Deltares
Enabling Delta Life

Informal mini-workshop on

Reduced complexity modelling

22nd May, 2015

Venue: Paviljoen building, Deltares, Rotterdamseweg 185, Delft, Netherlands (www.deltares.nl)

Rationale. Having recourse to the most complex model(s) is not always the best approach to address a practical issue. Sometimes, simpler models will do because, for instance, they are much less demanding in terms of CPU time while offering an acceptable level of accuracy. In addition, simple models, calibrated from the results of complex ones, may be used to understand the results of complex ones.

Nowadays intensive research is conducted on model complexity reduction. One can reduce the number of dimensions (i.e. go from 3D to 2D, 1D or 0D), concentrate on a couple of "modes" (hopefully the most significant ones) and/or reduce the number of state variables (e.g. simplifying an ecological model having a large number of state variables to an NPZD model). There is a wide spectrum of methods available to reduce the complexity/dimension/order of a model and/or determine the appropriate level of complexity of a model. At one end of the spectrum, there is handiwork guided by physical intuition, whilst at the other end of the spectrum more rational and sophisticated methods are being worked out.

Clearly, there are many types of methods and many reasons to build reduced-complexity models. Taking a look at all this should be the objective of this short and informal workshop.

Programme

Participants are welcome to buy a lunch at the Deltares restaurant (main Deltares building), which is open from 11:45 AM onward.

12:45 - 13:00 Welcome coffee

13:00 - 13:05 *Introduction*, E. Deleersnijder

13:05 - 13:25 *Model order reduction using Proper Orthogonal Decomposition*, A. W. Heemink

13:25 - 13:45 *An early warning indicator for atmospheric blocking events using transfer operators,* A. Tantet, F. R. van der Burgt, H. A. Dijkstra*

13:45 - 14:05 *Network science tools to simplify the output of a larval dispersal model in the Great Barrier Reef*, C. J. Thomas*, J. Lambrechts, E. Wolanski et al.

See overleaf.

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Programme (contd.)

14:05-14:25 *ONELAB: Bringing open-source simulation tools to industrial design and education*, C. Geuzaine, F. Henrotte*, J.-F. Remacle, R. V. Sabariego

14:25-14:45 *Modular conceptual modelling approach and software for river hydraulic simulations*, V. Wolfs*, P. Meert, P. Willems

14:45 - 15:05 *Model complexity and performance: How far can we simplify?*, M. Grégoire*, E. Delhez, C. Raick, K. Soetaert

15:05 - 15:25 *Coffee break*

15:25 - 15:45 *Metals fate and transport modelling: Implementation of empirical regression functions in a flexible-resolution transport model* , O. Gourgue*, A. de Brauwere, M. Elskens

15:45 - 16:05 *Assessing the consistency of ventilation in ocean biogeochemical models: oxygen solubility tracer*, A. Mouchet

16:05 - 16:25 *On diagnostic decomposition of salt fluxes and model reduction from 3D to 1D*, W. Kranenburg*, Y. Dijkstra, K. Kuijper, R. Uittenbogaard

16:25 - 16:45 *Simplified model of coastal resonance*, L. Maas

16:45 - 17:00 *Simple models to interpret the results of complex ones*, E. Deleersnijder*, E. Delhez, A. Mouchet

17:00 - 17:30 *Roundtable discussion*

17:30 - 18:30 *Drinks and snacks*

For additional information, contact Eric Deleersnijder (eric.deleersnijder@uclouvain.be).

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Abstracts

Model order reduction using Proper Orthogonal Decomposition

Arnold W. Heemink*

* *Delft University of Technology, Delft, The Netherlands*

Written in state space form, the dimension of numerical simulation models is of the order of the number of grid points and can be very high. As a result, these models are computationally very demanding, especially if many different scenarios has to be simulated. Proper Orthogonal Decomposition (POD) is based upon a limited number of simulations with the large-scale numerical model. By computing the covariance matrix of the model results, we obtain insight into the variability of the model behavior. Selecting the leading eigenvectors of this covariance matrix, we obtain the spatial patterns that represent the directions in state space where the model variability is dominant. The original numerical model can now be projected onto the reduced space spanned by the dominant spatial patterns. The result is a low dimensional model that is still able to reproduce the dominant model behavior. POD is very attractive for model calibration problems where many model simulations are required.

In the presentation we will describe the POD methodology, discuss a number of extensions and will illustrate the approach with a number of applications.

An early warning indicator for atmospheric blocking events using transfer operators

Alexis Tantet, Fiona R. van der Burgt, Henk A. Dijkstra*

* *Utrecht University, Utrecht, The Netherlands*

The existence of persistent midlatitude atmospheric flow regimes with time-scales larger than 5–10 days and indications of preferred transitions between them motivates to develop early warning indicators for such regime transitions. In this paper, we use a hemispheric barotropic model together with estimates of transfer operators on a reduced phase space to develop an early warning indicator of the zonal to blocked flow transition in this model. It is shown that the spectrum of the transfer operators of the reduction. The slowest motions are thereby found to have time scales of three to sixweeks and to be associated with meta-stable regimes (and their transitions) which can be

detected as almost-invariant sets of the transfer operator. From the energy budget of the model, we are able to explain the meta-stability of the regimes and the existence of preferred transition paths. Even though the model is highly simplified, the skill of the early warning indicator is promising, suggesting that the transfer operator approach can be used in parallel to an operational deterministic model for stochastic prediction or to assess forecast uncertainty.

Network science tools to simplify the output of a larval dispersal model in the Great Barrier Reef

Christopher J. Thomas*, Jonathan Lambrechts, Eric Wolanski, Vincent A. Traag, Vincent D. Blondel, Eric Deleersnijder, Emmanuel Hanert

** Université catholique de Louvain, Louvain-la-Neuve, Belgique*

The dispersal of coral larvae through the marine environment is a process of fundamental importance for coral reef ecosystems, but remains poorly understood and hard to gauge. Spatially explicit numerical modelling of marine currents and their transport of larvae has become a key tool used to understand and predict this process. However, the output from these Individual Based Models can be very complex, with millions of particles to consider, released from thousands of different habitats, potentially creating millions of inter-habitat connections. We will present a network science approach to identify spatial patterns in the output in order to simplify the results. By using community detection tools, clusters of highly-connected habitats can be identified, and key larval dispersal statistics can then be calculated for each cluster, allowing us to summarise larval dispersal patterns at the more useful cluster-scale rather than at the habitat-scale.

ONELAB: Bringing open-source simulation tools to industrial design and education

Christophe Geuzaine, François Henrotte*, Jean-François Remacle, Ruth V. Sabariego

** Université catholique de Louvain, Louvain-la-Neuve, Belgium*

Industry, research and education share a growing need for high-level scientific computation software. While the high licensing costs of commercial solutions are justified in large companies using them intensively, they cannot be afforded by the smaller and more occasional users. Open source software developed in universities and research centers (e.g. GetDP, Elmer, OpenFOAM, etc.) constitutes then an alternative. These codes are competitive with regard to both their capabilities and their performance, when compared to commercial equivalents. They have however a marginal penetration in small- and medium-size businesses, because of their lacking, in general, an easy-to-use interface, and because of a scarce (if not non-existent) documentation. On the other hand, industrial product developers need system-level simulation tools, i.e. tools with significant multi-physics capabilities, whereas specialized codes remain essentially mono-physics.

These observations tailored our design objectives for the ONELAB (Open Numerical Engineering LABoratory) software library. The results is a versatile, light-weight software library designed to interface arbitrary scientific simulation codes with each other. In particular ONELAB is

very useful to combine several models of increasing complexity of the same physical system into a single interactive simulation object. Thanks to the ONELAB interfacing, the sub-models communicate with a centralized database of geometrical and simulation parameters (they remains thus always consistent with each other), and they can be solved individually, or in chain, according to an arbitrary user-defined logic. ONELAB is freely available at http://onelab.info.

Modular conceptual modelling approach and software for river hydraulic simulations

Vincent Wolfs*, Pieter Meert, Patrick Willems

** KU Leuven, Leuven Belgium*

Numerous applications in river management require computationally efficient models that can accurately emulate the state of a river. Our research group developed a reduced complexity modelling approach based on the storage cell concept. Its modular design allows users to combine different model structures depending on the river dynamics and intended use. A semi-automatic software tool (Conceptual Model Developer, CMD) was developed to facilitate model set-up. To prevent instabilities during model simulations, a novel and highly efficient discrete calculation scheme is presented that employs a variable time step. To illustrate the effectiveness of the presented approach, the Marke River in Belgium was conceptualized based on simulation results of a detailed full hydrodynamic model. Results show that the derived conceptual model mimics the detailed model closely, while the calculation time is reduced by more than 2000 times. Finally, several applications are discussed that rely on conceptual models built according to the presented approach.

Model complexity and performance: How far can we simplify?

Marilaure Grégoire*, Eric Delhez, Caroline Raick, Karline Soetaert

** Université de Liège, Liège, Belgique*

Handling model complexity and reliability is a key area of research today. While complex models containing sufficient detail have become possible due to increased computing power, they often lead to too much uncertainty. On the other hand, very simple models often crudely oversimplify the real ecosystem and cannot be used for management purposes. Starting from a complex and validated 1D pelagic ecosystem model of the Ligurian Sea (NW Mediterranean Sea), we derived simplified aggregated models in which either the unbalanced algal growth, the functional group diversity or the explicit description of the microbial loop was sacrificed. To overcome the problem of data availability with adequate spatial and temporal resolution, the outputs of the complex model are used as the baseline of perfect knowledge to calibrate the simplified models. Objective criteria of model performance were used to compare the simplified models' results to the complex model output and to the available data at the DYFAMED station in the central Ligurian Sea. We show that even the simplest (NPZD) model is able to represent the global ecosystem features described by the complex model (e.g. primary and secondary productions, particulate organic matter export flux,

etc.). However, a certain degree of sophistication in the formulation of some biogeochemical processes is required to produce realistic behaviors (e.g. the phytoplankton competition, the potential carbon or nitrogen limitation of the zooplankton ingestion, the model trophic closure, etc.).

Metals fate and transport modelling: Implementation of empirical regression functions in a flexible-resolution transport model

Olivier Gourgue*, Anouk de Brauwere, Marc Elskens

** Vrije Universiteit Brussel, Brussels, Belgium*

Predicting metal concentrations in surface waters is an important step in the understanding and assessment of the ecological risk associated with metal contamination. In terms of risk an essential piece of information is the accurate knowledge of the metal partitioning between the dissolved and particulate phases, as the former species are generally regarded as bioavailable and thus potentially harmful. Yet, implementing mass balance differential equations describing metal speciation in a hydrodynamic transport model might be very complex: a wide range of assumptions, input parameters, data requirements, calibration and validation issues having to be addressed. To overcome on this situation, empirical regression functions describing the metal behavior in the dissolved and particulate phases were built up and introduced in a flexible-resolution transport model. Results of Cu and Cd fate and transport in the Scheldt Estuary are presented and discussed.

Assessing the consistency of ventilation in ocean biogeochemical models: oxygen solubility tracer

Anne Mouchet*

** LSCE, Institut Pierre Simon Laplace, Saclay, France*

Various techniques are used by modellers in order to overcome difficulties inherent to the modelling of biogeochemical cycles in global climate models. In several cases the numerical scheme used for the transport of tracers differs from that used for the active variables (temperature and salinity) in order to guarantee positivity or conservation. Some models also rely on asynchronous coupling of the dynamic and tracer modules. Off-line modelling is also common practice.

These techniques imply that the different scalars in the model may be subject to slightly different numerical transport equations. It is highly desirable that the chosen method be assessed as it might strongly affect the ocean ventilation rate as actually experienced by the tracers. In this work we show how a simple tracer, the solubility oxygen, offers an easy way of assessing the potential ventilation bias. It is straightforward to implement and provides at once information on surface as well as deep ocean processes without calling for additional experiments.

By comparing the solubility oxygen distribution to the theoretical oxygen saturation obtained from the model temperature and salinity fields a clear diagnostic of the ability of the tracer model to correctly reproduce the ventilation pathways of the OGCM may be drawn. Case studies with coupled circulation-biogeochemical ocean models are presented.

On diagnostic decomposition of salt fluxes and model reduction from 3D to 1D

Wouter Kranenburg*, Yoeri Dijkstra, Kees Kuijper and Rob Uittenbogaard

* *DELTARES, Delft, The Netherlands*

Salinity intrusion in estuaries is a difficult modelling issue. Many 3D mechanisms contribute to the salt transport. However, interpretation of 3D model output in terms of processes is complex. Next, there is a need for fast and simple models, e.g. to generate statistics in support of decision making. This role is often fulfilled by cross sectional averaged 1DH advection-dispersion models.

We will discuss recent and present work at Deltares on development of a new methodology for (1) diagnostic decomposition of salt-fluxes and (2) determination of equivalent dispersion formulations from 3D numerical model results. Firstly, we discuss the various contributions and their relation with the most relevant physical processes. Secondly, we describe the analytical method to determine the dispersion coefficient and illustrate the method from an idealized 2DV-model. Next, we discuss our present activities to couple the method to a 3D numerical model and conclude with a model reduction exercise for the Hollandsche IJssel, a case of high practical importance for fresh water supply in the Netherlands.

Simplified model of coastal resonance

Leo Maas*

* *Royal Netherlands Institute for Sea Research (NIOZ), Texel & Utrecht University, Utrecht, The Netherlands*

Tides are forced globally, on ocean-scales only. There, they acquire amplitudes of a few decimeter. In coastal seas, tides co-oscillate with the tide at the sea's entrance, and amplify when they resonate. Resonance occurs when the tidal period is close to one of the basin's eigen periods. A simple, zero-dimensional model shows that basins turn into nonlinear resonators, simply because of spatial variations in basin-depth. Thus coastal basins are to be perceived as forced and dissipative nonlinear resonators. As a consequence, tides may evoke multiple equilibria and even chaos. This is perhaps strongest expressed for tidal flows. Laboratory experiments and sea observations partially demonstrate these features and provide a challenge for sophisticated numerical models.

Simple models to interpret the results of complex ones

Eric Deleersnijder*, Eric Delhez and Anne Mouchet

* *Université catholique de Louvain, Louvain-la-Neuve, Belgium & Delft University of Technology, Delft, The Netherlands*

Numerical models routinely produce output files that are so large that making sense of all of these real numbers has become a challenging task. There exist a number of methods capable of reducing the amount of information submitted to the person in charge of producing a physical interpretation of the simulation results. One of them consists in developing a model that is much simpler than the

original one and is capable of reproducing to a satisfactory degree of accuracy indicators of the values of the key state variables of the complex model.

For tracer transport studies compartmental models, comprising no more than a couple of “boxes”, or one-dimensional “pipe” models are reduced-complexity tools than are easy to develop. Each of their key parameters usually has a clear physical meaning, such as a residence time, a transit time, a partioning coefficient, etc. Applications to semi-enclosed, shallow-water areas and the World Ocean show that the aforementioned simple models sometimes perform astonishing well, for reasons that yet to be entirely elucidated.

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ELSEVIER

Journal of Marine Systems 5 (1994) 401–410

JOURNAL OF MARINE SYSTEMS

Discussion paper

Do not use a simple model when a complex one will do

J.C.J. Nihoul

GHER, University of Liège, Belgium

Received and accepted 5 July, 1994

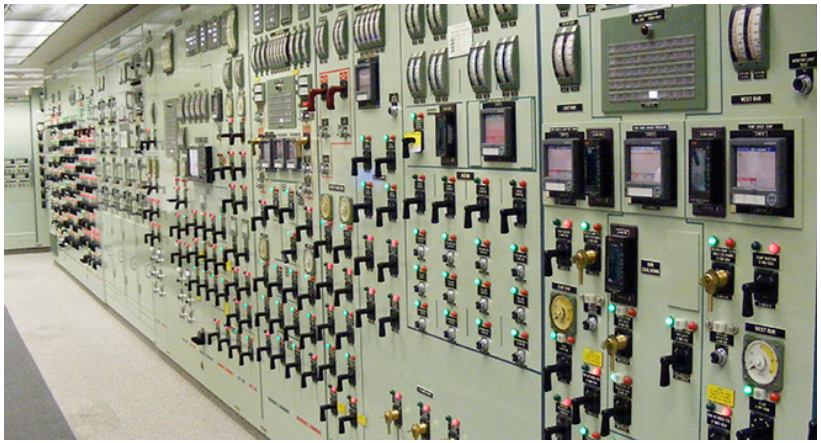
To hammer a nail into a piece of wood what kind of tool should we use?



a simple one?



a complex one?



Comment

On the complementary nature of observational data, scientific understanding and model complexity: the need for a range of models

A.M. Davies

Proudman Oceanographic Laboratory, Bidston Observatory, Birkenhead, Merseyside L43 7RA, UK

Alan Davies' 1994 response to this question (and Nihoul's statement) is a **complex** one, and, yet, it is probably the **good** one, pointing to issues that are still being dealt with nowadays.

Model complexity reduction

Why? Reduce CPU time and obtain an answer quicker
Interpret the results of complex models

What? Reduce the number of state variables
Reduce the number of dimensions (3D -> 2D, 1D or 0D)
Identify the most significant modes

How? Handiwork guided by physical intuition
Much more sophisticated and “rational” numerical methods
Smarter software architecture

And now, it is all yours!

Informal mini-workshop on ***Reduced Complexity Modelling***
Deltares, 22nd May, 2015

Simple models to interpret the results of complex ones

Eric Deleersnijder[1], Eric J.M. Delhez[2] and Anne Mouchet[3]

[1] *Université catholique de Louvain (Louvain-la-Neuve) & Delft University of Technology*

[2] *Université de Liège*

[3] *Laboratoire des Sciences du Climat et de l'Environnement (LSCE, Saclay, France)*

Dimension reduction

- Consider a system described by N ordinary differential equations:

$$\frac{d}{dt}\psi_n(t) = f_n(t,\vec{\psi}) \ , \quad n = 1,2...N, \quad \text{with} \quad \vec{\psi} = (\psi_1, \psi_2 ... \psi_N)$$

Let $g_i(t,\vec{\psi}),\ i = 1,2,...I$, with $I << N$, be a series of indicators of the state of the system.

- Consider another model of the same system with less equations:

$$\frac{d}{dt}\tilde{\psi}_m(t) = \tilde{f}_m(t,\vec{\tilde{\psi}}) \ , \quad m = 1,2...M, \quad \text{with} \quad \vec{\tilde{\psi}} = (\tilde{\psi}_1, \tilde{\psi}_2 ... \tilde{\psi}_M)$$

with $M < N$. Then, estimate the same indicators $\tilde{g}_i(t,\vec{\tilde{\psi}}),\ i = 1,2,...I$.

Dimension reduction is efficient if
$M << N$ **and** $\tilde{g}_i(t,\vec{\tilde{\psi}}) \approx g_i(t,\vec{\psi})$

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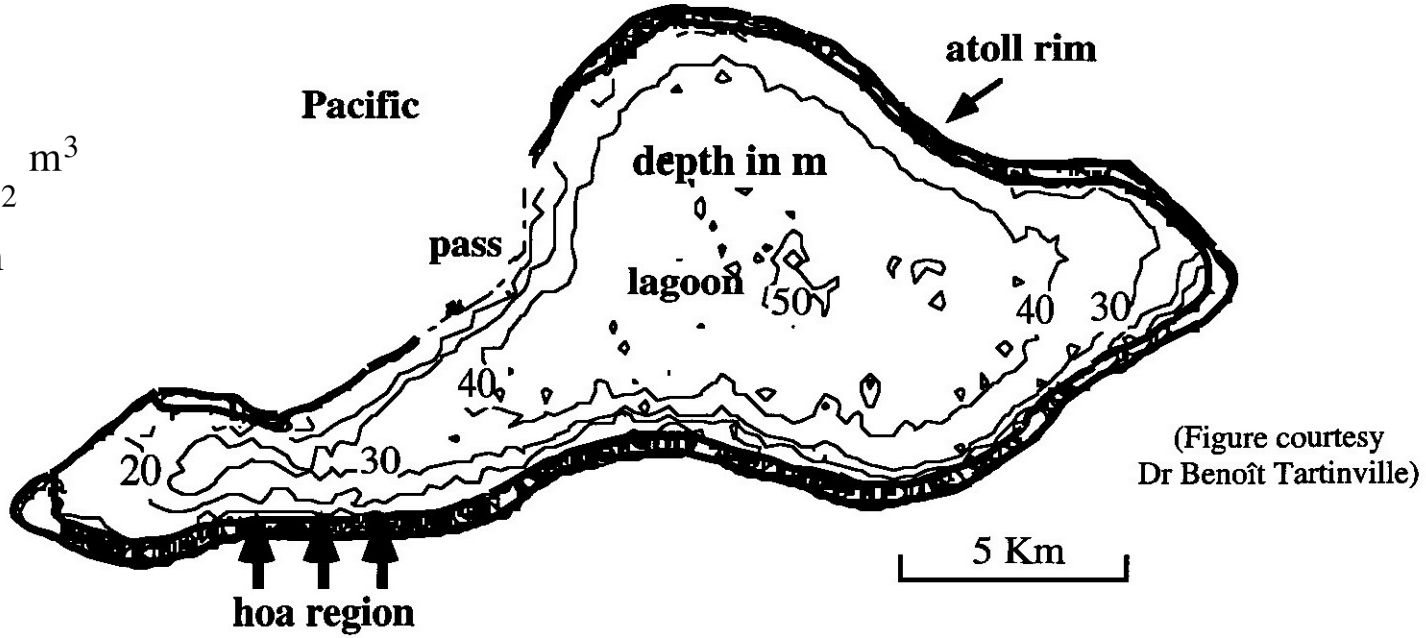
• Herein, no theoretical developments on dimension reduction; only handiwork, aiming at developing very simple models to help interpret the result of complex ones.

• Real-world applications dealt with:
  - Tracer transport from the Mururoa Atoll lagoon to the Pacific;
  - Tracer transport in the Prince William Sound, Alaska;
  - Ventilation of the World Ocean.

• The added value of the simple models (boxes, pipes), is two-fold:
  - their functioning is readily understood;
  - their parameters have a clear physical meaning, directly leading to interpretations in quantitative terms.

Mururoa Atoll lagoon (I)

Mouth/Pass:
length: ≈ 5 km
depth: ≈ 8 m
Lagoon:
volume: 4.7×10^9 m^3
area: 1.4×10^8 m^2
mean depth: 33 m



Mururoa Atoll is part of the Tuamotu Archipelago (French Polynesia) and is located in the Pacific at (longitude, latitude) = (138°55' E, 21°50° S).

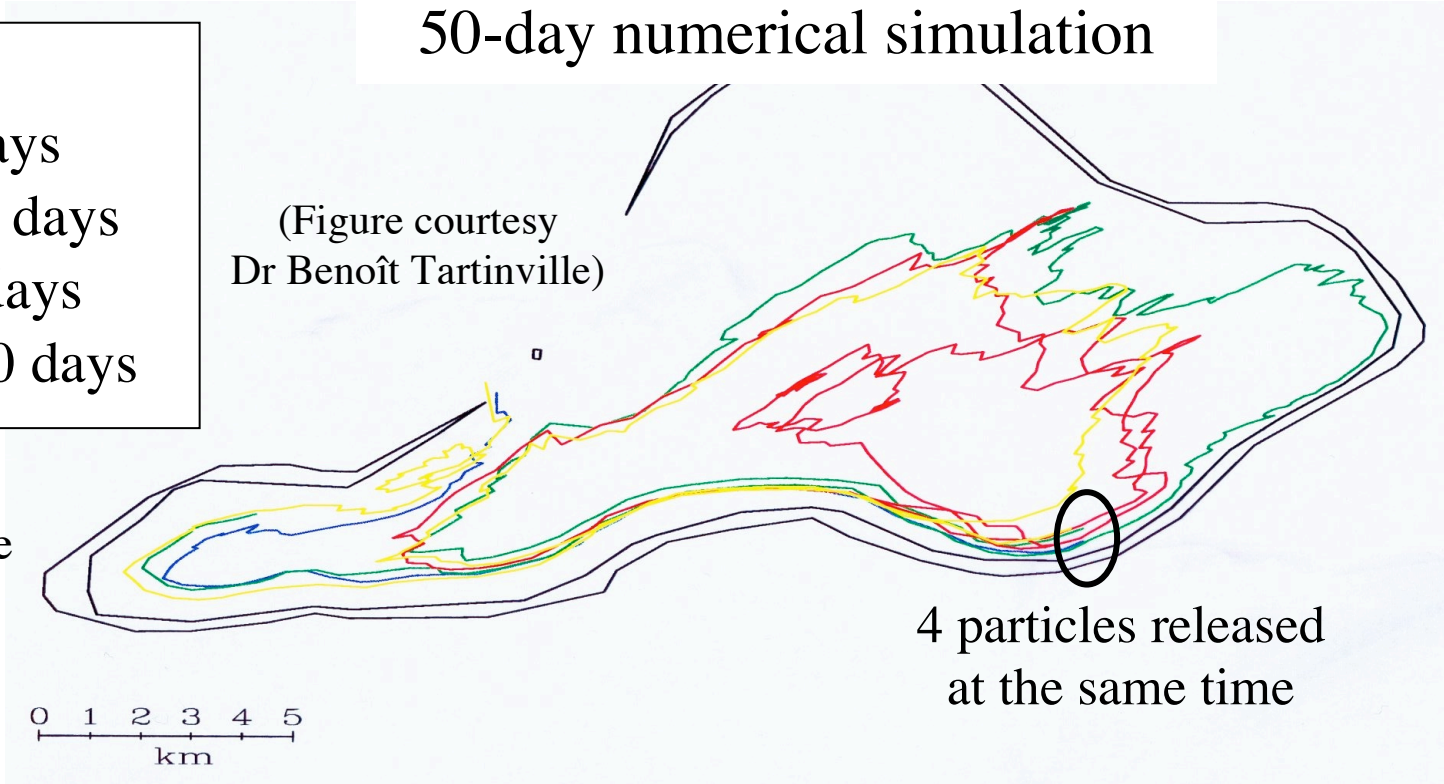
• How to model the flux of (radioactive) tracers (emerging from the lagoon's bottom) to the Pacific through the atoll mouth?

Mururoa Atoll lagoon (II)

• Tartinville et al. (1997) developed a 3D model for the lagoon hydrodynamics and tracer transport, with N=33,510 wet grid points.

Residence time*:
blue particle: 13 days
yellow particle: 38 days
red particle: > 50 days
green particle: > 50 days

*: time needed to leave the lagoon by crossing the pass



• This suggests that an intense mixing is taking place in the lagoon.

Mururoa Atoll lagoon (III)

- The tracer budget of the lagoon can be described as follows:

$$\frac{d}{dt}m(t) = \underbrace{\phi_{in}(t) - \phi_{out}(t)}_{\text{transport}} \underbrace{- \gamma m}_{\text{radioactive decay}} , \quad m(t) = \text{tracer mass}$$

with $\phi_{in(t)}$: incoming flux (through lagoon bottom)

$\phi_{out(t)}$: outgoing flux (through the atoll mouth)

$\log 2 / \gamma$: half-life of the tracer

- Key hypothesis: mixing throughout the lagoon is infinitely efficient.

$$\Rightarrow \quad \phi_{out} = \frac{m}{\theta}$$

where θ is the mean residence time, a timescale that is to be estimated by means of the 3D model.

Mururoa Atoll lagoon (IV)

• Then, the evolution of the tracer mass may be modelled by a "box model" consisting of a single ordinary differential equation (M=1):

$$\frac{dm}{dt} + \frac{m}{\theta^*} = \phi_{in} \ , \quad \text{with} \ \ \theta^* = \frac{\theta}{1+\gamma\theta}$$

General solution: $m(t) = m(0)\,e^{-t/\theta^*} + \int_0^t \phi_{in}(t')\,e^{-(t-t')/\theta^*}\,dt'$.

• How well does this mass estimate compare with that computed by means of the 3D model $m^{3D}(t)$?

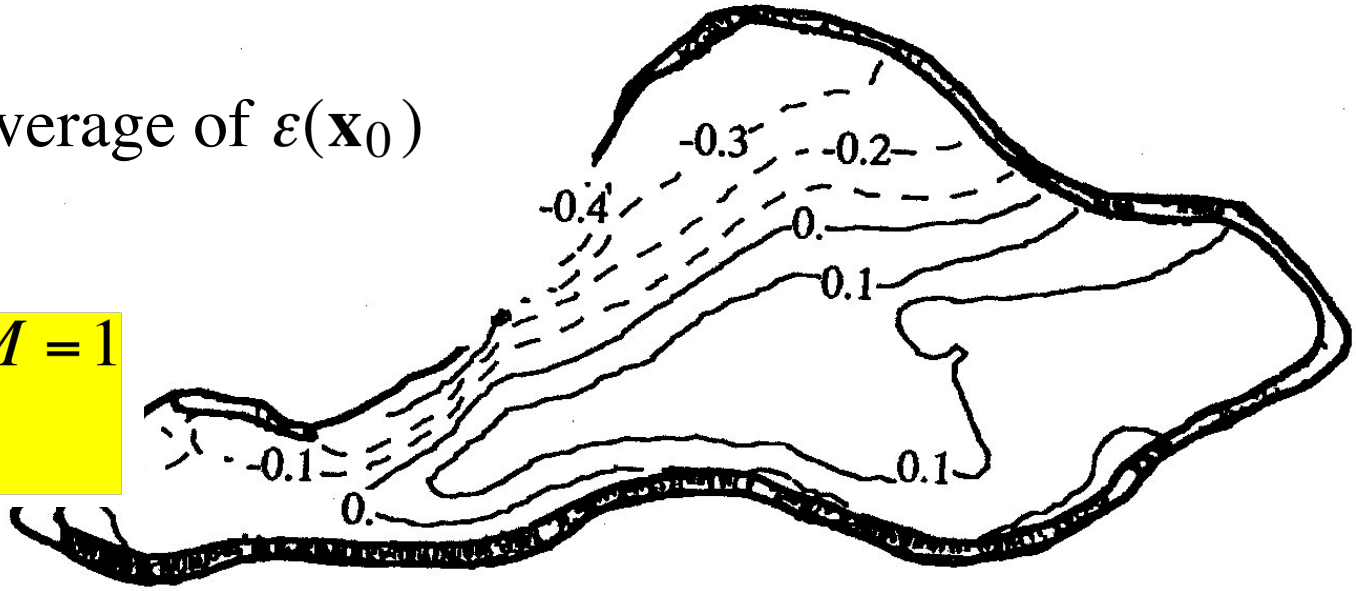
• Instantaneous, pointwise release of a unit mass of a passive tracer:

$$m(t) = e^{-t/\theta} \quad \text{and} \ \ m^{3D}(t) = \int_\Omega C(t,\mathbf{x})\,d\mathbf{x} \ \ \text{with} \ C(0,\mathbf{x}) = \delta(\mathbf{x}-\mathbf{x}_0)$$

Mururoa Atoll lagoon (V)

- Error measure: $\varepsilon(\mathbf{x}_0) = \frac{1}{\theta}\int_0^\infty [m^{3D}(t) - m(t)]\, e^{-t/\theta}\, dt$

depth-average of $\varepsilon(\mathbf{x}_0)$



$N = 33{,}510$ and $M = 1$

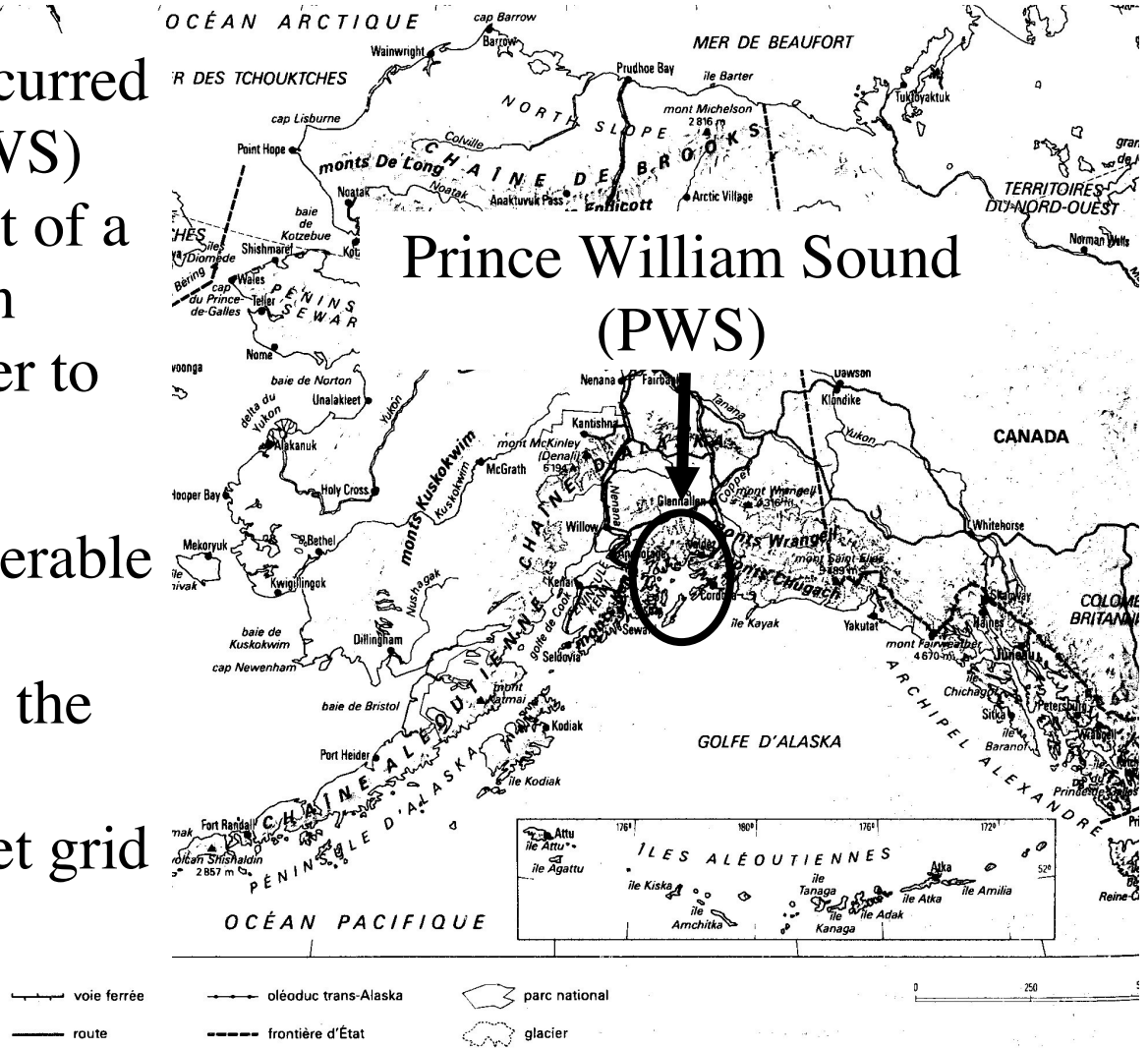
$|\varepsilon| \approx 12\%$

As $|\varepsilon(\mathbf{x}_0)| \approx 0.12$, the one-equation box model (M=1) provides a good estimate of the tracer mass — the indicator of the state of the system — simulated by means of a complex 3D model (N=33,510).

Prince William Sound, Alsaka (I)

America's largest oil spill occurred in Prince William Sound (PWS) on 24 March 1989, as a result of a navigation error of the Exxon Valdez, causing this oil tanker to run onto Bligh Reef.

Then, during the 90's, considerable efforts were devoted to the modelling of the PWS, using the 3D Princeton Ocean Model (POM), with $N = 83,762$ wet grid points.

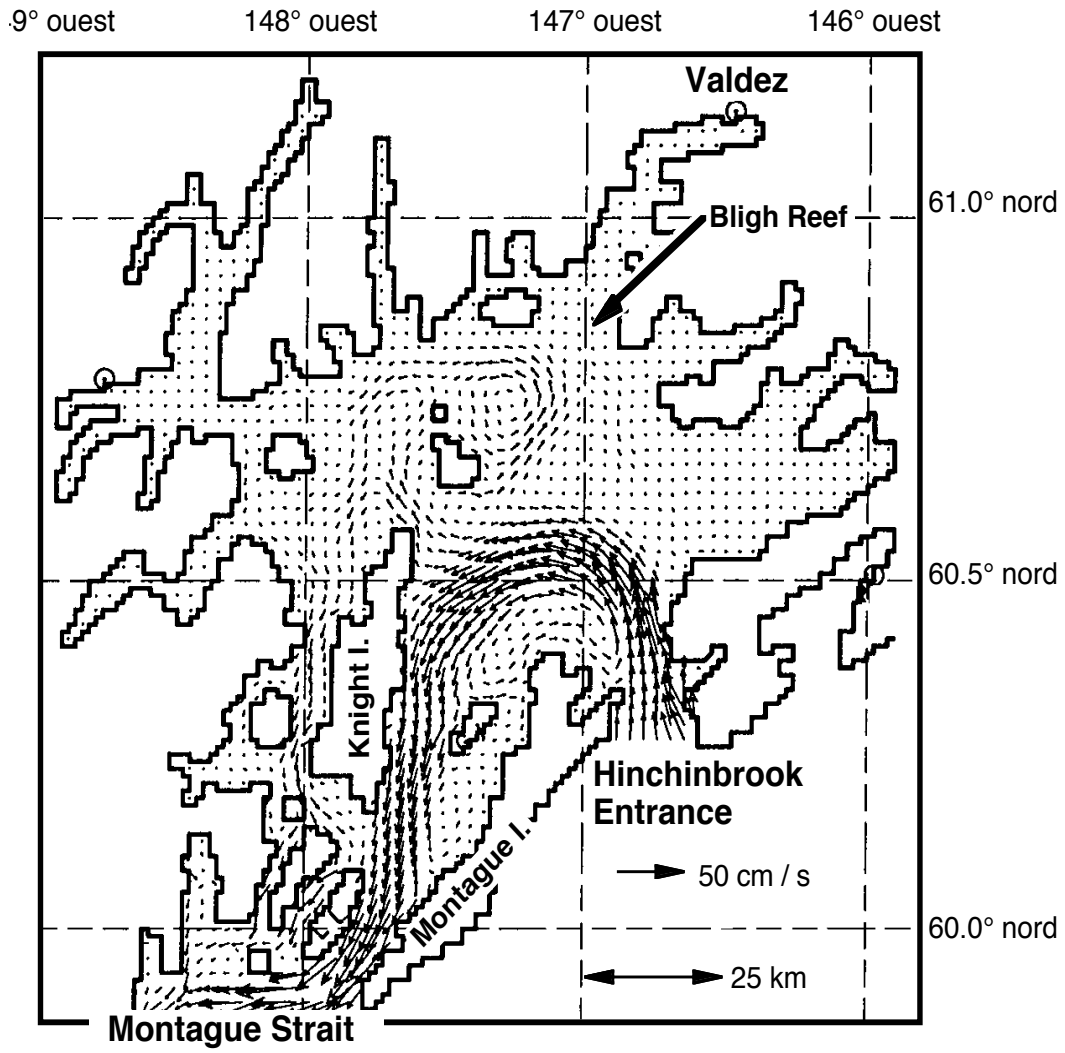


Prince William Sound, Alsaka (II)

Surface current in the PWS as simulated by POM in the zero-wind case (Mooers and Wang, 1998)

The hydrodynamics of the southern part of the domain is clearly different from that prevailing in the northern part.

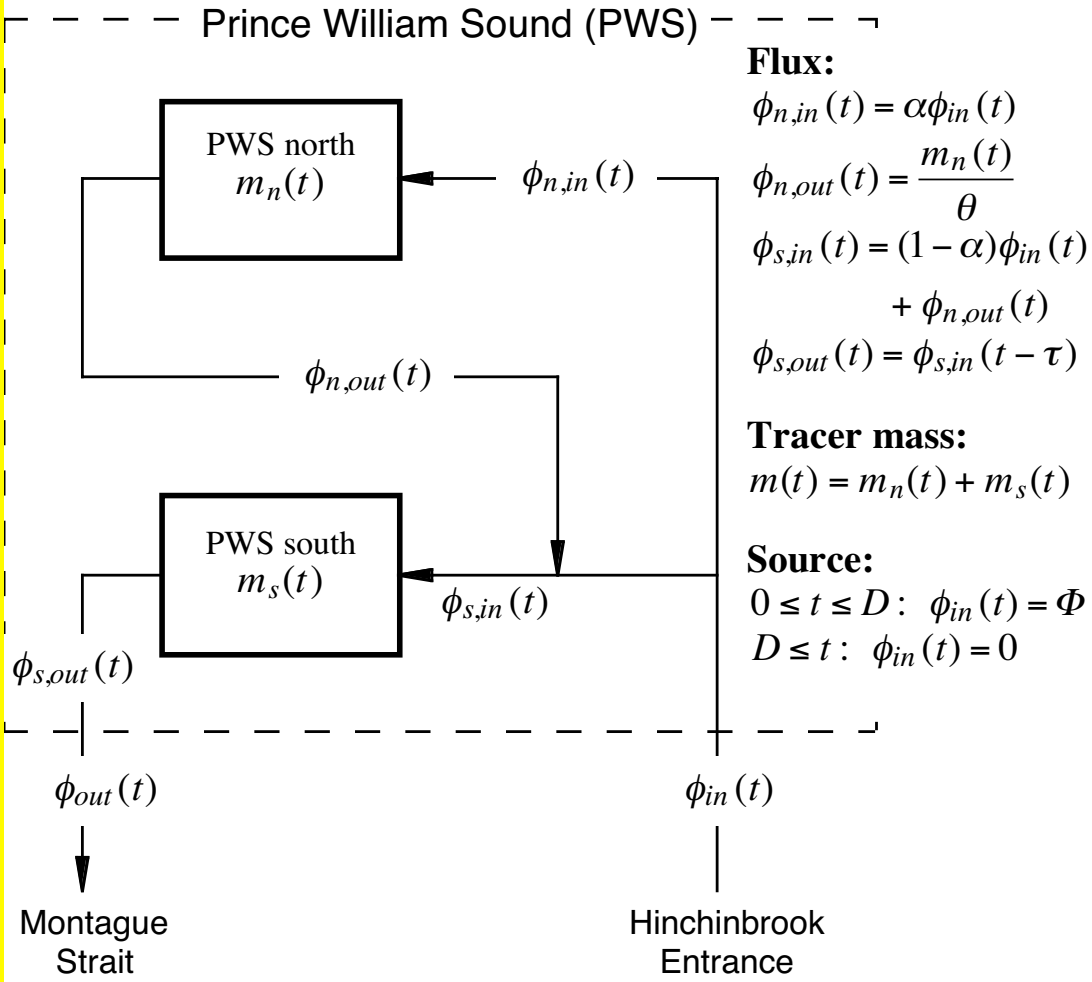
This suggest a way for achieving dimension reduction: dividing PWS into 2 boxes, i.e. northern PWS and southern PWS.



Prince William Sound, Alsaka (III)

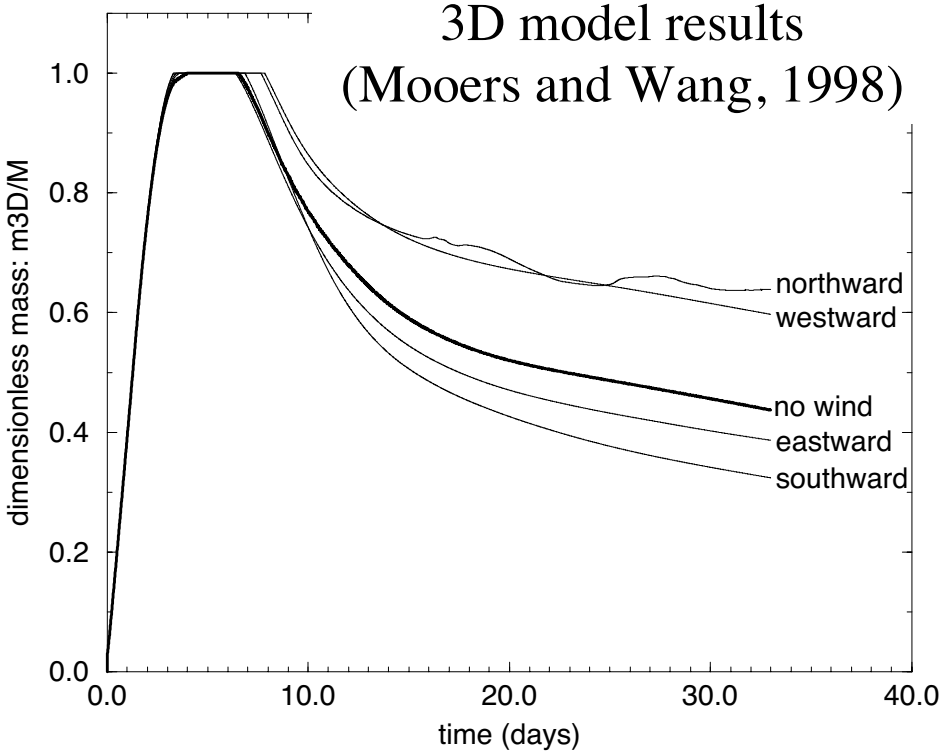
Key hypotheses:

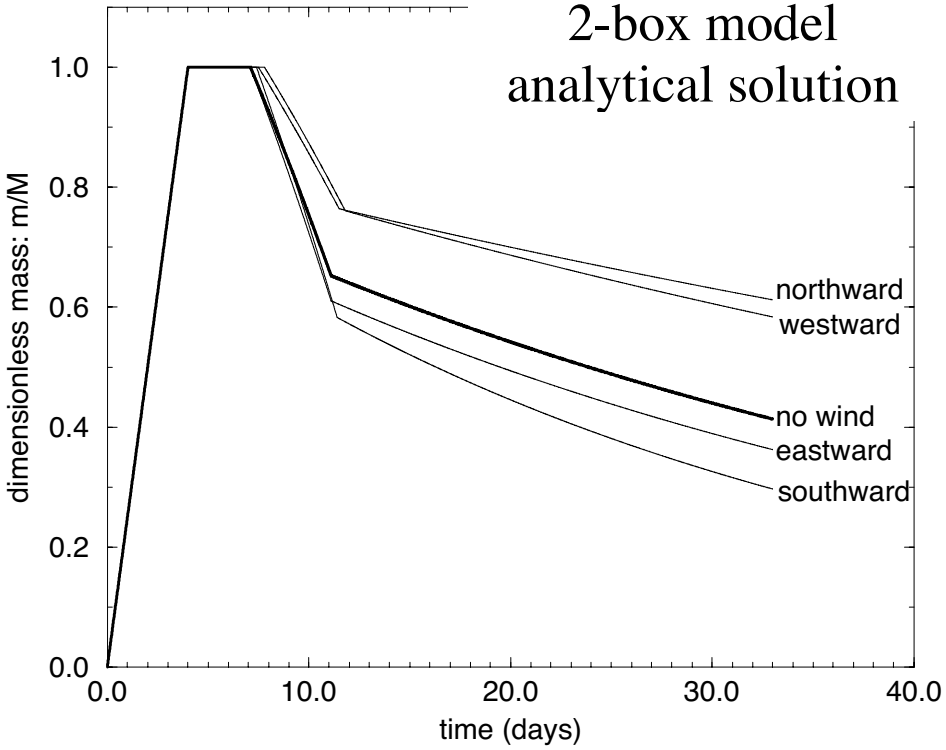
- Northern box is well-mixed, i.e. similar to Mururoa lagoon, with residence time θ.
- Southern box behaves as a pipe: $\phi_{s,out}(t) = \phi_{s,in}(t-\tau)$, with τ = transit time.
- α = fraction of the incoming flux at Hinchinbrook Entrance that enters directly the northern box.



Passive tracer source in Hinchinbrook Entrance active for $0 \le t \le D = 4$ days.

Prince William Sound, Alsaka (IV)





Parameters α, θ and τ have a clear physical meaning

$N = 83,762$ and $M = 2$

$\varepsilon \approx 7\%$

$$0 \le t \le D: \quad m(t) = \Phi t$$

$$D \le t \le \tau: \quad m(t) = \Phi D = M$$

$$\tau \le t \le D + \tau: \quad m(t) = \Phi(D + \tau - t) + \alpha\Phi\theta[1 - e^{-(t-\tau)/\theta}]$$

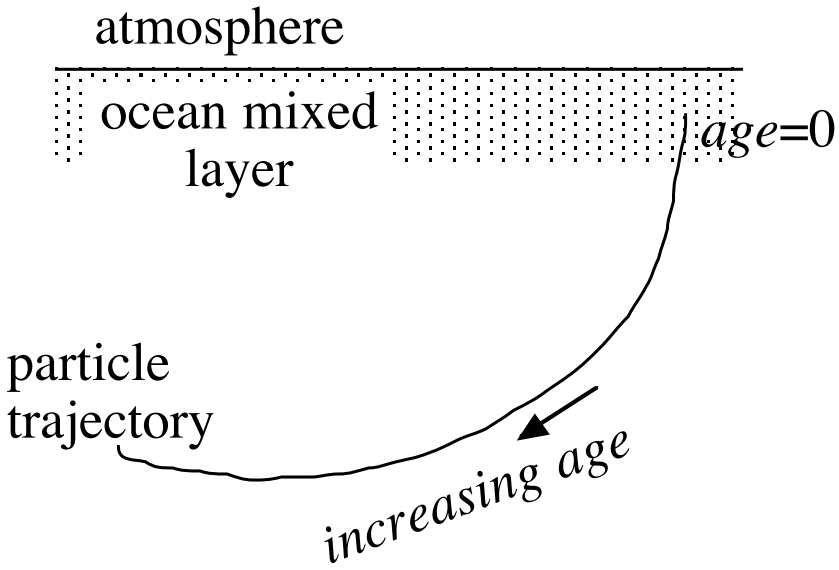
$$D + \tau \le t: \quad m(t) = \alpha\Phi\theta(e^{D/\theta} - 1)e^{-(t-\tau)/\theta}$$

Ventilation of the World Ocean (I)

According to England (JPO, 1995), the "World Ocean circulation at its largest scale can be thought of as a gradual renewal or ventilation of the deep ocean by water that was once at the sea surface."

Therefore, the age, a measure of the time since leaving the ocean upper mixed layer, is a popular diagnostic tool in the World Ocean.

estimating ocean ventilation rate



age = time elapsed since leaving surface mixed layer

Ventilation of the World Ocean (II)

• At a steady state, the *water age distribution* $c(\mathbf{x},\tau)$ is satisfies

$$\frac{\partial c}{\partial \tau} = -\nabla \bullet (\mathbf{u}c - \mathbf{K} \bullet \nabla c) \ , \ \left[c(\mathbf{x},\tau)\right]_{\Gamma} = \delta(\tau - 0) \ , \ \left[c(\mathbf{x},0)\right]_{\Omega} = 0$$

with τ = the age, Γ = the ocean surface and Ω = the ocean interior.

• *Global water age distribution* $\mu(\tau)$: the volume of the water whose age lies in the interval $[\tau, \tau + \Delta\tau]$ $(\Delta\tau \to 0)$ is $\Omega\mu(\tau)\Delta\tau$, with

$$\mu(\tau) = \frac{1}{\Omega}\int_{\Omega} c(\mathbf{x},\tau)\, d\mathbf{x}$$

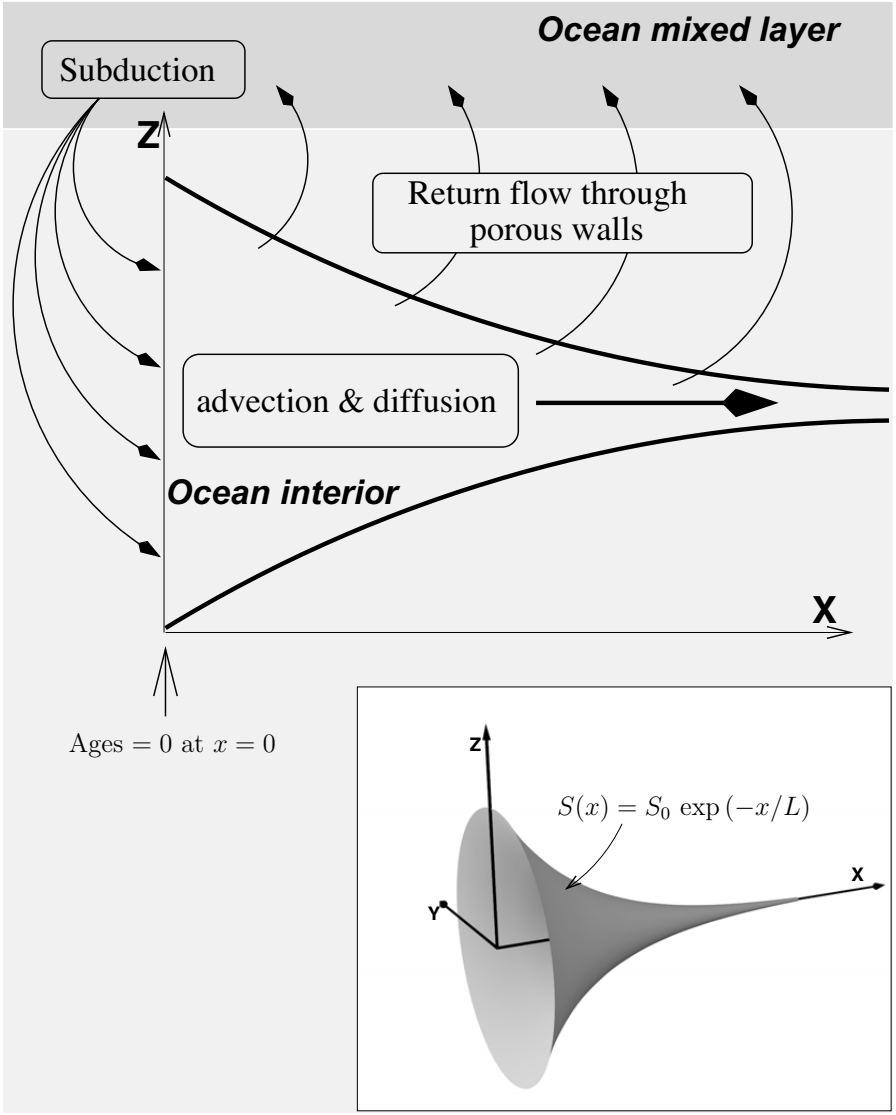
• *Global mean water age*: $\bar{a} = \int_0^{\infty} \tau\, \mu(\tau)\, d\tau = \frac{1}{\Omega}\int_0^{\infty}\int_{\Omega} \tau\, c(\mathbf{x},\tau)\, d\mathbf{x}\, d\tau$.

• The global water age distribution was derived from a coarse grid OGCM with N=50,863 wet grid points.

Ventilation of the World Ocean (III)

The leaky-funnel model,
a World Ocean idealization,
is based on the following
key assumption:

The horizontal circulation in the actual ocean may be thought to be a consequence of ***localized sinking*** *and* ***generalized upwelling***.
(Warren, 1981)



Ventilation of the World Ocean (IV)

- Parameters of the leaky funnel model:

U = water velocity, K = diffusivity

L = e-folding length scale for the section: $S(x) = S_0\, e^{-x/L}$

L is also the mean length of the water parcel trajectories in the funnel

- The leaky funnel water age distribution is (Mouchet et al. 2012)

$$\mu(\tau) = \sqrt{\frac{K}{\pi L^2 \tau}} \exp\left(-\frac{U'^2 \tau}{4K}\right) + \frac{1}{\theta}\left[1 + \mathrm{erf}\left(\frac{1}{\theta}\sqrt{\frac{L^2\tau}{K}}\right)\right] \exp\left(-\frac{U\tau}{L}\right)$$

with $Pe = UL/K$, $\frac{1}{\theta} = \frac{U'}{2L}\left(1 - \frac{2}{Pe'}\right)$, $Pe' = U'L/K$ and $U' = U + K/L$.

- The leaky funnel mean age is $\bar{a} = \frac{L}{U + K/L}$.

Ventilation of the World Ocean (V)

The parameters of the leaky funnel are optimised so as to minimise the following error measure:

$$\varepsilon = \frac{\int_0^\infty \left|\mu^{3D}(\tau) - \mu(\tau)\right| e^{-U\tau/L}\, d\tau}{\int_0^\infty \mu^{3D}(\tau)\, e^{-U\tau/L}\, d\tau}$$

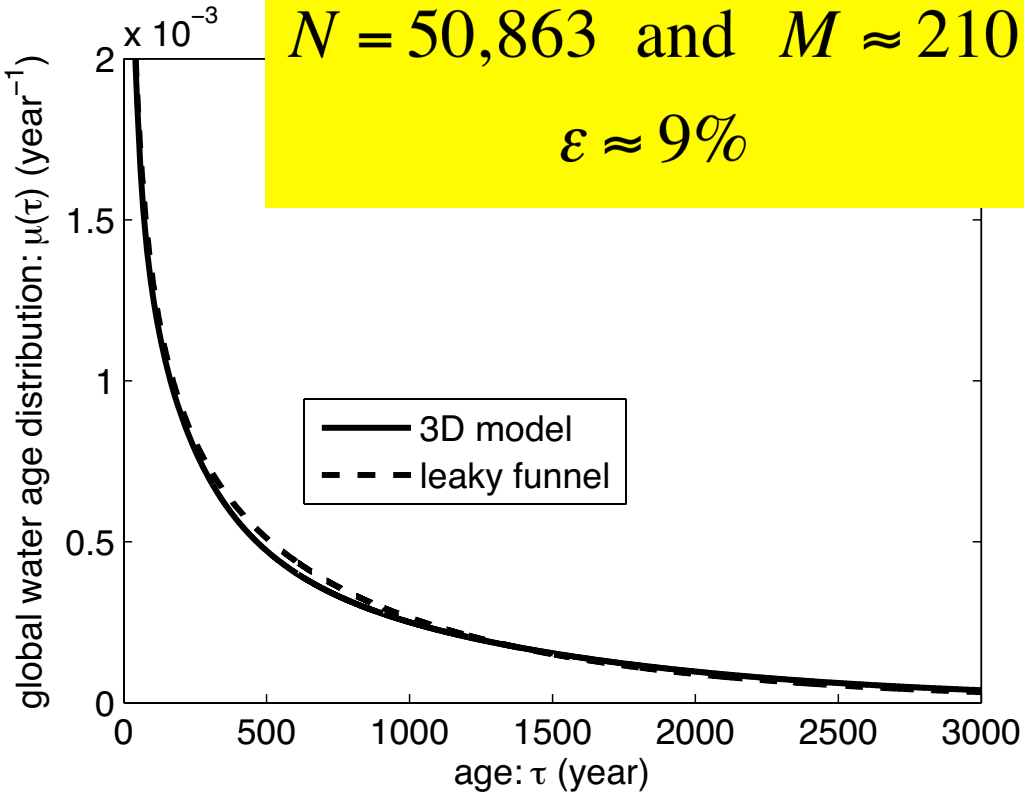
$$U \approx 3\times10^{-3}\,\mathrm{m\,s^{-1}} \;,\quad L \approx 10^{8}\,\mathrm{m}$$

$$K \approx 1.4\times10^{5}\,\mathrm{m^2\,s^{-1}}$$

$$\bar{a}^{3D} = 764\,\mathrm{y} \;,\quad \bar{a} = 721\,\mathrm{y}$$

$N = 50{,}863$ and $M \approx 210$

$\varepsilon \approx 9\%$



$$M \approx \frac{10\,L}{\Delta x} \;,\quad \text{with} \quad \Delta x \approx \frac{\min(L, K/U)}{10}$$

Conclusion and outlook

• Dimension reduction efficiency:

	N	M	N/M	ε
Mururoa	33,510	1	33,510	12%
PWS	83,762	2	41,881	7%
Ventilation	50,863	210	242	9%

• Dimension reduction is of use for the interpretation of the results of complex models, mainly because the parameters of the simple model have a clear physical meaning (residence time, transit time, etc).

• We are developing a more systematic or automatic way of designing a reduced-complexity, compartmental model. To do so, a generalisation of the concept of age is resorted to.

To get additional pieces of information about diagnostic timescales:

http://www.climate.be/cart