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Efficiency and redistribution in economies with hidden action

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Καί τώρα τί ϑά γένουμε χωρίς βαρβάρους.
Οἱ ἄνϑρωποι αὐτοί ἦσαν μιά κάποια λύσις.
(C. Kavafis)

Contents

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Introduction

In this thesis, we study the efficiency of competitive equilibrium with hidden action. We consider a pure exchange economy populated by a large number of risk averse consumers facing an idiosyncratic uncertainty on their endowments. They can choose at some cost an action that modifies the probabilities of the different idiosyncratic states. They can also buy insurance so as to smooth their consumption profile.

Given these characteristics, we investigate how the efficiency of a competitive outcome depends on the market structure. In particular, we consider both competitive anonymous markets, where consumers make their choices taking as given some relevant economic parameters, and competitive strategic markets, where consumers interact with financial intermediaries who explicitly take into account the choices of other economic agents.

In the first chapter, we start reviewing some known results on the efficiency of competitive equilibrium when there exists a complete set of markets for commodities with delivery contingent on the idiosyncratic state. Then we propose a detailed analysis of the efficiency properties of competitive equilibrium with strategic insurance intermediaries. In this case, we argue that the main problem as far as efficiency is concerned is whether consumers can be prevented from trading with more than one firm or not. In the former case, efficiency is easily obtained. In the latter case, it may not be possible to provide the proper incentives to consumers.

In the second chapter of the thesis, we consider a more general type of economy. In particular, we assume that there are multiple consumption goods and that the level of action affects the marginal benefit consumers get from consumption. For this economy, we consider two canonical market structures, one with a complete set of contingent-commodity markets, and one with complete financial markets, together with spot markets for consumption goods.

Not surprisingly, even under the more general assumptions of this chapter, the competitive equilibrium with complete contingent-commodity markets is efficient. We therefore ask whether the same is true for the equilibrium with complete financial markets.

When there is no hidden action, it is well known that the two market structures are equally efficient. It is therefore interesting to investigate if this equivalence holds under the case of asymmetric information we consider.

The main result we provide in this chapter is that under more general hypothesis than those usually considered in the literature, the equilibrium with financial markets is not efficient. This result depends essentially on the effects the possibility of trading on the spot market once action has been chosen and uncertainty is resolved has on consumers' incentives.

In the third chapter, we consider a rather different market structure. In particular, we assume that consumers can insure themselves by voluntarily committing to deliver part of their uncertain endowment to a common pool in exchange for a sure return from the pool itself.

In this type of market structure, we assume no strategic behavior. Consumer take as given the return of the pool, and make their choices according to their expectations. The return of the pool is then endogenously determined.

We show that equilibria with pool of promises exists, and they may imply either a low level of action and return of the pool, or a high level of both of them.

We finally show that if there exist heterogeneous groups of consumers, it is possible to form aggregate pool of promises, and that in this case welfare can be higher than the case of segregated pools.

Chapter 1

Efficiency of competitive equilibrium with hidden action: an overview

In this chapter, we review some known results concerning the efficiency of competitive equilibrium in pure exchange economies with hidden action and idiosyncratic uncertainty. After defining an appropriate efficiency concept, we consider different market structures and we propose a suitable equilibrium concept for each of them. We the ask whether the resulting equilibrium is efficient according to the above criterion.

Some simplifying assumptions are made in order to make the main arguments transparent. In particular, it is assumed that there is a single consumption good, that preferences are separable in the cost of action, and that only two action levels are possible.

In the first part of the chapter, we assume there exists a complete set of markets for commodities with contingent delivery. We propose an equilibrium concept based on the observation that prices should properly incorporate the optimal choice of action by consumers. Thanks to this property, we easily verify that the resulting equilibrium is efficient.

In the second part of the chapter, we consider a more complex market structure. In particular we assume that consumers trade with perfectly competitive insurance intermediaries selling insurance contracts. As a consequence, the equilibrium definition must incorporate the strategic interaction among intermediaries. This will in turn depend on whether it is possible for consumer to trade with more that one firm. We therefore analyze both an equilibrium with exclusivity, in which consumers can be restricted to buy only a single contract, and an equilibrium with non-exclusivity, in which such a possibility is excluded.

Among non-exclusive equilibria, we concentrate on the class of linear price equilibria, for it was long believed that liner prices and incentives were incompatible, in the sense that, with linear prices, only equilibria with too much insurance and too low a level of action were possible. Indeed, we show that this need not be the case by revising the characterization of high-action linear price equilibria.

Finally, we argue that under the assumption of non-exclusivity the efficiency criterion should be properly amended to take into account the possibility for consumers to make unverifiable trades. Hence, we discuss some type of non-exclusive equilibria that are efficient according to the modified criterion.

1.1 The economy

Consider a pure exchange economy with a single consumption good. The economy is populated by a large number of consumers all ex ante equal, and lasts two periods $t = 0, 1$.

There is no consumption in the first period, when consumers face an idiosyncratic uncertainty which realizes in the second period. At $t = 1$ each consumer may be in one out two states $s \in \mathcal{S} = \{1, 2\}$.

Uncertainty affects only the level of the single consumption good consumers are endowed with at the beginning of the second period. Individual endowments are denoted by the vector $w = (w_1, w_2) \in \mathbb{R}^2_+$. It is assumed that $w_1 > w_2 \geqslant 0$. With terminology consistent with the insurance set-up, state $s = 1$ will be referred to as the good state.

Consumers are risk averse, hence they wish to acquire insurance coverage against the uncertainty on their endowments. They can do so in formal insurance markets, which will be described later on.

Consumers can also choose an action $a \in \mathcal{A} = \{L, H\}$ affecting, on the one hand, the probabilities of the idiosyncratic states, and on the other hand the utility they get from consumption.

In what follows, π_a denotes the probability of state 1 when action a is chosen. It is assumed that $1 > \pi_H > \pi_L > 0$. The (dis-)utility of the action is equal to c_a. It is assumed that $c_H > c_L = 0$.

Preferences are represented by an expected utility function $v(x, a) : \mathbb{R}^2_+ \times \mathcal{A} \to \mathbb{R}$ given by:

$$v(x, a) := \pi_a u(x_1) + (1 - \pi_a)u(x_2) - c_a \,.$$

Because of the binary action set, preferences represented by v are inherently non-convex. On the other hand, we assume that the (Bernoulli) utility index u is a strictly increasing and strictly concave function.

It is assumed that the action chosen by a consumer remains unknown to anyone else in the economy, so that information is asymmetric. On the other hand, it is assumed that endowments are verifiable at $t = 1$, so that it is possible to determine in which idiosyncratic state consumers are.

1.2 Constrained efficient allocation

Suppose at $t = 0$ there exists a benevolent planner who wishes to maximize consumers' utility by assigning a consumption bundle x and prescribing an action a.

The planner is subject to a physical feasibility constraint, for he cannot redistribute more resources than those available. In the economy considered here, there is no aggregate uncertainty once action is chosen. This follows from the assumption of a large number of consumers. Aggregate resources at $t = 1$ when action a is chosen are equal to $\bar{w}_a = \pi_a w_1 + (1 - \pi_a)w_2$

As standard in models of asymmetric information, the planner is assumed to be subject on the same limits on information as all economic agents.

Hence, in addiction to the feasibility constraint, he has to take into account the fact that the only action consumers are going to choose is the optimal one given the prescribed consumption bundle. This latter constraint is known as the incentive compatibility constraint.

Let $\vec{\pi}$ denote the vector $(\pi, (1 - \pi)) \in \mathbb{R}^2_{++}$. Given the above observations, the planner's problem is given as follows:

$$
\begin{aligned}
\max_{x,a} \quad & v(x,a) \\
\text{s.t.} \quad & \vec{\pi}_a \cdot (x - w) \leqslant 0\,, && \text{(1.1a)} \\
& v(x,a') - v(x,a) \leqslant 0\,, \quad a, a' \in \mathcal{A}\,. && \text{(1.1b)}
\end{aligned}
$$

(1.1a) is the feasibility constraint, while (1.1b) is the incentive compatibility constraint. Notice that it is evaluated at the consumption bundle prescribed by the planner. It is therefore assumed that consumers do not engage in further trading once assigned the bundle x.

When (1.1b) holds with equality, it defines the locus of consumption bundles that make consumers indifferent between choosing high action and low action. In the literature it is usually referred to as the switching locus.[1] A portion of a typical switching locus is represented in the following figure:

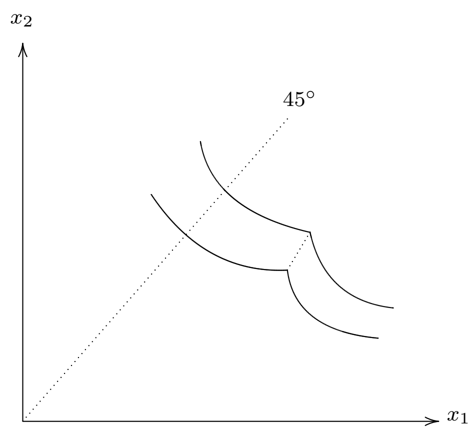

Figure 1

For consumption bundles below the switching locus, the incentive compatible level of action is $a = H$, while for consumption bundles above the switching locus, the incentive compatible level of action is $a = L$. For future reference, notice that independently of the actual position of

[1]The analysis of the properties of this locus is not pursued here. For our purpose it is enough to recall that it is an increasing curve and it lies below the 45° line whenever the cost of high action is higher than that of low action.

the switching locus, for a consumption bundle on or above the 45° line, the incentive compatible level of action is always $a = L$.

Provided that the cost of high action is not too high with respect to the cost of low action, and that the probability of the good state under high action is not too close to that under low action, the solution of the above maximization problem implies $a^* = H$ and and consumption bundle x^* which is at the intersection of the switching locus and the feasibility constraint, as illustrated in the following figure:

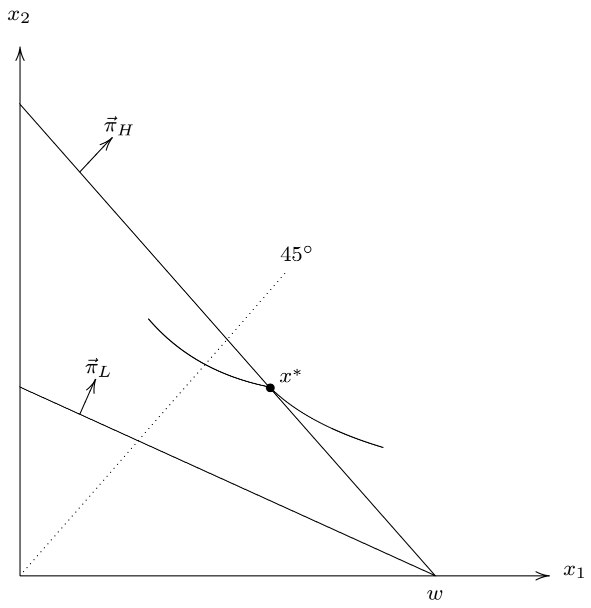

Figure 2

We now begin our investigation of the possible equilibrium concepts suitable for the economy just described. This will require describing the possibility of trades consumers have access to. We will also investigate the efficiency properties of the proposed equilibrium concepts. In particular, we will ask when an equilibrium allocation is efficient according to the criterion introduced above.

1.3 A model with contingent markets

In this section, we suppose there exists a complete set of markets for commodities with delivery contingent on the idiosyncratic state. Consumers trade in these markets, which are open at $t = 0$, by selling their state-contingent endowments and buying a bundle of consumption goods to be delivered only if the corresponding state is realized. This market structure then implies that consumers face a single budget constraint.

Once the bundle has been purchased, markets close. Consumers then choose their preferred action a. The uncertainty is resolved, and at $t = 1$ goods are delivered according to purchases.

In order to define an equilibrium concept for this market structure, we have to introduce an appropriate price system. Notice that there are $S = 2$ prices to be quoted.

Suppose for a moment that there is no hidden action, and maintain the hypothesis of absence of aggregate uncertainty. In this case, there exist restrictions on the structure of the price system such that a well defined equilibrium concept displaying satisfactory efficiency properties can be defined.

These restrictions essentially require the price of one unit of good delivered in state s to be equal the actual probability of that state. These prices are known in the literature as (actuarially) fair.

The presence hidden action does not alter the reasoning in an essential way. In this case, π_s depends on a. Hence we shall assume that the price of one unit of good with delivery contingent on state s when action a has been chosen is equal to $\pi_s(a)$.

The problem at this point is to consider the appropriate level of action. The only possible candidate is the incentive compatible one, which we recall satisfies (1.1b). It follows that the price of one unit of good to be delivered in state s cannot be quoted unless it is specified the consumption level of the good to be delivered in the other state.

Moreover, the price relative price for of good in state 1 in terms of good in state 2 differs according to the actual quantity purchased. In particular it is equal to $\pi_H/(1-\pi_H)$ if x is below the switching locus, and it is equal to $\pi_L/(1-\pi_L)$ if x is above the switching locus. For this reason, this price system is sometime referred to as non-linear.

1.3.1 Equilibrium with contingent markets

Given the market structure and the price system just described, the consumers' problem is equal to the planner's problem, where the feasibility constraint has to be interpreted as a budget constraint and the incentive compatibility constraint is required for the prices to be well defined.

Given this observation, we shall introduce an equilibrium concept suitable for this economy. Quite naturally, the equilibrium requires consumers to optimize given prices, and market to clear. These properties are summarized in the following:

Definition 1.3.1. *An equilibrium with markets for contingent commodities is* $(x, a, \vec{\pi}_a)$ *such that:*

(a) $(x, a) \in \arg\max \{v(x, a) \,|\, (x, a)$ satifies (1.1a) and (1.1b)$\}$,

(b) $\vec{\pi}_a \cdot (x - w) = 0$.

Given the extremely simplified economy here considered, the above definition displays redundancies and may appear overstated. Nonetheless, it has the advantage of elucidating the main characteristics of the equilibrium concept, which can be easily extended to more general environments.[2] Equally generalizable is the following proposition, which is apparent in the present context:

Proposition 1. *Every equilibrium with markets for contingent commodities is constrained efficient.*

1.4 A model with insurance intermediaries

In this section, we consider a different market structure. In particular, we assume that consumers can insure themselves by trading with intermediaries who sell insurance contracts. In this case, the equilibrium notion has to be modified to take into account the strategic behavior of insurance firms.

Suppose therefore there exists a countable number of intermediaries $j = 1, 2, ...$, offering insurance contracts to consumers.

An insurance contract is a set of transfers $\tau = (\tau_1, \tau_2) \in \mathbb{R}^2$ which allows consumers to transfers income from one idiosyncratic state to the other. A contract such that $\tau_1 < 0$ will be referred to as a positive-insurance contract.

[2] An illustrative example is given in the next chapter.

From an intermediary viewpoint, a positive-insurance contract τ guarantees revenues equal to $-\pi_a\tau_1 > 0$ and has costs equal to $(1-\pi_a)\tau_2 > 0$, hence it makes non-negative profits if $-\pi_a\tau_1 \geqslant (1-\pi_a)\tau_2$. This condition can be rewritten as follows:

$$\pi_a\tau_1 + (1-\pi_a)\tau_2 \leqslant 0\,, \tag{1.2}$$

Recall that $\vec{\pi}_a = (\pi_a, 1-\pi_a)$. The above condition can be rewritten as $\vec{\pi}_a \cdot \tau \leqslant 0$ and it implies that $\vec{\pi}_a$ and τ cannot make an acute angle if τ implies non-negative profits.

An important characteristic for the definition, and the properties, of an equilibrium with insurance firms is the number of intermediaries consumers are allowed to trade with. Two polar cases are usually considered in the literature. On the one hand, it is assumed that consumers can trade with at most one intermediary. This case is referred to as the exclusive case. On the other hand, it is assumed that consumers can trade with as many insurance companies as they wish. This case will be referred to as the non-exclusive case.

1.4.1 Exclusive equilibrium

In an exclusive equilibrium, it is implicitly assumed that an exclusivity covenant may be costlessly enforced. Hence, given a sequence of offered contracts $\tau_\circ = (..., \tau^j, ...)$ for $j = 1, 2...$, consumers choose a single contract τ^j, and then stop trading.

After the contract is signed, consumers choose an action a. The uncertainty is resolved, and at $t = 1$, consumers receive the payoff of the contract according to the state in which they are.

Consumers choose the intermediary j to trade with and the action so as to maximize utility. Hence, consumers' problem is described by the following:

$$\begin{aligned} \max_{x,a,j} \quad & \pi_a u(x_1) + (1-\pi_a)u(x_2) - c_a\,, \\ \text{s.t.} \quad & x = w + \tau^j\,, \quad a \in \{L, H\}\,. \end{aligned}$$

Let $\hat{g}(\tau_\circ)$ denote the solution set for this problem. Moreover let:

$$\hat{\gamma} : \tau_\circ \to \hat{\gamma}(\tau_\circ) = (x(\tau_\circ), a(\tau_\circ), j(\tau_\circ))$$

be a selection of $\hat{g}(\tau_\circ)$. Given $\hat{\gamma}$, profits of an insurance intermediary are equal to:

$$P^j_{\hat{\gamma}}(\tau_\circ) = \begin{cases} -\vec{\pi}_{a(\tau_\circ)} \cdot \tau^j & \text{if} \quad j = j(\tau_\circ) \\ \\ 0 & \text{if} \quad j \neq j(\tau_\circ) \end{cases} \tag{1.3}$$

At an equilibrium with exclusivity, all agents optimize given the choices of each other. These properties are summarized in the following:

Definition 1.4.1. *An equilibrium with exclusivity is* $(x, a, j, \tau_\circ)$ *such that:*

(a) $(x, a, j) = \hat{\gamma}(\tau_o)$,

(b) $P^j_{\hat{\gamma}}(\tau_\circ) \geq P^j_{\hat{\gamma}}(\tau^j, \tau_\circ^{-j})$ *for all* j ,

where $(\tau^j, \tau_\circ^{-j})$ is the new set of offered contracts when intermediary j substitutes $\tau^j_\circ$ with τ^j.

From (1.3) we notice that any intermediary whose contract is not bought at equilibrium makes zero profits. Therefore he has strong incentives to undercut the intermediary active in equilibrium, if the latter makes positive profits.

In an equilibrium with exclusivity competition among intermediaries looks then fierce enough equilibrium profits to zero. This intuition is indeed true, and its implication are even stronger, as suggested by the following:

Proposition 2. *The following statements are equivalent:*

(A) (x, a, j) *is an equilibrium with exclusivity;*

(B) (x, a) *is constrained efficient.*

Proof. (A) $\Rightarrow$ (B). Take an equilibrium with exclusivity and suppose (B) does not hold. Then there exists $(\bar{x}, \bar{a})$ satisfying (1.1b) such that $v(\bar{x}, \bar{a}) > v(x, a)$ and $\vec{\pi}_{\bar{a}} \cdot (\bar{x} - w) < 0$. Hence $P^i_{\hat{\gamma}}((\bar{x} - w), \tau_\circ^{-i}) > P^i_{\hat{\gamma}}(\tau_\circ)$ for any $i \neq j$, and (x, a, j) is not an equilibrium with exclusivity.

(B) $\Rightarrow$ (A) Let $\tau_\circ = ((x - w), \tau_\circ^{-1})$, where $\tau_\circ^{-1} = (0, 0)$ for $j = 2, 3, \ldots$. In this case there exist a selection $\gamma(\tau_\circ)$ such that $(x(\tau_\circ), a(\tau_\circ), j(\tau_\circ)) = (x, a, 1)$. Since it is straightforward to verify that there are no profitable deviations in this case, this is indeed an equilibrium with exclusivity. ■

1.4.2 Non-exclusive equilibrium

When the exclusivity covenant considered in the previous section cannot be enforced, the equilibrium concept must be properly amended. In particular, it is necessary to consider the possibility that consumers trade with more than one insurance firm.[3]

This possibility affects the profitability of each insurance contract, for the level of action consumers are going to make at equilibrium depends on the aggregate insurance purchases. Each insurance firm has to take this effect into account when offering contracts to consumers.

Given a sequence of offered contracts $\tau_\circ = (\ldots, \tau^j, \ldots)$ for $j = 1, 2 \ldots$, consumers choose a set of J contracts. After contracts are signed, consumers choose an action a. The uncertainty is resolved, and at $t = 1$, consumers receive the payoff of the contracts according to the state in which they are.

Consumers choose the set of intermediary J to trade with and the action so as to maximize utility. Hence, consumers' problem is described by the following:

$$\begin{aligned} \max_{x,a,J} \quad & \pi_a u(x_1) + (1 - \pi_a) u(x_2) - c_a \,, \\ \text{s.t.} \quad & x = w + \sum_{j \in J} \tau^j \,, \quad a \in \{L, H\} \,. \end{aligned}$$

In what follows, we let $\mathcal{C}$ denote the constraint set in the above problem. Let the solution set for this problem be $g(\tau_\circ)$. Moreover let:

$$\gamma : \tau_\circ \to \gamma(\tau_\circ) = (x(\tau_\circ), a(\tau_\circ), J(\tau_\circ))$$

[3]The analysis of the non-exclusive equilibrium presented here draws heavily from Hellwig [16]. In particular, proposition 3 and 4 correspond to his lemma 3.1 and 3.2, proposition 5 to his lemma A.1 and propositions $6 - 9$ to his propositions $3.3 - 3.6$.

be a selection of $g(\tau_\circ)$. Given γ, profits of an insurance intermediary are equal to:

$$P^j_\gamma(\tau_\circ) = \begin{cases} -\vec{\pi}_{a(\tau_\circ)} \cdot \tau^j & \text{if} \quad j \in J(\tau_\circ) \\ \\ 0 & \text{if} \quad j \notin J(\tau_\circ) \end{cases} \tag{1.4}$$

At a non-exclusive equilibrium, all agents optimize given the choices of each other. These properties are summarized in the following:

Definition 1.4.2. *An equilibrium without exclusivity is* $(x, a, J, \tau_\circ)$ *such that:*

(*a*) $(x, a, J) = \gamma(\tau_o)$,

(*b*) $P^j_\gamma(\tau_\circ) \geq P^j_\gamma(\tau^j, \tau_\circ^{-j})$ *for all* j .

Before proceeding with the analysis of the equilibria without exclusivity, we need to introduce a modified consumers' problem, whose solution set will be useful in what follows.

1.4.3 Compensated consumers' problem

Let $\hat{w}$ be such that $\hat{w}_1 > \hat{w}_2 \geqslant 0$. Recall that $\vec{\pi} = (\pi, (1-\pi)) \in \mathbb{R}^2_{++}$. We are interested in studying the solution set of the following maximization problem, which will be referred to as the compensated consumers' problem:

$$\max_{x_1, x_2, a} \quad \pi_a u(x_1) + (1-\pi_a) u(x_2) - c_a \, ,$$

$$\text{s.t.} \quad \vec{\pi} \cdot (x - \hat{w}) \leqslant 0 \, , \tag{1.5a}$$

$$x_1 - \hat{w}_1 \leqslant 0 \, , \tag{1.5b}$$

$$x_1 \geqslant 0 \, , \tag{1.5c}$$

$$a \in \{L, H\} \, . \tag{1.5d}$$

(1.5a) is a single budget constraint stating that consumers can purchase goods in the two states by selling their compensated endowments $\hat{w}$.

Trades occur at price $\vec{\pi}$. (1.5b) implies that consumers cannot transfers consumption from the bad to the good state. This constraint is referred to as non-negative insurance constraint.

In what follows, we let $\varphi(\vec{\pi}, w)$ be the solution set of the above maximization problem.

From the standard analysis of choice under uncertainty, it is well know that risk-averse consumers with state independent expected utility buy partial, over or full insurance when the price of insurance is respectively less than fair, more than fair or just fair. This behavior is not substantially affected by the presence of hidden action, as shown by the following:

Proposition 3. *For $\hat{w}_1 > \hat{w}_2 \geqslant 0$, $(x, a) \in \varphi(\vec{\pi}, \hat{w})$ implies $x_1 \lesseqgtr x_2$ as $\pi_a \lesseqgtr \pi$. Therefore $\pi_a \leqslant \pi$ implies $a = L$, hence $\vec{\pi}_a = \vec{\pi}_L$.*

Proof. See the Appendix. ■

Yet, when the price of insurance is very high and the cost of high action is very low, consumers may prefer underinsurance and high action, instead of full, or overinsurance, and low action, even if they can buy as much insurance as they wish at the price $\vec{\pi}_L$. This intuition is confirmed by the following:

Proposition 4. *For any given $(\vec{\pi}_H, \vec{\pi}_L, w)$ such that $1 > \pi_H > \pi_L > 0$ and $w_1 > w_2 \geqslant 0$ there exist $c > 0$ such that, for $c > c_H > 0$, $(x, a) \in \varphi(\vec{\pi}_L, w)$ implies $x_1 > x_2$ and $a = H$. In this case there exist a price of insurance $\vec{\pi}_*$, an overinsurance consumption bundle x_L and a partial insurance consumption bundle x_H such that $\{(x_H, H), (x_L, L)\} = \varphi(\vec{\pi}_*, w)$.*

Proof. See the Appendix. ■

The following figure illustrates the solution of the compensated consumers' problem mentioned in the above proposition:

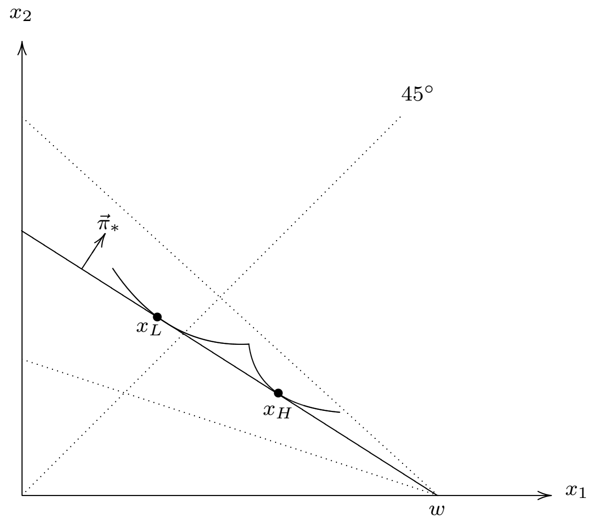

Figure 3

The following proposition describes some general properties of a non-exclusive equilibrium that will be relevant in the rest of the chapter:

Proposition 5. *Any non-exclusive equilibrium* $(x, a, J, \tau_\circ)$ *satisfies:*

(*a*) $P^j_\gamma(\tau_\circ) \geqslant 0\,,$

(*b*) $\vec{\pi}_L \cdot (x - w) \geqslant 0\,,$

(*c*) $(x, a) \in \varphi(\vec{\pi}_L, x)\,,$

(*d*) *for every* $\varepsilon > 0$ *there exist* j *such that* $P^j_\gamma(\tau_\circ) < \varepsilon$ *.*

Proof. (a): $P^j_\gamma(\tau_\circ) < 0$ implies $P^j_\gamma((0,0), \tau_\circ^{-j}) = 0$, which is not possible at equilibrium; (b): for $\varepsilon > 0$, let $\hat{\varepsilon} = \varepsilon/\pi_L$, $y = (x_1 + \hat{\varepsilon}, x_2)$ and $z = (x_1 + 2\hat{\varepsilon}, x_2)$. Contrary to the statement, let:

$$\vec{\pi}_L \cdot (x - w) < \vec{\pi}_L \cdot (z - w) \leqslant 0\,.$$

Using (d), the following chain of inequality gives the required contradiction:

$$P^j_\gamma(\tau_\circ) < \varepsilon \leqslant -\left(\vec{\pi}_L \cdot (y - w)\right) \leqslant P^j_\gamma((y - w), \tau_\circ^{-j})\,.$$

(c): a similar argument as the previous one gives the result. (d): (a) implies that the series $\sum_j P^j_\gamma(\tau_\circ)$ is non-negative and converges monotonically to $\vec{\pi}_{a(\tau_\circ)} \cdot (x - w)$. ■

We are ready to introduce and analyze an important class of non-exclusive equilibria called linear price equilibria.

1.4.4 Linear price equilibrium

A linear price equilibrium is a non-exclusive equilibrium with the additional property that the equilibrium consumption allocation belongs to a linear budget set. This property is formally stated in the following:

Definition 1.4.3. *An non-exclusive equilibrium $(x, a, J, \tau_\circ)$ is a linear price equilibrium there if exist $\vec{\pi}$ such that $\vec{\pi} \cdot \tau^j_\circ \leqslant 0$ for all j and $(x, a) \in \varphi(\vec{\pi}, w)$.*

Linear price equilibria may or may not exist. At a linear price equilibrium, consumers may choose low or high action. In the latter case, equilibrium profits may be positive. We investigate all these cases in the following section.

1.4.5 Low-action linear price equilibrium

A first type of linear price equilibrium is characterized by the following:[4]

Proposition 6. *If a linear price equilibrium is such that $(x, a) \in \varphi(\vec{\pi}, w)$ implies $x \neq w$ and $P^j_\gamma(\tau_\circ) = 0$ for all j, then $\vec{\pi} = \vec{\pi}_L$ and $(x, a) = (w_e, L)$.*

Proof. $P^j_\gamma(\tau_\circ) = 0$ for all j implies $\vec{\pi}_a \cdot \sum_{j \in J} \tau^j = \vec{\pi}_a \cdot (x - w) = 0$, since $(x, a, J) \in \mathcal{C}$. The equilibrium being linear implies $\vec{\pi} \cdot (x - w) = 0$. $x \neq w$ then implies $\vec{\pi}_a = \vec{\pi}$. From proposition 3 it follows that $x_1 = x_2$, hence $a = L$. This finally implies $\vec{\pi}_a = \vec{\pi}_L = \vec{\pi}$ and $x = w_e$. ■

This equilibrium will be referred to as the low-action linear price equilibrium. Its existence is established by the following:

Proposition 7. *The following statements are equivalent:*

(A) *there exists a linear price equilibrium such that $a(\tau^\circ) = L$;*

[4] In what follows, for a generic vector y, we let y_e denote the (full insurance) vector with components equal to $\vec{\pi}_L \cdot y$.

(B) *for all* $\hat{w}$ *such that:*

$$\hat{w}_1 > \hat{w}_2 \quad \text{and} \quad \vec{\pi}_H \cdot (\hat{w} - w) < 0 \leqslant \vec{\pi}_L \cdot (\hat{w} - w)\,, \tag{1.6}$$

it is true that $(\hat{w}_e, L) \in \varphi(\vec{\pi}_L, \hat{w})$.

Proof. (A) $\Rightarrow$ (B). (a) and (b) in proposition 5 imply respectively $\vec{\pi}_L \cdot \sum_{j\in J} \tau^j = \vec{\pi}_L \cdot (x-w) \leqslant 0$ and $\vec{\pi}_L \cdot (x-w) \geqslant 0$, hence $\vec{\pi}_L \cdot (x-w) = 0$. ($c$) in proposition 5 imply $(x, L) \in \varphi(\pi_L, x)$, hence $x_1 \leqslant x_2$.[5] Indeed, $x_1 < x_2$ is not possible, for otherwise there is a profitable deviation $\tau = y - w$ for some j, as the following figure shows:

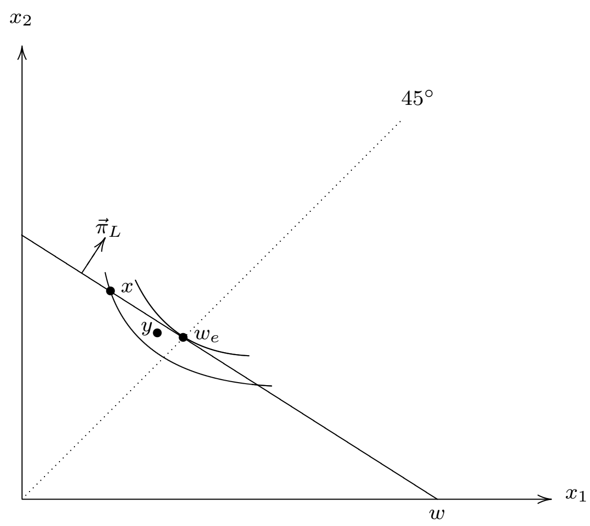

Figure 4

Therefore $x_1 = x_2 = w_e$, hence $x \neq w$. The equilibrium being linear implies that $\vec{\pi} \cdot (x - w) = 0$, hence $\vec{\pi} = \vec{\pi}_L$, and equilibrium profits are zero. Suppose now that (B) does not holds. Then there exist $\bar{w}$ satisfying (1.6) and $\hat{w}$ such that $(\hat{w}, H) \in \varphi(\vec{\pi}_L, \bar{w})$, hence $(\hat{w}, H) \in \varphi(\vec{\pi}_L, \hat{w})$. This implies $v(\hat{w}, H) \geqslant v(\hat{w}_e, L) > v(w_e, L)$. If offered, the contract $\hat{\tau} = \hat{w} - w$ would be accepted, since it gives higher utility, and it would then yield positive profits, as $\pi_H \cdot (\hat{w} - w) < 0$. The following figure illustrates these findings:

[5]This rules out the possibility that the equilibrium allocation is between the full insurance line and the switching locus.

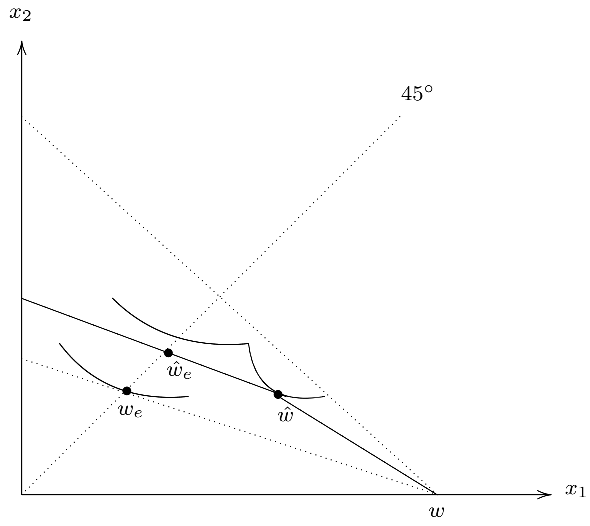

Figure 5

Formally, $(\hat{w}, H, j) = \gamma(\hat{\tau}^j, \tau_\circ^{-j})$ and $P_\gamma^j(\hat{\tau}^j, \tau_\circ^{-j}) > P_\gamma^j(\tau_\circ)$. This contradicts the hypothesis that $\tau_\circ$ was (part of) an equilibrium.

(B) $\Rightarrow$ (A). Let $\tau_\circ = ((w_e - w), \tau_\circ^{-1})$, where $\tau_\circ^{-1} = (w_e - w)/(j-1)$ for $j = 2, 3, \ldots$. In this case there exist a selection $\gamma(\tau_\circ)$ such that $(x(\tau_\circ), a(\tau_\circ)) = (w_e, L)$. To check that this is an equilibrium, suppose that j substitutes τ^j with some $\hat{\tau}^j = \hat{w} - w$, where $\hat{w}$ satisfies (1.6). If purchased, $\hat{\tau}^j$ would makes positive profits if consumers choose $a = H$. Since $(\hat{w}_e, L) \in \varphi(\vec{\pi}_L, \hat{w})$ by assumption and since consumers can supplement $\hat{\tau}^j$ with contracts in $\tau_\circ^{-j}$, it follows that $(x(\hat{\tau}^j, \tau_\circ^{-j}), a(\hat{\tau}^j, \tau_\circ^{-j})) = (\hat{w}_e, L)$. Hence $\hat{\tau}^j$ makes a loss if issued and it is not a profitable deviation. It is finally straightforward to verify that the equilibrium just constructed is indeed a linear price equilibrium with zero profits for each j. ■

Low-action linear price equilibria are consistent with the intuition that consumers cannot be given incentives to choose high action if they are not constrained in the amount of insurance they can buy, and prices at which they trade are linear.

Yet this intuition does not exhaust all possible equilibrium configurations. Indeed, it may well be the case that consumers choose at equilibrium partial insurance, and high action, even when they are not prevented from buying as much insurance as they wish at linear prices. This case will be analyzed in the following section.

1.4.6 High-action linear price equilibrium

Recalling proposition 4, we may argue that if $(x, a) \in \varphi(\vec{\pi}_L, w)$ implies $a = H$, then there exist high-action linear price equilibria. This conjecture is confirmed by the following:

Proposition 8. *The following statements are equivalent:*

(A) *there exists a linear price equilibrium such that* $a(\tau^\circ) = H$*;*

(B) (i) *there exists* $\vec{\pi}_L \leqslant_1 \vec{\pi}_* <_1 \vec{\pi}_H$ *such that:*

$$\{(x_H, H), (x_L, L)\} = \varphi(\vec{\pi}_*, w)\,, \tag{1.7}$$

and (ii) *for all* $\hat{w}$ *such that:*

$$\vec{\pi}_H \cdot (\hat{w} - w) < 0 < \vec{\pi}_* \cdot (\hat{w} - w)\,, \tag{1.8}$$

there exists $\hat{x}$ *such that* $(\hat{x}, L) \in \varphi(\vec{\pi}_*, \hat{w})$ *.*

In this case either $x_H = w$*, and equilibrium profits are zero, or* $x_H \neq w$*, and equilibrium profits are positive.*

Proof. (A) $\Rightarrow$ (B). Let $\vec{\pi}_*$ be the equilibrium price of insurance and set $x(\tau_\circ) = x_H$. Proposition 5 (b) implies $\pi_L \cdot (x_H - w) \geqslant 0$. The equilibrium being linear implies $\pi_* \cdot (x_H - w) = 0$. It follows that $\vec{\pi}_* \geqslant_1 \vec{\pi}_L$, as shown in the following figure:

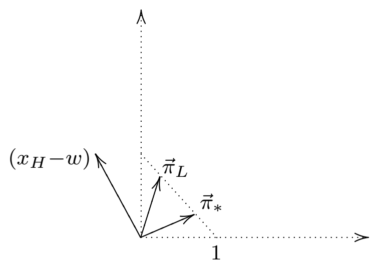

Figure 6

Since $a(\tau_\circ) = H$ is optimal in the compensated consumer problem, it follows that $x_1 > x_2$ and $\vec{\pi}_* \leqslant_1 \vec{\pi}_H$. Suppose (ii) does not hold. Then there exist $\bar{w}$ satisfying (1.8) and $\hat{w}$ such that $(\hat{w}, H) \in \varphi(\vec{\pi}_*, \bar{w})$, hence $(\hat{w}, H) \in \varphi(\vec{\pi}_*, \hat{w})$. Let $\hat{\tau} = \hat{w} - w$. Recall that the equilibrium being linear price implies that $\vec{\pi}_* \cdot \tau_\circ^j \leqslant 0$ for all j. If follows that $(\hat{w}, H, j) = \gamma(\hat{\tau}^j, \tau_\circ^{-j})$ and $P_\gamma^j(\hat{\tau}^j, \tau_\circ^{-j}) = \vec{\pi}_H \cdot \hat{\tau}^j > 0$ for all j. (d) in proposition 5 implies that there is j' such that $P^{j'}(\tau_\circ) < \vec{\pi}_H \cdot \hat{\tau}$. $\hat{\tau}$ is then a profitable deviation, which contradicts (A). Therefore (ii) holds. Now let $\{x_l\}$ be a sequence such that $x_h \to x_H$ and x_h satisfies (1.8) for all h. As (ii) holds, there exist x_l such that $(x_l, L) \in \varphi(\vec{\pi}_*, x_h)$ for all h. This implies in particular that x_l belongs to the constraint set of the compensated consumer problem, which is compact. Therefore the series x_l converges to some x_L as l tends to infinity. Since φ is upper-semi-continuous, it follows that $(x_L, L) \in \varphi(\vec{\pi}_*, x_H) = \varphi(\vec{\pi}_*, w)$. (i) then follows from the strict concavity of u. The following figure illustrates these findings:

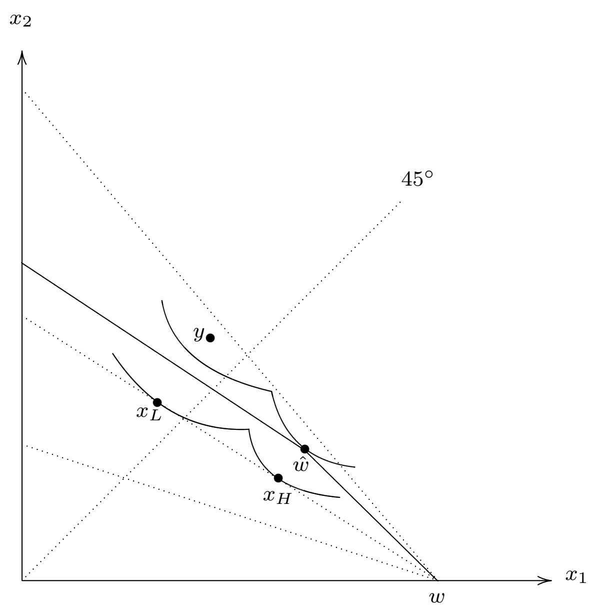

Figure 7

(B) $\Rightarrow$ (A). Let $\tau_\circ = (\tau_\circ^1, \tau_\circ^{-1})$, with $\tau_\circ^1 = (x_H - w)$ and $\tau_\circ^{-1} = (x_H - w)/(j-1)$ for $j = 2, 3, ...$ In this case there exists a selection $\gamma(\tau_\circ)$ such that $(x(\tau_\circ), a(\tau_\circ)) = (x_H, H)$. By the same reasoning as in the previous proposition, it is straightforward to prove that this is indeed an equilibrium. ■

Remark. *If in a linear price equilibrium it is admitted that $\vec{\pi} \cdot \tau_\circ^j > 0$ for some j, then it may be possible that despite (B) is violated, (A) is not. In particular, it may happen that once issued $\hat{w}$ is supplemented* *with another contract so as to reach a point like y, where the optimal action is L.*

In the insurance framework we are considering, the existence of high-action, positive-profit equilibria is linked to the non-convexity of the indifference curves. When the latter are convex, this type of equilibrium does not in general arise.

Convexity of the action set $\mathcal{A}$ is a necessary condition for indifference curves to be convex. In addition, specific assumptions on the shapes of the functions π_a and c_a has to be made.[6]

1.4.7 Non-existence of linear price equilibrium

According to last two propositions, linear price equilibria do exists under specific conditions. If these do not hold, a linear price equilibrium may not exist at all. This intuition is confirmed by the following:

Proposition 9. *The following statements are equivalent:*

(A) *no linear price equilibrium exists;*

(B) *for all $\vec{\pi}$ such that $\vec{\pi}_L \leqslant_1 \vec{\pi} \leqslant_1 \vec{\pi}_H$ either (i) $(x,a) \in \varphi(\vec{\pi}, w)$ implies $\vec{\pi}_a \cdot (x - w) > 0$, or (ii) for some x and some $\hat{w}$ such that*

$$\vec{\pi}_H \cdot (\hat{w} - w) < 0 < \vec{\pi} \cdot (\hat{w} - w) \,, \tag{1.9}$$

it is true that $(x, H) = \varphi(\pi, \hat{w})$.

For economies with preferences and endowments satisfying the assumptions made at the beginning of the chapter, existence is not an issue in case of constant absolute risk aversion (Bernoulli) utility index u.[7] With constant relative risk aversion (Bernoulli) utility index u, a linear price equilibrium may not exist.[8]

There is another potential source of non-existence of linear price equilibria: the possibility for consumers to buy negative insurance, so as to transfer income from the bad state to the good state. Consider the following figure:

[6] See for example Arnott [1].

[7] See Hellwig [15], proposition 5.b and lemma A.3.

[8] See Hellwig [16], proposition 3.8. This proposition, together with the companion proposition 3.7, shows that the conditions introduced in proposition 7, 8 and 9 are not empty.

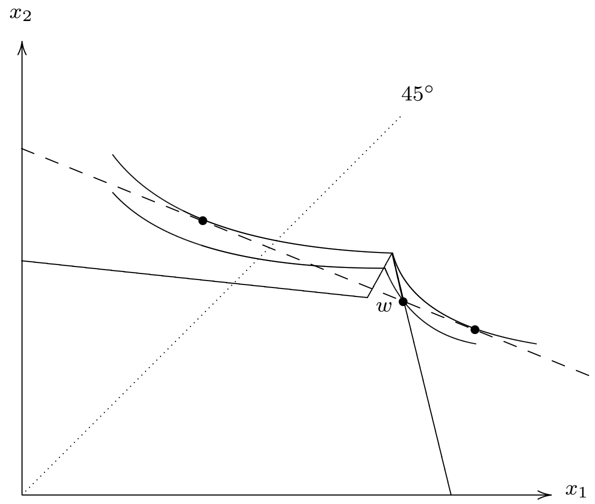

Figure 8

The solid line represent the boundary of the feasibility set. At price $\vec{\pi}_L$, consumers are not willing to buy positive insurance, as they would rather buy negative insurance. If they are allowed to do so, they will choose an unfeasible consumption bundle. In particular this implies that a negative-insurance contract offered at this price will be bought by consumers, and it will make negative profits.

As the price of insurance is reduced, consumers wish to buy less negative insurance, but the optimal bundle will still be unfeasible. At the price corresponding to the dashed line, consumers are indifferent between choosing a partial insurance consumption bundle, together with high action, and an overinsurance consumption bundle, together with low action, but none of the bundle is in the feasibility set. As the price of insurance is further decreased, consumers discontinuously switch to an overinsurance consumption bundle which is still outside the feasibility set. No equilibrium with intermediaries making non-negative profits can exist in this case.

This example makes clear the importance of the assumption of absence of negative insurance made in the previous sections.

Linear price equilibria are not the only class of non-exclusive equilibria with insurance intermediaries. In particular there are examples of non-exclusive equilibria such that the equilibrium allocation does not belong to a linear budget set. Equilibria in this class are still actively investigated, with results not completely settled. An example of an exclusive equilibrium in this class will be given in the following section.

1.4.8 Non-exclusive equilibrium and efficiency

When the constrained efficient allocation (x^*, a^*) is such that $a^* = H$, it is not possible to sustain it as an equilibrium with non-exclusivity. This follows from the fact that there are robust possibility of profitable deviations that can be exploited by intermediaries making at equilibrium sufficiently low profits. Formally one has:

Proposition 10. *Assume that (x^*, a^*) is a constrained efficient allocation such that $a^* = H$. In this case, if $(x, a, J, \tau_\circ)$ is an non-exclusive equilibrium, then $(x, a) \neq (x^*, a^*)$.*

Proof. Contrary to the statement, suppose there exists an non-exclusive equilibrium such that $(x, a) = (x^*, a^*) = (x^*, H)$. In this case there exists a contract $\hat{\tau} = \hat{x} - x^*$ which gives higher utility to consumers, and make strictly positive profits, as shown in the following figure:

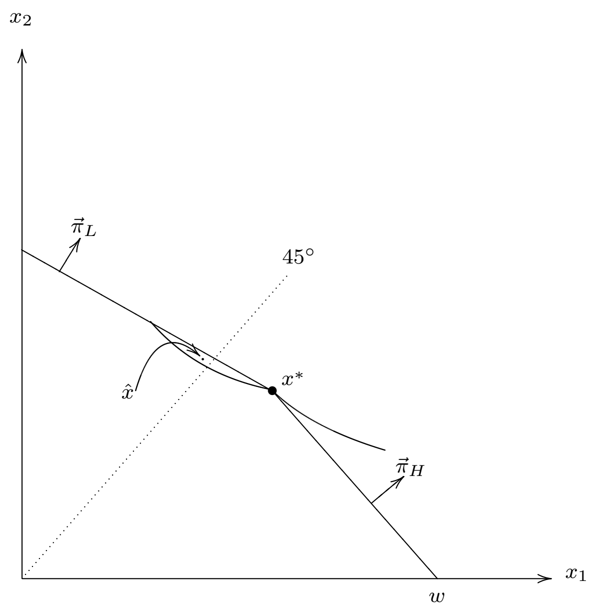

Figure 9

Using (d) in proposition 5, we can find an intermediary j such that $P^j_\gamma(\tau_\circ) < \varepsilon \leqslant P^j_\gamma((\hat{x} - x^*), \tau_\circ^{-j})$, a contradiction which proves the result. ■

Upon reflection, the constrained efficient allocation does not seem to be a suitable benchmark for evaluating the welfare properties of non-exclusive equilibria. Indeed, if we accept that the planner has to be subject to the same constraints on information as all economic agents, then the planner's problem should be amended to take into account the possibility for consumers to make unverifiable trades. The resulting constraint will in general depend on the market structure, for the latter determines the actual possibilities of trading for consumers.

Suppose that consumers can always trade with some intermediary offering insurance at fair prices conditional on low action. Since a contract traded at these prices always makes either zero or positive profits, this is assumption is compatible with the characteristics of an equilibrium with non-exclusivity.

If the planner cannot prevent consumers from trading at those prices, then its problem should be amended as follows:

$$\max_{x,a} \quad v(x,a)$$

$$\text{s.t.} \quad \vec{\pi}_a \cdot (x - w) \leqslant 0\,, \tag{1.10a}$$

$$v(x_e, L) - v(x,a) \leqslant 0\,. \tag{1.10b}$$

While (1.10a) is the traditional feasibility constraint, (1.10b) is the constraint that captures the fact that the planner can only choose among allocations that are be optimal given the possibility of further trading at fair prices conditional on low action.

According to the parameters of the economy, the solution of the problem may imply $a = L$ or $a = H$. If endowments are sufficiently skewed, then it is optimal to give consumers positive insurance up to the point where they are indifferent between a partial insurance bundle, and high action, and a full insurance bundle, and low action, so that (1.10b) holds with equality. This configuration is shown in the following figure:

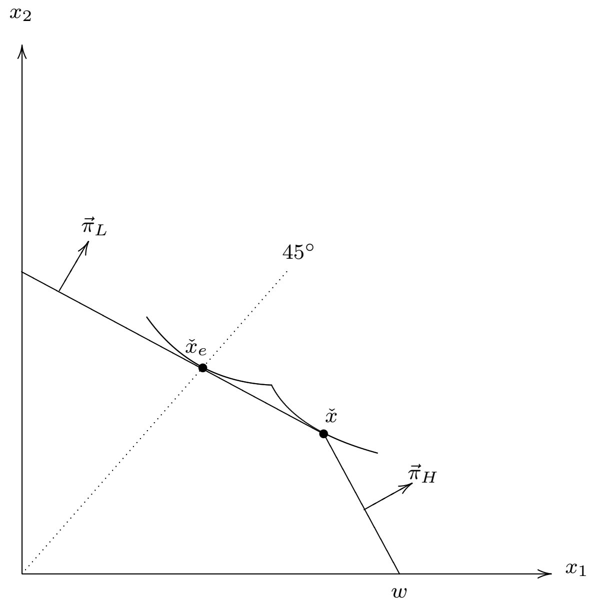

Figure 10

While an allocation like $(\check{x}, H)$ cannot arise in a linear price equilibrium, there exists notions of non-exclusive equilibrium that sustain it. As an illustration, assume consumers make offers to insurance firms, who may or may not accept them.

Since intermediaries anticipate that consumers can always ask for supplementary coverage, they are always willing to accept contracts at price $\vec{\pi}_L$, for these guarantees non-negative profits irrespective of trades consumers make. Yet they are also going to accept contract lower at prices, provided that it finances a consumption bundle satisfying (1.10b), for in this case they are sure consumers are not going to supplement it and switch to low action.

In this case, if consumers ask a contract $\check{\tau} = (\check{x} - w)$, there will be an insurance firm willing to accept it. This active intermediary makes zero profits, as well as all the other latent intermediary, who stand ready to accept any contract offer at prices $\vec{\pi}_L$. In is possible to show that in this case no profitable deviation exists: given the set of contract offers accepted by intermediaries, consumers maximize their utility. If any intermediary announces to be ready to accept contracts at lower price, consumers will then supplement this contract with those offered at price

$\vec{\pi}_L$ and will switch to low effort. The deviation is then not profitable for the intermediary.

1.5 Related literature

The contingent market equilibrium proposed above follows Kocherlakota [19], who considers a more general framework, in particular allowing for aggregate uncertainty. For the case of observable action, Marshall [22] proposed a similar concept, thus making clear that for the equilibrium to be (unconstrained) efficient, prices should be properly related to a even in absence of asymmetric information.

The analysis of the exclusive equilibrium dates back to the work of Helpman and Laffont [17]. The strategic version here considered follows Arnott and Stiglitz [5], [7]. In addition to the results recalled in this chapter, these authors observed that some of the properties of this type of equilibrium, and in particular the fact that it implies zero profits, do not necessarily hold when preferences are not separable in consumption and action. This observation has been subsequently analyzed by Bennardo and Chiappori [10], who characterize a class of positive profits exclusive equilibria.

The of linear price equilibrium was first analyzed by Pauly [23], who only considered the low-action, zero-profit configuration. Subsequently, Stiglitz [27], Hellwig [15], [16] and Arnott and Stiglitz [6] extended the analysis to the case of high action equilibria.

The non-existence problem recalled at the end of section 1.4.7 was first analyzed by Helpman and Laffont [17].

The efficiency criterion introduced at the end of the last section, as well as the concept of non-exclusive equilibrium that sustain allocations satisfying it, has been analyzed by Kahn and Mookherjee [18]. Thanks to the fact all the bargaining power is assigned to consumers, the sequential equilibrium concept they propose always entails zero profits for the active intermediaries.

Bisin and Guaitoli [11], following Hellwig [16], consider a class of equilibria with non-exclusivity similar to that of Kahn and Mookherjee. Yet they show that there exist equilibria with positive profits. This result is due the fact that they assume that insurance firms make offers to consumers, thus reverting the distribution of the bargaining power. The authors then argue that these equilibria are still efficient according to the same criterion proposed by Kahn and Mookherjee [18].

The analysis of models where exclusivity covenants cannot be enforced has been extended to cases where supplementary possibilities of trades do not come only from formal insurance markets, but also from informal ones, like the family (see Arnott and Stiglitz [8]), and from formal credit markets (see Bizer and De Marzo [12]).

1.6 Appendix

In this Appendix we prove proposition 3 and 4.

1.6.1 Proof of proposition 3

If $x \in \varphi(\vec{\pi}, \hat{w})$, then it satisfies the following first order conditions:

$$\pi_a u'(x_1) - \lambda\pi + \mu - \nu = 0\,, \tag{1.11a}$$

$$(1-\pi_a)u'(x_2) - \lambda(1-\pi) = 0\,, \tag{1.11b}$$

$$\lambda\left(\pi(x_1 - \hat{w}_1) + (1-\pi)(x_2 - \hat{w}_2)\right) = 0\,, \tag{1.11c}$$

$$\mu x_1 = 0\,, \tag{1.11d}$$

$$\nu\,(x_1 - \hat{w}_1) = 0\,, \tag{1.11e}$$

where $\lambda, \mu, \nu \geqslant 0$ are the multipliers associated with the inequality constraints. As (1.5a) always holds with equality, there are three cases left to consider: when (1.5b) holds with equality, when (1.5c) does, and when none of them does.

In the first case, $x_2 > x_1 = 0$, hence $u'(x_1) > u'(x_2)$ and $\mu \geqslant \nu = 0$. Therefore (1.11a) and (1.11b) imply:

$$\frac{\pi_a}{1-\pi_a} < \frac{\pi_a}{1-\pi_a}\frac{u'(x_1)}{u'(x_2)} \leqslant \frac{\pi}{1-\pi}\,. \tag{1.12}$$

In the second case, $x_1 = \hat{w}_1 > \hat{w}_2 = x_2$, hence $u'(x_1) < u'(x_2)$ and $\nu \geqslant \mu = 0$. Therefore (1.11a) and (1.11b) imply:

$$\frac{\pi_a}{1-\pi_a} > \frac{\pi_a}{1-\pi_a}\frac{u'(x_1)}{u'(x_2)} \geqslant \frac{\pi}{1-\pi}\,. \tag{1.13}$$

In the third case, $0 < x_1 < \hat{w}_1$ and $\mu = \nu = 0$, hence (1.11a) and (1.11b) imply:

$$\frac{\pi_a}{1-\pi_a}\frac{u'(x_1)}{u'(x_2)} = \frac{\pi}{1-\pi}\,, \tag{1.14}$$

which is satisfied whenever the following holds:

$$x_1 \lesseqqgtr x_2 \quad\Leftrightarrow\quad u'(x_1) \gtreqqless u'(x_2) \quad\Leftrightarrow\quad \frac{\pi_a}{1-\pi_a} \lesseqqgtr \frac{\pi}{1-\pi}\,. \tag{1.15}$$

From (1.12) and (1.15) we conclude that if $(x_1, x_2, a) \in \varphi(\vec{\pi}, \hat{w})$ and

$$\frac{\pi_a}{1-\pi_a} \leqslant \frac{\pi}{1-\pi}\,, \tag{1.16}$$

then $x_1 \leqslant x_2$, hence $a = L$.[9]

1.6.2 Proof of proposition 4

We first prove the first part of the proposition, then the second part. Let $\xi(x) := \pi_H u(x_1) + (1-\pi_H)u(x_2)$ and define:

$$\chi(\vec{\pi}_L, w) := \arg\max\left\{\xi(x) \,|\, \vec{\pi}_L \cdot (x - w) = 0\,, x_1 \geqslant 0\,, x_1 - w_1 \leqslant 0\right\}\,, \tag{1.17}$$

which is characterized by the following equations :

$$\pi_H u'(x_1) - \lambda\pi_L + \mu - \nu = 0\,, \tag{1.18a}$$

$$(1-\pi_H)u'(x_2) - \lambda(1-\pi_L) = 0\,, \tag{1.18b}$$

$$\lambda\left(\pi_L(x_1 - w_1) + (1-\pi_L)(x_2 - w_2)\right) = 0\,, \tag{1.18c}$$

$$\mu x_1 = 0\,, \tag{1.18d}$$

$$\nu\left(x_1 - w_1\right) = 0\,. \tag{1.18e}$$

[9] When marginal utility at zero is unbounded, the only relevant first order condition is (1.14). This implies in particular that a boundary solution is not optimal when $\hat{w}_2 = 0$.

$x_1 = 0$ cannot optimal, for in this case (1.18a)−(1.18b) yield a contradiction. When $x_1 = w_1 > w_2 = x_2$, then $u'(x_1) < u'(x_2)$ and $\nu \geqslant \mu = 0$. Therefore (1.18a) and (1.18b) imply that:

$$\frac{\pi_L}{1-\pi_L} \leqslant \frac{\pi_H}{1-\pi_H}\frac{u'(x_1)}{u'(x_2)} < \frac{\pi_H}{1-\pi_H}\,, \tag{1.19}$$

which is admissible. Finally, if $0 < x_1 < w_1$, then $\mu = \nu = 0$ and in this case (1.18a) and (1.18b) imply that:

$$\frac{\pi_L}{1-\pi_L} = \frac{\pi_H}{1-\pi_H}\frac{u'(x_1)}{u'(x_2)}\,. \tag{1.20}$$

Since $\pi_H > \pi_L$, the above condition holds when $u'(x_1) < u'(x_2)$, hence $x_1 > x_2$.[10] Summing up, we conclude that $x \in \chi(\pi_L, w)$ implies $x_1 > x_2$, hence $x \neq w_e$, where w_e is a vector with components all equal to $\pi_L \cdot w$. Since w_e satisfies the constraint set in (1.17), we conclude that $x \in \chi(\pi_L, w)$ also implies $\xi(x) > \xi(w_e)$, where the strict inequality follows from the strict concavity of ξ. For $x \in \chi(\pi_L, w)$, we let $\xi^* = \xi(x) - \xi(w_e)$. From the above analysis it follows that ξ^* it is strictly positive and from the Maximum Theorem it follows that it is continuous and therefore it reaches a minimum on the compact set $\mathcal{D} \subset \mathbb{R}^2_+ \times [0,1]^2$ such that $(w, \pi_H, \pi_L) \in \mathcal{D}$ implies $w_1 > w_2 \geqslant 0$ and $1 > \pi_H > \pi_L > 0$. Let $c > 0$ be this minimum. If follows that there exists $0 < c_H < c$ such that $c_H < \xi^*$, hence that $\xi(w_e) < \xi(x) - c_H$ with $x \in \chi(\pi_L, w)$. Therefore consumers prefer to choose high action and a partial insurance consumption bundle.

This concludes the proof of the first part of the proof. As for the second, we first recall that φ is upper semi-continuous by the Maximum Theorem. Now let Π be the set of $\vec{\pi}$ for which consumers choose full, or overinsurance, and low effort when $\hat{w} = w$. In symbols:

$$\Pi := \{\vec{\pi} \,|\, (x, L) \in \varphi(\vec{\pi}, w)\,,\ x_2 \geqslant x_1\}\ .$$

From the above analysis we know that $\vec{\pi}_H \in \Pi$, hence $\Pi \neq \varnothing$. Assume that $\vec{\pi}_L \notin \Pi$, i.e. assume that the parameters of the problem are such that consumers would rather choose underinsurance and high action when they can buy as much insurance as they wish at the price $\vec{\pi}_L$. Proposition 4 guarantees that this is possible.

[10] The previous remark on the optimality of boundary solutions when marginal utility at zero is unbounded applies also in this case.

Recall that $\vec{\pi} = (\pi, 1-\pi)$, $\pi \in (0,1)$ by assumption and that $\vec{\pi} \geqslant_1 \vec{\pi}_m$ whenever $\pi \geqslant \pi_m$. Moreover we let

$$\pi_* := \inf\{\pi \mid \vec{\pi} \in \Pi\}\,, \tag{1.21}$$

and $\vec{\pi}_* = (\pi_*, 1-\pi_*)$. Notice that $\pi_* > \pi_L$, hence $\vec{\pi}_* >_1 \vec{\pi}_L$. We know there exists x_H such that $(x_H, H) \in \varphi(\pi_L, w)$ and x_L such that $(x_L, L) \in \varphi(\pi_H, w)$. Let

$$\vec{\pi}_h := \left(1 - \frac{1}{h}\right)\vec{\pi}_* + \left(\frac{1}{h}\right)\vec{\pi}_L\,.$$

By construction, $\vec{\pi}_* >_1 \vec{\pi}_h >_1 \vec{\pi}_L$. Since $(x, a) \in \varphi(\vec{\pi}_h, w)$ cannot imply $a = L$ and $x_2 \geqslant x_1$, for otherwise (1.21) would be violated, and it cannot imply $a = L$ and $x_2 < x_1$ either, for otherwise (1.13) would be violated, we conclude that $(x, a) \in \varphi(\vec{\pi}_h, w)$ implies $a = H$ and $x_1 > x_2$.[11] Moreover $(x_h, H) \in \varphi(\vec{\pi}_h, w)$ implies that x_h belongs to the constraint set in the virtual consumer problem, which is compact. Therefore the series x_h converges to some x_H as h tends to infinity. Summing up, we have that $\vec{\pi}_h \to \vec{\pi}_*$ and $x_h \to x_H$ as $h \to \infty$, and $(x_h, H) \in \varphi(\pi_h, w)$ for every h. Since the Maximum Theorem guarantees that φ is upper-semi-continuous, it follows that $(x_H, H) \in \varphi(\vec{\pi}_*, w)$. Now let

$$\vec{\pi}_l := \left(1 - \frac{1}{l}\right)\vec{\pi}_* + \left(\frac{1}{l}\right)\vec{\pi}_H\,.$$

By construction, $\vec{\pi}_* <_1 \vec{\pi}_l <_1 \vec{\pi}_H$. It easy to see that $\vec{\pi}_l \in \Pi$. If not, either $\vec{\pi}_l \geqslant_1 \vec{\pi}$ for all $\vec{\pi} \in \Pi$ or $\vec{\pi}_l \leqslant_1 \vec{\pi}$ for all $\vec{\pi} \in \Pi$. The first case is false since $\vec{\pi}_l <_1 \vec{\pi}_H$, while the second is false for otherwise (1.21) would imply $\vec{\pi}_l \leqslant_1 \vec{\pi}_*$, which is impossible. It follows that $(x, a) \in \varphi(\vec{\pi}_l, w)$ implies $a = L$ and $x_2 \geqslant x_1$. Moreover $(x_l, L) \in \varphi(\vec{\pi}_l, w)$ implies that x_l belongs to the constraint set of the compensated consumer problem, which is compact. Therefore the series x_l converges to some x_L as l tends to infinity. Summing up, we have that $\vec{\pi}_l \to \vec{\pi}_*$ and $x_l \to x_L$ as $l \to \infty$, and $(x_l, L) \in \varphi(\vec{\pi}_l, w)$ for every l. Since the Maximum Theorem guarantees that φ is upper-semi-continuous, it follows that $(x_L, L) \in \varphi(\vec{\pi}_*, w)$.

As a final step we show that the map φ is discontinuous by showing that fails to be lower semi-continuous at $\vec{\pi}_*$. Formally this means that there

[11] $a = L$ and $x_1 > x_2$ means that the chosen bundle would be below the 45° line and above the switching locus.

exist a sequence $\vec{\pi}_n \to \vec{\pi}_*$ and a choice bundle $(x, a) \in \varphi(\vec{\pi}_*, w)$ such that there does not exist a sequence $(x_n, a_n) \to (x, a)$ and $(x_n, a_n) \in \varphi(\vec{\pi}_n, w)$ for all n. To see this, consider again the sequence $\vec{\pi}_h \to \vec{\pi}_*$ as above constructed and take $(x_L, L) \in \varphi(\vec{\pi}_*, w)$. If there exists a sequence $(x_h, a_h) \to (x_L, L)$ such that $(x_h, a_h) \in \varphi(\vec{\pi}_h, w)$ for all h, we could extract a sub-sequence such that $x_{h'} \to x_L$ and $(x_{h'}, L) \in \varphi(\vec{\pi}_{h'}, w)$. But this is impossible since from the above analysis we know that $(x_h, a_h) \in \varphi(\vec{\pi}_h, w)$ implies $a = L$.

Chapter 2

Constrained inefficiency of competitive equilibrium with hidden action

In this chapter, we consider a pure exchange economy with idiosyncratic uncertainty, hidden action and multiple consumption goods. For this economy, we consider two different market structures. On the one hand, we assume that there exists a complete set of markets for commodities with delivery contingent on the idiosyncratic state. On the other hand, we assume that there exists a complete set of financial markets, together with spot markets for consumption goods.

For each of the market structure considered, we propose a suitable equilibrium concept, and we evaluate the competitive equilibrium using an appropriate efficiency criterion. We then ask whether differences in the market structure result in differences in the efficiency of equilibrium.

It is well know that when there is no hidden action, assuming a complete set of contingent-commodity markets or a complete set of financial markets is irrelevant as far as efficiency is concerned. This result follows because the two market structures allow to reach the same allocations of consumption goods, provided that prices are related in an appropriate way.

When there is hidden action, the incentive compatibility constraint become an essential ingredient of the equilibrium, and it may interfere with that equivalence. This is because in a multiple-good setting, this constraint depends on optimal consumption bundles, and these in turns depend on the type of trades consumers can make.

In this chapter, we first show that any equilibrium with contingent-commodity markets is efficient in an appropriate sense.

Then, following the intuition of the previous paragraph, we show that the equilibrium with financial and spot markets is indeed inefficient, when assumptions on preferences more general than those usually considered in the literature hold.

2.1 The economy

Consider a pure exchange economy with $L \geqslant 1$ consumption goods. The economy is populated by a large number of consumers all ex ante equal, and lasts two periods $t = 0, 1$.

There is no consumption, when consumers face an idiosyncratic uncertainty which realizes in the second period. At $t = 1$ each consumer may be in one out two states $s \in \mathcal{S} = \{0, 1\}$.

Uncertainty affects only the level of goods consumers are endowment with at the beginning of the second period. Individual endowments are denoted by $w = (w_1, w_2) \in \mathbb{R}^{2L}_{++}$. It is assumed that $w_1 \gg w_2$, so that state $s = 1$ will be referred to as the good state.[1]

Consumers can choose an action $a \in \mathcal{A} = [0, 1]$ affecting, on the one hand, the probabilities of the idiosyncratic states, and on the other hand the utility consumers get from consumption.

Preferences are represented by an expected utility function $v(x_1, x_2, a) : \mathbb{R}^{2L}_{+} \times \mathcal{A} \to \mathbb{R}$ given by:

$$v(x_1, x_2, a) := \pi_1(a)\, u(x_1, a) + \pi_2(a)\, u(x_2, a)\,,$$

where $\pi_s(a) : \mathcal{A} \to (0, 1)$ is a function giving the probability of state s when action a is chosen. According to the above general formulation, the level of a affects the marginal utility of every consumption good. This case will be referred to as non-separable preferences.

If a does not affects the marginal utility of any consumption good, then we say that preferences are separable. Formally, this arises when the following assumption hold:

[1]Given two vectors $x, y \in \mathbb{R}^{L}_{++}$, $x \gg y$ means $x_l > y_l$ for all l.

Assumption S. (SEPARABLE PREFERENCES) *There exist utility indexes* $\tilde{u} : \mathbb{R}^L_+ \to \mathbb{R}$ *and* $c : \mathcal{A} \to \mathbb{R}$ *such that* $u(x_s, a) := \tilde{u}(x_s) - c(a)$ *.*

Intermediate cases are possible as well. In particular it may happen that a affects the marginal utility of all but one consumption goods, as stated in the following:

Assumption H. (HEMI-NON-SEPARABLE PREFERENCES) *There exist utility indexes* $f : \mathbb{R}_+ \to \mathbb{R}$ *and* $g : \mathbb{R}^{L-1}_+ \times \mathcal{A} \to \mathbb{R}$ *such that* $u(x_s, a) := f(x_{sj}) + g(x_s^{-j}, a)$ *for some consumption good* j *,*

where $x_s^{-j} \in \mathbb{R}^{L-1}_{++}$ is a bundle consumption goods obtained by eliminating good j.

It is assumed that the action chosen by a consumer remains unknown to anyone else in the economy, so that information is asymmetric. On the other hand, it is assumed that endowments are verifiable at $t = 1$, so that it is possible to determine in which idiosyncratic state consumers are.

2.2 Constrained efficient allocation

Suppose there exists a benevolent planner who knows all the fundamentals of the economy, in particular preferences and endowments. Suppose in addition that he is subject to the same limits on information as any economic agent, so that he cannot verify the level of action chosen by consumers.

Under these assumption the information available to the planner, though not complete, is indeed rather large. The admissibility of such an assumption, common to most of the literature on the subject, will not be discussed here.

The planner assigns a feasible distribution of consumption goods and prescribes an action so as to maximize consumers' utility. Because of the limits on information, the planner can only prescribe action that are optimal given the assigned consumption bundle. In the standard jargon, this means that the planner can only choose among the set of incentive compatible actions.

We assume that the planner observe actual consumption at $t = 1$, and as a consequence he can enforce a given distribution of consumption bundles. It is therefore excluded that consumers trade at $t = 1$ once received their bundles. This implies that the choice of a is the only hidden action consumers make.

From the above discussion it should be clear that the constrained efficient allocation (x_1, x_2, a) is the solution of the following problem:

$$\max_{x_1,x_2,a} \quad v(x_1, x_2, a)$$

$$\text{s.t.} \quad \pi_1(a)(x_1 - w_1) + \pi_2(a)(x_2 - w_2) \leqslant 0\,, \tag{2.1a}$$

$$a = \arg\max\, v(x_1, x_2, a)\,. \tag{2.1b}$$

(2.1a) is the feasibility constraint, which holds in expected value, while (2.1b) is the incentive compatibility constraint. It states that the only admissible level of action in the planner choice set are those which maximize consumers' utility, given prescribed consumption bundle.

In what follows, we let $\mathcal{B}$ denote the set of (x_1, x_2, a) that satisfy (2.1a) and (2.1b).

The solution of the planner's problem will be the benchmark for evaluating the efficiency of the different types of competitive equilibrium to be introduced in the following sections.

In the following sections, we introduce two different equilibrium concepts, and we compare the equilibrium allocations with those which are solution of the above problem.

2.3 A model with contingent markets

In this section, we suppose there exists a complete set of markets for commodities with delivery contingent on the idiosyncratic state. Consumers trade in these markets, which are open at $t = 0$, by selling their state-contingent endowments and buying a bundle of consumption goods to be delivered only if the corresponding state is realized. This market structure then implies that consumers face a single budget constraint.

Once the bundle has been chosen, markets close. Consumers then choose their preferred action a. The uncertainty is resolved, and at $t = 1$ consumption goods are delivered according to purchases.

In order to define an equilibrium concept for this economy, we have to introduce an appropriate price system. Given the complete set of contingent-commodity markets, SL prices has to be quoted. Suppose for a moment that there is no hidden action, and maintain the hypothesis of absence of aggregate uncertainty. In this case we know there exist restrictions on the structure of the price system such that a well defined equilibrium concept displaying satisfactory efficiency properties can be defined.

These restrictions essentially require the price of one unit of good l delivered in state s to be properly related to the actual probability of that state.

In particular, suppose that p_l is the price at $t = 0$ of one unit of good l with delivery at $t = 1$ no matter what the state is. Following Malinvaud [21], call it the price for sure delivery of good l.

Given that state s has probability π_s, we let the price at $t = 0$ of one unit of good l with delivery at $t = 1$ in state s be equal to $\pi_s p_l$.[2]

The presence hidden action does not alter the above reasoning in an essential way. In this case π_s depends on a. Hence we are naturally led to assume that the price of one unit of good l with delivery contingent on state s when action a has been chosen is equal to $\pi_s(a)p_l$.

The problem at this point is to consider the appropriate level of action. The only possible candidate is the incentive compatible one, which satisfies:

$$a = \arg\max v(x_1, x_2, a)\,. \tag{2.2}$$

From the above expression it is apparent that the incentive compatible level of a is well defined only once the the whole consumption bundle (x_1, x_2) is considered. Therefore the price of one unit of good l in state s cannot be quoted unless it is specified the consumption level of all the other goods.

Moreover, the price one unit of good l in state s may well be different when consumed in conjunction with different quantities of other consumption goods. For this reason, the above price system is sometime referred to as non-linear.

[2]Notice that the price system described in the text is the direct generalization to the case of multiple consumption good of the fair price system, which is well known in insurance economics.

2.3.1 CM consumers' problem

From the above discussion, it follows that at $t = 0$ consumers choose (x_1, x_2, a) so as to solve the following problem:

$$\max_{x_1,x_2,a} \quad v(x_1, x_2, a)$$

$$\text{s.t.} \quad \pi_1(a)\,(p \cdot (x_1 - w_1)) + \pi_2(a)\,(p \cdot (x_2 - w_2)) \leqslant 0\,, \tag{2.3a}$$

$$a = \arg\max\, v(x_1, x_2, a)\,. \tag{2.3b}$$

(2.3a) is the single budget constraint, while (2.3a) is the incentive compatibility constraint.

In what follows, $\mathcal{C}(p)$ denotes the set of (x_1, x_2, a) that satisfy (2.3a) – (2.3b).

2.3.2 CM Equilibrium

In a contingent-commodity markets (CM) equilibrium, consumers optimize and markets for goods clear. These properties are collected in the following:

Definition 2.3.1. *A CM equilibrium is (x, a, p) such that:*

1. $(x, a) = \arg\max \{v(x, a) \,|\, (x, a) \in \mathcal{C}(p)\}$,

2. $\pi_1(a)(x_1 - w_1) + \pi_2(a)(x_2 - w_2) = 0\,.$

The market clearing condition in the above definition is expressed in expected value because of absence of aggregate uncertainty.

The equilibrium just defined is a direct extension to the case of multiple consumption good of the equilibrium introduced in definition 1.3.1.

2.3.3 Efficiency of CM equilibrium

In this section we analyze the efficiency of the CM equilibrium. Efficiency is defined as usual: the equilibrium allocation of consumption goods, and the associated incentive compatible action, is constrained efficient if there does not exist a feasible allocation of consumption good, and an associated incentive compatible action, such that no consumer is worse off, and at least one consumer is better off.

In the economy under analysis all consumers are ex ante equal. Hence the last requirement implies that at the alternative allocation the utility of the (representative) consumer is higher than that at the equilibrium one.

Given this definition of efficiency, it is straightforward to verify the following:

Proposition 11. *Every CM equilibrium is constrained efficient.*

Proof. Take a CM equilibrium (x, a, p) and suppose that it is not constrained efficient. Then there exists $(\bar{x}, \bar{a}) \in \mathcal{B}$ such that $v(\bar{x}, \bar{a}) > v(x, a)$. Since $(\bar{x}, \bar{a}) \in \mathcal{B}$ implies $\bar{a} = \arg\max v(\bar{x}, a)$, if the following inequality holds:

$$\pi_1(a)\,(p \cdot (x_1 - w_1)) + \pi_2(a)\,(p \cdot (x_2 - w_2)) \leqslant 0\,,$$

then $(\bar{x}, \bar{a}) \in \mathcal{C}(p)$ and (x, a) cannot be an equilibrium, for it is not utility maximizing over the choice set. Therefore it must be true that:

$$\pi_1(a)\,(p \cdot (x_1 - w_1)) + \pi_2(a)\,(p \cdot (x_2 - w_2)) > 0\,,$$

so that:

$$p \cdot (\pi_1(a)(x_1 - w_1) + \pi_2(a)(x_2 - w_2)) > 0\,,$$

hence $(\bar{x}, \bar{a}) \notin \mathcal{B}$, a contradiction that concludes the proof. ■

The above proposition extends to the case of hidden action the well known result of efficiency of competitive equilibria for economies with uncertainty and complete contingent-commodity markets.

2.3.4 Characterization of CM equilibrium

In this section, we propose a characterization of the CM equilibrium through first order conditions that will be used later on in the chapter.

As a first step, we replace the incentive compatibility constraint (2.3b) with the corresponding first order condition:

$$\partial_a \, v(x_1, x_2, a) = 0 \, .$$

The CM consumers' problem may then be rewritten as follows:

$$\max_{x_1, x_2, a} \quad v(x_1, x_2, a)$$

$$\begin{aligned} \text{s.t.} \quad \pi_1(a)\,(p \cdot (x_1 - w_1)) + \pi_2(a)\,(p \cdot (x_2 - w_2)) \;&\leqslant\; 0 \, . && (2.4a) \\ \partial_a \, v(x_1, x_2, a) \;&=\; 0 \, . && (2.4b) \end{aligned}$$

An interior solution of the above problem is characterized by the following system of equations:[3]

$$\begin{aligned} \partial_{x_1} v - \hat{\gamma}\pi_1 p - \hat{\mu}\partial_{a x_1} v \;&=\; 0 \, , && (2.5a) \\ \partial_{x_2} v - \hat{\gamma}\pi_2 p - \hat{\mu}\partial_{a x_2} v \;&=\; 0 \, , && (2.5b) \\ \hat{\gamma}\hat{\sigma} + \hat{\mu}\partial_{aa} v \;&=\; 0 \, , && (2.5c) \\ \pi_1\,(p \cdot (x_1 - w_1)) + \pi_2\,(p \cdot (x_2 - w_2)) \;&=\; 0 \, , && (2.5d) \\ \partial_a \, v(x_1, x_2, a) \;&=\; 0 \, , && (2.5e) \end{aligned}$$

where $\hat{\gamma}$ is the multiplier for constraint (2.4a), $\hat{\mu}$ is the multiplier for constraint (2.4b) and:

$$\hat{\sigma} \;:=\; \partial_a \pi_1\,(p \cdot (x_1 - w_1)) + \partial_a \pi_2\,(p \cdot (x_2 - w_2)) \, .$$

Together with the market clearing conditions, (2.5a)$-$(2.5e) characterize the CM equilibrium.

For reasons that will be clear later on, it is useful to make explicit the equilibrium restrictions on the gradient of the (Bernoulli) utility index u. Using (2.4b), direct calculation gives:

$$\partial_{a x_s} v = \partial_a \pi_s \partial_{x_s} u + \pi_s \partial_{a x_s} u \, .$$

[3]To simplify notation, we write π_s instead of $\pi_s(a)$.

Substitute the above equation in (2.5a)−(2.5b) and collect terms to get:

$$\left(\frac{\pi_s - \hat{\mu}\partial_a \pi_s}{\pi_s}\right) \partial_{x_s} u = \hat{\gamma} p + \hat{\mu} \partial_{a x_s} u \,. \tag{2.6}$$

Any consumption bundle x_s which is part of a CM equilibrium must satisfy (2.6).

2.4 A model with financial markets

In this section we consider a different market structure. Instead of trading in contingent commodities, we assume there exist at $t = 0$ a complete set of financial markets. Through them, consumers can transfer income from one state to the other.

Once uncertainty is resolved, spot markets for consumption goods open. Consumers then use their real and financial income to buy goods with immediate delivery.

The financial instrument traded at $t = 0$ may be equivalently interpreted as an insurance contract, or as a portfolio of S Arrow-securities, each paying one unit of account if and only if idiosyncratic states s happens.

After having bought the financial instrument, consumers choose their preferred action. Subsequently uncertainty resolves, and trades on the spot markets take place.

To define a suitable equilibrium concept for the economy just described, $S + L$ prices must be considered, both for the financial instrument and the consumption goods.

Given the structure of trades, the price of the financial instrument must depend on the action chosen by consumers, which in turn depends on the optimal consumption at $t = 1$. The incentive compatibility constraint must then be adapted to consider the effects of trading on the spot markets.

2.4.1 FM consumers' problem

Let $p \in \mathbb{R}^L_{++}$ be the vector of prices for consumption goods traded on the spot markets. From the above discussion, it follows that at $t = 0$ consumers choose (x, a, τ) so as to solve:

$$\max_{x_1,x_2,a,\tau_1,\tau_2} \quad v(x_1,x_2,a)$$

$$\text{s.t.} \quad \pi_1(a)\tau_1 + \pi_2(a)\tau_2 \leqslant 0\,, \tag{2.7a}$$

$$p \cdot (x_1 - w_1) \leqslant \tau_1\,, \tag{2.7b}$$

$$p \cdot (x_2 - w_2) \leqslant \tau_2\,, \tag{2.7c}$$

$$a = \arg\max\, v(x_1,x_2,a)\,, \tag{2.7d}$$

$$x_1 = \arg\max\{u(x_1,a)\,|\,p\cdot(x_1-w_1)\leqslant \tau_1\}\,, \tag{2.7e}$$

$$x_2 = \arg\max\{u(x_2,a)\,|\,p\cdot(x_2-w_2)\leqslant \tau_2\}\,. \tag{2.7f}$$

(2.7a) is the budget constraint for the financial instrument, while (2.7b) and (2.7c) are the budget constraints for the consumption goods. As for (2.7d)$-$(2.7f) they constitute the (extended) incentive compatibility constraint. It implies in particular that the level of action considered is the optimal one given the optimal consumption at $t = 1$.

Recall that consumers do take p as given. Therefore in what follows, $\mathcal{D}(p)$ denotes the set of (x, a, τ) that satisfy (2.7a) $-$ (2.7f)

2.4.2 FM equilibrium

As in the previous section, the concept of financial market (FM) equilibrium we propose is standard. Indeed, in a FM equilibrium consumers maximize their utility and markets clear. These properties are summarized in the following:

Definition 2.4.1. *A FM equilibrium is (x, a, τ, p) such that:*

1. $(x,a,\tau) = \arg\max\{v(x,a)\,|\,(x,a,\tau)\in\mathcal{D}(p)\}$,

2. $\pi_1(a)(x_1 - w_1) + \pi_2(a)(x_2 - w_2) = 0\,.$

Here we just notice that the second equation in the above definition is the market clearing condition, expressed in expected value.[4]

[4]In the interpretation of τ as a portfolio of Arrow-securities, the market clearing condition on the asset market is automatically satisfied at the optimum.

2.4.3 Efficiency of FM equilibria

In this section, we analyze the efficiency of the FM equilibrium. Efficiency is defined as usual: the equilibrium allocation of consumption goods, and the associated incentive compatible action, is constrained efficient if there does not exist a feasible allocation of consumption good, and an associated incentive compatible action, such that the (representative) consumer is better off.

As claimed in the introduction, there are cases in which a FM equilibrium is constrained inefficient. A quick observation of the budget sets $\mathcal{C}(p)$ and $\mathcal{D}(p)$ reveals where the problem lies.

Since $(x, a, \tau) \in \mathcal{D}(p)$ implies that $(x, a) \in \mathcal{C}(p)$, it seems possible to reason as follows: take a CM equilibrium $(\hat{x}, \hat{a}, p)$. Construct transfers $\hat{\tau} := p \cdot (\hat{x} - e)$. Assume that $(\hat{x}, \hat{a}, \hat{\tau}) \in \mathcal{D}(p)$, but that $(\hat{x}, \hat{a}, \hat{\tau})$ is *not* a FM equilibrium.

In this case there exist $(\bar{x}, \bar{a}, \bar{\tau}) \in \mathcal{D}(p)$ such that $v(\bar{x}, \bar{a}) > v(\hat{x}, \hat{a})$. As $(\bar{x}, \bar{a}) \in \mathcal{C}(p)$, we get a contradiction with $(\hat{x}, \hat{a})$ being a CM equilibrium. Hence we shall conclude that $(\hat{x}, \hat{a})$ must be a FM equilibrium, which is then efficient.

Recall that this reasoning holds if $(\hat{x}, \hat{a}, \hat{\tau}) \in \mathcal{D}(p)$. The main question then is whether a CM equilibrium, together with the implied financial transfers, satisfies the budget set of the FM consumers' problem.

Inspection of $\mathcal{D}(p)$ reveals that it is possible that the required inclusion is not satisfied. The crux of the problem lies in equations (2.7e) and (2.7f). They require $\hat{x}$ be optimal at $t = 1$ − when trading takes place − given prices p, transfers $\hat{\tau}$ and the action $\hat{a}$ chosen at $t = 0$.

If there is only one consumption good, these equations do not add any constraint on the FM equilibrium, since there is no trading at $t = 1$. If preferences are separable, the level of action does not affect the level of ex post utility, hence even in this case we may suspect that these equation do not impose any further constraint on the FM equilibrium. In this two cases therefore we expect the above inclusion to be satisfied, hence the conclusion about the constrained efficiency of the FM equilibrium to be correct.

When preferences are not separable, consumers who are given enough income to buy $\hat{x}$ may wish to choose a different bundle. This happens when $\hat{x}$ does not satisfies (2.7e)−(2.7f), where a is taken as fixed. In this case here exists $\tilde{x}$ which costs not more that $\hat{x}$ at prices p and gives higher utility in at least one state s. Since $\tilde{x}$ would be feasible when action is $\hat{a}$, and since it gives higher utility in a least one state, it would be a natural candidate for a CM equilibrium. Yet there is no guarantee that $(\tilde{x}, \hat{a})$ satisfies the incentive compatibility constraint, hence that it is indeed a feasible candidate for a CM equilibrium.

While these observations highlight the main intuition for the inefficiency of the FM equilibrium, to verify this reasoning we are going to compare the first order conditions characterizing it with those characterizing the CM equilibrium.

2.4.4 Characterization of FM equilibrium

In this section, we derive the first order conditions characterizing the FM equilibrium. Instead of working directly with the consumers' problem as above defined, we first derive a reduced-form problem, so as to get a representation as close as possible to the CM consumers' problem. The corresponding first order conditions will then be easily comparable.

Starting with the constraints relevant at $t = 1$, we notice that (2.7e) and (2.7f) imply that any admissible x_s satisfies:

$$\max_{x_s} u(x_s, a) \quad \text{s.t.} \quad p \cdot (x_s - e_s) - \tau_s \leqslant 0 \,. \tag{2.8}$$

Let $x_s(\tau, a)$ denote the solution of the above problem.[5] It is characterized by the following equations:

$$\partial_{x_s} u - \lambda_s p = 0 \,, \tag{2.9a}$$

$$p \cdot (x_s - e_s) - \tau_s = 0 \,. \tag{2.9b}$$

Substitute $x_s(\tau, a)$ in the FM consumers' problem to get the following reduced-form:[6]

[5] It is apparent from (2.8) that consumption in state s depends only on the transfer in state s. To save on notation, we shall write $x_s(\tau, a)$ instead of $x_s(\tau_s, a)$. Moreover, since prices are always taken as given, we do not explicitly write them as an argument of the demand function.

[6] Here (2.9b) has been used to substitute for τ_s in (2.7a).

$$
\begin{aligned}
\max_{\tau,a} \quad & v(x_1(\tau,a), x_2(\tau,a), a) \\
\text{s.t.} \quad & \pi_1(a)\,(p\cdot(x_1(\tau,a)-w_1)) + \pi_2(a)\,(p\cdot(x_2(\tau,a)-w_2)) \leqslant 0\,, \\
& a = \arg\max v(x_1(\tau,a), x_2(\tau,a), a)\,. \qquad (2.10\text{a})
\end{aligned}
$$

Replace (2.10a) with the corresponding first order condition:

$$
\partial_{x_1} v \cdot \partial_a x_1 + \partial_{x_2} v \cdot \partial_a x_2 + \partial_a v = 0\,. \qquad (2.11)
$$

Since (2.9a)$-$(2.9b) imply that:

$$
\partial_{x_s} v \cdot \partial_a x_s = \pi_s \partial_{x_s} u \cdot \partial_a x_s = \pi_s \lambda_s (p \cdot \partial_a x_s) = 0\,,
$$

(2.11) reduces to:

$$
\partial_a v(x_1(\tau,a), x_2(\tau,a), a) = 0\,, \qquad (2.12)
$$

and the FM consumer's problem can be equivalently written as follows:

$$
\begin{aligned}
\max_{\tau,a} \quad & v(x_1(\tau,a), x_2(\tau,a), a) \\
\text{s.t.} \quad & \pi_1(a)\,(p\cdot(x_1(\tau,a)-w_1)) + \pi_2(a)\,(p\cdot(x_2(\tau,a)-w_2)) \quad \leqslant \quad 0\,, \\
& \partial_a v(x_1(\tau,a), x_2(\tau,a), a) \quad = \quad 0\,.
\end{aligned}
$$

This is the version of the FM consumers' problem we are interested in. It has the same structure as the CM consumers' problem, the only difference being that the admissible consumption bundles are those satisfying (2.8).

An interior solution of the above problem is characterized by the following equations:[7]

[7]In deriving (2.14a)$-$(2.14b) we used (2.9a) and the fact that $\partial_{\tau_2} x_1 = \partial_{\tau_1} x_2 = 0$, while in deriving (2.14c) we used the fact that $p \cdot \partial_a x_s = 0$.

$$
\begin{aligned}
\pi_1\lambda_1 - \gamma\pi_1 - \mu\left(\partial_{ax_1}v \cdot \partial_{\tau_1}x_1\right) &= 0\,, && (2.14a)\\
\pi_2\lambda_2 - \gamma\pi_2 - \mu\left(\partial_{ax_2}v \cdot \partial_{\tau_2}x_2\right) &= 0\,, && (2.14b)\\
\gamma\sigma + \mu\partial_{aa}v + \mu\left(\partial_{ax_1}v \cdot \partial_a x_1 + \partial_{ax_2}v \cdot \partial_a x_2\right) &= 0\,, && (2.14c)\\
\pi_1\left(p \cdot (x_1 - w_1)\right) + \pi_2\left(p \cdot (x_2 - w_2)\right) &= 0\,, && (2.14d)\\
\partial_a\, v(x_1, x_2, a) &= 0\,, && (2.14e)
\end{aligned}
$$

where γ is the multiplier for the first constraint, μ is the multiplier for the second constraint and:

$$
\sigma \;:=\; \partial_a\pi_1\left(p \cdot (x_1 - w_1)\right) + \partial_a\pi_2\left(p \cdot (x_2 - w_2)\right).
$$

Multiply (2.14a)$-$(2.14b) by p and use (2.9a) to get the following system of equations:

$$
\begin{aligned}
\partial_{x_1}v - \gamma\pi_1 p - \mu\left(\partial_{ax_1}v \cdot \partial_{\tau_1}x_1\right)p &= 0\,, && (2.15a)\\
\partial_{x_2}v - \gamma\pi_2 p - \mu\left(\partial_{ax_2}v \cdot \partial_{\tau_2}x_2\right)p &= 0\,, && (2.15b)\\
\gamma\sigma + \mu\partial_{aa}v + \mu\left(\partial_{ax_1}v \cdot \partial_a x_1 + \partial_{ax_2}v \cdot \partial_a x_2\right) &= 0\,, && (2.15c)\\
\pi_1\left(p \cdot (x_1 - w_1)\right) + \pi_2\left(p \cdot (x_2 - w_2)\right) &= 0\,, && (2.15d)\\
\partial_a\, v(x_1, x_2, a) &= 0\,. && (2.15e)
\end{aligned}
$$

To highlight the role of the assumptions on the fundamentals of the models, it is useful to rewrite the last addends in (2.15a)$-$(2.15c) in terms of the (Bernoulli) utility index u as follows:[8]

$$
\begin{aligned}
\partial_{x_1}v - \gamma\pi_1 p - \mu\partial_{ax_1}v + \psi_1 &= 0\,, && (2.16a)\\
\partial_{x_2}v - \gamma\pi_2 p - \mu\partial_{ax_2}v + \psi_2 &= 0\,, && (2.16b)\\
\gamma\sigma + \mu\partial_{aa}v + \psi_a &= 0\,. && (2.16c)
\end{aligned}
$$

[8]Calculations are in the Appendix.

where:

$$\psi_s \quad := \quad \mu\pi_s \left(\partial_{ax_s} u - \left(\partial_{ax_s} u \cdot \partial_{\tau_s} x_s\right) p\right) \,, \tag{2.17}$$

and:

$$\psi_a \quad := \quad \mu \left(\pi_1 \left(\partial_{ax_1} u \cdot \partial_a x_1\right) + \pi_2 \left(\partial_{ax_2} u \cdot \partial_a x_2\right)\right) \,. \tag{2.18}$$

Notice that (2.16a)−(2.16b) are together $2L$ equations, while (2.16c) is a single equation.

2.4.5 Comparing the first order conditions

Recall the system of equations that characterize the CM equilibrium:

$$\partial_{x_1} v - \gamma\pi_1 p - \mu\partial_{ax_1} v \quad = \quad 0\,, \tag{2.19a}$$

$$\partial_{x_2} v - \gamma\pi_2 p - \mu\partial_{ax_2} v \quad = \quad 0\,, \tag{2.19b}$$

$$\gamma\sigma + \mu\partial_{aa} v \quad = \quad 0\,, \tag{2.19c}$$

$$\pi_1 \left(p \cdot (x_1 - w_1)\right) + \pi_2 \left(p \cdot (x_2 - w_2)\right) \quad = \quad 0\,, \tag{2.19d}$$

$$\partial_a \, v(x_1, x_2, a) \quad = \quad 0\,. \tag{2.19e}$$

We shall compare these equations with those characterizing the FM equilibrium:[9]

$$\partial_{x_1} v - \gamma\pi_1 p - \mu\partial_{ax_1} v + \psi_1 \quad = \quad 0\,, \tag{2.20a}$$

$$\partial_{x_2} v - \gamma\pi_2 p - \mu\partial_{ax_2} v + \psi_2 \quad = \quad 0\,, \tag{2.20b}$$

$$\gamma\sigma + \mu\partial_{aa} v + \psi_a \quad = \quad 0\,, \tag{2.20c}$$

$$\pi_1((p \cdot (x_1 - w_1)) + \pi_2 \left(p \cdot (x_2 - w_2)\right) \quad = \quad 0\,, \tag{2.20d}$$

$$\partial_a \, v(x_1, x_2, a) \quad = \quad 0\,. \tag{2.20e}$$

[9]For simplicity we neglect the market clearing conditions, as they are not relevant for the present analysis.

As the last two equations are the same in both systems, any difference must come from (2.20a)−(2.20c). These equations in turns differs only for the ψ-terms. It is therefore necessary to study when $\psi_s = 0$ and $\psi_a = 0$. This happens if and only if the following $(2L+1)$ equations are satisfied:

$$\partial_{ax_1} u - (\partial_{ax_1} u \cdot \partial_{\tau_1} x_1)\, p = 0\,, \tag{2.21a}$$

$$\partial_{ax_2} u - (\partial_{ax_2} u \cdot \partial_{\tau_2} x_2)\, p = 0\,, \tag{2.21b}$$

$$\pi_1 (\partial_{ax_1} u \cdot \partial_a x_1) + \pi_2 (\partial_{ax_2} u \cdot \partial_a x_2) = 0\,. \tag{2.21c}$$

When there is a single consumption good and when preferences are separable, the above equations are *identities*.

Indeed, if there is a single consumption good, p is normalized to one, $\partial_{\tau_s} x_s \equiv 1$ and $\partial_a x_s \equiv 0$, so that the above system is identically satisfied.[10]

If preferences are separable, then $\partial_{ax_s} u \equiv 0$, so that the above system is again identically.

In the two above cases, the system of equations characterizing the CM and the FM equilibrium are identical. Hence, the equilibrium allocation of goods and action level must be the same. Given that every CM equilibrium is constrained efficient, so is every FM equilibrium.

The following proposition collects these findings:

Proposition 12. *Suppose that there is single consumption good or that assumption S holds. Then every FM equilibrium is constrained efficient.*

We now investigate the possibility that (2.21a)−(2.21c) are *not* satisfied, since in this case we could conclude that a FM equilibrium is not constrained efficient.

Let $\hat{\zeta}_s := (\partial_{ax_s} u \cdot \partial_{\tau_s} x_s)$. Then (2.21a)−(2.21b) imply:

$$\partial_{ax_1} u = \hat{\zeta}_1 p\,, \tag{2.22a}$$

$$\partial_{ax_2} u = \hat{\zeta}_2 p\,. \tag{2.22b}$$

[10] Notice that in this case $\partial_{ax_s} v$ is a scalar.

According to (2.22a)$-$(2.22b), for the system (2.21a)$-$(2.21c) to be satisfied it is necessary that $\partial_{ax_s} u$ and p are collinear, with scale factor $\hat{\zeta}_s \neq 0$. If this condition is violated, the system cannot be satisfied. The following proposition formalize this intuition:

Proposition 13. *Suppose there are multiple consumption goods and preferences are non-separable. Provided that $\partial_{ax_s} u \neq \zeta_s p$ for $\zeta_s \neq 0$, a FM equilibrium is not constrained efficient.*

The following proposition verifies that the condition in the above proposition is not empty:

Proposition 14. *Suppose that there are multiple consumption goods and that preferences satisfy assumption H. Then an FM equilibrium is not constrained efficient.*

Proof. The proof consists in verifying that $\partial_{ax_s} u \neq \zeta_s p$ for $\zeta_s \neq 0$. When assumption H holds, it follows that:

$$\partial_{ax_{sj}} u \equiv 0\,, \quad \text{and} \quad \partial_{ax_{sl}} u = \partial_{ax} g(x_{sl}, a) \quad \text{for} \quad l \neq j\,.$$

If it is true that $\partial_{ax_s} u = \zeta_s p$ for $\zeta_s \neq 0$, it must be true that:

$$0 = \zeta_1 p_j\,, \tag{2.23a}$$

$$\partial_{ax} g(x_{1l}, a) = \zeta_1 p_l \quad \text{for} \quad l \neq j\,, \tag{2.23b}$$

$$0 = \zeta_2 p_j\,, \tag{2.23c}$$

$$\partial_{ax} g(x_{2l}, a) = \zeta_2 p_l \quad \text{for} \quad l \neq j\,. \tag{2.23d}$$

Since the above system is inconsistent, we conclude that $\partial_{ax_s} u \neq \zeta_s p$. ∎

For a further intuition of the above result, compare the restrictions on $\partial_{x_s} u$ $-$ the gradient of the (Bernoulli) utility index u $-$ both in the FM and the CM equilibrium. According to (2.9a), in a FM equilibrium the consumption allocation must be such that $\partial_{x_s} u$ and p are collinear. According (2.6), this restriction is satisfied in a CM equilibrium provided that assumption S holds, or that preferences are non-separable and $\partial_{ax_s} u = \zeta_s p$ for $\zeta_s \neq 0$.

2.5 Related literature

The results in this chapter are closely linked to a series of papers that analyze the connection between hidden action, market structure and efficiency in perfectly competitive economies.

These models share some common assumptions. One of the most popular is that of large number of individuals, possibly grouped according to verifiable types, together with the one of idiosyncratic uncertainty, possibly augmented by aggregate uncertainty.

In this respect, these are generalizations to the case of hidden action of the model of Malinvaud [21], in which both types of market structure considered here are analyzed.[11]

The idea of the CM equilibrium dates back to the seminal paper of Prescott and Townsend [24]. Formally, their model is rather different from ours. To avoid non-convexities both in the utility functions, due to a discrete action set, and in the budget constraints, lotteries on consumption goods are introduced, and consumers are allowed to choose among them. While the objects of trades are different, the market structure and the nature of the price system are analogous to those considered here. As a result, an appropriate version of proposition 11 holds for their model too.

Closely related to the model of Prescott and Townsend [24] is that of Kocherlakota [19]. The main formal differences are the absence of lotteries, since non-convexities are eliminated by appropriate assumption on the utility function and on the action set, and the hypothesis of a single consumption good.

Rustichini e Siconolfi [26] have revised the model of Prescott and Townsend, generalizing the condition for existence and optimality of equilibrium.

In all the above models, consumption allocation is determined through trading in markets open at $t = 0$. This hypothesis, and its implications for efficiency, has been criticized in a series of papers by Arnott and Stiglitz [4], Greenwald and Stiglitz [14] and Arnott, Greenwald and Stiglitz [2].

[11] Given the absence of asymmetric information, the equivalence between the equilibrium with a complete set of contingent-commodity markets and the equilibrium with financial and spot markets is not an issue.

In these papers, it is assumed that a benevolent planner can redistribute income through insurance contracts assigned at $t = 0$, and tax or subsidize trades on the spot markets. It is then investigated when the planner finds it optimal to actually impose taxes or subsidies. Indeed, Arnott and Stiglitz in the working paper version [3] of their article [4] find that one of these cases arises if there are multiple consumption goods and preferences satisfy assumption H.

Clearly, the idea of the FM equilibrium as above proposed is meant to capture the importance of this difference in the structure of trades.

Under simpler assumptions, the FM equilibrium has been studied by Helpman and Laffont [17], who obtained an efficiency result analogous to proposition 12 for the case of a single consumption good, and by Lisboa [20], who obtained an efficiency result analogous to proposition 12 for the case of separable preferences.

2.6 Appendix

In this Appendix we derive equations (2.16a)−(2.16c). Recall that:

$$\partial_{ax_s} v = \partial_a \pi_s \partial_{x_s} u + \pi_s \partial_{ax_s} u\,.$$

Since $p \cdot \partial_{\tau_s} x_s = 1$, we get the following chain of equalities:

$$\begin{aligned}
(\partial_{ax_s} v \cdot \partial_{\tau_s} x_s)\, p &= ((\partial_a \pi_s \partial_{x_s} u + \pi_s \partial_{ax_s} u) \cdot \partial_{\tau_s} x_s)\, p \\
&= (\partial_a \pi_s (\partial_{x_s} u \cdot \partial_{\tau_s} x_s) + \pi_s (\partial_{ax_s} u \cdot \partial_{\tau_s} x_s))\, p \\
&= (\partial_a \pi_s \lambda_s (p \cdot \partial_{\tau_s} x_s) + \pi_s (\partial_{ax_s} u \cdot \partial_{\tau_s} x_s))\, p \\
&= \partial_a \pi_s \lambda_s p + \pi_s (\partial_{ax_s} u \cdot \partial_{\tau_s} x_s)\, p \\
&= \partial_a \pi_s \partial_{x_s} u + \pi_s (\partial_{ax_s} u \cdot \partial_{\tau_s} x_s)\, p \\
&= \partial_{ax_s} v - \pi_s \partial_{ax_s} u + \pi_s (\partial_{ax_s} u \cdot \partial_{\tau_s} x_s)\, p \\
&= \partial_{ax_s} v - \pi_s (\partial_{ax_s} u - (\partial_{ax_s} u \cdot \partial_{\tau_s} x_s)\, p)\,. \qquad (2.24)
\end{aligned}$$

Since $p \cdot \partial_a x_s = 0$, we also get the following chain of equalities:

$$
\begin{aligned}
(\partial_{ax_s} v \cdot \partial_a x_s) &= (\partial_a \pi_s \partial_{x_s} u + \pi_s \partial_{ax_s} u) \cdot \partial_a x_s \\
&= \partial_a \pi_s \left(\partial_{x_s} u \cdot \partial_a x_s \right) + \pi_s \left(\partial_{ax_s} u \cdot \partial_a x_s \right) \\
&= \partial_a \pi_s \lambda_s \left(p \cdot \partial_a x_s \right) + \pi_s \left(\partial_{ax_s} u \cdot \partial_a x_s \right) \\
&= \pi_s \left(\partial_{ax_s} u \cdot \partial_a x_s \right) \, . \qquad (2.25)
\end{aligned}
$$

Substituting (2.24) and (2.25) in (2.15a)$-$(2.15c) we get the equations in the text.

Chapter 3

Pool of promises and hidden action

(This is a joint work with C. Goulao, Core.)

Health, disability and longevity insurance is widely recognized as a necessity, but there is disagreement on how should it be organized. On one extreme, there are those who support a market-based system. On the other extreme, those supporting a compulsory public system. The debate is still far from being solved. Even recently, in the USA - one of the countries that rely the most on the private provision of health insurance - the Massachusetts governor announced the creation of a health insurance universal system to be implemented already in 2007.

In addition to the problem of the private or public nature of provision, there is the question of its compulsoriness. Should insurance against those risks be compulsory or voluntary? In developing countries, due to the lack of reliable systems of tax collection, it is frequently suggested that insurance should rely more on the market. The problem is that the population of these countries tends to be more often excluded from the market itself. As a consequence Pauly *et alii* [9] suggest that it is reasonable to think of insurance cooperatives as an adequate form of insurance organization in these countries.

Following this intuition, this paper studies a model in in which risk-averse consumers get insurance coverage against their uncertain endowment by voluntary participation in pools of promises. More precisely, consumers commit to contribute to the pool a fraction of their endowment, thus gaining the right to receive a fraction of the total return of the pool in the proportion of their contribution.

Consumers take the total return of the pool as given, and are unrestricted in the amount of positive promise they can make. Hence it may appear that the structure we think of is not compatible with the incentives consumers need to take an action level high enough. Therefore one may suspect that at equilibrium consumers choose too big an insurance coverage, and too low an action level. Indeed, this is a possible equilibrium configuration, but it is not the only one. In particular, there exists an equilibrium in which some consumers choose an high action level, thui raising the return of the pool.

Finally, we consider the case of heterogeneous groups of consumers, and we investigate whether it is possible to form an aggregate pool of promises, as opposed segregated pools. When consumers differ in their endowments, it may be conjectured that those from a wealthier group never find it acceptable to join a pool together with those from a poorer group, for the latter would lower the pool's return.

Contrary to this intuition, we provide an example showing that consumers from a rich group have no loss in welfare by joining a pool together with consumer from a poor group. Since the latter are better off than being segregated, the example shows the potential benefits of aggregated pools of promises among heterogeneous consumers.

3.1 The economy

Consider a pure exchange economy with a single consumption good. The economy is populated by a large number of consumers all ex ante equal, and lasts two periods $t = 0, 1$.

There is no consumption in the first period, when consumers face an idiosyncratic uncertainty which realizes in the second period. At $t = 1$ each consumer may be in one out two states $s \in \mathcal{S} = \{1, 2\}$.

Uncertainty affects only the level of the single consumption good consumers are endowed with at the beginning of the second period. Individual endowments are denoted by the vector $w = (w_1, w_2) \in \mathbb{R}^2_+$. It is assumed that $w_1 > w_2 \geqslant 0$. With terminology consistent with the insurance set-up, state $s = 1$ will be referred to as the good state.

Consumers are risk averse, hence they wish to acquire insurance coverage against the uncertainty on their endowments. They can do so in an organized insurance market, which will be described later on.

Consumers can also choose an action $a \in \mathcal{A} = \{L, H\}$ affecting, on the one hand, the probabilities of the idiosyncratic states, and on the other hand the utility they get from consumption.

In what follows, π_a denotes the probability of state 1 when action a is chosen. It is assumed that $1 > \pi_H > \pi_L > 0$. The (dis-)utility of the action is equal to c_a. It is assumed that $c_H > c_L = 0$.

Preferences are represented by an expected utility function $v(x, a) : \mathbb{R}^2_+ \times \mathcal{A} \to \mathbb{R}$ given by:

$$v(x, a) := \pi_a u(x_1) + (1 - \pi_a) u(x_2) - c_a \, .$$

Because of the binary action set, preferences represented by v are inherently non-convex. On the other hand, we assume that the (Bernoulli) utility index u is a strictly increasing and strictly concave function.

It is assumed that the action chosen by a consumer remains unknown to anyone else in the economy, so that information is asymmetric. On the other hand, it is assumed that endowments are verifiable at $t = 1$, so that it is possible to determine in which idiosyncratic state consumers are.

3.2 The pool of promises

Being risk-averse, consumers would like to smooth their consumption across idiosyncratic states. Instead of a markets for contingent commodities or a market for insurance contracts, in this model we assume that consumers can insure themselves through participation in a common pool of promises.

Participation to the pool is voluntary and it consists in making at $t = 0$ a binding promise of delivering a fraction θ of the realized endowment to the pool. Pooled deliveries are then redistributed to consumers in proportion of their contribution.

A pool can be costlessly formed, and it is assumed to accept unlimited positive promises. On the other hand, negative promises are forbidden. Hence the restriction $\theta \geqslant 0$ applies. In particular this implies that it is allowed to promise more than the actual endowment. This is not inconsistent, for deliveries to the pool are net of the share of each consumer is entitled to receive.

Since realized endowments are verifiable, we assume that promises are always honored when the realized endowment is positive. If endowment is zero, then it is allowed to default at no cost. This means that consumers who do not contribute because of lack of endowment are still entitled to receive their share of the pooled deliveries.

As an illustration, imagine that there are only two individuals, say i and j, who participate in the pool by promising respectively a fraction θ^i of endowment w^i and a fraction θ^j of endowment w^j. In this case aggregate deliveries to the pool are equal to $\theta^i w^i + \theta^j w^j$. As a consequence of the promises made, individual i, respectively i, is entitled to receive a fraction $\theta^i/(\theta^i + \theta^j)$, respectively $\theta^j/(\theta^i + \theta^j)$, of aggregate deliveries.

As usual in model of hidden action, the action consumers choose depends on the consumption stream, which in turn depends on promises made to the pool. Since we do not restrict promises to be equal across consumers, we cannot restrict consumers to all choose the same action either.

With consistent notation, we let θ_a be the promise of those consumers choosing action a. The return of the pool, that is the ratio of aggregate deliveries to aggregate promises, depends on both θ_a and q, for the action chosen influences the realization of endowments through π_a. This in turn influences total deliveries.

By letting $\bar{w}_a = \pi_a w_1 + (1 - \pi_a) w_2$ denote the average endowment when action a is chosen, the return of the pool is given by the following:

$$\kappa = \frac{q\theta_H \bar{w}_H + (1-q)\theta_L \bar{w}_L}{q\theta_H + (1-q)\theta_L} \,. \tag{3.1}$$

By construction, $w_2 < \bar{w}_L \leqslant \kappa \leqslant \bar{w}_H < w_1$ as $0 \leqslant q \leqslant 1$. In what follows, we say that the return κ of the pool is *consistent* if it satisfies (3.1).

3.3 Consumers' problem

Consumers take the return of the pool as given, and choose promises and action so as to maximize expected utility. Given the above discussion on the pool of promises, we shall writhe the consumers' problem as follows:

$$
\begin{aligned}
\max_{x,\theta,a} \quad & \pi_a u(x_1) + (1-\pi_a)u(x_2) - c_a\,, \\
\text{s.t.} \quad & x_1 = w_1 - \theta(w_1 - \kappa)\,, && \text{(3.2a)} \\
& x_2 = w_2 - \theta(w_2 - \kappa)\,, && \text{(3.2b)} \\
& \theta \geqslant 0\,, && \text{(3.2c)} \\
& a \in \mathcal{A}\,. && \text{(3.2d)}
\end{aligned}
$$

Consistently with the discussion in the previous section, (3.2b)−(3.2c) imply that if $w_2 = 0$, then consumers who promised $\theta > 0$ in the bad state do not deliver anything to the pool, and yet they receive $\theta\kappa > 0$.

Since $w_2 < \kappa < w_1$, participation to the pool with a positive promise allows to decrease consumption relative to the endowment in the good state and to increase it in the bad state, thus providing effective insurance.

In what follows, we let $\psi(\kappa, w)$ be the solution set of the above maximization problem.

3.4 Equilibrium

In an equilibrium with pool of promises, consumers maximize their utility by taking as given the return of the pool. The latter is then endogenously determined in a consistent way. Formally, we propose the following:

Definition 3.4.1. *An equilibrium with pool of promises is $(x, \theta, a, q, \kappa)$ such that:*

(*a*) $(x, \theta, a) \in \psi(\kappa, w)\,,$

(*b*) κ *is consistent*,

(*c*) q *satisfies:*

   (1) $q = 0$ *if* $(x, \theta, a) \in \psi(\kappa, w) \Rightarrow a = L\,,$

   (2) $q = 1$ *if* $(x, \theta, a) \in \psi(\kappa, w) \Rightarrow a = H\,,$

(3) $q \in (0,1)$ *otherwise*.

If at equilibrium $q \in \{0,1\}$, then we say that the equilibrium is with *uniform pool* of promises, while if $q \in (0,1)$, then we say that the equilibrium is with *mixed pool* of promises.

3.5 Existence of equilibrium

In this section we provide conditions under which both uniform-pool and mixed-pool equilibria exists. These conditions will be related to the existence of solutions of a modified consumers' problem analogous to that introduced in the first chapter.

This procedure will also allow an easy comparison of the equilibrium concept discussed in this chapter with the linear price equilibrium with strategic insurance intermediaries discussed in the first one.

3.5.1 The compensated consumers' problem

We shall modify the above consumers' problem as follows: eliminate θ by substituting (3.2a)−(3.2b) with the following equation:

$$x_2 = w_2 - \frac{\kappa - w_2}{w_1 - \kappa}(x_1 - w_1)\,. \tag{3.3}$$

Construct a vector $\vec{\pi} = (\pi, (1-\pi)) \in \mathbb{R}^2_{++}$ by setting:

$$\pi = \frac{\kappa - w_2}{w_1 - w_2}\,. \tag{3.4}$$

Finally rewrite the above consumers' problem as follows:

$$\max_{x,a} \quad \pi_a u(x_1) + (1-\pi_a)u(x_2) - c_a\,,$$

$$\text{s.t.} \quad \vec{\pi} \cdot (x - w) = 0\,, \tag{3.5a}$$

$$x_1 \geqslant 0\,, \tag{3.5b}$$

$$x_1 - w_1 \leqslant 0\,, \tag{3.5c}$$

$$a \in \{L, H\}\,. \tag{3.5d}$$

This in nothing but the compensated consumers' problem considered in the first chapter. With consistent notation, we therefore let $\varphi(\vec{\pi}, \kappa)$ denote its solution set.

The following apparent lemma will be useful for the characterization of the equilibria with pool of promises:

Lemma 3.5.1. *If $(x, a) \in \varphi(\pi, w)$, then there exists θ such that $(x, \theta, a) \in \psi(\kappa, w)$, with κ given by (3.4). Conversely, if $(x, \theta, a) \in \psi(\kappa, w)$, then $(x, a) \in \varphi(\pi, w)$, with π given by (3.4).*

For future reference, notice that when $\kappa = \bar{w}_L$, then (3.4) implies that $\vec{\pi} = \vec{\pi}_L$. In this case we shall also say that consumers can buy insurance at fair price conditional on low action.

3.5.2 Equilibrium with uniform pool

In this section we propose conditions for the existence of equilibria with uniform pool of promises. Recall that this are equilibria in which all consumers choose the same level of promises, hence the same action. First of all, we shall eliminate one of the cases of uniform pool by means of the following:

Proposition 15. *If $(x, \theta, a, q, \kappa)$ is an equilibrium with pool of promises, then $q \neq 1$.*

Proof. If $q = 1$, by definition of equilibrium $a = H$ and $\kappa = \bar{w}_H$. The latter implies $\pi = \pi_H$ by (3.4). From lemma 3.5.1, $(x, a) \in \varphi(\pi_H, w)$, and this implies $a = L$, a contradiction. ∎

By using the properties of the solution set of the compensated consumers' problem, we may then propose the following:

Proposition 16. *If $\{(\bar{w}_L, L)\} = \varphi(\vec{\pi}_L, w)$ then an equilibrium with uniform pool of promises exists.*

Proof. The argument is constructive. Given $\vec{\pi}_L$, (3.4) implies $\kappa = \bar{w}_L$. It follows that $(\bar{w}_L, 1, L) \in \varphi(\bar{w}_L, w)$, hence $q = 0$ and κ is consistent. ∎

For future reference notice that when a uniform pool arises at equilibrium, consumers are indeed exchanging consumption across idiosyncratic states at fair price conditional on low action.

3.5.3 Equilibrium with mixed pool

As shown in the first chapter, if the cost of action is low and the probability of the bad state under low action is high, it may not be optimal choose a full insurance consumption bundle, and $a = L$, when unlimited insurance can be purchased at a fair price condition on low action. This intuition has been verified by proposition 4, which we recall here:

Proposition 17. *For any given* $(\vec{\pi}_H, \vec{\pi}_L, w)$ *such that* $1 > \pi_H > \pi_L > 0$ *and* $w_1 > w_2 \geqslant 0$ *there exist* $c > 0$ *such that, for* $c > c_H > 0$, $(x, a) \in \varphi(\vec{\pi}_L, w)$ *implies* $x_1 > x_2$ *and* $a = H$. *In this case there exist a price of insurance* $\vec{\pi}_*$, *an overinsurance consumption bundle* x_L *and a partial insurance consumption bundle* x_H *such that* $\{(x_H, H), (x_L, L)\} = \varphi(\vec{\pi}_*, w)$.

In light of the analysis of the first chapter it should not come as a surprise that at points of discontinuity of the map φ equilibria with mixed pool of promises arises, as shown by the following:[1]

Proposition 18. *If there exists* $x_H \neq w$, x_L *and* $\vec{\pi}_L <_1 \vec{\pi}_* <_1 \vec{\pi}_H$ *such that:*

$$\{(x_H, H), (x_L, L)\} = \varphi(\vec{\pi}_*, w)\,, \tag{3.6}$$

then an equilibrium with mixed pool of promises exists.

Proof. Again, the argument is constructive. Given $\vec{\pi}_*$, (3.4) gives κ. Given κ and (x_H, x_L), (3.2a)$-$(3.2b) give (θ_H, θ_L). For κ to be consistent it must be that:

$$q = \left[1 + \frac{\theta_H\,(\kappa - \bar{w}_H)}{\theta_L\,(\bar{w}_L - \kappa)}\right]^{-1}. \tag{3.7}$$

Since $\vec{\pi}_L <_1 \vec{\pi}_* <_1 \vec{\pi}_H$ implies $\bar{w}_L < \kappa < \bar{w}_H$, $q \in (0, 1)$ and the proof is complete. ■

The following figure illustrates an equilibrium with mixed pool of promises:

[1] Recall that $\vec{\pi}_a \leqslant_1 \vec{\pi}_*$ means $\pi_a \leqslant \pi_*$, hence $(1 - \pi_a) \geqslant (1 - \pi_*)$.

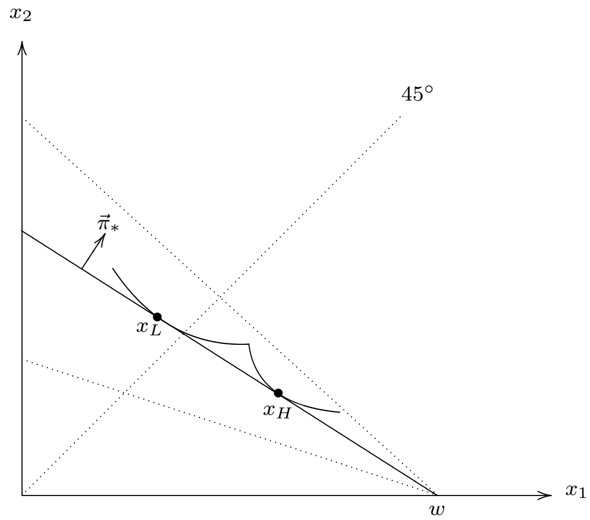

Figure 4

The condition for the existence of an equilibrium with mixed pool of promises is clearly not empty. To verify this, we propose two examples of equilibria with mixed pool of promises.

In both cases it is assumed that the (Bernoulli) utility index u belong to the constant-relative-risk-aversion class. In particular, it is assumed that $u(x) = x^\gamma/\gamma$ for $\gamma < 1$, and that $u(x) = \log x$ for $\gamma = 0$. This will allow a useful comparison with the linear price equilibria analyzed in the first chapter.

Example 1. *Let* $u(x) = x^\gamma/\gamma$ *with* $\gamma = 0.5$, $c_H = 0.163$, $(\pi_H, \pi_L) = (2/3, 1/3)$ *and* $w = (1, 0)$. *An equilibrium with mixed pool of promises exists in this case, and it is such that* $\kappa = 0.4$ *and* $q = 0.56$. *Equilibrium consumption and promises for those consumers choosing high action, respectively low action, are equal to* $(x_H, \theta_H) = ((0.85, 0.095), 0.23)$, *respectively to* $(x_L, \theta_L) = ((0.27, 0.48), 1.21)$. *For future reference, notice that the equilibrium utility level is* $v(x_H, H) = v(x_L, L) = 1.27$.

Example 2. *Let* $u(x) = \log x$, $c_H = 0.135$, $(\pi_H, \pi_L) = (2/3, 1/3)$ *and* $w = (1, 0)$. *An equilibrium with mixed pool of promises exists in this case, and it is such that* $\kappa = 0.4$ *and* $q = 0.3$. *Equilibrium consumption and promises for those consumers choosing high action, respectively low action, are equal to* $(x_H, \theta_H) = ((0.\bar{6}, 0.\bar{2}), 0.5)$, *respectively to* $(x_L, \theta_L) = ((0.\bar{3}\,, 0.\bar{4}), 1.1)$.

We close this section by proposing the following explicit expression for the optimal (interior) promise when action a is chosen, given that the (Bernoulli) utility function is of the constant-relative-risk-aversion class:

$$\theta(\kappa) = \frac{w_1 - \delta w_2}{(w_1 - \kappa)\left[\left(\frac{1-\pi_a}{\pi_a}\right)^{\frac{1}{\gamma-1}}\left(\frac{\kappa - w_2}{w_1 - \kappa}\right)^{\frac{\gamma}{\gamma-1}} + 1\right]},$$

where:

$$\delta = \left[\left(\frac{1-\pi_a}{\pi_a}\right)\left(\frac{\kappa - w_2}{w_1 - \kappa}\right)\right]^{\frac{1}{\gamma-1}}.$$

Indeed, with the help of the above expression, it is possible to verify the remark made at the end of section 3.5.1, since when $\kappa = \bar{w}_L$ and $a = L$, it follows that $\theta(\bar{w}_L) = 1$.

3.6 Linear price equilibrium revised

Comparing proposition 3.7 with proposition 8 in the first chapter reveals that the sufficient conditions for existence of a mixed pool equilibrium is weaker than that for a linear price equilibrium.

This suggests that the former type of equilibria may exist even when the latter do not. This intuition is indeed confirmed by the following:

Proposition 19. *There exists economies with equilibria with pool of promises, but no linear price equilibria.*

Proof. Consider the economy in example 1, which has a mixed pooling equilibrium. For the parameters of this economy, there exists $y \in [0, 2/3]$ such that the following inequality is satisfied:

$$\frac{1}{\gamma}\left[\pi_H\left(1 - \frac{y}{2}\right)^{\gamma} + (1-\pi_H)y^{\gamma} - \left(\bar{w}_L + \frac{y}{2}\right)^{\gamma}\right] - c_H > 0\,.$$

When the above condition holds, no linear price equilibrium exists, as shown in Hellwig [16], proposition 3.8. The proof is then complete. ■

The intuition for the above result is straightforward. A linear price equilibrium in this case do not exist because there exists a profitable deviation for one of the insurance intermediaries. In the present model, there are no intermediaries, hence no deviation can break the equilibrium allocation.

3.7 Extensions: heterogeneous consumers

The equilibrium concept just introduced may extended to the case of heterogeneous groups of consumers. In this case, it is interesting to investigate whether it is possible that a pool of promises is formed by all groups in the economy, or whether there can only exist segregated pools formed by consumers with homogeneous characteristics.[2]

For concreteness, we focus our attention to the case of two large groups of individuals, r and p, differing in preferences and endowments. Consumers in group r have higher expected endowment no matter what action is chosen, and a lower cost of high action, than consumers in group p. In symbols:[3]

$$\bar{w}_a^r > \bar{w}_a^p \quad \text{and} \quad c_H^p > c_H^r \,.$$

The first assumption qualifies consumers in group r as the rich, and those in group p as the poor. The assumption on the cost of high action is natural when interpreted in terms of access wealthier people have to accident-preventing practices.

3.7.1 The pool of promises

When groups are segregated, then the pools formed by consumers by each of them has a return given by (3.1), which we recall here:

$$\kappa^i = \frac{q^i \theta_H^i \bar{w}_H^i + (1 - q^i)\theta_L^i \bar{w}_L^i}{q^i \theta_H^i + (1 - q^i)\theta_L^i} \,. \tag{3.8}$$

for $i \in \{p, r\}$. On the other hand, when an aggregated pool is formed, its return will be given by the following expression:

$$\kappa = \frac{\sum_i q^i \theta_H^i \bar{w}_H^i + \sum_i (1 - q^i)\theta_L^i \bar{w}_L^i}{\sum_i q^i \theta_H^i + \sum_i (1 - q^i)\theta_L^i} \,, \tag{3.9}$$

for $i \in \{p, r\}$. Notice that promises are allowed to be different both within and between groups.

As before, we say that the return of an aggregate pool is consistent if it satisfies (3.9).

[2]It is maintained the assumption that within groups, all consumers are ex ante equal.

[3]It is maintained the assumption that $c_L^i = 0$ for $i \in \{p, r\}$.

3.7.2 Equilibrium

When groups are segregated, the equilibrium concept is analogous to the previous section. Each pool's return is given by (3.8), and for consumers in group i choices are optimal given κ^i.

When an aggregated pool is considered, the equilibrium definition must be amended to take into account that the pool's return is determined by (3.9) and that consumers' choices must be optimal given the same κ. These properties are summarized by the following:

Definition 3.7.1. *An equilibrium with aggregate pool of promises is* $(x^i, \theta^i, a^i, q^i, \kappa)$ *such that for* $i = \{p, r\}$*:*

(a) $(x^i, \theta^i, a^i) \in \psi^i(\kappa, w^i)$,

(b) κ *is consistent*,

(c) q^i *satisfies:*

(1) $q^i = 0$ *if* $(x^i, \theta^i, a^i) \in \psi^i(\kappa, w^i) \Rightarrow a^i = L$,
(2) $q^i = 1$ *if* $(x^i, \theta^i, a^i) \in \psi^i(\kappa, w^i) \Rightarrow a^i = H$,
(3) $q^i \in (0, 1)$ *otherwise*.

Consistently with the previous section, an equilibrium such that $q^i \in (0, 1)$ for $i \in \{p, r\}$ will be referred to as an equilibrium with mixed (aggregate) pool of promises.

3.7.3 Mixed aggregate pool: an example

In this section we verify that above the equilibrium concept is not empty by providing an explicit example of a mixed aggregate pool.

Suppose that there consumers in the rich group have preference and endowments such that:

$$(w_1^r, w_2^r) = (2, 0) \quad \text{and} \quad c_H^r = 0.2\,,$$

while consumers in the poor group have preference and endowments such that:

$$(w_1^p, w_2^p) = (1.5, 0) \quad \text{and} \quad c_H^p = 0.21\,.$$

Suppose also that $(\pi_H^i, \pi_L^i) = (2/3, 1/3)$ for $i = \{p, r\}$. In this case an equilibrium with aggregate mixed pool of promises exists, and it is such that $\kappa = 0.7$ and $(q^p, q^r) = (1, 0.8)$. Among the rich, equilibrium promises are equal to $(\theta_H^r, \theta_L^r) = (0.51, 1.02)$. Among the poor, equilibrium promise is equal to $\theta^p = 1.25$.

What is intriguing in the above example is that the equilibrium is welfare improving with respect to the equilibrium with segregated pools of promises. Indeed, in this case the two pools of promises have return respectively equal to $\kappa^r = 0.7$, with $q^r = 0.1$, and $\kappa^p = 0.52$, with $q^p = 0.1$. Since $\kappa = \kappa^r > \kappa^p$, it follows that consumers in the rich group are as well off in the aggregate pool as the segregated one, while consumer in the poor group are better off in the former.

The reason for this result is essentially due to the fact that in the mixed aggregate pool a large fraction of consumers in the rich group chooses the high action. This compensate the negative effect consumers from the poor group have on the return of the pool, both because they have lower endowments and they all choose the low action.

3.8 Related literature

The idea of the pool of promises has been first proposed by Dubey and Geanakoplos [13] for an economy with adverse selection. Their main goal was to overcome the existence problems of the competitive model with insurance intermediaries studied by Rothschild and Stiglitz [25]. The main conceptual difference with respect to our model is that in their case the proportion of high-risk consumers and low-risk consumers is an exogenous parameter. Hence the variable that corresponds to our q is given, while in our case it is an endogenous variable.

The linear price equilibrium described in the first chapter is a naturally related concept, and so is the literature cited there. It is apparent that in particular that the model of Hellwig [16] is a useful benchmark in elucidating the characteristics of the concept of equilibrium with pool of promises.

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