

Modeling of Wake Vortex Radar Detection in Clear Air Using Large-Eddy Simulation

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ABSTRACT

To detect wake vortices in all weather conditions, lidar and radar sensors are complementary. It is important to determine accurately the limitations of the radar detection in clear air and to determine the parameters influencing the detection. This paper is the first study to present the simulation and analysis of the radar signatures of wake vortices on the basis of state-of-the-art 3D large-eddy simulations. The setup of the large-eddy simulations is described and the evolution of the wake vortices is illustrated for the three simulated cases. A modified version of the model developed by Muschinski et al. is used here for the calculation of the radar-backscattered signal and the second moment of the Doppler spectrum that is used to estimate the wake vortex circulation. The simulator uses two different radar configurations for the retrieval of the circulation, which is compared with the circulation obtained directly from the large-eddy simulations. The retrieval of the circulation is fairly accurate if the level of backscattered signal is high enough above the detection threshold of the radar.

1. Introduction

After roll-up of the wake of an aircraft, a pair of counterrotating vortices is created. This phenomenon, called wake vortex (WV) turbulence, can constitute a hazard for any other aircraft behind the first, especially during takeoff and landing phases (Krause 2003). Indeed, the aircraft encountering a wake vortex system that has not dissipated enough may be subjected to a strong rolling moment (when encountering one of the vortices) or to a strong downwash velocity (when flying between the two vortices), causing a drop in lift.

To prevent such risks of encountering a wake, safety separations between aircraft were imposed by the International Civil Aviation Organization (ICAO) in the early 1970s. With the current exponentially growing air traffic, those safety separations have become too constraining for large and busy airports, limiting their capacity and flexibility, and causing many delays in departures and arrivals.

To optimize the airport/runway throughput while maintaining an acceptable level of safety, the separation minima were redefined in 2013 by splitting the ICAO heavy and medium categories into “upper” and “lower” aircraft parts, resulting in six categories instead of the four previous ones (Rooseleer and Treve 2013). This new categorization [European Wake Vortex Recategorisation (RECAT-EU)] has been deployed in France at Paris Charles de Gaulle airport since the spring of 2016 (Direction Générale de l’Aviation Civile 2016). A similar system (RECAT) has been implemented by the Federal Aviation Administration (FAA) in the United States (Cheng et al. 2016).

To support the development of new systems for safe decrease of aircraft spacing (such as weather-dependent separation systems), measurement tools that can provide information about the wake vortex situation around the airport in real time will also be required. The main sensors used for this issue are lidar and radar. Lidars operate in clear air but they require a sufficient concentration of aerosols in the air and they are blind in rain or fog conditions. Radars are able to see scattered electromagnetic waves on raindrops or fog particles but

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the power of the scattered electromagnetic waves is very weak in clear air around 10 GHz. To cover all weather conditions, both tools will likely be used together (Oude Nijhuis et al. 2018). For reliable detection, it is necessary to know what the limits of detectability for the sensors are under different weather conditions. The model presented in this paper proposes a new tool for the optimization of the radar parameters to detect WV in clear-air conditions. The analysis is based on using 3D fields from large-eddy simulations (LES) for generating the time series and the Doppler spectra of the scattered signal.

Detection of a wake vortex under clear-air conditions was demonstrated for VHF, UHF, L, S, and C bands (Noonkester and Richter 1980; Chadwick et al. 1984; Nespor et al. 1992; Gilson 1992; Shephard et al. 1994) and for X band (Shephard et al. 1994; Barbaresco et al. 2007, 2008), with an unsuccessful attempt reported in Marshall et al. (1997). The weak level of the backscattered power is the origin of the difficulties in detecting the WV with the radar at large distances. Radars provide remote sensing of the atmospheric phenomena at very large ranges in C and S bands. However, the long range is not the main requirement for the system aimed at the detection of the WV. Among the main advantages provided by the X band are high resolution relative to the C and S bands and a smaller sensor size. Therefore, the use of a wide bandwidth in the X band provides high-resolution measurements. Additionally, because of the smaller wavelength, it is possible to obtain a narrow beamwidth using a relatively small antenna.

Models describing the scattering mechanisms of the electromagnetic waves on the wake vortex are reported in the literature. They can be classified into two groups: the first one assumes the scattering on the spiral density pattern of the WV evolving with time (Myers et al. 1999; Shariff and Wray 2002; Li et al. 2010; Vanhoenacker-Janvier et al. 2012) and the second one assumes the scattering on the turbulence induced by the WV (Marshall and Myers 1996). The main limitations are that the models available are 2D models and most of them do not take into account the turbulent nature of the interaction between the WV and the surrounding turbulent atmosphere.

Vanhoenacker-Janvier et al. (2012) have developed an electromagnetic model for a radar cross section (RCS) due to wake vortices, as a function of meteorological and airplane parameters, using the Li et al. (2009, 2011) oscillatory integral method. This first model used a simplified 2D model to simulate the wake vortices. In this study, the electromagnetic simulator uses the stratified and turbulent wake vortices output

from a state-of-the-art LES, thus providing more realistic results. An improvement of the computing time for calculation of the backscattered signal time series is obtained, by using the Muschinski et al. (1999) model. The four main assumptions of the Muschinski et al. (1999) model are that 1) the size of the radar resolution volume is much larger than the LES grid resolution, 2) the size of the LES cells is much larger than the Bragg wavelength, 3) the turbulence is locally isotropic and homogeneous at the scale of the LES cell, and 4) the LES subfilter turbulence belongs to the inertial subrange of the refractive index spectrum on the Bragg scale. The first three assumptions are valid for the parameters of the modeled X-band radar and the LES resolution used in this study. However, with regard to the fourth assumption, it is expected that the Bragg scale lies in the dissipative range or in the transition zone from the inertial subrange to the viscous dissipative range for the X-band radar, which necessitates a more accurate modeling of the refractive index spectrum. To take this transition into account, the Hill (1978a) spectral model is applied in this study.

Section 2 of this paper describes the large-eddy simulation of the WV, the input parameters chosen, and illustrates the time evolution of various WV parameters and atmospheric fields. Section 3 describes the model developed for the calculation of the radar backscattering in clear air and further describes the hypotheses used for the application at X band. Section 4 analyzes the impact of the relative size of the smallest turbulent eddies with respect to the Bragg wavelength. Section 5 develops the method used for the calculation of the Doppler spectrum of the signal received by the radar, and section 6 demonstrates the estimated WV circulation using two methods: from the Doppler spectrum and from the estimated WV descent velocity and the distance between WV cores.

2. Large-eddy simulation of the wake vortices

The aircraft wake vortex system obtained after roll-up, as simulated here, consists of a pair of counter-rotating vortices with circulation Γ_0 and $-\Gamma_0$ and separated by a distance b_0 . The aim is to simulate those vortices in a stratified and turbulent atmosphere, and with the presence of a humidity gradient. This setup is similar to that of De Visscher et al. (2013), with the addition of the humidity field. The simulations were carried out using a pseudospectral solver. A homogeneous isotropic turbulence is first generated and converged to create a realistic background stratified turbulence. Next, the vortex pair generated by a heavy aircraft is superimposed over the turbulence.

a. Governing equations

Because the simulations are performed in a large periodic domain, the different variables must be decomposed into an imposed mean vertical profile and the deviations from this profile. The potential temperature $\Theta/T = (p_{\text{ref}}/p)^{(\gamma-1)/\gamma}$ (i.e., the temperature that a parcel of air would have after being brought isentropically from its pressure p to a reference pressure p_{ref}) and the specific humidity [i.e., the amount of water vapor per kilogram of humid air (g_v/kg_h)] are decomposed into mean linear vertical profiles $[\bar{\Theta}(z)]$ and $[\bar{\phi}(z)]$ and deviation fields $[\tilde{\Theta}(\mathbf{x}, t)]$ and $[\tilde{\phi}(\mathbf{x}, t)]$ around those means:

$$\Theta(\mathbf{x}, t) = \underbrace{\bar{\Theta}_g + \frac{d\bar{\Theta}}{dz}z}_{\bar{\Theta}(z)} + \tilde{\Theta}(\mathbf{x}, t) \quad \text{and} \quad (1)$$

$$\phi(\mathbf{x}, t) = \underbrace{\bar{\phi}_g + \frac{d\bar{\phi}}{dz}z}_{\bar{\phi}(z)} + \tilde{\phi}(\mathbf{x}, t), \quad (2)$$

where $\bar{\Theta}_g$ and $\bar{\phi}_g$ are the mean potential temperature (and thus also the mean static temperature because $p_{\text{ref}} = \bar{p}_g$) and mean humidity at the ground. The stability of the atmosphere is characterized by the Brunt–Väisälä frequency:

$$N = \left(\frac{g}{\bar{\Theta}_0} \frac{d\bar{\Theta}}{dz} \right)^{1/2}. \quad (3)$$

This frequency represents the number of oscillations per unit time that a parcel of air would perform around its equilibrium position if it was moved away from this equilibrium by a small perturbation.

The governing equations for the deviations are the incompressible Navier–Stokes equations supplemented by a subgrid-scale (SGS) model and the Boussinesq approximation for the buoyancy effects. A scalar convection–diffusion equation for the specific humidity with a SGS term is also solved. Assuming that $\mathbf{e}_z$ is the vertical direction, these equations are as follows:

$$\nabla \cdot \mathbf{u} = 0, \quad (4)$$

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla P - \frac{1}{\bar{\Theta}_0} \mathbf{g} \tilde{\Theta} + \nu \nabla^2 \mathbf{u} + \nabla \cdot \boldsymbol{\tau}^{\text{SGS}}, \quad (5)$$

$$\frac{\partial \tilde{\Theta}}{\partial t} + \nabla \cdot (\tilde{\Theta} \mathbf{u}) = -w \frac{d\bar{\Theta}}{dz} + \alpha \nabla^2 \tilde{\Theta} - \nabla \cdot \mathbf{q}^{\text{SGS}}, \quad \text{and} \quad (6)$$

$$\frac{\partial \tilde{\phi}}{\partial t} + \nabla \cdot (\tilde{\phi} \mathbf{u}) = -w \frac{d\bar{\phi}}{dz} + D^\phi \nabla^2 \tilde{\phi} - \nabla \cdot \mathbf{q}^{\phi, \text{SGS}}, \quad (7)$$

where $\mathbf{u} = (u, v, w)$ is the LES velocity field, P is the reduced pressure, ν and α are the kinematic viscosity and the thermal diffusivity, $\mathbf{g} = -g\mathbf{e}_z$ is the gravitational acceleration and $\boldsymbol{\tau}^{\text{SGS}}$ and $\mathbf{q}^{\text{SGS}}$ or $\mathbf{q}^{\phi, \text{SGS}}$ are the SGS stress tensor and SGS fluxes that model the influence of the unresolved scales on the resolved scales (LES fields). Here, $\bar{\Theta}_0$ represents the mean temperature at the initial position z_0 of the vortices. The D^ϕ coefficient is a mass diffusion coefficient, and $\mathbf{q}^{\phi, \text{SGS}}$ is a flux of water vapor.

The computational box size is set to $4b_0$, which corresponds to a few hundred meters for a heavy aircraft (i.e., b_0 on the order of 50 m). Regardless of the vortex wake simulated, the computational domain is too small to consider the Coriolis term (Rossby number $\ll 1$) in the momentum equation; therefore, it has been neglected.

From the deviation fields of potential temperature and humidity and the imposed mean profiles, we can reconstruct the global time-dependent profiles for the atmospheric fields, using Eqs. (1) and (2). The pressure field is also computed as the sum of a mean profile and the deviation around that mean:

$$p(\mathbf{x}, t) = \bar{p}(z) + \tilde{p}(\mathbf{x}, t) = \bar{p}(z) + P(\mathbf{x}, t)\bar{p}_0. \quad (8)$$

The deviation is easily obtained as the reduced pressure multiplied by the mean density at the initial position of the vortices: $\bar{p}_0 = \bar{p}(z = z_0)$. The variation of density is thus also neglected for the deviation part. The hydrostatic equilibrium yields

$$\frac{d\bar{p}}{dz} = -\bar{p}g = -\frac{\bar{p}g}{R\bar{\Theta}} \left(\frac{p_{\text{ref}}}{\bar{p}} \right)^{(\gamma-1)/\gamma}, \quad (9)$$

which, once integrated, provides the mean pressure profile:

$$\frac{\bar{p}}{p_{\text{ref}}} = \left\{ 1 - \frac{g}{c_p} \left(\frac{d\bar{\Theta}}{dz} \right)^{-1} \log \left[1 + \frac{1}{\bar{\Theta}_g} \left(\frac{d\bar{\Theta}}{dz} \right) z \right] \right\}^{\gamma/(\gamma-1)}, \quad (10)$$

where $c_p = [\gamma/(\gamma - 1)]R$ is the specific heat of air at constant pressure.

Once the global profiles for the potential temperature and pressure are computed, the static temperature can be obtained from the definition of the potential temperature:

$$T = \Theta(p/p_{\text{ref}})^{(\gamma-1)/\gamma}. \quad (11)$$

We finally compute the global density profile using the ideal gas law:

$$\rho = p/(RT). \quad (12)$$

b. Numerical method

To solve those equations, the same pseudospectral Navier–Stokes solver as in [De Visscher et al. \(2013\)](#) is used here with phase shift partial dealiasing ([Canuto et al. 2006](#); [Jeanmart 2002](#)). The governing equations are thus solved in a periodic domain. The main advantage of spectral solvers is that they are free of dispersion and diffusion errors. The time integration is carried out using [Williamson's \(1980\)](#) third-order, low-storage Runge–Kutta scheme. The SGS model used for the simulations is the regularized variational multiscale model ([Jeanmart and Winckelmans 2007](#)): it dissipates only at the smallest resolved scales of the LES, thus allowing the dynamics of the large and medium scales (the most important ones) to be preserved. The temperature and humidity SGS fluxes are obtained assuming unity SGS Prandtl and Schmidt numbers as in ([Sagaut 2006](#)).

c. Numerical setup

In the present simulations, atmospheric conditions encountered during approach are considered, that is, a very weakly/weakly stratified atmosphere combined with atmospheric turbulence ranging from weak to strong. Note that very strong unstable conditions may also be encountered in the lower layers of the atmosphere, yet the periodic setup used here does not allow us to simulate such unstable conditions.

To simulate the wake vortices in realistic conditions, background turbulent fields for each case have been generated first and converged statistically in the computational domain. Those fields were converged using the [Alvelius \(1999\)](#) forcing term, which is characterized by a random phase and an amplitude computed according to a prescribed injection spectrum ([Vanhoenacker-Janvier et al. 2014](#)). The integral of this spectrum corresponds to the amount of energy injected per unit time $dE_f/dt = \varepsilon_f$, and thus also to the dissipation rate when the stratified turbulence is statistically converged.

Once the stratified turbulence has been produced, the two counterrotating vortices (aligned with the x direction) are superimposed over the background turbulent velocity field. A low-order algebraic profile [also called Burnham–Hallock (B-H) in the aircraft community] is used as the initial profile of the vortices. The effective core size r_c (i.e., the radius of maximum induced velocity) is set to $r_c/b_0 = 0.05$, which is typical of what is used when modeling aircraft wake vortices with that model. The Reynolds number based on the vortex circulation is very high: $\text{Re}_\Gamma = \Gamma_0/\nu \simeq 10^7$.

The box size L is $4b_0$, with a uniform grid size Δ of resolution equal to $b_0/\Delta = 64$; $r_c/b_0 = 0.05$ has been

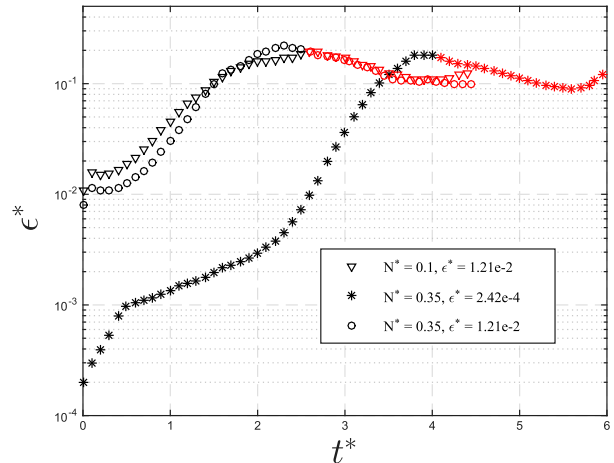


FIG. 1. Time evolution of the EDR for the three investigated cases. Red symbols highlight the intervals during which the simulations represent a real vortex system as obtained after roll-up.

chosen for the core size ratio, because such a resolution ensures that the most relevant dynamics of the vortices are well captured ($r_c/\Delta > 3$) ([De Visscher 2012](#)). For each case, the simulation was run for two additional characteristic times t_0 , after reaching the statistical turbulent equilibrium of the vortices with the background turbulence [=time of maximum eddy dissipation rate (EDR); [Fig. 1](#)].

This physical problem of adding two vortices in a stratified turbulent box can be characterized by two dimensionless parameters:

$$\varepsilon_f^* = \varepsilon_f b_0 / V_0^3 \quad \text{and} \quad N^* = N t_0, \quad (13)$$

where $V_0 = \Gamma_0/(2\pi b_0)$ and $t_0 = b_0/V_0$, the characteristic velocity and time scales of vortices, and L is the size of the domain. The first parameter characterizes the level of turbulence ε_f as scaled using vortex relevant quantities. The second parameter is an image of the level of stratification, scaling the Brunt–Väisälä frequency N with the descent time of the vortices. For all cases, the computational domain size is $L = 4b_0$ so that Crow instabilities ([Crow 1970](#)) cannot develop.

The three cases used in the radar simulator are summarized in [Table 1](#). The dimensional/physical parameters were here obtained by assuming a heavy aircraft ($\Gamma_0 = 400 \text{ m}^2 \text{ s}^{-1}$; $b_0 = 50 \text{ m}$), an initial height z_0 of 500 m, and typical atmospheric conditions; thus, $V_0 = 1.27 \text{ m s}^{-1}$ and $t_0 = 39.3 \text{ s}$.

d. Results

Once the vortices are introduced in the simulation, the vortex pair starts to descend at a velocity $V_0 = \Gamma_0/(2\pi b_0)$. The turbulence surrounding the vortices is affected by

TABLE 1. Simulation parameters of the three simulated cases used for the radar simulator. The subscripts g and 0 denote atmospheric factors at ground level ($z = 0$) and at the release height z_0 of the two vortices, respectively.

Dimensionless parameters			Dimensional/physical parameters									
Cases	N^* (–)	$\varepsilon_f^*(-)$	Γ_0 (m ² s ^{–1})	b_0 (m)	N (s ^{–1})	ε_f (m ² s ^{–3})	z_0 (m)	$\bar{\theta}_0$ (K)	$\frac{d\bar{\theta}}{dz}$ (K m ^{–1})	$\bar{p}_g$ (Pa)	$\bar{\phi}_g$ (g kg ^{–1})	$\frac{d\bar{\phi}}{dz}$ [g (kg m) ^{–1}]
1	0.1	1.21×10^{-2}	400	50	2.54×10^{-3}	5×10^{-4}	500	283	1.87×10^{-4}	101 325	6.25	-6×10^{-4}
2	0.35	2.42×10^{-4}	400	50	8.9×10^{-3}	1×10^{-5}	500	283	2.29×10^{-3}	101 325	6.25	-6×10^{-4}
3	0.35	1.21×10^{-2}	400	50	8.9×10^{-3}	5×10^{-4}	500	283	2.29×10^{-3}	101 325	6.25	-6×10^{-4}

two main phenomena. First, the background turbulence is stretched by the vortex system, generating intense transverse vortical structures. Second, due to the stratification, baroclinic vortical structures are generated at the border of the Rankine oval. This extra source term of vorticity arises from the difference in density between the warmer fluid carried by the vortices inside the Rankine oval and the cooler fluid outside. Subsequently, small-scale instabilities develop inside those structures, leading to a fully turbulent vortex system. The turbulence and the associated EDR increase until reaching a statistical equilibrium with the vortex system (Fig. 1). From that moment on, the flow becomes representative of a real vortex system as obtained after roll-up. This physical evolution is highlighted in red on the graphs. After reaching this equilibrium, the circulation of both vortices decreases smoothly (Fig. 2a), with the vortex system sustaining its own turbulence. Therefore, the descent velocity as well as the maximum tangential velocity (Fig. 2b) of the vortices also decreases. This equilibrium is reached at time $t^* = t/t_0 = 2.5$ for cases 1 and 3 and $t^* = 4$ for case 2.

In the LES, the total circulation of each vortex $\bar{\Gamma}_{\text{tot}}(t)$ is obtained by finding first, in each y – z slice, the local WV center. This localization is performed by locating the minimum of pressure in each slice. Those y – z planes are the planes perpendicular to the initial vortex tube direction. Afterward, the circulation profile $\Gamma(x_i, r, t)$ is computed in each slice (with r measured from the local WV center). Then we keep the maximum of the circulation $\Gamma_{\text{tot}}(x_i, t)$ in each slice. Last, these maximum values are averaged over all of the slices (256 slices) to get $\bar{\Gamma}_{\text{tot}}(t)$.

We see that the total circulation has already decayed greatly, as shown in Fig. 2a, which depicts the time evolution of the dimensionless vortex circulation $\bar{\Gamma}_{\text{tot}}^* = \bar{\Gamma}_{\text{tot}}(t)/\Gamma_0$. The intensification of the turbulence surrounding the vortices in case 3 is stronger than in case 2 with a weaker turbulent background; there, even though the atmosphere is equally stratified, the vortex tubes remain stronger and coherent for a longer time.

The stratification also creates baroclinic vortical structures. Those structures quickly interact with the turbulent vortices, enhancing the decay of the vortices compared to the less stratified case at same level of turbulence, that is, case 1 (Fig. 2a). This baroclinic vorticity also has an opposite sign relative to the primary vortices and thus tends to slow down the vortices as seen in Fig. 2c, which depicts the dimensionless vertical position of the vortex tubes: $\bar{z}_c^* = z_c(t)/z_0$.

For nonstratified or weakly stratified atmosphere, the circulation is proportional to the product of the descent

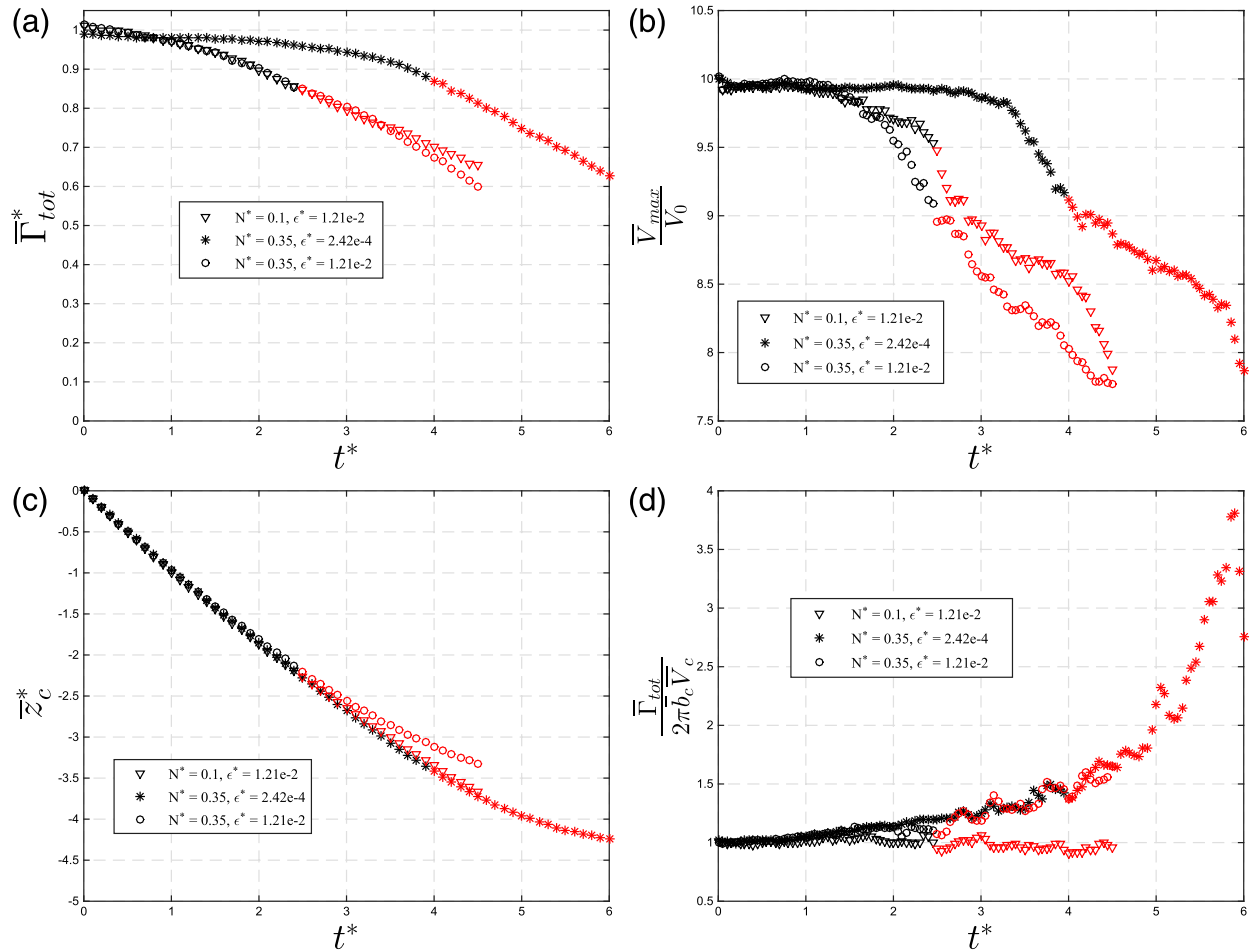


FIG. 2. Time evolution of: (a) $\bar{\Gamma}_{tot}^*$, (b) the maximum tangential velocity of the WV normalized by the initial descent velocity, (c) $\bar{z}_c^*$, and (d) the ratio between $\bar{\Gamma}_{tot}/(2\pi b_c)$ and the WV descent velocity $\bar{V}_c$. Each panel contains graphs for the three investigated cases. The reported values are averaged between the left and right wake vortices. Red symbols highlight the intervals during which the simulations represent a real vortex system as obtained after roll-up.

velocity V_c and the distance between the WV cores b_c . Figure 2d illustrates this ratio: for case 1 the circulation calculated from the WV velocity field is equal to that calculated from V_c and b_c , whereas for the more stratified cases 2 and 3 the ratio deviates significantly from unity.

Figure 5b shows the humidity field combined with the wind velocity (w, u) of case 1 at time $t^* = 3.75$ in a vertical cross section of the computational box. The black crosses represent the center of the vortex tubes. These were located by identifying the location of the minimum pressure. During its descent, the vortex system carries the lower humidity, which is situated at a high altitude, downward. Such a pattern in the humidity field (as well as in the temperature and pressure fields) may backscatter a characteristic return signal to the radar, which could be used to detect the vortex pair and possibly estimate its intensity as well.

3. Backscattering of the radar signal on wake vortices in clear air

The output of the LES of wake vortices described in the previous section is used for the simulation of the radar signal and its Doppler spectrum. The pulse Doppler radar used during the Seventh Framework Program (FP7) Ultrafast Wind Sensors (UFO; <http://www.ufo-wind-sensors.eu/home>) project was selected for use in this paper. However, the simulator can be used with other types of radars, in which case, only the scattered signal time series need to be modified in the calculation. Preliminary validations of the simulator were performed by extracting the EDR of the atmospheric turbulence under clear-air conditions with a radar wind profiler (Kovalev et al. 2016) and a scanning radar (Kovalev and Vanhoenacker-Janvier 2017) used during the UFO measurement campaign.

TABLE 2. Parameters of the simulated X-band radar.

Parameter name	Value
Frequency f_c	9.5 GHz
Pulse duration τ_{pls}	$2 \mu s$
Pulse repetition time T	$50 \mu s$
No. of pulses in a burst	512
Dwell time T_d	25.6 ms
Radar cell size in range Δr_{rc}	60 m; 6 m
Antenna beamwidth at 3 dB $\varphi_a = \theta_a$	3.1°

This section presents an improved version of the simulator, which was also used for the estimation of the circulation of the wake vortices.

a. Geometry and parameters of the radar used for the simulation

The main parameters of the radar are listed in Table 2. Figure 3 illustrates the structure of the radar signal: during the dwell time, the radar emits 512 pulses with a pulse repetition time of $50 \mu s$. The radar receives the pulses scattered by a given radar cell located at a distance of $c\tau/2$, where τ is the time necessary for an electromagnetic wave to travel from the radar to the scatterer and back and c is the speed of light. A pulse received at time $mT + \tau$, where m is the consecutive number of the pulses in the burst, corresponds to the pulse emitted at time mT and scattered back onto the refractive index inhomogeneities. For the sake of simplicity, the pulses in Fig. 3 are given in the form of a voltage that modulates the carrier frequency of the radar; the carrier frequency is not represented. It is assumed that the modulation of the pulses provides the needed range resolution.

Figure 4 illustrates the geometry used for the simulation: the radar is located at the side of the glide with the distance r to the runway axis equal to 1100 m. Two geometrical setups are used in the simulator. For setup A, the size of the radar resolution volume in range (Δr_{rc}) is 60 m and the elevation angle of the antenna beam is fixed at 5.3° to monitor the WV at the altitude of the glide ($h_1 = 100$ m). For setup B, the range resolution is 6 m and the radar performs a scan of the LES box in elevation (with a step of 0.27°) to track the WV descent.

b. Calculation of the radar signal

The electromagnetic waves emitted by the radar are backscattered by the refractive index irregularities of the clear air. The scattering occurs on inhomogeneities with a scale equal to the Bragg wavelength (Doviak and Zrnić 1984), corresponding to half the radar wavelength. The backscattered signal can be obtained by calculating of the so-called scattering integral:

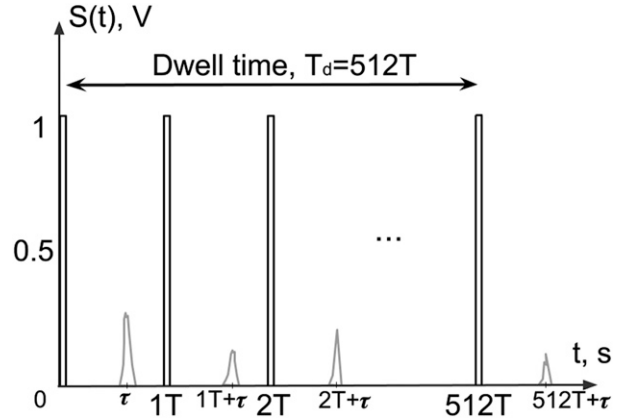


FIG. 3. Schematic representation of the train of emitted (black) and received (gray) pulses.

$$I(t) = A \int W(\mathbf{r}) f_{\theta\phi}^2(\mathbf{r}) n'(\mathbf{r}, t) e^{-jk_B r'} dV, \quad (14)$$

where $W(\mathbf{r})$ is the range weighting function, $f_{\theta\phi}^2(\mathbf{r})$ is the antenna pattern, $n'(\mathbf{r}, t)$ is the refractive index of the irregularities within the radar cell, $k_B = 4\pi/\lambda$ is the Bragg wavenumber, and A is an energetic constant that necessitates the knowledge of various radar parameters:

$$A = \frac{G}{\lambda r_0^2} \left(\frac{P_t}{2R} \right)^{1/2}, \quad (15)$$

where G is the antenna gain, λ is the radar wavelength, r_0 is the distance between the radar and the center of the radar resolution cell, P_t is the emitted power, and R is the receiver resistance. If some of the parameters are not available, this parameter can be evaluated using the signal backscattered by a calibrated target.

The discrete nature of the output of the LES imposes the transformation of the integral into a sum over the

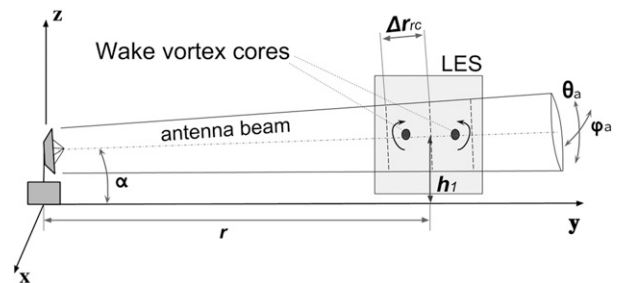


FIG. 4. Geometric configuration of the radar measurements simulation: the radar is located at the side of the glide with the distance r to the runway axis; Δr_{rc} is the radar cell size in range; the radar beam is pointed perpendicular to the glide at the elevation angle α ; h_1 is the altitude at which the center of the LES box is placed; φ_a and θ_a are the antenna beam widths at 3 dB in azimuth and elevation, respectively.

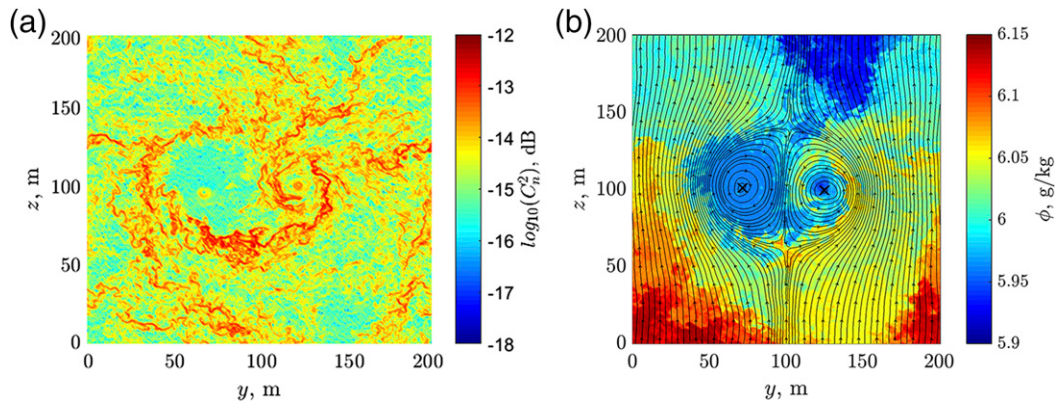


FIG. 5. (a) The C_n^2 distribution and (b) the humidity distribution combined with the wind velocity streamlines. Both are in a vertical cross section of the LES box for case 1 at $t^* = 3.75$. To obtain the streamlines in (b), v and w components of the velocity are averaged along the x axis and the WV descent velocity V_0 is added to the w component to show the Rankine oval formed by the WV velocity field. The black times signs in (b) correspond to the WV core centers averaged along the x axis.

radar cell volume for which the product $W(\mathbf{r})f_{\theta\phi}^2(\mathbf{r})$ is significant. The numerical calculation of the scattering integral necessitates the value of the refractive index irregularities $n'(\mathbf{r}, t)$ with a resolution better than the Bragg wavelength. For classical simulators using 2D models for the wake vortices, the resolution could possibly be adapted to the wavelength of the radar. The resolution of the present 3D LES suffices to capture the most relevant dynamics of the medium to large scales of the vortices and the associated turbulence, yet is not fine enough to capture the Bragg wavelength. This one is smaller than the LES resolution by a factor of 50, making a full calculation of the scattering integral impossible. Naturally, it is not possible to increase the LES resolution by a factor of 50, which would amount to 125 000 additional grid points.

In the absence of an atmospheric model with sufficient resolution, Muschinski et al. (1999) proposed a parameterized model for the calculation of the scattering integral and obtained the following time series of the scattered signal [Eq. (14)]:

$$I(t) = A(0.033k_B^{-11/3})^{1/2} \sum_{p=1}^N W_p \{ [C_n^2(t)]_p V_p \}^{1/2} e^{-j\varphi_p(t)}, \quad (16)$$

where C_n^2 is the refractive index structure parameter, W_p is the value of $W(\mathbf{r})f_{\theta\phi}^2(\mathbf{r})$ in the LES cell of volume V_p , $\varphi_p(t)$ is the phase of the signal backscattered by the p th grid cell, and N is the number of LES grid cells within the volume of a given radar cell. The calculation of the C_n^2 and $\varphi_p(t)$ parameters uses the LES output data and is described in sections 3c and 3d, respectively.

Because the LES output data are 3D snapshots of meteorological parameters at discrete time values, time t in Eq. (16) is replaced by the discrete values of kT , where T is the pulse repetition time of the radar.

Taylor's hypothesis of the "frozen turbulence" (Taylor 1938) is used: because of the relatively short dwell time (25.6 ms), it is assumed that the scattering pattern within the LES cell remains constant and is advected in time by the local velocity field; C_n^2 is assumed to be constant over the dwell time. If the dwell time is greater than the LES time step, proper use of Eq. (16) necessitates the interpolation of the values of n and the velocity field for the time corresponding to the radar pulses (Muschinski et al. 1999).

The Muschinski et al. (1999) model uses two main hypotheses:

- 1) First, the LES subgrid turbulence obeys the Kolmogorov scaling; that is, it belongs to the inertial subrange of the turbulence spectrum, at the Bragg scale.
- 2) Second, the initial phases φ_{0p} of the LES grid cell contributing to the sum over the radar cell are statistically independent.

Muschinski et al. (1999) proposed Eq. (16) for the simulation of a vertical wind radar profiler at 915 MHz.

TABLE 3. Estimation of the phase correlation time scale τ_ϕ , the inner scale l_0 , and the broadening of the Doppler spectrum $\Delta\sigma_v$ from the subgrid turbulence contribution.

LES case	τ_ϕ (ms)	l_0 (cm)	$\Delta\sigma_v$ (m s ⁻¹)
Cases 1 and 3 (initial turbulence)	69	0.9	0.23
Case 2 (initial turbulence)	254	2.4	0.06
Cases 1 and 3 (developed turbulent WV)	27–33	0.4–0.5	0.57–0.47
Case 2 (developed turbulent WV)	28–35	0.4–0.5	0.56–0.45

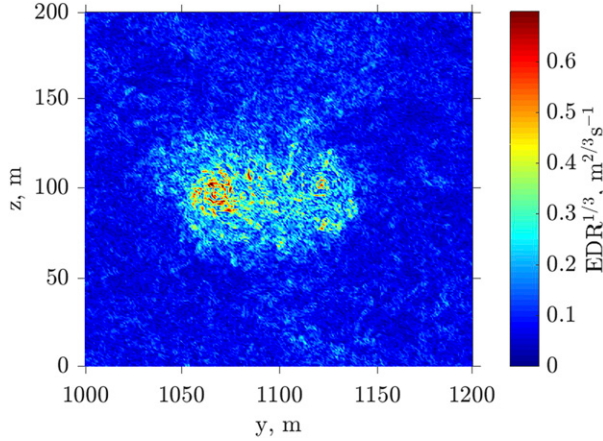


FIG. 6. Distribution of $\epsilon^{1/3}$ in a cross section of the LES for case 1 at $t^* = 3.75$.

For this frequency, the Bragg wavelength is on the order of 16 cm, which belongs to the inertial subrange of turbulence. For the X-band radar used in this study, one-half the radar wavelength is approximately 1.6 cm. This is on the order of magnitude of the smallest inhomogeneities of the refractive index in clear air that are generally considered as having dimensions between a few centimeters and a few millimeters. A solution to this problem will be proposed in section 4. The second hypothesis is verified because the distance between LES grid points is large with respect to the wavelength.

c. Calculation of n and C_n^2

The refractive index is calculated, using the pressure, temperature, and humidity fields provided by the LES and the formula proposed by Bean and Dutton (1966):

$$n = 1 + N \times 10^{-6} = 1 + \left[\left(\frac{77.6}{T} \right) \left(P + 4810 \frac{q}{T} \right) \right] \times 10^{-6}, \quad (17)$$

where N is the radio refractivity, P and q are the atmospheric pressure and the water vapor pressure in hectopascals, and T is the absolute temperature in kelvins.

The refractive index structure parameter C_n^2 represents the intensity of the turbulence [Eq. (18)] and directly influences the intensity of the scattered signal [Eq. (16)]. For all spatial inhomogeneities with scales belonging to the inertial subrange of the turbulence spectrum, the refractive index structure parameter C_n^2 can be calculated using the definition of the structure function for a turbulent field (Tatarskii 1971):

$$C_n^2 = \frac{\langle [n(\mathbf{r} + \boldsymbol{\delta}) - n(\mathbf{r})]^2 \rangle_r}{|\boldsymbol{\delta}|^{2/3}}, \quad (18)$$

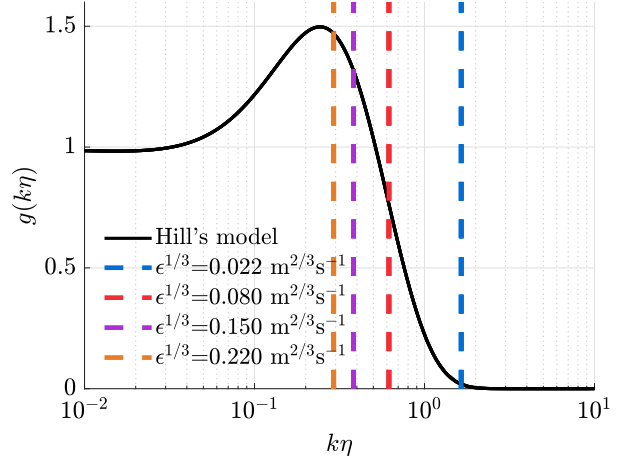


FIG. 7. Hill's bump as an attenuation factor for the refractive index spectrum for different values of ϵ . The vertical lines represent the Bragg wavenumbers for the radar wavelength of 3.2 cm for different levels of the energy dissipation rate.

where $\mathbf{r}$ is the position vector, $\langle \rangle_r$ serves for spatial averaging, and $\boldsymbol{\delta}$ is the vector defining the spatial separation between the points at which the spatial averaging is calculated.

The C_n^2 calculated at each LES grid node takes the six closest nodes (at a distance Δ equal to the size of the LES grid cell) and averages the square difference between values of n at plus and minus the LES resolution, taken two by two along the three axes. So, in this case, $|\boldsymbol{\delta}| = 2\Delta$.

An example of the distribution of C_n^2 in a vertical cross section of the LES box is provided in Fig. 5a for case 1. The humidity field combined with the wind velocity field is shown in Fig. 5b. The wake vortex is clearly visible, with high values of C_n^2 at the limit of the core and at the limit of the oval surrounding the vortices.

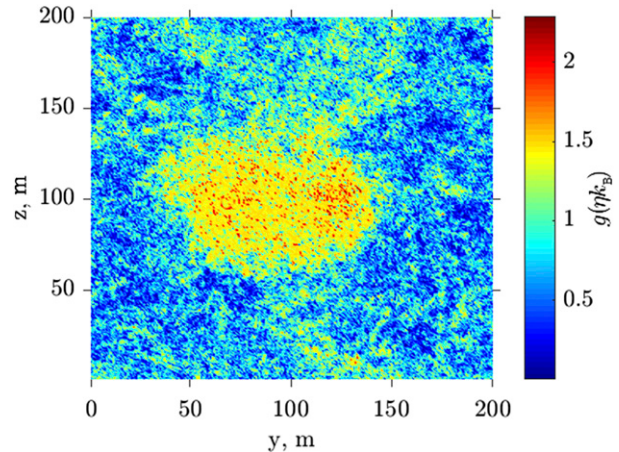


FIG. 8. Distribution of the attenuation factor in a cross section of the LES for case 1 at $t^* = 3.75$.

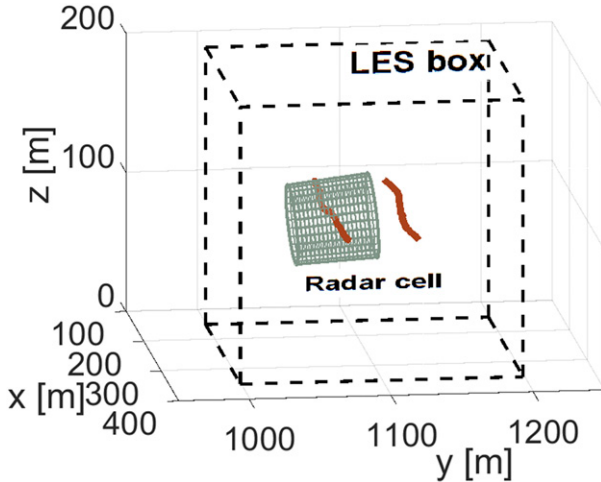


FIG. 9. Alignment of the radar cell relative to the wake vortex core. The black dashed lines indicate the borders of the LES box, red curves show the positions of the wake vortex cores, and gray delineates the borders of one radar cell.

d. Calculation of the phase

The phase contribution of the p th LES grid cell to the total sum in Eq. (16) for the k th pulse in the burst is calculated as follows:

$$\varphi_p(t_0 + kT) = \varphi_{0p} + \mathbf{k}_{Bp} \cdot \mathbf{v}_p(t_0)kT, \quad (19)$$

where t_0 is the time of the LES snapshot, φ_{0p} is the initial phase, $\mathbf{k}_{Bp}$ is the Bragg wavenumber vector directed from the radar antenna center to the p th LES grid node, $\mathbf{v}_p(t_0)$ is the local velocity at the LES grid node at t_0 , and kT is the time shift from the initial time of the LES snapshot.

The calculation of the time series of the scattered signal is based on the assumption that all Bragg's scatterers behave like point scatterers with an infinite lifetime. However, the refractive index irregularity pattern at the Bragg length scale $\lambda/2$ is reorganized after a time interval on the order of $\lambda/(\varepsilon\Delta)^{1/3}$ within the LES grid cell, where $(\varepsilon\Delta)^{1/3}$ is the characteristic turbulence velocity. Muschinski et al. (1999) estimated the upper boundary of this time, during which the Bragg scatterers pattern with a scale Δ remains coherent. This time is called the “phase correlation time scale” and is estimated as the time interval after which the RMS phase difference between two scatterers separated by a distance Δ is equal to π :

$$\tau_\varphi(\Delta) = 0.16 \frac{\lambda}{(\varepsilon\Delta)^{1/3}}. \quad (20)$$

Referring to the three LES cases (Table 1), with a spatial resolution of $\Delta \simeq 78$ cm and a wavelength of 3.16 cm, the

phase correlation time scale τ_φ corresponding to the forcing background turbulence is shown in Table 3 for the various cases.

As discussed before, the developed and turbulent WV have much higher EDR values than those used as the forcing background turbulence. For all cases, the mean value of the EDR increases to $3.9 \times 10^{-3} - 8 \times 10^{-3} \text{ m}^2 \text{ s}^{-3}$, which corresponds to τ_φ changing in the range from 35 to 27 ms. Proper use of Eq. (19) necessitates a random change of the phases of Bragg scatterers after time intervals of τ_φ .

e. Estimation of the subgrid turbulence contribution to the Doppler spectrum width

Because LES does not provide the information needed to calculate the contribution of the subgrid turbulence to the simulated Doppler spectrum, Muschinski et al. (1999) showed that this contribution to the Doppler spectrum width can be estimated as follows:

$$\Delta\sigma_v = (2\pi/\tau_\varphi)/k_B = \lambda/(2\tau_\varphi) = 3.12(\varepsilon\Delta)^{1/3}, \quad (21)$$

where k_B is the Bragg wavenumber.

Table 3 shows the broadening of the Doppler spectrum $\Delta\sigma_v$ caused by the subgrid turbulence. These estimates are given for the background forcing turbulence as well as for the strong increase of the EDR caused by the developed turbulent WV, which results in an increase of $\Delta\sigma_v$ for all cases.

The resolution of the Doppler spectrum is

$$\Delta\nu_r = \lambda/(2T_d), \quad (22)$$

where T_d is the dwell time used for the Doppler spectrum calculation. In comparing Eqs. (21) and (22), it can be concluded that, to keep the contribution of subgrid turbulence smaller than the Doppler spectrum resolution, the dwell time should be less than the phase correlation time scale. A further increase of the Doppler spectrum resolution is useless because the subgrid turbulence is not resolved by the LES.

4. Limits of detectability

The Bragg scattering is caused by variations of the clear-air refractive index having a spatial scale of half the radar wavelength (Tatarskii 1971). The smallest eddies in a turbulent flow are on the order of the Kolmogorov length scale:

$$\eta = (\nu^3/\varepsilon)^{1/4}. \quad (23)$$

The inner scale l_0 , characterizing the transition scale between the inertial subrange and the viscous dissipative

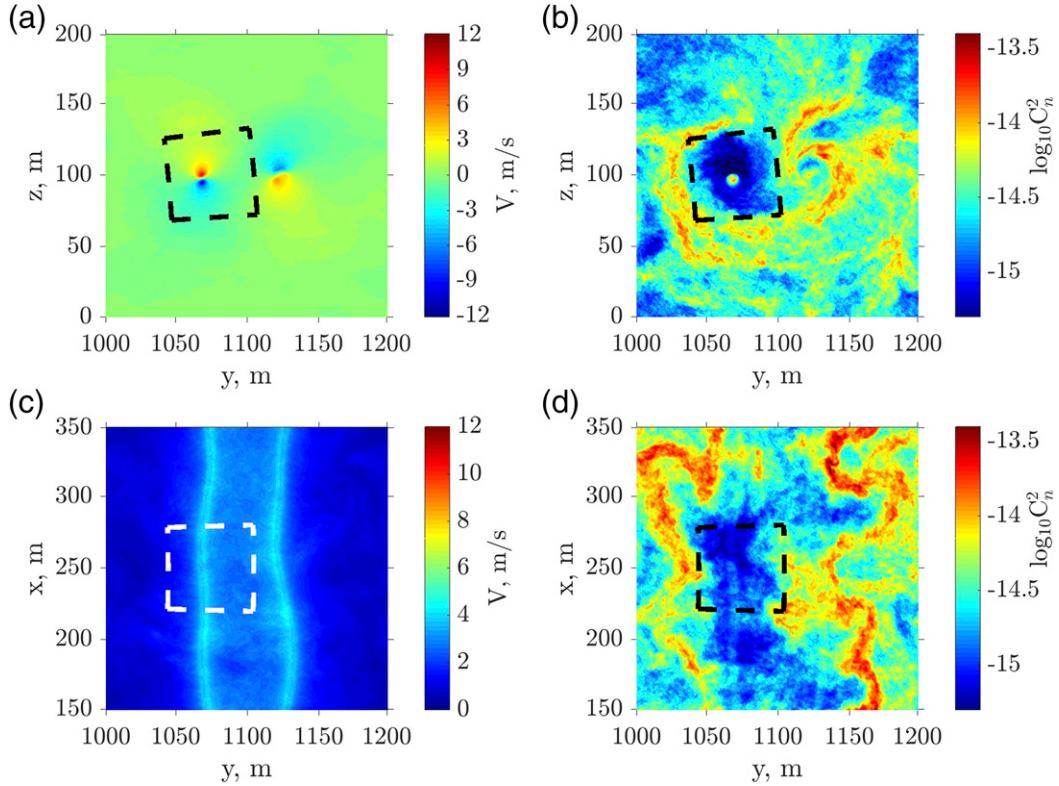


FIG. 10. Distribution of (b),(d) C_n^2 and (a),(c) the intensity of the velocity for case 1 at $t^* = 3.75$. The orders of the radar cell are indicated by the dashed lines.

range of the turbulence spectrum, was estimated by Hill and Clifford (1978) for the temperature spectrum of the turbulence:

$$l_0 \simeq 5.8\eta, \quad (24)$$

where 5.8 is the coefficient calculated for the Prandtl number $P_r = 1$. This inner scale is typically considered as a threshold for the radar detectability of the turbulence in clear air. Table 3 provides values of l_0 for different levels of the EDR.

To ensure backscattering on the turbulent eddies, the following condition should be met:

$$\lambda/2 \geq l_0. \quad (25)$$

For higher radar frequencies, there are no significant eddies with dimension on the order of the Bragg wavelength. The inner scales of the turbulence injected in the background stratified atmosphere for the LES used in this study are listed in Table 3. For the X-band radar used, the Bragg wavelength belongs to the transition region between the inertial subrange and the dissipative range. Therefore, the hypothesis used by Muschinski et al. (1999) to deduce Eq. (16) is not fulfilled for all the

three cases considered. However, the developed turbulent WV system has local EDR values in the WV oval that are much larger than that of the initial forcing turbulence. It is also the case in the background part, outside of the WV oval and downstream. An example of the

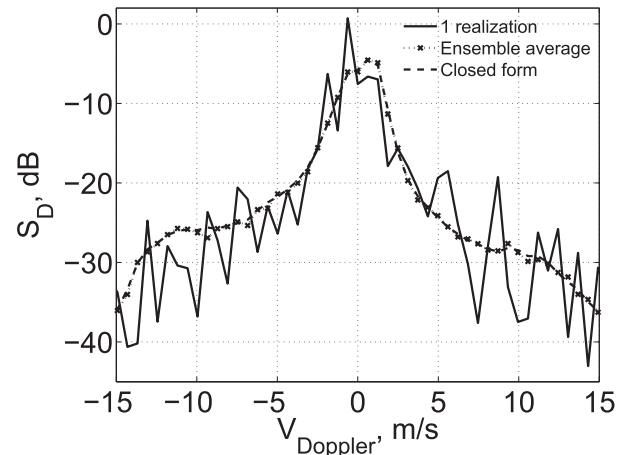


FIG. 11. Doppler spectrum of the scattered signal (case 1). The ensemble average is calculated over 100 realizations. The closed form represents the Doppler spectrum estimated with Eq. (28). All Doppler spectra are normalized to the area under their graphs.

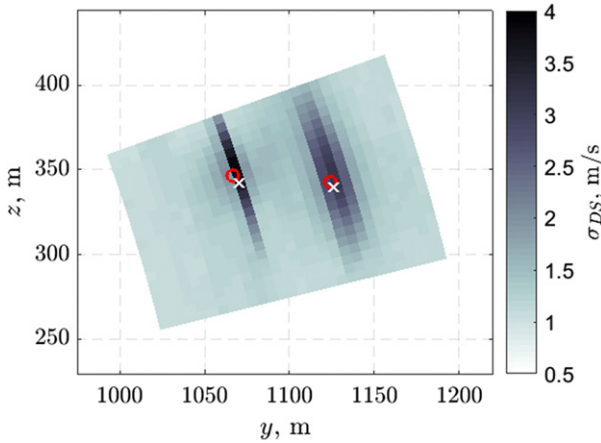


FIG. 12. Distribution of the Doppler spectra width in one elevation scan for case 1 at $t^* = 3.75$. The white times signs show the position of the WV cores as measured in the LES; the red open circles show those as estimated by the radar.

EDR values in a cross section of the LES box is provided for case 1 in Fig. 6. For an initial turbulence forcing with $\varepsilon_f^{1/3}$ of $0.08 \text{ m}^{2/3} \text{ s}^{-1}$, typical (median) values of 0.2 and as high as $1.7 \text{ m}^{2/3} \text{ s}^{-1}$ occur in the WV oval, and typical values of $0.1\text{--}0.14 \text{ m}^{2/3} \text{ s}^{-1}$ occur outside and downstream of the WV oval.

To take into account the attenuation of the spectrum level in the dissipation range, the Muschinski et al. (1999) formula is modified by introducing an attenuation factor in the refractive index spectrum (Hill 1978a) used to derive Eq. (16). Relying on the experimental research, Hill (1978b) discovered that, in the transition between the inertial subrange and the viscous dissipative range, the spectrum has a so-called bump. As shown by Muschinski and de Bruyn Kops (2015), “model 4” proposed by Hill (1978a) predicts the location of the bump accurately. The numerical solution of the Hill (1978a) model is approximated by Frehlich (1992) as

$$\Phi_n(k_B) = 0.033 C_n^2 k_B^{-11/3} g(\eta k_B), \quad (26)$$

where the attenuation factor $g(k)$ is

$$g(k) = \left(1 + \sum_{n=1}^K a_n k^n \right) \exp(-\delta k), \quad (27)$$

where, for $K = 4$, the values of the constants providing the best fit for the Hill spectral model are: $\delta = 1.1090$, $a_1 = 0.70937$, $a_2 = 2.8235$, $a_3 = -0.28086$, and $a_4 = 0.08277$. Figure 7 provides the dependence of $g()$ on the Bragg wavenumber scaled by Kolmogorov length scale η .

As seen from Fig. 7, for the values of ε typical for the WV oval, the Bragg wavelength of the X-band radar is in proximity to the bump value of the attenuation factor. For the lowest EDR, corresponding to the background forcing turbulence of case 2, the attenuation of the refractive index variance spectrum estimated by Hill’s and Tatarskii’s models are -17.5 and -11.5 dB, respectively. In this study, the contribution of each LES cell in Eq. (16) is weighted by the local factor $[g(\eta k_B)]^{1/2}$ to take into consideration the shape of the refractive index variance spectrum at the transition between the dissipative and the inertial range. In Fig. 8, an example of the factor $g(\eta k_B)$ is plotted for the snapshot in case 1 (Fig. 6). This figure clearly shows an increase of the refractive index variance spectrum in the WV oval relative to the background atmosphere.

5. Calculation of the Doppler spectrum

Two geometrical setups, as described hereinafter, are used to simulate the WV detection and its parameters estimation. First, the influence of the noise is not considered in the simulations so as to estimate the possibility of the WV circulation retrieval from the radar measurements. However, low RCS of the WV turbulence limits the detection of the WV on long distances. To evaluate this limitation, the signal-to-noise ratio (SNR) is estimated for the radar parameters used in this study (Table 2) in section 6c.

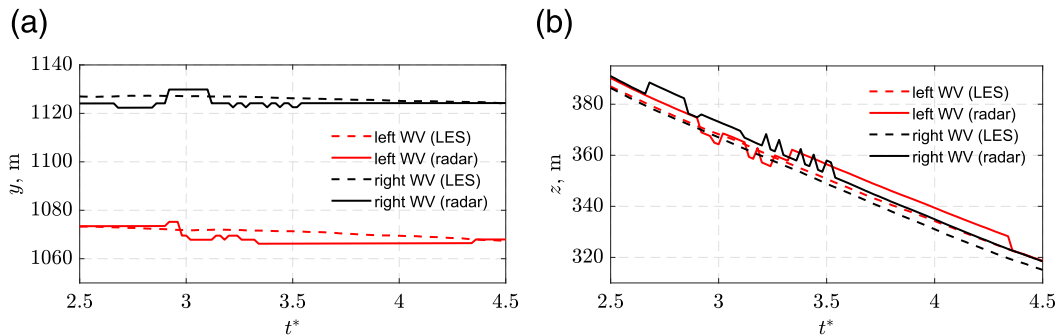


FIG. 13. Time evolution of (a) y and (b) z coordinates of the WV cores for case 1. Dashed lines show LES data, and solid lines are estimated from the Doppler spectrum width distribution.

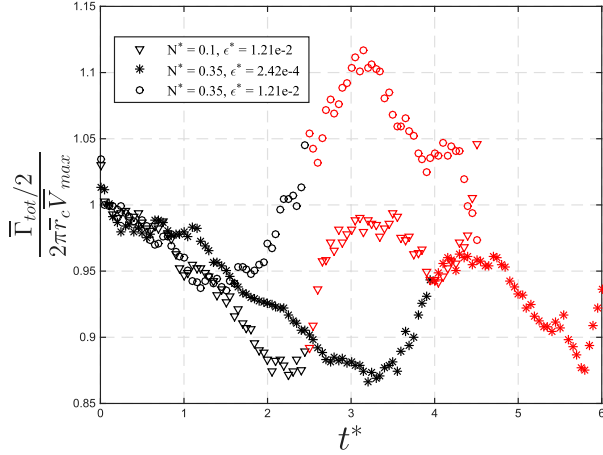


FIG. 14. Time evolution of the ratio between $\bar{\Gamma}_{tot}/2$ and $\bar{\Gamma}(r_c) = 2\pi\bar{r}_c\bar{V}_{max}$, as measured from the LES data.

a. Geometrical setup A

Figure 9 shows the details of geometrical setup A: the radar cell is aligned with the WV core center; the size of the radar cell in range is approximately 60 m; the half-power width of the antenna beam is 3.1° . A decision was made to position the WV core at the center of the radar cell here so as to be able to compare the WV circulation estimated from the Doppler spectrum to the exact value as measured in LES (section 6). At each LES time step, the LES box is placed at the same altitude as defined in section 3a. This compensation for the WV descent provides equivalent conditions for the estimation of the WV circulation.

Figure 10 represents a distribution of the local velocity (intensity) and C_n^2 in horizontal (Figs. 10c,d) and vertical cross sections of the LES (Figs. 10a,b). Each distribution is an average of 80 adjacent LES layers taken from the center of the LES, in the z and x directions for the horizontal and vertical cross sections, respectively. The borders of the radar cell centered at the initial position of the left WV are indicated by dashed lines.

Figure 11 shows an example of the Doppler spectrum of the simulated signal. The periodogram gives an

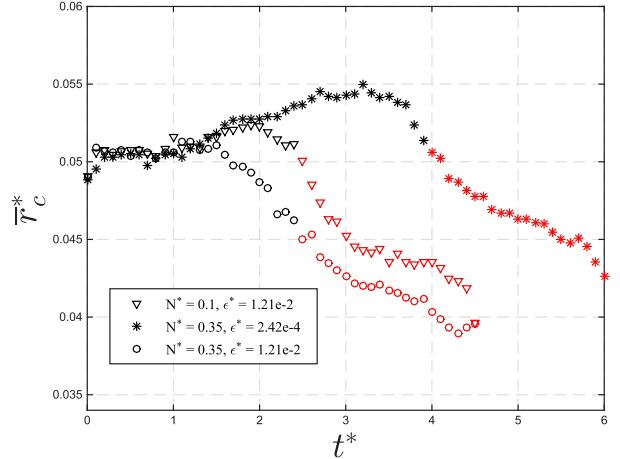


FIG. 15. Time evolution of the ratio $\bar{r}_c^* = \bar{r}_c/b_0$, as measured from the LES data.

estimate of the power spectral density based on a limited number of samples determined by a window (rectangular, Bartlett, Hamming, etc.). However, this estimator is not consistent because its variance does not converge to zero when the number of samples becomes large. Using a single realization of the radar scattered signal gives a very spiky spectrum (Blackman and Tukey 1958). To get statistically stable results, various Doppler spectra can be averaged for an ensemble of scattered signal realizations. Each realization is obtained with a randomly generated initial phase φ_{0p} entered in Eq. (19). The generation of an ensemble of scattered signal time series with Eq. (16) is computationally expensive. To speed up the simulation, the approach introduced by Muschinski et al. (1999) is used:

$$S(\omega_n) = \sum_{p=1}^N a_p^2 Z_{np} Z_{np}^*, \quad (28)$$

where $a_p = [(C_n^2)_p V_p g(\eta k_B)]^{1/2}$; $\omega_p = k_{Bp} v_p$; $\omega_n = 2\pi n/T_d$ for $n = \dots, -2, -1, 0, 1, 2, \dots$; and Z_{np} are the Fourier amplitudes

$$Z_{np} Z_{np}^* = \begin{cases} \frac{2}{[(\omega_p - \omega_n)T_d]^2} \{1 - \cos[(\omega_p - \omega_n)T_d]\}, & \omega_p \neq \omega_n \\ 1, & \omega_p = \omega_n \end{cases}. \quad (29)$$

The two Doppler spectrum estimators are compared: the Doppler spectrum calculated directly with the closed form Eq. (28) is compared with the ensemble average of the Doppler spectra calculated over

100 independent time series of the radar signal. These time series are generated with Eq. (16) using randomly generated initial phases φ_{0p} in Eq. (19) to simulate 100 realizations of time series. The Doppler spectra are

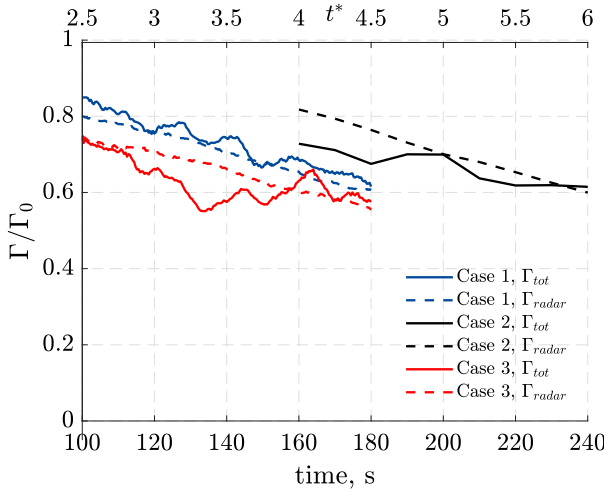


FIG. 16. Temporal evolution of the normalized circulation of the wake vortices retrieved from the radar simulator (solid) and obtained directly from the LES (dashed).

estimated for each realization and averaged. Figure 11 clearly shows that the ensemble-averaged Doppler spectrum is not distinguishable from the spectrum calculated with Eq. (28) and has a reduced variance with respect to a single realization. In the following sections, the closed-form Doppler spectrum is used.

b. Geometrical setup B

Geometrical setup B is similar to setup A but the altitude of the LES box tracks the altitude of the descending WV. The radar performs a scan of the LES box in elevation with a step of 0.27° . It is assumed that the position of the LES box does not change during each scan. The range resolution is 6 m. For each radar resolution volume, the Doppler spectrum is estimated with Eq. (28).

Figure 12 illustrates the distribution of the Doppler spectra width among radar cells in one elevation scan. The position of the WV cores is estimated by the local maxima of the Doppler spectrum width (Fig. 13). There are two causes of the discontinuities in estimated coordinates (y, z): 1) a relatively large radar range resolution (6 m) and 2) the fact that the radar cell with the maximum Doppler spectrum width does not necessarily correspond to the actual location of the WV core. To reduce the variance of the estimated WV coordinates, the width of the Doppler spectra in a given radar cell is obtained as an average over 25 closest radar cells in elevation and range (in an area of 5×5 of the radar resolution volumes centered at a given radar cell).

6. Estimation of the wake vortex circulation

Two methods are used to estimate the WV circulation. For geometrical setup A, the circulation of the WV is

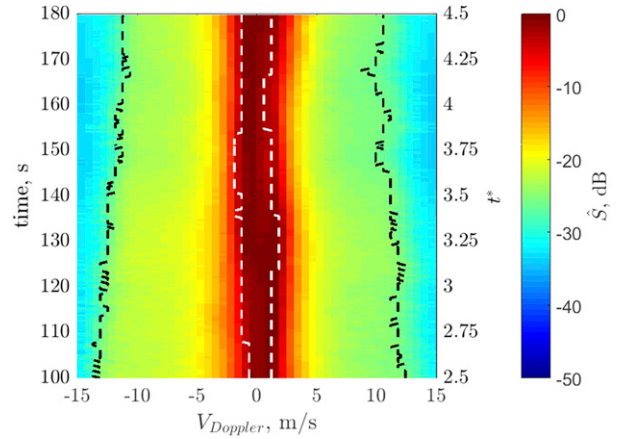


FIG. 17. Time evolution of the Doppler spectrum for case 1. The black curves correspond to the -26.5 -dB level (following approximately $V_{\max}$); the white curves correspond to the -3.5 -dB level.

estimated from the distribution of the radial velocities within the radar resolution volume. Those distributions are represented by the Doppler spectrum of the back-scattered signal. For geometrical setup B, the estimation is based on the tracking of the WV descent velocity and distance between WV cores.

a. Setup A

The estimation of the wake vortex circulation is based on the method proposed by Rubin (2000) that shows that the circulation is proportional to the second moment of the Doppler spectrum raised to the power $2/3$:

$$\Gamma_{\text{radar}} = k \left\{ \int_{-V_{\min}}^{-V_{\max}} V^2 [S(V)]^{2/3} dV + \int_{V_{\min}}^{V_{\max}} V^2 [S(V)]^{2/3} dV \right\}, \quad (30)$$

where $S(V)$ is the Doppler spectrum normalized to the area under its graph, $V_{\max} = \Gamma(r_c)/(2\pi r_c)$ is the maximum velocity induced by one WV (also close to the maximum found in the LES velocity field), $V_{\min}$ is chosen so as to exclude the broadening of the Doppler spectrum caused by the atmospheric turbulence, and $k = 2\pi r_c/V_{\min}$.

However, the Rubin model assumes that the wake vortex circulation Γ_{tot} is linked to $V_{\max}$ by $\Gamma_{\text{tot}} = 2\pi r_c V_{\max}$ and thus a top-hat wake vortex model (i.e., uniform vorticity within r_c and zero outside). In the B-H vortex model used as the initial condition in this study, $\Gamma(r)$ varies smoothly from 0 to Γ_{tot} and is equal to $\Gamma_{\text{tot}}/2$ at r_c . Although the circulation distribution of the developed turbulent WV no longer follows the simple B-H model, it is found that $\Gamma(r_c)$ is also roughly equal to $\Gamma_{\text{tot}}/2$, which holds fairly true for the representative time intervals; see the red parts of Fig. 14: $\Gamma(r_c)$ deviates from

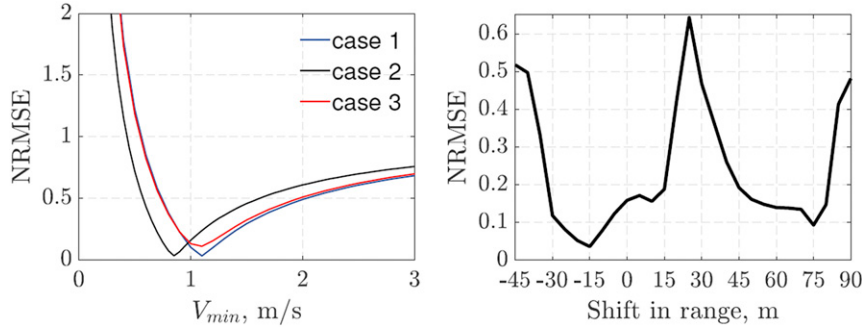


FIG. 18. Influence of (left) V_{min} and (right) the misalignment of the radar resolution volume relative to the WV core on the NRMSE (case 1).

$\Gamma_{tot}/2$ by approximately $\pm 12\%$ for all cases. To take this variation into account, it is proposed to modify Eq. (30) by using $\Gamma_{tot}/2 \simeq 2\pi r_c V_{max}$. The coefficient k was evaluated using $r_c = 0.05b_0 = 2.5$ m. The actual change of r_c over time with respect to the initial spacing between WV cores b_0 is shown in Fig. 15: r_c deviates from the initial value by from $+10\%$ to -20% depending on the LES case. As compared with Rubin (2000), the interval of integration in Eq. (30) is changed to $[-V_{max}, -V_{min}]$ and $[V_{min}, V_{max}]$ because the Doppler spectrum is not symmetric: the negative and positive parts of the spectrum represent different parts of the wake vortex system.

To compare the circulation retrieved from a given radar cell to the circulation measured from the LES velocity field, the radar cell center is aligned with the vortex core center. The LES reference $\Gamma(x_i, r_{max}, t)$ is sought at each x_i , where r_{max} is equal to one-half of the radar cell width in range. Then it is averaged over x_i (within the radar resolution volume) to obtain $\Gamma_{tot}(t)$. The total circulation shown in Fig. 16 is the average of the left and right wake vortex circulations, normalized by Γ_0 . The wake vortices decay in time, dissipating their energy by interacting with each other and with the surrounding turbulent background. Consequently, the WV circulation decreases in time as shown in Fig. 16 for the considered LES cases.

The radar estimates of the circulation at each LES time step are obtained from the radar cells centered at the initial positions of the left and right WV cores (at $t^* = 0$). Then the average between the left and right WV estimates is calculated. The results provided in Fig. 16 show a good agreement with the circulation calculated directly from the LES field. Note that in the LES $t^* = 0$ corresponds to the time of the injection of the wake vortices into the initial stratified turbulence. The moment at which the LES reaches a statistical equilibrium—the state at which the energy dissipation rate reaches its maximum—indicates the time after which the LES provides a physically meaningful simulation of

fully developed turbulent wake vortices in the atmosphere. Those moments differ among the LES cases, and consequently, the estimation of Γ_{tot} starts accordingly (the red parts in the previous figures, lasting for a total time of $2t_0$).

The limits of integration (V_{min}) in Eq. (30) significantly influence the accuracy of the estimation of Γ_{tot} . The lower limits V_{min} are chosen so as to exclude the influence of the Doppler spectrum broadening caused by the turbulence induced by the WV: 1.15 m s^{-1} for cases 1 and 3 and 1 m s^{-1} for case 2. The values for V_{min} are estimated by minimizing the error between the estimate Γ_{radar} and the reference circulation from the LES Γ_{tot} . However, in real systems, V_{min} should be estimated from the radar signal backscattered by the turbulence. For all cases, a constant upper limit $V_{max} = 12.8 \text{ m s}^{-1}$ is used. Nevertheless, the value of V_{max} at each time step can be accurately estimated from a total width of the Doppler spectrum (Fig. 17). The evolution of the Doppler spectrum is shown in Fig. 17 for case 1: the total width of the Doppler spectrum decreases with time; the width of the central part represents the velocity variance related to the turbulence.

The influence of V_{min} on the accuracy of the circulation estimation (with fixed V_{max}) is shown in the right panel of Fig. 18 in the form of a normalized root-mean-square error:

$$\text{NRMSE} = \left\{ \frac{\sum_{i=1}^N [\Gamma_{radar}(t_i) - \Gamma_{tot}(t_i)]^2}{\sum_{i=1}^N \Gamma_{tot}(t_i)^2} \right\}^{1/2}. \quad (31)$$

It first follows a type of hyperbolic law due to the presence of the $1/V_{min}$ factor in the k coefficient of Eq. (30). It then departs from the hyperbolic behavior and eventually reaches a minimum error for some optimal V_{min} value. These are the values mentioned above.

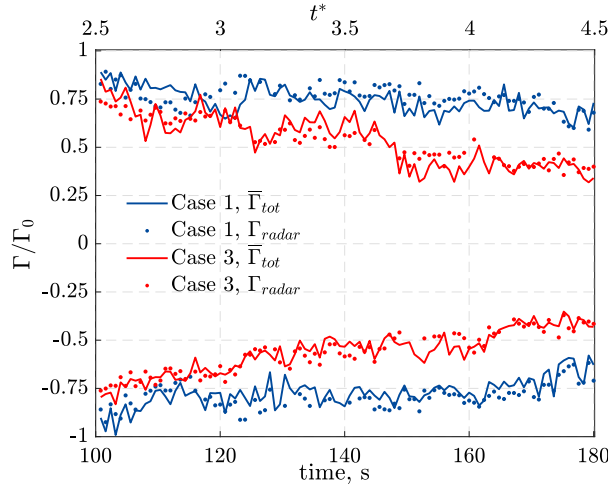


FIG. 19. Circulation estimated from the descent velocity of the WV.

After reaching the minimum, the curve reflects because of the quadratic form of the NRMSE definition.

To evaluate the influence of the misalignment of the radar resolution volume center relative to the WV core, the location of the radar cell center is changed in range from -45 to 90 m. For each shift in range, the circulation within a given radar cell is computed and Eq. (31) is used to calculate the error of the estimation (the right panel of Fig. 18). It can be seen that this error is minimal as long as the shift of the radar cell center is within ± 15 m relative to the center of the WV core. Deviations in the local minima of the NRMSE from the initial positions of the WV cores (range shifts of 0 and 50 m for the left and right WV, respectively) are due to the fact that the wake vortices do not remain symmetrical and centered at their initial positions as they evolve in the turbulent stratified atmosphere.

b. Setup B

In the absence of stratification, the relationship between the WV circulation and its descent velocity is defined as follows:

$$\bar{\Gamma}_{\text{tot}}(t) = 2\pi\bar{b}(t)\bar{V}_c(t), \quad (32)$$

where $\bar{V}_c(t)$ is the WV descent velocity and $\bar{b}(t)$ is the distance between the WV cores. This also works well for low stratification (Fig. 2d). However, it is no longer true for higher stratification as the WV are also subjected to a buoyancy effect. The validity of Eq. (32) is thus limited to the low stratification levels.

We note that De Visscher et al. (2013) showed that the influence of the stratification can be accurately modeled. For cases with significant stratification, the model they proposed could potentially be used for

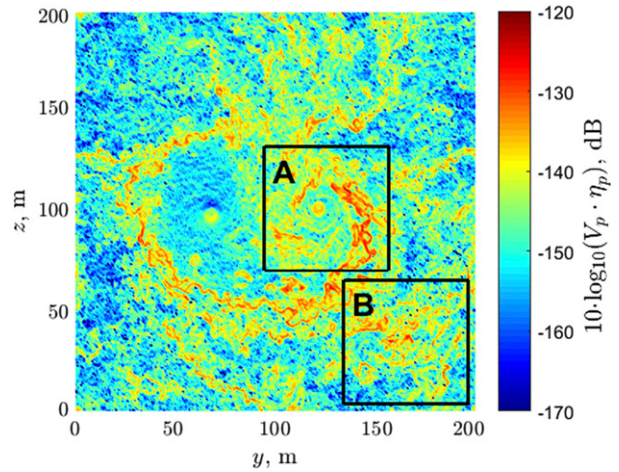


FIG. 20. Distribution of the RCS of the LES cells having a volume of approximately 0.5 m^3 .

estimating the WV circulation from the measurement of the WV descent velocity.

For geometrical setup B, the coordinates of the WV cores are estimated as explained in section 5b. Then the distance between the WV cores is computed and the descent velocity is estimated using three successive measurements for each WV core. Figure 19 shows the results of the circulation estimation obtained using Eq. (32). The parameters $\bar{V}_c$ and $\bar{b}_c$ of the descending WV are estimated with a reasonable accuracy (with some outliers due to the discontinuities in the estimation of the WV coordinates). As seen from Fig. 19, the method provides accurate estimates of $\bar{\Gamma}_{\text{tot}}$ for low stratification only (case 1). For higher stratifications, a significant deviation of the estimated value is observed, as expected.

c. Impact of the noise on the WV detection

Figure 20 provides an example of a distribution of the LES cells' RCS, calculated as $\eta_p V_p$, where η_p is the reflectivity (m^{-1}) calculated for the p th LES node having a volume V_p :

$$\eta_p = 0.38\lambda^{-1/3}(C_n^2)_p g(\eta k_B). \quad (33)$$

This distribution is given in a cross section of LES case 1. Two regions A (containing the left WV core) and B (outside the WV) are considered to illustrate the difference between the RCS calculated for the volumes with high and low EDR: the median values of $\varepsilon^{1/3}$ is $0.185 \text{ m}^{2/3} \text{ s}^{-1}$ for region A, and $0.075 \text{ m}^{2/3} \text{ s}^{-1}$ for region B. The regions are extended in the x -axis direction to form cubes having dimensions of $60 \times 60 \times 60 \text{ m}^3$, which roughly correspond to the size of the radar resolution volume used for geometrical setup A. The RCS of these

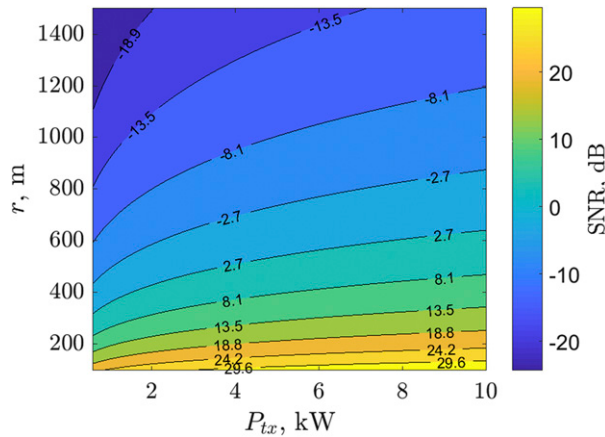


FIG. 21. Dependence of the SNR (estimated for a single pulse) on the distance between the radar and the WV and on the emitted power estimated for the WV RCS of -85 dB. The lowest limit of the emitted peak power range is 600 W.

volumes is -81.9 dB for A and -85.4 dB for B. The difference of 3.5 dB comes from the decrease of the attenuation function $g(\eta k_B)$ by a factor of approximately 2 corresponding to the $\varepsilon^{1/3}$ decrease from 0.185 to $0.075 \text{ m}^{2/3} \text{ s}^{-1}$ (Fig. 7).

Low RCS of the WV limits the radar detection on long distances. Figure 21 shows the SNR estimated for one pulse for different combinations of emitted peak power P_{tx} and range r . The value of -85 dB is taken for the WV RCS and the noise level of the receiver is -140 dB. The other parameters of the radar are given in Table 2. For the distance of 1050 m and the emitted peak power of 600 W, the single-pulse SNR is approximately -18 dB. However, it is estimated that 7–10 dB of the single-pulse SNR is needed for the accurate retrieval of the WV circulation from the Doppler spectrum (geometrical configuration A). For this SNR, the side parts of the Doppler spectrum (Fig. 11) are above the noise floor. So, for the emitted power of 600 W, the WV circulation can be retrieved at the distance of approximately 250 m. To increase the range, the emitted power should be increased correspondingly.

7. Conclusions

This paper presents a new application of the Muschinski et al. (1999) method and Hill's (1978b) spectral model to LES output data of wake vortices. The LES data serve as input for the simulation of the backscattered radar signal, based on the Bragg scattering by the turbulent field created by the WV. The ratio between the Bragg wavelength of the radar and the inner scale (itself proportional to the Kolmogorov scale) is an important parameter for the detectability of WV in clear air. The simulation of the radar signal backscattered by

the WV, and the associated turbulence, is used for the calculation of the Doppler spectrum, and two models for the retrieval of the circulation are tested. The Doppler spectrum components containing the useful information on the WV are located at approximately 20 dB below the peak of the spectrum. Therefore, the circulation can be estimated fairly accurately if the level of backscattered signal is high enough above the detection threshold of the radar. Moreover, for cases with low stratification, the circulation can also be estimated well using the measured descent velocity and spacing between the vortices. The developed platform and the tested models/methods, constitute a useful tool for the optimization of the radar configuration and parameters and for the improvement of the link budget. It could also be used, alongside other algorithms, for a benchmarking of methods for the estimation of WV circulations in clear air.

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