Uncertainties in a beam model belonging to a residential tower

L. Sgambi, E. Garavaglia, N. Basso

Department of Civil and Environmental Engineering, Politecnico di Milano, Milan, Italy

R. Pavani

Department of Mathematics "Francesco Brioschi", Politecnico di Milano, Milan, Italy

ABSTRACT: The starting point of this study are some student projects developed within the 2012 Mathematical Models in Architecture and Numerical Analysis Course, at Architecture of Building School, Politecnico di Milano. It deals with different models of a beam for a residential tall building floor. Assuming that every existing structure can be analyzed by a more or less extended model (completeness) with different accuracy (concreteness), the provided modeling points out some approximate solutions. The study is developed starting from the most simple solution of a beam model up to considering the complete frame of the building. Progressively improving the most simple model, the relevance of common modeling hypotheses within structural design is shown. Comparing the above-mentioned models, different solutions can be achieved. Some unusual but interesting questions are pointed out. The apparent statement that "the more complete and concrete is a numerical model, the closer it is to the truth" may be misleading: the accuracy of a model depends on the data input availability. Since uncertainty often affects experimental data and mechanical theories are based on assumptions, the more data provided by the input model, the more uncertain is the model. Thus, the most complex model doesn't always provide the optimal solution. Apparent correspondence "the more complete and concrete is a numerical model, the closer it is to truth" may be misleading: the accuracy of a model depends on the data input availability. Since uncertainty often affects experimental data and mechanical theories are based on assumptions, the more data input model provides, the more uncertain the model is. The most complex model isn't always the optimal solution.

1 INTRODUCTION

The study presented is based on a research carried out by a group of students within the 2012 Mathematical Models in Architecture and Numerical Analysis Course, Building School of Architecture at Politecnico di Milano. This research aims to highlight the model uncertainties that are always present when an engineer or an analyst chooses a particular numerical model to reproduce an aspect of reality.

1.1 *Some knowledge on numerical modeling*

Assuming the mentioned Bateson's considerations as "principles" (Bateson, 1979), numerical modeling and its errors and approximations have been explained:

1) Perception is the ability to feel the difference. Information contains the idea of "difference", and a threshold always limits the feeling of that difference.

2) The map is not the territory (coined by Alfred Korzybski), and the name is not the thing named. Bateson easily explains this multilevel idea through the comparison between coconuts and pigs as concrete items and their mental representation and categorization: "[…] and when we think of coconuts and pigs, there are no coconuts or pigs in the brain".

According to the Bateson's point of view, as well as for human reasoning, the above-mentioned ideas can be effectively applied both to design and numerical modeling. Considering the second point, a numerical model, is a representation of reality, as a map is. Naturally, a representation does not include any information present in the reality. Hence numerical models are always simplified views of the real problem (Sgambi 2005). Since an analyst does not have the suitable skills to handle all the information present in reality, a simplification process is needed. This means that numerical models are always approximated.

Experimental uncertainties involved in a structural problem make the related numerical model more complex (Bontempi and Giuliani 2010, Malerba et al. 2011). Concerning a generic structure, there are several kinds of approximation:

– the effective materials behavior is unknown. On the basis of simplified (less or more accurate) hypotheses (e.g. linear elastic material, plastic, etc.), mathematical formulations simulate the material behavior;

– the structural shape requires a complex solution. All structural mechanics theories geometrically simplify the problem (e.g. fundamental hypothesis of Eulero-Bernoulli's beam and Kirchhoff's slab). That's a real geometry conceptualization;
– the loads are affected by uncertainty. A comprehensive analysis of all possible load configurations over the structure's life is impossible. Hence, basing on specifications or experts' studies, load values are chosen.

Since so many approximations have to be dealt with, the best strategy is to perform several numerical models on a structural system. Then comparisons of different models allow to accept or reject our hypotheses and related approximations (Sgambi et al. 2006).

2 STRUCTURAL MODELLING

2.1 *Sample project*

In order to explain the two principles set out in §1.2, a residential tower developed over a yearly Architecture and Construction Design Studio (Professor Maria Grazia Folli) has been proposed as case study (Sgambi et al. 2013). The building is a practically square concentric plan tower. The vertical distribution system, in reinforced concrete, is represented by the core walls of the building (Figure 1 and 2).

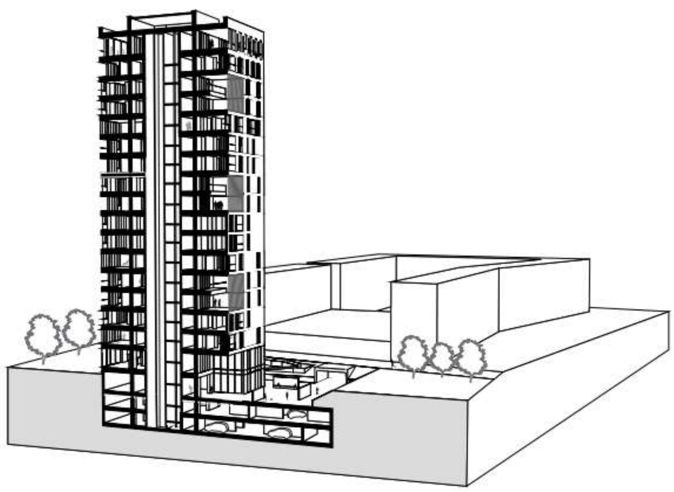

Figure 1. Perspective section of the building taken as an example.

Horizontal distribution is developed around that main core. A technical wall divides the circulation corridor and the apartments' water supply belt. Thus the plan flexibility is optimized. Some small exhibition areas are placed in the public lower ground floor (-3.00 m). Some ateliers lie on the first floor. The other floors have residential function, while collective areas are situated at the top of the building.
The purpose of this paper is apparently simple: the internal actions analysis of the red-highlighted beam (Figure 3). The beam's ends are weld-joined with a column and the staircase. Both the joints have been designed as stiff as fixed inner joint. The main direction of the floor runs in parallel to the beam analyzed. A couple of secondary beams sustains the floor and shares the load to the main beam.

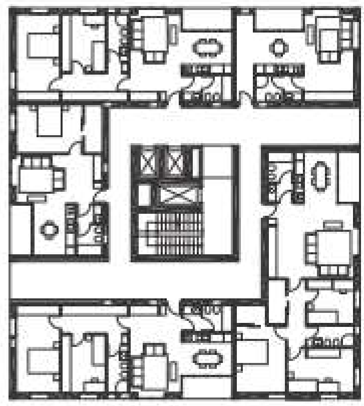

Figure 2. Distribution plan of a typical floor of the building.

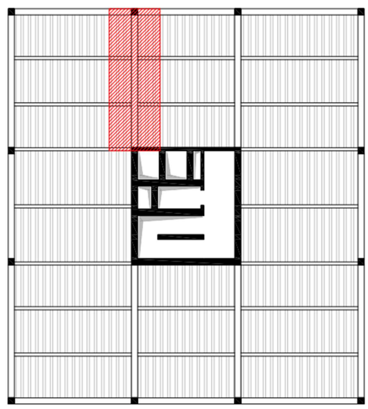

Figure 3. Structural plan of a typical floor of the building.

2.2 *Methods for modeling*

Some hypotheses have to be assumed for modeling. These hypotheses influence the accuracy of the numerical response.

Considering a simple steel beam of a building, an Euler-Bernoulli's beam model is the simplest representation of the real structural element. The analysis of the simple supported beam provides good approximation of the inner actions, leading the structural dimensioning. Simply supported beam leads no bending moment at the supports. If the joint between the beam and the column is bolted, the above-mentioned hypothesis might be considered correct even if it is unreal. A shell finite element analysis enables to model the beam. Setting constraints in bolts, joints can be carefully modeled. Thus bolts system partially shares bending moment. Although this is an approximate representation, it's a more realistic (concrete) than the previous one. Then a third modeling has been performed. Assuming the section beam weakness due to holes right in the joints, bolts deformability can be modeled. Once again the model is more concrete than the previous but it doesn't show the real structure system.

The above-mentioned remarks are valid in the case of bolted joints. On the other hand, if welded joints are considered, the behavior of the joint is stiffened and a greater amount of bending moment is transferred from the beam to the column and from the beam to the core walls. That amount of bending moment depends on the node deformability as well as on the size of the numerical model adopted.

Furthermore, the model extension has to be considered. It is clear that a numerical model may wholly or partially represent the structure. Referring to the case study, the numerical model may consider just the simple beam or it may belong to an equivalent frame system made from the structure (Figure 3). Since the columns in the latter configuration are fix, the beam rotation is partially avoided. Hence, some bending moment is led to the supports. This frame model seems to be more effective (complete). However since equivalent frame models assume system's symmetric configuration, column's stiffness is quite approximate. Considering a whole structure frame as well as the whole 3-dimension building structure, a better approximation might be gained. Then the model might be more complete.

Therefore the main question arises: which is the optimal numerical modeling solution?

Just relying on practical experience, an answer can be achieved. The *Mathematical Models in Architecture and Numerical Analysis* Course provides effective analysis tools and knowledge, leading students to understand numerical modeling assumptions. Considering 4 models with different completeness degree, the bending moment of a beam belonging to a residential tower has been analyzed:

1 Fixed-supported beam; it enables to approximately simulate the different stiffness between staircase and column.

2 Equivalent frame; it enables to approximate well columns stiffness.

3 Planar semi-frame; it enables to optimize columns stiffness modeling.

4 Planar frame; it enables to approximate well both columns and staircase stiffness.

In addition to these four models, two other models with a higher degree of concreteness have been introduced. These models are similar to models 1 and 2 presented but with a fine discretization by "shell" finite elements.
Comparing above-mentioned models, different solutions will be achieved. Some unusual but interesting questions will be pointed out.

3 DIFFERENT MODELS' RESULTS

The simplest model refers to a beam fixed at left and at right end. Right fix-joint represents the staircase high flexural stiffness; left fix-joint represents the stiffness of the welded joint between the beam and the column. Concerning completeness, that's the simplest model. It shows a high level of approximation due to entirely rigid joint (high flexural rigidity). It is an easy-to-manage model because results can be evaluated without any computational code.

The first model (Figure 4) has been analyzed by force method and computational code. Table 1, columns "Model N.° 1" show the bending moments provided by the two analysis approaches. The bending moment for the point load acting on the left side of the beam in column's simple-joint and the bending moment in staircase's fix-joint were calculated. Bending moment for each floor was the same.

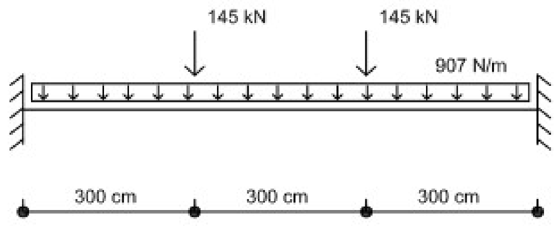

Figure 4. Geometrical representation of the Model N.° 1.

Using two semi-columns, the column's flexural stiffness in equivalent frame model (Figure 3, right side) has been modeled. Considering just the vertical loads influence and a tower of infinite floors, hinge joints in semi-columns' end represent the zero bending moment at the mid-column. Once again considering high flexural rigidity, a fix-joint was used to model the staircase. Table 2, columns "Model N.° 2" point out that model 2 is more complete than model 1.

Since model 2 assumes a virtually infinite tower, it supposes translational symmetry condition. Otherwise if the tower had finite floors, the different end-floors rigidity would have influenced the real solution both at first and top floors. But the equivalent frame model cannot represent this particular stiffened condition. Thence, referring to a middle-floors-beam, model 2 provides optimal solutions. While considering first-floors-beam as well as top-floors-beam, model 2 provides very approximate solutions. Changed the hinge joints' set-up on the column, columns flexibility would be modified and the system response might be improved. Since the right position of hinge joints is unknown, a further and more effective flexural rigidity modeling method may be introduced. It extends the numerical model over the whole tower.

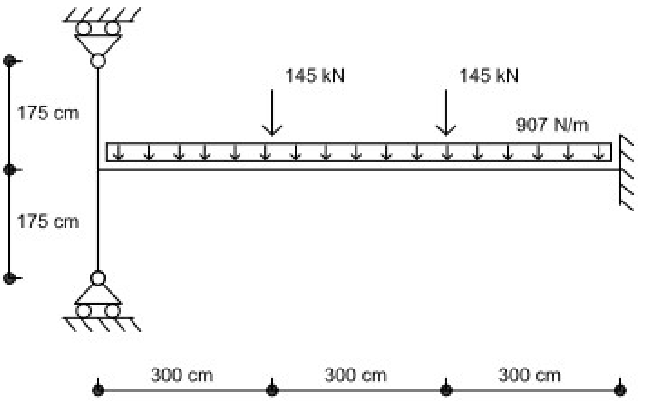

Figure 5. Geometrical representation of the Model N.° 2.

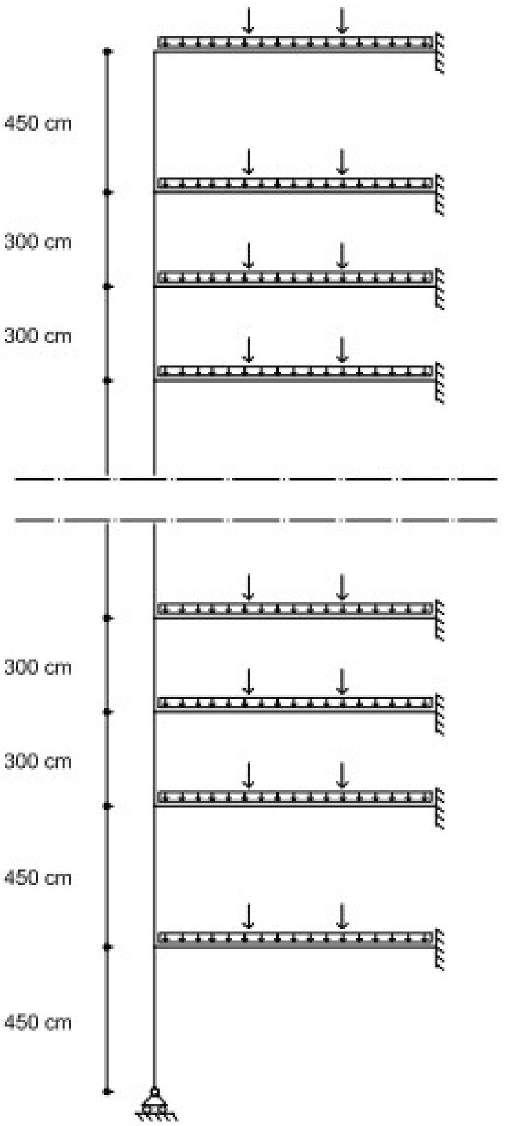

Figure 6. Geometrical representation of the Model N.° 3.

Basing on model 2, model 3 has been developed. The whole column has been modeled. First and top-floors-beams have a more accurate flexure joint. Staircase remains modeled as a fix-joint. Model 3 is more complete than model 2. Table 1, columns "Model N.° 3" show the results. Comparing model 3 with both model 1 and model 2, relevant difference (up to 50%) in bending moment values have been pointed out. Such a significant difference doesn't result only from the flexural rigidity modeling approach chosen. The whole reasoning here proposed is based on principle 1 (*perception is the ability to feel the difference*) stated in §1.2. The comparison between different models leads to new knowledge.

Although model 3 is more complete than model 2, it is more approximate. The column has been entirely modeled in order to optimize flexural stiffness modeling and the staircase has been modeled as a fix-joint. Considering flexural deformation, the third method is more efficient; while referring to axial deformation, the model introduces some errors. Model 2 has no axial deformation in both the end-joints, while model 3 has finite deformation of the column and no deformation in the staircase joint. Comparing with model 2, the relevant approximation in model 3 causes bending moments very different in values. Modeling the staircase axial deformation, model 3 may be optimized.

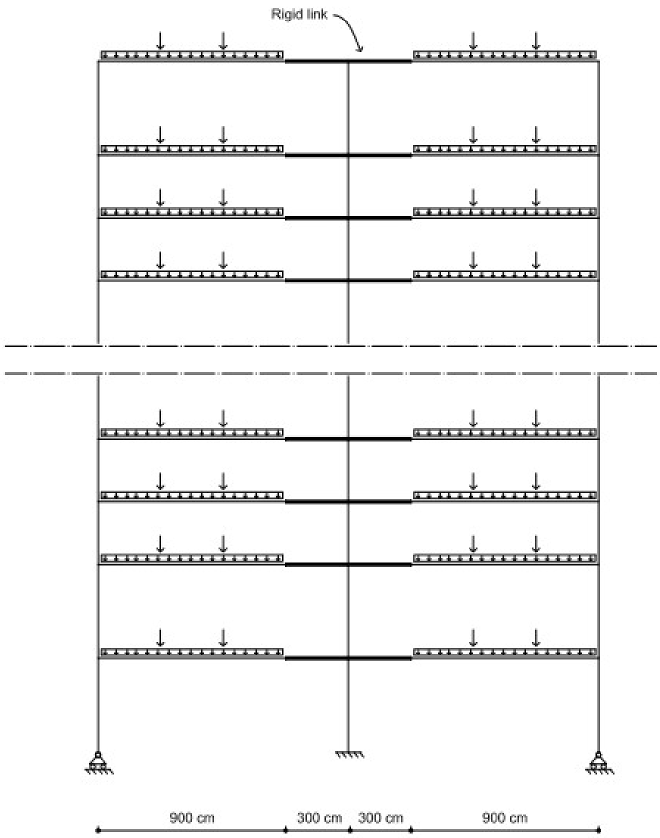

Figure 7. Geometrical representation of the Model N.° 4.

The whole staircase modeling has been implemented within model 4 evaluation. A Building's frame system has been performed. Bending moments provided by model 4 place themselves between model 2 and model 3 values. Referring to model 4, the average error in model 1 is 20% with peaks of 35% (top floors), the average error in model 2 is 13% with peaks of 25% (top floors).

Models from 1 to 4 explore the modeling space in the direction of completeness. However, as described in §2.2, the modeling space has at least two coordinate axes: completeness and concreteness.

Model N.° 5 and model N.° 6 (Figure 8 and Figure 9) explore the concreteness direction. Move along the direction of the concrete brings more information into the model. The joint between beam and column, in models 2-4 was simply modeled as continuity joint. The same joint in model 5 can be modeled using a fix restraint for each bolt of the joint. This representation is more realistic because it allows to investigate the trend of the tension near of the restraints (diffusive zone). Using a shell model, the exact position of each bolt can be modeled.

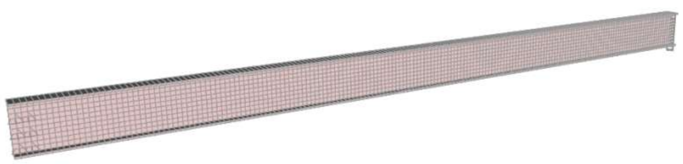

Figure 8. Discretization of the Model N.° 5.

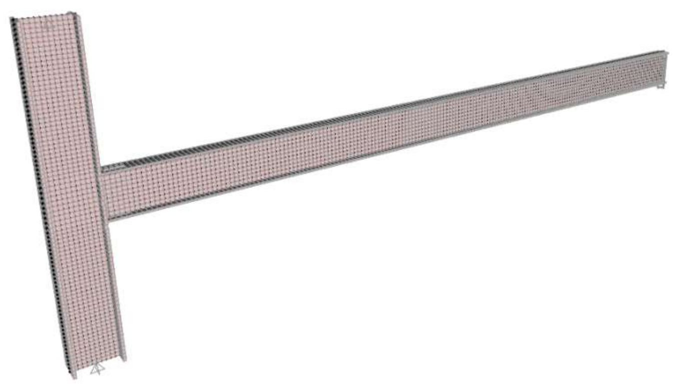

Figure 9. Discretization of the Model N.° 6.

Model 6 is similar to model 2, just more concrete. In this case, the bolted joint can't be modeled using restraints, because joined the with the column. To model the joint the mesh of the beam is kept separate from the mesh of the column and six constraints (equal displacement condition) have been introduced in correspondence of the six bolts. Another difference between model 2 and model 6 is on symmetry restraints. In the model 2 the column section on the hinge restraint remain plain for the assumptions of the Bernoulli-Navier beam. To simulate this assumption on the model 6, a rigid material was introduced in the element in the section close to the hinge point. Models 5 and 6 are the richest and the information to be introduced are many more than the previous ones. Not always a richer model is also more usable. In fact models 5 and 6 provide many more information than previous, but such information is not directly comparable. Tensions, provided in every point, must be integrated by a numerical integration to derive the value of bending moment present in the sections of the beam.

Bending moments, measured near of the column, in the central bay, and close to the lift shaft, for models 5 and 6 were lower than the analogous moments evaluated with models 1 and 2.

Figures 10, 11 and 12 summarize the comparisons among all the numerical models considered and for all floors of the building.

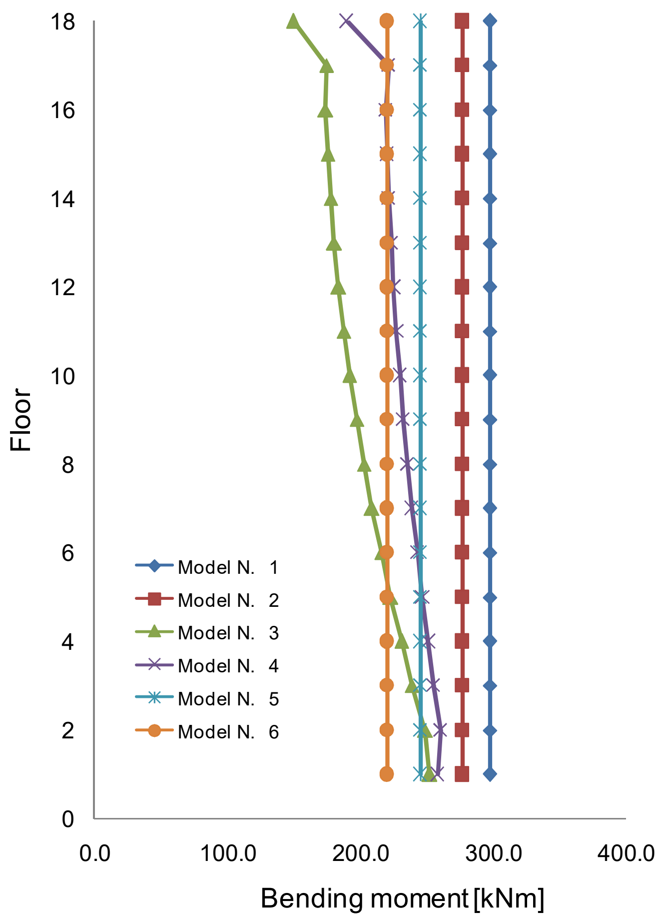

Figure 10. Comparison between the values of bending moment evaluated close to the column.

Comparisons show a very strong dispersion of values. Any model investigated could be used in a common design. Models 1 and 2 are probably the most widely used because they are more straightforward to use. In our case, these models always show values of bending moment higher than the other models.

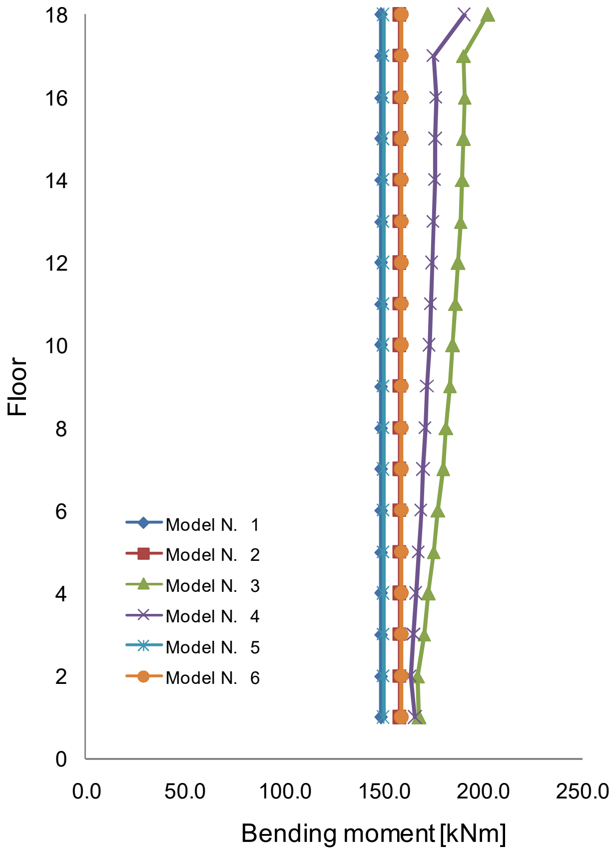

Figure 11. Comparison between the values of bending moment evaluated in central bay.

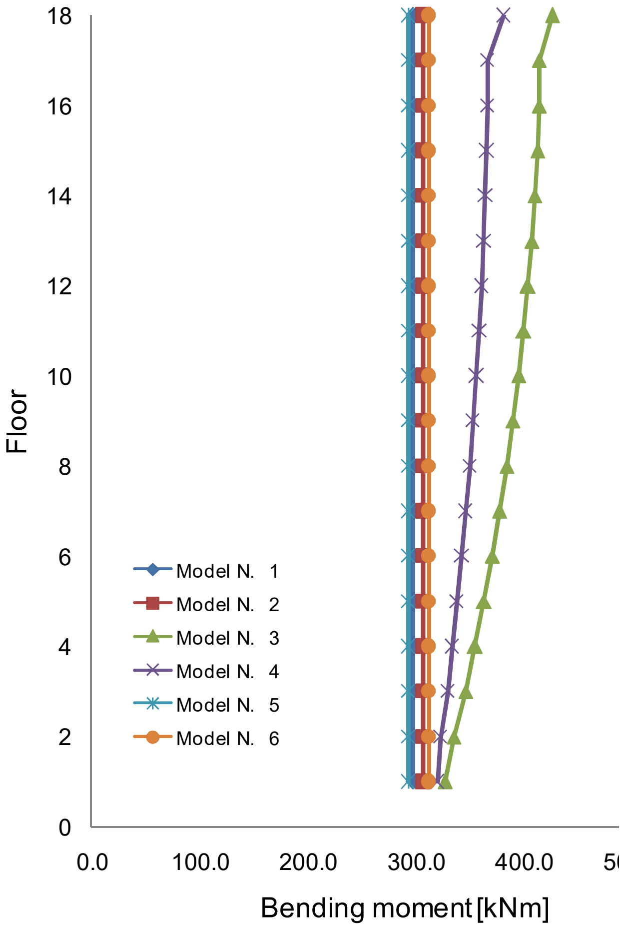

Figure 12. Comparison between the values of bending moment evaluated close to the lift shaft.

In order to estimate the dispersion of the results, for each floor, the standard deviation of the values of moment measured with all models was evaluated (Figure 13).

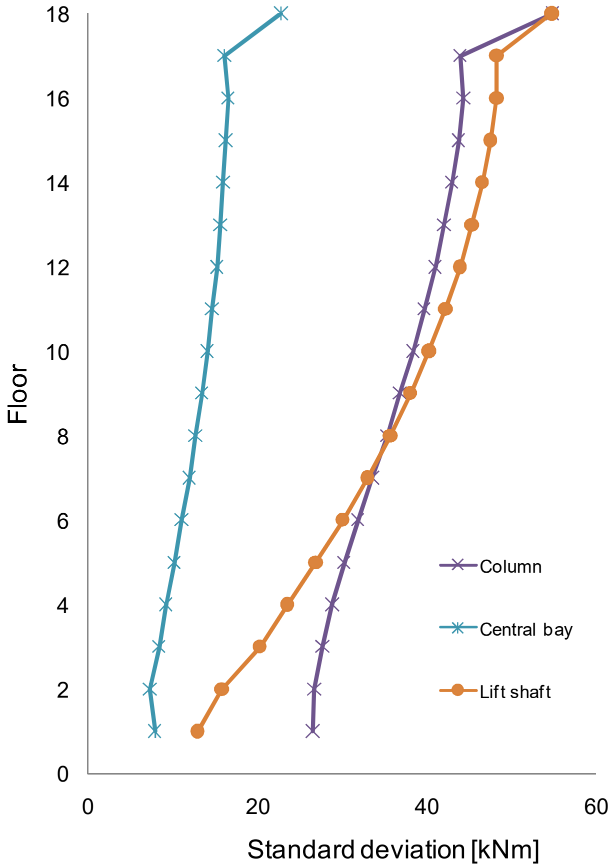

Figure 13. Standard deviation of the values of moment.

It is interesting to note that as the structure becomes more flexible the results provided by the models become more dispersed. The minimum dispersion occurs at the foundation level, where rigid hinge are present, while the maximum dispersion occurs close to the top floors.

4 FINAL REMARKS AND CONCLUSION

Considering the principle *The map is not the territory, and the name is not the thing named*, the bending moment of a beam in four numerical models has been analyzed. The models proposed represent the real system with different accuracy degree. Model 1 is an easy-to-apply and small-scale-map, even if it is approximate. From the second to the fourth model the map has been enhanced (e.g. columns flexural deformation, staircase axial deformation, etc.). *N*one of the four models proposed proves the real correspondence between real inner actions and values calculated. Although science provides hypotheses and improves them (e.g. from model 3 to model 2) or confutes them (e.g. from model 3 to model 4), correct description of reality may be only approached. Model 4 is the most complete among the models proposed. But as well as the other three models, model 4 is an approximate representation of the reality. Some of its main lacks are listed here below:

- Bottom restraints represent flexible joints leaning on the ground;
- Acting loads assessment result from an approximate method referring to influence area of beams;
- Since the real structure is a 3-dimension system, surely there exists any stiffness out of the plane considered;
- Frame joints don't behave as simple inner fixed joints.

Model 5 and model 6 are similar to models 1 and 2 but with a fine discretization by "shell" finite elements.

Considering the above-mentioned points, the numerical model may be improved. Since *Perception is the ability to feel the difference*, a "simple-to-complex" approach has to be preferred. Comparing solutions provided by means of different models, hypothesis relevance as well as results accuracy must be evaluated.

The apparent correspondence "the more complete and concrete (§2.2) is a numerical model, the closer it is to truth" may be misleading: the accuracy of a model depends on the data input availability. Since uncertainty often affects experimental data and mechanical theories are based on assumptions, the more data input model provides, the more uncertain the model is. The most complex model isn't always the optimal solution.

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