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Tax incidence on competing two-sided platforms: Lucky break or double jeopardy*

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Abstract

We consider the effects of taxes for competing two-sided platforms. We first detail how a platform passes a tax increase on its prices. Adding price competition, we study next how the tax affects profits. Because of the strategic implications of the cross-side external effects, the tax increase may end up increasing the profit of the taxed platform (lucky break) or, conversely, reducing it twice (double jeopardy).

Keywords: Two-sided platforms, taxation, pass-through

JEL-Classification: D43, L13, L86, 032

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1 Introduction

"Please, Tax Us!" is the message that Airbnb, the largest platform for short-term hosted accommodation, recently sent to mayors of US cities.[1] This surprising demand is generally presented as a quid pro quo for obtaining the same legitimacy as regular hotels and thereby, being free from harassment and fines. As other routes exist to gain such legitimacy (e.g., lobbying),[2] this appeal suggests that being subject to taxes may not be too detrimental for Airbnb's business. Actually, we show in this paper the theoretical possibility that a platform like Airbnb may even see a direct benefit in paying higher taxes.

The necessary ingredients for this counterintuitive result are price competition among two-sided platforms and asymmetric taxes. *Two-sided platforms* intermediate between two distinct groups of economic agents that benefit from interacting with one another but fail to organize this interaction by their own forces because of high transaction costs. Such platforms are active in a large variety of settings.[3] The main function of two-sided platforms is to internalize the various external effects that the interaction between the two groups generate. Of particular interest are the cross-side effects that make the well-being of one group depend on the participation of the other group; for instance, in the case of Airbnb, each group, hosts and guests, clearly benefits from a stronger participation of the other group.

[1] See *Airbnb Urges Mayors to 'Please Tax Us'* (New York Times, January 22, 2016; `http://nyti.ms/1ZRDtGW`; last consulted 15/03/2016) or *Please Tax Us: Airbnb Offers City \$21 Million* (The Huffington Post, June 25, 2014; `http://huff.to/1nKB7rL`; last consulted 15/03/2016).

[2] See *Airbnb Spends \$8 Million Lobbying Against San Francisco Ballot Initiative* (The Huffington Post, January 11, 2015; `http://huff.to/1MjKLYI`; last consulted 15/03/2016) or *Will Airbnb's \$21 Million Olive Branch Get It Legalized in New York?* (New York Magazine, April 18, 2014; `http://nym.ag/1YUtLiW`; last consulted 15/03/2016).

[3] *Peer-to-peer marketplaces*, like Airbnb, facilitate the exchange of goods and services between 'peers' (other examples are Uber, EatWith, TaskRabbit); *exchanges* help 'buyers' and 'sellers' search for feasible contracts and for the best prices (e.g., eBay, Booking.com, Cambridge University Press, edX); *hardware & software systems* allow applications developers and end users to interact (e.g., Mac OS, Android, PlayStation); *matchmakers* help members of one group to find the right 'match' within another group (e.g., Alibaba, Monster, Meetic); *crowdfunding platforms* allow entrepreneurs to raise funds from a 'crowd' of investors (e.g., Kickstarter, Indiegogo, LendingClub); *transaction systems* provide a method for payment to buyers and sellers that are willing to use it (e.g., Visa, Bitcoin, PayPal).

Platforms may be subject to two types of *taxes*, according to whether it is the access to the platform that is taxed or the transactions that are conducted on the platform. In the short-term hosted accommodation industry, regulations that force hosts, e.g., to install sprinklers in the rooms they rent fall in the first category ("access taxes"), while an occupancy tax that has to be paid per accommodated guest falls in the second category ("transaction taxes"). In the case of hardware/software systems, access taxes are taxes on digital devices (such as smartphones or game consoles), whereas transaction taxes are taxes on digital content or applications.

In our analysis, we focus on access taxes and we examine situations where one platform is subject to a larger tax than the competing platform.[4] Our objective is two-fold. First, we want to understand how taxes modify the platforms' equilibrium access fees. This issue is complex because of the two-sided nature of the market and because of price competition. When choosing its prices, a platform needs to reflect not only the cross-side external effects, but also the interactions with the rival platform. As a consequence, taxes will affect access prices for the two groups in a complex way that will depend on the relative strength of the cross-side effects and the relative intensity of competition on the two sides.

Knowing how taxes affect equilibrium prices, we turn next to equilibrium profits. Our main result here is that cross-side external effects affect the tax incidence through the strategic effect of taxes. By strategic effect, we mean the effect on one platform's profit that operates through the modification of the other platform's equilibrium prices. Absent cross-side external effects, we expect the strategic effect of higher taxes to be positive if firms compete in prices over substitutable services: a higher tax for firm A leads this firm to increase its price, which leads firm B to increase its price as well (because of strategic complementarity); this, in turn, raises firm A's profit, which contributes to attenuate the direct negative impact of taxes on profits.

The presence of cross-side external effects challenges the previous results in two major ways. First, cross-side external effects may *increase* the strategic effect and they may do so to such an extent that the strategic effect outweighs the negative direct effect; it follows that the net effect of higher taxes on profit becomes positive. The tax increase becomes thus a *lucky*

[4] This would be the case, for instance, if Airbnb was more present than its competitors in those cities that levy access taxes.

break, which may explain the 'Please, tax us!' attitude of some platforms. Second, in complete contrast with the previous case, external effects may *decrease* the strategic effect, even up to a point where it becomes negative; in the latter case, platforms would beg: 'Please, *do not* tax us!'. Indeed, higher taxes would put them in *double jeopardy*: platforms would first be hurt directly and next, indirectly, through the adjustment of the rival platform's equilibrium prices. We show that for either of these extreme cases to arise, cross-side effects must be large relatively to the intensity of competition on the two sides.

To the best of our knowledge, our analysis and our results are novel. This is partly explained by the fact that the literature analyzing the competition between two-sided platforms–following the seminal contributions of Caillaud and Jullien (2003), Rochet and Tirole (2003 and 2006), and Armstrong (2006)–has mostly considered symmetric platforms. Clearly, a symmetric setting is inappropriate to examine the impact of taxes on competing platforms.

There are, however, a few papers that consider the issue of taxation in two-sided markets. Yet, they do so in different settings or with different focus than ours. Kind, Koethenbuerger and Schjelderup (2008, 2009, 2010) are mainly concerned by comparing the impacts of ad valorem and unit taxes on tax revenues and on welfare in two-sided markets (with a specific focus on advertising-financed media). Some of their results, however, echo ours; for instance, in their (2010) paper, they show that a higher ad valorem tax on the user side does not necessarily induce the platform to raise the price on that side (we reach a similar result with a unit tax). Kind, Koethenbuerger and Stähler (2013) analyze the effects of taxes on newspaper differentiation. Kotsogiannis and Serfes (2010) address the issue of taxation of two-sided platforms in terms of tax competition between countries. Bloch and Demange (2015) focus on the effect of taxes on privacy protection (they model a monopolistic platform that collects data on users and make revenues either by exploiting this data or by selling it to third parties). Tremblay (2016) studies optimal taxation of a monopoly two-sided platform with two tax instruments (taxation on platform content and taxation on the platform itself). Finally, closer to our analysis, Bourreau, Caillaud and De Nijs (2015) assess the likely impacts of a tax on data collection and a tax on advertising on the pricing strategies of two-sided platforms and on fiscal revenues; in particular, in their duopoly model, they examine the impact of taxation on the platforms' profits; although differences in the models preclude a direct comparison between our results,[5] it is worth pointing that in their model, taxation always reduces the profits of the two competing platforms.

Our analysis also bears a clear connection with the (scarce) literature studying cost pass-through for multisided platforms or multiproduct firms (the unit tax we consider is indeed equivalent to a cost increase). Weyl (2010) analyzes cost pass-through for a monopoly two-sided platform, which is directly relevant to our analysis. However, our results cannot be compared as Weyl focuses on insulating tariffs (i.e., the platform is supposed to choose participation rates on the two sides rather than prices); the latter point makes a big difference as the effect of a cost (or tax) change on the price on side i is computed under the assumption that participation is kept fixed on side j, which is not the case in our analysis.

As the interaction between the two sides generates strong complementarities, two-sided platforms bear some resemblance with multiproduct firms.[6] The studies of cost pass-through for multiproduct firms are thus insightful for the first part of our analysis, i.e., the incidence of a unit tax on platform equilibrium prices. Moorthy (2005) analyzes a theoretical model where two competing retailers supply each two substitutable products to consumers, and examines how a cost increase for one firm affects this firm's prices, as well as its rival's prices. Alexandrov and Bedre-Defolie (2011), in contrast, suppose that the two retailers offer complementary products and that these products affect each other's demand in an asymmetric way (the price of one product influences the demand for the other product, but the reverse is not true); as will become clear below, our setting shares these two features. Armstrong and Vickers (2016) propose a general demand system for multiple products that yields simple formulas for the size and sign of own-cost and cross-cost passthrough relationships.

Finally, our result that a tax increase may raise profits of competing firms is not unheard of. For instance, this result is shown, e.g., by Hindriks and Myles (2006, Chapter 8) under Cournot competition and by Anderson, de Palma and Kreider (2001) under Bertrand competition and differenti-

[5] For instance, they allow the members of one group to multihome, while we impose singlehoming on both sides.

[6] Although, as Rochet and Tirole (2003) point out, end users internalize the corresponding externalities in a multiproduct setting but not in a multisided setting.

ated products. In the latter case (which is more relevant for this paper), the authors show that the profit increase can only happen for highly convex demands. As demands are linear in our model, the potential profit-enhancing effect of larger taxes clearly stems from a different channel.

The rest of the paper is organized as follows. Before examining tax incidence on prices on profits (Section 3), we derive the equilibrium of a pricing game between two asymmetric platforms (Section 2). We discuss our results in Section 4.

2 Price competition between asymmetric platforms

In this section, we extend the model of Armstrong (2006) with two-sided singlehoming by allowing for asymmetric costs across platforms. We first present the model and then solve for the price equilibrium.

2.1 The setting

Two platforms are located at the extreme points of the unit interval: platform U (for *Uppercase*, identified hereafter by upper-case letters) is located at 0, while platform l (for *lowercase*, identified by lower-case letters) is located at 1. Platforms facilitate the interaction between two groups of agents, noted a and b. Both groups are assumed to be of mass 1 and uniformly distributed on $[0, 1]$. We assume that agents of both sides can join at most one platform (so-called 'two-sided singlehoming'); in the real world, singlehoming environments may result from indivisibilities or limited resources.[7]

We define the net utility functions for an agent of group a and for an agent of group b, respectively located at x and $y \in [0, 1]$ as:

$$\begin{array}{ll} U_a(x) = \sigma_a N_b - \tau_a x - P_a & \text{if joining platform } U, \\ u_a(x) = \sigma_a n_b - \tau_a (1-x) - p_a & \text{if joining platform } l, \\ U_b(y) = \sigma_b N_a - \tau_b y - P_b & \text{if joining platform } U, \\ u_b(y) = \sigma_b n_a - \tau_b (1-y) - p_b & \text{if joining platform } l, \end{array}$$

where σ_j is the valuation for agents of group j of the interaction with an additional agent of the other group (i.e., it measures the strength of the cross-side external effect exerted on agents of group j), N_j (resp. n_j) is the mass of agents of group j that decide to join platform U (resp. l), τ_j is the

[7]For a discussion, see Case 22.4 in Belleflamme and Peitz (2015, p. 667).

'transport cost' parameter for group j, and P_j (resp. p_j) is the access fee that platform U (resp. l) sets for users of group j (with $j,\, k \in \{a, b\}$ and $j \neq k$).

Let $\hat{x}$ (resp. $\hat{y}$) identify the agent of group a (resp. b) who is indifferent between joining platform U or platform l; that is, $U_a(\hat{x}) = u_a(\hat{x})$ and $U_b(\hat{y}) = u_b(\hat{y})$. Solving these equalities for $\hat{x}$ and $\hat{y}$ respectively, we have

$$\begin{aligned} \hat{x} &= \tfrac{1}{2} + \tfrac{1}{\tau_a}\left(\sigma_a\left(N_b - \tfrac{1}{2}\right) - \tfrac{1}{2}\left(P_a - p_a\right)\right), \\ \hat{y} &= \tfrac{1}{2} + \tfrac{1}{\tau_b}\left(\sigma_b\left(N_a - \tfrac{1}{2}\right) - \tfrac{1}{2}\left(P_b - p_b\right)\right). \end{aligned}$$

In what follows, we implicitly assume that each platform provides the agents with an extra benefit that is sufficiently large to make sure that all agents join one platform.[8] Both sides are then fully covered, so that $N_j + n_j = 1$ ($j = a, b$). This entails the following equalities: $\hat{x} = N_a = 1 - n_a$ and $\hat{y} = N_b = 1 - n_b$. Using these equalities, we can solve the above systems of equations for N_a and N_b:

$$N_a = \frac{1}{2} + \frac{\tau_b}{2}\frac{p_a - P_a}{\tau_a\tau_b - \sigma_a\sigma_b} + \frac{\sigma_a}{2}\frac{p_b - P_b}{\tau_a\tau_b - \sigma_a\sigma_b}, \tag{1}$$

$$N_b = \frac{1}{2} + \frac{\tau_a}{2}\frac{p_b - P_b}{\tau_a\tau_b - \sigma_a\sigma_b} + \frac{\sigma_b}{2}\frac{p_a - P_a}{\tau_a\tau_b - \sigma_a\sigma_b}. \tag{2}$$

To ensure that participation on each side is a decreasing function of the access fee on this side, we assume that $\tau_a\tau_b > \sigma_a\sigma_b$. This assumption, which is common in the analysis of competition between two-sided platforms, says that the strength of cross-side external effects (measured by $\sigma_a\sigma_b$) is smaller than the strength of horizontal differentiation (measured by $\tau_a\tau_b$).

2.2 Equilibrium of the pricing game

Platforms simultaneously choose their access prices to maximize their profit, given by $\Pi = (P_a - T_a)\, N_a + (P_b - T_b)\, N_b$ and $\pi = (p_a - t_a)\, n_a + (p_b - t_b)\, n_b$. We assume that their costs per agent is limited to the tax they pay for admitting the agent on the platform; this tax may differ across sides and across platforms (T_a and T_b for platform U; t_a and t_b for platform l). For future reference, we define $\gamma_k \equiv T_k - t_k$ as the difference in taxes on side k between platforms U and l ($k = a, b$). The four first-order conditions for

[8] This could be done by adding a term R_i, $i \in (a, b)$ to the functions $U_i(x)$ and $u_i(x)$.

profit maximization can be written as:

$$\frac{\partial \Pi}{\partial P_a} = 0 \Leftrightarrow N_a + (P_a - T_a)\frac{\partial N_a}{\partial P_a} + (P_b - T_b)\frac{\partial N_b}{\partial P_a} = 0, \tag{3}$$

$$\frac{\partial \Pi}{\partial P_b} = 0 \Leftrightarrow N_b + (P_a - T_a)\frac{\partial N_a}{\partial P_b} + (P_b - T_b)\frac{\partial N_b}{\partial P_b} = 0, \tag{4}$$

$$\frac{\partial \pi}{\partial p_a} = 0 \Leftrightarrow n_a + (p_a - t_a)\frac{\partial n_a}{\partial p_a} + (p_b - t_b)\frac{\partial n_b}{\partial p_a} = 0, \tag{5}$$

$$\frac{\partial \pi}{\partial p_b} = 0 \Leftrightarrow n_b + (p_a - t_a)\frac{\partial n_a}{\partial p_b} + (p_b - t_b)\frac{\partial n_b}{\partial p_b} = 0. \tag{6}$$

The second-order conditions require $\tau_a\tau_b > \sigma_a\sigma_b$ and $\tau_a\tau_b > \frac{1}{4}(\sigma_a + \sigma_b)^2$. We note that $\frac{1}{4}(\sigma_a + \sigma_b)^2 - \sigma_a\sigma_b = \frac{1}{4}(\sigma_a - \sigma_b)^2 > 0$, which means that the second condition is more stringent than the first. We thus impose

$$\tau_a\tau_b > \tfrac{1}{4}(\sigma_a + \sigma_b)^2. \tag{7}$$

We now solve the system (3)-(6). To facilitate the exposition, we define $D \equiv 9\tau_a\tau_b - (2\sigma_a + \sigma_b)(\sigma_a + 2\sigma_b)$, which is positive according to Assumption (7). The equilibrium price of platform U on side a is found as

$$P_a^* = \overbrace{T_a + \tau_a}^{\mathbf{H}} \overbrace{-\sigma_b}^{\mathbf{A}} \overbrace{-\tfrac{1}{3}\gamma_a}^{\mathbf{V}} \overbrace{- \frac{(\sigma_a - \sigma_b)}{3D}\left[(2\sigma_a + \sigma_b)\gamma_a + 3\tau_a\gamma_b\right]}^{\mathbf{I}}. \tag{8}$$

We can decompose it as the sum of four components: (i) **H** is the classic *Hotelling* formula (marginal cost + transportation cost); (ii) **A** was identified by *Armstrong* (2006) as the price adjustment due to indirect network effects (the price is decreased by the externality exerted on the other side); (iii) **V** is the effect of *vertical differentiation*; (iv) the last term **I** results from the *interplay between vertical differentiation and cross-side external effects*. If platforms are symmetric ($\gamma_k = 0$) only **H** and **A** remain; absent external effects ($\sigma_k = 0$), only **H** and **V** remain. In the particular case where cross-side external effects are the same on the two sides ($\sigma_a = \sigma_b$), all terms but the last remain (we will extensively explain why below).

The equilibrium price of platform U on side b, as well as the equilibrium prices of platform l, are found by analogy

$$P_b^* = T_b + \tau_b - \sigma_a - \tfrac{1}{3}\gamma_b - \frac{(\sigma_b - \sigma_a)}{3D}\left[(2\sigma_b + \sigma_a)\gamma_b + 3\tau_b\gamma_a\right], \tag{9}$$

$$p_a^* = t_a + \tau_a - \sigma_b + \tfrac{1}{3}\gamma_a + \frac{(\sigma_a - \sigma_b)}{3D}\left[(2\sigma_a + \sigma_b)\gamma_a + 3\tau_a\gamma_b\right], \tag{10}$$

$$p_b^* = t_b + \tau_b - \sigma_a + \tfrac{1}{3}\gamma_b + \frac{(\sigma_b - \sigma_a)}{3D}\left[(2\sigma_b + \sigma_a)\gamma_b + 3\tau_b\gamma_a\right]. \tag{11}$$

We can now use the equilibrium prices to compute the equilibrium mass of agents of the two groups on the two platforms:

$$N_a^* = \tfrac{1}{2} - \tfrac{1}{2D}\left(3\tau_b\gamma_a + (\sigma_a + 2\sigma_b)\gamma_b\right),\ n_a^* = 1 - N_a^*,$$
$$N_b^* = \tfrac{1}{2} - \tfrac{1}{2D}\left(3\tau_a\gamma_b + (2\sigma_a + \sigma_b)\gamma_a\right),\ n_b^* = 1 - N_b^*.$$

To guarantee that the equilibrium mass is strictly positive and lower than unity, we impose the following restrictions on the space of parameters (which are trivially satisfied in the symmetric case where $\gamma_a = \gamma_b = 0$):

$$\left|3\tau_b\gamma_a + (\sigma_a + 2\sigma_b)\gamma_b\right| < D \text{ and } \left|3\tau_a\gamma_b + (2\sigma_a + \sigma_b)\gamma_a\right| < D. \tag{12}$$

Using the equilibrium values of prices and number of agents, we find the equilibrium profits

$$\begin{aligned}\Pi^* \quad = \quad & \frac{1}{2}(\tau_a + \tau_b - \sigma_a - \sigma_b) + \frac{1}{2D}\left(\tau_b\gamma_a^2 + \tau_a\gamma_b^2\right) + \frac{1}{2D}(\sigma_a + \sigma_b)\gamma_a\gamma_b \\ & - \frac{\gamma_a}{2D}\left(6\tau_a\tau_b + \tau_b(\sigma_a - \sigma_b) - (\sigma_a + \sigma_b)(2\sigma_a + \sigma_b)\right) \\ & - \frac{\gamma_b}{2D}\left(6\tau_a\tau_b - \tau_a(\sigma_a - \sigma_b) - (\sigma_a + \sigma_b)(\sigma_a + 2\sigma_b)\right),\end{aligned} \tag{13}$$

$$\begin{aligned}\pi^* \quad = \quad & \frac{1}{2}(\tau_a + \tau_b - \sigma_a - \sigma_b) + \frac{1}{2D}\left(\tau_b\gamma_a^2 + \tau_a\gamma_b^2\right) + \frac{1}{2D}(\sigma_a + \sigma_b)\gamma_a\gamma_b \\ & + \frac{\gamma_a}{2D}\left(6\tau_a\tau_b + \tau_b(\sigma_a - \sigma_b) - (\sigma_a + \sigma_b)(2\sigma_a + \sigma_b)\right) \\ & + \frac{\gamma_b}{2D}\left(6\tau_a\tau_b - \tau_a(\sigma_a - \sigma_b) - (\sigma_a + \sigma_b)(\sigma_a + 2\sigma_b)\right).\end{aligned} \tag{14}$$

Total profit at equilibrium is computed as

$$\Pi^* + \pi^* = \tau_a + \tau_b - \sigma_a - \sigma_b + \frac{1}{D}\left(\tau_b\gamma_a^2 + \tau_a\gamma_b^2 + (\sigma_a + \sigma_b)\gamma_a\gamma_b\right).$$

3 Incidence of taxes

In this section, we examine how a tax increase for one platform affects the equilibrium prices (Subsection 3.1) and profits (Subsection 3.2). In particular, we want to establish under which conditions situations of lucky break of or double jeopardy may emerge.

3.1 Tax pass-through

We want first to understand how platforms modify their equilibrium prices following a tax increase. This issue is complex because of the two-sided nature of the market and because of price competition. Each platform needs indeed to choose its prices to reflect not only the cross-side external effects, but also the interactions with the rival platform. In particular, differentiating the first-order conditions (3) with respect to (6) and borrowing the terminology of Moorthy (2005), we observe that the profit functions exhibit '*internal strategic substitutability*' and '*external strategic complementarity*'. The former property refers to the fact that the cross-partials of profits with respect to the two prices of the same platform are negative:[9]

$$\frac{\partial^2 \Pi}{\partial P_a \partial P_b} = \frac{\partial^2 \pi}{\partial p_a \partial p_b} = \frac{\partial N_a}{\partial P_b} + \frac{\partial N_b}{\partial P_a} = -\frac{1}{2}\frac{\sigma_a + \sigma_b}{\tau_a \tau_b - \sigma_a \sigma_b} < 0.$$

As for the latter property, it follows from the positive sign of the cross-partials of profits with respect to prices of different platforms (with $i, j \in \{a, b\}$, $i \neq j$):[10]

$$\begin{aligned} \frac{\partial^2 \Pi}{\partial P_i \partial p_i} &= \frac{\partial^2 \pi}{\partial p_i \partial P_i} = \frac{\partial N_i}{\partial p_i} = \frac{1}{2}\frac{\tau_j}{\tau_a \tau_b - \sigma_a \sigma_b} > 0, \\ \frac{\partial^2 \Pi}{\partial P_i \partial p_j} &= \frac{\partial^2 \pi}{\partial p_j \partial P_j} = \frac{\partial N_j}{\partial p_j} = \frac{1}{2}\frac{\sigma_i}{\tau_a \tau_b - \sigma_a \sigma_b} > 0. \end{aligned}$$

To track how platforms pass a tax increase on to the two groups of agents, we separate the internal and the external viewpoints. That is, we study first how a platform chooses to modify its prices assuming that the prices of the other platform are kept fixed. Next, we examine the price modifications that result from platforms best-responding to one another. Without loss of generality we consider how platform U reacts to a change in T_a.

For the **internal viewpoint**, we focus on platform U's best responses, $P_a(p_a, p_b)$ and $P_b(p_a, p_b)$, which are given by the solution of the system (3)-(4); deriving them with respect to T_a, we find[11]

$$\frac{\partial P_a(p_a,p_b)}{\partial T_a} = \frac{1}{2} - \frac{1}{2}\frac{(\sigma_a - \sigma_b)(\sigma_a + \sigma_b)}{4\tau_a\tau_b - (\sigma_a + \sigma_b)^2}, \; \frac{\partial P_b(p_a,p_b)}{\partial T_a} = \frac{\tau_b(\sigma_a - \sigma_b)}{4\tau_a\tau_b - (\sigma_a + \sigma_b)^2} \; . \tag{15}$$

[9] This follows from the positive cross-side external effects, which make the two sides complementary to one another (see Appendix 5.1 for the details). If one side exerted negative external effects, say $\sigma_a < 0$, then internal strategic substitutability would continue to prevail as long as $\sigma_b > -\sigma_a$ (i.e., the positive effect that side a exerts on side b must be stronger than the negative effect that side b exerts on side a). This is relevant, e.g., for advertising-based media platforms when viewers dislike ads.

[10] We also note here that this result partially depends on the positivity of the cross-side external effects. In the presence of negative effects, some cross-partials could be negative, thereby resulting in a mix of external strategic complementarity and substitutability.

[11] We can also obtain these results through an implicit derivation of the two first-order conditions. We describe this procedure in Appendix 5.1 so as to detail the various transmission channels.

To understand how the tax increase affects platform U's best responses, we take as a benchmark the case where cross-side effects are absent: $\sigma_a = \sigma_b = 0$. In this case, the two sides are independent and any change in the profitability on one side (e.g., an increase in T_a) does not affect the profitability on the other side: P_b is independent of T_a (the second derivative in (15) is nil) and only side a is affected. In particular, fifty percent of the increase in T_a is absorbed by an increase in P_a (the first derivative in (15) is equal to 1/2), which is the traditional result of a monopoly firm facing a linear demand with a slope equal to minus one.

The presence of cross-side external effects introduces three additional channels through which a tax affects prices. We call them the 'contamination', 'leverage' and 'ricochet' channels and we depict them in Figure 1.

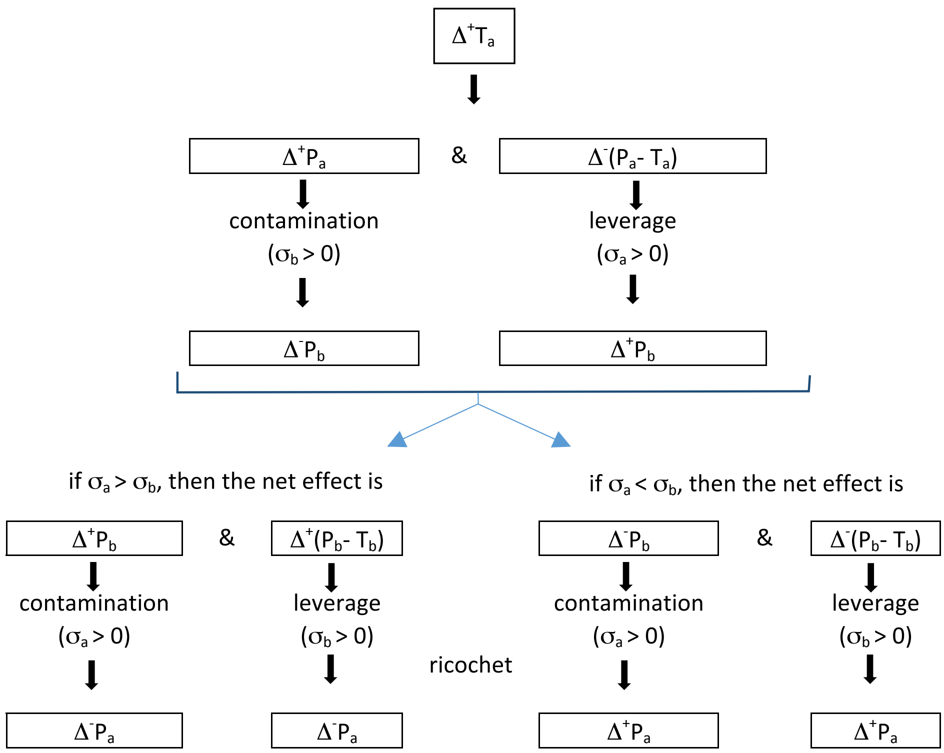

Figure 1: Channels of tax incidence on prices

The first two channels jointly affect the price that the platform sets on side b. The *contamination channel* pushes P_b down. Because agents on side b care about the interaction with agents on side a (i.e., $\sigma_b > 0$), the shock resulting from the increase in T_a contaminates side b through the following chain of events: the tax increase constrains the platform to reduce participation on side a, which affects negatively participation and, consequently, revenues on side b; the platform then reacts by lowering P_b so as to mitigate the propagation. In contrast the *leverage channel* pushes P_b up. Because agents on side a value the interaction with the other group (i.e., $\sigma_a > 0$), the platform is able to increase revenues on side a by lowering its price on side b (so as to attract more side-b users). Yet, exploiting this channel becomes less profitable as the tax increase reduces the margin that can be made on additional side-a users. This implies that the platform has lower incentives to reduce P_b.

The net effect of the previous two channels depends on the balance between σ_a and σ_b: there is more power in the leverage channel for $\sigma_a > \sigma_b$, and in the contamination channel otherwise (we observe indeed in (15) that $\partial P_b/\partial T_a > 0$ for $\sigma_a > \sigma_b$). So, unless $\sigma_a = \sigma_b$, the tax increase drives the platform to modify its price on side b, which, through a *ricochet channel*, induces the platform to adjust P_a. The shock we consider now is the change in P_b instead of the increase in T_a; we have the same two transmission channels as before but they now push in the same direction. If (say) $\sigma_a > \sigma_b$, P_b goes up and the platform has two reasons to decrease P_a: the increase in P_b not only reduces participation on side b and, thus, on side a (contamination), but also increases the margin on side b and so, the incentive to decrease P_a (leverage). Hence, compared to the benchmark, a lower fraction of the tax will be passed on to P_a. This is clear in (15) where we see that $\partial P_a/\partial T_a < 1/2$ when $\sigma_a > \sigma_b$. The opposite reasoning can be made when $\sigma_b > \sigma_a$.

We consider now the **external viewpoint** and examine the best responses of platform l. To this end, we solve the system (5)-(6) and express $p_a\left(P_a, P_b\right)$ and $p_b\left(P_a, P_b\right)$. We want to evaluate how platform l, which is not directly affected by the tax increase, will modify its prices after the change in P_a and P_b. We find

$$\begin{array}{ll} \frac{\partial p_a(P_a,P_b)}{\partial P_a} = \frac{1}{2} + \frac{1}{2}\frac{(\sigma_a-\sigma_b)(\sigma_a+\sigma_b)}{4\tau_a\tau_b-(\sigma_a+\sigma_b)^2}, & \frac{\partial p_a(P_a,P_b)}{\partial P_b} = \frac{\tau_a(\sigma_a-\sigma_b)}{4\tau_a\tau_b-(\sigma_a+\sigma_b)^2}, \\ \frac{\partial p_b(P_a,P_b)}{\partial P_a} = -\frac{\tau_b(\sigma_a-\sigma_b)}{4\tau_a\tau_b-(\sigma_a+\sigma_b)^2}, & \frac{\partial p_b(P_a,P_b)}{\partial P_b} = \frac{1}{2} - \frac{1}{2}\frac{(\sigma_a-\sigma_b)(\sigma_a+\sigma_b)}{4\tau_a\tau_b-(\sigma_a+\sigma_b)^2}. \end{array} \tag{16}$$

Let us start with the special case where $\sigma_a = \sigma_b$. We know from the previous discussion that platform U only modifies its fee on side a; we now observe that platform l does so as well since $\partial p_b / \partial P_a = 0$. Hence, in the special case where interaction is valued equally on both sides, both platforms pass on the increase in T_a only to side a (fees on side b are left unchanged).

When the cross-side external effects are different, it is easy to show that platform l always raises its fee on side a: (i) if $\sigma_a > \sigma_b$, then platform U increases both P_a and P_b, and both $\partial p_a / \partial P_a$ and $\partial p_a / \partial P_b$ are positive; (ii) if $\sigma_b > \sigma_a$, then platform U raises P_a and reduces P_b, while $\partial p_a / \partial P_a > 0$ and $\partial p_a / \partial P_b < 0$. Using expressions (15) and (16), we check that the combined effect on p_a of the changes in P_a and P_b is equal to

$$\frac{dp_a}{dT_a} = \frac{\partial p_a}{\partial P_a}\frac{\partial P_a}{\partial T_a} + \frac{\partial p_a}{\partial P_b}\frac{\partial P_b}{\partial T_a} = \frac{\tau_a \tau_b - \sigma_a \sigma_b}{4\tau_a \tau_b - (\sigma_a + \sigma_b)^2} > 0.$$

As for the change in p_b, it is a priori ambiguous as the variations of P_a and P_b may have opposite effects. Yet, comparing expressions (15) and (16), we observe that $\partial p_b (P_a, P_b) / \partial P_a = -\partial P_b (p_a, p_b) / \partial T_a$ and $\partial p_b (P_a, P_b) / \partial P_b = \partial P_a (p_a, p_b) / \partial T_a$. It follows that the combined effect of the changes in P_a and P_b on p_b is nil:

$$\frac{dp_b}{dT_a} = \frac{\partial p_b}{\partial P_a}\frac{\partial P_a}{\partial T_a} + \frac{\partial p_b}{\partial P_b}\frac{\partial P_b}{\partial T_a} = -\frac{\partial P_b}{\partial T_a}\frac{\partial P_a}{\partial T_a} + \frac{\partial P_a}{\partial T_a}\frac{\partial P_b}{\partial T_a} = 0.$$

This means that platform l does not modify p_b as a direct reaction to the change in platform U's fees. Yet, platform l still needs to adjust p_b to respond to its own modification of p_a, an effect that is not taken into account in this partial analysis.

As just noted, the analysis we performed so far is incomplete as it just looked at two rounds of price reactions after the tax increase. It was, however, insightful, as it allowed us to understand the crucial importance of the relationship between the intensities of the cross-side effects, σ_a and σ_b. To evaluate the total effect of the tax increase, we need to differentiate the equilibrium prices with respect to T_a. Using expressions (8)-(11) and recalling that $\gamma_a = T_a - t_a$, we find

$$\frac{\partial P_a^*}{\partial T_a} = \frac{2}{3} - (\sigma_a - \sigma_b)\frac{(2\sigma_a + \sigma_b)}{3D}, \tag{17}$$

$$\frac{\partial P_b^*}{\partial T_a} = (\sigma_a - \sigma_b)\frac{\tau_b}{D}, \tag{18}$$

$$\frac{\partial p_a^*}{\partial T_a} = \frac{1}{3} + (\sigma_a - \sigma_b)\frac{(2\sigma_a + \sigma_b)}{3D} = 1 - \frac{\partial P_a^*}{\partial T_a}, \tag{19}$$

$$\frac{\partial p_b^*}{\partial T_a} = -(\sigma_a - \sigma_b)\frac{\tau_b}{D} = -\frac{\partial P_b^*}{\partial T_a}. \tag{20}$$

As already noted, in the special case where $\sigma_a = \sigma_b$, the tax increase only affects prices on side a: the taxed platform passes $2/3$ of the tax increase on to side-a agents;[12] the other platform reacts by increasing its price by $1/3$ of the tax increase. In comparison, in the case where $\sigma_a > \sigma_b$ (interaction is more valuable for side-a agents), the taxed platform transfers part of the pass-trough from side a to side b, while the other platform raises its price further on side a but reduces its price on side b. It is important to note that the taxed platform may even choose a form of 'negative pass-through' as its optimum could be to *decrease* its price on side a.[13] Using the value of D, we have that $\partial P_a^* / \partial T_a < 0$ if and only if $6\tau_a\tau_b < (\sigma_a + \sigma_b)(2\sigma_a + \sigma_b)$; it can be checked that the latter condition is compatible with the second-order condition (7) if $\sigma_a > \sigma_b$.

The opposite situation prevails when $\sigma_b > \sigma_a$. Here (still in comparison with the case $\sigma_a = \sigma_b$), the taxed platform intensifies the pass-through on side a as it chooses to reduce its price on side b, while the other platform responds by lessening its price increase on side a and by increasing its price on side b. In this case, it is even possible that the other platform ends up *decreasing* its price on side a. We have that $\partial p_a^* / \partial T_a < 0$ if and only if $3\tau_a\tau_b < \sigma_b (2\sigma_a + \sigma_b)$, which is compatible with condition (7) when $\sigma_b > \sigma_a$.

Note finally that, irrespective of the balance between σ_a and σ_b, the sum of the two platforms' prices on side a increases by the amount of the tax increase, while the sum of prices on side b is left unchanged:

$$\frac{\partial P_a^*}{\partial T_a} + \frac{\partial p_a^*}{\partial T_a} = 1 \text{ and } \frac{\partial P_b^*}{\partial T_a} + \frac{\partial p_b^*}{\partial T_a} = 0.$$

Figure 2 identifies the regions of parameters where the different price variations are observed (see Appendix 5.1 for the proof). We note that cross-side effects need to be strong–i.e., close the limit imposed by the second-order condition (7)–to have that prices decrease on side a after the tax increase (i.e., $\partial P_a^* / \partial T_a < 0$ when $\sigma_a > \sigma_b$ or $\partial p_a^* / \partial T_a < 0$ when $\sigma_b > \sigma_a$).

[12] We can compute the total effect of T_a on P_a by combining the internal and external viewpoints. We have that $dP_a(p_a, p_b)/dT_a = dp_a(P_a, p_b)/dP_a = 1/2$; it can be shown that $dP_a(p_a, p_b)/dp_a$ is also equal to $1/2$. Hence, the total change is equal to $\frac{1}{2}\left(1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots\right) = \frac{2}{3}$.

[13] An example could be the following: a game platform that is imposed a larger tax on game consoles ends up decreasing the price of its console while increasing the fee it charges to game developers.

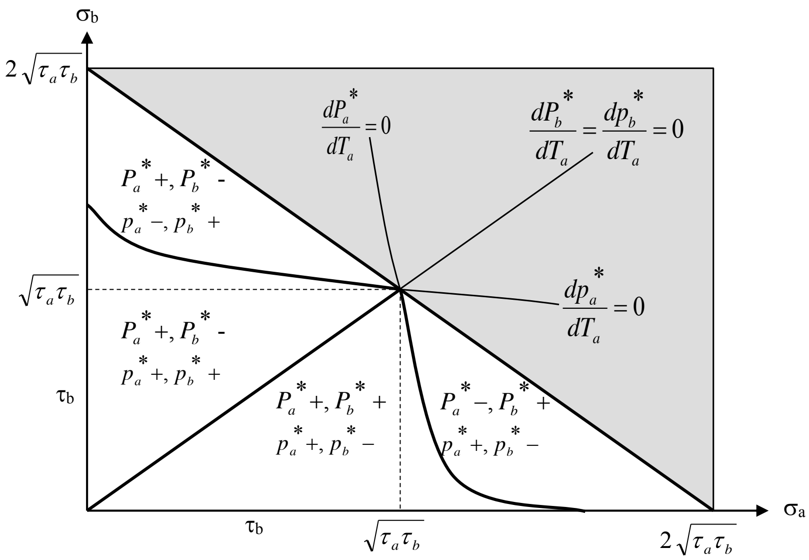

Figure 2: Tax passthrough

3.2 Incidence on profits

Now that we have a clear mapping of the effects of the tax increase on the equilibrium prices of the two platforms, let us examine how the price changes translate into profit changes. As above, we consider that platform U has to pay a higher tax for each agent it admits on side a: the tax increases from T_a to $T_a + x$, with $x > 0$. In what follows, we use the superscripts 0 and x to denote, respectively, the initial situation (with a tax T_a) and the new situation (with a tax $T_a + x$). The profit change can be decomposed into three effects (see Appendix 5.2 for the derivation):

$$\begin{aligned}\Pi^x - \Pi^0 \quad = \quad & -xN_a^x - \left(\frac{3\tau_b}{2D}\frac{\partial P_a^*}{\partial T_a} + \frac{2\sigma_a + \sigma_b}{2D}\frac{\partial P_b^*}{\partial T_a}\right)x^2 \\ & +\frac{1}{2}\left(1 - \frac{(\sigma_a + 2\sigma_b)\gamma_b + 3\gamma_a\tau_b}{D}\right)\frac{\partial p_a^*}{\partial T_a}x \\ & +\frac{1}{2}\left(1 - \frac{(2\sigma_a + \sigma_b)\gamma_a + 3\gamma_b\tau_a}{D}\right)\frac{\partial p_b^*}{\partial T_a}x.\end{aligned} \tag{21}$$

The first term is the *direct effect* and is clearly negative: the profit decreases as the platform has to pay the extra tax x for all agents it now admits on side a (N_a^x). The second term reflects what could be called the *own-price effect*, as it describes how the taxed platform affects its profit by adjusting its own prices.[14] Using expressions (17) and (18), we see that the own-price effect is always negative as the sum of the second and third terms is equal to $-\left(\tau_b x^2\right)/D < 0$. Finally, the combination of the third and fourth terms gives the *strategic effect*: the tax change leads the rival platform to modify its prices, which affects in turn the profit of the taxed platform. Using expressions (19) and (20), we compute that the strategic effect is equal to

$$SE = \frac{x}{6} - \frac{x}{2D}\tau_b\gamma_a - \frac{x}{2D}\sigma_b\gamma_b + \frac{x}{6D}\left(\sigma_a - \sigma_b\right)\left(2\sigma_a + \sigma_b - 3\tau_b\right). \tag{22}$$

Following Fudenberg and Tirole (1984), we expect the strategic effect to be positive because of external strategic complementarity. That is, the tax increase should lead platform l to increase its prices, thereby affecting positively platform U's profit. This is indeed what we obtain in 'one-sided markets', i.e., when cross-side effects are absent: setting $\sigma_a = \sigma_b = 0$ in the above expression (and recalling that $D = 9\tau_a\tau_b - \left(2\sigma_a + \sigma_b\right)\left(2\sigma_b + \sigma_a\right)$), we see that $SE = x\left(3\tau_a - \gamma_a\right)/\left(18\tau_a\right)$, which is positive by virtue of condition (12) that guarantees an interior equilibrium. Note that an increase in γ_a reduces the strategic effect: as platform U's disadvantage with respect to platform l increases, its market share decreases and thereby the sensitivity of its profit to a change in l's price. In the presence of cross-side effects, an increase in γ_b also reduces the strategic effect, as shown by the third term in (12). We can thus state that *the strategic effect of a tax increase is less pronounced for platforms with smaller market shares.*

As for the last term in (22), it measures how the intensity of the cross-side effects impacts the strategic effect. In particular, we see that if $\left(\sigma_a - \sigma_b\right)\left(2\sigma_a + \sigma_b - 3\tau_b\right) > 0$ (resp. < 0), the strategic effect is larger (resp. smaller) than in the case where cross-side effects are nil. This can lead to two striking situations. On the one hand, cross-side effects may make the strategic effect grow so large that it eventually outweighs the direct effect of the tax increase; as a result, the platform *benefits* from the tax increase and we talk of a *lucky break*. In contrast, cross-side effects may depress the strategic effect to such an extent that it eventually becomes negative; then, the tax increase hurts the platform twice, first directly and next through the price reaction of the other platform; there is thus *double jeopardy*. We now want to identify the regions of parameters where these two situations may occur.

[14] If we were considering an infinitesimal change of the tax, these two terms would vanish (envelope theorem).

Lucky break. We talk of a lucky break if $\Pi^x > \Pi^0$, i.e., if platform U's profit grows after an increase in T_a. Using (21), we derive a necessary and sufficient condition for platform U's profit to grow after an increase in T_a as

$$6\tau_a\tau_b - (2\sigma_a + \sigma_b)(\sigma_a + \sigma_b) + (\sigma_a - \sigma_b)\tau_b < 2\tau_b\gamma_a + (\sigma_a + \sigma_b)\gamma_b + x\tau_b.$$

It is immediate that the latter condition is more likely to be satisfied the larger platform U's initial cost disadvantage of (i.e., for larger γ_a or γ_b) and the more important the tax increase (i.e., for larger x). To emphasize the role of cross-side effects, let us focus on the case where platforms are initially symmetric ($\gamma_a = \gamma_b = 0$) and take x close to zero (which guarantees that condition (12) for an interior equilibrium is satisfied). Then, the condition for $\Pi^x - \Pi^0 > 0$ amounts to

$$6\tau_a\tau_b - (2\sigma_a + \sigma_b)(\sigma_a + \sigma_b) + (\sigma_a - \sigma_b)\tau_b < 0. \tag{23}$$

We can show that the latter condition is equivalent to $\partial(P_a^* + P_b^*)/\partial T_a < 0$ (see Appendix 5.2). We therefore observe that *if platforms are initially symmetric, then a small tax increase raises the profit of the taxed platform if and only if it reduces the sum of the platform's two equilibrium prices.*

Double jeopardy. The double jeopardy situation arises when the strategic effect becomes negative. Then, adding insult to injury, the direct negative effect of the tax is made worse by the aggressive price reaction of the rival platform. As already stressed, this result is unexpected given that the two platforms are substitutes and compete in prices. Using (22), we find the following necessary and sufficient condition for this situation to arise:

$$3\tau_a\tau_b - (\sigma_a - \sigma_b)\tau_b - \sigma_b(2\sigma_a + \sigma_b) < \sigma_b\gamma_b + \gamma_a\tau_b.$$

As in the previous case, an initial cost disadvantage for platform U (i.e., positive values of γ_a or γ_b) makes the latter condition more likely to be satisfied. Again, we want to focus on cross-side effects only and we thus set $\gamma_a = \gamma_b = 0$. Then, the condition for the strategic effect to be negative becomes

$$3\tau_a\tau_b - (\sigma_a - \sigma_b)\tau_b - \sigma_b(2\sigma_a + \sigma_b) < 0. \tag{24}$$

We can show that condition (24) is equivalent to $\partial(p_a^* + p_b^*)/\partial T_a < 0$ (see Appendix 5.2). Hence, *if platforms are initially symmetric, then a small* *tax increase makes the strategic effect negative if and only if it decreases the sum of the two equilibrium prices of the rival platform.*

We further show in Appendix 5.2 that conditions (23) and (24) are both compatible with the second-order condition (7), which establishes the existence of configurations of parameters where a lucky break or a double jeopardy situation arises. Figures 3 and 4 depict these configurations. In both figures, the admissible range of parameters is delimited by the second-order condition, and the regions identified with a sign '+' (resp. '-') comprise the parameters for which the strategic effect is larger (resp. smaller) when cross-side effects are present. Within these regions, we note with a '++' the lucky break situations and with a '- -' the double jeopardy situations.

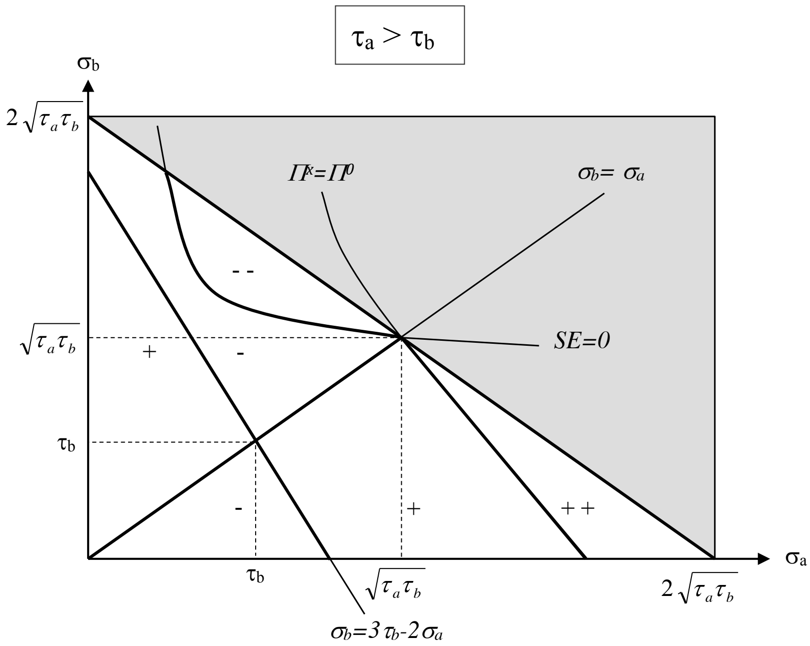

Figure 3: Tax incidence on profits ($\tau_a > \tau_b$)

We make the following observations. First, a necessary condition to encounter either of the two extreme situations is that the combination of the cross-side effects be very large (with respect to the transportation costs). Second, the double jeopardy situation requires the intensity of competition and of cross-side effects to be aligned across sides; that is, if competition is the strongest on side i ($\tau_i < \tau_j$), then it is also on side i that interaction is

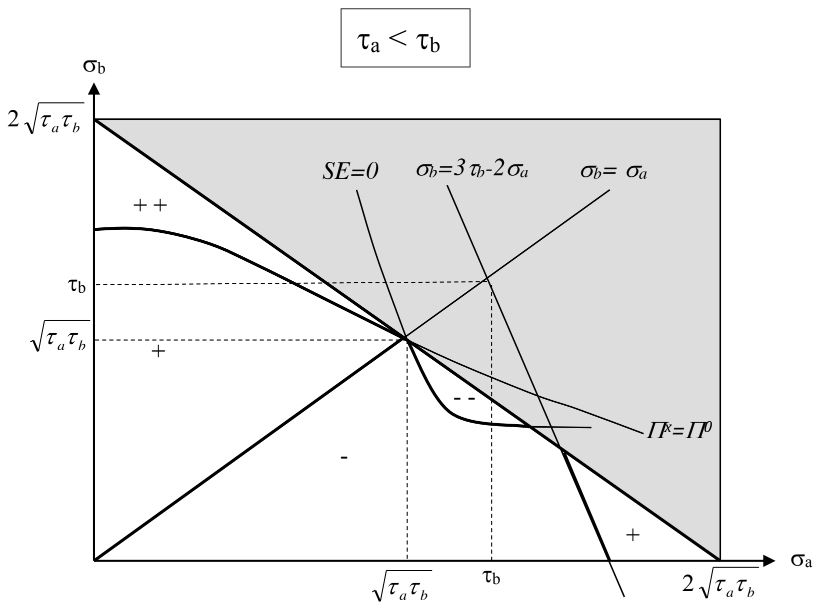

Figure 4: Tax incidence on profits ($\tau_b > \tau_a$)

valued the most ($\sigma_i > \sigma_j$).[15] In other words, double jeopardy requires that one group be very sensitive both to its own price (because this group sees the platforms as close substitutes) and to the other group's price (because it values the interaction a lot). Third, the lucky break situation requires the reverse: the side with the strongest intensity of competition must be the side with the *lowest* cross-side effects ($\tau_a > \tau_b$ and $\sigma_a > \sigma_b$, or $\tau_b > \tau_a$ and $\sigma_b > \sigma_a$). If one group is very sensitive to its own price, it is less sensitive to the other group's price.

Recall that Figures 3 and 4 are drawn under the assumptions that $\gamma_a = \gamma_b = 0$ and x is small. As we noted above, the lucky break and double jeopardy regions expand when $\gamma_a > 0$ and/or $\gamma_b > 0$, and when x increases. That is, these extreme situations are more likely to occur when the taxed platform has initially a smaller market share than its rival on either side, and when the tax increase is relatively large.

[15]We see indeed that in Figure 3 (where $\tau_b < \tau_a$), double jeopardy is observed above the diagonal (where $\sigma_b > \sigma_a$), while the opposite applies in Figure 4.

4 Discussion

So far, we focused on the tax incidence for one platform, assuming that only that platform was taxed on a particular side. Here, we briefly address a number of related issues.

On which side would a platform prefer to be taxed? In our analysis, we have arbitrarily considered a unit tax on side a. Suppose now that the tax authority lets the platform choose whether the tax should be levied on the number of side a or side b users. What would the platform choose? To simplify the analysis, suppose again that platforms are initially symmetric ($\gamma_a = \gamma_b = 0$). Defining Δ_x^k as the difference between platform U's profit after and before being imposed a unit tax on side k (see expression (21) for Δ_x^a), we compute the following:

$$\begin{aligned}
&\Delta_x^a - \Delta_x^b = \tfrac{x}{2D}\left[x\left(\tau_b - \tau_a\right) + \left(\sigma_b - \sigma_a\right)\left(\tau_a + \tau_b - \left(\sigma_a + \sigma_b\right)\right)\right], \\
&\Delta_x^a > 0 \Leftrightarrow 6\tau_a\tau_b < x\tau_b - \left(\sigma_a - \sigma_b\right)\tau_b + \left(2\sigma_a + \sigma_b\right)\left(\sigma_a + \sigma_b\right), \\
&\Delta_x^b > 0 \Leftrightarrow 6\tau_a\tau_b < x\tau_a + \left(\sigma_a - \sigma_b\right)\tau_a + \left(2\sigma_b + \sigma_a\right)\left(\sigma_a + \sigma_b\right).
\end{aligned}$$

As condition (7) implies that $\tau_a + \tau_b > \sigma_a + \sigma_b$, the top line reveals, for instance, that if $\tau_b > \tau_a$ and $\sigma_b > \sigma_a$, then $\Delta_x^a > \Delta_x^b$. That is, *if platform differentiation and cross-side effects are both stronger on side b than on side a, then the platform would prefer to be taxed on side a.* Combining the last two lines, we also see that it is perfectly possible that both Δ_x^a and Δ_x^b are positive, meaning that *there exists configurations of parameters for which the platform would enjoy a lucky break irrespective of the side on which the tax is levied.*[16]

How is the other platform affected? Regarding the impact of the tax on the other platform's profit, we would expect it to be positive. However, the previous analysis taught us not to trust our hunches. We check indeed that $\pi^x < \pi^0$ if and only if

$$6\tau_a\tau_b - \left(2\sigma_a + \sigma_b\right)\left(\sigma_a + \sigma_b\right) + \left(\sigma_a - \sigma_b\right)\tau_b < -\left(2\tau_b\gamma_a + \left(\sigma_a + \sigma_b\right)\gamma_b + x\tau_b\right),$$

[16]Take for instance $\tau_a = 4.5$, $\tau_b = 2$, $\sigma_b = 2$ and $x = 1$. The second-order condition imposes $\sigma_a < 4$; $\Delta_x^a > 0 \Leftrightarrow \sigma_a > 3.8$ and $\Delta_x^b > 0 \Leftrightarrow \sigma_a > 3.59$. So, for $3.8 < \sigma_a < 4$, we have that the platform's profit if T_a is increased to $T_a + x$ or if T_b is increased to $T_b + x$.

which may well be possible. For the sake of comparison, we reproduce the condition for $\Pi^x > \Pi^0$:

$$6\tau_a\tau_b - (2\sigma_a + \sigma_b)(\sigma_a + \sigma_b) + (\sigma_a - \sigma_b)\tau_b < 2\tau_b\gamma_a + (\sigma_a + \sigma_b)\gamma_b + x\tau_b.$$

We observe that the LHS of the two inequalities are the same, while the RHS have opposite values, which shows that there exist configurations of parameters for which the two conditions are met. In other words, *the tax increase may end up benefiting the taxed platform* ($\Pi^x > \Pi^0$) *while hurting the untaxed platform* ($\pi^x < \pi^0$).[17]

What is the impact of symmetric taxes? When both platforms are taxed in the same way (i.e., on the same side and for the same amount), it is easy to see that, in this model, profits are left unchanged for both platforms. This is an artefact of the double Hotelling setting that we use: because demand is inelastic and markets are supposed to be covered, profits depend on the difference of the marginal costs across platforms, which does not change if both T_a and t_a are increased by the same amount. As a result, the tax increase is entirely passed on to consumers. In an alternative model with elastic demands, this would no longer be the case and we expect thus platforms' profits to be affected even when both platforms are taxed.

To conclude, our objective in this paper was just to highlight the potential counterintuitive effects of taxes for competing two-sided platforms. We therefore kept our model as simple as possible. Naturally, a more exhaustive analysis would require us to extend the model in a number of directions. First, we could allow users on one side to multihome. Second, we could prevent platforms to set negative fees as such fees are generally not feasible in practice. Third, we could model the transactions among users and, thereby, give a micro-foundation of the users' utilities; this would allow us to consider the effects of transaction taxes. This is broadly the road map for our future research.

[17]Combining the two inequalities, we see that initial asymmetries make this less likely to happen.

5 Appendix

5.1 Tax incidence on prices

We derive the system (3)-(4) of platform U's reaction first-order conditions with respect to T_a:

$$\begin{cases} \frac{\partial^2 \Pi}{\partial P_a^2}\frac{\partial P_a}{\partial T_a} + \frac{\partial^2 \Pi}{\partial P_a \partial P_b}\frac{\partial P_b}{\partial T_a} = -\frac{\partial^2 \Pi}{\partial P_a \partial T_a}, \\ \frac{\partial^2 \Pi}{\partial P_a \partial P_b}\frac{\partial P_a}{\partial T_a} + \frac{\partial^2 \Pi}{\partial P_b^2}\frac{\partial P_b}{\partial T_a} = \frac{\partial^2 \Pi}{\partial P_b \partial T_a}. \end{cases}$$

Solving for $\partial P_a/\partial T_a$ and $\partial P_b/\partial T_a$, we find

$$\begin{aligned} \frac{\partial P_a}{\partial T_a} &= \frac{1}{K}\left(\frac{\partial^2 \Pi}{\partial P_b \partial T_a}\frac{\partial^2 \Pi}{\partial P_a \partial P_b} - \frac{\partial^2 \Pi}{\partial P_a \partial T_a}\frac{\partial^2 \Pi}{\partial P_b^2}\right), \\ \frac{\partial P_b}{\partial T_a} &= \frac{1}{K}\left(\frac{\partial^2 \Pi}{\partial P_a \partial T_a}\frac{\partial^2 \Pi}{\partial P_a \partial P_b} - \frac{\partial^2 \Pi}{\partial P_b \partial T_a}\frac{\partial^2 \Pi}{\partial P_a^2}\right), \end{aligned}$$

where $K \equiv \frac{\partial^2 \Pi}{\partial P_a^2}\frac{\partial^2 \Pi}{\partial P_b^2} - \left(\frac{\partial^2 \Pi}{\partial P_a \partial T_a}\right)^2$ is positive by the second-order conditions.

Computing the second-order derivatives of profits, we find

$$\begin{gathered} \frac{\partial^2 \Pi}{\partial P_a^2} = 2\frac{\partial N_a}{\partial P_a}, \frac{\partial^2 \Pi}{\partial P_b^2} = 2\frac{\partial N_b}{\partial P_b}, \frac{\partial^2 \Pi}{\partial P_a \partial P_b} = \frac{\partial N_a}{\partial P_b} + \frac{\partial N_b}{\partial P_a} \\ \frac{\partial^2 \Pi}{\partial P_a \partial T_a} = -\frac{\partial N_a}{\partial P_a}, \frac{\partial^2 \Pi}{\partial P_b \partial T_a} = -\frac{\partial N_a}{\partial P_b}, \end{gathered}$$

Plugging these values into the above expressions and simplifying, we have

$$\begin{aligned} \frac{\partial P_a}{\partial T_a} &= \frac{1}{K}\left(-\frac{\partial N_a}{\partial P_b}\left(\frac{\partial N_a}{\partial P_b} + \frac{\partial N_b}{\partial P_a}\right) + 2\frac{\partial N_a}{\partial P_a}\frac{\partial N_b}{\partial P_b}\right) \\ \frac{\partial P_b}{\partial T_a} &= \frac{1}{K}\left(-\frac{\partial N_a}{\partial P_a}\left(\frac{\partial N_a}{\partial P_b} + \frac{\partial N_b}{\partial P_a}\right) + 2\frac{\partial N_a}{\partial P_b}\frac{\partial N_a}{\partial P_a}\right) = -\frac{1}{K}\frac{\partial N_a}{\partial P_a}\left(\frac{\partial N_b}{\partial P_a} - \frac{\partial N_a}{\partial P_b}\right). \end{aligned}$$

Finally, using the 'demand functions' (1) and (2), we compute

$$\begin{aligned} \frac{\partial N_a}{\partial P_a} &= \frac{-\tau_b}{2(\tau_a\tau_b - \sigma_a\sigma_b)}, \frac{\partial N_b}{\partial P_b} = \frac{-\tau_a}{2(\tau_a\tau_b - \sigma_a\sigma_b)}, \\ \frac{\partial N_a}{\partial P_b} &= \frac{-\sigma_a}{2(\tau_a\tau_b - \sigma_a\sigma_b)}, \frac{\partial N_b}{\partial P_a} = \frac{-\sigma_b}{2(\tau_a\tau_b - \sigma_a\sigma_b)}. \end{aligned}$$

It follows that

$$\begin{aligned} sgn\left(\frac{\partial P_a}{\partial T_a}\right) &= sgn\left(2\tau_a\tau_b - \sigma_a\left(\sigma_a + \sigma_b\right)\right), \\ sgn\left(\frac{\partial P_b}{\partial T_a}/\right) &= sgn\left(\sigma_a - \sigma_b\right). \end{aligned}$$

We now show how Figure 2 is drawn. The thick diagonal line depicts the limit imposed by the second-order condition (7): $\tau_a\tau_b = \frac{1}{4}\left(\sigma_a + \sigma_b\right)^2$ or $\sigma_b = 2\sqrt{\tau_a\tau_b} - \sigma_a$. The loci $\partial P_b^*/\partial T_a$ and $\partial p_b^*/\partial T_a$ are obvious. As for $\partial P_a^*/\partial T_a$, we compute:

$$\frac{\partial P_a^*}{\partial T_a} = 0 \Leftrightarrow \sigma_b = \frac{1}{2}\sqrt{\sigma_a^2 + 24\tau_a\tau_b} - \frac{3}{2}\sigma_a \equiv \lambda\left(\sigma_a\right),$$

with $\lambda''(\sigma_a) = 12\tau_a\tau_b / \left(\sigma_a^2 + 24\tau_a\tau_b\right)^{3/2} > 0$, $\lambda^{-1}(0) = \sqrt{3}\sqrt{\tau_a\tau_b} < 2\sqrt{\tau_a\tau_b}$, and $\lambda(\sqrt{\tau_a\tau_b}) = \sqrt{\tau_a\tau_b}$. That is, the locus $\partial P_a^*/\partial T_a = 0$ is a convex function of σ_a, which lies under the SOC in the area where $\sigma_a > \sigma_b$ and crosses the SOC at $\sigma_a = \sigma_b = \sqrt{\tau_a\tau_b}$.

Finally, we have

$$\frac{\partial p_a^*}{\partial T_a} = 0 \Leftrightarrow \sigma_b = \sqrt{\sigma_a^2 + 3\tau_a\tau_b} - \sigma_a \equiv \eta(\sigma_a),$$

with $\eta''(\sigma_b) = 3\tau_a\tau_b / \left(\sigma_a^2 + 3\tau_a\tau_b\right)^{\frac{3}{2}} > 0$, $\eta(0) = \sqrt{3}\sqrt{\tau_a\tau_b} < 2\sqrt{\tau_a\tau_b}$ and $\eta(\sqrt{\tau_a\tau_b}) = \sqrt{\tau_a\tau_b}$. That is, the locus $\partial p_a^*/\partial T_a = 0$ is a convex function of σ_a, which lies under the SOC in the area where $\sigma_a < \sigma_b$ and crosses the SOC at $\sigma_a = \sigma_b = \sqrt{\tau_a\tau_b}$.

5.2 Tax incidence on profits

5.2.1 Derivation of the strategic effect

Recalling that $\Pi = (P_a - T_a) N_a + (P_b - T_b) N_b$, we can write

$$\Pi^x - \Pi^0 = -xN_a^x + \Delta_{P_a} N_a^x + \left(P_a^0 - T_a\right)\Delta_{N_a} + \Delta_{P_b} N_b^x + \left(P_b^0 - T_b\right)\Delta_{N_b},$$

where, using expressions (1) and (2) and the tax incidence on prices,

$$\begin{aligned}
\Delta_{N_a} &\equiv N_a^x - N_a^0 = \tfrac{\tau_b(\Delta_{p_a} - \Delta_{P_a}) + \sigma_a\left(\Delta_{p_b} - \Delta_{P_b}\right)}{2(\tau_a\tau_b - \sigma_a\sigma_b)}, \\
\Delta_{N_b} &\equiv N_b^x - N_b^0 = \tfrac{\tau_a\left(\Delta_{p_b} - \Delta_{P_b}\right) + \sigma_b(\Delta_{p_a} - \Delta_{P_a})}{2(\tau_a\tau_b - \sigma_a\sigma_b)}, \\
\Delta_{P_a} &\equiv P_a^x - P_a^0 = \tfrac{\partial P_a^*}{\partial T_a}x,\ \Delta_{P_b} \equiv P_b^x - P_b^0 = \tfrac{\partial P_b^*}{\partial T_a}x, \\
\Delta_{p_a} &\equiv p_a^x - p_a^0 = \tfrac{\partial p_a^*}{\partial T_a}x,\ \Delta_{p_b} \equiv p_b^x - p_b^0 = \tfrac{\partial p_b^*}{\partial T_a}x.
\end{aligned}$$

Grouping terms, we have

$$\begin{aligned}
\Pi^x - \Pi^0 \quad = \quad & -xN_a^x + \left(N_a^x - \tfrac{\tau_b\left(P_a^0 - T_a\right)}{2(\tau_a\tau_b - \sigma_a\sigma_b)} - \tfrac{\sigma_b\left(P_b^0 - T_b\right)}{2(\tau_a\tau_b - \sigma_a\sigma_b)}\right)\Delta_{P_a} \\
& + \left(N_b^x - \tfrac{\sigma_a\left(P_a^0 - T_a\right)}{2(\tau_a\tau_b - \sigma_a\sigma_b)} - \tfrac{\tau_a\left(P_b^0 - T_b\right)}{2(\tau_a\tau_b - \sigma_a\sigma_b)}\right)\Delta_{P_b} \\
& + \left(\tfrac{\tau_b\left(P_a^0 - T_a\right)}{2(\tau_a\tau_b - \sigma_a\sigma_b)} + \tfrac{\sigma_b\left(P_b^0 - T_b\right)}{2(\tau_a\tau_b - \sigma_a\sigma_b)}\right)\Delta_{p_a} \\
& + \left(\tfrac{\tau_a\left(P_b^0 - T_b\right)}{2(\tau_a\tau_b - \sigma_a\sigma_b)} + \tfrac{\sigma_a\left(P_a^0 - T_a\right)}{2(\tau_a\tau_b - \sigma_a\sigma_b)}\right)\Delta_{p_b}.
\end{aligned}$$

The expression in the text is obtained from the previous expression by inserting the equilibrium prices (8) to (11), and

$$N_a^x = \tfrac{1}{2} - \tfrac{1}{2D}\left(3\tau_b(\gamma_a + x) + (\sigma_a + 2\sigma_b)\gamma_b\right).$$

5.2.2 Infinitesimal analysis

The total effect on profit of an infinitesimal increase in T_a can be written as

$$\frac{d\Pi}{dT_a} = \frac{\partial \Pi}{\partial T_a} + \underbrace{\frac{\partial \Pi}{\partial P_a}}_{=0} \frac{dP_a^*}{dT_a} + \underbrace{\frac{\partial \Pi}{\partial P_b}}_{=0} \frac{dP_b^*}{dT_a} + \underbrace{\left(\frac{\partial \Pi}{\partial p_a} \frac{dp_a^*}{dT_a} + \frac{\partial \Pi}{\partial p_b} \frac{dp_b^*}{dT_a} \right)}_{SE}$$

The first term is the direct effect; the second and third terms are nil by the envelope theorem; the fourth term is the strategic effect.

Recall that the equilibrium profit is equal to $\Pi^* = (P_a^* - T_a) N_a^* + (P_b^* - T_b) N_b^*$. The direct effect is computed by ignoring the effect on equilibrium prices and quantities. It is thus equal to $-N_a^*$. As for the strategic effect, we know that

$$\begin{aligned} \frac{\partial \Pi}{\partial p_a} &= (P_a - T_a) \frac{\partial N_a}{\partial p_a} + (P_b - T_b) \frac{\partial N_b}{\partial p_a} \\ &= -\left[(P_a - T_a) \frac{\partial N_a}{\partial P_a} + (P_b - T_b) \frac{\partial N_b}{\partial P_a} \right] = N_a^* \end{aligned}$$

where the second equality uses the fact that $\partial N_a / \partial P_a = -\partial N_a / \partial p_a$, $\partial N_b / \partial P_b = -\partial N_b / \partial p_b$ by (1) and (2), and where the third equality uses platform U's first-order condition for profit maximization on side a ($\partial \Pi / \partial P_a = 0$). By analogy, $\partial \Pi / \partial p_b = N_b^*$. Hence

$$SE = N_a^* \frac{dp_a^*}{dT_a} + N_b^* \frac{dp_b^*}{dT_a}.$$

If platforms are initially in a symmetric situation ($\gamma_a = \gamma_b = 0$), then $N_a^* = N_b^* = 1/2$ and we have that $SE < 0 \Leftrightarrow \partial (p_a^* + p_b^*) / \partial T_a < 0$, as claimed in the text.

Putting the direct and strategic effect together, we have

$$\frac{d\Pi}{dT_a} = N_a^* \left(\frac{dp_a^*}{dT_a} - 1 \right) + N_b^* \frac{dp_b^*}{dT_a} = -\left(N_a^* \frac{dP_a^*}{dT_a} + N_b^* \frac{dP_b^*}{dT_a} \right),$$

where the second equality follows from $dP_a^* / dT_a = 1 - dp_a^* / dT_a$ and $dP_b^* / dT_a = -dp_b^* / dT_a$. Again, in an initial symmetric situation, $N_a^* = N_b^* = 1/2$ and $d\Pi / dT_a > 0 \Leftrightarrow \partial (P_a^* + P_b^*) / \partial T_a < 0$, as claimed in the text.

5.2.3 Lucky break and double jeopardy

Figures 3 and 4 are drawn according to the following computations.

First, condition (7) is equivalent to

$$\sigma_b < 2\sqrt{\tau_a \tau_b} - \sigma_a \equiv S(\sigma_a),$$

where $S(\sigma_a)$ is a line with an intercept $2\sqrt{\tau_a \tau_b}$ and a slope -1. The relevant area lies in the south-east of this line.

Second, condition (23) is equivalent to

$$\Pi^x > \Pi^0 \iff \sigma_b > \frac{1}{2}\sqrt{(\sigma_a + 5\tau_b)^2 + 24\tau_b(\tau_a - \tau_b)} - \frac{1}{2}(3\sigma_a + \tau_b) \equiv X(\sigma_a)$$

Drawing precisely the function $X(\sigma_a)$ is more tedious. However, we can easily show that it crosses $S(\sigma_a)$ only once, at $\sigma_a = \sigma_b = \sqrt{\tau_a \tau_b}$. For $\Pi^x > \Pi^0$ to be relevant, we need that $X(\sigma_a) < S(\sigma_a)$, which is equivalent to

$$\sigma_a\left(\sqrt{\tau_a} - \sqrt{\tau_b}\right) > \sqrt{\tau_a \tau_b}\left(\sqrt{\tau_a} - \sqrt{\tau_b}\right)$$

Hence $S(\sigma_a)$ and $X(\sigma_a)$ are equal if and only if $\sigma_a = \sqrt{\tau_a \tau_b} = \sigma_b$; this is their unique intersection. If $\tau_a > \tau_b$, there exists values of σ_a and σ_b such that $\Pi^x > \Pi^0$ if and only if $\sigma_a > \sqrt{\tau_a \tau_b}$; by contrast, if $\tau_a < \tau_b$, such values exist if and only if $\sigma_a < \sqrt{\tau_a \tau_b}$. Those two conditions are represented by Figure 3 and 4 respectively. Note that it is not possible that $\Pi^x > \Pi^0$ if $\tau_a = \tau_b$.

Third, the influence of cross-side effects on the strategic effect is easily drawn. Indeed, from (22), we know that if $(\sigma_a - \sigma_b)(2\sigma_a + \sigma_b - 3\tau_b) > 0$ (resp. < 0), the strategic effect is larger (resp. smaller) than in the case where cross-side effects are nil. The lines $\sigma_b = \sigma_a$ and $\sigma_b = 3\tau_b - 2\sigma_a$ intersect at $\sigma_a = \tau_b = \sigma_b$, which is lower than the intersection of $S(\sigma_a)$ and $X(\sigma_a)$ if and only if $\tau_a > \tau_b$.

Fourth, to draw easily the strategic effect, we draw σ_a as a function of σ_b, rather than the opposite. The strategic effect is negative if and only if

$$\text{SE} < 0 \iff \sigma_a > \frac{3\tau_a \tau_b + \sigma_b(\tau_b - \sigma_b)}{2\sigma_b + \tau_b}$$

Condition (7) requires that $\sigma_a < 2\sqrt{\tau_a \tau_b} - \sigma_b$. Hence, there exists values of σ_a such that the strategic effect is negative and (7) is met if and only if

$$2\sqrt{\tau_a \tau_b} - \sigma_b > \frac{3\tau_a \tau_b + \sigma_b(\tau_b - \sigma_b)}{2\sigma_b + \tau_b} \iff \left(\sigma_b - \sqrt{\tau_a \tau_b}\right)\left(\sigma_b - \left(3\sqrt{\tau_a \tau_b} - 2\tau_b\right)\right) < 0$$

It is readily checked that $3\sqrt{\tau_a \tau_b} - 2\tau_b > \sqrt{\tau_a \tau_b} \iff \tau_a > \tau_b$. It can also be checked that $3\sqrt{\tau_a \tau_b} - 2\tau_b > 2\sqrt{\tau_a \tau_b} \iff \tau_a > 4\tau_b$, and $3\sqrt{\tau_a \tau_b} - 2\tau_b >$

$0 \iff \tau_b < \frac{9}{4}\tau_a$. Consider first $\tau_a > \tau_b$. In the non empty interval $\sigma_b \in \left(\sqrt{\tau_a \tau_b}, \min\left\{3\sqrt{\tau_a \tau_b} - 2\tau_b, 2\sqrt{\tau_a \tau_b}\right\}\right)$, there exists values of σ_a such that the strategic effect is negative. Consider next $\tau_a < \tau_b$. There exists values of σ_a such that the strategic effect is negative in the non empty interval $\sigma_b \in \left(\max\left\{0, 3\sqrt{\tau_a \tau_b} - 2\tau_b\right\}, \sqrt{\tau_a \tau_b}\right)$.

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