

CHAPTER 1

A review of the literature

Introduction

Numbers are extremely flexible tools that can be used in a variety of contexts. For instance, number 3 can indicate as many different things as the number of children in a family (cardinal context), the rank of the sportsman receiving the bronze medal in a competition (ordinal context), the temperature of a winter day, in degrees Celsius (measure context), one of Paris subway lines running through the City from East to West (nominal, non-numerical context), and so on. Hence, numbers¹ refer to specific meanings when used in particular contexts.

Few authors have provided a detailed account for the different meanings numbers may convey. Fuson (1988, 1992) adopted a developmental approach and examined children's elaboration of numerical skills in a variety of contexts, and their construction of number meanings. She proposes, in fact that number words come to attain specific meanings by their use in particular contexts. She distinguishes numerical contexts (cardinal, measure, ordinal), sequence context, and non-numerical contexts, and places particular emphasis on sequence recitation (or sequence knowledge)² as one of the basic skills underlying the later development of numerical meanings. Wiese (2003) provided another interesting approach to numerical concepts that integrates views from psychology, linguistics and philosophy. She distinguishes the numerical concepts of quantity, rank and label, as determined by the assignment of numbers in cardinal, ordinal and nominal contexts, respectively.

¹ Let us note that the term "number" is used here, and unless otherwise specified (e.g. when we speak about the "mental representation of numbers" or about "the number concept"), in reference to its symbolic representation (i.e., in the Arabic or verbal codes; see Fayol & Seron, 2004, for a discussion about number representations). In fact, while non-symbolic representations of number (e.g. 000, for the numerosity 3) do convey quantity information, the way they would be used in ordinal, or even measure and nominal contexts is far from clear.

² Note that Fuson sometimes speaks about "sequence meaning" when referring to the use of numbers in sequence contexts; we prefer using the terms "sequence knowledge", instead, when referring, for instance, to sequence recitation, as this skill might not require any semantic elaboration.

In a cardinal context (cardinal assignment), the number indicates “how many”, as in 3 children and 3 degrees Celsius, and refers to the number of elements in a set. In an ordinal context (ordinal assignment), a number will tell us the relative position of an element within a set, such as the position of a sportsman in a competition; unlike in cardinal assignments, the number does not apply to a set, but to an individual element of a set or sequence. There is no quantification involved in ordinal assignments, thus any non-numerical ordered sequence (e.g. the letters of the alphabet) could also be used to provide meaningful ordinal assignments (Wiese, 2003). In a non-numerical context (nominal assignment), a number will merely serve to identify an object within a set and recover a purely labelling function, as in subway line #3. Fuson (1988) considers this kind of assignment as non-numerical, because numbers could perfectly be replaced by non-numerical stimuli, such as proper names or nouns, to indicate the referent object. Take, for instance, Paris and London subway systems: the former uses numbers to indicate the different lines (from 1 to 14), while the latter uses names (e.g. the Piccadilly line, the Central line, etc.), here numbers or names are merely equivalent. Nominal assignments are independent from the ordering property of numbers; they are exclusively related to language and found only in linguistic humans. Because numbers do not bear an essential (numerical) role in nominal assignments, they will not be discussed further in the remainder of this chapter.

Among numerical meanings, quantity has usually been considered as the most prominent one (e.g. Dehaene, 1997; Butterworth, 1999), with other number meanings often neglected. The Number Module described by Butterworth (1999), for instance, is merely devoted to ‘categorize the world in terms of numerosities – the number of things in a collection’, and accounts for our grasp of quantities. Likewise, Dehaene’s (1997) ‘number sense’ is a domain-specific ability that provides us with an apprehension of quantities, and that we share with higher animal species. Hence, ‘number’ in these views only refers to numerical quantity.

It might not be surprising then that most studies of numerical cognition placed particular emphasis on examining how numerical quantity is represented, processed and neurally implemented (see Dehaene, Piazza, Pinel, & Cohen, 2003; Nieder, 2005, for reviews). However, numbers don’t only

refer to quantities and may also be used in non-quantitative ordinal assignments to describe the position of an item within an ordered set. We still know relatively little about the way numerical order³ is processed (but see Tzelgov & Ganor-Stern, 2004; see also Nieder, 2005) and about the brain regions it recruits. Yet, this is a fundamental question since “sequential order” (i.e. the fact that each element -number- has a fixed position within the sequence) is, according to Wiese (2003), the crucial and central property of numbers. It is, in fact, because they form an ordered progression of well-distinguished entities that we may use numbers in both cardinal and ordinal assignments.

Hence, understanding how numerical order is processed is a theoretically relevant, though largely unknown, issue. Besides, the nature of the relationship between numerical order (i.e., the relative position of a number in the sequence) and numerical quantity (i.e., the quantity that number refers to) is also critical. In fact, while order might be disjoined from quantity in non-numerical contexts (e.g. the letters of the alphabet), they are linked up with each other in the number domain: the larger the quantity a number refers to, the later it is located in the conventional sequence of number words (Wiese, 2003; Wynn, 1992). Consequently, examining whether processing numerical quantity and processing the relative position of a number in the sequence rely on a common underlying mechanism, or whether numerical quantity and numerical order address (partially) distinct processes is also relevant for a comprehensive approach to the number domain.

These two issues will be addressed in the present thesis. On the one hand, we will seek for a better understanding of numerical order processing. We will do so in comparing subjects’ performance (and the mechanisms involved) when processing (1) the relative position of a number in the conventional sequence of number words and (2) the relative position of a non-numerical item (e.g. a letter) in its corresponding ordered series (the alphabet). In both

³ In the present dissertation, we will use the terms “numerical order” (or “order”, when opposed to “quantity”) to refer to the order of numbers in the sequence and, more precisely, to the “relative position of a number within the conventional sequence of number words”. Note that different authors use different terms (e.g. “sequential order”, “ordinal numerical knowledge”, “internally-order information”, or “rank order”) to refer to the order of the verbal sequence as the underlying structural characteristic of numbers.

cases, information about the conventional order of the items is held in long-term memory and must be retrieved to perform the task. Potential differences and similarities in the underlying mechanisms, according to the kind of material to be processed, will be examined. On the other hand, the question of the relationship between numerical quantity and numerical order will be addressed through the direct comparison of the mechanisms involved in processing (1) the quantity a number refers to and (2) the relative position of that number in the conventional sequence. To fulfil these aims, we asked a brain-injured patient, CO, and groups of healthy adult individuals to process identical numerical stimuli according either to quantity (e.g. in pair 2 5, which number is smaller?) or to order instructions (e.g. in pair 2 5, which number comes before in the sequence?). We compared their performance in these tasks in order to examine whether the processing of numerical quantity and the processing of numerical order rely on a common or on distinct cognitive mechanisms. The patient and healthy individuals were also asked to process the relative position of non-numerical stimuli (e.g. letters) in their corresponding sequence (e.g. in pair B E, which letter comes before in the alphabet?) and their performance in these tasks was compared to performance in the corresponding numerical (order) tasks. These questions were addressed in a single-case study, in an Event-Related brain Potentials (ERPs) study and in two behavioural experiments.

The present introductory chapter will be divided in three main sections. In the first section, we will examine how concepts of numerical quantity and numerical order (or rank) relate to one another, and on which grounds they differ. This will be done taking Wiese's (2003) approach as a main reference frame. Additionally, since sequence knowledge is also an important feature of the number domain, the work done by Fuson (1988) with respect to the development of the number sequence will also be presented. In the second section, we will review the experimental evidence available so far that is consistent with the view that processing quantity and processing the order of (numerical and non-numerical) elements in their corresponding series involve some common cognitive mechanism. Evidence from behavioural, neuroimaging and neuropsychological studies will be discussed in turn. In

the third section, we will examine how developmental, neuropsychological and behavioural data argue, instead, for potentially distinct mechanisms underlying (numerical) quantity processing and processing of the order of elements in a sequence.

1. Cardinal and ordinal number assignments

In this first section, we will describe how numbers can be assigned to assess specific properties of empirical objects with equal efficiency in cardinal or ordinal contexts, and mostly refer to the approach of Wiese (2003). We will then review the evidence for the developmental course leading to the elaboration of the concepts of numerical quantity and numerical rank, and highlight the important role played by sequence knowledge in this process of elaboration. Doing this, we will refer to research done in the field of developmental cognitive psychology, and in particular, to the work done by Fuson (1988). In fact, whereas Wiese (2003) will refer to sequential order as a crucial property of numbers, she doesn't provide a comprehensive approach to the acquisition of sequence knowledge, whereas Fuson (1988) does. The approaches of Wiese (2003) and Fuson (1988) will thus be considered as complementary⁴. Both authors will consider the sequence of number words as a non-referential tool, whose knowledge underlies the use of number words in a variety of contexts.

1.1. Cardinal and ordinal number assignments: the framework proposed by Wiese (2003)

Wiese (2003) provides a comprehensive account for the use of number words in various contexts. She refers to numerical meanings as 'numerical

⁴ One difference, however, between these approaches is with respect to what the term 'ordinal' encompasses. Fuson (1988) applies this term to the use of ordinal number words (i.e. first, second, third, instead of one, two, three). She will refer, in fact, to an ordinal context as a situation in which we use an ordinal number word to tell the relative position of one object within an ordered set. Wiese (2003) doesn't make such a strong restriction about the use of ordinal number words in ordinal assignments; we might, indeed, use conventional (or 'cardinal') number words equally well to refer to the rank of an object within a sequence (e.g. as when we say that page 4 comes before page 5).

assignments' since numbers are used in various contexts to assess different, and specific, properties of empirical objects (individual objects or sets). Number assignments are primarily about relations because we associate relations between numbers with relations between empirical objects⁵. When we say, for instance, that '3 children in a family is more than 2', we associate the numerical ordering relation '>' ('3 > 2') with the empirical relation (between sets of children) 'has more elements than'. Specific numerical relations will then be used to reflect the specific properties we want to assess about objects. In an ordinal assignment, for instance, the ordering relation '>' might reflect the empirical relation 'comes after' and assess the position of an element within a sequence⁶. Number assignments are thus seen as mappings from numbers to empirical objects, with numbers and objects not being correlated as individuals, but as elements of two systems. The association between these two systems of relations will be called "(systems)-dependent linking". Wiese (2003) further suggests that it is our language faculty that enabled us to link relations between numbers with relations between objects and thus provided us with a systematic number concept⁷.

It is clear from this perspective that number words are not confined to cardinality but can be used to identify cardinal, ordinal and nominal relations alike. Hence, numbers will assess different properties of empirical objects according to whether they are used in cardinal or ordinal assignments.

In cardinal assignments, the empirical objects numbers refer to are sets (not individuals) and the property we want to assess is their numerical quantity. The number identifies the cardinality of a set and tells us "how many" elements are in the set. It can both refer to the discrete size of sets (e.g. 3 children) or to continuous measures (e.g. 3 degrees Celsius). In the

⁵ The features that make number assignments meaningful were analysed within the general framework of the Representational Theory of Measurement (RTM, see Stevens, 1946).

⁶ Note that the ordering relation represented by '>' will thus bear a different meaning (respectively 'has more elements than' vs. 'comes after') according to whether it is used in a cardinal or in an ordinal context, and according to the nature of the ordering that exists on the empirical objects (see Fuson, 1988, for a similar approach).

⁷ The (full-blown or systematic) 'number concept' is what enables us to apply the sequence of number words not only to cardinal, but also to ordinal and nominal number assignments (i.e., to assess different properties of empirical objects).

case of measures, the number doesn't tell us the cardinality of the measured objects (e.g. the air) but instead the cardinality of a certain unit of measurement (e.g. degrees Celsius). In this sense measures are a special case of cardinal number assignments. Yet, for this reason, and because measure contexts were not thoroughly studied in the present thesis, only general cardinal assignments will be discussed further, leaving aside the particular case of measure (but see Wiese, 2003, for a detailed discussion).

The most straightforward way to determine the cardinality of a set is in counting its elements (i.e., in establishing a one-to-one mapping between the elements of the empirical set and the numbers from 1, up to the last counted element, in sequential order)⁸ and then assigning the last number used in counting to the entire set (this is also known as the “cardinality principle” of counting; Gelman & Gallistel, 1978, see details below). Hence, a crucial feature allowing numbers to represent cardinalities is their sequential order. In fact, it is because numbers have a fixed position within the sequence that 2 sets comprising the same number of elements will end up having the same cardinality. Thus, it is because cardinal numbers are well-ordered by magnitude that they enable us to judge not only the numerical equivalence between sets, but also to identify the set with a larger or smaller numerosity (i.e. containing more or less items)⁹.

In ordinal number assignments, the empirical objects numbers refer to are individuals placed in a certain order. We use a number to identify the rank or relative position of one individual element within a well-defined totally ordered set, in which the ordering relation has a specified initial point. The crucial question for ordinal assignments is “Which one” (e.g. which month is June?). Note that there is no quantification involved in ordinal assignments. If we consider the case of runners in a race, assigning a runner a higher number (e.g. 4) than another (e.g. 2) is not proportional to the distance between the runners, it just means that the runner in position 4 arrived after (i.e. ran slower than, but not twice as slow as) the runner in position 2. Hence, the distance between two runners is not the same as the

⁸ Note that the order that is important here is the order of the number words, that must be fixed, while the order in which the elements are counted in the set is irrelevant.

⁹ The numerosity refers to the discrete number of items in a set.

distance between the respective numbers we assign them (i.e., runners in positions 2 and 4 might not be equidistant to the runner in position 3). Accordingly, any other ordered sequence with an initial point (e.g. the letters of the alphabet) could be used to provide meaningful ordinal assignments.

The rank of an element in a set is determined by the one-to-one mapping between the empirical objects placed in a certain order, and the sequence of natural numbers, starting with 1. A crucial difference between this mapping and the counting procedure used to assign the cardinality of a set is that the order in which objects are counted is crucial in ordinal number assignments, while it is irrelevant in cardinal number assignments (this latter feature is known as the “order irrelevance” principle of counting in cardinal contexts; Gelman & Gallistel, 1978, see details below).

Being elements of a sequence is thus an essential property of numbers in cardinal and ordinal contexts alike. Consequently, learning the sequence of number words that is, acquiring sequence knowledge is a necessary step towards the assignment of numbers in various contexts to assess specific properties of empirical objects.

Wiese’s (2003) account for cardinal and ordinal number assignments is summarized in Table 1.

Table 1. Wiese’s (2003) account for cardinal and ordinal number assignments.

Number assignment	Cardinal	Ordinal
<i>Empirical object</i>	Set	Individual element of an ordered set
<i>Empirical property</i>	Cardinality of a set (numerical quantity)	Rank or relative position of an individual element
<i>Question</i>	“How many”	“Which one”
<i>Procedure to establish number assignment</i>	One-to-one mapping (counting) <i>Order of objects is irrelevant</i>	One-to-one correlation between sequential position of numbers and position of objects <i>Order of objects is crucial</i>
<i>Crucial feature</i>	Sequential order of numbers	Sequential order of numbers (but the sequence can be non- quantitative)

As we will see in the remainder of this section, cardinal and ordinal assignments don't only differ with respect to the empirical objects and properties they refer, they were also acquired through different developmental sequences. Wiese (2003) will argue that the use of counting words in cardinal assignments is the 'gateway to number' that will later enable ordinal (and nominal) assignments to develop. She proposes that children's early apprehension of cardinal assignments first comes from their prelinguistic grasp of quantity (through object-files and analogue magnitudes) that enables them to address the relevant property in cardinal contexts (namely, cardinality). Children will later learn the number word sequence and perceive the sequential order of counting words (i.e. the fact that they have a fixed position within the sequence) as a prominent and salient feature. Relying on fingers when counting will further emphasize the stable order of counting words and also provide children with an iconic representation of cardinality. Finally, counting principles, and in particular the 'cardinality principle', will open the way to non-iconic representations of cardinality, and to a systematic number concept. Wiese (2003) will further argue that it is language that will enable children to associate sequential relations between counting words (e.g. the relation '>', as in $5 > 3$) with quantitative relations between sets (in the e.g. (5) 'has more elements than' (3)); this pattern of system-dependent linking being later generalised to non-cardinal contexts, giving rise to ordinal number assignments and to a full-blown number concept (Wiese's proposal of how the human language faculty contributed to the development of systematic numerical thinking is presented in Box 1 below).

In the remainder of this section, we will first describe, in turn, the various steps that drove to the construction of cardinal number assignments, from an initially prelinguistic grasp of cardinalities, to the achievement of the cardinality principle. We will then present similar evidence for the development of ordinal assignments.

1.2. Cardinal number assignments as a 'gateway to number'

1.2.1. Pre-verbal grasp of cardinality in infants

Before children learn to use counting words in cardinal number assignments, they can already distinguish the numerosity of small sets as a relevant empirical property by way of subitizing, and they can estimate the size of larger sets.

**** The facts***

Research in the field of developmental psychology has shown increasing evidence that infants exhibit early quantitative capacities, well before they acquire language (see Feigenson, Dehaene, & Spelke, 2004, for review)¹⁰. We will describe in turn the evidence for numerical abilities over small and larger sets.

On the one hand, converging evidence from different paradigms have established that human infants of several months of age can precisely represent and keep track of small numbers of individual objects and are sensible to the numerosity of small sets (e.g. Feigenson & al., 2004). Studies using the habituation-dishabituation paradigm reported successful discriminations of 2 vs. 3 and sometimes 1 vs. 2, for visually presented individuals (dots: Antell & Keating, 1983; Starkey & Cooper, 1980; pictures of objects varying in shape, size and position: Strauss & Curtis, 1981; Starkey, Spelke, & Gelman, 1983), for three-dimensional objects (Feigenson, Carey, & Spelke, 2002), and for moving shapes undergoing occlusion (Van Loosbroek & Smitsman, 1990). Similar results were obtained with sequentially presented auditory stimuli, like syllables (Bijeljac-Babic, Bertoni, & Melher, 1991), and with successive actions (Wynn, 1996). Infants were also able to detect numerical correspondence between small sets of entities presented in different modalities (e.g. Starkey, Spelke, & Gelman, 1990; but see Mix, Cohen Levine & Huttenlocher, 1997, for

¹⁰ Research with higher animals further suggests that we share these capacities with other species (see Davis & Perusse, 1988, and Brannon, 2004, for reviews).

different results in older children). These abilities were restricted to very small sets that did not exceed 3 individuals (infants often failed to discriminate 3 vs. 4; Strauss & Curtis, 1981, and they consistently failed with 4 vs. 5, and 4 vs. 6; Starkey, Spelke, & Gelman, 1990; Strauss & Curtis, 1981; Starkey & Cooper, 1980). Infants were further found able to anticipate the result of simple addition and subtraction problems, as revealed by the use of violation-of-expectation paradigms: babies were found to look longer at (i.e. to judge the event as unexpected) the impossible outcome of 1 for the addition $1+1$, than at the possible outcome of 2 (Wynn, 1992). Infants were shown to expect exactly 2 objects in the $1+1$ situation, as they looked longer at the impossible outcome of 3 in a $1+1 = 2$ vs. 3 event (Wynn, 1992), and they also succeed at $2 - 1 = 2$ vs. 1, while success with larger numbers is less robust (e.g. Uller & Leslie, 2000). These data were replicated in many laboratories (Feigenson, Carey, & Spelke, 2002; Koechlin, Dehaene, & Melher, 1997; Uller, Huntley-Fenner, Carey, & Klatt, 1999, but see Simon, Hespos, & Rochat, 1995, for a non-numerical interpretation). Recent studies have challenged, however, the numerical nature of infants' reported abilities. In fact, because most of these studies did not adequately control for non-numerical perceptual variables that are correlated with number, they may not preclude that infants responded to some continuous quantitative dimension, instead of number itself. This was reported, in fact, in a recent study showing that when number is pitted against other continuous variables, or when other continuous variables are strictly controlled for, infants show no evidence for a response to the cardinal values of small sets of objects, while they reacted to a change in continuous perceptual variables (e.g. Feigenson, Carey, & Spelke, 2002, Clearfield & Mix, 1999; but see Feigenson and Carey, 2003, for a genuine evidence, using a manual search paradigm in which continuous perceptual variables were carefully controlled, for infants' exact representation of small numerosities up to 3). Although it is controversial whether infants relied on the discrete number of objects or on continuous quantitative dimensions and only later develop discrete representations, these studies indicate that babies show some sensitivity to the numerosity of small sets before language development.

On the other hand, recent studies have reported that preverbal infants are able to discriminate larger sets of elements provided that their ratio is large enough. In an habituation experiments, Xu and Spelke (2000; Xu, Spelke & Goddard, 2005; see also Brannon, Abbott, Lutz, 2004) showed that, with perceptual variables carefully controlled for, 6-months old infants were able to discriminate between numerosities as large as 8 vs. 16 dots, and 16 vs. 32 dots (ratio of 1/2), but they failed for 8 vs. 12 dots, and 16 vs. 24 dots (ratio of 2/3). Equivalent results were successfully replicated with auditory sequences (Lipton & Spelke, 2003) and further extended to sequences of actions (puppet jumps; Wood & Spelke, 2005). Yet, infants' ability to discriminate between large numerosities is highly imprecise; with nonetheless increasing precision over development (Lipton & Spelke, 2003; Wood & Spelke, 2005). All these data suggest that infants have some kind of representation of numerical quantity, in the form of analogue-magnitudes. Furthermore, similar patterns of performance in discriminating visual-spatial arrays, auditory-temporal sequences, and action sequences, suggest that a single abstract system might underlie infants' representation of approximate cardinal values of large numbers of entities.

Taken together, research studies on number representation in infancy suggest that infants might rely on two different systems when processing small and large sets: respectively, an object-tracking system, and a system for representing numerosities as analogue-magnitudes (see Xu, 2003, for a direct evidence for the co-existence of these two systems in infancy). Among these systems, the former might play a more central role in the development of discrete numerical representations (Wiese, 2003). We will describe each of these systems in turn.

**** The underlying mechanisms***

Object-files are used in early numerical reasoning to represent and keep track of small numbers of objects through the assignment of temporary markers (also known as "object-files" or "tokens"; Kahneman, Treisman & Gibbs, 1992; Trick & Pylyshyn, 1994) to individual objects as they move and change (e.g. Simon, 1997; Xu, 2003; Feigenson, Carey & Hauser, 2002).

This spatio-temporal system is precise, but has a limited number of “markers” that allow to represent only a limited number of items (up to about 3) at once. Object-files representations were proposed to explain infants’ ability to discriminate between sets of up to 3 objects. Infants would represent each object in a set by one of these markers that would be stored in memory when the set disappears. Numerosity discrimination would then result from the mismatch in one-to-one correspondence between mentally stored tokens and objects of the new set. This mechanism was proposed to underlie infants’ ability to choose the larger of two small quantities (e.g. 1 vs. 2 and 2 vs. 3, Feigenson & al., 2002; Feigenson & Carey, 2003), and their performance in the simple addition/subtraction tasks (longer looking times for the unexpected/impossible outcome were due to a numerical mismatch in the one-to-one correspondence between the object-files stored in memory and the actual visible outcome; Simon, 1997; Simon, Hespos, & Rochat, 1995, see also Uller, & al., 1999; see Feigenson, Carey & Spelke, 2002, for a revised object-file model coding also for the physical properties of objects). The object-file system is also operative in adults as evidenced by their ability to subitize small sets, that is, to rapidly grasp the numerosity of up to 4-item sets without counting (e.g. Mandler & Shebo, 1982; Trick & Pylyshyn, 1994); the numerical percepts of “twoness” and “threeness” might thus not be generated by any form of counting (Butterworth, 1999).

Yet, the primary role of this object-tracking system is not quantitative, and numerical information can only be derived implicitly, through a one-to-one correspondence mechanism. Hence, infants would represent each member of a set with a symbol or “marker”, but no single symbol can serve to represent the numerosity of the set. This system can thus only provide infants with an iconic representation of cardinality (e.g. a set with 2 objects will be represented as “one object, another object”; Feigenson & Carey, 2003; Wiese, 2003).

Sets with a larger numerosity were thought to be processed through the approximate representation of analogue magnitudes. The mechanism generating analogue magnitudes has been initially described as an accumulator (e.g. Meck and Church, 1983). The accumulator model assumes that when a discrete number of elements are presented to the organism, each

element will add a roughly equal amount of energy at a constant rate to an internal device (the accumulator). The total amount of energy accumulated at the end of this process is proportional to (and thus represents in an iconic way) the numerosity of the array. However, because the amount of energy entering the accumulator slightly varies from trial to trial, the resulting numerosity representation will only be approximate¹¹. Note that in this model, a continuous representation is derived from a discrete number of elements.

Hence, both object-files and analogue magnitudes represent the size of sets via the representation of its elements, that is, they provide us with an iconic representation of numerosity. In fact, each element in a set either corresponds to a distinct object-file, or to an increment of the analogue magnitude. This representation does not rely on dependent-linking, since the size of an empirical set is represented by the cardinality of another set (a set of object-files) or by an accumulated continuous quantity (the size of the analogue magnitude being directly related to the number of elements in the empirical set). Early numerosity representation therefore suggests that iconic stages develop before the emergence of dependent-linking.

1.2.2. The acquisition of the counting sequence and its use in cardinal contexts

The evidence reviewed thus far suggests that humans possess a grasp of numerical quantities and their interrelations that is independent from the language faculty. Hence, understanding the empirical properties (e.g. cardinality) we assess with numbers in various modalities has a non-linguistic basis. However, there is more to the number concept than a grasp of iconic cardinalities. We will now see how children acquire a full-blown concept of number, by first learning the sequence of counting words, and then progressively applying it in meaningful (e.g. cardinal) contexts, after their language faculty enabled them to understand the pattern of dependent-linking.

¹¹ The accumulator model is one kind of magnitude-coding number line models that will be described in the following section of this chapter.

Beside subitizing and estimation, verbal counting is another way we can assess the cardinality of a set (assessing cardinality might even be the primary role of counting; Fuson, 1988). Counting consists in the one-to-one mapping between the elements of a set and the initial sequence of number words. This mechanism enables the precise assessment of numerosities since exactly as many number words are employed as there are objects. Nonetheless, before the verbal counting ability develops, the child must learn to master the sequence of number words.

The acquisition of the number word sequence is a long-lasting process that starts around the age of 2 and takes the child up to about the age of 6. Fuson (1988) has provided a detailed account for the way children acquire and elaborate the number word sequence and then use it in counting situations to derive the cardinality of a set (and the relative position of an element in an ordering). Fuson (1988; see also Fuson, Richards, & Briars, 1982; Fuson & Hall, 1983) proposes to distinguish between two different, though overlapping, phases in the development of the number word sequence: an early acquisition phase in which number words are learned as part of a connected sequence, and an elaboration phase in which relations between individual words become more salient.

In the acquisition phase, the child learns that there are special words that refer to numbers, by their use in particular (e.g. counting) contexts. The most common form of the sequence produced by children during this phase is that of a stable conventional portion (the child correctly and consistently repeats the beginning part of the conventional sequence), followed by a stable non-conventional portion (that deviates from the conventional sequence but which is produced in a consistent manner by a given child), followed by a non-stable (it differs from trial to trial for a given child) non-conventional sequence portion. After the number word sequence is acquired, it first functions as a unidirectional whole structure, from which internal words cannot be produced independently (e.g. “onetwothree”, string level). Importantly, the sequential character of the counting sequence is a prominent feature during the acquisition phase: number words are first and foremost represented as elements of a progression, which has initially to be produced entirely (number words not being differentiated at the string level) in numerical contexts.

Additional evidence for the prominence of the sequential character of counting words comes from the kind of deviations we find in children's early counting sequences: when producing 'stable non-conventional portions' of the sequence, children leave out certain counting words, but they do not violate their sequential order (i.e. children will not produce a larger number, say 'nine', before a smaller one, say 'five'; Fuson, 1988). Besides, children sometimes produce elements from another sequence, the letters of the alphabet, in counting situations, both having a salient sequential nature (Fuson & al., 1982), while intrusions of 'non-sequential' material was not reported. The sequential character of counting words is thus salient early on, and it will be essential for the use of counting words in number assignments (Wiese, 2003).

The elaboration of the counting sequence is a lengthy process of differentiating the words in the sequence and establishing relations upon these words. This period of elaboration has been divided by Fuson & al. (1982, 1983) into five levels, each of which is characterized by new abilities mastered by the child¹². At the *string level*, number words are undifferentiated and form a whole unidirectional structure that can only be recited starting from 1. At the *unbreakable chain level*, the sequence words are distinguishable, but they are still closely connected and the sequence can only be produced starting from the beginning. The differentiation of words at this level allows the child to establish one-to-one correspondence between words and counted entities (counting meaning) and to use counting to find out "how many" or "what position", that is, in cardinal and ordinal contexts. At the *breakable chain level*, the connecting links between the words in the sequence are understood by the child. He can count up from any given number word and he can also produce the sequence backwards. At the *numerable chain level*, the number words are distinct units; the child can ascertain their numerosity, count them and match them to a set of items of known numerosity (e.g. five fingers). He can also use his fingers to keep track of counted entities at this stage (e.g. when counting n steps from a

¹² Note that different portions of the sequence might be in different phases of development at the same time.

given number). Note that the use of fingers will provide the child with an iconic representation of the cardinality of a set, each object being associated with one finger and one counting word. At the *bidirectional chain level*, the sequence is strongly automatized in the forward and backward directions. The words are not only used for counting objects, but become themselves the items which are counted for arithmetic and relational purposes (what Wiese, 2003, refers to as ‘abstract cardinalities’).

Children will initially use the number words in their conventional order in purely recitation situations that is, with no reference to external entities. This is what Fuson (1988; Fuson & Hall, 1983) refers to as the use of the conventional sequence of number words in sequence contexts. Children will then progressively use the sequence of number words in counting situations that is, in correspondence to external entities. The next step will be the use of counting words to tell ‘How many’ objects are in a set: children will associate one counting word with each external entity (one-to-one mapping) and end up with a set of counting words of the same cardinality than the set of empirical objects. Later, children use fingers along with words when counting and the association of one finger to each counted entity and each counting word will end up again with an iconic representation of cardinality. Yet, the early use of number words in counting situations long provides the child with iconic representations of cardinality. The step to non-iconic representations will come with the acquisition of the cardinality principle: at this stage, the child will be able to associate *one* number word with a *whole* set of counted entities.

The acquisition of cardinal assignments is made possible once the child adheres to a series of ‘counting principles’ (Gelman & Gallistel, 1978)¹³ when learning to count.

¹³ Contrary to the position of Gelman & Gallistel (1978), we agree with Wiese (2003) with respect to the fact that the ‘counting principles’ are not necessarily innate, but may arise from the way counting words are introduced as meaningful tools in cardinal number assignments.

1.2.3. The counting principles and the development of non-iconic cardinality representations

The principles of counting are first acquired in cardinal number assignments and only later generalize, with some crucial adaptations though, to ordinal number assignments. The *One-One Principle* refers to the matching between exactly one counting word and each object in the empirical set. This requires individualising the single items to be counted, to tag or correlate words with the items, and to distinguish the items already counted from the rest of the set (partitioning; Wiese, 2003). These steps often require some kind of keeping track procedure, usually finger counting, to coordinate the verbal counting procedure with the tagging and partitioning of the items to be counted (see Alibali & DiRusso, 1999). Hence, the child can compare the cardinality of two sets by linking up their elements in a one-to-one correlation, but this is not yet a cardinal assignment. The *Stable-Order Principle* refers to the use of counting words in a fixed order, a feature that is salient early on during development. Note that the sequential aspect of counting might be enhanced during development by the use of fingers along with words during counting routines (Wiese, 2003; Butterworth, 1999). In fact, fingers have an invariant spatial arrangement on each hand, and we generally use them in a stable order. The *Order-Irrelevance Principle* is specific to cardinal assignments and refers to the fact that the elements in the empirical set can be counted in any order, without affecting the cardinality of the set. The *Abstraction Principle* indicates that any collection of things, in any modality, can be counted. The *Cardinality Principle* refers to the use of the last word to identify the cardinality of the set. Children master this principle at about 3 ½ years and make the step from an iconic to a non-iconic representation of cardinality. This is, in fact, the first evidence for a genuine cardinal number assignment (Wiese, 2003). A counting word is assigned to a set such that its position in the counting sequence reflects the set's cardinality. This way of matching counting words and sets is based on dependent-linking: it associates a (sequential) relational structure between numbers with a (quantitative) relational structure between sets. Wiese (2003) proposes that it is the human language faculty that provides such a cognitive pattern of dependent-linking, and that allows us to develop (from initially

iconic representations) a systematic number concept (see Box 1 for Wiese's, 2003, account of how the language faculty contributed to the emergence of a systematic numerical thinking in humans).

Let us sum up Wiese's (2003) account of children's route to cardinal assignments. Before they master the cardinality principle, children represent the cardinality of sets in an iconic way, initially through mental tokens in subitizing, then using as many fingers, and later as many counting words, as there are objects in the set. With the progressive mastery of the counting sequence, the child acquires the cardinality principle and is then able to associate one number with a whole set of elements. What enables the child to identify the relevant property in cardinal number assignments is his prelinguistic grasp of cardinalities through object-files and analogue magnitudes. This precedes the development of a full-blown number concept and further enables its acquisition. It is his language faculty that will later enable the child to recognize systematic relationships between counting words (with their salient sequential relational structure) and to associate them with relationships between objects. Children's prenumerical concept of cardinality will thus be progressively integrated into a concept of numerical quantity that is based on the application of the counting sequence in cardinal assignments. Hence, the crucial step from iconic (one counting word associated to each entity in the set) to non-iconic representations of cardinality (the last counting word associated to the whole set of counted entities) is the initial step that will later open the way to systematic number assignments: the pattern of dependent-linking will not be restricted then to cardinal contexts, but will be transferred to ordinal and nominal situations giving rise to a full-blown concept of number. In this sense, the initial application of counting words in cardinal assignments is considered, by Wiese (2003) as the 'gateway to number'.

Box 1. Wiese's account of the language legacy to numerical cognition

Wiese (2003) places particular emphasis on the human language faculty, as being the necessary condition for the development of a full-blown systematic number concept. What is important about human language is that it is a system of symbolic reference. Symbolic reference refers to the arbitrary, conventional association between a sign (e.g. the sentence 'I write a thesis') and a referent (i.e., the meaning of that sentence, 'I' being the agent of the action of writing, and 'thesis' the object)¹⁴. Yet, what is crucial in this association is that the link we establish is not between an individual symbol and an individual referent, but between a relational structure of signs and a relational structure of the objects they refer to. Thus, the links between a symbol (a sentence) and its referent (the meaning of that sentence) will rely on the elements' respective positions in the system (e.g. the position of the words in the sentence), and be called 'system-dependent links'. In the human language, the connection between sound (sign) and meaning (referent) is mediated by the syntactic system which correlates linear order from the phonological system (a word 'comes before/ after' the verb in the sentence) with hierarchical order in the semantic-conceptual system (that word 'is agent of/patient of' the action/verb). It is this pattern of (system) dependent-linking we have described that, according to Wiese (2003), prepared the way to a systematic number concept. Indeed, because our language faculty supports a routine of non-iconic, but system-dependent, links between words and objects, it provides a cognitive pattern for number assignments. Number assignments are, in fact, primarily about relations: we associate specific relations between numbers with specific relations between empirical objects, according to the properties we want to assess (e.g. the numerical relation '>' will be associated with the empirical relation 'has more elements than', in a cardinal assignment; and with the empirical relation 'comes after', in an ordinal assignment). Yet, what is crucial in these mappings between numbers and empirical objects is that numbers and objects are not correlated as individuals, but as elements of two systems (via 'system-dependent links'). Crucially, it is because numbers are part of a system that they can be used to identify cardinal, ordinal (and nominal) relations alike.

Yet, beside the cognitive pattern of dependent-linking, another way language contributed to numerical thinking is by means of recursive rules that open the way to discrete infinity. Like language, in fact, counting words are based on recursive rules and form an infinite progression. Finally, the use of "convention" to relate signs and referents (i.e., the fact that the sign-object relation is arbitrary) is a third way the linguistic system contributed to numerical thinking. Just like language (e.g. the visual or phonological form of a word shares no specific features with its meaning), the numerical function of a number (number word and Arabic numeral) is based on a conventional system, and is unrelated with its particular form (but is related to its position inside the sequence).

¹⁴ Note that in iconic reference, the link between sign and referent is not arbitrary, as the sign shares certain features with its referent (i.e., the sign is similar to the object it refers to).

1.2.4. Abstract cardinalities

Once the concepts of cardinality and numerical quantity have developed in cardinal assignments as referential signs (i.e. they denote properties of sets), they can later be generalised and abstracted from particular sets. Cardinality thus evolves from the property of a particular set, to the property of a set in general (e.g. ‘3’ denotes the set of all sets with three elements) and number words are now ‘non-referential’. That is, they are not only used to denote empirical objects, but can be abstracted from concrete cardinalities and refer to the concept of numerical quantity. Abstract cardinalities then serve as the basis for our mathematical thinking and lays the grounds for the integration of new mathematical entities, such as negative numbers, “0”, fractions, and so on (Butterworth, 1999; Wiese, 2003).

1.3. The development and acquisition of ordinal number assignments

It stems from our discussion thus far, that cardinal number assignments could be the basis for a systematic number concept and allow for the later use of numbers in non-cardinal contexts. The use of number words in ordinal number assignments, however, might not purely derive from their use in cardinal contexts. In fact, before they master the cardinality principle, children have an early grasp of sequential order as a salient property of numbers by their use in counting routines. Besides, animals (the picture is less clear for preverbal infants) were shown to possess a grasp of serial order and an apprehension of an object’s position within a progression. These early (pre/nonverbal) abilities might well underlie the concept of numerical rank. We will now present some of the data in the animal and, to a lesser extent, the infant literature.

1.3.1. Grasp of serial order

A preverbal apprehension of serial order has been reported in many animal species (e.g. birds, rats, monkeys) using various experimental tasks.

For instance, list learning was employed to test animal's ability to encode and then retrieve an arbitrary list of items in the correct order. In one study (Chen, Swartz, & Terrace, 1997), monkeys learned 4 lists each containing 4 photographs in a certain order. Photographs from these original lists were then re-assembled to create derived lists in which the item's original ordinal position was either maintained (i.e., the item appeared in the same position in the original and derived lists), or not. Interestingly, monkeys learned the derived lists in which item's original ordinal position was maintained rapidly and without error, whereas derived lists in which the ordinal position was changed were as difficult to learn as new lists. These results indicate that monkeys can encode the ordinal position of list items (see also Terrace, Son, & Brannon, 2003 for similar results). In another study (Orlov, Yakovlev, Hochstein, & Zohary, 2000), monkeys were shown three-item lists in a fixed temporal order and then tested on a delayed sequence recall task: after the sequential presentation of the three items (abstract images) in a list, a test stimulus that showed the same three items and a distractor (an item from another learned list) was presented, and monkeys were required to touch the three items in their original order, without touching the distractor. Monkeys' most common error was to touch the distractor when it had the same ordinal position in its own list as the correct item. This suggests that monkeys categorize items in a list according to their ordinal position.

Beside their ability to reproduce ordered lists of arbitrary stimuli, monkeys were also shown to represent cardinalities of sets in ascending order. Brannon and Terrace (1998, 2000, 2002) trained rhesus monkeys to respond to exemplars of the numerosities 1, 2, 3, 4 in ascending numerical order (i.e., monkeys had to touch the set with 1 element first, then the set with 2 elements, and so on). After the training phase, monkeys were able to order novel exemplars of the numerosities 1-4 in ascending order. In subsequent experiments, monkeys were found able to transfer their ordering rule to novel numerosities (5-9). On the contrary, monkeys that were trained on descending order (4, 3, 2, 1) could not extrapolate their ordering rule to novel numerosities, and attempts to train monkeys on numerically random orders (e.g. 3, 1, 4, 2) failed. These results were thus interpreted as evidence that monkeys can represent numerosities in an ordinal sequence (see Biro &

Matsuzawa, 1999, for evidence of a Chimpanzee's ability to order three Arabic numerals from the range 0-9 in ascending order).

Altogether these studies suggest that some apprehension of the serial order of items (whether these items are photographs, images, sets of different numerosities or Arabic numerals) can be achieved in the absence of language. Wiese (2003) further suggests that the non(pre)linguistic ability to apprehend an object's position within a sequence might serve as a precursor for the concept of sequential rank, and play a role in the later ability to grasp the empirical property that we identify with numbers in ordinal assignments.

With respect to the infant literature, studies did not address the question of the apprehension of serial order by preverbal infants, but focused, instead, on their ability to order or compare the numerosity of different sets and to grasp *quantitative relationships* between them. For instance, Brannon (2002) investigated what she called 'ordinal numerical knowledge' (which is, in fact, knowledge of the order of numerosities) using an habituation paradigm in which 9- and 11-months old infants had to discriminate sequences of arrays with ascending numerosity from sequences with descending numerosity (e.g. 3-6-12 vs. 12-6-3). Older infants were able to perform the discrimination, while younger infants failed. Yet, these results say little about infants' ability to encode the relative position (or rank) of an item within an ordered list, since judgments in these tasks could be based on numerosity-discrimination mechanisms, rather than serial order. Hence, further research is needed to investigate infants' ability to process the sequential position of items within an ordered set.

Little is known so far with respect to the underlying mechanisms that allow for a preverbal grasp of serial order. Wiese (2003) has suggested that this mechanism might be an ordinal counterpart of the subitizing ability. This mechanism would provide accurate representation for initial positions in a sequence and fuzzier representation for higher ranks. It would work by individualising positions in a progression, up to the object whose rank must be identified, and it provide 'ordered object-files' representations. Although the spatio-temporal nature of the object-tracking mechanism (that codes objects with their relative position in space as they move and change) might

lend some support to this view, it still needs to be empirically tested.

Overall, the empirical data suggest that the ability to apprehend the position of objects within a sequence is independent of a systematic concept of number. Wiese (2003) proposes that it is the early grasp the serial order of objects that might later enable the child to identify the relevant empirical property in ordinal assignments. It is once he is able to relate the sequential position of objects to the sequential position of number words, that the child acquires a concept of numerical rank. This concept will then be integrated, together with the earlier concept of numerical quantity, into the cognitive number domain and will provide the child with a full-blown systematic concept of number.

Nonetheless, the early grasp of serial order is not sufficient for the development of a concept of numerical rank. Wiese (2003) suggests that children further need to acquire certain counting principles that differ in some respects from those for cardinal assignments. These principles were described by Wiese (2003) and will be presented below in relation to the ‘cardinal principles of counting’.

1.3.2. Principles of ordinal assignments

The *One-One Principle* is similar in both cardinal and ordinal number assignments. In both cases we assign exactly one number word to one empirical object. One difference in ordinal, with respect to cardinal, assignments though, is that the objects must be ordered in a certain way. Besides, not all objects in the ordering need to receive one number word, but only those objects up to the final element of which we want to assess the relative position. This is because numbers in ordinal contexts are assigned to individual elements in the ordering and not to the whole ordered set. In cardinal assignments, no ordering is needed for the objects in the set, but they all need to receive one tag (number word) to assess the cardinality of the set. The *Stable-Order Principle* is the same as in cardinal assignments and refers to the use of number words in their stable sequential order. The *Order Relevance Principle* states that empirical objects also need to be counted in a certain order (whereas this order is irrelevant in cardinal

assignments; see also Fuson & Hall, 1983). The *Ordinality Principle* refers to the use of counting, not just as a subroutine to identify the cardinality of a set (a mechanism that could be replaced by subitizing for small sets in cardinal assignments), but as *the* ordinal number assignment. The one-to-one correlation between empirical objects (in a predetermined order) and the counting words determines exactly the position (or rank) each object occupies in the set. Finally, the *Abstraction Principle* restricts ordinal assignments to empirical objects that are a progression, or that can be ordered according to some property. Thus, while cardinal number assignments apply to sets without restriction, ordinal assignments only refer to the subgroup of (potentially) ordered sets.

Yet because research so far has concentrated on cardinal assignments, we still know relatively little about the different stages in the acquisition of ordinal number assignments, and in which order these principles are acquired. The knowledge of ordinal number assignments appears nonetheless to lag behind the acquisition of cardinal number assignments; likewise, ordinal number words (e.g. *first*) are acquired later than cardinal number words (*one*; e.g. in English, Fuson & Hall, 1983). This suggests that ordinal number principles might build on cardinal number principles. Consequently, concepts of numerical order might also be acquired later than the concept of quantity. Evidence thus far points to a primacy in the acquisition of cardinal (quantity) over ordinal (rank) number concepts in individual development (possibly also because of the greater emphasis placed on cardinal as opposed to ordinal number assignments when children learn to apply counting words to empirical objects) suggesting that number assignments start first and foremost as representations of cardinality. Other application of numbers might be included in the concept only later, and in particular, after the step from iconic to non-iconic representations has been made and number words are used in cardinal number assignments (i.e., after children master the Cardinality Principle; Wiese, 2003). Ordinal assignments can then be based on a generalization: they are acquired as an additional domain in which counting words can be used. However, the empirical evidence is scarce and inconclusive, thus no definite assumption can be drawn about the acquisitional sequences of cardinal versus ordinal number

assignments (see also the later discussion about Fuson's approach, 1988, and the developmental controversy about the primacy of order relations in cardinal or sequence contexts).

1.4. Cardinal and ordinal assignments: how do they relate?

We have seen in this first section that the relationship between cardinal and ordinal assignments are much complicated than it appears at first glance. Yet, it is not because adults easily shift from one number meaning to another with no effort, and even unconsciously one might think, that these meanings do not refer to different empirical properties, that they are acquired at different times during development and through different sequences.

We have reviewed the evidence (based on Wiese's approach) that number words can be used with equal efficiency to assess the cardinality of sets or the relative position of individual objects within ordered sets. Each number assignment refers to a specific empirical property (respectively cardinality and relative position), but both are grounded in a pattern of dependent-linking, that associates specific relations between numbers (example for a cardinal assignment: the relation smaller/larger) with the relations between empirical objects that we want to assess ('has more/less elements than'). It is the language faculty that supports the application of verbal elements in this pattern of dependent-linking. Yet, in both cardinal and ordinal assignments the crucial property of number words is their sequential order, a grasp of which is acquired early on during development. In both numerical assignments, counting can be used to assess the empirical property of objects, with the order in which the objects are correlated to the number words being irrelevant in cardinal assignments, but crucial in ordinal assignments. Besides, each numerical subdomain can build on early prenumerical concepts that enable the child to grasp the property that is relevant in each number assignment: subitizing, based on object-files, and estimation, based on analogue-magnitudes, support a grasp of cardinality; 'ordered object-files' might allow to grasp ranks in serial order. With respect to their sequential development, some evidence suggests that the identification of the cardinality of sets might be the prime function of

numbers, followed by the identification of relative positions (or ranks) of individual elements. Concepts of numerical quantity might thus be the core numerical meaning, upon which other meanings (e.g. numerical order) develop and are later related. The developmental primacy of cardinality still needs, however, more empirical evidence (but see Dehaene & Changeux, 1993). Besides, developmental psychologists (e.g. Fuson, 1988; Wynn, 1995) have put forward an alternative view in which numerical order is available from the start and cardinal relations develop only later (this view will be developed later in this chapter).

In the present section, we have emphasized the grounds upon which cardinal and ordinal number knowledge could be distinguished, mainly focusing on their developmental sequences. Yet, the fact that number meanings have distinct developmental courses doesn't rule out the hypothesis that, after their acquisition during childhood, the concepts of numerical quantity and numerical order get inextricably and possibly irreversibly linked up in a full-blown concept of number, as that found in adults. In fact, concepts of numerical rank are related to concepts of numerical quantity by their common numerical basis. Furthermore, while empirical properties like rank and quantity are disjoined in non-numerical contexts, they are linked up with each other in the cognitive number domain: they are associated as two kinds of properties that can be assessed with numbers. Hence, using number words in cardinal and ordinal assignments alike might cause them to be strongly related in numerical processing, and to rely on some common underlying mechanism.

We will now turn to the empirical evidence that supports the hypothesis for the involvement of a common underlying mechanism when processing quantity and numerical order (i.e., the relative position, or rank, of an item in the sequence).

2. Similarities in processing quantity and numerical order

In this second section, we will review the evidence suggesting that numerical quantity and numerical order might be processed through similar cognitive mechanisms. First evidence for similarities in processing comes from the behavioural effects of distance and SNARC that were equally reported in processing numbers and non-numerical ordered series. We will also present how models of numerical processing using the number line metaphor can account for these effects. We will then turn to neuroimaging evidence suggesting that quantity and serial order processing might recruit some common underlying neural substrate in the parietal cortex. Finally, neuropsychological evidence of conjointly spared, or impaired, processes of numerical quantity and non-numerical ordered series will be presented.

2.1. Behavioural evidence

The first line of evidence suggesting that processing numerical quantity and processing the order of elements in a sequence might rely on shared functional processes comes from the distance and SNARC effects. These were reported, in fact, in the processing of both numbers and non-numerical ordered series, such as the letters of the alphabet. We will present the experimental evidence for each effect in turn and then ask whether they could derive from the coding of sequential order. We will finally present two classes of number line models that account for the distance and SNARC effects and that could also provide some kind of order coding.

2.1.1. The distance effect

The distance effect has been consistently reported in various numerical processing tasks, such as comparison. The distance effect refers to the empirical finding that the ability to discriminate between two numbers improves (in both time and accuracy) as the numerical distance between them increases (e.g. RTs are faster and subjects are more accurate in comparing numbers 5 and 9, than 5 and 6). Moyer and Landauer (1967) were

the first to report a distance effect in adults' speeded comparisons of Arabic digits. After this landmark study, a distance effect was consistently reported in comparison studies with symbolic numbers, such as Arabic digits or number words (e.g. Dehaene, Dupoux, & Melher, 1990); it was not restricted to single digits and was also reported with 2-digit numerals (Dehaene & al., 1990). This effect was further found when comparing non-symbolic numerosities, such as dot patterns (Buckley & Gillman, 1974).

The distance effect was usually taken as evidence that numerals activate an analogue magnitude representation, that is, an abstract representation of the numbers' corresponding quantity. This representation was proposed to take the form of a left-to-right oriented continuum or 'number line' (Dehaene, 1989; 1992). This internal quantity representation appears to be automatically accessed when processing numbers (see Tzelgov & Ganor-Stern, 2004, for review). A numerical distance effect was reported, in fact, when subjects merely had to say whether two presented numbers were the same or different that is, in the absence of explicit quantity processing (Dehaene & Akhavein, 1995). In a related vein, an effect of the distance between numbers (in the reverse direction, though) was found in priming experiments in which the mere presentation of an Arabic digit or a number word facilitated the subsequent processing of a numerically close target number (Reynvoet & Brysbaert, 1999; Naccache & Dehaene, 2001).

This effect is not specific to numbers, and was also reported when processing non-numerical quantifiable dimensions, such as the size of objects (Holyoak, 1977; Moyer, 1973; Paivio, 1975). These data were interpreted as evidence that quantitative information might converge onto a shared analogue representation of magnitude (Dehaene, Dehaene-Lambertz, & Cohen, 1998; Fias, Lammertyn, Reynvoet, Dupont, & Orban, 2003).

Nonetheless, the distance effect was not restricted to quantitative stimuli (like numbers), but was also reported when subjects processed non-numerical ordered series such as the letters of the alphabet (Gevers, Reynvoet, Fias, 2003; Hamilton, & Sanford, 1978; Jou, Aldridge, 1999; Taylor, Kim, Sudevan, 1984) and the months of the year (Gevers & al., 2003). Distance effects have thus been observed not only when the linear order is intrinsic to the stimuli (e.g. in the case of animal sizes), but also

when it is conventionally determined (as in the case of letters or any other arbitrary ordered series).

Overall, it appears that the distance effect could well be explained by the pure sequential (or ordinal, Tzelgov & Ganor-Stern, 2004) representation of the numerals themselves. Hence, these data suggest that processing numerical and non-numerical ordered series may rely on a common underlying representation, coding for relative positions along a continuous (quantifiable or non-quantifiable) dimension.

2.1.2. The SNARC effect

The SNARC (Spatial Numerical Association of Response Codes, Dehaene, Bossini, & Giraux, 1993) is a second psychophysical effect common to the processing of numbers and non-numerical ordered series. The original SNARC effect was thought to reflect the association between number magnitude and spatial response preference with faster left- than right-side responses to small numbers and faster right- than left-side responses to large numbers (Dehaene & al., 1993). This effect was also found in tasks that did not require any quantity processing of the digits, such as parity judgment (Dehaene & al., 1993), phoneme monitoring (Fias, Brysbaert, Geypens, & d'Ydewalle, 1996), mere detection of numerical stimuli (Fisher, Castel, Dodd, & Pratt, 2003), orientation-discrimination of a shape superimposed on a digit (Fias, Lauwereyns, Lammertyn, 2001)¹⁵, and was taken as evidence for the automatic activation of an internal magnitude representation in the form of a left-to-right oriented number line (with small numbers on the left and large numbers on the right) whenever numbers are processed (see Fias & Fischer, 2004 for review). Because the SNARC effect was found with stimuli in various input modalities and with different output effectors (e.g. manual response, pointing, saccade, or grip aperture; Andres, Davare, Pesenti, Olivier, & Seron, 2004; Calabria & Rossetti, 2005; Fisher, 2001; 2003; Schwarz & Keus, 2004), the experimental data point to a central

¹⁵ This latter study suggests, however, that not all non-semantic tasks (e.g. colour discrimination) involving mere presentation of numbers elicit a SNARC effect, and that the interaction between numbers and space might depend on the overlap of neural circuits.

locus of interaction between numbers (small/large) and space (left/right, respectively) at the level of their abstract representations (see Hubbard, Piazza, Pinel & Dehaene, 2005, for review).

However, a SNARC effect has recently been reported with the ordered series of letters of the alphabet, months of the year and days of the week (Gevers & al., 2003; 2004) in both order relevant (e.g. judging the position of a letter as coming before or after a given standard) and order irrelevant tasks (e.g. consonant-vowel classification). These results suggest that the internal representation of ordered sequences is spatially coded, just as numbers, and that this spatial code is automatically activated.

2.1.3. May sequential order account for the distance and SNARC effects?

Taken together, these data suggest that processing numbers, even in non-quantitative tasks, and ordered series elicit the occurrence of distance and SNARC effects. These effects were further taken as evidence for the automatic activation of an abstract representation in the form of a left-to-right oriented continuum. Accordingly, we may ask on which grounds the number sequence might be related with the (non-quantitative) ordered series of letters, days and months (see Fias & Fischer, 2004, for a discussion on this point). One common feature is their sequential order, i.e. the fact that the items in these series have a fixed position -order- within the sequence¹⁶. With respect to the series of letters, days and months, their sequential structure is a (the) fundamental characteristic (see Jou, 2003, for a discussion about letters), and the elements of these series can be easily defined according to their relative position within the sequence. With respect to the number sequence, it derives from the above discussion on cardinal and ordinal number assignments (Wiese, 2003) that sequential order is also a general and crucial feature of numbers. Consequently, and contrary to what is usually claimed in number processing studies, distance and SNARC

¹⁶ Note that for the series of numbers, letters, days and months, the order of the items within the sequence is well-known by healthy adult subjects and held in long-term-memory.

effects might not reflect the activation of a quantity representation, but could simply arise from the direct comparison of positions along the sequence (see Wiese, 2003, for a similar interpretation of the data). The hypothesis that numerical distance and SNARC effects would reflect the automatic activation of, according to their terms, “ordinal information” has been recently proposed by Tzelgov and Ganor-Stern (2004). In their view, however, “ordinal information” was tightly linked to magnitude information, and did not reflect a purely non-quantitative sequence order. Our view is that the occurrence of distance and SNARC effects when processing numbers and other ordered series could be explained by the pure sequential nature of the series involved, with no additional necessary activation of cardinalities. Accordingly, the representation underlying distance and SNARC effects would also need to code for the sequential order of the items (see Wiese, 2003), whether or not additionally coding for cardinalities¹⁷.

The way purely sequential order is represented has, so far, been the object of little consideration and still remains unclear. Certain clues with respect to the nature of this representation can, nonetheless, be derived from the distance and SNARC effects, which suggest that it might be spatially coded and take the form of a left-to-right oriented continuum. Consequently, using the “number line” metaphor¹⁸ might be a suitable way to represent the internal coding of sequential order. Hence, while the “number line”, has usually been used thus far as a metaphor for the mental representation of magnitude¹⁹, we make the hypothesis that it could equally, and possibly more suitably, be used as a medium for the internal representation of sequential order (of numbers and non-numerical series). Consequently, the number line would be primarily devoted to the representation of sequential order and, in the case of numbers, would additionally code for magnitude

¹⁷ Note that although order seems to be the crucial feature underlying the effects of distance and SNARC, when these effects occur in numerical tasks, they don’t preclude the additional activation of quantities.

¹⁸ This must be considered as a metaphor since no topographic ordering was anatomically reported among number-sensitive neurons in the monkey parietal cortex (Nieder & Miller, 2004).

¹⁹ Other accounts for the representation of number magnitude have also been proposed. For instance, McCloskey (1992) proposes that the (abstract) representation of magnitude consists in a place-value system based on powers of ten. Sequential order of number words is not spatially coded in this semantic representation, but should be derived from the lexicon.

(quantity) information. Thus, an internal left-to-right oriented continuum would also be accessed to merely derive the order, or the relative position, of items in the sequence in the absence of any associated quantity (e.g. when processing non-numerical ordered series, such as the letters of the alphabet). We will further develop our interpretation of how quantity and order information would be represented and derived from a number line representation in the General Discussion.

In the following section we will describe two main classes of number line models. Yet these models were developed to account for the representation of quantity (magnitude) information and we will present them along these lines. We will only ask whether these number line models also allow for some kind of representation of sequential order and how the relative position of items in the sequence might be derived from these models.

2.1.4. Models of the number line

Two main classes of number line models can be distinguished: place-coding and magnitude-coding models.

**** Place-coding models***

Place-coding models (Dehaene, 1992; Verguts, Fias & Stevens, 2005; Nieder & Miller, 2004) refer to the assumption that each number will strongly activate a specific position (or place) on the number line. This activation peaks at the target position but also spreads to the neighbouring positions (i.e. neighbouring numbers) with decreasing strength as a function of distance. For instance, as shown in Figure 1 (panel A), the number 5 will maximally activate its corresponding position on the number line, with some activation spreading to the position of adjacent numbers (4 and 6), and decreasing activation attaining numbers farther apart from 5. In this way, a number only activates a (restricted) portion of the number line and acts as a band-pass filter. This kind of coding scheme has also been described as “barcode magnitude representation” because number magnitude is coded as a moving bar of activation on a topographic scale (Zorzi, Stoianov, & Umiltà, 2004). Note that

the distance effect is explained in these models by the amount of overlap between two number representations (close numbers overlap more and are thus more difficult to discriminate than numbers farther apart).

A place-coding representation of magnitude has been proposed by several authors (e.g. Dehaene, 1992, 2001; Verguts, Fias & Stevens, 2005), though models differ with respect to other properties such as the organization of the number line (logarithmic, Dehaene, 2001; linear, Verguts & al., 2005), the mapping from the number line to output mechanisms (non-linear in Verguts & al., 2005), but also the nature of the input stimuli (symbolic, Arabic numerals in Verguts & al., 2005; non-symbolic numerosities in Dehaene, 1992, 2001). These properties drive to different accounts of the size effect and propose either an exact (e.g. Verguts & al., 2005) or approximate (Dehaene, 1999, 2001) representation of numbers.

Recent support for place-coding models has come from the discovery by Nieder and colleagues (Nieder, Freedman, & Miller, 2002; Nieder & Miller, 2004) of number-selective neurons in the monkey prefrontal and parietal cortices during a delayed-match-to-numerosity task. These neurons showed a typical place-coding number line characteristic since a neuron that responded maximally to one numerosity also responded to close numerosities, but with decreasing strength with increasing distance.

With respect to sequential order, some kind of ordering relation might be derived in these models from the spreading of activation from the target position to the position of neighbouring numbers on the mental line. Yet, the order would only be coded for a circumscribed portion of the items around the target (and mostly for adjacent numbers), but not for the entire sequence. Imagine, for instance, that number 8 is presented and activates its corresponding position and that of its two immediate neighbours (7 and 9), if number 3 is presented just after, it will also activate the numbers adjacent to it (2 and 4), but the position of number 3 with respect to number 8 (i.e. whether 3 comes before or after 8) might not be easily derived, since numbers 3 and 8 might activate non-overlapping portions of the number line (2-3-4 and 7-8-9, respectively). Thus, order coding might not be derived beyond adjacent numbers in this kind of model.

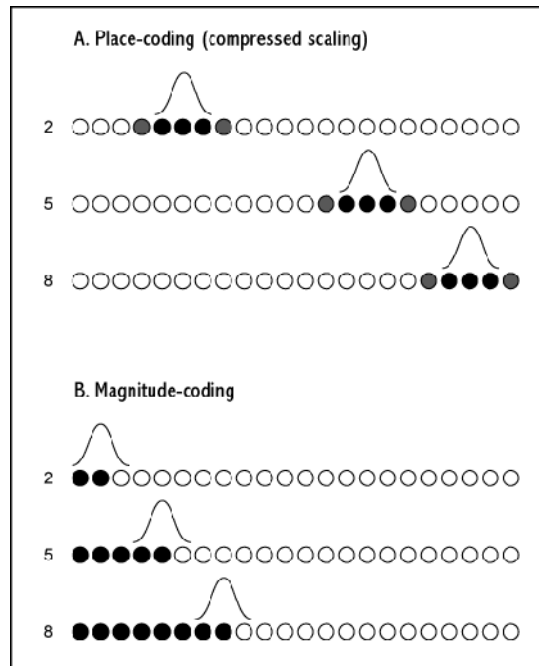


Figure 1. Graphical representation of (A) place-coding and (B) magnitude coding 'number line' models, adapted from Verguts, Fias & Stevens (2005).

* *Magnitude-coding models*

In magnitude-coding models, a number will activate a portion of the number line which, unlike place-coding, is not restricted to a region around the target number but includes the complete range of numbers up to the target number (see Figure 1, panel B), much like a thermometer or an accumulator (e.g. Gallistel & Gelman, 1992; Meck and Church, 1983; Zorzi & Butterworth, 1999). A consequence for this representation is that a smaller number will activate a subset of number line units that are also activated by a larger number, thus providing a more direct representation of cardinality and its inclusion property (e.g. number 4 will activate numbers 1, 2 and 3, which means that number 3 is included in the representation of number 4). The mental code for a number will thus be analogous to the magnitude it represents²⁰. With this kind of magnitude-coding, number comparison is

²⁰ This is what Wiese (2003) refers to as an iconic representation of cardinality.

similar to physical magnitude discrimination, which is robustly characterized by distance and size effects. The distance effect arises from the higher similarity in the internal magnitude codes for two close numbers, than for two far numbers.

The models that assume a magnitude-coding of number differ however in some of their features, such as the way the number line is organized, with scalar variability (Gallistel & Gelman, 1992; Meck and Church, 1983) or no variability (Zorzi & Butterworth, 1999), the mapping between the mental line and the output units (non-linear in the model of Zorzi & Butterworth, 1999), and whether they lead to approximate (Gallistel & Gelman, 1992; Meck and Church, 1983) or exact representations of a number's magnitude (possibly Zorzi & Butterworth, 1999).

Magnitude-coding might then allow for an exhaustive representation of the order of numbers up to the target number, since all smaller numbers are concomitantly activated. However, this order must be purely derived from magnitude information (or cardinality): to tell that '3' comes before '5', one must know that 3 is included in the representation of 5, i.e. that it is smaller than 5, and then derive its position in the sequence ('before 5') from this cardinal relation. It is unclear then how a non-quantitative order (i.e., an ordering that would not be determined by the relative quantity conveyed by its constituent elements) would be derived from this kind of representation.

Thus, none of the existing models of number representation appear to provide a comprehensive representation of sequential order. Moreover, whether quantity and order information are represented on, and derived from, a single abstract representation, or whether they rely on distinct underlying mental representations, is still an open question that should be addressed in future research.

2.2. Neuroimaging evidence

Neuroimaging studies provide a second line of evidence for the hypothesis that processing quantity and coding for the order of elements in a

sequence might involve some common underlying mechanism. This evidence is only indirect however, since neuroimaging studies of number processing have mostly focused on the neural circuitry underlying the representation of quantity, with little attention paid to numerical order. Evidence for the neural localisations of order coding will thus be gathered from a few non-numerical studies that might be directly relevant to our question. We will first present the neural substrate for numerical quantity together with two other regions of parietal cortex that respond to number; we will then turn to the brain regions activated when processing the order of non-numerical items; finally, we will see, for each region of parietal cortex that responded to number, whether it could be a potential host for the representation of sequential order.

2.2.1. Quantity

Extensive evidence has now shown that number processing activates specific regions of parietal cortex. Among these, the (bilateral) intraparietal sulcus (IPS) has been proposed to play a central role in number magnitude representation (i.e., the ‘number line’ representation discussed above; e.g. Piazza, Izard, Pinel, LeBihan, & Dehaene, 2004; Dehaene, Spelke, Stanescu, Pinel, Tsivkin). This region was found to be activated in tasks of number comparison and simple addition (e.g. Chochon, Cohen, van de Moortele, Dehaene; Pesenti, Thioux, Seron, De Volder; Pinel, Dehaene, Riviere, LeBihan), with increasing activation as the task put greater emphasis on quantity processing. A right-hemispheric advantage was found for number comparison and tasks requiring the abstraction of numerical relations (Chochon, & al., 1999; Langdon & Warrington, 1997; Rosselli & Ardila, 1989). The IPS was also recruited in a non-quantitative (spoken and written) number detection task (Eger, Sterzer, Russ, Giraud, Kleinschmidt)²¹, which suggests that numerical activation in the IPS might be automatic (task-independent), supramodal (for visual and auditory numerals) and notation-

²¹ In this study, number detection was compared with a letter detection task and showed a higher activation for numerical than alphabetical stimuli in the IPS.

independent (i.e., whether numbers are presented in the Arabic code or as verbal numerals; see also Naccache & Dehaene, 2001). The IPS was further found to be engaged in more general magnitude judgments, such as judging the physical size of digits (Pinel, Piazza, Le Bihan, & Dehaene, 2004) and comparing non-symbolic quantities like angles and lines (Fias, & al., 2003). Note that in this latter study, magnitude judgments on numerical and non-numerical stimuli were shown to activate a common locus in parietal areas.

Beyond the encoding of quantity information, Dehaene and colleagues (2003) have reported the involvement of two additional areas in number processing. The left angular gyrus, in connection with other left-hemispheric perisylvian areas, might be related to linguistic processing and support the manipulation of numbers in the verbal form (i.e. the auditory-verbal code in the Triple-Code model, Dehaene & Cohen, 1995). This area would be engaged in tasks making strong demands on the verbal coding of numbers (e.g. multiplication and arithmetic fact retrieval, Cohen & Dehaene, 2000). The last circuit, recruiting bilateral posterior superior parietal areas, would support spatial attentional orientation and would not be specific to the number domain. Activation of these regions during counting (because of shifts in spatial attention when sequentially enumerating objects) and spatial working memory tasks suggests that they might be involved in the selection of locations in space.

2.2.2. Non-numerical order

Activation of parietal areas is not restricted to numerical (quantity) processing yet, since similar brain regions were shown to be involved in serial order processing tasks (Cabeza, Mangels, Nyberg, Habib, Houle, McIntosh, Tulving, 1997; Marshuetz, Smith, Jonides, DeGutis, Chenevert, 2000). The study of Marshuetz & al. (2000) is particularly relevant and points to a common neural substrate underlying (numerical) quantity and (non-numerical) order processing. These authors found, in fact, that judging the order (vs. the identity) of two letters in a previously memorized array activated similar regions in the left and right parietal cortices, than those regions that were previously shown to be recruited by number comparison

(Chochon & al., 1999). This led the authors to suggest that ‘the underlying representation of order and numbers may share a common process’. Nonetheless, because the serial order task was far more difficult than the letter identity task, a contribution of task complexity to this pattern of activation may not be excluded. In a related vein, performing a transitive inference task (i.e., selecting in a pair of visual shapes the one coming later in a previously learned ordered sequence) was also shown to recruit parietal areas in the two hemispheres (Acuna, Eliassen, Donoghue, Sanes, 2002). In an ERP study of order judgment on letters (e.g. deciding if a letter comes before or after a given target in the alphabet), the distance effect was found to modulate the amplitude of a left parietal component (Szűcs, & Csépe, 2004). And in a neuroimaging study of sequence recitation, reciting the series of months backwards was further found to increase activation in left posterior parietal regions (Wildgruber, Kischka, Ackermann, Klose, & Grodd, 1999).

Overall, these studies suggest that parietal areas were also recruited when processing the order of non-numerical stimuli, whether this order was temporarily stored in short-term memory (Marshuetz & al., 2000), had to be processed over a newly learned sequence (Acuna & al., 2002), or was derived from a well-known sequence held in long-term-memory (Szűcs, & Csépe, 2004; Wildgruber & al., 1999). Altogether, these data with the evidence of parietal activations in quantity processing point to some common underlying neural basis for (non-numerical) order and magnitude judgments. However, none of these studies directly addressed the question of the underlying mechanism and neural substrate underlying quantity and (serial or sequence) order processing using equivalent material and the same subjects (in fact, in Marshuetz, & al, 2000, the regions of interest -ROIs- recruited by order judgments were compared to the ROIs reported by Chochon & al., 1999, in number comparison; that is, different tasks, materials, subjects, and methods were compared, which is far from being a direct evidence). Such long missing direct evidence has been recently provided, at last, in an fMRI study (Caessens, Fias, & Orban, 2005): the authors compared magnitude judgments on symbolic (number size) and non-symbolic stimuli (angle size), with the processing of order information also

on symbolic (judging the relative position of a letter in the alphabet) and non-symbolic stimuli (judging the orientation of a dot on a circle) and found that the IPS was activated by the conjunction of these tasks. Hence, these data suggest that the IPS is not only recruited when processing numerical quantity (or magnitude at large), but also when processing the relative position of items in an ordered series (e.g. letters in the alphabet), and even when judging a nonsymbolic order. Thus, IPS activation is not specific to numbers and might extend to other categories that share with numbers a strong spatial or serial component.

2.2.3. Numerical order

Because sequential order (though being a central property of numbers) has deserved, so far, little consideration, the issue of the potential cerebral locus of numerical order representation (i.e., representation of the relative position of numbers in the sequence) is still largely unresolved. At least two hypotheses can be derived from the review above. One possibility is that the order of numbers is coded on the same representation as numerical quantity, which might be expected by the direct link between a number's position in the sequence and its corresponding magnitude (Wynn, 1990). If this is true, numerical order should be coded in or close to the IPS, which hosts the abstract quantity representation. The recent observation of IPS activation when processing the order of letters in the alphabet (Caessens & al., 2005) might lend some support to this assumption.

Another hypothesis is that numerical order would recruit a neural substrate that was shown to be involved in number processing, but is not the locus of numerical quantity representation. In this case, the angular gyrus might be one potential candidate for the representation of sequential order (see Hubbard, & al., 2005). This region was found, in fact, to be involved in automatic recitation, which could be required in sequence processing tasks, and in verbal working memory, that might also be essential in processing serial order. Bilateral posterior parietal areas would be another potential candidate for sequential order representation. In fact, because of their involvement in sequential enumeration (i.e. counting) and in spatial

attention, these regions might be recruited when processing the sequential order of items along a mental or real oriented line. Moreover, these regions were found to be involved in spatial coding and might be recruited when processing sequential positions of elements in an ordering. Thus, the recently reported spatial coding for ordered series and numbers (Gevers, & al., 2003; Zorzi, Priftis, & Umiltà, 2002) might provide some further support to a posterior parietal locus hosting the representation of sequential order. Further research is needed, however, to better understand how the order of numbers in the sequence is processed and which cerebral regions it recruits.

2.3. Neuropsychological evidence

A third line of evidence for a common basis underlying quantity and sequence order processing comes from the association of impairments or spared processes in brain damaged patients. The evidence is poor yet, since processing the order of elements in a sequence (whether numbers or non-numerical sequences) has only been rarely investigated in acalculic patients. Two cases show, however, the crucial associations, in opposite directions (associated impairment and associated preservation).

Patient CG (Cipolotti, Butterworth & Denes, 1991) had a severe deficit in processing numbers (she was totally unable to deal with numbers above 4) and was also impaired in processing stimuli belonging to ordered series (she could not correctly recite the series of letters, days and months nor give the next item in the sequence). On the contrary, patient NM (Thioux, Pillon, Samson, de Partz, Noel, Seron, 1998) was severely impaired in several semantic tasks (he showed severe anomia for all categories of words), while his performance with numbers and non-numerical ordered sequences was largely preserved (he was flawless, for instance, in answering ‘what comes before/next’ questions with numbers, and with the series of days and months, though his performance was slightly less accurate with letters). Hence, these case studies suggest that processing numbers and non-numerical ordered series might rely on a common underlying basis.

2.4. Summary

In the present section, we have reviewed the evidence in favour of some common mechanism underlying quantity processing and processing of the relative order of items in a sequence. At a behavioural level, the distance and SNARC effects were reported with both numbers and non-quantitative ordered series. We made the hypothesis that these behavioural effects might come from the sequential nature of the stimuli involved. We then reviewed number lines models that account for these effects and sought how they might code for sequential order. Neuroimaging evidence for a common neural substrate underlying quantity and order coding was also described. Only weak conclusions can be drawn from the available data since only one study directly addressed the comparison of quantity and (non-numerical sequence) order processing mechanisms (e.g. using the same subjects and methods). Finally, two neuropsychological studies have shown so far that abilities to process numbers and non-numerical ordered series might be conjointly impaired or spared after cerebral lesion. Altogether, the evidence for some common underlying mechanism recruited by the processing of quantity or the processing of items' relative position in a sequence is mostly indirect and inconclusive as to the precise relationship between numerical quantity and numerical order processing.

In the following section, we will discuss the opposite line of evidence and show how developmental, neuropsychological and behavioural data suggest that mechanisms underlying quantity processing and the coding of items' (e.g. numbers) order in a sequence might be (partially) distinct.

3. Differences in processing quantity and numerical order

In the first section of this chapter, we have emphasized the differences between cardinal and ordinal number assignments (Wiese, 2003). Beyond their common crucial feature of sequential order, in fact, these number assignments were shown to differ with respect to their developmental acquisition, to the specific empirical properties to which they refer and to the contexts in which they are used. These distinctions, some of which trace back to childhood, and even before, might cause cardinal and ordinal number assignments and, at an abstract level, numerical quantity and numerical order (or rank), to rely on (at least partially) distinct processing mechanisms that remain separate up to adulthood.

A few additional sources of evidence speak in favour of (partially) distinct mechanisms underlying the processing of quantity and the processing of numbers' relative position in the sequence. At a developmental level, Fuson (1988) has described the way children acquire the counting sequence and showed that they can recite the sequence of number words well before they know the exact quantity associated with each number. Neuropsychological dissociations between cardinal meaning and sequence knowledge that corroborate the evoked developmental distinction will constitute our second line of evidence in favour of (potentially) dissociated mechanisms underlying numerical quantity and numerical order processing. Finally, at a behavioural level, beyond the distance and SNARC effects that we reported as evidence for some commonality of mechanisms, two specific behavioural markers were found in non-numerical order judgments and were not (or less consistently) reported with numbers: the pair-order effect and the reverse distance effect.

3.1. Developmental evidence

Some evidence for the distinction between cardinal meaning and sequence knowledge comes from developmental studies on children acquisition of the number concept. Fuson (1988) has reported, in fact, that while they learn to recite the sequence of number words, children understand

order relations in sequence and cardinal contexts at different times (see Box 2 for a description of these order relations). More generally, sequence recitation will provide the child with some knowledge of sequential order well before he understands the cardinal meaning of numbers (Wynn, 1992). We will start with a presentation of Fuson's (1988) account for the developmental stages at which children become able to process order relations in sequence and cardinal situations. As we will see, these abilities will be closely related to the child's mastery of the counting sequence.

Fuson (1988; Fuson & al., 1982; Fuson & Hall, 1983) proposes that number meanings are first learned separately, as context-dependent, and only later integrated into a mature number concept. She will place particular emphasis on the role of sequence recitation and 'sequence knowledge'²² in the later connection of cardinal and ordinal number meanings. Learning to recite the sequence of number words starts around the age of 2 and progresses up to the age of about 6 years. As we have reviewed earlier in this chapter, Fuson (1988) distinguishes 5 stages in the elaboration of the counting sequence, each associated with the child's achievement of specific and increasingly complex recitation skills (in both the forwards and backwards directions). The elaboration of recitation skills will progressively enable the child to understand order relations in sequence (and later in cardinal) contexts. 'Just After' is the first order relation mastered by the child: he can understand this relation at the unbreakable chain level, and he is able to produce the next element in the sequence at the later breakable chain level. The later ability to recite the sequence backwards will enable the child to produce the number 'Just Before', which is a much longer process (than producing the next element in the sequence) which starts at the breakable chain level. 'After' (e.g. 5 comes after 3) and 'Before' are two other order relations that will be later derived from the sequence ordering. Order relations are not restricted however to sequence contexts but have a cardinal counterpart ('One Greater Than/Smaller Than' for the 'Just After/Before' relation, and 'More Than/Fewer Than' for 'After/Before' relations; Fuson,

²² Let us note that although Wiese (2003) does not refer to a particular sequence knowledge she places crucial importance on the role of counting and learning the counting sequence, enabling the later step from iconic to non-iconic number assignments.

Box 2. Fuson's (1988) distinction between ordinal situation, order relation (in cardinal, sequence and ordinal contexts) and ordering.

Fuson (1988) provides an interesting distinction among ordinal situation, order relation and ordering.

An *ordinal situation* is one in which the ordinal number words (first, second, third, etc.) will apply to define the relative position of one entity with respect to the other ordered entities.

Order relations are not restricted to ordinal situations; they involve a comparison between two non-equivalent cardinal, ordinal, or sequence situations. For instance, in a cardinal context we will say that '3 is *less than* (or *smaller than*) 5', and apply the order relation between 3 and 5 (i.e. the fact that 3 comes before 5 in the counting sequence) to the cardinality of sets; thus, numbers refer to their cardinal or quantity meaning (i.e., 3 refers to a set with 3 elements, and 5 to a set with 5 elements). On the contrary, in a sequence context, the order relation between 3 and 5 will be expressed as '3 *comes before* 5' (in the sequence). The order relation will thus purely apply to the relative position of numbers within the counting sequence, with no additional reference to quantities or cardinalities (as when we say that 'page 3 comes before page 5'). Hence, order relations between number words in a sequence context are similar to the order relations that can be made on letters of the alphabet (e.g. letter C comes before letter E in the alphabet). Let us highlight that order relations involve the same number words in both cardinal and sequence contexts, but they will have a different meaning in each context (and either refer to cardinalities, or to relative positions -or ranks- in the sequence, respectively). Hence, sequence recitation is sufficient to solve questions about order relations in sequence contexts (e.g. 'Just Before/After', 'Before/After' relations), whereas an additional knowledge of the quantities numbers refer to is necessary to understand order relations in cardinal contexts (e.g. 'Fewer Than/ More Than', 'Smaller/Larger' relations).

With respect to order relations in ordinal contexts, they will involve the use of other (namely, ordinal) number words (first, second, third, etc.) that are specific to ordinal situations. In an ordinal context, the order relation will thus be expressed as 'the third comes before the fifth'. These number words will refer to specific positions within the sequence and convey ordinal meaning. In this perspective, sequence recitation and counting are not ordinal situations, since we don't use the sequence of ordinal number words for counting.

Repeated use of a given order relation on all the elements of a given finite set results in an *ordering*. An ordering can (but doesn't have to) be defined in a cardinal situation, while it must be defined for a situation to be ordinal, since without ordering it is not possible to define the relative position of one entity with respect to all the others.

Let us highlight that most of the empirical evidence we have collected and will discuss in the present dissertation with respect to numerical order processing refers to the processing of order relations among numbers in sequence contexts, and not (unless specified, as in the single-case study) to the processing of ordinal number words. Besides, to investigate numerical quantity processing we often resorted to a number comparison task, which corresponds to the processing of what Fuson (1988) calls 'order relations in cardinal contexts' (e.g., 3 5, which is larger?).

1988). Whether processing cardinal order relations derives from the understanding of the corresponding relations in sequence contexts, or whether cardinal and sequence relations develop independently, is still largely unclear (Fuson & Hall, 1983). Some developmental evidence suggests that once the sequence relations 'Just After/Before' are acquired they are later used to answer similar questions in cardinal contexts ('One Greater Than/ Smaller Than'; Fuson & al., 1982). The opposite was described, however, for the sequence relations 'After/Before' that appeared to develop only after the equivalent capacity in cardinal contexts was achieved (i.e. 'More Than/Fewer Than'), but this was only true for number words below 10, with the reverse pattern for number words between 10 and 20 (Fuson, 1988). It is not clear then whether 'After/Before' relations on sequence words are derived from those on cardinal contexts or whether the sequence order relations involve a different (although possibly similar) representation and an independent processing.

Further evidence for dissociated acquisition of sequence knowledge and cardinal number meaning comes from studies conducted by Wynn (1992). She has reported, in fact, that children first learn to recite the sequence of number words without knowing which quantity they refer to. Understanding the cardinal meaning of number words will not be mastered until the age of 3 ½, that is until children learn that each position of a word in the sequence is directly related to its quantity. During the process of sequence elaboration, number words will thus initially provide information about a number's relative position in the ordered list, without any related cardinal meaning. Hence, a child might know at this stage that 5 comes just after 4, but not that 5 is larger than 4, nor which quantity each number refers to. For the child to acquire cardinal meanings, he must not only be able to recite the sequence of number words, he also needs to understand the direct relationship between a word's position in the ordered list and the quantity it refers to (i.e., that the farther along a number word occurs in the list, the greater the quantity it refers to; Wynn, 1990; 1992). A developmental distinction appears then between the early knowledge of a *position* of a number word within the counting sequence, and the later knowledge of the *quantity* it refers to, together with its cardinal relationships with other number words. Hence,

children can count from more than a year before they learn what the words in the count sequence mean: i.e., before they learn how “one, two, three, four,...” represent numbers.

Hence, knowledge about the cardinal meaning of symbolic numbers is achieved later than knowledge about their position in the sequence. Nonetheless, recent studies investigating the processing of numerical quantity in young children using non-symbolic stimuli (collections of items) found that some understanding of quantity information can be achieved independently from (and even prior) to the acquisition of the counting sequence. In one of these studies (Rousselle, Palmers, & Noël, 2004), the relationship between counting skills and performance in magnitude comparison tasks (i.e., in making ‘More Than/Fewer Than’ judgments) was investigated in 3-year old children and revealed that pure sequence recitation skills were not related to performance in magnitude comparison tasks. Thus, some understanding of cardinal order relations can be achieved independently of sequence knowledge (see also the study by Feigenson, Carey, & Hauser, 2003, about preverbal infants’ ability to choose the numerically larger of two sets of 1 vs. 2 and 2 vs. 3 food items when amount of food was confounded with number). Nonetheless, when perceptual variables were carefully controlled for in the comparison task, children who possessed minimal cardinality knowledge²³ were shown to have a better performance in finding the collection containing more items (Rousselle & al., 2004). Hence, skilled counting abilities involving some cardinal knowledge are not necessary for, but ensure better processing of, cardinal order relations.

The evidence reviewed thus far suggests that sequence knowledge and cardinal meaning are acquired at different times during development, and that there might be no relationship between pure sequence recitation skills and the performance in judging cardinal order relations (‘More Than/Fewer Than’ relations) on non-symbolic stimuli. Yet, these are distinctions between

²³ This was assessed by performance in a ‘Give a number task’ in which children were presented with 10 items and asked to give the examiner a specific number of items. Children were considered to have minimal cardinality knowledge if they could give at least one correct answer in this task.

sequence knowledge and cardinal meaning, but what about the distinction between cardinal and *ordinal* meanings? We still know relatively little about the developmental course of children's acquisition of ordinal number meaning: is it acquired before, after or in concomitance with cardinal meaning? How is it related to sequence knowledge? These are some of the largely unresolved questions. Studies provide some information about the acquisition of ordinal number words; they suggest that ordinal number words might not be learned as a sequence, but from their isolated use in ordinal contexts, for the first several words, and other words would be derived from the standard number sequence (see Fuson & Hall, 1983). Thus, the production of ordinal number words lags behind production of the standard sequence of counting words, but the reason for this is not clear (is it due to a lack of experience with ordinal number words in everyday life, or to additional features that must be understood in an ordinal context, like the ordering; Fuson, 1988). Furthermore, using the ordinal number words "second" and "third" to identify the corresponding positions in an ordered set, was found to be more difficult for 5- to 7-year old children than simply producing "second" and "third" in an ordinal word sequence (Beilin, 1975, reported by Fuson & Hall, 1983). Yet, this doesn't mean that children have no idea of what numerical rank is before the age of seven, since Fuson and Hall (1983) reported that 5-year old children who did not understand questions with ordinal number words (e.g. 'Is the yellow car *third* in the race?') did however understand the same questions with the corresponding counting words ('Is the yellow car *number three* in the race?'). Consequently, identifying the relative position (or rank) of an object in an ordering might be independent of, and arise prior to, the understanding of how to apply the ordinal number words to ordinal situations.

With respect to the developmental relationship between the acquisition of cardinal and ordinal number meanings, the evidence is vague. Some authors proposed a developmental primacy of 'ordinality over cardinality', but they used misleading tasks: ordinality was tested, for instance, by asking children to process physical orders (e.g. seriate bars according to their physical size, Brainerd, 1979; Kingma & Koops, 1981), and not ordinal positions; and cardinality was tested with visual one-to-one correspondence

tasks. Of course, nothing like an ordinal number meaning is tested in these tasks, and no conclusion can be drawn from these experiments about the relationship between cardinality and ordinality. Piaget (1941) proposed that cardinal and ordinal aspects of number are constructed simultaneously by the child through the parallel understanding of class inclusion and seriation (order relations), and refuted the idea of separated constructions. We would expect, in fact, as also suggested by Fuson (1988), that at some point during children's development cardinal and ordinal number meanings become integrated into a full-blown number concept (i.e., the concept found in healthy adults) but the question of approximately *when* this integration takes place, is still open²⁴. Thus, the question of how children's understanding of numerical rank relates to their understanding of cardinality (or numerical quantity) is largely unresolved, and more research is needed in this direction in the future (note that to avoid a bias in the results due to children's later understanding of ordinal number words, this issue might be better addressed using the conventional sequence of number words in both cardinal and numerical order tasks).

The same caveats hold for neuropsychological studies of number processing since ordinal number meaning, as defined by Fuson, was not investigated in the majority of studies, and thus could not be compared to cardinal number meaning. With respect to sequence knowledge, it was usually only briefly tested in neuropsychological examinations. In the following section we will describe a patient's performance in tasks assessing sequence knowledge and compare it with his abilities to process numerical quantity.

²⁴ Moreover, another unresolved question related to this is whether, though being integrated into a full-blown number concept, the cardinal and ordinal aspects of numbers (i.e. numerical quantity and numerical rank) remain potentially separate during adulthood, or whether they get inextricably linked up.

3.2. Neuropsychological evidence

One neuropsychological study of a brain-lesioned patient corroborates to some extent the developmental distinction between the mastery of the number sequence and the understanding of cardinal number meaning and provides a second line of evidence for dissociated processes underlying numerical quantity and sequence knowledge.

Patient SE (Delazer & Butterworth, 1997) presented with a (transient) inability to access the cardinal meaning of numbers from the Arabic code after a left frontal infarct. Not only quantity meaning but also sequence knowledge were investigated in this patient and revealed an interesting pattern of dissociation. Quantity processing tested three weeks after infarct, revealed that SE showed a reverse distance effect in Arabic number comparison (he was slower in comparing far than close numbers) and he was unable to solve simple addition and subtraction problems with Arabic numerals (even in a multiple-choice procedure). Nonetheless, SE was able to process numerosities up to certain level: he could select the bigger numerosity of dot arrays, he could order dot arrays according to their numerosity, and perform simple additions with arrays of dots. While these achievements (e.g. recognizing that four dots is more than three) don't necessarily entail understanding of the cardinal meaning conveyed by the corresponding Arabic numerals (4 is more than 3), SE's further preserved abilities (1) to represent Arabic numerals with arrays of dots or chips and (2) to count the number of dots or chips corresponding to an Arabic numeral, showed some genuine understanding (and preserved representation) of cardinality. This was corroborated by the patient's correct answers to 'How many' questions. However, he was very slow in this latter task, which suggests that his performance in cardinal tasks might have relied on sequence recitation, instead of automatically accessing the cardinality of numbers. Sequence knowledge was found, in fact, to be better preserved: SE could count from 1 to 20, both orally and in the Arabic code (while counting by 2 or 3 was impossible), and he could name or write the number coming just after a given Arabic numeral (between 6 and 20). Hence, he might have used preserved (forward) sequence recitation abilities (we know nothing

about his backward recitation skills) when processing certain cardinal tasks (e.g. those involving dot patterns). Altogether, SE's pattern of performance was proposed to reflect impaired automatic access from Arabic numerals to magnitude representations, in the face of preserved sequence knowledge, that was sometimes used to perform cardinal tasks. The patient's impaired access to cardinal meaning was only transient, however, and the comparison distance effect soon recovered its typical pattern (slower processing for close than far numerals), few weeks after infarct. Although SE's cardinal meaning impairment was only transient (and partial), this case-study was the first attempt to describe a dissociation between (automatic) quantity processing and sequence knowledge after cerebral damage.

Hence, this single-case study suggests that processing the quantity or the sequential order of numbers might rely on distinct and thus potentially dissociable mechanisms, which could be selectively impaired or spared after cerebral lesion. However, the dissociation was not pure, and incomplete. Patient SE showed a transient impairment in the automatic access from Arabic numerals to numerical quantity representations, but he was largely preserved in processing non-symbolic magnitude (e.g. conveyed by dot patterns). Besides, his ability to process sequential order was only partially investigated, since we know nothing about backwards counting skills, nor tasks like 'give a number before'; and non-numerical ordered series were not examined in this patient. Anyhow, this case study provides some important evidence that numerical quantity and sequence knowledge might rely on partially distinct mechanisms.

3.3. Behavioural evidence

Behavioural experiments provide a third line of evidence for potentially dissociated mechanisms underlying the processing of numerical quantity and numerical (and non-numerical) order. The evidence relies on the presence of peculiar (order-specific) effects when processing the order of elements in their corresponding sequence that were not (or less consistently) reported when processing numerical quantity.

Pairwise judgment tasks have been systematically used with number stimuli (e.g. 2 5, which is larger?) to investigate the behavioural markers of numerical quantity processing. Similar paradigms were also frequently employed with non-numerical ordered series, and in particular with the letters of the alphabet (e.g. B E, which comes later in the alphabet?), to identify the markers of order processing²⁵ (see Leth-Steensen & Marley, 2000, for review). As we will see, comparative judgment tasks with numbers and letters showed certain common behavioural effects (e.g. the distance effect), whereas other effects were specifically reported with one or the other kind of stimuli. We will now review these specific behavioural effects (common effects were already discussed earlier in this chapter). We will then turn to a comparison of the series of numbers and letters and see what are their main differences and similarities, and how sequential order is coded in each series.

3.3.1. Serial position, pair-order and reverse distance effects in order judgments

Numerical quantity processing in normal individuals has often been investigated using comparative judgment tasks. In these tasks, subjects were either presented with a pair of numbers (e.g. 2 5) and asked to choose the numerically larger (or smaller) number (selection paradigm), or they were presented with one number (e.g. 2) and had to tell whether it was smaller or larger than a standard of reference (e.g. 5; classification paradigm, see Dehaene, 1989). In both paradigms, classic behavioural effects include the distance effect (which we discussed about earlier in this chapter) and the size effect. The size effect refers to the finding that, for equal numerical distance, discrimination of two numbers is slower and performance is less accurate as their numerical size increases (e.g. it is more difficult to choose the larger number between 8 and 9, than between 2 and 3). This, together with the distance effect, was taken as evidence for the activation of an analogue magnitude representation in comparative judgments about numbers

²⁵ That is, the markers reflecting the processing of the relative position of items in an ordered series.

(Dehaene & al., 1990; see also Verguts & al., 2005, for a review of how number line models account for the size effect).

Because little is known, thus far, about numerical order processing, we will seek for behavioural markers of order processing from studies with non-numerical sequences. These have typically used pairwise judgment tasks in which people were asked either to judge (1) which of two items (e.g., letters) came earlier or later in the (e.g., alphabetic) sequence (analogous to number comparison; Jou & Aldridge, 1999; Parkman, 1971), or (2) whether a pair of items (e.g., B C) was presented in the conventional (e.g., alphabetic) or non-conventional order (Grenzebach & McDonald 1992; Hamilton & Sanford, 1978; Lovelace & Snodgrass, 1971). A standard distance effect was found with both paradigms, and is similar to the distance effect reported in number comparison tasks. Serial position effects were also observed and refer to faster and more accurate responses to items near or at the two ends of a series than to items in the middle of the series (Jou & Aldridge, 1999; see also Jou, 1997 for an extensive investigation of this effect with letters). The serial position effect might be considered as a kind of size effect (which is also determined by the position of numbers on the mental continuum), although the origin of the two effects is much different. The serial position effect, in the case of letters, is thought to reflect the accessibility of a letter in memory, according to its position along the (alphabetic) sequence (Jou & Aldridge, 1999). The memory accessibility of the alphabetic letters decreases, in fact, from A onward as a function of the increase in alphabetic position (Klahr, Chase, & Lovelace, 1983; Lovelace, Powell, & Brooks, 1973). Serial order effects are also explained by the finite nature of the memorized ordered series (since the effect affects both the beginning *and the end* of the sequence), as well as by its length and by the mastery of its recitation (only forwards, or forwards and backwards). With respect to the numerical size effect, it received many different interpretations according to the various ‘number line’ models: it was either taken as an evidence for the compression of the ‘number line’ towards larger numerals (Dehaene, 1992), or was explained by the scalar variability in the mapping from the input units to the number line (Gallistel & Gelman, 1992), or by the magnitude-coding of numbers (Zorzi & Butterworth; 1999), or it was alternatively explained by

a non-linear mapping from the number line to the output units (Verguts, & al., 2005). Hence, this effect did not simply originate from the mastery of the number sequence recitation. Nonetheless, pure serial position effects (i.e., not confounded with a size effect) were reported by Schwarz and Stein (1998) in a number comparison task. Subjects were presented with vertically aligned pairs of digits from the range 3-7 and asked to indicate the location of the numerically larger digit. One peculiarity of the experiment was that the digits of the pair were presented asynchronously. This entailed a serial position effect i.e., faster responses when the end terms of the digit range used (3 and 7) were presented first (Experiment 1). This could be explained as probabilistic effect (when 3 or 7 were presented first, subjects did not need to wait for the second digit to provide the correct response). Serial position effects disappeared, in fact, when 3 and 7 were followed by a smaller or a larger number with equal probability (Experiments 2 and 3). Thus, serial position effects in number comparison are only observed under peculiar experimental conditions, whereas they are more consistently reported with alphabetic stimuli.

Two unique and specific effects were further reported in relative-order judgment tasks. First, pairs presented in the conventional, ascending order were processed faster when adjacent (e.g., B C) than non-adjacent (e.g., B D) in the sequence, thus presenting a reverse distance effect. Second, pair-order affected reaction times (RTs), which were faster for pairs presented in the conventional ascending order (e.g., B C) relative to the descending order (e.g., C B; Hamilton & Sanford, 1978; Lovelace & Snodgrass, 1971; Grenzebach & McDonald 1992). Pair-order and reverse distance effects might be specific behavioural markers of order processing. In fact, while they were often reported in order judgment tasks involving non-quantitative stimuli, they were rarely found to affect numerical quantity judgments. For instance, Parkman (1971) showed no effect of pair-order in number comparison tasks (whether subjects had to choose the larger or the smaller number). Nonetheless, Brysbaert (1995) reported a pair-order effect in a task of two-digit number comparison. He interpreted faster RTs to ascending, relative to descending number pairs, as a congruity effect with the left-to-right orientation of the internal number line (this effect could also be

considered as an instance of the SNARC effect, since subjects responded faster to smaller numbers when presented on the left-side of the pair, and to larger numbers when presented on the right side; see also Schwarz and Stein, 1998, for a similar spatial congruity effect; and see Zebian, in press, for a reversal of this spatial congruity effect in Arabic Monoliterate subjects using a same/different judgment task).

With respect to the reverse distance effect, this was recently reported to affect number processing too (Jou, 2003), but only when comparison involved arrays of more than 2 numbers. When participants had to choose the middle number in a three- or five-item array, their RTs were faster for arrays of consecutive numbers (e.g., choosing 5 in 4-5-6) relative to non-consecutive numbers (e.g., 3-5-7), thus leading to a reverse distance effect. This reversal was however limited to consecutive-number arrays (distance 1) when compared to arrays of numbers having a distance of 2, whereas arrays with more distant numbers (distance of 3: 2-5-8) were processed faster than both consecutive number and distance 2-number arrays. Besides, when subjects had to select the numerically smallest or largest number of the 3- or 5-item array, no reverse distance effect was observed, and extreme items showed a standard distance effect, instead. A similar pattern was reported with the letters of the alphabet in that study.

Taken together, the results from comparative judgments about numbers and letters showed, beside a common standard distance effect, certain specific behavioural markers that were (more) consistently reported in order judgment tasks (as compared to numerical/quantity tasks). The serial position effect could be explained by the finite sequential nature of the ordered series (or range) examined, whereas the pair-order effect and the reverse distance effect could point to the involvement of specific mechanisms (e.g. serial search) when processing the order of stimuli. Yet, the occurrence of both a standard and a reverse distance effect suggest the potential involvement of two qualitatively different cognitive processes in order judgments. The standard distance effect would be explained by a size-based comparison mechanism, similar to that involved in numerical comparison tasks (see Birnbaum & Jou, 1990). In contrast, the reverse distance effect for conventionally-ordered ascending pairs implies a serial

search process in which the time taken to establish the order of two items is determined by the number of items intervening in the sequential series (Jou, 1997). Because a reverse distance effect was recently reported in comparison of multiple number arrays (Jou, 2003), a similar serial-search process (much like a counting or sequence recitation mechanism) might be recruited when processing numerical stimuli, depending on the task (see the General Discussion for a more detailed account of the comparison and serial search processes).

Beside the potential involvement of specific mechanisms when processing numerical quantity and serial order, another difference between judgments about numbers and about other ordered series is the nature and structure of the series themselves. If we take the case of numbers and letters, they differ at many levels: lexical, syntactic and semantic, one series is infinite, the other is finite (but not cyclical, contrary to days and months), and so on. We will now briefly present some grounds upon which numbers and letters may differ.

3.3.2. Comparing the number sequence and the alphabet

The number sequence and the alphabet are two basic (ordered) systems in which the order of their respective elements is determined by convention and must be learned by heart during childhood. In adults, these ordered series are overlearned, held in long-term-memory and commonly used. Beside their general similarity as being ordered series, they bear a few important differences, however. In fact, while the number system is a rule-governed open-ended order, the alphabet is an arbitrary and closed order. We will describe each in turn.

Numbers are special in many ways. They constitute an autonomous lexical domain in language, and the distinction between number and non-number words is acquired very early during development (Fuson & al., 1982). Number words form a system which has a limited lexicon (the lexical primitives and multipliers), a simple syntax (the rules of combination) and a clear semantics (the quantity, or cardinality of a set in a cardinal context, and

the rank or relative position of an item in an ordinal context). The symbols representing numbers can be expressed in two main notations: the verbal code (number words) and the Arabic code. The verbal system is constituted by a finite set of lexical primitives that include units (from 'one' to 'nine'), teens (from 'eleven' to 'nineteen' in English, but they vary from one language to another), tens (from 'ten' to 'ninety'), the multipliers ('hundred', 'thousand' and 'million') and 'zero'. The primitives of the Arabic system are the units from 1 to 9, plus 0. The lexical primitives of the number lexicon are organised in a sequential order, and this sequence has to be learned by rote during childhood (see our discussion above about Fuson, 1988, and the construction of the counting sequence by children); an educated subject is then able to recite the sequence in the conventional order both forwards and backwards. A crucial point is that the sequential order of numbers is semantically determined since the number names (e.g. when used in counting activities) mirror the increase of the corresponding quantities, and of the corresponding rank (or relative position) of counted items. Importantly, Arabic and verbal symbols can be combined to express an infinite set of numerosities or ranks, following certain simple governing rules (additive or multiplicative rules; see Power & Longuet-Higgins, 1978, and Power & Dal Martello, 1990).

The infiniteness of the counting sequence is, precisely, an important aspect, which is induced by recursive rules. Wiese (2003) defines the number word sequence as a non-terminating progression of non-referential well-distinguished entities. There are morphosyntactic rules that generate infinitely many counting words, based on a finite set of lexical primitives and a few 'base' elements (the multipliers) that are used in combination with the lexical primitives. As a result, the number sequence is formed by an infinite set of counting words, and the elements of this set have an internal structure that forms the basis for their sequential order.

The number series is thus a special order system in that it uses a set of systematic generative rules to code the ordinal relationship between the elements of the series. That is, for the Arabic code, once the order of the nine units is learned, together with the place-holding role of zero and the place-value for powers of ten, with the acquisition of a few simple (additive and

multiplicative) rules, the order of any two numbers can be deduced from the memorized order of the Arabic numbers 0 to 9. Jou (2003) will further argue that the ordering is explicit in the number system since the main purpose of this coding system is to denote the rank or quantity of things.

The alphabet is a serial system that has no generative syntactic rules by which additional terms could be generated from a basic set of elements. Thus, the entire alphabetic order of the 26 letters has to be learned and memorized by rote. Besides, the order information is implicit in this system (Jou, 2003) since the primary role of the alphabet is not for coding an order, but to transcode sounds (phonemes) into graphemic representations (letters or graphemes); and letters are then combined to form meaningful words. The alphabet is a closed order in the sense that it has a clear beginning and a clear ending. Finally, contrary to digits, letters do not denote anything by themselves, but must be included in words to be semantically relevant.

Given these differences in the order structures of numbers and letters, retrieving the order information from these two linear order systems might thus be different in several respects (see Jou, 2003). First, retrieval should be faster from the numeric than the alphabetic order (see Parkman, 1971, for such evidence). This might be explained by the increased difficulty to enter the alphabet from an arbitrary, intermediate, letter (see Klahr, Chase, & Lovelace, 1983; Lovelace, Powell, & Brooks, 1973), whereas entering the number sequence is easy from any point for normal adults (Fuson, 1988). Second, because the number sequence in adults is a bi-directional chain, that is, it can be easily recited backwards too, while it is not the case for the alphabet, a stronger pair-order effect (i.e. stronger advantage for ascending relative to descending pairs, or a stronger directional bias, see Jou, 2003, 1997) is expected with letters compared to numbers. Good knowledge of the number word sequence in both directions might explain why a pair-order effect was less consistently reported with numbers in comparative judgments.

Thus, the way order is coded in the sequence (i.e., whether it derives from quantities or not) and to what extent the order relationship is mastered forwards and backwards by normal adults, might influence the underlying mechanisms involved in processing the order of numbers and letters. Yet,

because numbers also refer to magnitudes, and because magnitudes appear to be automatically activated whenever numerals are merely detected (see Eger & al., 2003), processing the sequential order of numbers might not show the expected order-specific behavioural effects because these are overruled by interfering, closely connected, quantity information.

3.4. *Summary*

In the present section, we have reviewed the evidence suggesting that the processing of quantity and numerical (or alphabetic) order might rely on distinct underlying mechanisms. With respect to developmental and neuropsychological data, however, the main evidence was that of a dissociation between sequence knowledge and cardinal meaning of numbers, since purely ordinal number knowledge was either poorly investigated and biased by the later knowledge of ordinal number words compared to counting words (in developmental studies), or not tested at all (in neuropsychological studies). With respect to the behavioural data, numerical quantity tasks have never been contrasted, thus far, with numerical order tasks. Consequently, we only have indirect evidence suggesting a potential distinction between mechanisms involved in numerical quantity processing and in processing numbers' relative positions in the sequence. However, order processing was investigated in behavioural experiments using either order verification tasks (e.g. B-C, are they presented in the conventional, alphabetic order?) or relative-order judgments (B-C, which comes earlier in the sequence?)²⁶ and was found to entail two specific (order-related) effects: a reverse distance effect and a pair-order effect. Consequently, beside the differences between the sequence of number words and the alphabet, if these behavioural effects were observed when judging the relative order of numbers in the sequence they might reflect the involvement of (numerical) order-specific processing mechanisms.

²⁶ Yet, none of these tasks addressed the retrieval of ordinal information (e.g., which is the second letter of the alphabet?).

Overall, what is astonishing about the literature on order processing is that “order” can recover many different aspects and be tested through various tasks, all possibly addressing slightly different aspects of the ordered series representation. Overall, the evidence suggests that processing the property of numbers that sequential order is might involve distinct mechanisms than processing the quantity meaning of numbers, whether sequential order is tested in a relative-order judgment task, using pair-order verification, through the retrieval of serial (ordinal) positions, and so on. These tasks might all summon, in fact, the activation of the mental representation of the ordered sequence in long-term-memory. Nonetheless, to gather more direct evidence about the relationship between quantity and “order” processing mechanisms, one aim of our thesis will be to compare order judgment tasks with the equivalent quantity tasks. Besides, we will also try to better understand how the order of elements in a sequence is processed. To fulfil these objectives, we will use two different “order” tasks with healthy subjects: judging the relative position of a number in the sequence (does 3 come before or after 5 in the counting sequence?) and an order verification task (2-3, are they presented in the conventional, ascending, order?). These tasks will be compared with equivalent quantity-judgment tasks (is 3 smaller or larger than 5? 2-3, which is larger?). Only in the neuropsychological study will we briefly investigate our patient’s knowledge of ordinal number words.

4. Aims of the present thesis

We have overviewed, in the present chapter, the available empirical data about potential similarities and differences in processing numbers according to either their quantity meaning, or their relative position (or rank) in the sequence. Numbers are particularly challenging, since the quantity a number refers to is intimately linked to its relative position within the sequence of number words: a number can be used, in fact, to identify the *cardinality* of a set because it occupies a certain and fixed *position* within the number sequence (Wiese, 2003). Thus, the question of the potential dissociation of the mechanisms underlying the processing of numerical quantity and what we termed ‘numerical order’ is tricky.

We started our review by presenting the position of Wiese (2003) who enabled us to tell apart cardinal and ordinal number assignments. Numbers were found in these contexts to refer to different empirical properties of objects, to rely on different prelinguistic skills, and to follow distinct developmental sequences, until they are ultimately integrated into a full-blown number concept. The reviewed evidence argued for an earlier mastery of cardinal assignments that constituted then for the child a ‘gateway to number’. Such a primacy of numerical quantity might partially legitimate why a vast majority of studies on numerical cognition were devoted to the study of numerosity, and numerical quantity at large. Nonetheless, we also placed particular emphasis in our review on the crucial importance of sequential order in the development of a mature number concept. Yet, number sequence knowledge and processing of numerical rank have been largely neglected thus far, and deserve better consideration. It is to a better understanding of numerical order processing, and to its peculiar relationship with numerical quantity that the present thesis is devoted.

The available empirical data is, at present, inconclusive as to whether numerical quantity and numerical order (i.e. processing the relative position of numbers in the sequence) rely on some common underlying mechanism, or whether they refer to (at least partially) distinct processes. We have shown, in fact, that empirical data might support each hypothesis. On the one hand, the hypothesis of a common process is supported by similar behavioural distance and SNARC effects when processing numbers and

other ordered series; by the activation of similar parietal regions when performing numerical quantity tasks and order judgments on non-numerical material; and by the concomitant impairment or preservation of numbers and other ordered series after brain damage. On the other hand, the evidence for (partially) distinct processes underlying numerical quantity and “order” at large comes from a developmental distinction between early sequence knowledge and later acquisition of the cardinal meaning by children; from a dissociation, after brain damage, between impaired numerical quantity processing and preserved sequence knowledge; and, finally, from the occurrence of specific behavioural effects in relative-order judgment tasks with ordered series, that were only rarely reported in numerical (quantity) tasks. Overall however, none of these data is clear enough to make a strong claim about a potential association or dissociation between quantity and “order” processing mechanisms. We see one major reason for this: none of these studies was specifically devoted to examining the relationship between (numerical) quantity and (numerical) order processing mechanisms. Thus, similarities and differences can only be grabbed from different studies, using different methods, different subjects and materials and that we tentatively put together. The best evidence, then, for a more clear picture about the relationship between numerical quantity and numerical order would be to compare them directly, that is, at best, using the same methods, the same subjects and the same materials, and only changing the instructions to focus subjects’ attention on either the quantity meaning of numbers or on their relative position in the sequence. Furthermore, to attain a better understanding of the mechanisms involved in processing the sequential order of numbers, numerical stimuli and non-numerical ordered series (such as the letters of the alphabet) might be included in similar order judgment tasks so as to highlight potential similarities between their underlying mechanisms. This is the way we devised our approach.

Our experimental investigation will be divided in three main parts:

(1) First, as we mentioned in the foreword of this chapter, investigation about numerical abilities of a brain-damaged patient, CO, drove our attention to the existence of a potential dissociation between numerical quantity and sequence knowledge. CO's performance in quantity and sequence/ordinal number tasks was thus thoroughly examined.

(2) Second, we will present an electrophysiological study that directly addressed the questions of the relationship between numerical quantity and numerical order, and between numerical order and alphabetic order. Event-related potentials were recorded while participants performed a number comparison task with quantity instructions ('Is the target number smaller or larger than 5?'), with order instructions ('Does the target number come before or after 5 in the counting sequence?') and an alphabetic relative order judgment task ('Does the target letter come before or after M in the alphabet?'), on the same material, same display and with the same subjects.

(3) Finally, the hypothesis of potentially dissociated mechanisms when processing numbers according to their quantity meaning or to their relative order in the sequence was further investigated in two behavioural experiments, one of which also included a letter order judgment task. Subjects were again presented with the same pairs of numbers (e.g. 2 3) and asked either to choose the smaller or larger (quantity task), or to tell whether the numbers were presented in the conventional counting order (order task). An order judgment task with letters was also included (B C, are the letters shown in the alphabetical order?).

We will present these three studies in detail in the following sections. We will then try to integrate the data collected into a coherent framework that we will introduce in the General Discussion.

5. References

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