Variable reduction for interior-point methods using partial minimization

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1 Motivation

In this paper we consider interior-point methods for convex optimization problems consisting in minimizing a linear function over a convex set P for which no closed-form self-concordant barrier is available. However, we do assume that set P is the projection of some other set Q, i.e.

$$y \in P \subseteq \mathbb{R}^n \Leftrightarrow \exists v \in \mathbb{R}^m \text{ s.t.}(y,v) \in Q \subseteq \mathbb{R}^{n+m}$$

for which a computable self-concordant barrier is known.

Consider for example the situation where we wish to optimize over the set $P = \{y \in \mathbb{R}^n \mid \sum_{i=1}^n \exp(b_i - a_i^\mathrm{T} y) \leq 1\}$, which occurs in geometric programming. No explicitly computable self-concordant barrier is known for this set. However it is clear that $y \in P$ if and only there exists a vector v such that

$$\exp(b_i - a_i^\mathrm{T} y) \leq v_i, \; i = 1, \ldots, n$$
$$\sum_{i=1}^n v_i = 1.$$

defining a set for which an explicit self-concordant barrier is known, namely

$$F(y,v) = -\sum_{i=1}^n \log\bigl(\log(v_i) - \bigl(b_i - a_i^\mathrm{T} y\bigr)\bigr) - \sum_{i=1}^n \log(v_i)$$

However, we see that price for getting this barrier is the introduction of artificial variables v, that are not of direct interest, and which increase the problem size and therefore the computational effort required to solve the problem. The number n of artifical variables v can even in some cases be be much higher than the number m of original variables y. In this talk, we show how to deal efficiently with these artificial variables, and reduce the memory storage and numerical cost of each iteration.

2 Partial Minimization

We make use of a not so widely known result by Nesterov [2], which asserts that the partial minimization of a self-concordant barrier with respect to some of its variables preserves self-concordance.

However, this result has only limited practical use because the partial minimization subproblem cannot often be solved analytically. Therefore we consider the situation where the subproblem is solved only approximately using an iterative scheme.

3 Main results

Polynomial complexity is preserved
We show how to embed approximate partial minimization into interior-point methods while preserving the global polynomial-time complexity of the algorithm. This variable reduction technique can be seen to be equivalent to restricting the iterates to affine subspaces that are tangent to the surface of partial minimizers.

Partial minimization is beneficial
We show that the use of partial minimization is beneficial, as compared to the full formulation, when the number of artificially introduced variables is high. We present two problem classes that fit that setting and show how this new technique can be applied to solve them more efficiently.

Extension to primal-dual framework
Our dual path-following method with approximate partial minimization is phrased in conic terms, and it is possible to extend the dual algorithm to a non-symmetric primal-dual method. We suggest a modification of Nesterov's non-symmetric primal-dual predictor-corrector method [1] suitable for our situation.

References

[1] Y. Nesterov, "Towards nonsymmetric conic optimization", *CORE Discussion Paper*, 28, 2006.

[2] Y. Nesterov, "Parabolic target space and primal-dual interior-point methods", *Discrete Applied Mathematics*, 156(11): p. 2079-2100, 2008.