Multibody modelling of a travelling wave linear piezomotor

Jean-François Collard[1], Christophe Vloebergh[1], Bruno Dehez[1], Paul Fisette[1]
[1]Université catholique de Louvain, Center for Research in Mechatronics
Place du Levant 2 – 1348 Louvain-la-Neuve
emails: jf.collard@uclouvain.be, christophe.vloebergh@uclouvain.be,
bruno.dehez@uclouvain.be, paul.fisette@uclouvain.be

Abstract—**To predict the performance of a travelling wave linear piezoelectric actuator, the deformations of the beam through which the wave propagates is modelled via a rigid-multibody approach: the beam is considered as a succession of rigid segments linked by revolute joints with a given stiffness. Contrary to a finite-element modelling of the beam, this finite-segment model enables faster simulations and moreover an easier implementation of the contact model between the beam and a slider moving on it. In this work, it has been proposed to interpolate in a smooth manner between the positions and velocities of the rigid segments in order to smooth the contact model and therefore avoid numerical discontinuity problems. The simulation results are very promising and an accurate identification of the model parameters with the experimental results of a prototype will allow us to optimize the performance of the actuator.**

Keywords— **piezoelectric actuator, linear piezomotor, finite-segment modelling, multiphysic modelling.**

I. Introduction

TRAVELLING wave linear piezoelectric actuators use the limited deformations of piezoelectric ceramics to produce linear displacements of large amplitude. The working principle of these actuators is the following: when a travelling wave propagates through a beam, each material point on the beam surface describes an elliptic motion. Therefore, when a rigid slider is placed in contact with the beam, these material points successively enter into contact with the slider and transfer to it, by friction, a motion in a direction opposite to that of the travelling wave.

Obviously, the effective generation of a travelling wave is the key point for the efficiency of the driving system. There exist two main ways of generating such a travelling wave in a finite-length elastic beam.

The first one consists in locating two piezoelectric actuators at both ends of the beam. One is used to generate a wave while the other is used to absorb it in order to prevent the appearance of standing waves due to the reflection. However, in practice, it is difficult to find the right impedance of the absorbing actuator because it varies with the load, the temperature and the frequency (see [5]).

Another way of generating a travelling wave consists in using two piezoelectric actuators located at a specific distance from the tips of the beam. These distances are chosen in order to preferentially excite two similar standing waves space-shifted and time-shifted of a quarter period, their superposition producing a travelling wave [3]. The main advantage of this method is its easiness. For instance, the motion direction can be reversed by simply changing the phase shift from 90° to -90°. Nevertheless, it does not work at a resonant frequency of the beam, the amplitude of oscillations being therefore severely limited. In this work, this second method is implemented.

One of the main issues of this linear actuator system is how to model the friction contact between the flexible beam and the slider. The behaviour of the flexible beam can be simulated using analytical or finite-element models but these approaches cannot easily deal with the contact model when simulating the displacement of the slider on the beam [1]. This is why a model for a travelling wave linear piezomotor has been developed using a finite-segment multibody approach (see [2]) combined with an interpolation smoothing technique to solve the geometrical contact problem. The kinematic and dynamical equations of this model have been automatically generated using the *Robotran*© software, which provides the equations in a symbolic form using recursive algorithms [4]. The generated code is thus compact and efficient for simulations of large systems such as the finite-segment formulation used here.

In the following, the multibody model of the flexible beam and its interaction with the slider are first described. Then, the simulation results of the slider displacement on the beam are presented. Finally, some prospects will given in conclusion.

II. Multibody Model

The finite-segment model itself is described first before giving the details of the contact model between the beam and the slider.

A. Finite-Segment Model of the Flexible Beam

As proposed in [2], the basic idea of this multibody model is to consider the beam as a succession of small rigid segments. To model the bending motion of the beam, these segments are linked by revolute joints with stiffness parameter k_{ij} between two segments i and j (see Fig. (1)). It is also possible to consider structural damping effects

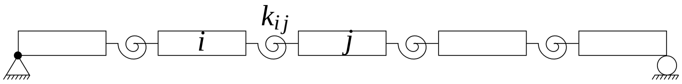

Fig. 1. Finite-segment model

via lumped dashpots. The stiffness parameter k_{ij} is given by:

$$k_{ij} = \frac{2E_iE_jI_iI_j}{E_iI_jl_j + E_jI_il_i}, \tag{1}$$

where E_i and E_j are the Young's moduli, I_i and I_j are the geometrical moments of inertia of the cross-sections, l_i and l_j are the lengths of the respective segments. In this case of identical segments, Eq. (1) reduces to:

$$k_{ij} = \frac{E\,I}{l}, \tag{2}$$

where $E = E_i = E_j$, $I = I_i = I_j$ and $l = l_i = l_j$.

The formalism chosen to model the multibody system uses relative generalized coordinates q to describe the relative joint variables between each body. To simulate the multibody system, the equations of motion are integrated after the reduction of the differential-algebraic equations using the *Coordinate Partitioning method* [6]. This partitioning between independent and dependent coordinates must be carefully performed since it may greatly affect the behaviour of the integrator in the case of closed-loop systems with long kinematic chains. The finite-segment approach provides excellent results for such a system with small deformations: the results of modal analysis are very close to the analytical solutions even for higher normal modes, providing that the number of segments is sufficient. For example, it has been shown in [1] that 49 segments are needed to obtained the right frequency of the 22^{nd} mode (4467 Hz) with a good accuracy (relative error less than 1%).

B. Friction Contact Model

The main advantage of the multibody formulation of this system is to be able to consider pretty easily the friction contact between the beam and a slider moving over it. Computing the contact force at a given instant requires:

- finding the contact point or contact surface;
- determining the constitutive law applying the contact force to each body.

In this preliminary model, it is assumed that the slider is a point with mass m_s and the chosen constitutive law of the tangential dry friction force F has the following form:

$$F = -\mu\ m_s\ g\ \tanh\left(K\left(\dot{q}_t^{slider} - \dot{q}_t^{beam}\right)\right), \tag{3}$$

where μ is the coefficient of kinetic friction, g is gravity, $\dot{q}_t^{slider}$ and $\dot{q}_t^{beam}$ are, respectively, the velocity of the slider and the velocity of the beam, along the tangential direction with respect to the beam, and K is the slope of the hyperbolic tangent at origin. Coefficients μ and K will be identified experimentally.

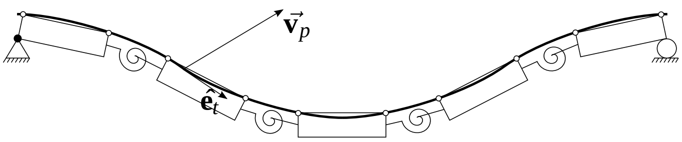

Fig. 2. Spline interpolation of the contact surface of the beam

Since the beam is modeled as a succession of rigid segments, the contact line is thus piecewise linear with a discontinuous tangential velocity and may lead to numerical difficulties, like in the case of cam-follower contact modelling [7]. Therefore, it is proposed here to make the shape and the slope of the beam continuous when determining the contact point and the tangential direction. A possibility is to use a spline interpolation between the upper-corner point positions of each segment as shown in Fig. (2). Assuming that the slider always remains on the beam, the contact point is therefore taken at the longitudinal coordinate of the slider on the correct spline section. The tangential velocity of the beam $\dot{q}_t^{beam} = \vec{\mathbf{v}}_p \cdot \hat{\mathbf{e}}_t$ (see Fig. (2) is also computed thanks to a spline interpolation on the joint point velocities projected along the tangential direction.

III. Simulation

In order to produce a travelling wave, the piezoelectric actuators must vibrate at a given frequency f and must be located at a given distance d from both ends of the beam as explained in [1]. In this example, since the beam is made of aluminum and is 500 mm long, 20 mm large and 1 mm thick, the corresponding values of f and d are: $f = 4276$ Hz and $d = 11.6$ mm. For this frequency, 49 segments are sufficient to obtain a reasonable accuracy on the closest normal frequencies as explained in [1].

As a first result (see Fig. (III) and Fig. (III)), the displacement of the slider was simulated from 0 to 1 millisecond, considering only the tangential contact force acting on the slider by a segment of the beam. The slider is assumed to stick to the spline-interpolated beam. Figure (III) shows that the slider moves from the center of the beam (longitudinal position = 0 m) towards the left (negative position). Its mean longitudinal velocity is 1.5 cm/s. In Fig. (III), it is observed that the tangential contact force varies abruptly between its saturation values of $\pm 2 \cdot 10^{-3}$ N, according to Eq. (3). However its mean value is negative as is the mean tangential velocity of a beam surface contact point.

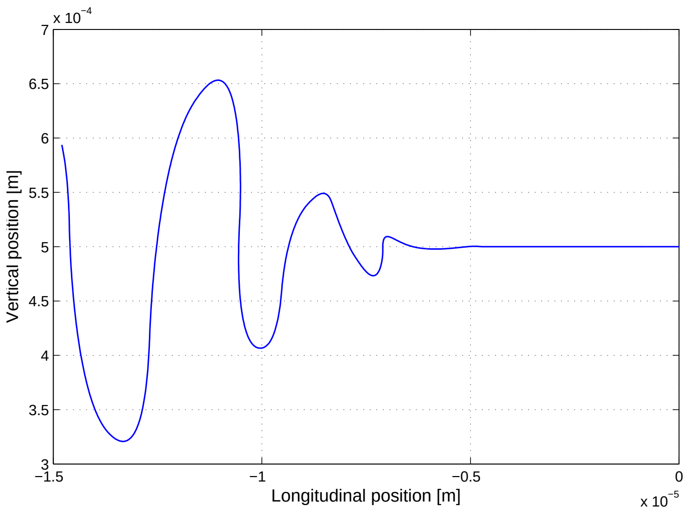

Fig. 3. Simulation results of slider trajectory from 0 to 1 ms when the beam is excited at 4276 Hz

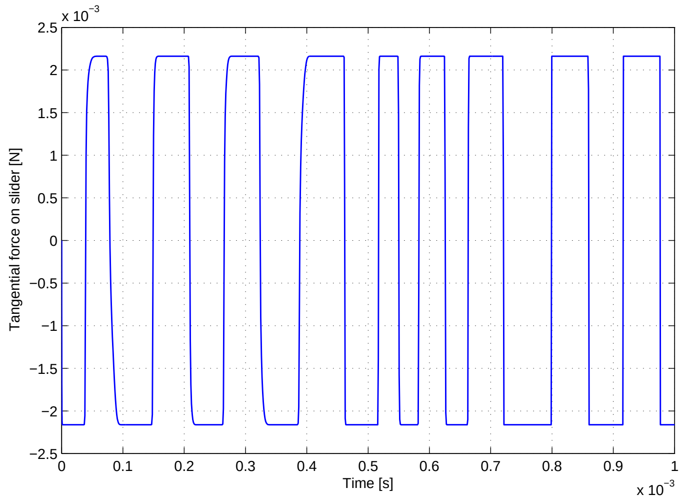

Fig. 4. Simulation results of the tangential contact force on slider when the beam is excited at 4276 Hz

IV. Conclusion and Prospects

This paper presents a rigid-multibody modelling approach to simulate the behaviour of a linear piezoelectric actuator made of a flexible beam. With respect to a finite-element modelling approach, this method is less time-consuming and enables a pretty easy implementation of a contact model between the beam and a slider body moving on it. This contact model has been implemented with a smoothing interpolation of the contact line to satisfy the real physical system geometry and to avoid numerical difficulties. The simulation of this model has shown very promising results to predict the displacement of the slider. A particular attention has been paid to time-integration accuracy and efficiency.

To refine the model, multiple contact points between the beam and the slider will be considered, as should be the case. The design of a prototype will also be made to identify the model parameters with the experimental results. Another improvement will be the implementation of an electromechanical model of the piezoelectric actuators themselves, called Mason's model [8]. This will lead to a full integrated multiphysics model of the system, including the multibody model of the beam, the electromechanical model of the piezoelectric actuators and a controller model. Thanks to simulations of this multiphysic model after identification with the experimental prototype, the slider motion could be predicted and the actuator performance could be optimized.

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