is already eliminated). Now suppose that the $l=0$ axis is attained. In this case, because $c > 0$ and $\dot{c} = \eta c(1+c) > 0$, c will be increasing - such a point is obviously not a steady state.

¹² Either $c_1=0$, and there is no motion, therefore $c_1=\bar{c}=\bar{l}=0$, or $c_1 > 0$ and, as proved in appendix A, $\bar{c}=\bar{l}=0$.

¹³ Consider $c_1 > 0$. Linearizing about the equilibrium point $(0,0)$ does not introduce any alteration in the previous analysis.

¹⁴ At $c=0$, $\dot{c}=-\theta l$, $c > 0$, $l > 0$ imply $\dot{c}=0$.

5. CORRUPT AGENTS AMONGST THE DECISION MAKERS

In this section we consider that corrupt individuals can influence policy decisions, either in its formulation or its execution, in such a way that their utility functional should be taken into account. Before we have assumed that the level of activity of these individuals is constant (at v); henceforth, we will consider that their instantaneous utility- denoted by V - depends only on the risk of being caught and punished, which can be assumed to be a decreasing function of c ¹⁵. Accordingly, the economic authority now maximizes:

$$\max \int_0^{\infty} U(\alpha - 1 - \psi c) \cdot \exp(-\beta t) \cdot dt + \Psi \int_0^{\infty} V(c) \cdot \exp(-\beta t) \cdot dt$$

where Ψ denotes the relative strength of the two populations in the decision process. If the corrupt agents play no role in this process, $\Psi=0$ and the previous analysis applies; in the other extreme $\Psi=\infty$, $l=0$ and c will be increasing ($\dot{c}=\eta c(1+c)$) until the extinction of the non-corrupt population ($C=P$). Here the intermediate case is of more interest, and we will restrict our analysis to interior solutions¹⁶.

Proceeding, as before, to derive the long run equilibrium of the state and control variables, we have:

$$\dot{c} = (\eta c - \theta l)(1 + c) \quad (12)$$

$$\dot{l} = [(\eta + \psi \theta - \psi \theta V'/U')(1 + c) - \beta]U'/U'' - \psi \dot{c} \quad (13)$$

that implies:

¹⁵ The higher the number of corrupt individuals the lower the likelihood that a particular one is the one who gets caught. also assume that, for a given variation in c , variations in the likelihood of being caught are smaller the higher the initial level of c . Therefore, the function V has positive first derivative, since an increase in c , reducing the risk of being caught, increases their utility level; Also $V'' < 0$, because as c increases, reductions in the risk level (hence increases in utility) become smaller.

¹⁶ In this case, $\beta > \eta + \theta(\psi - \psi V'/U') > 0$. This implies $\bar{c} > 0$ (see appendix C).