

Strain rate dependence of the contribution of surface diffusion to bulk sintering viscosity

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Abstract

Modeling of bulk sintering viscosity usually neglects the contribution of pore surface diffusion with respect to grain-boundary diffusion. This approximation is questionable at the high densification rates used today in advanced fast sintering techniques. A two-dimensional analysis of the problem shows that the influence of surface diffusion on bulk viscosity at high strain rate can be decomposed as the sum of two terms: a term linked to the change in pore surface curvature and a term linked to the change in grain-boundary size. The computational procedure relies on the partition of pore profile evolution into a transient component accounting for non-densifying phenomena and an asymptotic component accounting for strain-rate-controlled phenomena. The largest impact of surface diffusion is found to arise from the change in grain-boundary size. It follows a transition from Newtonian viscosity at low strain rate to non-Newtonian viscosity which, during densification, increases nearly linearly with strain rate. In some conditions, viscosity can then reach more than twice the value estimated when neglecting pore surface diffusion. Reversely, expansion is accompanied by a decrease in grain-boundary size which causes a decrease in viscosity and can lead to grain separation at high strain rate.

KEYWORDS

diffusion, grain coordination, sintering, viscosity

1 | INTRODUCTION

For the prediction of the evolution of dimension and shape during sintering, use is commonly made of a formulation in which the macroscopic strain rate tensor, $\dot{\mathbf{E}}$, is a linear function of the macroscopic stress tensor, $\mathbf{S} = \mathbf{S}_\infty - \Sigma \mathbf{I}$ where $\mathbf{S}_\infty$ is the remotely applied stress, $\mathbf{I}$ is the identity tensor, and Σ is the so-called sintering stress arising from the thermodynamic forces driving free sintering.^{1,2} The linear relationship between $\dot{\mathbf{E}}$ and $\mathbf{S}$ defines a macroscopic viscosity tensor which, for an isotropic body, reduces to only two scalar components—"bulk viscosity", K , and "shear viscosity", G ,—that characterize the resistance to deformation under purely hydrostatic deformation and purely shear deformation, respectively. Numerical

simulations rely on constitutive laws for the dependence of K and G on grain size, relative density and strain rate.

This work considers the mechanism of sintering governed by diffusion along pore surfaces and grain boundaries. Much effort has been invested in literature with the objective to derive K for this mechanism by modeling dissipative phenomena arising from diffusion fluxes.^{3–5} As sketched in Figure 1, two types of diffusion fluxes along pore surface have to be distinguished:

1. the diffusion fluxes feeding the reversible exchange of matter between grain boundaries and pores: these fluxes are driven by sintering stress and external loading, and bring about macroscopic straining, ie, densification or expansion, possibly coupled with shear;

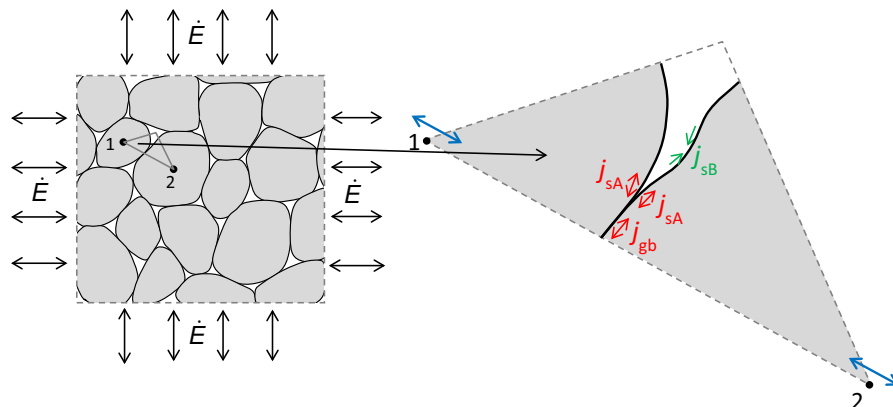


FIGURE 1 Two types of surface diffusion fluxes: fluxes denoted j_{sA} feed grain-boundary diffusion fluxes j_{gb} , the divergence of which brings about relative displacement of the centroids of grains 1 and 2; fluxes denoted j_{sB} are driven by small-scale curvature gradient and are not coupled to macroscopic strain rate

2. the diffusion fluxes feeding the transient, irreversible transfer of matter from surface to surface at a scale smaller than grain size: these fluxes are driven by local curvature gradients and do not bring about macroscopic strain, ie, they do not contribute to sintering viscosity.

Only diffusion fluxes of type (1) have an impact on K . This work focuses on the influence of grain-boundary diffusion on bulk viscosity K under the high strain rate conditions used in the various fast sintering methods that are becoming increasingly practiced nowadays. It is assumed that the same sintering mechanism operates throughout the whole range of strain rate that is considered. The influence of grain-boundary diffusion on K then depends on the relative size of grain boundaries and pores, and on the ratio $\Delta = \frac{\delta D_{gb}}{\delta D_s}$ of grain-boundary diffusivity to surface diffusivity. As Δ increases with homologous temperature, dissipative phenomena are dominated by grain-boundary diffusion at low homologous temperature. In that case, it is commonly considered acceptable to neglect the contribution of surface diffusion to K and G . This so-called “quasi-equilibrium approximation” amounts to assuming that pore shape and grain-boundary size remain undistinguishable from static equilibrium shape and size.^{3,5–8} Models based on this approximation predict a Newtonian viscosity, ie, a viscosity independent of strain rate. The quasi-equilibrium approximation is implicit in finite element codes resting on physically based constitutive laws.^{9,10} Actually, when $\dot{\epsilon}$ increases, grain-boundary size differs from equilibrium size and curvature gradients driving surface diffusion increase. Moreover, distinction between strain-rate-controlled fluxes (1) and transient fluxes (2) becomes difficult if strain rate is comparable to the rate of damping of transient fluxes. Hence, unless Δ is very small, the validity of the quasi-equilibrium approximation breaks down at high strain rate. In literature, the sole example of analysis of the contribution of surface diffusion to K is the work of Svoboda and Riedel¹¹ who, via numerical simulations of a 2D hexagonal array of identical rods,

derived an analytical approximation valid close to the quasi-equilibrium limit. These authors did not contemplate the influence of high strain rate on K , nor the distinction between transient and strain-rate-controlled diffusion fluxes. No clear picture stands out about the non-Newtonian character that can be anticipated if surface diffusion significantly contributes to dissipation.

Like former literature on the subject, our analysis is based on a two-dimensional model, which provides insight into the essential physics (transition toward a final sintering stage with closed pores is thus meaningless). The paper is divided into two sections dealing successively with basic equations (Section 2) and with computational results (Section 3). Section 2.1 presents the representative volume element: model parameters are the initial grain radius R_G , the relative density ρ , the dihedral angle ψ (defined as $\psi = 2\arccos(\gamma_{\text{grain boundary}}/2\gamma_{\text{surface}})$), the average grain coordination Z , and the diffusivity ratio Δ (for which the range is limited to $0 < \Delta < 1$). Section 2.2 shows that bulk viscosity K can be expressed as a sum of two terms: a term linked to pore surface curvature gradients and a term linked to grain-boundary size. Section 2.3 summarizes the computational procedure, which relies on the results of a companion paper in which the evolution of pore surface is partitioned into a transient component accounting for non-densifying diffusion phenomena and an asymptotic component accounting for strain-rate-controlled diffusion phenomena.¹² The concept of sintering viscosity is valid only in conditions warranting that the asymptotic behavior dominates the overall evolution of the system. Computational results presented in Section 3 show that the largest impact of surface diffusion on K is due to the change in grain-boundary size. In particular, results reveal a transition from Newtonian viscosity at low strain rate to non-Newtonian viscosity which, during densification, increases nearly linearly with strain rate. In some conditions, the value of K accounting for surface diffusion can then be more than twice the value predicted in the framework of the quasi-equilibrium approximation. The reverse behavior occurs during expansion.

2 | MODEL AND EQUATIONS

2.1 | Representative volume element for arbitrary average grain coordination

We consider a randomly isotropic bundle of infinitely elongated grains undergoing purely radial densification or expansion (ie, plane strain deformation). The model represents the grains as having identical size equal to the average grain size, of which the measure is taken to be the radius R_G of a cylinder of same volume. The in-plane component, $\dot{E}$, of the macroscopic strain rate is negative in the case of densification and positive in the case of expansion. The macroscopic stress, which is thus also radial in the plane, writes $S = S_\infty - \Sigma$ (ie, $\dot{E} = 0$ when $S_\infty = \Sigma$). During sintering, the average grain coordination, Z , increases monotonously with relative density ρ . As illustrated in Figure 2, the RVE of a system with arbitrary (non-integer) value of Z may be limited to a triangle (containing the part of the solid grain in gray shade) characterized by its opening angle

$$\beta = \frac{\pi}{Z}, \quad (1)$$

and by its height H ,

$$H = \sqrt{\frac{\pi}{Z\rho \tan \beta}} R_G = \sqrt{\frac{\beta}{\rho \tan \beta}} R_G. \quad (2)$$

The macroscopic strain rate is

$$\dot{E} = \frac{\dot{H}}{H} = -\frac{1}{2} \frac{\dot{\rho}}{\rho}. \quad (3)$$

In Figure 2, dihedral angle ψ was taken high enough for the pore curvature to be convex (ie, for the solid surface to be concave): the transition from convex to concave pore curvature occurs when $\beta = \frac{\psi}{2}$. Point A is the centroid of a grain, point J the middle of a grain boundary, point T the triple line, point O the centroid of the pore and point M the middle of the pore profile. Axis y is taken parallel to OA. The curve connecting M to T represents the actual pore profile during densification whereas the dashed curve in black represents the static equilibrium profile at the same relative density. The latter is an arc of circle of radius denoted R_e with center of curvature located by vector $\mathbf{P}_e$ parallel to axis y . $\kappa_e = \frac{1}{R_e}$ is the curvature of the static profile whereas κ_T denotes the curvature of the profile at triple line T. T_e and T_e are the positions of points T and M at static equilibrium. The azimuthal angle spanning the static profile from M_e to T_e is

$$\varphi_{Te} = \frac{\psi}{2} - \beta. \quad (4)$$

Hence, $\varphi_{Te} > 0$ in case of convex pore and $\varphi_{Te} < 0$ in case of concave pore. R_e is related to ρ , Z , H and ψ via

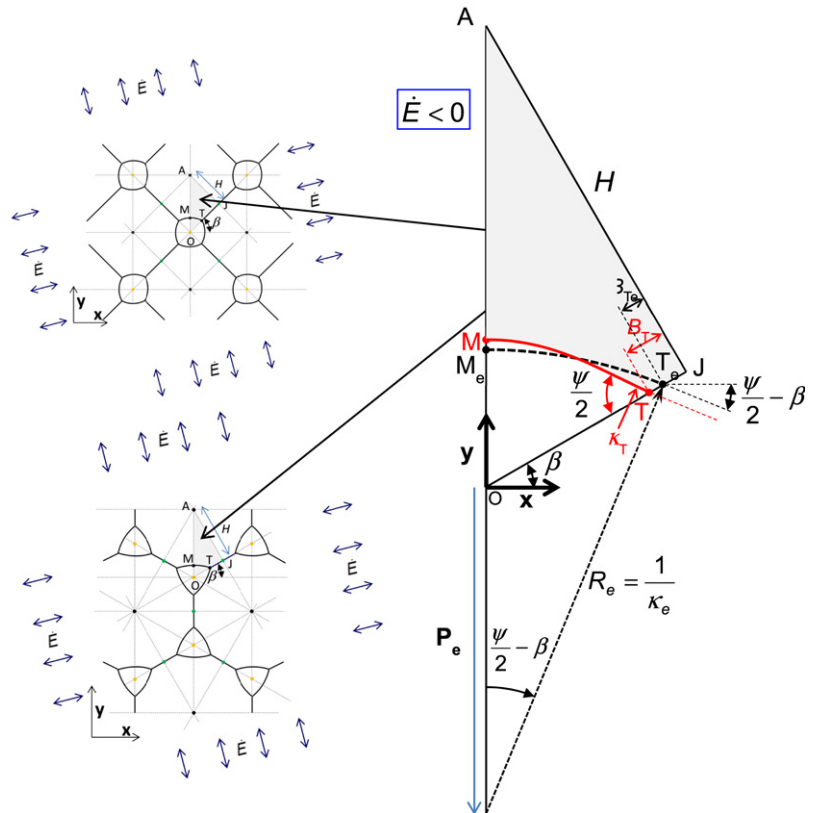


FIGURE 2 RVE in the case $\dot{E} < 0$ with convex pore curvature and arbitrary average coordination number Z

$$1 - \rho = \frac{1}{H^2 \tan \beta} \left(\int_0^{\frac{\psi}{2} - \beta} R_e^2(\varphi) d\varphi - R_e^2 \frac{\cos \frac{\psi}{2}}{\cos \beta} \sin \left(\frac{\psi}{2} - \beta \right) \right) \\ = \frac{R_e^2}{H^2 \tan \beta} \left[\left(\frac{\psi}{2} - \beta \right) - \frac{\cos \frac{\psi}{2} \sin \left(\frac{\psi}{2} - \beta \right)}{\cos \beta} \right] \quad (5)$$

(the expression between the brackets is positive when $\frac{\psi}{2} \neq \beta$). We denote B the distance from point J along grain boundary: $B = 0$ at J and $B = B_T$ at T. B_T thus denotes half of the grain-boundary length. At static equilibrium, $B_T = B_{Te}$ with

$$B_{Te} = H \tan \beta - R_e \frac{\sin(\frac{\psi}{2} - \beta)}{\cos \beta} \quad (6)$$

In the case represented in Figure 2, $\dot{E} < 0$ which means that $B_T > B_{Te}$ and $\kappa_T < \kappa_e = \frac{1}{R_e}$ because matter diffuses along pore surface from T to M. The reverse applies if $\dot{E} > 0$. Equation (6) is valid for convex pores as well as for concave pores if one takes by convention $R_e > 0$ for a convex pore and $R_e < 0$ for a concave pore.

2.2 | Bulk viscosity K

2.2.1 | K as a function of κ_T

In plane strain, the definition bulk viscosity is

$$K = \frac{1}{2} \frac{S}{\dot{E}} \quad (7)$$

The link between macroscopic strain rate $\dot{E}$ and macroscopic stress S derives from the coupling of surface and interface diffusion fluxes at the triple line. The underlying theory has been worked out by several authors.^{11,13,14} Let us, by convention, take diffusion fluxes, j_s and j_{gb} , along pore surface and grain boundary as positive when oriented from T to M and from T to J. j_{gb} is related to the local gradient of the stress component normal to the grain boundary, $\sigma_{nbg} = \mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n}$, as¹⁵:

$$j_{gb} = - \frac{\delta D_{gb}}{kT} \frac{\partial \sigma_{nbg}}{\partial B} \quad (8)$$

If grain boundary is assumed to remain plane, the divergence of j_{gb} is constant, ie, for the RVE of Figure 2,

$$\frac{2}{\Omega} \dot{H} = \frac{\partial j_{gb}}{\partial B} \\ = - \frac{\delta D_{gb}}{kT} \frac{\partial^2 \sigma_{nbg}}{\partial B^2} \quad (9)$$

where Ω is the volume of diffusing species. With the boundary conditions $\frac{\partial \sigma_{nbg}}{\partial B} = 0$ in $B = 0$ and $\sigma_{nbg} = \sigma_s = \gamma_s \kappa_T$ in $B = B_T$, Equation (9) yields

$$j_{gb} = - \frac{2}{\Omega} \dot{H} B \quad (10)$$

and

$$\sigma_{nbg} = \gamma_s \kappa_T + \frac{kT}{\Omega \delta D_{gb}} \dot{H} (B_T^2 - B^2). \quad (11)$$

As the macroscopic stress S_∞ exerted by external loading is purely radial, the resultant forces F_{AJ} , F_{TJ} , and F_{AM} exerted on the sides AJ, TJ, and AM of the RVE (Figure 2) have no shear component. Static equilibrium implies that

$$F_{AM} = \frac{F_{AJ}}{\cos \beta} = \frac{S_\infty H}{\cos \beta}, \quad (12)$$

i.e.,

$$F_{TJ} = F_{AM} \sin \beta = S_\infty H \tan \beta \quad (13)$$

The integral of σ_{nbg} (Equation (11)) on side TJ is balanced by the sum of F_{TJ} plus the component, in the direction normal to the grain boundary, of the force exerted at T by the surface tension γ_s :

$$\gamma_s \kappa_T B_T + \frac{2}{3} \frac{kT}{\Omega \delta D_{gb}} B_T^3 \dot{H} = - \gamma_s \sin \frac{\psi}{2} + S_\infty H \tan \beta \\ = - \gamma_s \sin \frac{\psi}{2} + H \tan \beta (S + \Sigma) \quad (14)$$

At static equilibrium, $S_\infty = \Sigma$ and pore surface curvature is uniform: $\kappa_{Te} = \kappa_e$. Hence, according to Equation (14), sintering pressure is

$$\Sigma = \frac{\gamma_s}{H \tan \beta} \left(\sin \frac{\psi}{2} + B_{Te} \kappa_e \right) \quad (15)$$

Alternatively, substitution of B_{Te} via Equation (6) yields

$$\Sigma = \gamma_s \left(\frac{\cos \frac{\psi}{2}}{H} + \kappa_e \right) \\ = \frac{\gamma_{gb}}{2H} + \gamma_s \kappa_e \quad (16)$$

It is shown in Appendix 1 that Equation (16) can also be derived from the thermodynamic definition of Σ in terms of partial derivative of interface energy with respect to the volume of the system.^{16,17}

Combining Equations (7), (14), and (15), bulk viscosity is obtained as

$$K = \frac{1}{2} \frac{1}{\tan \beta} \left[(B_T \kappa_T - B_{Te} \kappa_e) \frac{1}{\dot{E}} \frac{R_G}{H} \frac{1}{t_{gb}^3} + \frac{2}{3} \left(\frac{B_T}{R_G} \right)^3 \right] \frac{kTR_G^3}{\Omega \delta D_{gb}} \quad (17)$$

with

$$t_{gb}^3 = \frac{kTR_G^4}{\Omega \delta D_{gb} \gamma_s} \quad (18)$$

Multiplying by $2\dot{E}$ the first term between the brackets in Equation (17) yields the expression of the stress needed to induce the gradient of surface curvature that drives the diffusion flux j_s . Conversely, multiplying by $2\dot{E}$ the second term yields the expression of the stress needed to induce the diffusion flux j_{gb} along the grain boundary.

If $\Delta = \frac{\delta D_{gb}}{\delta D_s} \cong 0$, one may approximate $B_T \cong B_{Te}$ and $\kappa_T \cong \kappa_e$. Equation (17) then reduces to the quasi-equilibrium estimate of K :

$$K_{qe} = \frac{1}{3} \frac{1}{\tan \beta} \frac{kT}{\Omega \delta D_{gb}} B_{Te}^3 \quad (19)$$

K_{qe} involves only dissipation at grain boundaries. In contrast to K , K_{qe} does not depend on straining rate: non-Newtonian viscosity can arise only from the contribution of surface diffusion. Equations (14), (16) and (19) agree with the equations developed by Svoboda and Riedel for the case $Z = 6$.¹¹ Equation (19) can also be retrieved from the equations derived by Mullins⁷ and Zhang et al¹³

2.2.2 | K as a function of j_s

K can be obtained via an alternative route by making use of the relationship

$$K = \frac{\dot{Q}}{4\dot{E}^2} \quad (20)$$

where $\dot{Q}$ is the dissipation rate per unit volume, which, considering a unit thickness of the RVE of Figure 2, expresses as

$$\dot{Q} = \frac{2}{H^2 \tan \beta} \frac{\Omega kT}{\delta D_{gb}} \left(\Delta \int_0^{S_T} j_s^2 dS + \frac{1}{2} \int_0^{B_T} j_{gb}^2 dB \right) \quad (21)$$

where S denotes the current coordinate along the pore profile, which spans the range $S_M = 0 \leq S \leq S_T$. Hence,

$$K = \frac{1}{2\dot{E}^2} \frac{1}{H^2 \tan \beta} \frac{\Omega kT}{\delta D_{gb}} \left(\Delta \int_0^{S_T} j_s^2 dS + \frac{1}{2} \int_0^{B_T} j_{gb}^2 dB \right) \quad (22)$$

The fact that the second term at the right hand side is identical in Equations (17) and (22) is consistent with Equation (10). The equality of the first terms in Equations (17) and (22) implies

$$B_T \kappa_T - B_{Te} \kappa_e = \frac{1}{\dot{E}} \frac{1}{H} \frac{\Omega kT}{\gamma_s \delta D_{gb}} \Delta \int_0^{S_T} j_s^2 dS \quad (23)$$

which follows from the law of dependence of j_s on curvature gradients (eg, Ref.¹⁸):

$$j_s = -\frac{1}{\Delta} \frac{\delta D_{gb} \gamma_s}{kT} \frac{\partial \kappa}{\partial S} \quad (24)$$

2.3 | Computation of κ_T , B_T , and j_s

Computation of Equations (17) and (22) requires computation of the dependence of B_T , κ_T , and j_s on ρ , Z , $\dot{E}$, and Δ . As detailed in companion paper,¹² the evolution of the pore profile under a given macroscopic strain rate is the solution of a partial differential equation (PDE) of the fourth order which can be resolved numerically via the finite difference method. Inherently, different solutions are obtained depending on the function chosen for the initial profile. No unique relationship $K(\rho, Z, \dot{E}, \Delta)$ can thus be derived from these solutions. As shown in Ref.,¹² the non-uniqueness of the solution can be circumvented by partitioning the full solution of the PDE into an asymptotic component non-dependent on initial conditions and a transient component dependent on initial conditions. These two components correspond to the two types of diffusion fluxes mentioned in the Introduction Section (Figure 1). As sintering viscosity arises only from dissipation linked to diffusion fluxes governed by strain rate, only the asymptotic component of the solution is pertinent for the present purpose. It is shown in Ref.¹² that, via some approximation, asymptotic solutions can be calculated via a semi-analytical procedure and that, whatever initial conditions, full numerical profiles converge with good accuracy toward these solutions when strain increases. Accordingly, in the present paper, the dependence of B_T , κ_T , and j_s on ρ , Z , $\dot{E}$, and Δ is calculated solely on the basis of asymptotic solutions calculated via the semi-analytical method. For the details of the computational procedure, the reader is referred to Section 2.3.2 of Ref.¹²

Using a criterion based on the rates of evolution of transient and asymptotic components, an upper limit of the product $|\dot{E}||\Delta|$ was identified beyond which the concept of sintering viscosity as a material property is meaningless because the evolution of the system would then be dominated by transient diffusion fluxes driven by small-scale curvature gradient.¹² By referring to the graphs presented in Figures 10 and 12 of Ref.¹² the reader can verify that the $|\dot{E}||\Delta|$ values used for the computational results presented in Section 3 hereunder are lower than the upper bound of the domain of existence of the asymptotic behavior.

3 | RESULTS AND DISCUSSION

In order to enlighten the effect of surface diffusion on K , we will focus on the relative correction, $\frac{K - K_{qe}}{K_{qe}}$, that must be brought to the quasi-equilibrium estimate of K (Equation (19)). This correction involves two contributions corresponding to the two terms in Equations (17) and (22):

$$\frac{K - K_{qe}}{K_{qe}} = \frac{K_{js}}{K_{qe}} + \frac{K_{BT}}{K_{qe}} \quad (25)$$

The correction $\frac{K_{js}}{K_{qe}}$ arises from the neglect of the dissipation due to j_s : according to Equations (17) and (19),

$$\frac{K_{js}}{K_{qe}} = \frac{3}{2} \frac{1}{\dot{E} t^{gb}} \frac{R_G}{H} (B_T \kappa_T - B_{Te} \kappa_e) \left(\frac{R_G}{B_{Te}} \right)^3, \quad (26)$$

whereas, according to Equations (18), (19), and (22)

$$\frac{K_{js}}{K_{qe}} = \frac{3}{2} \frac{1}{\dot{E}} \frac{1}{H^2} \frac{\Omega^2}{B_{Te}^3} \Delta \left(\int_0^{S_T} j_s^2 dS \right) \quad (27)$$

(both equations will be helpful in the following discussion). The correction $\frac{K_{BT}}{K_{qe}}$ arises from the neglect of the effect of surface diffusion on grain-boundary size B_T : according to Equation (17)

$$\frac{K_{BT}}{K_{qe}} = \frac{1}{R_G^3} (B_T^3 - B_{Te}^3) \left(\frac{R_G}{B_{Te}} \right)^3 \quad (28)$$

3.1 | Assessment of computational precision for $\frac{K_{js}}{K_{qe}}$ and comparison with literature

A difference may exist between $\frac{K_{js}}{K_{qe}}$ calculated via Equation (26) or via Equation (27) because the approximations involved in the semi-analytical procedure of computation of the asymptotic profile may cause Equation (23) not to be exactly verified. The departure between Equations (26) and (27) will be assessed while comparing our results with the work of Svoboda and Riedel.¹¹ On the basis of a hexagonal array of grains ($Z = 6$), these authors proposed an analytical expression for the discrepancy between K and K_{qe} in conditions close to the quasi-equilibrium limit, which brings them to adopt the approximation $B_T = B_{Te}$. They denote the correction $\frac{K - K_{qe}}{K_{qe}} \equiv A\Delta$: for consistency with Equation (25), we will write $A\Delta \equiv \frac{K_{SR}}{K_{qe}}$ where K_{SR} denotes the bulk modulus according to Svoboda and Riedel. Owing to the approximation $B_T = B_{Te}$, $\frac{K_{SR}}{K_{qe}}$ must be compared to $\frac{K_{js}}{K_{qe}}$. The function A is given by Equation (25) of Ref¹¹: according to this equation, $\frac{K_{SR}}{K_{qe}}$ does not depend on strain rate.

As an illustration of the difference between K_{SR} and K_{js} , Figure 3 compares, for $Z = 6$, $\psi = 105^\circ$, $\Delta = 0.1$, and densification rates $\dot{E} = -10$ or -500 ($t^{gb})^{-1}$, the relative difference $\frac{K_{js} - K_{SR}}{K_{SR}}$ in the range $0.8 \leq \rho \leq 1$ calculated using Equations (26) or (27) for $\frac{K_{js}}{K_{qe}}$ and Equation (25) of Ref¹¹ for $\frac{K_{SR}}{K_{qe}}$.

- The curves calculated via Equation (27) are slightly lower than the curves calculated via Equation (26). The difference is null at full density and increases when ρ

decreases and when $|\dot{E}|$ increases: the maximum at $\rho = 0.8$ is merely 0.2% when $\dot{E} = -10$ ($t^{gb})^{-1}$ and 5% when $\dot{E} = -500$ ($t^{gb})^{-1}$. This difference results from the approximations involved in the analytical integration of j_s along the pore profile.¹² Its very small value supports the validity of the mathematical procedure. As a consequence, we will present in the following only $\frac{K_{js}}{K_{qe}}$ values obtained via Equation (26).

- The agreement between K_{js} and K_{SR} is excellent at $\dot{E} = -10$ ($t^{gb})^{-1}$: it was verified that K_{js} tends to K_{SR} when strain rate tends to zero. At low strain rate, the present model is thus consistent with the model of Svoboda and Riedel. In contrast to K_{SR} , K_{js} does depend on strain rate: the difference $K_{js} - K_{SR}$ increases when $|\dot{E}|$ increases and when porosity increases, reaching about 10% (depending whether Equation (26) or (27) is used) at $\rho = 0.8$ when $\dot{E} = -500$ ($t^{gb})^{-1}$. The small amplitude of this strain rate dependence can be justified by consideration of Equation (27): as j_s is expected to vary linearly with $\dot{E}$, the dependence of $\frac{K_{js}}{K_{qe}}$ on $\dot{E}$ can arise only from two secondary factors: the increase in the profile length S_T with $\dot{E}$ and the increase in j_s with B_T . These two factors were neglected by Svoboda and Riedel.¹¹

3.2 | Relative correction on K_{qe} as a function of relative density at given densification rate

Let us first consider the densification ($\dot{E} < 0$) of a hypothetical system in which grain coordination remains $Z = 6$ independently of relative density. Figure 4 compares, for the case $\Delta = 0.1$, the dependence on relative density of $\frac{K_{js}}{K_{qe}}$ (Equation (26)), $\frac{K_{BT}}{K_{qe}}$ (Equation (28)), and of the total of the two contributions in the range $0.65 \leq \rho \leq 1$. The ordinate scale is logarithmic. Three dihedral angles, $\psi = 180^\circ$, 105° , and 30° , and two straining rates, $\dot{E} = -10$ ($t^{gb})^{-1}$

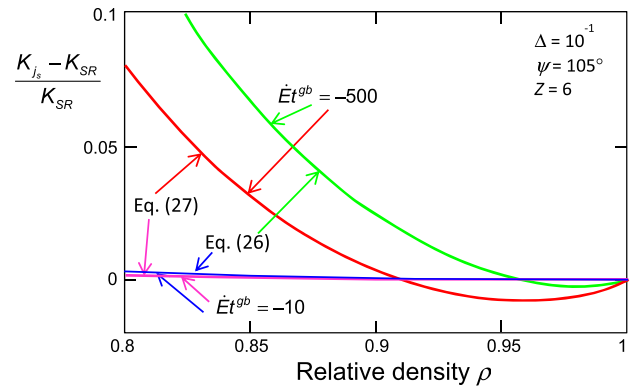


FIGURE 3 Relative difference $\frac{K_{js} - K_{SR}}{K_{SR}}$ calculated using Equations (26) or (27) for $\frac{K_{js}}{K_{qe}}$ and Equation (25) of Ref¹¹ for $\frac{K_{SR}}{K_{qe}}$. The parameters are $\Delta = 0.1$, $\psi = 105^\circ$, $Z = 6$ and $\dot{E} = -10$ or -500 ($t^{gb})^{-1}$

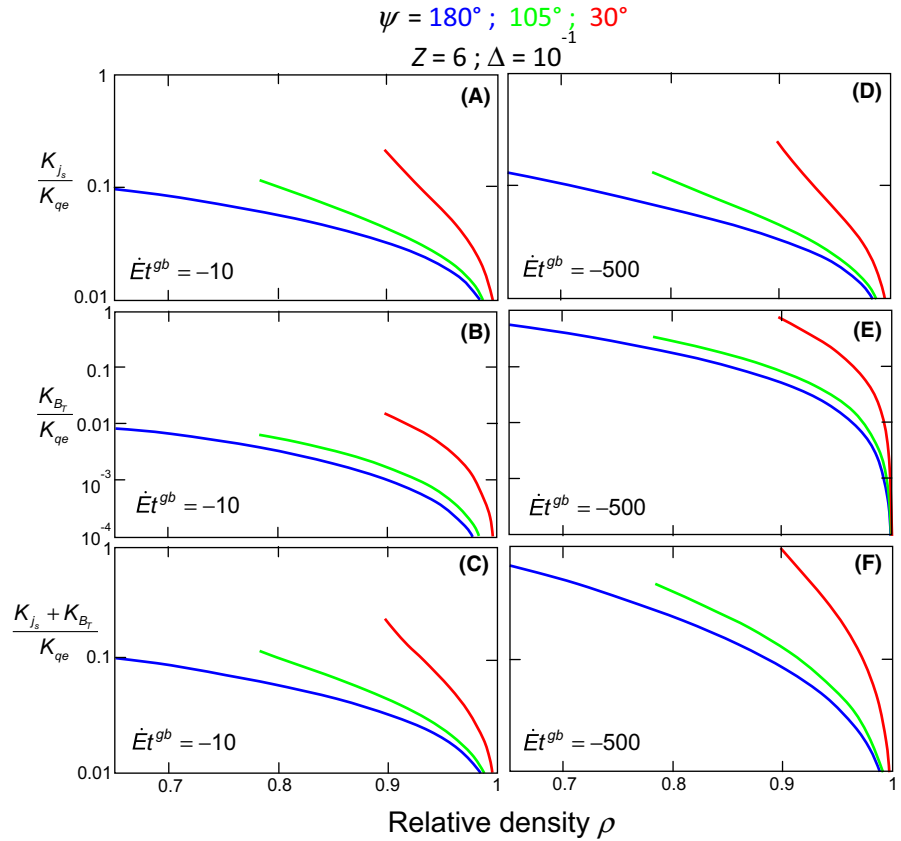


FIGURE 4 Dependence on relative density of $\frac{K_{js}}{K_{qe}}$ (A and D), $\frac{K_{B_T}}{K_{qe}}$ (B and E), and the total of the two corrections (C and F) for three dihedral angles, two densification rates, and $Z = 6$

and $\dot{E} = -500 (t^{gb})^{-1}$, are considered. The dihedral angles have been chosen for illustrating the behavior of systems with either convex pores ($\psi = 180^\circ$ and 105°) or concave pores ($\psi = 30^\circ$). The curves for $\psi = 105^\circ$ and 30° cover a limited density range because, if $Z = 6$, the maximum porosity (perfect cylinders in simple contact) is then lower than 0.35. Consistently with Equations (25) and (27), both $\frac{K_{js}}{K_{qe}}$ and $\frac{K_{B_T}}{K_{qe}}$ are positive (the positive sign of $\frac{K_{js}}{K_{qe}}$ could not have been anticipated a priori from Equation (26) because, when $\dot{E} < 0$, $B_T > B_{Te}$ and $\kappa_T < \kappa_e$).

Figure 4 shows that $\frac{K_{js}}{K_{qe}}$ and $\frac{K_{B_T}}{K_{qe}}$ strongly increase when density decreases and are larger at given porosity when ψ is smaller. This reflects the fact that pore surface area is larger when ρ is smaller and when ψ is smaller. As shown already in Figure 3, an increase by a factor of 50 in densification rate has only a quite small effect on $\frac{K_{js}}{K_{qe}}$ (Figure 4A,D). In contrast, an increase by a factor of 50 in densification rate causes an increase in $\frac{K_{B_T}}{K_{qe}}$ by a factor of about 50 (Figure 4B,E). This increase reflects the increase in B_T (Figure 2) when densification rate increases. In total, this brings about a strong dependence of K on densification rate, ie, a strongly non-Newtonian viscosity (Figure 4C,F). For $Z = 6$ and $\Delta = 0.1$, $\frac{K_{js}}{K_{qe}}$ is larger than $\frac{K_{B_T}}{K_{qe}}$ when $\dot{E} = -10 (t^{gb})^{-1}$ whereas the reverse applies when $\dot{E} = -500 (t^{gb})^{-1}$. When $\dot{E} = -500 (t^{gb})^{-1}$ (Figure 4F), the total correction to be applied to K_{qe} reaches nearly 100% close to the maximum porosity.

In order to be as much as possible relevant to real systems, the model should account for the evolution of average grain coordination with relative density. From a compilation of an extensive set of experimental data in real systems, German¹⁹ proposed as best fit the empirical law

$$Z = 2 + 11\rho^2 \quad (29)$$

In 2-D, Z must converge to 6 at full density. We will make use in the following of a phenomenological law inspired by Equation (29):

$$Z = 0.2 + 5.8\rho^2 \quad (30)$$

As shown in Ref.¹² this law is consistent with the progressive growth of grains boundaries. Computations using Z varying according to Equation (30) show that, during densification, asymptotic pore profiles remain either convex (for $\psi = 180^\circ$ and 105°) or concave (for $\psi = 30^\circ$) from $\rho = 0.75$ to $\rho = 1$.

Figure 5 is the equivalent of Figure 4 when use is made of Equation (30). At the same density, Z is lower in Figure 5 than in Figure 4. In order to better enlighten the effect of the change in Z , the curves for $\psi = 180^\circ$ and $Z = 6$ of Figure 4A,B,D,E have been reproduced as dashed curves in Figure 5A,B,D,E (comparison is not given with the curves for $\psi = 105^\circ$ and 30° in Figure 4 because the observations are identical). The difference between full and

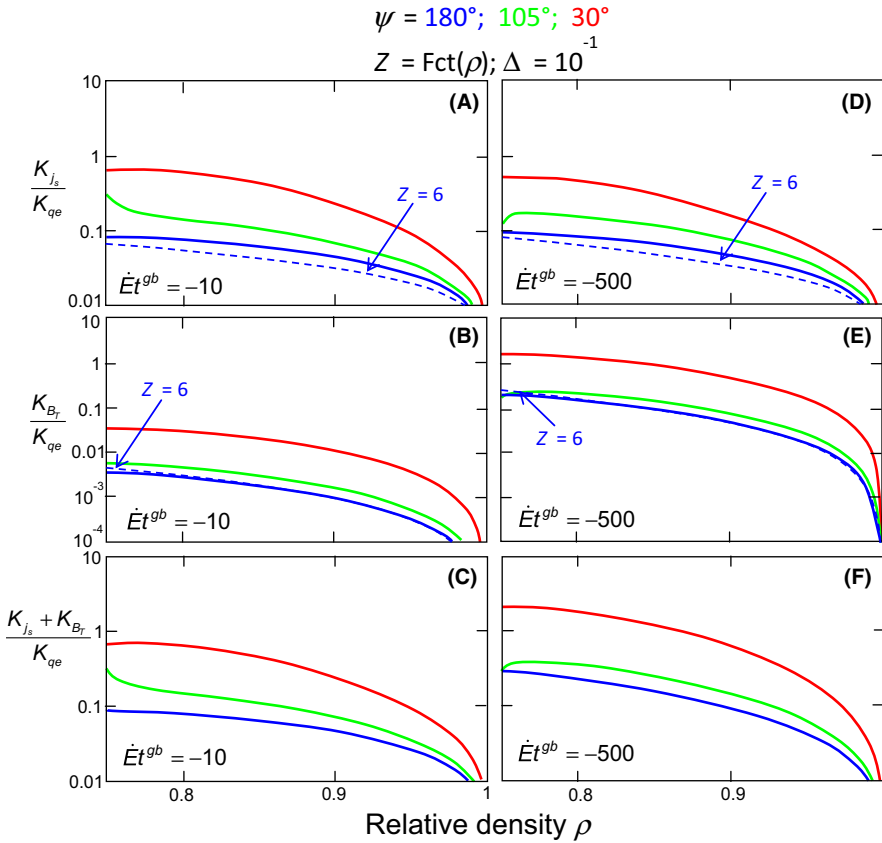


FIGURE 5 Dependence on relative density of $\frac{K_{js}}{K_{qe}}$ (A and D), $\frac{K_{Br}}{K_{qe}}$ (B and E), and the total of the two corrections (C and F) for three dihedral angles, two densification rates, and Z varying according to Equation (30). Dashed curves reproduce the curves for $Z = 6$ and $\psi = 180^\circ$ shown Figure 4

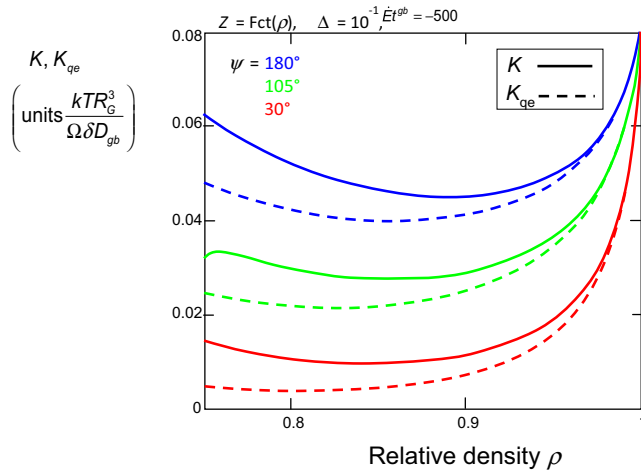


FIGURE 6 Variation in K (Equation (17)) and K_{qe} (Equation (19)) for the same conditions as for Figure 5F

dashed curves is not large because a decrease in Z causes an increase in size of both pores and grain boundaries. As the latter increase brings about a strong increase in K_{qe} when Z decreases,⁶ the decrease in Z brings about an increase in $\frac{K_{js}}{K_{qe}}$ by only about 20% (Figure 5A,D) and no significant change in $\frac{K_{Br}}{K_{qe}}$ (Figure 5B,E). Nevertheless, the total of the two contributions to the correction to be

brought on K_{qe} becomes larger than 100% when $\psi = 30^\circ$ with $\Delta = 0.1$ and $\dot{E} = -500$ (t^{gb})⁻¹.

In order to illustrate in absolute terms the contribution of surface diffusion, the curves on Figure 6 compare K according to Equation (17) (full curves) to K_{qe} according to Equation (19) (dashed curves) for the same conditions as in Figure 5F. At low ρ , the decrease in K and K_{qe} when ρ increases reflects a decrease in B_T and B_{Te} . Indeed, as shown in literature (Figure 1 in Ref.⁵; Figure 7 in Ref.⁴; Figure 2B in Ref.⁶), an increase in Z at given ρ brings a decrease in B_T , causing a decrease in K . The increase in B_T and B_{Te} due to the increase in ρ is thus counteracted by a decrease due to the increase in Z .²⁰ The shape of the curves in Figure 6 is due to the fact that, when using Equation 30, the effect of the increase in Z overcomes the effect of the increase in ρ until $\rho \geq 0.9$.

3.3 | Relative correction on K_{qe} as a function of densification rate at given relative density

Figure 7 compares, for the case $\rho = 0.80$ and $Z = 3.91$ (according to Equation (30)), the dependence on strain rate of $\frac{K_{js}}{K_{qe}}$, $\frac{K_{Br}}{K_{qe}}$, and of the total of the two corrections in the range $1 t^{gb-1} \leq -\dot{E} \leq 10^3$ (t^{gb})⁻¹. The curves are computed for the same three dihedral angles as in Figures 4 and 5 using two diffusivity ratios: $\Delta = 10^{-2}$ (A-C) and $\Delta = 10^{-1}$ (D-F). Comparison of Figure 7A,D shows that,

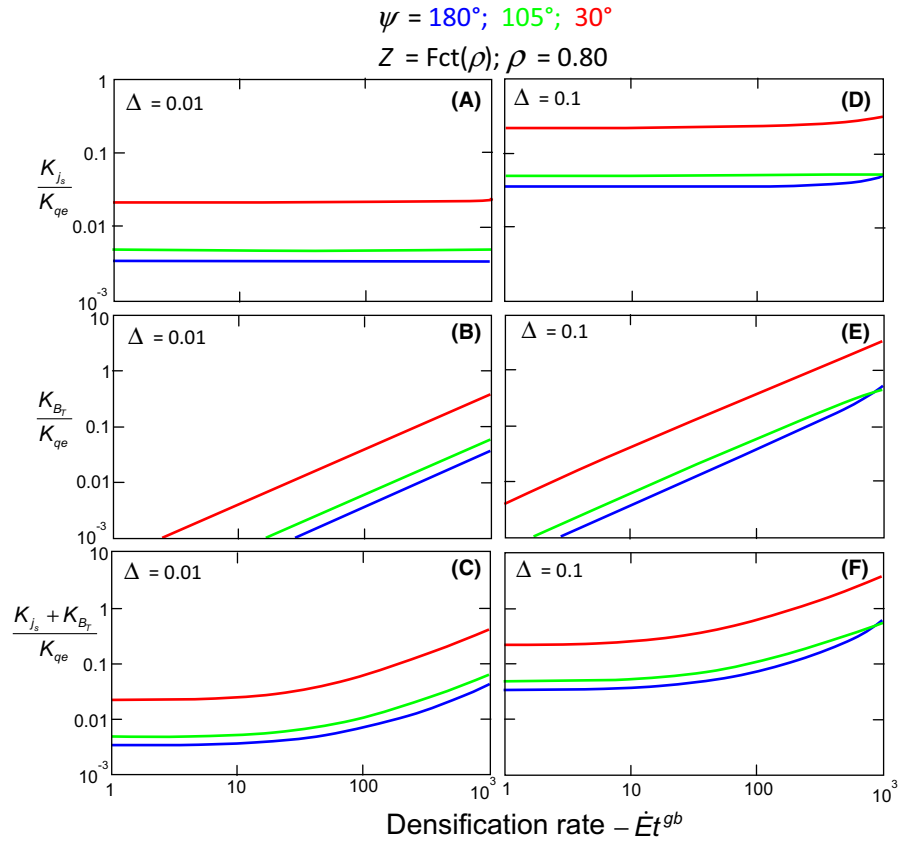


FIGURE 7 Dependence on densification rate of $\frac{K_{js}}{K_{qe}}$ (A and D), $\frac{K_{BT}}{K_{qe}}$ (B and E), and the total of the two contributions (C and F) when $\rho = 0.8$ and $Z = 3.91$ for three dihedral angles and two diffusivity ratios

consistently with Equation (27), $\frac{K_{js}}{K_{qe}}$ increases nearly linearly with Δ and shows little dependence on strain rate as long as $-\dot{\epsilon} t^{gb}$ is lower than ~ 100 : when $\Delta = 0.1$, the increase in $\frac{K_{js}}{K_{qe}}$ with strain becomes significant only when $-\dot{\epsilon} t^{gb} > 100$. In contrast, Figure 5B,E show that $\frac{K_{BT}}{K_{qe}}$ increases nearly linearly both with Δ and with $||\dot{\epsilon}||$. Although it was anticipated that $B_T - B_{Te}$ increases when densification rate increases, it could not be anticipated a priori that $B_T^3 - B_{Te}^3$ would vary nearly linearly with $\dot{\epsilon}$ (Equation (28)). It follows that the sum of the two corrections (Figure 7C,F) reveals different behaviors depending on strain rate: viscosity is Newtonian if $-\dot{\epsilon} < 10 (t^{gb})^{-1}$ and becomes non-Newtonian beyond this threshold. A nearly linear dependence of K on $||\dot{\epsilon}||$ is observed once $-\dot{\epsilon} > 100 (t^{gb})^{-1}$. Computations for other relative densities than $\rho = 0.80$ indicate that the threshold strain rate at which viscosity changes from Newtonian to non-Newtonian slightly increases when ρ increases. As illustrated in Figure 7F, the effect of surface diffusion on viscosity can be predominant when the product $\Delta ||\dot{\epsilon}||$ is large.

3.4 | Relative correction on K_{qe} in the case of expansion ($\dot{\epsilon} > 0$)

The sign of strain rate very much affects the corrections to be brought on K_{qe} because it causes a change of sign of

both $(\kappa_T - \kappa_{Te})$ and $(B_T - B_{Te})$. The consequence is illustrated in Figure 8 which shows the influence of relative density, strain rate, and dihedral angle on the corrections when $\dot{\epsilon} > 0$ (Z and ρ being linked by Equation (28)). Only results for $\Delta = 0.1$ are presented because the influence of Δ was found to be identical under expansion or compression. Figure 8A-C present the dependence on relative density for an expansion rate $\dot{\epsilon} = 500 (t^{gb})^{-1}$ while Figure 8D-F present the dependence on strain rate for a relative density $\rho = 0.80$ (hence, except for the change of sign of $\dot{\epsilon}$, Figure 8A-C correspond to Figure 5D-F whereas Figure 8D-F correspond to Figure 7D-F). Notice, however, that the ordinate scale is negative in Figure 8B,C,E,F and is linear in Figure 8C,F. The curves $\frac{K_{js}}{K_{qe}}$ for $\psi = 105^\circ$ show a divergence at $\rho = 0.825$ (Figure 8A). This divergence is a computational artefact linked to the fact that, when $\dot{\epsilon} = 500 (t^{gb})^{-1}$ and $\psi = 105^\circ$, the average curvature of the asymptotic pore profile changes from convex when $\rho > 0.825$ to concave when $\rho < 0.825$. Due to the approximations involved in the computation procedure, the precision on j_s and κ_T (hence on $\frac{K_{js}}{K_{qe}}$) degrades in the vicinity of this transition.

Figure 8 shows that whereas $\frac{K_{js}}{K_{qe}}$ has remained positive (consistently with Equation (27)), $\frac{K_{BT}}{K_{qe}}$ has become negative as a result of the change of sign of $(B_T^3 - B_{Te}^3)$ (Equation (28)). The decrease in $\frac{K_{js}}{K_{qe}}$ observed when $\dot{\epsilon} > 100 (t^{gb})^{-1}$ (Figure 8D) reflects the decrease in j_s when B_T

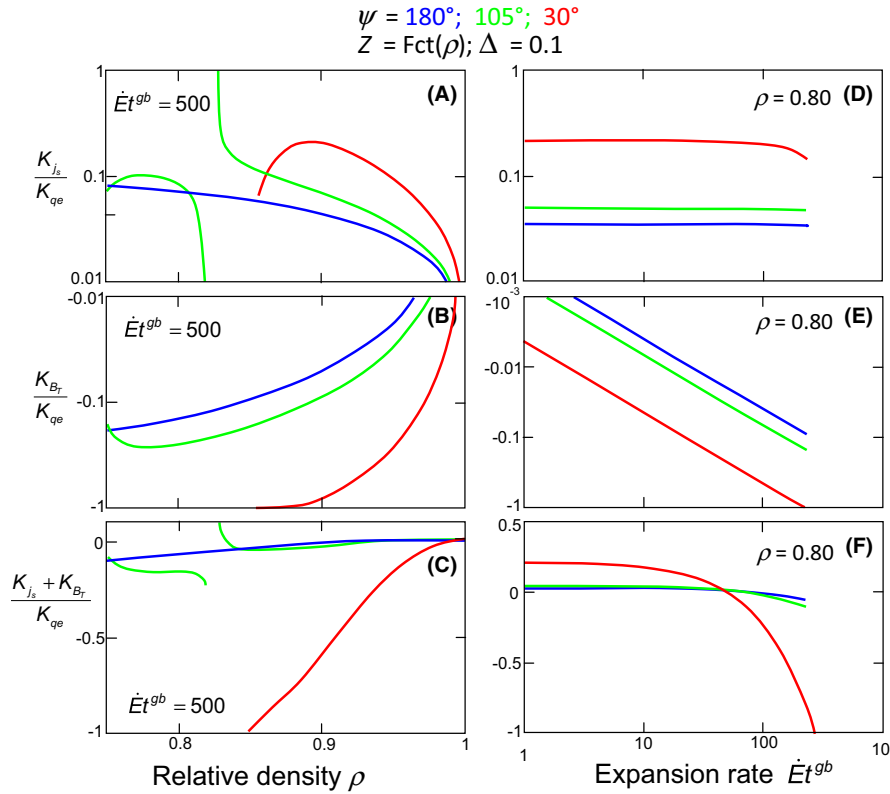


FIGURE 8 Dependence of $\frac{K_{js}}{K_{qe}}$ (A and D), $\frac{K_{Br}}{K_{qe}}$ (B and E), and of the total of the two corrections to K_{qe} (C and F) during expansion for three dihedral angle with $\Delta = 0.1$ and Z defined according to Equation 3.2. (A), (B), (C): dependence on relative density when $\dot{E} \approx 500 \text{ (t}^{gb})^{-1}$; (D), (E), (F): dependence on expansion rate when $\rho = 0.8$

decreases. The magnitude of the softening effect due to the negative sign of $\frac{K_{Br}}{K_{qe}}$ increases when ρ decreases and when $\dot{E}$ increases. $\frac{K_{Br}}{K_{qe}}$ reaches -1 when $B_T = 0$ (Equation 28) ie, when grains separate from one another. When $\psi = 30^\circ$, grain separation occurs at $\rho = 0.85$ when $\dot{E} = 500 \text{ (t}^{gb})^{-1}$ (Figure 8B) or at $\rho = 0.80$ when $\dot{E} = 230 \text{ (t}^{gb})^{-1}$ (Figure 8E). When $\psi = 180^\circ$, extrapolation of the curve of Figure 8E, predicts a rate of $\sim 2000 \text{ (t}^{gb})^{-1}$ for grain separation to occur at $\rho = 0.8$. Of course, these conditions for grain separation may not be used for the prediction of creep fracture by growth of cavities at grain boundaries because this analysis does not consider the strain localization induced by the softening accompanying porosity expansion. The influence of pore expansion on strain localization and on the ensuing creep fracture was analyzed by Chuang et al²¹ and Hsueh and Evans.²² Figure 8F shows that, whatever the dihedral angle, viscosity decreases when strain rate increases above $\dot{E} \approx 50 \text{ (t}^{gb})^{-1}$. Hence, it can be anticipated that, in addition to the softening due to pore expansion, this strain rate softening behavior further contributes to strain localization during expansion at high strain rate.

Under purely radial loading, surface diffusion fluxes j_s are everywhere positive (ie, oriented from T to M in Figure 2) during densification and everywhere negative during expansion. In contrast, under pure shear loading, the sign of diffusion fluxes depends on the orientation of the grain boundary with respect to the stress tensor: matter diffuses

away from grain boundaries undergoing compression and toward grain boundaries undergoing tension. The present work shows that, depending on the sign of the diffusion flux, accounting for surface diffusion brings about either an increase or a decrease in $\frac{K_{Br}}{K_{qe}}$. Under shear loading, local effects at the scale of individual grain boundaries and pores should overall balance one another. It can thus be anticipated that surface diffusion affects very much less shear viscosity G than to bulk viscosity K : shear viscosity is expected to remain Newtonian even at high strain rate.

4 | CONCLUSION

The quasi-equilibrium estimate of bulk viscosity, K_{qe} , neglects both the dissipation due to surface diffusion fluxes and the presence of curvature gradients along pore surface. The paper shows that the correction to be applied to K_{qe} in order to account for surface diffusion involves two terms: $K = K_{qe} + K_{js} + K_{Br}$ where K_{js} accounts for the dissipation due to surface fluxes and K_{Br} accounts for the change that surface curvature gradients bring to the dissipation due to grain-boundary fluxes. The influence of strain rate on the dependence of these two terms on microstructural parameters has been analyzed by calculating the asymptotic evolution of pore profile under a controlled macroscopic strain rate. At low densification rate, K_{js} agrees with former model of literature. Most conspicuously, the paper reveals that

K_{B_T} is positive in densification and negative in expansion, and increases nearly linearly with strain rate. This brings bulk viscosity in densification to become strongly non-Newtonian at high strain rate: in some conditions, $\frac{K_{B_T}}{K_{qe}}$ can then amount to more than 100%. In expansion, the softening effect due to the decrease in grain-boundary size can lead to grain separation at high strain rate.

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APPENDIX 1

SINTERING STRESS AS PARTIAL DERIVATIVE OF INTERFACE ENERGY

At equilibrium, the free energy excess due to interfaces is, per unit thickness of the RVE of Figure 2,

$$\begin{aligned} E_{\text{interface}} &= \gamma_s A_s + \gamma_{\text{gb}} \frac{A_{\text{gb}}}{2} \\ &= \gamma_s \left[A_s + \cos\left(\frac{\psi}{2}\right) A_{\text{gb}} \right] \\ &= \gamma_s H \left[\left(\frac{\psi}{2} - \beta\right) r + \cos\left(\frac{\psi}{2}\right) \frac{B_{\text{Te}}}{H} \right] \end{aligned} \quad (\text{A.1})$$

where A_s and A_{gb} are the areas of pore surface and grain boundary included in the RVE. For concision in the following equations, use is made of the notation

$$r = \frac{R_e}{H} = \frac{1}{H\kappa_e} \quad (\text{A.2})$$

The definition of sintering stress as a thermodynamic driving force for densification is¹⁷

$$\Sigma = \left(\frac{\partial E_{\text{interface}}}{\partial V} \right)_{V_s} \quad (\text{A.3})$$

where V is the volume of the system and V_s is the volume of solid. We can write successively

$$\Sigma = \frac{\partial E_{\text{interface}}}{\partial V} = \frac{\partial E_{\text{interface}}}{\partial \rho} \frac{\partial \rho}{\partial V} \quad (\text{A.4})$$

$$\frac{\partial \rho}{\partial V} = -\frac{\rho^2}{V_s} = -\frac{2\rho}{H^2 \tan \beta} \quad (\text{A.5})$$

and

$$\begin{aligned} \frac{\partial E_{\text{interface}}}{\partial \rho} = \gamma_s \left\{ \left[\left(\frac{\psi}{2} - \beta \right) r + \cos \left(\frac{\psi}{2} \right) \frac{B_{\text{Te}}}{H} \right] \frac{\partial H}{\partial \rho} \right. \\ \left. + H \left[\left(\frac{\psi}{2} - \beta \right) + \cos \left(\frac{\psi}{2} \right) \frac{\partial}{\partial r} \left(\frac{B_{\text{Te}}}{H} \right) \right] \frac{\partial r}{\partial \rho} \right\} \end{aligned} \quad (\text{A.6})$$

From Equation (2)

$$\frac{dH}{d\rho} = -H \frac{1}{2\rho}, \quad (\text{A.7})$$

from Equation (6)

$$\frac{\partial}{\partial \gamma} \left(\frac{B_{\text{Te}}}{H} \right) = \left(\sin \frac{\psi}{2} - \tan \beta \cos \frac{\psi}{2} \right) \quad (\text{A.8})$$

and from Equation (5)

$$\frac{dr}{d\rho} = -\frac{1}{2} \frac{r}{1-\rho} \quad (\text{A.9})$$

hence, using Equations (A.5) to (A.9), Equation (A.4) becomes

$$\begin{aligned} \Sigma = \frac{2\rho}{H^2 \tan \beta} \gamma_s \left\{ \left[\left(\frac{\psi}{2} - \beta \right) r + \cos \left(\frac{\psi}{2} \right) \frac{B_{\text{Te}}}{H} \right] H \frac{1}{2\rho} \right. \\ \left. + H \left[\left(\frac{\psi}{2} - \beta \right) - \cos \left(\frac{\psi}{2} \right) \left(\sin \frac{\psi}{2} - \tan \beta \cos \frac{\psi}{2} \right) \right] \frac{1}{2} \frac{r}{1-\rho} \right\} \end{aligned} \quad (\text{A.10})$$

which, via Equations (5) and (6) yields

$$\begin{aligned} \Sigma = \frac{\rho}{H^2 \tan \beta} \gamma_s \left\{ \left[\left(\frac{\psi}{2} - \beta \right) r + \cos \left(\frac{\psi}{2} \right) \frac{B_{\text{Te}}}{H} \right] H \frac{1}{\rho} \right. \\ \left. + H \tan \beta \frac{1}{r} \right\} \\ = \frac{\rho}{H^2 \tan \beta} \gamma_s \left\{ \left[\left(\frac{\psi}{2} - \beta \right) r \right. \right. \\ \left. + \tan \beta \frac{1-\rho}{r} - r \left(\frac{\psi}{2} - \beta \right) + \cos \frac{\psi}{2} \tan \beta \right] H \frac{1}{\rho} + H \tan \beta \frac{1}{r} \right\} \\ = \gamma_s \left\{ \frac{\cos \frac{\psi}{2}}{H} + \kappa_e \right\} \end{aligned} \quad (\text{A.11})$$

Equation (A.11) is identical to Equation (16).