Optimal Abstraction-based Control with Local Affine Controllers.

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1. Symbolic control

- Discretization of input $\mathcal{U}$ and state $\mathcal{X}$ spaces, leading to graph-based representations [1, 2, 3]. 😮
- Computationally heavy. ☹️
- Easily deals with complex constraints. 😀
- Incremental stability of open-loop system often needed. ☹️
- Usually lead to non-deterministic representations. 😕
- Leverage graph-based algorithms to solve control problem. 😀

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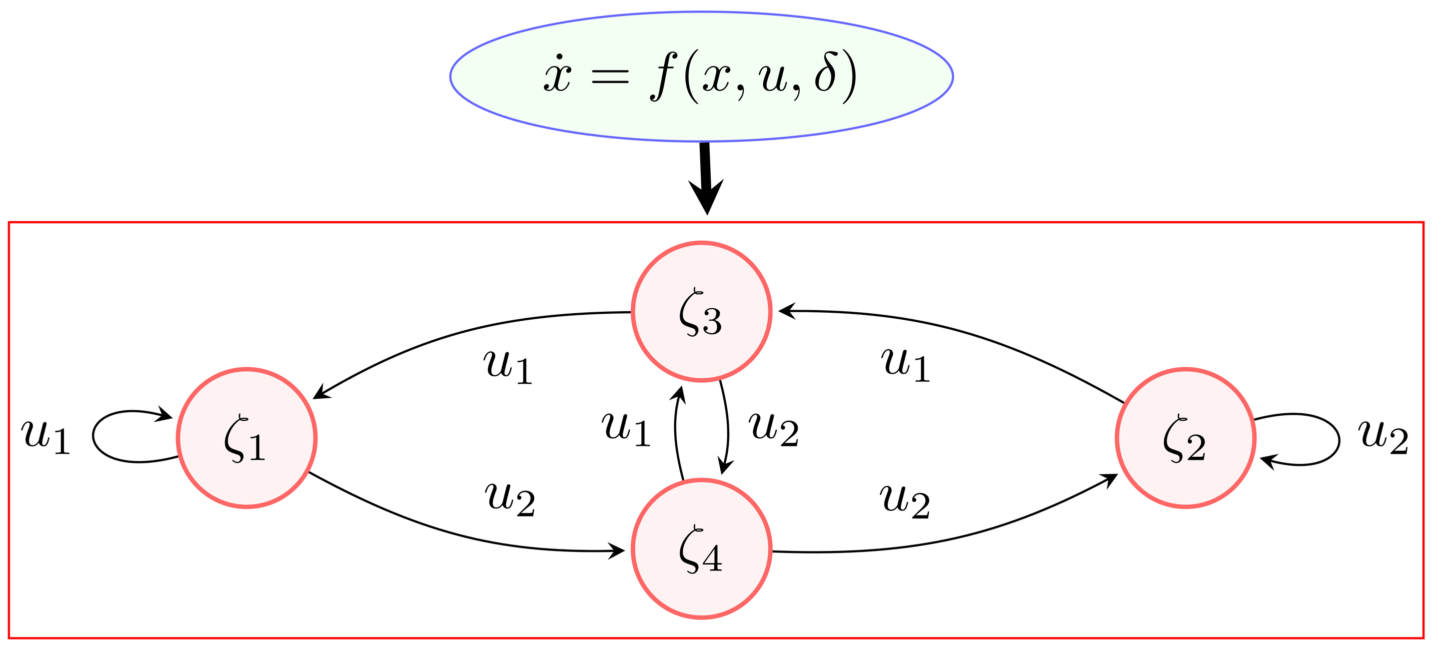


2. PWA models

Non-deterministic piecewise-affine systems

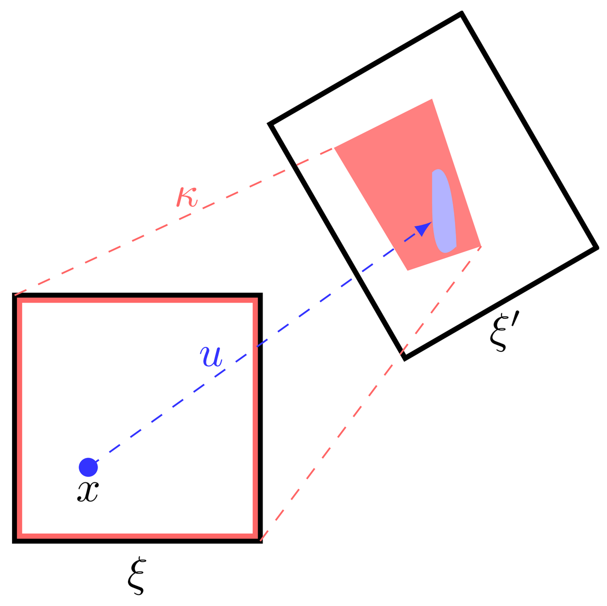
$$x(k+1) \in F(x(k), u(k)), \; x(0) = x_0, \qquad (1)$$

$$F(x, u) = \{A_{\psi(x)}x + B_{\psi(x)}u + g_{\psi(x)}\} \oplus \Omega_{\psi(x)},$$

- $\psi : \mathcal{X} \to \mathcal{I}_p = \{1, \ldots, N_p\}$ is a given state-dependent switching law.
- $\Omega_i \subset \mathbb{R}^n$, $0 \in \Omega_i$, characterizes both exogenous input and uncertainties.
- (A_i, B_i, g_i) defines the i-th subsystem.
- Optimal control problem, taking x from $\mathcal{X}_0$ to $\mathcal{X}_\star$ while avoiding obstacles and minimizing cost.

3. Symbolic abstractions with affine-feedback controllers

- Local feedback controllers $\kappa(x) = K(x - c) + \ell$.
- Using ellipsoidal sets as discrete states.
- Transitions computed using necessary and sufficient linear matrix inequalities.
- Lyapunov-like functions can be obtained.



Considering quadratic starting and final quadratic sets

$$\mathbb{B}_s = \{x \in \mathbb{R}^n \; : \; (x-c)'P(x-c) \le 1\}, \quad \mathbb{B}_f = \{x \in \mathbb{R}^n \; : \; (x-c_+)'P_+(x-c_+) \le 1\} \qquad (2)$$

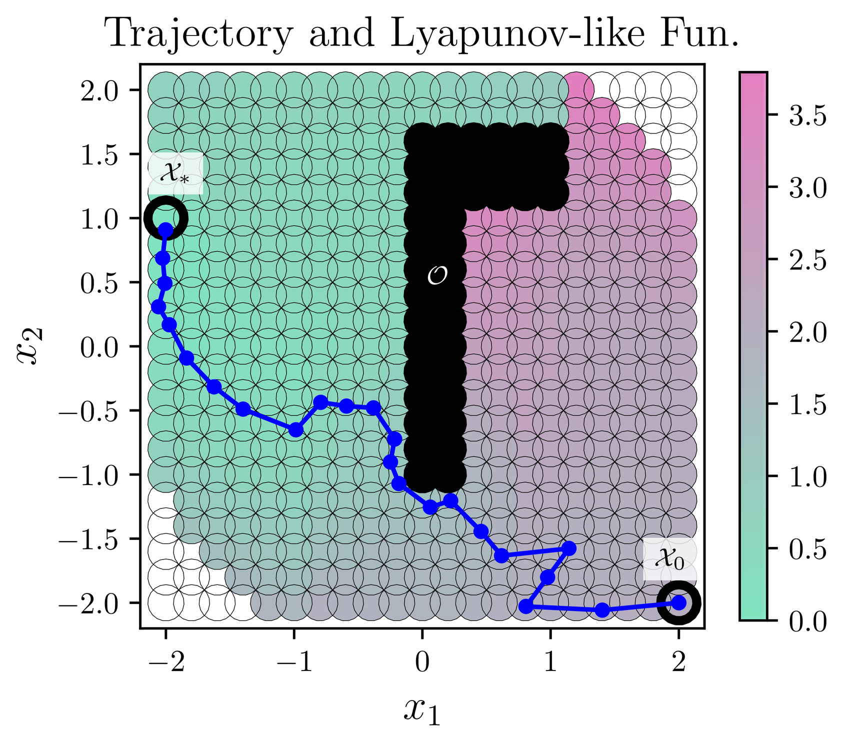
affine controllers of the form $\kappa(x) = K(x - c) + \ell$ can be computed by means of linear matrix inequalities, leading to tight upper-bounds on transition costs [4].

4. A numerical example

System $\mathcal{S} := (\mathcal{X}, \mathcal{U}, \rightsquigarrow_F)$ with $F(x, u)$ defined by

$$A_1 = \begin{bmatrix} 1.01 & 0.3 \\ -0.1 & 1.01 \end{bmatrix}, \; B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; g_1 = \begin{bmatrix} -0.1 \\ -0.1 \end{bmatrix},$$

$A_2 = A_1^\top$, $A_3 = A_1$, $B_2 = B_3 = B_1$, $g_2 = 0$ and $g_3 = -g_1$, additive-noise sets $\Omega_1 = \Omega_2 = \Omega_3 = [-0.05, 0.05]^2$, the control-input set $\mathcal{U} = [-0.5, 0.5]^2$ and the state space $\mathcal{X} = [-2, 2]^2$, with partitions $\mathcal{X}_1 = \{x \in \mathcal{X} \; : \; x_1 \le -1\}$, $\mathcal{X}_3 = \{x \in \mathcal{X} \; : \; x_1 > 1\}$, and $\mathcal{X}_2 = \mathcal{X} \setminus (\mathcal{X}_1 \cup \mathcal{X}_3)$. Use of **Dijkstra's** algorithm.

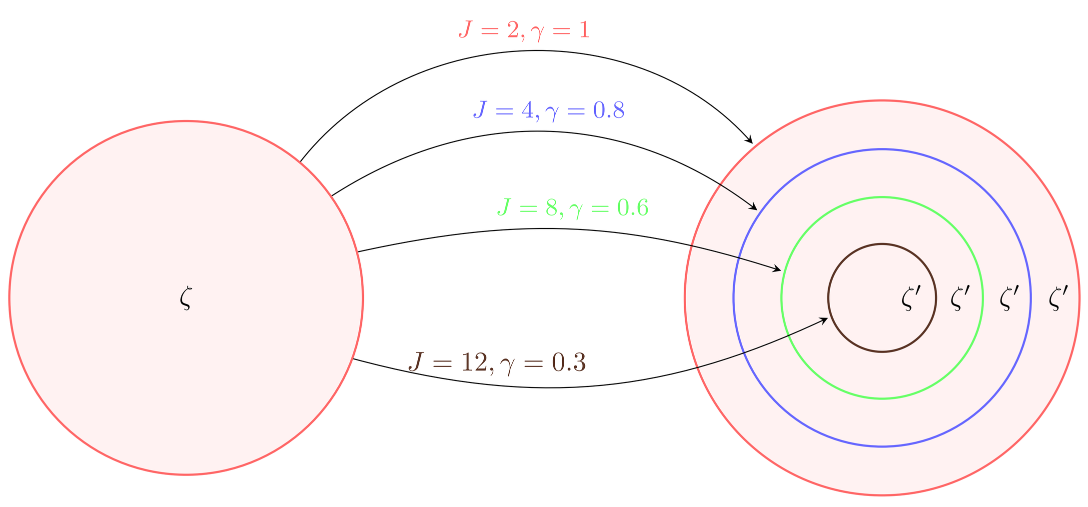


5. Conclusions

- Deterministic graph even when original system is non-deterministic.
- Correct-by-design local feedback controllers satisfying transition requirements and constraints.
- Lyapunov-like functions obtained solving convex optimization problems.

6. Future work

- Iterative approach also designing the cells (centers and shapes), i.e., grid is not fixed a priori.
- Contraction rates weighting the edges can lead to a optimization problem in the global level, guiding the construction of the abstraction.



7. References

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