

MODELLING THE FLOW AROUND ISLANDS IN RIVERS USING A ONE-DIMENSIONAL APPROACH

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ABSTRACT

One-dimensional models are widely used for the simulation of flows in rivers. Around islands, the flow is divided and the discharge is distributed in the different branches. Such flow in a bifurcation is a typical two-dimensional feature, and representing the accurate partition of discharge might be a challenge in one-dimensional models depending on the flow conditions. In the present paper, a one-dimensional model is presented where the connections between the main river and the sub-channels are modeled as inner boundary conditions. These conditions are defined considering characteristic paths and conservation of the water head. Four flow configurations are considered: (i) a subcritical junction, (ii) a subcritical bifurcation, (iii) a supercritical junction and (iv) a supercritical bifurcation. For the supercritical bifurcation, an additional condition is required and it is proposed here to partition the discharge in proportion to an equivalent cross-section width. The new one-dimensional model has been validated by comparing the results with those from a two-dimensional model for the four configurations. The comparison showed that the one-dimensional model predicts correctly the discharge and the water depth in each branch. Finally, the one-dimensional model was used to simulate the flow in a reach of the Meuse River (Belgium), showing good agreement with field data.

1. INTRODUCTION

One-dimensional models for river flow simulations are still widely used and often constitute the only realistic option when results are expected in a very short time. In addition, in many cases, especially for smaller streams, no detailed two-dimensional bathymetric data are available and the setup of a detailed two-dimensional model is not possible. In these cases, one-dimensional models are often preferred, as they are faster and simpler to build. However, one limit of using 1D models is that they are, of course, unable to capture 2D phenomena that can occur for example at junctions, bifurcations, or around islands where these two cases are combined. There, the channel is divided into two sub-channels around the island before merging again.

To represent this behavior in a one-dimensional model, several options are available. First, the island can be simply ignored by either combining the cross-sections on both sides of the island or creating a large section englobing the island and the two sub-channels. In these approaches, the flow velocity is considered as being the same in the two branches around the island, which constitutes a limitation when the flow in each branch is significantly different. Secondly, the junction can be modeled using a local 2D model (Soares-Frazão and Zech 2002, Ghostine et al. 2015). This method is also used to model the interactions between flood plains and main channel during a flood (Finaud-Guyot et al. 2011; Morales-Hernández et al. 2013). Finally, the junction can be represented using an “internal boundary condition”, which allows staying in a purely one-dimensional framework. In this latter case, corresponding to the approach adopted here, the river reach containing an island is divided into four linked parts: (i) the upstream channel, (ii-iii) the two sub-channels on both sides of the island and (iv) the downstream channel.

This approach has been first used by Chow (1959) for steady subcritical flows: an iterative procedure is developed for the discharge repartition in the sub-channels until an equality of the water level is reached in the upstream channel. Instead of using the water level equality, Wylie (1972) proposed the use of the water head (or energy) equality. His model, also, was developed for steady subcritical flows. For unsteady flows, Garcia-Navarro and Saviron (1992) developed a model to solve the convergence (or junction) in subcritical and supercritical flows using the characteristic paths. The model developed here uses the characteristic paths method and the conservation of water head to represent both subcritical connections (junction and bifurcation) and supercritical junctions. For the supercritical bifurcation, where one equation is missing to close the system, it is proposed to partition the discharge according to the available width of each sub-channel.

The research focuses on the flow around islands in a one-dimensional framework such as presented in Figure 1a. Therefore, the bifurcation at the upstream end of the island is represented as in Figure 1b, where only the case with equal and small angles ($\alpha_1 = \alpha_2 < 90^\circ$) will be considered. This contrasts with other works presented in the literature focusing on the flow in crossroads (e.g. Mignot et al. 2011;) where the angles are equal to 90° or in confluences and bifurcations (e.g. Ghostine et al. 2012) where both angles are different. In order words, the approach proposed here focuses on the discharge distribution in the branches around an island assuming that the global flow direction is one-dimensional along the river thalweg.

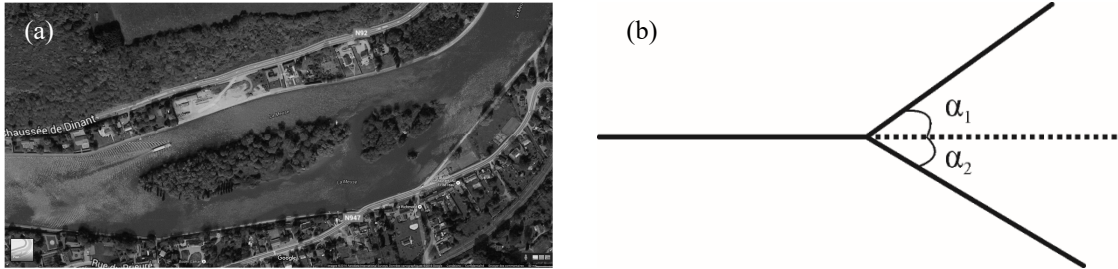


Figure 1: (a) Islands near Yvoir, Belgium (credits Google); (b) Description of the bifurcation upstream of an island

2. GOVERNING EQUATIONS AND NUMERICAL SCHEME

2.1 One-dimensional Saint-Venant equations in conservative form

The one-dimensional Saint-Venant equations are written in conservative form that is best suited for flows in arbitrary topographies featuring sudden changes in the cross-section shape (Petaccia et al., 2013). Therefore, the equations read

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad (1)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{Q^2}{A} + gI_1 \right) = g \left(I_2 + A(S_0 - S_f) \right) \quad (2)$$

where Q is the discharge, A the wetted area, $S_0 = -dz_b/dx$ the bed slope with z_b the bed level, more precisely the thalweg level. The term $gI_1 = g \int_0^h (h-\eta)b \, d\eta$ (with η a local variable for the depth integration and $b(x, \eta)$ the width of the channel at a determined depth as sketched in Figure 2) represents the hydrostatic pressure thrust while $gI_2 = g \int_0^h (h-\eta) \frac{\partial b}{\partial x} d\eta$ represents the longitudinal component of the lateral pressure due to the longitudinal width changes. The friction is modeled using the Manning equation as $S_f = n^2 Q^2 P^{-4/3} A^{-10/3}$ with the Manning roughness coefficient n and the wetted perimeter P .

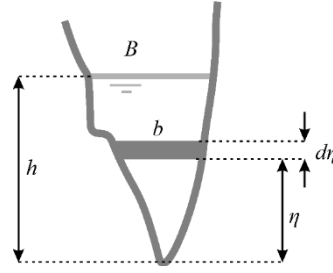


Figure 2: Definition of the hydrostatic pressure

The system of equations (1) and (2) can be written in vector form as

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = \mathbf{S} \quad (3)$$

with $\mathbf{U} = [A; Q]^T$ the vector of conserved variables, $\mathbf{F} = [Q; Q^2/A + gI_f]^T = [Q; \Sigma]^T$ the vector of fluxes and $\mathbf{S} = [0; gI_f + gA(S_0 - S_f)]$ the source terms.

2.2 Finite-volume resolution of the equations

The first-order finite-volume discretization of (3) is given by

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n + \frac{\Delta t}{\Delta x} (\mathbf{F}_{i-1/2}^* - \mathbf{F}_{i+1/2}^*) + \mathbf{S} \Delta t \quad (4)$$

where the superscript n refers to the time step and the subscript i to the spatial step. To compute the fluxes $\mathbf{F}_{i+1/2}^*$, the Augmented Roe solver with energy balanced (Murillo and Garcia-Navarro, 2014) was used. This model, developed for arbitrary cross-sections, was validated against a series of test cases by Franzini and Soares-Frazão (2016) and showed good results especially in terms of discharge conservation in steady flows. The particular feature of this solver is that all source terms, i.e. topographical and friction terms, are included in the fluxes computation. The water head (or energy) conservation is included in the model by combining linearly two expressions of the topographical sources terms.

3. INTERNAL BOUNDARY CONDITIONS

This section presents the different internal boundary conditions used to represent the flow at junctions and bifurcations. These boundary conditions are expressed using the characteristics, along which the Saint-Venant equations can be expressed as ordinary differential equations (Mahmood and Yevjevich 1975). The negative and positive characteristics are defined by

$$C^\mp \equiv \frac{dx}{dt} = u \mp c \quad (5)$$

Along these curves, the Saint-Venant equations become (6) and (7), also known as compatibility equations, written here without source terms

$$C^\mp \equiv \frac{dQ}{dt} - (u \pm c) \frac{dA}{dt} = 0 \quad (6)$$

$$C^\mp \equiv \frac{d\Sigma}{dt} - (u \pm c) \frac{dQ}{dt} = 0 \quad (7)$$

The internal boundary conditions will be expressed using these compatibility equations. As these expressions are based on the same principles of propagation of information as in Roe scheme, these conditions are fully compatible with the expressions for the fluxes used outside the junctions and bifurcations. Depending on the flow conditions, sub- or supercritical, information is conveyed in the system

from both upstream and downstream of the junction, or only from upstream. So, the four possible flow situations described below are subcritical junction, subcritical bifurcation, supercritical junction and supercritical bifurcation. This latter case requires special attention as only the upstream information is available from the characteristics, so one information is missing and additional conditions are required.

A typical situation of bifurcation in two sub-channels, as it can occur upstream of an island, is represented in Figure 3a, while a junction is sketched in Figure 3b. Based on these sketches, the following subscripts will be used throughout the paper: M for the main channel, $S1$ and $S2$ for the two sub-channels. As for all following cases, the unknowns are the fluxes of mass and momentum for each branch. There are thus six unknowns in total and therefore six equations are needed, based on the characteristics as sketched in Figure 4.

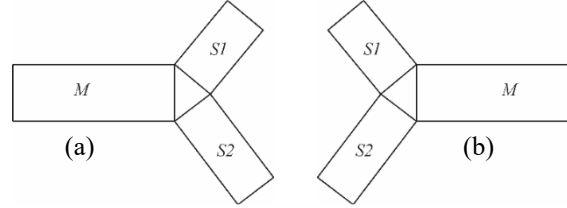


Figure 3: Representation of the connections: (a) Bifurcation and (b) Junction

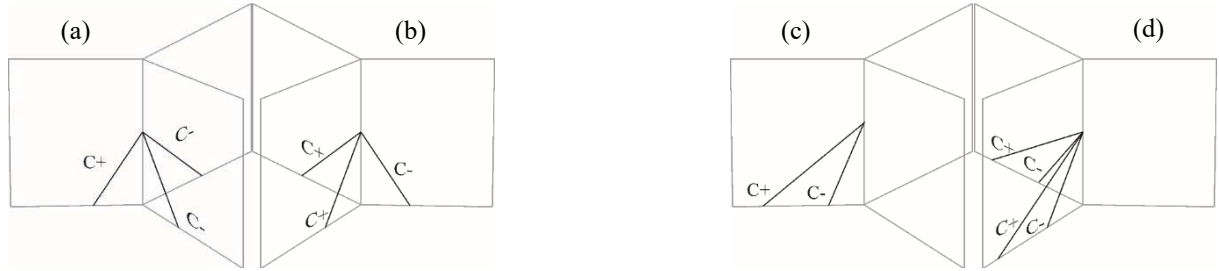


Figure 4: Characteristics at (a) the subcritical bifurcation, (b) the subcritical junction, (c) the supercritical bifurcation and (d) the supercritical junction

3.1 Subcritical junction

For the particular case of the subcritical junction (Figure 3b and Figure 4b), the compatibility equations (6) and (7) are discretized at the interface using the expressions for the positive characteristic for the sub-channels $S1$ and $S2$ and the negative characteristic for the main channel M to convey the information towards the interface. So, the mass fluxes can be expressed as

$$Q_{Sj}^* = Q_{Sj}^n + (\bar{u} - \bar{c})(A_{Sj}^* - A_{Sj}^n) \quad (8)$$

$$Q_M^* = Q_M^n + (\bar{u} + \bar{c})(A_M^* - A_M^n) \quad (9)$$

The superscript $*$ refers to the value at the interface, the other values being taken in the cells and $\bar{u}$ and $\bar{c}$ are the arithmetic average of the velocities and celerities, respectively. As only two equations (8) for the subchannels and one equation (9) are available with the characteristics, three other relations are needed to close the system. First, the conservation of discharge at the interface is imposed:

$$Q_{S1}^* + Q_{S2}^* = Q_M^* \quad (10)$$

Then, the last two equations are the conservation of the water head between $S1$ and M , and between $S2$ and M . They are written as

$$z_{b,Sj} + \frac{A_{Sj}^*}{B_{e,Sj}} + \frac{u_{Sj}^2}{2g} = z_{b,M} + \frac{A_M^*}{B_{e,M}} + \frac{u_M^2}{2g} + S_{f,Sj \rightarrow M} \Delta x \quad (11)$$

With B_e the equivalent, average width of the channel, defined as $B_e = A/h$, with h the water depth, and $S_{f,Sj \rightarrow M} \Delta x$ the friction loss between the sub-channel j and the main channel. By combining these six

equations, the wetted area at the interface of the main channel A_M^* can be determined. The other wetted areas A_{S1}^* and A_{S2}^* are then simply computed using the energy balances. Finally, using the compatibility equations (6) and (7), the mass and momentum fluxes are written as

$$Q_{Sj}^* = Q_{Sj}^n + (\bar{u} - \bar{c})(A_{Sj}^* - A_{Sj}^n) \quad (12)$$

$$Q_M^* = Q_M^n + (\bar{u} + \bar{c})(A_M^* - A_M^n) \quad (13)$$

$$\Sigma_{Sj}^* = \Sigma_{Sj}^n + (\bar{u} - \bar{c})(Q_{Sj}^* - Q_{Sj}^n) \quad (14)$$

$$\Sigma_M^* = \Sigma_M^n + (\bar{u} + \bar{c})(Q_M^* - Q_M^n) \quad (15)$$

3.2. Subcritical bifurcation

A subcritical bifurcation into two sub-channels, before an island, is represented in Figure 3a and Figure 4a. Here, the three available characteristic equations are the positive characteristic for the main channel M and the negative characteristic for the sub channels $S1$ and S . The compatibility equations are thus written as

$$Q_{Sj}^* = Q_{Sj}^n + (\bar{u} + \bar{c})(A_{Sj}^* - A_{Sj}^n) \quad (16)$$

$$Q_M^* = Q_M^n + (\bar{u} - \bar{c})(A_M^* - A_M^n) \quad (17)$$

To complete the system of equations, the same equations as for the junction are used: discharge conservation (18) and water head conservation (19), with only a change in sign for the friction terms.

$$Q_M^* = Q_{S1}^* + Q_{S2}^* \quad (18)$$

$$z_{b,M} + \frac{A_M^*}{B_{e,M}} + \frac{u_M^2}{2g} = z_{b,Sj} + \frac{A_{Sj}^*}{B_{e,Sj}} + \frac{u_{Sj}^2}{2g} + S_{f,M \rightarrow Sj} \Delta x + J_E \quad (19)$$

The energy loss J_E in (19) is due to the expansion of the river that can occur at the bifurcation, where often the total available flow section downstream of the bifurcation is larger than the available section before. This head loss is computed using a modified Borda equation (Guinot and Soares-Frazão, 2006) as

$$J_E = \frac{\left(\frac{Q_{S1} + Q_{S2}}{A_{S1} + A_{S2}} \right)^2}{2g} \left[\left(\frac{B_{e,S1} + B_{e,S2}}{B_{e,M}} - 1 \right)^2 + \frac{1}{9} \right] \quad (20)$$

By combining these six equations (16)- (19), the wetted area at the bifurcation seen from the main channel A_M^* can be determined. The other wetted areas A_{S1}^* and A_{S2}^* are then simply computed using the head balances. Finally, the mass and momentum fluxes can be written as

$$Q_{Sj}^* = Q_{Sj}^n + (\bar{u} + \bar{c})(A_{Sj}^* - A_{Sj}^n) \quad (21)$$

$$Q_M^* = Q_M^n + (\bar{u} - \bar{c})(A_M^* - A_M^n) \quad (22)$$

$$\Sigma_{Sj}^* = \Sigma_{Sj}^n + (\bar{u} + \bar{c})(Q_{Sj}^* - Q_{Sj}^n) \quad (23)$$

$$\Sigma_M^* = \Sigma_M^n + (\bar{u} - \bar{c})(Q_M^* - Q_M^n) \quad (24)$$

3.3 Supercritical junction

For the case of the supercritical junction, the characteristics provide four equations, as all the information is coming from upstream (Figure 3b and Figure 4d)

$$Q_{Sj}^* = Q_{Sj}^n + (\bar{u} + \bar{c})(A_{Sj}^* - A_{Sj}^n) \quad (25)$$

$$Q_{Sj}^* = Q_{Sj}^n + (\bar{u} - \bar{c})(A_{Sj}^* - A_{Sj}^n) \quad (26)$$

Therefore, only two more equations are needed to close the system. The four equations (25)-(26) can be solved directly for the mass fluxes towards the two sub-channels Q_{S1}^* and Q_{S2}^* , yielding (27) **Erreur ! Source**

du renvoi introuvable.. To determine the mass flux coming from the main channel, the conservation of the mass fluxes (28) is used. Then, using (29)-(30), the momentum fluxes in the sub-channels and in the main channel can be obtained. The six fluxes are thus

$$Q_{Sj}^* = Q_{Sj}^n \quad (27)$$

$$Q_M^* = Q_{S1}^* + Q_{S2}^* \quad (28)$$

$$\Sigma_{Sj}^* = \Sigma_{Sj}^n \quad (29)$$

$$\Sigma_M^* = \Sigma_{S1}^* + \Sigma_{S2}^* \quad (30)$$

3.4. Supercritical bifurcation

This case is the most complex one as the characteristics only provide two useful equations (Figure 3a and Figure 4c)

$$Q_M^* = Q_M^n + (\bar{u} + \bar{c})(A_M^* - A_M^n) \quad (31)$$

$$Q_M^* = Q_M^n + (\bar{u} - \bar{c})(A_M^* - A_M^n) \quad (32)$$

Four more conditions are thus needed. Two of them are, as before, the conservation of the water head:

$$z_{b,M} + \frac{A_M^*}{B_{e,M}} + \frac{u_M^2}{2g} = z_{b,Sj} + \frac{A_{Sj}^*}{B_{e,Sj}} + \frac{u_{Sj}^2}{2g} + S_{f,M \rightarrow Sj} \Delta x + J_E \quad (33)$$

with J_E , the energy loss due to a widening computed using the same modified Borda equation (20) as for the subcritical bifurcation. The last two equations are provided considering a partition of the discharge proportional to the sub-channels average widths $B_{e,S1}$ and $B_{e,S2}$.

$$Q_{Sj}^* = Q_M^* \frac{B_{e,Sj}}{B_{e,S1} + B_{e,S2}} \quad (34)$$

As explained before, the average width B_e is defined as the ratio A/h . It is important to note that these two equations (34) ensure the conservation of the discharge. As the flow is supercritical, the wetted area at the interface downstream of the main channel is equal to the wetted area in the cell, because all information come from upstream. The wetted area at the other interfaces are defined by the water head conservation. Finally, the fluxes are written as,

$$Q_M^* = Q_M^n \quad (35)$$

$$Q_{Sj}^* = Q_M^* \frac{B_{e,Sj}}{B_{e,S1} + B_{e,S2}} \quad (36)$$

$$\Sigma_M^* = \Sigma_M^n \quad (37)$$

$$\Sigma_{Sj}^* = \bar{u} Q_{Sj}^* + g I_{1,Sj}^* \quad (38)$$

4. RESULTS AND DISCUSSION

First, the results of this one-dimensional model are compared to those of a full two-dimensional model in a number of numerical test cases. For each case, the water levels in the different branches and the discharge partition are compared. The two-dimensional model used as reference is the model developed by Soares-Frazão and Zech (2002). Then, the one-dimensional model is used to simulate the flow in the Meuse River in Belgium (Dal Cin *et al.* 2005) and compared to field measurements.

For the numerical cases, different configurations of channels with rectangular cross-sections are tested (R-I to R-V), as summarized in Table 1. In addition, one configuration where the cross-sections are trapezoidal (T), yielding more complex junctions and bifurcations, is considered. The trapezoidal cross-sections are defined in the main channel with the bank slope $Z = 0.25$. The sub-channels are defined as semi-trapezoidal cross-sections with one vertical wall and one sloping bank ($Z = 0.25$). For each configuration, the four flow cases corresponding to the four internal boundary conditions are considered. The main channel keeps the same characteristics for all the tests: a bottom width (B_0) of 75 m, a Manning coefficient (n) of $0.025 \text{ s/m}^{1/3}$ and a slope of 0.3% and 5% for the subcritical and supercritical, respectively. In the equivalent

2D model, the two branches of the junction or bifurcation make an angle of 60° with the main channel. This choice is arbitrary, as the angle at the connection is not taken into account in the flux expressions of the 1D model. For each type of internal boundary condition, the water depths results are shown for configurations R-I, R-III and R-IV. The plots show the comparison between the 1D water depth and the cross-section averaged value of the 2D water depth across the cross-section. The discharges are compared for all the tests.

Table 1. Key parameters of the different numerical test cases: bottom width (B_0), slope (S_0) and Manning coefficient (n). The bold number highlight the differences between the test and the classical symmetrical test R-I

Test case		Sub-channel S1			Sub-channel S2		
		B_0	S_0	n	B_0	S_0	n
		(m)	(%)	(s/m ^{1/3})	(m)	(%)	(s/m ^{1/3})
R-I	Sub	37.5	0.3	0.025	37.5	0.3	0.025
	Super	37.5	5	0.025	37.5	5	0.025
R-II	Sub	37.5	0.3	0.025	37.5	0.3	0.045
	Super	37.5	5	0.025	37.5	5	0.045
R-III	Sub	37.5	0.3	0.025	37.5	0.1	0.025
	Super	37.5	5	0.025	37.5	8	0.025
R-IV	Sub	50	0.3	0.025	25	0.3	0.025
	Super	50	5	0.025	25	5	0.025
R-V	Sub	60	0.3	0.025	15	0.3	0.025
	Super	60	5	0.025	15	5	0.025
T	Sub	49.75	0.3	0.025	24.75	0.3	0.025
	Super	49.75	5	0.025	24.75	5	0.025

4.1 Subcritical

For configuration R-I, as shown in Figure 5, the 1D model is able to provide the correct water depth far from the connections but is not able to capture the increased elevation and the depression. These features, that depend on the angle at the connection, can only appear in the 2D model (Figure 6) as the flows coming from each sub-channel have opposite velocities along the y -axis, and so their mixing results in an area of higher depth at the junction. Also, as the flows are directed towards the center of the main channel, two area of shallow depth appear close to the wall. However, besides these very local 2D phenomena, the water depth remains uniform along the whole 2D channel as predicted by the 1D model. The discharges are compared in Table 2, showing an accurate prediction by the 1D model.

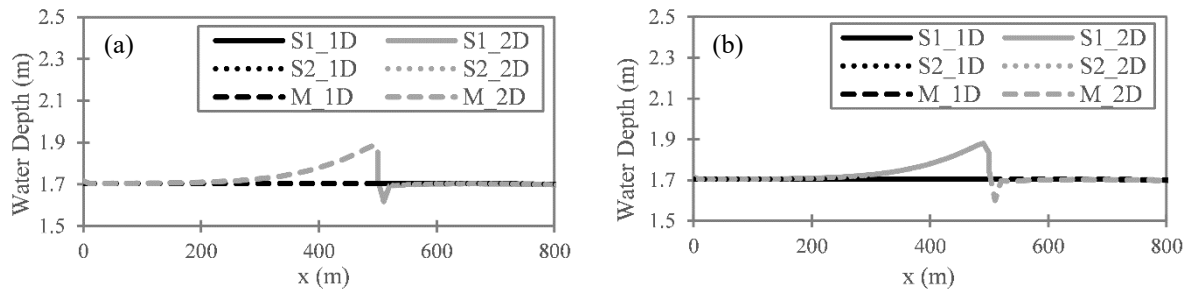


Figure 5: (a) Subcritical bifurcation and (b) subcritical junction: Comparison of the water depths obtained using 1D and 2D models for test R-I (symmetrical)

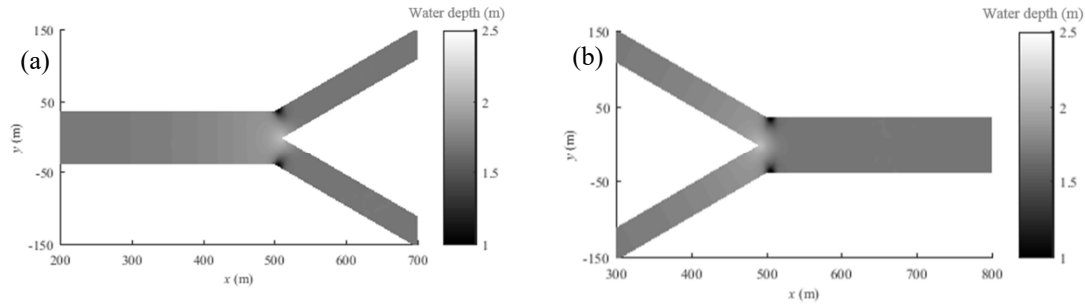


Figure 6: (a) Subcritical bifurcation and (b) subcritical junction: 2D water depth for test R-I

For configuration R-II, the Manning coefficient is increased to $0.045 \text{ s/m}^{1/3}$ in one of the sub-channel. As in the previous case, the water depths are well predicted except for local differences at the junction where two-dimensional features appear when the two flows merge. Table 2 shows that the discharge is also correctly computed. Then, in configuration R-III, the slope in one sub-channel is decreased to 0.1 %. It can be seen in Figure 7 that the 1D model correctly predicts the water depths except for the exact position of the connection, in a similar way than for configuration R-I and R-II. Again, the discharge partition is well predicted (Table 2).

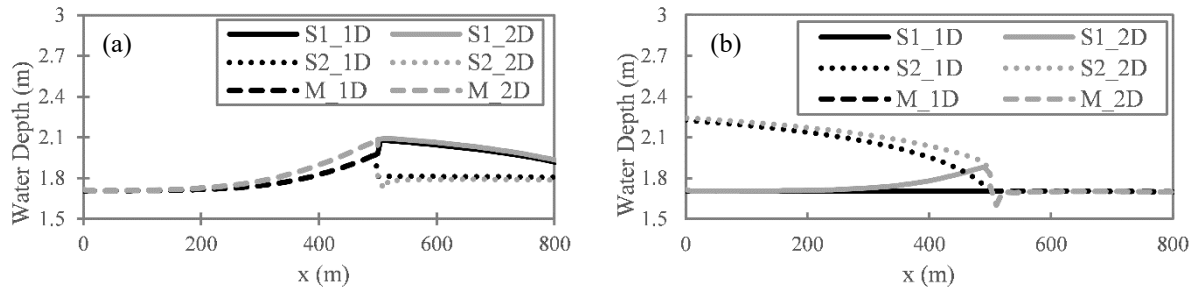


Figure 7: (a) Subcritical bifurcation and (b) subcritical junction: Comparison of the water depths obtained using 1D and 2D models for test R-III (different slopes: 0.3% for M and S1 and 0.1% for S2)

The sub-channel width is investigated in configurations R-IV (50 m – 25 m) and R-V (60 m – 15 m). Figure 8 shows the water depth results for R-IV with a subcritical junction. The results for R-V are very similar and are not illustrated. As for the other configurations, the overall water depth and the discharge (Table 2) are correctly predicted except are the location of the junction.

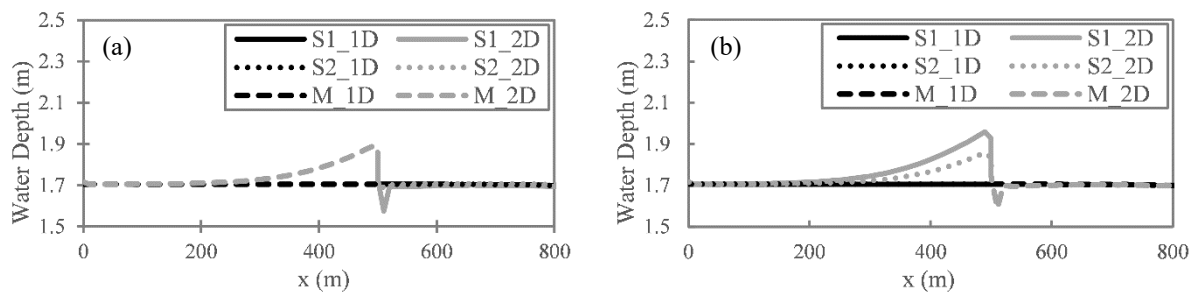


Figure 8. (a) Subcritical bifurcation and (b) subcritical junction: Comparison of the water depths obtained using 1D and 2D models for test R-IV (different width: 75 m for M, 50 m for S1 and 25 m for S2)

Finally, in configuration T (trapezoidal channel), the main channel consists of a full trapezoidal cross-section and the sub-channels are defined with semi-trapezoidal cross-sections, with one vertical wall and one sloping bank. The bank slope is the same for both sub-channels and for the main channel. Each sub-channel has its own, different, bottom width, 49.75 m and 24.75 m, respectively. The results are only shown for the discharge (Table 2), as the water depth conclusions are similar to the rectangular cases. Table 2 shows that the partition of the discharge between the two sub-channels corresponds to the values obtained by the two-dimensional model. Furthermore, the one-dimensional model conserves correctly the discharge (the sum of the discharges in the sub-channels is equal to the discharge in the main channel). In addition, the partition of

the discharges between the two sub-channels is proportional to the equivalent width even though this proportionality is not imposed as a condition in the subcritical flux model.

4.2 Supercritical

For configuration R-I, as observed in Figure 9, the water depth is correctly represented far from the connections. At the connections, a peak is observed in the 2D model results because of the velocity distribution (Figure 14). However, besides these very local 2D phenomena, the water depth remains uniform along the whole 1D and 2D channel. The discharge is also accurately predicted (Table 3).

Table 2. Subcritical bifurcation: Comparison of the discharges (in m³/s) obtained using the 1D and 2D models

Test	$Q_{M, 1D}$	$Q_{M, 2D}$	$Q_{SI, 1D}$	$Q_{SI, 2D}$	$Q_{S2, 1D}$	$Q_{S2, 2D}$
R-I	400	400	200	200	200	200
R-II	400	400	229.4	225.6	170.6	174.4
R-III	400	400	221.3	217.5	178.7	182.5
R-IV	400	400	266.7	266	133.3	134
R-V	400	400	320	319.2	80	80.8
R-VI	400	400	200	199.9	200	200.1
T	400	400	261.7	262.5	138.3	138.7

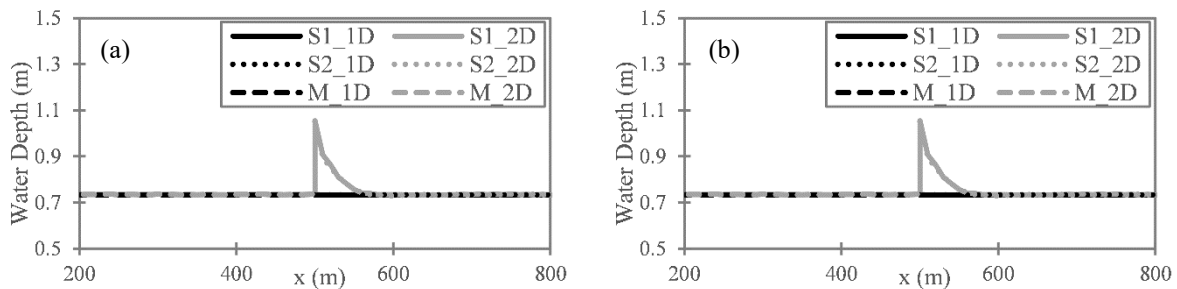


Figure 9: (a) Supercritical bifurcation and (b) Supercritical junction: Comparison of the water depths obtained using 1D and 2D models for test R-I (symmetrical)

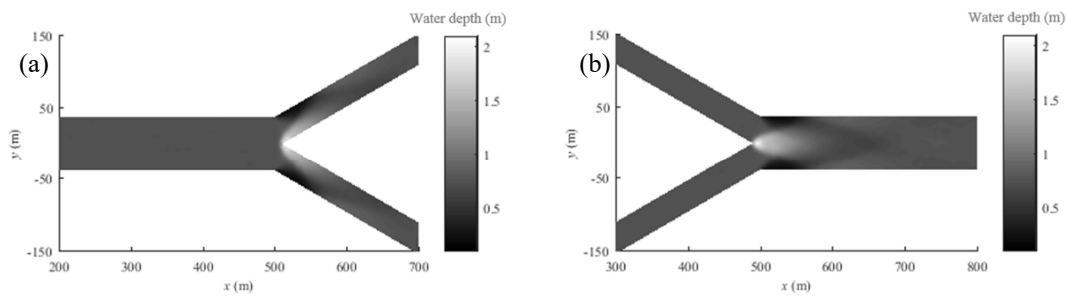


Figure 10: (a) Supercritical bifurcation and (b) Supercritical junction: 2D water depth for test R-I

For configuration R-II, the Manning coefficient is increased in one of the sub-channel to 0.045 s/m^{1/3}. Table 3 shows that the discharge is correctly computed. For configuration R-III, the slope in one of the sub-channel is increased to 8 %. The results are similar to those of configuration R-I and R-II, the water depth is correct but the peak is not captured (Figure 11). The discharge is also accurately predicted (Table 3).

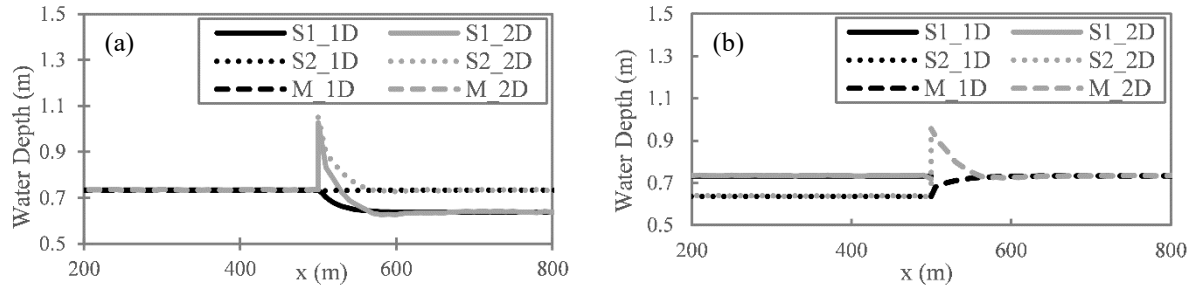


Figure 11: (a) Supercritical bifurcation and (b) Supercritical junction: Comparison of the water depths obtained using 1D and 2D models for test R-III (different slopes: 5% for M and S1, 8% for S2)

Then, the impact of the sub-channel width is investigated for configurations R-IV (50 m– 25 m) and R-V (60 m – 15 m). The water depth results are only shown for configuration R-IV. As for the previous tests, the water depths and the discharges are correctly predicted, only the local peak being not captured (Figure 12 and Table 3).

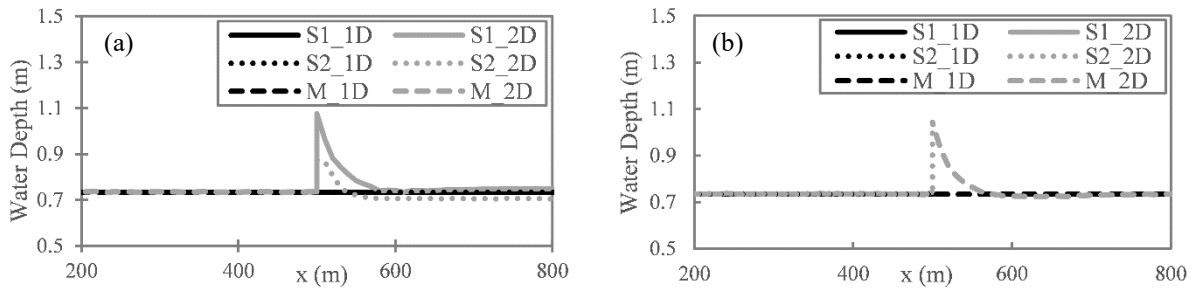


Figure 12: (a) Supercritical bifurcation and (b) Supercritical junction: Comparison of the water depths obtained using 1D and 2D models for test R-IV (different widths: 75 m for M, 50 m for S1, 25 m for S2)

For configuration T, as for the other configurations, the one-dimensional model predicts correctly the overall water depth. The partition of the discharge is also close to the partition predicted by the two-dimensional model (Table 3). However, it is important to note that the latter presents important fluctuations of the computed discharge due to an imperfect treatment of the wet-dry interface on the banks of the trapezoidal cross-sections.

Table 3: Supercritical bifurcation: Comparison of the discharges (in m^3/s) obtained using the 1D and 2D models

Test	$Q_{M, 1D}$	$Q_{M, 2D}$	$Q_{S1, 1D}$	$Q_{S1, 2D}$	$Q_{S2, 1D}$	$Q_{S2, 2D}$
R-I	400	400	200	200.2	200	199.8
R-II	400	400	200	200.1	200	199.9
R-III	400	400	200	199.9	200	200.1
R-IV	400	400	266.67	275.4	133.33	124.6
R-V	400	400	320	327.5	80	72.5
R-VI	400	400	200	200.2	200	199.8
T	400	400	264.6	262	135.4	138.1

4.3 Application to the Meuse River

Finally, the one-dimensional model is compared to a real case, the simulation of the flow in one reach of the Meuse River in Belgium featuring an island surrounded by two sub-reaches of different widths (Figure 13): subreach S_1 is much wider than subreach S_2 and they present a very different bed elevation. The 4 km reach is located between the cities of Houx and Hun in the south of Belgium. The reach has been divided into 45 cross-sections as represented on Figure 13. Measurement of the water level 200 m upstream of the city of Houx are compared to the numerical simulations for one almost complete year of measurements, from the 1st of January 2016 to the 22nd of December 2016, computed from the measured hydrograph represented in Figure 14a, with discharges ranging from $30 \text{ m}^3/\text{s}$ to $780 \text{ m}^3/\text{s}$. The model correctly predicts the measured

water level upstream of the island. Figure 14b summarizes the results for the complete test, comparing the hourly measured and computed water depths (more than 8000 points of comparison). It shows that the computed water depth is within 5 cm of the measured water depth for the most of the predictions.

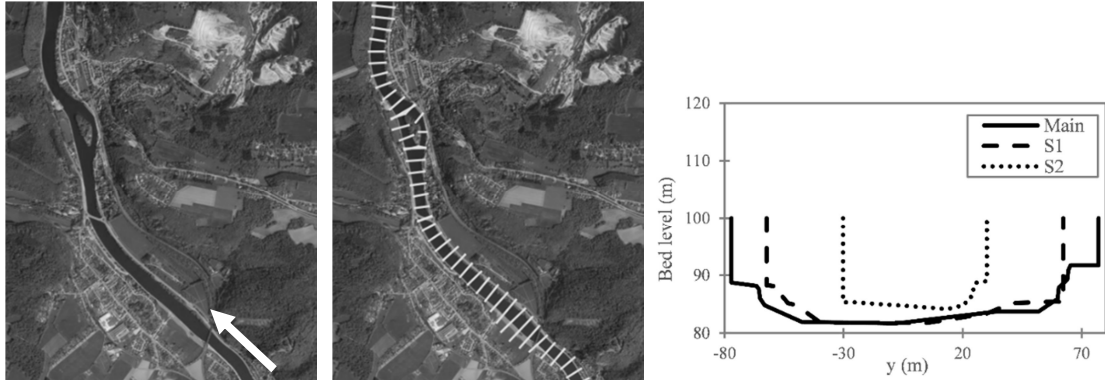


Figure 13: (a) Aerial view of the Meuse River reach simulated and (b) cross-sections representation (credits Bing Maps) and (c) Bifurcation of the river at the island location: Comparison of the three cross-sections

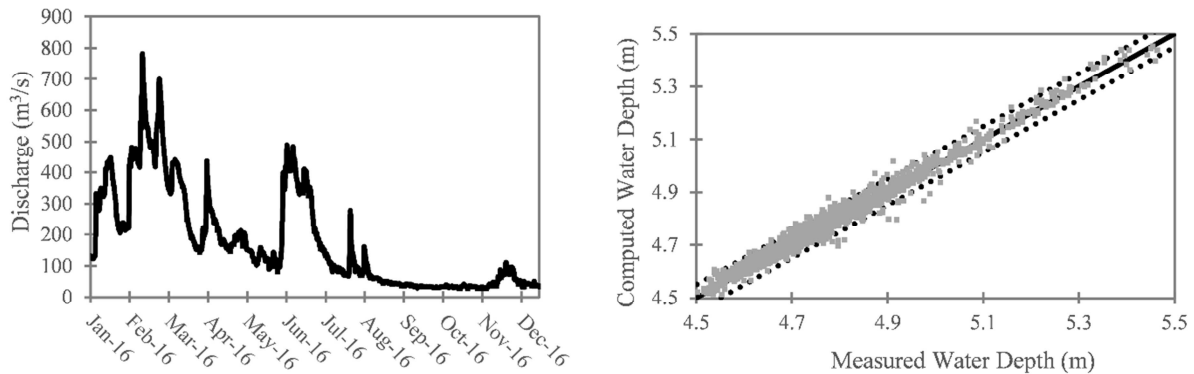


Figure 14: (a) Discharge imposed upstream, (b) Comparison between the computed and measured water depths for the Meuse River. The solid line represent the exact prediction and the dash lines an error of 5 cm

5. CONCLUSION

A one-dimensional model to simulate the flow around islands in a river has been presented. The model uses the characteristic equations and the water head balance to compute the fluxes before and after the island, i.e. at bifurcations and junctions. It has been compared to a 2D model for rectangular and trapezoidal cross-sections. It showed good results in predicting the discharge partition according to the equivalent width. However, the water depth at the junction/bifurcation showed some differences between the 1D and 2D models. These differences are caused by the non-consideration of the transversal velocities or the constriction in the one-dimensional model that makes it impossible to capture the reflections on the walls of the channel. Nevertheless, the water depth results provided by the 1D model are correct, when neglecting these local 2D phenomena with, for example, a correct continuity for the symmetrical cases (Figure 5 and Figure 9). Furthermore, the water depth was correct away from the junction / bifurcation and, in real rivers, the transitions and divisions of the flow are usually smoother than the one tested here, creating a much reduced local peak of the water level. In addition, the computations with the one-dimensional model were substantially faster than the two-dimension model. Finally, the one-dimensional model has been used to simulate the flow in a reach of a real river, the Meuse River in the south of Belgium. The results showed good agreement with the measurements for a year of predictions.

Future work should focus on the computation of the sediment transport in the connections and should treat the particular issue of wetting/drying of a sub-channel. These improvements would allow the model to be used to predict the flood in river with intense sediment transport.

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