



Université Catholique de Louvain
Institute of Mechanics Materials and Civil
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von Karman Institute for Fluid Dynamics
Turbomachinery and Propulsion Department

Development of Methods For the Characterization and Uncertainty Assessment of Turbulent Quantities in High Speed Compressible Flows by Hot Wires and Fast Response Pressure Probes

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List of Acronyms

A

C

CCA	Constant Current Anemometry
CFD	Computational Fluid Dynamics
CTHWA	Constant Temperature Hot Wire Anemometry
CVA	Constant Voltage Anemometry

D

DNS	Direct Numerical Simulation
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F

FFT	Fast Fourier Transform
-----	------------------------

H

HPT	High Pressure Turbine
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L

LES	Large Eddy Simulation
LPT	Low Pressure Turbine

M

MC	Monte Carlo
MBB	Moving Block Bootstrap

O

ODE	Ordinary Differential Equation
-----	--------------------------------

P

PDE	Partial Differential Equation
PLA	Phase Locked Averaging

R

RANS	Reynolds Averaged Navier Stokes
RPM	Rotations per Minute

T

TCF	Turbine Center Frame
TMTF	Turning Mid Turbine Frame
TSM	Taylor Series Method
TVF	Turbine Vane Frame

V

VKI	von Karman Institute
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Abstract

While numerical simulations are a fundamental analysis but also design tool in the turbomachinery industry, experimental validation is still as relevant as ever. Turbulence modelling and the lack of well defined boundary conditions are two of the main limitations in the application of CFD in turbomachinery flows, with the characteristics of the inlet turbulent flow being usually the biggest unknown. Moreover, as turbulence plays a crucial role in the behaviour of all the components (affecting heat transfer, boundary layer transition, losses), its characterization is fundamental in all levels of wind tunnel testing (from linear cascade testing to complete component testing). To meet these demands, more reliable turbulence measurements are needed. Nevertheless, the experimental characterization of turbulence is a challenging task, as it requires demanding measurements in terms of quality and resolution. This task is even more challenging in high speed and complex flows like turbomachinery flows, due to the limitations of measurement techniques in such environments.

In this context, the goal of this thesis is to provide tools for the enhancement of turbulence measurements in challenging flow environments, typical of industrial applications. The focus is placed on two well-established measurement techniques with widespread application in both industry and research: the Constant Temperature Hot Wire Anemometry technique (CTHWA) and fast response aerodynamic pressure probes.

The application of CTHWA in complex flows is reexamined. Despite being the most popular tool for turbulence measurements, its application is still limited. By evaluating its sensitivity to all flow properties, the reliability of CTHWA to measure turbulence in different flow regimes (compressible, low-density flows) is evaluated and non-dimensional calibrations and multi-wire configurations are addressed. The dynamic response of CTHWA and its effect on the measurement of turbulence parameters is investigated in detail. Non-optimal adjustment of the response, heat conduction effects, the inherent non-linearity of the system and bandwidth limitations are identified as the potential errors sources. The first two can be avoided and/or corrected with an appropriate design of the probe and the application of a transfer function. Using a synthetic turbulence signal, an evaluation of the error on the turbulence measurements due to the latter is provided, as a function of the nature of the turbulent flowfield.

Fast response pressure probes are extensively used for unsteady measurements in turbomachinery flows since they are more robust and thus more adapted to harsh environments. The detailed calibration steps (static, aerodynamic and dynamic

calibration) required for reliable measurements are provided. Particular attention is placed on the dynamic calibration of probes in shock tubes. A methodology for the extraction of a transfer function is proposed, considering the disturbances present in such shock tubes. An uncertainty estimation on this dynamic calibration method is presented as well. Finally, the case of the estimation of turbulent quantities with single-sensor probes is treated, critically discussing underneath assumptions.

As any reliable measurement should be accompanied by uncertainty bounds, an uncertainty estimation methodology for turbulence properties is presented. The measurement uncertainty stemming from the instruments and measurement methodology is computed using a Monte Carlo method. The statistical uncertainty, defined as the uncertainty due to the limited measurement time, is estimated using the Moving Block Bootstrap method, a resampling algorithm.

The thesis concludes with two test cases. Firstly, the uncertainty quantification methodology is applied to turbulence measurements performed at the inlet of a short-duration turbine test rig. The confidence intervals of the spanwise distributions of mean velocity, turbulence intensity and length scales are provided, as well as the contributions of the two main uncertainty components. Finally, a test campaign for the turbulence characterization of the inlet and outlet flowfield of an inter-turbine duct at an industrial test rig is presented.

1

Introduction

1.1 Research Motivation

A rigorous characterization of flow turbulence (i.e. turbulence intensity, time and length scales, even spectrum) is needed to better understand loss mechanisms and heat transfer phenomena in complex turbomachinery flows. In high pressure turbines (HPT), the freestream turbulence is one of the main parameters defining the heat transfer process and magnitude to the vanes [1]. In low pressure turbines (LPT), the freestream turbulence intensity and length scales are critical for the boundary layer transition and/or separation along the blade surfaces, and therefore for the component's losses [2]. An illustration of the importance of the whole turbulence spectrum for the prediction of turbulence effects on laminar boundary layer transition and heat transfer has been given by Mayle et al. [3]: they demonstrated that the effects of free-stream turbulence are dependent on its spectral distribution, and not just its level, or its level and length scale, but its full spectrum. Detailed and accurate information on turbulent quantities in relevant conditions is thus significant for the successful, reliable and cost-effective design of these components.

With the advances in computer performance, CFD has become a primary design tool for turbomachinery components. Considering the high specific cost of experimental facilities (manufacturing, operation), it is also becoming a widely used investigation tool, substituting more and more experimental investigations. Nevertheless, as discussed by Denton [4], there are still significant limitations on

the use of CFD, especially for complex turbomachinery flows, which underline the need for experimental validation. Among those limitations, turbulence modelling and unknown or incomplete boundary conditions are included. The calibration of turbulence models is very often based on simple laboratory cases which do not represent the real flow conditions, imposing the tuning and adjustment of the model constants for specific applications. Though the use of high-fidelity CFD, like Large Eddy Simulations (LES), may mitigate the errors due to modelling, the quality and validity of simulations still depend on the accuracy of the imposed boundary conditions. From the point of view of turbulence, these normally include the turbulent kinetic energy and length scales at the inlet of the computational domain, information which is rarely completely known.

It is interesting to compare two review articles on the trends in turbulence modelling and simulations for turbomachinery flows at a distance of 6 years (with one common author). Tucker, in his review article in 2013 [5], highlighted the limitations of turbulence modelling in the case of RANS (Reynolds Averaged Navier Stokes), as well as the challenges in the application of eddy-resolving methods. He underlined the need for relevant experimental data: *“There is the need for improved measurements, with well-defined boundary conditions, that supply Reynolds stress and even spectral information. This, ideally, needs to be at Reynolds numbers that connect with typically powerful turbomachinery systems. These measurements are needed to refine both RANS and eddy resolving strategies.”* Tyacke et al., in their 2019 article [6], stress on the capabilities of eddy-resolving methods for a wide range of flows and encourage its gradual integration to the design cycle by the turbomachinery industry. Due to the high computational cost of LES, it is a common strategy to use RANS-LES zones where appropriate inlet turbulence has to be generated at the domain boundaries and the interfaces between modelling zones. They stress on the lack of complete experimental inflow turbulence data (other than the turbulence level) that can be fed to synthetic turbulence generators or reduced order models. The severe lack of data for validation is also highlighted. Based on these reviews, it is clear that despite the progress of numerical simulations for turbomachinery applications in the last decade, the need for detailed experimental turbulence data is as relevant as ever.

The problem of the validation of advanced CFD methods has been posed by Sagaut & Deck [7]. They presented a classification of the validation level for unsteady CFD data in six grades, starting from integral forces up to high order statistics and time-frequency analysis. These validation levels are given in Table 1.1. The validation level should normally be proportional to the level of fidelity achieved by the simulation method. Nevertheless, validation levels higher than 2-3 (first and second order statistics) are rare in the literature. Tyacke & Tucker [8] reviewed more than 200 publications containing validation data for turbomachinery applications, as can be seen in Fig. 1.1. Only 17% of the works contained

grade 3 validation level data and only 7% grade 4. This percentage is much lower compared to data available for airframe zones. Experimental research at industrial conditions is therefore needed, in order to provide reliable data for CFD input and validation.

Grade	Level of validation
1	integral forces (lift, drag and pitch)
2	mean aerodynamic field (velocity or pressure profiles)
3	second-order statistics (r.m.s. quantities)
4	one-point spectral analysis (power spectral densities)
5	two-point spectral analysis (correlation, coherence and phase spectra)
6	high-order and time–frequency analysis (time–frequency, bicoherence spectra)

Table 1.1 – Validation levels for simulation techniques, from Sagaut & Deck [7].

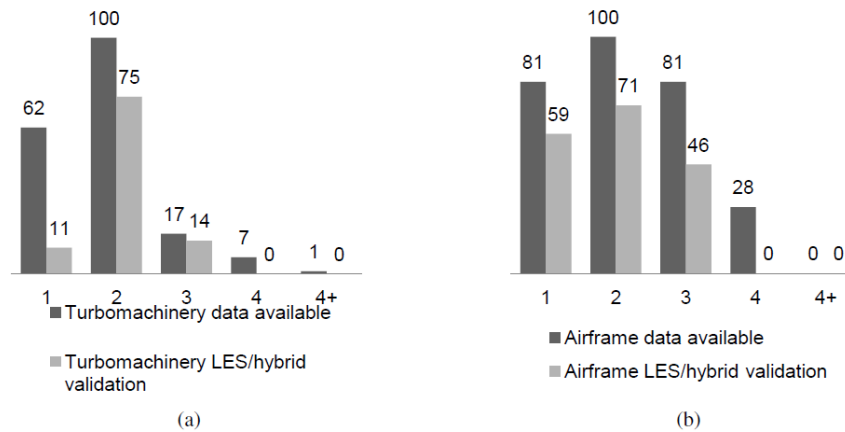


Figure 1.1 – Review of validation levels in the literature for a) turbomachinery, and b) airframe zones, by Tyacke et al. [8].

For the case of turbomachinery flows, Tyacke et al. [6] attributed this lack of data to problems of limited access and flow disturbances that prevent the performance of measurements. In addition to that, the flow conditions through a turbomachine vary significantly, reaching high values of temperature, pressure and Mach numbers, while the flow is highly unsteady and three-dimensional. At the same time, the complex nature of turbulence imposes demanding measurements to obtain sufficient information: high frequency response, high spatial resolution and sufficient measurement time are required. Due to the limitations of currently established experimental methods, turbulence data in such conditions are very limited in literature.

The aim of this thesis is to provide an extension of the applicability of the most commonly employed intrusive measurement techniques for the characterization of turbulence in both industrial and academic applications: the Constant Temperature Hot Wire Anemometry technique (CTHWA) and fast response aerodynamic pressure probes. Despite its fragility, CTHWA is one of the most common tools for turbulence research and often the reference technique against which other methods are validated. Nevertheless, as a consequence of its mainstream usage, the peculiarities of its application are often overlooked. This can lead to poorly mastered experimental data and post-processing, with a general lack of fidelity on the final results, for example when fluctuations on the flow properties other than the velocity are not considered. Fast response aerodynamic pressure probes are more robust and are increasingly used for time-resolved measurements in turbomachinery flows. In the case of turbulence measurements, their main limitation is that their response to velocity fluctuations is not straightforward. In the following, both techniques are analyzed in terms of calibration, application and processing of the acquired data and a thorough uncertainty analysis procedure is provided. Finally, a set of guidelines for their robust application in relevant complex flows is provided.

1.2 State of the art

1.2.1 Turbulence characterization in turbomachinery flows

Experimental turbomachinery studies containing data up to grade 4 validation level are summarized here. Camp & Shin [9] were one of the first to provide turbulence data in a turbomachinery application. They measured the turbulence intensity and integral length scale in a low-speed multistage compressor rig using hot-wire anemometry. Measurements were performed at the inlet and outlet of the stator of an embedded stage. Oro et al. [10] analyzed the structure of turbulence at a single stage, low speed axial fan by performing hot wire measurements. They measured the turbulence intensity and length scales at three axial stations (inlet to the stage, rotor exit, stator exit). Maunus et al. [11] retrieved the turbulent kinetic energy, dissipation and integral length scale from structure functions in a turbofan stage, by hot wire measurements between the rotor and the fan exit guide vanes. More recently, Odier et al. [12] acquired the evolution of the turbulence intensity and integral length scale through the fan stage of a small geared turbofan, using hot wires. Looking at relevant studies in a turbine environment, Axelsson et al. [13, 14] measured the turbulence intensity and length scales at the exit of an HPT (inlet of an inter-turbine duct) in a low-speed facility using hot wires.

Other relevant studies in turbines are limited to statistics up to grade 3 validation level (turbulence intensity values). Porreca et al. [15] performed turbulence

measurements with fast response pressure probes in a two-stage turbine test rig. Chasoglou et al. [16] performed turbulence measurements in the same test rig with a multi-hole fast response pressure probe, evaluating also the anisotropy level of the flowfield. Lengani et al. [17] retrieved the turbulence intensity downstream of an LPT rotor using a single-sensor fast response pressure probe. Bauinger et al. [18] performed turbulence measurements with hot wires and fast response pressure probes in a two-stage two-spool turbine test rig, downstream of the LPT rotor. They provided a comparison of the turbulence intensity computed by the two techniques.

It is clear from the literature review that even integral length scale information is rare in experimental turbomachinery test cases (with the exception of fan characterization, for which a higher number of studies have been reported). Such data in engine representative conditions (in terms of Mach and Reynolds number) are even more rare, as many studies have been carried out at low-speed facilities (e.g. [13, 14]). This lack of data can be attributed to the difficulties in applying suitable measurement techniques in such environments but also to the complexity of the required post-processing methods when deterministic fluctuations are also present in the signal.

Looking into studies measuring the inflow turbulence for experimental test cases, more complete characterizations can be found. In the case of HPT testing, turbulence intensity and integral length scales at the inlet of the test rig are provided by different researchers, e.g. [19–21]. For LPT testing, experimentally obtained inlet turbulence intensities and integral length scales are reported e.g. by Schwarzbach et al. [22].

What is common for both boundary conditions and validation data is that the uncertainty of turbulence parameters is rarely stated. This issue has been raised by Favier et al. [23], who classified the uncertainty sources into two macro categories: the measurement and the statistical uncertainty. The first includes errors related to the instruments and experimental method, while the second stems from the averaging over a finite signal duration to obtain time-averaged statistics. The second item is also relevant for high-fidelity simulations, where the simulation time is often limited due to the computational cost. Indeed, no uncertainty values on the turbulence parameters are provided for the studies reported above. Odier et al. [12] who compared the experimental data with LES simulations, pointed out the issue of comparing statistics averaged over very different time periods, to be further treated in their future works.

1.2.2 Experimental methods

It can be seen that CTHWA is the main measurement technique used in the aforementioned studies. It is one of the most popular measurement techniques for

turbulence research, thanks to its high frequency response, high spatial resolution and good signal to noise ratio [24]. Despite being extensively used for many years, (the first works date back to the beginning of the 20th century, e.g. Boussinesq [25] and King [26]), its application to certain flow regimes like high subsonic and low density flows is still limited. Extensions to the applicability of CTHWA in such conditions have been proposed by researchers like Cukurel et al. [27] and Boyle et al. [28], but further studies are required before a general measurement methodology can be established. For those reasons, few studies exist in high subsonic/transonic and subsonic slip flows where many turbomachinery flows lie.

Due to its fragility (wire diameters of the order of 5-10 μm), CTHWA can only be used in clean flows. This led to the development of fast response aerodynamic pressure probes for time-resolved pressure measurements. When dealing with harsh measurement conditions, they have demonstrated a high degree of robustness and a wide range of applicability [29–31]. The first attempt to measure turbulence with fast response pressure probes was realized by Heneka [32] already in 1983 using a four-hole wedge pressure probe. Since then, researchers have focused mainly on single- or double-sensor probes in order to reduce the measurement volume and increase the accuracy. Wallace and Davies [33] proposed a methodology to relate velocity fluctuations to total pressure for incompressible and compressible flows, assuming isotropic turbulence. This methodology has been used in most of the studies in turbomachinery flows previously reported.

Due to the complex nature of turbulence, demanding measurements are required for its characterization. Sufficient measurement time, high spatial and temporal resolution are required. This requirement is even more important in high speed flows where higher frequencies are typically present. The instruments dynamic response is therefore crucial. For the case of CTHWA, the dynamic response is optimized using the ‘square wave test’. It is an electronic testing method, embedded in most commercial units, which estimates the cut-off frequency of the instrument assuming that the response remains flat up to this limit. This commonly used practice neglects potential error sources due to the dynamic effects. Moreover, while the effect of the finite spatial resolution on the measurement of turbulence measurements has been treated by different researchers (e.g. [34, 35]), the same has not been done for the temporal resolution, to the author’s knowledge.

As far as the dynamic response of fast response pressure probes is concerned, it depends on the geometry and configuration of each probe. For reasons of probe size minimization and spatial resolution reduction, the piezoresistive sensors are often placed inside the probe body. The acoustic behaviour of the line-cavity system connecting the sensor surface to the flow defines, in this case, the dynamic response, which in turn defines the useful bandwidth of the probe (which is usually a certain fraction of the resonance frequency). Two approaches are found in the literature. In the first case, the resonance frequency of the probe cavity was estimated

by measurements in a well-known flow and since it was found to be much higher compared to the desired frequency bandwidth, no compensation was deemed necessary by the authors. This approach has been followed by [36] and [16]. It has to be noted though that even when the energy amplification is negligible, there will still be a phase delay in the signal, which is not taken into account with this approach. In the second case, the transfer function of the probe is obtained by an experimental method, usually shock tube testing, as the fast rising steps in shock tubes allow the calibration up to frequencies of 500 kHz. This method has been followed by Persico et al. [37] for a single-sensor cylindrical probe. The transfer function was used to extend the useful bandwidth up to 80 kHz for a cavity resonance frequency of 49 kHz. Gaetani et al. [38] investigated the flowfield in low-pressure shock tubes (shock tubes operated at low pressure ratios, of the order of 1.3) which are typically used for turbomachinery applications. They observed the presence of non-ideal phenomena due to the partial opening of the diaphragm which resulted in the presence of strong oscillations in the signal, after the leading shock. They developed an analytical model to describe this behaviour. Persico et al. [37] had observed the contamination of the calibration signal due to these oscillations. They proposed a post-processing method for the computation of the transfer function in order to minimize the post-shock oscillations effect: cutting the probe under calibration signal shortly after the leading shock and assuming as input to the transfer function an ideal pressure step. Despite the current interest in the so-called ‘traceability’ of dynamic measurements and the effort to develop traceable dynamic calibration procedures [39], no standards yet exist for the computation of uncertainty in shock tube tests. Few studies exist in the literature on this subject, like [40] and [41], both of which assume an ideal step input.

1.3 Research objectives

The objective of the present work is to promote a more robust experimental approach to turbulence measurements in turbomachinery applications by providing an extended applicability of standard measurement techniques and a more thorough analysis of the results. The developments target a reduction of the experimental errors associated with the operation of a certain technique, together with the definition of a complete methodology for the computation of the associated uncertainty. When possible, a description of data processing algorithms is also presented for the sake of completeness and for future reference. The ambition is to generate a series of tools that could facilitate the achievement on a larger scale of grade 4 to grade 5 validation level in turbomachinery cases, therefore strongly supporting the development of high-fidelity numerical codes. The main contributions of this work are listed below.

The novel contributions to CTHWA focus on:

- A precise analysis of the hot wire sensitivities to the various flow properties, leading to a series of guidelines for its calibration and application based on the expected flow conditions.
- The modelling of the dynamic behaviour of constant temperature anemometers. This study contributes to the evaluation of the related experimental errors and to the definition of a strategy for the mitigation of such errors. At its final extent, the availability of such models allows the definition of the optimal geometrical parameters in the early design phase. A novel evaluation of the errors as a function of the existing turbulence flowfield, employing synthetic turbulence generation, should be particularly pointed out.

In the field of fast response probes, the following original contributions are hereafter described:

- The definition of a detailed post-processing method for the computation of the transfer function of fast response pressure probes. This method is tailored to the non-ideal flowfield in low-pressure shock tubes and further extends the work of Persico et al. [42].
- The evaluation of the dynamic calibration uncertainty of fast response probes in shock tubes considering also non-ideality. The uncertainty of the amplitude and phase of the transfer function is provided as a function of frequency.

The development of an uncertainty quantification methodology for turbulence measurements is another main contribution in this work. It takes into account both the measurement uncertainty and the statistical uncertainty budgets and it can provide the evaluation of confidence intervals for all quantities of interest.

Finally, two test cases are presented, showcasing the implementation of some of the developed concepts in a practical application and demonstrating the performance of turbulence measurements in a complex environment, also in terms of data reduction and processing methodologies.

- The first test case consists of the application of the uncertainty quantification methodology to turbulence statistics measured by CTHWA at the inlet of a short duration turbine test rig. The spanwise distributions of the turbulence intensity, skewness and length scales are provided with the 95% confidence bounds, while the relative contributions of the measurement and statistical uncertainties are reported, following the categorization of Favier et al. [23]. The implementation of the Monte Carlo method used for the quantification of the measurement uncertainty and that of the Moving Block Bootstrap method, used for the statistical uncertainty, are presented step by step, to act as a reference for similar studies in the future.

- In the second test case, turbulence measurements are performed with a hot wire and a fast response probe at the inlet and outlet of an inter-turbine duct, in an industrial test rig. The duct is placed downstream of an HPT component, providing a realistic inlet flowfield, while the tests are performed at engine-representative conditions of Reynolds and Mach number. The objective of this work is to provide the turbulence intensity and integral length scale at the inlet and outlet of the duct in order to support the performance of high fidelity simulations on the same configuration. It has to be noted that the only similar study found in the literature performed in such a configuration is, to the author's knowledge, the works by Axelsson et al. [13, 14], that were performed in a low-speed test rig. At the same time, this test case will provide an analysis of the required post-processing methods for the computation of such statistics in an environment with strong periodic unsteadiness. For the fast response pressure probe in particular, the aim is to examine its capability to provide turbulence quantities other than the turbulence intensity, usually found in the literature.

The complete list of publications that resulted from the present work can be found in Appendix A.

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2

Constant Temperature Hot Wire Anemometry

In this chapter, the basics of the operation and performance of Constant Temperature Hot Wire Anemometry (CTHWA) are covered. Different types of data reduction methods are presented, including the non-dimensional calibration method employed in the present work. The performance of CTHWA in different flow regimes is analyzed based on its sensitivity to different flow properties.

2.1 Introduction

2.1.1 Operating principle

The operation of Hot Wire Anemometry is based on the convective heat transfer from a heated sensor to the surrounding fluid. The CTHWA system is composed of the hot wire probe and an electronic unit which supplies and controls the operation of the sensor.

A single-wire straight hot wire probe is presented in Fig. 2.1. The sensor is typically a fine wire (diameters range between 5 and 10 μm) supported by two prongs. The sensor material should feature a high temperature coefficient of resistance a_w , with common materials being tungsten and platinum-based alloys (e.g. Platinum-Rhodium(90%Pt/10%Rh) and Platinum-Nickel (90%Pt/10%Ni)). The sensor resistance R_w at a temperature T_w is then given by the following expres-

sion:

$$R_w = R_{ref}(1 + a_w(T_w - T_{ref})) \quad (2.1)$$

where R_{ref} is the sensor resistance at a reference temperature T_{ref} .

A schematic of a simplified CTHWA circuit is presented in Fig. 2.2. The hot wire probe is connected to the bottom left leg of the Wheatstone bridge, with R_w the resistance of the wire and R_L the resistance of the prongs, probe stem and cable. The resistance R_{dec} is adjustable and it is used to set the desired overheat ratio. During the system operation, the resistance R_w is maintained constant by the feedback amplifier, which adjusts the current to compensate for resistance fluctuations. In this way, the resistance and therefore the temperature of the sensor are maintained constant.

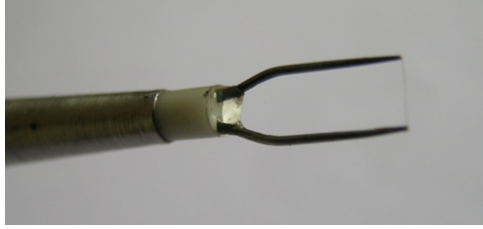


Figure 2.1 – VKI single-sensor hot wire probe.

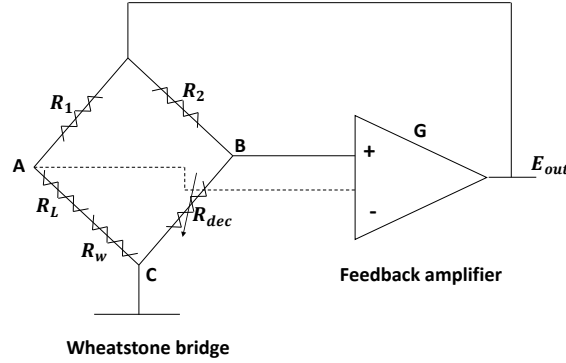


Figure 2.2 – Schematic of a simplified CTA circuit.

The hot wire is sensitive to the convective power of the flow. In incompressible, isothermal flows the output voltage of the anemometer can be directly related to the flow velocity and well known relationships like King's Law can be used. However, in the most general case, the convective heat transfer is expressed by the Nusselt

number Nu defined as:

$$Nu = \frac{h d_w}{k}, \quad (2.2)$$

where h is the heat transfer coefficient, d_w is the wire diameter and k the thermal conductivity of the fluid. The Nusselt number depends on the following parameters [1, 2]:

$$Nu = f(Re, M, Pr, Gr, \frac{l}{d_w}, \tau_{wr}, \alpha, \beta), \quad (2.3)$$

where Re , M , Pr and Gr are the Reynolds, Mach, Prandtl and Grasshof numbers of the flow respectively. l and d_w are the length and diameter of the sensor and α , β the two flow angles. τ_{wr} is the overheat ratio which is defined as $\tau_{wr} = (T_w - T_r)/T_r$, where T_r is the recovery temperature of the flow.

For Constant Temperature Anemometry (CTA), with a given probe geometry ($l/d_w = \text{const}$), in a specific fluid with moderate flow temperature variations ($Pr = \text{const}$), and by neglecting the natural convection (when $Re > Gr^{1/3}$) [1], the relationship reduces to the following equation [3]:

$$Nu = f(Re, M, \alpha, \beta). \quad (2.4)$$

The particularities and limitations of the HWA application in different flow conditions are well summarized in a review paper by Stainback & Nagabushana [2], where the following flow regimes are distinguished (in terms of Mach number):

- subsonic incompressible flow ($M < 0.3$)
- subsonic compressible, transonic and low supersonic flow ($0.3 < M < 1.4$)
- high supersonic flow ($1.4 < M$)

within which, one can distinguish the following sub-regimes (in terms of Knudsen number):

- continuum flow ($Kn < 0.01$)
- slip flow ($0.01 < Kn < 0.1$)
- free molecular flow ($0.1 < Kn$)

The Knudsen number is defined as the ratio of the molecular mean free path length to a characteristic length of the flow (the wire diameter in the Hot Wire case) and it expresses the deviation from the continuum flow. It can be related to Mach and Reynolds numbers by:

$$Kn = \frac{M}{Re} \sqrt{\frac{\gamma\pi}{2}}. \quad (2.5)$$

In incompressible continuum flows the Nusselt number is a unique function of Reynolds. The same has been observed for supersonic continuum flows with $M > 1.4$ [2, 4], so in this case, Eq. 2.4 reduces to $Nu = f(Re)$ (if we neglect the flow direction effect, assuming wires mounted normal to the mean flow). In high speed flows there is a strong Mach number effect, as expressed in Eq. 2.4. Moreover, it has been observed by different researchers, that in low density flows this compressibility effect can extend much lower than the typical incompressibility limit of $M = 0.3$ [2]. Spangenberg [5] was the first to attribute this effect to gas rarefaction: at low densities the HW often operates in slip flow conditions due to its small diameter. Thus, high subsonic/transonic continuum flows and subsonic slip flows can be treated in the same way, as the same heat transfer relationship applies to both: $Nu = f(Re, M, \tau_{wr})$. Since Re , M and Kn are linked, as shown in Eq. 2.5, any two of the three parameters can be used to describe the non-dimensional heat transfer relationship. So $Nu = f(Re, M)$ is equivalent to $Nu = f(M, Kn)$ and $Nu = f(Re, Kn)$. Different sets of variables have been used by different researchers, as described in [2].

The recovery factor η which defines the ratio of the recovery temperature T_r to the total temperature of the flow T_0 is also a function of the flow conditions [1]. While in incompressible continuum flows its effect is negligible ($T_r \approx T_0$), it should instead be taken into account in compressible and slip flows. An empirical correlation with a wide range of validity was proposed by Dewey [6], in the form of $\eta = f(M, Kn)$. It is based on a collection of experimental data by different researchers and can be used for all Mach numbers, while extending from the free molecular limit to the high Reynolds continuum limit.

Taking into account the recovery factor effect, the heat transfer balance over the wire is given by:

$$I^2 R_w = \pi l k (T_w - \eta T_0) Nu \quad (2.6)$$

where I is the current supplied to the wire and k is the thermal conductivity of the fluid. By expressing the power in terms of the anemometer bridge output voltage, the Nusselt number can be rearranged as:

$$Nu = \frac{E_b^2}{k(T_w - \eta T_0)} \frac{R_w}{\pi l (R_1 + R_L + R_w)^2} \quad (2.7)$$

where R_1 is the resistance on the top left leg of the Wheatstone bridge (see Fig. 2.2) and E_b is the bridge output voltage.

2.1.2 Sensitivities method

Kovazsnay [4] and Morkovin [7] put the basis of the application of HWA for fluctuation measurements in supersonic and transonic flows respectively. By assuming small perturbations, Morkovin [7] related the fluctuations of the output voltage of

the anemometer to the fluctuations of density, velocity and total temperature for Constant Current Anemometry (CCA). For Constant Temperature Anemometry (CTA) and straight wires mounted parallel to the flow, the following relationship stands:

$$\frac{e_b'}{E_b} = S_\rho \frac{\rho'}{\bar{\rho}} + S_u \frac{u'}{\bar{u}} + S_{T_0} \frac{T_0'}{\bar{T}_0} \quad (2.8)$$

where S_ρ , S_u and S_{T_0} are the sensitivities to density, velocity and total temperature respectively, defined as follows:

$$S_\rho = \left(\frac{\partial \log E_b}{\partial \log \rho} \right)_{u, T_0, \alpha, \beta = ct} \quad (2.9)$$

$$S_u = \left(\frac{\partial \log E_b}{\partial \log u} \right)_{\rho, T_0, \alpha, \beta = ct} \quad (2.10)$$

$$S_{T_0} = \left(\frac{\partial \log E_b}{\partial \log T_0} \right)_{u, \rho, \alpha, \beta = ct} \quad (2.11)$$

The sensitivities can be expressed in terms of non-dimensional parameters [2]:

$$S_\rho = \frac{1}{2} \left(\frac{\partial \log Nu}{\partial \log Re} + \frac{\partial \log \eta}{\partial \log Re} \right) \quad (2.12)$$

$$S_u = S_\rho + \frac{1}{2m} \left(\frac{\partial \log Nu}{\partial \log M} - \frac{1}{\tau_{wr}} \frac{\partial \log \eta}{\partial \log M} \right) \quad (2.13)$$

$$S_{T_0} = \frac{1}{2} \left[n_t + 1 - m_t \frac{\partial \log Nu}{\partial \log Re} - \frac{(\theta - \eta)}{\theta} - \frac{1}{\tau_{wr}} \left(-\frac{1}{2m} \frac{\partial \log Nu}{\partial \log M} \right) \right] \quad (2.14)$$

where $n_t = \frac{\partial \log k}{\partial \log T_0}$, $m_t = \frac{\partial \log \mu}{\partial \log T_0}$, $m = \frac{1}{1+M^2 \frac{\gamma-1}{2}}$ and $\theta = \frac{T_w}{T_0}$.

In incompressible and supersonic flows $S_u = S_\rho = S_{\rho u}$, since the Mach number does not affect the heat transfer process ($Nu \neq f(M)$). In this case, the HW is sensitive to mass flux and total temperature fluctuations and Eq. 2.8 can be simplified to:

$$\frac{e_b'}{E_b} = S_{\rho u} \frac{\rho u'}{\bar{\rho} \bar{u}} + S_{T_0} \frac{T_0'}{\bar{T}_0}. \quad (2.15)$$

In high subsonic/transonic and subsonic slip flows though, there is a strong Mach (or Knudsen) effect on the heat transfer process, and $S_\rho \neq S_u$. In this case, Eq. 2.8 can only be solved instantaneously by using at least three wires with sufficiently different sensitivities (e.g. different overheats) to solve a system with three equations and three unknowns.

In literature, there are discrepancies concerning the behavior of the sensitivities. Horstman & Rose [8] found that for high overheat ratios ($\tau_{wr} > 0.8$) and

high Reynolds numbers ($Re_d > 20$), $S_u = S_\rho$. Nevertheless this was not supported by other studies, where the sensitivity to density was always higher than the sensitivity to velocity [9, 10].

The yaw angular sensitivity S_α for inclined wires in a planar flow field was first introduced by Motallebi [11] and expressed in the following form:

$$S_\alpha = \frac{1}{2} \left(\frac{1}{\tau_{wr}} \frac{\partial \log \eta}{\partial \log \alpha} - \frac{\partial \log Nu}{\partial \log \alpha} \right). \quad (2.16)$$

for three or more inclined wires in a three dimensional flow field, the pitch angle sensitivity S_β should also be included:

$$S_\beta = \frac{1}{2} \left(\frac{1}{\tau_{wr}} \frac{\partial \log \eta}{\partial \log \beta} - \frac{\partial \log Nu}{\partial \log \beta} \right) \quad (2.17)$$

and the sensitivity equation takes the following form:

$$\frac{e_b'}{\overline{E_b}} - S_\alpha \frac{\alpha'}{\overline{\alpha}} - S_\beta \frac{\beta'}{\overline{\beta}} = S_\rho \frac{\rho'}{\overline{\rho}} + S_u \frac{u'}{\overline{u}} + S_{T_0} \frac{T_0'}{\overline{T_0}}. \quad (2.18)$$

The yaw and pitch angles can be defined with respect to any reference system. Usually, they are expressed with respect to the wire coordinates system, as will be explained in Section 2.2. Eq. 2.18 could be solved instantaneously using a probe with at least five wires with different sensitivities, in order to transform it in a system of five equations and five unknowns.

The sensitivities can be directly obtained through a systematic calibration, where one parameter is varied while keeping the others constant. This method requires extensive, lengthy calibrations in closed loop facilities. In the present study, a non-dimensional calibration will be used to compute the sensitivities from their non-dimensional expressions (Eq. 2.12 - 2.14).

2.1.3 Objectives of the present work

It is clear from the above that the application of CTHWA in all cases other than isothermal, incompressible flows is not straightforward, due to the nature of this measurement technique. In this chapter, the following points are covered:

- An existing non-dimensional calibration methodology for CTHWA application in compressible and non-isothermal flows is reviewed and compared to classical calibration methods.
- An analysis of the performance of CTHWA in different flow regimes of interest to turbomachinery applications is presented, based on the computation of their sensitivities. The aim is to clearly define the bounds of applicability.

2.2 Non-dimensional calibration methodology

The method used for the calibration is based on the calibration methodology for X-wire probes developed by Cukurel et al. [3]. This method consists in the use of an effective wire temperature and of empirical correlations to eliminate the dependency of the calibration on total temperature and Mach number of the flow respectively, which allows establishing a unique $Nu-Re$ calibration curve. More details can be found in Ref. [3] but the procedure is also presented hereunder for completeness.

The Nusselt number is computed by Eq. 2.7, while the Reynolds number is defined by:

$$Re = \frac{\rho U d_w}{\mu}. \quad (2.19)$$

The viscosity μ and the thermal conductivity k of the flow are evaluated at the total temperature of the flow.

The static calibration is divided into two steps: the low-speed calibration and the mass-flux calibration. The low-speed calibration is performed to identify the effective wire temperature $T_{w,eff}$. $T_{w,eff}$ is defined as the wire temperature which collapses a set of $Nu-Re$ data obtained at different flow T_0 on a single curve. The low-speed calibration is performed in incompressible continuum flow, therefore, since there is no compressibility effect, the Nusselt number is a unique function of Reynolds. Based on that, an iterative procedure takes place, where the wire temperature is systematically varied. The Nu is recomputed for each iteration and plotted against Re . The $Nu-Re$ curve is fitted with a 4th order polynomial and the R^2 of the fit is used as the selection criterion: the wire temperature which results in the best fit (highest R^2) is selected as the temperature that better represents the convective heat transfer process from the wire to the flow. The low-speed calibration procedure is depicted in Fig. 2.3. In Fig. 2.3a, a set of data obtained at different flow temperatures is presented. When plotted in terms of voltage and velocity, a different curve is created for each temperature level. But when plotted in terms of Nu and Re with the effective wire temperature in Eq. 2.7, all the data collapse in a single curve. In Fig. 2.3b the R^2 of the 4th order polynomial fit is presented as a function of the wire temperature and the selected $T_{w,eff}$, corresponding to the maximum R^2 value is indicated.

The mass-flux calibration is then performed in the range of flow conditions indicated by the test requirements. The data are plotted in the non-dimensional form $Nu-Re$, using the $T_{w,eff}$ in the computation of the Nusselt number. In the compressible region, a correlation proposed by Dewey [6] is used to eliminate the Mach number dependency:

$$Nu_{corr}(Re, M) = \frac{Nu(Re, M)}{\phi(Re, M)} \quad (2.20)$$

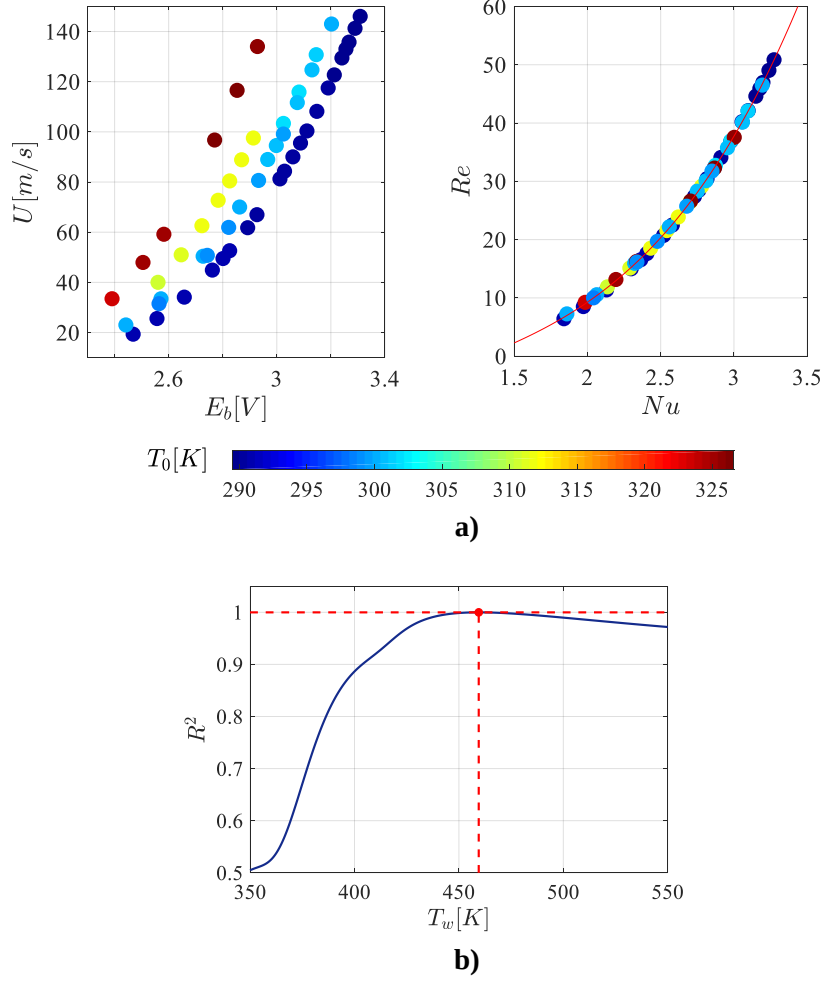


Figure 2.3 – Hot wire low speed calibration: a) Voltage-velocity and Nu-Re curves for different flow temperatures, b) Selection of optimal wire temperature.

where

$$\phi(Re, M) = 1 + A(M) \left[1.834 - 1.634 \left(\frac{Re^{1.109}}{2.765 + Re^{1.104}} \right) \right] \times \left[1 + \left(0.3 - \frac{0.0650}{M^{1.670}} \right) \left(\frac{Re}{4 + Re} \right) \right] \quad (2.21)$$

and

$$A(M) = \frac{0.6039}{M} 1 + 0.5701 \left[\left(\frac{M^{1.222}}{1 + M^{1.222}} \right)^{1.659} - 1 \right] \quad (2.22)$$

The correction term $\phi(Re, M)$ is used to compute a corrected Nusselt number which is only dependent on Reynolds. The correlation is valid for $M > 0.3$ and for all the Reynolds numbers. The correlation has been developed based on data from infinitely long wires placed normal to the flow, therefore the Mach number used in the correlation should correspond to the velocity component normal to the wire. The compressibility effect on the calibration and its correction are illustrated in Fig. 2.4. When the correction is applied, a single Nu_{corr} - Re calibration curve is retrieved.

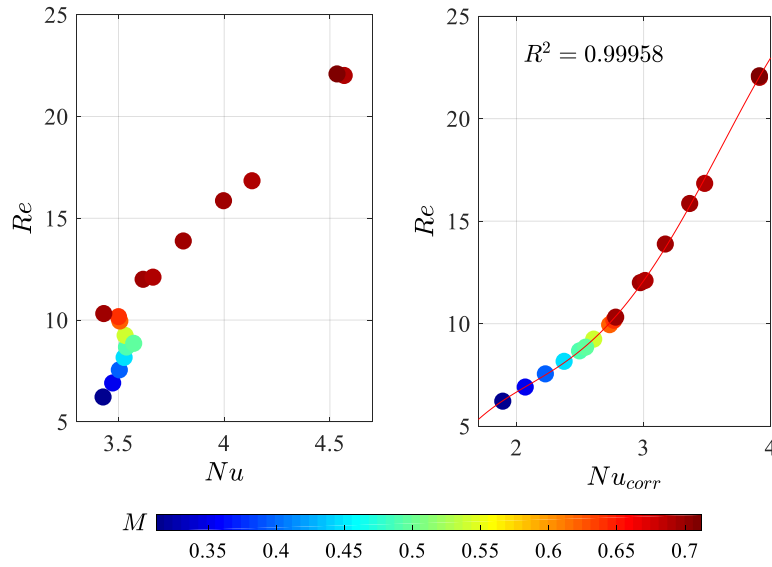


Figure 2.4 – a) Compressibility effect on the hot-wire calibration curve and b) the corrected calibration curve using Dewey's correlation.

The application of the Mach number correction to a dataset covering a Mach number from 0.1 to 0.55 is presented in Fig. 2.5a. Since this correlation is valid for $M > 0.3$, two different curves appear in this case:

- Curve A: For $M > 0.3$, Dewey's correlation is applied.
- Curve B: $M < 0.3$, no correction applied, $Nu = Nu_{corr}$.

This behavior can create difficulties when measuring in Mach numbers around 0.3. It is preferable to have a single calibration curve covering all the Mach number range. For this reason, a correlation proposed by Klopfer [12] is employed, which constitutes an extension to Dewey's correction to cover the range $0 < M < 0.4$. Klopfer uses a weighted logarithmic average between the corrected

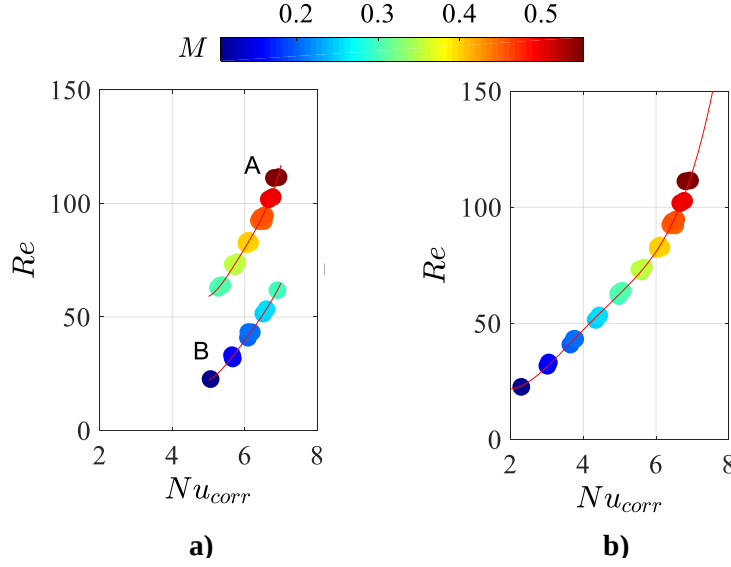


Figure 2.5 – Non-dimensional hot wire calibration curve with a) application of Dewey's correlation, b) addition of Klopfer's extension.

Nusselt number at $M = 0.4$ by Dewey and the Oseen solution at $M = 0$ by Cole and Roshko [13]. The corrected Nusselt number Nu_{corr} is given by:

$$Nu_{corr} = 10^{F \log(Nu(Re,0)) + (1-F) \log(Nu_{corr}(Re,0.4))} \quad (2.23)$$

where

$$F = [0.618 + (2.5M)^{0.9}]^{-1} - 0.618 \quad (2.24)$$

and

$$Nu(Re, 0) = 0.19 + Re^{0.45} \quad (2.25)$$

This extension was validated on a set of experimental data ranging over $0.1 < Re < 300$. In Fig. 2.5b, the extended correlation is applied, creating a continuous calibration curve for the whole Mach number range:

- For $M \geq 0.4$: Dewey's correlation
- For $M < 0.4$: Klopfer's extension

The velocity vector U can be decomposed in three velocity components in the wire coordinate system, as presented in Fig. 2.6a: the velocity component normal to the wire U_N , the component tangential to the wire U_T and finally the binormal component U_B . The yaw angle α is defined in the normal-tangential plane as described in Fig. 2.6b and the pitch angle β is defined in the normal-binormal

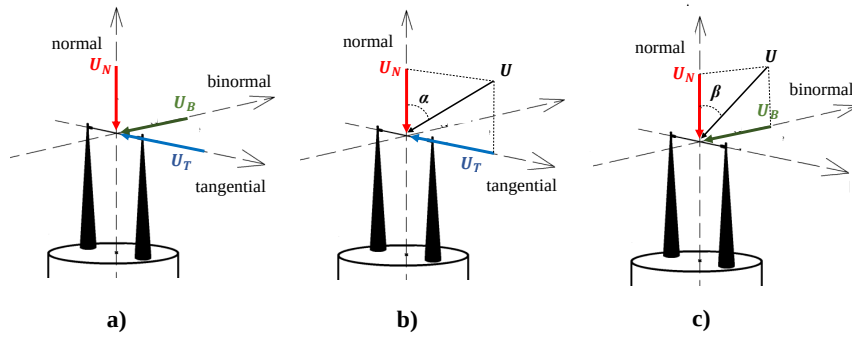


Figure 2.6 – Directional sensitivity of a hot wire: a) Definition of the wire coordinate system, b) Yaw angle defined in the normal-tangential plane, c) Pitch angle defined in the normal-binormal plane.

plabe as shown in Fig. 2.6c. The wire effective velocity, defined as the velocity to which the wire is sensitive, is usually expressed as:

$$U_{eff}^2 = U_N^2 + k^2 U_T^2 + h^2 U_B^2. \quad (2.26)$$

k and h are coefficients obtained by a directional calibration or, in some cases, provided by the probe manufacturer. Typical values of k^2 are of the order of 0.01-0.02 while for h^2 of the order of 1.02-1.05. For single wires, the different sensitivity to the normal and binormal components is due to the presence of the prongs which cause a local acceleration of the flow, resulting in the h^2 coefficient being higher than 1.

In the case of X-wire probes, a directional calibration should be applied, that allows the performance of 2D measurements. An example of an X-wire probe is given in Fig. 2.7. The design consists of two slanted wires positioned on two parallel planes at a small distance (less than 1 mm). The two wires are slanted in different directions with an angle γ , creating therefore a cross in the xy plane (see Fig. 2.7a). The measurement point is the cross-section point of the two wires.

The velocity vector decomposition for a velocity of magnitude U and yaw angle ϕ , is also presented in Fig. 2.7a. The angle ϕ is the yaw angle in the probe frame of reference. The transformation from the probe coordinate system to the wire coordinate system is as follows:

$$U_N = U \cos(\phi - \gamma) \quad (2.27)$$

$$U_T = U \sin(\phi - \gamma) \quad (2.28)$$

The angle $\phi - \gamma$ corresponds to the yaw angle α in the normal-tangential plane of the wire, as described in Fig. 2.6b.

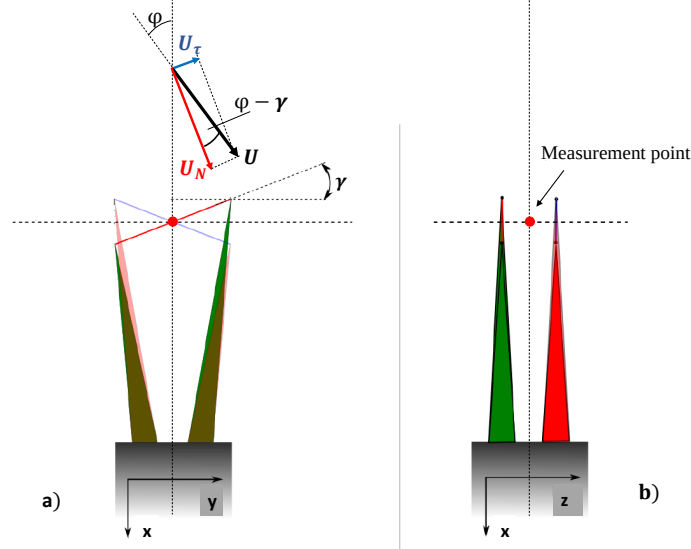


Figure 2.7 – Schematic of an X-wire probe.

In this work, the calibration methodology of Cukurel et al. [3] is used also for the directional calibration. The procedure is illustrated in Fig. 2.8. The calibration is performed by placing the hot wire in a uniform flow of known properties and tilting it at a known yaw angle. The calibration illustrated in Fig. 2.8 has been performed for a $\pm 25^\circ$ angle range. Re_1 and Re_2 is the Reynolds number felt by Wire 1 and 2 respectively, computed by applying the static calibration law at the voltage acquired for each flow angle. The ratio of Re_1 and Re_2 is a unique function of the yaw angle, as can be seen in Fig. 2.7a. Re in Fig. 2.7b is the magnitude of the flow Reynolds number. Cukurel et al. [3] found that the angular calibration curves were independent of the Mach and Reynolds numbers in the tested range.

The data reduction using this methodology in the measurement case, is performed as follows:

- The instantaneous voltages of the two wires Eb_1 and Eb_2 are obtained.
- The respective Nusselt numbers are computed by Eq. 2.7 (knowledge of the mean flow properties is required).
- Using the mass-flux calibration, the instantaneous Reynolds numbers Re_1 and Re_2 are obtained.
- By the ratio Re_1/Re_2 the instantaneous yaw angle is acquired from the angular calibration curve in Fig. 2.8a.

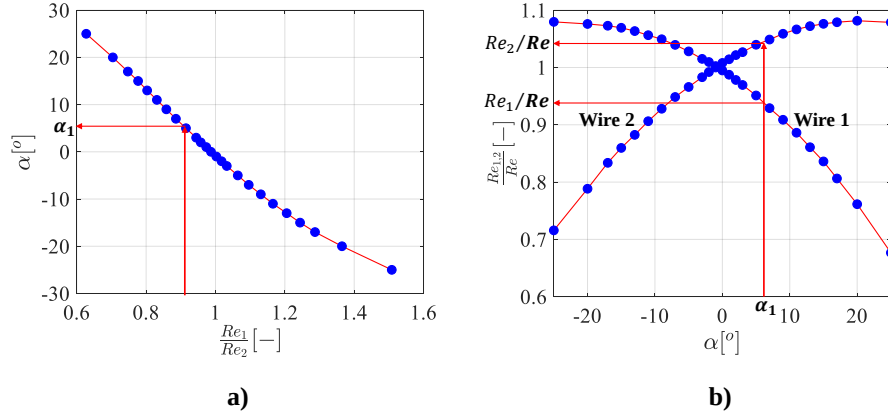


Figure 2.8 – Directional calibration example for a X-wire probe, in non-dimensional numbers.

- The acquired yaw angle value is inserted in the curve of Fig. 2.8b and the ratios Re_1/Re and Re_2/Re are acquired. Since Re_1 and Re_2 are known, the magnitude of the flow Reynolds number is retrieved.
- With the time series of Re and α , the desired 2D statistics can finally be computed.

2.3 Comparison between data reduction methods

In this section, a comparison between different data reduction methods will be performed for a low-speed, incompressible, isothermal case. This case is selected in order to compare the non-dimensional calibration and the sensitivities method against a classical data reduction case like the King's law. A low-turbulence case is selected, as the sensitivities method is valid for small fluctuations.

The so-called King's law is a power law relating the anemometer output voltage E to the flow velocity U as:

$$E^2 = A + BU^n \quad (2.29)$$

where A , B and n are constants. A is equal to the squared voltage at zero flow E_0^2 .

The two calibration curves for the same dataset are presented in Fig. 2.9. The calibrations have been performed in situ, in a low-speed windtunnel. The $T_{w,eff}$ for the $Nu - Re$ calibration has been defined at a previous step and the current calibration points have been acquired at an almost constant flow temperature (less than one degree variation), as can be seen in Fig. 2.9b.

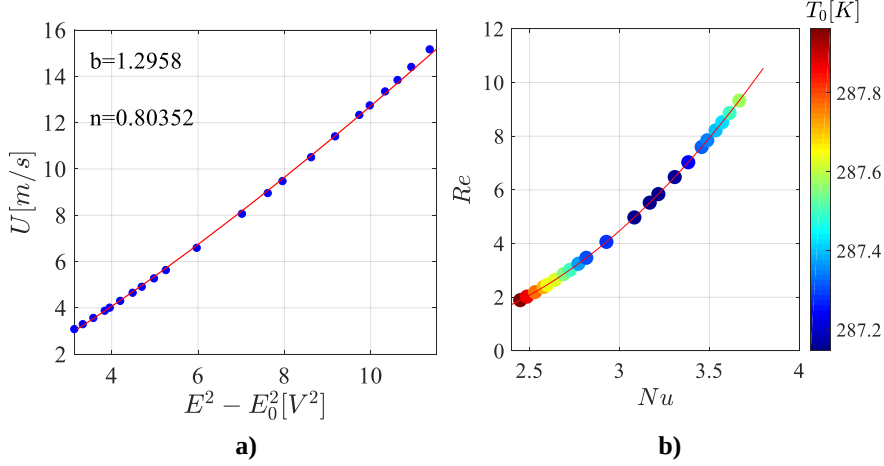


Figure 2.9 – Comparison between hot wire calibration methods: a) King's law, b) Nu-Re calibration.

The turbulence intensity computed by the different methods will be examined. The instantaneous velocity can be acquired by inserting the instantaneous voltage in the calibration equations. The third method to be tested is the sensitivities method. For the case examined, where density and total temperature fluctuations are considered negligible, the following equation stands:

$$\frac{e'}{E} = S_{\rho u} \frac{u'}{\bar{U}}, \quad \text{with } S_u = S_\rho = S_{\rho u} \quad (2.30)$$

In order to compute the sensitivities, the set of equations 2.12 -2.14 are used. The logarithmic derivatives of the recovery factor to Re and Mach ($\frac{\partial \log \eta}{\partial \log Re}$, $\frac{\partial \log \eta}{\partial \log M}$) can be analytically computed from the empirical correlation proposed by Dewey [6]. The logarithmic derivatives of Nusselt to Re and Mach can be transformed to:

$$\frac{\partial \log Nu}{\partial \log Re} = \frac{Re}{\phi} \frac{\partial \log \phi}{\partial \log Re} + \frac{\partial \log Nu_{corr}}{\partial \log Re} \quad (2.31)$$

$$\frac{\partial \log Nu}{\partial \log M} = \frac{M}{\phi} \frac{\partial \log \phi}{\partial \log M} \quad (2.32)$$

The terms $\frac{\partial \log \phi}{\partial \log Re}$ and $\frac{\partial \log \phi}{\partial \log M}$ are the partial derivatives of the correction term $\phi(Re, M)$. The only term depending on the wire properties is $\frac{\partial \log Nu_{corr}}{\partial \log Re}$, which can be directly obtained by the calibration.

In the present case, since the flow is incompressible and Nusselt is a unique function of Reynolds:

$$\frac{\partial \log Nu}{\partial \log Re} = \frac{\partial \log Nu_{corr}}{\partial \log Re} \quad (2.33)$$

This term represents the slope of the calibration curve in Fig. 2.9b. It can be therefore directly obtained from the calibration, but different methods can be used which will be compared hereunder.

- Method 1: The derivative is obtained analytically, by differentiation of the calibration polynomial's expression (retrieved by a 4th order least squares regression).
- Method 2: The derivative is computed numerically by the calibration grid points, with a central difference method.
- Method 3: The derivative is computed numerically by the calibration grid points with Fornberg's method [14]. This algorithm generates the optimal weights for the computation of derivatives in arbitrarily spaced grids. This is suitable for the present case, as the calibration points are not equally spaced.

The mass-flux sensitivity computed by the three methods is given in Fig. 2.10 for all measurement points. It can be seen that methods 1 and 3 agree well far from the calibration boundaries, while method 2 differs in the Reynolds range 3.5 - 6, where the calibration grid is coarser, as expected. This confirms that method 3 should be preferred when numerically computing the sensitivity derivatives.

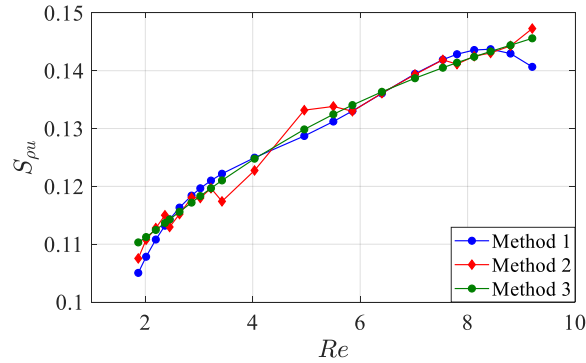


Figure 2.10 – Sensitivity to mass-flux computed with the three different methods for the computation of the sensitivities.

The turbulence intensity computed by the King's law, the Nu-Re calibration and the sensitivities method (computed by method 1) is given in Fig. 2.11. Differences between the King's law and the non-dimensional calibration are attributed to the better fitting of the calibration curve in the second case (reduced scatter), as observed in Fig. 2.9. The sensitivities method results agree with the Nu-Re calibration, with a maximum difference of 5.1% at the extreme regions of the calibration curve.

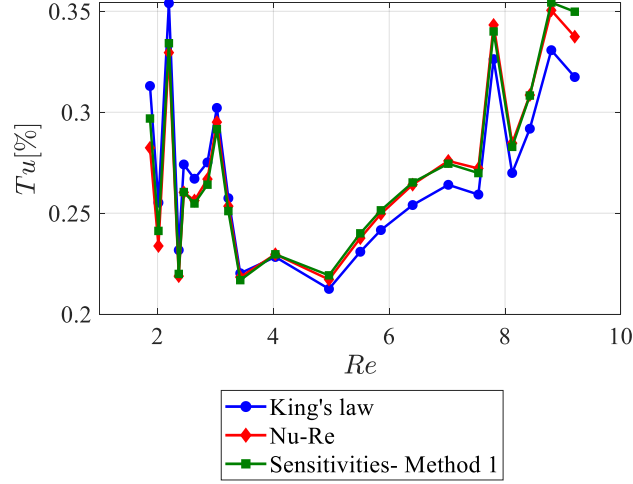


Figure 2.11 – Comparison of turbulence intensity level computed using three different data reduction methods: the King's law, the Nu-Re calibration and the sensitivities method.

The performance of the different sensitivity computation methods is given in Fig. 2.12. The difference in the turbulence intensity value obtained by the sensitivities method with respect to the one obtained by the $Nu - Re$ calibration is presented. The relative error is defined as:

$$\delta_{Tu} = \frac{abs(Tu_{NuRe} - Tu_{sens})}{Tu_{NuRe}}. \quad (2.34)$$

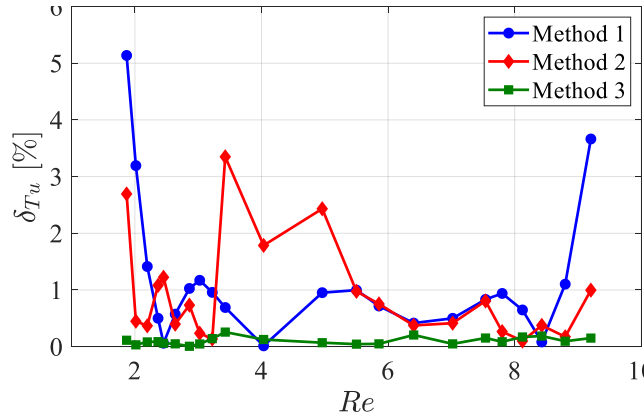


Figure 2.12 – Error on turbulence intensity computed by sensitivities methods with respect to the Nu-Re method.

It can be seen that method 3 features the best agreement, with a maximum error of less than 0.2%. Method 1 deviates at the extremes of the calibration range, where the derivative is underestimated. The same behaviour is observed for method 2, with the addition of higher error for the points where the calibration grid is coarser. It is clear that a weighted method should be used for the correct estimation of the derivatives (and therefore the sensitivities), in the case where the sensitivities method is used for data reduction. Weighted polynomial regression methods could be investigated for the case of analytical derivation (method 1).

2.4 Performance in different flow regimes

In this section, the performance of three hot wire probes tested at different flow regimes is examined. The analysis is based on the computation of their sensitivities to the different flow properties. All probes feature tungsten wires of $9\ \mu\text{m}$ diameter. A three-wire probe with slanted wires, designed for three-dimensional measurements, is tested in incompressible flow, while two single-wire probes placed normal to the flow are tested in compressible flow. Directional effects will be thus discussed only for the first case. Finally, the sensitivities are compared with experimentally obtained values given by Nagabushana & Stainback in Ref. [10].

In order to compute the sensitivities, the set of equations 2.12 -2.14 are used. As explained before, all terms in these equations can be computed analytically by correlations. The term $\frac{\partial \log Nu_{corr}}{\partial \log Re}$ is computed by Method 1 presented in the previous section, i.e. analytically, by the differentiation of the calibration law. In the previous section, it was demonstrated that computing the sensitivities by Method 3 (finite difference scheme adapted for arbitrarily spaced grids) is the best option. Nevertheless, since in this section the sensitivities are used to showcase trends in the CTHWA behaviour and not for data reduction, Method 1 was selected for simplicity.

2.4.1 Case 1: Incompressible continuum flow including angular effects

A hot wire probe with three slanted wires operated at different overheat ratios was tested in incompressible continuum flow. The flow conditions and wire characteristics are given in Table 2.1.

In this case, according to the theory, there is no Mach number effect on the heat transfer process and the sensitivity to velocity is equal to the sensitivity to density, $S_u = S_\rho$. Calculated by Eq. 2.12, the sensitivity to velocity for all three wires seems to be almost independent of the Reynolds number of the flow, as can be seen in Fig. 2.13. Moreover, it is only slightly affected by the operating wire temperature, which results in small differentiation of its value for the three

wires. More specifically, the sensitivity to velocity is almost equal for wires 1 and 3 ($S_{u1} = 0.161$, $S_{u3} = 0.1611$), despite having the biggest difference in T_w . A higher sensitivity ($S_{u2} = 0.1662$) is computed for wire 2, although it is operated at an intermediate temperature. In Fig. 2.14, the sensitivity to velocity as a function of the overheat is presented. Since the wire temperature is constant for each wire, overheat variations reflect variations in the flow total temperature. It can be seen that the overheat has no effect on the velocity sensitivity values for a given wire.

$d_{w1,2,3}$	9 μm
T_{w1}	491 K
T_{w2}	462 K
T_{w3}	431 K
M	0.01 - 0.26
Re	5 - 44
Kn	0.0071 - 0.0079
ρ	1.11 - 1.22 kg/m ³
U	1 - 90 m/s
T_0	291 - 315 K

Table 2.1 – HW characteristics and test flow conditions for Case 1.

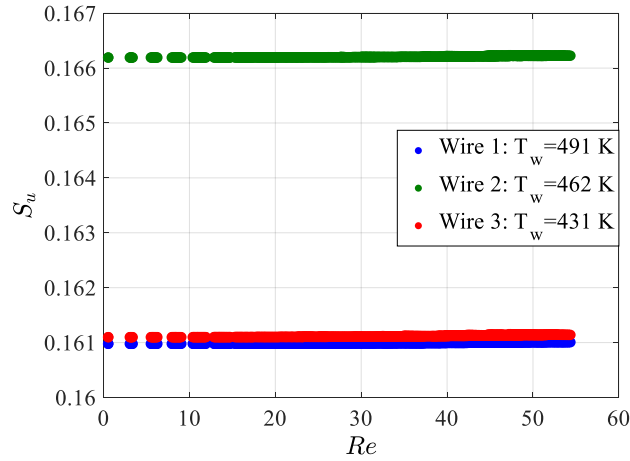


Figure 2.13 – Case 1: Sensitivity to velocity as a function of Reynolds number for the three wires.

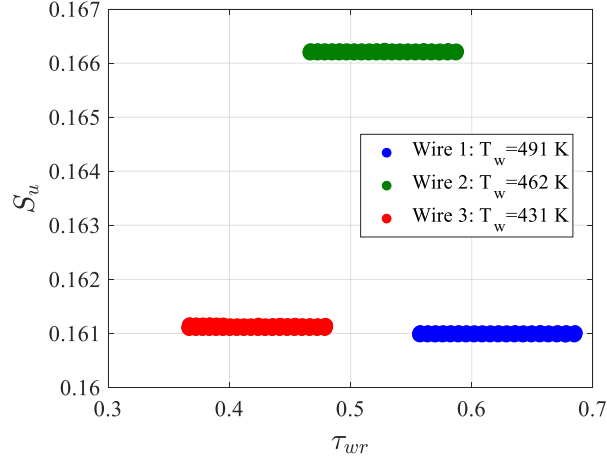


Figure 2.14 – Case 1: Sensitivity to velocity as a function of the overheating parameter for the three wires.

In this range of conditions, the total temperature sensitivity seems to be independent of the Reynolds number and a unique function of the temperature difference between the heated wire and the flow. This is clearly illustrated in Fig. 2.15, where the sensitivities of the three wires are plotted as a function of the overheat τ_{wr} . The sensitivities are always negative, as an increase of the flow temperature results in a decrease of the heat transfer rate. As the overheat increases, the wires become less sensitive to total temperature fluctuations. It should be noted that the absolute value of the total temperature sensitivity is significantly higher to that of the velocity sensitivities, even for the highest overheat. Hence, the assumption of negligible total temperature fluctuations should be treated with caution in such conditions, as it could lead to errors.

Since $S_u = S_\rho$ and the hot wire is sensitive to all three velocity components, the sensitivity equation for this case is written as:

$$\frac{e'}{E_i} - S_{\alpha,i} \frac{\alpha'}{\alpha} - S_{\beta,i} \frac{\beta'}{\beta} = S_{\rho u,i} \frac{\rho u'}{\rho u} + S_{T_0,i} \frac{T'_0}{T_0} \quad (2.35)$$

The sensitivities to the flow angles S_α and S_β are a function of the flow Reynolds number and the flow direction. In Fig. 2.16, they are presented as a function of the angle for a range of $\pm 30^\circ$, where $\alpha, \beta = 0^\circ$ corresponds to the case where the flow is aligned to the probe body. The angles in this case are defined with respect to the probe coordinate system, therefore the sensitivity of the wires to the yaw and pitch angle depends on their orientation. It can be seen that wire 1 is more sensitive to the pitch angle, while wire 2 is more sensitive to the yaw angle.

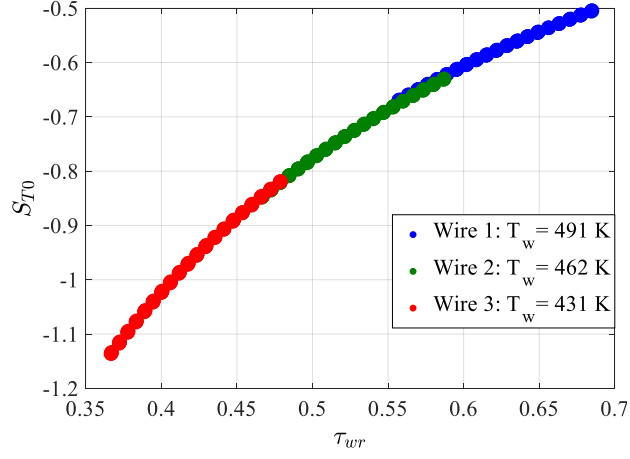


Figure 2.15 – Case 1: Total temperature sensitivity as a function of the overheating parameter for the three wires.

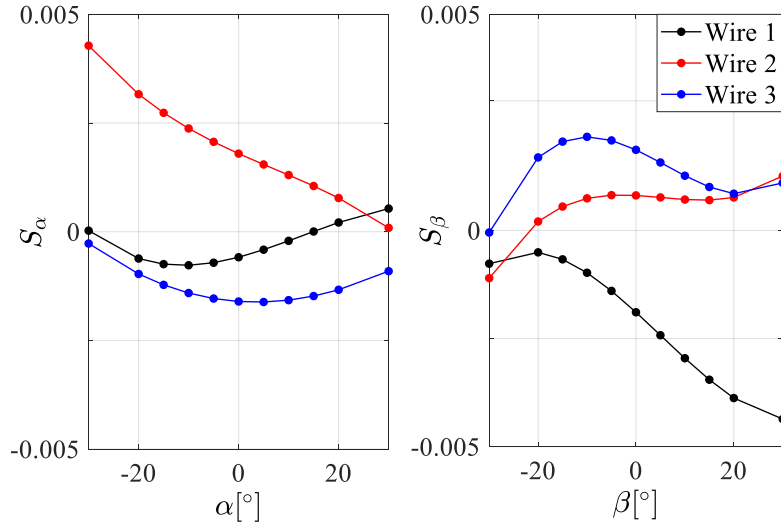


Figure 2.16 – Case 1 Angular sensitivities: Yaw sensitivity as a function of the yaw angle for $\beta = 0^\circ$ and $Re = 8.5$ (left) and Pitch sensitivity as a function of the pitch angle for $\alpha = 0^\circ$ and $Re = 8.5$ (right).

The fluctuating angles can be directly obtained by an angular calibration, so the angular terms are considered known and are moved to the left hand side of

Eq. 2.35. Subtracting the angular terms from the voltage term in Eq. 2.35, results in the correlation of the voltages of the three wires, as can be seen in Fig. 2.17. Difference in the amplitude of the fluctuations are attributed to different S_u values. This behavior has been observed by Cukurel et al. [3] for an X-wire probe, only sensitive to the yaw angle. By compensating for the angular sensitivity, the problem of the three slanted wires is equivalent to that of three parallel wires normal to the flow. Thus only two unknowns remain $\frac{\rho u'}{\rho u}$ and $\frac{T'_0}{T_0}$ and the system can be solved by employing any set of two wires. It is reasonable in this case to choose the two wires with the highest and lowest overhear, in order to maximize the difference in the total temperature sensitivity values.

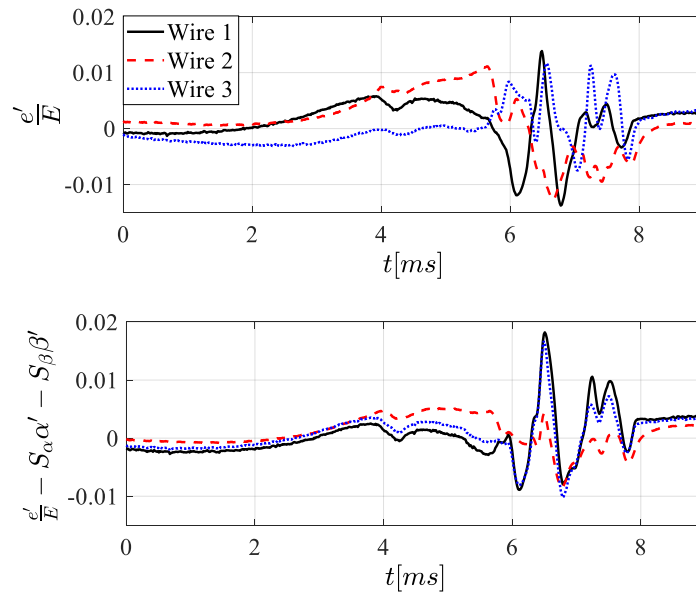


Figure 2.17 – Case 1: Time-resolved voltage and angular compensation for the three wires.

2.4.2 Case 2: High subsonic slip flow.

This hot wire was tested in high subsonic ($M = 0.605 - 0.67$) slip flow ($Kn = 0.0567 - 0.1139$). The hot wire characteristics and test flow conditions are presented in Table 2.2. In this range, the sensitivities to density and velocity are expected to be different and three wires with different sensitivity values will be needed to solve the sensitivity equation system.

d_w	9 μm
T_w	475 K
M	0.605 - 0.67
Re	8 - 17
Kn	0.0567 - 0.1139
ρ	0.0806 - 0.1629 kg/m ³
U	197 - 216 m/s
T_0	289 - 295 K

Table 2.2 – HW characteristics and test flow conditions for Case 2.

The sensitivities to velocity and density are presented in Fig. 2.18. They are plotted in terms of Mach and Knudsen numbers in order to highlight the gas rarefaction effects. High Knudsen numbers correspond to low densities. Both sensitivities are highly dependent on Knudsen and Mach, even for the narrow Mach number range that was tested in this case. They both increase with increasing Mach number and decrease with increasing Knudsen number. The sensitivity to density is always higher than the sensitivity to velocity, with the difference being amplified for the higher Knudsen numbers (lower densities), as can be seen in Fig. 2.19. The sensitivities to density and velocity are not affected by the small total temperature variations that were present in this case.

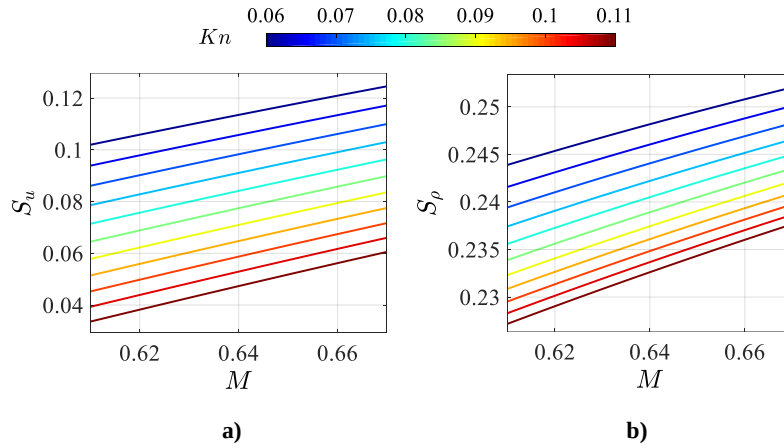


Figure 2.18 – Case 2: a) Sensitivity to velocity, and b) Sensitivity to density as a function of Mach and Knudsen numbers.

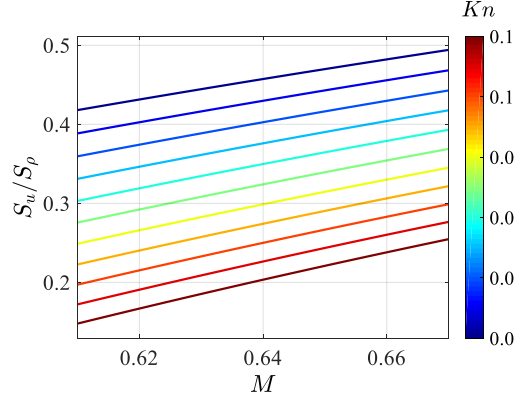


Figure 2.19 – Case 2: Sensitivity to velocity over sensitivity to density ratio, as a function of Mach and Knudsen numbers.

The total temperature sensitivity is a function of the Mach, Knudsen and total temperature of the flow. Its absolute value increases with increasing Mach and decreasing Reynolds numbers, as can be seen in Fig. 2.20a. As observed in the previous section, the total temperature sensitivity is increasing with increasing total flow temperature, due to the decrease of overheat. This behaviour is illustrated in Fig. 2.20b.

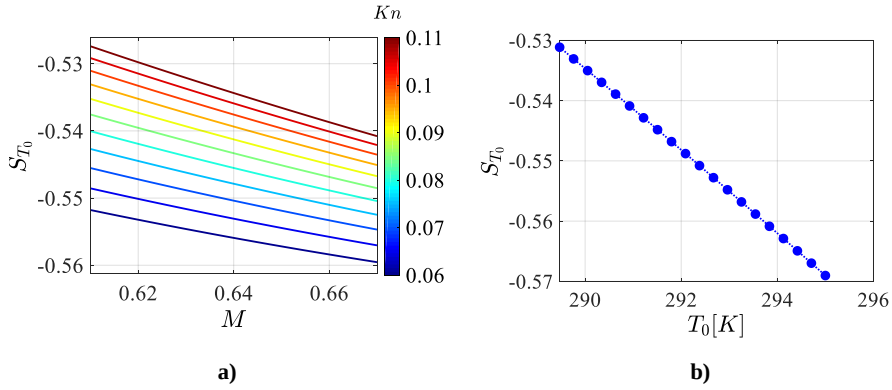


Figure 2.20 – Case 2: Sensitivity to total temperature a) as a function of Mach and Knudsen numbers, for $T_0 = 292$ K, b) as a function of total temperature for $M = 0.61$ and $Kn = 0.075$.

It can be seen that in the range of conditions of the present case, the sensitivities feature a linear behaviour. This indicates that for a limited range of conditions where such a condition stands, the calibration effort can be significantly reduced.

2.4.3 Case 3: High subsonic continuum flow

In this case, a hot wire was tested in continuum flow ($Kn < 0.01$) in a Mach number range from 0.3 to 0.55. The wire characteristics and the flow conditions are summarized in Table 2.3. Since the flow is in the compressible region, the correlation of Dewey is used for the Mach correction.

d_w	9 μm
T_w	459 K
M	0.1 - 0.55
Re	22.6 - 111.5
Kn	0.007 - 0.0072
ρ	1.2 - 1.23 kg/m ³
U	36 - 180 m/s
T_0	280 - 287 K

Table 2.3 – HW characteristics and test flow conditions for Case 3.

The sensitivities are presented in Fig. 2.21 - 2.22. The sensitivity to density presents similar behavior with case 2: it increases with increasing Mach number. It also increases slightly with decreasing Knudsen number: this effect is very small for all sensitivities, since there is only a small variation of density for this case.

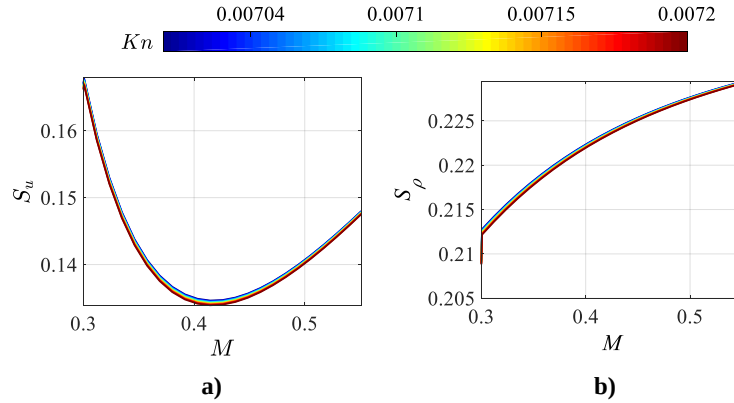


Figure 2.21 – Case 3: a) Sensitivity to velocity, b) Sensitivity to density.

The sensitivity to velocity increases with the Mach number, as in case 2, for $M > 0.42$. But a sharp increase is observed towards lower Mach numbers. This behavior is caused by the term $\frac{\partial \log Nu}{\partial \log M}$ (Eq. 2.13) which is computed by Eq. 2.32, so is solely based on the correlation. Finally, as it can be seen in Fig. 2.22, the

sensitivity to velocity is always lower than the one of density, even at the proximity of the incompressibility limit of $M = 0.3$.

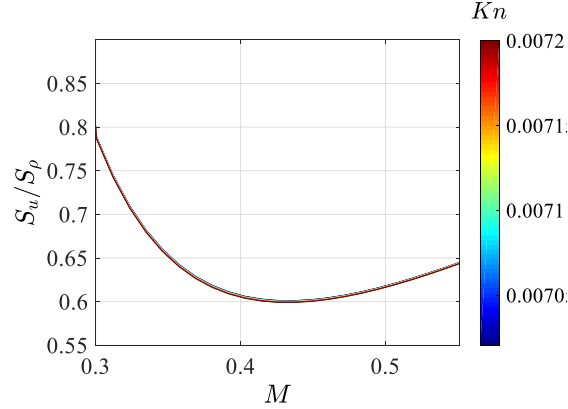


Figure 2.22 – Case 3: Sensitivity to velocity over sensitivity to density.

2.4.4 Comparison with literature

In this section, the data presented before are compared with the data of Nagabushana and Stainback [10]. It is a collection of experimental data obtained in subsonic slip flow conditions with CTA anemometers, where the sensitivities to temperature, velocity and density are computed with two different methods. The results that will be included here were obtained using a single wire placed normal to the flow, in the conditions presented in Table 2.4. The sensitivities presented were computed based on the functional dependency of the voltage to velocity, density and total temperature $E_b = f(\rho, u, T_0)$ and not by using non-dimensional values. The total temperature sensitivity data of Nagabushana & Stainback have significant scatter and no trends could be observed. Hence, they would not be included in the following comparison.

The sensitivities to density for Cases 2 and 3 are plotted together with the ones of Nagabushana & Stainback [10] in Fig. 2.23a. For all cases, the sensitivity is increasing with increasing Mach number. In the case of Nagabushana & Stainback, there seems to be no effect of the Knudsen number, contrary to cases 2 and 3. Nevertheless, for Case 3 the Knudsen number varies in a very small range, while Case 2 data are obtained at higher Mach and Knudsen numbers. Thus, further investigation is required before a conclusion can be reached. It can be seen though that the values of case 3 are close to the values obtained by Nagabushana & Stainback.

d_w	$3.8 \mu\text{m}$
T_w	583 K
M	0.05 - 0.49
Re	2 - 132
Kn	0.0022 - 0.0388
ρ	$0.57 - 9.5 \text{ kg/m}^3$
U	$17.4 - 122 \text{ m/s}$
T_0	$299 - 324 \text{ K}$

Table 2.4 – HW characteristics and test flow conditions, Stainback & Nagabushana [10].

The sensitivities to velocity are presented in Fig. 2.23b. The data of Nagabushana & Stainback feature significant scatter and it is difficult to discern clear patterns. Nevertheless, it seems that the sensitivity decreases for increasing Knudsen numbers (lower densities) as is also observed for Case 3. Moreover, there seems to be a decreasing trend with increasing Mach number which matches the behavior of Case 3 for $M > 0.42$. It seems, therefore, that the use of the Mach number correlation leads to physical trends of the velocity sensitivity.

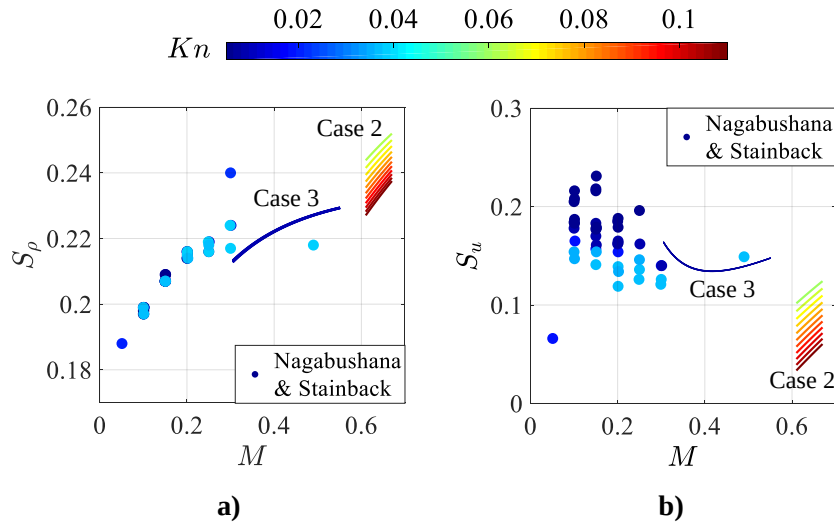


Figure 2.23 – Comparison of cases 2 & 3 with literature data: a) Sensitivity to density, b) Sensitivity to velocity.

In Fig. 2.24, the ratios of the sensitivity to velocity over the sensitivity to density are presented. Except for a few points of Nagabushana & Stainback, the sensitivity to velocity is always lower than the sensitivity to density. With the correlation in use, the sensitivity to velocity can never be higher than the sensitivity

to density for $M > 0.3$, as the term added to S_ρ in Eq. 2.13 is always negative. The ratio decreases with increasing Knudsen (lower velocities) for all cases, implying that at low density flows a hot wire is significantly more sensitive to density than to velocity. Thus, this should be considered when measuring in such kind of flows.

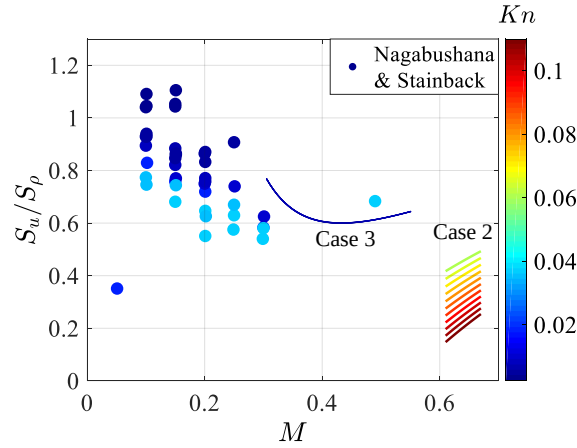


Figure 2.24 – Comparison of cases 2 & 3 with literature data: Sensitivity to velocity over sensitivity to density ratio.

2.5 Conclusions

This chapter is dedicated to the analysis of the operation of the Hot Wire Anemometry technique in Constant Temperature mode. As the operating principle is based on the heat transfer from the heated wire to the flow, the output of the instrument is sensitive to velocity, but also to temperature and density. In isothermal and incompressible flows, simple voltage-velocity calibration laws can be retrieved, but more elaborate schemes are required for all other cases.

The non-dimensional calibration methodology developed by Cukurel et al. [3] has been described in this chapter. This method consists in expressing the heat transfer law in non-dimensional numbers and using correlations to eliminate the compressibility effect. The resulting relationship is $Nu = f(Re)$. The methodology description has been included in this chapter, as the method is employed in the work presented in the next chapters of the thesis. An extension to this methodology has been added, by employing the correlation of Klopfer [12] to eliminate possible rarefaction effects in low Mach numbers.

The non-dimensional calibration has been compared to the King's law calibration and the sensitivities method for a simple, low speed case for which the assumptions of all three methods are met. A low-turbulence case is selected to satisfy the small fluctuations assumption implied for the sensitivities method. The turbulence intensity measured by the two first methods presents absolute variations on the order of 0.02%, which are attributed to the better fitting of the calibration curve for the non-dimensional method. The mass-flux sensitivity is computed by the non-dimensional calibration using three methods: a) by differentiation of the calibration polynomial's analytic expression and numerically, b) using a central difference scheme and c) using a finite difference scheme adapted for arbitrarily spaced grids. The third method, being a weighted method, was found to have the best performance, especially for points located at the extremes of the calibration curve.

The HWA sensitivities to density, velocity and total temperature have been studied in different flow conditions. For all the cases tested, it is clear that the sensitivity to total temperature is mainly dependent on the overheat, while slightly affected by Knudsen and Mach. Thus, by varying the temperature of the wire, very different sensitivities can be achieved. On the contrary, the overheat seems to have a small influence on the sensitivities to density and velocity, even when it is significantly varied (Case 1). As a result, using multiple wires operated at different temperatures might not be adequate in order to solve the sensitivity equation system. Other solutions should be investigated, like using different wire diameters (different Knudsen numbers), since the Knudsen number seems to have an important effect, at least on the velocity sensitivity.

The absolute value of the total temperature sensitivity is always more than 50% higher than the ones of velocity and density. The common assumption of negligible total temperature fluctuations should thus be treated with caution for each application, unless much higher overheats are used. In addition, the sensitivity to density is significantly higher than the sensitivity to velocity for low density flows, a fact that should be taken into account in such applications. In order to evaluate the methodology, the results are compared with experimentally computed sensitivities by Nagabushana & Stainback [10]. Unfortunately these data cover a small range of conditions and there is limited overlap with the current cases. Nevertheless, similar trends are observed and reasonable sensitivity values. Finally, while in general the sensitivity trends are non-linear, they feature linear behaviour in limited ranges. When such a case is considered, the calibration effort can be thus significantly reduced by minimizing the required number of calibration points.

To conclude, the most important remarks concerning the application of CTHWA are repeated here:

- When using typical overheating parameters of 0.7-0.8, S_{T0} is more than 50% higher than S_ρ and S_u . Hence, total temperature fluctuations can have

an important effect on the measurement.

- In slip flows and high subsonic flows S_ρ is significantly higher than S_u , so density fluctuations can strongly influence the measurement.
- Different overheats do not suffice to differentiate S_ρ and S_u in order to decouple the velocity and density fluctuations using the sensitivity equation. Other solutions should be investigated, like using different wire diameters.

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3

CTHWA Dynamic Response

This chapter focuses on the investigation of the dynamic response of Constant Temperature Hot Wire Anemometry (CTHWA), with the aim of understanding and minimizing the errors potentially inflicted on the measurement of turbulence parameters by the dynamic characteristics of the technique. The different phenomena affecting the dynamic response are examined using a numerical model, while an experimental method is proposed for the a posteriori correction of the measured turbulence spectra. Finally, a synthetic turbulence signal is used to perform a systematic study on the evaluation of errors as a function of the anemometer characteristics and the turbulent field properties. The focus is set on conditions relevant to turbomachinery flows.

3.1 Introduction

As every measurement technique, CTHWA is subject to different sources of errors including, but not limited to, calibration errors, errors due to mean fluid temperature variations and spatial resolution errors. The first two sources of errors have been extensively investigated, e.g. in Ref. [1] and [2]. The filtering effect on the measured fluctuations due to the limited spatial resolution of the hot wire sensor has been extensively treated as well, and a review of different studies considering

both isotropic and anisotropic turbulence can be found in Ref. [3].

Another source of errors is the dynamic response. A CTHWA is composed of a hot wire probe and an electronic control unit, which consists of a Wheatstone bridge (the hot wire is part of the active leg of the bridge) and a feedback loop which adjusts the current to keep the temperature of the wire constant. The dynamic response of the system is, therefore, defined by the properties of the wire and the electronics. The popular method to test the frequency response of a CTHWA is the well-known "square-wave" test. It consists in injecting a pulsating current in the circuit (usually at the diagonal of the Wheatstone bridge) and monitoring the output voltage response in an oscilloscope, as depicted in Fig. 3.1. A square wave testing unit is implemented in most commercial anemometers. The system is then tuned in order to obtain an optimal response, resembling to a second order system. According to Freymuth [4], the optimal response shape is achieved when the response presents an undershoot of 15% relative to the maximum (as can be seen in Fig.3.2). Most anemometer systems include at least two control parameters that allow the user to adjust the response. In the Dantec system used in the present study, these controls are the gain and filter setting of the feedback amplifier and the cable compensation, which balances the capacitance and inductance on both legs of the Wheatstone bridge. Freymuth [4] proposed the following expression for the estimation of the cut-off frequency f_{cut} of a well-adjusted system:

$$f_{cut} = \frac{1}{1.3\tau_w} \quad (3.1)$$

where τ_w is defined as the time when the amplitude is decreased at 3% of the peak value, as depicted in Fig. 3.2. f_{cut} is defined as the frequency at which the amplitude is attenuated by -3 dB. This criterion is valid for a system optimized in order to achieve the highest cut-off frequency with maximally flat frequency response. It is suggested that the tuning of the system should be performed for the highest velocity expected in the measurements (or for the condition with the highest convective power of the flow when also density and temperature can vary), since operation at higher velocities can lead to instabilities and self-oscillations.

The common way to use CTHWA is to optimize the dynamic response for one operating point using the well-known square wave test and use static calibration laws to retrieve the instantaneous velocity, assuming a flat frequency response up to the cut-off frequency. This practice neglects the following sources of potential errors:

- The attenuation of fluctuations due to the heat conduction to the wire supports.
- The fact that the square wave test doesn't guarantee a flat frequency response up to the cut-off frequency and can result in amplification or damping of fluctuations.

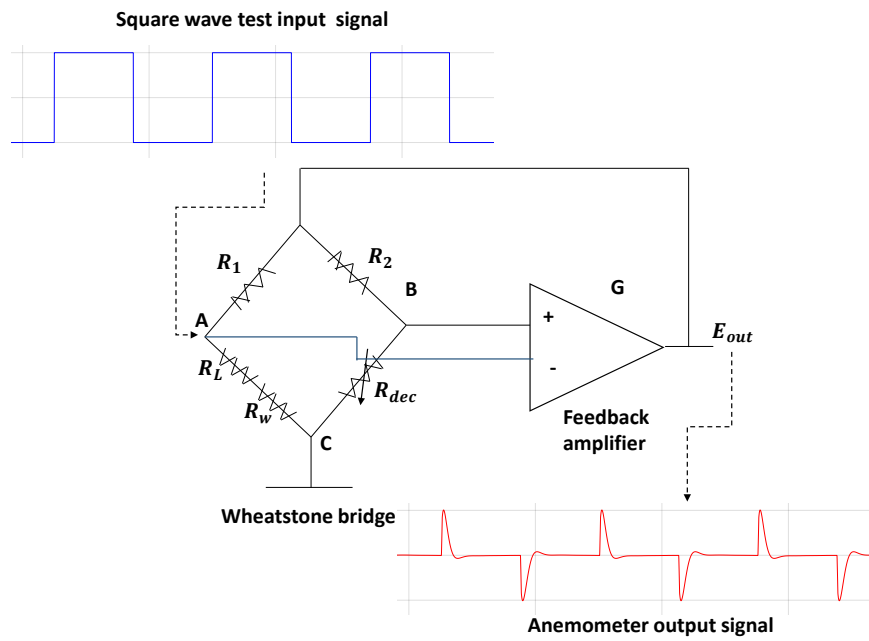


Figure 3.1 – Schematic of square wave test.

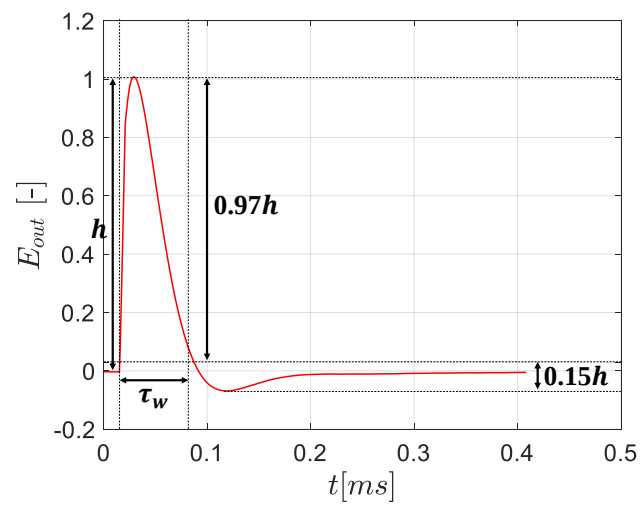


Figure 3.2 – Anemometer time response to square wave test: optimal shape.

- The non-linear nature of the governing equations.

These sources are analyzed in the following subsections.

Another fundamental matter is whether the square wave testing is really equivalent of a velocity perturbation, as contradicting results can be found in the literature. This issue will also be treated in the following.

3.1.1 Heat conduction effects

The heat conduction from the wire to the prongs leads to a non-uniform temperature distribution along the wire, resulting in the attenuation of fluctuations due to thermal transient effects. This phenomenon is presented in Fig. 3.3. This attenuation occurs at frequencies lower than the cut-off frequency of the system, since the time constant of thermal effects is longer compared to the time constant of the system. This means that even for an optimally adjusted anemometer, the response is not flat up to the cut-off frequency. Freymuth [5], Morris & Foss [6] and Li [7], among others, have investigated this phenomenon. The attenuation starts at a frequency f_1 , which according to Freymuth [5] can be estimated as:

$$f_1 = \frac{40D_w}{2\pi l^2} \quad (3.2)$$

where D_w is the thermal diffusivity of the wire material and l is the wire length. As the frequency increases, the attenuation reaches a plateau at a frequency f_2 . The value of the final attenuation factor e_1 is a function of the Biot number of the wire, which can be estimated by [5]:

$$Bi = \frac{l}{d} \left(\frac{k_f}{k_w} Nu \right)^{1/2} \quad (3.3)$$

The Biot number is a function of the wire aspect ratio l/d , the Nusselt number and the ratio of the thermal conductivity of fluid and wire.

The conduction losses to the wire supports, and consequently the attenuation, clearly decrease for wire materials of low conductivity, for high Nusselt numbers and for large length-to-diameter ratios. It should be noted, however, that large wire lengths decrease the spatial resolution of hot-wires, since fluctuations smaller than the wire are averaged over the wire length, a phenomenon called spatial filtering. Therefore, while it would be convenient to increase the wire length in order to achieve large l/d ratios without decreasing the diameter (and thus making the wire more fragile), this comes at the expense of spatial resolution. In their well-known study, Ligrani & Bradshaw [8] suggested that for accurate measurements of turbulence in the viscous sublayer of a turbulent boundary layer, the length should be 20 times smaller than the viscous length scale and the length-to-diameter ratio larger than 200. While the criterion of $l/d > 200$ has been used ever since as a

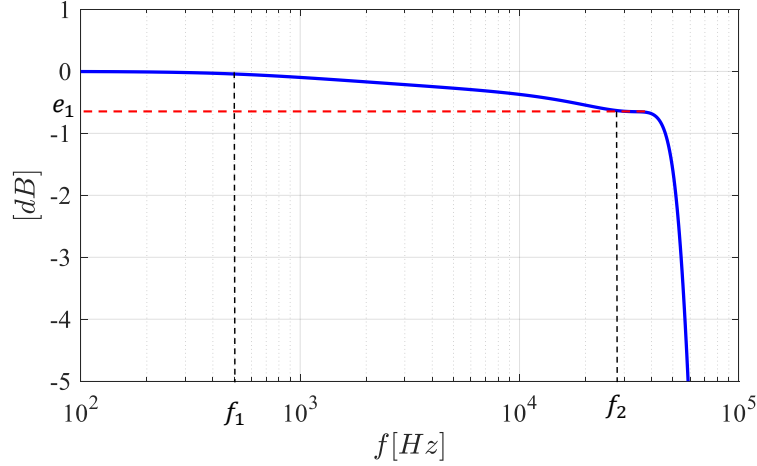


Figure 3.3 – Modelled CTHWA transfer function with attenuation at low frequencies due to heat conduction.

rule of thumb to design probes for low conduction effects, the limit value should depend on the flow conditions and the wire material, as expressed by Eq. (3.3). In this context, Hultmark et al. [9] proposed a new criterion for negligible attenuation due to heat conduction which takes into account the aforementioned parameters:

$$\Gamma = \frac{l}{d} \sqrt{4\alpha(k_f/k_w)Nu} \quad (3.4)$$

where k_f is the thermal conductivity of the fluid evaluated at film temperature, k_w is the conductivity of the wire material and α the thermal coefficient of resistance of the wire material. For $\Gamma > 14$ conduction effects are considered negligible. Li [10], on the other hand, proposed a correction scheme using a digital filter to correct the effect of the attenuation a posteriori. Considering the above, it is clear that the optimal design for a hot wire probe, in terms of temporal and spatial resolution, depends on the expected flow conditions.

3.1.2 Non-optimal adjustment of the dynamic response

Despite being easy to perform, the square wave test is not a very reliable way to determine the dynamic response. Optimal adjustment is not easy to achieve by monitoring the time response, as can be seen in Ref. [11]. Moreover, as the dynamic response of the CTHWA depends on the flow conditions, it is clear that Eq. (3.1) is valid only for the velocity at which the tuning was performed. Assuming a flat response up to the cut-off frequency can lead to over- or underestimation of

the turbulent energy if the anemometer is not optimally adjusted. Fig. 3.4 is used to showcase this point, where the transfer function of three anemometers with the same cut-off frequency but different adjustments is presented. One is optimally adjusted (therefore its response is as flat as possible before the cut-off frequency), one is underdamped (therefore with amplification of the energy before the cut-off frequency) and the other is overdamped (meaning that the energy is damped before the cut-off frequency). In order to overcome these problems, Weiss et al. [12] proposed to compute the complete transfer function of the system by the Fourier transform of the time response to the square wave voltage. The transfer function was used in the post-processing to correct the amplitude of the turbulent spectra. When different operating conditions are tested, a different transfer function should be obtained for each condition. This method is therefore suitable to correct turbulence spectra acquired at a uniform flow but can not be applied in flows with strong transients.

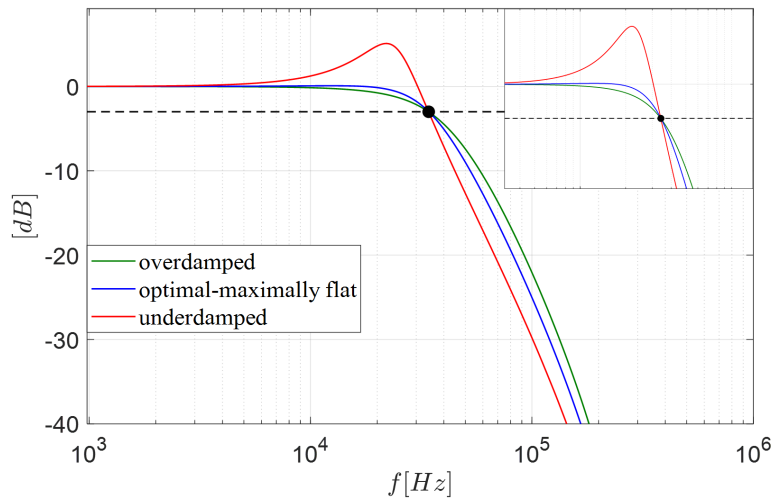


Figure 3.4 – Transfer function of optimally and non-optimally adjusted CTA systems with the same cut-off frequency.

3.1.3 Non-linearity effects

The issue of non-linearity is probably the most overlooked. Freymuth [4] developed the non-linear theory for CTHWA and demonstrated that for a well-tuned system and small fluctuations, their behavior can be sufficiently described by a third order linear equation. Thereafter, most investigations have focused on the

linear theory and non-linear effects are usually neglected even for large fluctuations (see for example Ref. [13]). Freymuth [14] demonstrated analytically that non-linear effects become important when the amplitude or the frequency of the fluctuations increases. Fluctuations as large as 50% of the mean velocity fall in the linear regime when their frequency is lower than 10% of the cut-off frequency of the anemometer. The non-linearity effect consists in the generation of harmonics, as depicted in Fig. 3.5, where the spectra of an input sinusoidal signal and the anemometer's output signal are presented. The presence of higher harmonics is evident in the spectrum of the latter. The non-linearity has a small effect on the intensity of the fluctuations but affects significantly odd moments like the skewness.

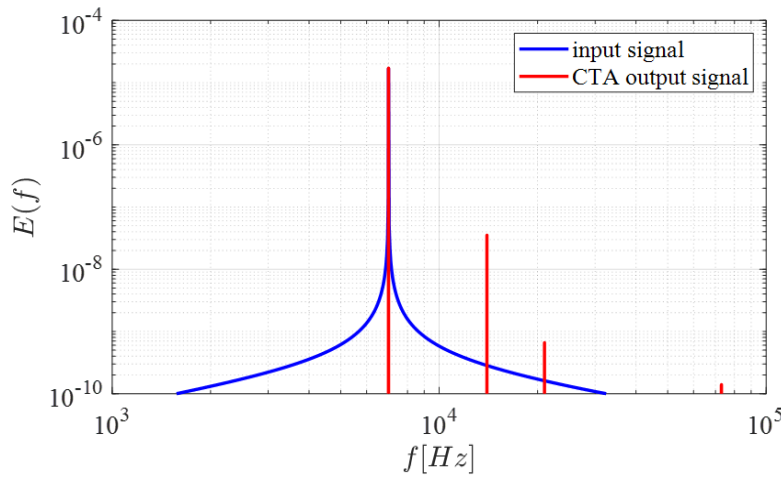


Figure 3.5 – Non-linear anemometer response to a sinusoidal input signal: generation of harmonics.

Weiss et al. [15] verified Freymuth's [14] analytical results by solving numerically the system of non-linear equations and comparing with experimental results by electronic testing for three different anemometers. This non-linear behavior has been investigated for Constant Current Anemometry (CCA) by Comte-Bellot & Schon [16] and for Constant Voltage Anemometry (CVA) by Berson et al. [17]. Berson et al. [17] applied a post-processing algorithm to correct for nonlinearities by inversely solving the differential equations characterizing the CVA's response. No similar method is available for CCA or CTA to the author's knowledge.

3.1.4 Square wave test suitability

In theory, the dynamic response of a CTHWA should be directly tested by imposing a known flow disturbance on the wire. In practice, it is difficult to create a velocity fluctuation of adequately high known frequency and amplitude and this led to the popularity of the square wave test. Nevertheless, there is disagreement in the literature on whether the square wave test is really equivalent to the imposition of a velocity perturbation on the wire. The equivalence of electronic testing to direct experimental testing is supported by theoretical investigations, like the work of Freymuth [18], who was the first to propose it, and Weiss et al. [12] who reconfirmed Freymuth's result. The few attempts to validate the square wave test by direct testing with flow perturbations produced contradicting results. Khoo et al. [19] compared voltage-perturbation testing to velocity-perturbation testing using a chopped rotating disc and reported a cut-off frequency of almost one order of magnitude lower for the latter, suggesting that the square wave test leads to significant overestimation of the cut-off frequency. Hutchins et al. [20] proposed a direct method to test the temporal resolution of CTHWA using the capabilities of the Princeton Superpipe to produce pipe flows with the same Reynolds number and turbulent energy in different frequency ranges. Assuming Reynolds similarity and scaling the spectra, they attributed differences at the high frequency range to limited frequency response. They concluded that the square wave test correctly estimated the dynamic behavior for two of the three anemometers tested. Li [21] conducted a numerical investigation of the response of a CTHWA under different types of perturbations. More specifically, he compared the response to a velocity impulse to the response when step voltage perturbations are introduced at the Wheatstone bridge and at the offset voltage of the amplifier. The voltage perturbations injection point seems to affect the response, with the response to offset voltage perturbations being closer to the one obtained by a velocity impulse perturbation.

A much less discussed point in the literature is that the amplitude of the square wave test signal has an effect on the response shape. Most commercial units do not foresee the test signal adjustment by the user, therefore this effect is not easily observed in practice. The characteristic time t_w in Fig. 3.2 will depend on the intensity of the test signal if this is the case. It has to be noted that in most numerical studies of the anemometer response (e.g. [21]), the use of small voltage perturbations is reported.

3.1.5 Objectives of the present work

Considering the literature review previously presented, it is clear that the dynamic response of CTHWA is a multifaceted issue. The aim of the present work is to propose a strategy to best face all these aspects for more accurate turbulence mea-

surements, while addressing gaps in the literature. More specifically, the following points are addressed:

- A practical experimental method to compute the transfer function using the square wave test is presented, based on the method of Weiss et al. [12]. The transfer function should be obtained for the operating condition of the experiment and is then used to correct the measurements a posteriori. This method allows the correction of non-optimal adjustment problems.
- The non-linear model of Freymuth [4] and Weiss et al. [15] is modified to consider also the heat conduction to the wire's prongs. This model is then used to investigate the effect of design parameters on the dynamic response.
- This enhanced version of the model is used to re-examine differences between the response to a square wave voltage perturbation with respect to a velocity (or Reynolds number) perturbation. The sensitivity of the system's response to different square voltage amplitudes is also investigated.

Even when an anemometer is optimally adjusted (with maximally flat frequency response) and with a design that minimizes conduction effects, an error will still be introduced in the measurement of turbulence statistics due to its finite temporal resolution. Moreover, possible non-linear effects will still be present as they are inherent to the system. Temporal resolution effects have not been as covered as spatial resolution effects in the literature. Moreover, non-linear effects have only been evaluated for sinusoidal fluctuations [15] and not for turbulent signals. These two issues are targeted by the last point:

- The error on the measurement of turbulence parameters due to the dynamic response of the anemometer is investigated using the non-linear model. An optimally adjusted anemometer is considered and heat conduction is neglected, in order to independently evaluate the effect of bandwidth limitations and non-linearity. Synthetic turbulence signals are employed as inputs to the model.

3.2 Experimental transfer function

3.2.1 Methodology

The experimental transfer function is obtained using the square wave test, following the method of Weiss et al. [12]. In this case, the embedded square wave generator of the Dantec Streamline anemometer, used for the operation of the hot wire, is employed for the square wave test. The square wave test is performed with

the head of the hot wire probe exposed at a uniform flow of constant velocity. The procedure is explained step by step below, using an example.

A hot wire with a tungsten sensor of $9\text{ }\mu\text{m}$ diameter and 1 mm length, operated at a wire temperature of 455 K is used. The probe head is placed inside the potential core of a jet subject to a constant flow of $M = 0.25$ and then $M = 0.6$. The square wave test is performed for both conditions and the response is optimally adjusted for each (the gain, filter and cable compensation settings have been tuned in order to achieve a response with 15% underdamped as indicated in Fig. 3.2). The anemometer output voltage is acquired for 1 s with a sampling frequency of 1.25 MHz . The response to the square wave voltage is superimposed to the response to flow fluctuations, therefore phase averaging is performed to obtain a clean signal. In Fig. (3.6) the raw output voltage and the phase-averaged voltage over 900 events are presented.

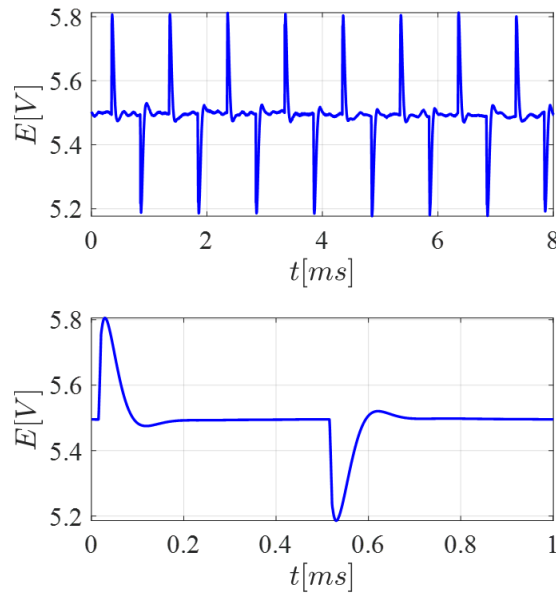


Figure 3.6 – Experimental time response to square wave test: Raw output voltage signal(top) and phase-averaged output voltage (bottom).

In order to obtain the response in the frequency domain, the following steps are performed:

- The time response to the positive or negative step (from the bottom figure 3.6) is extracted.

- The signal is artificially elongated, in order to increase the spectral resolution Δf in the frequency domain. The elongation time T_e is computed by:

$$T_e = \frac{1}{f_s} \left(\frac{f_s}{\Delta f} - N \right) \quad (3.5)$$

where f_s is the sampling frequency and N the number of samples of the original signal. In this example, Δf is set equal to 25 Hz and the signal is elongated by adding a vector with the mean voltage level at the end of the time series.

- The Fourier transform of the time series is performed. To obtain the transfer function, the amplitude of the Fourier transform is non-dimensionalized by dividing it with the amplitude value at a frequency where the response should be equal to 1. The frequency used for non-dimensionalization should be low enough to be considered unaffected by the cut-off of the system but also by the attenuation due to heat conduction.

The frequency response computed by this method is presented in Fig. 3.7 for both flow conditions. The cut-off frequency, computed as the frequency where the amplitude has been attenuated by -3 dB, is shown to be 9.67 kHz and 9.89 kHz, for Mach number 0.25 and 0.6 respectively. It can be seen that although the response was adjusted in both cases following the guidelines, the amplitude is not completely flat up to the cut-off frequency. There is a small amplification for both cases which indicates that the system is slightly underdamped. This result confirms that the square wave test does not always result in optimal adjustment of the dynamic response. In Fig. 3.8, the time response to the square wave test is given for both conditions and the time constants are indicated on the graph. The cut-off frequency can be estimated using the criterion of Freymuth (Eq. 3.1). The results are given in Table 3.1. The square wave test overestimates the cut-off frequency by approximately 1 kHz with respect to the Fourier transform method.

	Square wave		Fourier transform
	τ_w [ms]	f_{cut} [kHz]	f_{cut} [kHz]
M = 0.25	0.0721	10.67	9.67
M = 0.6	0.0704	10.93	9.89

Table 3.1 – Dynamic response evaluation with classical square wave test and Fourier transform method.

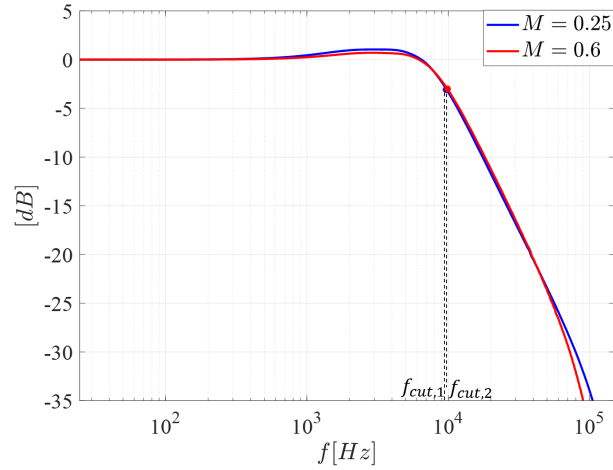


Figure 3.7 – Experimental frequency response to square wave test for two flow conditions.

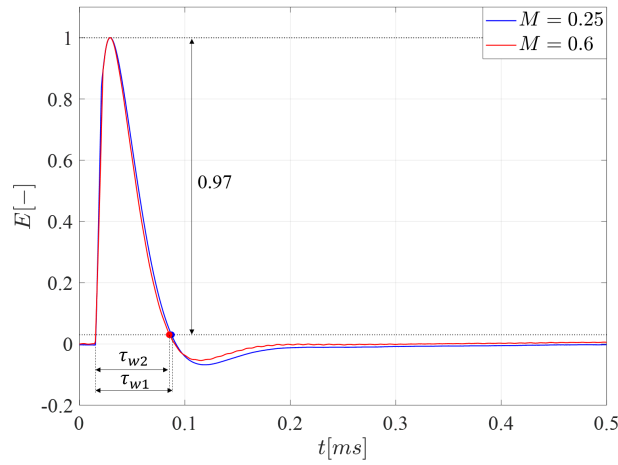


Figure 3.8 – Time response to square wave test for two flow conditions.

3.2.2 Effect of flow conditions and sensor properties

The effect of the flow conditions on the dynamic response is showcased in Fig. 3.9, where the experimental transfer function of a hot wire is presented for three different Mach numbers. In this case, the same dynamic response adjustments (gain, filter and cable compensation settings) are applied to all three tests. It can be seen that a higher velocity leads to a higher cut-off frequency (the values are

given in Table 3.2), but also to higher energy amplification. This indicates that higher velocity (higher convective power of the flow) leads the system towards a more unstable condition (more prone to oscillations).

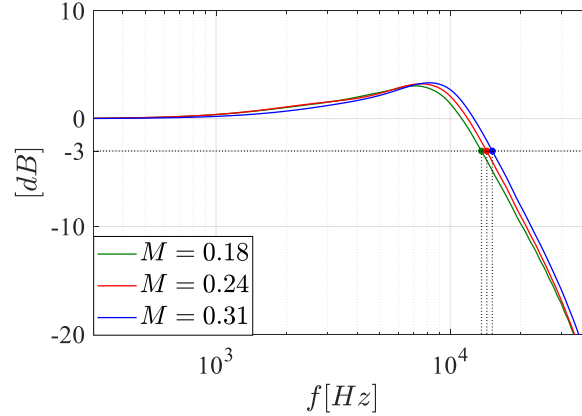


Figure 3.9 – Experimental frequency response to square wave test, Mach number effect.

M	f_{cut} [kHz]
0.18	13.6
0.24	14.3
0.31	15.13

Table 3.2 – Mach number effect on cut-off frequency.

The effect of the sensor properties on the dynamic response is instead presented in Fig. 3.10, where the experimental transfer function of three wires with different parameters is given, for a Mach number of 0.25. The wire properties and the cut-off frequency for optimal adjustment are presented in Table 3.3. The sensors feature different diameters d_w , different operating temperatures T_w and different materials, resulting in different temperature coefficients of resistance α_w : sensors 1 and 3 are tungsten wires, while sensor 2 is of Platinum-Nickel alloy (90%Pt/10%Ni).

The difference in the bandwidth can be explained using the concepts of the time constant of the wire and the system. Freymuth [4] demonstrated that the optimized cut-off frequency for a CTHWA system is given by:

$$f_{cut} = \left[\frac{G}{M''M} \right]^{1/3} \quad (3.6)$$

where M is the time constant of the wire, while G and M'' are the gain and the

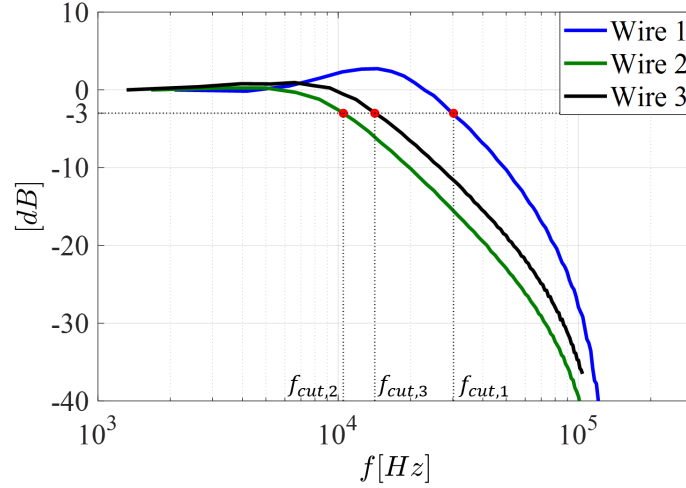


Figure 3.10 – Experimental frequency response to square wave test, effect of sensor properties.

	$d_w [\mu\text{m}]$	$T_w [\text{K}]$	Material	$\alpha_w [\%/K]$	$f_{\text{cut}} [\text{kHz}]$
Wire 1	5	435	Tungsten	0.36	30.2
Wire 2	9	540	Pt/Ni	0.13	10.5
Wire 3	9	470	Tungsten	0.36	14.5

Table 3.3 – Sensor properties effect on cut-off frequency.

second order time constant of the feedback amplifier respectively. The wire time constant can be computed as:

$$M = \frac{1}{2} \frac{(R_1 + R^* + R_L)^2}{R_1} \frac{m_w c_w}{f(Re) \pi l_w k (R^* - R_0) \alpha_w R_{ref}} \quad (3.7)$$

where R^* , R_{ref} , R_0 , R_1 and R_L are the operating wire resistance, the wire resistance at a reference temperature (usually 20 °C), the wire resistance at the flow temperature, the top resistance of the Wheatstone bridge and the resistance of the probe stem and cable respectively. m_w , c_w , l_w and α_w are the wire mass, heat capacity, length and temperature coefficient of resistance respectively. It can be seen that an increase in the wire diameter leads to a higher mass m_w of the wire and a larger time constant. This explains why in Fig. 3.10, Wire 1, with the smallest diameter of 5 μm , features the highest cut-off frequency. On the other hand, a larger temperature coefficient of resistance leads to a decrease of the time constant, therefore the Pt/Ni Wire 2, with $\alpha_w = 0.13\%$, features the lowest cut-off frequency. The

time constant is also a function of the Nusselt number and therefore the Reynolds number, since $Nu = f(Re)$. Higher Reynolds leads to a decrease of the time constant, explaining the increase of the cut-off frequency for increasing velocity in Fig. 3.7 and 3.9.

3.2.3 Compensation for the dynamic response

The examples presented above make clear that optimal adjustment of the dynamic response is not easy to achieve using the square wave test. As can be seen e.g. in Fig. 3.7 and 3.10, the response is often not flat before the cut-off frequency leading to over- or underestimation of the spectral energy. The amplitude transfer function $I(f)$ computed with the method presented in Section 3.2 can be used to correct the measured spectra and extend the useful bandwidth of the probe. It has to be noted that only the amplitude of the spectra can be corrected with this method, as only the amplitude transfer function has been computed. Information about the phase is not available.

The procedure used for the correction is the following:

- The Fourier transform of the raw signal is performed and its amplitude A and phase ϕ are computed.
- The transfer function is downsampled to match the frequency resolution of the signal.
- The amplitude is corrected as follows: $A_{corr}(f) = 10^{\frac{20 \log A(f) - I(f)}{20}}$.
- The corrected signal is then computed by: $z = A_{corr}(\cos \phi + i \sin \phi)$ and transformed to the time domain using an inverse Fourier transform.

The outcome of this procedure is depicted in Fig. 3.11, where the transfer function is used to correct the power spectral density (PSD) measured by a single hot wire in a jet flow. The transfer function computed for the same flow conditions is presented as well. It can be seen that the corrected spectrum shows a sharp increase after the cut-off frequency. This behaviour is due to the presence of electronic noise, as has been demonstrated by Weiss et al. [12], which limits the compensation range. The attenuation due to the dynamic response hides the presence of noise and can mislead one to believe that the anemometer is capable of resolving the spectrum up to higher frequencies. As the signal to noise ratio becomes lower than one at frequencies higher than the cut-off, a low-pass filter will be applied to the corrected signal at f_{cut} . This demonstrates that the dynamic response compensation can not extend the frequency bandwidth of a hot wire higher than its cut-off, but it is rather used to correct the amplitude up to this frequency.

It has to be noted that in the example presented in Fig. 3.11, the transfer function is flat, meaning that the anemometer is optimally adjusted. The need for

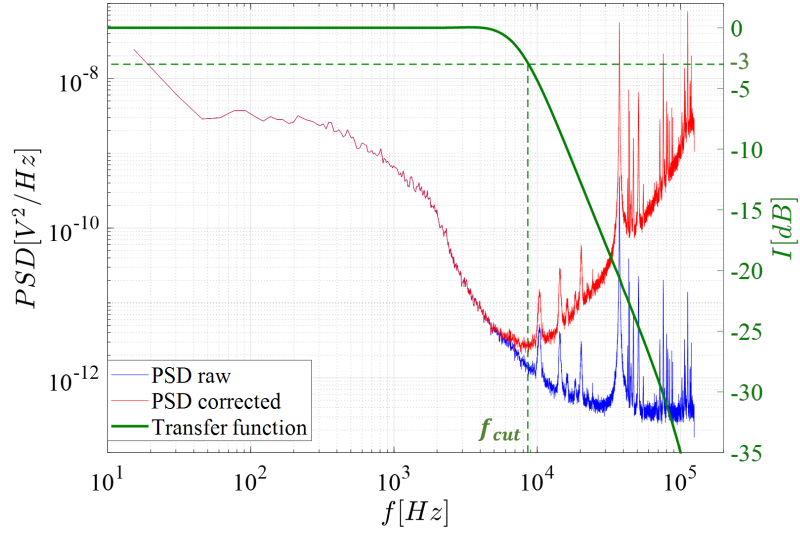


Figure 3.11 – Spectra correction for dynamic response

correction would be stronger in the case of underdamped or overdamped response. Similarly, the correction effect would be more important for a turbulence spectrum with energy content at higher frequencies, as in the present example the cut-off frequency falls in the dissipation range.

3.3 Dynamic response model

3.3.1 Governing equations

A schematic of a simplified CTHWA circuit is presented in Fig. 3.1. The hot wire probe is connected at the bottom left leg of the Wheatstone bridge, with R_w the resistance of the wire and R_L the resistance of the prongs, stem and cable. The resistance R_{dec} is adjustable and it is used to set the desired overheat ratio. The resistance R_w is maintained constant by the feedback amplifier, which adjusts the current to compensate for resistance fluctuations.

As previously mentioned, the model of Freymuth [4] and Weiss et al. [15] is used to simulate the dynamic behavior of a CTHWA system. Three equations are needed: one equation for the Wheatstone bridge, one for the feedback amplifier and one for the wire.

The equation representing the behavior of the Wheatstone bridge is the following:

$$\delta = \frac{R_1(R^* - R_w)}{(R_1 + R^* + R_L)(R_1 + R_w + R_L)}E + M_B \frac{dE}{dt} + \frac{R_1(R_w + R_L)}{R_1 + R_w + R_L} \frac{E_t}{R_t} \quad (3.8)$$

where δ is the voltage difference at the bridge diagonal ($\delta = V_B - V_A$, see Fig. (3.1)), R^* is the value of the operating wire resistance for a balanced bridge (the set value), E is the bridge output voltage and M_B a time constant. The last term in Eq. (3.8) is present only in the case of electronic testing, when a square wave voltage perturbation is injected at point A of the bridge. E_t is the square wave voltage and R_t is the resistance placed in series of the square wave voltage generator. During flow measurements this term is set to 0.

The equation simulating the behavior of the amplifier is a second order linear equation:

$$M'' \frac{d^2 E}{dt^2} + M' \frac{dE}{dt} + E = G\delta + E_b \quad (3.9)$$

where M'' and M' are the first and second order time constants of the amplifier and G is the gain. E_b is the offset voltage of the amplifier which can be adjusted to ensure a stable operation.

The wire equation has been modified to take into account a non-dimensional calibration law for the convective heat transfer ($Nu = f(Re)$ instead of $E = f(U)$). The equation can be expressed as follows:

$$\frac{m_w c_w}{\alpha_w R_{ref}} \frac{dR_w}{dt} = \frac{E^2 R_w}{(R_1 + R_L + R_w)^2} - Nu \pi l_w k_f \alpha_w R_{ref} (R_w - R_0) \quad (3.10)$$

The left hand side of the equation expresses the heat accumulation in the wire, while on the right hand side, the first term expresses the Joule heating and the second term the convection losses. R_w is the instantaneous wire resistance, R_{ref} is the wire resistance at a reference temperature of $T_{ref} = 20^\circ\text{C}$ and R_0 is the resistance of the wire when it is placed unheated in the flow. It is defined as: $R_0 = R_{ref}(1 + a_w(T_0 - T_{ref}))$ where T_0 is the total temperature of the flow and a_w is the wire material's temperature coefficient of resistance. Equations 3.8–3.10 form a system of non-linear ODEs with unknowns E , δ and R_w , which is solved with a Runge-Kutta method. The forcing function is the Nusselt number $Nu(t)$ which is related to the flow conditions using the following correlation [22]:

$$Nu(t) = 0.42Pr^{0.2} + 0.57Pr^{0.33}Re(t)^{0.5} \quad (3.11)$$

For a constant Prandtl number Pr (given fluid and small temperature variations), Nusselt is a unique function of Reynolds: $Nu(t) = f(Re(t))$. The Reynolds number is based on the wire diameter. Compressibility effects are not considered in this analysis for simplification, since they do not influence the dynamic response.

3.3.2 Model including conduction losses

In order to take into account thermal transient effects, the unsteady temperature distribution along the wire must be taken into account. In this case, a heat conduction term is added in Eq. (3.10). In order to solve the system, the methodology proposed by Morris & Foss [6] is followed, where the wire is discretized in $n - 1$ elements along its length, as can be seen in Fig. 3.12. A node is placed at the center of each element and at the two edges of the wire.

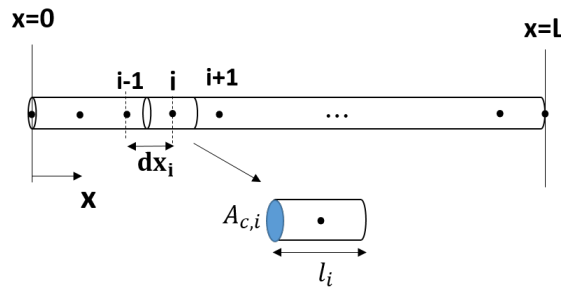


Figure 3.12 – Wire discretization

Eq. 3.10 is then transformed into the following Partial Differential Equation (PDE) for one hot wire element:

$$\Delta m_w c_w \frac{\partial T_w}{\partial t} = \frac{E^2 \Delta R_w}{(R_1 + R_L + R_w)^2} - Nu \pi l_w k_f (T_w - \eta T_0) - k_w A_c \Delta x \frac{\partial^2 T_w}{\partial x^2} \quad (3.12)$$

where Δm_w , ΔR_w , Δx and A_c are the mass, resistance, length and cross-section of the element. By discretizing the second order spatial derivative with a central difference, the wire equation is written for each node as follows:

$$\Delta m_{w,i} c_w \frac{dT_{w,i}}{dt} = \frac{E^2 \Delta R_{w,i}}{(R_1 + R_L + R_w)^2} - Nu \pi l_{w,i} k_f (T_{w,i} - \eta T_0) - k_{w,i} A_{c,i} \Delta x \frac{T_{w,i+1} - 2T_{w,i} + T_{w,i-1}}{\Delta x^2} \quad (3.13)$$

In this way, the PDE is transformed into an ODE for each node, so the system is now a system of $n+2$ ODEs (where n is the number of nodes) and it can be solved using a Runge-Kutta method.

The wire temperature that the circuit "feels" at each time is the spatial average of the temperature along the wire length:

$$T_w = \frac{1}{L} \int_0^L T_w(x) dx \quad (3.14)$$

The wire resistance R_w corresponding to this temperature is the input at Eq. (3.8) and (3.9):

$$R_w = R_{ref}(1 + a_w(T_w - T_{ref})) \quad (3.15)$$

where a_w is the temperature coefficient of resistance of the wire material and R_{ref} the wire resistance at a reference temperature T_{ref} .

In Fig. (3.12) the case of an unplated wire is presented, but Eq. (3.13) can also be applied to plated wires. In this case, the conductivity of the plating material and the corresponding diameter should be considered for the nodes in the plated length.

The wire is discretized in 101 nodes, and a time step of $10^{-7} s$ is used, resulting from a parametric study. The boundary condition at the edge of the wire is $T_{w,0} = T_{w,L} = T_{0,flow}$, assuming that the prongs remain at the average total temperature of the flow. This assumption is based on the fact that the mass of the prongs is much larger compared to the mass of the wire, leading to a higher thermal inertia, and on the fact that their larger surface results in higher convection.

3.4 Square wave test vs velocity perturbations

While the square wave test is the most popular method to test the dynamic response of a CTHWA, there is disagreement in the literature on whether it is equivalent to the imposition of a flow perturbation, as is the case during measurements. This will be investigated in this section using the dynamic response model.

- To test the response to a flow perturbation, the last term in Eq. 3.8 is set to 0 and $Nu(t)$ in Eq. 3.10 is the forcing function. More specifically, an impulse perturbation of 10% of the mean Reynolds value is imposed, and using Eq. 3.11, Nu is computed as $Nu(t) = f(Re(t))$.
- To simulate the response to the square wave test, the full Eq. 3.8 is used and a mean Nu number is input in Eq. 3.10, corresponding to the mean flow Reynolds number considered. A sensitivity analysis to the amplitude of the voltage E_t is first performed.

3.4.1 Model parameters estimation

The model parameters should be adjusted in order to represent correctly the electronic circuit and the hot wire considered. In this study, the experimental case described in Section 3.2.1 will be simulated, for $M = 0.25$. Since a commercial anemometer system is used in this study, some parameters from Eq. (3.8) and (3.9) are not known and should be estimated. The resistances R_1 and R_2 of the bridge are known, the resistances R_w and R_L depend on the hot wire probe used and

the resistance R_{dec} is adjusted accordingly, to fix the desired overheat ratio. The unknown parameters are therefore M_B , E_b , G , M'' and M' .

Weiss et al. [15] encountered the same problem and proposed to compute these parameters assuming an optimally adjusted anemometer and specifying the cut-off frequency of the system using the experimental result. This methodology will not be followed here for two reasons: firstly, as can be seen in Section 3.2.1, the response obtained from the experimental square wave test is not optimal but slightly underdamped. Moreover, the methodology of Weiss et al. [15] assumes a constant amplifier gain G and adjusts the parameters M_B , E_b . On the other hand, the Streamline system allows three modifications during the square wave test: the gain and filter of the feedback amplifier and the compensation for the cable inductance. The gain of the amplifier is therefore adjustable, and the filter setting f_{limit} is used to set the amplifier bandwidth limit. The compensation for the cable inductance is achieved by varying a set of capacitors or by varying the inductance of a set of coils placed in the bottom right leg of the bridge. No adjustment is available for the bridge voltage E_b .

The time constant M_B is used to take into account the high frequency effects of reactive elements in the bridge. According to Freymuth [4], if the cable inductance λ_c and an inductance λ_L in series with the resistance R_{dec} are considered, M_B can be computed as:

$$M_B = \frac{\lambda_L/R_2 - \lambda_c/R_1}{(n+1)^2} \quad (3.16)$$

where n is the bridge ratio. Since the compensation is not perfect, a finite small value is selected, $M_B = 10 \times 10^{-10}$ s. Since the same cable is assumed for all simulations, the value of M_B will be kept constant.

The offset value voltage E_b is assumed to be constant as well, as no adjustment is provided by the anemometer. As the parameter E_b increases the damping of the system, Perry & Morrison [23], suggested that when its adjustment is not available, a high value should be used to ensure the stability of the system at different conditions. Therefore, a constant value of $E_b = 5V$ is used.

Finally, since the parameters G , M'' and M' can not be known, they are adjusted in order to better match the response to the experimental square wave test.

3.4.2 Sensitivity of the system to square wave test excitation voltage

The commercial anemometer system used in the present work offers an embedded square wave generator which does not provide information on the circuit or the amplitude of the test signal injected to the Wheatstone bridge. The values of E_t and R_t in Eq. 3.8 are therefore unknown. The value $R_t = 993$ Ohm is selected for the resistor, as reported in Weiss et al. [15]. The response of the system to

different amplitudes of square wave voltage E_t has been evaluated and the results of the time response are presented in Fig. 3.13, where E_{out} is the output voltage of the anemometer, non-dimensionalized with the peak amplitude. It is clear that the amplitude of the square wave voltage has an important effect on the dynamic response of the system. The τ_w from Eq. 3.1 is clearly decreasing for increasing test voltage amplitudes, indicating that this criterion would estimate different values of cut-off frequency for each amplitude.

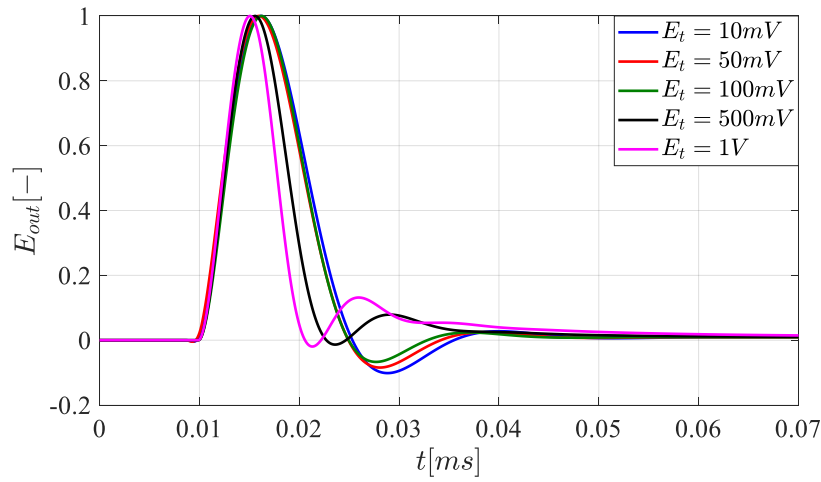


Figure 3.13 – Time response to square wave test with different amplitudes of the test voltage.

Looking in the literature for typically used square wave voltage amplitude values, an amplitude of 25 mV is used in [15] and a value of 50 mV is reported in [6]. Indeed, in Fig. 3.13 we can see that the effect on the response for an E_t amplitude of 10 mV-100 mV is rather small. An amplitude voltage value of 50 mV is therefore selected for the following analysis. Nevertheless, the results of this section suggests that the estimation of the frequency response with the square wave voltage should be treated with caution.

3.4.3 Results

The comparison between the response to the impulse in Reynolds and the square wave test is given in Figure 3.14 for the frequency domain and in Figure 3.15 for the time domain. The frequency response matches well for the two cases, with differences appearing at a frequency significantly higher than the cut-off. In the time domain, we can see that the response to the square-wave does not decay to 0

but instead stabilizes in a slightly higher value. This is typical behaviour for the square wave test, as mentioned also by Li [21], who notes that oscilloscopes with ac coupling that remove the dc voltage are usually used to monitor the square wave test response. This happens because the square wave is a step signal, and a higher voltage value is present in the circuit after the rising step.

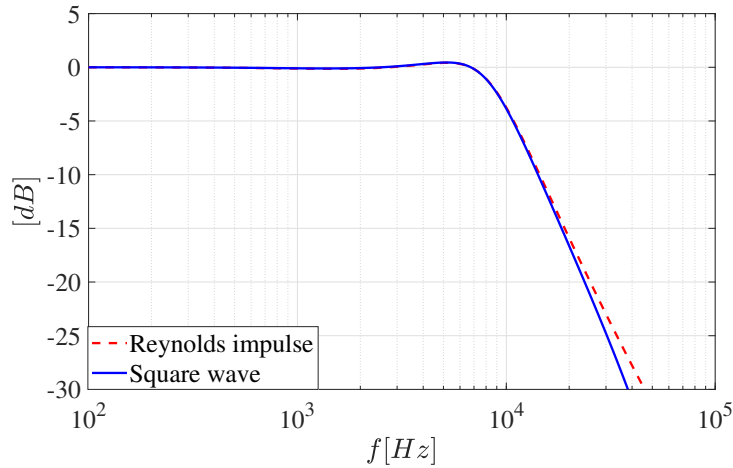


Figure 3.14 – Comparison of the simulated frequency response to the square wave test and to a Reynolds perturbation.

The results demonstrate that the square wave test with an amplitude of 50 mV simulates considerably well the response to a flow perturbation. The inverse transfer function could be used to correct the amplitude of turbulence spectra (as e.g. in Weiss et al. [12]), since differences occur for frequencies higher than the cut-off. These results do not agree with the results of Li [21], who observed bigger differences, especially when the square wave was injected in the bridge diagonal. He thus suggested to inject the square wave voltage at the offset of the last stage amplifier. In our case, the square wave generator used is implemented in the commercial anemometer unit and injects the voltage at the bridge diagonal. Therefore, this was the only case simulated.

3.5 Effect of design parameters on heat conduction

Heat conduction to the wire prongs results in a non-uniform temperature distribution along the wire and to amplitude attenuation. The mean wire temperature profile for the case presented in the previous section, when the wire is exposed to a uniform flow of $M = 0.25$, is presented in Figure 3.16. The resulting spatially

averaged wire temperature is $T_w = 450K$.

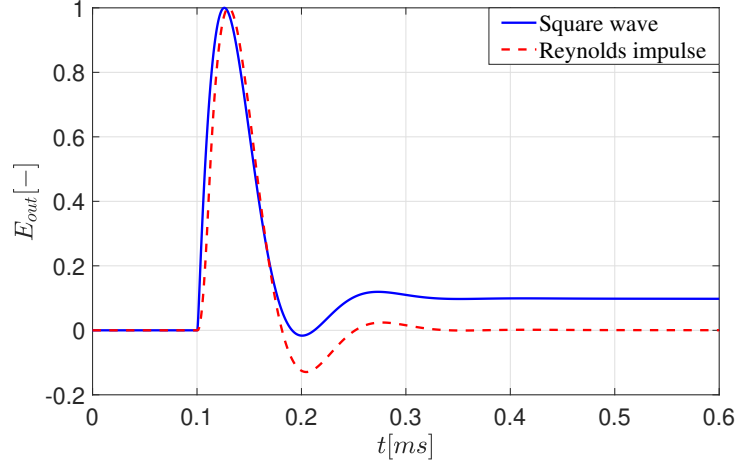


Figure 3.15 – Comparison of the simulated time response to the square wave test and to a Reynolds perturbation.

The wire aspect ratio in this case is $l/d = 111$, so lower than 200 which is assumed to be the limit value for negligible attenuation by some researchers (see section 3.1.1). Indeed, if we zoom in the low frequency part of Fig. 3.14 (presented at Fig. 3.17), we can see that the amplitude attenuation starts at a frequency of $f_l = 150Hz$ and extends up to a frequency of at least 1.4 kHz. This attenuation is present at both plots. The attenuation seems to be slightly stronger for the square wave test, but this difference can be considered negligible, therefore the square wave test could be used with an acceptable accuracy to estimate the attenuation effect. Li [21] arrived to the same conclusion.

For frequencies higher than 1.4 kHz, the amplification due to the system's dynamics does not allow to observe when the attenuation would reach the plateau. In order to focus on this phenomenon, the model's amplifier parameters are adjusted in order to move the system's cut-off to higher frequencies:

- $G = 1000$;
- $M' = 5 \times 10^{-5} \text{ s}$
- $M' = 10 \times 10^{-12} \text{ s}$

and the wire's length is varied in order to examine the length to diameter ratio's effect. The results are presented in Figure 3.18, where it can be seen that the cut-off has moved to much higher frequencies with the new parameters. The effect

of l/d on the wire temperature profile is presented in Fig. 3.19. It can be seen that the profile flattens with increasing l/d values, which is due to the fact that the convection effect becomes more important than the conduction effect.

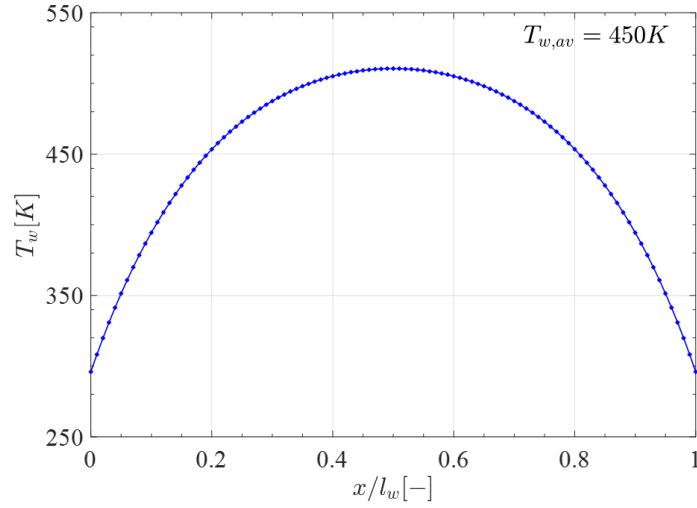


Figure 3.16 – Wire temperature profile.

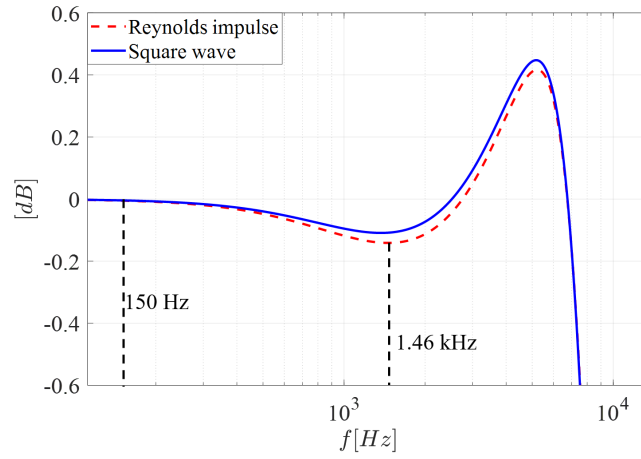


Figure 3.17 – Amplitude attenuation due to heat conduction.

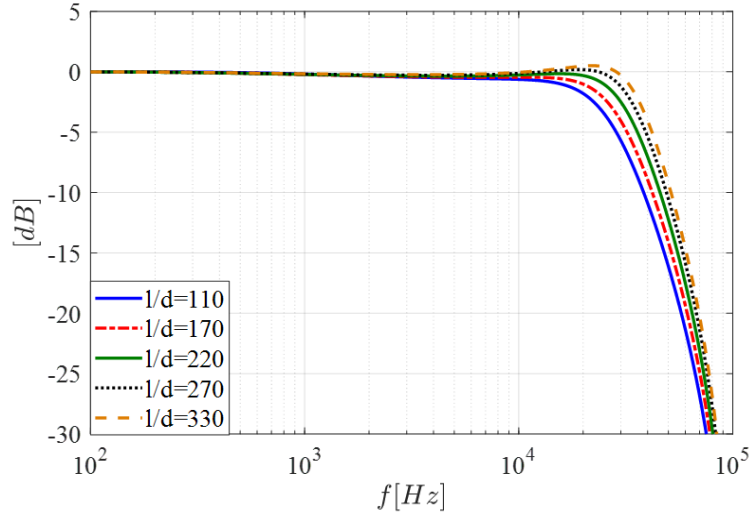


Figure 3.18 – Length to diameter ratio's effect on the amplitude transfer function.

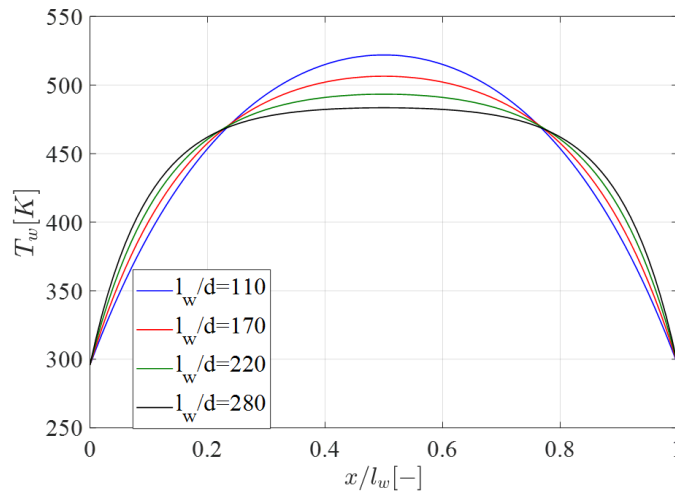


Figure 3.19 – Length to diameter ratio's effect on the wire temperature profile.

Focusing on the low-frequency part of the amplitude spectrum, given in Figure 3.20, it can be seen that the l/d has a strong effect on the attenuation. There is also an effect on the system dynamics: with increasing l/d (increasing l in this case) the convection term, which is the forcing function, increases and the system becomes underdamped. Nevertheless, this effect is present at high frequencies and should

not affect the attenuation. As expected, the final attenuation value decreases with increasing l/d values. Thus, high l/d ratios should be used in order to minimize this error. This can be achieved either by increasing the wire length at the expense of spatial resolution, or by decreasing the diameter compromising the mechanical strength. The optimal design should be decided depending on the requirements of each application. Finally, it can be observed by Figure 3.20 that the attenuation is present for a wide range of l/d , even for the highest value of 330. This indicates the importance of applying a correction.

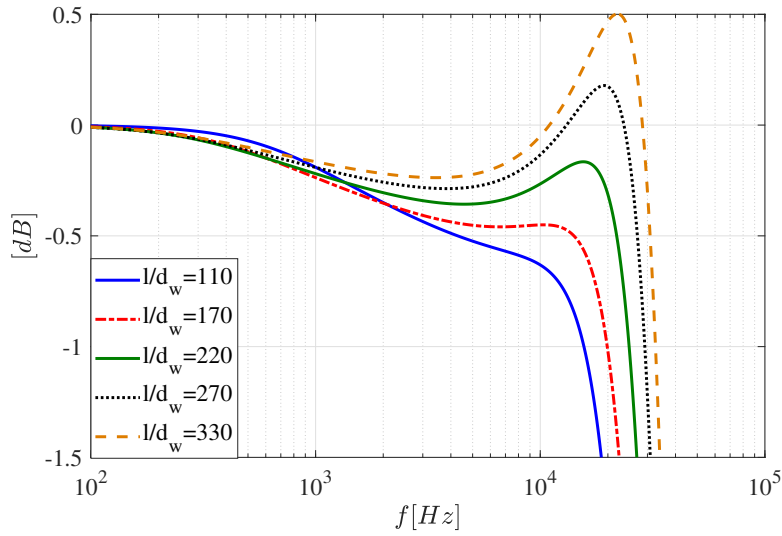


Figure 3.20 – Length to diameter ratio's effect on the amplitude attenuation due to heat conduction, zoom in the affected region.

3.6 Effect of bandwidth limitations and non-linearity on turbulence measurements

In this section, the non-linear model of Freymuth [4] and Weiss et al. [15] is employed to estimate the errors arising in the computation of turbulence quantities due to the limited bandwidth of the anemometers and potential non-linearity effects. Unlike the previous works for CTA, where the effects of non-linearity were evaluated using sinusoidal fluctuations as input signal, synthetic turbulent flow fluctuations with varying levels of turbulence intensity and length scales are used in the present work. The emphasis is placed on flow conditions that are of inter-

est for turbomachinery applications which are characterized by large fluctuations and high turbulence levels. Even in laboratory testing with grid-generated turbulence, the trend is to increase the inlet turbulence intensity to more realistic levels, reaching values up to 10% and 20%. In these conditions, it is probable that the small-fluctuations assumption is not valid anymore and that errors can affect the turbulence measurements due to the non-linearity of the CTHWA dynamic response. Optimally adjusted anemometers are considered in this study, in order to isolate the effects of bandwidth limitation and non-linearity. Moreover, this makes the study more general, applicable to any well-adjusted anemometer. The objective is to provide a way to estimate errors present in relevant studies (e.g. for uncertainty computation) but also to provide guidelines for the design of improved experiments.

3.6.1 Synthetic turbulence signal

In order to create a realistic turbulent velocity signal as input to the dynamic response model, a theoretical model for the turbulence energy spectrum is used. Pope's [24] model is employed, which expresses the 3D turbulent kinetic energy E_{33} as a function of wave number κ :

$$E_{33}(\kappa) = C\epsilon^{2/3}\kappa^{-5/3}f_L(\kappa L)f_\eta(\kappa\eta) \quad (3.17)$$

$C\epsilon^{2/3}\kappa^{-5/3}$ defines the inertial subrange part of the spectrum, where $C = 1.5$ is the Kolmogorov constant and ϵ is the dissipation. f_L and f_η are functions defining the shape of the energy-containing and dissipation range respectively and are defined as:

$$f_L(\kappa L) = \left(\frac{\kappa L}{[(\kappa L)^2 + c_L](1/2)} \right)^{(5/3+p_0)} \quad (3.18)$$

$$f_\eta(\kappa\eta) = \exp(-\beta([\kappa\eta]^4 + c_\eta^4)^{1/4} - c_\eta) \quad (3.19)$$

where L and η represent the integral and Kolmogorov length scale respectively. $\beta = 5.2$ and $p_0 = 2$ are empirically defined constants, while c_η and c_L are constants defined so that the following requirements are satisfied:

$$k = \int_0^\infty E_{33}(\kappa) d\kappa \quad (3.20)$$

$$\epsilon = \int_0^\infty 2\nu\kappa^2 E_{33}(\kappa) d\kappa \quad (3.21)$$

where ν is the kinematic viscosity of the fluid. For high Reynolds numbers $c_\eta \approx 0.4$ and $c_L \approx 6.78$.

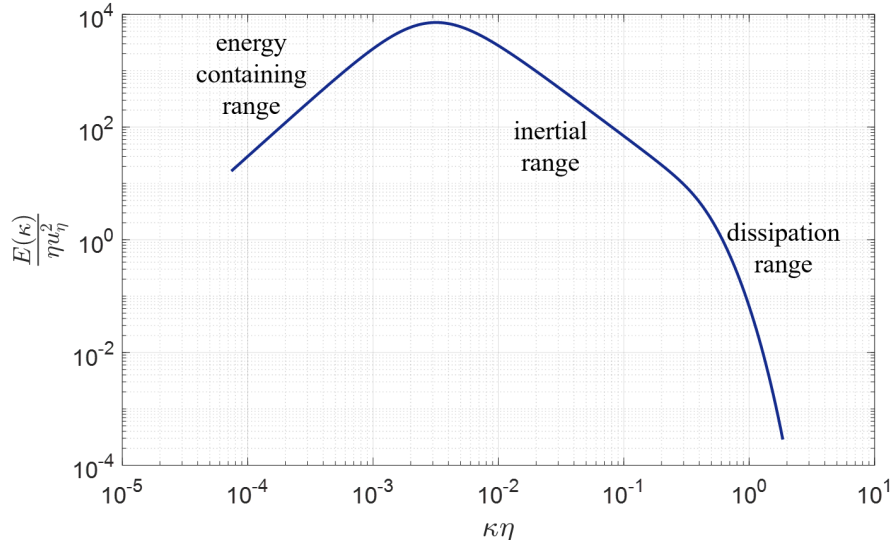


Figure 3.21 – 3D non-dimensional model spectrum for $R_\lambda = 230$.

The 3D model spectrum can be normalized by the Kolmogorov scales, in which case its shape is a unique function of the Taylor Reynolds number Re_λ . The non-dimensional model spectrum is presented in Fig. 3.21 for $R_\lambda = 230$, with the three distinctive ranges highlighted.

The turbulence intensity Tu (based on the longitudinal velocity) the integral length scale L and the kinematic viscosity ν can be linked to the turbulent kinetic energy k , the dissipation ϵ and the Kolmogorov length scale η , assuming isotropic turbulence and using scaling laws:

$$Tu = \frac{2k/3}{\bar{U}} \quad (3.22)$$

$$L = \frac{k^{2/3}}{\epsilon} \quad (3.23)$$

$$\frac{\eta}{L} = Re_L^{-3/4} \quad (3.24)$$

where $Re_L = \frac{k^{1/2} L}{\nu}$ is the Reynolds number based on the integral length scale. The Taylor scale Reynolds number Re_λ is related to Re_L by:

$$Re_\lambda = \left(\frac{20}{3} Re_L\right)^{1/2}. \quad (3.25)$$

For isotropic turbulence, the 1D turbulence spectrum can be computed as:

$$E_{11}(\kappa_1) = \int_{\kappa_1}^{\infty} \frac{E_{33}(\kappa)}{\kappa} \left(1 - \frac{\kappa_1^2}{\kappa^2}\right) d\kappa \quad (3.26)$$

while the longitudinal integral length scale is given by:

$$L_{11} = \frac{3\pi}{4k} \int_0^{\infty} \frac{E_{33}(\kappa)}{\kappa} d\kappa. \quad (3.27)$$

Using Taylor's [25] frozen vortex hypothesis, the spectrum can be expressed in terms of frequency ($f = (\kappa_1 U)/2\pi$, where U is the mean velocity). The velocity time signal can then be retrieved by the inverse FFT of the complex signal $z = r(\cos(\phi) + i\sin(\phi))$. The amplitude r is retrieved by the energy spectrum and random phase values are generated, uniformly distributed between 0 and 2π .

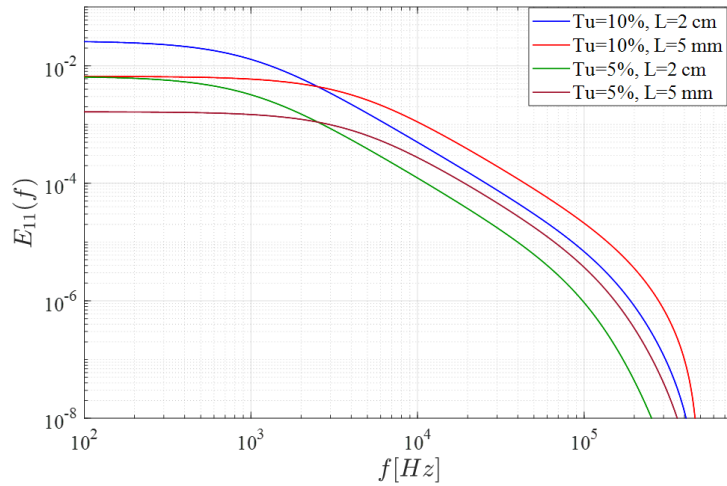


Figure 3.22 – 1D spectrum for different Tu and L .

Examples of the resulting 1D turbulence spectrum and velocity signal for different values of turbulence intensity and length scales are presented in Fig. 3.22 and Fig. 3.23. All the cases presented consider a constant kinematic viscosity ν of $1.567 \times 10^{-5} \text{ m}^2/\text{s}$, corresponding to ambient values of pressure and temperature and a constant Mach number of 0.2. It can be seen that for the same turbulence intensity and smaller length scale, the energy containing range of the spectrum has a lower energy level but extends to higher frequencies.

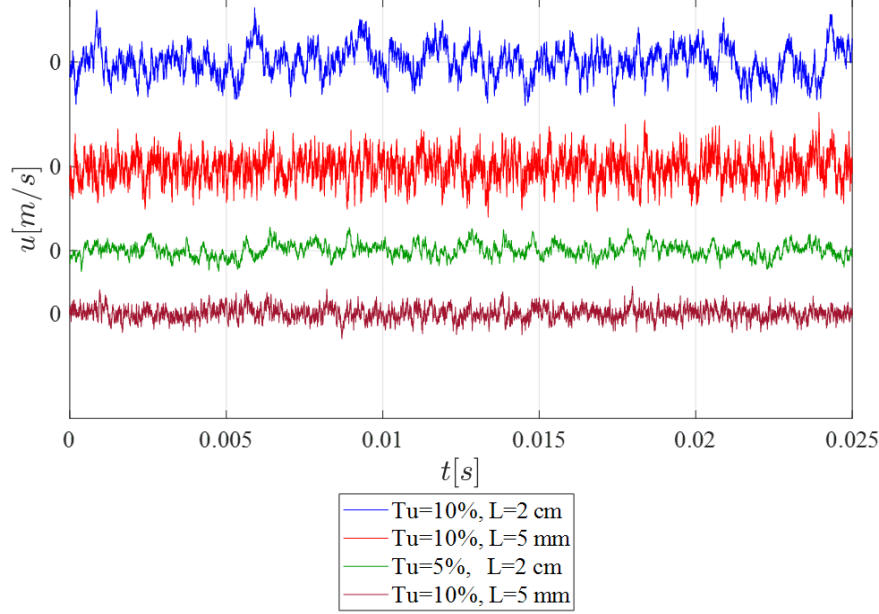


Figure 3.23 – Velocity fluctuations for different Tu and L .

The goal of this analysis is to define the properties that affect the shape of the spectrum and generate a population of synthetic velocity signals for a certain range of these properties that will be used to evaluate the behaviour of the anemometer in these conditions. A uniform flow is assumed along the wire, as only the temporal resolution effect will be analyzed. Spatial filtering effects due to integration over the sensor length are not considered in this study. The shape of the 1D spectrum can be uniquely defined by the properties k , ϵ and ν or alternatively Tu , L_{11} and a Reynolds number divided by the characteristic length $Re/c = \frac{U}{\nu}$, for a constant mean velocity. The second set of properties is preferred in this case, as Tu and L_{11} values are more frequently encountered in the literature compared to k and ϵ and are easier to estimate a priori during the design of experimental campaigns. Similarly, the Reynolds number is one of the main parameters used to characterize experiments. The range of conditions tested in this analysis is presented in Table 3.4: four ν levels, nine Tu values and seven L values, resulting in a $4 \times 9 \times 7$ test matrix.

3.6.2 CTHWA dynamic response model

In this analysis, three optimally adjusted anemometers with different bandwidths will be considered. The non-linear model of Freymuth [4] and Weiss [15] de-

ν [m ² /s]	Re/c [m ⁻¹]	Tu [%]	L ₁₁ [mm]
3.14×10^{-5}	2.20×10^6	0.5	1
1.96×10^{-5}	3.53×10^6	1	3
1.57×10^{-5}	4.41×10^6	3	5
1.05×10^{-5}	6.62×10^6	7	8
-	-	10	10
-	-	13	30
-	-	16	50
-	-	19	-
-	-	22	-

Table 3.4 – Flow conditions test matrix.

scribed in section 3.3.1 is used. The conduction losses are neglected in this case to isolate the bandwidth and non-linearity effects.

The model parameters are defined by the wire properties and the circuit adjustment. A tungsten wire with a diameter d_w of 5 μm and length l_w of 1 mm is considered. The corresponding mass m_w , temperature coefficient of resistance α_w , heat capacity c_w and reference resistance R_{ref} are 3.78×10^{-10} kg, 0.0036 K^{-1} , $133.9 \text{ J/kg} \cdot \text{K}$ and 3 Ohm respectively. The resistances R_1 and R_2 depend on the anemometer. In this case, a top resistance R_1 of 20 Ohm and a bridge ratio BR of 20 are used, leading to $R_2 = 400$ Ohm. The overheat ratio OHR ($\text{OHR} = (R^* - R_{ref})/R_{ref}$) is set to 0.58 which corresponds to an operating wire resistance of $R^* = 4.748$ Ohm and an operating temperature of $T_w = 455$ K. The resistance of the prongs, stem and cable is set to $R_L = 1.63$ Ohm and the decade resistance is set as: $R_{dec} = 20(R_L + R^*) = 127.56 \text{ Ohm}^1$.

The parameters G , M'' , M' , M_b and E_b determine the dynamic behavior of the system for fixed wire properties. According to Freymuth [14] and Weiss et al. [15], the cut-off frequency of the anemometer is determined by the time constant $[\frac{MM''}{G}]^{1/3}$, where M is the wire time constant defined as:

$$M = \frac{1}{2} \frac{(R_1 + R^* + R_L)^2}{R_1} \frac{m_w c_w}{f(Re) \pi l_w k (R^* - R_0)} \quad (3.28)$$

and M'' and G are characteristics of the anemometer's amplifier. In our case, since M is fixed, M'' and G are varied to change the cut-off frequency of the system. Assuming that the amplifier consists of two identical stages, the first order time constant is computed as $M' = 2\sqrt{M''}$. M_b and E_b can be adjusted to optimize the dynamic response. For a maximally flat frequency response, $M_{b,opt}$ and $E_{b,opt}$

¹These values refer to properties of an existing probe but were selected as typical values for a hot wire application

are computed as proposed in Freymuth [14] and Weiss et al. [15]:

$$M_{b,opt} = \frac{M'}{G} + \frac{\gamma M''}{GM_w} - \frac{2}{M} \left(\frac{MM''}{G} \right)^{2/3} \quad (3.29)$$

$$E_{b,opt} = E_0 \left[2 \frac{G}{M} \left(\frac{MM''}{G} \right)^{1/3} - \gamma \frac{M' - GM_{b,opt}}{M_w} \right] \quad (3.30)$$

In the present case, the anemometers are optimized for the mean flow conditions of each case. Three different anemometer circuits are considered to evaluate the effect of the system's cut-off frequency to the results. Their parameters are given in Table 3.5. The cut-off frequency is the frequency corresponding to a signal attenuation of -3 dB. The system's transfer function is evaluated by its response to a Reynolds impulse. The corresponding transfer functions for $\nu = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$ are presented in Fig. 3.24.

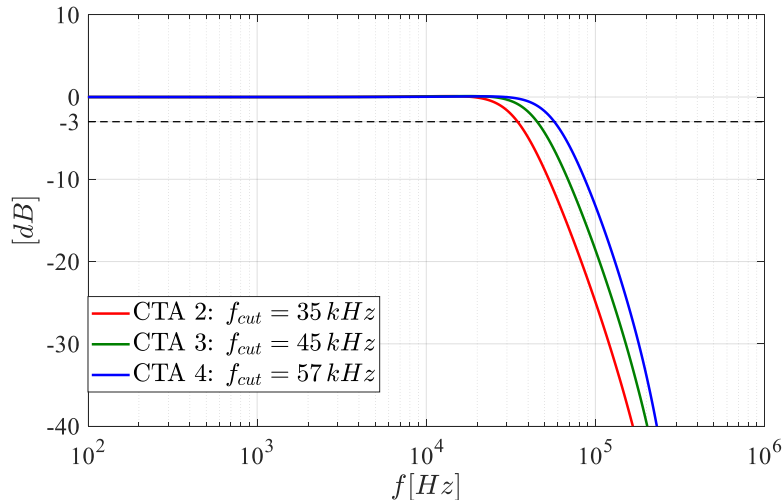


Figure 3.24 – Transfer function of three optimally-adjusted CTA with different bandwidth for $\nu = 1.57 \times 10^{-5}$.

3.6.3 Results

Synthetic turbulence fluctuations generated with the method previously explained are superimposed to a uniform mean flow of $M = 0.2$ and $T_0 = 300 \text{ K}$. The flow static pressure is varied between 500 and 1500 mbar for the different kinematic viscosity cases. The turbulence parameters test matrix is summarized in Table 3.4. The synthetic velocity signal is used to create a synthetic Reynolds number $Re(t)$,

		f_{cut} [kHz]	G [-]	M'' [s]
$\nu = 3.14 \times 10^{-5} \text{ m}^2/\text{s}$	CTA 1	34.7	732	6.43×10^{-12}
	CTA 2	44.5	955	3.72×10^{-12}
	CTA 3	57.0	1250	2.2×10^{-12}
$\nu = 1.96 \times 10^{-5} \text{ m}^2/\text{s}$	CTA 1	34.2	660	6.44×10^{-12}
	CTA 2	43.5	851	3.73×10^{-12}
	CTA 3	55.5	1110	2.23×10^{-12}
$\nu = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$	CTA 1	34.6	640	6.55×10^{-12}
	CTA 2	45.2	850	3.73×10^{-12}
	CTA 3	57.2	1100	2.23×10^{-12}
$\nu = 1.05 \times 10^{-5} \text{ m}^2/\text{s}$	CTA 1	34.4	585	6.54×10^{-12}
	CTA 2	45.0	780	3.72×10^{-12}
	CTA 3	56.5	1000	2.24×10^{-12}

Table 3.5 – CTA circuit parameters.

and using Eq. 3.11, the forcing function $Nu(t)$. The system of equations 3.8 - 3.10 is then solved for all three anemometers and the output signal is compared to the input. The errors on the measured fluctuations' rms, integral length scale and skewness due to the CTHWA dynamic response are evaluated. The synthetic signals are generated with a sampling frequency of 200 kHz and a duration of 0.4 s. To ensure that the results are independent of the signal duration and number of samples, the case of CTA 2 and $\nu = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$ was rerun for durations of 1 s and 2 s, yielding identical results.

The root mean square of the velocity fluctuations, u_{rms} is defined as:

$$u_{rms} = \left[\frac{1}{\Delta T} \int_{t_n}^{t_0 + \Delta T} (U(t) - \bar{U})^2 dt \right]^{1/2} \quad (3.31)$$

In temporal measurements, the integral length scale can be computed using Taylor's frozen vortex hypothesis [25] by multiplying the integral time scale by a convective velocity, which is usually considered equal to the local mean velocity. The integral time scale T expresses the time interval for which the velocity signal is correlated and can be computed as:

$$T = \int_0^\infty \rho(\tau) d\tau \quad (3.32)$$

where $\rho(\tau)$ is the autocorrelation coefficient:

$$\rho(\tau) = \frac{\overline{u(t)u(t+\tau)}}{\overline{u^2(t)}} \quad (3.33)$$

The longitudinal integral length scale is finally computed as:

$$L_{11} = \bar{U}T \quad (3.34)$$

The third order central moment, or skewness, is a measure of the lack of symmetry of the distribution and is zero for a symmetric distribution. It is computed as:

$$S_u = \frac{1}{(\overline{u^2})^{3/2}} \frac{1}{\Delta T} \int_{t_0}^{t_0+\Delta T} (U(t) - \bar{U})^3 dt \quad (3.35)$$

The relative error for a certain parameter x is defined as:

$$\delta_x = 100\% |x_{in} - x_{out}| / x_{in} \quad (3.36)$$

while the absolute error as:

$$y_x = x_{in} - x_{out} \quad (3.37)$$

3.6.3.1 Errors as function of Tu and L_{11} inputs

In this section, the errors on the measured turbulence quantities are evaluated as a function of the input turbulence level and integral length scale for a given anemometer and Reynolds number. The case $\nu = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$ and CTA 1 is selected.

The absolute and relative errors on the velocity rms are presented in Fig. 3.25 and Fig. 3.26 respectively. It can be seen that the velocity rms is underestimated for smaller integral length scales. As previously observed, this is explained by the extension of the energy containing range of the spectrum to higher frequencies. The absolute error is increasing linearly with increasing turbulence intensity. The relative error is also a function of the turbulence intensity for small values but reaches a plateau for $Tu > 2.5\%$. To better understand this behavior, the input and output energy spectrum for an integral length scale of 3 mm and the first three turbulence levels are presented in Fig. 3.27. The amplitude transfer function for CTA 1 is given as well. The input and output spectra match well, until the frequency for which attenuation starts, as indicated by the transfer function. The underestimation of u_{rms} is therefore due to the limited bandwidth of the CTA system which leads to attenuation of the output signal at high frequencies.

The absolute and relative errors on the longitudinal length scale are presented in Fig. 3.28 and Fig. 3.29 respectively. It can be seen that for the cases examined, the integral length scale is overestimated by the CTHWA. The relative error trend is similar to the u_{rms} error: higher for smaller length scales and increasing with Tu until $Tu > 2.5\%$. The longitudinal integral length scale for isotropic flow is directly related to:

$$L_{11} = \frac{\pi E_{11}(0)}{2u_1'^2} \quad (3.38)$$

Therefore, the underestimation of u_{rms} by the CTHWA due to limited bandwidth results in an overestimation of the integral length scale, as reflected in Fig. 3.28.

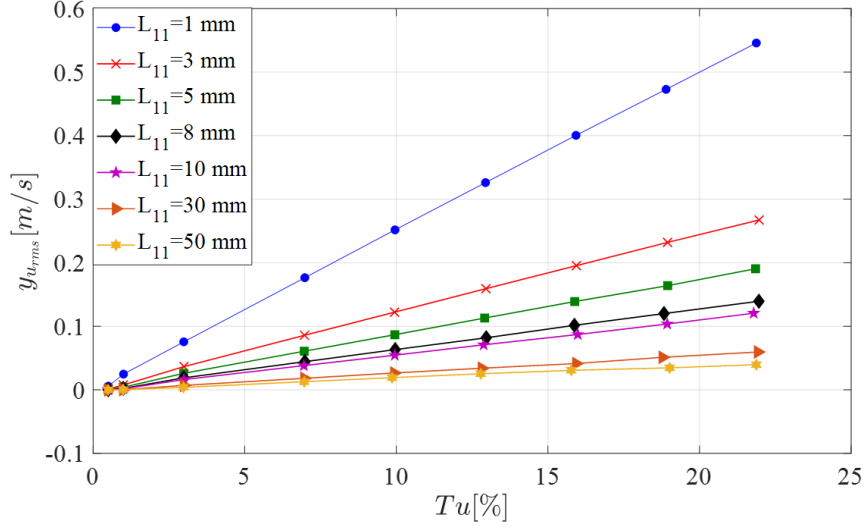


Figure 3.25 – Absolute error on the velocity rms for $\nu = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$ and CTA 1, as a function of turbulence intensity and integral length scale.

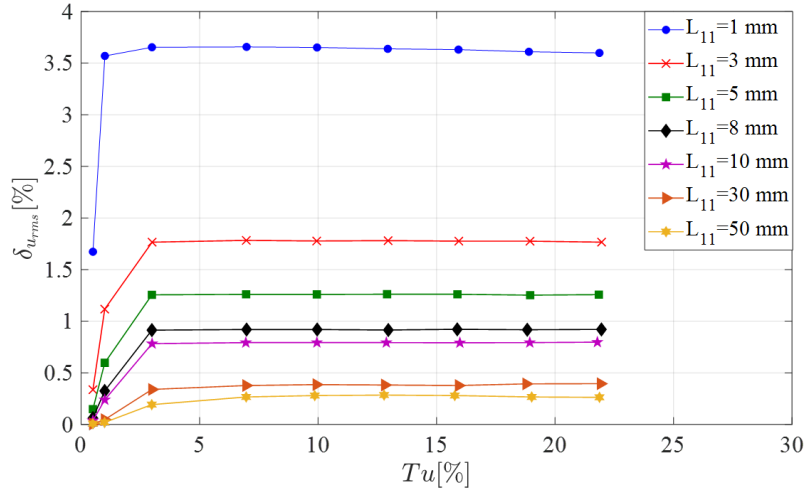


Figure 3.26 – Relative error on the velocity rms for $\nu = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$ and CTA 1, as a function of turbulence intensity and integral length scale..

The absolute error for the skewness is presented in Fig. 3.30. The relative error is omitted due to the input skewness value being close to zero. Moreover, the absolute error can be compared with the results of Weiss et al. [15]. It is verified that the skewness is increased by the presence of large fluctuations at high

frequencies, as a clear dependency on Tu and L_1 is observed.

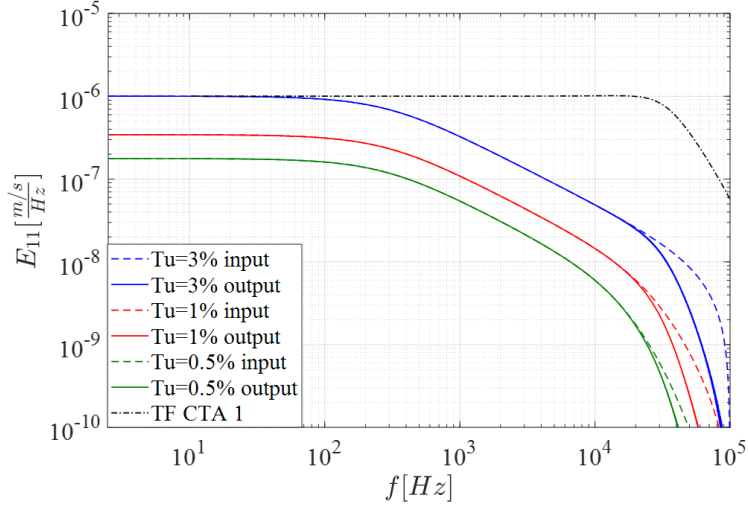


Figure 3.27 – Input and output turbulence energy spectrum for $L_{11} = 3\text{mm}$ and three turbulence levels, for $\nu = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$ and CTA 1.

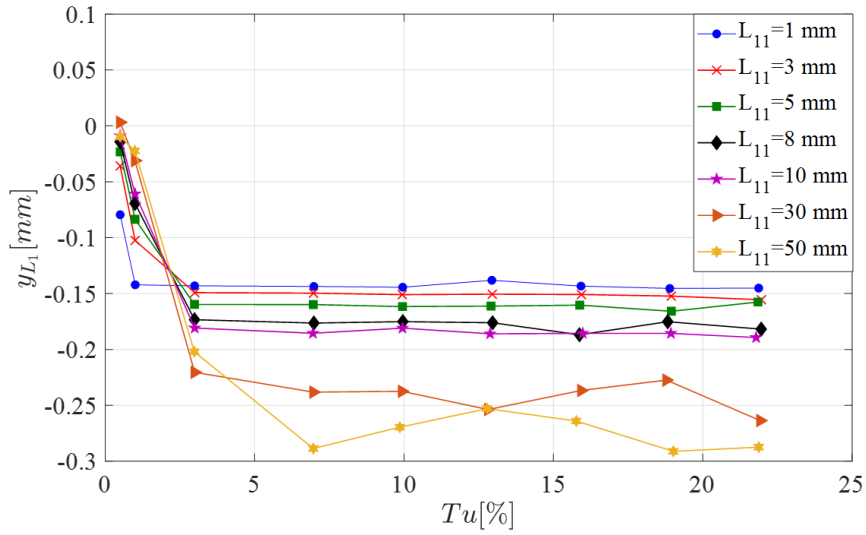


Figure 3.28 – Absolute error on the integral length scale for $\nu = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$ and CTA 1, as a function of turbulence intensity and integral length scale.

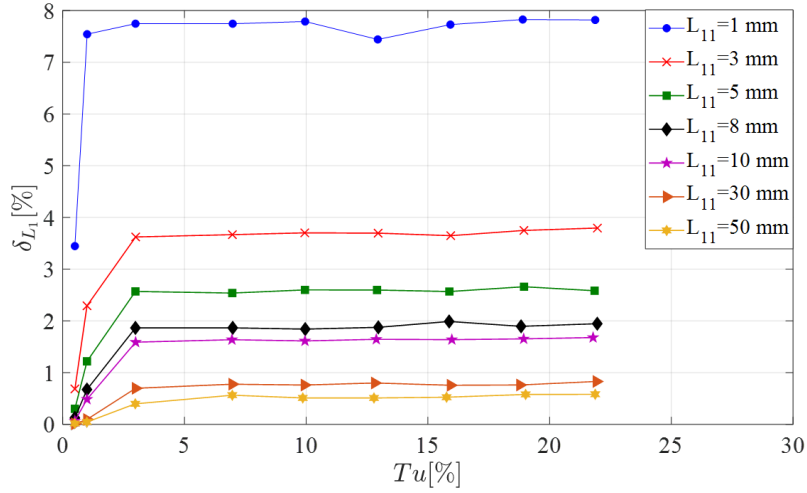


Figure 3.29 – Relative error on the integral length scale for $\nu = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$ and CTA 1, as a function of turbulence intensity and integral length scale.

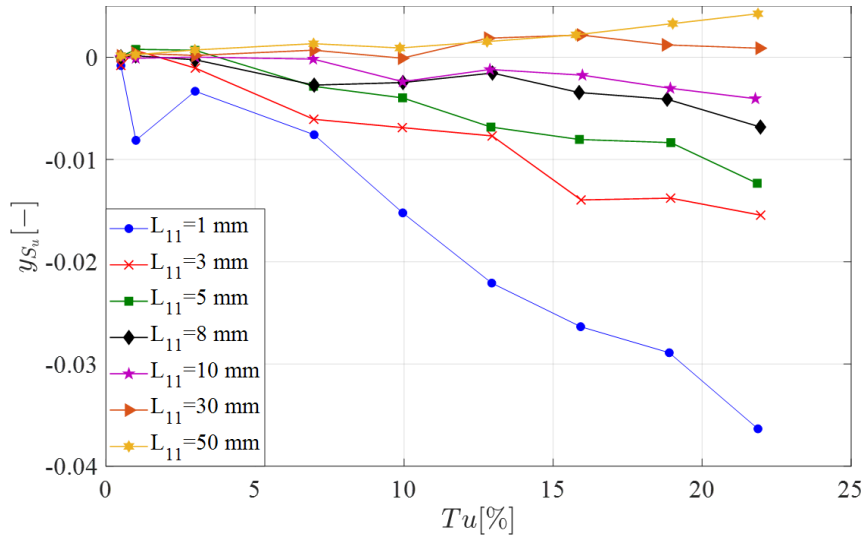


Figure 3.30 – Absolute error on the skewness for $\nu = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$ and CTA 1, as a function of turbulence intensity and integral length scale.

3.6.3.2 Errors as a function of the anemometer's cut-off

In this section, the results of all the anemometers are considered. As expected, the anemometers with higher cut-off frequency perform better and feature reduced er-

rors. The critical parameters are the position of the cut-off relative to the shape of the turbulence spectrum and the turbulence level. Therefore, the errors are plotted as a function of Tu and f_L/f_{cut} , where $f_L = \frac{\bar{U}}{L_{11}}$ is a characteristic frequency of the spectrum corresponding to the large, energy-containing eddies. This approach is in line with other studies, see Ashok et al. [26], who expressed the bandwidth requirements for minimum temporal resolution errors, in their case, as a function of the "kolmogorov frequency" $f_k = \frac{\bar{U}}{\eta}$. The frequency f_L related to the longitudinal integral length scale was considered more appropriate in this case, as the integral length scale value is easier to be estimated in practical experiments. The relative errors on u_{rms} and L_{11} , as well as the absolute error on the skewness are presented in Fig. 3.31 – 3.33. The same trend is observed overall as in the previous section for CTA 1. Therefore, we can use these results to evaluate the magnitude of errors for turbulence measurements with optimally adjusted anemometers (for maximally flat frequency response).

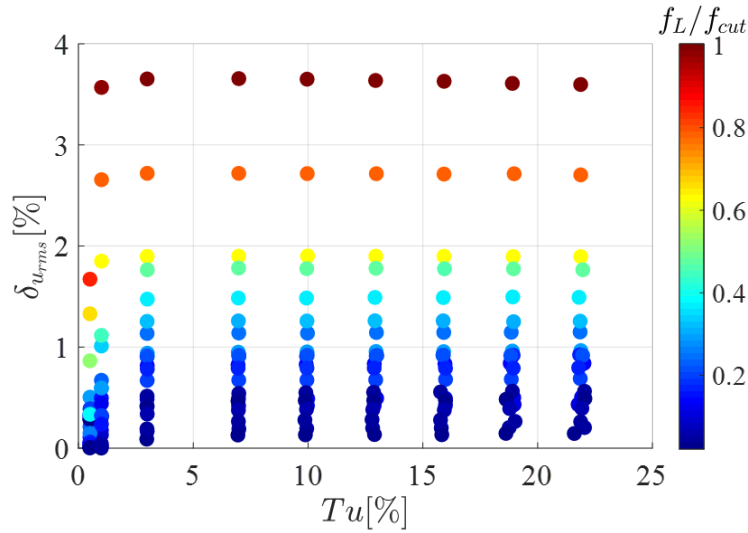


Figure 3.31 – Relative error on u_{rms} for $\nu = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$ and all CTA, as a function of turbulence intensity and f_L/f_{cut} .

3.6.3.3 Reynolds number effect

To better understand the effect of the Reynolds number (kinematic viscosity) on the spectrum, the input and output spectrum for CTA 2 is presented in Fig. 3.34, for the case $Tu = 5\%$ and $L_{11} = 3\text{mm}$. It can be seen that increasing the Reynolds number results in an extension of the inertial frequency range, therefore the dissipation range shifts to higher frequencies. This results in a decrease of the energy

level in the energy containing range, as the area under the curve should be equal for the same turbulence intensity level.

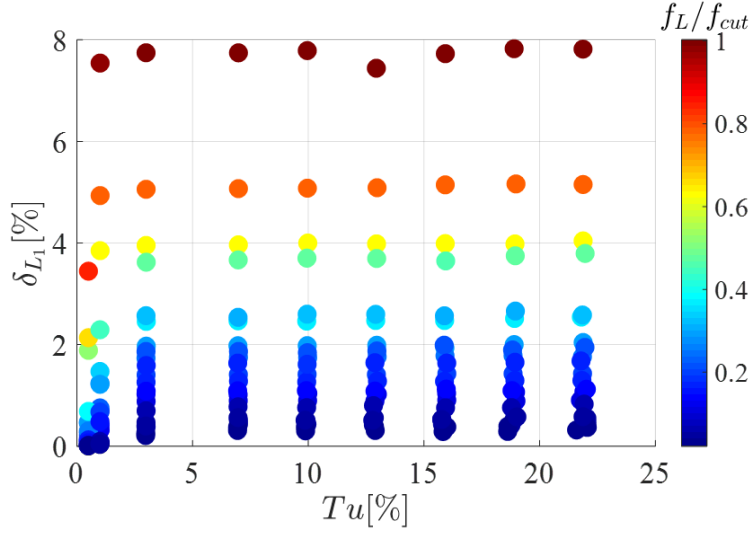


Figure 3.32 – Relative error on L_{11} for $\nu = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$ and all CTA, as a function of turbulence intensity and f_L/f_{cut} .

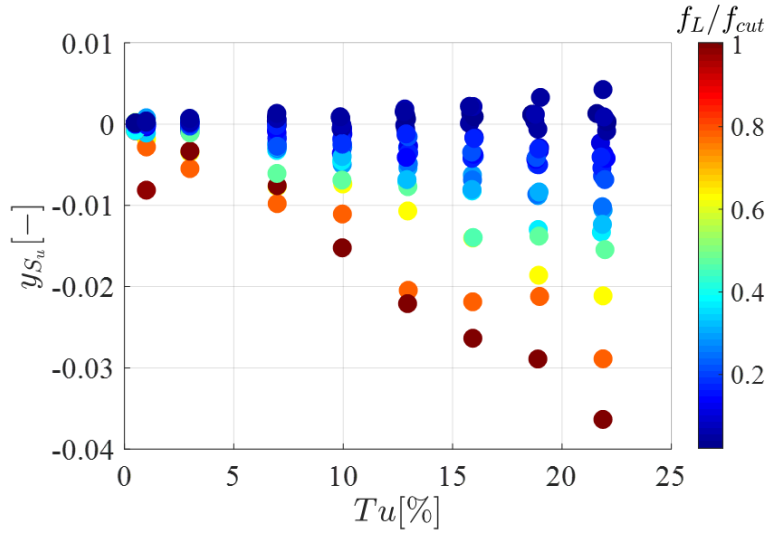


Figure 3.33 – Absolute error on the skewness for $\nu = 1.57 \times 10^{-5} \text{ m}^2/\text{s}$ and all CTA, as a function of turbulence intensity and f_L/f_{cut} .

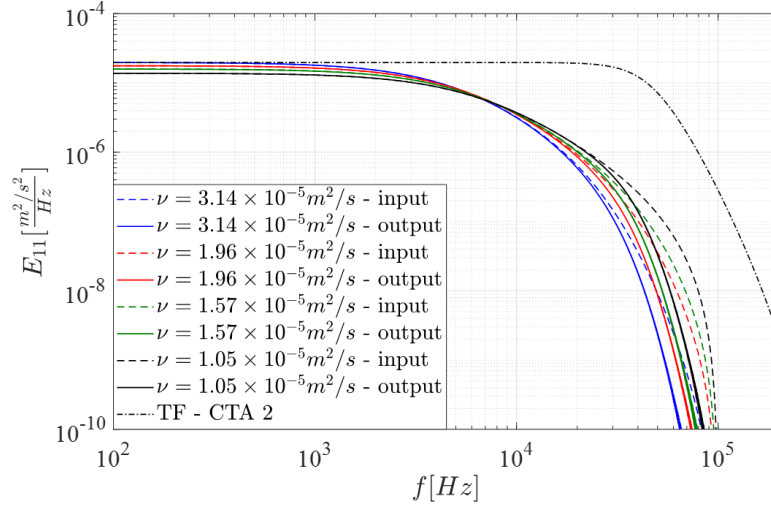


Figure 3.34 – Effect of Reynolds number on the input and output spectrum for $Tu = 5\%$ and $L_{11} = 3mm$.

The effect of the kinematic viscosity on the relative error of u_{rms} is illustrated in Fig. 3.35. It can be seen that the error is decreasing with lower ν (higher Re). The same effect is observed for the L_{11} error in Fig. 3.36. This behaviour is in line with the remark of Hutchins et al. [20] who expected a decrease in the relative errors for increasing Reynolds number for their boundary layer measurements. The opposite effect is observed for the skewness error in Fig. 3.37. This can be explained by the fact that, for increasing Reynolds number, large energy fluctuations are present at higher frequencies.

3.6.3.4 Application examples

The results of the previous analysis can be used to define the instrument's bandwidth requirements for a specific application or they can be employed in the framework of an uncertainty analysis after measurements have been performed. This will be demonstrated for two typical examples of turbomachinery laboratory flows. The first case is high-intensity, large-scale inlet turbulence for wind tunnel testing of high pressure turbine blades. For representative heat transfer measurements, the turbulence field at the combustor exit must be simulated, which is characterized by high turbulence intensity and large length scales. Many works exist in the literature, for example Ames & Plesniak [27] and van Fossen & Bunker [28].

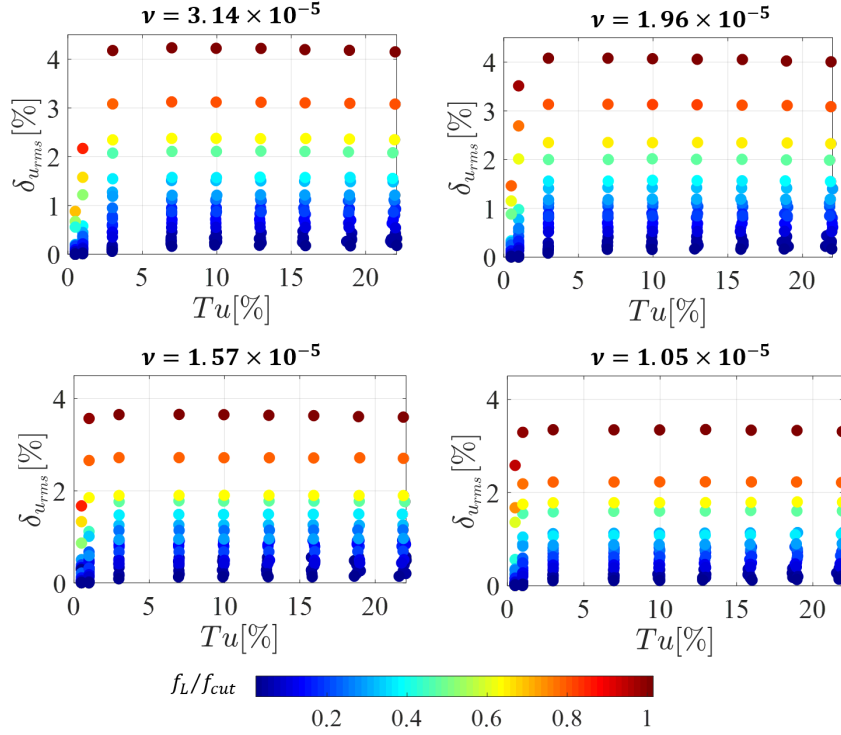


Figure 3.35 – Effect of Reynolds number on the u_{rms} relative error.

The turbulent field characteristics for these works are: $Tu = 28.5\%$ - $L = 3.3$ cm and $Tu = 12\%$ - $L = 3.3$ cm respectively. Assuming three different inlet Mach numbers of 0.1, 0.15 and 0.2 and ambient temperature, the characteristic frequency f_L can be computed for each condition as 1 kHz, 1.5 kHz and 2 kHz respectively. In this case, even for a CTHWA with a low cut-off frequency of 15 kHz, the parameter f_L/f_{cut} ranges from 0.067–0.13. As can be seen from Fig. 3.35 and 3.36, the error is less than 1% for both u_{rms} and L_{11} , for both turbulence levels and all Reynolds numbers considered. From Fig. 3.37, we can see that the error in the skewness is negligible. Despite the high turbulence levels, this case is rather favorable. Thanks to the large length scales and the low mean velocities, the high energy content is restricted to the low frequency range and the errors due to the dynamic response are minimized.

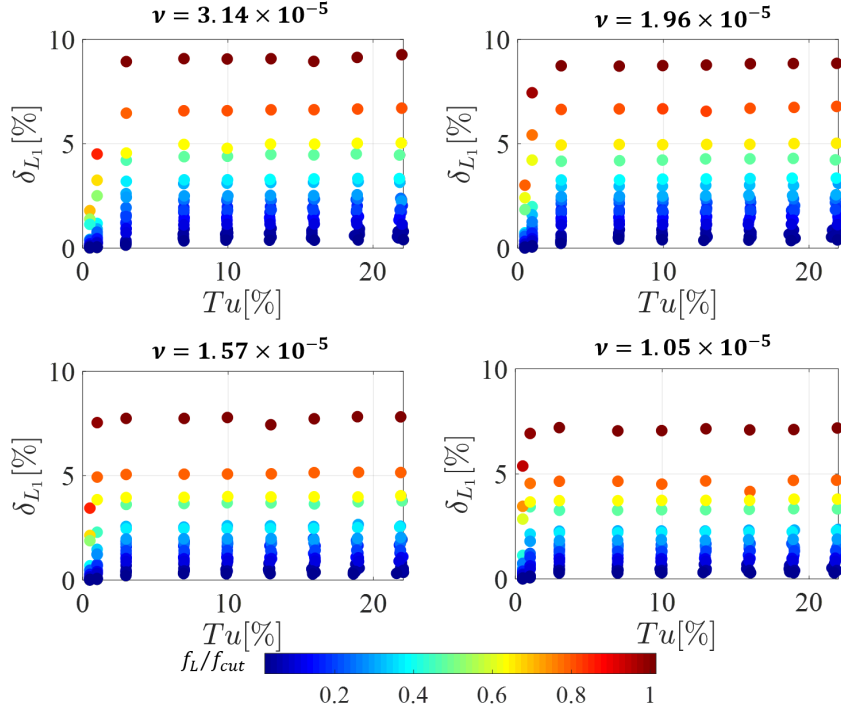


Figure 3.36 – Effect of Reynolds number on the L_{11} relative error.

For the second case, turbulence measurements inside a blade wake are considered. This case is characterized by high turbulence intensity (in the range of 10 – 15%, e.g. in Ref. [29]) but small length scales, of the order of magnitude of the trailing edge thickness. The Mach number can vary significantly depending on low-speed or high-speed testing. For a turbulence intensity level of 10%, a length scale of 2 mm and a Mach number of 0.3, the characteristic frequency f_L is equal to 49.5 kHz. From Fig. 3.35 and 3.36 it can be seen that to keep the error on u_{rms} less than 2%, f_L/f_{cut} must be lower than 0.6, resulting in an f_{cut} of at least 82.5 kHz. For the same f_{cut} , the error on L_{11} is around 4-5%. Finally, the error on the skewness is around -0.01 from Fig. 3.37. This case is more problematic due to the small length scales, as a significant part of the spectrum extends in the high frequency range. Anemometers with high frequency response should be used and the magnitude of possible errors should be evaluated.

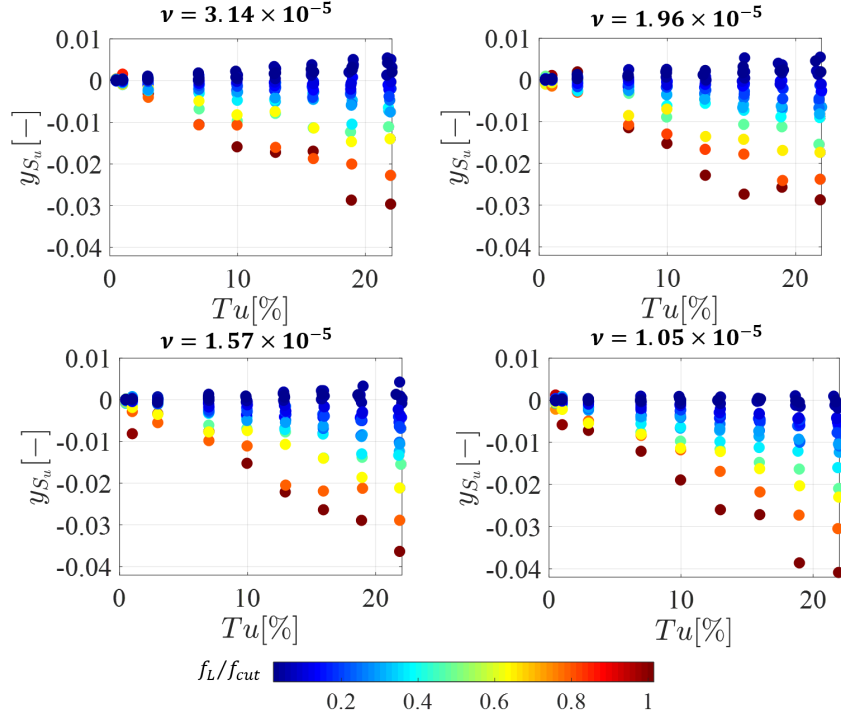


Figure 3.37 – Effect of Reynolds number on the S_u error.

3.6.4 Discussion

The results previously presented are, as explained, based on a theoretical model for the turbulence spectrum and a model for the dynamic response of CTHWA. Their validity is therefore directly linked to the validity of the models. The dynamic response model simulates the case of an optimally adjusted anemometer following the theory of Freymuth [4]. Weiss et al. [15] used this model in comparison to experimental data from three different anemometers acquiring similar results. The synthetic turbulence model was selected in an effort to give a physical approach to dynamic response errors, considering the nature of the turbulent flow. Similar analyses have been performed for the evaluation of spatial filtering effects, like the work of Wyngaard [30] who used a Pao spectrum and the work in Ref. [3] featuring a Karman–Saffman–Pao spectrum, resulting in the expression of errors on the measurement of turbulent quantities as a function of L/l and η/l . The use of the spectrum model implies isotropic flow and is not necessarily valid for all type of flows, especially for the energy-containing range which is strongly affected by the mean flow characteristics. Nevertheless, it is useful to provide general trends and

an estimation of the magnitude of this bias. Another advantage of an approach of this type is that it allows to perform a parametric study examining the effect of different parameters, while isolating the dynamic response effects which in practice can be difficult to distinguish from other types of errors like the spatial resolution (Ashok et al. [26]). Indeed, the literature research reveals that there are limited experimental studies quantifying errors on turbulence measurements due to CTHWA temporal resolution, like the work of Ashok et al. [26] for grid generated turbulence and the work of Hutchins et al. [20] for wall-bounded turbulence. In the first case, where low speed cases were considered, the temporal resolution effect turned out to be negligible with respect to spatial resolution filtering. In the second case, where spectra with different energy contents were considered, the errors on the velocity fluctuations variance were significant for non-optimally adjusted anemometers. The present study demonstrates that temporal resolution errors can be important in flow conditions of interest for turbomachinery flows. Non-linearity errors on the other hand are minimal in the case of a uniform mean flow with superimposed turbulence fluctuations. Possible continuation of this work could include rotor flows with different turbulence contents, to evaluate the dynamic response errors at the more challenging conditions of rotating wake flow.

3.7 Conclusions

In this chapter, a detailed analysis of the dynamic behaviour of CTHWAs has been performed and the different sources of errors affecting the measurement of turbulence parameters have been identified. A non-linear third order model has been used for the dynamic response, which has been modified to also include the heat conduction from the wire to the prongs. Experimental square wave tests have also been performed, using the embedded square wave generator of a commercial anemometer system. Synthetic turbulence generated based on the spectral model of Pope [24] is used as an input to the CTHWA model in an effort to quantify the dynamic response errors based on the characteristics of the turbulent flow.

The results of this study sum up in a series of guidelines for the best use of CTHWA in order to minimize dynamic response errors:

- The amplitude attenuation due to heat conduction can begin at a frequency as low as $150Hz$, and therefore affect almost all the useful range for turbulence measurements, and it can be present even for large length to diameter ratios. The specific flow conditions should be taken into account in the design or selection phase of the instrument, in order to minimize heat conduction errors. An appropriate criterion like the one of Hultmark et al. [9] (Eq. 3.4) should be used. In cases where this criterion might be difficult to meet, e.g. when a single probe is meant to be used for a range of experiments or when

spatial resolution restrictions limit the l/d ratio, a correction could be applied. Li [10] has proposed such a scheme for a posteriori correction. The present work suggests that this attenuation is also present at the response to the square wave, indicating that the square wave might be used as a transfer function to correct for the attenuation, strongly simplifying the procedure.

- The numerical study on the sensitivity of the system response to different values of square wave test voltage amplitude has shown a strong effect. This creates doubt on whether the square wave response can be reliably used for the estimation of the frequency response. Nevertheless, by using a commonly applied value for the amplitude (according to literature), the numerical comparison between electronic testing and direct testing (Reynolds impulse perturbation) yields minimal differences between the two methods for the frequency range of interest. Based on that, we can conclude that the experimental transfer function obtained by the square wave test can be considered representative of direct testing, when a suitable square wave test voltage is used. Still, it is not clear from this study how such a value can be defined. Moreover, given that this value is often unknown when commercial systems are used, further research is required to conclude on that issue.
- In the case where the transfer function from the square wave test is considered representative, it could be used for the correction of the signal, following the method explained in detail in 3.2. Compensating for the dynamic response is crucial to correct errors due to non-optimal adjustment of the anemometer leading to amplification or attenuation of the turbulent fluctuations before the cut-off frequency. Due to low signal to noise ratio, the frequency can not be extended higher than the f_{cut} corresponding to a -3 dB attenuation. After compensating for the dynamic response, the corrected spectra will correspond to those that would have been obtained with an optimally-adjusted anemometer with the same cut-off frequency. Depending on the nature of the flow, the results of Section 3.6 can be used to quantify the final errors on the measurement of the relevant turbulent quantities.

The parametric synthetic turbulence study is one of the most significant contributions in this work. It is one of the few works attempting to quantify the errors on the measurement of turbulence due to CTHWA dynamic response. Despite a significant number of works using turbulence spectrum models for spatial resolution errors (summarized in Ref. [3]), this is the first approach of this type for temporal resolution errors to the author's knowledge. The results show that the errors in the measurement of the rms of the turbulence fluctuations and the integral length scale, mainly occur due to the limited bandwidth of the instrument and are

a function of the ratio f_L/f_{cut} , where f_L is a characteristic frequency of the turbulence spectrum and f_{cut} the cut-off frequency of the anemometer. The error on the skewness is affected both by f_L/f_{cut} and the turbulence level, and stems from the non-linearity of the system. The magnitude of the latter is nevertheless found to be negligible for uniform mean flow with turbulent fluctuations, even for high turbulence levels. Increasing Reynolds number leads to a decrease in the percent errors of u_{rms} and L_{11} . Two turbomachinery flow examples are used to illustrate the importance of this analysis: high-intensity, large-scale turbulence typical of the inlet flow for high pressure turbine wind tunnel testing, and blade wake turbulence, characterized by high intensity and small length scales. The second case is more critical for errors stemming from the dynamic response, due to the small-scale turbulence where the high energy range extends to higher frequencies. This analysis can be used to evaluate the errors stemming from the dynamic response of CTHWAs.

The set of models developed in this chapter correspond to a powerful tool for a robust design of experiments, as they allow to retrieve optimal probe design parameters for given characteristics of the target turbulence flowfield.

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4

Fast response aerodynamic pressure probes

In this chapter, the use of fast response pressure probes for turbulence measurements in turbomachinery flows is examined. The operating principle and different calibration steps are explained in detail. Particular attention is given to the dynamic calibration of such probes in a shock tube. More specifically, a detailed experimental and post-processing methodology has been developed in order to improve the quality of the resulting transfer function, considering the flow physics in low-pressure shock tubes. An uncertainty estimation analysis of the dynamic calibration is also presented. Finally, the particular case of measuring turbulence with single-sensor probes is examined and the underlying correlations are critically investigated.

4.1 Introduction

Thanks to their robustness, simplicity and high frequency response, fast response aerodynamic pressure probes are one of the most commonly employed techniques in turbomachinery applications [1]. Their advantages are high sensitivity, small size and good dynamic behavior. Fast response pressure probes can be catego-

rized based on different criteria, like the number of sensors (single-sensor or multi-sensor), the positioning of the sensor (sub-surface or flush-mounted), their external geometry and the mode of operation (real or virtual multi-hole mode). Extensive reviews of different designs can be found in Refs. [2, 3] and more recently in Ref. [4].

Single-sensor probes have the advantage of smaller size, therefore better spatial resolution and minimum blockage, and usually higher frequency response, as the less demanding geometrical constraints allow for a more effective optimization of the sensors cavity dimension and shape. Some examples of high-performance single-sensor probes can be found in [5–7]. Single-sensor and two-sensor cylindrical probes can be operated in virtual multi-sensor mode by rotating the probe around its axis [6]. This technique allows to retrieve two-dimensional (2D) or three-dimensional (3D) phase-resolved measurements with reduced flow blockage and high spatial resolution. Multi-sensor configurations should be instead used when instantaneous directional measurements are required [8–12]. The commercially available Kulite[®] FAP-250 [11] and the four-sensor probe of Chasoglou et al. [12] are two of the most recent examples in the literature. The former has a hemispherical tip of 6.35 mm diameter and five flush-mounted sensors, while the latter is cylindrical, with a hemispherical tip of 4 mm diameter.

With the smallest commercial transducers available featuring a diameter between 1.4 mm and 2 mm, reducing the probe size to minimize blockage at the measurement point often requires the sensor placement inside the probe body. Spatial resolution requirements lead to the same approach, in order to minimize the sensing area of the taps. In this case, the sensor surface is connected to the flow through a line-cavity system. The natural frequency of the sensors can range between 200 and 350 kHz, depending on the exact type and pressure range. The useful frequency bandwidth of the probe is, therefore, limited by the acoustic behaviour of the line-cavity system, so that its dynamics should be characterized in order to define its cut-off frequency. Pfau et al. [6] performed measurements in a jet with their two single-sensor probes to identify their resonance frequencies at 34 kHz and 44 kHz respectively. As the desired response was estimated up to 15 kHz, no compensation was deemed necessary by the authors. Chasoglou et al. [12], using the same method, computed the resonance frequencies in the range of 80–85 kHz for the holes positioned in the yaw plane and at 65 kHz for the pitch sensitive hole. Persico et al. [7] characterized in detail the dynamic response of a single-sensor cylindrical probe with shock tube tests, numerical simulations and analytical models. The natural frequency of the cavity was found at 49 kHz and the transfer function was used to extend the bandwidth up to 80 kHz. In the case of flush-mounted sensors, the bandwidth limit is imposed by the natural frequency of the sensor membrane. For the Kulite[®] FAP-250 [11], the resonance frequencies of the sensors are at 300 kHz with the response being flat up to 100 kHz. How-

ever, the probe configuration featuring surface-mounted transducers reduces the measurement spatial resolution and constrains the aerodynamic design of the head shape.

The geometrical design of the probe is known to affect both the steady and unsteady aerodynamic performance. Dynamic effects induced by the unsteady flow have been investigated by Humm et al. [13] and Miller and Ainsworth [14]. They were found to be directly related to the probe size. Effects due to the probe body, like wake unsteadiness and vortex shedding, have been numerically investigated by Liu and Paniagua [15] for different geometries and flow angles.

Fast response pressure probes can be used to measure turbulence quantities. The first attempt was realized by Heneka [8], already in 1983, using a four-hole wedge pressure probe. Since then, researchers have focused mainly on single- or double-sensor probes in order to reduce the measurement volume and increase the accuracy. Porreca et al. [16] demonstrated the use of a double-sensor probe in virtual four sensor mode to measure turbulence in incompressible flows. Wallace and Davies [17] proposed a methodology to measure turbulence intensity with single-sensor probes by relating velocity fluctuations to total pressure for incompressible and compressible flows, assuming isotropic turbulence. The methodology of Wallace and Davies has since been used by various researchers like Persico et al. [18] and Lengani et al. [19] in turbomachinery flows, providing satisfying qualitative results, but without verifying the validity of the assumptions at their conditions.

4.1.1 Objectives of the present work

In the present work, the following aspects of the operation of fast response pressure probes will be treated:

- The required calibration steps for high-quality measurements, namely static, aerodynamic and dynamic calibration, are presented in detail.
- Particular attention is paid to the dynamic calibration of fast response probes in shock tubes. A detailed post-processing methodology is proposed, taking into account the complex flowfield present in low-pressure shock tubes, typically used for the calibration of such probes. The outcome is a robust methodology that minimizes the effect of disturbances on the transfer function.
- The uncertainty analysis of the dynamic calibration in shock tubes is addressed, considering the non-ideal flowfield.
- The measurement of turbulence with single-sensor probes is re-examined. More specifically, the suitability of correlations about the magnitude of static pressure fluctuations employed in previous works is addressed.

4.1.2 Employed probes description

Two probes are employed in this chapter to demonstrate the concepts.

Five-sensor probe: 5S The 5S probe in Fig. 4.1 is a five-sensor probe with a hemispherical head, designed for instantaneous 3D measurements in turbomachinery flows. More details about this probe can be found in Ref. [20].

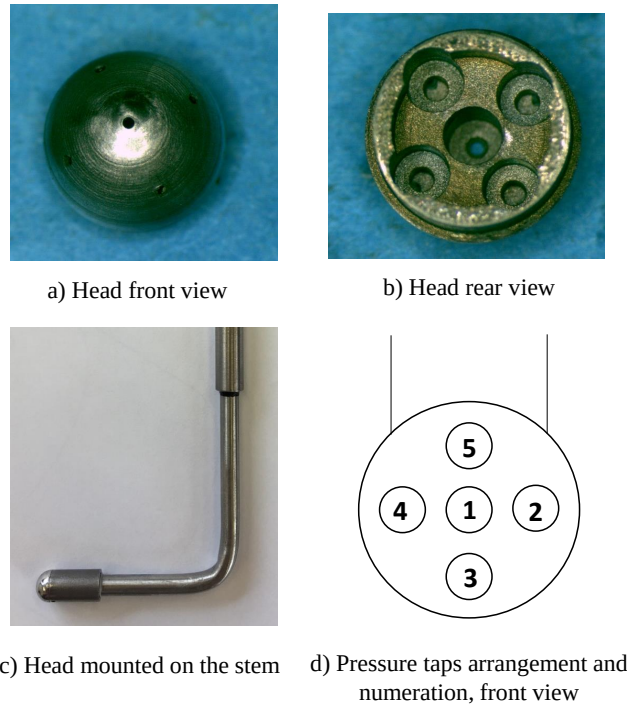


Figure 4.1 – 5S probe components and pressure taps numeration.

The 5S probe features sub-surface mounted sensors which are connected to the flow with two different line-cavity systems: one for the central sensor and one for the four lateral ones. A transducer with 6 mm length was selected for the central tap, while four transducers of 2.5 mm length were used for the lateral taps. The line-cavity system designs are presented in Fig. 4.2. The cavity volume of the central hole has been minimized by positioning the sensor as close as possible to the external surface. The resulting line length is 0.142 mm with a diameter of 0.4 mm. The diameter was selected as a trade off between spatial resolution and frequency response. The cavity is characterized by a length of 0.1 mm and a diameter of 0.6 mm. This arrangement exposes most of the sensing membrane

to the flow. The lateral holes have a line length of 0.26 mm and a diameter of 0.4 mm. The height of the cavity is 0.8 mm and the diameter 0.7 mm. A 2D cut of the probe head depicting the line-cavity systems of the central and lateral holes and the placement of the sensors is presented in Fig. 4.2. The line-cavity systems of the central and lateral holes are presented in details A and B respectively. Once the pressure sensors are installed into the head support, the inner side of the head is filled up with a layer of glue that has the purpose of sealing the inner volume of the probe from the wetted surface and of strengthening the head-sensor assembly. The presence of sensor chip recess is also depicted in Fig. 4.2: in some cases the transducer sensing membrane is not flush mounted inside the transducer package, but it is recessed with respect to the transducer packaging edge. This results in an additional cavity volume which can have important effects in the dynamic response performance of the probe.

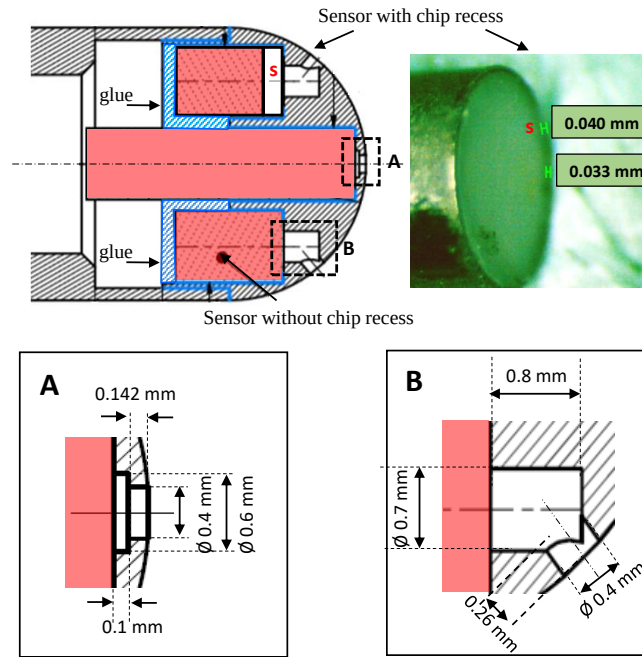


Figure 4.2 – Line-cavities system design of 5S probe.

Kiel probe: PT-K The PT-K probe in Fig. 4.3 is a Kiel-type probe with an L-shaped stem. Both probes feature Kulite[®] XCQ-062 miniature transducers. These transducers have an average diameter of 1.6 ± 0.05 mm, while the length of the packaging can vary. The PT-K probe head design is presented in Fig. 4.4.

It features a Kulite sensor with a protective screen (Fig. 4.3c) retreated inside the probe head, in order to allow the recovery of the total pressure for a wider range of angles (see e.g. Ref. [1]). The cavity in front of the sensor has a length of 0.2 mm and a diameter of 1.5 mm. The connection to the flow is realised through a conical pipe with a length of 0.4 mm and a big diameter of 3 mm.

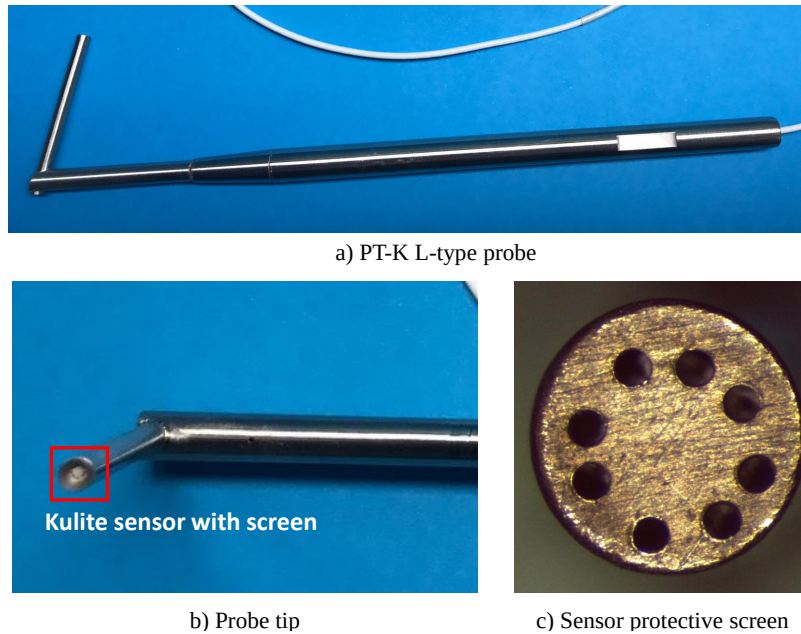


Figure 4.3 – PT-K probe.

4.2 Operating principle and static calibration

Fast response pressure probes for aerodynamic measurements typically feature silicon piezoresistive sensors. Four strain gages arranged in a Wheatstone bridge form are placed on the silicon diaphragm. The deflection of the diaphragm leads to the compression/extension and change of resistance of the strain gages and to the generation of an output voltage proportional to the pressure applied on the membrane. The disadvantage of this sensor type is the sensitivity to temperature variations, as the resistance of the sensor elements is also a function of temperature. In order to compensate for this effect, a passive or active compensation method can be used. Passive compensation consists in providing a voltage already corrected for the temperature effect, usually using external resistors to reduce the sensitivity of the bridge to temperature effects. Passive compensation modules

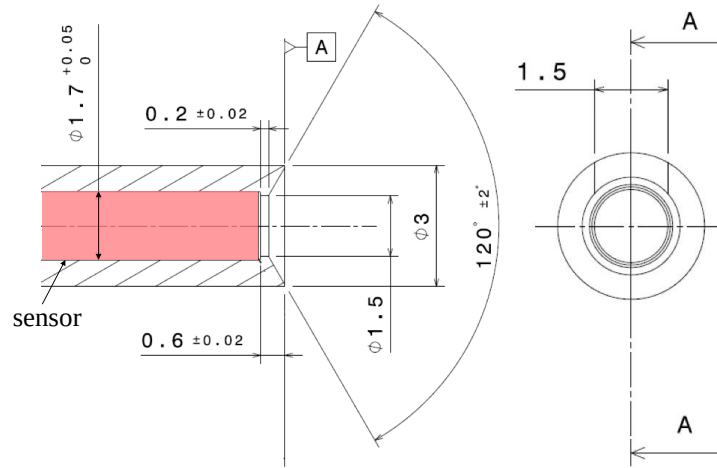


Figure 4.4 – Sketch of the PT-K probe head.

are typically provided by the transducer manufacturers. Active compensation, on the other hand, consists in characterizing the sensor sensitivity to temperature and correcting for it numerically during the post-processing. This can be realized by placing the Wheatstone bridge inside an external Wheatstone bridge, whose output voltage V_s is proportional to the sensor temperature. A similar simplified solution is the use of a resistor parallel to the sensor bridge. The two approaches are depicted in Fig. 4.5. The voltage across the resistor is again proportional to the sensor temperature. More details on the sensor operation and temperature compensation methods is given by Sieverding et al. [1].

4.2.1 Static calibration

In the present work, an active temperature compensation method is used, which requires the calibration of the sensor in both pressure and temperature.

The calibration setup is presented in Fig. 4.6. The probe under calibration is placed inside a sealed cylinder together with a PT100 platinum resistance thermometer. The cylinder is connected through a brass tube with a Druck DPI 610 pressure calibrator. The assembly is then placed inside a calibration oven. The oven, which can reach a temperature up to 200°C, heats up the cylinder and its content. Once the desired temperature has been stabilized in the inside of the cylinder, as indicated by the PT100 thermometer, the pressure calibrator is used to vary the pressure inside the cylinder. The output voltages of the sensor (the sensor bridge output V_p and the temperature-sensitive voltage V_s) are registered for each pressure and temperature level.

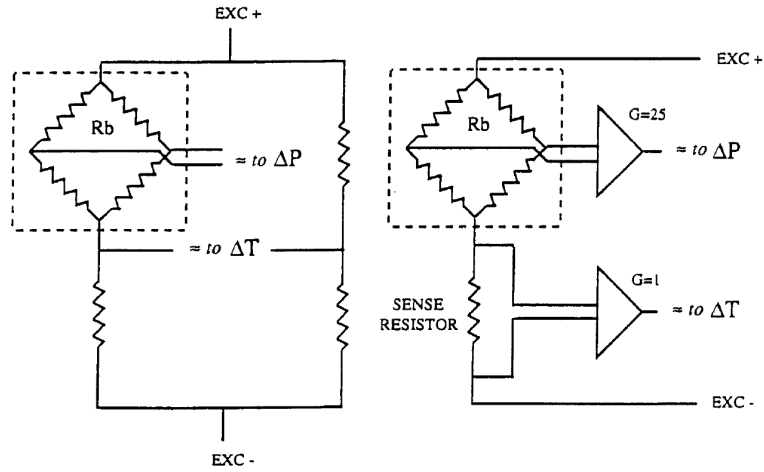


Figure 4.5 – Active compensation configurations: Wheatstone bridge (left) and sense resistor in series with the bridge (right) [1].

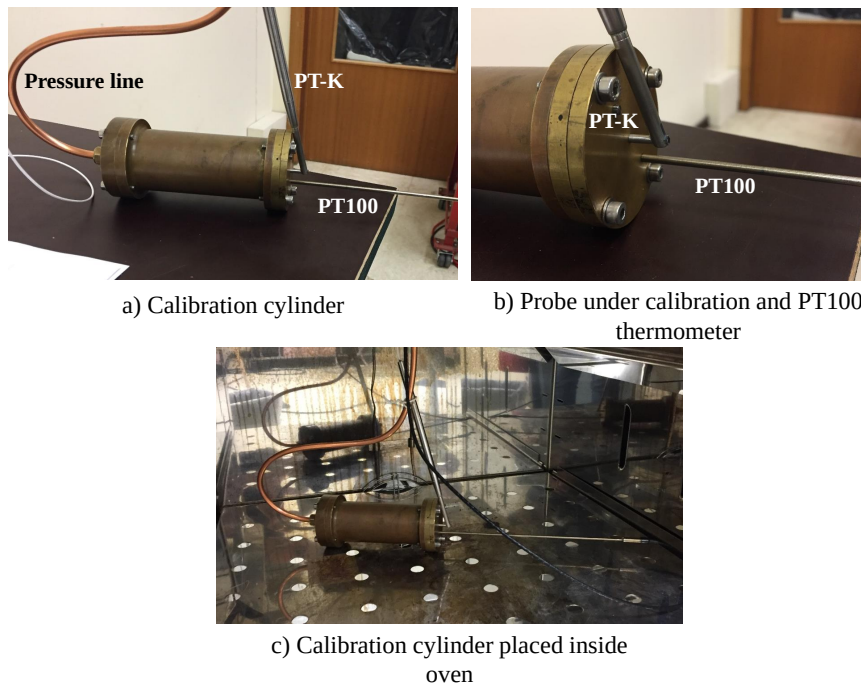


Figure 4.6 – Static calibration experimental setup.

The absolute pressure and temperature calibration maps for probe PT-K are presented in Fig. 4.7. The temperature is varied between 20 and 150°C, while the absolute pressure between 200 and 1650 mbar¹. The nominal range of the sensor is 1700 mbar. The calibration surfaces are fitted with the following polynomial types:

$$P_{abs}(V_p, V_s) = p0 + p10 * V_p + p01 * V_s + p11 * V_p * V_s + p02 * V_s^2 \quad (4.1)$$

$$T(V_p, V_s) = c0 + c10 * V_p + c01 * V_s + c11 * V_p * V_s + c02 * V_s^2 \quad (4.2)$$

expressing a linear dependence on the V_p and a second order dependence on V_s for both pressure and temperature. The best polynomial type has been defined by trying different orders and selecting the one exhibiting the best fitting. It has to be noted that while a calibration polynomial for the sensor temperature is given, the measurement of temperature with this type of sensors should be treated with caution as the temperature sensitivity is much lower compared to the pressure one. Its use must be thus considered for correction purposes only.

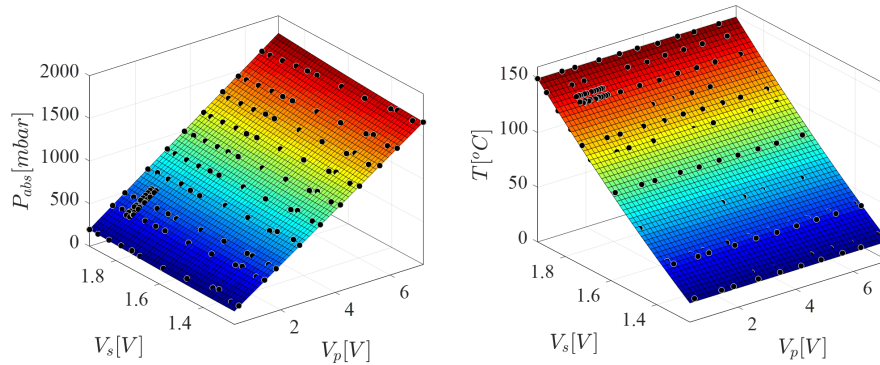


Figure 4.7 – Static calibration maps of PT-K probe.

4.3 Aerodynamic performance and dynamic effects

The aerodynamic performance of the probes depends on the design of the measuring head and its ability to recover the flow properties for different flow conditions. The aerodynamic performance is evaluated with the so-called aerodynamic calibration which allows to retrieve the flow properties by the pressure measured on the surface of the probe. The procedure and the output for probes PT-K and 5S

¹The cluster of calibration points present in the maps of Fig. 4.7 is due to a finer calibration performed for the pressure and temperature range expected in a certain application.

are presented in this section. The presence of dynamic effects related to the probe design (e.g. vortex shedding from the probe stem) is treated as well in this section.

4.3.1 Aerodynamic calibration

The calibrations took place in the VKI high-speed calibration facility C4 (see Fig. 4.8). The calibration bench consists of a circular convergent nozzle characterized by a contraction ratio of 14.75 with an outlet diameter of 50 mm. The discharge Mach number at the nozzle exit can be varied between 0.1 and sonic conditions. The jet Mach number can be regulated by means of a needle valve that adjusts the total pressure upstream of the nozzle, the outlet static pressure being equal to atmospheric condition.

The C4 rig is equipped with total temperature and total pressure instrumentation uniformly spread over the nozzle inlet circumference, downstream of a series of mesh-wires screens. The nozzle inlet total temperature is acquired using Platinum Resistance Thermometers (RTDs - PT100) and set by means of a PID-controlled heater. The facility features a two axis traversing system on which the probe is fixed by means of a probe holder. The latter was designed in such a way that the probe tip keeps in a fixed position during the traversing (center of rotation). More details about the calibration facility can be found in [21].

The probe 5S has been calibrated varying the pitch angle β and yaw angle α between $\pm 20^\circ$ and $\pm 30^\circ$ respectively, by steps of 2° . The probe PT-K has been calibrated for a yaw angle of $\pm 10^\circ$ and a pitch angle of $\pm 15^\circ$, by steps of 1° . The angle sign conventions for both probes are given in Fig. 4.8b.

4.3.1.1 Probe PT-K

Probe PT-K is a single-sensor probe used for the measurement of the unsteady total pressure. The aerodynamic calibration, therefore, consists in the computation of the total pressure coefficient C_{p_0} for different flow angles and conditions. The coefficient C_{p_0} is defined as:

$$C_{p_0} = \frac{P_1}{P_0} \quad (4.3)$$

where P_1 is the pressure measured by the probe and P_0 is the total pressure of the jet flow, measured by total pressure transducers at the inlet of the calibration nozzle.

The total pressure coefficient as a function of yaw angle is presented in Fig. 4.9 for four different Mach numbers. It can be seen that the coefficient remains almost constant in this angle range, satisfying the design targets of the Kiel head. The same can be observed for the pitch angle effect in Fig. 4.10.

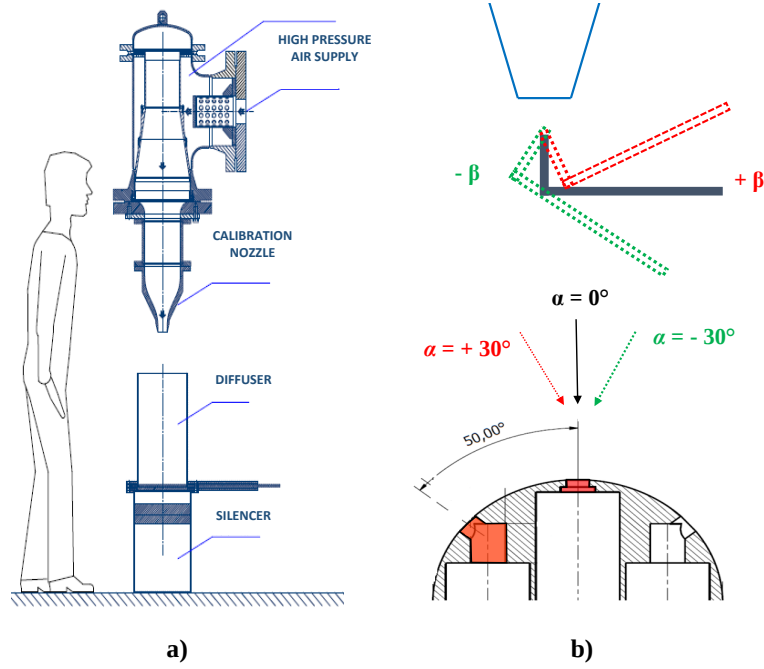


Figure 4.8 – a) VKI C4 high speed calibration nozzle, b) sign conventions for pitch angle (top) and sign convention for yaw angle for probe 5S (bottom).

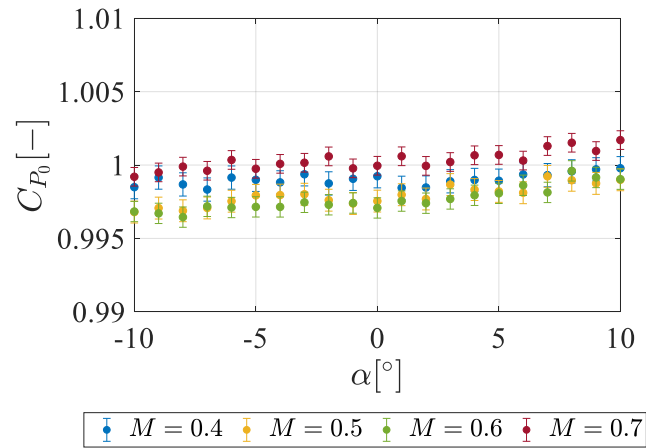


Figure 4.9 – Total pressure coefficient distribution for the PT-K probe, as a function of the yaw angle and different Mach numbers.

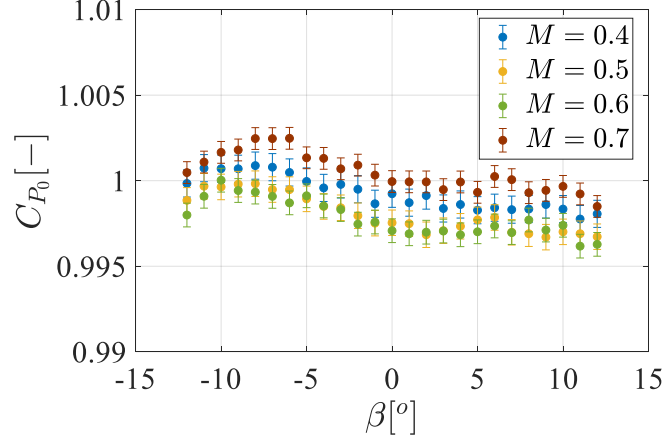


Figure 4.10 – Total pressure coefficient distribution for the PT-K probe, as a function of the pitch angle and different Mach numbers

4.3.1.2 Probe 5S

The aerodynamic calibration depends on both the Mach and Reynolds number of the flow. The angular range of operation and the angular sensitivity of the probe are particularly influenced. The angular range is limited by the maximum flow angle for which the flow remains attached on the surface of the probe, at the location of the lateral taps.

The probe 5S is employed here to evaluate these effects. Two Mach numbers ($M = 0.4$ and $M = 0.5$) and two total temperatures ($T_0 = 293$ K and $T_0 = 313$ K) have been investigated in order to assess independently the Mach and Reynolds number effect. The Reynolds number of the probe diameter Re_D is the critical parameter. The corresponding test matrix is given in Table 4.1. Test pairs one/two and three/four are used to evaluate the Reynolds influence.

By exploiting the fully symmetrical design of the head, its angular sensitivity can be inferred by simply computing the yaw calibration coefficient Cp_{yaw} as:

$$Cp_{yaw} = \frac{P_2 - P_4}{P_0 - P_{av}} \quad (4.4)$$

where P_{av} is the average of readings P_2 and P_4 (the readings of the lateral taps #2 and #4, located at the yaw plane). Due to a failure of lead connections of the right sensor (sensor 4) during the calibration phase, P_4 has been computed by mirroring the P_2 distribution with respect to the axis of the head. Still, the retrieved calibration maps preserve their representativeness of the nominal angular sensitivity of the probe. On the same line, in order to isolate the effect of the stem proximity on the results, only the 0° pitch angle is investigated in detail. A contour

Test #	M [–]	T_o [K]	Re_D [–]
1	0.5	293	6.7×10^4
2	0.5	313	6.1×10^4
3	0.4	293	5.4×10^4
4	0.4	313	5.0×10^4

Table 4.1 – Test matrix for the angular calibration of probe 5S.

plot of the coefficient $C_{p,yaw}$ as a function of yaw and pitch angles is presented in Fig. 4.11 for test #1. The non-symmetrical map with respect to the pitch angle is due to the potential field of the probe stem.

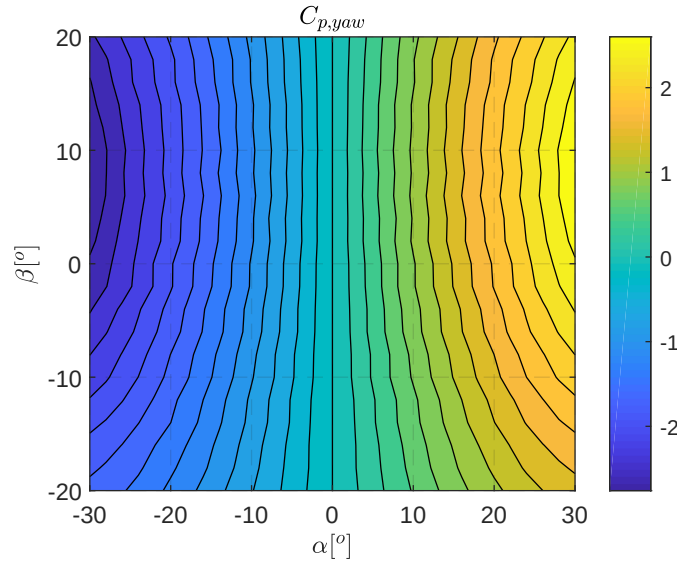


Figure 4.11 – Aerodynamic yaw coefficient for the probe 5S, as a function of the yaw angle α and pitch angle β (test # 1).

To quantify the effect of the Reynolds number on the angular sensitivity of the probe, the sensitivity to the yaw coefficient has been computed as the slope of the curves in Fig. 4.12. The sensitivity values for the different tests at null yaw are given in Table 4.2. The sensitivity of the yaw coefficient reduces by 27.5% when the Reynolds number decreases by 8.9% at $M = 0.5$. At a lower Mach number (test #3 and test #4), the sensitivity reduces by 6.7% for a reduction in Re_D number of 7.8%.

The Reynolds number effect is usually due to the pressure distribution on the head surface. Strong evidence of sensitivity to the Reynolds number indicates the presence of flow regions that are highly sensitive to Reynolds, usually zones

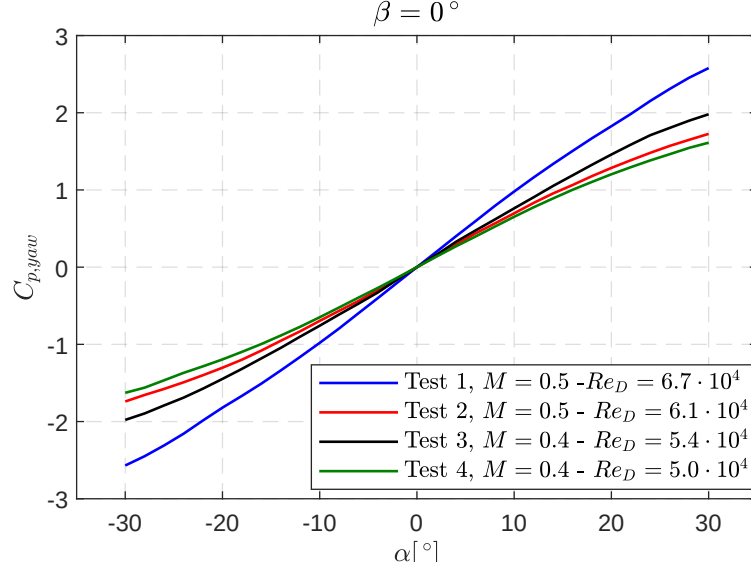


Figure 4.12 – Effect of Reynolds and Mach number on the yaw coefficient of the probe 5S for pitch angle $\beta = 0^\circ$.

Test #	$Re_D [-]$	$M [-]$	$d_{cp,yaw}/d_\alpha [-]$
1	6.7×10^4	0.5	0.098
2	6.1×10^4	0.5	0.071
3	5.4×10^4	0.4	0.073
4	5.0×10^4	0.4	0.0682

Table 4.2 – Test matrix with the sensitivity of the yaw coefficient of the probe 5S to the yaw angle, for pitch angle $\beta = 0^\circ$.

with flow separation. Hemispherical probes are in general considered to be less susceptible to the Reynolds effect compared to conical or pyramidal geometries [22, 23].

The presence of dynamic effects on the pressure measurement are evaluated experimentally by computing the energy spectra of the central and lateral tap #2 for three incidence angles. The computation is performed for $\beta = 0^\circ$ and three yaw angles α : -30° , 0° , $+30^\circ$ for tests #2 and #4. The signals were sampled at 500 kHz and analog low-pass filtered at 125 kHz. Considering the sign conventions presented in Fig. 4.8, the lateral tap is more exposed to the flow at a yaw angle of $+30^\circ$. The results are presented from Fig. 4.13 to Fig. 4.14. Two peaks at 3.1 kHz and 2.4 kHz are observed for both holes, for test #2 and test #4 respectively. These frequencies correspond to Strouhal numbers of 0.10 - 0.13. For the central tap, the peak amplitude grows at a negative and positive yaw angle of 30° ,

while for the lateral tap only at $+30^\circ$. At a yaw angle of $+30^\circ$, the lateral tap is facing the flow with an angle of 20° (as it is placed at an angle of 50° with respect to the probe axis.) The peak is not present at the lateral tap signal for a yaw angle of -30° , i.e. when the hole is facing the flow at an angle of 80° . Moreover, at this angle the lateral tap measures larger stochastic fluctuations as suggested by a rather flat evolution of the spectrum in the low frequency range and by its high energy content. The cavity resonance of the lateral hole #2 is present at 25.5 kHz for both Mach numbers (the cavity resonance frequency has been identified by shock tube testing presnted in the next section).

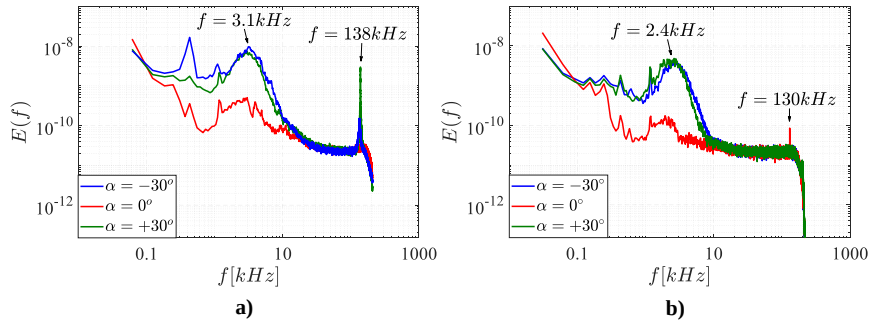


Figure 4.13 – Probe 5S: Energy spectra of the central tap at different incidence, for a) test #2, and b) test #4.

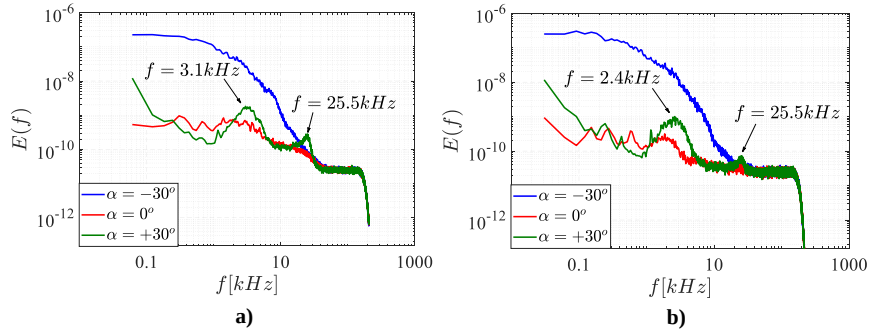


Figure 4.14 – Probe 5S: Energy spectra of the lateral tap #2 at different incidence for a) test #2, and b) test #4.

Two high-frequency peaks are observed at 130 kHz and 138 kHz in the spectrum of the central hole. The peaks' frequency is too high to be related to vortex shedding from the probe stem, while their presence was not recorded during the shock tube tests. Moreover, while for test #2 the peak is present for large incidence

angles, in test #4 it only appears at 0° incidence. These considerations suggest that the high frequency peaks are certainly related to the aerodynamic field generated by the probe and, as such, they are directly dependent on the Reynolds number (highest for test #2). In particular, as shown in Fig. 4.15 for test #2, the high frequency peak appears at an incidence of 24° , with amplitude increasing as the incidence increases to 30° . Basing on the latter result, it is postulated that the origin of this phenomenon is the flow separation at the edge of the central hole, a conclusion that cannot be verified for test #4, as peaks at large incidence angles are not present in the observed frequency range.

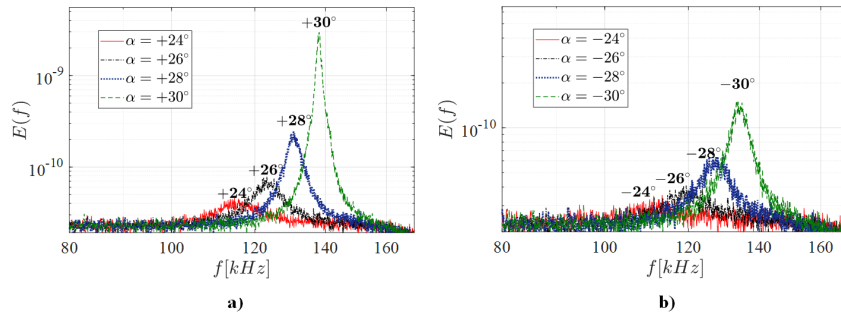


Figure 4.15 – Probe 5S: High frequency peaks for the central hole for test #2 at a) positive incidence, b) negative incidence.

It can be concluded from this analysis that the probe geometry can induce dynamic effects. A small degree of asymmetry seems to generate peaks of different amplitudes in the spectra at different angles. In this case, the peak attributed to separation at the edge of the central hole is present at a high frequency, which might be out of the range of interest for most experiments. Moreover, due to the significantly higher energy level at lower frequencies, these peaks are not expected to contribute to the overall unsteadiness level. Nevertheless, it is suggested that for time-resolved measurements the spectra should be at least checked prior to the experiment, in the expected angle range, to allow the identification of dynamic effects due to the probe itself from dynamic effects related to the experiment or e.g. the facility.

4.4 Dynamic response

The careful design and characterization of line-cavity systems is of big importance to achieve the desired dynamic behavior. Analytical models can be used during the design stage to estimate the frequency response of the line-cavity systems and lead to an improved configuration. Experimental characterization is usually performed

a posteriori to identify the actual response of the probe. In this stage, the transfer function can be computed and then be used for the correction of the measurements, extending the useful frequency bandwidth of the probe (digital compensation).

4.4.1 Analytical models for line-cavity systems

Various theoretical models are available for the prediction of the dynamic response of line-cavity systems. The appropriate model should be selected based on the flow conditions and the geometry under investigation. One of the simplest approaches is the Helmholtz resonator model, valid for systems where the volume of the line V_L is negligible compared to the volume of the cavity V_c . The model considers inertial effects to be confined to the line, whereas elasticity effects to the cavity [9]. The natural frequency f_n of the system is given by:

$$f_n = \frac{c}{2\pi} \sqrt{\frac{\pi D_l^2}{4LV_c}} \quad (4.5)$$

where c is the speed of sound and D_l and L are the diameter and length of the line respectively.

The Hougén model [24] is applicable to gas systems with a line volume comparable to the cavity volume ($V_c/V_L \sim 1$). When the volume of the line becomes a significant fraction of the total volume of the system, compressibility effects in the line are non-negligible. In this case, the system's natural frequency can be predicted by:

$$f_n = \frac{c}{2\pi L \sqrt{\frac{1}{2} + V_c/V_L}} \quad (4.6)$$

When the line length decreases, Eq. 4.5 and Eq. 4.6 overpredict the natural frequency. In practice, an amount of air outside the end of the line participates in the acoustic oscillation, resulting in a natural frequency which corresponds to a line length longer than the physical length. To compensate for this effect, an end correction can be added to the length L . The corrected length L_e , which can replace L in the previous equations, is given by [25]:

$$L_e = L + \frac{8}{3\pi} D_l \quad (4.7)$$

4.4.2 Experimental characterization

Shock tube testing is the main method used for the dynamic calibration of fast-response pressure probes for turbomachinery applications, thanks to the fast rising pressure steps that allow the calibration up to frequencies of ~ 500 kHz. The post-processing methodology to retrieve the corresponding transfer functions is presented in detail.

4.4.2.1 Shock tube testing

The experiments are performed in the VKI shock tube, presented in Fig. 4.16a. It has a total length of 8 m with an internal diameter of 100 mm. The measurement station (Fig. 4.16b and 4.16d) can be placed at two axial positions: at a distance of 2.4 m or 3.4 m from the diaphragm. It can accommodate up to three probes simultaneously: two probes under calibration and a reference probe. The latter is an L-shaped probe of 8 mm diameter with a flush-mounted (no cavity present in front of the sensor) Kulite[®] XCQ-062 transducer. The reference sensor is unscreened and its natural frequency has been experimentally defined to be about 300 kHz. The length of the reference probe, shown in Fig. 4.16c, can be modified in order to align the reference sensor tip to the axial position of the tested probe with an uncertainty of about 0.1 mm. The probes are inserted into the shock tube and fixed at a radial distance of 5 mm from the tube axis. This position was selected on the basis of previous investigations conducted at VKI. It was observed that moving the probe from the walls towards the tube axis, there exists a radial position (5 mm from the axis) after which no change in the estimated transfer function is observed. Such disturbances are attributed to the wave pattern inside the shock tube and the deflection of the shock close to the walls.

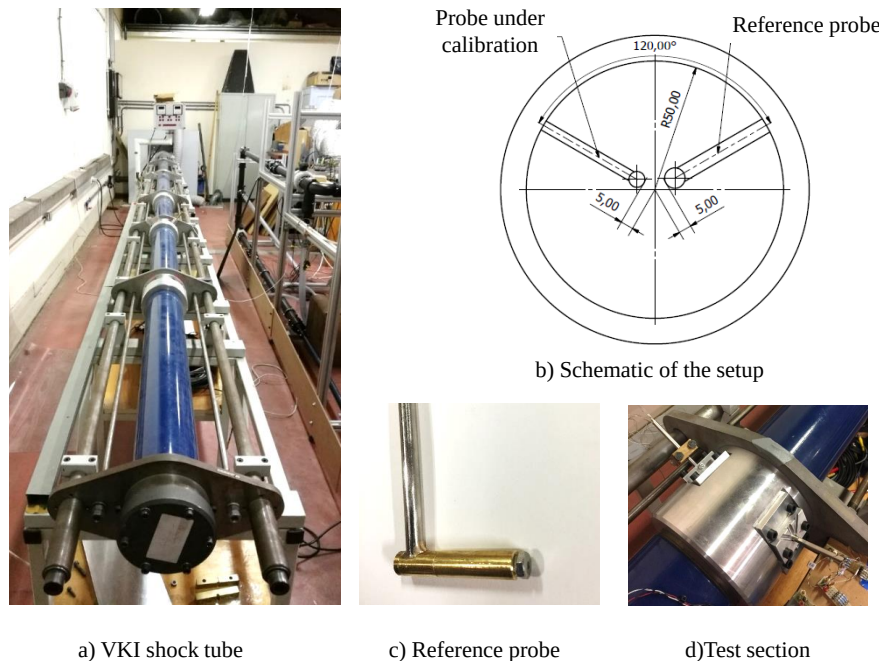


Figure 4.16 – Shock tube setup.

Tests were performed with the measurement station at both axial positions. Considering the axial length of the reference probe, the non-dimensional distance from the diaphragm x/D_t is 23.85 and 33.45 respectively. The signals were sampled at $f_s = 1$ MHz and low-pass filtered at 250 kHz.

4.4.2.2 Shock tube flowfield

The shock tube consists of two chambers which are initially isolated by a diaphragm. The low-pressure chamber is known as driven section and the high-pressure chamber as driver section. The two sections are kept at constant conditions of pressure and temperature of state (1) and (4) for the driven and driver section respectively. When the diaphragm bursts, a shock wave is created that travels towards the driven section at a speed c_s , while an expansion wave travels towards the driver section. As the shock front propagates in the driven section, the probes under calibration are exposed to a pressure step P_2/P_1 , where (2) stands for the conditions in the driven section behind the shock front. More details about shock tube flows and the theoretical relationships describing the ideal case can be found in Ref. [26].

In the case of real low-pressure shock tubes, the diaphragm is rarely completely opened, which leads to a significant deviation from the ideal behaviour. This case has been studied in detail by Gaetani et al. [27]. When the diaphragm is partially opened, the initial shock wave partially diffracts close to the tube walls and forms an annular shock wave, while the central portion of the shock wave remains a normal shock. A rarefaction wave is created at the diaphragm edge which collides with the normal shock, reducing its intensity. The diffracted portion of the initial shock wave and the rarefaction waves reflect at the tube walls creating a complex wave pattern. Gaetani et al. [27] concluded that for measurement stations sufficiently distant from the diaphragm, the main effects are a reduction in the intensity of the leading shock and the presence of strong pressure oscillations after the passage of the shock front, hereby referred to as post-shock oscillations. They proposed an analytical model for the post-shock oscillations, based on a simple model for wave propagation inside the shock tube, which they validated with experimental and numerical results. The wave reflection scheme at a certain instant after the diaphragm burst is presented in Fig. 4.17, where the point S_0 represents the origin of the leading shock wave W_0 , located on the tube axis at the diaphragm location. The leading shock is modelled as a spherical wave originating from S_0 and moving at a speed σ_0 . The first shock reflection W_1 is simulated as a spherical wave originating at a radial distance D_t from S_0 , where D_t is the diameter of the shock tube. The j th reflection W_j is simulated as a spherical wave originating at a radial coordinate $(-1)^j j D_t$, propagating at the acoustic wave speed $u + c$, where u and c are values of the flow velocity and the speed of sound post-shock. Consid-

ering a measurement station placed at a distance L from the diaphragm, the time instant of the arrival of the j th wave is given by:

$$t_j = \frac{L}{(u + c) \sin \alpha_j} \quad (4.8)$$

where $\alpha_j = \arctan(L/(jD_t))$. The frequency of the j th wave can be computed as the inverse of the time span between two successive waves:

$$f_j = \frac{f_\infty}{2j+1} \left[\sqrt{(j+1)^2 + \left(\frac{L}{D_t}\right)^2} + \sqrt{j^2 + \left(\frac{L}{D_t}\right)^2} \right]. \quad (4.9)$$

The local frequency of the perturbations is shown to tend to an asymptotic value $f_\infty = (u + c)/D_t$. The local frequencies are bounded by f_∞ and a maximum value f_{max} corresponding to the local frequency of the first post-shock wave.

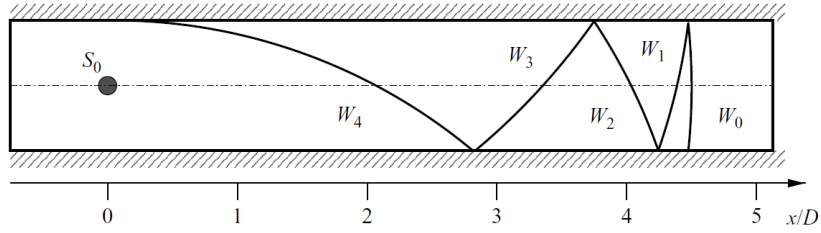


Figure 4.17 – Shock tube wave reflection scheme from [27].

In light of the above, pressure measurements obtained by the reference probe in the VKI shock tube are examined to identify these effects. Three measurement cases are considered, whose conditions are given in Table 4.3. The shock wave Mach number M_s is computed by the measured calibration ratio P_2/P_1 (pressure ratio across the shock front) and normal shock wave theory. The local frequency limiting values f_∞ and f_{max} are computed by the model previously presented.

Test #	$x/D_t [-]$	$P_2/P_1 [-]$	$M_s [-]$	$f_\infty [\text{kHz}]$	$f_{max} [\text{kHz}]$
1	23.85	1.101	1.042	3.76	11.2
2	23.85	1.150	1.061	3.88	10.5
3	33.45	1.095	1.039	3.74	12.5

Table 4.3 – Flow conditions in the shock tube for the three tests.

The time-frequency plots of pressure fluctuations amplitude (computed by a Continuous Wavelet Transform (CWT)) for the three tests are presented in Fig. 4.18a -4.18c. The time instant $t = 0$ corresponds to the moment when the shock

arrives at the measurement point. The analytical model results are also presented with the white dashed line. It can be clearly seen that post-shock oscillations are present in the test results. The model slightly underpredicts the frequency values of the disturbances, but, qualitatively, describes quite well their behaviour. This discrepancy can be attributed to the uncertainty in the computation of the shock wave velocity. A direct measurement would require the accommodation of two reference sensors at two different axial positions during the same test, which was not possible with the current experimental setup. Nevertheless, a precise evaluation of the shock Mach number is not expected to have a significant impact on the present findings. The presence of post-shock oscillations is also evident in the pressure spectra of Test # 3, given in Fig. 4.18d. The maximum amplitude of the oscillations is computed as: 0.6%, 0.5% and 0.5% of the post-shock mean pressure level P_2 for tests 1 to 3 respectively.

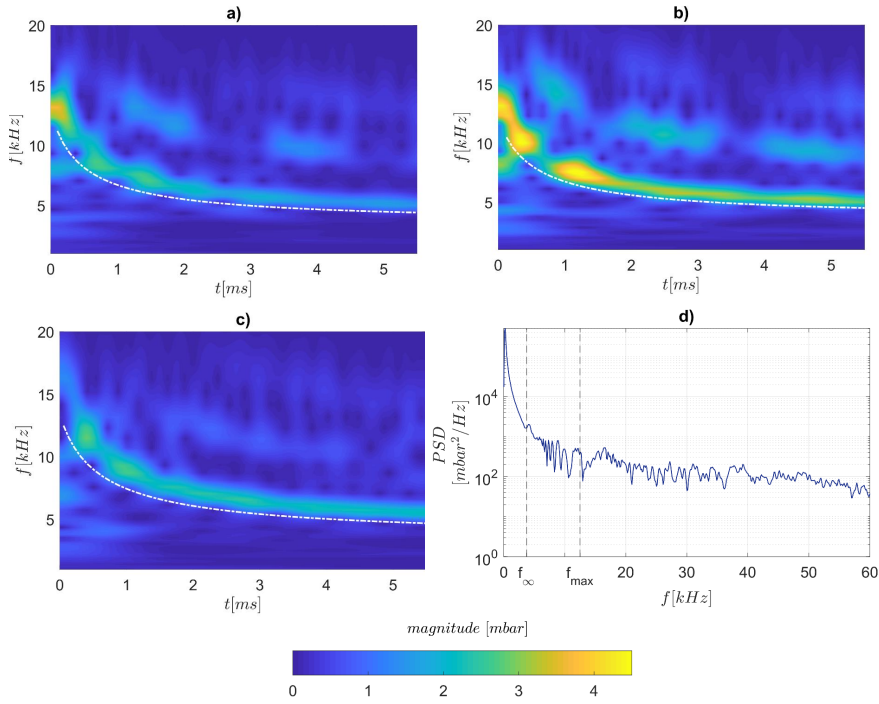


Figure 4.18 – Time-frequency plots of the pressure oscillations magnitude in the shock tube acquired by the reference sensor, for a) Test #1, b) Test #2, c) Test #3, and d) Pressure spectra of Test #3.

4.4.2.3 Dynamic calibration processing methodology

The transfer function $H(f)$ of a system is defined as the complex ratio between the Fourier transform of the output response $Y(f)$ and that of the input $X(f)$:

$$H(f) = \frac{Y(f)}{X(f)} \quad (4.10)$$

In this case, $Y(f)$ is the Fourier transform of the probe response to the pressure step. The response of the reference sensor (flush-mounted sensor) is often used as $X(f)$, considering that its response is much faster with respect to the response of a probe with a line-cavity system. Nevertheless, the post-shock disturbances examined in the previous section can introduce distortions in the computed transfer function. As these oscillations have been demonstrated to be non-uniform in the radial direction, signals with different frequency content might be acquired by the two probes (reference probe and probe under calibration), even if they are placed in the same axial station. For this reason, the approach of Persico et al. [28] is adopted, that uses an ideal pressure step as the input $X(f)$. In this case, the reference sensor is used only to define the rising point of this pressure step. With this approach, the result is independent from distortions of the reference signal. Moreover, to eliminate the effect of post-shock disturbances on the probe signal, the signal should be cut shortly after the step rise, as long as the dynamics of the probe have been completed. The step-by-step procedure for the experimental identification of the transfer function is presented in detail.

Firstly, the signals of all tests are normalized using the pressure levels before and after the shock. This step is required to eliminate small differences due to the test-to-test repeatability. Secondly, the signals of different tests are synchronized to ensure that the rising step occurs always at the same time instant ($t = 0$ s). The rising point of the reference signal is used to create the ideal pressure step. The signal is cut t_s seconds after the pressure step and the remaining part of the signal is discarded. The selection of the optimal post-step length (parameter t_s) will be explained in detail in the following section. An example of non-dimensional pressure signals P_{nd} for a shock tube test are presented in Fig. 4.19. The ideal pressure step rising time is imposed by the digital resolution of the experimental signals ($dt = 10^{-6}$ s).

Digital filtering is then applied to the signals to remove the high frequency oscillations due to the sensor membrane resonance. The cut-off frequency of the low-pass filter (LPF) should be well above the expected natural frequency of the line-cavity system. In this case, the signals were low-pass filtered at 150 kHz, except for the signal of the central tap of probe 5S, which was filtered at 250 kHz.

The acquired time-series are artificially extended to achieve the desired spec-

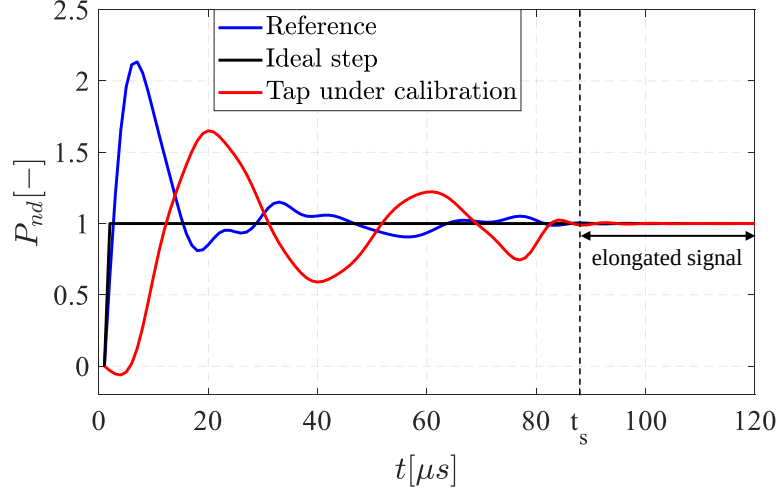


Figure 4.19 – Time response to a shock tube pressure step: reference sensor, pressure tap under calibration and ideal pressure step.

tral resolution. The required elongation time T_e is computed as:

$$T_e = \frac{1}{f_s} \left(\frac{f_s}{\Delta f} - N \right) \quad (4.11)$$

where Δf is the desired frequency resolution and N is the number of samples of the original signal. $\Delta f = 100$ Hz is set in this case. The signal is elongated after the instant t_s using the mean pressure level value at the end of the time-series, as demonstrated in Fig. 4.19. A half-Hanning window is applied at the elongation point to render the signal periodic and reduce spectral leakage. The Fourier transform of each signal is computed and the spectra are averaged in the frequency domain to reduce the noise in the resulting transfer function $H(f)$. The experimental $H(f)$ is computed by Eq. 4.10 and is plotted in terms of amplitude I and phase ϕ .

The line-cavity system can be modelled as a Linear Time Invariant (LTI) second order system. The experimental transfer function is used to compute the characteristics of the linear system: the natural frequency ω_n and the damping ζ . The peak amplitude I_p of the transfer function and the corresponding angular resonance frequency ω_p are identified, as indicated in Fig. 4.20. The damping coefficient is then computed by:

$$I_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \quad (4.12)$$

The angular natural frequency ω_n , (the resonance frequency of the system if the

damping were nil), can be computed by:

$$\omega_p = \omega_n \sqrt{1 - \zeta^2} \quad (4.13)$$

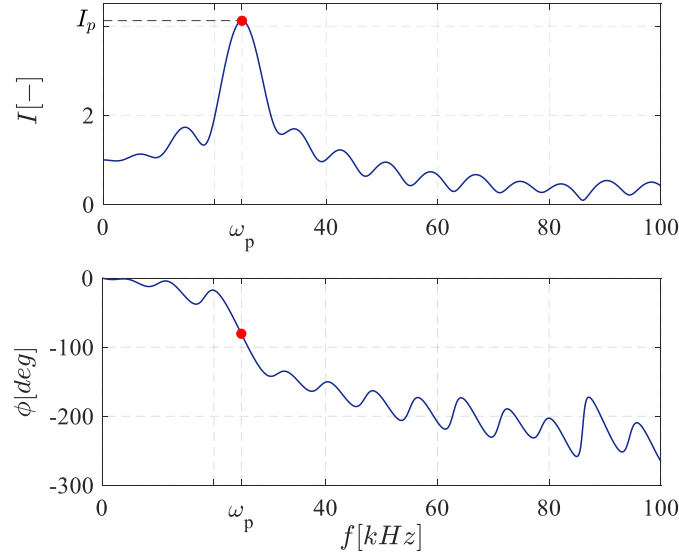


Figure 4.20 – Identification of LTI system parameters (resonance peak and resonance frequency) from the experimental transfer function of a fast response pressure probe.

4.4.2.4 Effect of post-shock oscillations on the transfer function

To better understand the effects of the post-shock oscillations on the transfer function and optimize the post-processing methodology, a parametric study using analytical models to simulate both the transfer function and the input signal has been performed. The case $f_n > f_{max}$ will be considered, where f_n is the natural frequency of the probe's line-cavity system and f_{max} is the maximum post-shock oscillations frequency, as explained in section 4.4.2.2. As the frequencies computed by the analytical model and observed in the experiments range between 3–15 kHz, they fall in the frequency range of interest for turbomachinery flows. Therefore, it is important that the transfer function is well-defined in this range. For probes used in turbomachinery flows, f_n is usually higher than 14 kHz, in order to be able to resolve several harmonics of the Blade Passing Frequency (BPF). The higher the natural frequency of the probe, the smaller would be the effect of the disturbances. Two cases will be considered in this analysis: a line-cavity system with $f_n = 24$ kHz and a more favorable case of a system with $f_n = 38$ kHz, hereby

referred to as Probe 1 and Probe 2 respectively. The first case is representative of the dynamic behaviour of the lateral sensors of the 5S probe, while the second case corresponds to the behaviour of the PT-K probe.

Model and methodology The line-cavity system of the probe is modelled as a second order LTI system with natural frequency f_n and damping ζ . It has been demonstrated in previous works [20, 29] that the dynamic behaviour of line-cavity systems is well-represented by such models. The analytical transfer function of the probe is computed as:

$$H_{in}(f) = \frac{1}{-\frac{(2\pi f)^2}{(2\pi f_n)^2} + 2i\zeta \frac{2\pi f}{2\pi f_n} + 1}. \quad (4.14)$$

The parameters of the two line-cavity systems considered in this analysis are given in Table 4.4. The settling time T_s is also included in the table, defined as the time for which the amplitude of the step response of the system remains within 2% of the final value. This parameter is therefore indicative of the time required to complete the dynamics of the system. It can be computed by:

$$e^{-\omega_n \zeta T_s} = 0.02 \sqrt{1 - \zeta^2}. \quad (4.15)$$

Probe	f_n [kHz]	ζ [-]	T_s [ms]
1	24	0.1	0.26
2	38	0.15	0.11

Table 4.4 – Line-cavity systems parameters.

The conditions of Test #3 (Table 4.3) will be considered in this analysis. Based on its good qualitative agreement with the experimental results, the model of Gaetani et al. [27] is used for modelling the post-shock disturbances. The input signal X_{sim} is simulated as an ideal step with superimposed sinusoidal oscillations, whose amplitude is 0.5% of P_2 and whose frequency ranges between f_∞ and f_{max} (Table 4.3). As the analytical model does not provide information on the intensity of the oscillations but only on their frequency, the maximum amplitude of oscillations observed in the experiments is used. The time instant of the arrival of the first oscillation is defined by the model. The simulated input signal is shown in Fig. 4.21. From the time-frequency plot in Fig. 4.21b, the variation of the post-oscillations frequency with time can be clearly seen, as imposed by the analytical model presented with the white dashed line. In Fig. 4.21c, the presence of the post-shock oscillations in the expected frequency range can be seen.

The transfer function H_{in} is used to compute the response of the probe Y_{sim}

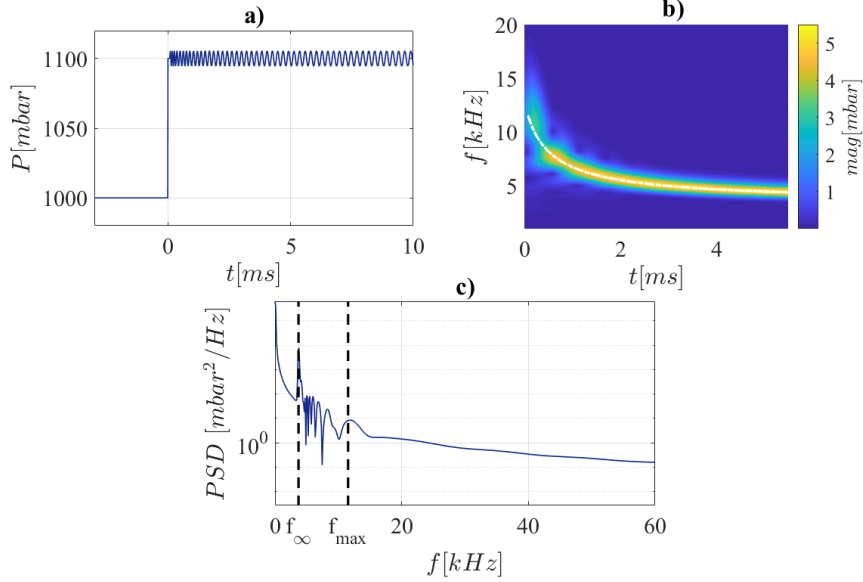


Figure 4.21 – Simulated shock tube input signal X_{sim} a) in time, b) in time and frequency, c) in frequency.

to the simulated input signal X_{sim} , as

$$Y_{sim}(f) = H_{in}(f) * X_{sim}(f) \quad (4.16)$$

Then, Y_{sim} is used to compute the transfer function H_{out} as

$$H_{out}(f) = \frac{Y_{sim}(f)}{X_{step}(f)} \quad (4.17)$$

where $X_{step}(f)$ is the Fourier transform of an ideal pressure step. The methodology presented in section 4.4.2.3 is used to compute the transfer function H_{out} : to minimize the effect of the post-shock oscillations, the signals Y_{sim} and X_{step} are cut at a time instant t_s after the step rising point. They are then artificially extended to achieve the desired frequency resolution and windowed to reduce spectral leakage.

In the following, the output transfer function is computed for different values of the parameter t_s . Once the transfer function has been computed, the parameters of the second order model, ω_n and ζ , can be recomputed as well.

Results In Fig. 4.22, the effect of the cutting point of the signal t_s on the output values of ζ and f_n is presented for Probes 1 and 2. The nominal values of the parameters are indicated with the red dashed line. Moreover, the settling time

parameters T_{s1} and T_{s2} are indicated with the dashed vertical lines, as well as the parameter t_{j1} , the time instant for which the first wave arrives at the measurement station according to the analytical model. It can be seen in Fig. 4.22b that for a clean step signal, the values converge to the ‘real’ input value. Nevertheless, in the presence of post-shock oscillations (Fig. 4.22a) both parameters feature an oscillating behaviour as t_s increases. It can be observed that the damping parameter requires a higher t_s value to converge, which coincides with the settling time, even for the case with post-shock oscillations. From this graph, it can be concluded that the minimum value of t_s to correctly identify the second order system parameters is of the order of magnitude of the settling time.

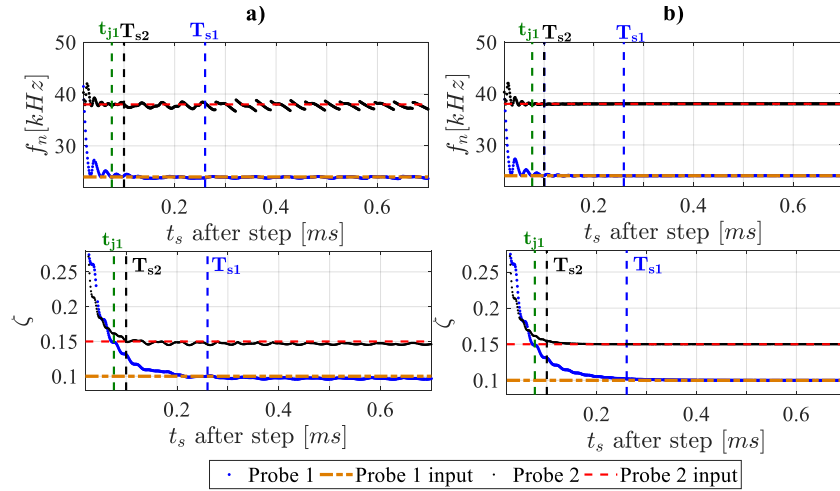


Figure 4.22 – Recomputed ζ and f_n values as a function of t_s for Probes 1 and 2: a) signal with post-shock oscillations, b) clean signal.

To better understand this effect on the transfer function, the H_{out} computed for three t_s values is presented in Fig. 4.23 along with H_{in} . The three t_s values considered are $t_{s1} = 0.136$ ms, $t_{s2} = 0.41$ ms and $t_{s3} = 1$ ms. These values correspond to 52%, 157% and 384% of the settling time for Probe 1, and 123%, 372% and 909% of the settling time for Probe 2 respectively. The mean deviation of amplitude and phase from the ideal transfer function is given in Tables 4.5 and 4.6 for Probes 1 and 2. The mean deviation is computed over the low frequency part (0-15 kHz for both probes) and over the region of the resonance peak (15-30 kHz for Probe 1 and 15-50 kHz for Probe 2). This is to examine independently the effect on the post-shock oscillations region and the region of the resonance peak, used for the computation of the LTI system parameters.

For Probe 1, it can be seen that for $t_{s1} < T_{s1}$, the error is bigger at the resonance peak zone and smaller at the low-frequency range. More specifically,

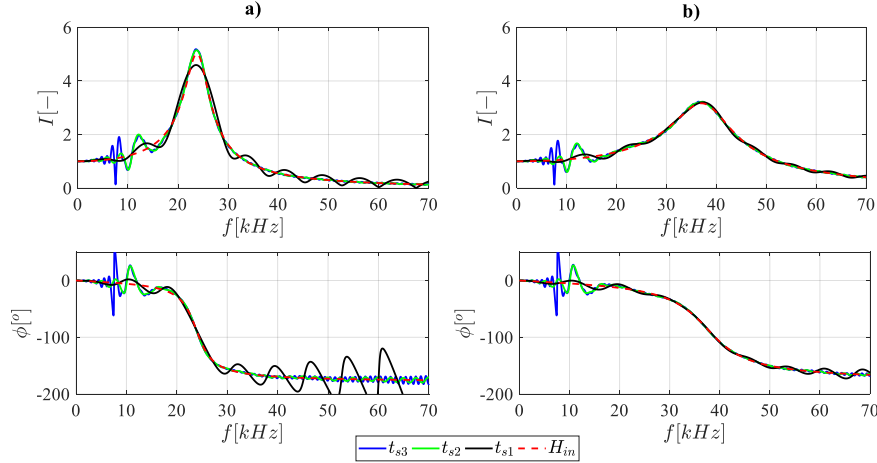


Figure 4.23 – Amplitude and phase plots of transfer function H_{out} for different t_s values and comparison with H_{in} for a) Probe 1 and b) Probe 2.

f range	Amplitude			Phase		
	t_{s1}	t_{s2}	t_{s3}	t_{s1}	t_{s2}	t_{s3}
0-15 kHz	5.8%	11.9%	17.5%	65%	135%	236%
15-30 kHz	8.5%	3.8%	3.8%	16.4%	7.3%	6.5%

Table 4.5 – Mean deviation of H_{out} from H_{in} for different t_s , probe 1.

the resonance peak is not correctly captured, leading to the overestimation of the damping. The trend is inversed for t_{s2} and t_{s3} , which are both higher than the settling time. The transfer function is well captured at the area of the resonance peak, leading to the correct estimation of the second order system parameters. Nevertheless, the effect of post-shock oscillations is present at the lower frequency part of the transfer function for t_{s2} , getting more intense for t_{s3} . It is thus concluded that while the low frequency part of the transfer function might be contaminated by the presence of post-shock oscillations, the parameters for the second order model can still be computed with a good accuracy. On the other hand, cutting the signal too soon does not allow the probe dynamics to be completed. The advantage of Probe 2 can easily be observed for case t_{s1} : having a lower settling time, the probe dynamic has been already completed by t_{s1} , allowing the accurate estimation of the model parameters, while the low frequency part of the transfer function is not yet significantly affected by the post-shock oscillations.

In an attempt to provide a quantification of this effect, the absolute error x is presented in Fig. 4.24 for both probes, defined as the difference between the computed values of ζ and f_n and the input values. It can be seen that the error

f range	Amplitude			Phase		
	t_{s1}	t_{s2}	t_{s3}	t_{s1}	t_{s2}	t_{s3}
0-15 kHz	4.2%	11.8%	17.4%	57%	148%	256%
15-50 kHz	3.4%	2.3%	2.3%	10%	6.7%	6.1%

Table 4.6 – Mean deviation of H_{out} from H_{in} for different t_s , probe 2.

on the damping parameter converges to a value lower than the input ζ . On the other hand, in the case of f_n the error fluctuates around a constant value, while the amplitude of the oscillations gets higher as t_s increases.

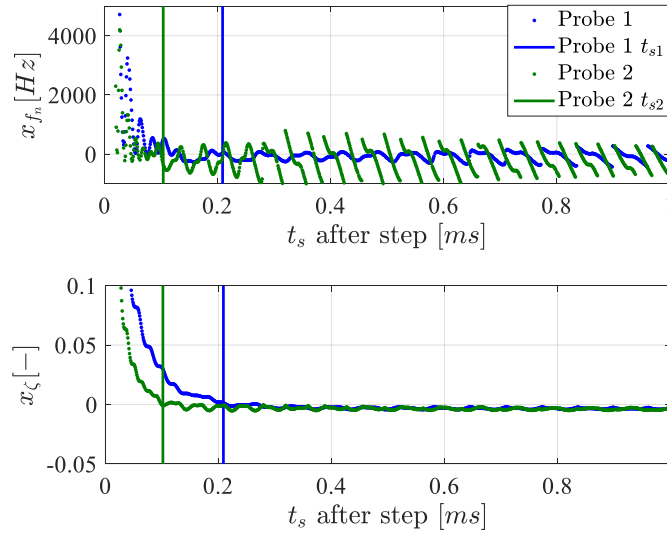


Figure 4.24 – Probe 1 & 2 absolute error for f_n and ζ as a function of t_s .

The aforementioned behaviour leads to the definition of a criterion for the selection of t_s based on the convergence of ζ . The signal is cut at the t_s for which ζ has reached the first plateau, after the rapid decrease at the beginning. The selected t_s is 0.209 ms and 0.101 ms for Probe 1 and 2, corresponding to $f_n = 24.03$ kHz, $\zeta = 0.1014$ and $f_n = 37.94$ kHz, $\zeta = 0.149$ respectively. As the same criterion is going to be used in the experimental procedure, an attempt is made to evaluate the uncertainty related to this process. The error values for ± 0.05 ms around the selected point are examined for both f_n and ζ . In Fig. 4.24, the selected t_s is represented by the solid lines for the two probes. The minimum, mean and maximum error values for the selected range are given in Table 4.7.

In Fig. 4.25, the effect of the system's ζ is presented for constant $f_n = 24$ kHz.

Error	min	mean	max
Probe 1 x_{f_n} [Hz]	-263	-112	124
Probe 1 x_{ζ} [-]	-0.013	0.0023	0.0077
Probe 2 x_{f_n} [Hz]	-703	-113	431
Probe 2 x_{ζ} [-]	-0.052	0.005	0.0311

Table 4.7 – Error values for f_n and ζ in the selected range

It can be seen that for bigger damping coefficients smaller t_s is required to minimize the error, as expected, since bigger ζ translates to a smaller settling time. Nevertheless, systems with smaller damping converge to a smaller absolute error value.

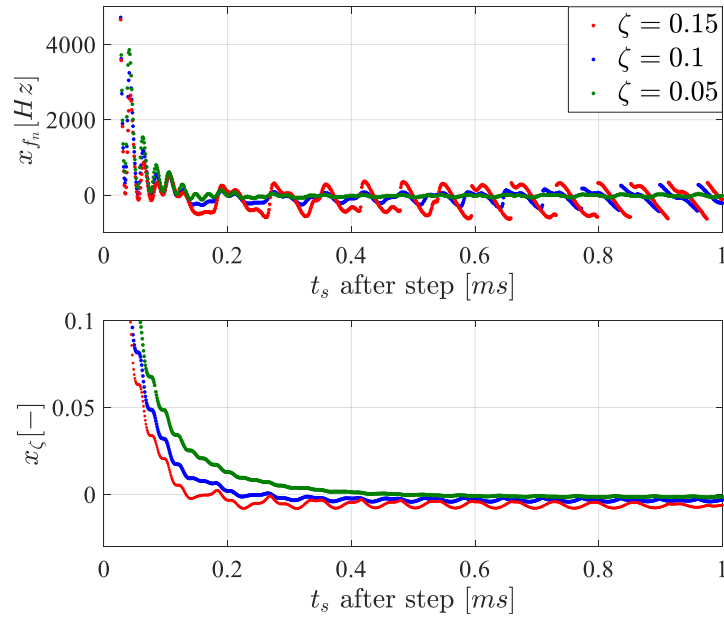


Figure 4.25 – Absolute error on the computed damping and resonance frequency values as a function of t_s , for different system damping.

4.4.3 Dynamic response results

In this section, the experimental dynamic response results for probes 5S and PT-K are presented. The shock tube test conditions are similar for the two probes and are presented in Table 4.8. The test conditions are similar with those of Test #3

in Table 4.3, for which the numerical analysis of the previous section has been performed.

Probe	$x/D_t[-]$	$T_1[K]$	$P_2/P_1[-]$	$M_s[-]$	$f_\infty[kHz]$
PT-K	33.45	293	1.1	1.042	3.76
5S	33.45	293	1.095	1.039	3.74

Table 4.8 – Shock tube test conditions.

Selection of optimal t_s The methodology presented in section 4.4.2.3 has been used for the computation of the transfer function for different values of the parameter t_s , which represents the time instant at which the signals are cut after the step rising. The computed parameters of the second order system for probe PT-K, the natural frequency f_n and the damping ζ , are presented in Fig. 4.26 as a function

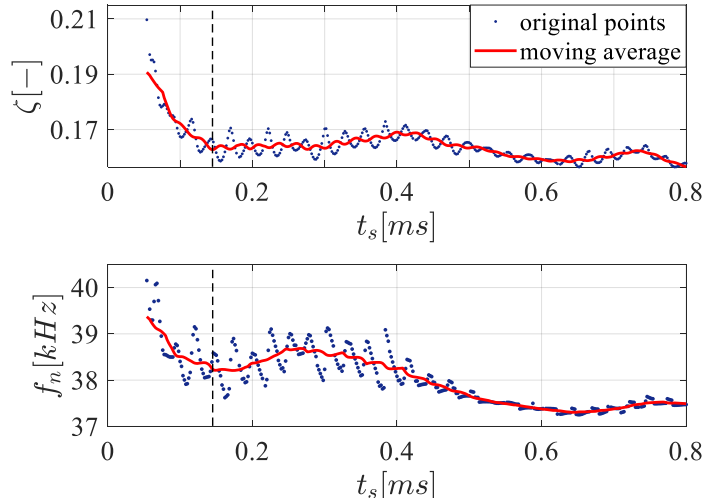


Figure 4.26 – Probe PT-K: natural frequency and damping as a function of t_s .

of t_s . Following the conclusions of the analytical study, the time instant when the damping reaches the first plateau is selected as the optimal t_s . From Fig. 4.26, $t_s = 0.144$ ms, corresponding to a damping $\zeta = 0.1645$ and a natural frequency $f_n = 38.3$ kHz. The transfer function for three different t_s values is presented in Fig. 4.27. It can be seen that for the two higher t_s values the shape of the transfer function at the peak region is similar, with the higher t_s case featuring more noise. For $t_s = 0.05$ ms (34% of the selected t_s), however, the transfer function features a significantly different shape at the peak region, indicating that the probe dynamic

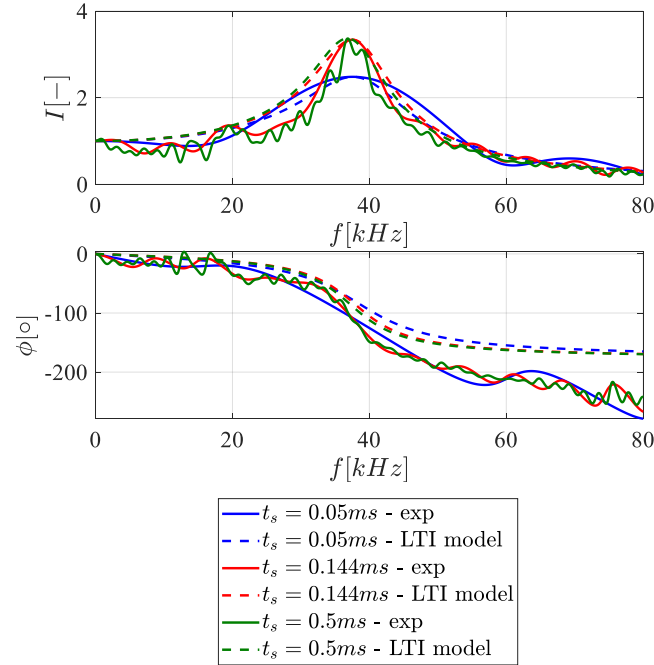


Figure 4.27 – Probe PT-K: transfer function as a function of t_s .

has not been completed yet.

Another index for the quality of the transfer function is the coherence function between the input signal x and the output signal y of a linear system, defined as:

$$C_{xy}(f) = \frac{|G_{xy}(f)|^2}{G_{xx}(f)G_{yy}(f)} \quad (4.18)$$

where G_{xy} are the cross spectral densities between $x(t)$ and $y(t)$, while G_{xx} and G_{yy} is the auto spectral density of $x(t)$ and $y(t)$ respectively. The coherence expresses the similarity in the frequency content of the two signals and takes values between 0 and 1, where 1 expresses two perfectly similar signals. The coherence is reduced due to non-linearities of the system and due to noise that is not common in both the input and output of the system. The coherence can therefore also be used to evaluate the signal to noise ratio and the frequency up to which the system identification can be performed.

The coherence as a function of frequency for different t_s values is presented in Fig. 4.28a for Probe PT-K. In this case the input signal $x(t)$ is the ideal pressure step and the output signal $y(t)$ is the output of the probe. The coherence is decreasing with frequency for all cases, due to the decrease of signal-to-noise ratio.

For higher t_s , the coherence features a decrease in the range 5–30 kHz due to the presence of the post-shock oscillations only at the probe signal. The average value of the coherence function for frequencies up to 60 kHz is presented in Fig. 4.28b as a function of t_s . The 60 kHz limit is selected by Fig. 4.28a, as the coherence maintains a high value up to this frequency, indicating a good signal-to-noise ratio and the possibility of dynamic compensation. It is clear that the mean coherence is decreasing with t_s . This further justifies the selection of the lowest possible t_s , defined by the line-cavity system dynamics.

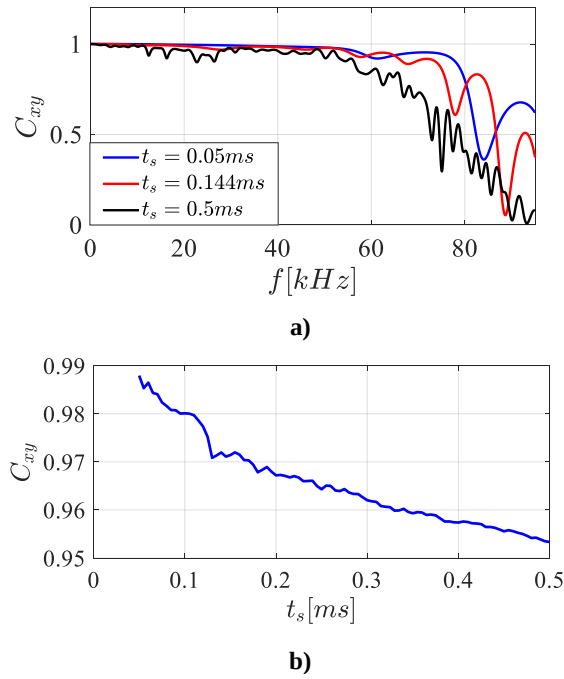


Figure 4.28 – Probe PT-K: a) Coherence as a function of frequency, b) Mean coherence up to 60 kHz as a function of t_s .

Final transfer functions and comparison with analytical models Increasing iteratively the parameter t_s and using the previously defined criteria, the optimal transfer functions for probes PT-K and probe 5S have been computed. The transfer functions are presented in Fig. 4.29 for PT-K and in Fig. 4.30 for the lateral tap #2 of probe 5S (since all four lateral taps feature the same design). The corresponding second order system parameters are given in Table 4.9. It can be seen that the amplitude response of probe PT-K is simulated very well by the LTI system up to a frequency of 80 kHz. The phase is slightly underestimated, which

could be related to alignment issues between the probe and the reference probe during the shock tube tests. For the lateral tap of probe 5S the LTI model matches the amplitude well up to 28 kHz, while a small deviation is present for higher frequencies. This can be attributed to the higher t_s required in this case (longer post-step length) which allows the appearance of disturbances that can contaminate the signal. Indeed, a peak is present at a frequency of around 16 kHz, which can be attributed to the post-shock oscillations. The frequency range over 28 kHz should not be affected by the primary post-oscillations predicted by the model of Gaetani et al. [27]. Nevertheless, the experimental time-frequency results in Fig. 4.18 reveal traces of other disturbances at higher frequencies, albeit of much lower magnitude. It can be observed that for both cases the phase of the experimental transfer function keeps decreasing at higher frequencies, deviating from the 2nd order system behaviour (asymptotically converging to -180°). Fitting with models of higher order could allow to capture better this behaviour. Nevertheless, the advantage of the 2nd order model is that it provides a simple representation, as it can be only described by the damping and natural frequency. When compensation up to a higher frequency is required, the experimental transfer function can be directly applied to the measurement data.

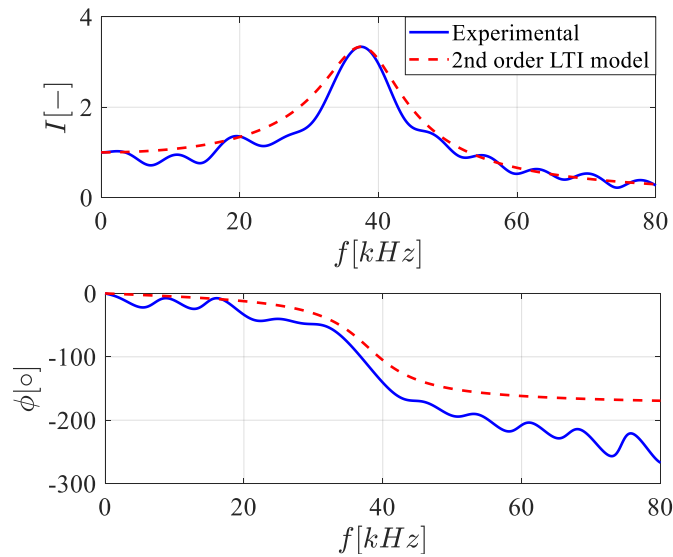


Figure 4.29 – Probe PT-K: Final transfer function.

The transfer function of the central tap (tap #1) of 5S is presented in Fig. 4.31. The central tap features a much higher frequency response and a much higher amplification with respect to the lateral tap, as the sensor is much closer to the external

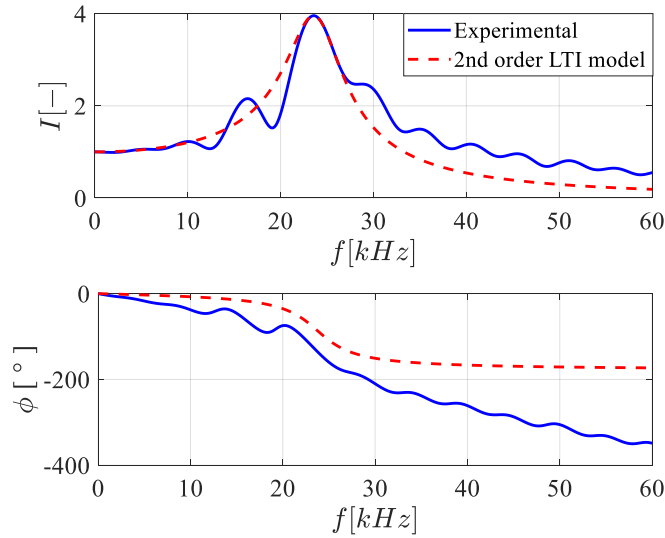


Figure 4.30 – Probe 5S-tap# 2: Final transfer function.

Probe	t_s [ms]	f_n [kHz]	ζ [-]	T_s [ms]
PT-K	0.144	38.3	0.1645	0.101
5S- lateral tap #2	0.205	23.9	0.127	0.2054

Table 4.9 – 2nd order LTI system parameters for probes PT-K and the lateral tap #2 of probe 5S.

surface. More specifically, the measured resonance frequency is $f_p = 202$ kHz. In this case, the second order model does not simulate adequately the experimental behavior. Nevertheless, thanks to the high bandwidth, no digital compensation is required for the typical requirements in turbomachinery measurements (up to 50-60 kHz).

At this point, a comparison can be performed with the analytical models used during the design phase. The comparison in terms of natural frequency is given in Table 4.10 for probe 5S. It has to be noted that possible sensor recess can affect significantly the performance, if not taken into account. Since the presence of chip recess was observed for all the sensors, the recess depth of each sensor was measured under a microscope. The results are summarized in Table 4.10, considering that the estimated measurement accuracy of the chip recess is rather low (around $\pm 25 \mu\text{m}$). The flow conditions in the shock tube (Table 4.8) have been used for the theoretical computations. Concerning the lateral holes, the Hougen CL model features the best prediction of the natural frequency. For the taps #2 #3 and #5, the

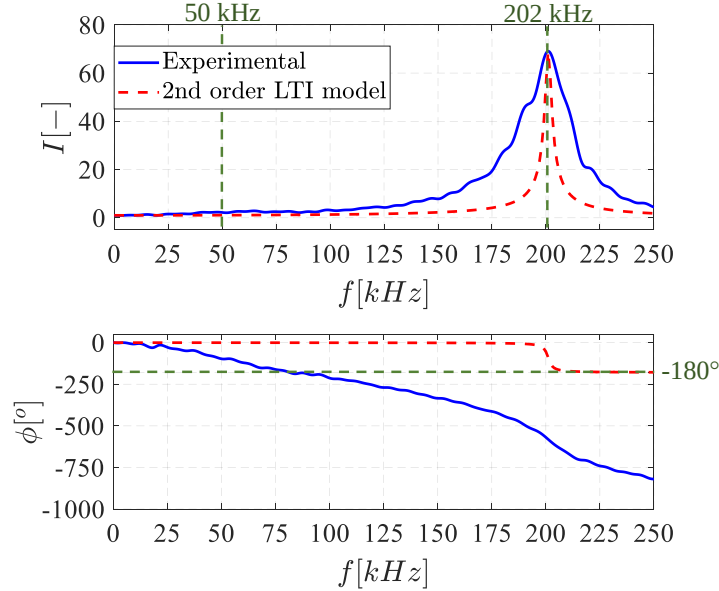


Figure 4.31 – Probe 5S-central tap# 1: Final transfer function.

prediction is accurate within 3 kHz, 4 kHz and 1 kHz respectively. Given that the design geometry is the same for all three line-cavity systems, the discrepancies are believed to be due to manufacturing/assembly uncertainty. The recess was measured on the encapsulated sensor before the probe assembly. Thus, any variations in the line-cavity system due to the actual settlement of the sensors inside the probe head could not be accounted for. The most accurate prediction for the central tap geometry is provided by the model of Hougen, which overpredicts the measured tap response by 30 kHz.

For probe PT-K, a prediction with analytical models is more complicated due to its design deviating from the classical line-cavity system. The ‘line’ in this case has a conical shape. Moreover, its diameter and cross-section are comparable to the ones of the cavity, which excludes a priori the application of the Helmholtz model (see Section 4.4.1). The Hougen model should be more appropriate for this case and its comparison with the experimental results is given in Table 4.11. The Hougen model (Eq. 4.6) requires the following geometrical parameters for the computation of the natural frequency: the cavity volume V_c , the line volume V_L and the line length L . The actual volume and length of the tapered line are considered as V_L and L . The line diameter D_l is additionally required for the length correction CL (Eq. 4.7). In this case, the cases $D_l = D_{max}$ and $D_l = D_{min}$ are computed. It can be seen that the Hougen model with CL clearly overestimates the natural frequency. The case with CL for which $D_l = D_{min}$ in the computation

Model	Natural frequency [kHz]			
	Central	Lateral 2	Lateral 3	Lateral 5
	Recess	Recess	Recess	Recess
	0.040 mm	0.142 mm	0.418 mm	0.300 mm
Helmholtz	262.2	63.9	55.5	58.3
Hougen	236.8	62.5	54.5	57.3
Helmholtz with CL	142.4	42.1	36.5	38.4
Hougen with CL	69.8	27.1	23.6	24.8
Experimental	201.7	23.9	27.0	25.7

Table 4.10 – Probe 5S: Comparison of experimental f_n to theoretical predictions. Predictions computed for $T_2 = 301$ K, temperature expected in the shock tube after the shock front.

of the corrected length gives the value closer to the experimental one.

Model	Natural frequency [kHz]
Hougen	162.3
Hougen with CL ($D_l = D_{max}$)	22.2
Hougen with CL ($D_l = D_{min}$)	38.8
Experimental	38.3

Table 4.11 – Probe PT-K: Comparison of experimental f_n to theoretical predictions. Predictions computed for $T_2 = 301$ K, temperature expected in the shock tube.

4.5 Dynamic calibration uncertainty

Despite the current interest in the so-called ‘traceability’ of dynamic measurements and the effort to develop traceable dynamic calibration procedures [30], no standards yet exist for the computation of uncertainty in shock tube tests. Few studies exist in the literature on this subject, like [31] and [32], both of which assume an ideal step input. In the present section, the aim is to perform an investigation of the error sources affecting the dynamic calibration of pressure probes in low-pressure shock tubes and an evaluation of its propagation to the obtained transfer function. The analysis of the flowfield and the effect of the post-shock disturbances on the transfer function previously presented are taken into account. The dynamic calibration of Probe PT-K will be used in this analysis. The test conditions are given in Table 4.8.

The potential error sources affecting the dynamic calibration, which will be separately addressed hereafter, are:

- the presence of post-shock oscillations,
- the position of the rising step,
- the uncertainty of the sensor.

4.5.1 Uncertainty due to post-shock oscillations

The effect of post-shock oscillations on the dynamic calibration of probe PT-K has been analyzed extensively in Section 4.4.3. The second order model parameters obtained are $f_n = 38.3$ kHz and $\zeta = 0.1645$. Their uncertainty can be estimated based on the results of the preliminary analysis in Section 4.4.2.4. Aiming to be conservative, we can estimate the error to range between the minimum and maximum values given in Table 4.7 for Probe 2 (Probe 2 features the same LTI parameters as Probe PT-K), following a uniform distribution.

From the preliminary analysis, and in particular considering the results of Table 4.6, it can be seen that for the optimum t_s (selected in order to minimize the error on the computation of the second order model parameters), the error is high in the low frequency range due to the presence of the post-shock oscillations. This error is minimized for low t_s values. Based on that, we consider that the error is negligible in the low-frequency region (up to 25 kHz in this case) for $t_s = 0.05$ ms (presented in Fig. 4.27). Therefore, we consider the post-shock oscillations effect as a bias error that can be accounted for by applying a correction factor. This correction factor is the difference between the transfer function for $t_s = 0.144$ ms and that of $t_s = 0.05$ ms in the low frequency range (0 to 25 kHz).

4.5.2 Effect of the step rising point

In the method followed for the transfer function computation, the signal of the flush-mounted reference sensor is used to trigger the generation of the ideal pressure step used as input in the calibration. The two probes (reference probe and probe under calibration) are placed in the same axial position and are thus expected to receive the shock simultaneously. An uncertainty in the axial distance between the probe tips is therefore translated to an uncertainty in the position of the rising step. Considering uncertainties in the manufacturing and the alignment of the probes, the uncertainty on the axial distance Δx is estimated to be ± 0.3 mm. The leading shock speed for these conditions is calculated as $c_s = 360$ m/s. This leads to an uncertainty in the step rising point definition of $\Delta t = \pm 4.2 \cdot 10^{-7}$ s, which is smaller than the digital resolution of our signal (10^{-6} s for sampling frequency of 1 MHz). In this case, this uncertainty contribution could be assumed conservatively equal to the digital resolution. This case is presented in Fig. 4.32,

where the nominal step and the steps rising at $\pm\Delta t$ are given with respect to the probe signal.

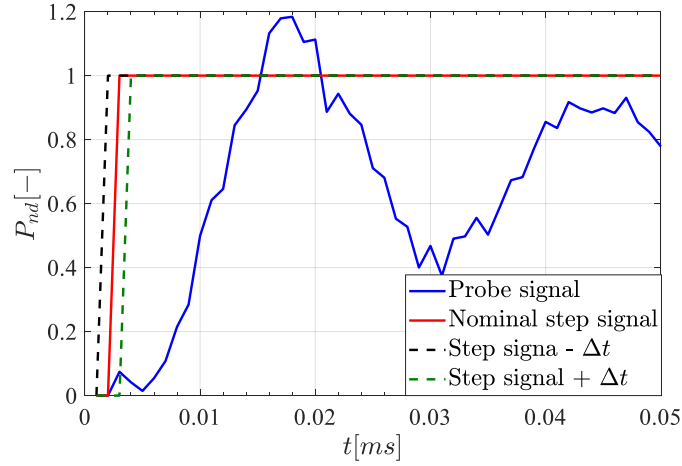


Figure 4.32 – Step rising points considered.

The effect of the step rising point uncertainty on the transfer function is given in Fig. 4.33. The transfer function for the nominal step rising point and the two extreme cases are presented. For simplicity, it is assumed that the uncertainty for each frequency band (frequency resolution) follows a uniform distribution between the two extreme curves. It can be seen that the uncertainty increases for higher frequencies and is more important for the phase rather than the amplitude. The step rising point effect on the analytical model (presented with dashed lines) is very small, since the parameters ζ and f_n are computed by the resonance peak amplitude and position (see Eq. 4.12 and 4.13). Similarly, it is considered that the uncertainty of ζ and f_n follows a uniform distribution between the two extreme values: $f_n : [38.327 - 38.41 \text{ kHz}]$, and $\zeta : [0.158 - 0.1704]$.

4.5.3 Measurement uncertainty

The uncertainty originating from the sensor and measurement chain should also be accounted for. Considering the sensor manufacturer's specifications and the noise levels measured inside the shock tube in no-flow conditions, this error source is modelled as a normal distribution with zero mean and a standard deviation of 0.3% of P_2 . Since this error source is expected to cause random fluctuations to the time series, its propagation to the transfer function is implemented with a Monte Carlo (MC) method. An alternative method of computing the random uncertainty would

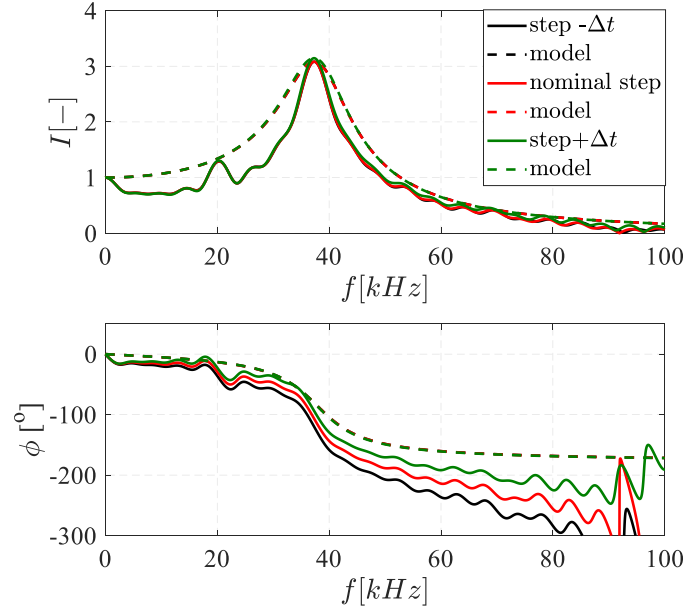


Figure 4.33 – Effect of step rising point uncertainty on the transfer function.

be by computing the variability of the transfer function based on a number of repeatable tests. The disadvantage of this method is that it would require a larger number of tests and the results would also include variations due to differences in the post-shock oscillations between the different tests. By estimating the random uncertainty in the pressure measurement and propagating it to the output, the measurement uncertainty component can be independently evaluated.

For each iteration i of the MC loop, a new time series $P_{t,i}$ is created, where each sample $P_t(k, i)$ is defined as:

$$P_t(k, i) = \overline{P_t(k)} + s_{k,i} \quad (4.19)$$

where $\overline{P_t(k)}$ is the expected value for this time instant k and $s_{k,i}$ is a random sample from the error distribution.

The transfer function computation is performed for each MC iteration, resulting in the transfer function accompanied by uncertainty bounds, presented in Fig. 4.34. The mean transfer function is given with the red solid line while the 95% confidence interval lies in the gray shaded area. It can be observed that the uncertainty is increasing for higher frequencies, in particular for frequencies higher than the resonance. This trend is common in the amplitude and phase diagrams but the uncertainty magnitude is much higher for the phase.

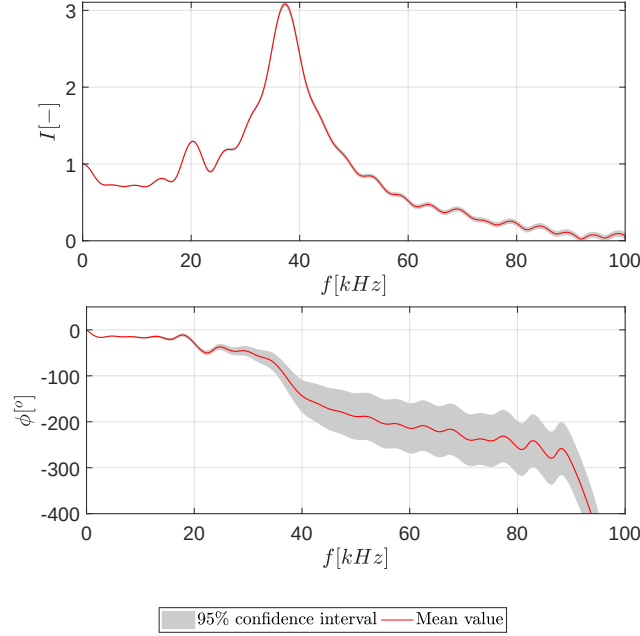


Figure 4.34 – Transfer function with 95% confidence interval due to measurement uncertainty.

The resulting natural frequency and damping parameters are: $f_n = 38.37 \text{ kHz} \pm 0.034$, and $\zeta = 0.1645 \pm 0.0013$, following a normal distribution. As expected, the uncertainty on the definition of this parameters is small, as it is based only on the amplitude diagram.

4.5.4 Total uncertainty

The total uncertainty is computed as follows:

$$u_{tot,f}^2 = u_{meas,f}^2 + u_{step,f}^2 \quad (4.20)$$

where $u_{meas,f}$ and $u_{step,f}$ are the measurement uncertainty and the uncertainty in the step rising point at a frequency f , computed at standard deviation level (66.67 % confidence level). This computation is performed independently for the amplitude and phase of the transfer function, over finite frequency bands. The frequency bands in this case are defined by the frequency resolution $\Delta f = 20 \text{ Hz}$. The post-shock oscillations effect is not included in the computation since, as explained before, it should be considered as a known bias error which can be corrected for.

In Fig. 4.35 and 4.36, the total uncertainty of the transfer function is presented, with uncertainty bands at 95% confidence level. The corrected mean trans-

fer function is given with the dashed line. The relative total uncertainty and the two uncertainty sources at 95% confidence level are given. The total uncertainty of the amplitude is less than 3% up to a frequency of 50 kHz. The two uncertainty sources considered have a comparable contribution for all the frequency range of interest. On the other hand, the total uncertainty of the phase is over 20% for the bigger part of the frequency range, while reaches values up to 50% locally. The two uncertainty sources have a similar value up to 20 kHz, while the measurement uncertainty becomes more important for higher frequencies. Overall, it can be clearly seen that the uncertainty is much higher on the computation of the phase compared to that of the amplitude. This justifies the choice of Equations 4.12 and 4.13 for the computation of f_n and ζ , only based on the identification of the resonance peak from the amplitude graph. This method is expected to reduce the uncertainty on the computation of the parameters of the line-cavity system.

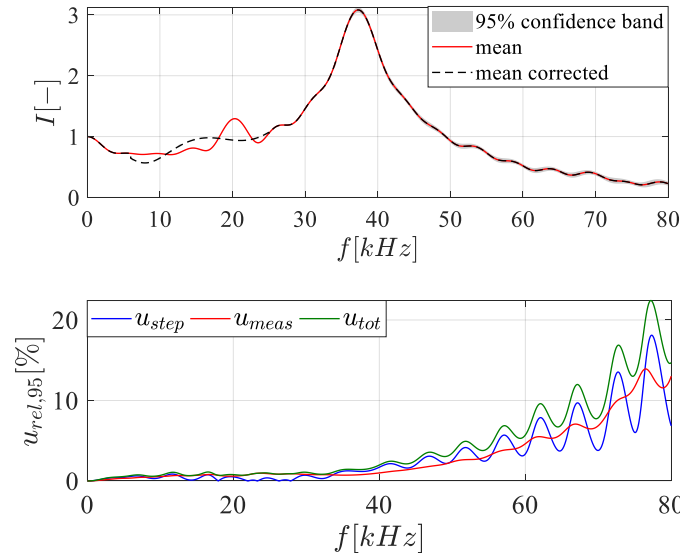


Figure 4.35 – Amplitude transfer function with 95% total uncertainty band (top), and relative uncertainty contributions (bottom).

Similarly, the total uncertainty on ω_n and ζ is computed as:

$$u_{f_n,totl}^2 = u_{f_n,meas}^2 + u_{f_n,step}^2 + u_{f_n,nd}^2 \quad (4.21)$$

$$u_{\zeta,tot}^2 = u_{\zeta,meas}^2 + u_{\zeta,step}^2 + u_{\zeta,nd}^2 \quad (4.22)$$

where $u_{f_n,nd}$ and $u_{\zeta,nd}$ is the uncertainty due to nonideality (post-shock oscillations). The total uncertainty and the individual contributions, described in the

previous sections, are given in Table 4.12. It can be seen that the biggest contribution comes from the post-shock oscillations effect. For the other two sources, the uncertainty is low, in line with the small uncertainty in the amplitude computation presented before.

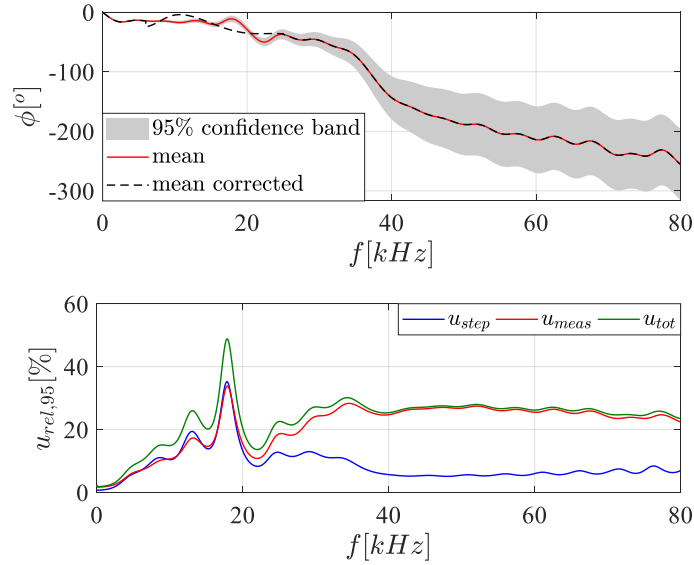


Figure 4.36 – Phase transfer function with 95% total uncertainty band (top), and relative uncertainty contributions (bottom).

	source	distribution	absolute	relative
f_n	u_{meas}	normal	34 Hz	0.09%
	u_{step}	uniform	50 Hz	0.13%
	u_{nd}	uniform	655 Hz	1.7%
	u_{tot}	normal	657 Hz	1.71%
ζ	u_{meas}	normal	0.0013	0.8%
	u_{step}	uniform	0.0068	4.1%
	u_{nd}	uniform	0.048	29.1%
	u_{tot}	normal	0.0484	29.5%

Table 4.12 – Uncertainty sources and total uncertainty of f_n and ζ at 95% confidence level.

4.6 Turbulence measurements with single-sensor pressure probes

The measurement of turbulence with fast-response multi-sensor probes consists in the ability of these probes to measure the instantaneous velocity vector, as performed e.g. by Chasoglou et al. [12]. Similarly, single-sensor or double-sensor probes used in virtual mode can measure the periodic velocity components and rms values of the stochastic velocity fluctuations (e.g. in Ref. [16]). A particular case is the measurement of turbulence with single-sensor pressure probes (total-pressure probes), since it requires the establishment of a correlation between the measured total pressure fluctuations and the velocity fluctuations. The advantages of this approach is the use of simple probe designs with minimal size (since only one sensor is needed) and reduced testing time, since no probe rotation is required (as in the virtual multi-sensor mode). This approach has been used by Wallace & Davies [17], Persico et al. [18, 33] and Lengani et al. [19].

Wallace & Davies [17] derived a relationship between the root mean square of the total pressure fluctuations and the velocity fluctuations for compressible flow. Their approach is summarized here for completeness. Starting from the isentropic equation for compressible flow and the perfect gas law, the following yields (considering $\gamma = 1.4$ for air):

$$\frac{P_0}{P_s} = \left(1 + \frac{\rho U^2}{7P_s}\right)^{\frac{7}{2}} \quad (4.23)$$

Applying a Reynolds decomposition (expressing variables as the addition of a mean and fluctuating component), neglecting terms of order higher than third and assuming that the density fluctuations are negligible, results in the following equation:

$$p'_0 = A p'_s + B u' \quad (4.24)$$

which in terms of mean square values becomes:

$$\overline{p_0'^2} = A^2 \overline{p_s'^2} + AB \overline{(p_s' u')} + B^2 \overline{u'^2} \quad (4.25)$$

where $A = 1 - 0.175M^4$ and $B = \rho U(1 + 0.5M^2)$.

To get a direct relationship between total pressure and velocity fluctuations, the following correlation for the static pressure fluctuations by Hinze [34] is used:

$$\overline{p_s'} = \sqrt{0.49\rho u'^2} = 0.7\rho u' \quad (4.26)$$

which is derived assuming isotropic and homogeneous turbulence. Wallace & Davies state that this correlation can also be used in instantaneous form. Therefore, introducing Eq. 4.26 in 4.24 and 4.25, the following relationships are acquired:

$$p'_0 = (1 - 0.175M^4)\rho u'^2 + (1 + 0.5M^2)u' \quad (4.27)$$

$$\begin{aligned} \overline{p_0^2} = & 0.49\rho^2(1 - 0.175M^4)^2\overline{u^4} + \rho^2U^2(1 + 0.5M^2)^2\overline{u^2} \\ & + 0.7\rho(1 - 0.175M^4)(1 + 0.5M^2)\overline{u^3} \end{aligned} \quad (4.28)$$

Assuming that $\overline{u^3} = 0$, Persico et al. [18] further simplified Eq. 4.28 and used it to estimate the turbulent kinetic energy downstream of a turbomachinery rotor with a single-sensor fast response pressure probe. Comparison with LDV measurements shows qualitative agreement, with quantitative differences higher in the wake and vortex regions. Lengani et al. [19] used the same approach as Persico et al. [18] to estimate the turbulence intensity contour downstream of a low-pressure turbine rotor.

In a later study, Persico et al. [33] performed a similar analysis to the one of Wallace & Davies [17] starting from the Bernoulli equation. Using always the correlation of Hinze [34] (Eq. 4.26), they derived the following expression for instantaneous velocity fluctuations:

$$u'^2 + \frac{U}{1.2}u' - \frac{p'_0}{1.2\rho} = 0 \quad (4.29)$$

A kiel-type fast response total pressure probe was used in this study, due to its insensitivity to the flow angle, allowing to recover total pressure fluctuations in a wide flow angle range. Measurements were performed with this probe and a single hot-wire across a circular jet, in different downstream positions. The authors partially attributed discrepancies between the two techniques to the fact that the large sensing area of the total pressure probe recovers all three turbulence components, while the hot wire measures only the streamwise one. Applying a correction factor for this effect assuming isotropic turbulence, results in a good agreement at the measured turbulence intensity values for axial positions close to the jet exit. Comparison of velocity spectra for the same downstream positions reveals a good match as well.

Moss & Oldfield [35] derived an expression starting from the Bernoulli equation and considering the second order velocity fluctuation terms negligible, they acquired a linear relationship between the velocity and the total pressure fluctuations:

$$\frac{u'}{U} = \frac{1}{2} \frac{p'_0}{P_0 - P_s} \quad (4.30)$$

They remarked a good agreement between turbulence spectra measured by the fast response probe and a hot wire, when correcting the former for the dynamic response and the spatial resolution. The same approach was used more recently by Lubbock & Oldfield [36] to measure turbulent kinetic energy and length scales in a gas turbine exit flow.

The common point in all previous studies is the use of a correlation to model static pressure fluctuations, in order to retrieve a unique relationship between total pressure and velocity fluctuations. In the present work, the aim is to better understand the origin and impact of this correlation by getting deeper in the theoretical

analysis, and to make a decision on the methodology to use for the fast response pressure probe case, which will be presented later in Chapter 7.

4.6.1 Pressure fluctuations in turbulent flows

Hinze [34] retrieved a relationship between the double correlation for pressure fluctuations at two neighbouring points and the velocity correlation, starting from the Poisson equation for incompressible flow and assuming isotropy and homogeneity. Using the Millionshchikov hypothesis (which relates the fourth order velocity correlation to the second order one assuming that the joint distribution of velocities at two neighbouring points is Gaussian) and using an empirical correlation for the shape of the longitudinal velocity correlation for large Reynolds numbers, the expression for the intensity of the pressure fluctuations is retrieved (Eq. 4.26). This analysis had been previously performed by Batchelor [37], who used an experimentally defined shape for the velocity correlation at large Reynolds number turbulence. He resulted in the following expression for the intensity of the pressure fluctuations:

$$\overline{p_s} = \sqrt{0.34} \rho u^2 = 0.58 \rho u^2 \quad (4.31)$$

With the same analytical approach, Uberoi [38] estimated the root mean square pressure fluctuations by the measured double velocity correlations in isotropic flow of different turbulence Reynolds numbers R_λ . For the highest Reynolds number reported, the coefficient is close to 0.7, as in Eq. 4.26 derived by Hinze [34]. Kraichnan [39] treated the case of anisotropy, for which he concluded that the coefficient would be reduced compared to the isotropic case.

Kraichnan [39] also considered the pressure fluctuations at a flow with a constant mean shear and they identified two mechanisms contributing to the generation of pressure fluctuations: the turbulence-turbulence interaction and the interaction of the turbulence with the mean flow. The first contribution is identical to the ones previously mentioned. George et al. [40] further investigated the case of pressure fluctuations in free shear flows and derived spectral models for the pressure fluctuations, as well as expressions for the mean square pressure fluctuations due to both the turbulence-turbulence interaction and the turbulence-mean shear interaction. The analysis of George et al. [40] starts from the Poisson equation for a unidirectional flow with constant mean shear and retrieves a pressure spectrum function $\pi_p(\kappa)$ which is composed by three parts:

$$\pi_p(\kappa) = \pi_{s2}(\kappa) + \pi_{s3}(\kappa) + \pi_t(\kappa) \quad (4.32)$$

where $\pi_{s2}(\kappa)$ is the second moment turbulence-mean shear interaction, $\pi_{s3}(\kappa)$ is the third moment and $\pi_t(\kappa)$ is the turbulence-turbulence interaction. Employing dimensional arguments, the following forms are retrieved for each contribution in

the inertial range:

$$\frac{1}{\rho^2} \pi_{s2}(\kappa) = \alpha_2 K^2 \epsilon^{\frac{2}{3}} \kappa^{-\frac{11}{3}} \quad (4.33)$$

$$\frac{1}{\rho^2} \pi_{s3}(\kappa) = \alpha_3 K \epsilon \kappa^{-3} \quad (4.34)$$

$$\frac{1}{\rho^2} \pi_t(\kappa) = \alpha_p \epsilon^{\frac{4}{3}} \kappa^{-\frac{7}{3}} \quad (4.35)$$

where α_2 , α_3 and α_p are universal constants, ϵ is the turbulence dissipation rate and K is the mean shear. It can be seen that the slope of the pressure spectrum in the inertial range will depend on which term is more dominant for each contribution.

Assuming isotropy, the third moment contribution is zero (Eq. 4.34), and the turbulence-mean shear contribution to the 1D pressure spectrum results in:

$$\frac{1}{\rho^2} F_{ps}^1(\kappa_1) = K^2 \left[\kappa_1^2 \int_{\kappa_1}^{\infty} \frac{E(\kappa)}{\kappa^5} d\kappa + \kappa_1^4 \int_{\kappa_1}^{\infty} \frac{E(\kappa)}{\kappa^7} d\kappa \right] \quad (4.36)$$

where $E(\kappa)$ is the velocity spectrum function. In order to calculate the 1D pressure spectrum due to the turbulence-mean shear interaction, an empirical model for $E(\kappa)$ can be used. Using the von Karman spectrum, the following expression is retrieved:

$$\frac{1}{\rho^2} F_{ps}^1(\kappa_1) = 0.307 \alpha K^2 \epsilon^{\frac{2}{3}} \kappa_1^{-\frac{11}{3}} \quad (4.37)$$

The mean square pressure fluctuations due to the turbulence-mean shear interaction are subsequently computed as:

$$\frac{1}{\rho^2} \overline{p_s'^2} = 0.33 K^2 l^2 \overline{u'^2} \quad (4.38)$$

where the coefficient depends on the spectral model used.

George et al. [40] demonstrated that in the inertial subrange the turbulence-mean shear pressure spectrum features a slope of $\kappa^{-11/3}$, while the turbulence-turbulence interaction spectrum a slope of $\kappa^{-7/3}$.

4.6.2 Discussion

The previous review aims to shed some light in the physical background of the correlation between velocity and static pressure fluctuations, not usually reported in the frame of turbulence measurements. It can be seen that the same correlation has been derived by different researchers, albeit with different coefficients. It has been decided that the Hinze [34] correlation with a coefficient of 0.7 will be used in the present work, as it is the one used by all researchers in the field of turbulence measurements. Moreover, it is in agreement with the only experimental

work by Uberoi [38]. As has been demonstrated by George et al. [40], this correlation reflects only the static pressure fluctuations due to the turbulence-turbulence interaction and an additional term should be considered for the contribution due to the turbulence interaction with the mean shear flow. However, the latter requires knowledge of the mean flow shear and mixing length, which is difficult to estimate in three dimensional flows. It would be nevertheless worthy to consider in future studies for 2D shear flows like jet experiments or linear cascade wakes.

Considering the different methodologies to relate total pressure to velocity fluctuations, the Wallace & Davies approach will be preferred as it is the only one taking into account the compressibility effect. The equation 4.24, in instantaneous form will be employed, in order to perform a spectral analysis of the measurements, apart from the computation of the turbulence intensity values.

In future work, it would be interesting to consider modelling the spectral distribution of the static pressure fluctuations using a spectral model, instead of modelling the mean square pressure value. The work of George [40] provides the background for such a study, which, to the author's knowledge, has never been realized in the context of turbulence measurements.

4.7 Conclusions

In this chapter, different aspects of the operation and application of fast response aerodynamic probes have been addressed.

Firstly, a description of the different calibration steps has been given, namely static, dynamic and aerodynamic calibration. A detailed static calibration in both pressure and temperature has to be performed in order to correct for the temperature effects on the sensor output. The aerodynamic calibration is adapted to each probe configuration and aims to characterize the effect of the aerodynamic field on the measurements. It has been demonstrated that Reynolds and Mach number effects can be important in this case, while dynamic effects stemming from the flow interaction with the probe geometry should be considered as well. More specifically, the probe spectra in the expected angle range should be analyzed in order to identify peaks that risk to contaminate the measurements.

A big part of this chapter has been dedicated to the dynamic calibration of fast response pressure probes in shock tubes. A detailed post-processing methodology has been developed to allow the computation of the transfer function with the best accuracy, taking into account non-ideal phenomena in the shock tube flowfield. Analytical models used for the prediction of the dynamic response at the design phase are also presented and compared with the experimental results.

An uncertainty computation method for the dynamic calibration has been presented. The uncertainty bounds as a function of frequency have been computed for the amplitude and phase of the transfer function. It has been found that the

absolute uncertainty increases significantly after the resonance frequency for both amplitude and phase. Nevertheless, the relative uncertainty for the amplitude is less than 5% up to a frequency of 60 kHz (for a resonance frequency at 38 kHz), while for the phase the value is on average around 20% at the same range. This indicates that the amplitude is computed with a better accuracy, suggesting that when modelling the system as an LTI 2nd order model, the damping and natural frequency should preferably be computed by the amplitude plot.

The chapter concludes with a review and critical analysis on the measurement of turbulence with single-sensor pressure probes. This practice assumes a correlation between the total pressure fluctuations and the velocity fluctuations. The underlying assumptions in such correlations are rarely elaborated in the context of turbulence measurements, therefore it was deemed worthy to dedicate a section on that issue.

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5

Uncertainty analysis

This chapter is dedicated to the uncertainty on the computation of turbulence statistics. The uncertainty sources are divided into two categories: measurement and statistical uncertainty. The two macro uncertainty sources are investigated and methods for their computation are proposed.

5.1 Introduction

As suggested by Favier [1], two macro uncertainty sources can be identified for experimental data: the measurement related uncertainty and the uncertainty associated with the statistical data reduction. The first item addresses the bias and random errors that result from the application of a particular instrument and measurement methodology (e.g. sensor, calibration, installation, flow conditions, data reduction, etc.). The second item refers to the significance of the sample size when computing statistical quantities. For the case of turbulence parameters, it is equivalent to the observation time (i.e. the sampling duration) with respect to the period of the phenomenon. In other words, in the case of turbulence research, the first item refers to the uncertainty of each single sample, while the second to the uncertainty due to the statistical treatment of the finite time series. Different terms are used in the literature to characterize this uncertainty source: statistical uncer-

tainty [2, 3], sampling uncertainty [4] and random uncertainty [5]. In the present work, this item will be referred to as statistical uncertainty, while the first item as measurement uncertainty.

The measurement uncertainty budget is normally evaluated by two widely accepted standards, the ASME Measurement Uncertainty [6] and the ISO Guide to the Expression of Uncertainty in Measurements (GUM) [7]. The two methods differ in terms of classification of uncertainty sources but are consistent in terms of results and computation methodology: the final uncertainty is derived from the propagation of all the uncertainty terms through a first order approximation of a Taylor's series expansion. Such a procedure implicitly assumes the linearization of the measurement model and that the input and output quantities would follow a normal or a scaled and shifted t-distribution. According to the GUM [7], when the model features significant non-linearity, the inclusion of higher order terms in the Taylor series expansion is required. This procedure can be algebraically complex and the assumption of the symmetry of the distributions should still be met. When this assumption is not valid, a Monte Carlo Method (MCM) is suggested to correctly propagate the probability distributions of the variables. The GUM supplement [8] provides guidelines for the application of MCM in measurement uncertainty computations.

The aforementioned works refer to single-sample experiments where only the statistical uncertainty of the population mean is included. In turbulence research, the statistical uncertainty of higher order statistics, due to the sampling of the random process, is also of interest. In turbulent flows, the samples are considered independent when the sampling rate is less than $1/(2T_{int})$, with T_{int} being the integral time scale of the turbulent field [9, 10]. When higher sampling rates are used, the uncertainty should be estimated based on the number of independent samples N_{eff} [11]. As the uncertainty of the time-averaged quantities decreases with rate $1/\sqrt{N_{eff}}$ [2, 11], the minimization of the uncertainty requires the elongation of the averaging time T (i.e. the signal duration), which can often be bounded by limitations imposed by the experimental setup (e.g. in short-duration wind tunnel tests). The averaging time required for statistical converge can also be expressed in terms of eddy turnover time in order to take into account differences between experiments. Antonia et al. [12] observed that the required averaging time varies depending on the statistical quantity to be computed (e.g. mean, rms, skewness) and potentially with other parameters of the flow like the Reynolds number and the distance from the wall for boundary layer measurements.

Error estimates for central moments up to the fourth order and Reynolds stresses can be found in Benedict & Gould [2], for both normal and arbitrary distributions. This is an essential requirement for the treatment of velocity distributions in turbulent flows that are not (necessarily) normal. Turbulent velocity derivatives are known to be inherently non-Gaussian, as a manifestation of the intermittency in the

inertial and dissipation turbulence ranges [13, 14]. Although, the PDF of the velocity distribution is often assumed to be Gaussian (e.g. [15]), Jimenez [16] demonstrated that the PDF of the velocity fluctuations is dominated by the behaviour of the largest structures, for turbulent flows with spectra steeper than κ^{-1} . Therefore, in the general case, including flows whose spectrum has a slope of $\kappa^{-5/3}$ in the inertial range, the PDF of the velocity fluctuations should not be expected to be Gaussian but it will rather present the characteristics of these large eddies.

In the present work, the use of a non-parametric resampling algorithm like the moving block bootstrap method (MBB) will be considered for the computation of the statistical uncertainty. This method allows for the computation of the uncertainty of all statistics without requiring the derivation of complex formulas (see e.g. the expressions for the error estimates of r -th order moments in [2]). Compared to other resampling methods, MBB allows the extraction of overlapping blocks from the original time series, therefore allowing to obtain a larger number of blocks to be resampled. This is particularly important when the original series is not very long. The MBB can be applied to correlated time series, allowing to obtain uncertainty estimates also for the turbulent time and length scales. Finally, it does not require any assumption about the underlying probability distribution of the population, making it suitable for use in turbulent flows.

5.1.1 Objectives of the present work

This chapter aims at defining a complete approach to the computation of the uncertainty of turbulence parameters (i.e. statistics). The methods presented can nevertheless be applied to any measured quantity, with the statistical uncertainty treatment being generally valid, regardless if the data have a numerical or an experimental origin.

- The measurement uncertainty is treated in the first section. Methods to categorize and estimate the elemental uncertainty sources are presented. The elemental uncertainties can be propagated to the data reduction algorithm with the Taylor Series or the Monte Carlo method. The two methods are described and are applied to a simple measurement case in order to illustrate their differences and compare the results.
- The statistical uncertainty is examined in the second section. Analytical expressions for error estimates are presented for statistics up to the fourth order. The MBB algorithm for the computation of uncertainty estimates is also presented and experimentally obtained velocity signals are used to demonstrate its application in comparison to analytical expressions.

5.2 Measurement uncertainty

5.2.1 Uncertainty estimation methods

As mentioned before, the nomenclature of the ASME methodology [17] is followed in the present study, and the uncertainty sources are categorized as ‘systematic’ and ‘random’. Random sources vary randomly during the measurement period and cause scatter in the measurement data, while systematic sources cause a constant deviation from the true value and they do not cause any variability in the measurements.

For a finite number of measurements N of a value X , the sample mean and the sample standard deviation can be calculated by Eq. 5.1 and 5.2 respectively:

$$\bar{X} = \frac{1}{N} \sum_{i=1}^N X_i \quad (5.1)$$

$$s_X = \left[\frac{1}{N-1} \sum_{i=1}^N (X_i - \bar{X})^2 \right]^{1/2}. \quad (5.2)$$

The central limit theorem states that, independently of the distribution of the variable X , when a large number of independent samples N is used, the sample mean will follow a normal distribution with mean value $\bar{X}$ and standard deviation $\overline{s_X}$, where

$$\overline{s_X} = \frac{s_X}{\sqrt{N}}. \quad (5.3)$$

When the variable X is instead normally distributed and a small sample size N is used, the sample mean will follow a t-distribution with $N - 1$ degrees of freedom ν .

The random standard uncertainty sources are computed by Eq. 5.3. As systematic sources do not cause any variability in the measurement data, their effect can not be estimated by repetitive measurements. Therefore, they are evaluated using available data (e.g. manufacturing specifications, calibration data) and engineering judgment.

When all systematic b_{X_i} and random s_{X_i} standard uncertainties have been evaluated, the total standard uncertainty of X can be calculated by:

$$u_X = \left(\sum_{i=1}^{N_b} b_{X_i}^2 + \sum_{i=1}^{N_s} s_{X_i}^2 \right)^{1/2}. \quad (5.4)$$

5.2.1.1 Taylor Series Method (TSM)

When the TSM is used for the propagation of uncertainties, the first order terms of a Taylor series expansion are considered, and the propagation takes place at the

standard uncertainty level. A detailed derivation of the TSM propagation equation can be found in [18].

If a variable f is a function of two variables x and y ($f = f(x, y)$), then the standard uncertainty in f is calculated by the following equation:

$$u_f = \left[\left(\frac{\partial f}{\partial x} \right)^2 b_x^2 + \left(\frac{\partial f}{\partial y} \right)^2 b_y^2 + 2 \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} b_{xy} + \left(\frac{\partial f}{\partial x} \right)^2 s_x^2 + \left(\frac{\partial f}{\partial y} \right)^2 s_y^2 \right]^{(1/2)} \quad (5.5)$$

where b_x and b_y are the systematic standard uncertainties of x and y , and s_x and s_y the random standard uncertainties. The third term in Eq. 5.5 is the correlated systematic uncertainty term which contains the covariance factor b_{xy} , when the systematic uncertainties of x and y are not independent and share common error sources. An equivalent term for the random uncertainties is not included, since random uncertainties are usually considered uncorrelated [18].

In order to expand the standard uncertainty u_f at a specified confidence level, we need to use an appropriate coverage factor. According to the central limit theorem, when f is affected by multiple independent error sources, the resulting error distribution for f will be normal, independently of the distributions of the original error sources. Therefore, for most engineering applications the appropriate Student's t coverage factor can be used for the desired confidence level. The expanded standard uncertainty will be then calculated as:

$$U_f = t_{\%} u_f \quad (5.6)$$

In order to choose the t value for any confidence interval, the degrees of freedom for the combined system must be known. For most applications, the degrees of freedom can be considered high enough, in order to have a constant t for any confidence level. For the 95% confidence level, $t = 2$ when the degrees of freedom are higher than 30 [17, 18].

5.2.1.2 Monte Carlo Method (MCM)

The MCM for uncertainty propagation consists in propagating probability distributions instead of standard uncertainty values. Initially, the probability distribution functions (PDFs) for all uncertainty sources are defined. Random sources are usually assumed to have a Gaussian distribution, while for systematic sources the PDFs are defined based on available data and engineering judgment. In order to define the input variables, a sample is randomly chosen from each PDF and added to the expected value (which is usually the measured value). This procedure is illustrated in Fig. 5.1 for two variables X_1 and X_2 , which are affected by uncertainty sources s_1, b_1, b_2 and b_2, b_3 respectively. For each iteration i , a random sample is selected from each PDF and the input variables for this iteration are defined as: $X_{1i} = \bar{X}_1 + s_{1i} + b_{1i} + b_{2i}$ and $X_{2i} = \bar{X}_2 + b_{2i} + b_{3i}$, where $\bar{X}_1$

and $\bar{X}_2$ are the expected values. The values X_{1i} and X_{2i} constitute the inputs to the measurement model $Y = f(X_1, X_2)$ for this iteration. Common error sources are taken into account as depicted in Fig. 5.1, where the uncertainty source b_2 is common to both variables. An advantage of the MCM is that when input uncertainty sources are correctly defined, the implementation of correlated uncertainties is straightforward.

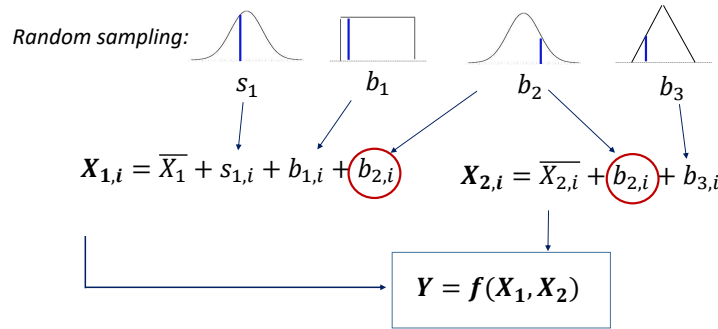


Figure 5.1 – Definition of input variables

Once the input variables are defined, they are fed into the measurement model and the output quantities are computed. This process is repeated M times and results in a certain distribution of the output quantities. Then, the uncertainty values at the desired confidence level (e.g. 95%) can be easily computed. This procedure is described schematically in Fig. 5.2.

Increasing number of M simulations ensures a better approximation of the desired uncertainty values but also increases significantly the simulation time. According to the guidelines of the GUM supplement [8], the number of samples M can be either selected adaptively or a priori. In the first case, the number of samples is continuously increased until the results stabilize in the statistical sense. In the latter case, which is employed in the current study, the number of samples is selected a priori, following the empirical criterion suggested by [8]:

$$M \geq 10^4 \frac{1}{1-p} \quad (5.7)$$

where p is the desired confidence level. For a confidence level of 95%, M should be equal or higher to 2×10^5 .

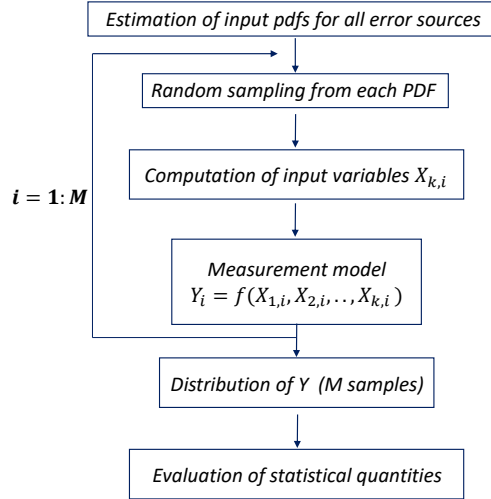


Figure 5.2 – MCM procedure

5.2.2 Uncertainty quantification example: HWA measurements

The uncertainty assessment of turbulence statistics measured by HWA is presented in this section. The measurements were performed in the core of a subsonic, non-isothermal jet.

5.2.2.1 Experimental setup and methodology

A single Hot-Wire probe was used during the measurement campaign, featuring a platinum-coated tungsten wire of $9\ \mu\text{m}$ diameter and 1 mm active sensor length. The Hot-Wire Anemometer was operated by the Dantec Streamline 90N10 Frame. The Frame includes 4 CTA modules, each of them consisting of a CTA bridge and a signal conditioner. The system is configured and operated with the Dantec StreamWare software.

The measurements have been carried out at an open-jet facility, presented in Fig. 5.3. The nozzle is connected to a 7 bar pressure supply through a heater, which is used to vary the flow temperature. The head of the HW probe was placed inside the core of the jet. The total temperature and total pressure inside the nozzle are measured by means of a thermocouple and a Validyne transducer.

The non-dimensional calibration methodology described in Chapter 2.2 is used, in order to account for the temperature variations. The same setup has been used for calibration and measurements.

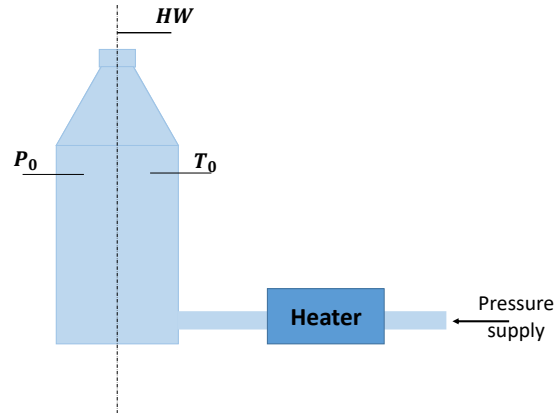


Figure 5.3 – Schematic of the experimental setup.

5.2.2.2 Uncertainty analysis

In this section, the uncertainty computation is presented step by step. The final uncertainties are expressed at a 95% confidence level. The number of trials M for the MCM is selected as 2×10^5 , following the criterion proposed by the GUM (Eq. 5.7). It has to be noted that only instrumentation uncertainties are considered in the present analysis and other uncertainty sources like environmental effects and probe positioning are not taken into account.

Total temperature and total pressure uncertainty The total temperature and total pressure of the jet are measured by a K-type thermocouple and a Validyne transducer respectively. The calibration curves of both instruments have been obtained by a linear regression of the calibration data and are given in Fig. 5.4. The uncertainty analysis for the total temperature and total pressure measurements is similar. The uncertainty computation of the thermocouple calibration is presented in detail as an example.

The thermocouple calibration curve is given by the following equation:

$$T = m \cdot V + c \quad (5.8)$$

where T is the temperature, V the voltage and m , c the slope and intercept respectively. The temperature T of the calibration points is measured by a PT100 thermometer, whose accuracy is noted by the manufacturer to be $\pm 0.1^\circ\text{C}$. As no more information is provided, the error is assumed to follow a uniform (rectangular) distribution and the related systematic standard uncertainty is computed

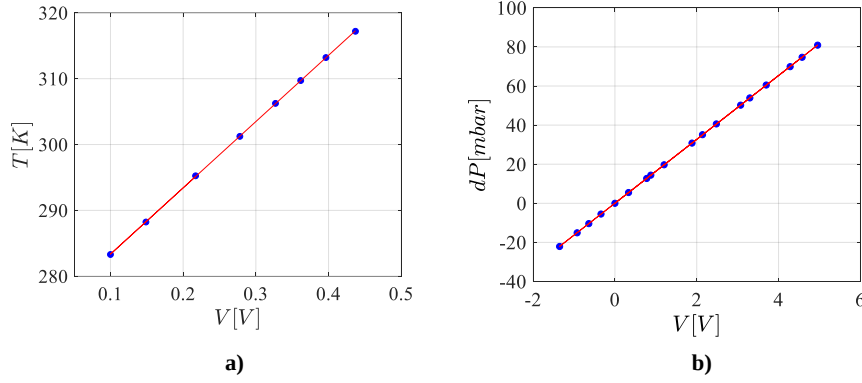


Figure 5.4 – a) Thermocouple, and b) Validyne pressure transducer calibration curve.

as:

$$b_T = \frac{0.1}{\sqrt{3}} = 0.058 \quad (5.9)$$

The random uncertainty of the temperature measurement is not taken into account, due to lack of sufficient successive measurements ($s_T = 0$). The systematic standard uncertainty of the voltage measurement is again estimated considering the manufacturer's accuracy and a uniform distribution, resulting in $b_V = \pm 0.0128 \text{ V}^1$. The systematic standard uncertainty b_V is present also during the measurements, since the same instrument is used in both cases. The random standard uncertainty s_V is computed using Eq. 5.3.

When the thermocouple is used during the experiment, Eq. 5.8 takes the form:

$$T_0 = m \cdot V_{meas} + c \quad (5.10)$$

where T_0 is the measured total temperature value and V_{meas} is the voltage measurement. To calculate the uncertainty in T_0 , both the uncertainty of the calibration and the uncertainty of the measured voltage V_{meas} should be considered. For the calibration uncertainty (linear regression uncertainty), a simplified approach is often used, where the standard error of regression (SEE) can be computed as:

$$SEE = \left[\sum_{i=1}^N \frac{(Y_i - mX_i - c)^2}{N - 2} \right]^{1/2} \quad (5.11)$$

and the total temperature curve fit uncertainty is estimated as:

$$s_{T_0} = \frac{SEE}{\sqrt{N}}. \quad (5.12)$$

¹This includes the gain error, offset error and digital resolution error provided by the manufacturer of the data acquisition device.

In this case, s_{T_0} expresses the curve fit uncertainty of T_0 (retrieved by the calibration curve) for a given value V , considering no uncertainty in the V values. This uncertainty can be added to the systematic component in order to compute the total uncertainty in T_0 .

A more comprehensive approach can be used when all random and systematic uncertainties should be considered. The calibration uncertainty consists of the uncertainty of the calibration points $(V, T)_i$ and of the uncertainty due to the regression. Correlated uncertainties between the calibration points (since they are all obtained by the same instrument) and between calibration and experiment should also be considered. In this case, V_{meas} is obtained with the same acquisition device as the calibration points, therefore their uncertainties are correlated. Using the procedure proposed by Coleman & Steele [18], the standard uncertainty of T_0 for each measurement point is given by:

$$\begin{aligned}
 u_{T_0} = & \left[\sum_{j=1}^N \left(\frac{\partial T_0}{\partial T_j} \right)^2 (s_{T_j}^2 + b_{T_j}^2) + \sum_{j=1}^N \left(\frac{\partial T_0}{\partial V_j} \right)^2 (s_{V_j}^2 + b_{V_j}^2) \right. \\
 & + 2 \sum_{j=1}^{N-1} \sum_{k=j+1}^N \frac{\partial T_0}{\partial T_j} \frac{\partial T_0}{\partial T_k} b_{T_j T_k} + 2 \sum_{j=1}^{N-1} \sum_{k=j+1}^N \frac{\partial T_0}{\partial V_j} \frac{\partial T_0}{\partial V_k} b_{V_j V_k} \\
 & + \left(\frac{\partial T_0}{\partial V_{meas}} \right)^2 (b_{V_{meas}}^2 + s_{V_{meas}}^2) \\
 & \left. + 2 \sum_{j=1}^N \left(\frac{\partial T_0}{\partial V_{meas}} \right) \left(\frac{\partial T_0}{\partial V_j} b_{V_{meas} V_j} \right) \right]^{1/2} \quad (5.13)
 \end{aligned}$$

The first two terms account for the measurement uncertainty of the calibration data (N pairs of (V_j, T_j)). The third and fourth terms account for the correlated uncertainties between calibration points, since all points were acquired using the same instruments. The fifth term expresses the uncertainty due to the voltage measurement during the experimental campaign and, finally, the last term expresses the correlated uncertainty between the voltage of the calibration data and the experimental measurement. The derivatives $\frac{\partial T_0}{\partial T_j}$, $\frac{\partial T_0}{\partial V_j}$ are computed numerically.

For the MCM, the procedure for one measurement point is illustrated in Fig. 5.5. The input variables are the N calibration point pairs (V_j, T_j) and the measured voltage V_{meas} . The input uncertainty sources are randomly sampled for each iteration and the input variables for this iteration are defined, as illustrated in Fig. 5.1. It is reminded that correlated uncertainties are automatically considered. The calibration is included in the measurement model and a distribution for the slope and intercept of the calibration curve is retrieved. The output is a distribution for each T_0 value comprised of M samples. Then, the uncertainty for the desired confidence level can be computed numerically. The distribution of T_0 for one measurement point is presented in Fig. 5.6, with the 95% confidence interval.

The distribution is Gaussian, which validates the assumption used in the TSM.

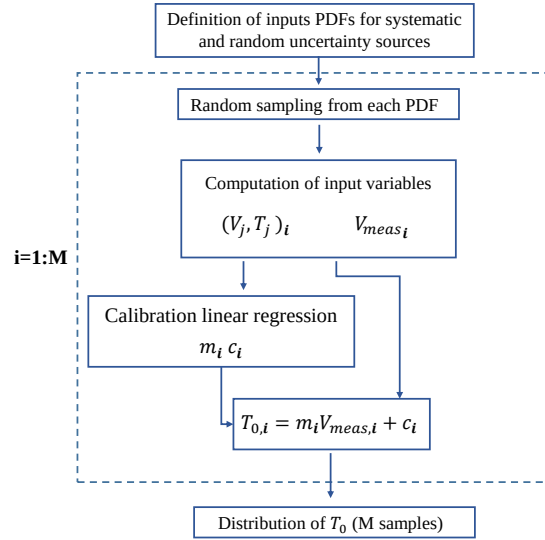


Figure 5.5 – MCM procedure for the computation of the uncertainty in the total temperature measurements.

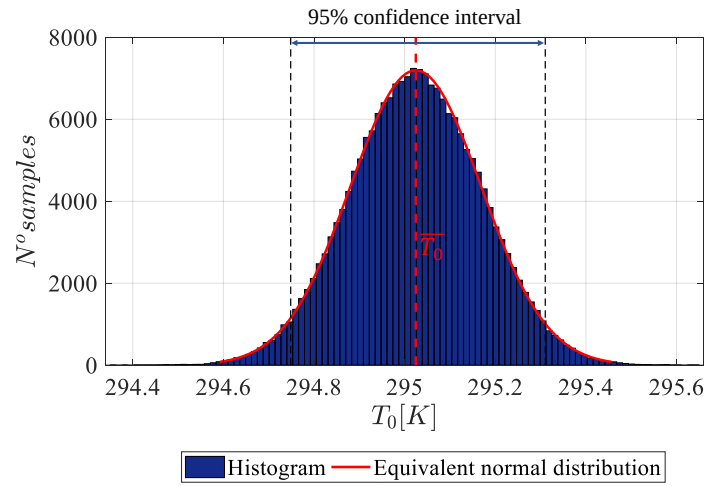


Figure 5.6 – T_0 distribution for one measurement point.

The expanded uncertainties at the 95% confidence level computed both by the comprehensive TSM approach (Eq. 5.13) and the MCM are presented in Fig. 5.7.

The two methods give similar results, as was expected, since the TSM assumptions are valid for linear models. The TSM seems to overestimate the uncertainty by approximately 0.01°C , corresponding to $\sim 3\%$ of the total uncertainty. The uncertainty calculated by the TSM is between $\pm 0.29 - 0.3^\circ\text{C}$ and the uncertainty calculated by the MCM is between $\pm 0.28 - 0.29^\circ\text{C}$ for all measurement points.

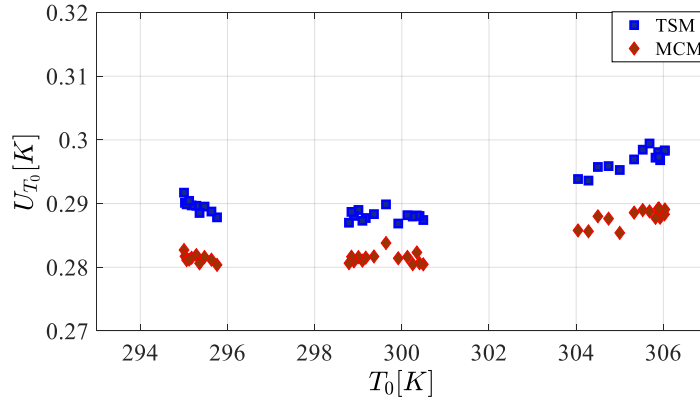


Figure 5.7 – 95% uncertainty of T_0 for all measurement points computed by the TSM and the MCM.

The total pressure for each measurement point is calculated by applying the Validyne calibration (Fig. 5.4b) to the measured voltage and adding the ambient pressure P_{amb} :

$$P_0 = dP + P_{amb} = (m_{val}(V - V_0) + c_{val}) + P_{amb} \quad (5.14)$$

The uncertainty of dP is computed following the procedure described for the thermocouple. The pressure dP of the calibration points is measured by a Druck 30 psi (2 bar) pressure calibrator, whose accuracy is provided by the manufacture as $\pm 0.025\%$ of the full scale, i.e. ± 0.5 mbar. A uniform distribution is considered as no more information is provided, resulting in a systematic standard uncertainty b_{dP} of 0.29 mbar. No random uncertainty is considered for the pressure calibrator. The voltage of the calibration is the output of a Validyne transducer with a 65 mbar membrane. According to manufacturer's specifications, the accuracy is $\pm 0.05\%$ of the full scale. Assuming a uniform distribution, the systematic standard uncertainty in the voltage measurement $b_{V,dP}$ is 0.012 V. The random standard uncertainty $s_{V,dP}$ is computed by Eq. 5.3 for each point. The uncertainty on the measured dP , u_{dP} , is computed by Eq. 5.13. The ambient pressure P_{amb} is measured with a systematic standard uncertainty b_{amb} of 0.24 mbar. The standard

uncertainty on P_0 is computed as:

$$u_{P_0} = \sqrt{u_{dP}^2 + b_{P_{amb}}^2} \quad (5.15)$$

The expanded uncertainties at the 95% confidence level computed both by the TSM and the MCM are presented in Fig. 5.8. The two methods give again similar results. The TSM seems to underestimate the uncertainty by approximately 0.1 mbar (12.5%-9% underestimation).

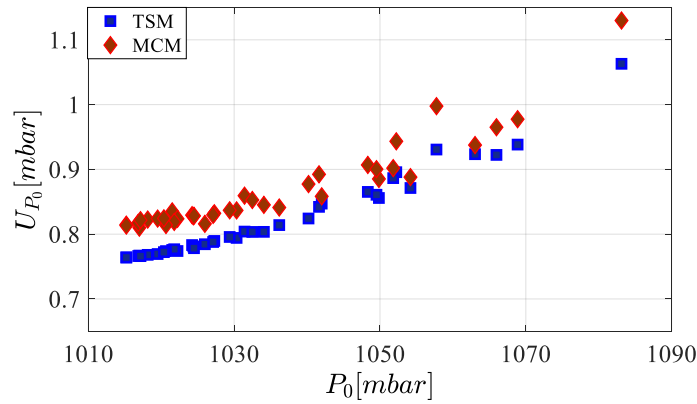


Figure 5.8 – 95% uncertainty of P_0 for all measurement points computed by the TSM and the MCM.

Wire temperature As described in section 2.2, the first step of the calibration methodology is to define the effective temperature of the wire as the temperature that collapses the $Nu - Re$ calibration data on a single curve, with the best fitting (highest R^2). The value of T_{wire} is a constant during calibration and measurement, and its contribution to the uncertainty of the results is included in the calibration curve uncertainty. The T_{wire} value has, in fact, been selected in order to minimize the standard error of regression ((Eq. 5.11)).

Velocity measurement uncertainty The data reduction procedure to compute the instantaneous velocity U' is presented in Fig. 5.9. The instantaneous velocity is retrieved by the Reynolds number. The Nusselt and Reynolds numbers are computed by Eq. 2.7 and 2.19 respectively. The elemental uncertainty sources considered in this study are the uncertainties due to the measurement of the total temperature (T_0) and the total pressure (P_0) presented in the previous section, the ambient pressure (P_{amb}) and the output voltage of the HWA bridge. For the HW

voltage, the random uncertainty is estimated by Eq. 5.3, for measurements in no-flow conditions. No systematic standard uncertainty is considered due to lack of information from the manufacturer. These elemental uncertainties are propagated in the computation of Nu , Re' and eventually U' .

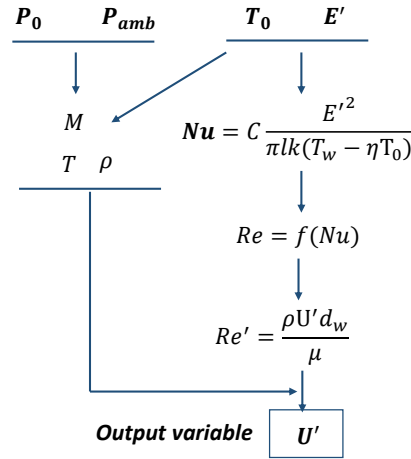


Figure 5.9 – Data reduction algorithm.

The uncertainty is computed following the procedure outlined for the thermocouple in section 5.2.2.2. The uncertainty of the output Reynolds number is due to the uncertainty of the calibration data (N pairs of (Nu_j, Re_j)), the uncertainty due to the measurement of Nusselt Nu_{exp} during the experiment, and the uncertainty due to the 4th order polynomial regression. An equation similar to Eq. 5.13 is used, where the derivatives expressing the sensitivities are computed numerically. The calibration polynomial with the 95% confidence interval, computed by the TSM and the MCM methods, is presented in Fig. 5.10. The 95% uncertainty values computed by both methods are also given in Fig. 5.10. It can be seen that the calibration uncertainty is higher in the lower velocity range. The MCM method predicts a higher uncertainty in the lower calibration range, up to the point where Nusselt number is equal to 9 (this point corresponds to a calibration velocity of about 60 m/s).

The velocity is computed from the Reynolds number obtained by the calibration curve. The 95% expanded uncertainty computed by both methods is presented in Fig. 5.11. The TSM underestimates the uncertainty at the lower velocity range, while for higher velocities (from 60 m/s) the results demonstrate a good agreement. The velocity uncertainty trends for both methods reflect the behaviour of their respective calibration uncertainty. It seems therefore that any difference in the uncertainty estimates of the mean velocity originate from the difference in

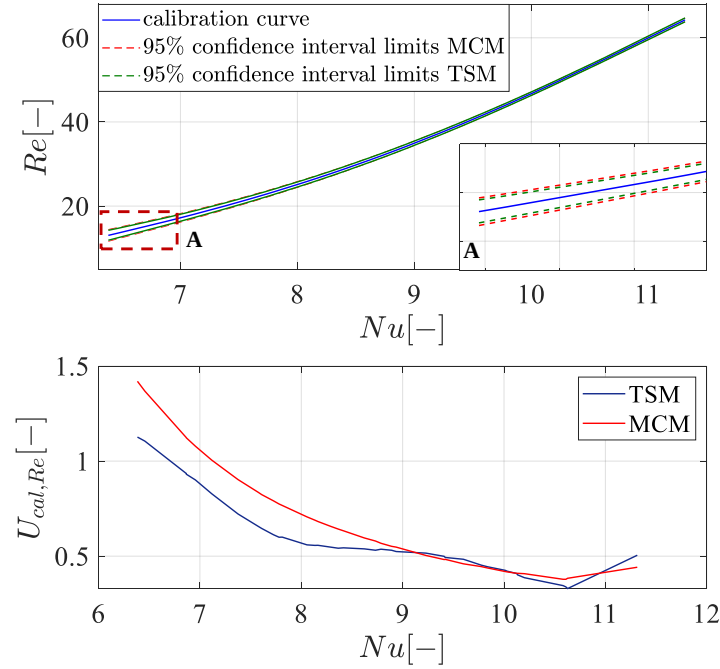


Figure 5.10 – Hot wire calibration curve with 95% confidence interval computed by the TSM and the MCM methods (top), 95% calibration curve fit uncertainty computed by the TSM and the MCM (bottom).

the uncertainty of the calibration. Anyway, the differences are rather small, with 0.5 m/s difference for the lowest velocity.

The probability distributions of two different operating points, Point A and Point B, are presented in Fig. 5.12, as computed by the MCM. Point B is at the high velocity range ($\bar{U} = 106.5$ m/s), where there is a good agreement between the uncertainty results computed by the two methods and its distribution is almost normal. For the low velocity Point A though ($\bar{U} = 23$ m/s), the PDF exhibits a higher deviation from the normal distribution (skewness of -0.11). This implies that the assumptions of the TSM are no more valid in this case, and the method fails to predict correctly the confidence intervals. This might also contribute to the disagreement of the two methods in the lower velocity range.

When instantaneous voltage measurements are used for the computation of the Nusselt number (Eq. 2.7), the instantaneous Reynolds number is acquired from the calibration and the instantaneous velocity is computed by:

$$U' = \frac{Re' \mu}{\rho d_w} \quad (5.16)$$

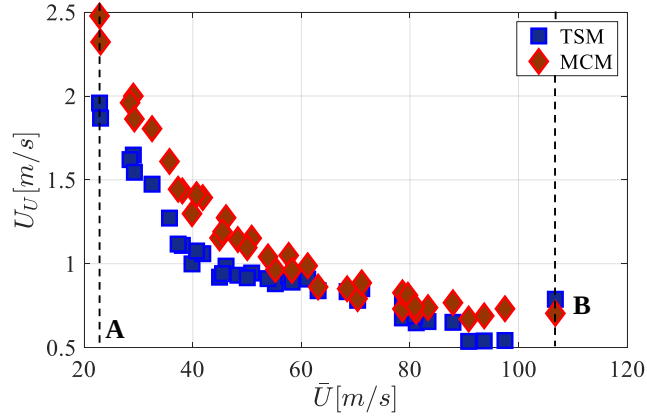


Figure 5.11 – 95% uncertainty of the mean velocity computed by the TSM and the MCM methods.

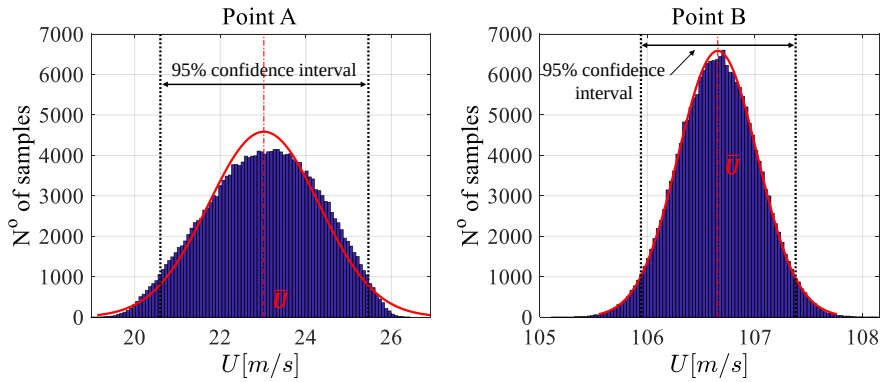


Figure 5.12 – Probability distributions of mean velocity for measurement points A and B.

The uncertainty of the instantaneous velocity is the same as the uncertainty of the mean velocity when considering only the uncertainty sources present in the current study and ignoring effects related to high turbulence intensity, probe geometry etc.

5.2.3 Turbulence intensity uncertainty with HWA

The difference in the velocity uncertainty computed by the TSM and MCM implies that the same difference will appear in the turbulence intensity uncertainty. For this reason, only the uncertainty computed by the MCM is presented, since it is considered more reliable.

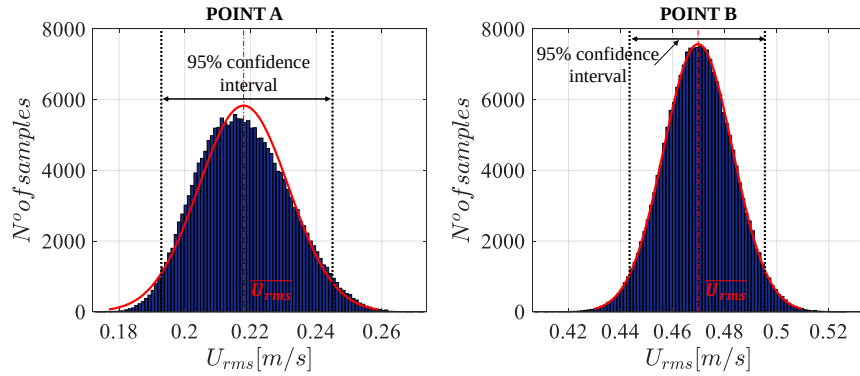


Figure 5.13 – Probability distribution of velocity rms for Points A and B

In Fig. 5.12, 5.13 and 5.14 the probability distributions for the mean velocity, the velocity rms and the turbulence intensity for the operating points A and B are presented respectively. For the Point A, the distributions for all three quantities deviate from the normal distribution. The highest deviation is observed for the turbulence intensity PDF, which has a right skewness of 0.4. This implies that the assumption of a normal distribution employed by the TSM is not valid, which makes the method unreliable in this case. For Point B, on the other hand, all three distributions are very close to normal.

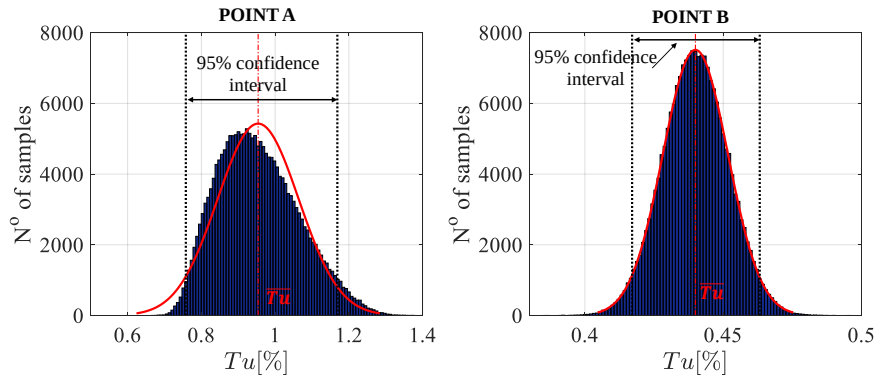


Figure 5.14 – Probability distribution of turbulence intensity for Points A and B

It should be noted that although the absolute uncertainty is high for the mean velocity (4.8 m/s for Point A and 1.5 m/s for Point B), the uncertainty of the U_{rms}

is significantly lower (0.054 m/s and 0.045 m/s for Point A and B respectively). This can be explained by the definition of this quantity:

$$U_{rms} = \sqrt{\overline{u^2}} \quad (5.17)$$

where $u = U' - \bar{U}$ is the fluctuating velocity component. Due to the subtraction of the mean velocity from the instantaneous velocity, the systematic uncertainty is cancelled out and only the random uncertainty affects the output.

The velocity rms and the turbulence intensity values as a function of the mean flow velocity are presented in Fig. 5.15 and 5.16, with 95% uncertainty bounds. It can be seen that the absolute uncertainty decreases with increasing mean velocity for both quantities.

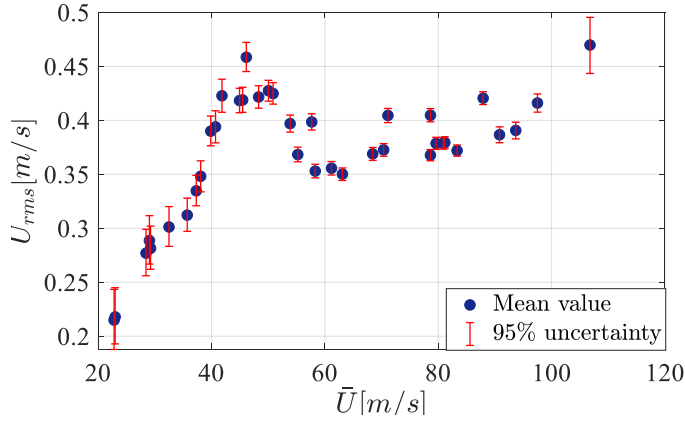


Figure 5.15 – Velocity rms and 95% uncertainty bounds as a function of the mean velocity.

5.2.4 Convergence of output quantities

As mentioned in Section 5.2.1.2, the MCM iterations were selected a priori equal to $2 \cdot 10^5$, following the guidelines of the GUM supplement [8]. The number of iterations is adequate, when the output quantities of interest have converged to the final value. In this case, the quantities of interest are the 95% confidence interval limits of the turbulence intensity estimate. In Fig. 5.17, the evolution for the upper and lower interval limits for Point B are presented, with increasing number of samples. It can be seen that the values converge for the selected number of samples.

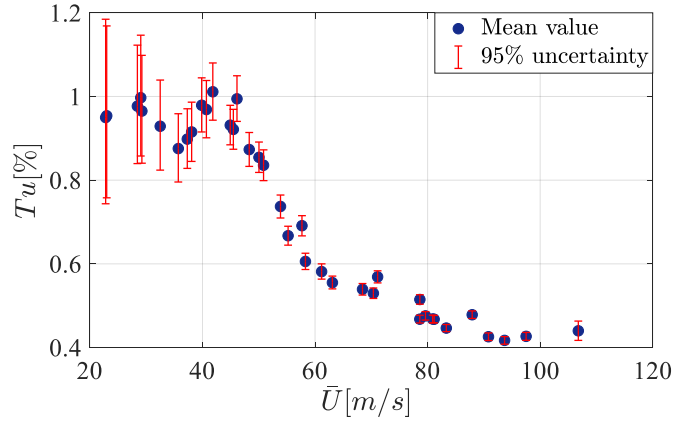


Figure 5.16 – Turbulence intensity and 95% uncertainty bounds as a function of the mean velocity

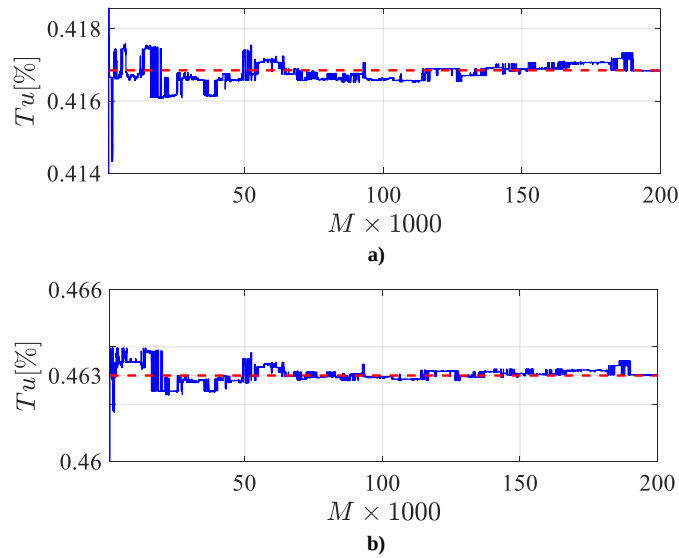


Figure 5.17 – Convergence of 95% confidence interval limits for the Tu , Point B: a) Lower limit, b) Upper limit.

5.3 Statistical uncertainty

The statistical uncertainty refers to the level of statistical convergence achieved when computing averaged quantities over a finite signal duration. In the case of turbulence, statistics up to the third order are commonly computed. Benedict &

Gould [2] report expressions for error estimates of various statistics for both normal and arbitrary distributions. These expressions are given in Table 5.1, where N refers to the number of independent samples in the signal. $\bar{U}$, $\sqrt{\overline{u^2}}$, $\overline{uv}$, $\overline{u^2}$, $\overline{u^3}$ and $\overline{u^4}$ are the mean velocity, the velocity rms, the Reynolds stress and the second, third and fourth central moments respectively.

Statistic	Valid for any distribution	Valid for normal distribution
$\bar{U}$	$\overline{u^2}/N$	$\overline{u^2}/N$
$\sqrt{\overline{u^2}}$	$[\overline{u^4} - (\overline{u^2})^2]/4N\overline{u^2}$	$\overline{u^2}/2N$
$\overline{uv}$	$[u^2v^2 - (\overline{uv})^2]/N$	$(1 + R_{uv}^2)(\overline{u^2})(\overline{v^2})/N$
$\overline{u^2}$	$[\overline{u^4} - (\overline{u^2})^2]/N$	$2(\overline{u^2})^2/N$
$\overline{u^3}$	$[\overline{u^6} - (\overline{u^3})^2 - 6(\overline{u^4})(\overline{u^2}) + 9(\overline{u^2})^3]/N$	$6(\overline{u^2})^3/N$
$\overline{u^4}$	$[\overline{u^8} - (\overline{u^4})^2 - 8(\overline{u^5})(\overline{u^3}) + 16(\overline{u^3})^2\overline{u^2}]/N$	$96(\overline{u^2})^4/N$

Table 5.1 – Estimator variances by Benedict & Gould [2].

It is clear from the expressions in Table 5.1 that the estimates depend on the number of samples N . If the signal is correlated and the complete number of samples N is used, the uncertainty will be underestimated. The estimation of the sampling rate to provide independent samples requires an estimation of the integral time scale of the signal. The optimal sampling rate from the uncertainty minimization point of view should be defined a priori (which might be too low for a spectral analysis of the signal) or the N number in the uncertainty computation should be properly adjusted to the number of independent samples.

When dealing with non-normal distributions, the expressions for the error estimates of turbulence statistics can become complicated (as seen in Table 5.1). Moreover, statistics for which analytical expressions of the error estimates are not available might be of interest (e.g. length scales, dissipation of the turbulence kinetic energy). In this case, resampling algorithms can be used, which allow the random uncertainty estimation of any quantity, at the expense of increased computational time. Benedict & Gould [2] applied two algorithms, the jackknife and the bootstrap, to Laser Doppler Anemometry (LDA) data. Comparison with analytical expressions showed a good agreement and verified that for non-normal velocity distributions the expressions assuming normality underestimate the uncertainty. The jackknife and the bootstrap algorithm can only be applied to time series with independent samples. In the present work, the Moving Block Bootstrap method (MBB) has been proposed, which extracts overlapping blocks of data from the original time series and can be applied to correlated time series. The method will be described in detail in the following section.

5.3.1 Moving Block Bootstrap method

The bootstrap method is a resampling algorithm that allows computing the statistical inference of a group of samples when its size is statistically too small (i.e. non-representative) or when the probability distribution is unknown. The method, developed originally by Efron [19] in an econometric framework, consists in the generation of B sample series that are constructed by randomly resampling the original data series x . The statistic of interest θ_i is then computed for each sample series, leading therefore to the generation of a population θ_B for which a mean value μ_B and a variance σ_B can then be computed.

Although the bootstrap technique provides a good estimation of the sampling distribution [20], its direct application to turbulence measurements would be inconsistent since, by randomly resampling the original data series, the intrinsic correlation of turbulent phenomena is automatically degraded. In order to overcome this issue, the moving block bootstrap (MBB) methodology by Kunsch [21] is adopted. The procedure firstly splits the original data series x , consisting of N samples, into $N - c + 1$ overlapping blocks. N/c blocks will then be extracted and randomly concatenated, generating therefore a bootstrapped data series $x_{b,i}^*$ of roughly the same size of the original one. The process is then repeated B times in order to create a population of the statistic parameter of interest. A principle sketch of the moving block bootstrap method is shown in Figure 5.18. The MBB method results in a larger number of blocks compared to other block bootstrap methods that do not allow overlapping, and is therefore preferred for time signals with small/average duration.

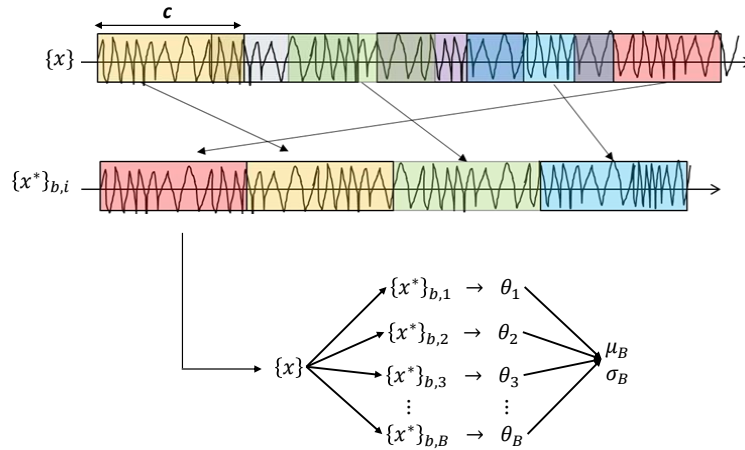


Figure 5.18 – Moving block bootstrap principle.

Politis & White [22] suggested an automatic procedure for the selection of the

block length c , based on the autocorrelation function of the signal $R(\tau)$. $R(\tau)$ represents the degree of correlation between the fluctuating velocity components u at two instants separated by a time interval τ :

$$R(\tau) = \overline{u(t)u(t+\tau)} \quad (5.18)$$

The autocorrelation coefficient $\rho(\tau)$ can also be defined as:

$$\rho(\tau) = \frac{\overline{u(t)u(t+\tau)}}{\overline{u^2(t)}} \quad (5.19)$$

The parameter m is defined as the smallest integer for which $\rho(k\Delta t)$ becomes negligible. Then, for $M = 2m$, the optimal block length is computed as follows:

$$c_{opt} = \left(\frac{2G^2}{D} \right)^{1/3} N^{1/3} \quad (5.20)$$

where

$$D = \frac{4}{3} g^2(0) \quad (5.21)$$

$$g(0) = \sum_{k=-M}^M \lambda(k/M) R(k\Delta t) \quad (5.22)$$

$$G = \sum_{k=-M}^M \lambda(k/M) |k| R(k\Delta t) \quad (5.23)$$

and

$$\lambda(k/M) = \begin{cases} 1, & 0 \leq |k/M| \leq 0.5 \\ 2(1 - |k/M|), & 0.5 \leq |k/M| \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad (5.24)$$

It should be noted however that the application of bootstrap techniques to correlated time series is not as straightforward as dealing with randomly samples. Due to the correlation in the original dataset, the MBB produces estimates with a large bias error for the variance and higher order moments [23]. The bias of the variance estimator is of the order of $O(c^{-1})$: it therefore decreases for increasing block length (the optimal block length increases with increasing sample size). Improved methods have been proposed, like the tapered block bootstrap method, which leads to variance estimates with a bias of the order of $O(c^{-2})$, decreased by an order of magnitude [24]. In this work, the original MBB has been applied, since it has been used for the uncertainty estimation of turbulence data before with good results. Garcia et al. [3] validated the MBB method both analytically and experimentally. They first used synthetic turbulence signals with a wide range of conditions to validate the error estimate for the sample mean and variance against

analytical expressions. Then, they used 80 repeatable tests and compared the variance of different parameters with that computed by applying the MBB on one test. They reported a good agreement on the computed uncertainty of the turbulence parameters and they concluded that the MBB method is capable of quantifying the statistical errors that produce scatter in repeated measurements. Based on this study, we consider the statistical uncertainty computed by the MBB method to take into account also random errors that produce scatter from sample-to-sample, e.g. noise in the HW voltage.

5.3.2 Statistical uncertainty computation example

In this section, an example for the statistical uncertainty computation of turbulence parameters is presented. Two experimental velocity signals with different characteristics are employed to demonstrate the procedure.

5.3.2.1 Experimental signals

The velocity signals have been acquired by CTHWA in the VKI S-1 high-speed wind tunnel. It is a closed circuit facility which accommodates a linear turbine cascade test section. A schematic of the test section is presented in Fig. 5.19. A turbulence generating grid is placed upstream of the cascade to raise the inlet turbulence level, while two measurement planes allow the traversing of probes at the inlet and outlet of the cascade. The HW probe has been traversed downstream, at Plane 2, covering one blade pitch s . The two measurement points considered in this study are depicted on the mean velocity profile, in Fig. 5.20: point 1 at the wake center and point 2 at the wake vicinity. The signals have been acquired at a sampling frequency f_s of 330 kHz for a total duration of 10 s.

In order to obtain a first estimation of the degree of time correlation in the signal, the integral time scale is calculated as the integral of the autocorrelation coefficient:

$$T_{int} = \int_0^{\infty} \rho(\tau) d\tau \quad (5.25)$$

Since experimentally obtained autocorrelation coefficients often feature wide oscillations around zero, the integration for the complete signal duration can produce erroneous results. Therefore, different options have been proposed for the upper integration limit of Eq. (5.25): the time lag for which the amplitude of the function decreases to $e-1$; or the time lag corresponding to the first root (see for example [25]). In this case, the autocorrelation function is fitted with a Gaussian fit up to the first zero crossing. The fitted function is then integrated, up to the time instant corresponding to the first zero crossing. This procedure is illustrated in Fig. 5.21. Using this approach, the integral time scale for Points 1 and 2 can be computed. They are reported in Table 5.2 along with the mean velocity $\bar{U}$, the

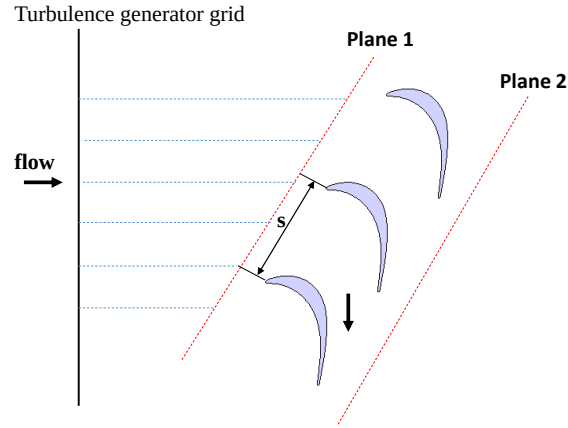


Figure 5.19 – Schematic of the test section.

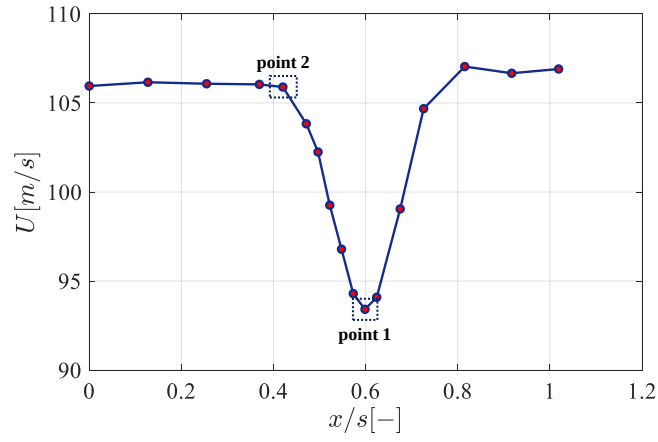


Figure 5.20 – Mean velocity profile.

velocity variance $\overline{u'^2}$ and the skewness S_u . The number of independent samples N_{eff} is also stated, computed as:

$$N_{eff} = \frac{T}{2T_{int}} \quad (5.26)$$

where T is the total signal duration. The velocity distributions for the two points

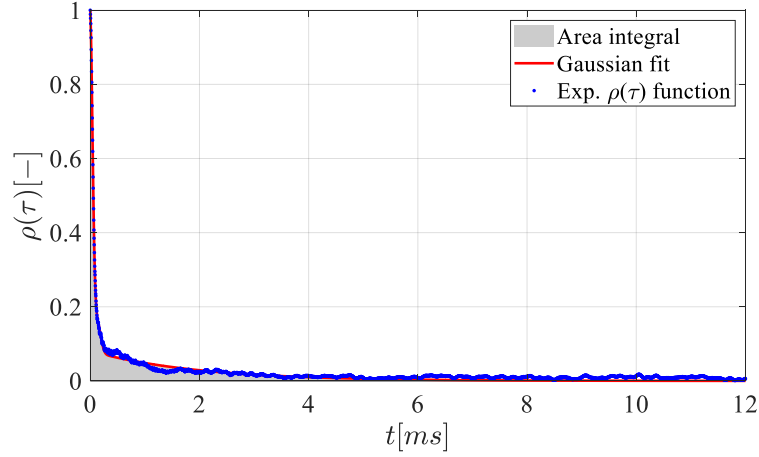


Figure 5.21 – Autocorrelation coefficient for point 1.

are depicted in Fig. 5.22. It can be seen that the two points selected feature different time scales (degrees of correlation), resulting in the number of independent samples for point 1 being higher by almost a factor of 2.

	Point 1	Point 2
$\bar{U}[\text{m/s}]$	93.4	105.9
$\overline{u'^2}[\text{m}^2/\text{s}^2]$	9.34	5.92
S_u	9.34	0
$T_{\text{int}}[ms]$	0.217	0.404
N_{eff}	23085	12364

Table 5.2 – Signal characteristics for points 1 and 2.

Using the Taylor's frozen vortex hypothesis [26], the integral length scale L_{int} can be computed multiplying the integral time scale by a convective velocity, usually considered equal to the local mean velocity. An alternative method has been proposed by Roach [27] where the integral scale is computed from the power spectrum:

$$L_{int} = \left[\frac{E(f)\bar{U}}{4\overline{u'^2}} \right]_{f \rightarrow 0} \quad (5.27)$$

This method will be followed here to avoid the ambiguity on the computation of the integral time scale, as explained before.

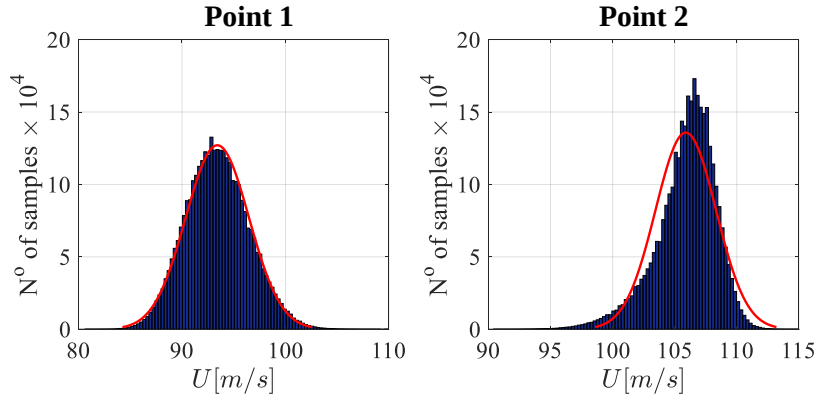


Figure 5.22 – Histograms of velocity distribution for points 1 and 2.

5.3.2.2 Uncertainty analysis

For the application of the MBB method, the optimal block length for each signal is computed using the method of Politis & White [22] described in Section 5.3.1. In regards to the optimal number of bootstrap series B , Efron & Tibshirani [20] stated that 200 should be enough to estimate an error variance in most cases. Garcia et al. [3] argued that 1000 iterations are instead needed for the estimation of confidence intervals. In this case, the bootstrap series number is set to 5000. The resulting bootstrapped distributions for all turbulent quantities of interest are given in Fig. 5.23 and 5.24 for points 1 and 2 respectively. The mean values and 95% confidence intervals are indicated on each plot. It can be seen that the bootstrapped quantities follow a normal distribution, also for the case of point 2, for which the original velocity distribution features a high degree of skewness.

The effect of the signal duration on the uncertainty of the computed parameters is investigated by truncating the signal at different instants from 0.05 s to 10 s. The MBB method is then applied to the truncated signal. The optimal block length is recomputed for every iteration, as it is a function of the signal duration. The series number B is set to 5000. The evolution of the mean value and the 95% confidence interval as a function of the time duration are presented in Fig. 5.25 - 5.28 for the mean velocity, the variance, the skewness and the integral length scale respectively. It can be seen that increasing the signal duration results in a decrease of the confidence interval (uncertainty) computed by the MBB method. Moreover, it can be seen that the uncertainty is in all cases lower for Point 1, due to the lower integral time scale, resulting in a higher number of independent samples for the same signal duration.

In Fig. 5.29 and 5.30 the uncertainty at standard deviation level computed by

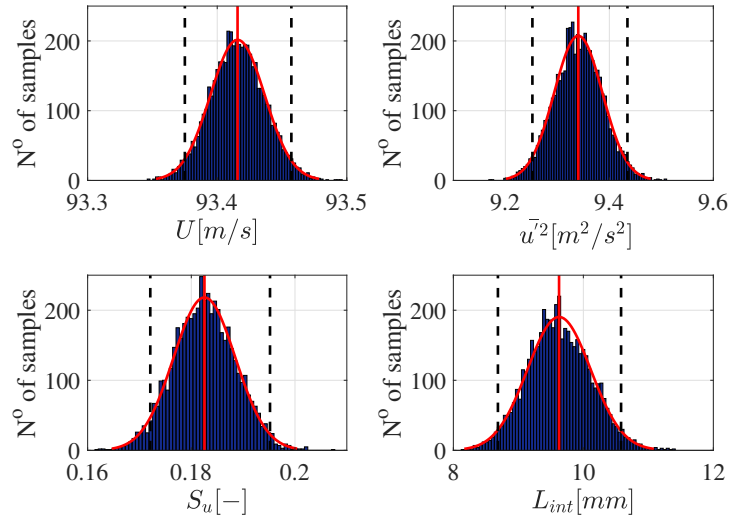


Figure 5.23 – Histograms of bootstrapped parameters point 1: Mean velocity, variance, skewness, integral length scale.

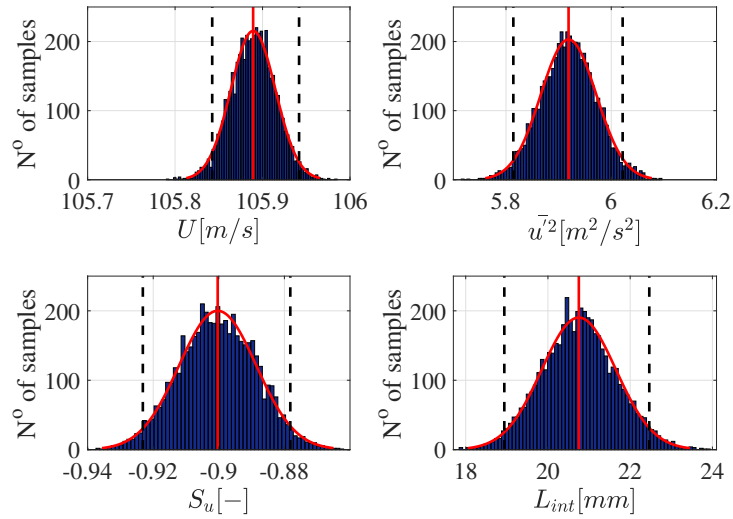


Figure 5.24 – Histograms of bootstrapped parameters point 2: Mean velocity, variance, skewness, integral length scale

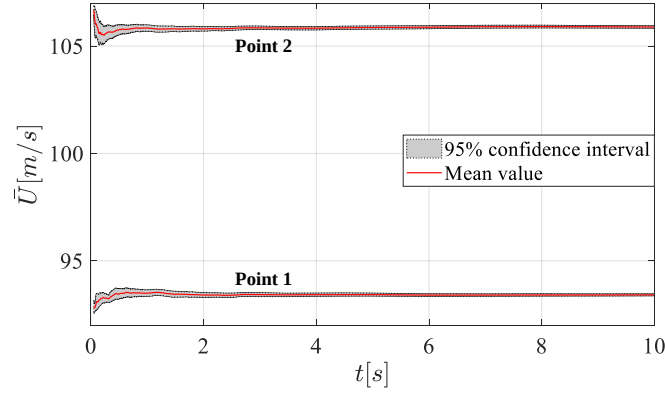


Figure 5.25 – Mean velocity by MBB: Evolution of the mean value and confidence interval with the signal duration.

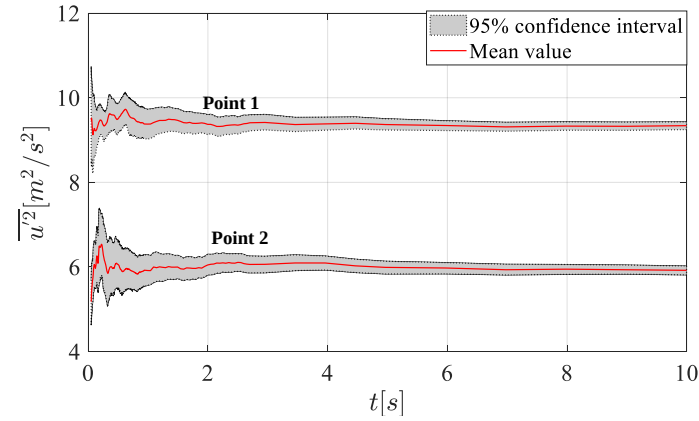


Figure 5.26 – Variance by MBB: Evolution of the mean value and confidence interval with the signal duration.

the MBB and the analytical expressions is compared, for the mean velocity and the variance respectively. The analytical expressions are reported in Table 5.1. For the mean velocity, the same expression stands for both normal and arbitrary distributions. For the variance, both expressions are computed and reported in Fig. 5.30. It can be seen that the uncertainty computed by the analytical expressions follows the same trend as the MBB: decreasing with increasing signal duration. For the mean velocity, the two methods differ by less than 0.01 m/s for $t > 2$ s for both points. For the variance uncertainty, different observations can be made for points 1 and 2. Interestingly, the analytical error estimations for normal and

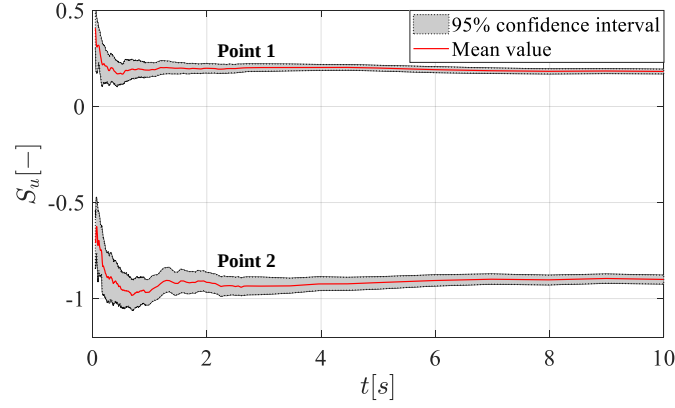


Figure 5.27 – Skewness by MBB: Evolution of the mean value and confidence interval with the signal duration.

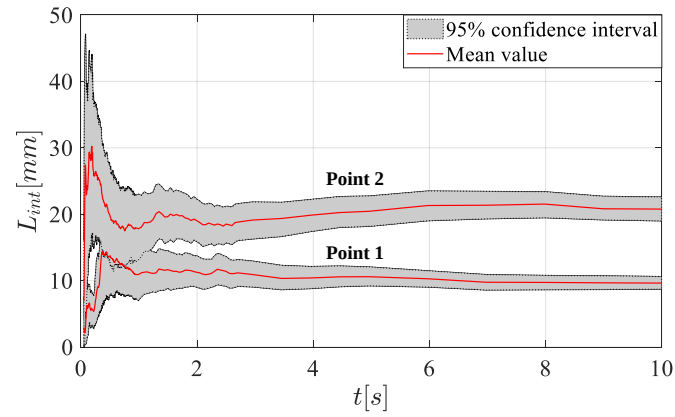


Figure 5.28 – Integral length scale by MBB: Evolution of the mean value and confidence interval with the signal duration.

arbitrary distributions give more similar results for point 2 with respect to point 1, although the velocity distribution of point 2 features a bigger deviation from normality. The MBB method in this case, gives an uncertainty estimation lower by $0.05 \text{ m}^2/\text{s}^2$ for point 2 and $t > 0.3 \text{ s}$ and $0.03 \text{ m}^2/\text{s}^2$ for point 1 and $t > 0.7 \text{ s}$,

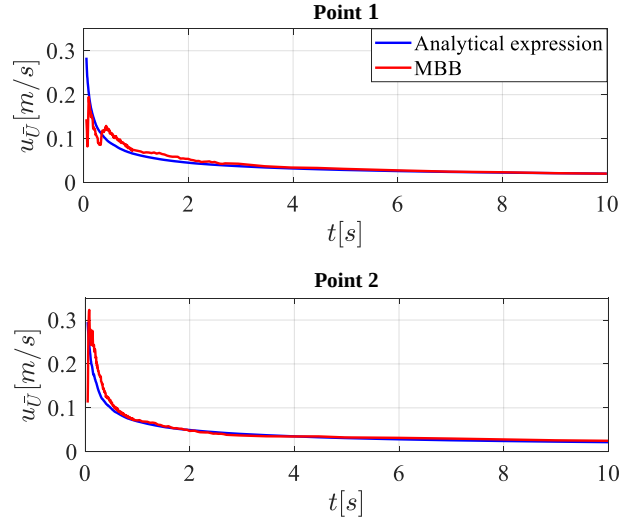


Figure 5.29 – Mean velocity: Standard uncertainty comparison between MBB and analytical expressions.

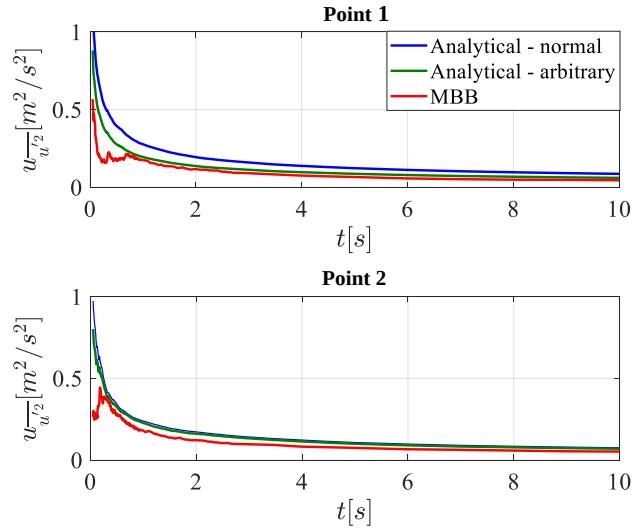


Figure 5.30 – Variance: Standard uncertainty comparison between MBB and analytical expressions.

5.4 Conclusions

In this chapter, the two macro uncertainty sources affecting the computation of turbulence statistics (measurement and statistical uncertainty) are examined and the

methods available for their computation are presented through simple applications.

For the propagation of the measurement uncertainty, the TSM and the MCM methods are considered and compared. The TSM is the baseline method, consisting in the propagation of uncertainty values at a standard deviation level. The MCM can be a useful tool for uncertainty assessment in certain cases. As demonstrated in this work, when complex data reduction procedures are used, the assumptions of the Taylor Series Method may no longer be applicable. Indeed, for certain operating points the turbulence intensity computed by the MCM demonstrates a rightly skewed distribution, which makes the assumption of a normal distribution employed by the TSM no longer valid. Moreover, the application of the MCM is quite straightforward and even simpler than the TSM when correlated uncertainties need to be considered. The main drawback of the method is the computational time, which increases with increasing number of samples (and desired confidence level).

Analytical expressions and the MBB method are presented and compared for the computation of the statistical uncertainty. Two experimental velocity signals acquired over a long period of time (10 s) and at a high sampling rate (330 kHz) are employed in the study, allowing the evaluation of the statistical uncertainty as a function of the signal duration, as well as demonstrating the application of analytical expressions in the case of correlated samples. The statistical uncertainty decreases significantly with the time duration for all parameters examined in this work. Comparison of the MBB method with analytical expressions for the mean velocity and the velocity variance shows a good agreement between the two methods for a time duration longer than approximately 0.5 s, in particular when the expressions for arbitrary distributions are used. Nevertheless, the MBB has the advantage of allowing the uncertainty estimation of any parameter, without the derivation of complex expressions (e.g. the integral length scale).

A straightforward application of the MBB has been performed in this work without considering corrections for the bias errors in the different estimates. Good agreement with analytical expressions has been nevertheless achieved, possibly thanks to the long time series used. Indeed, for short signal durations the MBB is found to underestimate the uncertainty estimates. This issue deserves more attention in future work and comparison with more advanced bootstrap methods like the tapered bootstrap should be considered.

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6

Test case 1: Uncertainty quantification of turbulence statistics

In this chapter, a practical application for the uncertainty quantification of turbulence statistics in a complex case is presented. CTHWA measurements have been performed at the inlet of a high-pressure turbine test rig to compute the turbulence characteristics of the inlet flow. The non-dimensional calibration presented in Chapter 2.2 is used to take into account pressure and temperature variations. Due to the complex data reduction algorithm, the Monte Carlo Method (MCM) is selected for the computation of the measurement uncertainty, as explained in Chapter 5.2. The tests have been performed at the CT3 compression tube facility at the von Karman Institute, a short-duration test rig, with useful testing times of approximately 0.3-0.5 s. In these conditions, the statistical uncertainty is important and is computed with the Moving Block Bootstrap Method (MBB), introduced in Chapter 5.3. Finally, the spanwise profiles of the turbulence intensity and length scales are provided, accompanied by the 95% uncertainty bounds. The contributions of each uncertainty source to the total are also presented. The procedure is described step by step, in order to serve as a guideline for future works.

6.1 Experimental methods and setup

6.1.1 The CT3 Compression Tube facility

The turbine tests were conducted in the rotating turbine rig of the von Karman Institute (VKI). This short duration facility operates on the principle of a compression tube and allows testing a high pressure turbine stage at a larger scale while preserving the actual engine Mach and Reynolds numbers and gas-to-wall temperature ratio [1]. The layout of the facility is presented in Fig. 6.1.

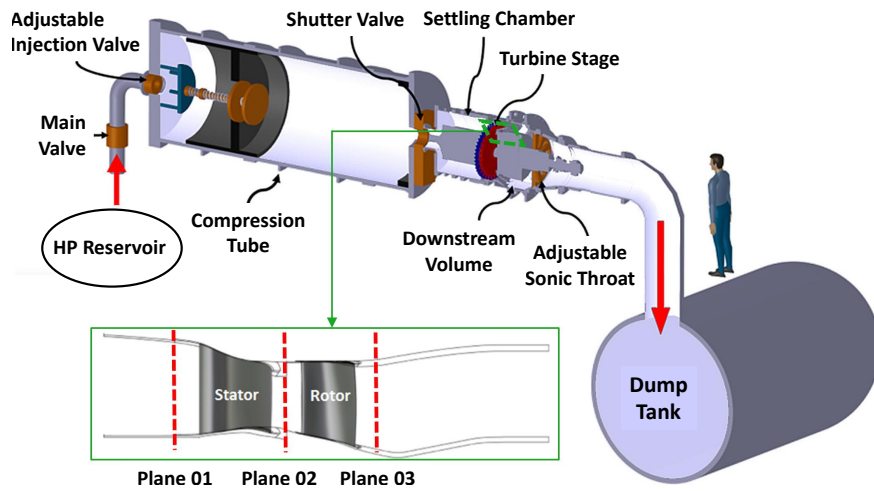


Figure 6.1 – Layout of the facility.

The duration of the test is around 0.3 s. In the present experimental campaign, one operating point has been investigated, whose operating conditions are given in Table 6.1.

Quantity	Value	Units
Turbine inlet Mach number	0.10	—
Inlet total pressure P_{01}	1040	mbar
Inlet total temperature T_{01}	463	K
Wall-to-gas temperature ratio	0.68	—
Non-dimensional rotation speed	281.3	—
Total-to-static turbine pressure ratio	2.21	—
Chord-based Reynolds number at plane 2	6.93×10^5	—

Table 6.1 – Experimental test conditions at mid-span.

6.1.2 The Measurement Plane

The test section in this experimental campaign hosts a single high-pressure stage consisting of a stator with 38 blades and a rotor with 48 blades. The measurements are performed at the inlet of the stage, half a stator axial chord upstream of the stator leading edge. The span at this position is 74.71 mm. The instrumentation used at the measurement plane and their setup are presented in Fig. 6.2. A rake of four Kiel type pressure probes is used to measure the total pressure $P_{0,1}$ along the span and a rake of four thermocouples is used to measure the total temperature $T_{0,1}$. Two reference probes consisting of a thermocouple and a Kiel probe are used to measure $T_{0,1}$ and $P_{0,1}$ at mid-span. All probes are placed at different circumferential positions. The static pressure is measured at the tip and hub endwalls with five static pressure taps distributed over one vane pitch.

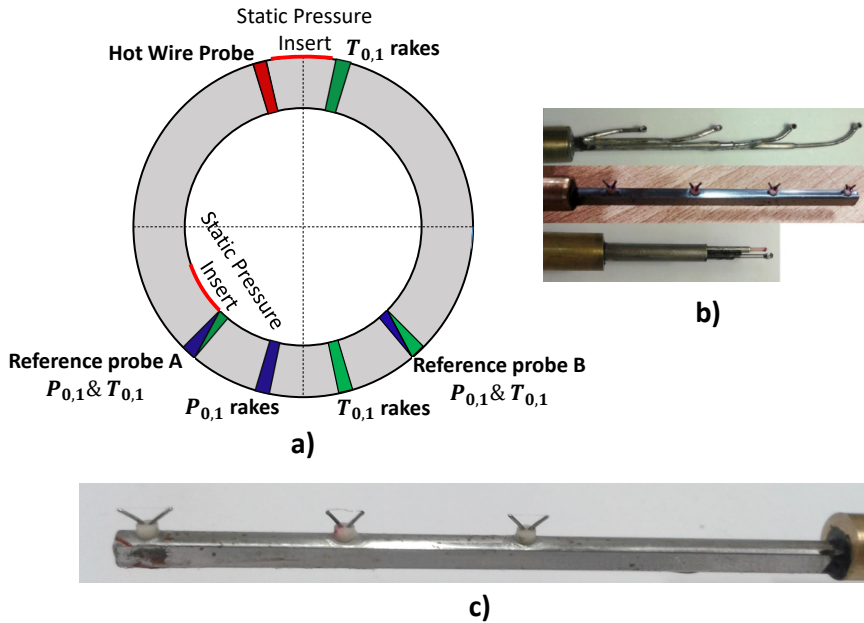


Figure 6.2 – a) Instrumentation and setup at measurement plane, b) total pressure and thermocouple rakes, reference probes, c) HW probe.

Fig. 6.2a shows the employed HW probe as mounted in the test section. It features two HW heads, placed at half span distance from each other. The double-head configuration allows to sweep the channel span (by traversing radially the probe) with a reduced number of tests. The probe is also equipped with a thermocouple in order to measure the total temperature between the HWs. Due to the high inlet total temperature (~ 440 K), the operating temperature of the HWs had to be increased to reduce the sensitivity to the flow temperature. For this reason,

a Platinum and Nickel alloy (90%Pt/10%Ni) was selected as wire material. This allowed using an operating temperature T_w of ~ 540 K and ~ 570 K, for heads 1 and 2 respectively. The Pt-Ni alloy was chosen as it offers high resistance to oxidation compared to conventional Platinum-plated tungsten wires, whose operating temperature limit is ~ 550 K. The wires are of $9\ \mu\text{m}$ diameter and 2 mm long. The actual sensing length of the HW is 1 mm, considering that copper plated stubs, used to eliminate the interference with the prongs, cover the wire along 0.5 mm at each extremity.

6.1.3 Hot-Wire Calibration Method

In the frame of the present investigation, the non-dimensional HW calibration based on Nusselt (Nu) and Reynolds (Re) numbers is used, in order to eliminate the effect of the local flow conditions on the HW output [2, 3]. The methodology uses an effective wire temperature and empirical correlations to establish a unique $Nu - Re$ calibration curve. The procedure has been described in detail in Chapter 2.2.

For the present test conditions ($Mach < 0.3$), the Nusselt number that expresses the convective heat transfer from the wire to the flow can be considered a unique function of the Reynolds number of the flow:

$$Nu = f(Re) \quad (6.1)$$

The Nusselt number is computed as:

$$Nu = \frac{E_b^2 R_w}{\pi l_w k (T_w - \eta T_0) (R_s + R_{top} + R_w)^2} \quad (6.2)$$

where R_{top} and R_s are constant resistances which depend on the circuit, R_w is the wire resistance, E_b is the bridge voltage, l_w the active wire length, T_w the wire temperature and T_0 the total temperature of the flow. The recovery factor r can be assumed to be equal to 1 provided the low Mach number of the present application. The wire Reynolds number is defined as

$$Re = \frac{\rho U d_w}{\mu} \quad (6.3)$$

This calibration methodology eliminates the need for an in-situ calibration. Therefore, a simple heater-equipped open jet facility can be employed for the calibration of the Hot Wires. The calibration is performed at a different temperature and velocity compared to the measurement conditions, but at the same Reynolds number as the turbine experiment. The resulting calibration curve for HW head 2 is presented in Fig. 6.3, with the effective operating wire temperature computed at 572 K.

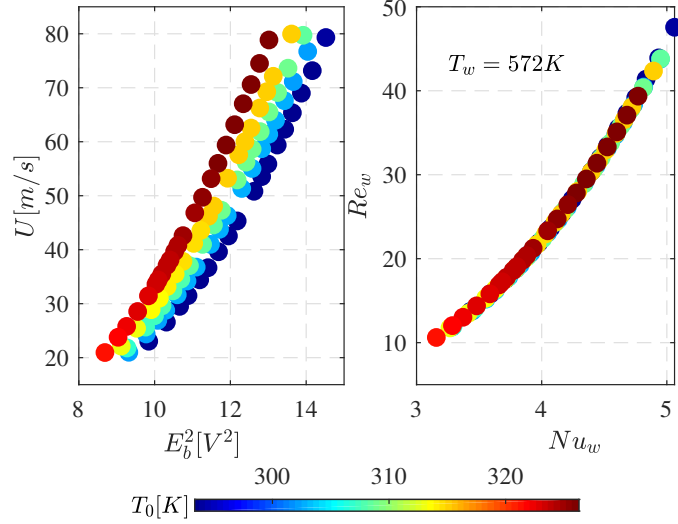


Figure 6.3 – Effect of temperature on the absolute HW calibration (left), non-dimensional HW calibration (right).

6.1.4 Processing Methodology

The first step in the measurement methodology is to detect the interval for which the flow is stabilized and extract the corresponding signals. In Fig. 6.4, the extracted signal parts are indicated for the inlet total temperature $T_{0,1}$, the inlet total pressure $P_{0,1}$, the static pressure at the tip $P_{s,tip}$ and for the HW signal. For this time interval, which lasts approximately 100-150 ms, the $P_{0,1}$ and $P_{s,tip}$ signals are stabilized. The $T_{0,1}$ signal presents some low-frequency fluctuations. The flow temperature variations with time are reflected in the HW raw voltage signal, because of the HW dependency on temperature.

Based on the calibration methodology, the total temperature and the total and static pressure values should be known at the HW location in order to retrieve the instantaneous velocity from the HW. Therefore, the data obtained for all the tests for this operating point are used to build radial profiles of P_0 , P_s and T_0 . The measurements of T_0 and P_0 are normalized by the reference value at mid-span for each test. Considering the uncertainty in the positioning of the probe, the values are averaged over 2% of the span and fitted with a Fourier fit. The measurements of P_s at the endwalls are normalized by the P_0 value at mid-span for each test and the mean values are fitted with a linear curve to estimate the static pressure values along the span. The non-dimensional profiles and the mid-span values of T_0 and P_0 are used to determine the local values at each HW measurement po-

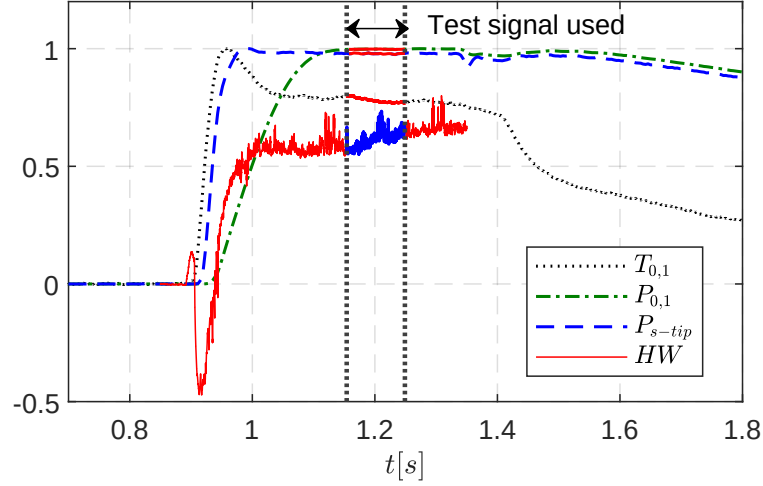


Figure 6.4 – Normalized signals of total pressure, total temperature, static temperature and hot wire voltage during one test, with the signal part used in the post-processing indicated.

sition. A mean value of the pressure measurements is considered for each test. This operation is valid since the flow pressure values remain very stable ($\pm 0.1\%$ of the mean) throughout the entire duration of the turbine run. The fluctuations in total temperature registered during the test duration are taken into account using the time-resolved signal from the thermocouple of the HW probe.

The HW instantaneous voltage is transformed into Nusselt number applying Eq. (6.2) and the instantaneous T_0 signal. The instantaneous wire Reynolds number is then retrieved from the Nu-Re calibration law. The instantaneous velocity is computed by Eq. (6.3), where the density and the dynamic viscosity of the fluid are estimated using the local temperature and static pressure values. The velocity time series is further processed to compute turbulence statistics.

6.1.5 Turbulence statistics

The statistical moments of turbulence are computed over the signal time duration ΔT . Since only the longitudinal velocity $U(t)$ is measured, all the following equations refer to this component. The first central moment is the mean velocity, which is computed as:

$$\bar{U} = \frac{1}{\Delta T} \int_{t_0}^{t_0 + \Delta T} U(t) dt. \quad (6.4)$$

The second central moment is the variance. In turbulence research the root mean square of the velocity fluctuations is most often used, which is the square

root of the variance:

$$u_{rms} = \left[\frac{1}{\Delta T} \int_{t_0}^{t_0 + \Delta T} (U(t) - \bar{U})^2 dt \right]^{1/2}. \quad (6.5)$$

The square root of the variance defines the turbulence intensity of the flow according to:

$$Tu = \frac{u_{rms}}{\bar{U}}. \quad (6.6)$$

The third central moment, the skewness, is

$$S_u = \frac{1}{(\bar{u}^2)^{3/2}} \frac{1}{\Delta T} \int_{t_0}^{t_0 + \Delta T} (U(t) - \bar{U})^3 dt. \quad (6.7)$$

The skewness is a measure of the lack of symmetry of the distribution and is zero for a symmetric distribution.

To obtain information about the structure of the turbulent flow field, the time autocorrelation coefficient $\rho(\tau)$ is used, which represents the degree of correlation between the fluctuating velocity components u at two instants separated by a time interval τ :

$$\rho(\tau) = \frac{\overline{u(t)u(t+\tau)}}{\overline{u^2(t)}}. \quad (6.8)$$

The integral time scale gives an estimate of the time interval for which the velocity signal is correlated. This parameter is calculated as:

$$T_{int} = \int_0^\infty \rho(\tau) d\tau. \quad (6.9)$$

Using the Taylor's frozen vortex hypothesis [4], the integral length scale L_{int} can be computed multiplying the integral time scale by a convective velocity, usually considered equal to the local mean velocity. Since experimentally obtained autocorrelation coefficients often feature wide oscillations around zero, the integration for the complete signal duration can produce erroneous results. Therefore, different options have been proposed for the upper integration limit of Eq. (6.9): the time lag for which the amplitude of the function decreases to e^{-1} ; or the time lag corresponding to the first root (see for example [5]). The presence of oscillations leads to an unreliable computation of the integral time scale, especially for short time intervals. An alternative method has been proposed by Roach [6] where the integral scale is computed from the power spectrum:

$$L_{int} = \left[\frac{E(f)\bar{U}}{4\overline{u'^2}} \right]_{f \rightarrow 0}. \quad (6.10)$$

The Taylor frozen vortex hypothesis is assumed also in this case.

The micro length scale η , also known as Kolmogorov length scale, is considered representative of eddies that dissipate turbulent energy. It can also be computed following Roach [6] as

$$\eta = \left[\frac{\overline{U}^2 \overline{u'^2}}{2\pi^2 \int_0^\infty f^2 E(f) df} \right]. \quad (6.11)$$

6.2 Uncertainty Analysis

In this section the uncertainty analysis methodology and the uncertainty sources are presented in detail.

6.2.1 Measurement Uncertainty Sources

The measurement uncertainty consists of two major sources: (a) the uncertainty of the HW calibration polynomial and (b) the uncertainty of the local flow properties at the HW measurement position. The effect of the temporal and spatial resolution of the HW is briefly discussed as well.

6.2.1.1 HW calibration uncertainty.

The uncertainty stemming from the HW calibration can be categorized into two parts: the uncertainty of the calibration points and the uncertainty due to the fitting of the calibration curve. The HW voltage, the total temperature, the total and static pressure measurements are used to compute the calibration points (Nu, Re) . The points are then fitted with a 4th order polynomial. For each Monte Carlo iteration i , the distributions of the input variables E_b , T_0 , P_0 and P_{amb} are randomly sampled, the i -th set of calibration points $(Nu, Re)_i$ is computed and finally this set is fitted with a 4th order polynomial p_i . At the end of M iterations, a distribution of M values is obtained for each coefficient of the polynomial p . If the model used for the fitting correctly represents the physical phenomenon, any fitting errors should be attributed to random errors in the measurements that cause scatter in the calibration points. Therefore, if the random uncertainty in the input variables (in this case E_b , T_0 , P_0 and P_{amb}) is correctly estimated, then the curve fitting error should be automatically accounted for by the uncertainty of the polynomial.

Initially, the random uncertainties for the input variables are estimated based on the variance of their output at flow-off conditions (instrument noise). The systematic uncertainties are evaluated based on the instrument' documentation and calibration. The total pressure and temperature are measured inside the calibration nozzle by means of a K-type thermocouple and a Validyne differential pressure transducer respectively. The static pressure at the measurement point is the ambient pressure, measured by a Druck DPI 150 pressure indicator. The thermocouple

and the pressure transducer have been calibrated against a PT100 thermometer and a Druck DPI 610 pressure indicator respectively. The elemental uncertainties originating from the instruments are summarized in Table 6.2. The uncertainty values s represent the uncertainty at the standard deviation level for normal distributions, while for uniform distributions the values a and b represent the lower and upper limits.

Property	Type	Distribution	Value
HW voltage	random	normal	$s = 0.001 \text{ V}$
Ambient pressure	systematic	uniform	$b = -a = 5 \text{ Pa}$
PT100	systematic	uniform	$b = -a = 0.1 \text{ K}$
Thermocouple	random	normal	$s = 0.0015 \text{ K}$
Thermocouple	systematic	uniform	$b = -a = 1.1 \text{ K}$
Druck DPI 610	systematic	uniform	$b = -a = 17.5 \text{ Pa}$
Validyne	random	normal	$s = 0.0012 \text{ V}$
Validyne	systematic	uniform	$b = -a = 32.5 \text{ Pa}$

Table 6.2 – Elemental uncertainties of instruments used in the HW calibration chain

The calibration curve with the uncertainty of the polynomial is presented in Fig. 6.5. The confidence interval corresponds to the 95% upper and lower confidence intervals of the calibration-predicted Reynolds number $Re_{fit} = p(Nu)$, for each value of Nu . No uncertainty is considered in the Nu values, only the uncertainty in the polynomial p . The original calibration points are also presented in the Figure with their 95% uncertainty bars. It can be seen in Fig. 6.5b, that for some calibration points the deviation from the curve is higher than their measurement uncertainty. The relative curve fitting error is shown in Fig. 6.6 to better showcase this issue, computed on the average values. It seems that the fitting error is not taken into account automatically by the MC. Hölle et al. [7] encountered the same problem when evaluating the measurement uncertainties of pneumatic multi-hole pressure probes. The authors concluded that the method cannot be used when the fitting error is not negligible compared to the calibration uncertainty. To overcome this limitation, a different approach is applied in the present case. Since the curve fitting error is the result of random errors producing scatter in the calibration points, this error is used as a measure of the random uncertainty and no random uncertainty is assigned to the input values to avoid accounting for it twice. It has to be noted that this approach assumes that the scatter in the calibration points is exclusively due to random errors in the Reynolds values. The distribution of the fitting error (which is computed as $Re_{cal} - Re_{fit}$) has a mean value of zero. Thus, it does not affect the average value of the calibration output, but only its variability. Therefore, a normal distribution is assigned to the fitting error with the computed mean and standard deviation values. The fitting error is therefore treated as an additional error source added to the uncertainty of the measured Reynolds number. The measured Reynolds number for one iteration i is then defined by the following

equation:

$$Re_i = p_i(Nu_i) + s_i \quad (6.12)$$

where Nu_i contains the uncertainty of the measured Nusselt number, p_i the uncertainty of the calibration polynomial parameters and s_i the uncertainty due to the fitting error. The uncertainty s_i is a systematic term for the measured Reynolds number, as the calibration curve is fixed. The parameter s has a mean value of zero and a standard deviation of 0.0273 and 0.1176 for HW1 and HW2 respectively.

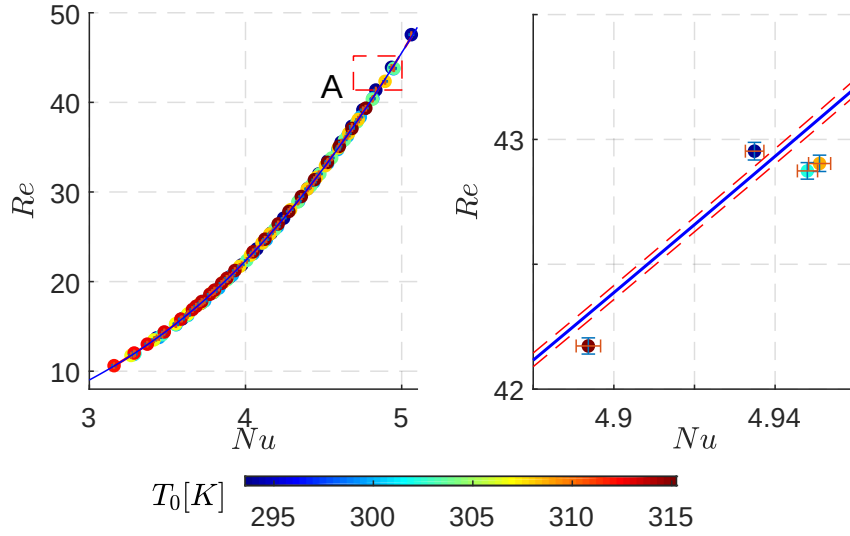


Figure 6.5 – HW calibration curve with uncertainty bars (left), insight view into the calibration curve (right).

6.2.1.2 Uncertainty of local flow properties.

The flow properties at the HW measurement position are used to retrieve the velocity from the HW calibration. These flow properties are the total temperature T_0 , the total pressure P_0 and the static pressure P_s . As previously explained, the profiles of P_0 and T_0 along the span are defined by measurements with different instruments. The values are normalized by the value at mid-span for each test. The non-dimensional profiles and measurement at mid-span at each specific test are used to obtain the local flow data at each HW measurement position. The uncertainty therefore stems from the uncertainty in the determination of the profile and the bias error due to the measurement at mid-span during each test.

The uncertainty of the profiles is computed considering the test-to-test variability of the measurements. Since the measurements are performed with different instruments, the bias errors are not considered separately: they are taken into account

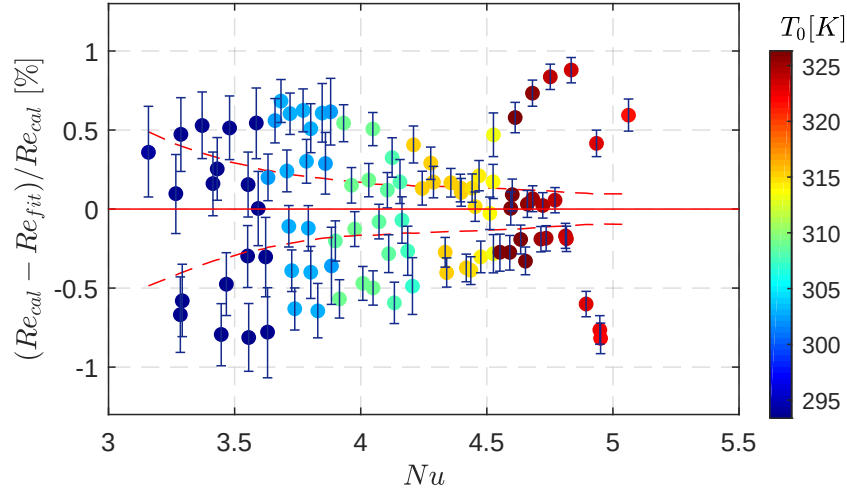


Figure 6.6 – Relative calibration curve fitting error. The uncertainty of the calibration polynomial is indicated by the red dashed line.

in the variability. The non-dimensional measurements are averaged over 2% of the span considering the uncertainty in the positioning of the probes ($\sim \pm 0.5$ mm). The mean value and the standard deviation are computed at each spanwise cluster. The standard deviation for total pressure and temperature profiles is presented in Fig. 6.7. Since there is no distinct trend in the distribution of the standard deviation along the span, the mean value of the standard deviation std_{mean} is considered representative of the uncertainty in the measurement of T_0 and P_0 due to the test-to-test repeatability and the bias of the different instruments. Since there are not enough measurement points to evaluate the distribution per cluster, the normality assumption is taken. Therefore, the input value of T_0 for one span position k at one MC iteration i is defined as follows:

$$T_{0_{k,i}} = \overline{T_{0_k}} + s_{T_{0_i}} \quad (6.13)$$

where $\overline{T_{0_k}}$ is the mean value at a certain span location and $s_{T_{0_i}}$ is drawn from a normal distribution with zero mean value and standard deviation equal to std . For each MC iteration i , the values for all H span positions $[T_{0_{k,i}}]_{k=1:H}$ are fitted with a Fourier model to get the spanwise profile $T_{0_{fit,i}}$. The same procedure applies to the measurements of total pressure. An analogue procedure is followed for P_s : the values at hub and tip are computed and the profile fit is obtained by linear interpolation between these two values. The uncertainty of the profiles at the 95% confidence level is shown in Fig. 6.8.

The bias error of the T_0 measurement b_{T_0} is assumed to follow a normal distri-

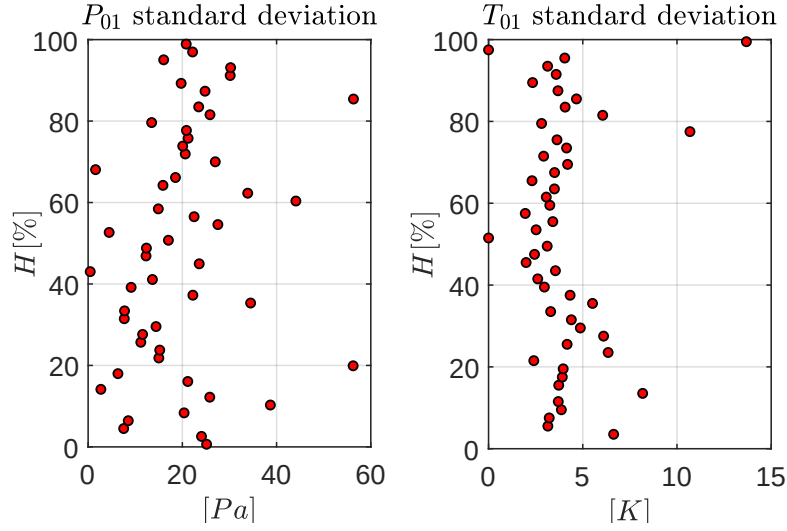


Figure 6.7 – Test-to-test variability of $P_{0,1}$ and $T_{0,1}$ along the span.

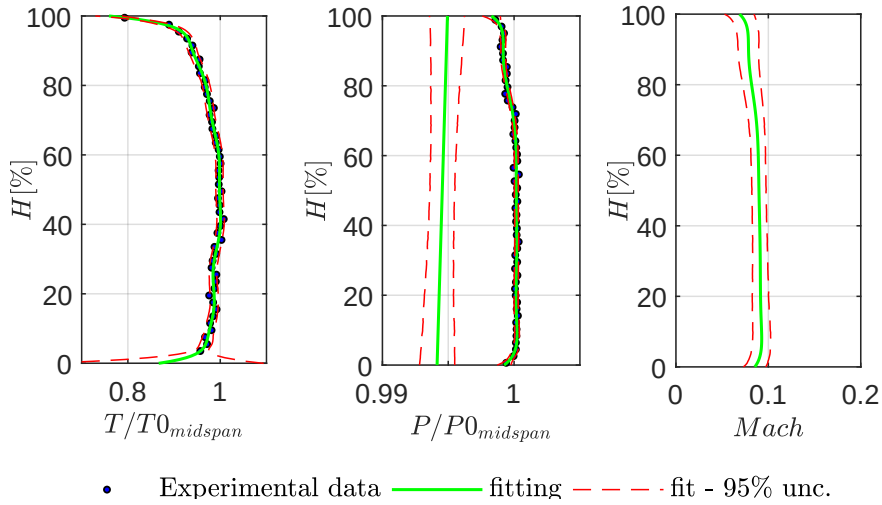


Figure 6.8 – Uncertainty of spanwise profiles at 95% confidence level.

bution with zero mean and standard deviation equal to 0.25 K. The uncertainty of the P_0 measurement, being a differential measurement, originates from two error sources: the error in the measurement of the differential pressure b_{P_0} and the error in the measurement of the ambient pressure $b_{P_{amb}}$. b_{P_0} is estimated to follow a normal distribution with zero mean and standard deviation equal to 0.25 mbar.

$b_{P_{amb}}$ follows a uniform distribution with limits at ± 0.05 mbar (instrument's resolution). The methodology to compute the flow properties at the HW span position for one MC iteration is schematically presented in Fig. 6.9. $T_{0_{ms}}$ and $P_{0_{ms}}$ are the values of the mid-span measurement for the specific test.

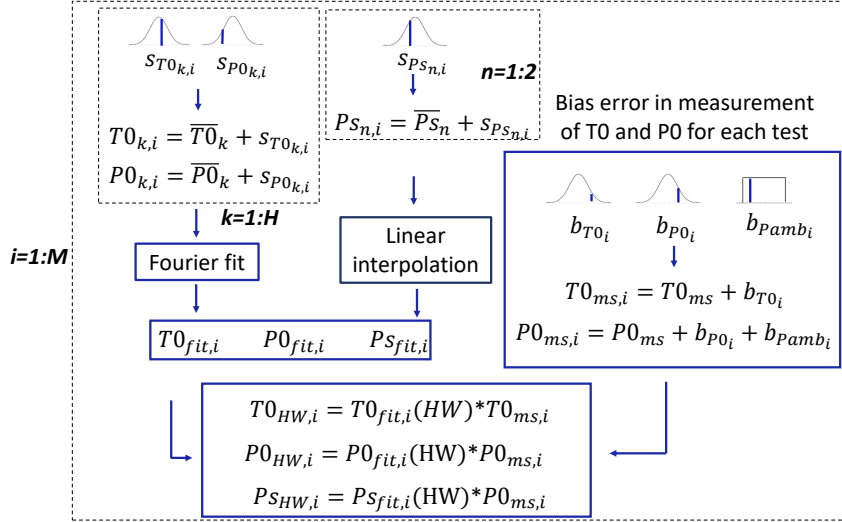


Figure 6.9 – Methodology for the computation of the flow properties T_0 , P_0 and P_s at the spanwise position of the HW probe.

6.2.1.3 Temporal and spatial resolution effects.

The measurement of turbulence statistics is affected by errors arising from the instrument's temporal and spatial resolution. The HW's cut-off frequency (corresponding to an attenuation of the signal of -3 dB) has been estimated to be between 17-20 kHz based on the result of an anemometer's embedded square-wave test. The anemometer has been adjusted to feature an optimally-damped second order response (see Chapter 3) which corresponds to a maximally flat frequency response up to the system's cut-off frequency. In other words, the response of the HW resembles the behavior of a low-pass filter, resulting in attenuation of the energy of the turbulent fluctuations over a certain frequency.

The HW's finite length can lead to spatial filtering when the turbulent flow scales are comparable or smaller than the heated sensor size [8, 9]. This means that the magnitude of the turbulent fluctuations is attenuated over a certain wavenumber due to spatial resolution issues, independently of the frequency response. The temporal and spatial resolutions are thus inherently connected.

To examine the range of frequencies that affect the value of the velocity fluctu-

ations RMS (and thus the value of the turbulence intensity), u_{rms} is alternatively computed as

$$u_{rms} = \sqrt{\int_0^{f_{limit}} PSD(f) df} \quad (6.14)$$

where PSD is the power spectral density of the velocity fluctuations. Fig. 6.10 shows that u_{rms} converges to its final value at around $f_{limit} = 8.5 \text{ kHz}$ within 0.002 m/s .

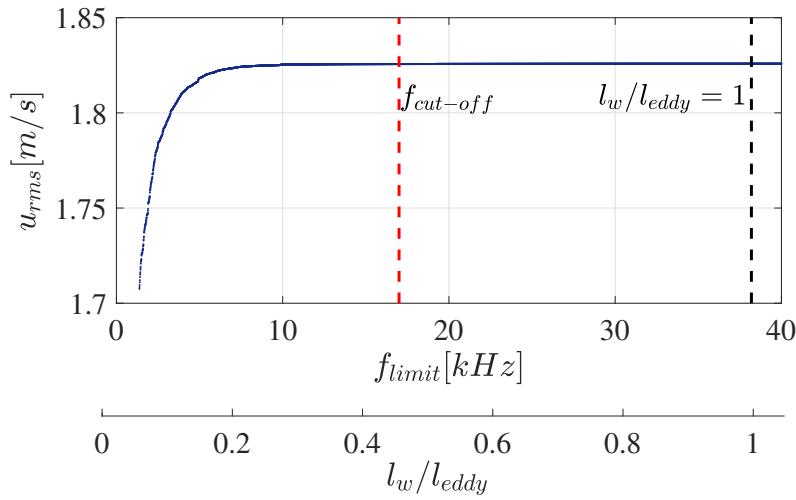


Figure 6.10 – Effect of the upper frequency limit of the HW system on the u_{rms} value.

As the limit frequency is less than half of the estimated HW cut-off frequency, we can conclude that the frequency range which contributes to the u_{rms} value is not affected by the system's finite temporal resolution and this uncertainty source can be safely neglected.

To assess the effect of the spatial resolution, Taylor's frozen vortex hypothesis [4] is used to compute the eddy streamwise length scale corresponding to each frequency ($l_{eddy} = U_{conv}/f$). In Fig. 6.10, the second x-axis represents the ratio of the wire length to the local eddy length scale. The dashed (black) vertical line identifies the frequency for which this ratio is equal to 1. This frequency is approximately 38 kHz for the flow under investigation, four times higher than the limit frequency of 8.5 kHz. Spatial filtering initiates at frequencies lower than those corresponding to $l_{eddy}/l_w = 1$, depending on the nature of the turbulent flow field. This effect is more severe with increasing anisotropy of the flow, see e.g. Cameron [10]. Since only single-wire measurements were conducted in this work,

there is no information on the degree of anisotropy of the flow-field. Although the flow is expected to feature a certain degree of anisotropy, it is assumed that the spatial resolution has a negligible contribution to the uncertainty on u_{rms} in the present case.

The errors due to the HW temporal and spatial resolution can be considered negligible for the velocity RMS, the turbulence intensity and the integral length scale. However, this is not the case for other quantities such as the Kolmogorov length and time scales and the skewness. The Kolmogorov length and time scales correspond to a part of the velocity spectrum that is likely attenuated by the spatial filtering and by the wire finite time response even at frequencies below the system cut-off frequency. In Chapter 3, it has been demonstrated that the non-linear response of the anemometer on the skewness value is negligible for uniform mean flow, even for high turbulence levels. The skewness can nevertheless be affected by spatial filtering [11].

6.2.2 Uncertainty of the time series

The uncertainty inputs are inserted in the measurement model to retrieve the velocity time series for each test. For each MC iteration i , each instantaneous velocity sample j is computed as follows

$$U_{i,j} = [p_i(Nu_{ij}) + s_i] \frac{\mu_{ij}}{\rho_{ij} d_w} \quad (6.15)$$

where

$$Nu_{i,j} = \frac{E_{b_j}^2 R_w}{\pi l k_{ij} (T_w - \eta T_{0_{ij}}) (R_s + R_{top} + R_{wire})^2} \quad (6.16)$$

All the turbulence statistics of interest are also computed at each iteration i . After M iterations, the output of the MC loop is the distribution G_Y (composed by M points) for each turbulence parameter Y and a distribution for each sample of the velocity time series. The measurement uncertainty is defined for each parameter by computing the confidence intervals at the 95% level. The methodology is presented in Fig. 6.11. The velocity time series with 95% confidence intervals is presented in Fig. 6.12 and the histograms for two measurement samples A and B are presented in Fig. 6.13. The distributions are slightly skewed and the median value of the velocity is a rather more suitable parameter than the mean value. The same degree of skewness is observed for both points. This is because the systematic uncertainty sources considered mainly affect the ensemble of the time series by introducing a bias. The only factor causing variability is the calibration polynomial uncertainty p_i .

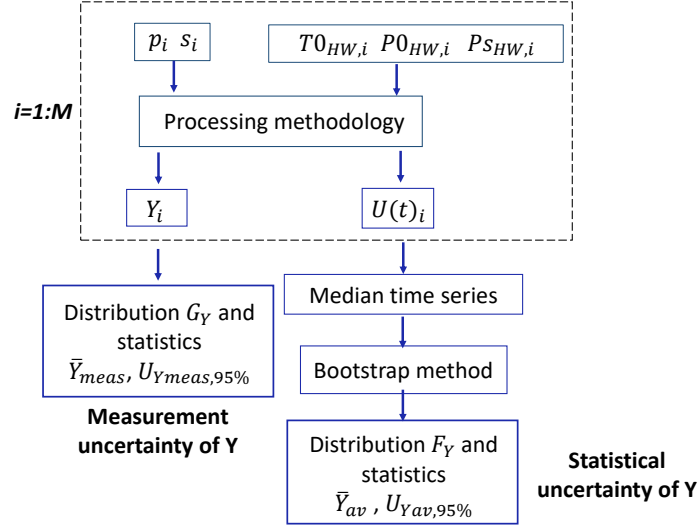


Figure 6.11 – Overall methodology.

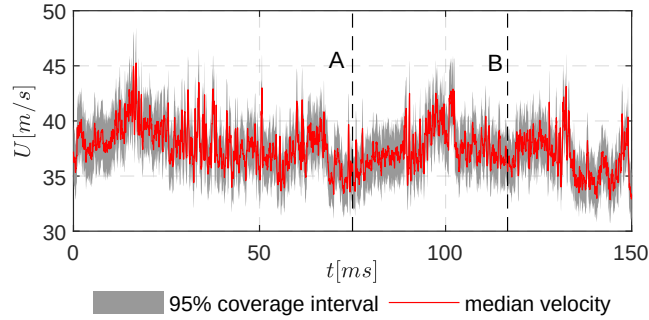


Figure 6.12 – Velocity time series with 95% confidence interval.

6.2.3 Statistical uncertainty

The statistical uncertainty of the turbulence statistics is computed by applying the MBB to the median velocity time series (Fig. 6.12). The output is a distribution F_Y for each turbulence parameter and the computed confidence intervals express the statistical uncertainty at the 95% confidence level. The statistical uncertainty is independent of the measurement uncertainty, as the latter only considers systematic uncertainty sources. Applying the bootstrap to the mean velocity time series

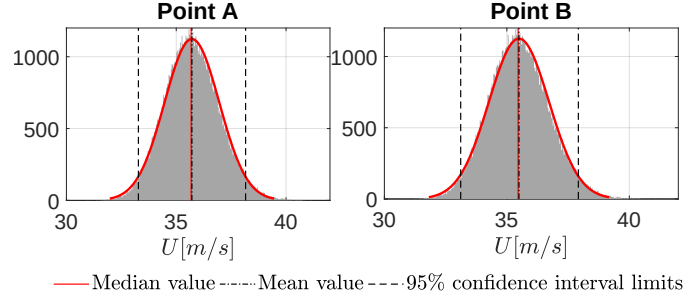


Figure 6.13 – Histograms of points A and B.

or to any times series $U_i(t)$ (corresponding to the i MC iteration) yields the same statistical uncertainty, only with a small variation on the mean value.

The histogram of the velocity time series for one test is presented in Fig. 6.14. The distribution deviates from normality, featuring positive skewness. This reinforces the necessity to use a non-parametric method which does not require a priori assumptions on the probability function of the velocity distribution.

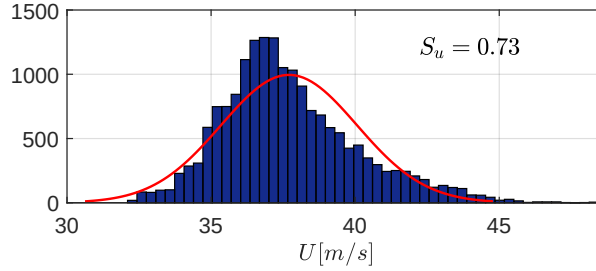


Figure 6.14 – Histogram of the median velocity time series for one test.

The application of the MBB requires the selection of two parameters: the number of the bootstrap series B and the optimal block length c_{opt} for which the intrinsic correlation of the signal is maintained. c_{opt} was selected by the algorithm presented in Chapter 5.3. B is selected by monitoring the convergence of all the turbulence statistics of interest. Indicatively, the convergence for the turbulence intensity and the integral length scale are reported in Fig. 6.16. It can be seen that the computation of the standard deviation requires a smaller number of iterations compared to the computation of the confidence interval limits. Therefore, the computation of a confidence interval is computationally more expensive compared to the computation of the variance. Moreover, the number of bootstrap series B required for convergence is higher for the integral length scale (78000 series) than for the convergence on the turbulence intensity (35000 series). Considering the

convergence histories of all the statistics, $B = 10^5$ was selected.

The histograms of the bootstrapped turbulence quantities for the same test are displayed in Fig. 6.17. The empirical distributions are well approximated by Gaussian distributions.

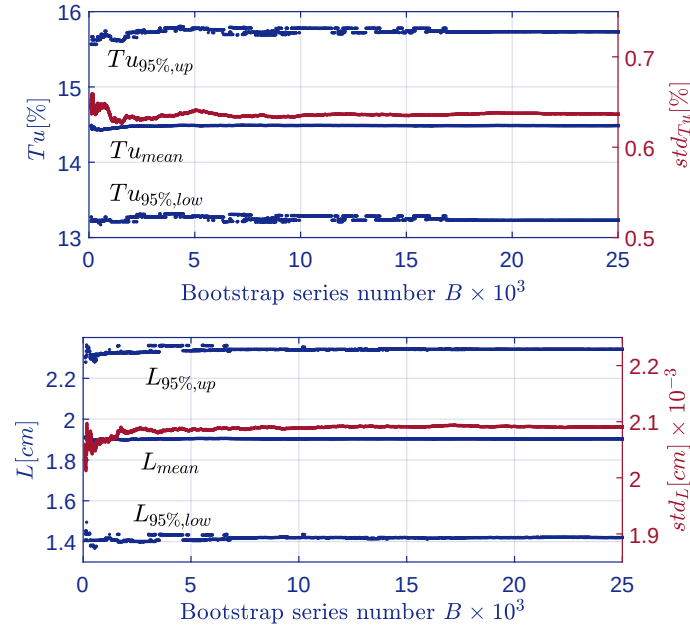


Figure 6.16 – Mean, standard deviation and confidence interval limits of the turbulence intensity (top) and integral length scale (bottom), convergence with bootstrap series number B .

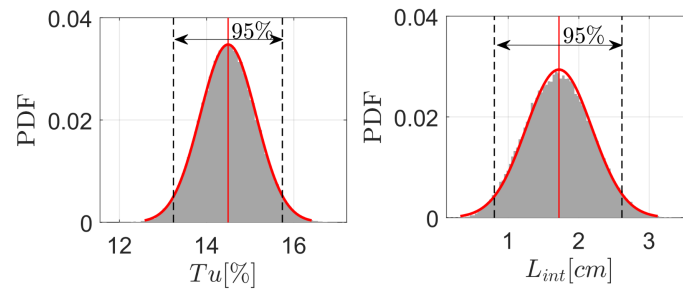


Figure 6.17 – Histogram of bootstrapped turbulence intensity (left) and integral length scale (right).

The total uncertainty on a parameter Y is finally computed as:

$$u_Y^2 = u_{meas,Y}^2 + u_{stat,Y}^2 \quad (6.17)$$

where $u_{meas,Y}$ and $u_{stat,Y}$ are the measurement and statistical uncertainty for Y at the standard deviation level. The expanded uncertainty at the 95% confidence level is then computed as:

$$U_Y = t_{95,\nu} u_Y \quad (6.18)$$

where $t_{95,\nu}$ is the 95% coverage factor from the t-distribution for ν degrees of freedom. Following the recommendation in [12], the degrees of freedom are considered large enough, so that $t_{95} = 2$.

6.3 Results and Discussion

6.3.1 Uncertainty of turbulence statistics

The spanwise distributions of the turbulence statistics measured at the turbine inlet plane are presented in Fig. 6.18 - Fig. 6.22. The measured moments are bounded by error bars that indicate the total uncertainty at the 95% confidence interval. These distributions are also accompanied by bar graphs that showcase the contributions of each macro-uncertainty source for three selected tests with the HW probe at different span positions.

The turbulence intensity features values of 8-10% at mid-span and increases towards the endwalls, reaching $\sim 20\%$ and $\sim 25\%$ close to the hub and tip respectively. These values are in agreement with measurements performed in the past in the same plane by Yasa et al. [13]. The length scales distributions feature the opposite trend, with length scales being larger at mid-span and gradually decreasing towards the walls. The skewness presents positive values for most span positions, with almost-Gaussian values observed very close to the endwalls. As explained by Jimenéz [14], the shape of the PDF of turbulent velocity fluctuations is dominated by large-scale motion. Therefore, the positive skewness between 10% and 90% of the span can be attributed to the presence of large coherent structures in the channel, while the close to Gaussian values are observed inside the hub and tip boundary layers.

The measurement uncertainty is the prevalent uncertainty source on the computation of the mean velocity. This contribution is significant due to the propagation of the uncertainty of all the flow properties on the HW signal. For the other statistics, the statistical uncertainty prevails. It can be seen that the measurement uncertainty is important for the length scales as the mean velocity value is required in their computation (Eq. (6.10) and (6.11)). The measurement uncertainty of the corresponding time scales is significantly smaller, as they depend mainly on the

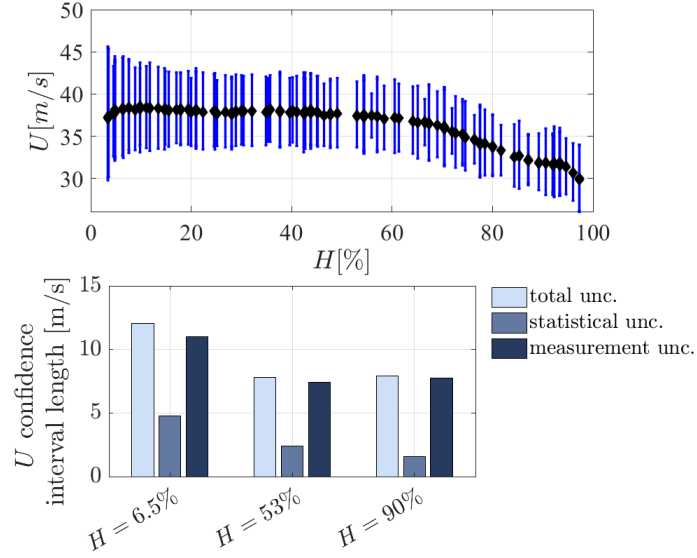


Figure 6.18 – Spanwise distribution of the mean velocity with 95% uncertainty (top), uncertainty budget at three spanwise positions (bottom).

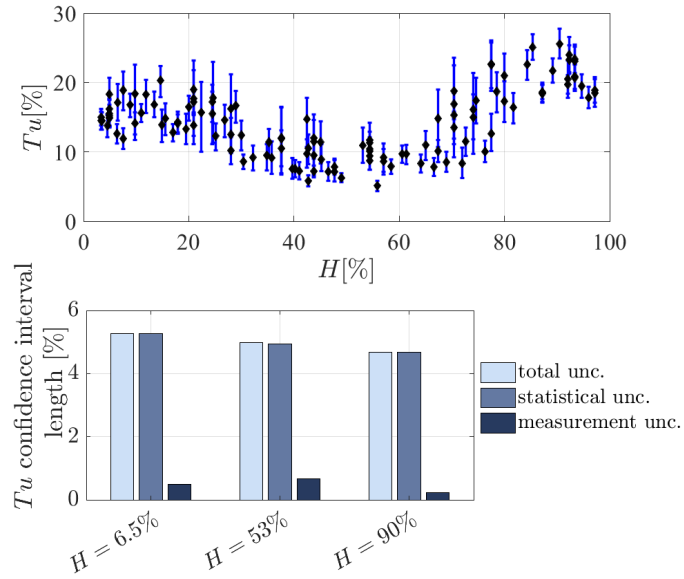


Figure 6.19 – Spanwise distribution of the turbulence intensity with 95% uncertainty (top), uncertainty budget at three spanwise positions (bottom).

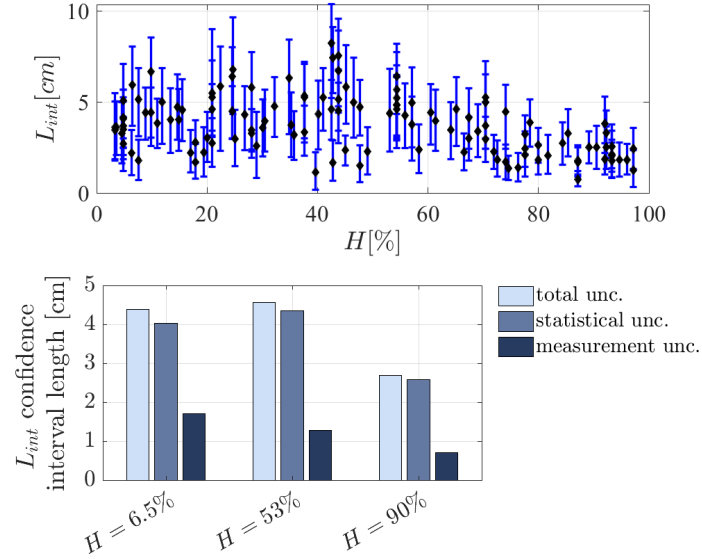


Figure 6.20 – Spanwise distribution of the integral length scale with 95% uncertainty (top), uncertainty budget at three spanwise positions (bottom).

shape of the power spectrum. Similarly, the measurement uncertainty is almost negligible for the skewness as only the fluctuating velocity component is needed for its computation (Eq. 6.7). Finally, the integral length scale is more affected by the statistical uncertainty compared to the kolmogorov scale, as the short signal duration limits the convergence of the power spectrum at the low-frequency range.

As the measurement uncertainty does not have a significant effect on the computation of the velocity fluctuations, the measurement uncertainty of the turbulence statistics can be reduced by reducing the uncertainty of the mean velocity. One possibility could be the use of a second measurement of the time-averaged flow velocity by an other instrument featuring lower uncertainty over the range of test flow conditions (e.g. pneumatic pressure probes). The statistical uncertainty can be reduced by increasing the duration of the velocity signal, which is limited by the test duration in short duration facilities.

6.3.2 Discussion on quantities provided

The turbulence intensity and integral length scale are the main inlet turbulent properties reported in experimental studies. Since they are key parameters for the evolution of a number of phenomena, many correlations, e.g. for the prediction of heat transfer or boundary layer losses, are based on them. They are therefore

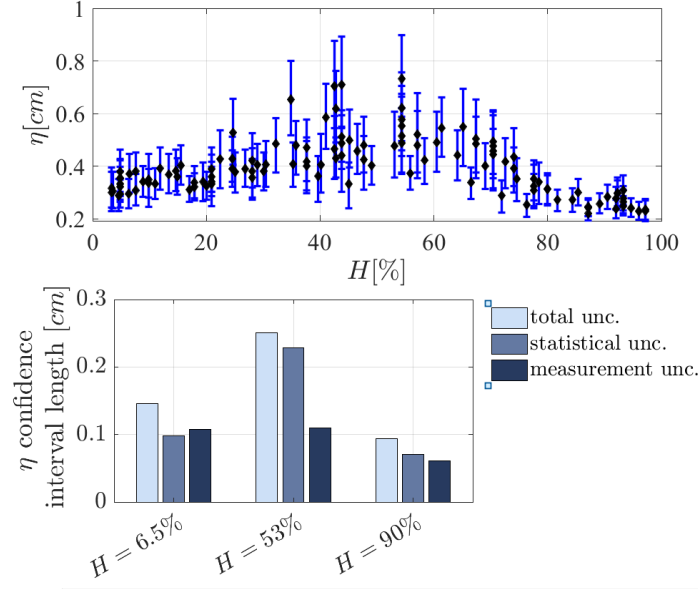


Figure 6.21 – Spanwise distribution of the kolmogorov length scale with 95% uncertainty (top), uncertainty budget at three spanwise positions (bottom).

used as reference values to compare different experiments. In this context, the turbulence intensity and integral length scale are the properties reported also in the present study. Nevertheless, from the point of view of providing data for setting up or validating numerical simulations, the turbulent kinetic energy and rms of the velocity fluctuations can be provided instead of the turbulence intensity, since it is clear from the discussion above that this would lead to a significant reduction of the measurement uncertainty. Similarly, the integral time scale should be provided instead of the length scale. This is even more crucial, since the length scales have been computed based on the Taylor's hypothesis (see Section 6.1.4) which includes some strong assumptions. The Taylor's hypothesis assumes that the turbulent eddies propagate downstream with a certain convection velocity. Using this approximation is common practice in experiments, assuming a convection velocity close to the local mean velocity, due to the lack of a better alternative. However, it has been demonstrated that the convection velocity depends on the eddy frequency and wavenumber and neglecting this dependence can lead to errors [15, 16]. The error related to the frozen vortex hypothesis is not quantified in the present study and is therefore not included in the uncertainty estimates of the length scales given in Fig. 6.20 and 6.21.

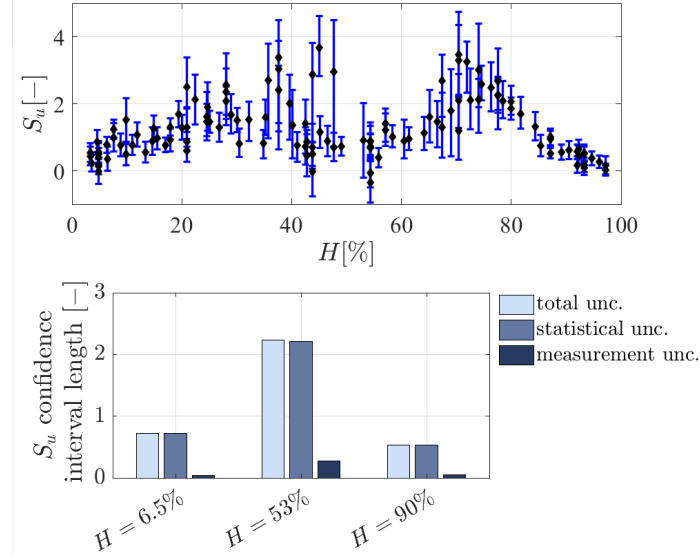


Figure 6.22 – Spanwise distribution of the integral length scale with 95% uncertainty (top), uncertainty budget at three spanwise positions (bottom).

6.4 Conclusions

In this work, a method for the quantification of the uncertainty of turbulence statistics is presented. The method considers both the measurement uncertainty stemming from the experimental process and the statistical uncertainty originating from the statistical treatment of the velocity time series. It is applied to HWA measurements performed at the inlet of a transonic turbine stage, in a short duration wind tunnel. The application is presented in detail, in order to serve as an example for the uncertainty computation in this type of facilities or, more in general, for any time-constrained fluid dynamics experiment.

Firstly, the method consists in identifying the measurement uncertainty sources and propagating them through the measurement model using a Monte Carlo method. These sources are the HW calibration uncertainty and the uncertainty of the local flow properties at the measurement location, used to retrieve the velocity from the HW signal. The local flow properties are computed based on data retrieved by a number of tests, and are therefore also affected by the repeatability of the facility. The HW calibration uncertainty considers separately the uncertainty of the calibration polynomial's coefficients and the uncertainty because of the calibration curve's fitting error, as the latter is not negligible compared to the former and it is not automatically taken into account by the MC. The propagation of these uncertainty sources results in the computation of the velocity time series' uncertainty

and of the measurement uncertainty of the turbulence statistics.

The aforementioned uncertainty does not consider the added uncertainty due to the finite time signal duration. This contribution, the statistical uncertainty, is computed by applying the Moving Block Bootstrap, a non-parametric resampling algorithm, to the median velocity time series. The strongly non-Gaussian values of the skewness measured along the span, confirm the need of a non-parametric method that does not require a priori assumptions on the distribution of the velocity time series.

The uncertainty budget of the computed statistics reveals that the measurement uncertainty is the prevalent source only for the mean velocity, while for the other parameters the statistical uncertainty is the major contributor. The measurement uncertainty is important only for turbulence statistics that require the mean velocity value for their estimation. On the other hand, the statistical uncertainty depends mainly on the duration of the velocity time signal that is bounded by the time constraints imposed by the experimental setup.

To conclude, this work presents a practical application of an uncertainty quantification method for turbulence statistics that can be applied to any type of experiment and measurement technique. In this context, an exhaustive list of uncertainty sources affecting HWA in particular, (eg. turbulence anisotropy, velocity gradients etc.) has not been included in this work. The important statistical uncertainty contributions observed in this analysis confirm the need to consider this type of uncertainty in turbulence experiments, in particular for short-duration tests. The proposed method has the advantage of being non-parametric, thus requiring no a priori assumptions on the probability distribution of the turbulent velocity. The computation of the measurement uncertainty requires a reliable estimation of the elemental uncertainties and their probability distributions, but thanks to the MC propagation, it avoids the computation of complex sensitivity derivatives that are required for the common uncertainty propagation based on the Taylor series method. Therefore, besides providing a complete estimation of the uncertainty, the method is also easy to implement and can prove to be a useful tool in turbulence research.

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7

Test case 2: Turbulence characterization of an inter-turbine duct

In this chapter, turbulence measurements in an industrial test rig are presented, employing the techniques and concepts presented in the previous chapters. The aim is to characterize the turbulent flow at the inlet and outlet of an inter-turbine duct equipped with guide vanes. Hot wire anemometry measurements have been performed at the inlet of the duct (downstream of a two-stage high pressure turbine), while measurements with a total pressure fast response probe have been performed at the duct exit. The turbulence intensity and integral length scale have been characterized in both measurement planes. Particular attention has been paid to the post-processing methodology required to extract the stochastic component from the signal in the presence of strong deterministic fluctuations (stemming from the blade rotation).

7.1 Introduction

Inter-turbine ducts are diffusers that connect the high pressure turbine (HPT) to the low pressure turbine (LPT). The role of these components is to guide the flow to a larger diameter at a higher radius, leading to their characteristic S-shape. The radial offset becomes higher for engines operating at high bypass ratios, as the LPT operates at a larger mean diameter. The need to reduce their weight leads to more aggressive designs by shortening the length of the duct for a fixed radial displacement. Inter-turbine ducts are usually equipped with struts for structural reasons (load-bearing) but also operational reasons, as pathway for the piping of the engine oil and lubrication systems [1]. The struts can be turning or non-turning, i.e. aerodynamically loaded or unloaded. In the first case, they further diffuse the flow and provide a certain swirl as input to the LPT. This configuration is known as Turbine Vane Frame (TVF) or Turning Mid Turbine Frame (TMTF). In the second case, they are aerodynamically optimized to minimize losses but do not turn the flow. This concept is often referred to as Turbine Center Frame (TCF). Examples of studies featuring the first and second configurations can be found in Ref. [2] and [3] respectively. An overview of research on the aerodynamics of inter-turbine diffusers and their evolution can be found in Ref. [4].

In the present work, turbulence measurements have been performed at the inlet and outlet of the TVF of an inter-turbine duct, placed downstream of a two-stage HPT. A schematic of the test setup is given in Fig. 7.1. Hot-Wire measurements have been performed at the inlet of the duct (Plane 0), which coincides with the outlet of the rotor of the second HPT stage. Measurements with a fast response total pressure probe have been performed at the outlet of the duct (Plane 1). The tests have been performed at engine-representative conditions in terms of Mach number and Reynolds number. Combined with the 3D flowfield, this test case provides a challenging measurement environment to apply the concepts and methodologies developed in the previous chapters.

7.2 Instrumentation design

In this section, the design concept of the instrumentation will be presented.

7.2.1 Hot-wire probe design concept and calibration

An X-type hot wire probe has been designed for 2D measurements. The 2D velocity statistics at the blade-to-blade (axial/tangential) plane are targeted. The input test conditions based on which the probe was designed are the following:

- Total pressure level: ~ 600 mbar

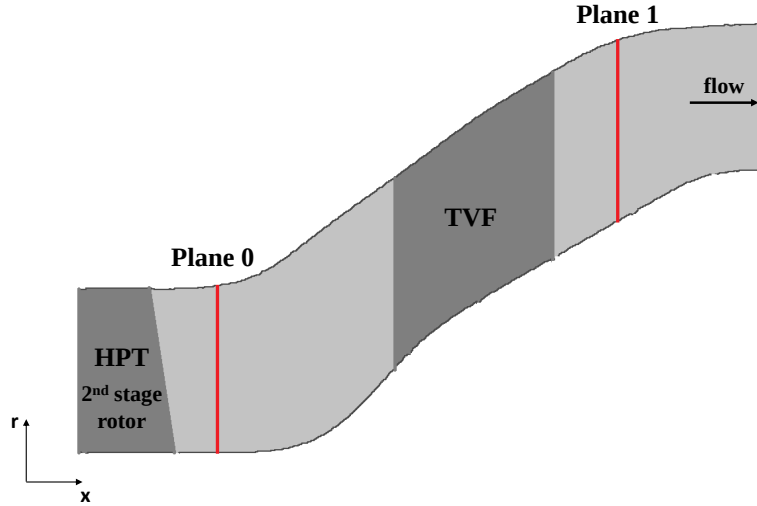


Figure 7.1 – Schematic of the measurement setup (meridional plane).

- Total temperature level: ~ 300 K for the cold condition and ~ 420 K for the hot condition
- Mach number: up to high subsonic
- Pitch angle variation (angle in meridional (axial/radial) plane with respect to the mass averaged value in the plane): $\pm 5^\circ$
- Yaw angle variation (blade-to-blade plane): $\pm 20^\circ$

The hot wire probe will be mounted on a traversing system that allows the rotation around the axis, in order to align it with the mean flow direction on the blade-to-blade plane. The expected yaw angle variation in the aforementioned list refers to the unsteady angle variation around the mean value. The probe design should allow the directional measurement in this range. The design of the probe head is given in Fig. 7.3. The design consists of two slanted wires positioned on two parallel planes at a distance of less than 1 mm. The two wires are slanted in different directions with an angle of 35° , creating therefore a cross in the blade-to-blade plane (see Fig. 7.3(a)). The measurement point is considered to be located at the intersection of the two wires, as indicated in Fig. 7.3. The slanted angle was chosen according to the expected range of variation of the flow angle ($\pm 20^\circ$) while the prongs are designed to reduce flow-induced vibrations.

The hot wire probe is depicted in Fig. 7.2, while the measurement head design is given in Fig. 7.3. As can be seen in Fig. 7.2, the probe stem is L-shaped and the

probe head is designed with an angle imposed by the endwall contouring, in order to allow measurements close to the wall. Due to the fragility of hot wires, a detachable head configuration was decided. This allowed the design and manufacturing of two heads that could be easily substituted in case of wire breakage. A fastening system consisting of a fixing screw and a blocking screw was adopted, that allows a fast, secure and precise connection. The fastening mechanism is hidden inside the carter during the measurements, with only the thinner part of the stem entering the channel to minimize blockage.



Figure 7.2 – X-wire probe.

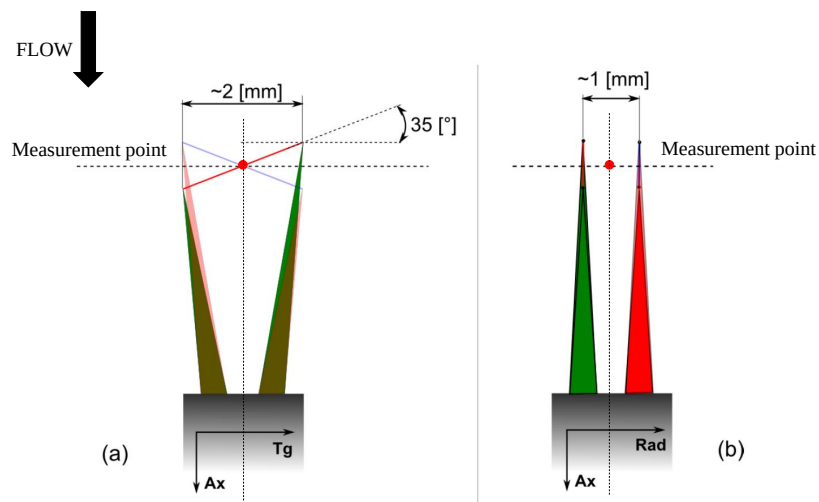


Figure 7.3 – X-wire head configuration.

The sensor properties were selected considering the operating conditions and dynamic response requirements. As two operating conditions were initially fore-

seen, at low and high flow temperature, two different materials were selected for each. The measurement head for the hot condition was equipped with Platinum-Nickel wires which have a reduced oxidation rate and allow the sensor operation at a higher temperature. The measurement head for the cold condition was instrumented with platinum-plated tungsten wires instead, for which the operating temperature limit is lower (~ 550 K) but are more mechanically resistant. A sensor diameter of $5\text{ }\mu\text{m}$ was selected for both heads aiming at a high bandwidth.

7.2.2 Calibration

The Hot Wires have been calibrated with the non-dimensional methodology described in section 2.2 aiming to achieve similarity between the Reynolds number in the facility and the Reynolds number in the calibration jet. The range of the expected Reynolds number for the Hot and Cold Nominal points are given in Table 7.1. The static calibration curves for both measurement heads are given in Fig. 7.4. The Reynolds range for the two operating conditions is also marked in the figure. Both measurement heads have been calibrated in order to cover the Reynolds range of both conditions, so that they can be used as a backup in case of wire breakage. The calibration range has been extended over the minimum and maximum Reynolds limits to avoid extrapolation in the case of large fluctuations.

Quantity	Hot condition		Cold condition	
	Min	Max	Min	Max
Total pressure [mbar]	485	523	600	650
Total temperature [K]	373	401	293	308
Mach number	low subsonic	high subsonic	low subsonic	high subsonic
Reynolds based on HW diameter	11	22	18	38

Table 7.1 – Operating conditions range.

The properties of the two measurement heads are summarized in Table 7.2. The mentioned wire temperature is the effective wire temperature retrieved by the methodology in section 2.2. The frequency bandwidth has been obtained by the square wave test, as explained in section 3. It can be seen that the cut-off frequency is lower for the Pt/Ni wires, for the exact same geometry. This is because the lower temperature coefficient of resistance α_w for the Pt/Ni results in an increase of the time scale and therefore a lower cut-off frequency. This effect has been observed and explained in section 3.3.2.

The angular calibration has been carried out according to the procedure described in Section 2.2.

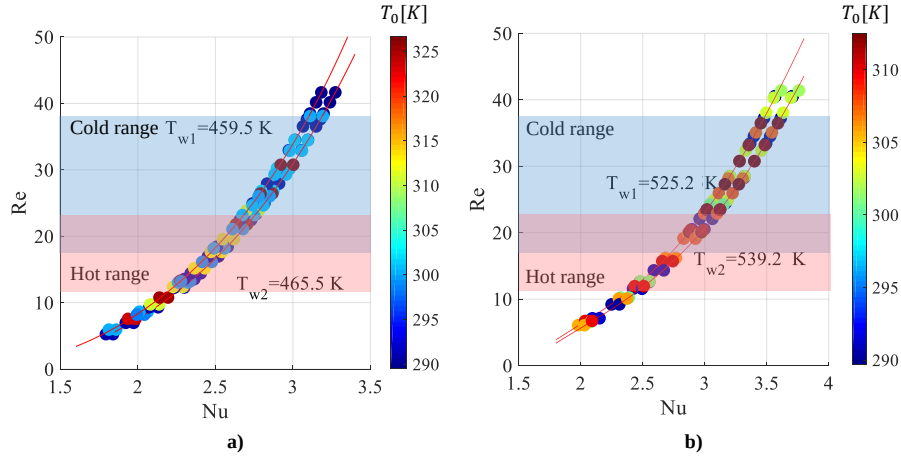


Figure 7.4 – Static calibration of the two wires for a) Head 1 and b) Head 2.

	Head 1	Head 2
Diameter	5 μm	5 μm
Material	Tungsten	90%Pt/ 10%Ni
α_w	0.36%	0.13%
Mechanical strength	Higher	Lower (more fragile)
Frequency bandwidth	65-70 kHz	35-40 kHz
Wire temperature	460-465 K	525-540 K
Target	cold condition	hot condition (backup for cold condition)

Table 7.2 – Properties of measurement head 1 and 2.

7.2.3 Fast response probe design concept and calibration

The fast response probe employed in this study is the PT-K probe, whose design and calibration have already been partially presented in Chapter 4. More details on the design conception are given hereafter. The probe has been designed for the following input conditions:

- Maximum total pressure level of ~ 850 mbar.
- Maximum total temperature level of ~ 470 K for the hot condition.
- Pitch angle variation (angle in meridional (axial/radial) plane) w.r.t. to mean value: $\pm 12^\circ$.
- Yaw angle variation (blade-to-blade plane) w.r.t. to mean value: $\pm 7^\circ$.

The probe will be mounted on a traversing system that allows the rotation around the axis, in order to align it with the mean flow direction on the blade-to-blade plane. Since the measurement plane is located at a certain distance from the traversing axis, an L-shaped design was selected. Given the expected angle variations, a Kiel-type probe head was considered as it provides an angle insensitivity range of about $\pm 10^\circ$. A picture of the probe is given in Fig. 7.5a. A sketch of the Kiel head principle is presented in Fig. 7.5b: the sensor is retreated inside the probe head, allowing the recovery of the total pressure for a wider angle range. An angle γ between the measuring head and the stem is selected in order to adapt to the strong contouring of the endwall, allowing measurements closer to the wall. Moreover, the positive inclination of the measurement head allows the spanwise flow pitch angle variation to fall well within the probe's insensitivity range.

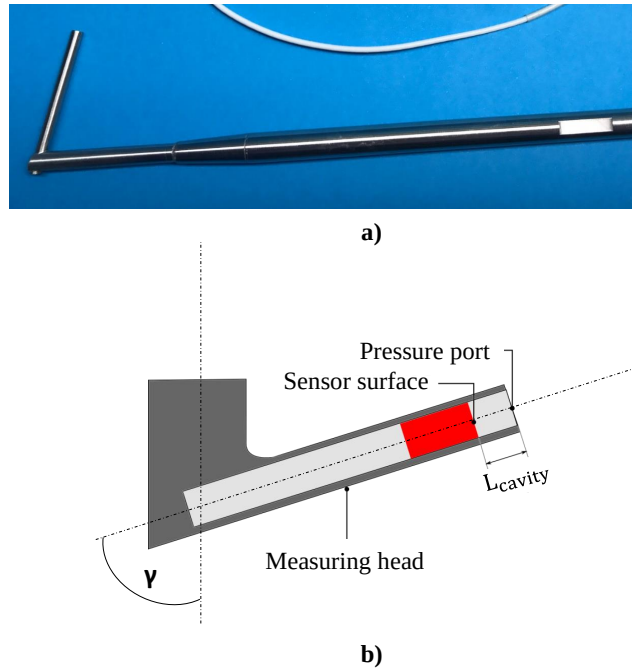


Figure 7.5 – PT-K probe: a) Probe assembly, b) Measurement head sketch.

7.2.3.1 Calibration

The probe has been subjected to static, aerodynamic and dynamic calibration as described in detail in Chapter 4. The acquired calibration functions and their application to the test data are presented below.

Static calibration The static calibration map of the PT-K probe is given in Fig. 7.6. The measured total pressure P_0 is computed as a function of the two output voltages V_p and V_s with the following calibration law:

$$P_0 = f(V_p, V_s) \quad (7.1)$$

which takes into account the temperature effects through the voltage V_s .

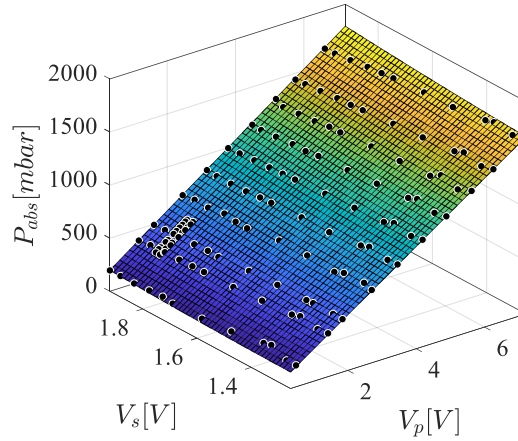


Figure 7.6 – PT-K probe: static pressure calibration map.

Aerodynamic calibration The ability of the probe to recover the flow total pressure is expressed through the total pressure coefficient C_{p0} . The coefficient is a function of the flow angles and of the Mach number. As has been demonstrated in Section 4.3.1, the Mach number has a more important effect on the value of the coefficient with respect to the flow angles, in the calibration range examined ($\pm 10^\circ$ for the yaw angle and $\pm 12^\circ$ for the pitch angle). Since the probe will be aligned with the mean flow direction on the blade-to-blade plane, the total pressure coefficient is expected to be insensitive to the yaw angle. The coefficient is then expressed as a function of the pitch angle and the Mach number, as can be seen in Fig. 7.7. The acquired data should be corrected for the recovery factor as follows:

$$P_{0,corr} = \frac{P_0}{C_{p0}} \quad (7.2)$$

where P_0 is the total pressure acquired using the static calibration law. To apply the aerodynamic coefficient to the acquired test data, an iterative procedure has been implemented. The static pressure P_s at the measurement point is known, but not the Mach number.

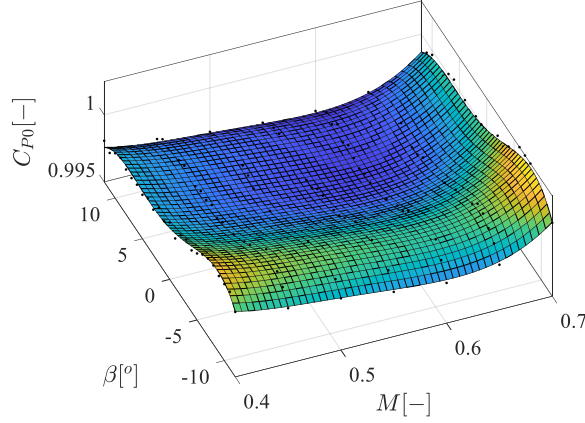


Figure 7.7 – PT-K probe: aerodynamic calibration map.

- The initial Mach number is computed by the isentropic relationship:

$$M = \sqrt{\frac{2}{\gamma - 1} \left[\frac{P_s}{P_0}^{\frac{\gamma}{\gamma - 1}} - 1 \right]} \quad (7.3)$$

- The recovery factor is computed $C_{p0} = f(M, \beta)$, where $\beta = 0^\circ$.
- The corrected total pressure is computed: $P_{0,corr} = \frac{P_0}{C_{p0}}$.
- A new Mach number is computed as: $M = g(P_{0,corr}, P_s)$.
- A convergence criterion for the Mach number is introduced: $abs(M_{new} - M) < 10^{-3}$. When this criterion is not met, the procedure is repeated from step 2 with $M = M_{new}$.
- Once the convergence criterion is met, the procedure is completed and M_{new} and $P_{0,corr}$ are the final values of Mach number and total pressure.

The procedure was initially planned to take into account also the mean pitch angle β at each position. These data were finally not available, therefore the total pressure coefficient for $\beta = 0^\circ$ was used. The probe head design has been designed in order to be aligned with the mean flow direction (obtained by numerical simulations) at the meridional plane (radial). Therefore, the expected pitch angle range should fall inside the calibration range ($\pm 12^\circ$). The variation of C_{p0} in this range for a constant Mach number is at the order of 0.002. Considering a pressure level of 600 mbar, this variation results in a 1.2 mbar difference in pressure.

Dynamic calibration The probe has been subjected to shock tube testing in order to experimentally characterize its dynamic behavior and obtain its transfer function. The experimental transfer function has been modeled by a 2nd order linear model with natural frequency $\omega_{n,1}$ and damping ζ_1 . The transfer function depends on the thermodynamic conditions of the flow. If the thermodynamic conditions in the shock tube are indicated with 1 and the conditions in Plane 1 are indicated with 2, the model parameters are recomputed as :

$$\frac{\omega_{n1}}{\omega_{n2}} = \frac{c_1}{c_2} \quad (7.4)$$

$$\frac{\zeta_1}{\zeta_2} = \frac{\mu_1 c_2 \rho_2}{\mu_2 c_1 \rho_1} \quad (7.5)$$

where c is the speed of sound, μ is the dynamic viscosity and ρ is the density of the flow. The transfer function $H(f)$ corrected for the thermodynamic quantities is then computed as:

$$H(f) = \frac{1}{-\left(\frac{2\pi f}{\omega_{n,2}}\right)^2 + 2i\zeta_2 \frac{2\pi f}{\omega_{n,2}} + 1} \quad (7.6)$$

The transfer function is then applied to the instantaneous pressure signal to compensate for the dynamic response, as depicted in Fig. 7.8.

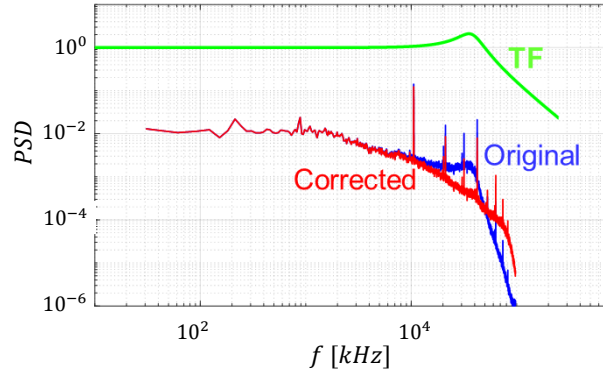


Figure 7.8 – Application of the transfer function to the pressure signal: original and corrected spectra.

7.3 Experiment description

In this section, the details of the experiment are provided.

7.3.1 Experimental data and test matrix

The following sets of measurements have been performed for two different operating points of the test rig:

- Set 1: Measurements were performed with the PT-K probe in Plane 1. The measurement grid consisted of 16 radial positions x 30 azimuthal positions, covering one pitch (the angular sector between two consecutive blades) of the TVF. The operating point is 95% nominal speed and 96% of nominal specific work.
- Set 2: Measurements were performed simultaneously with the HW in Plane 0 and the PT-K in Plane 1, on a coarse grid of 3 radial positions x 10 azimuthal positions. The azimuthal range covered corresponds to two pitches of the second stator of the HPT at Plane 0 and two pitches of the TVF at Plane 1. The operating point was a nominal cold operation point (100% nominal speed and 100% nominal specific work).

The output voltages of the HW (E_1 and E_2 for each of the two wires), the two output voltages of the PT-K (V_p and V_s), the three positions of the PT-K probe and the Top Acquisition trigger signal have been acquired. The three position signals correspond to the radial and azimuthal position of the probe and its rotation angle (in the blade-to-blade plane). The Top Acquisition signal was used to trigger the acquisition when the probes moved to a new position. The RPM signal acquisition was foreseen but finally not available during the tests. The signals were sampled at 500 kHz for an acquisition time of 3 s per point. The position of the HW probe was reconstructed using the correspondence with the position of the PT-K probe.

7.3.2 Additional data employed during the post-processing

The following data were provided by the industrial partner and were employed in the post-processing.

Experimental data The thermodynamic quantities of the flow in the measurement planes are required for the post-processing. These quantities have been acquired by measurements at the same conditions as the experiment but in different tests. They are summarized hereafter:

- In Plane 0, the total pressure P_t , the static pressure P_s , the total temperature T_t , and the flow angles (α the azimuthal and ϕ the radial) are provided.
- In Plane 1, the static pressure P_s is provided. Since the total temperature was not available, it was assumed to be equal to the total temperature in Plane 0, as no work extraction takes place between the two planes.

Numerical data 3D RANS simulation results at Plane 0 have been provided. The simulations include the full HPT. The Mach number, total and static pressure, total temperature and velocity components are given over three rotor pitches.

7.3.3 In situ HW calibration

Some problems were encountered during the tests, leading to the HW sensors breaking. More specifically, the HW sensors of both heads were damaged during the insertion of the probes in the test section. The sensors were repaired on the spot but there was no possibility to reperform a calibration, therefore the measurements were performed with a non-calibrated hot wire with the characteristics of Head 1, as reported in Table 7.2. Thus, a methodology had to be developed in order to calibrate the HW in situ, using the current measurement data and measurements of the flow properties in Plane 1.

Initially, the mean HW voltages for each measurement point and the thermodynamic flow properties in Plane 1 were used to compute the Nusselt and Reynolds numbers for every measurement point and reconstruct the calibration curve given in Fig. 7.9. The problem with this approach is that the calibration points generated do not cover a wide range of conditions. In fact, the instantaneous fluctuations for one measurement point exceed the calibration range, as it can be seen in Fig. 7.9b. Using this calibration curve would therefore lead to extrapolation and inaccurate results.

In order to improve the calibration curve, more calibration points have to be generated. The data in the rotor frame of reference were employed for this purpose. The phase-locked averaged (PLA) HW voltages were used together with the RANS results (the procedure to obtain the PLA HW voltage will be described in the following section). The methodology followed is described step by step hereafter:

- The simulation data include three rotor passages. Since the simulation is periodic, the mean blade passage is computed for all the quantities of interest. An example for span=50% is given at Fig. 7.10.
- To take into account differences between experimental data and simulations, each simulated quantity X is recomputed as follows:

$$X(t) = \bar{X} + X' \quad (7.7)$$

where X' is the variation of the quantity around its mean value computed by the simulations and $\bar{X}$ is the mean experimental value for each measurement point.

- The PLA HW voltage is computed for each measurement point. The mean rotor revolution is computed first (Fig. 7.11a) and then the mean blade pas-

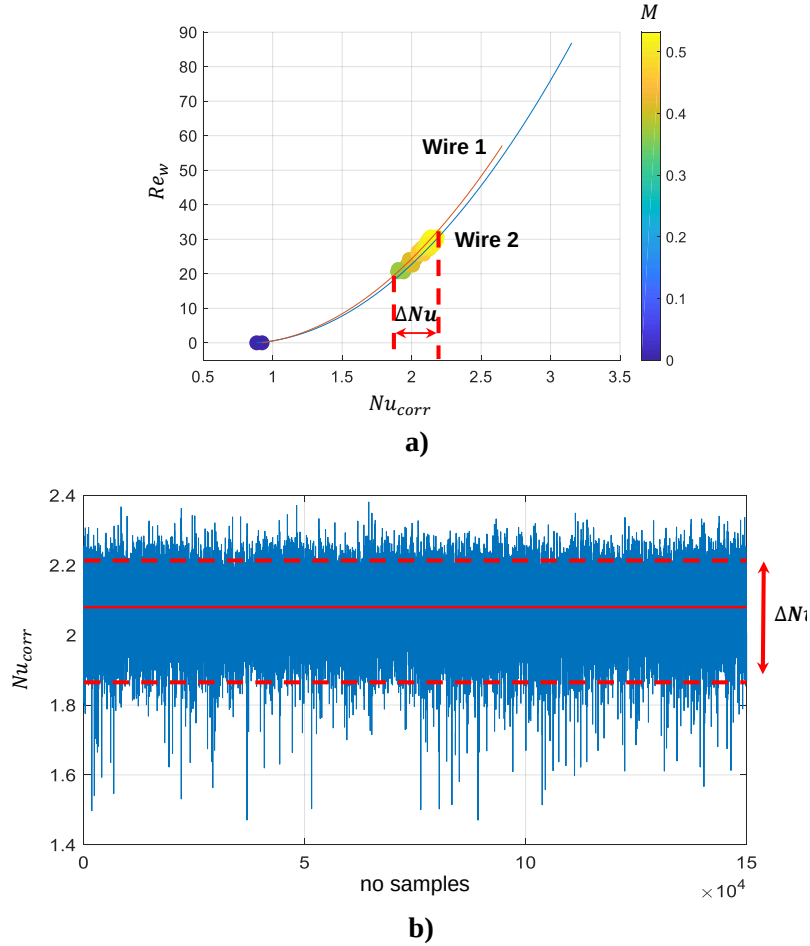


Figure 7.9 – a) HW calibration curve using mean flow properties for each measurement point, b) Instantaneous Nusselt number for one measurement point.

sage (Fig. 7.11b). Using the angular rotational speed, the time axis is transformed to rotor angular sector, as can be seen in Fig. 7.11c.

- From the HW theory, the HW voltage should be proportional to the quantity $Re_w / (T_w - \eta T_0)$. Using the ‘target’ temperature that was set at the HW setup ($T_w = 480K$), the aforementioned quantity is computed for each measurement point. A cross-correlation is performed between the simulated quantity and the HW PLA mean blade passage voltage in order to define the correct phasing between the two signals.

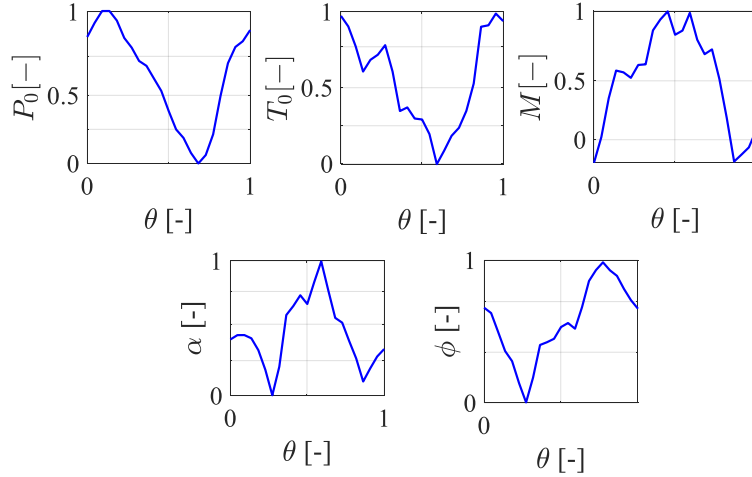


Figure 7.10 – NS3D data: Mean blade passage variations for all non-dimensional quantities over one rotor pitch, at span=50%.

- Once the correct phase has been defined, the signals are re-phased and the Nusselt and Re values for the mean blade passage are computed. By discretizing these signals in N rotor angular positions, N new calibration points are generated for each measurement position.
- As explained before, the wire temperature in this calibration methodology is not a known input. While the approximate wire temperature is defined during the setup of the system, the actual temperature distribution along the wire is unknown and defined with an extended calibration at different temperature levels. In this particular case, since this was not possible, the wire temperature was selected by minimizing the mean velocity error. More specifically, the new extended calibration curve was computed for different values of the wire temperature and the mean velocity measured by the Hot Wires was computed and compared to the one defined by the input data. As can be seen in Fig. 7.12, the mean velocity error was minimized for $T_w = 470K$.

Since the angular calibration depends predominantly on the geometry of the probe head and the position of the needles that support the wires, it was assumed that the angular calibration would not change significantly by replacing the broken wires. Therefore, the original angular calibration curve was applied during the post processing. However, a mismatch in the mean velocity computations (Reynolds computation) was observed when applying the angular decomposition. For wires 1 and 2, the mean Reynolds number is overestimated leading to the overestimation

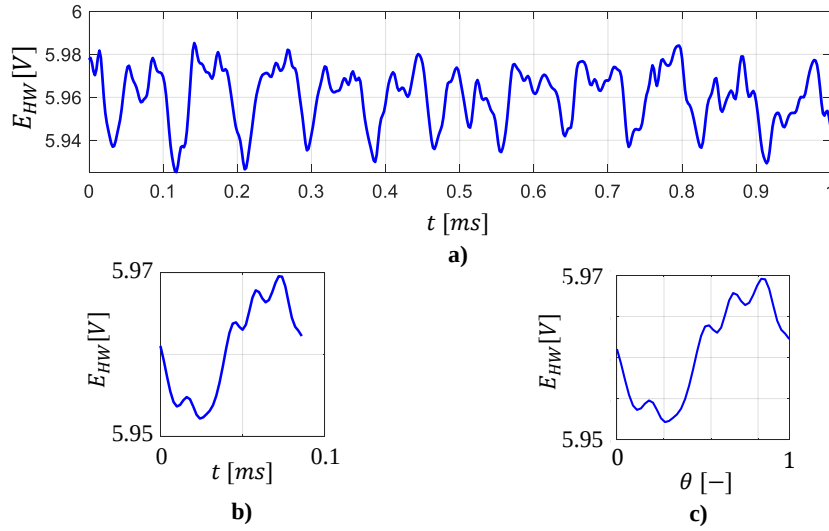


Figure 7.11 – PLA of HW voltage: a) part of mean rotor revolution, b) mean blade passage, c) mean blade passage as a function of rotor angular sector.

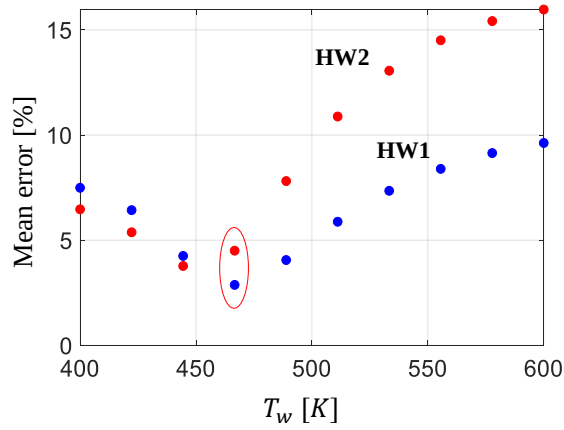


Figure 7.12 – Mean velocity error as a function of input wire temperature for Hot Wire sensors 1 and 2.

of the mean velocity and an underestimation of the turbulence intensity. Different approaches have been tested (using an « old » angular calibration, trying to re-compute the angular calibration on the base of the alignment of the probe dur-

ing measurements, systematic manual variation of the angles for best matching on mean velocity, etc.). None of the approaches gave satisfying results so far. It must be moreover highlighted that, given the alignment of the probe with respect to the mean flow direction during the experiments, results also exhibit a very pronounced sensitivity to the chosen angle: small variations of the latter lead to drastic variations of the recomputed velocity, making the task particularly challenging. Therefore, the angular decomposition could not be realized and all the results presented hereafter are reported for each wire independently and are expected to contain contributions for both the longitudinal and lateral velocity components.

7.4 Time-series post-processing methodology

7.4.1 Phase averaging

Since the flow in multistage turbomachines is characterized by deterministic and stochastic unsteadiness, one of the challenges in this study is to extract the stochastic fluctuations from the signal in order to compute the turbulence quantities. In the case of rotating bladerows, this decomposition is performed by the so-called phase-averaging or phase locked averaging (PLA), resulting in the phase-averaged flow field, which is periodic over one rotor revolution or over one blade passage (when the flow between different passages is considered periodic). This decomposition can be expressed as follows:

$$u = \tilde{U} + u' \quad (7.8)$$

where u is the instantaneous velocity, $\tilde{U}$ is the phase-averaged velocity (deterministic component) and u' is the fluctuating component (stochastic). The phase-averaged velocity is computed as:

$$\tilde{U}(\theta) = \frac{1}{M} \sum_{k=1}^M u(k, \theta) \quad (7.9)$$

where M is the number of recorded rotor revolutions and θ is the rotor angular position. The phase-averaged velocity can then be subtracted by the instantaneous component to retrieve the stochastic fluctuations.

This method results in the extraction of all deterministic components related to the rotor revolution and the Blade Passing Frequency (BPF). Nevertheless, other sources of deterministic unsteadiness might still be present in the signal, related e.g. to aerodynamic phenomena like vortex shedding, blade misalignment, or test rig operation. In such situations, different methods have been used in the literature. Fourier filtering has been used by different researchers like [5–7]. In this case, the deterministic peaks are identified in the frequency domain, their amplitude is set

to zero and the signal is then transformed to the time domain by an inverse Fourier transform. In other cases, low-pass or bandpass filters are used to remove periodic fluctuations uncorrelated with the blade rotation.

In this study, low-frequency oscillations have been observed in the signal, too low to be related to the rotor revolution. These fluctuations are attributed to the facility operation. A high-pass filter with a cut-off frequency set at 2 kHz is applied to eliminate these oscillations, and its effect on the phase averaged signal is presented in Fig. 7.13.

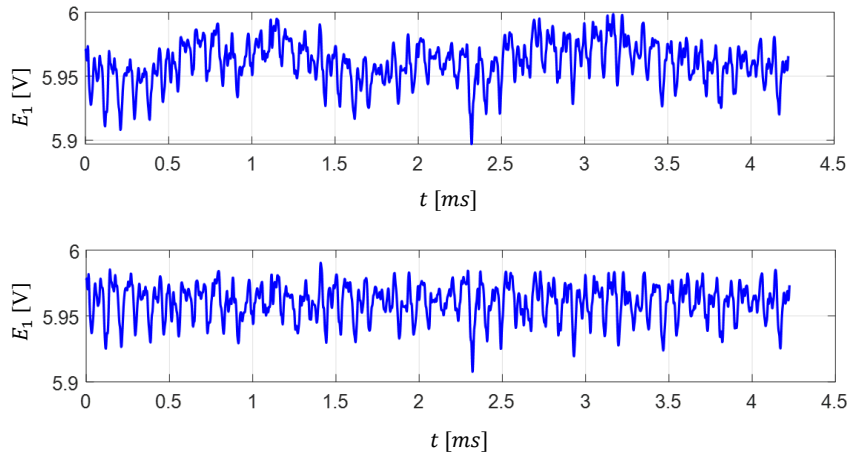


Figure 7.13 – Phase averaging on HW voltage without high-pass filter (top) and with high-pass filter at 2 kHz (bottom).

Once the low-frequency unsteadiness has been removed, the phase averaging is performed. Two different procedures have been considered in this work, as described hereafter.

Phase averaging method 1 (based on RPM signal) In the first method, the RPM signal information provided a posteriori is employed. For each measurement point, the mean RPM value is used, as the sampling frequency of the RPM signal is much lower compared to the one of the instruments. Moreover, since the RPM standard deviation for one measurement point is of the order of 0.04%, the rotational speed is considered stable.

The mean RPM value is used to compute the number of samples per rotation and divide the signal in rotor revolutions. 709 complete rotor revolution have been recorded for each measurement point. Then, the desired statistics (mean velocity, rms and turbulence intensity, etc) can be computed for each rotor angular position. Similarly, considering the number of rotor blade channels, the statistics

for the mean rotor blade channel can be computed. The effect of applying the phase averaging to the instantaneous voltage is presented in Fig. 7.14.

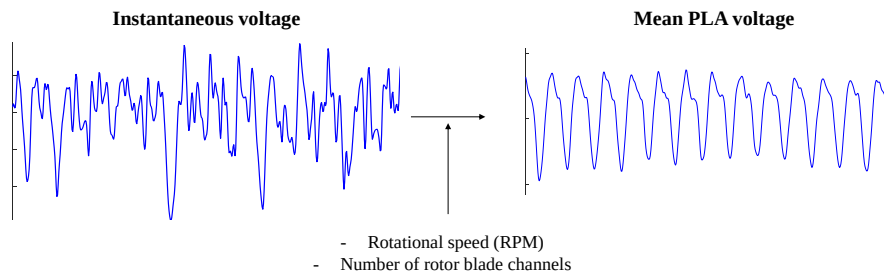


Figure 7.14 – Phase averaging application.

Phase averaging method 2 (selective phase averaging) The second phase averaging approach was initially developed at the absence of the RPM data. It is based on an automatic detection of the blade passages from the HW signal and the extraction of the most frequent blade rotation. This approach is based on the following steps:

- From the spectral analysis the blade passing frequency (BPF) was detected at 11.36 kHz.
- A bandpass filter between 11.3 and 11.4 kHz is applied to the original signal and the local peaks are detected, corresponding to the blade passages, as can be seen in Fig. 7.15.
- Using the number of rotor blades, the signal can then be divided in rotor revolutions. One rotor revolution is defined as the part of the signal containing N complete blade passages (N peaks), where N is the number of rotor blades.
- M revolutions are extracted, each with a different number of samples, due to variations in the detected blade passing frequency. Since this detection is based on the wake passage detection, differences in frequency can be attributed to aerodynamic oscillations of the wake (given that the rotational speed seems to be very stable).
- The revolutions with the most frequent number of samples are selected (as shown in Fig. 7.16) and the phase averaging is performed on them.

The mean blade passage computed with the selective phase averaging method and method 1 is presented in Fig. 7.16. It can be seen that with the first method the

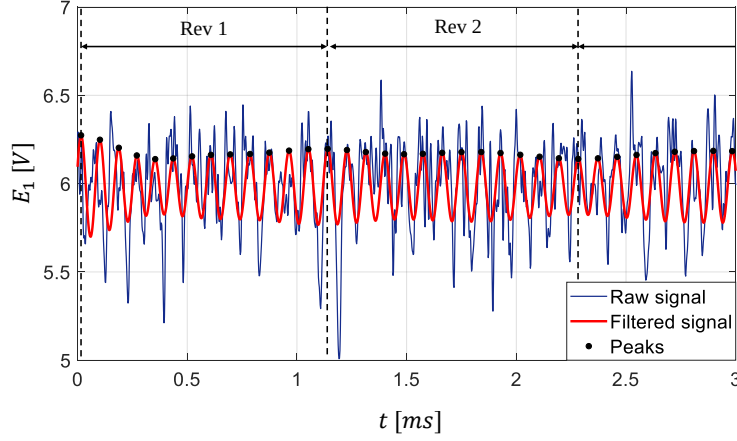


Figure 7.15 – Selective phase averaging application: automatic blade passage detection.

wake depth is significantly underestimated, indicating the superiority of the selective PLA method. Further comparison can be performed in terms of turbulence intensity. In Fig. 7.17, the turbulence intensity in Plane 0 is presented, obtained using PLA methods 1 and 2 (To have a meaningful comparison, the values have been non-dimensionalized using the min and max values by both methods). The measurements have been performed in three span positions over an azimuthal sector s covering approximately two HPT stator pitches p . It can be seen that higher values of turbulence intensity are obtained by PLA method 1 for all span positions. PLA method 2 leads to lower turbulence intensity values by 0.4% to 3%. This indicates that PLA method 1 doesn't completely remove the contribution of the periodic component, further supporting the superiority of the selective PLA method. Therefore, PLA method 2 will be used for all results presented hereafter.

The PLA method selected will be applied to the data of both the Hot Wire and the PT-K probe since the BPF signature is present also in the spectra of the PT-K probe. Nevertheless, due to the wake mixing, the automatic blade passage detection does not perform as well on the PT-K signal. As the measurements of Set 2 include simultaneous measurements of the HW and the PT-K, the blade passage detection and signal division in rotor revolutions will be performed using the HW signal as reference. For the measurements of Set 1, since no HW measurements have been performed, phase averaging will not be applied. The effect of rotating wakes in Plane 1 is anyway smaller due to the mixing taking place between Planes 0 and 1.

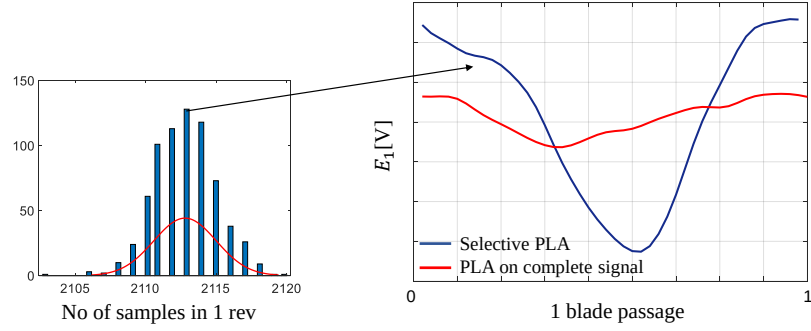


Figure 7.16 – Selection of the most frequent number of samples per revolution (left), comparison between the mean blade passage computed with the selective phase averaging method and the phase averaging on the complete signal (right).

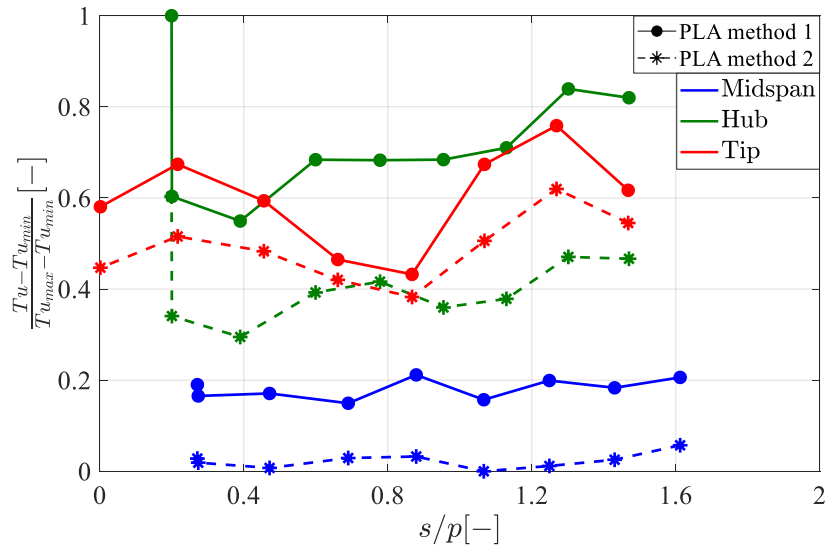


Figure 7.17 – Comparison of non-dimensional turbulence intensity at Plane 0 obtained by PLA methods 1 and 2 for all measurement points.

7.4.2 Turbulence measurement with the PT-K probe

To convert the total pressure fluctuations measured by the PT-K probe to velocity fluctuations, the following expression proposed by Wallace & Davies [8] will be

used:

$$p'_0 = 0.7\rho(1 - 0.175M^4)u'^2 + \rho\bar{U}(1 + 0.5M^2)u' \quad (7.10)$$

A review on the different methods used for the measurement of turbulence with pressure probes has been provided in Section 4.6. The Wallace & Davies equation has been chosen here as it provides a correction for the compressibility effect.

The finite size of the Kulite sensor (diameter of 1.6 mm) limits its spatial resolution and is expected to cause a spectral roll-off at high frequencies (decrease of the measured amplitude), due to its inability to resolve the smaller turbulence eddies. This phenomenon is considered negligible for the Hot Wire probe thanks to its small diameter (5 μm). Lubbock & Oldfield [9, 10] calibrated their probe for this effect and the dynamic response effect against spectra acquired by a Hot Wire. They retrieved a transfer function $G(\kappa)$ for the spatial resolution, as a function of wavenumber κ and a transfer function $F(f)$ for the dynamic response as a function of frequency. In the present case, the dynamic response has been corrected with the transfer function obtained with shock tube testing and corrected for the thermodynamic conditions in the test rig. No calibration has been performed for the spatial resolution, but some considerations can be made based on the literature. Lubbock & Oldfield [10] identified the highest spatial resolution limit of the probe at a wavenumber $\kappa = \frac{2\pi}{d}$, where d is the diameter of the sensor. In the present case, this corresponds to a wavenumber of $\sim 3900 \text{ m}^{-1}$. Some attenuation is nevertheless expected to happen earlier. Looking at the $G(\kappa)$ transfer function of Lubbock & Oldfield [9], it can be seen that deviation from 1 results at a wavenumber corresponding to a value of $\sim \frac{2\pi}{2.5d}$. This corresponds to a wavenumber of $\sim 1500 \text{ m}^{-1}$ in our case. These considerations can be transformed to frequency using the frozen vortex hypothesis as $f = \frac{\bar{U}\kappa}{2\pi}$, with $\bar{U}$ being the mean velocity. More details are given in Section 7.5.1.

7.4.3 Computation of turbulent quantities

The turbulent quantities of interest in this work are the turbulence intensity and the integral length scale. The turbulence intensity is computed as:

$$Tu = \frac{\overline{u'}}{\bar{U}} \quad (7.11)$$

where $\overline{u'}$ is the rms of the velocity fluctuations and $\bar{U}$ is the mean velocity. When phase averaging is applied, $\bar{U}$ is the phase averaged mean velocity $\tilde{U}$, while $\overline{u'}$ is the phase averaged velocity rms. An example of the phase averaged turbulence intensity $\tilde{T}u$ over the period of one rotor revolution T is presented in Fig. 7.18. The phase averaged mean velocity and the instantaneous velocity for one revolution are also presented at the top of the figure. As, expected the turbulence intensity is higher inside the wake. In order to compute the mean value for one measurement

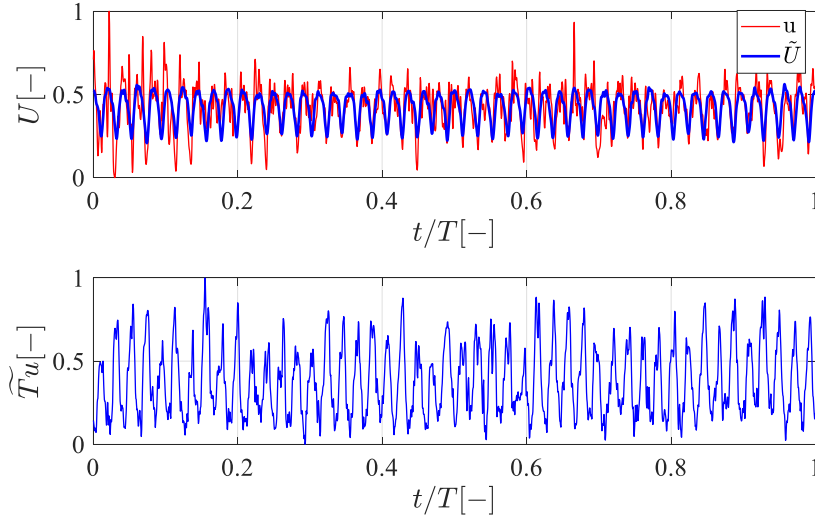


Figure 7.18 – Phase averaged mean velocity (top) and phase averaged turbulence intensity (bottom) over one rotor revolution.

point, the average of the phase-averaged quantities is performed.

Two methods are available for the computation of the integral length scale and are analyzed below. The common point is that both methods compute the integral time scale T_{int} and obtain the length scale L_{int} employing Taylor's frozen vortex hypothesis: $L_{int} = \bar{U} \cdot T_{int}$.

Method 1 The integral time scale can be defined as:

$$T_{int} = \int_0^\infty R_{uu} dt \quad (7.12)$$

where R_{uu} is the time autocorrelation of the velocity signal, presented in Fig. 7.19. As it can be seen, the experimental autocorrelation signal tends to oscillate around zero, therefore instead of integrating the whole signal, the time lag of the first root is often considered as the upper integration limit. This is the method employed here. In the presence of blade rotation, the signal autocorrelation is dominated by the BPF, as is demonstrated in Fig. 7.19. Therefore, in this case, the autocorrelation should be applied to the stochastic fluctuations of the signal. The methodology is the following:

- With the PLA, the signal is divided in rotor revolutions and the mean PLA velocity is computed

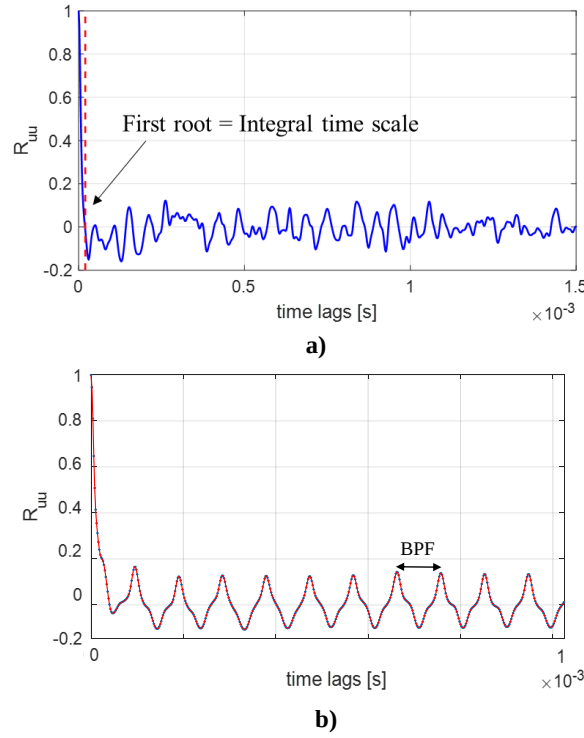


Figure 7.19 – a) Time autocorrelation of the velocity signal, b) time-autocorrelation dominated by the BPF.

- The mean PLA velocity is subtracted from the instantaneous velocity and the stochastic fluctuations are decomposed for each revolution, as depicted in Fig. 7.20.
- The integral time scale is then computed by the autocorrelation of the signal of each revolution.
- The mean integral time scale is finally computed and then the integral length scale is retrieved applying the Taylor's hypothesis.

Method 2 Roach [11] proposed a method to compute the length scale using the turbulent energy spectra. The integral length scale is computed by:

$$L_{int} = \left[\frac{E(f)\bar{U}}{4\bar{u}^2} \right]_{f \rightarrow 0} \quad (7.13)$$

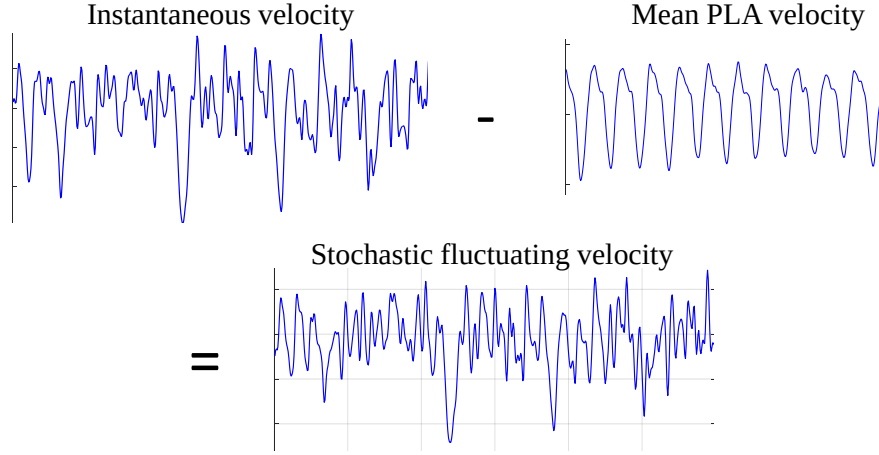


Figure 7.20 – Decomposition of stochastic fluctuations, actual data.

The spectral energy $E(f)$ is extrapolated to zero frequency. Roach [11] proposed as well the following expression for the Kolmogorov length scale η , which is representative of the smallest turbulence scales:

$$\eta = \left[\frac{\overline{U}^2 \overline{u^2}}{2\pi^2 \int_0^\infty f^2 E(f) df} \right] \quad (7.14)$$

The two methods have been compared using the data of Set 2 acquired with the PT-K probe at the outlet of the TVF. The results are given in Figures 7.21 and 7.22. The measurements have been performed for three span positions h over an angular sector s covering two TVF pitches g . The integral length scale values presented have been non-dimensionalized as:

$$L_{int,nd,j} = \frac{L_{int,j} - \min(L_{int,1}, L_{int,2})}{\max(L_{int,1}, L_{int,2}) - \min(L_{int,1}, L_{int,2})}, \text{ where } j = 1 : 2. \quad (7.15)$$

where the indices 1 and 2 refer to Methods 1 and 2 respectively.

It can be seen in Fig. 7.21 that the two methods result in the same integral length scale pattern over the measurement plane. The differences in the computed value are better visible in Fig. 7.22. It can be seen that the differences lie for the most part in the range of $\pm 20\%$, with the maximum difference being 33% for one point. These differences might be on one hand related to the assumptions of the second method, which assumes that the flow is isotropic and homogeneous. On the other hand, they might also be related to the uncertainty in the computation of the

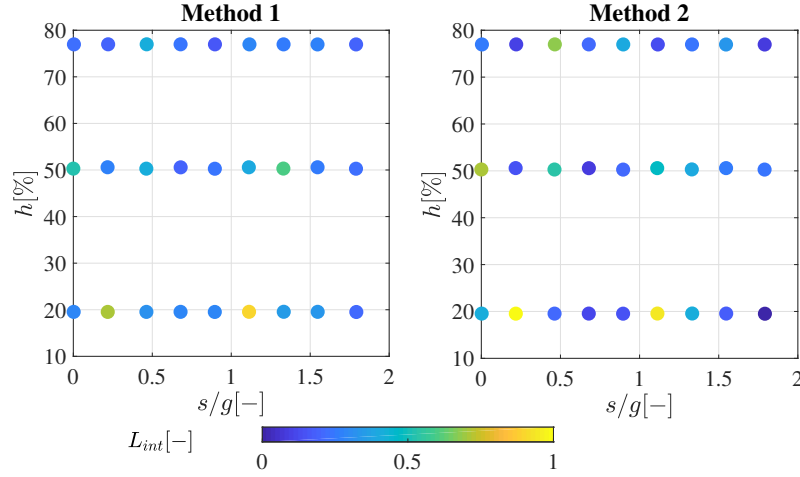


Figure 7.21 – Non-dimensional integral length scale at the TVF outlet, comparison between Methods 1 and 2.

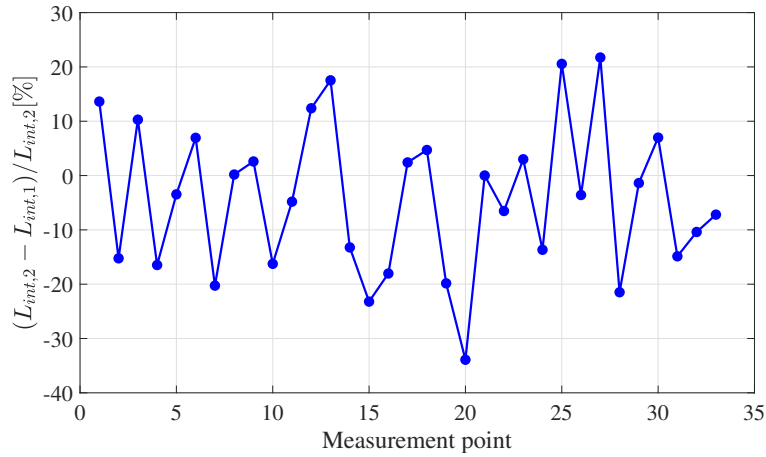


Figure 7.22 – Relative difference in integral length scale values between Methods 1 and 2.

integral time scale in Method 1. Theunissen et al. [12] compared the integral time scale value for different upper integration limits of Eq. 7.12, commonly used in the literature (the first root, the 1/e value, the minimum value). They observed differences up to 50% in the integral time scale value between the different methods. Therefore, we can conclude that the two methods used in this case give consistent results. Method 1 will be used for the data of Set 2 and Method 2 will be used for the data of Set 1. This is because for Set 1 no hot wire measurements were per-

formed, whose signal is used to apply the selective PLA method. The application of Method 1 is therefore not possible, as the autocorrelation is contaminated by the periodic component.

7.5 Results and discussion

7.5.1 Set 1: TVF outlet

The results of Set 1, obtained at the outlet of the TVF (Plane 1) with the PT-K probe are first presented. The fine measurement grid used in this case allows a better insight on the flowfield. The measurement grid covers one pitch g of the TVF at the azimuthal direction. All the results presented hereafter are non-dimensionalized with the minimum and the maximum values at each plane.

The mean total pressure and the rms of the total pressure fluctuations are presented in Fig. 7.23. The wake of the TVF vane is present between 15% and 50% of the pitch, indicated by the low mean total pressure region. A region of high total pressure fluctuations can be observed close to the hub at the one side of the wake.

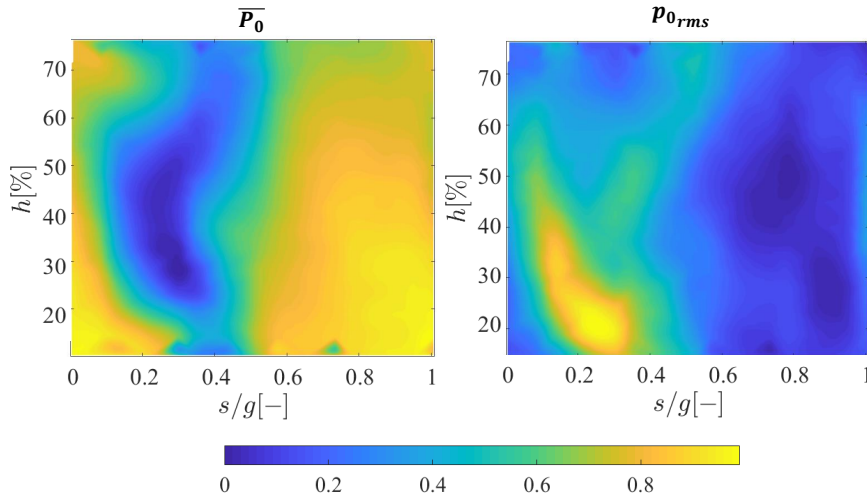


Figure 7.23 – Mean total pressure (left) and total pressure rms (right).

The rms of the velocity fluctuations and the turbulence intensity are presented in Fig. 7.24. The high turbulence intensity region coincides with the wake region. The maximum value of the rms can be observed at the suction side of the wake, due to the high velocity gradients that promote turbulence generation. The integral and kolmogorov length scales are presented in Fig. 7.25. It is reminded that

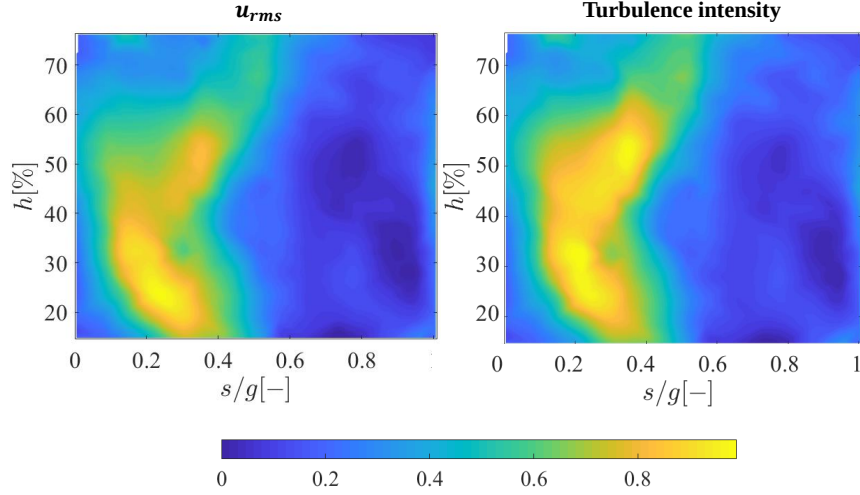


Figure 7.24 – Velocity rms (left) and turbulence intensity (right).

these length scales have been computed by the turbulence spectra, as explained in Section 7.4.3. The computation of the Kolmogorov length scale involves the high frequency part of the spectrum, which can be strongly affected by spatial and temporal resolution limitations of the instrument. Therefore, little confidence is given to this computation, but the results are presented for completeness. In an effort to evaluate the physical meaning of the integral length scale distribution, it is superimposed to the total pressure distribution in Fig. 7.26. This plot clearly shows small values of the integral length scale inside the wake and larger values at the side of the wake region. These large values indicate the presence of highly correlated structures at the wake sides.

Finally, the spectra are presented for two measurement points in Fig. 7.27: one inside the wake (Point B) and one in the clean flow region (Point A). The spectra of the velocity fluctuations, the spectra of the total pressure fluctuations and the PT-K transfer function are plotted together. The BPF and its harmonics can be observed in the spectra, as no phase-averaging has been performed for this dataset. It can be seen that despite the compensation for the dynamic response, there is still some small amplification due to the probe cavity resonance present at the corrected spectra. This can be attributed to the uncertainty in the total temperature in Plane 1. Since no temperature measurements have been performed at this plane, a uniform total temperature is assumed, equal to the mean value in Plane 0. Differences in the mean total temperature will result in different thermodynamically corrected natural frequency and damping and therefore different transfer function, as described in

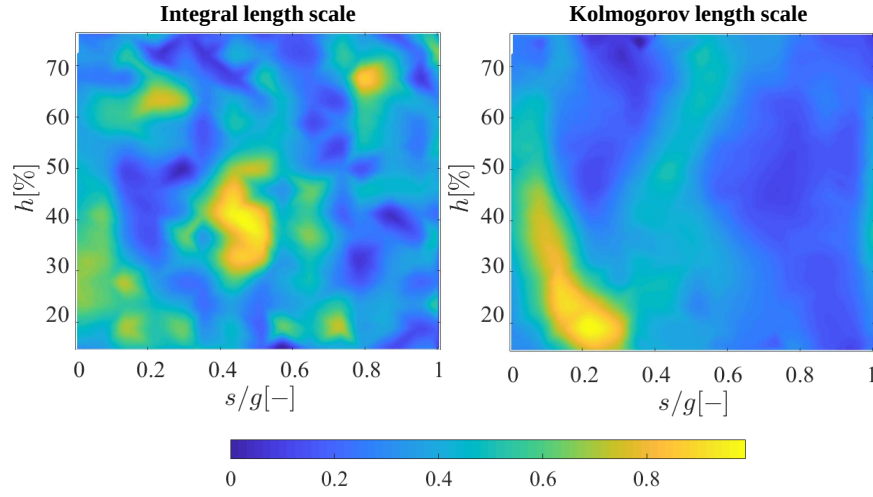


Figure 7.25 – Integral length scale (left) and kolmogorov length scale (right).

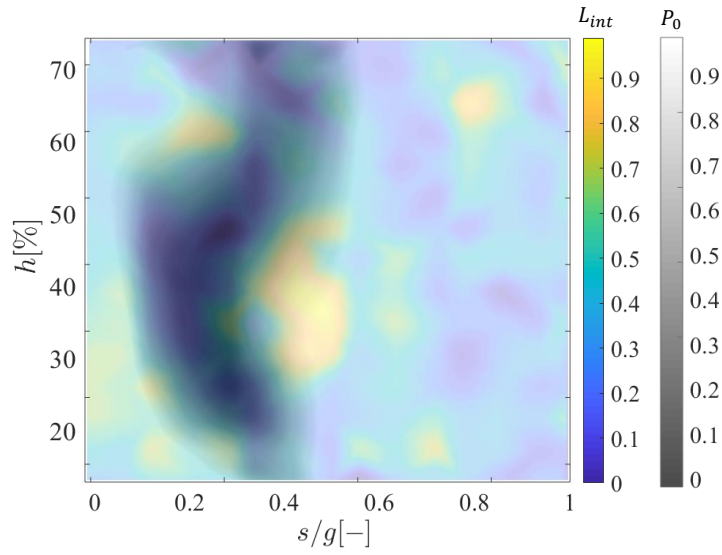


Figure 7.26 – Superposition of total pressure and integral length scale maps.

Section 7.2.3.1.

The spectra can be employed to examine the spatial resolution effect due to the

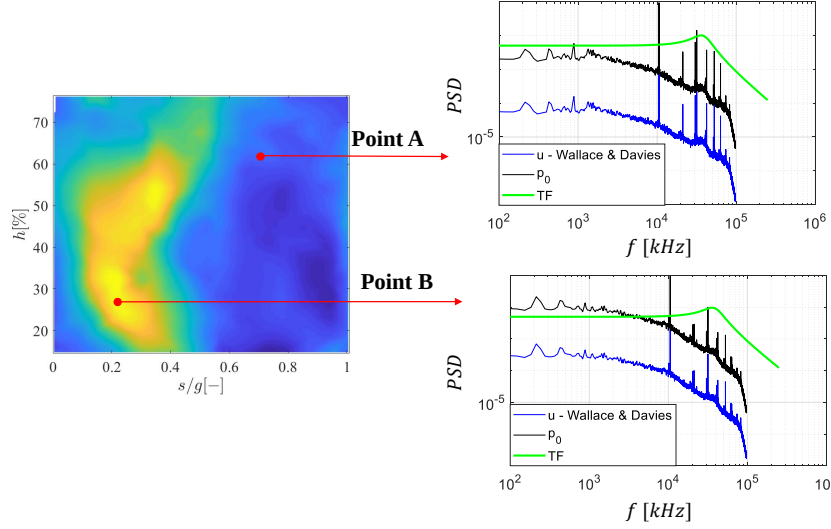


Figure 7.27 – Spectra for two measurement points: *clean flow*(top) and *wake region* (bottom).

finite size of the sensor surface. Following the considerations developed in Section 7.4.3, the frequency for which the spectra roll-off is expected to begin based on the literature is indicated in Fig. 7.28 with the vertical dashed lines. The frequency is computed as $f_{att} = \frac{\kappa_{att} \bar{U}}{2\pi}$. It has to be noted that Fourier filtering has been applied in the spectra, in order to remove the deterministic peaks.

The difference in f_{att} between the two points is therefore due to the mean velocity difference. The velocity rms can be computed as:

$$u_{rms} = \sqrt{\int_0^{f_{limit}} PSD(f) df} \quad (7.16)$$

where PSD is the power spectral density of the velocity fluctuations. By varying the upper integration limit, the contribution of different parts of the spectrum to the total energy is computed. The results are presented in Fig. 7.29. The attenuation frequencies are given with the dashed vertical lines. It can be seen that $\bar{u}'$ has reached 95.9% of its final value for point A and 93.5% for point B by f_{att} . Therefore, the u_{rms} might be affected by spectral roll-off due to the spatial resolution in this case. The computation of the integral length scale, on the other hand, is based on the low-frequency energy plateau and should be therefore unaffected by

the spatial resolution.

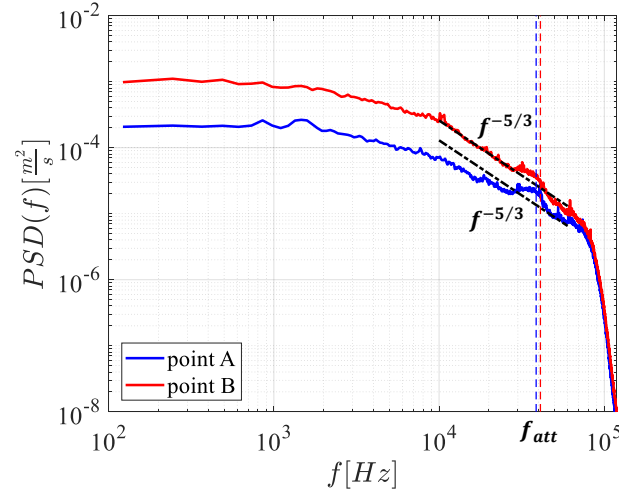


Figure 7.28 – Spectra for points A and B, with attenuation frequencies indicated.

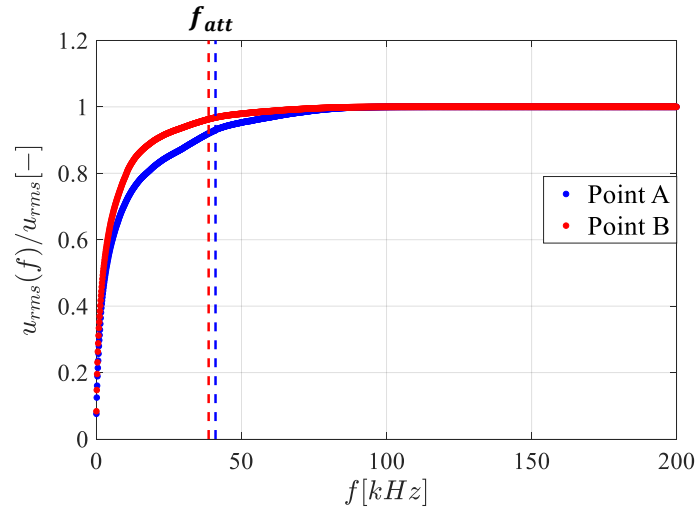


Figure 7.29 – Effect of frequency range considered in the velocity rms value for points A and B.

In Fig. 7.28 the inertial range, characterized by a slope of $f^{-5/3}$ is indicated. For the wake spectra, a clear inertial range can be observed for frequencies higher

than 10 kHz. For the spectra of point A, the existence of this range is not so evident, due to the resonance peak distortion. The presence of the inertial subrange indicates that turbulence in these frequencies is independent of the periodic flow motion, since the spectral energy flux is by definition constant in the inertial subrange. Axelsson & George [13] estimated the dissipation ϵ using the following model for the inertial subrange:

$$F_{11}(\kappa) = \frac{9}{55} \alpha \epsilon^{2/3} \kappa^{-5/3} \quad (7.17)$$

where $F_{11}(\kappa)$ is the 1D wavenumber spectrum and α is the Kolmogorov constant, usually taken equal to 1.5. The same approach will be followed here. The wavenumber spectra are computed as $F_{11}(\kappa) = \frac{\bar{U}}{2\pi} PSD(f)$ where $\bar{U}$ is the mean velocity for each point. The spectra are then plotted multiplied by $\kappa^{5/3}$, as can be seen in Fig. 7.30. Plotting the spectra in this way, allows to better identify the inertial subrange region, which is characterized by a plateau, before the sharp decrease at the dissipation range. Due to the remaining effect of the cavity resonance, the

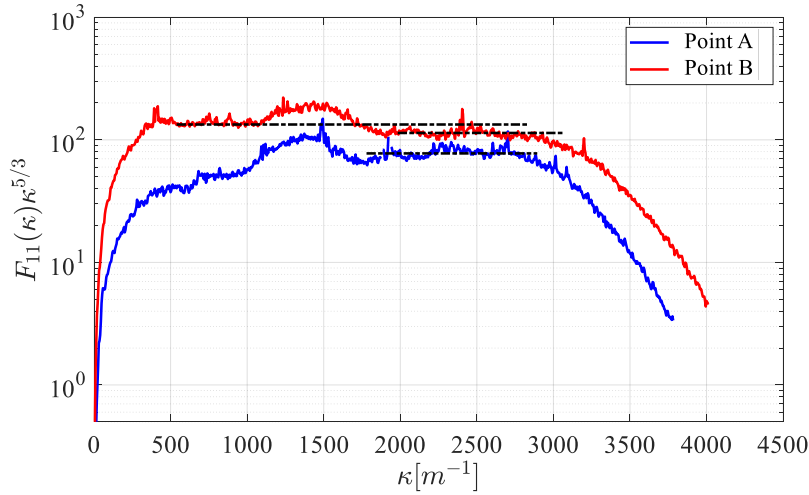


Figure 7.30 – Wavenumber spectra with inertial subrange plateau for points A and B.

spectra exhibit some distortion in the range of interest. For point B, the plateau seems to start at a wavenumber of $\sim 500 \text{ m}^{-1}$. After the amplification due to the resonance, a second plateau can be identified at a lower level. The dissipation will be computed for both cases, since it is not clear which value should be chosen. For point A, the plateau is present at higher wavenumbers between $2000\text{--}2700 \text{ m}^{-1}$. The dissipation ϵ is computed as $5600 \text{ m}^2/\text{s}^3$ for point A and as $12\,000 \text{ m}^2/\text{s}^3$ and $9800 \text{ m}^2/\text{s}^3$ for the wake point, using the first and the second plateau respectively.

7.5.2 Set 2: TVF inlet and outlet

During this test, measurements have been performed simultaneously at the inlet and outlet of the TVF, using the HW and the PT-K probe respectively. The measurements have been performed on a coarse grid, covering two pitches of the second stator of the HPT at the inlet and two pitches of the TVF at the outlet. Therefore, these results do not provide an insight on the flowfield but allow the computation of the turbulence quantities in both planes, to support future numerical simulations on this configuration. In order to allow an evaluation of the turbulence evolution along the duct, the results presented hereafter will be non-dimensionalized with the minimum and maximum value of each quantity considering both planes.

As far as the HW measurements are concerned, spectral analysis revealed the presence of a strong peak at a high frequency for wire 1, as indicated in Fig. 7.31. This peak is present for all measurement points and it is attributed to vibrations of the sensor/prongs, maybe as a result of the wake interaction between the two sensors. This peak has a strong effect on the spectra of sensor 2 that can not be corrected by filtering. The results of sensor 2 are therefore considered more reliable and will be presented in this section.

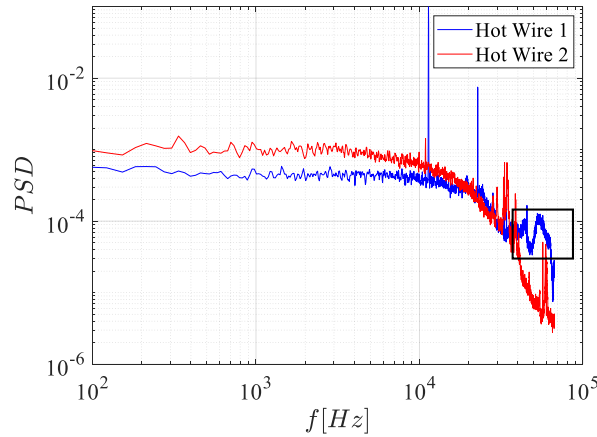


Figure 7.31 – Raw hot wire spectra for both sensors.

The turbulence intensity and integral length scale are presented Fig. 7.32 and 7.33 respectively. It can be seen that both the turbulence intensity and length scale decrease from the inlet to the outlet of the TVF. At the inlet plane, the turbulence intensity is lower at mid-span and increases at hub and tip, with the highest values observed at the hub. For the integral length scale no clear trend can be identified based on its span-wise or azimuthal distribution.

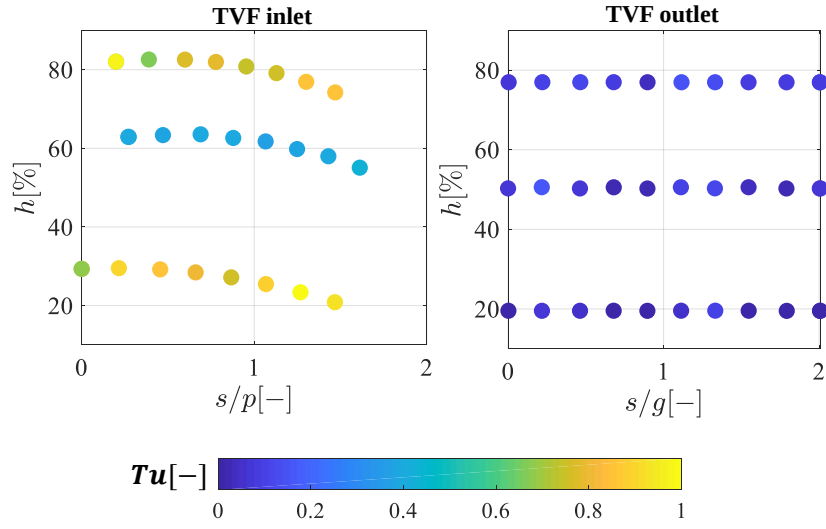


Figure 7.32 – Turbulence intensity at the inlet and outlet of the TVF.

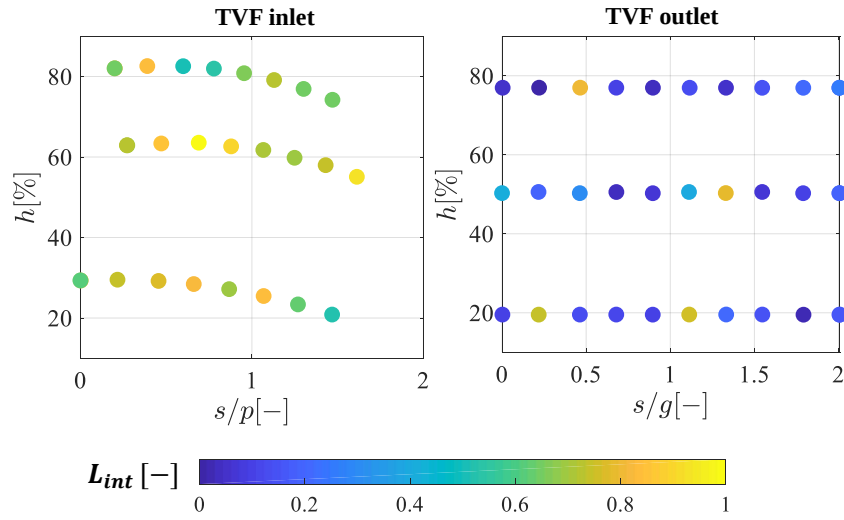


Figure 7.33 – Integral length scale at the inlet and outlet of the TVF.

7.6 Conclusions

In this chapter, an experimental campaign consisting of measurements of turbulence quantities at an industrial test rig has been presented. Turbulence measure-

ments have been performed at the inlet and outlet of an inter-turbine duct with guide vanes with a Hot Wire probe and a fast response pressure probe respectively. The duct is placed downstream of a high pressure turbine stage, providing an engine-realistic inflow at the duct. The design concept of the instrumentation has been explained and their calibration procedure has been described.

Due to the sensors of the HW breaking during the installation of the probe in the facility, the measurements were performed with a non-calibrated Hot Wire. Since there was no possibility to re-perform a calibration, an in-situ calibration procedure was developed, employing time-averaged measurements of the thermodynamic quantities at the measurement plane. CFD data at the rotor frame of reference were also employed to enrich the calibration curve. The minimization of the error on the mean velocity was used as criterion to define the wire temperature, which is a critical parameter in the static calibration procedure. The angular decomposition could not be performed and the 2D statistics on the blade to blade plane have not been retrieved. Nevertheless, a turbulence intensity and integral length scale value have been acquired, containing contributions of both the longitudinal and the lateral component.

Since the flow downstream of the rotor is highly periodic, phase-averaging needs to be performed in order to extract the deterministic component from the signal and retrieve the stochastic fluctuations. At the absence of a signal indicating the rotor revolutions (1/revolution signal), a selective phase-averaging method was developed, based on the automatic detection of the blade passing from the hot wire signal. This allows the division of the signal in rotor revolutions. To account for changes in the rotor speed only the most similar rotations in terms of period have been selected. Comparison with a phase-averaging approach based on the RPM revealed the superiority of the selective phase-averaging approach.

The computation of the integral length scale at the presence of periodic unsteadiness is not straightforward. As has been demonstrated, the autocorrelation of the signal is contaminated by the BPF in this case. Two different methods have been used in the present work to confront this problem: using the low frequency part of the turbulence spectrum and using the autocorrelation function of the stochastic signal after phase-averaging. Comparison between the two methods gives a difference up to $\pm 20\%$, which is less than the variation found in literature using the second method with different integration limits for the autocorrelation. It can be thus concluded that the two methods can be used interchangeably, with the application of the spectral method being favored when phase-averaging can not be performed.

The turbulence intensity and integral length scale have been acquired at the TVF inlet, while the distribution of the mean total pressure and turbulence quantities has been obtained at the TVF outlet, providing an insight in the flowfield. The effect of the spatial resolution of the PT-K probe is evaluated using literature data,

suggesting that the attenuation starts at a wavenumber $\sim \frac{2\pi}{2.5d}$. 93 to 95% of the total spectral energy is present up to this limit, implying a small effect on the intensity of the fluctuations. The integral length scale computation is not affected by the spatial resolution. In future works, a better investigation of this effect should be performed and the possibility to reduce the sensing area of the probe should be considered. A spectral analysis of the results reveals the presence of the inertial subrange in the turbulence spectrum, characterized by a slope of $f^{-5/3}$. This allows the estimation of the turbulent kinetic energy dissipation rate ϵ , by the fitting of a spectral model function to the 1D wavenumber spectrum.

In conclusion, this work presents a useful example of turbulence measurements in a complex environment from the point of view of post-processing methodology, and also an example of detailed turbulence measurements with fast response pressure probes. The turbulence intensity, integral length scale and dissipation have been provided, demonstrating the capability of fast response pressure probes to measure turbulence. A complementary numerical activity is currently taking place, consisting of high fidelity simulations of the present configuration. The experimental results will be used to support and validate the simulations, leading to a description of the mechanisms driving the turbulence evolution through the duct.

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8

Conclusions

8.1 Overview

In this work, a series of considerations related to the difficulties in performing reliable turbulence measurements in complex flows have been developed. The focus is placed on intrusive measurement techniques which are of wide application in both industry and academia: Constant Temperature Hot Wire Anemometry and fast response pressure probes. The aforementioned techniques have been analyzed in terms of calibration, application and post-processing methods, while particular focus is placed on the uncertainty computation of turbulence statistics. The thesis concludes with two test cases of turbulence measurements in turbomachinery flows, serving as practical guides for the application of the proposed concepts. The aim of the thesis is to propose a series of tools to improve the quality of turbulence measurements in industrial flow applications, providing valuable data for the support of high-fidelity simulations.

The theoretical background on the operation of CTHWA has been provided and different types of calibration methods have been presented. The static response of the CTHWA technique has been analyzed in terms of its sensitivity to the different flow parameters. This behaviour has been examined in different flow regimes, by computing the sensitivities from the non-dimensional calibration curves.

The different aspects of the dynamic response of CTHWA (low-frequency attenuation due to heat conduction, non-optimal adjustment, non-linearity) have been analyzed and a model considering all effects has been proposed. A strategy to reduce the experimental errors due to the dynamic response is presented. Finally, an evaluation of the errors on the turbulence measurements due to the dynamic response of the anemometer is performed. A synthetic turbulence generation method is employed to reproduce a realistic turbulent input signal and evaluate the errors as a function of the input turbulence parameters.

The complete calibration procedure (static, aerodynamic and dynamic calibration) of fast response pressure probes has been presented in detail. Particular attention has been paid to the development of a robust post-processing methodology for the computation of the transfer function by shock tube tests, taking into account also non-ideality effects in the shock tube flowfield. Moreover, a method for the estimation of the uncertainty in the transfer function has been developed, considering sensor uncertainty, non-ideality effects and alignment issues.

The macro uncertainty sources contributing to the uncertainty of turbulence statistics, categorized as measurement and statistical uncertainty, have been examined. Classical methods typically used for their quantification, the Taylor series method and analytical error estimates respectively, are compared with more advanced methods, namely the Monte Carlo method and the Moving Block Bootstrap method. Simple measurement cases are employed to demonstrate their application and showcase the differences.

The complete uncertainty quantification methodology has then been applied to CTHWA measurements performed at the inlet of a short duration turbine test rig. The spanwise distributions of the mean velocity, turbulence intensity, length scales and skewness factor have been presented, with the 95% confidence intervals. The contributions of the two uncertainty components for each quantity have been evaluated as well. The application of the method is presented step by step, in order to provide a reference for its application in complex cases in the future.

Finally, turbulence measurements in an engine-representative test case are presented with both CTHWA and fast response probes. The turbulence flowfield at the inlet and outlet of an inter-turbine duct has been characterized. Particular focus has been placed on the required data processing methods at the presence of highly periodic unsteadiness: at the absence of a 1/rotor revolution signal, a selective phase averaging method has been developed based on the selection of the most similar rotations. Special emphasis is placed on the capability of the fast response probe to measure turbulence parameters other than the turbulence intensity.

8.2 Conclusions

It has been clearly demonstrated in this work that the performance of reliable turbulence measurements requires attention to all steps of the process. The major conclusions drawn are outlined hereafter, in an effort to provide a succinct take-away message for the reader.

As the response of CTHWA depends on the flow regime, the computation of its sensitivities to velocity, density and total temperature provides an evaluation of the feasibility of the technique application in each case. In the present work, it has been shown that the sensitivities can be computed by the non-dimensional calibration without the need of varying independently every variable, which would require lengthy calibrations in specially-designed facilities. The measurement of turbulence parameters is more difficult in certain cases, like low-density flows, where the sensitivity to density can be higher than the sensitivity to velocity by a factor of 10.

It has been demonstrated that the dynamic response of CTHWA can have a dramatic effect on the measurement of turbulence when not properly accounted for. It is suggested that the dynamic response transfer function is acquired when performing the square wave test and used to correct the measurement data during the post-processing. The detailed method is provided in Chapter 3. This procedure corrects the spectra for attenuation or amplification due to non-optimal adjustment. This method is based on the assumption that the transfer function obtained by the square wave is representative, an issue that has not been resolved in this study and should be further investigated in future work. The errors on the measured turbulence parameters due to the finite anemometer bandwidth can then be evaluated as a function of the parameter f_L/f_{cut} (representing the ratio of a frequency related to the integral length scale and the anemometer's cut-off) and the flow parameters. The non-linearity has a negligible effect for uniform flow with superimposed turbulence, even when high turbulence levels (up to 22%) are considered.

It has been shown that the transfer function computation from shock tube testing is subject to contamination due to post-shock disturbances that exist in a non-ideal shock tube flow. The effect of such disturbances can be minimized following the proposed post-processing method, valid for probes featuring a resonance frequency higher than the frequency of the disturbances. The uncertainty of the transfer function has also been evaluated and it has been found that its amplitude can be computed with a much lower uncertainty with respect to the phase. It is thus suggested that when the transfer function is modelled as a LTI system, methods for which the damping and natural frequency are computed by the amplitude plot should be preferred.

Concerning the uncertainty computation, it has been shown that the Monte Carlo method should be favored for the computation of the measurement uncer-

tainty when possible, as non-normal input distributions and non-linear data reduction algorithms can lead to deviations when the Taylor series method is used. Concerning the statistical uncertainty, methods that do not assume a priori normal distributions should be favored, as the distribution of the velocity fluctuations does not need to be normal. The Moving Block Bootstrap method is therefore proposed for the uncertainty computation for time series of arbitrary distributions and for any parameter of interest without the need of deriving complex expressions. The application of the complete uncertainty quantification methodology to the inlet turbulence characterization of a short duration turbine test rig reveals the importance of the statistical uncertainty for short-duration tests: the statistical uncertainty for the turbulence intensity and integral length scale represents $\sim 95\%$ of the total uncertainty in this case. The method is non-parametric and easy to implement and applicable to any type of experiment and measurement technique.

The performance of turbulence measurements in a turbomachinery environment poses additional challenges due to the large levels of periodic unsteadiness. In the present work, at the absence of an once-per-revolution reference signal to allow the division of the signal in rotor revolutions, a signal based method has been developed automatically detecting the blade passages and selecting the most similar rotations in terms of period (number of samples in each rotation). The advantageous performance of this method with respect to a division based on the RPM value has been showcased, resulting in 3% lower levels of turbulence intensity. A comparison on the available methods to compute the integral length scales in such conditions has also been performed. It has been demonstrated that using the autocorrelation method based on the stochastic signal or using a spectral based method on the complete signal results in differences up to 20%, which is less than differences due to different upper integration limits in the autocorrelation method as stated in the literature. It is thus concluded that the two methods can be used interchangeably.

It is suggested that with an appropriate calibration procedure, fast response pressure probes can be used to provide also spectral information and thus grade 4 validation level data in addition to turbulence intensity levels (grade 3 validation level) as provided by most studies. The probe spectra in the test case 2 provide a physical turbulence cascade characterized by the $f^{-5/3}$ slope at the inertial sub-range. The integral length scale and dissipation estimates are provided. It should be nevertheless noted that due to the sensor size, spatial resolution effects require further attention for the case of turbulence measurements with fast response pressure probes.

In both test cases presented in this study, the turbulence intensity and integral length scale parameters have been provided. Nevertheless, it has been clearly demonstrated that the measurement errors are lower for turbulent properties that do not use the mean velocity in their computation, like the turbulent kinetic en-

ergy and the time scales. It is therefore concluded that these properties should be preferred for validation of numerical simulations. This is particularly important for the time scales, since they are free from the error related to the Taylor's frozen vortex hypothesis, which is used to retrieve the length scales.

As a closing remark, it is concluded that CTHWA and fast response pressure probes are valuable tools for the measurement of turbulence in complex environments. It has been demonstrated that with proper calibration and post-processing treatments they can produce results with increased confidence. In comparison to optical techniques like Laser Doppler Velocimetry (LDV), they demonstrate the advantage of easier application to different types of test rigs without the requirements of seeding and optical access. This is particularly important for industrial applications where access to probes is anyway foreseen for standard instrumentation (e.g. for performance characterization), allowing the performance of turbulence measurements with no additional facility modifications.

8.3 Perspectives

As a general recommendation, it is the author's opinion that the improvement of the quality of turbulence measurements in complex flows can only be achieved with a systematic design of experiments approach, targeting all aspects of the campaign: from the instrumentation design and the definition of the calibration procedures to the tuning of the test campaign parameters and the data processing algorithms. Such an approach requires well-defined measurement targets (turbulent quantities to be measured and the accepted uncertainty level), as well as an a priori definition or estimation of the expected flow conditions, including an approximation of the turbulence flowfield parameters. Moreover, this procedure implies the existence of models for each step of the process. The different tools developed in the present work could be implemented at such extent e.g. to model the static and dynamic behaviour of hot wire anemometers in order to define the optimal design for a specific requirement. Similarly, the test campaign parameters like sampling frequency and acquisition time should be defined in order to reach the accepted statistical uncertainty, while respecting limitations related to available testing time and number of experiments. In this framework, specific points worthy of further investigation are analyzed below.

It has been shown that the CTHWA technique can be significantly affected by total temperature and density fluctuations, depending on the flow regime. In such a case, computation of the intensity of these fluctuations by e.g. DNS, could be used in order to evaluate the magnitude of this effect and possibly correct for it. This would be a situation where simulations performed prior to the experiment can be used to improve the experimental campaign.

One point that has not received enough attention is the effect the turbulence

flowfield has on the measurements, both on CTHWA and fast response pressure probes but also on other measurement techniques. Looking in the literature, only turbulence intensity effects are usually considered:

- The effect of turbulence intensity from the point of view of increased unsteady angular fluctuations that may exceed the directional range of measurement of a probe has been analyzed.
- In the case of CTHWA measurements, it has been suggested that large values of turbulence intensity may imply the invalidity of the small fluctuations assumption.
- When performing pneumatic pressure measurements, it has been demonstrated that the mean pressure values acquired might be artificially increased at the presence of high turbulence intensity.

All the aforementioned examples consider the effect of turbulence in terms of large amplitude fluctuations, while its spectral characteristics are commonly neglected. An effort should be done to consider such effects, especially for high frequency measurements. The dynamic response error analysis as a function of the inlet turbulence parameters described in Chapter 3.6 is a step towards this direction. The use of synthetic turbulence methods, like the one used in the present study, could be further employed in the preliminary stages of the instrumentation design process to assess the effects of a realistic turbulence flowfield. Eddy-resolving simulations, like LES, could be then used at the advanced design stages for a more accurate evaluation.

The dependence of the statistical uncertainty on the flowfield parameters is an area requiring further investigation. The required signal duration to reach a certain level of statistical convergence can be estimated based on the statistical parameter to be computed and the degree of correlation of the signal. It is unclear if the turbulence structure and other flow parameters, like the skewness and kurtosis values or the Reynolds number, may also affect the convergence rate. The investigation of such dependencies and, eventually, the provision of correlations would be imperative for the better estimation of the statistical convergence rate and the optimization of the experimental campaigns with a view to uncertainty minimization. Different methods to evaluate the statistical uncertainty should also be examined, aiming to decrease the bias errors when correlated time series are treated.

Considering the performance of turbulence measurements with single-sensor fast response pressure probes, all research performed up to now, including the present work, consists in the use of correlations to model the mean square pressure fluctuations. Future work could consider the use of spectral models for the pressure fluctuations, thus modelling the distribution of the static pressure fluctuations energy in the frequency domain. Such an approach has been used in the

field of acoustics, but has never been considered in the turbulence measurement framework, to the author's knowledge.



List of publications

Journal articles

- E. Boufidi, S. Lavagnoli, and F. Fontaneto, 2020. *An Uncertainty Quantification Method for Turbulence Statistics Measured by Hot Wire Anemometry in a Short Duration Wind Tunnel*, Journal of Engineering for Gas Turbines and Power, 142(3).
- E. Boufidi, M. Alati, F. Fontaneto, and S. Lavagnoli, 2019. *Design and Testing of a Miniaturized Five-Hole Fast Response Pressure Probe With Large Frequency Bandwidth And High Angular Sensitivity*. Journal of Engineering for Gas Turbines and Power, 141(10).

Conference papers

- E. Boufidi, and F. Fontaneto, *The Dynamic Calibration Uncertainty of Fast Response Pressure Probes*, ASME Turbo Expo 2020, 21-25 September 2020, Virtual, Online, GT2020-14680.
- E. Boufidi, M. Cottes, and F. Fontaneto, *CTHWA Dynamic Response Effects on Turbulence Measurements in Turbomachinery Flows*, XXIV Symposium on Measuring Techniques in Turbomachinery, 29-31 August 2018, Prague, Czech Republic.

- E. Boufidi, and F. Fontaneto, *Towards a More Reliable Application of Hot-Wire Anemometry in Complex Compressible Flows*, XXIII Symposium on Measuring Techniques in Turbomachinery, 1-2 September 2016, Stuttgart, Germany.

Poster presentations

- E. Boufidi, and F. Fontaneto, *A Methodology for Advanced Turbulence Measurements in Turbomachinery Flows*, ASME Turbo Expo 2018, 11-15 June 2018, Oslo, Norway.