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Application of graphic statics and parametric force diagrams for the design of reinforced concrete structures

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Abstract
This paper extends the theoretical developments of the ongoing research on 2D internally and externally statically indeterminate networks, using graphic statics. Specifically, parametric force diagrams are used for the analysis and optimization of strut-and-tie networks inscribed inside continuous structures. A simple wall is used to exemplify a series of optimization strategies that can be pursued while designing reinforced concrete structures. These help designers to reduce the overall cost of the structure while minimizing the overall quantity of reinforcement and optimizing the distribution of the compressive forces in the concrete struts.

Keywords: Graphic statics, plastic theory, static indeterminacy, structural design, reinforced concrete, strut-and-tie modeling.

1. Introduction
"The theory of plasticity provides a rational, logical, consistent and simple method for investigating the static behavior and strength of reinforced concrete structures" (Muttoni et al. [1]). More specifically, the lower bound theorem of limit analysis states that, for a material presenting a ductile behavior, any distribution of internal forces in equilibrium that respects both boundary and yield conditions is valid. These distributions of forces can advantageously be represented by strut-and-tie networks inscribed within the structural envelope. Networks are easily analyzed and manipulated using graphic statics' form and force diagrams, as they provide simultaneous and visual insight into the geometry and the magnitude of the associated internal stresses. For statically indeterminate structures like multi-supported monolithic reinforced concrete elements, the method requires to manipulate force diagrams that cannot be explicitly defined for a given loading. Rondeaux et al. [2, 3] proposed two procedures for building parametric force diagrams in the case of internal or external static indeterminate structures, which are summarized and combined in Section 2. In Section 3, the case of a simple reinforced concrete wall then illustrates a series of optimization procedures of the parametric force diagram that address several design questions.

2. Internally and externally statically indeterminate network
In previous works, the authors have developed a graphical methodology to construct parametric force diagrams for the analysis of statically indeterminate pin-jointed networks. A first contribution (Rondeaux et al. [2]) is exclusively focused on internal static indeterminacy, while a second one (Rondeaux et al. [3]) deals with external static indeterminacy. Both can be combined in a general methodology, as resumed in the 4 following steps:

1. The degree of static indeterminacy n of the structure is obtained by adding its degrees of external and internal indeterminacies. These are respectively found by counting the number n_e of excess reaction forces and the one n_i of independent self-stress states.

2. The n_e associated external force diagrams $\mathbf{F_{ej}}^*$ and the n_i force diagrams $\mathbf{F_{ik}}^*$ corresponding to the self-stress states are drawn at an arbitrary scale.
3. By performing a vector addition, the isostatic diagrams $\mathbf{F_{ej}}^*$ and $\mathbf{F_{ik}}^*$are assembled with the force diagram of the applied loads $\mathbf{F_q}^*$ to form the global force diagram $\mathbf{F}^*$. As each of these diagrams can be drawn at an independent scale, they correspond to degrees of freedom of $\mathbf{F}^*$.
4. $\mathbf{F}^*$ is consequently constructed as a parametric force diagram $\mathbf{F_p}^*$, in which each degree of freedom corresponds to an offset of $\mathbf{F}^*$. $\mathbf{F_p}^*$ then depends on parameters x_n characterizing the position of nodes $\mathrm{P_n}^*$ along the line of action of one of the forces $\mathbf{r_j}^*$ or $\mathbf{f_k}^*$ in $\mathbf{F_p}^*$.

Figure 1 illustrates the described graphical methodology for a simple truss with one degree of external static indeterminacy and one of internal indeterminacy.

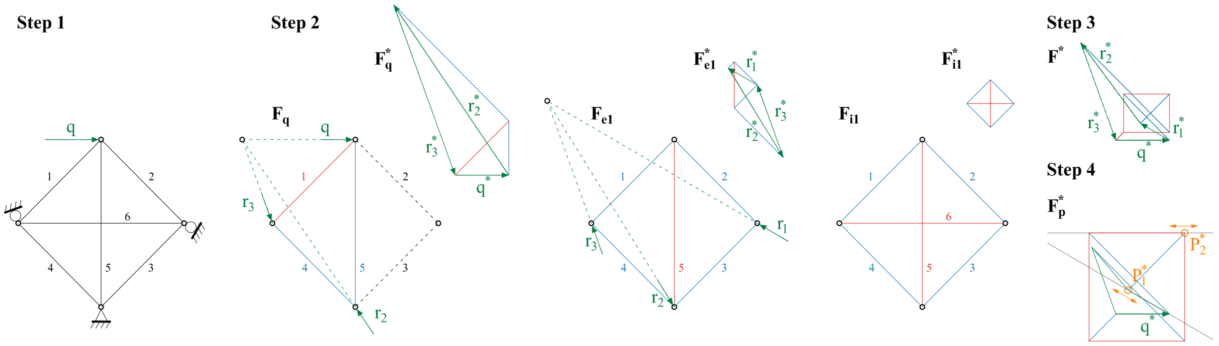

Figure 1: 4-step procedure to construct the parametric force diagram of a simple truss with $n_i = n_e = 1$ (compression elements in blue and tension ones in red)

3. Optimization criteria for reinforced concrete structures

Under the hypothesis that reinforced concrete is a material with sufficient ductile properties, strut-and-tie networks can be used to model its mechanical behavior. An orthogonal grid of nodes fixing the endpoints of the network is then defined inside the geometry of the structure, as in the *Ground Structure Method* (Dorn et al. [4]). However, as deformations between compressed and tensed parts of reinforced concrete structures have to be compatible (Eurocode 2 [5]), a common practice is to consider angles between struts and ties larger than 25°. Furthermore, in order to simplify the on-site assembly of the structure, ties modeling rebars are preferably set at an angle of 0 or 90° (and 45° if possible).

A simple 2D reinforced concrete wall of 2 m by 2 m (Figure 2) is loaded on the top by a horizontal point load q = 2.3 MN. The wall is supported by a fixed support and two rollers arbitrarily oriented. A 1 m by 1 m grid of nodes is inscribed within the structure; six topologies for the network connecting these nodes have been chosen for the purpose of the analysis. Each network has one degree of internal static indeterminacy as well as an external one. The optimization of the parametric force diagram $\mathbf{F_p}^*$ corresponding to each network is then performed according to four concerns relevant for the design of reinforced concrete structures: minimization of the total volume of material (Section 3.1), minimization of the volume of ties (Section 3.2), minimization of the volume of struts (Section 3.3), and standardization of the compression forces (Section 3.4).

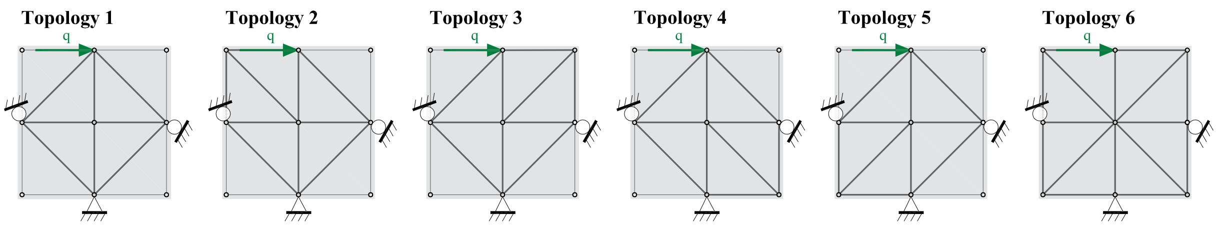

Figure 2: six strut-and-tie networks topologies modeling the behavior of a simple reinforced concrete wall

3.1. Minimization of the total volume of material

Maxwell's theorem reformulated by Baker et al. [6] states that the load path *LP* of any strut-and-tie network, which gives its efficiency in terms of volume of material, can be calculated using:

$$LP = V_T\sigma_T + V_C\sigma_C = \sum_i V_{Ti}\,\sigma_{Ti} + \sum_i V_{Ci}\,\sigma_{Ci} = \sum_i f_{Ti}\,f^*_{Ti} + \sum_i f_{Ci}\,f^*_{Ci} \tag{1}$$

with V the necessary volume of material, and σ the material strength considering a rigid-plastic behavior with an infinite ductile plateau. The T and C indices refer to bars in tension and compression respectively. For a given topology, the formula states that minimizing the load path is equivalent to minimizing the sum of the product of each bar's length f_i in the form diagram by its respective length f_i* in the force diagram, giving a result that does not take material strength into account . Values for the minimal load path LP_{min} corresponding to the six considered topologies are given in the first row of Table 1.

3.2. Minimization of the volume of ties

The minimal volume of ties, corresponding to the minimal load path in tension $LP_{min,T}$, is obtained by applying Maxwell's theorem to bars in tension only. Corresponding values are given in the second row of Table 1.

3.3. Minimization of the volume of struts

Minimal load paths in compression $LP_{min,c}$ are obtained similarly, with corresponding values given in the third row of Table 1.

3.4. Standardization of the compression forces

For each topology, the mean value of the compressive forces C_{mean} is obtained using:

$$C_{mean} = \sum_i |\boldsymbol{f}^*_{Ci}|/n_C \tag{2}$$

with n_C the number of bars in compression. The tolerance value t is then obtained by penalizing each bar accordingly to the difference between the magnitude of its corresponding compression force $|\mathbf{f}_{Ci}^*|$ and the mean compressive force C_{mean}:

$$t = \Sigma(|\boldsymbol{f}^*_{Ci}|.(|\boldsymbol{f}^*_{Ci}| - C_{mean})) \tag{3}$$

Values for t are minimized for each topology, thus yielding force diagrams with elements in compression having the most similar length possible. Values for C_{mean} associated to the minimum tolerance t_{min} are given in the last row of Table 1.

Table 1: values for the minimal volumes (total, ties, struts) and the standardized compression forces for each of the six considered topologies, with optimal values in bold

Topology	1	2	3	4	5	6
LP_{min} [MNm]	**8.19**	9.79	12.79	**8.19**	**8.19**	12.79
$LP_{min,T}$ [MNm]	**2.30**	3.63	4.60	**2.30**	2.83	5.31
$LP_{min,C}$ [MNm]	**5.19**	5.99	7.50	**5.19**	**5.19**	7.50
C_{mean} [MN] (t_{min})	**1.75 (0.01)**	1.46 (1.76)	1.28 (3.65)	1.22 (1.64)	1.59 (0.12)	1.27 (3.10)

3.5. Comparison of results

Whatever the chosen design criterion, topology 1 shows the best performance. In fact, this configuration is the one that makes use of the smallest number of bars in the corresponding strut-and-tie network. The four optimized parametric force diagrams $\mathbf{F}_{opt,1}^*$, $\mathbf{F}_{opt,2}^*$, $\mathbf{F}_{opt,3}^*$ and $\mathbf{F}_{opt,4}^*$, corresponding to the four aforementioned criteria applied to topology 1, are depicted in Figure 3. These help to design the optimal reinforcement layout in order to reach the chosen design performances.

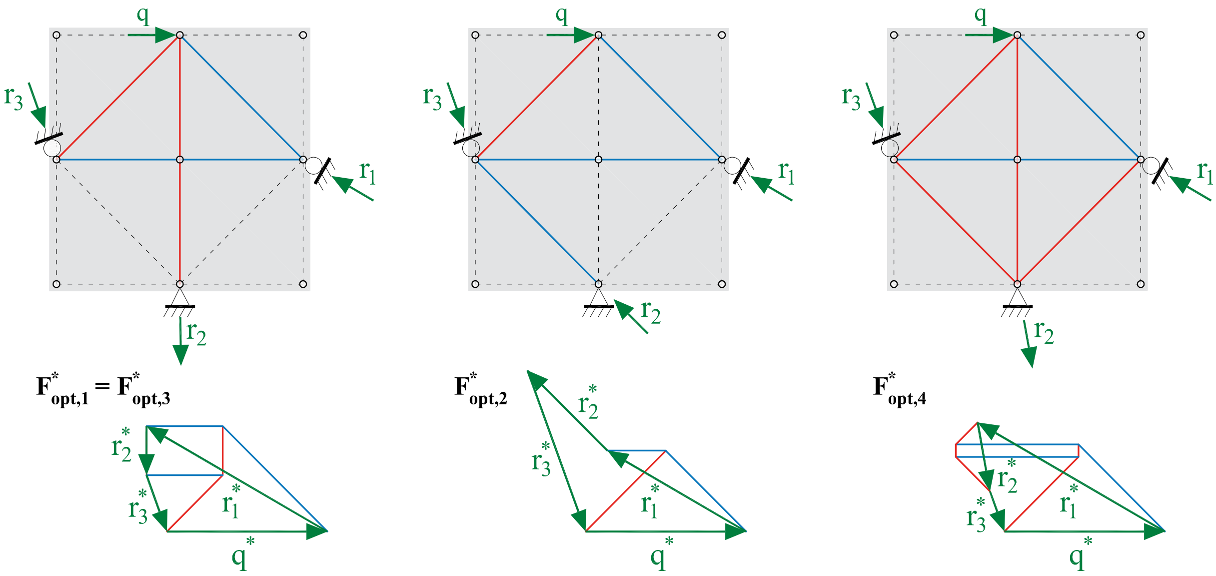

Figure 3: optimized parametric force diagrams $\mathbf{F}_{\mathbf{opt}}^{*}$ linked to topology 1 and corresponding strut-and-tie models – minimum total and strut volumes (left), minimum tie volume (center), mean compression forces (right)

4. Conclusion and perspectives

Based on strut-and-tie modeling and graphic statics' reciprocal diagrams, this paper presents a geometrical approach to optimize statically indeterminate networks according to several structural and building requirements. In particular, when applied to the design of reinforced concrete walls, the method can be used to help practitioners to choose the reinforcement layout according to specific parameters. The approach has great advantages for design purposes, as it is visual and interactive.

In a following step, the methodology could be automated to manage more strut-and-tie topologies for a fixed grid of nodes. It could also be extended for the design of more complex reinforced concrete structures (in 2D or 3D), with a finer grid of nodes to better represent real case scenarios. The multiplicity of load paths associated to the number of possible strut-and-tie topologies could also be exploited to get insight on the level of robustness of reinforced concrete structures.

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