HARMONIZATION, MUTUAL RECOGNITION OR NATIONAL TREATMENT: A MELITZ APPROACH

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Harmonization, Mutual Recognition or National Treatment: a Melitz approach

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Abstract

This paper builds on a Melitz model to compare the welfare effects of three classic legal frameworks used in trade agreements: national treatment, mutual recognition, and harmonization. I specifically deal with two countries setting quality standards in a world where love-for-quality is heterogeneous across country. My results show that harmonization is the best choice in terms of national welfare when exporters are confronted with both lower and higher foreign standards. In addition, with a higher foreign standard, harmonization improves competition in a better way than mutual recognition[1]

I. Introduction

The most recent and most developed multilateral trade agreements largely introduce non-tariff measures, such as sanitary and phyto-sanitary measures (SPS) as well as technical barriers to trade (TBT), which can concern horizontal standards (for example, electrical plug types) as well as vertical standards (for example, CO2 emission limits). Often, those measures are heterogeneous across partners and slow down trade. Indeed, the numerous amounts of trading firms need to update their production to each destination market. Moreover, the impact is likely to be varying across firm's size. Consequently, present, and future partners manage to find common grounds on these legal clauses under which goods and services are traded. Three types of legal framework are commonly and largely used to deal with that, although they are not welfare equivalent. Knowing which one is the most efficient is increasingly crucial since those agreements become more frequent and more complex. A handful of papers already compare their welfare impacts, but none approach them with both many heterogeneous firms and vertical standards. By adapting the Melitz model to quality standards, I wish to remedy this issue. In addition to firm heterogeneity, I take account of specific preferences for each country to deal with this question. This specifically gives new and more precise insights not only on welfare and market size effect, but also on the quality update that occurs when countries make a trade agreement. I will now illustrate the three classical legal trade frameworks with examples from the car industry.

In the national treatment scheme, all firms from all countries selling in the same destination market must follow the same destination rules for each similar product and service. It protects from discrimination; without this, exporters might be treated with harsher conditions. For a government, it is the "simplest" framework among the three usual ones because there is a uniform ruling and certification process. However, foreign exporters encounter adaptation costs to meet each country's rules and this might influence their price. They must adapt their product to each market's standard, even if standard and certification processes in different economic regions mainly aim at a similar goal, such as security and health. A historical example of different standards in two economic regions for a similar good is the Citroën DS car headlamp for French and US markets. Between 1940 and 1983, the US regulation only allowed "sealed beam" headlamps

[1]I would like to thank gratefully for their helpful comments and readings Pr. Vannoorenberghe, Pr. Gilson and for their remarks and questions Pr. Gaigné, Pr. Parenti, Pr. Mayneris. Also for their discussion at the PSE Summer School Pr. Disdier and Pr. Fontagné.

while the European regulation was banning them, forcing Citroën producer to adapt to both markets, although the functionality of headlamps remained identical.

A second legal framework is mutual recognition. Contrarily to national treatment, each country recognizes its partner's standards, and this allows exporters to enter a foreign market with no adaptation costs. While this facilitates trade, it also discriminates between foreign exporters and domestic firms. In the potentially future Transatlantic Trade and Investment Partnership (TTIP) between the US and the European Union, the will is to recognize each other's car safety standards. For example, exported tires made in the US will not need to be certified in Europe. This scheme is also concretely done in the Comprehensive Economic and Trade Agreement (CETA) through the recognition of the certification of good manufacturing practices (GMP) in many industrial sectors such as the car industry, in machinery, electrical goods, and in the pharmaceuticals industry. While this advantages exporters, non-exporters are confronted to a stricter competition.

Finally, the third legal framework considered in this paper is harmonization. Countries must agree on common international standards, in a way that is beneficial for them. All exporters from all origins follow the same rules. It takes some of the pros and cons of national treatment and mutual recognition. On the one hand, is often easier and more transparent for exporters to adapt to a common standard. On the other hand, in some respects, it is still discriminatory. Indeed, the costs are asymmetric for each country since they do not start from the same level of standards: some of them gain, some lose. An example of harmonization is also present in the CETA. Canada has recognized several standards from the United Nations global forum for the harmonization of automotive standards through the "companion standardization body" (CSB)[2]. These standards are internationally set by an external board. Both partners also created a work program on the convergence of safety standards for motorists.

None of the different legal frameworks presented above seems to be indisputably the best policy. As discussed above, they have cons, and they are all used today, even for closely related standards. They also seem to concern only exporters willing to trade in the other partner's market, but they indirectly affect the domestic producers. Taking the standardization of norms as a goal, one can see the evolution going from national treatment towards mutual recognition and then harmonization as if it were linearly improving welfare. However, other players such as non-exporters and consumers are also impacted in different ways. Thus, it is necessary to dive deeper in the economic analysis to better sort the welfare impacts and to understand why one framework should be used more than another given specific determinants (which will be discussed in the results). In this paper, I bring more insights than the past literature by analyzing those three frameworks with the famous Melitz trade framework and with the angle of quality standards.

The welfare analysis of free trade in the literature has not been unidirectional (Arkolakis et al., 2012). In line with my results in this paper, introducing a more complex model such as the Melitz model brings more understanding of the complex problem of trade. The first models in the homogeneous firms' literature[3] present the positive effect of free trade. Indeed, those models of free trade demonstrate that globalization creates more varieties which in turn induces higher competition and higher welfare (through representative consumer's utility and firm's profit). Even more refined models show a global improvement of surpluses for firms (more profits, more market shares, more exports) and for consumers (more varieties, and higher quality) (Amiti and Khandelwal, 2013; Bas and Berthou, 2017; Fan et al., 2018; Hallak and Sivadasan, 2013; Mangelsdorf et al., 2012). But, later, the Melitz model(Melitz, 2003) has brought new discernments on the potentially negative impacts of globalization. It implements heterogeneous firms and follows the increasing availability of data at the time[4]. This data improvement led economist to

[2]https://www.cencenelec.eu/intcoop/Agreements/PSB/Pages/default.aspx

[3]Authors using Krugman's monopolistic competition framework (Krugman, 1979).

[4]At the start of our century, computer developments made available larger databases ; they became cheaper and much

analyse deeper firm's characteristics, such as quality. There are three main results from free trade. First, free trade expands market size and in turn decreases the price threshold[5]: it encourages the entry of low productive firms (the so-called "market-size effect") (Melitz and Ottaviano, 2008)[6]. Second, inequality among firms grows after globalization, decreasing the consumer surplus because of higher prices in the market[7]. Third, with the introduction of quality into the Melitz model along with the study of free-trade agreements, liberalization increases the costs in research and administrative work (Beghin and Bureau, 2001). This encourages companies to discriminate among consumers and destination countries: for example, a new standard on quality is expected to improve consumer's surplus because of higher quality but it might also increase development costs to get to this quality. Thus, it lowers producer's surplus. Moreover, those bad effects can push firms to produce for the less demanding consumers only and to set arbitrary discrimination measures towards developing countries which do not create those high-quality standards (Clerides and Hadjiyiannis, 2008; Disdier, 2018). Also, regulations seem to hurt small exporters the most and Fontagné et al. (2015), Disdier et al. (2018) specify that high-quality product firms are more hit by standardization if they are too small because they now encounter less quality differentiation against their competitors and this lowers the average quality level.

The aim of my paper is to cover the issue of legal frameworks in new trade agreements and to understand whether previous results in the literature are modified by the introduction of a Melitz model accounting for quality standards. It is only recently that welfare is examined with both quality standards and the legal framework of trade agreements. Below, I review the existing literature on legal schemes in trade agreements. To start with, Costinot (2008) builds a game-theory framework with two symmetric countries in order to compare two legal settings, namely mutual recognition (MR) and national treatment (NT). Considering vertical standards (i.e.: standards on the quality level), this author points out that MR makes the governments produce too few standards because they do not account for externalities brought in the exporting market. Lutz and Pezzino (2012) propose a model with two asymmetric countries. In comparison with Costinot (2008), they enlarge the range of agreements by adding a harmonization legal setting (H) to MR and NT. Those authors conclude that MR is always better to improve welfare for two countries of different sizes that cannot agree upon common rules. Finally, in their working paper, Grossman et al. (2019) try to rank NT and MR given their welfare level. They generate a Venables model in order to assess for difference in taste-for-variety across countries, without taking quality into account[8]. Without externalities, they conclude that mutual recognition is the best choice among the three legal frameworks presented above.

As I explain above, I differ from this literature by introducing a Melitz framework with quality standards. From the start, governments can set a quality standard for several reasons. Most of the time, it is set up to solve market distortions such as asymmetries of information and environmental or health externalities. Less evidently, Clerides and Hadjiyiannis (2008) show that it can be used to boost the domestic market. Finally, in an oligopolistic world, quality standards can help to reduce market power asymmetries. In my model, the market distortion is due to the presence of firm heterogeneity in the production of quality which creates a misallocation. This channel has been highlighted in recent papers (Gaigné and Larue (2016), Macedoni and Weinberger (2019), Baqaee and Farhi (2020), Edmond et al. (2018), Weinberger (2020)). It brings a market distortion such that a quality standard improves the optimal situation without standards in terms of welfare under certain conditions. More precisely, those authors find that quality standards allow a reallocation of production from low to high productivity firms. In Gaigné and Larue (2016), the positive welfare

easier to deal with.

[5]More firms in the market increases competition, and lowers the price level.

[6]Thus, continuing this reasoning, it would be more optimal in terms of productivity and efficiency to have few large firms. But this increases market power.

[7]It is observed in more complex Melitz models such as in Parenti (2018), etc.

[8]Venables (1987) builds the first model with homogeneous firms but different market shares in different countries.

impacts are mitigated by the value of the elasticity of fixed cost with respect to quality which needs to be lower than 1 to obtain this result. Macedoni and Weinberger (2019) get rid of the fixed cost to upgrade quality and find no more ambiguity on the positive effect of quality standards on welfare. Coming back to the use of the fixed cost to upgrade quality, I also find a unambiguous positive impact of quality standards on welfare because of different willingness-to-pay for quality across countries (see below). In addition to that, I show that the choice of a quality standard (QS) by the government modifies the distribution of firms in terms of variety, price and quality and that it lowers the extensive margin[9].

To my knowledge, I am the first to make the following set of three assumptions: heterogeneous firms produce different quality for each market, productivity is divided into two categories, and love-for-quality is heterogeneous across countries. These hypotheses have already been studied separately but never together. I will now detail each of these hypotheses.

My first assumption allows firms to produce two levels of quality to sell "different" goods in the two different markets. In the literature, this is similar to Costinot (2008), Lutz and Pezzino (2012), and Macedoni and Weinberger (2019). Product differentiation in quality was first used in a Melitz model by Kugler and Verhoogen (2012) who try to explain why quality is observed in some firms and not others, using R & D and advertisements expenditures as fixed quality investments. This quality feature generates a correlation between firm size and input prices, which was not present in the original Melitz model. Consequently, they find that quality is a major determinant of the size and export status of a firm (see also Curzi and Olper (2012)). Adding the vertical dimension to my problem also fits better the problem of NTMs since most of them concerns quality issues.

My second assumption is the distinction between firm size and quality level in the production function. I introduce two productivity types: process productivity (φ, the cost productivity) and product productivity (the production of quality). In this way, as in Hallak and Sivadasan (2013), I am able to explain why some small-size high-quality firms produce, contrarily to models which do not take this assumption into account. Indeed, Hallak and Sivadasan (2013) noticed an important issue in the previous Melitz models using only the standard productivity function (Melitz (2003), and Chaney (2008a) only consider cost productivity): using only the process productivity does not allow to discriminate according to firm size. The observed exporter premia[10] could just be correlated with firm size; but tools to proxy size are also used to average wage, capital intensity, output and input prices. With this two-dimensions productivity, Hallak and Sivadasan (2013) show that exporters sell higher quality products than domestic firms, conditional on size. Using this two-productivity assumption is also used in some important paper from the most recent literature on quality standards (Feenstra and Romalis (2014), Gaigné and Larue (2016)).

My last hypothesis in this set of three is the different willingness-to-pay for quality across countries. Intuitively, this is a must in order to fit the idea that countries set different standards initially(Fajgelbaum et al., 2015). Heterogeneity in taste between and among countries is explored in several empirical works. For example, those divergences in taste are studied for the industry of beer and wine by Aizenman and Brooks (2008). Although wine is more consumed in countries where it is produced, they show how culture explains a significant part of the trade patterns[11]. Auer (2010) have the same conclusion: international differences in taste affect the pattern of trade in differentiated varieties (see also Bernard et al. (2012), Crozet et al. (2012), Holloway (2014), Disdier et al. (2010b), Disdier et al. (2010a), etc.). As said above concerning the literature of legal frameworks, only Grossman et al. (2019) try to explain difference in taste by using strictly a difference in willingness-to-pay; but they concentrate on variety (horizontal differentiation) while I concentrate on quality (vertical differentiation) which tends to make more sense in the recent trade agreement occurring with growing SPS and TBT measures[12].

[9] The number of exporters diminishes.

[10] A firm has a premia in exporting if it is more productive, it sets higher wages, etc. than the non-exporting firm.

[11] Also, they manage to link this observation with the creation of habits.

[12] I assume that consumers from different countries (or cultures) should differ in taste-for-quality as well as in taste-for-

The rest of the paper is structured as follows. First, I solve the model when the quality standard is set to 0. Firms can remain domestic or export their production. As a result, I find the different productivity cutoffs. Then, quality standards become greater than 0. I discuss their welfare impacts which are positive and explain why a world without quality standards is sub-optimal. Next, three legal frameworks are studied to see their economic impact with this divergence in standards across countries. Finally, I discuss my results and conclude by giving some policy repercussions and paths for future research.

II. Model

The background story of this model is similar to Hallak and Sivadasan (2013). Before the beginning of the game, each firm receives two independent productivities: in terms of cost efficiency (process productivity) and in terms of quality efficiency (product productivity). Also, each country has a different willingness-to-pay for quality. In the first stage, governments set quality standards and make a trade agreement. They choose among NT, MR, or H to maximize national welfare. In the second stage, firms adjust to that new scheme. They enter or not, paying the entry sunk cost f_e. Following Lugovskyy and Skiba (2015), I also assume that all firms can produce one quality level among a continuous interval of quality $\left[\underline{\theta};\overline{\theta}\right]$. There are two possible lines of product for each firm, one by destination (home or foreign market). Each firm maximizes its profit on quality and price for each line of product.

I first establish the structure of demand and supply. Then, firms (selling domestically and exporting) and consumers maximize their objective function and find a common optimal solution for price and quality level without and then with a national regulatory framework. The levels at which firms start producing and exporting are also calculated.

I. Demand

As in Melitz (2003), consumers follow CES preferences for the differentiated products and a homogeneous aggregate good is available. Both types of good are dispatched into a Cobb-Douglas utility function. As explained in the introduction, there is empirical proof of the existence of common cultural taste into a given cultural region, here limited at the country level(Aizenman and Brooks, 2008). Consequently, I assume that countries differ in their valuation of quality: I add the "willingness-to-pay for quality" inside the consumer preferences[13]. Intuitively, this plays a major role in the setting of the level of quality standards and on the optimal quality set by each firm.

Consumers utility U in a given country i is:

$$U_i \; = q_0^{1-\mu}\left(\int_{k\in\Omega_{\mathrm{i}}}\left(\theta_k^{\beta_i}q_k\right)^{\frac{\sigma-1}{\sigma}}dk\right)^{\frac{\mu\sigma}{\sigma-1}} \tag{1}$$

where Ω_i is the set of varieties available in this country: it includes varieties either imported from foreign to domestic country or locally produced. $\sigma > 1$ is the constant elasticity of substitution between varieties; μ is strictly between 0 and 1. q_k is the quantity of the heterogeneous goods attached to variety k and q_0 is the quantity of the homogeneous good whose price is normalized to 1. Also, note that $\theta \in \{\underline{\theta};\overline{\theta}\}$ is the quality level which relates positively with the consumer's utility with a minimum and a maximum level respectively. A change in quality standard will change this minimum quality level.

variety.

[13]In a way, cultural, institutional, educational, or revenue differences could explain why a firm selling the same quality in two countries will face different outcomes.

Each country has a different “willingness-to-pay for quality” drawn from $\mathbb{B}$, a set of continuously distributed β’s: $\beta_i > 0$ for domestic country and $\beta_j > 0$ for foreign country. For this analysis, I assume both cases with $\beta_i \neq \beta_j$ and $\beta_i = \beta_j$.[14].

II. Cost structure

As in Hallak and Sivadasan (2013), I consider two productivity types[15]. Process productivity ($\varphi > 0$) is the ability to reduce production costs, while product productivity ($\xi > 0$) is the ability to improve quality. Both are independent with each other and specific to each firm k. They are set exogenously by a Pareto-distribution (as in Chaney (2008b), Feenstra and Romalis (2014) and Hallak and Sivadasan (2013))[16]. φ_a, ξ_a are the lowest bounds and γ, ι are the dispersion parameters of those productivity distributions (assumed identical across countries for analytical simplicity). Therefore, my cumulative distribution functions are:

$$G(\varphi) = 1 - \left(\frac{\varphi_a}{\varphi}\right)^{\gamma} for\ \varphi_a \leq \varphi \tag{2}$$

$$V(\xi) = 1 - \left(\frac{\xi_a}{\xi}\right)^{\iota} for\ \xi_a \leq \xi \tag{3}$$

For each firm k (or variety k), I account for the fixed costs as well as the variable costs linked to domestic or foreign destinations. They are all "multi-products" producers in the sense that they sell one variety with two qualities: one of quality $\theta_{ii} \geq 0$ (if they sell domestically) and one of quality $\theta_{ij} \geq 0$ (if they export)[17]. In the rest of my paper, the notation is simplified to θ_i and θ_j.

Also, notice that the problem is symmetric (θ_i for domestic and θ_j for foreign) in both market destinations except for the transportation cost induced when exporting to foreign.

II.1 Fixed costs

I set three fixed costs: the standard fixed transportation costs f_x and a fixed cost f necessary to produce a positive quantity. Also, I add a specific cost that depends on quality: ω. This is the cost for technology and labor training necessary to upgrade quality θ to a certain level for a given variety k[18]. For simplicity, the upgrading cost must be paid for each quality of a variety k and, thus, it is country specific. Each firm pays an additional upgrading fixed cost for building a new product line for each country-specific quality.

Intuitively, ω is inversely related to the firm specific product productivity level ξ. The quality-elasticity of upgrading fixed cost is α. This cost function is the following:

$$\omega = \frac{\theta^{\alpha}}{\xi} \tag{4}$$

[14] If both β’s are equal, it results in a homogeneous countries model.

[15] See Hallak (2006) or the appendix in Feenstra and Romalis (2014) for a model with these two productivity types in a similar framework.

[16] Following previous ideas in Feenstra and Romalis (2014) and Verhoogen (2008), I could assume that ξ is ’endogenous’. However, it would add unnecessary complexity to the model since it depends on the entrepreneur’s ability which is exogenous (Verhoogen, 2008).

[17] This is standard in the endogenous quality literature.

[18] It is a close definition to the λ in Verhoogen (2008) which represents "entrepreneurial ability or technical know-how". This is proxied in their data by "log domestic sales deviated from industry mean". This means that it can be easily measured only with domestic sales data for each firm compared to the industry mean.

Equivalently, quality depends on the product productivity but also on the upgrading fixed cost. My product productivity function is the following and I assume diminishing returns to quality[19]: $\frac{1}{\alpha}\epsilon]0;1]$, thus $\alpha \in [1;\infty[$.

$$\theta = (\xi\omega)^{\frac{1}{\alpha}} \tag{5}$$

II.2 Variable costs

Second, I set a variable marginal cost c which depends negatively on the firm process productivity φ and positively on an indexation of quality θ^{ε}. $\varepsilon \in [0;1]$ is the quality-elasticity of the marginal cost.

$$c = \frac{\theta^{\varepsilon}}{\varphi} \tag{6}$$

Finally, I propose a variable transportation cost which relies upon ad valorem and good specific components such as c.i.f. price[20].

Including these elements leads to using a cost function which is like an iceberg transportation cost, widely employed by trade economists.

$$t = 1 + a + b \geq 1 \tag{7}$$

t is the iceberg trade cost and is the quantity of variety k paid to ship one unit of the good from i to j. If no transportation costs, $t = 1$; otherwise, $t > 1$. a and b are respectively the ad valorem cost[21] and the c.i.f. price. This trade cost also implies that there will always be two different qualities for the two destinations, justifying the obligation to pay a new ω for each destination.

At the end total cost for one variety k in one market takes this form:

$$C_j = c_j t q_j + \omega_j + F \tag{8}$$

where $j = i$ for the domestic costs and j for the exporting costs. Also, $F = f + \mathbb{I}_j f_x$ with $\mathbb{I}_j = 1$ if the firm is exporting to country j.

III. Supply

This is a monopolistic competition environment: every firm produces a specific variety for each destination market. Following Melitz's framework (2003), I assume that the only factor of production is labor, which is paid at a wage rate of 1 for simplicity. Also, there is a constant return to scale and perfect competition for the numeraire sector for which both countries produce a positive amount.

The profit function of a firm producing a variety k in one market is:

$$\pi_j = p_j q_j - c_j t q_j - \omega_j - F \tag{9}$$

[19]This is similar to Feenstra and Romalis (2014).

[20]Cost, insurance and freight (c.i.f.) prices include all the transportation charges incurred by the sector until the buyer receives the good. They depend on the type of traded products.

[21]Such as VAT, it is a tax put on the transported good given its value.

IV. Firm's optimal choices

I now solve the firm and consumer problems to give a complete solution for the profit as well as the demand system. The goal is also to find the production and exporting cut-off functions[22]. The quality standards are set to 0 for comparative purpose with the next section which introduces positive quality standards. In this latter section, I give more details on the quality standards process. In this first section, all firms can produce one quality level among a continuous interval of quality $\{0;\overline{\theta}\}$. In the next section, a positive QS truncates this interval and becomes the lower bound $\underline{\theta}$ in this interval.

IV.1 Entry

In each country, the production occurs after each firm learns its productivity by paying a fixed entry cost f_e. The cost is paid only if the expected post-entry profit is positive.

$$\overline{\pi} = \sum_{i}^{j} \int_{0}^{\infty} \int_{0}^{\infty} \pi\left(\varphi,\ \xi\right) v(\xi) g(\varphi) d\xi d\varphi = f_e \tag{10}$$

IV.2 Equilibrium demand

Deriving utility, I find the following demand at the equilibrium for a variety produced and sold in market i (the case from the point of view of country j is symmetric.):

$$q_i = \theta^{\beta_i(\sigma-1)}\ p_i^{-\sigma} L_i P_i^{\sigma-1} \tag{11}$$

where

$$P_i^{1-\sigma} = \int_{k\in\Omega_i} \theta_k^{\beta_i(\sigma-1)} p_{i_k}^{1-\sigma} dk = \sum_{k}^{K} \int_{\varphi}^{\infty} \int_{\xi}^{\infty} \theta_{ki}^{\beta_i(\sigma-1)} p_{ki}^{1-\sigma}\left(\varphi\right) M_{ki} \varphi_{kk}^{\gamma} \xi_{kk}^{\iota} g(\varphi) v(\xi) d\xi d\varphi \tag{12}$$

is the price index in i [23], and L_i is the number of consumers in country i (which is allowed to differ across countries). M_{ki} is the mass of firms producing k and selling in i; $g(\varphi) = \gamma\varphi_i^{-\gamma-1}$ and $v(\xi) = \iota\xi_i^{-\iota-1}$ are the ex-post distribution of productivities if the firm exist and sell in i. As in Gaigné and Larue (2016), note the negative relationship between the price index and quality because $\sigma > 1$.

The expenditure function is the following for country i, while this is symmetric for country j:

$$r = \mathbb{I}_i \theta^{\beta_i(\sigma-1)}\ p_i^{1-\sigma} L_i P_i^{\sigma-1} \tag{13}$$

The representative firm maximizes profit according to two elements: f.o.b. price and quality (i.e.: Rodriguez (1979); Feenstra and Romalis (2014)).

In order to obtain a feasible result (i.e.: a non-negative quality), I assume that $\varepsilon < \beta \in B$. It means that the willingness-to-pay for quality is always strictly greater than the quality-elasticity of the marginal cost. Indirectly, it forces the model to assume the fixed cost $F = f + \mathbb{I}_j f_x > 0$ to obtain a feasible result.

I now focus on the domestic producers (from i) selling in country j (i.e.: the exporters); the non-exporters case present symmetric results but with no trade costs.

[22]The solution is found by optimizing the firm and consumer functions.

[23]The price index is endogenously determined and is as a constant in monopolistic competition.

IV.3 Exporting firm

A priori, two types of exporting firms exist: (a) those selling both in i and in j and (b) those selling only in j although producing in i. Case (a) exists if the corresponding profit is above the productivity cut-offs (to produce or to export). Case (b) means that, with heterogeneous β's and quality standards, it may happen that the choice of selling domestically costs more than exporting (i.e.: the negative effect of the transportation costs is lower than the costs induced by an expensive combination of β_j and θ_{sj}). In the short term, a firm could decide to produce domestically while selling only in the foreign destination. In the long term, this firm changes the location of her production to country j and second case (b) does not occur[24]. In conclusion, (a) is the only case of interest in a long-term analysis. The first order optimal price that I obtain is $p_j = \frac{1}{\varphi}\frac{\sigma}{\sigma-1}t\theta_j^{\varepsilon}$. I find the optimal quality:

$$\theta_j^* = \left[\frac{\beta_j - \varepsilon}{\alpha}\left[\frac{\sigma-1}{\sigma}\right]^{\sigma}\varphi^{\sigma-1}\xi t^{1-\sigma}L_j P_j^{\sigma-1}\right]^{\frac{1}{\alpha-\Lambda_j}} \tag{14}$$

Where $\alpha - \Lambda_j = \alpha - (\sigma-1)\left(\beta_j - \varepsilon\right) > 0 \iff \alpha - \Lambda_j < \alpha \; since \; \sigma > 1 \; and \; \varepsilon < \beta_j$ by assumption.

As expected, a higher φ leads to a lower marginal cost c and a higher optimal quality. A higher α (the elasticity of the upgrading cost) lowers the θ_j necessary to upgrade quality and pushes firms to adopt a higher optimal quality (since it costs less). Notice also that a higher willingness-to-pay for quality β_j implies a higher optimal quality because companies want to meet consumers' desire for quality. Finally, a higher sensitivity of the marginal cost ε in direction of quality leads to lower quality, as expected.

In its final form, the optimal price is deduced from the optimal quality:

$$p_j{}^* = \left(\frac{\beta_j - \varepsilon}{\alpha}\right)^{\frac{\varepsilon}{\alpha-\Lambda_j}}\left(\frac{\sigma-1}{\sigma}\right)^{\frac{\beta_j(\sigma-1)+\varepsilon-\alpha}{\alpha-\Lambda_j}}\xi^{\frac{\varepsilon}{\alpha-\Lambda_j}}\varphi^{\frac{\beta_j(\sigma-1)-\alpha}{\alpha-\Lambda_j}}\left(t^{1-\sigma}L_j P_j^{\sigma-1}\right)^{\frac{\varepsilon}{\alpha-\Lambda_j}} \tag{15}$$

As in Hallak and Sivadasan (2013), I cannot say whether price increases or decreases with a change in φ given ξ because it depends on the sign of $\beta_j(\sigma-1)-\alpha$. Two opposite effects are at play when φ increases: the first-order price diminishes thanks to productivity gains and the optimal quality increases because, everything else being equal, more investment can be made in quality upgrading. Everything rests on the willingness to pay for quality, the elasticity of substitution and the quality-elasticity of the upgrading cost. Also, ceteris paribus, a higher ξ given φ generates a higher price and a higher quality. Both impacts welfare in an opposite way; adjusting for quality given willingness-to-pay for quality is needed to obtain a more refined interpretation (if setting $\widetilde{p} = \frac{p}{\theta^{\beta_i}}$):

$$\widetilde{p}_j = A_j \eta_j^{\frac{-1}{\sigma-1}}\left(t^{1-\sigma}L_j P_j^{\sigma-1}\right)^{\frac{\varepsilon-\beta_j}{\alpha-\Lambda_j}} \tag{16}$$

With $\widetilde{p}_j = \frac{p_j}{\theta_j^{\beta_j}}$; $A_j \equiv [\frac{\sigma-1}{\sigma}]^{\frac{-\alpha-\beta_j+\varepsilon}{\alpha-\Lambda_j}}[\frac{\beta_j-\varepsilon}{\alpha}]^{\frac{\varepsilon-\beta_j}{\alpha-\Lambda_j}}$; $\eta_j \equiv (\varphi^{\frac{\alpha}{\alpha-\Lambda_j}}\xi^{\frac{\beta_j-\varepsilon}{\alpha-\Lambda_j}})^{\sigma-1}$

It is clear that $\widetilde{p}_j \searrow$ if$\varphi \nearrow$, $\xi \nearrow$. Higher productivities lead to a lower price adjusted for quality. This means that, keeping a fixed quality, more productive firms are more able to lower their optimal price.

Given quality and price, the optimal revenue after a few calculation steps using 14 and 15 is:

[24] However, it is still possible that the cost of switching the country where the good is produced is too high and the firm remains in the domestic country. I assume it is not the case.

$$r_j^* = H\eta(t^{1-\sigma}L_jP_j^{\sigma-1})^{\frac{\alpha}{\alpha-\Lambda_j}} = \frac{\omega\sigma\alpha}{\Lambda_j} \tag{17}$$

I show that $r_j{}^* \nearrow if \varphi \nearrow,\ \xi \nearrow, \beta_j \nearrow$.

Finally, the optimal profit is:

$$\pi_j^* = \eta JN\left(t^{1-\sigma}L_jP_j^{\sigma-1}\right)^{\frac{\alpha}{\alpha-\Lambda_j}} - F = \frac{\omega\left(\alpha-\Lambda_j\right)}{\Lambda_j} - F \tag{18}$$

Here, a higher β_j gives a higher profit since it positively impacts each part of the profit function. I also get that $\pi_j{}^* \nearrow if \varphi \nearrow,\ \xi \nearrow$. I assume that $\sigma,\ \alpha,\ \beta_j,\ \varepsilon,\ P_j$ are the same in similar industries. In that case, firms only diverge in profit or revenue if they differ in summarized productivity η_j. Firms might have a distinct combination of $(\varphi,\ \xi)$ to get the same η_j: this enables to get a different optimal quality and a different optimal price for the same $\eta_j,\ \pi_j,\ r_j$. Therefore, price is not related to size. This result, similar to Hallak and Sivadasan (2013), departs from the conclusion in Macedoni and Weinberger (2019) where there is no distinction between firm size and quality level. Therefore, a firm decides to export if and only if $\frac{\omega\left(\alpha-\Lambda_j\right)}{\Lambda_j} \geq F$. The production cutoff is determined similarly but for $t = 1$ and $f_x = 0$. I also find the minimum productivity threshold to export:

$$\underline{\varphi}^{\sigma-1} = \theta_j^{-\Lambda_j}\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{\sigma\left(\frac{\theta_j^{\alpha}}{\xi} + F\right)}{t^{1-\sigma}L_jP_j^{\sigma-1}} \tag{19}$$

Contrarily to Crozet et al. (2012), there is a level of quality productivity at which the firm can start exporting. Finally, the price index which takes the optimal price into account is rewritten like this:

$$P_j = F^{\frac{\alpha-\Lambda_j}{(\sigma-1)\alpha}}\left[\left(\frac{\Lambda_j}{\alpha}\right)^{\Lambda_j} - \left(\frac{\Lambda_j}{\alpha}\right)^{\alpha}\right]^{\frac{-1}{(\sigma-1)\alpha}}\left[\frac{1}{\sigma-1}\right]^{\frac{-\alpha}{(\sigma-1)\alpha}}\xi^{\frac{-\Lambda_j}{(\sigma-1)\alpha}}\left[\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\varphi^{\sigma-1}t^{1-\sigma}L_j\right]^{\frac{-\alpha}{(\sigma-1)\alpha}}$$

III. Introducing a positive quality standard

Each country sets an exogenous quality standard greater than 0 and denoted θ_s. The main purpose is to improve the allocation of the production of quality products among firms, as explained by Gaigné and Larue (2016) and Macedoni and Weinberger (2019). A distortion appears due to monopolistic competition with firm heterogeneity in the production of quality. It causes competitors to ignore the consequences of their quality choice on each other and this misallocate production. A policy such as a quality standard reallocate production to improve welfare. Those minimum qualities are attained by paying at least the minimum fixed upgrading cost ω_{sj}. If the productivity of a firm is below this "s" cut-off, she exits the market. In other words, the new standard truncates the left part of the combined productivity distribution[25]. I measure the impact of a change in the QS. The introduction of a positive QS creates a new price index based on the surviving firm with the lowest productivity (denoted below 'lsj' for "Lowest productivity firm" with the "Standard" in country "j"). This new price index changes the optimal price and quality level. Thus, firms with different productivities are not similarly impacted by a QS (Gaigné and Larue, 2016; Hallak, 2006).

[25] The combined productivity distribution summarizes both productivity types: two firms can have a similar combined productivity even though they are different in both productivity types.

I deal with this QS introduction using a substitution method. This separates the problem into two categories of firms facing the standard: constrained (c-firms) and unconstrained (u-firms). Since the standard is positive, I assume QS is restrictive for at least one firm and not restrictive for at least one firm. Initially, a c-firm produces a quality which is lower than the standard; given its post-standard expected profit, a c-firm decides to exit or to upgrade quality to meet the standard. A u-firm already produces a quality equal or above to the new QS but is confronted to a new price index which takes the standard into account. I start the analysis by studying the reactivity of exporting firms towards a new foreign standard. The domestic firm holds a symmetric result, once converting j into i indices, setting $t = 1$ and $f_x = 0$.

A new QS is set in the foreign country where domestic firms export: θ_{sj} (where s stands for "standard", j for country "j"). Since I want to analyze how c-firms and u-firms react to this, I look for their characteristic equations: total sales and optimal price. As a first step, I find the price index, which is identical in both cases. I need the minimum profit function for the exporting firms which leads to the price index, as I demonstrate below. The minimum exporting profit for both c-firms and u-firms is summarized as follows when I introduce a QS and when I use the optimal price found in 15:

$$\pi_j = \theta_j^{\beta_j(\sigma-1)} p_j^{1-\sigma} L_j P_j^{\sigma-1} - \frac{\theta_j^{\varepsilon}}{\varphi} t \theta_j^{\beta_j(\sigma-1)} p_j^{-\sigma} L_j P_j^{\sigma-1} - \frac{\theta_j^{\alpha}}{\xi} - F \tag{20}$$

This is the profit of the firm with the lowest process and product productivities available to reach the QS in the market j (i.e.: φ_{lsj} and ξ_{lsj}, respectively). It is a c-firm since the constraint imposing a new minimum quality is binding and she must now produce the quality level of the standard θ_{sj} (i.e.: the lowest quality available on the market j). In other words, she cannot choose her late optimal quality anymore. The minimum profit function varies positively with the quality level of the standard θ_{sj}. I find the new price index either by isolating it from the profit function when it equals 0 or using equation 12 with the standard and the optimal price 15.

$$P_j^{\sigma-1} = \sigma \varphi^{1-\sigma} t^{\sigma-1} \left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \theta_{sj}^{-\Lambda_j} \frac{1}{L_j} \left(\frac{\theta_{sj}^{\alpha}}{\xi_{lsj}} + F\right) \tag{21}$$

On the condition that the fixed cost F is low, a higher standard raises the price index since $\alpha > \Lambda_j$ is sufficient to show that $\frac{\partial P_j^{\sigma-1}}{\partial \theta_{sj}} > 0$. However, a sufficiently high F leads to a negative impact of the standard on the price index; this case is not standard in the literature and I depart from it for the rest of my analysis. Now, I look at the divergence between both types of firms.

I. Sales, prices, and profit for the c-firm

A c-firm has no choice but to adopt the standard to continue to exist on the destination market. To do this, she must invest in her technology and labor training, enough to improve the quality: the upgrading cost $\omega_{sj} = \frac{{\theta_{sj}}^{\alpha}}{\xi}$ and the marginal cost $c = \frac{{\theta_{sj}}^{\varepsilon}}{\varphi}$ both increase. Her profit is lower than before because of this. Indeed, the old optimal quality brought the best profit. However, c-firms still make a positive profit, it is simply no longer optimal. Other less productive firms (i.e.: with lower φ and ξ combinations than c-firms) stop producing and exit the market because the quality standard level is too cost-expensive to make a positive profit. The "lsj" c-firm makes no-profit (the non-exiting firm with the lowest combined productivity).

The first-order equilibrium price is:

$$p_j = \frac{\sigma t}{\sigma - 1} \frac{\theta_{sj}^{\varepsilon}}{\varphi} \tag{22}$$

This equation only depends on φ in terms of productivity since ξ does not influence the fixation of the QS θ_{sj}. A higher QS increases the price.

Under a QS limitation, the sales of a c-firm are found using the new price index 21 and the previous revenue equation 13 divided by the new first-order equilibrium price 22:

$$q_j^c = \frac{\sigma - 1}{t} \frac{\varphi^{\sigma}}{\varphi_{ls}^{\sigma-1}} \frac{\theta_s^{\alpha-\varepsilon}}{\xi_{ls}} - \frac{F\varphi(\sigma-1)}{\sigma t \theta_s^{\varepsilon}} \tag{23}$$

The sales are conditioned on the lowest productivity boundaries φ_{ls} and ξ_{ls}. As expected, they increase with the φ of the c-firm and decrease with the trade cost t (again, this assumes that F is low). Sales do not depend on the willingness-to-pay for quality in the market, contrarily to the case without standard. The sales also vary with the QS depending on the size of α and ε. This result clearly shows that higher QS does not especially bring higher sales[26]. The relation is positive if and only if $\alpha > \varepsilon$. This means that the quality-elasticity of the upgrading cost needs to be strictly greater than the quality-elasticity of the marginal cost. A change of quality is brought only with a change of standard in the case of c-firms. Thus, the change of standard must impact the upgrading fixed cost more importantly than the variable cost for it to impact sales positively. This analysis amounts to distinguishing between increasing and decreasing costs to scale.

Using 26 and 27, the exporting profit of the c-firms turns out to be:

$$\pi_j^c = \theta_s^{\alpha} \left[\frac{1}{\xi_{ls}} \left(\frac{\varphi}{\varphi_{ls}} \right)^{\sigma-1} - \frac{1}{\xi} \right] - F \left(\frac{\varphi^{\sigma-1}}{\sigma} \left(\frac{t\sigma}{\sigma-1} \right)^{1-\sigma} \theta_s^{\Lambda} L_j + 1 \right) \tag{24}$$

Assuming a low F, a higher standard always increases the optimal profit, which is positive or equal to zero for the lsj-firm. Indeed $\left[\frac{1}{\xi_{ls}} \left(\frac{\varphi}{\varphi_{ls}} \right)^{\sigma-1} - \frac{1}{\xi} \right] > 0$. This is mainly due to the exit of low productivity firms which decreases competition. Remember that the additional cost due to a fixed quality standard is higher than the optimal quality for each c-firm. The imposed quality standard leads towards a second-best profit level; c-firms remain in the market if π is not strictly negative. The profit also depends on the distance of both productivities over their lowest limits. Firms that are too far left in the combined productivity distribution exit. The lowest productive c-firm (ls) is making a zero profit but influences the profit of the other c-firms which make positive profits. Since the process productivity φ and the product productivity ξ are distributed independently, some low-quality firms survive due to their high φ, while some high-quality firms with a φ that is too low exit. Consequently, it is difficult to know intuitively whether the average quality increases with a new standard[27]. Indeed, being good in product productivity (ξ) only gives an advantage in terms of fixed cost. However, the firm with bad process productivity (φ) loses profit for each new output. Intuitively, the combination of a lower quality differentiation and a higher average quality is less important to survival than price level differentiation. Notice also that the previous relation between β_j and π_j which was ambiguous in the case with a zero-level standard, is present in the fixed cost term of the profit $F\left(\frac{\varphi^{\sigma-1}}{\sigma} \left(\frac{t\sigma}{\sigma-1} \right)^{1-\sigma} \theta_s^{\Lambda} L_j + 1 \right)$. A higher preference for quality slightly impact the c-firms profit in a negative direction. Indeed, since c-firms cannot change their quality level, they cannot follow this β if it changes.

II. Sales, prices, and profit for the u-firm

For the u-firms, I find a result situated between that of the c-firms and the case with a zero standard. As in the case with a zero standard, they can choose an optimal quality but, as in the

[26]This result is similar as in Gaigné and Larue (2016)

[27]This result is present in Disdier et al. (2018) and in Gaigné and Larue (2016); norms have been shown to advantage medium-quality high productivity firms.

c-firm case, they follow the new price index linked to the lsj-firm. The optimal quality is also found maximizing profit in terms of quality and using equation 15.

$$\theta_j^* = \left[\frac{\Lambda}{\alpha}\theta_s^{-\Lambda}\left(\frac{\theta_s^{\alpha}}{\xi_{ls}}+F\right)\xi\left(\frac{\varphi}{\varphi_{ls}}\right)^{\sigma-1}\right]^{\frac{1}{\alpha-\Lambda}} \tag{25}$$

The optimal quality increases with better productivity draws and with a higher β_j (through the Λ_j). Intuitively, a higher quality standard pushes u-firms to increase their quality if the fixed cost F is not too important[28]. This means that a higher standard can lead to a global improvement of the average quality level (even for the firms that are not directly concerned by it such as the c-firms, as shown above).

The price and the sales are found in a similar way as for the c-firms. I find the following equations:

$$p_j^u = \frac{\sigma t}{(\sigma-1)}\left[\frac{\Lambda}{\alpha}\theta_s^{-\Lambda}\left(\frac{\theta_s^{\alpha}}{\xi_{ls}}+F\right)\xi\left(\frac{1}{\varphi_{ls}}\right)^{\sigma-1}\right]^{\frac{\varepsilon}{\alpha-\Lambda}}\varphi^{\frac{(\sigma-1)\beta_i-\alpha}{\alpha-\Lambda}} \tag{26}$$

$$q_j^u = \frac{\alpha(\sigma-1)}{\Lambda\xi t}\left[\frac{\Lambda}{\alpha}\theta_s^{-\Lambda}\left(\frac{\theta_s^{\alpha}}{\xi_{ls}}+F\right)\xi\left(\frac{1}{\varphi_{ls}}\right)^{\sigma-1}\right]^{\frac{\alpha-\varepsilon}{\alpha-\Lambda}}\varphi^{\frac{\sigma\alpha-(\sigma-1)\beta_i}{\alpha-\Lambda}} \tag{27}$$

Notice first that the price equation depends on ξ: a higher performance in the creation of high quality is impacting the price positively. It was not the case with c-firms. Also, α (the upgrading fixed cost elasticity) is impacting the sales in a more direct way than in the c-firms case since it is not only present in the exponent. Both equations 26 and 27 are very alike because 27 builds on 26. Both are influenced by the optimal quality found in 25. Still, several differences emerge between those two equations. While a higher β_j (present through the Λ) clearly increases the weight of φ on price, the impact is less strong on sales because of the negative sign in front of β_j. Also, the QS is positively related to sales if $\alpha > \varepsilon$ and conversely. As explained for the sales of c-firms, the quality-elasticity of the upgrading cost needs to be strictly greater than the quality-elasticity of the marginal cost. But, the relation between QS and the price is positive without ambiguity. The profit of the u-firm is found as in the c-firm case by using the new price index 21 and 27:

$$\begin{aligned}\pi^u = \theta_s^{\alpha}&\left[\left(\frac{\xi\Lambda}{\alpha}\right)^{\frac{\Lambda}{\alpha-\Lambda}}\left(\frac{1}{\xi_{ls}}\right)^{\frac{\alpha}{\alpha-\Lambda}}\left(\frac{\varphi}{\varphi_{ls}}\right)^{\frac{(\sigma-1)\alpha}{\alpha-\Lambda}}-\frac{1}{\xi}\right]\\ &\qquad - F\left[\frac{\varphi^{\sigma-1}}{\sigma}\left(\frac{t\sigma}{\sigma-1}\right)^{1-\sigma}\theta_s^{\Lambda}L_j+1\right]\\ &+\left[\frac{\xi\Lambda}{\alpha}\theta_s^{-\Lambda}F\left(\frac{\varphi}{\varphi_{ls}}\right)^{\sigma-1}\right]^{\frac{\Lambda}{\alpha-\Lambda}}\left(\frac{\theta_s^{\alpha-\Lambda}}{\xi_{ls}}\left(\frac{\varphi}{\varphi_{ls}}\right)^{\sigma-1}-F\frac{\varphi^{\sigma-1}}{\sigma}\left(\frac{t\sigma}{\sigma-1}\right)^{1-\sigma}L_j\right)\end{aligned} \tag{28}$$

Thus, the only difference between u-firms and c-firms in terms of profit is the presence of $\frac{\xi\Lambda}{\alpha}$ in the first term and a third term depending on F for u-firms. Assuming that F is low enough, it marginally influences profit. A higher willingness-to-pay for quality β_j increases the profit through $\frac{\xi\Lambda}{\alpha}$. This result is not so straightforward since the β_j negatively impacts sales when $\varepsilon > \alpha$. A higher β_j allows u-firms to increase their profit margin mostly through price. As a result,

[28]Indeed, $\alpha > \Lambda_j$.

and assuming low F, only highly productive firms (i.e.: u-firms) can enjoy the heterogeneous characteristic of markets in willingness-to-pay for quality and improve their profit level. Regarding the quality-elasticity of fixed cost (α), there is an opposite reaction to β_j in $\frac{\xi\Lambda}{\alpha}$. The more the change in quality affects the upgrading fixed cost, the less the u-firm makes profit. However, looking at all the other α present in the exponents of the first term, the analysis is less clear. In conclusion, α impacts the profit in an unclear direction. It is obvious but it is not observed in the case of the c-firm. In the next section, I find the level of productivity necessary to become a u-firm.

III. Limit firm

The level of productivity for a firm indifferent between the standard and her optimal quality is found by equalizing $\pi_j^c = \pi_j^u$ and assuming that this limit firm is indifferent between θ_s and θ_j:

$$\frac{\widehat{\varphi}}{\xi} = \left(\frac{\Lambda}{\alpha}\right)^{\frac{1}{1-\sigma}} \frac{\varphi_{ls}}{\xi_{ls}} \tag{29}$$

Although ξ and φ are independently distributed, a higher capacity for high quality products (ξ) influences more the profit of u-firms. As a result, this limit firm can only be determined by the ratio of both productivities.

IV. Labor market clearing, mass of firms, and welfare with a positive QS

A positive QS that is imposed by the regulator impacts both the labor market clearing condition and the mass of firms. Once they are modified, I find how the QS also changes the welfare compared to a zero-level QS case. The mass of domestic producers M_i (the successful entrants) is found using the labor market clearing condition (LMC) in i. This sectoral LMC ensures that profit is precisely facing the cost of entry. The LMC is:

$$L_i = l_i^A + l_i^B + M_i\varphi_i^\gamma \xi_i^\iota f_e + M_i\varphi_i^\gamma \xi_i^\iota \sum_j^i \varphi_j{}^{-\gamma}\xi_j{}^{-\iota-1}\left(\omega_j + F\right) \tag{30}$$

It is divided into 4 parts: l_i^A and l_i^B are the variable mass of workers allocated to the production in the c-firms and u-firms, respectively. $M_i\varphi_i^\gamma \xi_i^\iota f_e$ is the cost of entry paid by firms surviving entry (i.e.: the number of workers to survive entry), $M_i\varphi_i^\gamma \xi_i^\iota \sum_j^i \varphi_j{}^{-\gamma}\xi_j{}^{-\iota-1}\left(\omega_j + F\right)$ is the cost of production paid by the producing firms which entered the domestic market i or the exporting market j according to the index. I also define the expected profit by the probability that a firm is constrained or not. Remember that both distributions of ξ and φ are independent. Thus, $h\left(\varphi,\ \xi\right) = v(\xi)g\left(\varphi\right)$ [29]. Using $\hat{\varphi}$, I find the following expected profit:

$$\overline{\pi}_i = f_e = \varphi_i^{-\gamma}\xi_i^{-\iota}\sum_j^i \varphi_{lsj}^{-\gamma}\xi_{lsj}^{-\iota-1}\overline{\Delta_j} \tag{31}$$

with $\overline{\Delta_j} = \theta_{sj}^\alpha\psi_j + F\left(\varphi_{lsj}^{\sigma-1}\xi_{lsj}\phi_j t^{1-\sigma}L_j\left(\rho_j\left(\theta_{sj}^{\Lambda_j} - \theta_j^{\Lambda_j}\right) + \theta_{sj}^{\Lambda_j}\right) - \xi_{lsj}\right)$. This is also the new free-entry condition[30].

Finally, using the LMC and isolating M_i, I find the mass of firms M_i. I assume that $\sigma - \gamma - 1 < \frac{-\gamma\Lambda}{\alpha}$. This also implies that $\sigma - 1 < \gamma$, the usual assumption about the Pareto distribution

[29]Reminder: $v\left(\xi\right) = \iota\xi^{-\iota-1}$; $V\left(\xi\right) = -\xi^{-\iota}$; $g\left(\varphi\right) = \gamma\varphi^{-\gamma-1}$; $G\left(\varphi\right) = -\varphi^{-\gamma}$

[30]In this equation, $\psi_j > 0$, $-1 < \rho_j =< 0$ and $\phi_j < 0$. Proofs are in the appendix.

(Arkolakis et al., 2008). There is an equivalent assumption for the other distribution dispersion parameter ι, $\frac{\Lambda}{\alpha-\Lambda} < \iota$.

$$M_i = L_i \varphi_i^{-\gamma} \xi_i^{-\iota} \sum_j^i \varphi_{lsj}^{\gamma} \xi_{lsj}^{\iota+1} \Delta_j^{-1} \tag{32}$$

where $\Delta_j = \theta_{sj}^{\alpha}\left[\mathrm{Y}_j + \frac{1}{\xi_{lsj}}\right] + F\xi_{lsj}\left[\varphi_{lsj}^{\sigma-1}\phi_j t^{1-\sigma} L_j \left(\rho_j\left(\theta_{sj}^{\Lambda_j} - \theta_j^{\Lambda_j}\right) + \theta_{sj}^{\Lambda_j}\right) + \varpi_j + \xi_{lsj}^{\frac{\gamma\alpha+\Lambda_j}{\alpha-\Lambda_j}}\chi_j\right]$, $\mathrm{Y}_j > 0$, and $\varphi_i^{\gamma}\varphi_j^{-\gamma} = \frac{G(\varphi_j)}{G(\varphi_i)}$ is the proportion of exporting firms. I analyze the impact of a change in θ_{sj} on M_i[31]. A direct effect is determined by the presence of θ_{sj} in the two terms of the Δ_j. First, $\theta_{sj}^{\alpha}\left[\mathrm{Y}_j + \frac{1}{\xi_{lsj}}\right]$ leads to a negative impact on M_i because of the negative exponent on the Δ_j and the positive elements in the square brackets. Second, the second term multiply θ_{sj} by $F\xi_{lsi}\varphi_{lsi}^{\sigma-1}\phi_i t^{1-\sigma} L_i \Lambda_i (\rho_i + 1)$ with $-1 < \rho < 0$ and $\phi_i < 0$. Thus, this impacts positively M_i (because of the negative exponent on the Δ_j). Using the implicit function theorem, there is an indirect effect due to the two productivity determinants ξ_{lsj} and φ_{lsj} which both depend on θ_{sj}. As it is explained in more detail in the appendix, all those elements varying with a change in θ_{sj} will act in one direction or the other depending on whether the fixed cost F is high or low. Since I assumed earlier that F is very low but positive, a change in θ_{sj} lowers M_i. The welfare can be found using this mass of firms into equation 12. I find:

$$\mathbb{W}_i = \frac{\theta_{si}^{\frac{\Lambda}{\sigma-1}} L_i^{\frac{\gamma-\sigma+1}{\gamma(\sigma-1)}} \sigma^{\frac{-1}{\sigma-1}} (\sigma-1)^{\frac{1}{\gamma}}}{[\gamma-\sigma+1]^{\frac{1}{\gamma}}} \left(L_k \sum_j^i \varphi_{lsj}^{\gamma} \xi_{lsj}^{\iota+1} t_{ki}^{-\gamma} \left(\frac{\theta_{si}^{\alpha}}{\xi_{lsi}} + F\right)^{\frac{-\gamma+\sigma-1}{\sigma-1}} \xi_{ki}^{-\iota} \Delta_j^{-1}\right)^{\frac{1}{\gamma}} \tag{33}$$

However, it has been shown in the literature that the welfare is the inverse of the price index 21(Melitz and Redding, 2014). This simpler and equivalent expression is:

$$\mathbb{W}_i = \frac{\sigma-1}{t\sigma}\left[\frac{\sigma\theta_{si}^{-\Lambda_i}}{L_i}\left(\frac{\theta_{si}^{\alpha}}{\xi_{lsi}(\theta_{si})} + F\right)\right]^{\frac{1}{1-\sigma}} \varphi_{lsi}(\theta_{si}) \tag{34}$$

This version of the domestic welfare only includes the firm with the lowest productivity. This firm is following the lowest standard available (depending on whether she exports or not). In the next section, I discuss the variation of welfare depending on the legal framework chosen by the two trading economies.

V. Legal trade frameworks

For the sake of the analysis, I assume that each government chooses a different standard. Consequently, there are different fixed and variable costs for the two markets. I compare two cases: $\theta_{sj} < \theta_{si}$, or $\theta_{sj} > \theta_{si}$. A $\theta_{sj} < \theta_{si}$ ($\theta_{sj} > \theta_{si}$) pushes low quality firms to sell more (less) in the foreign market.

The choice of the legal framework is determinant to impact welfare. Three frameworks are defined as such:

- **National Treatment:** domestic producers and foreign exporters receive the same standard. There is no discrimination among firms in the same destination market. Domestic exporters stick to the foreign standard.

[31] A more detailed analysis is available in the appendix.

- **Mutual Recognition:** exporters are allowed to take the lowest standard among domestic and foreign ones. It removes the constraint of a domestic standard upon foreign exporters and they can benefit from it. Thus, a standard only limits domestic producers. On the government perspective, a stricter standard under MR does not especially brings higher quality and can hurt domestic firms in terms of cost of quality and unequal competition between domestic producers and foreign exporters.
- **Harmonization:** a unique standard will be chosen for all exporters from both countries, different from national standards. I arbitrarily take the middle ground[32]: $\frac{\theta_{si}+\theta_{sj}}{2}$.

I measure the changes in the mass of firms and in terms of welfare. I use the results found above for the exporting firms (constrained or not) which correspond to the national treatment case by default. The welfare function is specific to the less productive firm. Consequently, I must separate the result into two categories: when the domestic standard is larger than the foreign standard versus the opposite situation. The mass of firm is written under a more general form because it considers firms selling in both destinations (domestic only or both domestic and foreign markets). However, it will be important to separate the result in the same way. When a firm can choose the best standard among domestic and foreign (in terms of profit optimization), she will. Indeed, having two different standard implies that the lowest productive firm is among non-exporters sometimes and among exporters sometimes. In this last case, it means that all non-exporters exit the market. In other words, all domestic firms trade internationally. This might not be the case for foreign firms. First, I take the simplest case: mutual recognition with $\theta_{sj} > \theta_{si}$. Then, the other cases are discussed.

V.1 Mutual Recognition with a higher foreign standard

In this MR case, the constrained domestic exporters do not pay an "adaptation" cost to the foreign standard. Their home standard is giving them an advantage on the foreign market. There is not much difference with the results found in the case with a zero-level standard. The unconstrained firms are proceeding as usual. The mass of firm is:

$$M_{i,MR1} = L_i \varphi_i^{-\gamma} \xi_i^{-\iota} \sum_j^i \varphi_{lsj}^{\gamma} \xi_{lsj}^{\iota+1} \Delta_i^{-1} \tag{35}$$

With $\Delta_i = \bar{\theta}_{si}^{\alpha}\left[Y_j + \frac{1}{\xi_{lsj}}\right] + F\xi_{lsj}\left[\varphi_{lsj}^{\sigma-1}\phi_j t^{1-\sigma} L_j\left(\rho_j\left(\bar{\theta}_{si}^{\Lambda_j} - \theta_j^{\Lambda_j}\right) + \bar{\theta}_{si}^{\Lambda_j}\right) + \varpi_j + \xi_{lsj}^{\frac{\gamma\alpha+\Lambda_j}{\alpha-\Lambda_j}}\chi_j\right]$. As discussed in the previous section, the mass of firms varies negatively with the standard because of the very low F. There is no difference between the NT case and the MR case when $\theta_{sj} > \theta_{si}$. The welfare is:

$$\mathbb{W}_{i,MR1} = P_j^{-1} \Longrightarrow \mathbb{W}_j = \frac{\sigma-1}{t\sigma}\left[\frac{\sigma\bar{\theta}_{si}^{-\Lambda_i}}{L_i}\left(\frac{\bar{\theta}_{si}^{\alpha}}{\xi_{lsi}} + F\right)\right]^{\frac{1}{1-\sigma}} \varphi_{lsi} \tag{36}$$

The welfare in this MR case is equivalent to the NT case since the firm with the lowest productivity follows the domestic standard in both cases.

[32]It is also possible to take a weighted average, to account for the country size.

V.2 Mutual Recognition with a higher domestic standard

Domestic firms are worse off when they follow the domestic standard. The adaptation to the foreign standard is beneficial to them.

$$M_{i,MR2} = L_i \varphi_i^{-\gamma} \xi_i^{-\iota} \sum_j^i \varphi_{lsj}^{\gamma} \xi_{lsj}^{\iota+1} \Delta_j^{-1} \tag{37}$$

Since lsj is the lowest productive exporter, the mass of firm will follow the foreign standard and this leads to a higher mass of firms than in the initial case (i.e.: NT). For the welfare, I find the following equation:

$$\mathbb{W}_{i,MR2} = \frac{\sigma-1}{t\sigma} \left[\frac{\sigma \bar{\theta}_{sj}^{-\Lambda_i}}{L_i} \left(\frac{\bar{\theta}_{sj}^{\alpha}}{\xi_{lsi}} + F \right) \right]^{\frac{1}{1-\sigma}} \varphi_{lsi} \tag{38}$$

I measure the impact of the chosen foreign standard compared to the higher domestic standard. How does welfare react with a change of one unit of the standard? I take the quality-elasticity of welfare[33]:

$$\frac{\theta_{sj}}{\mathbb{W}_j} \frac{\partial \mathbb{W}_j}{\partial \theta_{sj}} = \alpha - \Lambda_j - \frac{\xi_{lsj}}{\theta_{sj}^{\alpha}} \Lambda_j F \tag{39}$$

The result is positive if and only if F is very small.

Equation 39 shows that a country with a higher willingness-to-pay for quality β_j is less impacted by a change in the level of standard. On the contrary, the quality-elasticity of fixed cost α influences positively the impact of standard on welfare: the more the cost increases with quality, the more the welfare is impacted. In a much complex analysis, if the standard is already high, ξ_{lsj} increases more than the determinant θ_{sj}^{α}. Indeed, $\frac{\partial \xi_{lsj}}{\partial \theta_{sj}} > 0$ as I demonstrate in the appendix. Thus, $\frac{\xi_{lsj}}{\theta_{sj}^{\alpha}}$ increases with a higher standard and, in turn, the impact of the standard on welfare diminishes with the size of the standard. This welfare repartition is not ambiguous contrarily to the rest of the literature (as far as I know). For example, Gaigné and Larue (2016) find an ambivalent impact of standard on welfare. To conclude with the analysis of the mutual recognition case when the foreign standard is lower than the domestic one, the welfare is lower than with national treatment.

V.3 Harmonization

Harmonization proposes a middle ground: $\theta_{sh} = \frac{\theta_{si}+\theta_{sj}}{2}$. For simplicity, I take the mean of both standards but the analysis is identical in any proportion between both extremes. Contrarily to mutual recognition, all exporters must follow the harmonized standard. The mass of firms is decomposed into two parts: one for the non-exporters which follow only the domestic standard, one for the exporters which follow the harmonized standard. As explained in the beginning of this section on legal frameworks, the ls-firm will choose the lowest among domestic and international standard and this decides whether there exists non-exporters or not.

$$M_i^h = L_i^{-1} \varphi_i^{\gamma} \xi_i^{\iota+1} \left(\varphi_{lsj}^{-\gamma} \xi_{lsj}^{-\iota-1} \Delta_h^{-1} + I_d \varphi_{lsi}^{-\gamma} \xi_{lsi}^{-\iota-1} \Delta_i^{-1} \right) \tag{40}$$

[33] as in Gaigné and Larue (2016)

with $I_d = 1$ when the ls-firm is a non-exporter, $I_d = 0$ when the ls-firm is an exporter (i.e.: the international standard is higher than the domestic one). Also, $\Delta_h = \bar{\theta}_{sh}^{\alpha}\left[\mathrm{Y}_j + \frac{1}{\xi_{lsj}}\right]$
$+ F\xi_{lsj}\left[\varphi_{lsj}^{\sigma-1}\phi_j t^{1-\sigma} L_j\left(\rho_j\left(\bar{\theta}_{sh}^{\Lambda_j} - \theta_j^{\Lambda_j}\right) + \bar{\theta}_{sh}^{\Lambda_j}\right) + \varpi_j + \xi_{lsj}^{\frac{\gamma\alpha+\Lambda_j}{\alpha-\Lambda_j}}\chi_j\right]$
and $\Delta_i = \theta_{si}^{\alpha}\left[\mathrm{Y}_i + \frac{1}{\xi}\right] + F\xi_{lsi}\left[\varphi_{lsi}^{\sigma-1}\phi_i t^{1-\sigma} L_i\left(\rho_i\left(\theta_{si}^{\Lambda_i} - \theta_i^{\Lambda_i}\right) + \theta_{si}^{\Lambda_i}\right) + \varpi_i + \xi_{lsi}^{\frac{\gamma\alpha+\Lambda_i}{\alpha-\Lambda_i}}\chi_i\right]$. The welfare is such that, when taking the shorter version of it, the result changes depending on which standard is the highest between the domestic and the foreign one and if the distance between both is not too big: $f_x < (\theta_{si} - \theta_{sh})$. If the domestic standard is the lowest standard, the non-exporting ls-firm takes it, and the welfare is the same as in the NT framework. If the foreign standard is the lowest one, the now exporting ls-firm takes rather the harmonized standard, which is lower than the domestic one and it provides a higher profit.

$$\mathbb{W}_{j,H} = \frac{\sigma - 1}{t\sigma}\left[\frac{\sigma\bar{\theta}_{sh}^{-\Lambda_i}}{L_i}\left(\frac{\bar{\theta}_{sh}^{\alpha}}{\xi_{lsi}} + F\right)\right]^{\frac{1}{1-\sigma}}\varphi_{lsi} \tag{41}$$

V.4 Comparing masses of firms

The impact of the different legal frameworks can be summed up according to the relationship between the domestic standard and the foreign standard. However, it should also be reminded that the welfare varies positively with the change of standard only up to a certain point. Indeed, the quality-elasticity of welfare decreases with $\frac{\xi_{lsj}}{\theta_{sj}^{\alpha}}$ (i.e.: in equation 39) and, at a certain point, it exceeds $\alpha - \Lambda_j$ (for any F previously fixed, large or small) because a higher θ_{sj} increases the weight of $\frac{\xi_{lsj}}{\theta_{sj}^{\alpha}}$[34]. Thus, assuming the level of θ_{sj} still gives a positive impact on welfare, the impact of the different legal frameworks on welfare is summarized as such:

- with $\theta_{sj} > \theta_{si}$, $\mathbb{W}_{MR} < \mathbb{W}_{H} < \mathbb{W}_{NT}$
- with $\theta_{sj} < \theta_{si}$, $\mathbb{W}_{NT;MR} < \mathbb{W}_{H}$

Those relationship are reversed when θ_{sj} is too high.

Also, the impact of a choice of legal framework on the mass of firms can be recapped as follows:

- with $\theta_{sj} > \theta_{si}$, $M_i^{MR} > M_i^{H} > M_i^{NT}$
- with $\theta_{sj} < \theta_{si}$, $M_i^{MR} = M_i^{NT} > M_i^{H}$

First, one observation is made regarding welfare: harmonization is a prudent decision by default; it is the "maximin". Also, it always provides a more interesting welfare than mutual recognition. On the contrary, mutual recognition is never the best solution. A second observation is made about the mass of firms: the market is more competitive with mutual recognition than with harmonization. However, eliminating the less performing firms results in better average quality. A lower foreign standard is a nuisance in terms of domestic welfare and product quality. This only favors the less successful exporting companies.

[34] As explained above, $\frac{\partial\xi_{lsj}}{\partial\theta_{sj}} > 0$.

IV. Discussion

Intuitively, the discrimination between domestic and foreign firms, present in the mutual recognition framework, is not leading to the best deal. This discrimination is less present in the harmonization framework and not at all in the national treatment framework (my initial case), as one could expect from the EU's 4 principles[35]. MR is clearly advantaging exporters compared to non-exporters. This result comforts Marette and Beghin (2016) who indicate the negative effect of foreign exporters when adding a new standard. However, H is not always the best option when accounting for the mass of firm in the industry. Indeed, when the foreign standard is lower than the domestic one, harmonization might encourage an economy with fewer firms. It might be due to the reduction in the number of varieties (it allows only goods which are standardized to the new normative level).

As several other similar works (Gaigné and Larue (2016), Disdier et al. (2018)), the quality standards impact the industrial structure in two directions. Setting a standard gives (1) an ambiguous impact on average quality and (2) hurts more the fixed cost advantage of domestic firms. More specifically, the choice of QS by the government modifies the distribution of firms in terms of variety, price, and quality. Indeed, firms see their variable and fixed costs increase with higher quality (either imposed for restricted firms or chosen for non-restricted ones). However, the increase in the variable cost might be mitigated by adapting the price level, while nothing can be done to lower the impact of the higher fixed cost. As a result, when a firm has a relatively higher product productivity than its process productivity, the rising variable cost is more painful than the fixed cost increase. Consequently, it might lower the highest quality available and increase the lowest quality available, with ambiguous impact on average quality[36]. In a way this is as if a quality standard lowers the "inequality" in the production of quality. A higher quality standard also diminishes the number of firms in the market (i.e.: the mass of firms decreases - the extensive margin is lower). This is made through a reallocation from low productivity firms towards high productivity firms and, intuitively, this last effect increases welfare because the average quality is higher.

A series of problems comes out of the fact that I use a CES utility function and a non-homothetic framework. First, CES is not the most down-to-earth hypothesis (Manova and Yu, 2017, p.131) and my result could lead to unrealistic outcomes. But, in terms of modelling, CES has the advantage of being highly tractable[37]. Also, as I find in my analysis just above, the impact of standards on welfare are intuitive when a few initial assumptions are made. Second, the use of quality in a homothetic Melitz framework brings issues. Homotheticity implies that, in cross-sections, the rich and the poor consume goods in the same proportions (Matsuyama, 2002). But in fact, it is not the case when inserting quality because poor consumers will make more expenses on low quality goods than rich consumers. While I do not want to make my model too complex and less tractable (Holloway, 2014), I partly respond to that by inserting different willingness-to-pay for quality across country, similarly as Feenstra and Romalis (2014) (see also Baldwin and Harrigan (2011); Fajgelbaum et al. (2015).

V. Conclusion

In the first part of my model, I check the impact of standard on welfare. Nowadays, new trade policies focus on non-tariff barriers such as quality standards. Indeed, standards are often viewed

[35] However, in Marette and Beghin (2016), this 'non-discrimination' principle in NT results is called 'anti-protectionist', meaning that " foreign producers are much more efficient than domestic producers".

[36] The firm's fixed cost is higher with QS, but it has more power to decrease the impact of the variable cost at a lesser cost (for example, by producing less or by increasing the price).

[37] especially concerning the price index

as non-discriminatory and are believed to push trade in contrast to tariffs. In the second part of my model, I compare harmonization, mutual recognition and national treatment. Harmonization and mutual recognition are both used to deal with legal aspects of trade measures. Intuitively, two effects tend to suggest that none is clearly better than the other. On some aspects, H is expected to boost trade more than MR: common standards lead to higher homogeneity and substitutability between products, as well as lower information costs for both firms and consumers. Also, MR discriminates by advantaging the exporting firms which can prefer to lower their quality to sell in the lowest standard country[38]. On other aspects, mutual recognition puts a high point on unnecessary obstacles to trade, allowing exporting firms from each market to choose their national standards. Harmonization can have a negative impact on trade that can be avoided through MR: one trade partner might be more impacted by the new harmonization and it may generate compliance (i.e.: adaptation) costs that vary for different countries.

The specificity of my model comes from the integration of both different love-for-quality in a two countries setup and two productivities à la Hallak and Sivadasan in a Melitz framework. Because of those assumptions, my model has various implications. First, after the introduction of a positive quality standard, some firms exit the market and all remaining firms see either a status quo or an improvement of their profit. Second, the remaining firms which produced a lower quality than the standard adopt the standard but do not produce with regards to the heterogeneous love-for-quality across country (contrarily to non-constrained firms). Third, placing two productivities in my model leads to results different from those obtained in the literature (Grossman et al., 2019; Lutz and Pezzino, 2012; Petropoulou, 2012): I find a level of quality productivity at which firm start exporting (like for the product productivity in a standard Melitz model). This contradicts a previous empirical measure from Crozet et al. (2012) and calls for an empirical review in a future paper. Also, thanks to the introduction of quality productivity, I depart from Macedoni and Weinberger (2019) who find positive impact of standard only for specific non-optimal behavior of so-called "low-quality firms". My result identifies a positive impact of standard on welfare up until some level, with no distinction among firms' behavior. At some point, the standard becomes too high and welfare decreases. Fourth, in terms of legal framework impact, I show that, given low fixed production and transportation costs and when the quality standard is not too high, harmonization is better than mutual recognition in terms of welfare but it decreases the number of firms in the market. Note that this result does not contradict the fact that, according to Lamy (2015); Sykes (2000), the degree of harmonization should depend on the degree of heterogeneity among culture.

In future research, it could be interesting to link quality output, market structure and international trade in a more empirical work (particularly with data from trade agreements like CETA which started on September 21st, 2017 when and if data is available). For example, it could concern how cultural heterogeneity impacts the legal framework choice, especially by making a distinction between TBT and SPS measures. Economic regions which have been through different legal frameworks can be compared. This requires finding specific moments in time where standards have improved in one country and not for its trading partners. Finally, some other sub-projects concerning quality standards and legal frameworks of trade agreements can be developed and reveal themselves to be primary. Among several ideas, I sort out the distinction between developed versus developing countries, the integration of middle to high revenue consumers in a similar model, or the inclusion of the GTP utility framework to account for different willingness-to-pay for quality across consumers, as Macedoni and Weinberger (2019) do.

[38]In addition, being promoted by the WTO, it allows entry into the whole regional market to 3rd countries while it is not the case for MR agreements. Even more, MR creates trade-diversion effects for 3rd countriesGaigné and Larue (2016)

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I. Appendix

II. Utility, demand and expenditures

The domestic utility specific to the consumers of one country:

$$\max_{q_k;p_k} U_i = \left(\int_{k\in\Omega_i} \left(\theta_k^{\beta_i} q_k\right)^{\frac{\sigma-1}{\sigma}} dk \right)^{\frac{\sigma}{\sigma-1}}$$

$$s.c.: L_i = E_i = \int_{k\in\Omega_i} p_k q_k dk \theta \in \underline{\theta}; \overline{\theta}$$

Deriving it by $q_k; p_k$ using a Lagrangian, I find the following demand and expenditures equations:

$$q_i = \frac{\theta_i^{\beta_i(\sigma-1)} \left[\int_{k\in\Omega_i} \left(\theta_k^{\beta_i} q_k\right)^{\frac{\sigma-1}{\sigma}} dk \right]^{\frac{\sigma}{\sigma-1}} p_k^{-\sigma}}{\lambda^{\sigma}}$$

$$p_k q_k = \frac{\theta_k^{\beta_i(\sigma-1)} \left[\int_{k\in\Omega_i} \left(\theta_k^{\beta_i} q_k\right)^{\frac{\sigma-1}{\sigma}} dk \right]^{\frac{\sigma}{\sigma-1}} p_k^{1-\sigma}}{\lambda^{\sigma}}$$

Plugging the expenditure in the budget constraint and isolating the marginal utility of income, I find:

$$\lambda^{\sigma} = \frac{1}{L_i} \int_{k\in\Omega_i} \theta_k^{\beta_i(\sigma-1)} p_k^{1-\sigma} \left[\int_{k\in\Omega_i} \left(\theta_k^{\beta_i} q_k\right)^{\frac{\sigma-1}{\sigma}} dk \right]^{\frac{\sigma}{\sigma-1}} dk$$

Plugging this expression back in the expenditure equation, I find:

$$p_k q_k = L_i \theta_k^{\beta_i(\sigma-1)} p_k^{1-\sigma} P_i^{\sigma-1}$$

$\int_{k\in\Omega_i} \theta_k^{\beta_i(\sigma-1)} p_{i_k}^{1-\sigma} dk$ is the price index $P_i^{1-\sigma}$ which is derived from the well-known properties of the CES demand function. To be able to measure the impacts on the number of firms later, I complete this expression as follows:

$$P_i^{1-\sigma} = \int_{k\in\Omega_i} \theta_{ki}^{\beta_i(\sigma-1)} p_{ki}^{1-\sigma} dk = \sum_{k}^{K} \int_{\varphi}^{\infty} \int_{\xi}^{\infty} \theta_{ki}^{\beta_i(\sigma-1)} p_{ki}^{1-\sigma}(\varphi) M_{ki} \varphi_{kk}^{\gamma} \xi_{kk}^{\iota} g(\varphi) v(\xi) d\xi d\varphi$$

Where M_{ki} is the mass of firms producing k and selling in $i; g(\varphi) = \gamma \varphi_i^{-\gamma-1}$ *and* $v(\xi) = \iota \xi_i^{-\iota-1}$ are the ex-post distribution of productivities if the firm exist and sell in i. As in Gaigné & Larue (2016), note the negative relationship between the price index and quality because $\sigma > 1$. For simplicity of notation, I get rid of the variety k index.

III. Demand

I get the following domestic demand:

$$q_i = L_i \theta \beta_i (\sigma-1) p_i^{-\sigma} \left(P_i^{1-\sigma}\right)^{-1} = \theta \beta_i (\sigma-1) p_i^{-\sigma} L_i P_i^{\sigma-1}$$

By symmetry, foreign demand is the following:

$$q_j = \theta \beta_j (\sigma-1) p_j^{-\sigma} L_j P_j^{\sigma-1}$$

And total demand is the following if a firm sells in both market (in i+j):

$$q_{ij} = L_i\theta\beta_i\left(\sigma-1\right)p_i^{-\sigma}P_i^{\sigma-1} + L_j\theta\beta_j\left(\sigma-1\right)p_j^{-\sigma}P_j^{\sigma-1}$$

Notice that prices are different in each market, depending on the quality chosen for each of them.

IV. Revenue

Given the demand, the domestic revenue is:

$$r_i = p_i q_i = \theta\beta_i\left(\sigma-1\right)\, {p_i}^{1-\sigma}L_iP_i^{\sigma-1}$$

Thus, the exporting revenue is:

$$r_j = \theta^{\beta_j(\sigma-1)}p_i(1-\sigma)L_jP_j^{\sigma-1}$$

Total revenue for the firm selling in both markets is:

$$r_T = r_i + r_j = p_iq_i + p_jq_j = \theta\beta_i\left(\sigma-1\right)p_i^{1-\sigma}L_iP_i^{\sigma-1} + \theta\beta_j\left(\sigma-1\right)p_j^{1-\sigma}L_jP_j^{\sigma-1}$$

Costs:

$$C = \frac{\theta_j\varepsilon}{\varphi}tq_j + \frac{\theta_j\alpha}{\xi} + F$$

Where $F = f + \mathbb{I}_j f_x$ Profit:

$$\pi_i = p_iq_i - c_itq_i - \omega_i - F$$

V. Exporting profit maximization

Exporting profit:

$$\pi_j = p_jq_j - \frac{\theta_j\varepsilon}{\varphi}tq_j - \frac{{\theta_j}^{\alpha}}{\xi} - F = \theta_j^{\beta_j(\sigma-1)}p_j^{1-\sigma}L_jP_j^{\sigma-1} - \frac{\theta_j\varepsilon}{\varphi}t\theta_j\beta_j\left(\sigma-1\right)p_j^{-\sigma}L_jP_j^{\sigma-1} - \frac{{\theta_j}^{\alpha}}{\xi} - F$$

Max in terms of p_j

$$\max_{p_j}\pi_j \Longleftrightarrow {\pi_j}' = 0 \Leftrightarrow p_j = \frac{1}{\varphi}\frac{\sigma}{\sigma-1}t\theta_j^{\varepsilon}$$

Max in terms of θ_j Maximize with respect to quality

$$\max_{\theta_j}\pi_j \Longrightarrow {\pi_j}' = 0$$

$${\pi_j}' = \beta_j\left(\sigma-1\right)\theta_j^{\beta_j(\sigma-1)-1}p_j1-\sigma L_jP_j^{\sigma-1} - \left(\varepsilon+\beta_j\left(\sigma-1\right)\right)\frac{1}{\varphi}t\theta_j^{\varepsilon+\beta_j(\sigma-1)-1}p_j{-}\sigma L_jP_j^{\sigma-1} - \alpha\frac{\theta_j^{\alpha-1}}{\xi}$$

$$\theta_j = \left[\frac{\beta-\varepsilon}{\alpha}\left[\frac{\sigma-1}{\sigma}\right]^{\sigma}\varphi^{\sigma-1}\xi t^{1-\sigma}L_jP_j^{\sigma-1}\right]^{\frac{1}{\alpha-\Lambda}} \tag{42}$$

If $\beta_j - \varepsilon \leq 0$, the profit is decreasing with quality. Firms will always produce the minimum quality; if $\beta_j - \varepsilon > 0$, a firm increases its operating profit (the left part of the inequality) by increasing its quality. However, it is not necessarily the case if I take the cost of upgrading quality into account. Particularly above the optimum quality, $\frac{\partial \pi}{\partial \theta} < 0$

$$\beta_j (\sigma - 1) \theta_j^{\beta_j(\sigma-1)} p_j 1 - \sigma L_j P_j^{\sigma-1} - \alpha \frac{\theta_j^{\alpha}}{\xi} = \left(\varepsilon + \beta_j (\sigma - 1)\right) \frac{1}{\varphi} t \theta_j^{\varepsilon+\beta_j(\sigma-1)} p_j - \sigma L_j P_j^{\sigma-1} \tag{43}$$

Putting 42 into 43, I get this final expression:

$$\Longleftrightarrow \theta_j^* = \left[\frac{\beta_j - \varepsilon}{\alpha} \left[\frac{\sigma - 1}{\sigma} \right]^{\sigma} \varphi^{\sigma-1} \xi t^{1-\sigma} L_j P_j^{\sigma-1} \right]^{\frac{1}{\alpha - \Lambda_j}}$$

$$\Longleftrightarrow \theta_j^* = \left[\frac{1}{\sigma} \frac{\Lambda_j}{\alpha} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma-1} \xi \varphi^{\sigma-1} t^{1-\sigma} L_j P_j^{\sigma-1} \right]^{\frac{1}{\alpha - \Lambda_j}}$$

The optimal price is found:

$$\Longleftrightarrow p_j^* = \left[\frac{\beta_j - \varepsilon}{\alpha} \right]^{\frac{\varepsilon}{\alpha-\Lambda_j}} \left[\frac{\sigma - 1}{\sigma} \right]^{\frac{\beta_j(\sigma-1)+\varepsilon-\alpha}{\alpha-\Lambda_j}} \xi^{\frac{\varepsilon}{\alpha-\Lambda_j}} \varphi^{\frac{\beta_j(\sigma-1)-\alpha}{\alpha-\Lambda_j}} \left[t^{1-\sigma} L P^{\sigma-1} \right]^{\frac{\varepsilon}{\alpha-\Lambda_j}}$$

The price adjusted to quality is:

$$\widetilde{p_j^*} = A \eta^{\frac{-1}{\sigma-1}} \left(L_j P_j^{\sigma-1} t^{1-\sigma} \right)^{\frac{\varepsilon - \beta_j}{\alpha - \Lambda_j}}$$

With $\eta_i (\varphi, \xi) = \xi^{\frac{\Lambda_j}{\alpha-\Lambda_j}} \varphi^{\frac{\alpha(\sigma-1)}{\alpha-\Lambda_j}}$; $A_j \equiv \left[\frac{\sigma-1}{\sigma} \right]^{\frac{-\alpha-\beta_j+\varepsilon}{\alpha-\Lambda_j}} \left[\frac{\beta_j-\varepsilon}{\alpha} \right]^{\frac{\varepsilon-\beta_j}{\alpha-\Lambda_j}}$ The exporting revenue is:

$$r_j = H \eta \left(t^{1-\sigma} L_j P_j^{\sigma-1} \right) \frac{\alpha}{\alpha - \Lambda_j}$$

Where: $H = \left[\frac{\sigma-1}{\sigma} \right]^{\frac{(\sigma-1)\alpha+\Lambda_j}{\alpha-\Lambda_j}} \left[\frac{\beta_i-\varepsilon}{\alpha} \right]^{\frac{\Lambda_j}{\alpha-\Lambda_j}}$ Finally, the optimal profit:

$$\pi_j^* = \eta J N \left(t^{1-\sigma} L_j P_j^{\sigma-1} \right)^{\frac{\alpha}{\alpha-\Lambda_j}} - F$$

With:

$$J \equiv \left[\frac{\sigma - 1}{\sigma} \right]^{\frac{\sigma\alpha}{\alpha-\Lambda_j}} \left[\frac{\beta_j - \varepsilon}{\alpha} \right]^{\frac{\alpha}{\alpha-\Lambda_j}} ; N \equiv \frac{\alpha t^{\sigma} (\sigma t^{-1} - \sigma + 1) - \Lambda_j}{\Lambda_j}$$

Small simplifications of foc from my results:

$$r_j = \left[\frac{\beta_j - \varepsilon}{\alpha} \right]^{\frac{\alpha}{\alpha-\Lambda_j}} \left[\frac{\sigma - 1}{\sigma} \right]^{\frac{\sigma\alpha}{\alpha-\Lambda_j}} \eta \left(t^{1-\sigma} L_j P_j^{\sigma-1} \right)^{\frac{\alpha}{\alpha-\Lambda_j}} \left[\frac{\sigma}{\sigma - 1} \right] \left[\frac{\alpha}{\beta_j - \varepsilon} \right] \tag{44}$$

$$\eta (\varphi, \xi) = \xi^{\frac{\Lambda_j}{\alpha-\Lambda_j}} \varphi^{\frac{\alpha(\sigma-1)}{\alpha-\Lambda_j}} \tag{45}$$

$$\theta_j^* = \left[\frac{\beta_j - \varepsilon}{\alpha} \left[\frac{\sigma - 1}{\sigma} \right]^{\sigma} \varphi^{\sigma-1} \xi t^{1-\sigma} L_j P_j^{\sigma-1} \right]^{\frac{1}{\alpha-\Lambda_j}} \tag{46}$$

$$\Longrightarrow r_j = \xi^{-1}\theta_j^{\alpha}\left[\frac{\sigma}{\sigma-1}\right]\left[\frac{\alpha}{\beta_j-\varepsilon}\right]$$

$$\Longleftrightarrow p_j q_j = \frac{\omega\sigma\alpha}{\Lambda_j}$$

$$\pi_j^* = \left[\frac{\beta_j-\varepsilon}{\alpha}\right]^{\frac{\alpha}{\alpha-\Lambda_j}}\left[\frac{\sigma-1}{\sigma}\right]^{\frac{\sigma\alpha}{\alpha-\Lambda_j}}\eta\left(t^{1-\sigma}L_jP_j^{\sigma-1}\right)^{\frac{\alpha}{\alpha-\Lambda_j}}\left(\frac{\alpha-\Lambda_j}{\Lambda_j}\right)-F$$

$$\Longleftrightarrow \pi_j^* = \frac{\omega_j\left(\alpha-\Lambda_j\right)}{\Lambda_j}-F$$

as a result, a firm decides to export if and only if $\frac{\omega_j\left(\alpha-\Lambda_j\right)}{\Lambda_j}\geq F$

VI. Exporting cutoff

$$\pi_j^* = \frac{\omega_j\left(\alpha-\Lambda_j\right)}{\Lambda_j}=F$$

Since $\alpha-\Lambda_j>0$ because of $\alpha'>0$, $\underline{\theta_j}=\left(\frac{F\Lambda_j\xi}{\left(\alpha-\Lambda_j\right)}\right)^{\frac{1}{\alpha}}$ is the only condition to get to the production cutoff and represent the lowest quality to export. Similarly, the zero-profit condition for the domestic firm is found by setting $t=1$ and using β_i. Here below, I find the welfare based on the price index and this zero-profit condition. Melitz and Redding use the zero-profit condition:

$$\Rightarrow P^{-1}=\left(\frac{L}{\sigma f_d}\right)^{\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\varphi_d$$

$$r_j = H\eta\left(t^{1-\sigma}L_jP_j^{\sigma-1}\right)^{\frac{\alpha}{\alpha-\Lambda_j}}$$

$$r_j=\left[\frac{\sigma-1}{\sigma}\right]^{\frac{(\sigma-1)(\alpha-1)+\Lambda_j}{\alpha-\Lambda_j}}\left[\frac{\Lambda_j}{\alpha}\right]^{\frac{\Lambda_j-1}{\alpha-\Lambda_j}}\sigma^{\frac{1}{\alpha-\Lambda_j}}\left[\frac{1}{\sigma-1}\right]^{\frac{\Lambda_j}{\alpha-\Lambda_j}}\xi^{\frac{\Lambda_j-1}{\alpha-\Lambda_j}}\varphi^{\frac{(\alpha-1)(\sigma-1)}{\alpha-\Lambda_j}}\left(t^{1-\sigma}L_jP_j^{\sigma-1}\right)^{\frac{\alpha-1}{\alpha-\Lambda_j}}$$

$$\theta_j^*\theta_j^{\Lambda_j}L_jP_j^{\sigma-1}t^{1-\sigma}\varphi^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}$$

total costs are found using the optimal quality:

$$\theta_j^{\Lambda_j}t^{1-\sigma}\varphi^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}L_jP_j^{\sigma-1}+\frac{\theta_j^{\alpha}}{\xi}+F$$

What is the minimum productivity threshold? (when not taking the optimal quality)

$$\theta_j^{\Lambda_j}L_jP_j^{\sigma-1}t^{1-\sigma}\varphi^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}=\theta_j^{\Lambda_j}t^{1-\sigma}\varphi^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}L_jP_j^{\sigma-1}+\frac{\theta_j^{\alpha}}{\xi}+F$$

$$\Longleftrightarrow \underline{\varphi}^{\sigma-1}=\theta_j^{-\Lambda_j}\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{\sigma\left(\frac{\theta_j^{\alpha}}{\xi}+F\right)}{t^{1-\sigma}L_jP_j^{\sigma-1}}$$

What is the price index? (using both optimal price and optimal quality)

$$\Longleftrightarrow P_j = (F)^{\frac{\alpha-\Lambda_j}{(\sigma-1)\alpha}} \left[\left(\frac{\Lambda_j}{\alpha}\right)^{\Lambda_j} - \left(\frac{\Lambda_j}{\alpha}\right)^{\alpha} \right]^{\frac{-1}{(\sigma-1)\alpha}} \left[\frac{1}{\sigma-1}\right]^{\frac{-\alpha}{(\sigma-1)\alpha}} \xi^{\frac{-\Lambda_j}{(\sigma-1)\alpha}} \left[\left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \varphi^{\sigma-1} t^{1-\sigma} L_j \right]^{\frac{-\alpha}{(\sigma-1)\alpha}}$$

VII. Labor market clearing and mass of firms under a zero-QS

$$L_i = l_i + M_e f_e + \sum_j M_i \varphi_i^{\gamma} \xi_i^{\iota} \varphi_j^{-\gamma} \xi_j^{-\iota} F$$

The probability of entry f_e is:

$$\overline{\pi_j} = \underline{\varphi_i}^{\gamma} \underline{\xi_i}^{\iota} \int_{\xi_j}^{\infty} \left[\int_{\varphi_j}^{\infty} \left[\left(\frac{\varphi_j}{\underline{\varphi_j}}\right)^{\frac{(\sigma-1)\alpha}{\alpha-\Lambda_j}} F - F \right] g(\varphi)\, d\varphi \right] v(\xi) d\xi = 0 \Longleftrightarrow f_e = \underline{\varphi_i}^{-\gamma} \underline{\xi_i}^{-\iota}$$

$$\Longrightarrow f_e = \frac{(\sigma-1)\alpha}{\gamma(\alpha-\Lambda_j) - (\sigma-1)\alpha} F \underline{\varphi_j}^{-\gamma} \underline{\xi_j}^{-\iota}$$

$$l_i = \int_{\xi_j}^{\infty} \left[\int_{\varphi_j}^{\infty} M_j \left[\frac{\sigma-1}{\sigma} p_j q_j + \frac{\theta_j^{\alpha}}{\xi} \right] g(\varphi)\, d\varphi \right] v(\xi) d\xi = M_i \underline{\varphi_i}^{\gamma} \underline{\xi_i}^{\iota} \left(1 + \frac{\Lambda_j}{\alpha(\sigma-1)}\right) \frac{\gamma f_e}{\alpha}$$

Also (and finally),

$$\sum_j M_i \varphi_i^{\gamma} \xi_i^{\iota} \varphi_j^{-\gamma} \xi_j^{-\iota} F = M_i \varphi_i^{\gamma} \xi_i^{\iota} f_e \frac{\gamma(\alpha-\Lambda_j) - (\sigma-1)\alpha}{(\sigma-1)\alpha}$$

$$\Longrightarrow L_i = M_i \underline{\varphi_i}^{\gamma} \underline{\xi_i}^{\iota} f_e \frac{\gamma}{\alpha} \left(\frac{\Lambda_j + \alpha(\alpha-\Lambda_j)}{\alpha(\sigma-1)} + 1 \right)$$

Thus,

$$M_i = \frac{L_i(\sigma-1)\alpha}{\underline{\varphi_i}^{\gamma} \underline{\xi_i}^{\iota} f_e \gamma} \frac{\alpha}{\alpha(\sigma-1+\alpha-\Lambda_j) + \Lambda_j}$$

VIII. Constrained and unconstrained firms

Constrained firms have optimal qualities below or equal to the standard but are still making non-negative profit; unconstrained firms have optimal qualities above the standard and choose their optimal quality. Their case is similar as when the standard is 0. In a world with both types of firms, I need to find a specific price index which accounts for this standard. I can also find the mass of firms accounting for both types of firms. To solve the mass of firms, I need to make the hypothesis that there are no fixed costs. In Disdier (2018), the fixed cost only depends on quality. They assume that $f = 0$ or is a product of the other fixed cost. Lugovskyy & Skiba (2015) also ignores the fixed transportation cost. For simplicity, since I study the case of the exporting firm, I omit several index j $(\Lambda = \Lambda_j,\ \beta = \beta_j,\ \theta_s = \theta_{sj},\ \varphi_{ls} = \varphi_{lsj},\ and\ \xi_{ls} = \xi_{lsj})$. The impact of a QS on quality and prices using substitution The exporting profit for a firm is:

$$\max_{p_j,\theta_j} \pi_j = \theta_j^{\beta_j(\sigma-1)} p_j^{1-\sigma} L_j P_j^{\sigma-1} - \frac{\theta_j^{\varepsilon}}{\varphi} t \theta_j^{\beta_j(\sigma-1)} p_j^{-\sigma} L_j P_j^{\sigma-1} - \frac{\theta_j^{\alpha}}{\xi} - F$$

Under constraint: $\theta_s \leq \theta_j$ At the limit, one firm is indifferent between the standard and its optimal quality.

Binding constraint However, some firms are not able to optimize their profit according to quality, those for which the constraint is binding.

Thus, when the constraint is binding, the profit maximization is:

$$\max_{p_j} \pi_j = \theta_{sj}^{\beta_j(\sigma-1)} \; p_j^{1-\sigma} L_j P_j^{\sigma-1} - \frac{\theta_{sj}^{\varepsilon}}{\varphi} t \theta_{sj}^{\beta_j(\sigma-1)} p_j^{-\sigma} L_j P_j^{\sigma-1} - \frac{\theta_{sj}^{\alpha}}{\xi} - F$$

$$\frac{\partial \pi}{\partial p_j} = 0$$

$$\iff p_j = \frac{1}{\varphi} \frac{\sigma}{\sigma-1} t \theta_{sj}^{\varepsilon}$$

Thus, the optimal profit for the representative constrained firm is:

$$\pi_j^* = \left(\frac{1}{\varphi} \frac{\sigma}{\sigma-1} t \theta_{sj}^{\varepsilon} - \frac{{\theta_{sj}}^{\varepsilon}}{\varphi} t \right) {\theta_{sj}}^{\beta_j(\sigma-1)} \left(\frac{1}{\varphi} \frac{\sigma}{\sigma-1} t \theta_{sj}^{\varepsilon} \right)^{-\sigma} L_j P_j^{\sigma-1} - \frac{{\theta_{sj}}^{\alpha}}{\xi} - F$$

$$= \frac{\varphi^{\sigma-1}}{\sigma} \left(\frac{t\sigma}{\sigma-1} \right)^{1-\sigma} {\theta_{sj}}^{\Lambda_j} L_j P_j^{\sigma-1} - \omega_j - F$$

The constrained firm with the lowest profit has a profit of zero. Thus, the Price Index is the following:

$$P_j^{\sigma-1} = \sigma \varphi^{1-\sigma} t^{\sigma-1} \left(\frac{\sigma}{\sigma-1} \right)^{\sigma-1} \theta_{sj}^{-\Lambda_j} \frac{1}{L_j} \left(\frac{\theta_{sj}^{\alpha}}{\xi_{lsj}} + F \right)$$

Sales An increase in the QS has a positive effect on revenues. Substituting for the price index which applies to all firms:

$$p_j^c q_j^c = \sigma \left(\frac{{\theta_s}^{\alpha}}{\xi_{ls}} \right) \left(\frac{\varphi}{\varphi_{ls}} \right)^{\sigma-1} - F$$

Which I divide with equilibrium price (I get the same result as Gaigné & Larue, 2016) to find the sales:

$$q_j^c = \frac{(\sigma-1)}{t} \frac{\varphi^{\sigma}}{\varphi_{ls}^{\sigma-1}} \frac{\theta_s^{\alpha-\varepsilon}}{\xi_{ls}} - \frac{F}{\frac{\sigma t}{\sigma-1} \frac{1}{\varphi} \theta_s^{\varepsilon}}$$

The profit

$$\pi^c = \theta_s^{\alpha} \left[\frac{1}{\xi_{ls}} \left(\frac{\varphi}{\varphi_{ls}} \right)^{\sigma-1} - \frac{1}{\xi} \right] - F \left(\frac{\varphi^{\sigma-1}}{\sigma} \left(\frac{t\sigma}{\sigma-1} \right)^{1-\sigma} \theta_s^{\Lambda} L_j + 1 \right)$$

Non-binding constraint For unconstrained firms, a standard should be below or equal to the optimal quality (when profit is max).

$$\left[\frac{\beta_j - \varepsilon}{\alpha} \left[\frac{\sigma-1}{\sigma} \right]^{\sigma} \varphi^{\sigma-1} \xi L_j {P_j}^{\sigma-1} \right]^{\frac{1}{\alpha-\Lambda_j}} \geq \theta_s$$

The unconstrained firms make an optimal quality similar as in the case with a standard of zero but with respect to the new price index.

$$\Longrightarrow \theta_j^* = \left[\frac{\Lambda}{\alpha} \theta_s^{-\Lambda} \left(\frac{\theta_s^{\alpha}}{\xi_{ls}} + F \right) \xi \left(\frac{\varphi}{\varphi_{ls}} \right)^{\sigma-1} \right]^{\frac{1}{\alpha-\Lambda}}$$

The optimal profit for the representative unconstrained firm is the following (taking back the optimal profit for the firm with a standard of zero:

$$\pi_j^* = \Lambda_j^{\frac{\Lambda_j}{\alpha-\Lambda_j}} \xi^{\frac{\Lambda_j}{\alpha-\Lambda_j}} \left[\frac{1}{\alpha} \theta_s^{-\Lambda} \left(\frac{\theta_s^\alpha}{\xi_{ls}} + F \right) \left(\frac{\varphi}{\varphi_{ls}} \right)^{\sigma-1} \right]^{\frac{\alpha}{\alpha-\Lambda}} (\alpha - \Lambda_j) - F$$

And the revenue is:

$$p_j q_j = \Lambda_j^{\frac{\Lambda_j}{\alpha-\Lambda_j}} \xi^{\frac{\Lambda_j}{\alpha-\Lambda_j}} \left[\frac{1}{\alpha} \theta_s^{-\Lambda} \left(\frac{\theta_s^\alpha}{\xi_{ls}} + F \right) \left(\frac{\varphi}{\varphi_{ls}} \right)^{\sigma-1} \right]^{\frac{\alpha}{\alpha-\Lambda}} \sigma\alpha$$

Below, I find the price, quantities, sales and then the number of constrained firms needed to measure welfare.

Sales Using the same method as for the constrained firms, I find the sales. The optimal revenue is (profit max):

$$p_j q_j = \frac{\sigma\alpha}{\Lambda\xi} \theta_j^{*\alpha} = \frac{\sigma\alpha}{\Lambda\xi} \left[\frac{\Lambda}{\alpha} \theta_s^{-\Lambda} \left(\frac{\theta_s^\alpha}{\xi_{ls}} + F \right) \xi \left(\frac{\varphi}{\varphi_{ls}} \right)^{\sigma-1} \right]^{\frac{\alpha}{\alpha-\Lambda}}$$

The optimal price is:

$$p_j = \frac{\sigma t}{(\sigma-1)} \left[\frac{\Lambda}{\alpha} \theta_s^{-\Lambda} \left(\frac{\theta_s^\alpha}{\xi_{ls}} + F \right) \xi \left(\frac{1}{\varphi_{ls}} \right)^{\sigma-1} \right]^{\frac{\varepsilon}{\alpha-\Lambda}} \varphi^{\frac{[\sigma-1]\beta_i - \alpha}{\alpha-\Lambda}}$$

Thus, the quantity is:

$$q_j = \frac{\alpha(\sigma-1)}{\Lambda_{\text{¸}} t} \left[\frac{\Lambda}{\alpha} \theta_s^{-\Lambda} \left(\frac{\theta_s^\alpha}{\xi_{ls}} + F \right) \xi \left(\frac{1}{\varphi_{ls}} \right)^{\sigma-1} \right]^{\frac{\alpha-\varepsilon}{\alpha-\Lambda}} \varphi^{\frac{\sigma\alpha - [\sigma-1]\beta_i}{\alpha-\Lambda}}$$

The profit

$$\begin{aligned}
\pi^u = \theta_s^\alpha & \left[\left(\frac{\xi\Lambda}{\alpha} \right)^{\frac{\Lambda}{\alpha-\Lambda}} \left(\frac{1}{\xi_{ls}} \right)^{\frac{\alpha}{\alpha-\Lambda}} \left(\frac{\varphi}{\varphi_{ls}} \right)^{\frac{(\sigma-1)\alpha}{\alpha-\Lambda}} - \frac{1}{\xi} \right] \\
& - F \left[\frac{\varphi^{\sigma-1}}{\sigma} \left(\frac{t\sigma}{\sigma-1} \right)^{1-\sigma} \theta_s^\Lambda L_j + 1 \right] \\
& + \left[\frac{\Lambda}{\alpha} \theta_s^{-\Lambda} F \xi \left(\frac{\varphi}{\varphi_{ls}} \right)^{\sigma-1} \right]^{\frac{\Lambda}{\alpha-\Lambda}} \left(\frac{\theta_s^{\alpha-\Lambda}}{\xi_{ls}} \left(\frac{\varphi}{\varphi_{ls}} \right)^{\sigma-1} - F \frac{\varphi^{\sigma-1}}{\sigma} \left(\frac{t\sigma}{\sigma-1} \right)^{1-\sigma} L_j \right)
\end{aligned}$$

Limit firm The productivity for a firm at the limit between being constrained and unconstrained is the following:

$$\begin{aligned}
\pi^c &= \pi^u \\
\iff \hat{\varphi} &= \left(\frac{\Lambda}{\alpha} \right)^{\frac{1}{1-\sigma}} \varphi_{ls} \frac{\xi}{\xi_{ls}}
\end{aligned} \tag{47}$$

IX. On entry and exit (mass of firms)

The expected profits are summarized by the probability to adopt the public standard or to be above it, I call this form of profit the mixed expected profit. I make the hypothesis that both distributions of ξ and φ are independent to make $h(\varphi,\ \xi) = v(\xi)g(\varphi)$. A specification will be made to distinguish home and foreign destinations at the end of the calculation. Here also, for simplicity, I omit several index j ($\Lambda = \Lambda_j,\ \beta = \beta_j,\ \theta_s = \theta_{sj},\ \varphi_{ls} = \varphi_{lsj},\ and\ \xi_{ls} = \xi_{lsj}$).

$$[1 - G(\varphi)][1 - V(\xi)]\,\overline{\pi_i} = \sum_1^2 \int_{\xi_{lsj}}^{\infty} \left[\int_{\varphi_{lsj}}^{\hat{\varphi}} \pi_j^c(\theta_{sj})\, g(\varphi)\, d\varphi + \int_{\hat{\varphi}}^{\infty} \pi_j^u(\theta_j)\, g(\varphi)\, d\varphi \right] v(\xi) d\xi$$

$$= \sum_j^i \varphi_i^{\gamma} \xi_i^{\iota+1} \varphi_{lsj}^{-\gamma} \xi_{lsj}^{-\iota-1} \left(\theta_{sj}^{\alpha} \psi_j + F\left(\varphi_{lsj}^{\sigma-1} \xi_{lsj} \phi_j t^{1-\sigma} L_j \left(\rho_j \left(\theta_{sj}^{\Lambda_j} - \theta_j^{\Lambda_j} \right) + \theta_{sj}^{\Lambda_j} \right) - \xi_{lsj} \right) \right)$$

with

$$\psi_j = \left(\frac{\alpha}{\Lambda_j} \right)^{\frac{\sigma-\gamma-1}{\sigma-1}}$$

$$\left(\frac{\gamma(\alpha - \Lambda_j)}{(\sigma - \gamma - 1)\alpha + \gamma\Lambda_j} \frac{\iota(\alpha - \Lambda_j)}{(\sigma - \gamma - \iota - 1)\alpha + \Lambda_j(\gamma + \iota + 1)} - \frac{\gamma}{\sigma - \gamma - 1} \frac{\iota}{\sigma - \gamma - \iota - 1} \right)$$
$$- \frac{\gamma}{\sigma - \gamma - 1} - \frac{\iota}{\iota + 1} > 0$$

and $-1 < \rho_j = \left(\frac{\Lambda_j}{\alpha} \right)^{\frac{\sigma-\gamma-1}{1-\sigma}} \frac{\iota}{\sigma-\gamma-\iota-1} < 0$ and $\phi_j = \frac{\gamma}{(\sigma-\gamma-1)\sigma} \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} < 0$.

Proofs: $\psi_j > 0 \frac{\gamma(\alpha-\Lambda_j)}{(\sigma-\gamma-1)\alpha+\gamma\Lambda_j} \frac{\iota(\alpha-\Lambda_j)}{(\sigma-\gamma-\iota-1)\alpha+\Lambda_j(\gamma+\iota+1)} > 0, \frac{\gamma}{\sigma-\gamma-1} \frac{\iota}{\sigma-\gamma-\iota-1} > 0, \frac{\gamma}{\sigma-\gamma-1} < 0, \frac{\iota}{\iota+1} > 0.$ I prove that $\frac{\gamma(\alpha-\Lambda_j)}{(\sigma-\gamma-1)\alpha+\gamma\Lambda_j} \frac{\iota(\alpha-\Lambda_j)}{(\sigma-\gamma-\iota-1)\alpha+\Lambda_j(\gamma+\iota+1)} > \frac{\gamma}{\sigma-\gamma-1} \frac{\iota}{\sigma-\gamma-\iota-1}$, that $\frac{\iota}{\iota+1}$ is small and belongs to the interval $[0;1]$ at the limits of both parameters $\iota,\ \gamma$. Finally, I prove that $\frac{\gamma}{\sigma-\gamma-1} > \frac{\iota}{\iota+1}$.

- $\frac{\gamma(\alpha-\Lambda_j)}{(\sigma-\gamma-1)\alpha+\gamma\Lambda_j} \frac{\iota(\alpha-\Lambda_j)}{(\sigma-\gamma-\iota-1)\alpha+\Lambda_j(\gamma+\iota+1)} > \frac{\gamma}{\sigma-\gamma-1} \frac{\iota}{\sigma-\gamma-\iota-1}$ because $1 - \sigma < 0$ and $\Lambda > 0$.
- $\frac{\gamma}{\sigma-\gamma-1} > \frac{\iota}{\iota+1}$ because $\sigma - \gamma - 1 < 0$, $\sigma - 1 > 0$, and $\frac{\iota}{\iota+1} < 1$

The labor market condition is:

$$L_i = l_i^A + l_i^B + M_i \varphi_i^{\gamma} \xi_i^{\iota+1} f_e + M_i \varphi_i^{\gamma} \xi_i^{\iota} \sum_j^i \varphi_j{}^{-\gamma} \xi_j{}^{-\iota-1} (\omega_j + F)$$

where $M_i \varphi_i^{\gamma}$ is the mass of firms in i, l_i^A is the mass of workers allocated to the production in the constrained firms and l_i^B is the unconstrained firm's mass of workers. I assume that $M_i = M_i^A + M_i^B$. Among that, it is important to distinguish also firms selling domestically versus those exporting only. As in Demidova & Rodríguez-Clare (2013), I can say that constrained firms might be able to export if foreign standard is lower than the domestic one. where

$$l_i^A = \sum_j^i \int_{\xi_{lsj}}^{\infty} \left[\int_{\varphi_{lsj}}^{\hat{\varphi}} M_j^A \left[(\sigma - 1) \left(\frac{\theta_{sj}^{\alpha}}{\xi_{lsj}} \right) \left(\frac{\varphi}{\varphi_{lsj}} \right)^{\sigma-1} + \frac{\theta_{sj}^{\alpha}}{\xi} \right] g(\varphi)\, d\varphi \right] v(\xi)\, d\xi$$

$$l_i^B = \int_{\xi_{lsj}}^{\infty} \left[\int_{\hat{\varphi}_{ij}}^{\infty} M_j^B \left[\frac{\theta_j^{\varepsilon} t_j q_j(\varphi)}{\varphi} + \frac{\theta_j^{\alpha}}{\xi} \right] g(\varphi)\, d\varphi \right] v(\xi) d\xi$$

In conclusion:

$$M_i = L_i \varphi_i^{-\gamma} \xi_i^{-\iota} \sum_j^i \varphi_{lsj}^{\gamma} \xi_{lsj}^{\iota+1}$$

$$\left[\theta_{sj}^{\alpha} \left[\Upsilon_j + \frac{1}{\xi_{lsj}} \right] + F\xi_{lsj} \left[\varphi_{lsj}^{\sigma-1} \phi_j t^{1-\sigma} L_j \left(\rho_j \left(\theta_{sj}^{\Lambda_j} - \theta_j^{\Lambda_j} \right) + \theta_{sj}^{\Lambda_j} \right) + \varpi_j + \xi_{lsj}^{\frac{\gamma\alpha+\Lambda_j}{\alpha-\Lambda_j}} \Xi_j \right] \right]^{-1}$$

Where

$$\Upsilon_j = -(\sigma-1) \frac{\gamma}{\sigma-\gamma-1} \left(\left(\frac{\Lambda_j}{\alpha} \right)^{\frac{\sigma-\gamma-1}{1-\sigma}} \left(\frac{\iota}{\sigma-\gamma-\iota-1} \right) + 1 \right)$$

$$- \left(\frac{\iota}{-\iota-1} - \left(\frac{\Lambda_j}{\alpha} \right)^{\frac{-\gamma}{1-\sigma}} \left(\frac{\iota}{-\gamma-\iota-1} \right) \right)$$

$$+ \frac{\iota \left(\alpha - \Lambda_j \right)}{\alpha \left(\sigma - \iota - \gamma - 1 \right) - \Lambda_j \left(-\iota - \gamma - 1 \right)} \frac{\gamma \left(\alpha - \Lambda_j \right) \left((\sigma-1)\alpha + \Lambda_j \right)}{\left((\sigma-\gamma-1)\alpha + \gamma\Lambda_j \right) \Lambda_j} \left(\frac{\Lambda_j}{\alpha} \right)^{\frac{\gamma}{(\sigma-1)}} + \psi_j > 0$$

when accounting for the previous assumptions on the different parameters and assuming that $l^A > 0$.

Where $\Xi_j = \left(\frac{\Lambda_j}{\alpha} \right)^{\frac{-\gamma}{1-\sigma}} \left(\alpha - \Lambda_j \right) \frac{\iota}{\alpha(\sigma-\iota-\gamma-1)-\Lambda_j(-\iota-\gamma-1)} \frac{\gamma\left((\sigma-1)\alpha+\Lambda_j\right)}{\left((\sigma-\gamma-1)\alpha+\gamma\Lambda_j\right)\Lambda_j}$;
$\varpi_j = \left(\frac{\Lambda_j}{\alpha} \right)^{\frac{-\gamma}{1-\sigma}} \left(\alpha - \Lambda_j \right) \frac{\iota}{-\gamma-\iota} < 0$; $X_j > \varpi_j$; $\rho_j = \left(\frac{\Lambda_j}{\alpha} \right)^{\frac{\sigma-\gamma-1}{1-\sigma}} \frac{\iota}{\sigma-\gamma-\iota-1} < 0$ and
$\phi_j = \frac{\gamma}{(\sigma-\gamma-1)\sigma} \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} < 0$.

X. Reaction to standards

How does the mass of firms react with θ_{si} ?

$$\frac{\partial \xi_{lsi}}{\partial \theta_{si}} = -\frac{\frac{\partial M_i}{\partial \theta_{si}}}{\frac{\partial M_i}{\partial \xi_{lsi}}}$$

$$\frac{\partial \varphi_{lsi}}{\partial \theta_{si}} = -\frac{\partial M_i}{\partial \theta_{si}} / \frac{\partial M_i}{\partial \varphi_{lsi}}$$

In conclusion, the sign will depend on the inequality:
$\alpha \theta_{si}^{\alpha} \left[\Upsilon_i + \frac{1}{\xi_{lsi}} \right] \lesseqgtr F\xi_{lsi} \varphi_{lsi}^{\sigma-1} \phi_i t^{1-\sigma} L_i \Lambda_i \left(\rho_i + 1 \right) \theta_{si}^{\Lambda_i}$

$$\frac{\theta_{si}}{M_i} \frac{\partial M_i}{\partial \theta_{si}} = \frac{-\left(\alpha \theta_{si}^{\alpha} \xi_{lsi}^{-1} \left[\Upsilon_i + \frac{1}{\xi_{lsi}} \right] + F \left[\varphi_{lsi}^{\sigma-1} \phi_i t^{1-\sigma} L_i \Lambda_i \theta_{si}^{\Lambda_i} \left(\rho_i + 1 \right) \right] \right)}{\theta_{si}^{\alpha} \xi_{lsi}^{-1} \left[\Upsilon_i + \frac{1}{\xi_{lsi}} \right] + F \left[\varphi_{lsi}^{\sigma-1} \phi_i t^{1-\sigma} L_i \left(\rho_i \left(\theta_{si}^{\Lambda_i} - \theta_i^{\Lambda_i} \right) + \theta_{si}^{\Lambda_i} \right) + \varpi_i + \xi_{lsi}^{\frac{\gamma\alpha+\Lambda_i}{\alpha-\Lambda_i}} \Xi_i \right]}$$

$$+ \frac{\theta_{sj}}{\varphi_{lsj}} \frac{\partial \varphi_{lsi}}{\partial \theta_{si}} + \frac{\theta_{sj}}{\xi_{lsj}} \frac{\partial \xi_{lsi}}{\partial \theta_{si}}$$

XI. Welfare

Without standards,

Using Melitz and Redding (2013).

$$\mathbb{W}_j = \left[\left(\frac{\Lambda_j}{\alpha}\right)^{\Lambda_j} - \left(\frac{\Lambda_j}{\alpha}\right)^{\alpha}\right]^{\frac{1}{(\sigma-1)\alpha}} \left(\frac{t^{1-\sigma}L_j}{F^{\frac{\alpha-\Lambda_j}{\alpha}}\sigma}\right)^{\frac{1}{\sigma-1}} \left(\frac{\sigma-1}{\sigma}\right)\xi^{\frac{\Lambda_j}{(\sigma-1)\alpha}}\varphi$$

Using Gaigné and Larue (2016) (Using the initial price index that accounts for the mass of firms).

$$P_i^{1-\sigma} = \int_{k\in\Omega_i} \theta_{ki}^{\beta_j(\sigma-1)} p_{ki}^{1-\sigma} dk = \sum_k^K \int_\varphi^\infty \int_\xi^\infty \theta_{ki}^{\beta_i(\sigma-1)} p_{ki}^{1-\sigma}(\varphi) M_{ki}\varphi_{kk}^{\gamma}\xi_{kk}^{\iota} g(\varphi) v(\xi)\, d\xi d\varphi$$

$$P_i^{1-\sigma} = \frac{\theta_i^{\Lambda_i}\gamma}{\gamma-\sigma+1}\sum_k^K \left(\frac{\sigma}{\sigma-1}t_{ki}\right)^{1-\sigma}\left(\frac{M_{ki}}{\varphi_{kk}^{-\gamma}\xi_{kk}^{-\iota}}\right)\varphi_{ki}^{\sigma-\gamma-1}\xi_{ki}^{-\iota}$$

Now using $\varphi_{ki}^{\sigma-1} = \theta_i^{-\Lambda_i}\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{\sigma\left(\frac{\theta_i^{\alpha}}{\xi_{ki}}+F\right)}{t_{ki}^{1-\sigma}L_iP_i^{\sigma-1}}$ and $M_k = \frac{L_k(\sigma-1)\alpha}{\varphi_{kk}^{\gamma}\xi_{kk}^{\iota}f_e\gamma}\frac{\alpha}{\alpha(\sigma-1+\alpha-\Lambda_k)+\Lambda_k}$, I find:

$$\mathbb{W}_i = P_i^{-1} = \frac{\theta_i^{\frac{\Lambda}{\sigma-1}} L_i^{\frac{\gamma-\sigma+1}{\gamma(\sigma-1)}} \sigma^{\frac{-1}{\sigma-1}} (\sigma-1)^{\frac{2}{\gamma}}\alpha^{\frac{2}{\gamma}}}{[\gamma-\sigma+1]^{\frac{1}{\gamma}} f_e^{\frac{1}{\gamma}}\gamma^{\frac{1}{\gamma}}}$$

$$\left(\sum_k \left(\frac{L_k}{\alpha(\sigma-1+\alpha-\Lambda_{ki})+\Lambda_{ki}}\right) t_{ki}^{-\gamma}\left(\frac{\theta_i^{\alpha}}{\xi_{ki}}+F\right)^{\frac{-\gamma+\sigma-1}{\sigma-1}}\xi_{ki}^{-\iota}\right)^{\frac{1}{\gamma}}$$

With standards:

Using Melitz and Redding (2013):

$$\mathbb{W} = \frac{\sigma-1}{t\sigma}\left[\frac{\sigma\theta_{si}^{-\Lambda_i}}{L_i}\left(\frac{\theta_{si}^{\alpha}}{\xi_{lsi}(\theta_{si})}+F\right)\right]^{\frac{1}{1-\sigma}}\varphi_{lsi}(\theta_{si})$$

Using Gaigné and Larue (2016): as in the case without standards, I take the price index that accounts for the mass of firms. Since the mass of firms consider constrained and unconstrained firms, so does the welfare. The form is the same as in the case without standard ($\theta_s = 0 \iff$ no constrained firms). Only the mass of firms differ, as well as the optimal price which accounts for the standard because the price index is based on the lowest productive firm (constrained):

$$\mathbb{W}_i = \frac{\theta_{si}^{\frac{\Lambda}{\sigma-1}} L_i^{\frac{\gamma-\sigma+1}{\gamma(\sigma-1)}}\sigma^{\frac{-1}{\sigma-1}}(\sigma-1)^{\frac{1}{\gamma}}}{[\gamma-\sigma+1]^{\frac{1}{\gamma}}}\left[\sum_k\left(\frac{M_{ki}}{\varphi_{kk}^{-\gamma}\xi_{kk}^{-\iota}}\right)t_{ki}^{-\gamma}t_{ki}^{-\gamma}\left(\frac{\theta_{si}^{\alpha}}{\xi_{lsi}}+F\right)^{\frac{-\gamma+\sigma-1}{\sigma-1}}\xi_{ki}^{-\iota}\right]^{\frac{1}{\gamma}}$$

XI.1 Alternative welfare comparison including mass of firms

Another clear direct effect is the impact of the standard on the distribution of firms due to two principal factors: the marginal cost and the fixed cost. Comparing the different welfare can be done for the more complex welfare function that takes the mass of firms into account

Then, the more complex definition of welfare considers the mass of firms. How does the mass of firm react with a change of standard?

$$\frac{\theta_{si}}{M_i}\frac{\partial M_i}{\partial\theta_{si}} = -\left(\alpha\theta_{si}^{\alpha}\left[Y_i + \frac{1}{\xi_{lsi}}\right] + F\xi_{lsi}\left[\varphi_{lsi}^{\sigma-1}\phi_i t^{1-\sigma}L_i\Lambda_i\theta_{si}^{\Lambda_i}\left(\rho_i+1\right)\right]\right)\Delta_i^{-1} + \frac{\theta_{sj}}{\varphi_{lsj}}\frac{\partial\varphi_{lsi}}{\partial\theta_{si}} + \frac{\theta_{sj}}{\xi_{lsj}}\frac{\partial\xi_{lsi}}{\partial\theta_{si}} \quad (48)$$

The sign of this quality-elasticity of mass of firms is either positive or negative depending on the sign of $\alpha\theta_{si}^{\alpha}\xi_{lsi}^{-1}\left[Y_i + \frac{1}{\xi_{lsi}}\right] + F\left[\varphi_{lsi}^{\sigma-1}\phi_i t^{1-\sigma}L_i\Lambda_i\theta_{si}^{\Lambda_i}\left(\rho_i+1\right)\right]$ which greatly depend on whether the upgrading cost is higher or lower than the other fixed cost F. If F is low although different from zero leads to a negative result which is intuitive: a higher standard diminishes the number of firms on the market.

From this basic analysis, it confirms that harmonization seems to be a good framework (even when not taking information asymmetries or externalities into account).