

REPLY TO EDWARD FURMAN, YISUB KYE, AND JIANXI SU ON THEIR DISCUSSION ON THE PAPER TITLED “SIZE-BIASED RISK MEASURES OF COMPOUND SUMS”

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Reply to Edward Furman, Yisub Kye, and Jianxi Su on Their Discussion on the Paper Titled “Size-Biased Risk Measures of Compound Sums”

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I am grateful to Furman, Kye, and Su for providing a very interesting discussion of my paper. The overview of the applications of weighted distributions across disciplines is illuminating. In addition, the extension of the size-biased transform of univariate distributions to higher order and dimension in Definition 1 and the results derived in this more general setting are nice additions to the actuarial literature.

It turns out that together with my colleague Christian-Yann Robert (Ecole Nationale de la Statistique et de l'Administration Economique [ENSAE], Paris), we recently extended some of the results involving conditional tail expectation (CTE) and conditional mean risk sharing (Denuit and Dhaene 2012; Denuit 2019) to random vectors. The tool needed in that respect is precisely transform (2) in the discussion with $h_i = 1$ and $h_j = 0$ for $j \neq i$. See equation (2.6) in Denuit and Robert (2020b), referring to Arratia, Goldstein, and Kochman (2019). Proposition 2.2 in Denuit and Robert (2020b) then allows the actuary to derive a useful representation for the CTE of sums of correlated risks. Denuit and Robert (2020a) applied these results to dependent risks with a correlation structure induced by graphical models.

Among the examples considered in Denuit and Robert (2020b), the inverted Dirichlet and the Liouville distributions both fall in the Multiplicative Background Risk Model (MBRM) setting. It turns out that similar results hold true in the common mixture model, of which MBRM is a particular case. Precisely, proposition 4.2 in Denuit and Robert (2020b) extends proposition 1 in the discussion in the special case $h_+ = 1$. As an application, Denuit and Robert (2020b) consider random vectors of compound sums with claim frequencies obeying a Poisson common mixture model. General mixtures are also studied there, extending the results to imperfectly correlated claim frequencies.

The special case of (2) with $h_i = 1$ and $h_j = 0$ for $j \neq i$ naturally appears in the extensions of the results contained in the present paper to correlated risks. The allowance for general values h_i is mathematically elegant, but it is not clear whether it can lead to deeper results for CTE and conditional mean risk sharing.

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