Search in the Product Market and the Real Business Cycle

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Search in the Product Market and the Real Business Cycle*

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Abstract

Empirical evidence suggests that most firms operate in imperfectly competitive markets. We develop a search-matching model between wholesalers and retailers. Firms face search costs and form long-term relationships. Price bargain results in both wholesaler and retailer markups, depending on firms' relative bargaining power. We simulate the model to explore the role of product market search frictions in business cycles. We show that the way search costs are modelled is crucial to provide a realistic picture of firms' business environment and improve the cyclical properties of an otherwise standard real business cycle model.

Keywords: Business cycle, Frictions, Product market, Price bargain

JEL classification: E10, E31, E32

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1 Introduction

Empirical evidence suggests that most firms operate in imperfectly competitive markets, where they have some power of setting prices themselves, and form long-term relationships with their customers, which are predominantly other firms. These relationships are typically governed by implicit or explicit contracts. Available evidence also shows that firms produce substantial effort, typically in form of advertising or marketing, to find new customers and sell their products to. For instance, in the US total advertising expenditures have been close to 2.5% of GDP over the last 10 years. This effort may also cause an economic chain reaction by increasing sales, consumption and employment. In the standard Walrasian real business cycle model (hereafter RBC, see for instance King and Rebelo (1999) for an in depth exposition), the product market is perfectly competitive and adjustments occur without frictions. Given the above stylised facts, this paper aims to provide a more realistic story of business relationships and price formation mechanism than in the RBC model, and explores whether this can play a significant role in the propagation of shocks.

More precisely, we replace the Walrasian product market of the standard business cycle model with a product market with frictions by following Pissarides (2000) and the associated search-matching literature. In our model, downstream producers act as wholesalers and have long-term relationships with upstream retailers, who in turn sell to final consumers. Retailers act as intermediaries between producers and consumer; they alleviate the search costs for final consumers. We believe that this is fair characterization of most product markets in industrialized economies. Only in very special markets do producers sell directly to consumers without intermediaries. Wholesalers produce effort (e.g. advertising or marketing) to find retailers to sell their products to. Retailers produce effort (e.g. by employing purchasing managers) to find wholesalers to buy their products from in order to refill their stores and enlarge their selection. The amount of products exchanged therefore depends on their respective search efforts. Moreover, every buyer-seller contact generates a surplus over which the wholesaler and the retailer bargain. We therefore provide a story how wholesalers and retailers meet in the market and for the subsequent price formation mechanism between them. Still, our model makes use of a simple representation and remains very close to the standard RBC model.[1]

The role of marketing frictions (usually consumer search frictions) have been analyzed to explain industry and firm dynamics (Fishman and Rob (2003)) and in the literature in international business cycles to explain price differences between countries (Alessandria (2004, 2009)) or the behaviour of imports, exports and the terms of trade (Drozd and Nosal (2008)). In standard (closed economy) RBC models, most papers introducing imperfections in the product

[1] We acknowledge that the model in this paper provides a too simplistic view of what in reality can be considered a rather complex relationship between buyer and seller. Clearly, contracting parties do not only bargain prices, but also quantities, discounts, after sales services, etc... However, this is beyond the scope of this paper.

market assume monopolistic wholesalers, as Blanchard and Giavazzi (2003), Messina (2006) or the recent DSGE literature (see for instance Christiano et al. (2005) or Smets and Wouters (2003)). Alternatively, in Fagnart et al. (2007), wholesalers experience privately observed and uninsured idiosyncratic shocks, which generates a sub-optimal equilibrium. But none of these approaches allow for bilateral relationships and negotiations between wholesalers and retailers. Very recent related approaches that aim at providing better descriptions of customer-firm relationships are those by Hall (2008), Arsenau and Chugh (2007) and Kleshchelski and Vincent (2007). Hall (2008) explores customer search and seller recruiting by adapting principles of the labour market search and matching models to the product market. In his model, producers invest heavily in attracting final customers, as they receive a large share of the surplus. Hall's approach is concerned with retail markets, and there are frictions but no bargaining between customers and sellers. Arsenau and Chugh (2007) extend Hall's model and explore the effects of different bargaining assumptions. They specifically set out to analyse how the distributive role of prices through the notion of fairness affects price dynamics. In Kleshchelski and Vincent (2007), customers incur switching costs. Customers and firms form long-term relationships and idiosyncratic marginal shocks are only incompletely passed through into prices. However, all three approaches are concerned with the relationship between retail firms and final consumers, whereas our model provides a story of firms' relationships and of the price formation process. Moreover, we provide an in depth exploration of the RBC and welfare properties and compare simulation results to real US data.

Our key findings are as follows: First, the price bargain results in one markup for wholesalers and another markup for retailers. Respective markups depend on the relative bargaining power of the wholesalers and the retailers. Markups are procyclical with a productivity shock and countercyclical with "friction shocks". Second, what is important for the results are not frictions *per se* but the way search costs are modelled.[2] In particular, convex search costs are able to produce hump-shaped dynamics for all variables, a highly persistent output and a realistic representation of the product market variables (search and prices). Third, if the total size of the markups (because directly linked to search costs, *cf.* above) is important for the whole results, the way total markups are split between wholesalers and retailers is only distributive and therefore only affect product market variables. Fourth, welfare costs of frictions are quantitatively sizeable with a reasonable calibration.

Section 2 provides some selective evidence on the product market functioning and further motivation of this paper. Sections 3 and 4 develop and discuss the search-matching model with frictions in the product market and price bargain. Sections 5 and 6 present the calibration and some numerical simulations for US data. Section 7 computes the welfare costs of the different

[2]The labour market literature also recently stressed this kind of results (role of vacancy opening costs for the volatility of labour market variables), see for instance Yashiv (2006) or Fujita and Ramey (2007).

inefficiencies and section 8 concludes.

2 On firms' business environment

It is widely accepted that most firms operate in markets, which are governed by imperfections or frictions. By providing search effort firms try to overcome market imperfections. This is motivated by recent evidence, in particular from recent firm surveys. Following the lead by Blinder et al. (1998) firm surveys have been conducted for several industrial countries/areas over the last decade, which improved our understanding of firms' business environment and price setting practices.[3] According to those surveys, it seems that a fair characterisation of a typical or representative firm's business environment is that it operates an imperfectly competitive market and uses some form of mark up pricing above marginal cost as its predominant form of price-setting practice, thereby implying that it is able exert some market power.[4] This firm engages in business-to-business (B2B) rather than business-to-consumer (B2C) relationships, where it typically does most business with repeat customers and forms long-term relationships with them. A long-term relationship, which can also be regarded as one form of an implicit contract based on principles of trust and fairness, is an effective way to reduce search cost, which the firm otherwise would have to bear. Together, these surveys provide a strikingly coherent set of empirical results and a challenge to many modelling assumptions usually employed in standard Walrasian macroeconomic models.

To substantiate the above points, consider that in the US, 85% of firms surveyed by Blinder et al. (1998) indicate that they mainly engage in long-term relationships with their customers. 77% of their main customers are other firms. The corresponding figures for other industrialised economies are of similar magnitudes (EA 70% and Sweden 86% for the share of long-term customers and 75 and 70% for the share of other firms as main customers). Furthermore, these long-term relationships are mainly governed by contracts. 50% of US firms responded that they have 60% or more of their sales covered by explicit or written contracts, which according to Blinder et al. (1998) is estimated to correspond to 38% of US GDP. The contracts typically last one year (both the median and the mode are 12 months). Furthermore, this theory is of more importance for firms that are primarily engaged in B2B relationships. Surveys for other industrialised countries generally corroborate these findings (see Amirault et al. (2004), Apel et al. (2005) and Fabiani et al. (2006, 2007)).

Since the product market is necessarily imperfect, there is a need for search of customers, and

[3]See Amirault et al. (2004) for Canada, Fabiani et al. (2006, 2007) for 9 euro area (EA) countries, Nakagawa et al. (2000) for Japan, Apel et al. (2005) for Sweden and Hall et al. (2000) for the UK.

[4]For example, 54% of euro area firms answered that they use a mark up pricing strategy. 73% of euro area firms said that their main market is the domestic market. In Canada, the corresponding figure is 81%.

thus advertising and marketing effort, and a need for search of suppliers. The need for advertising, marketing, promotions, public relations, and sales managers provided almost 600,000 jobs in 2006. Similarly, roughly an equally large number of people were engaged in purchasing and buying occupations (Bureau of Labor Statistics, 2007). This represents almost 0.5% of total US employment for each. And still in the US, total annual expenditures in advertising in all the media represented on average 2.4% of GDP over the last decades. In other words, advertising expenditures amounted to 271 billions US dollars in 2005. Figure 1 also shows that over the economic cycle, advertising expenditures are positively correlated with GDP, have a higher volatility than GDP, especially over the last years, and are very persistent. This figure includes spending for advertising in newspapers, magazines, radio, television, direct mail, billboards and displays, Internet, and other forms, and thus also includes the advertising that is directly targeted towards final consumers, such as car manufacturers' or pharmaceuticals' television adverts.[5] Although producers may directly target consumers, consumers buy via intermediaries and hence advertising towards consumers indirectly affects retailers.

[Insert figure 1 about here]

3 Model

There are three types of agents in the economy: households, wholesale firms and retail firms. Goods are produced by wholesale firms and consumed by households. However, conversely to the standard real business cycle literature, we do not assume that products are directly exchanged between producers and consumers; instead we introduce retailers as intermediaries. Retailers buy from producers, who act as wholesalers, and sell to households. Trade frictions are present in the product market between wholesalers and retailers, and we provide an explicit theory of price determination since every wholesaler-retailer cont(r)act generates a surplus over which firms bargain. More precisely, the product market consists of a two-sided search market between sellers (wholesale firms) and buyers (retail firms). Let T_t be the number of contracts between wholesale-retail pairs at period t, a contract meaning that both parties agree to exchange one unit of output. These contracts terminate and the pairs separate at the exogenous rate $0 < \chi < 1$. The contract duration is, thus, on average given by $d = 1/\chi$. This results in a continuous depletion of the stock of contracts, and thus trade volume, and consequently a need to refill it. In order to do so, wholesale firms provide a search effort S_t (marketing or advertising expenditures) to find new buyers; and retail firms provide a search effort D_t (by purchasing agents) to find new sellers. The number of new matches between sellers and buyers is increasing and concave in the search efforts, and assumed to be generated by

[5]Source: www.galbithink.org/ad-spending.htm.

a standard Cobb-Douglas matching function:

$$M_t = \bar{m} S_t^{\gamma} D_t^{1-\gamma}, \tag{1}$$

where $\bar{m} > 0$ and $0 < \gamma < 1$. In analogy to the labour market, the relationship between the search effort of wholesalers and retailers can be regarded as a product market equivalent of the "Beveridge curve". It has search effort of wholesalers on the vertical axis and search effort of retailers on the horizontal. It slopes downwards as wholesalers produce higher effort (advertising) when retailers are reluctant buying goods. Downward and upward shifts in this curve would signify structural improvements and deteriorations in the efficiency of the matching process, respectively. Movements along the curve, in contrast, imply a cyclical adjustment without alteration of the matching efficiency.

The trade volume evolves according to:

$$T_t = (1-\chi) T_{t-1} + M_t. \tag{2}$$

3.1 Households

The economy is populated by a large number of infinitively lived households. Their time endowment is normalized to 1 and split between work N_t and leisure $1 - N_t$. Their current utility is defined as:

$$\mathcal{U}(C_t, 1-N_t) = \log(C_t) + \frac{\theta}{1-\eta}\left((1-N_t)^{1-\eta} - 1\right), \tag{3}$$

where C_t represents consumption. Utility is assumed to be concave in its arguments and specified as in King and Rebelo (1999): $\theta \geq 0$ and $\eta \geq 0$ is the parameter governing the labor supply elasticity. Households receive an income from lending capital to wholesale firms at interest rate $r_t + \delta$, and from working at a wage rate w_t. In each period, they choose the size of the capital investment I_t and labor supply N_t, in order to maximize the present discounted value of their life-time utility[6]:

$$W_t^H = \max_{I_t, N_t} \left\{ \mathcal{U}(C_t, 1-N_t) + \beta W_{t+1}^H \right\}, \tag{4}$$

subject to the constraints:

$$\begin{aligned} C_t + I_t &= w_t N_t + (r_t + \delta) K_t + \Pi_t, & (5)\\ I_t &= K_{t+1} - (1-\delta) K_t, & (6) \end{aligned}$$

where β denotes the discount factor. Equation (5) is the budget constraint. Households own both the wholesale and the retail firms and ultimately receive their profits Π_t. Equation (6) is

[6] One has to bear in mind that all future variables are actually conditional expectations based on the information available at time t. For instance, Z_{t+j} stands for $\mathrm{E}_t(Z_{t+j})$, where Z_t may be any variable or combination of variables. Our simplified notation is however easier to read.

the capital accumulation equation and δ denotes the exogenous capital destruction rate. The first order conditions are:

$$\begin{aligned}\frac{1}{C_t} &= \beta(1+r_{t+1})\frac{1}{C_{t+1}}, &(7)\\ \frac{\theta}{(1-N_t)^{\eta}} &= \frac{w_t}{C_t}. &(8)\end{aligned}$$

3.2 Wholesale firms

The economy is composed of a continuum of identical wholesale firms using capital K_t and labor N_t to produce tradable products T_t, through a Cobb-Douglas production function:

$$T_t = \epsilon_t K_t^{\alpha} N_t^{1-\alpha}, \tag{9}$$

where ϵ_t is a productivity shock and $0 < \alpha < 1$. Given the selling price P_t, the firms choose their optimal search effort, i.e. level of advertising expenditures S_t to find new buyers, as well as the optimal capital-labor ratio to produce the output level T_t. They take as given q_t^S, the rate at which every effort leads to a new match. The rate is defined as:

$$q_t^S = M_t/S_t. \tag{10}$$

Hence, the problem faced by each firm can be summarized by the following dynamic programming problem:

$$W_t^W = \max_{S_t, N_t} \left\{ P_t T_t - w_t N_t - (r_t+\delta)K_t - \kappa\frac{S_t^{\mu}}{\mu} + \beta_t W_{t+1}^W \right\}, \tag{11}$$

subject to the constraints (2), (9) and (10). w_t and $r_t + \delta$ are respectively the labor and the capital costs. We have $\kappa \geq 0$ and assume a convex search cost $\mu \geq 1$.[7] The discount factor β_t is compatible with the pricing kernel of the consumers-shareholders:

$$\beta_t = \beta\frac{\mathcal{U}_{C_{t+1}}}{\mathcal{U}_{C_t}}. \tag{12}$$

The first order condition for the search intensity is:

$$\frac{\kappa S_t^{\mu-1}}{q_t^S} = P_t - \Lambda_t + \beta_t(1-\chi)\frac{\kappa S_{t+1}^{\mu-1}}{q_{t+1}^S}, \tag{13}$$

[7] It is worth noting that $\mu > 1$ is a kind of backdoor way to have diminishing returns in the matching function. To show this, let us define a net matching function as $t(S_t, D_t) = M(S_t, D_t) - \kappa S_t^{\mu}/\mu$. When $\mu = 1$, $t(aS_t, aD_t) = a\,t(S_t, D_t)$, that is we have constant returns to scale. When $\mu > 1$, $t(aS_t, aD_t) < a\,t(S_t, D_t)$, that is we have diminishing returns to scale. Alternatively, we could keep $\mu = 1$ and directly introduce diminishing returns to scale into the matching function, but in this case we loose the Pareto optimality (Hosios) condition (see section 4).

where Λ_t is the real marginal cost and given by:

$$\Lambda_t = \frac{1}{\epsilon_t}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t+\delta}{\alpha}\right)^{\alpha}. \tag{14}$$

The optimal capital-labor ratio is:

$$\frac{K_t}{N_t} = \frac{\alpha}{1-\alpha}\ \frac{w_t}{r_t+\delta}. \tag{15}$$

3.3 Retail firms

The economy is also composed of a continuum of identical retail firms buying tradable products T_t, and selling them to households. At given buying price P_t, the firms choose their optimal search effort D_t, i.e. by setting aside the necessary number of purchasing and buying employees, to find and bargain with new wholesalers. They take as given q_t^D, the rate at which every effort leads to a new match. The rate is defined as:

$$q_t^D = M_t/D_t. \tag{16}$$

Hence the problem faced by each firm can be summarized by the following dynamic programming problem:

$$W_t^R = \max_{D_t}\left\{T_t - P_tT_t - \kappa\frac{D_t^{\mu}}{\mu} + \beta_t W_{t+1}^R\right\}, \tag{17}$$

subject to the constraints (2) and (16). We impose the same search cost and convexity as for wholesale firms.[8] The discount factor β_t is still defined by (12). The first order condition for the search intensity is:

$$\frac{\kappa D_t^{\mu-1}}{q_t^D} = 1 - P_t + \beta_t(1-\chi)\frac{\kappa D_{t+1}^{\mu-1}}{q_{t+1}^D}. \tag{18}$$

3.4 Price formation and markups

Each product market match yields pure economic rents equal to the expected search costs for wholesalers and retailers (including foregone profits). The agreed price is such that these rents are shared and in addition each party is compensated for its incurred costs of forming the match. We follow the labour market literature (see for instance Pissarides (2000)) and assume that the rent sharing is a solution to a Nash (1950) bargaining problem. More precisely, prices are (re-)negotiated between wholesalers and retailers at the beginning of every period through a Nash bargain over the surplus resulting from the match.[9] Because all firms are identical, the

[8] See the calibration and simulation sections for a discussion.

[9] Note that P_t represents a *real* producer price (price of final/consumer products are still normalized to 1 as in a standard real business cycle model). The marginal mark up is equal to the average mark up because all wages are (re-)negotiated every period. In section 6, we discuss the role of price rigidities.

price is the same throughout the economy:

$$\max_{P_t} \left(\frac{\partial W_t^W}{\partial T_t}\right)^{1-\lambda} \left(\frac{\partial W_t^R}{\partial T_t}\right)^{\lambda}, \tag{19}$$

where $0 \leq \lambda \leq 1$ is the retailer bargaining power. This gives:

$$\lambda \, \frac{\partial W_t^W}{\partial T_t} \;=\; (1-\lambda)\, \frac{\partial W_t^R}{\partial T_t}, \tag{20}$$

that simplifies to:

$$P_t \;=\; \lambda \Lambda_t + (1-\lambda). \tag{21}$$

Prices are therefore a weighted average of the marginal cost and 1, and increasing (resp. decreasing) in the bargaining power of the wholesalers (resp. retailers). If wholesalers have no bargaining power, the price is equal to their marginal cost. If retailers have no bargaining power , the price is equal to 1.

Wholesaler and retailer markups

The model generates two markups. The wholesaler markup is the markup above the marginal cost and the retailer markup is the markup above the producer price. They are respectively defined as:

$$\varphi_t^W \;=\; P_t - \Lambda_t = (1-\lambda)(1-\Lambda_t), \tag{22}$$

$$\varphi_t^R \;=\; 1 - \Lambda_t = \lambda(1-\Lambda_t). \tag{23}$$

The wholesaler markup is therefore a fraction $1-\lambda$ (wholesaler bargaining power) of the total surplus Λ_t of the match, and the retailer markup is a fraction $1-\lambda$ (retailer bargaining power) of the total surplus of the match.

We linearize markup equations to look at their cyclicality. This gives:

$$\hat{\varphi}_t^W \;=\; -\left(1 - \frac{\lambda\Lambda}{P}\right)\hat{\Lambda}_t, \tag{24}$$

$$\hat{\varphi}_t^R \;=\; -\frac{\lambda\Lambda}{P}\hat{\Lambda}_t. \tag{25}$$

It is straightforward the two markups are negatively correlated with the marginal costs. With a productivity shock, output increases, marginal cost decreases and the markups are procyclical. We obtain a similar correlation with a government shock (higher output and lower marginal costs because lower wages) and a preference shock for leisure (lower output and higher marginal cost because higher wages). On the other hand, more frictions in the product market (negative shock on $\bar{m}$ or positive shock on κ) decrease the marginal cost and output, and therefore imply countercyclical markups.

3.5 Equilibrium definition

Given initial conditions on K_t, an equilibrium of this economy is a sequence of prices $\{\mathcal{P}_t^r\}_{t=0}^{\infty} = \{r_t, w_t, P_t\}_{t=0}^{\infty}$ and a sequence of quantities $\{\mathcal{Q}_t\}_{t=0}^{\infty} = \{C_t, K_{t+1}, S_t, N_t, D_t\}_{t=0}^{\infty}$ such that:

- given a sequence of prices $\{\mathcal{P}_t^r\}_{t=0}^{\infty}$, $\{C_t, K_{t+1}\}_{t=0}^{\infty}$ are solutions to the household solution (7) and the product market law of motion (2)
- given a sequence of prices $\{\mathcal{P}_t^r\}_{t=0}^{\infty}$, $\{S_t, N_t\}_{t=0}^{\infty}$ are solutions to the wholesaler solutions (13) and (15)
- given a sequence of prices $\{\mathcal{P}_t^r\}_{t=0}^{\infty}$, $\{D_t\}_{t=0}^{\infty}$ is solution to the retailer solution (18)
- given a sequence of quantities $\{\mathcal{Q}_t\}_{t=0}^{\infty}$, $\{r_t, w_t\}_{t=0}^{\infty}$ clear the capital market (5) and the labour market (8)
- the price $\{P_t\}_{t=0}^{\infty}$ is set according to the Nash bargain solution (20)

4 Inefficiency sources

The economy we describe is characterised by two sources of inefficiency. The first source is search costs that induce an inefficiently low level of output. Proposition 1 shows that when search costs disappear, the steady state tends to the Walrasian steady state: labor and capital are priced at their respective marginal productivity, firms make no profit and output is maximised[10].

Proposition 1 *(Search costs and Walrasian output)*
When $\kappa \longrightarrow 0$ (no search costs), the steady state solution tends to the Walrasian one.

Proof. When $\kappa \longrightarrow 0$, first order conditions (13) and (18) simplify to $P = \Lambda$ and $P = 1$. Combining $\Lambda = 1$ with equations (14) and (15), we obtain $w = (1-\alpha)\epsilon(K/N)^{\alpha}$ and $r + \delta = \alpha\epsilon(K/N)^{\alpha-1}$. This means that prices are normalized to 1, wages are equal to the marginal productivity of labor and interest rates (incl. depreciation) are equal to the marginal productivity of capital. Moreover, by replacing P, w, $r+\delta$ in equations (11) and (17) and using (9), we see that profits of wholesalers and retailers are equal to zero. This solution is therefore equivalent to the Walrasian one. ■

The second source of inefficiency results from the search externalities. In a decentralized economy, search process exhibits externalities, and in most cases, the decentralized equilibrium is

[10] Although the steady state tends to the Walrasian one, still the dynamics are different from the dynamics of a standard Walrasian real business cycle.

different from the social planner's equilibrium. With search frictions in the labor market, Hosios (1990), in a static environment, and Merz (1995), in a dynamic environment, show that an efficiency condition (workers' bargaining power equal to unemployed's elasticity in the matching function) exists such that the externalities are internalized and the decentralized outcome is strictly equivalent to the social planner's outcome. In proposition 2, we show that a similar condition exists when the search frictions are in the product market.

Proposition 2 *(Externalities in the decentralized economy)*
When $\lambda = 1 - \gamma$, the decentralized equilibrium is strictly equivalent to the social planner's problem.

Proof. The social planner's problem is solved in Appendix A and the equivalence between the two solutions is proved. ■

In section 7, we quantify the size of these respective inefficiencies.

5 Calibration

The technology shock is the exogenous driving force and is assumed to be AR(1):

$$\log(\epsilon_t) = \rho \ \log(\epsilon_{t-1}) + u_t^{\epsilon}, \tag{26}$$

where ρ is the autoregressive parameter and $u_t^{\epsilon} \sim N(0, \sigma_{\epsilon}^2)$.

We consider two versions of the model. We first remove all the frictions in the product market to obtain a standard Walrasian real business cycle model, where labor and capital are priced at their respective marginal productivity and prices are normalized to 1 (the standard Walrasian real business cycle is presented in Appendix B, "RBC" hereafter). We then add frictions. In this case, labor and capital are priced below their respective marginal productivity, prices are endogenous and lower than unity and firms make profits.

We calibrate our RBC model parameters $\{\beta,\ \delta,\ \alpha,\ \theta,\ \eta,\ \rho,\ \sigma_{\epsilon}\}$ on quarterly data to reproduce some stylized facts for the US economy. We fully follow King and Rebelo (1999). The discount factor is calibrated to yield an average return to capital of 6.5% per annum: $\beta = 1/(1 + 0.065/4)$. To match a capital-output ratio of 8, the rate of capital depreciation set to $\delta = 0.025$. We set $\alpha = 1/3$, which is the standard value for the long run capital income share. We assume that $\eta = 1$ in order to obtain a double log utility function: $\mathcal{U}(C_t, 1 - N_t) = \log(C_t) + \theta \log(1 - N_t)$. We choose $\theta = 3.3$ to match $N = 0.20$, which means that 20% of total available time is used for work. Finally, we set $\rho = 0.979$ and $\sigma_{\epsilon} = 0.0072$ to obtain realistic standard deviation and autocorrelation of output. Table 1 summarizes the calibration (line

"RBC"; similar calibration is found in King and Rebelo (1999), table 2, p. 955).

[Insert table 1 about here]

We also have the RBC parameters in the model with frictions. However, because of these frictions, giving the same values to the RBC parameters changes the steady state. In particular, the capital-output ratio and the employment level decrease (see proposition 1). To keep a similar steady state, we re-calibrate some of these parameters. More precisely, we change δ and θ to keep a capital-output ratio of 8 and an employment level of 0.2. The TFP process is similar in the RBC model and the model with frictions (same ρ and σ_ϵ). We check in the next section how a similar TFP process introduced in different models (RBC *vs.* frictions) changes - or not - the output volatility and persistence. The other parameters $\{\lambda,\ \gamma\ \ \chi,\ \kappa,\ \bar{m}\}$ are specific to the model with search frictions.

The "Hosios' parameters"

If retailers (wholesalers) have no bargaining power, *i.e.* no market power, the wholesaler (retailer) appropriates all rents from the contractual relationship. In reality, the bargaining power of retailers and wholesalers is in-between these extreme cases. It may also vary across markets and depend for instance on the relative size of buyers and sellers (and for example whether firm-specific investments have to be undertaken, which gives rise to the classic hold up problem). Since we do not have any priors and data on "economy-wide" bargaining power, we assume that wholesalers and retailers have the same bargaining power: $\lambda = 1 - \lambda = 0.5$. In order to have a Pareto optimal outcome (see proposition 2), we impose $\gamma = 1 - \lambda = 0.5$. The Pareto optimality assumption is fairly standard in the labour matching literature[11], but we agree this is certainly arbitrary. We therefore conduct a sensitivity analysis on these parameters in section 6.

Search costs and total markup

In our model, prices can be adjusted every period and in any case χ governs the average length of an actual relationship (not a price contract) between wholesalers and retailers. Empirical evidence (see section 2) show that the length of business relationships is on average one year (or alternatively the fraction of business relationships that are new is about 25%) and we set $\chi = 0.25$. Data (see section 2) say that advertising expenditures represent on average 2.4% of output. We have no information for retailers' search costs and assume they are equivalent to wholesalers' costs. Total search costs are therefore:

$$costs = \frac{\kappa}{\mu}\left(S^{\mu} + D^{\mu}\right) = 4.8\%\ T.$$

Another important parameter is the convexity μ of search costs. From equations (13) and (18),

[11] See for instance Merz (1995) and Andolfatto (1996) for such a calibration.

we have:

$$\frac{1-\Lambda}{1-\beta(1-\chi)} = \mu \, \frac{costs}{M}. \tag{27}$$

The term on the left-hand side is the expected discounted surplus of a relationship. The term on the right-hand side is the total search costs per new relationship multiplied by μ. Linear search costs ($\mu = 1$) mean that costs per new relationship must be exactly covered by expected surplus. Strictly convex search costs ($\mu > 1$) mean that costs per new relationship must be more than covered by expected surplus. Our calibration exercise therefore implies a strict relationship between search costs, convexity and total markup. In the next section, we run simulations with alternatively linear and quadratic search costs, at given steady state for total search costs (*cf.* above). The former case leads to $\Lambda \cong 0.95$, *i.e.* the total markup needed is 0.05, whereas the latter case leads to $\Lambda \cong 0.90$, *i.e.* the total markup needed must increase to 0.1. However, given the uncertainty around the estimation of total search costs, we conduct a sensitivity analysis on this variable in section 6.

Other parameters

A result of the calibration exercise is that the value of κ is unimportant for the simulations. Indeed, at given total costs and at given μ, a higher κ will only implies lower steady state values for S and D, and a higher implied value for $\bar{m}$ (matching efficiency parameter), leaving unchanged $\kappa S/q^S$ and $\kappa D/q^D$ and hence the linearized equations.[12] We simply choose κ to have matching probabilities close to 0.5. Another direct implication is that assuming different κ's for wholesalers and retailers would not change our simulation results.

The parameters for each version of the model with frictions (linear *vs.* quadratic search costs) are displayed in table 1 (lines "$\mu = 1$" and "$\mu = 2$").

6 Simulations

We use the autoregressive productivity shock and simulate three different models: (i) the standard Walrasian real business cycle model (RBC), (ii) the model with frictions in the product market presented in section 3 and linear search costs ($\mu = 1$), and (iii) the same model but with quadratic search costs ($\mu = 2$). We compare results to the business cycle characteristics of US data (see Appendix C). The simulation results as well as the US statistics are reported in table 2 and figure 2 (similar results for the RBC version are found in King and Rebelo (1999), table 3

[12] As already explained, this is because parameters are calibrated to match some steady state values. In section 7 (welfare cost of inefficiencies), we instead fix parameters leaving the steady state free to move. As we will see, changes in κ have then obviously steady state and dynamic effects.

p. 957 and figure 10 p. 968).

[Insert table 2 and figure 2 about here]

It is well known that although the Walrasian RBC model does a good job in reproducing consumption and investment behaviour, it suffers from some weaknesses: (i) employment is not volatile enough and wages are too procyclical, (ii) the autocorrelation of output is too weak although the autocorrelation of the shock is close to one (not enough endogenous persistence in the model), and (iii) there are no smooth impulse responses (except for consumption and wages).

The results with linear search frictions are very close to the RBC results. The only improvement comes from the - slightly - smoother reaction of GDP and investment due to the search and matching process on the product market. We also see that frictions dampen the absolute volatility of output (numbers between brackets in table 2). Looking at variables specific to the model with frictions, we see that the advertisement expenditures are procyclical and highly volatile as in data. However, the reaction is very short-lived (autocorrelation close to zero). The short-lived costs combined with $\mu = 1$ make that surplus (markups) do not increase that much (see equation (27) in section 5), which in turn implies an almost negligible fall in the producer price (relative standard deviation of 0.01 with respect to 0.59 in data).

The story is completely different with quadratic search costs. Although the initial reaction of advertisement expenditures is similar, the persistence is much higher. Combined with $\mu = 2$, it explains why surplus (markups) must strongly increase (see equation (27) in section 5). This in turn allows for a much stronger fall in producer prices (relative standard deviation increases from 0.01 with $\mu = 1$ to 0.24 with $\mu = 2$). As displayed in figure 2, quadratic search costs also quite remarkably produce hump-shaped reactions for all variables. Indeed, although linear and quadratic costs generate similar initial increases in *search costs*, the initial increases in *search efforts* (S_t and D_t) are much smaller with a quadratic relationship between efforts and costs. As a result, only few new relationships are initially created which in turn affects investment and labour. Afterwards the hump-shaped reaction follows from the higher persistence in search costs and therefore in search effort. We see that the first-order autocorrelation of output strongly increases and is close to what observed in data. Obviously, this also dampens further the absolute volatility of output. Finally, we observe that search frictions do not modify much (with respect to the standard RBC model) moments related to consumption, investment and the factor prices.

From the above analysis, we see that product market frictions *per se* do not really improve results with respect to the standard real business cycle model. What is important to obtain a hump-shaped reaction is to generate persistent search efforts and this can be achieved through an adequate modelisation of search costs. Moreover, convex search costs require a strong reaction in markups which in turn allows for a high (and realistic) volatility of producer prices. We now discuss further some of our parameters (we use the version with $\mu = 2$ but similar conclusions would be reached with the version $\mu = 1$), and compare our results with other existing attempts to generate hump-shaped reactions.

Bargaining power

The Hosios condition ($\gamma = 1 - \lambda = 0.5$) holds in the benchmark calibration. We look at the effects of changes in the retailers' bargaining power λ. A different λ does not change anything but the way the surplus is split between wholesalers and retailers. The higher the parameter λ, the higher is the surplus share the retailers receive, *i.e.* the lower is the producer price, the lower is the wholesaler markup and the higher is the retailer markup. As a result, the price volatility increases with λ.[13] For instance, moving from $\lambda = 0.1$ to $\lambda = 0.9$ increases the relative standard deviation of P_t from 0.05 to 0.45 (0.59 in data). It is also worth noting that higher retailer (resp. lower wholesaler) markups allow them to bear higher (resp. lower) search costs. In other words, the calibration still assumes total search costs representing 4.8% of output, but these search costs are no more evenly distributed with $\lambda \neq 0.5$.

Search costs

Total search costs themselves are uncertain. We associate search costs to advertising expenditures for wholesalers and assume similar costs for retailers. This is certainly arbitrary and maybe too restrictive. For instance, total marketing budgets (including all needed resources as labour, capital, etc.) could be more informative for wholesalers. We let the total search costs vary from 2.5% to 7.5% of output. It implies that the marginal cost moves from 0.95 to 0.85 (see equation (27) in section 5), or equivalently that total surplus (or total markup) moves from 0.05 to 0.15. We can show that increasing search costs obviously decreases their volatility and increases the volatility of producer prices (because of their lower steady state). Output is also less volatile but more persistent. Moments for the other variables are also affected but only marginally. In conclusion, higher frictions (search costs) combined with higher markups for the retailers improve further the product market representation (higher price volatility and lower cost volatility) but this improvement is not really dramatic. Similarly, we could fine-tune the results by allowing for different μ's for wholesalers and retailers.

Price rigidities

In the model, prices are bargained every period without any cost. However, prices may be subject to convex adjustment costs (see for instance Rotemberg (1982, 1983)) or staggered contracts (see for instance Taylor (1999) or Calvo (1983)). In the extreme case (constant prices), the volatility of the total markup would be unchanged but completely due to the volatility of the

[13] This is obvious from equation (25): $\hat{P}_t = -\hat{\varphi}^R_t = \frac{\lambda\Lambda}{P}\,\hat{\Lambda}_t$.

wholesaler markup (the retailer markup remaining unchanged). As a result, the volatility of the advertising expenditures also strongly increases. In other words, price rigidities destroy (with respect to data) the cyclical properties of the product market. Again, since price rigidities only change the split of the surplus between wholesalers and retailers, other cyclical properties remain unchanged.

Comparison with other approaches

One success of the model (frictions with quadratic search costs) is to generate hump-shaped reactions for all variables. There are of course other existing explanations for such dynamics. For instance, labour market search as in Merz (1995) or Andolfatto (1996) will lead to employment being hump-shaped, but nor investment neither output. On the other hand, capital adjustment costs produces hump-shaped investment but no hump-shaped labour and output. Combining the two may therefore lead to a dynamics similar to ours. Multi-sector models also have interesting properties. Benhabib et al. (2006) show that a three-sector model has a strong propagation mechanism under conventional parameterizations and lead to hump-shaped consumption, output and investment. It however comes at the expenses of a too high labour volatility. We instead generate hump-shaped reactions with only search in the product market without relying upon any extra features.[14]

7 Welfare cost of inefficiencies

In the previous section, we conduct sensitivity analysis (μ, λ and *costs*) keeping constant some steady state values. We now fix instead the parameters and see how changes in one of these parameters affect the steady state, the dynamics and hence the welfare of the economy. In particular, we are interested to compute the welfare cost of the two product market inefficiencies (see propositions 1 and 2 in section 4) to ultimately understand which inefficiency would be meaningful to correct in order to efficiently improve welfare.

We follow Lucas (1987) and calculate the welfare cost as a fraction of the consumption a household would agree to give up each period in return for moving to the efficient situation. We define the expected welfare of an agent in the efficient situation as:

$$W_t^e = \log(C_t^e) + \theta \log(1 - N_t^e) + \beta W_{t+1}^e. \tag{28}$$

Similarly, we define the expected welfare of an agent in the inefficient situation as:

$$W_t^i = \log(C_t^i) + \theta \log(1 - N_t^i) + \beta W_{t+1}^i. \tag{29}$$

If the welfare cost of living in the inefficient economy is ψ, equation (29) can be rewritten as:

$$W_t^i = \log((1-\psi)C_t^e) + \theta \log(1 - N_t^e) + \beta W_{t+1}^i. \tag{30}$$

[14]However, the multisector model is able to amplify shocks, which is not the case with the search model.

By subtracting equation (28) from equation (30), we obtain the welfare cost of the inefficiency:

$$\psi = 1 - \exp((1-\beta)(W_t^i - W_t^e)). \tag{31}$$

We use a second order approximation of equation (31) to avoid the certainty equivalence property and take the model with quadratic search costs ($\mu = 2$ but $\mu = 1$ would lead to similar qualitative results). We first consider the search cost inefficiency. The efficient situation W_t^e is considered when the search cost parameter κ is equal to zero (Walrasian steady state, see proposition 1). We then increase κ and compute W_t^i for each value of κ. The function $\psi(\kappa)$ increases in κ, as displayed in figure 3. With our calibration ($\kappa = 0.35$, see table 1), an household would be willing to give up 10% of her consumption each quarter to live in the efficient/Walrasian world. Such a high number is not surprising. Total search costs themselves already represent 4.8% of output, that is 6.1% of consumption.

We then consider the search externality inefficiency. We show in proposition 2 that when the retailer's bargaining power λ is equal to her search elasticity $1-\gamma$ in the matching function, the decentralised solution is equivalent to the social planner's solution. This is our efficient solution W_t^e. We then move the bargaining power λ from 0.1 to 0.9 and compute W_t^i for each value of λ. We obtain $\psi(\lambda)$, as displayed in figure 4. We see that $\psi(\lambda) = 0$ when $\lambda = 1-\gamma = 0.5$ (see calibration in table 1). The welfare cost increases when the distance between the bargaining power and the matching elasticity increases. For instance, with a bargaining power of 0.2 or 0.8, an household would be willing to give up 3% of her consumption each quarter to live in the social planner's world. This welfare cost of search externalities is therefore small relative to the welfare cost of the search costs.

[Insert figures 3 to 4 about here]

8 Conclusion

This paper develops a theoretical model, where both wholesale and retail firms provide search effort (*i.e* through advertising expenditures and employment of sales and purchasing managers) to meet their customers in a product market with search frictions. Firms form long-term contractual relationships and downstream producers or wholesalers bargain over prices with upstream retailers, who in turn sell to the final consumers. We show that the way search costs are modelled is crucial for the results. In particular, convex search costs generate hump-shaped reactions for all variables and provide nice statistical properties (compared to real data).

Our model remains simple and could be extended along several dimensions. For instance, introducing inventories would break the one for one relationship between production and trade. In good time, firms produce more and build inventories as long as new trade relationships are not formed. Also, introducing a monetary dimension (see Smets and Wouters (2003) or Christiano et al. (2005)) and comparing with the standard monopolistic competition New Keynesian set up would be an exciting research programme. Moreover, the aim of this paper is rather analytic (introduction of product market frictions and effects) but so far nothing is said on the normative implications of our findings. For instance, given the strong adverse effects of search frictions in the product market, policies aiming at reducing these imperfections (lower entry barriers, role of subsidies, taxation, trade associations, ...) might prove powerful. Finally, we could use this setup to discriminate between product market and labour market regulations (see for instance Messina (2006) or Fang and Rogerson (2007) for models with monopolistic competition). We leave these extensions to future research.

References

Alessandria, G. (2004). International deviations from the law of one price: the role of search frictions and market share. *International Economic Review 45*(4), 1263–1291.

Alessandria, G. (2009). Consumer search, price dispersion, and international relative price volatility. *International Economic Review, Forthcoming*.

Amirault, D., C. Kwan, and G. Wilkinson (2004). A survey of the price-setting behaviour of Canadian companies. *Bank of Canada Review Winter 2004-2005*, 29–40.

Andolfatto, D. (1996). Business cycles and labor-market search. *American Economic Review 86*(1), 112–132.

Apel, M., R. Friberg, and K. Hallsten (2005). Microfoundations of macroeconomic price adjustment: Survey evidence from Swedish firms. *Journal of Money, Credit and Banking 37*(2), 313–338.

Arsenau, D. and S. Chugh (2007). Bargaining, fairness, and price rigidity in a DSGE environment. International Finance Discussion Papers, No. 900, Board of Governors of the Federal Reserve System.

Benhabib, J., R. Perli, and P. Sakellaris (2006). Persistence of business cycles in multisector real business cycle models. *International Journal of Economic Theory 2*(3-4), 181–197.

Blanchard, O. and F. Giavazzi (2003). Macroeconomic effects of regulation and deregulation in goods and labor markets. *Quarterly Journal of Economics 118*(3), 879–907.

Blinder, A., E. Canetti, D. Lebow, and J. Rudd (1998). *Asking About Prices: A New Approach to Understanding Price Stickiness*. Russell Sage Foundation: New York.

Bureau of Labor Statistics (2007). Occupational employment and wages, 2006. http://www.bls.gov/oes.

Calvo, G. (1983). Staggered prices in a utility maximizing framework. *Journal of Monetary Economics 12*, 383–398.

Christiano, L., M. Eichenbaum, and C. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1-45), 346–353.

Drozd, L. and J. Nosal (2008). Understanding international prices : Customers as capital. Federal Reserve Bank of Minneapolis, WP 411.

Fabiani, S., M. Druant, I. Hernando, C. Kwapil, B. Landau, C. Loupias, F. Martins, T. Mathä, R. Sabbatini, H. Stahl, and A. Stokman (2006). What firms' surveys tell us about price-setting behavior in the Euro Area. *International Journal of Central Banking 2*(3), 3–47.

Fabiani, S., C. Loupias, F. Martins, and R. Sabbatini (2007). *Pricing Decisions in the Euro Area: How Firms Set Prices and Why*. New York: Oxford University Press.

Fagnart, J., O. Pierrard, and H. Sneessens (2007). Microeconomic uncertainty and macroeconomic indeterminacy. *European Economic Review 51*, 1564–1588.

Fang, L. and R. Rogerson (2007). Product market regulation and market work: A benchmark analysis. NBER Discussion Paper 12981.

Fishman, A. and R. Rob (2003). Consumer inertia, firm growth and industry dynamics. *Journal of Economic Theory 109*, 24–38.

Fujita, S. and G. Ramey (2007). Job matching and propagation. *Journal of Economic Dynamics and Control 31*(11), 3671–3698.

Hall, R. (2008). General equilibrium with customer relationships: A dynamic analysis of rent-seeking. mimeo.

Hall, S., M. Walsh, and A. Yates (2000). Are UK companies' prices sticky? *Oxford Economic Papers 52*(3), 425–446.

Hosios, A. (1990). On the efficiency of matching and related models of search and unemployment. *Review of Economic Studies 57*, 279–298.

King, R. and S. Rebelo (1999). Resuscitating real business cycles. In J. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, Volume I B, Chapter 14. Amsterdam: North-Holland.

Kleshchelski, I. and N. Vincent (2007). Market share and price rigidity. manuscript.

Lucas, R. (1987). *Models of Business Cycles*. Basil Blackwood: Oxford.

Merz, M. (1995). Search in the labor market and the real business cycle. *Journal of Monetary Economics 36*, 269–300.

Messina, J. (2006). The role of product market regulations in the process of structural changes. *European Economic Review 50*(7), 1863–1890.

Nakagawa, S., R. Hattori, and I. Tagagawa (2000). Price setting behaviour of Japanese companies. Bank of Japan Working Paper.

Nash, J. (1950). The bargaining problem. *Econometrica 18*(2), 155–162.

Pissarides, C. (2000). *Equilibrium Unemployment Theory*. The MIT Press: Cambridge.

Rotemberg, J. (1982). Monopolistic price adjustment and aggregate output. *The Review of Economic Studies 49*(4), 517–531.

Rotemberg, J. (1983). Aggregate consequences of fixed costs of price adjustment. *The American Economic Review 73*(3), 433–436.

Smets, F. and R. Wouters (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association 1*(5), 1123–1175.

Taylor, J. (1999). Staggered price and wage setting in macroeconomics. In J. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*. Amsterdam: North-Holland.

Yashiv, E. (2006). Evaluating the performance of the search and matching model. *European Economic Review 50*(4), 909–936.

A The social planners's problem

Social planner

The social planner's maximization problem is:

$$W_t = \max_{C_t, S_t, D_t} \left\{ \log(C_t) + \frac{\theta}{1-\eta} \left((1-N_t)^{1-\eta} - 1 \right) \right\},$$

subject to the constraints:

$$\begin{aligned} T_t &= C_t + K_{t+1} - (1-\delta)K_t + \kappa \frac{S_t^{\mu}}{\mu} + \kappa \frac{D_t^{\mu}}{\mu}, \\ T_t &= F(K_t, N_t) \ = \ \epsilon_t K_t^{\alpha} N_t^{1-\alpha}, \\ T_t &= (1-\chi) T_{t-1} + M(S_t, D_t). \end{aligned}$$

The three first order conditions are:

$$\frac{1}{C_t} = \beta \left[\frac{1-\delta}{C_{t+1}} + \frac{\theta}{(1-N_{t+1})^{\eta}} \ \frac{\alpha}{1-\alpha} \ \frac{N_{t+1}}{K_{t+1}} \right], \tag{P1}$$

$$\frac{\kappa S_t^{\mu-1}}{C_t} \ \frac{1}{M_{S_t}} = \frac{1}{C_t} - \frac{\theta}{(1-N_t)^{\eta}} \ \frac{1}{F_{N_t}} + \beta(1-\chi) \ \frac{\kappa S_{t+1}^{\mu-1}}{C_{t+1}} \ \frac{1}{M_{S_{t+1}}}, \tag{P2}$$

$$\frac{\kappa D_t^{\mu-1}}{C_t} \ \frac{1}{M_{D_t}} = \frac{1}{C_t} - \frac{\theta}{(1-N_t)^{\eta}} \ \frac{1}{F_{N_t}} + \beta(1-\chi) \ \frac{\kappa D_{t+1}^{\mu-1}}{C_{t+1}} \ \frac{1}{M_{D_{t+1}}}. \tag{P3}$$

Decentralised equilibrium

In section 3, the three similar first order conditions are:

$$\frac{1}{C_t} = \beta(1+r_{t+1}) \frac{1}{C_{t+1}}, \tag{D1}$$

$$\frac{\kappa S_t^{\mu-1}}{q_t^S} = P_t - \frac{w_t}{F_{N_t}} + \beta_t(1-\chi) \frac{\kappa S_{t+1}^{\mu-1}}{q_{t+1}^S}, \tag{D2}$$

$$\frac{\kappa D_t^{\mu-1}}{q_t^D} = 1 - P_t + \beta_t(1-\chi) \frac{\kappa D_{t+1}^{\mu-1}}{q_{t+1}^D}. \tag{D3}$$

Using equations (8) and (15), we can rewrite equation (D1) as:

$$\frac{1}{C_t} = \beta \left[\frac{1-\delta}{C_{t+1}} + \frac{\theta}{(1-N_{t+1})^{\eta}} \ \frac{\alpha}{1-\alpha} \ \frac{N_{t+1}}{K_{t+1}} \right]. \tag{D4}$$

Using equations (10), (12), (8) and (21), we can rewrite equation (D2) as:

$$\frac{\kappa S_t^{\mu-1} \gamma}{M_{S_t}} = (1-\lambda) \left(1 - \frac{C_t \theta}{(1-N_t)^{\eta}} \ \frac{1}{F_{N_t}} \right) + \beta \frac{C_t}{C_{t+1}} (1-\chi) \ \frac{\kappa S_{t+1}^{\mu-1} \gamma}{M_{S_{t+1}}}. \tag{D5}$$

Using equations (16), (12), (8) and (21), we can rewrite equation (D3) as:

$$\frac{\kappa D_t^{\mu-1}(1-\gamma)}{M_{D_t}} = \lambda \left(1 - \frac{C_t\theta}{(1-N_t)^{\eta}} \frac{1}{F_{N_t}}\right) + \beta \frac{C_t}{C_{t+1}}(1-\chi) \frac{\kappa D_{t+1}^{\mu-1}(1-\gamma)}{M_{D_{t+1}}}. \tag{D6}$$

Equivalence

The central planner's equilibrium is equivalent to the decentralized equilibrium if and only if the first order conditions (P1)-(P2)-(P3) are equivalent to the first order conditions (D4)-(D5)-(D6). We see that equations (P1) and (D4) are always identical. We also see that $\gamma = 1 - \lambda$ is a sufficient and necessary condition to ensure that the system of equations (P2)-(P3) is equivalent to the system of equations (D5)-(D6).

B The standard Walrasian real business cycle model

The equations of the standard Walrasian real business cycle model are:

$$\begin{aligned}
\frac{1}{C_t} &= \beta(1+r_{t+1})\frac{1}{C_{t+1}}, \\
\frac{\theta}{(1-N_t)^{\eta}} &= \frac{w_t}{C_t}, \\
\epsilon_t K_t^{\alpha} N_t^{1-\alpha} &= C_t + K_{t+1} - (1-\delta)K_t, \\
w_t &= \epsilon_t(1-\alpha)\left(\frac{K_t}{N_t}\right)^{\alpha}, \\
r_t + \delta &= \epsilon_t \alpha \left(\frac{K_t}{N_t}\right)^{(\alpha-1)}.
\end{aligned}$$

C Quarterly US data

From 1971:q1 to 2006:q1.

Real producer price: Monthly PPI deflated by the monthly CPI. The monthly data are transformed into quarterly ones. Source: BLS. Logged and HP-filtered with a 1600 smoothing weight.

Advertising expenditures: Sum of quarterly advertising expenditures in newspaper (source: http://www.naa.org/, seasonally adjusted using X12) and quarterly advertising expenditures in internet (source: http://www.iab.net/resources/ad_revenue.asp). The sum is GDP-deflated, logged and HP-filtered with a 1600 smoothing weight.

GDP: Quarterly gross domestic product. Source: BEA. Logged and HP-filtered with a 1600 smoothing weight.

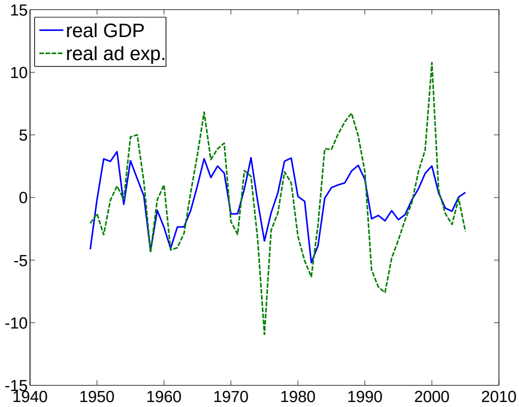
Consumption: Quarterly total private consumption. Source: BEA. Logged and HP-filtered with a 1600 smoothing weight.

Investment: Quarterly total private investment. Source: BEA. Logged and HP-filtered with a 1600 smoothing weight.

Employment: Quarterly employment in the non farm business sector. Source: BLS. Logged and HP-filtered with a 1600 smoothing weight.

Wages: Quarterly hourly compensation in the non farm business sector. Source: BLS. Logged and HP-filtered with a 1600 smoothing weight.

Interest rate: Monthly 3-month Treasury bill nominal rate. Nominal rates are deflated by the realized 3-month inflation rate. The monthly data are transformed into quarterly ones. Source: Federale Reserve Bank of St Louis. HP-filtered with a 1600 smoothing weight.



Sources: www.galbithink.org/ad-spending.htm. The yearly nominal series are GDP-deflated, logged and HP-filtered ($\lambda = 100$) to extract the business cycle components.

Figure 1: Cyclical fluctuations of real advertising expenditures

	β	δ	α	θ	η	ρ	σ_ϵ
RBC	0.984	0.025	0.33	3.34	1	0.979	0.0072
$\mu = 1$	0.984	0.023	0.33	3.30	1	0.979	0.0072
$\mu = 2$	0.984	0.021	0.33	3.06	1	0.979	0.0072
	λ	γ	χ	κ	$\bar{m}$		
$\mu = 1$	0.5	0.5	0.25	0.05	0.52		
$\mu = 2$	0.5	0.5	0.25	0.35	0.51		

Table 1: Parameter values

	relative standard deviation			
	US data	RBC	$\mu = 1$	$\mu = 2$
P_t	0.59		0.01	0.24
ad_t	2.57		2.89	4.53
GDP_t	(1.56) 1.00	(1.39) 1.00	(1.34) 1.00	(0.98) 1.00
C_t	0.81	0.44	0.44	0.50
I_t	3.41	3.38	3.47	3.62
N_t	1.00	0.49	0.51	0.49
w_t	0.64	0.54	0.55	0.56
r_t	0.31	0.03	0.03	0.02

	first-order autocorrelation				contemporaneous correlation with output			
	US data	RBC	$\mu = 1$	$\mu = 2$	US data	RBC	$\mu = 1$	$\mu = 2$
P_t	0.80		0.71	0.34	-0.17		-0.94	-0.37
ad_t	0.88		0.02	0.43	0.77		0.58	0.55
GDP_t	0.88	0.72	0.78	0.90	1.00	1.00	1.00	1.00
C_t	0.88	0.79	0.79	0.76	0.86	0.94	0.95	0.92
I_t	0.92	0.71	0.79	0.88	0.94	0.99	0.99	0.97
N_t	0.94	0.71	0.70	0.58	0.83	0.97	0.96	0.69
w_t	0.81	0.76	0.76	0.86	0.24	0.98	0.98	0.98
r_t	0.55	0.71	0.71	0.87	-0.06	0.96	0.84	0.94

All variables have been logged (with the exception of the real interest rate) and detrended with the HP filter. US data: see Appendix C; RBC: standard Walrasian real business cycle model *à la* King and Rebelo (1999) presented in Appendix B; $\mu = 1$ and $\mu = 2$: models with frictions in the product market presented in section 3. Numbers between brackets: absolute standard deviations. P_t : real producer price, $ad_t = \kappa S_t^{\mu}/\mu$: advertising expenditures, $GDP_t = C_t + I_t$: gross domestic product, C_t : consumption, I_t : investment, N_t : employment, w_t : wage, r_t : interest rate.

Table 2: Steady state and cyclical properties

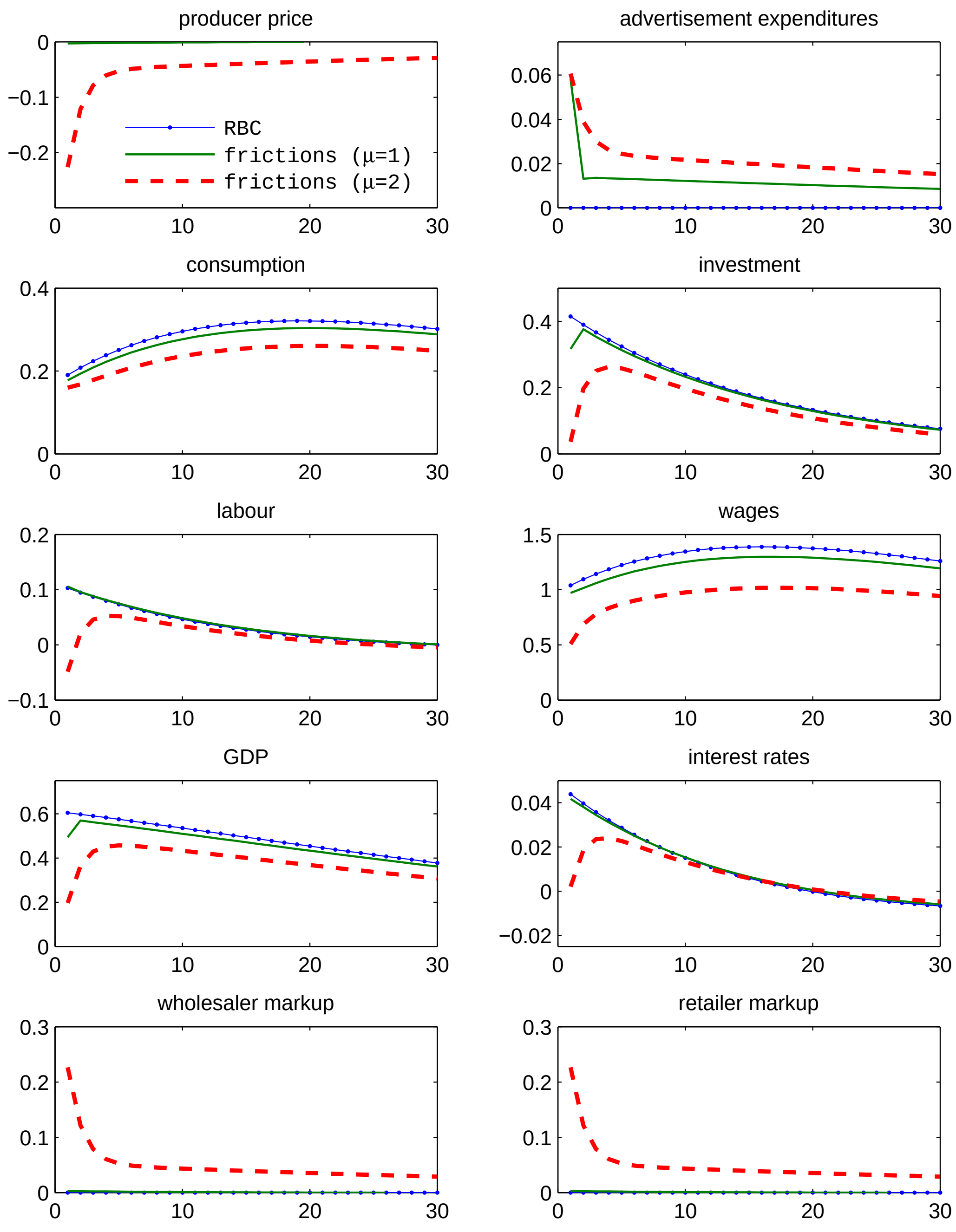


Figure 2: Impulse response functions to a productivity shock

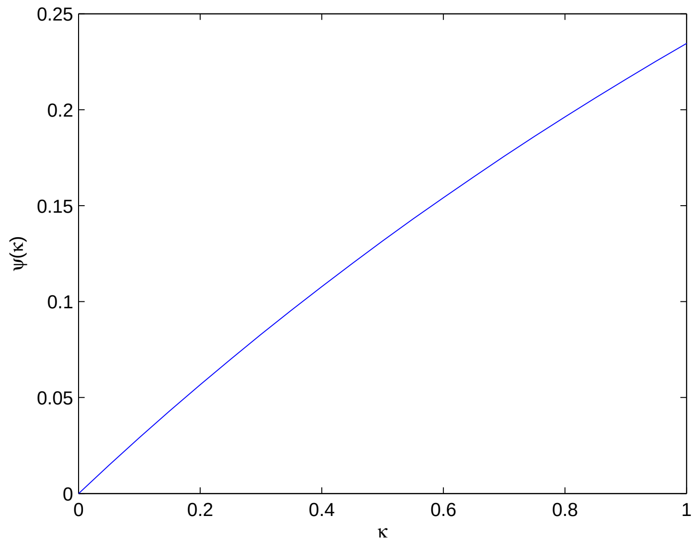


Figure 3: Welfare cost of search frictions

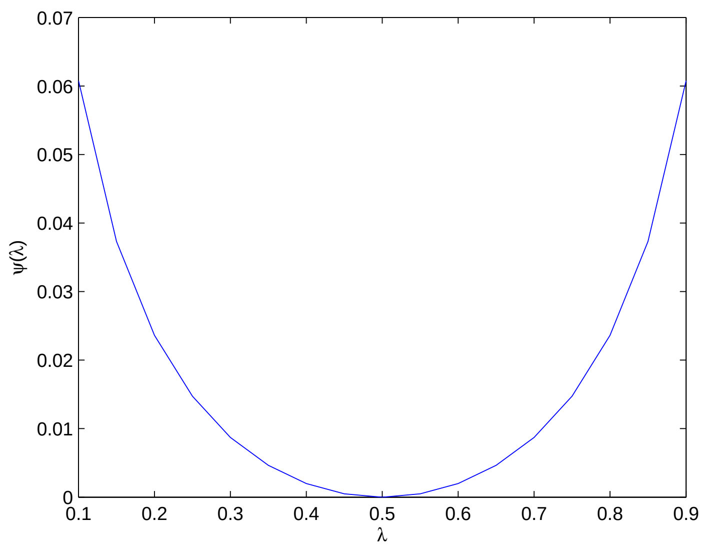


Figure 4: Welfare cost of matching externalities

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