

Signature of Phobos' interior structure in its gravity field and libration

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ABSTRACT

The interior of the Martian moon Phobos has not been precisely determined yet, in spite of space missions sent at close distance to the body. The current measurements (imagery, astrometric, etc.) lead to controversial conclusions about the level of heterogeneity inside Phobos. Yet, the inside mass distribution is a signature of the conditions prevailing at its formation as well as of its evolution. Here, we study possible heterogeneous mass distributions based on internal models built from surface observables and available bulk density and shape measurements. We identify four different families of mass distribution involving rocky material, (macro-) porosity and ice. Mass heterogeneities correspond to either an excess of porosity or a compaction of material under Stickney crater, or a deficit of porosity in the upper layers of Phobos, or a concentration of ice either in depth inside Phobos or in shallow layers. We then discretize the shape of Phobos using 500m-length cubes to fit its volume and total mass. We compute the possible distribution of these cubes for each family of internal mass heterogeneity model. We deduce the possible values of the principal moments of inertia as well as of the geodetic observables such as the libration amplitude and the gravity field anomalies (up to degree and order 10) associated with these models. A comparison of these computed observables between the different heterogeneity models and with their expected homogeneous mass distribution values allow us to quantify the possible heterogeneity degree within Phobos. The computed heterogeneity observables can depart by tens of percent from the homogenous values. The most striking departures are from the interior model with mass excess (less porosity) or deficit (more porosity) beneath Stickney crater. In turn, measurements of libration amplitude and low degree coefficients of the gravity field at a precision better than 5% can allow to identify such kinds of heterogeneities mainly located beneath Stickney. Mass excess or deficit can therefore be also distinguished, which is of importance to identify whether Phobos was already porous or monolithic before the formation of Stickney. The current measurements of libration amplitude and degree-two gravity coefficient are quite uncertain, but seem to reject models with higher porosity under Stickney, favoring a pre-impact porous body. The icy models depart less from the homogenous signatures, hence requiring more precise measurements of the geodetic observables and of the shape of Phobos. The improvement of these measurements by the Mars Moon Explorer mission for instance could thus allow for better constraining our model of Phobos' interior and bring further constraints for its formation.

1. Introduction

Phobos' origin is still mysterious despite the numerous missions already sent to the Martian system within the last decades. Where does Phobos come from? How did it form? Learning whether Phobos and Deimos co-accreted with Mars, were captured from the outer solar system by the proto-Mars disk, or coalesced after a giant impact into Mars would provide a key piece to the puzzle of Mars accretion. It would also reveal the true nature of these bodies, which are targets for sample return and will likely be the first ports for human explorers in the 2030s. To unveil the mystery of their origin, the science community is pushing the space agencies to dedicate a mission to the Martian moons. The mission concepts recently proposed by different groups across the world: Pandora (Raymond et al., 2015), PADME (Lee et al., 2015), MERLIN (Murchie et al., 2015), DePhine (Oberst et al., 2018) and the recently selected Japanese Phobos sample return mission named Martian Moons eXploration (MMX) (Fujimoto et al., 2017; Kuramoto et al., 2017; 2018); all agree with the fact that the origin of Phobos can be revealed by combining different kinds of observation

(e.g. imagery, spectroscopy, radio-science, sample return, etc.).

It is not straightforward to relate interior properties to the origin, but it is easy to agree that interior properties of any body have to be accounted for to explain its formation and evolution. What event could explain such an internal structure? Where should this object have formed to present such interior characteristics and composition? Interior properties constitute important benchmarks against which origin models must be gauged. Because of its small size and mass, Phobos could have very puzzling interior structure, which could affect differently its geodetic observables, like the libration amplitude and the gravity field with respect to the expected values for homogeneous mass distribution. We can reasonably expect larger signatures for larger heterogeneous mass distribution. Therefore, probing the interior structure of Phobos by precisely determining its gravity field and libration should allow us to ascertain whether large scale heterogeneity exist inside the satellite and perhaps trace back its origin.

Radio-science data, sensitive to the mass distribution inside the body, are commonly used to determine the gravity field of a celestial body through precise orbit determination of spacecraft around the body

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or just flying by it. In addition to gravity, these data¹ are sensitive to the rotational motion of the body (e.g. Konopliv et al. (2018)). It has been shown for the particular case of Phobos, that a hypothetical spacecraft performing 2-polar flybys at about 10 km from the surface of Phobos and orbiting it during one week in a low-latitude retrograde orbit at an altitude ranging from 2 to 10 km would allow determining the gravity field up to degree and order ten and the libration amplitude with a precision better than 1% (Le Maistre et al., 2015; 2017). The rotation state of Phobos can also be determined with even more accuracy from a radio-science experiment onboard a landed probe (Le Maistre et al., 2013). Moreover, if equipped with a gravimeter, such a hypothetical lander mission would allow for a determination of the surface acceleration from which the density anomaly of the landing site could be inferred and the libration amplitude estimated.

This paper aims at answering the question of what we can say about the interior structure of Phobos from the analysis of geodesic data provided by a future mission to Phobos. We first describe the method used to model the interior of Phobos and compute the geodesic observables (Section 2). Section 3 presents the interior properties of a homogeneous Phobos, used as a reference for the interpretation of the heterogeneous models observables computed in Section 5. In Section 4 we discuss the current knowledge on Phobos' interior and define the specific interior model families considered in our analysis. Section 5 shows how the moments of inertia, the libration amplitude, the offset between the center of mass and the center of figure, the degree-2 gravity coefficients, the 10×10 gravity field and the surface acceleration vary as a function of heterogeneous interior structures of Phobos. The last Section 6 gathers the main results and concludes the paper.

Note that we often refer to the “homogeneous values or model” in the present paper to talk about parameters derived from a constant-density shape model.

2. Method

The gravitational potential at a field-point of coordinates (r, ϕ, λ) , where r is the distance to Phobos' center and ϕ, λ the latitude and longitude of the point respectively, is usually represented as a spherical harmonic expansion characterized by the $C_{l,m}$, $S_{l,m}$ coefficients (Kaula, 1966) according to:

$$U(r, \phi, \lambda) = \frac{GM}{r} \sum_{l=0}^{\infty} \sum_{m=0}^l \left(\frac{R}{r} \right)^l \bar{P}_{l,m}(\sin \phi) (\bar{C}_{l,m} \cos m\lambda + \bar{S}_{l,m} \sin m\lambda), \quad (1)$$

where G is the gravitational constant, M is the total mass of the body and R is a reference radius, usually equal to the radius of the Brillouin sphere (encompassing the whole body). $P_{l,m}$ is the Legendre function of degree l and order m . The bar symbol denotes normalized quantities obtained as follows

$$P_{l,m} \begin{bmatrix} C_{l,m} \\ S_{l,m} \end{bmatrix} = N_{l,m} P_{l,m} \begin{bmatrix} C_{l,m}/N_{l,m} \\ S_{l,m}/N_{l,m} \end{bmatrix} = \bar{P}_{l,m} \begin{bmatrix} \bar{C}_{l,m} \\ \bar{S}_{l,m} \end{bmatrix}, \quad (2)$$

with $N_{l,m} = \sqrt{(2 - \delta_{0,m})(2l+1)(l-m)!/(l+m)!}$ where the Kronecker delta $\delta_{0,m}$ is equal to 1 for $m=0$, and $\delta_{0,m}=0$ otherwise. Dimensionless coefficients of degree l and order m , $C_{l,m}$ and $S_{l,m}$, encode information about the mass distribution into the celestial body at a given scale, showing thereby their great geophysical interest. This scale is typically half of the wavelength (λ_h) of the considered harmonic

$$\lambda_h = \frac{2\pi R}{l} \quad (3)$$

¹ Radio-science data are commonly combined with optical data to determine the rotational motion of celestial bodies (e.g. Konopliv et al. (2002, 2014, 2018)).

Given the above rule of thumb, one can expect for instance for a 10×10 gravity field of Phobos, a spatial resolution at the 3–4 km level.

The degree-2 gravity coefficients condition the inertia tensor according to

$$\begin{aligned} I_{xx}/MR^2 &= I/MR^2 + C_{2,0}/3 + 2C_{2,2}, \\ I_{yy}/MR^2 &= I/MR^2 + C_{2,0}/3 - 2C_{2,2}, \\ I_{zz}/MR^2 &= I/MR^2 - 2C_{2,0}/3, \\ I_{xy}/MR^2 &= -2S_{2,2}, \\ I_{xz}/MR^2 &= -C_{2,1}, \\ I_{yz}/MR^2 &= -S_{2,1}, \end{aligned} \quad (4)$$

with $I = (I_{xx} + I_{yy} + I_{zz})/3$, while the main moments of inertia (I_{xx}, I_{yy}, I_{zz}) drive the amplitude of the physical librations, which are periodic variations around a uniform rotational motion of the body. Indeed, the amplitude of the librational motion of Phobos depends on the distribution of mass inside the moon, consequently bearing the signature of its internal structure, according to (e.g. Duxbury and Callahan (1989), Willner et al. (2010))

$$\theta = \frac{2e}{1 - \frac{I_{zz}}{3(I_{yy} - I_{xx})}}, \quad (5)$$

where $e = 0.01511$ is the orbital eccentricity of Phobos, assuming the off-diagonal moments close to zero.

In order to study the effect of Phobos's internal mass distribution on its gravity field, rotation, and moments of inertia, we build a 3-dimensional model of its interior by discretizing its volume in a set of cubes of equal size following the method first proposed by Rosenblatt et al. (2010) and used later on by Pätzold et al. (2014b). A precise determination of the shape and volume of Phobos have recently been obtained by Willner et al. (2014) (see Fig. 1). The number of cubes, or equivalently the cube edge length should be chosen such that the difference between the approximate volume from our cubic discretization and Phobos' volume from its shape is below the measured uncertainty of Phobos' volume of 35 km³ or a relative error of about 0.6% (Willner et al., 2014). As can be seen in Fig. 2 already for a cube edge length of 1400 m the relative error on the volume is smaller than 0.5%. A further constraint for the cube edge length is provided by the measured libration amplitude of Phobos if it is assumed that Phobos' interior has a uniform density. The actual relative error on this measurement is about 3% (Burmeister et al., 2018), which is larger than that of a model which uses cubes with an edge length of 1350m (see Fig. 2). In this paper the cube length is chosen to be equal to $d = 500$ m, which implies the use of about $N_{tot} \simeq 46,000$ cubes in total to fill the volume of Phobos. With such a cube length, the relative errors on the $C_{l,m}$ and $S_{l,m}$ up to degree and order five are all smaller than 3–5% of the coefficients computed with 50 m cubes, except for $C_{5,3}$ (15%), and $S_{3,2}$ (10%), $S_{5,2}$ (30%) and $S_{5,4}$ (25%). Stokes coefficients of degree and order larger than five of our models with cube edge length of 500 m can have relative errors of several tens of percent if compared to a model which has cubes of 50 m edge lengths. Since all models considered in the present study are affected by the same systematic errors they qualify for a comparative study also for degrees larger than five. However, when real data will be available at degree above five, smaller cubes are required to compare predicted gravity fields to observations. The discretization of the volume of Phobos is being done using the NAIF-JPL SPICE toolkit (Acton, 1996) and in particular using the so-called DSK model of Phobos matching the shape of Willner et al. (2014).

In order to compute models of mass heterogeneity inside the discretized volume of Phobos, we proceed as follows:

1. First we choose the proportion of porosity (i.e. N_v the number of cubes of void with $\rho_v = 0$ kg/m³) and water ice content (i.e. N_{ice} the number of cubes of constant density $\rho_{ice} = 940$ kg/m³) from which the number of cubes of rock $N_r = N_{tot} - N_v - N_{ice}$ is deduced to best

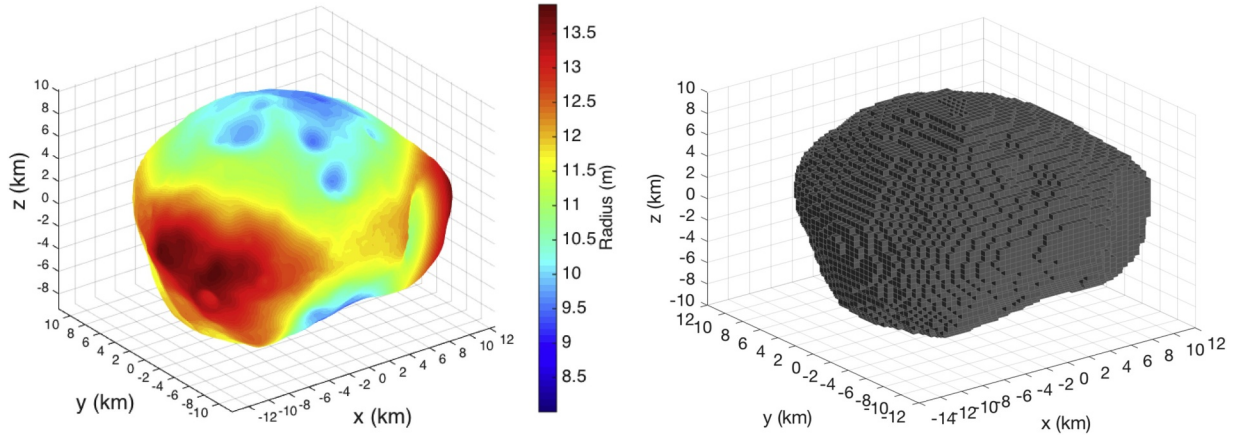


Fig. 1. Phobos volume from Willner et al. (2014) and discretization of volume (cube edge length: 500 m).

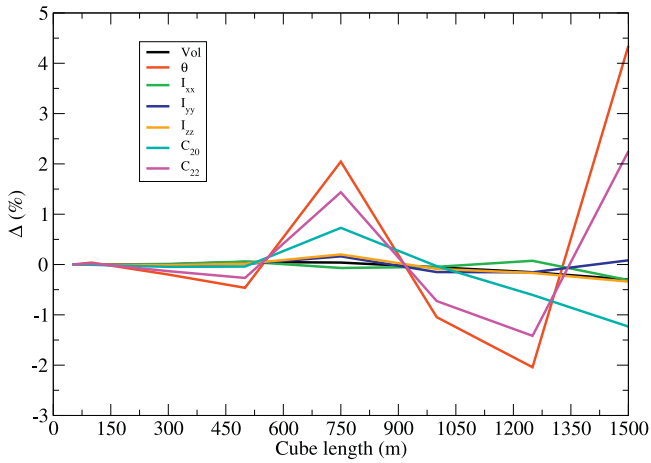


Fig. 2. Relative error resulting from the volume discretization as a function of the cube edge length on the volume Vol, the libration amplitude θ , the principal moments of inertia I_{xx} , I_{yy} , and I_{zz} , and the second degree-order zero and second degree-order two gravity field coefficients, $C_{2,0}$ and $C_{2,2}$ respectively. It is assumed here that Phobos is of uniform density.

fit the shape of Phobos (Willner et al., 2014) and the density of the rocky material ρ_r is calculated exactly to match the total mass of Phobos $M = 1.06 \times 10^{16}$ kg (Pätzold et al., 2014b) according to

$$\rho_r = \frac{M - \rho_{ice} d^3 N_{ice}}{d^3 N_r}. \quad (6)$$

- Then the position of each cube of void, ice and rock is randomly chosen while respecting the characteristics of the interior model family being investigated (more or less porosity under Stickney for example, see description of the interior families in Section 4).
- Finally the physical characteristics (gravity field, libration amplitude and moment of inertia) are computed for this particular internal mass distribution as explained below.

In step 1, N_v and N_{ice} are picked up within the ranges [0,46]% and [0,64]%² of N_{tot} respectively in order to get $\rho_r \in [2250, 3500]$ kg/m³ as proposed by Rosenblatt (2011). For a given set of (N_v , N_{ice}) steps 2 and 3 are repeated 500 times to allow making statistics, i.e. to characterize the parameter distributions for each family. Note that depending on the

² Note that this range extends much more than the 0–20% obtained by Pätzold et al. (2014b) who kept the porosity within $30 \pm 5\%$.

ranges of proportions of ice and porosity considered and depending on the chosen sampling pitch in these ranges, the number of models per family varies from 3000 to 10,000.

There are different ways to compute the spherical harmonic coefficients $C_{l,m}$ and $S_{l,m}$ corresponding to a given interior model, i.e. for a given cube distribution inside Phobos. We can (1) compute the coefficients of each block and sum them to predict the gravity coefficients of the whole body or (2) compute the gravitational potential by summing the potential of each cube and then express the so-obtained gravity potential of the whole Phobos in the form of spherical harmonic expansion. Here we decide to use the first procedure based on the recurrence relationships proposed by Werner (1997). This method allows us to compute the Stokes normalized coefficients $\bar{C}_{l,m}^i$ and $\bar{S}_{l,m}^i$ of the i th cube according to:

Sectorial ($l = m$):

$$l = 0: \begin{bmatrix} \bar{C}_{0,0}^i \\ \bar{S}_{0,0}^i \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad (7)$$

$$l = 1: \begin{bmatrix} \bar{C}_{1,1}^i \\ \bar{S}_{1,1}^i \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} x_i/R \\ y_i/R \end{bmatrix}, \quad (8)$$

$$l > 1: \begin{bmatrix} \bar{C}_{l,l}^i \\ \bar{S}_{l,l}^i \end{bmatrix} = \frac{2l-1}{\sqrt{2l(2l+1)}} \begin{bmatrix} x_i/R & -y_i/R \\ y_i/R & x_i/R \end{bmatrix} \begin{bmatrix} \bar{C}_{l-1,l-1}^i \\ \bar{S}_{l-1,l-1}^i \end{bmatrix}, \quad (9)$$

Subdiagonal ($m = l - 1$):

$$\begin{bmatrix} \bar{C}_{l,l-1}^i \\ \bar{S}_{l,l-1}^i \end{bmatrix} = \frac{2l-1}{\sqrt{2l+1}} \frac{z_i}{R} \begin{bmatrix} \bar{C}_{l-1,l-1}^i \\ \bar{S}_{l-1,l-1}^i \end{bmatrix}, \quad (10)$$

Vertical (for a given m):

$$\begin{bmatrix} \bar{C}_{l,m}^i \\ \bar{S}_{l,m}^i \end{bmatrix} = (2l-1) \sqrt{\frac{2l-1}{(2l+1)(l+m)(l-m)}} \frac{z_i}{R} \begin{bmatrix} \bar{C}_{l-1,m}^i \\ \bar{S}_{l-1,m}^i \end{bmatrix} - \sqrt{\frac{(2l-3)(l+m-1)(l-m-1)}{(2l+1)(l+m)(l-m)}} \left(\frac{r_i}{R}\right)^2 \begin{bmatrix} \bar{C}_{l-2,m}^i \\ \bar{S}_{l-2,m}^i \end{bmatrix}, \quad (11)$$

where (x_i, y_i, z_i) denote the cartesian coordinates of the center of the i th cube and $r_i^2 = x_i^2 + y_i^2 + z_i^2$. For each of our interior models, the center location, the mass (m_i), the volume (d^3) and consequently the density (ρ_i) of each cube is well known. Thus, the gravitational potential of the i th cube at the exterior field point of coordinates (r, ϕ, λ) expanded into a spherical harmonic series of above-computed coefficients $\bar{C}_{l,m}^i$ and $\bar{S}_{l,m}^i$ can be computed according to:

$$U_i(r, \phi, \lambda) = \frac{Gd^3\rho_i}{r} \sum_{l=0}^n \sum_{m=0}^l \left(\frac{R}{r}\right)^l \bar{P}_{l,m}(\sin\phi) (\bar{C}_{l,m}^i \cos m\lambda + \bar{S}_{l,m}^i \sin m\lambda), \quad (12)$$

where n is the maximum degree of the expansion. It is then straight forward to come up with the following gravitational potential U of the entire Phobos

$$U(r, \phi, \lambda) = \sum_{i=1}^{N_{tot}} U_i(r, \phi, \lambda), \quad (13)$$

where N_{tot} is the number of cubes. From Eqs. (1), (12) and (13), the $\bar{C}_{l,m}$, $\bar{S}_{l,m}$ normalized coefficients of the entire body can be computed from the parameters of our interior models :

$$M\bar{C}_{l,m} = \sum_{i=1}^{N_{tot}} d^3\rho_i \bar{C}_{l,m}^i, \quad (14)$$

$$M\bar{S}_{l,m} = \sum_{i=1}^{N_{tot}} d^3\rho_i \bar{S}_{l,m}^i. \quad (15)$$

The main moments of inertia can equivalently be derived from the so-obtained-degree-2 unnormalized gravity coefficients (Eq. (4)) or directly from the cube positions and densities according to

$$I_{xx} = \sum_{i=1}^{N_{tot}} (y_i^2 + z_i^2)\rho_i d^3, \quad I_{xy} = \sum_{i=1}^{N_{tot}} x_i y_i \rho_i d^3, \quad (16)$$

$$I_{yy} = \sum_{i=1}^{N_{tot}} (x_i^2 + z_i^2)\rho_i d^3, \quad I_{yz} = \sum_{i=1}^{N_{tot}} x_i z_i \rho_i d^3, \quad (17)$$

$$I_{zz} = \sum_{i=1}^{N_{tot}} (x_i^2 + y_i^2)\rho_i d^3, \quad I_{yz} = \sum_{i=1}^{N_{tot}} y_i z_i \rho_i d^3. \quad (18)$$

For each cube model we then compute the libration amplitude according to Eq. (5).

3. Uniform density model of Phobos

From their shape analysis, Willner et al. (2014) have deduced a colatitude Θ and longitude λ dependent spherical harmonic expansion of the surface of Phobos up to degree $l = 45$. The function R_{topo} describing the surface can be written as

$$R_{topo}(\Theta, \lambda) = \sum_{l=0}^{45} \sum_{m=0}^l P_{l,m}(\cos\Theta) (A_{l,m} \cos(m\lambda) + B_{l,m} \sin(m\lambda)), \quad (19)$$

where the values of the coefficients $A_{l,m}$ and $B_{l,m}$ are given in Willner et al. (2014), m is spherical harmonic order, and $P_{l,m}$ are the associated Legendre polynomials. They are related to the degree l Legendre polynomials, P_l , through the following relation:

$$P_{l,m}(x) = (-1)^m (1 - x^2)^{m/2} \frac{d^m}{dx^m} P_l(x). \quad (20)$$

With the radius function R_{topo} , the volume can be written as

$$V = \frac{1}{3} \int_0^\Theta \sin\Theta d\Theta \int_0^{2\pi} d\lambda R_{topo}^3(\Theta, \lambda). \quad (21)$$

The volume of Phobos computed by Willner et al. (2014) is $5741 \pm 35 \text{ km}^3$. From the volume and from the latest estimate of Phobos' gravitation parameter ($GM = 0.7084 \pm 0.0007 \cdot 10^{-3} \text{ km}^3/\text{s}^2$, Pätzold et al. (2014a)) the average density ρ_b of Phobos is calculated to be $1848.6 \pm 11 \text{ kg/m}^3$. The volume averaged radius equals 11,108.2 m. Hereafter, we choose $R_0 = 11.1 \text{ km}$ for reference radius.

Under the assumption that Phobos has a uniform density, the center of mass and the moment of inertia tensor can be calculated from R_{topo} . The center of mass (CoM) slightly offset compared to the center of

figure (CoF), such that CoM coordinates in the reference frame centred on Phobos' figure are³

$$\mathbf{CM} = (-66.4, 63.0, -84.3) \text{ m} \quad (22)$$

The moment of inertia tensor $\mathbf{I}$ is

$$\mathbf{I} = MR_0^2 \begin{pmatrix} 0.353657 & 3 \times 10^{-8} & -1 \times 10^{-7} \\ 3 \times 10^{-8} & 0.418499 & 2 \times 10^{-7} \\ -1 \times 10^{-7} & 2 \times 10^{-7} & 0.490239 \end{pmatrix}, \quad (23)$$

The principal moments of inertia A , B , and C are the eigenvalues of $\mathbf{I}$. Since the off-diagonal elements are significantly smaller than the diagonal elements, they can be neglected and the principal moments are then: $A = I_{x,x}$, $B = I_{y,y}$, and $C = I_{z,z}$. The uncertainty on the volume induces uncertainties on $\mathbf{I}$. Willner (pers.com.) claimed that a very conservative estimate for the errors on the radii ($13.03 \times 11.40 \times 9.14 \text{ km}$) stated in Willner et al. (2014) is $\pm 25 \text{ m}$ on each radius. The resulting uncertainties are $(\sigma_A, \sigma_B, \sigma_C)/MR^2 = (0.0012, 0.0013, 0.0014)$, corresponding to about (0.35, 0.30, 0.30)% uncertainties on (A, B, C) ⁴. From the principal moments of inertia and from their associated uncertainties the degree two unnormalized gravity field coefficients $C_{2,0}$ and $C_{2,2}$, and the libration amplitude θ can be determined according to Eqs. (4) and (5), respectively. They are

$$C_{2,0} = -0.10416 \pm 0.0016 (1.5\%), \quad C_{2,2} = 0.016211 \pm 0.0004 (2.7\%), \quad \theta = -1^\circ.139 \pm 0^\circ.052 (4.5\%). \quad (24)$$

Using the method described in the previous section, the gravity coefficients of Phobos can be computed assuming a constant density $\rho_i = \rho_b = 1848.6 \text{ kg/m}^3$ (Eqs. (12) and (13)). Table 1 shows the so-obtained gravity field for a homogeneous Phobos computed up to degree and order ten. Unlike the above $C_{2,0}$ and $C_{2,2}$ which are computed with a reference radius R_0 , for the coefficients of Table 1 we use $R = 14 \text{ km}$. Both quantities are related to each other by $(R/R_0)^2$ (see Eq. (1)).

4. Heterogeneous models of Phobos' interior

Small solid bodies with no macro-porosity in their interior should be rare in the Solar system, involving rather large asteroids, or maybe co-accreted moons. Moreover, the bulk density of Phobos (1848.6 kg/m^3) is lower than the density of any known meteoritic material⁵ (typically ranging in $[2250; 3500] \text{ kg/m}^3$) and Phobos has a cratered surface with one major crater of $0.8 \times R_0$ wide, called Stickney. These characteristics of Phobos led scientists to propose heterogeneous Phobos' interior models in composition and mass repartition. In particular, it is expected that the low density is due to the presence of significant macro-porosity and/or water ice in its interior (e.g. Rosenblatt (2011) and references in there). Macro-porosity suggests a second-generation body, with a different interior aspect today than that of the primitive body, because resulting from a re-accretion process, a tidal disruption or from damages occurred following a major impact. This hypothesis of a second-generation body in terms of interior structure is strongly supported by the large Stickney crater that shall have had some major consequences on the moon's interior, unless the original body were made of very porous material, or contained a lot of water, enabling it totally to absorb the impact energy and relax.

³ Note that the CoM-CoF offset reported in Eq. (22) is in good agreement with Shi et al. (2012) values. It has been computed from the spherical harmonic expansion of Phobos' shape (Eq. (19)) and not from the DSK model since the latter is not suitable for degree and order one (i.e. CoM-CoF relative position) as acknowledged by Willner et al. (2014). Table 1 is also derived from the spherical harmonic expansion of Phobos' shape.

⁴ The uncertainties on the off-diagonal moments of inertia are several orders of magnitude smaller than those on the diagonal and are therefore negligible.

⁵ Except for the Tagish Lake material, which has a density lower than Phobos' one, see Rosenblatt (2011).

Table 1

Gravity coefficients for a homogeneous Phobos obtained with the method described in Section 2 with cubes of 500 m-length. The reference radius is $R = 14$ km and the total mass is 1.06×10^{16} kg.

l	m	$C_{l,m}$	$S_{l,m}$	$C_{l,m}$	$S_{l,m}$
0	0	1.000000	0.000000	1.000000	0.000000
1	0	0.003585	0.000000	0.006209	0.000000
1	1	0.002818	-0.002856	0.004881	-0.004947
2	0	-0.029243	0.000000	-0.065390	0.000000
2	1	0.000084	0.000072	0.000108	0.000093
2	2	0.015664	-0.000020	0.010111	-0.000013
3	0	-0.002222	0.000000	-0.005878	0.000000
3	1	-0.002450	0.001399	-0.002646	0.001511
3	2	0.004268	-0.000537	0.001458	-0.000183
3	3	0.000917	-0.006642	0.000128	-0.000926
4	0	0.002693	0.000000	0.008078	0.000000
4	1	-0.001469	0.000402	-0.001394	0.000382
4	2	-0.000920	-0.000555	-0.000206	-0.000124
4	3	0.001263	-0.001218	0.000076	-0.000073
4	4	-0.000032	0.000088	-0.000001	0.000002
5	0	-0.000498	0.000000	-0.001652	0.000000
5	1	-0.000424	-0.000324	-0.000363	-0.000277
5	2	-0.000482	-0.000010	-0.000078	-0.000002
5	3	0.000179	0.000823	0.000006	0.000027
5	4	0.000457	-0.000190	0.000004	-0.000001
5	5	-0.000444	-0.001082	-0.000001	-0.000003
6	0	-0.000348	0.000000	-0.001253	0.000000
6	1	0.000432	-0.000136	0.000340	-0.000107
6	2	0.000089	0.000267	0.000011	0.000033
6	3	-0.000377	0.000422	-0.000008	0.000009
6	4	-0.000028	0.000034	-0.000000	0.000000
6	5	0.000190	-0.000103	0.000000	-0.000000
6	6	-0.000996	-0.000126	-0.000000	-0.000000
7	0	0.000391	0.000000	0.001514	0.000000
7	1	0.000453	0.000057	0.000332	0.000042
7	2	-0.000063	0.000021	-0.000006	0.000002
7	3	-0.000087	-0.000082	-0.000001	-0.000001
7	4	-0.000091	0.000012	-0.000000	0.000000
7	5	0.000240	0.000374	0.000000	0.000000
7	6	-0.000089	-0.000170	-0.000000	-0.000000
7	7	-0.000590	0.000171	-0.000000	0.000000
8	0	0.000073	0.000000	0.000300	0.000000
8	1	-0.000010	-0.000016	-0.000007	-0.000011
8	2	-0.000095	-0.000150	-0.000008	-0.000012
8	3	0.000049	0.000011	0.000000	0.000000
8	4	0.000022	-0.000007	0.000000	-0.000000
8	5	-0.000029	0.000051	-0.000000	0.000000
8	6	0.000228	0.000023	0.000000	0.000000
8	7	0.000115	0.000040	0.000000	0.000000
8	8	-0.000272	0.000372	-0.000000	0.000000
9	0	-0.000200	0.000000	-0.000872	0.000000
9	1	-0.000156	-0.000024	-0.000102	-0.000016
9	2	0.000084	-0.000025	0.000006	-0.000002
9	3	0.000020	0.000041	0.000000	0.000000
9	4	0.000022	0.000016	0.000000	0.000000
9	5	-0.000034	-0.000092	-0.000000	-0.000000
9	6	0.000005	0.000006	0.000000	0.000000
9	7	0.000175	0.000050	0.000000	0.000000
9	8	-0.000039	-0.000018	-0.000000	-0.000000
9	9	-0.000054	0.000278	-0.000000	0.000000
10	0	-0.000019	0.000000	-0.000089	0.000000
10	1	-0.000008	0.000020	-0.000005	0.000012
10	2	0.000039	0.000064	0.000002	0.000004
10	3	-0.000006	-0.000057	-0.000000	-0.000000
10	4	-0.000034	-0.000010	-0.000000	-0.000000
10	5	0.000025	0.000008	0.000000	0.000000
10	6	-0.000061	-0.000048	-0.000000	-0.000000
10	7	-0.000033	0.000032	-0.000000	0.000000
10	8	0.000131	-0.000077	0.000000	-0.000000
10	9	-0.000009	0.000062	-0.000000	0.000000
10	10	0.000135	0.000207	0.000000	0.000000

Macro-porosity is a crucial quantity to characterise the internal structure of small bodies like Phobos. Indeed, [Britt et al. \(2002\)](#) divided asteroid structures into three general groups based on their macro-

porosity⁶: (1) the solid (coherent) objects with very low or no macro-porosity, (2) the heavily fractured bodies with macro-porosity $\sim 20\%$, and (3) the asteroids with macro-porositities $> 30\%$ that are loosely consolidated “rubble pile” structures. Given this classification, Phobos could be either a fractured body or a rubble-pile, but an outer-solar-system body, not accounted for in this classification, cannot be ruled out.

We review in this section models of heterogeneous interior of Phobos proposed in the literature on the basis of its bulk physical properties and surface observables, defining by the way four different families of interior for Phobos sketched in [Figs. 4–7](#). In this paper we focus on these four families (and a couple of sub-families), looking for their signatures in the geodesic observables in order to conclude on the ability for a dedicated mission to Phobos to distinguish between these particular families of interior models. The characteristics of the interior structure of these families, as considered in the calculations performed here, are reported in [Table 2](#), shown in [Fig. 3](#) and described in the following subsections.

4.1. Family 1: the rubble pile

Phobos’ interior could be composed of blocks of two kinds of rocky materials with voids in-between and with interstitial water ice ([Murchie et al., 1991](#)). This model is based on the so-called red and blue units identified in the reflectance spectra and the images acquired by the Phobos-2 mission and interpreted as two different material composing Phobos, given the density of possible meteoritic analogs to Phobos. The voids and water ice have been added in order to account for the low bulk density of Phobos. However, only an estimate of porosity has been given (from 10% to 50%, depending on the density of the rocky materials, [Murchie et al. \(1991\)](#)) and not of the content of water ice. In addition, the true composition of the Phobos’ materials cannot be unambiguously identified from the reflectance spectra (e.g. carbonaceous or silicate material, [Murchie et al. \(1991\)](#), see also [Rosenblatt \(2011\)](#)) and the difference between the spectra of the red and blue units could totally reflect space weathering effects on a same single material (e.g. [Murchie and Erard \(1996\)](#)). Therefore, we consider herein as the rubble pile Phobos the simplified version of the above that proposes a moon made of blocks of one single rocky material with a volume fraction of voids greater than 30% and possible presence of water ice up to about 20%, resulting in a density larger than ~ 2650 kg/m³ for the rocky material. This loosely consolidated rubble pile body could have formed from the re-accretion of debris in Mars’ orbit ([Andert et al., 2010; Rosenblatt, 2011](#)). Our models are created by randomly assigning one of the 3 densities to each of the cubes in the entire volume of Phobos with the elements’ proportions taken every 5% within ranges shown in green in [Fig. 3](#). For each proportion of ice and porosity considered, 500 models are built. One model among 6000 considered for this family is shown on the top right panel of [Fig. 4](#), next to a sketch of the Family 1 models. [Fig. 4](#) bottom left panel shows a sketch of a sub-family of dry models which exhibit a regolith zone with no porosity. This sub-family is based on the scenario of a formation of Phobos after disruption of a primary body ([Britt et al., 2002](#)). The largest pieces would then re-accrete first, creating large void inside the body, but with no void in the surface layer which would have been filled by the smaller pieces in the second stage of the re-accretion process. One of the 4500 dry models of rubble pile considered in this paper is shown in [Fig. 4](#) bottom right panel.

⁶ Note that no distinction is made in our approach between macro- and micro-porosity which are summed up in the bulk porosity, modelled by massless cubes. Micro-porosity can also be seen as an interdependent quantity with rock density, since the former lowers the latter.

Table 2

Interior structure configuration of each 500 m-cubes-models family. The characteristics of each subregion used to build each model of a given family are provided. The rubble pile models, representing Family 1 models in a general way, are not mentioned herein since the cube distribution for them is purely random (no distinction by regions as for the others). All the other families and sub-families are described here, namely: the Disrupted and Reaccreted sub-family (DAR-F1), the Heavily Fractured family (HF-F2), the Porous Compressed family (PC-F3), the Icy Evenly Distributed sub-family (IED-F4) and the Icy Surface Concentrated sub-family (ISC-F4). The DAR-F1 subregion goes from the herein mentioned depth until Phobos' center. The IED-F4 subregion goes from Phobos' center until the herein mentioned depth. The ISC-F4 subregion goes from the depth limit of 1.5 km (from the surface) until the herein mentioned depth.

Subregion characteristics	DAR-F1	HF-F2	PC-F3	IED-F4	ISC-F4
Ellipsoid center coordinates (km)		(7, −9.5, 0)	(7, −9.5, 0)		
Ellipsoid semi-axes (km)		(8, 8, 4)	(7.5, 7.5, 3.5)		
Subregion's depth (± x, ± y, ± z) (km)	(1, 1, 1)			(1, 1, 0.5)	(1, 1, 0.5)
Number of inside cubes	38,136	3784	2965	39,720	9852
Inside volume fraction (%)	83	8.2	6.45	86.4	21.4
Range of bulk porosity (%)	30–46	18–20	30–46	10–40	10–40
Range of porosity inside (%)	36.1–55.4	50.4–52.4	0	1.7–45.5	18.6–56.3
Range of porosity outside (%)	0	15.1–17.1	32–49	2.6–62.8	7.6–35.6
Range of bulk ice proportion (%)	0	0	0	5–50	5–15
Range of ice proportion inside (%)	0	0	0	6–58	23.3–70
Range of ice proportion outside (%)	0	0	0	0	0

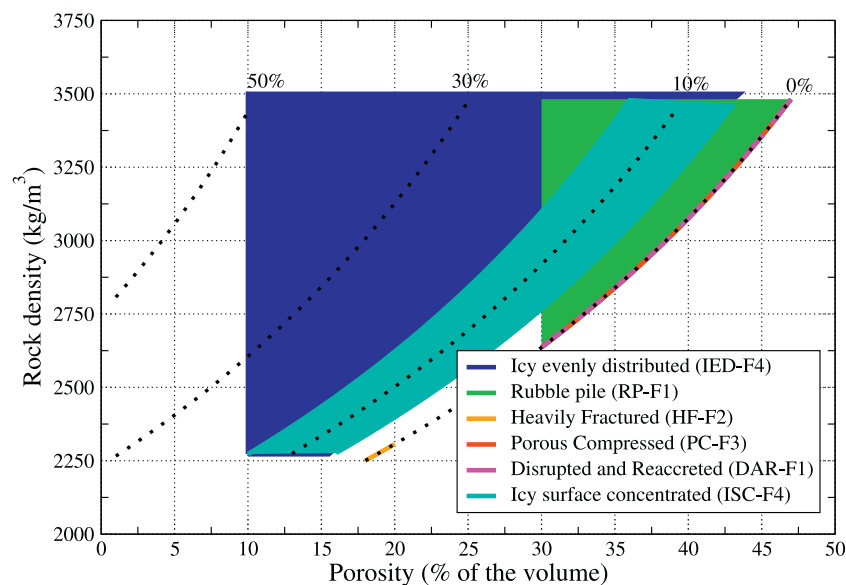


Fig. 3. Ranges of porosity versus rocky material density ranges for the four Phobos' interior model families and their sub-families. Dotted lines show the ice volume proportion. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

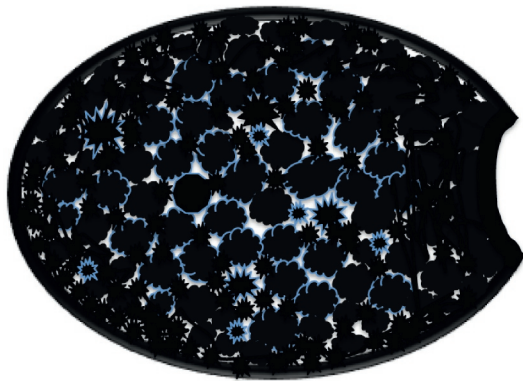
4.2. Family 2: the heavily fractured Phobos

If Phobos was initially a solid monolith, the impact forming Stickney should have produced cracks throughout the cratered hemisphere and at the antipode (Asphaug et al., 2002). Asphaug and Melosh (1993) have shown that the porosity subsequent to the impact is sufficient to explain the low bulk density of the Martian moon. It has also been proposed that the grooves on Phobos might be the surface expression of deep-seated fractures, although other explanations proposed for the origin of these grooves do not require large fractures inside the body (Ramsley and Head III, 2013; Wilson and Head, 2015). The large inside fractures could decrease the bulk density of Phobos by an amount of 15–20%, which is in agreement with Phobos' bulk density if the material analog to Phobos has the same bulk density as the carbonaceous meteoritic material (CI/CM class, see Rosenblatt (2011)) of about 2250 kg/m³. Because of the small range of porosity allowed by this second family, only 3000 models are considered here, one of them being plotted in the right panel of Fig. 5, next to a schematic view of Family 2 models. The dimensions of the ellipsoidal region (see Table 2) confining most of the damage induced by the impact are taken such that the volume fraction of this zone is around 8.5% of the total volume of Phobos, as predicted by the simulations published by Bruck Syal

et al. (2016) for an initially non-porous Phobos.

4.3. Family 3: the porous moon

The porous nature of a body has a strong impact on its ability to absorb the energy delivered by a major impact. Asphaug et al. (2002) have shown that a porous target is a poor transmitter of impact stress such that collisional effects are confined to the impacted hemisphere ('creating a crater and little more'). The presence of Stickney, a very large crater (9 km across) with respect to Phobos' dimensions (≤ 26 km), suggests that Phobos is able to survive a major impact, meaning that it should be able to dissipate a lot of energy. If Phobos is highly porous, the energy of the short-term stresses induced by the collision should have been absorbed via compression, with little to no tensile wave developed in the structure (Richardson et al., 2002). This family is in some sense also a sub-family of rubble-piles (see Fig. 3). As for the mother family, the porosity of the bodies falling in our porous moon family is ≥ 30 %, allowing for rock's density to be greater than ~ 2650 kg/m³ in order to fit the global mass and volume of Phobos. However, this category of bodies does not contain ice in their interior and they all exhibit a compression zone, under Stickney, where the porosity is reduced to almost zero. The characteristics of this higher-

Rubble Pile body

Porosity $\geq 30\%$
Possible presence of ice $\leq 20\%$
(Murchie et al. 1991)

Disrupted and reaccreted body

The largest pieces reaccreted
first, creating large void
spaces in the interior.
The fine material stayed in
the regolith zone.
(Britt et al. 2002)

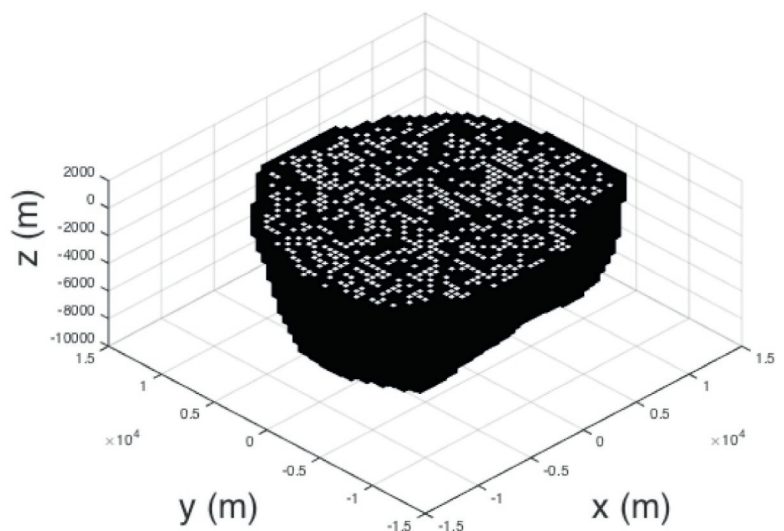
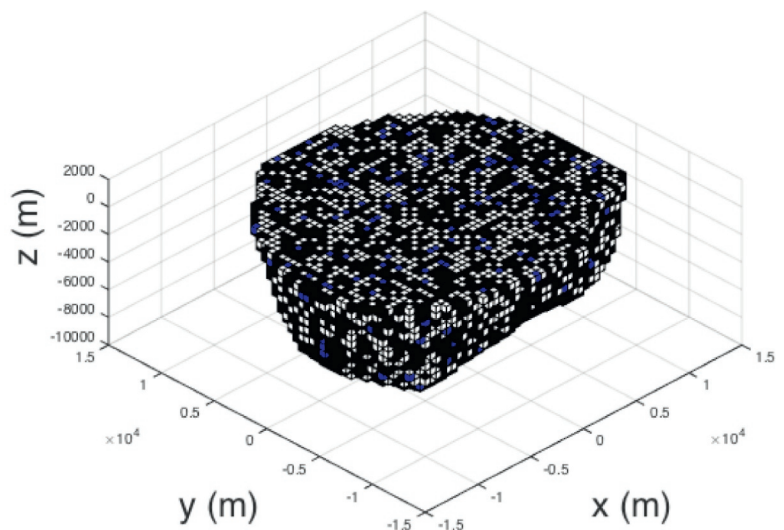


Fig. 4. Family 1 interior structures. Left panels show schematic cross-sections of Phobos for typical interior structure of the Rubble Piles (RP-F1) models as well as for the sub-family, called Disrupted and Reaccreted (DAR-F1), consisting in dry models with a regolith zone. Right panels show one particular cube model of each sub-family considered here.

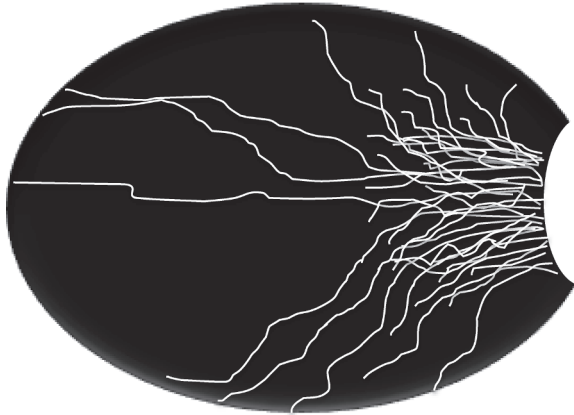
density zone, described in Table 2, are chosen such that the volume fraction of the damaged (or compressed) zone is around 6.5% of Phobos' total volume (Bruck Syal et al., 2016). This subregion is smaller and shallower than the heavily fractured subregion chosen for the Family 2 in accordance with Asphaug et al. (2002) conclusions. 4500 models are considered hereafter for this family, with porosity level taken every 2% within the allowed range of 30–46% (see Fig. 3). Fig. 6 left panel shows a schematic view of the interior structure of the Family 3 models, whereas the right panel shows one model belonging to this third family.

4.4. Family 4: the icy body

For this family, Phobos is considered to be a mixture of rock (one single material) with interstitial void and water ice in deeper depth (Fanale and Salvail, 1989; 1990). This kind of model is based on the interpretation of reflectance spectra of the surface which argues in favor

of a composition similar to the one of primitive asteroids (expected to be volatile-rich) and on computation of stability of water ice inside Phobos at Mars' orbit. For this family, supporting the capture scenario, the bulk volume fraction of water ice can reach in principle 64%, depending on the density of the rocky material (see Fig. 3). We reduce the allowed models for this family to those matching the ones proposed by Fanale and Salvail (1990). These authors have built models depending on the microporosity (10%–50%), the mass-fraction of ice originally in Phobos (0.1%–10%, i.e. 0.2–20% of the volume), and the pore size (1–10 microns).

As per Fanale and Salvail (1990) study, we consider for this family two sub-families with different water ice distribution scenarios. The first one considers the water ice initially evenly distributed in the body (blue curves in the graphs of the present paper) whereas the second one assumes that the water ice is concentrated in a near-surface layer (cyan curves in the graphs of the present paper). The latter is considered by the authors to be more likely than the former. For both scenarios,

Heavily fractured body

Porosity ~15–20% of the volume induced by major event like impacts (Britt et al. 2002)

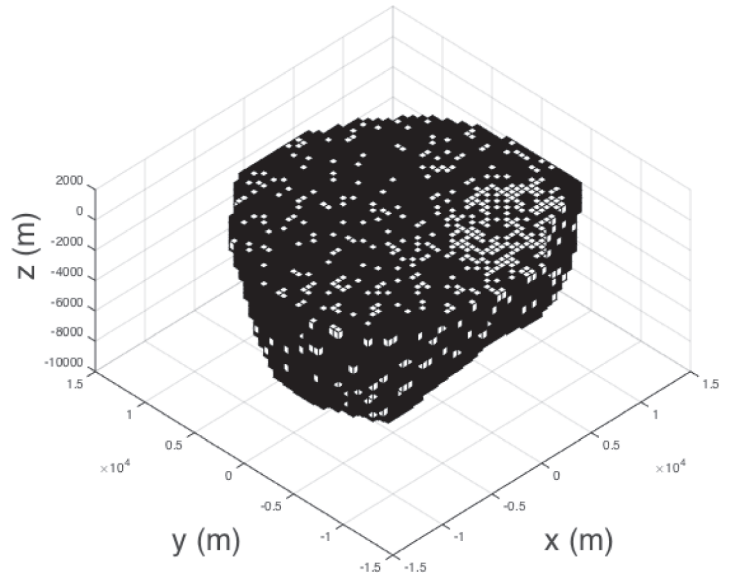
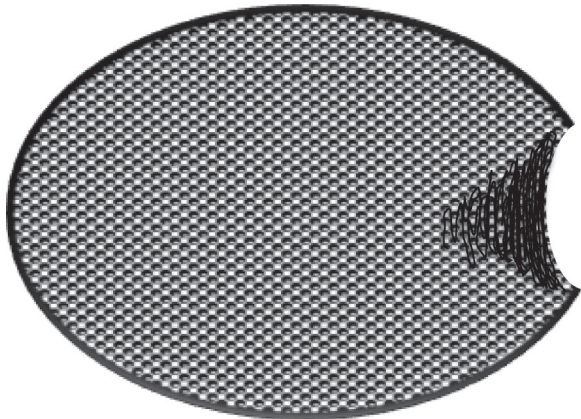


Fig. 5. Family 2 interior structure. Left panel shows a schematic cross-section of Phobos for typical interior structure of Heavily Fractured (HF-F2) models defining the Family 2. Right panel shows one particular cube model of Family 2.

Compressed Porous body

Porosity $\geq 30\%$
Compaction and crushing allow to damp impact energy (Richardson et al. 2002)

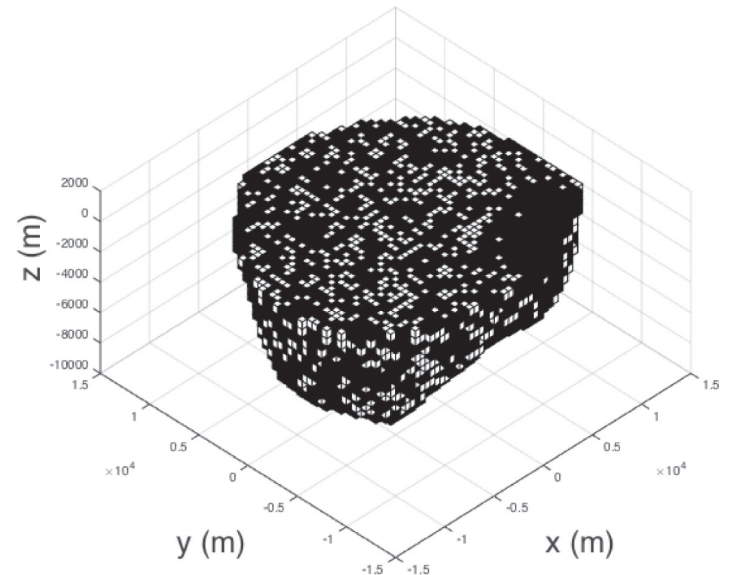


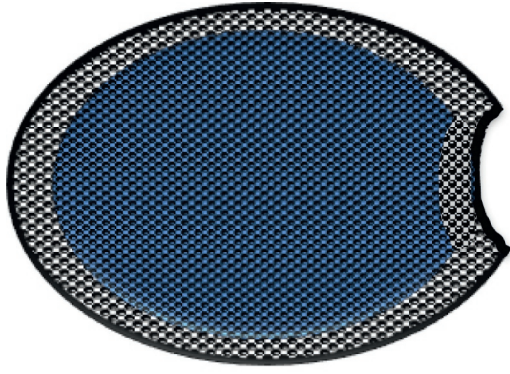
Fig. 6. Family 3 interior structure. Left panel shows a schematic cross-section of Phobos for typical interior structure of Porous Compressed (PC-F3) models defining the Family 3 considered here. Right panel shows one particular cube model of Family 3.

Fanale and Salvail (1990) predict that, million years after Phobos' formation, one should find the ice buried under Phobos' regolith, deeper at the equator (at the kilometer level) than at the pole (a the tens meter level). The water at the surface would have escaped due to solar insolation and replaced by interstitial void in the surface layer (increasing the porosity just below the surface). For the surface concentrated case, these authors predict also a lower limit under which no ice would be found. This depth limit is determined by the specified initial fraction of water ice allocated to Phobos assuming the pore spaces to be filled with ice. Accurately to match their models in terms of burial depth of the ice under the surface, we would need to reduce the cube length to about 50 m. This would lead to models made of about 45 million cubes, which is too much time-consuming given our current computer capabilities. Our icy body models thus are only roughly

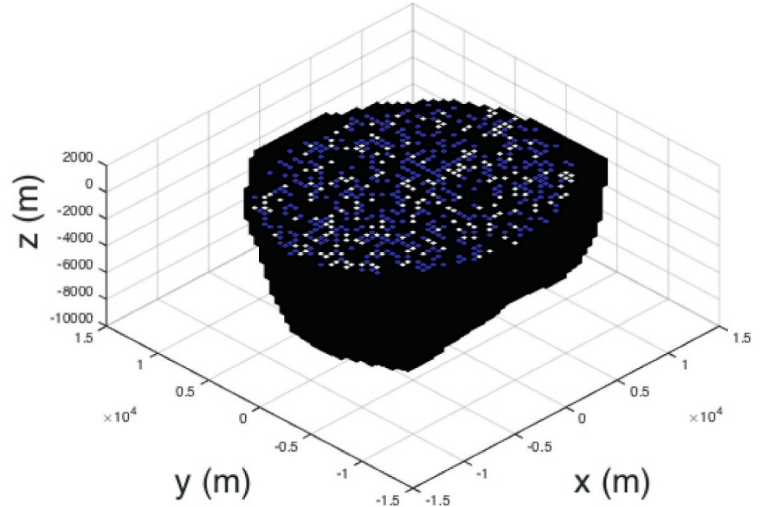
matching Fanale and Salvail (1990)'s models, with the ice maintained below 1 km at the equator and below 500 m at the pole, and with a depth limit of 1.5 km for the surface concentrated models. Family 4 models have porosity ranging from 10% to 40%. The Icy Evenly Distributed (IED) models exhibit ice ranging from 5% to 50% of the volume, while the Ice Surface Concentrated (ISC) models keep the ice content between 5% and 15% of the volume (see Fig. 3). By sampling these ranges every 10% for the porosity and every 5% for the ice proportion, 10,000 models are selected for IED and 4000 for ISC⁷, with no

⁷ Note that half of the IED models allowed by these ranges (i.e. 10–40% porosity and 5–50% ice) are not compliant with the total mass of Phobos and the range of rock density ([2250; 3500] kg/m³) considered in this paper. For the same reasons, about 10% of the ISC models are not compliant.

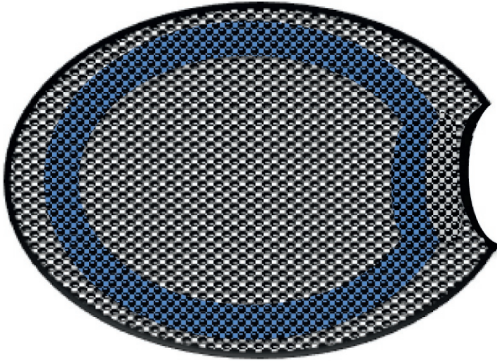
Evenly distributed Icy body



Porosity $\approx 10\text{--}50\%$
Ice $\leq 50\%$ and buried until
Phobos center, but deeper
at the equator than it is at
the poles
(Fanale and Salvail, 1990)



Surface-concentrated Icy body



Porosity $\approx 50\%$
The ice ($\leq 15\%$) is buried
deeper at the equator than it is
at the poles with no ice below
1.5km under the surface
(Fanale and Salvail, 1990)

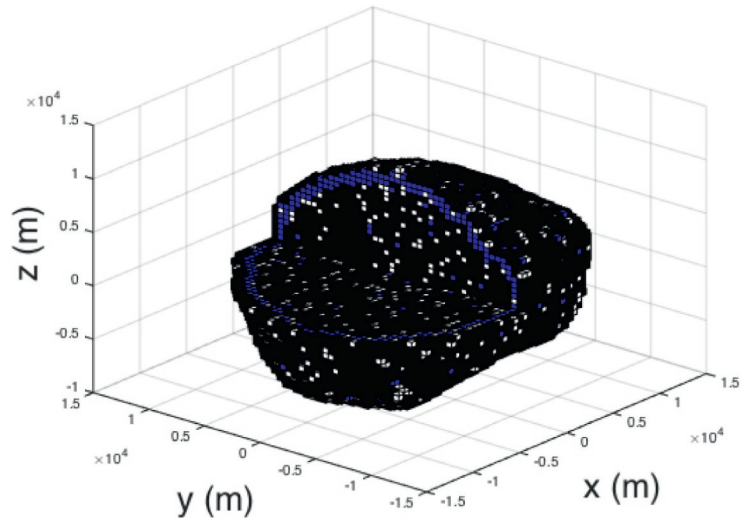


Fig. 7. Family 4 interior structures. Left panels show schematic cross-sections of Phobos for typical interior structures of the two candidates retained to define the Family 4, namely: the Icy Evenly Distributed (IED-F4) sub-family (upper panels) and the Icy Surface Concentrated (ISC-F4) sub-family (lower panels). Right panels show one particular cube model of each sub-family.

ice at the surface of Phobos for both sub-families. The interior structure characteristics of these models are reported in Table 2. Fig. 7 shows schematic cross-sections of both sub-families of the icy models considered in the present paper (left panels) along with a representative model of each sub-family (right panels).

5. Geodesic observables

In this section, we compute the signature of each of our families (and sub-families) of interior models in different geodesic observables in order to see if they can be distinguished from the homogeneous Phobos and from each other.

5.1. Moments of inertia

The moments of inertia (MOI) are related to the radial mass distribution inside Phobos. This is a precious information to constrain the interior structure of the satellite. Unfortunately, the moments of inertia

are not directly measurable, but are usually determined from measurement of the spin quantities like the precession rate, or the obliquity, or the libration amplitude of the body considered, combined with a measurement of its gravity field degree-2 coefficients. In the case of Phobos, locked in spin-orbit resonance, the amplitude of libration at the orbital period (θ) and the degree-2 order-2 gravity coefficient ($C_{2,2}$) are certainly the most appropriated observable quantities to be used to infer the polar moment of inertia (I_{zz}) according to the following equation derived from Eqs. (4) and (5):

$$\frac{I_{zz}}{MR_0^2} = 12C_{2,2} \left(1 - \frac{2e}{\theta} \right). \quad (25)$$

We compute the main MOI of each interior model of all of the four families and sub-families according to Eqs. (16)–(18). The so-obtained distributions are plotted in Fig. 8 and compared to the MOI obtained assuming a constant density inside Phobos (see Section 3). Note that these homogeneous MOI values are shown in black on Fig. 8 and have been computed with our 500 m-cube models. They differ from those

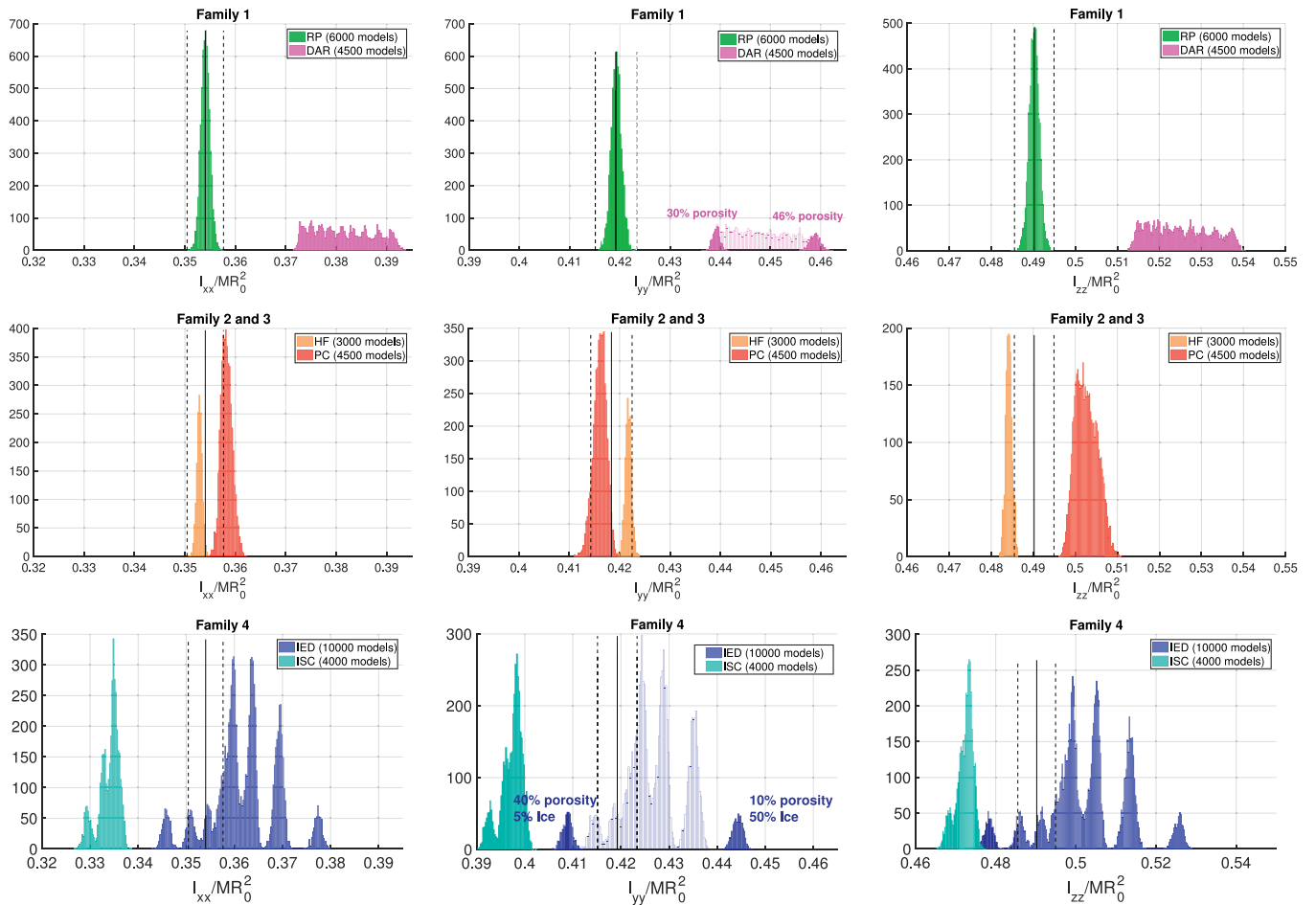


Fig. 8. Distributions of the principal normalized moments of inertia (I_{xx} (left) < I_{yy} (middle) < I_{zz} (right)) computed from thousands of randomly-chosen models for each interior model family. The black solid vertical line points out the homogeneous value computed with our 500-m cube model. The black dashed vertical lines define the 1σ uncertainty regions due to uncertainties in Phobos' shape and volume.

published by Willner et al. (2014) because of the radius chosen for the normalisation ($R_0 = 11.1$ km in the present study against $R = 10.993$ km for Willner et al. (2014)).

Generally speaking, whatever the family considered, the level of departure of I_{xx} , I_{yy} and I_{zz} due to heterogeneity in Phobos' interior are all smaller than 15% of their respective homogeneous value. However, even if the MOI are inferred with a precision significantly better than 15%, it will be difficult to discriminate between the different candidates of interior structure. Indeed, as shown on Fig. 8, most of the predicted MOI values in the 15% range around the homogeneous values match at least two different families. Only the ice surface concentrated (cyan) and (perhaps) the highly-porous disrupted (magenta) models could be identified unequivocally.

More specifically, one can conclude from Fig. 8 the following:

- The rubble pile models have MOI undistinguishable from that of a constant density Phobos (green histograms embedded in the black homogeneous domain). This was expected since the rubble pile models are built by randomly distributing the cubes of ice, void and rock in the volume. This ends up with well-mixed-interior models having in average MOI equal to that of a constant density Phobos.
- The disrupted and re-accreted (DAR) models (magenta) have MOI significantly larger than the homogeneous model (black). This was also predictable since DAR models exhibit a non-porous regolith zone (see Fig. 4 lower panels), making the moon denser at the surface than at the center. The range of MOI allowed by this sub-family is outside the 3σ uncertainty regions of the homogeneous

model. In addition, we see on Fig. 8 top-middle panel that the disrupted Phobos' moments of inertia strongly depend on the porosity of Phobos. The larger are I_{xx} , I_{yy} and I_{zz} , the larger is the void proportion in the Martian moon and the larger is the rocky material density.

- The MOIs of the heavily fractured (orange) and porous compressed (red) models behave in an opposite way with respect to homogeneous MOI. One increases when the other decreases. Given the parameterization chosen for those families (see Table 2), one could distinguish between MOI of an HF-F2 model and MOI of a PC-F3 model since their histograms do not intersect. However, I_{xx} and I_{yy} of both families are largely intersecting the homogeneous regions making them nearly impossible to distinguish from the homogeneous ones. This is not the case for I_{zz} which stands outside 1σ uncertainty region of the homogeneous case as shown on Fig. 8 middle-right panel. Note that larger sub-regions than those defined here above in Table 2 would increase the degree of departure of these families' MOI, making them easier to distinguish from homogeneous.
- The ice surface concentrated models (cyan) display MOI always smaller than the homogeneous ones by more than 3 times the uncertainty of the latter. This is because the icy layer (20% of the total volume), maintained between 1.5 km and 0.5 km from the surface, decreases the average density of the outer part of the body, since ice is less dense than any rocks considered here.
- The models made of ice buried and evenly distributed in the volume (blue) offer the largest ranges of allowed MOI. These ranges include

the homogeneous ranges for the three MOI: I_{xx} , I_{yy} and I_{zz} . As for the disrupted models, these models exhibit multiple local maxima in the MOI distributions which are related to the proportion of ice and void. The larger is the porosity and the lower is the ice proportion, the smaller will be the MOI. The larger is the ice proportion and the lower is the porosity, the larger will be the MOI (see bottom-middle panel of Fig. 8). This is explained by the non-uniform distribution of ice inside Phobos as used in this model.

5.2. Libration amplitude

The way the moon reacts to external periodic forcing (mainly caused by the variations of the moon-Mars distance) is characterized by its proper modes, themselves depending on the moments of inertia. Phobos has a proper frequency of libration in longitude close to the forced libration at orbital frequency (Rambaux et al., 2012; Le Maistre et al., 2013). Therefore, the latter periodic spin rate variations have amplitudes which depend on the moments of inertia (see Eq. (5)). Measuring such variations in the rotation rate of the moon should therefore provide insight to its internal structure (Rambaux et al., 2012; Le Maistre et al., 2013). This is what is tested in this section.

Phobos' internal heterogeneities, as defined in the present paper, can shift the libration amplitude of Phobos from its homogeneous value by up to 35% of the latter (see Fig. 9). Although this relative range is twice larger than the relative range of shifts in the MOI, four of the six families and sub-families considered here can not assuredly be distinguished from the one of a homogeneous moon or from each other. Actually, unlike the MOI, Family 2 and 3 are the ones affecting the most the libration, both being distinguishable from each other and from the homogeneous value. Indeed, these models exhibit MOI close to the homogeneous model, but have libration amplitudes significantly different from the constant-density Phobos model. Conversely, the disrupted models (DAR-F1) have MOI strongly depending on the porosity of Phobos and very much differing from their homogeneous homologues, but their libration signal is actually not depending on the porosity and not distinguishable from the homogeneous libration as shown in magenta in Fig. 9. This is also true for the icy models (IED-F4 and ISC-F4) for which the distribution of allowed libration amplitudes seen in the right panel of Fig. 9 are also Gaussians and overlapping the homogenous domain. The weak signature in the libration amplitude of DAR-F1, IED-F4 and ISC-F4 models that yet most affect the MOI can be explained by the fact that the libration amplitude is a function of $(I_{yy} - I_{xx})/I_{zz}$. This relative MOI ratio remains confined while the MOI are individually spread out.

It is interesting to mention here that the libration amplitude of Phobos has been estimated by different groups with different methods and different data. Basically, there are those estimating θ based on imagery (cross-points network) analysis (e.g. Duxbury and Callahan (1989), Willner et al. (2010), Oberst et al. (2014),

Burmeister et al. (2018)) and those estimating θ , based on astrometric and radiometric data, via the impact of libration on Phobos' trajectory (e.g. Jacobson (2010) and Lainey et al. (2019)). The most recent estimate obtained with the imagery technique ($-1^\circ.14 \pm 0^\circ.03$ Burmeister et al. (2018)) is equal to the homogeneous value (see Section 3) and greater, in absolute value, than the most recent one provided by the dynamical technique ($-1^\circ.04 \pm 0^\circ.02$ Lainey et al., 2019) which supports a non-homogeneous Phobos. Despite the non-agreement at the 1σ level between these solutions, one could conclude given our model predictions that the hypothesis of a Phobos heavily fractured below Stickney is quite unlikely (see Fig. 9 central panel). Family 2, assuming a second-generation body born from a solid monolithic mother body with almost no porosity, seems indeed to be rejected by the current libration amplitude estimates from both techniques. Such a claim still needs to be validated via additional observations since the determinations of θ can both be contested because they could present important bias (i.e. larger true errors than the formal errors associated with the estimates).

5.3. Center of mass - center of figure

The degree-1 gravity field coefficients position the center of mass (CoM) with respect to the center of figure (CoF) of the body, the latter being determined from images, according to

$$\begin{aligned} x_{com} &= R C_{1,1} \\ y_{com} &= R S_{1,1} \\ z_{com} &= R C_{1,0} \end{aligned} \quad (26)$$

where R is the reference radius used to compute the gravity coefficients. Therefore, these coefficients can be used to determine the displacement of the center of mass that could be related to a density modification induced by the Stickney impact for instance. The difference between CoM and CoF for all the heterogeneous models considered in this paper are computed according to Eq. (26) and compared to the one for a uniform density model. The results are shown in Fig. 10. We see that Family 2 and 3 are the only ones displaying significant CoM-CoF offset, both at the level of a couple of hundred meters or more along the x and y axis of Phobos. Moreover, as one could expect, the heavily fractured models exhibiting a decrease in density below the crater (see Fig. 5), pushed the center of mass away from Stickney's hemisphere, whereas the porous models, involving a compression in this region, induce an increase in density below the crater (see Fig. 6), which moves the CoM closer to Stickney. None of the considered families significantly change the CoM location along the spin axis (right panel of Fig. 10). A determination of Phobos' degree-1 gravity coefficients with an uncertainty of 0.01 (on $C_{1,1}$ and $S_{1,1}$ in particular) would be sufficient to identify if Phobos interior structure belongs to Family 1 or 2, possibly confirming the rejection of Family 2 by the libration estimates.

It is important to mention that the CoM-CoF reference offset used

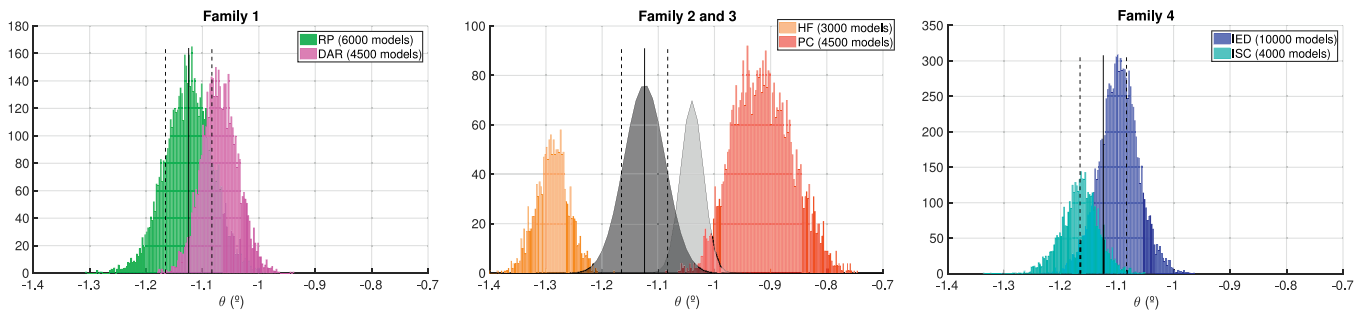


Fig. 9. Distributions of the libration amplitudes (θ) in degrees, computed from thousands of randomly-chosen models for each interior model family. The black solid vertical line points out the homogeneous value computed with our 500-m cube model. The black dashed vertical lines define the 1σ uncertainty regions due to uncertainties in Phobos' shape and volume. The central panel displays the heterogeneous HF and PC models distribution along with the homogeneous distribution (dark grey, more or less corresponding to Burmeister et al. (2018) solution) as well as the solution of Lainey et al. (2019) (light grey).

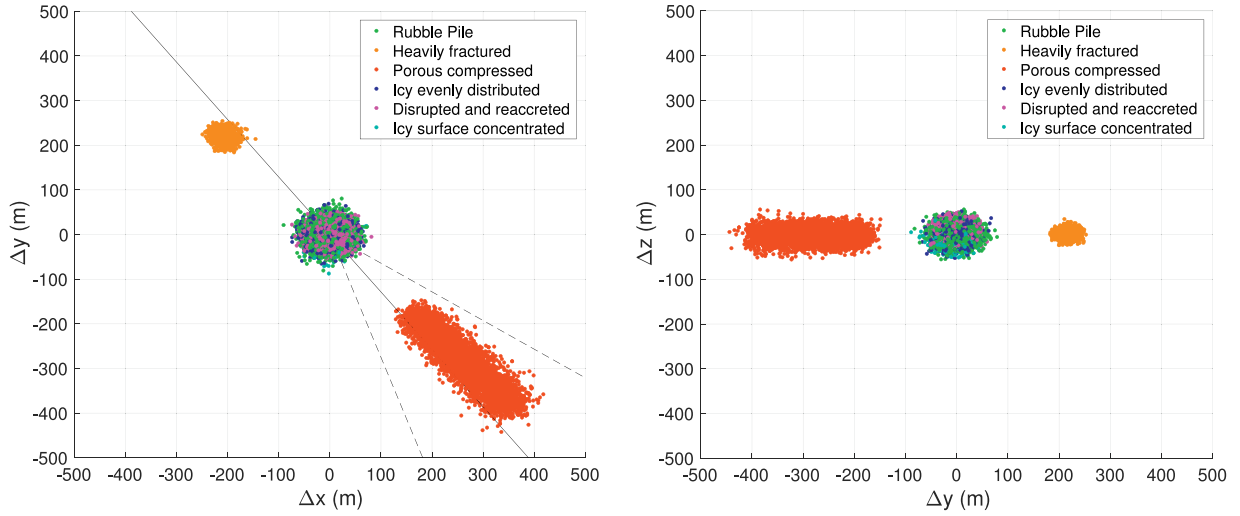


Fig. 10. Shift between the center of Mass and the center of figure in meters. The zeros are set on the homogeneous (x_{com} , y_{com} , z_{com}) values and the Δ s are computed by removing the latter coordinates to their heterogeneous counterparts. The Stickney crater is located at the surface in the $(+x, -y, 0)$ direction with a crater center around $(+7 \text{ km}, -9 \text{ km}, 0 \text{ km})$ (along the solid-black line) while the crater's edges are pointed by the dashed-black lines.

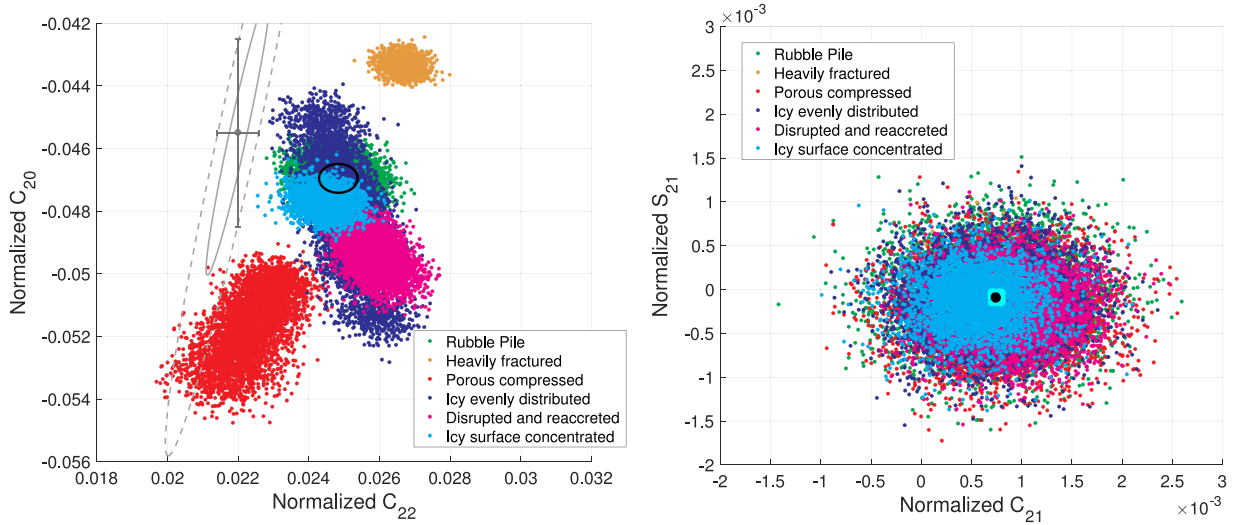


Fig. 11. Distributions of the degree-2 $C_{2,0}$ versus $C_{2,2}$ coefficients (left) and $S_{2,1}$ versus $C_{2,1}$ (right) computed from thousands of randomly-chosen models for each interior model family. Black ellipses are the 1σ -uncertainty regions computed from the shape model assuming a constant density in Phobos and $\sim 0.6\%$ of uncertainty on Phobos' volume. Note that the numerical values of these gravity coefficients are consistent with $R_0 = 11.1 \text{ km}$. The $C_{2,0}$ and $C_{2,2}$ estimated values (Jacobson and Lainey, 2014) are reported in grey in left panel, along with the 1σ (solid grey) and 3σ (dashed grey) covariance error ellipses resulting from the 96% correlation between $C_{2,0}$ and $C_{2,2}$ estimated values. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

here is actually not the one reported in Section 3, Eq. (22), which has been computed from the spherical harmonic expansion of Phobos' shape. The homogeneous CoM-CoF, here derived from the DSK shape model, is: $x_{com} = -306.6 \text{ m}$, $y_{com} = -383.5 \text{ m}$, $z_{com} = 113.3 \text{ m}$. The large bias of the CoM position w.r.t. the position reported in Eq. (22) is due to some limitations of DSK model to get degree-1 as explicitly stated by Willner et al. (2014). Nevertheless, the conclusions of the comparative study drawn here are not jeopardised by such limitation since the systematic errors affect all models the same way.

5.4. Degree-2 gravity coefficients

In this section, we take a look at the signature of the interior heterogeneities into the degree-2 gravity coefficients. Degree-2 is well known to be of great interest to probe the interior of a celestial body since it conditions the inertia tensor according to Eq. (4). $C_{2,1}$, $S_{2,1}$ and $S_{2,2}$ are nulls when the gravity field is expressed along the axes of

principal moments of inertia. The degree-2 order-1 gravity coefficients can be used to locate the orientation of the polar principal axis of inertia. Fig. 11 right panel showing the couple $(C_{2,1}, S_{2,1})$ for each model of each family, can then be interpreted as the displacement of the rotation axis due to heterogeneous mass distribution inside Phobos. As one can see on this figure, while most of the models⁸ differ significantly from the homogeneous case (and are even outside the latter 1σ -uncertainty region in black on the figure), none of the Families exhibit noticeable departure of their polar principal axis of inertia with respect to the homogeneous ones and to each other. A determination of the actual orientation of Phobos' polar principal axis of inertia appears thus to be useless to distinguish between the moon's interior structures considered in this study.

This is not the same for $C_{2,0}$ and $C_{2,2}$, for which the distributions

⁸ One model corresponds to one point on the graphs.

are more scattered in the parameter space (Fig. 11 left panel). Once again the current error on the volume/shape induces errors on the homogeneous value of $C_{2,0}$ and $C_{2,2}$ (black ellipse in Fig. 11 left panel) smaller than the signature of most of the different heterogeneities considered here, but only Family 2 (orange in Fig. 11) and 3 (red in Fig. 11) are unambiguously distinguishable from the homogeneous and from the other families. A determination of $C_{2,0}$ and $C_{2,2}$ both at the 5% level or better would corroborate the conclusions drawn from the libration estimates.

Using two different methods, Pätzold et al. (2014b) and Jacobson and Lainey (2014) attempted to estimate the degree-2 gravity coefficients $C_{2,0}$ and $C_{2,2}$ of Phobos. The former tried to estimate these two parameters along with the GM of Phobos from the orbit determination of Mars Express during its 77-km distance flyby in 2010. Given the rather large distance of the flyby, they were not able to determine these parameters accurately enough. Jacobson and Lainey (2014) fitted $C_{2,0}$ and $C_{2,2}$ together with the Phobos and Deimos ephemeris and GMs using all Earth-based and spacecraft based observations from 1877 to 2007. From the fit of these observational data, these authors could not obtain reasonable estimates for both the libration amplitude and the gravity harmonics. They chose to fix the libration to the $-1^\circ.2$ observed value published by Willner et al. (2010). Re-normalized using our reference radius R_0 , the solutions they obtained are $\bar{C}_{2,0} = -0.0455 \pm 0.003$ and $\bar{C}_{2,2} = 0.022 \pm 0.0006$. This $(\bar{C}_{2,0}, \bar{C}_{2,2})$ solution does not match any of our model-predicted values (see Fig. 11 left panel). However, Jacobson and Lainey (2014) estimates of $(\bar{C}_{2,0}, \bar{C}_{2,2})$ are not independent from each other (with a correlation of 96%) and are consistent with a libration of $-1^\circ.2$ of amplitude. When considering the $(\bar{C}_{2,0}, \bar{C}_{2,2})$ confidence region resulting from the 96% correlation between them, then only some porous compressed (PC-F3) models are in agreements with the observations at 3σ (see grey ellipses in Fig. 11 left panel). Therefore, $(\bar{C}_{2,0}, \bar{C}_{2,2})$ estimates support the conclusions drawn from the libration estimates (Section 5.2) claiming that a heavily fractured moon is unlikely. They also suggest that Phobos is not homogeneous but could be a porous compressed body, presenting an increment of density below Stickney. Once again, this conclusion is to be taken with caution since θ , $C_{2,0}$ and $C_{2,2}$ are highly correlated and thus certainly affected by large errors. Nevertheless, this conclusion seems to be in good agreement with the conclusions drawn for the Moon claiming that a preimpact body porosity greater than $\sim 7\%$ produces positive gravity anomalies whose magnitude is greater for impacted surfaces with higher initial porosity (Milbury (2015).)

5.5. 10×10 gravity field

From thousands of models of each family we compute first the mean value of each harmonic coefficient. Assuming these mean values as representative of their family, we can get an idea of the order of magnitude of the departure of the $\bar{C}_{l,m}$, $\bar{S}_{l,m}$ with respect to their homogeneous analogs.

Figs. 12 and 13 show respectively the mean absolute deviation in percent of these mean- $\bar{C}_{l,m}$ and mean- $\bar{S}_{l,m}$ coefficients with respect to the coefficients of the homogeneous Phobos⁹. We see in these figures that the rubble pile models are very close to the homogeneous case, typically shifted by less than 5–10% from the latter's coefficients. All the other models exhibit departure from homogeneous by several tens of percent depending on the coefficient considered. Ignoring the coefficients for which the homogeneous reference values are small (e.g. $\bar{C}_{6,4}$, $\bar{C}_{7,4}$, $\bar{C}_{9,3-9}$ or $\bar{S}_{7,2}$, $\bar{S}_{9,2}$, $\bar{S}_{9,6}$ etc.) and have thus great chances to be largely shifted (relatively speaking), there are two typical patterns seen

in Figs. 12 and 13. On one hand, all the coefficients are almost equally affected at the 10–40% level. This is the case of the two icy sub-families as well as the disrupted models (three lower panels of Figs. 12 and 13). In such cases, all the $\bar{C}_{l,m}$, $\bar{S}_{l,m}$ are similarly shifted (in absolute value), with a slightly larger deviation for high degree and order coefficients, because the models are built assuming a different material distribution (and thus a different average density) in a subregion which is defined following the topography of Phobos (see Table 2). The two icy sub-families (IED-F4 and ISC-F4) exhibit layer(s) at the surface with lower density than in the center, whereas DAR-F1 has a denser outer layer. On the other hand, the heavily fractured models and the porous compressed models display dissymmetrical mass distributions which are responsible for the shifts in the tesseral and sectoral coefficients. Indeed, Figs. 12 and 13 upper panels show that HF-F2 (middle) and PC-F3 (right) sectoral coefficients (with $l = m$) are strongly shifted (sometimes by more than 200%), as are the sub-subdiagonal tesserals with $m = l - 2$.

Even if the mean models considered here above are representative of their own family, and even if the shift we computed are not artefacts due to the size of the cubes as explained above (see Eq. 3 and Fig. 2), it is difficult to conclude about the detectability of a given heterogeneity based on the analysis of only one model per family. To be able to provide such conclusions with some degree of confidence we now consider thousands of models per family (the exact number of model depending on the ranges of porosity and ice proportion, see Section 4), randomly defined but respecting their family characteristics (see Section 4). Then, in order to visualize a statistically-representative deviation of the gravity coefficients for each family and be able to see if such a deviation is significant and can be observed in the gravity data, we compute and plot in Figs. 14 and 15 what we call the overlapping factor. This factor is defined as follow

$$F = \frac{|p_1 - p_2|}{\sqrt{\sigma_1^2 + \sigma_2^2}}, \quad (27)$$

where p_i is the parameter mean value for the thousands models of family i and σ_i the standard deviation of the parameter p_i distribution. $F = k$ means that p_1 and p_2 are adjacent at $k\sigma$. Therefore $F > 1$ suggests a possible distinction between model 1 and 2 assuming a good-enough precision of the determination of the parameter p , whereas an $F \geq 3$ implies a very likely distinction between the two considered model families.

Fig. 14 shows the F 's values assuming the $\bar{C}_{l,m}$ of the homogeneous Phobos model are well known (i.e. $\sigma_{\bar{C}_{l,m}} = 0$ for the homogeneous model). This assumption is obviously not realistic as discussed in Section 3 mostly because of the uncertainty on the shape of Phobos. However, it is reasonable to assume that a mission to Phobos will reduce the shape and volume uncertainties by at least one order of magnitude, reducing the inferred $\sigma_{\bar{C}_{l,m}}$ to 1% or less, which is much smaller than the typically shifts due to heterogeneities as shown previously on Figs. 12 and 13.

The rubble pile family appears to be indistinguishable from the homogeneous Phobos in its gravity field expansions since all its harmonic coefficient distributions overlap at 1σ (i.e. the overlapping factor $F \leq 1$) as shown in Fig. 14 for the $\bar{C}_{l,m}$ (same behaviors are seen on the $\bar{S}_{l,m}$ not shown here). This is not surprising since the distribution of the latter, based on a random process without constraint on the location of the porosity corresponds to a well-mixed interior. At a scale larger than $d = 500$ m, the rubble pile models are homogeneous in mass distribution, that is to say in structure.

Fig. 14 top-middle panel shows the strong changes of the heavily fractured Phobos coefficients compared to the homogeneous nominal values. The $\bar{C}_{l,m \geq l-2}$ are either not affected or strongly distinguishable from those of the homogeneous model with $F \geq 3$, following a checkerboard-like distribution. The departure of the porous compressed moon's $\bar{C}_{l,m}$ (and $\bar{S}_{l,m}$ even not shown here) from the homogeneous

⁹ Note that the coefficients listed in Table 1 are not exactly those used as reference here since Table 1 coefficients have been computed from the spherical harmonic expansion of Phobos' shape and not from the DSK model used in the rest of the paper.

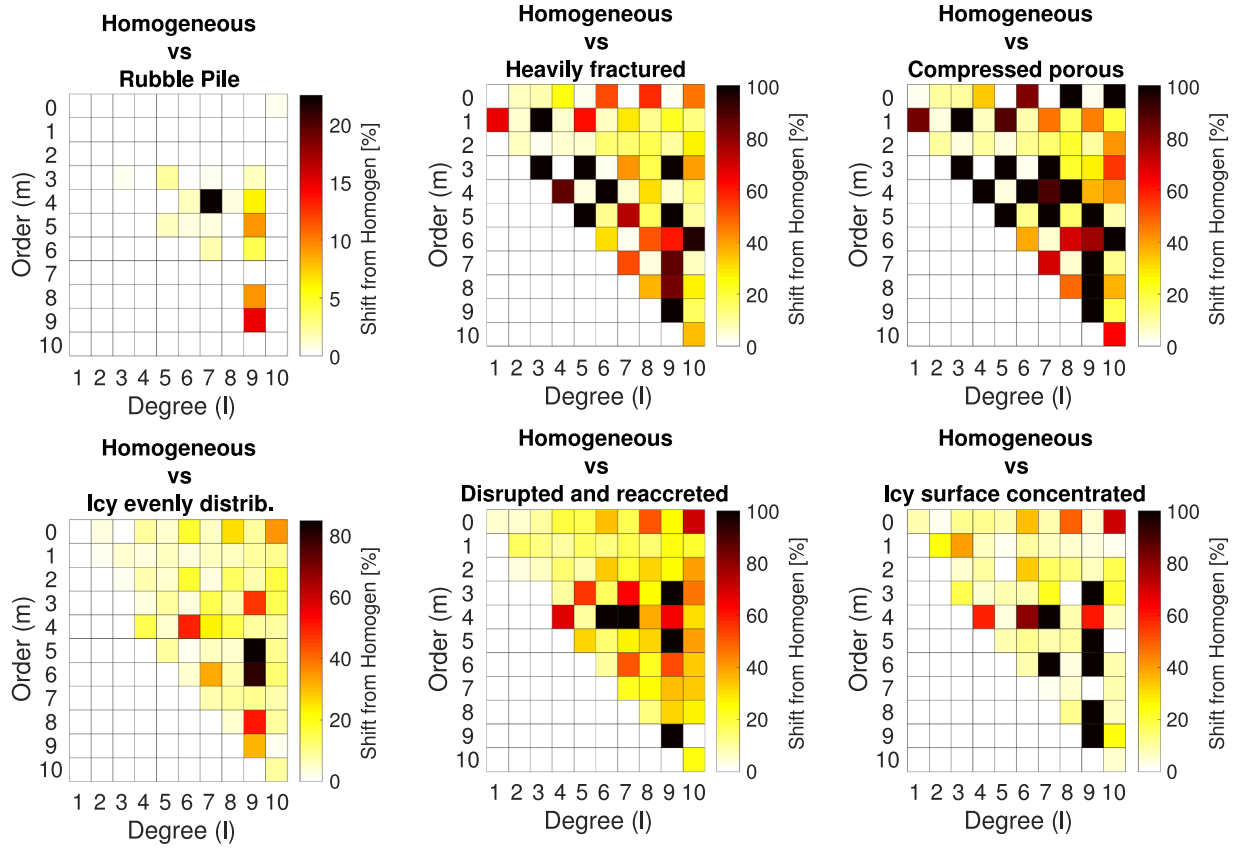


Fig. 12. Mean absolute deviation of the gravity spherical harmonic cosine coefficients ($C_{l, m}$) of each family from the coefficients of the homogeneous Phobos. Values greater than 100% have been set to 100%.

values (Fig. 14 top-right panel) are very similar to that of the heavily fractured models. The disrupted models family also exhibits numerous overlapping factors greater than two or three, especially for the zonal coefficients (Fig. 14 bottom-middle panel), as do the icy surface concentrated models to a minor extent (Fig. 14 bottom-right panel). On the contrary, the icy evenly distributed models can not be distinguished from the homogeneous nominal values (Fig. 14 bottom-left panel) since $F \leq 1$ for all the coefficients of this family.

Up to now we have shown that four of the six families considered here, namely DAR-F1, HF-F2, PC-F3 and ISC-F4, could be distinguished from the homogeneous Phobos assuming the gravity coefficients determined at a good-enough precision level and assuming the volume accuracy improved by one order of magnitude or more. The question is now, can we distinguish these heterogeneous models from each other? Fig. 15 allows us to answer this question. Indeed, the heavily fractured and the porous compressed Phobos, which perturb the gravity field in a similar way (same coefficients affected with a comparable order of magnitude in absolute) have actually very different gravity field coefficients, easily distinguishable from each other because departing from the homogeneous values by the same amount but opposite sign as suggested by Fig. 15 left panel. As said above, DAR-F1 and the icy models (IED-F4 and ISC-F4) show similar patterns in the mean absolute deviation matrices (see Figs. 12 and 13), suggesting an inability to distinguish between them. Since IED-F4 can not be distinguished from the homogeneous model anyway, we only compute the overlapping factors between DAR-F1 and ISC-F4 (Fig. 15 right panel), which appear to be larger than one or two sometimes, making possible the distinction between the two families. Finally, since DAR-F1 and ISC-F4 modify the gravity coefficients in a quite different way from HF-F2 and PC-F3, many of the overlapping factors computed between DAR-F1 or ISC-F4, and HF-F2 or PC-F3 are greater than two, three or even more. This is not shown here but one might guess so looking at Figs. 12–14.

5.6. Surface gravity

Surface gravity measurements have been proposed by several teams to probe the interior of small bodies like Phobos (e.g. Carroll et al. (2016)). Relying on a gravimeter measuring the local surface acceleration, such an experiment would allow the determination of local gravity anomalies (related to subsurface cavities for instance), providing a density map of the landing subsurface region, and bringing information about the body's porosity. Moreover, in situ gravity time series could be obtained, providing information on the rotational motion of the body. Surface acceleration is the gradient of the gravity potential U_{tot} . $U_{tot} = U_{grv} + U_{rot} + U_{tid}$ is the sum of the gravitational potential, the rotational potential and the tidal potential, respectively.

We compute the surface gravitational potential of Phobos according to Newton's law of universal gravitation

$$U_{grv}(p) = G \iiint \frac{\rho_c}{|\vec{r} - \vec{r}_c|} dz dy dx, \quad (28)$$

where p is the surface point of latitude ϕ , longitude λ and distance r from Phobos' center, G is the gravitational constant, ρ_c the density of the cube centered at (x_c, y_c, z_c) and distant from the body's center by $r_c = \sqrt{x_c^2 + y_c^2 + z_c^2}$. Given the cubic decomposition of Phobos' volume performed here, assuming heterogeneous density inside the body, the upper equation can be approximated by the following expression:

$$U_{grv}(p) = \sum_{i=1}^{N_{tot}} \frac{G \rho_i d^3}{\sqrt{(x_p - x_i)^2 + (y_p - y_i)^2 + (z_p - z_i)^2}}, \quad (29)$$

where (x_i, y_i, z_i) are the coordinates of the i th cube center and (x_p, y_p, z_p) the cartesian coordinates of p . ρ_i is the density of the i th cube and d is the length of each of the N_{tot} cubes used to fill Phobos' shape.

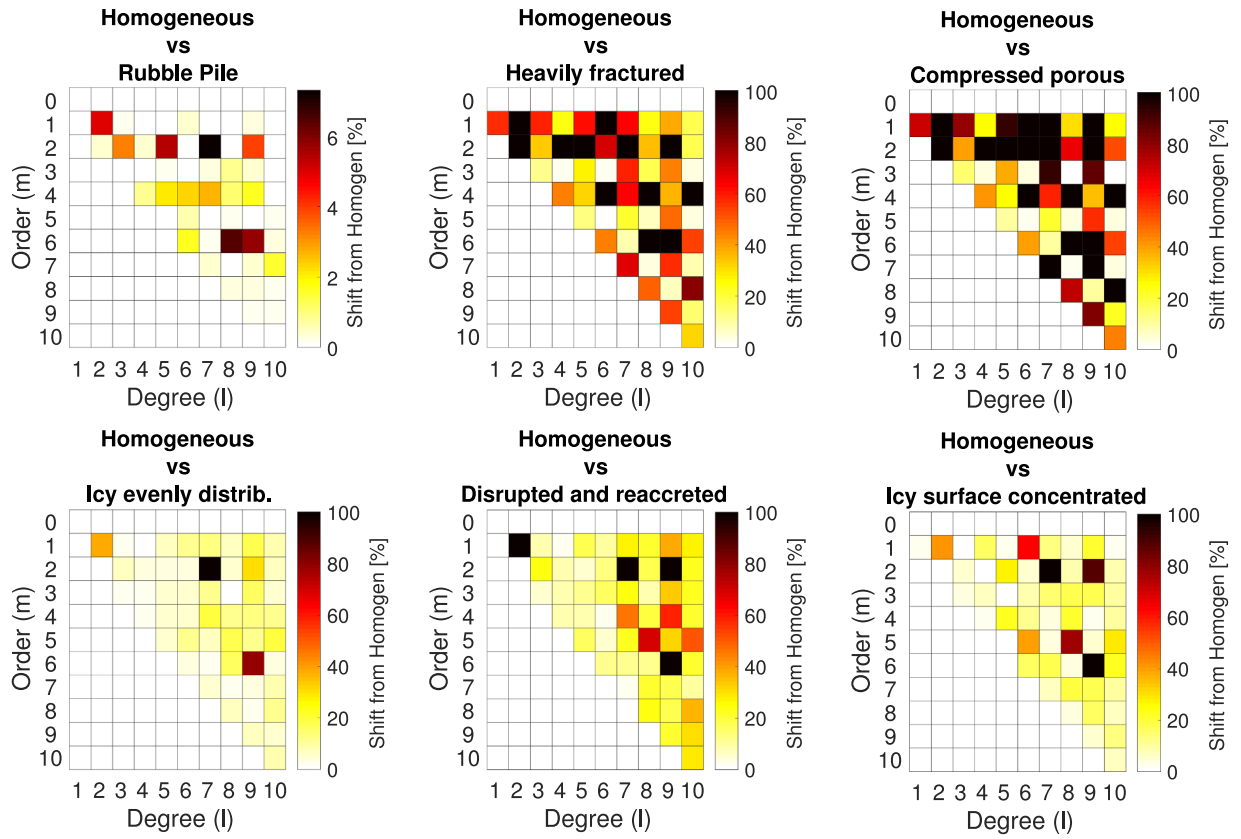


Fig. 13. Mean absolute deviation of the gravity spherical harmonic sine coefficients ($S_{l,m}$) of each family from the coefficients of the homogeneous Phobos. Values greater than 100% have been set to 100%.

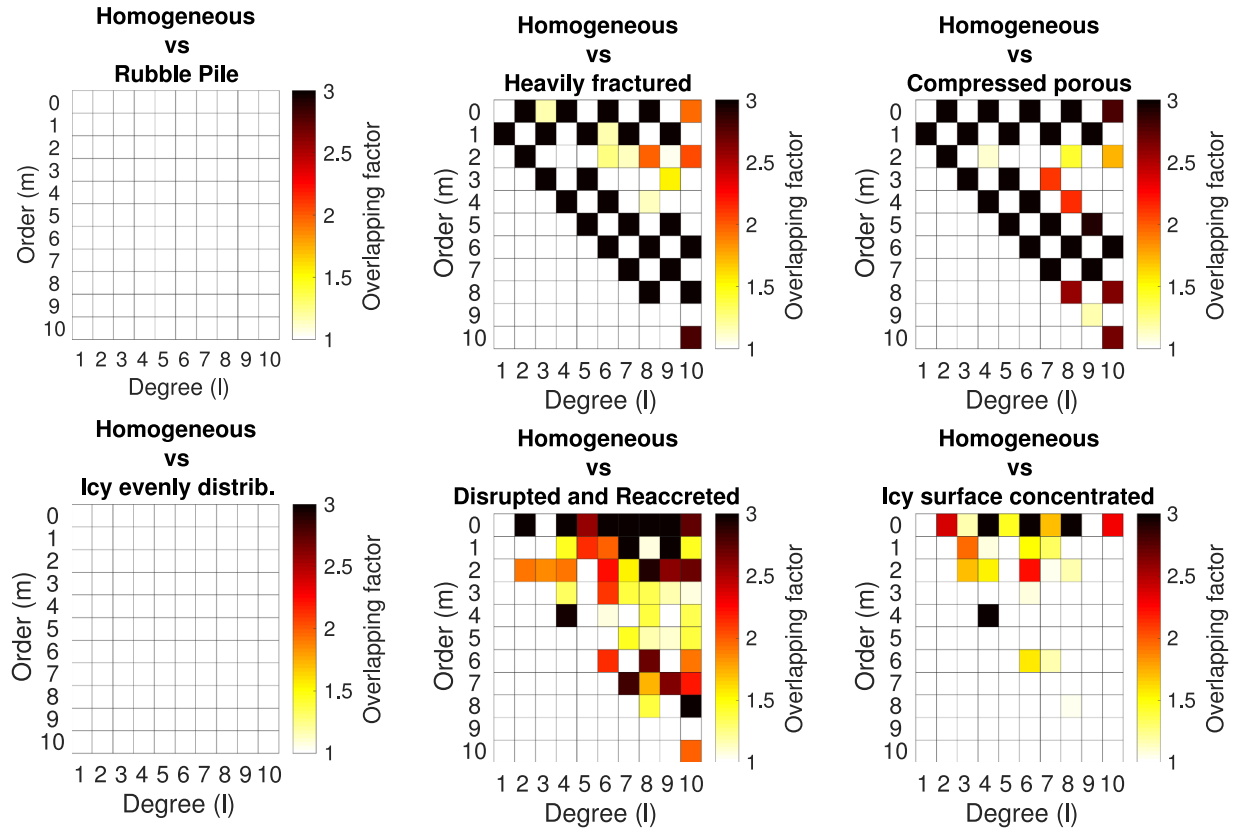


Fig. 14. $C_{l,m}$ overlapping factors corresponding to the multiplicative factors to apply to the $C_{l,m}$ standard deviation of a given family in order to include the homogeneous $C_{l,m}$ values. Values below 1 have been set to 1 and values greater than 3 have been set to 3.

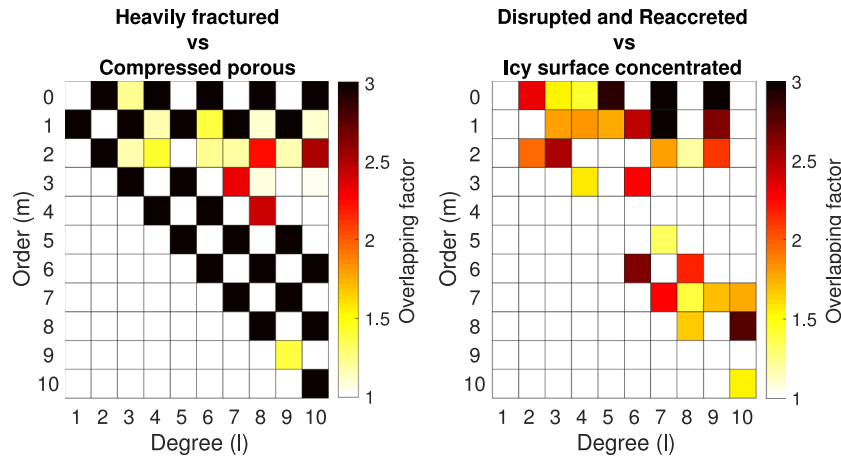


Fig. 15. $C_{l,m}$ Overlapping factors between different heterogeneous families. Values below 1 have been set to 1 and values greater than 3 have been set to 3.

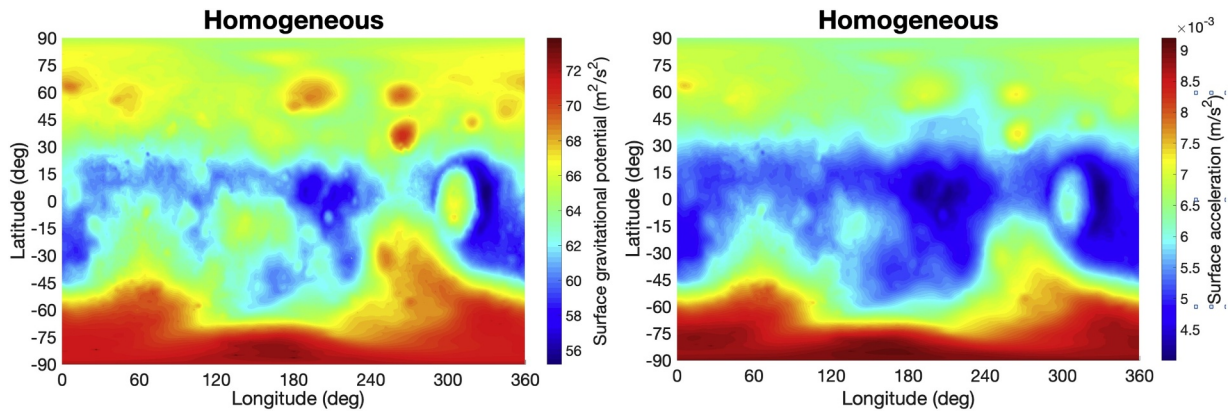


Fig. 16. Surface gravity potential (left panel) and acceleration (right panel) for a homogeneous Phobos with a constant density equal to 1845.4 kg/m³.

Using Eq. (29) we compute, for a representative model of each family considered in this paper, the gravitational potential and its corresponding surface acceleration. These two quantities are mapped in Fig. 16 for the uniform density model of Phobos. The difference between the surface acceleration of this homogeneous model and two heterogeneous models are shown on Fig. 17. A heavily fractured model is on the left panel and a compressed porous model is on the right panel. We displayed in Fig. 17 only the models from families exhibiting distinguishable patterns with deviations from the homogeneous case larger than the $1, (\mu/s^2)$ ($< 0.1\%$) sensitivity of typical gravimeters (e.g. Karatekin et al. (2016)) as could be proposed for Phobos mission. Indeed, HF-F2 and PC-F3 both deviate at the 1–10% level of the acceleration of a homogeneous Phobos and exhibit clear patterns in their surface gravity with respect to the homogeneous Phobos one. These variations of the surface acceleration could actually be observed in the measurements of a gravimeter landed in the Stickney crater and correctly interpreted. The other heterogeneous models do not display distinguishable and measurable patterns. Note that we assumed here the gravimeter is able to provide measurements with an absolute precision of $1, \mu/s^2$, since the signal plotted on Fig. 17 corresponds to spatial variations in the surface acceleration, which are constant with time. If the instrument only provides relative measurements at such a precision with an absolute precision two orders of magnitude worse, the plotted signal will not be observed.

The rotational potential at p is a function of the rotation rate (ω) and the distance to the spin axis ($r \cos \phi$):

$$U_{rot}(p) = \frac{1}{2} \omega^2 r^2 \cos^2 \phi. \quad (30)$$

The rotation rate depends on the librations since $\omega = \omega_0 - \theta \omega_0 \sin(\omega_0 t)$, where ω_0 is the average rotation rate. Therefore, U_{rot} can be used to infer the libration amplitude θ . The amplitude of the surface acceleration induced by the librations depends on p 's latitude ϕ and distance to the body center r according to

$$A_{\Delta a_s} = 2\omega_0^2 r \theta \cos^2 \phi. \quad (31)$$

Fig. 18 (left panel) shows the time variation of the surface acceleration (Δa_s) due to Phobos' librations with an amplitude obtained from Eq. (31) for a gravimeter located at the equator, with $\phi = 0^\circ$ and $r = R_0 = 11.1$ km. The time variable surface acceleration signals shown on this figure correspond to the 1σ uncertainty regions of the two most recent solutions of libration amplitude propagated to the surface acceleration and the one expected from measurements provided by a gravimeter of $1, \mu/s^2$ accuracy assuming $\theta = -1^\circ$. 14 (Burmeister et al., 2018). The precision of the libration estimate itself from the gravimeter equals 0.05 (using Eq. (31)), which is at the same order of magnitude as the precision of the current libration estimates (see Fig. 18 left panel). If a determination of θ using a $1, \mu/s^2$ -accuracy gravimeter is not going to reduce the current uncertainty in the libration amplitude, such a new and independent measurement of θ would definitely help to identify the expected bias in the current libration solutions as clearly shown on Fig. 18 (right panel)¹⁰.

¹⁰ Note that the gravimeter VEGA (Carroll et al., 2016) can provide surface acceleration measurements two orders of magnitude more precise than the $1, \mu/s^2$ considered here, which would allow the determination of θ with 0.0005 of uncertainty only.

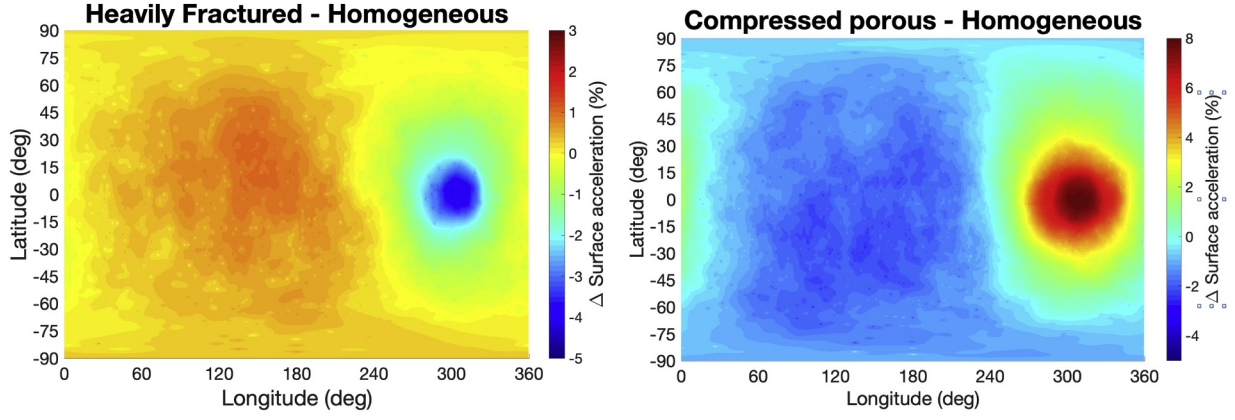


Fig. 17. Differences between the surface acceleration of a homogeneous Phobos and two heterogeneous Phobos: a heavily fractured model with 19% of porosity (top-left) and a porous compressed model with 38% of porosity (top-right).

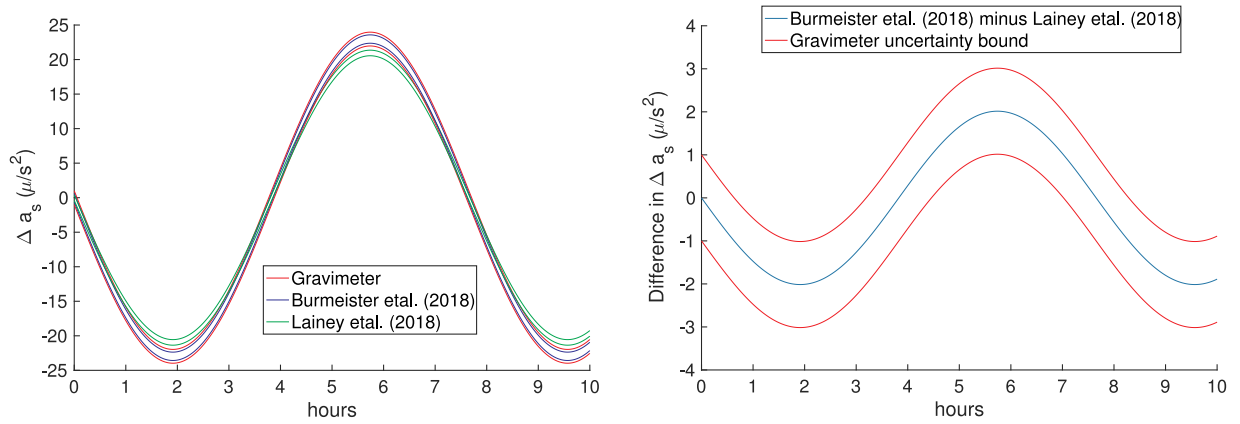


Fig. 18. Left: Comparison between the 1σ uncertainty region of the current solutions of libration amplitude propagated to the surface acceleration and the expected precision provided by a gravimeter of $1, \mu/s^2$ accuracy assuming $\theta = -1^\circ$. 14. Right: Difference between libration amplitude of [Burmeister et al. \(2018\)](#) and of [Lainey et al. \(2019\)](#) superimposed with the 1σ uncertainty region provided by a gravimeter.

The tidal potential at the surface point p of a synchronous rotator like Phobos, developed to degree 2, is a function of the mass of the primary, here Mars (M_σ), and the coordinates (r, ϕ, λ) of p according to:

$$U_{tid}(p) = GM_\sigma \frac{r^2}{4D^3} (2P_{2,0}(\sin \phi) - P_{2,2}(\sin \phi) \cos(2\lambda)). \quad (32)$$

The distance from Mars to Phobos $D = a(1 - e^2)/(1 + e \cos \nu)$ is minimum at pericenter, when the true anomaly $\nu = 0$. D is then equal to $D_{\min} = a(1 - e)$ and the tidal potential is maximum. $a = 9375$ km is Phobos orbit's semi-major axis and $e = 0.01511$ its eccentricity. Symmetrically, the tidal potential is minimum when $D = a(1 + e) = D_{\max}$ is maximum ($\nu = 180^\circ$). Raised to the power of -3 , $D_{\min}$ and $D_{\max}$ become to first order in eccentricity $D_{\min}^{-3} = a^{-3}(1 + 3e)$ and $D_{\max}^{-3} = a^{-3}(1 - 3e)$. According to the latter expressions of $D_{\min}$ and $D_{\max}$, and given the definition of the associated Legendre functions $P_{2,0}$ and $P_{2,2}$, one can compute from Eq. (32) the amplitude of the tidal potential variations in p , $A_{\Delta U_{tid}}(p) = |U_{tid}^{\max}(p) - U_{tid}^{\min}(p)|/2$, leading to

$$A_{\Delta U_{tid}}(p) \simeq GM_\sigma \frac{9e}{4a} \left(\frac{r}{a} \right)^2 \left| \cos(2\lambda) \cos^2 \phi - \sin^2 \phi + \frac{1}{3} \right|. \quad (33)$$

This variation of the external tidal potential is due to the variation in the tidal torque because of Phobos' orbital eccentricity. It does not depend on Phobos' interior properties. Conversely to the tidal potential (or torque), the tidal response of the non-rigid satellite, i.e. the way Phobos will deform under such a tidal torque, will depend on the in-

terior structure of the moon. This has been quantified to be very small (e.g. [Le Maistre et al. \(2013\)](#)), inducing radial displacements likely much smaller than 10^{-2} meters at the surface. Such a displacement of the masses inside Phobos would periodically modify the gravitational potential proportionally to the external tidal potential evaluated at the surface. The coefficient of proportionality is the dynamic Love number, which depends on the density and rheology, as well as of the frequency of the excitation. At the orbital frequency, the degree-two Love number k_2 has been calculated by [Le Maistre et al. \(2013\)](#) to be smaller than 5×10^{-4} . Assuming k_2 to be significantly larger than the higher-degree Love numbers $k_{l \geq 3}$, one can compute from Eq. (33) the order of magnitude of the amplitude of the time variable surface acceleration due to the tides raised by Mars at p : $A_{\Delta a_s}(p) = k_2 \nabla A_{\Delta U_{tid}}(p)$. Assuming k_2 is

maximum possible value, $A_{\Delta ds}(p) \leq 10^{-2} \mu/s^2$. This is the amplitude of the signal of geophysical interest, which would allow us to give constraints on Phobos interior and composition if measured. However, such a signal won't be observed because it is 10^3 times smaller than the expected sensitivity of the gravimeter. Note that such a tiny variation corresponds to a modulation of the surface acceleration amplitude at the $< 0.001\%$ level. This modulation is solely induced by the variation in the Mars-Phobos distance (D in Eq. (1)) and Phobos' ability to deform (k_2). Even if we were able to measure this tidal acceleration with high precision¹¹, we wouldn't be able confidently to take it out from the noise induced by the current errors in the distance D . Indeed, the relative variations in U_{tid} due to the 1-km errors on the radial distance D in Phobos' ephemeris (Lainey et al., 2019; Jacobson, 2010) is 0.03% leading to uncertainties on $A_{\Delta U_{tid}}$ at a 0.01% level, which is one order of magnitude greater than the geophysical signal. Moreover, even if the level of errors in Phobos' ephemeris was drastically reduced, such a signal would be impossible to distinguish from the librational signal shown in Fig. 18 (left panel) since both have the same period.

6. Discussions and conclusions

The present paper predicts the range of plausible values of geodetic quantities related to the interior of the largest moon of Mars, Phobos. In particular, ranges for the moments of inertia, the libration amplitude, the shift between center of mass and center of figure, the gravity coefficients up to degree and order 10 and the surface acceleration are provided for six different characteristics of Phobos' interior structure. The signature of each of the six interior model families into the mentioned geodetic observables is computed based on a finite element approach. Generally speaking, the levels of departure of the moments of inertia (MOI) due to heterogeneity in Phobos' interior are shown to be smaller than 15% of their respective homogeneous values. However, even if the MOI are estimated with a precision significantly better than 15%, it will be difficult to discriminate between the different candidates of interior structure. Phobos' internal heterogeneities, as defined in the present paper, can shift the libration amplitude from its homogeneous value by up to 35% of the latter. We found that two of the heterogeneous families considered here, the heavily fractured (see Fig. 5) and the porous compressed body (see Fig. 6), exhibit distinguishable deviations of libration amplitude from the homogeneous value and are displaying significant displacement of CoM with respect to CoF, both at the level of a couple of hundred meters or more along the x and y axes of Phobos. These same two families also exhibit the most significant deviations in the 10×10 gravity signal and in particular in the $C_{2,0}$ and $C_{2,2}$ coefficients. It is finally shown that the surface acceleration could be measured with a gravimeter with enough precision likely to provide an independent estimate of the libration amplitude and thereby help to constrain the interior structure of Phobos. A measurement of the surface acceleration could also help identifying an increment or lack of density under the Stickney region if the gravimeter lies in the crater.

Given the classification of Britt et al. (2002), Phobos could be either a fractured body or a rubble-pile, but an outer-solar-system body, not accounted for in this classification, cannot be ruled out. Based on some initial assumption about the possible interior properties of Phobos, our analysis shows that the $(C_{2,0}, C_{2,2})$ estimates provided by Jacobson and Lainey (2014) and the latest estimates of the libration amplitude of Phobos (Burmeister et al. (2018), Lainey et al. (2019)) tend to exclude the heavily fractured body, pushing more in favor of a globally porous body, which would have higher density below Stickney due to the impact responsible for the latter giant crater. Heavily fractured bodies have most likely been created by impacting an initially solid monolithic

body. Therefore, this study makes us think that Phobos' original body was not a solid monolith. This conclusion is however very preliminary since $C_{2,0}$, $C_{2,2}$ and the libration amplitude are expected to be affected by true errors larger than their 1σ uncertainties.

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