The impact of line integrals in full-wave modeling of coax-fed antennas

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***Abstract*—Special attention is required when applying the MoM to coaxial-fed printed antennas. First, the junction between vertical and horizontal conductors must be carefully considered. In this paper, we realize this transition thanks to broken rooftops which have the merit of being elementary basis functions, thus allowing a fine representation of the feeder structure. In this case, an additional line of charge distributions appear due to the difference between scalar potentials associated to vertical and horizontal conductors. The impact of these lines of charges is studied in this paper. Second, the feeding technique needs to be carefully selected. Here, we compare the accuracy of the delta-gap excitation to the one based on an impressed magnetic frill of current. The importance of both the line of charges and the technique of excitation is highlighted by confronting our results against the ones obtained from the CST software.**

***Index Terms*—Method of Moments, coaxial-fed antennas, magnetic frill, broken rooftops.**

I. Introduction

Accurate numerical analysis is of paramount importance towards the design of high performance antenna arrays. Full-wave numerical methods such as the Method of Moments (MoM) have proven their versatility and their accuracy over the past few decades. In particular, the MoM has the advantage of only requiring surface meshing while the configuration of the layered medium can be accounted for through dedicated dyadic Green's functions. This can be done for planar layered media, with the hypothesis that the layers are infinite in the horizontal direction and finite in the vertical direction. While edge effects are neglected, this model has proven highly reliable for structures printed on large dielectric substrates. This means that the MoM is an excellent candidate for studying large arrays printed on a grounded dielectric slab. In this paper, we tackle the accurate analysis of the unit element of a large connected array which, in free-space appears as a self complementary structure. Self complementary structures [1] exhibit constant impedance irrespective of the frequency. While this property is of high interest, in practice, it is required to provide a support for the array: for example, a grounded dielectric substrate, thus partly breaking the self complementary aspect of the structure. That being said, the array can still exhibit a large bandwidth if the thickness of the substrate layer is well chosen. Several layers with different permittivities can also improve the bandwidth [2]. We consider here the connected array depicted in Fig. 1 and we focus on the feeding and the calculation of the input impedance of the unit element. A delta gap excitation located on a planar elementary basis function has been employed to feed the array in the past [3]. This modeling has the advantage of keeping the structure planar and thus, the Green's function only needs to be evaluated on the air-substrate interface. However, this model neglects the presence of feeding probes and thus only approximates the input impedance. In this paper, the vertical probes feeding the antenna across the dielectric substrate are represented with elementary basis functions. Hence, the precise geometry of the probe can be accounted for. Several precautions need to be taken when analyzing probe-fed antennas with the MoM. Half basis functions need to be connected to the ground plane and adequate Green's functions of vertical sources in layered media need to be calculated. The feeding technique, which can be based for example on a gap-voltage or a magnetic frill of current [4], needs to be carefully studied [5]. Moreover, the transition from vertical conductors to horizontal conductors at the air-dielectric interface requires special attention. Sub-domain [6] or entire-domain [7] attachment modes have been studied in the past. However, they often entail approximations. In the present case, we realize the transition from the probe to the patch using bent (or broken) rooftops [8], which remains quite flexible.

In the mixed-potential integral-equation (MPIE) formulation [9], the Green's functions $\mathbf{G}_A$ and $\mathbf{G}_V$, associated with the vector and scalar potentials, respectively, need to be calculated. When using a classical approach, the scalar potential associated with horizontal source differs from the one associated with vertical sources. This difference gives rise to a line of charges [8] located at the air-dielectric interface. In the present paper, we study the impact of the line of charges on the isolated element of the connected array as well as the accuracy of the delta-gap excitation versus the magnetic frill excitation.

II. Modeling

The antenna element and the feeding probe are shown in Figs. 2 and 3, respectively. As can be seen, the probe is described with rooftop basis functions while the patch is described with Rao-Wilton-Glisson (RWG) basis functions [10]. The broken rooftop (depicted in red in Fig. 3) is composed of a horizontal half basis function connected to a vertical half basis function. In this case, an octagonal cross section has been chosen for the probe, meaning that the probes are connected to the patch trough eight bent rooftops. The bowtie antenna is printed on a single-layered dielectric substrate backed by a ground plane. Two feeding techniques are considered here. The simplest one is a delta gap feeding at the bottom of the

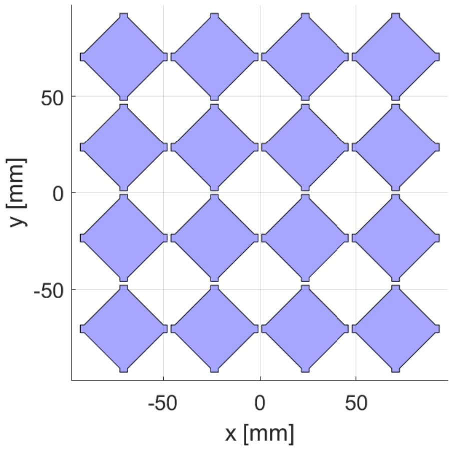

Fig. 1. Example of a connected array.

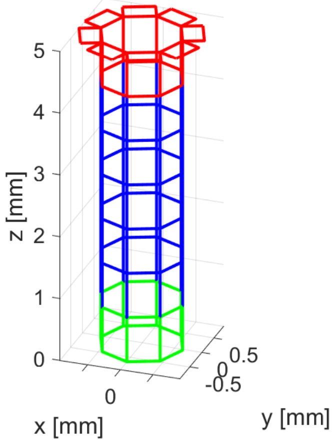

Fig. 3. Mesh of the feeding probe.

probe, meaning that a an infinitesimal voltage gap is placed at the bottom of each half basis function connected to the ground plane. The second feeding technique uses a magnetic frill of current derived from the waveguide mode supported by the coaxial cable [4]. This excitation allows accounting for the outer conductor of the coaxial cable while the delta gap excitation does not. The accuracy of both feeding methods will be studied in the next section.

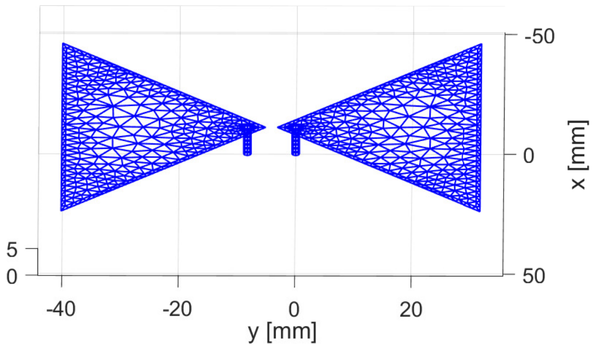

Fig. 2. Mesh of the element, obtained with Gmsh [11].

III. Line integrals

In Fig. 3, it is clear that an octagonal cross section probe supports eight bent rooftops and thus eight additional line charge distributions appear due to the difference between scalar potentials associated to vertical and horizontal sources [8]. The interaction between the line of charges supported by a given bent rooftop $\boldsymbol{f}_b$ and a given basis function $\boldsymbol{f}_t$ can be written as

$$\oint_{\partial S} \hat{n}(\boldsymbol{r}) \cdot \boldsymbol{f}_b(\boldsymbol{r}) \left(\iint_{S'} \nabla' \cdot \boldsymbol{f}_t(\boldsymbol{r'}) \left(\mathbf{G}_V^h(R) - \mathbf{G}_V^v(R) \right) \, \mathrm{d}S' \right) \mathrm{d}l(\boldsymbol{r}) \qquad (1)$$

where $\boldsymbol{r'}, \boldsymbol{r}$ denote the source and test positions, $R = |\boldsymbol{r} - \boldsymbol{r'}|$, and $\mathbf{G}_V^h$, $\mathbf{G}_V^v$ are the scalar potentials associated with horizontal and vertical sources, respectively. The domain surfaces of the bent rooftop and of the testing function are denoted by S and S', respectively. $\hat{n}$ is the outer normal to surface S. It has been shown in [8], [12] that, while both scalar potentials exhibit a singular behavior as the distance of interaction R tends to zero, the difference between the two potentials is singularity-free.

IV. Numerical results and validation

Let us consider the element depicted in Fig. 2 and evaluate its input impedance. The input impedance is simulated with an in-house MoM solver where we include and then omit the additional line integrals to observe their effect on accuracy. The antenna is also simulated using the frequency domain solver of CST [13], for validation purposes. In this case, the dielectric substrate is finite as can be seen in Fig.. 4. The antenna feeding in CST relies on waveguide ports.

First, we feed the antenna with an infinitesimal delta-gap excitation at the bottom of the probe. The results are shown in Fig. 5, along with the ones from CST. A relatively bad accuracy of the MoM solver is observed with this technique of feeding. We can see for example that resonant frequencies are significantly shifted.

Next, a magnetic frill of current is used as an excitation in the MoM solver. The input impedance is calculated from simulations including line integrals and is compared to results from CST in Fig. 6, where an excellent agreement can be observed. In Fig. 7 the line integrals are omitted. The corresponding S parameters, with 50 Ohms reference impedance, are shown in Fig. 8. A lack of accuracy can be clearly observed when omitting the line integrals, especially at higher frequencies, i.e. when the dielectric substrate becomes electrically thicker. Hence, the line integrals need to be accounted for to avoid this lack of accuracy.

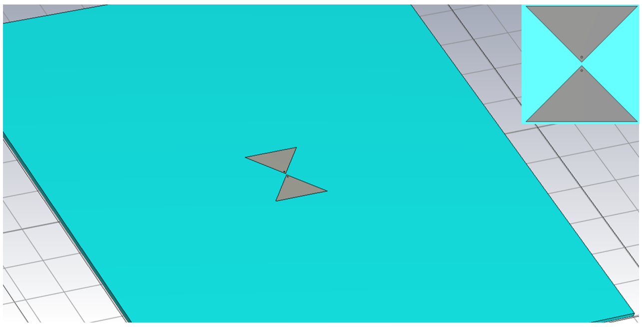

Fig. 4. Antenna in CST software.

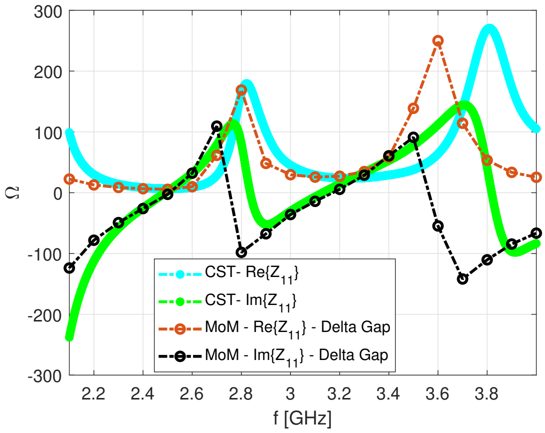

Fig. 5. Comparison between real and imaginary parts of the antenna input impedance, obtained with the MoM code and with CST. A delta-gap excitation is selected for the MoM solver.

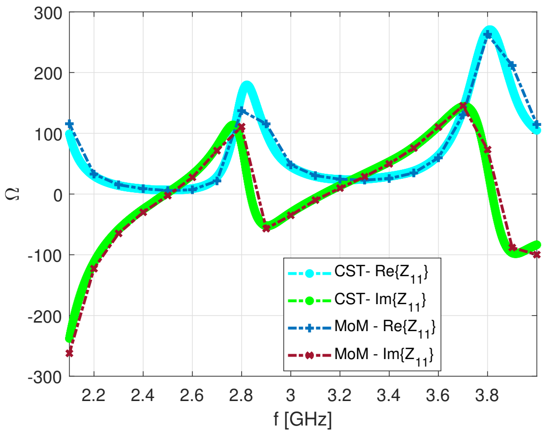

Fig. 6. Comparison between real and imaginary parts of the antenna input impedance, obtained with the MoM code and with CST.

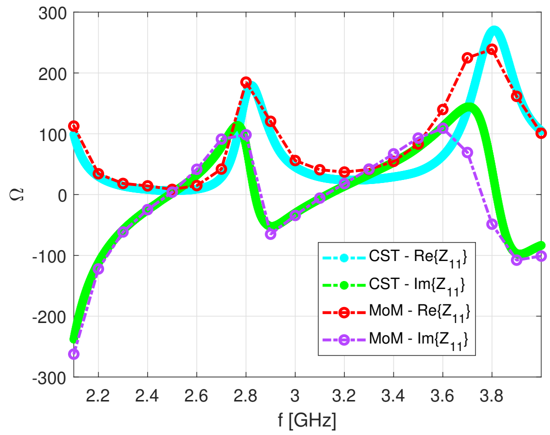

Fig. 7. Comparison between real and imaginary parts of the antenna input impedance, obtained with the MoM code and with CST. Line integrals are omitted in the MoM solver.

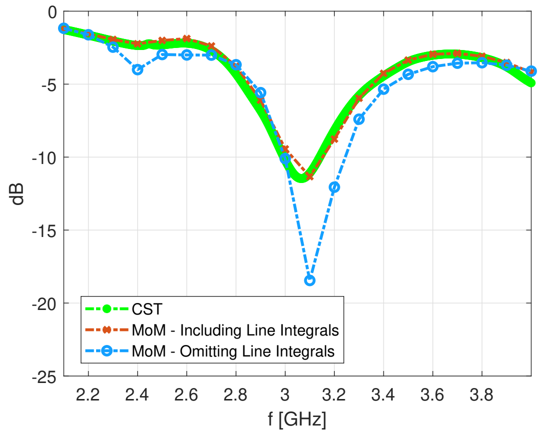

Fig. 8. Comparison between S parameters, obtained with the MoM code and with CST.

V. Conclusion

In this paper, we carefully studied a coaxial-fed antenna with the MoM. The special features of coaxial-fed antennas include the judicious choice of the excitation technique and the treatment of the junction between vertical and horizontal conductors. Here we chose to employ broken rooftop basis functions. This choice allows one to accurately reproduce the feeder geometry. However, additional line integrals appear due to the difference between scalar potentials associated to vertical and horizontal sources. We have shown here the critical impact of the line integrals on a relevant example. We concluded that these integrals are necessary for good accuracy, especially at higher frequencies. In addition, we confronted the results obtained when selecting a delta gap excitation as opposed to a magnetic frill of current. A dramatic loss of accuracy has been observed when selecting the former.

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REFERENCES

[1] Y Mushiake, “Self-Complementary Antennas,” *IEEE Antennas Propag. Mag.*, vol. 34, pp. 23-29, Dec. 1992.

[2] M. Gustafsson, “Use of dielectric sheets to increase the bandwidth of a planar self-complementary antenna array,” *IEEE Antennas Propag. Soc. Int. Symp.*, Albuquerque, NM, USA, pp. 2413-2416, 2006.

[3] X. Dardenne and C. Craeye, “Simulation of the effects of a ground plane on the radiation characteristics of self complementary arrays,” *Proc. of the 2003 IEEE Antennas Propag. Soc. Symp.*, Columbus, Ohio, June 2002.

[4] L. Tsai, “A numerical solution for the near and far fields of an annular ring of magnetic current,” IEEE Trans. Antennas Prop., vol. 20, no. 5, pp. 569-576, Sep. 1972.

[5] H. Xu, D. R. Jackson and J. T. Williams, “Comparison of models for the probe inductance for a parallel-plate waveguide and a microstrip patch,” *IEEE Trans. Antennas Propag.*, vol. 53, no. 10, pp. 3229-3235, Oct. 2005.

[6] J. T. Aberle and D. M. Pozar, “Analysis of infinite arrays of probefed rectangular microstrip patches using a rigorous feed model,” *Proc. Inst. Elect. Eng.*, vol. 136, no. 2, pp. 110-119, Apr. 1989.

[7] G. Mayhew-Ridgers, J. W. Odendaal and J. Joubert, “Entire-domain versus subdomain attachment modes for the spectral-domain method of moments analysis of probe-fed microstrip patch antennas,” *IEEE Trans. Antennas Prop.*, vol. 52, no. 6, pp. 1616-1620, Jun. 2004.

[8] M. Grzegorczyk and J. R. Mosig, “Line charge distributions arising in the integral equation treatment of bent scatterers in stratified media,” *Microw., Antennas Propag.*, vol. 148, no 6, p. 365-368, 2001.

[9] J. R. Mosig, “Arbitrarily shaped microstrip structures and their analysis with a mixed potential integral equation,” *IEEE Trans. Microw. Theory Techn.*, vol. 36, no. 2, pp. 314-323, Feb. 1988.

[10] S. M. Rao, D. R. Wilton, and A. W. Glisson, “Electromagnetic scattering by surfaces of arbitrary shape ,” *IEEE Trans. Antennas Propag*., vol. 30, no. 3, pp. 409-418, 1982.

[11] C. Geuzaine and J.-F. Remacle, “Gmsh: a three-dimensional finite element mesh generator with built-in pre- and post-processing facilities,” *Int. J. Numer. Meth. Eng.*, vol. 79, no. 11, pp. 1309-1331, 2009.

[12] J. Cavillot, “Fast electromagnetic analysis of large objects involving planar layered media: Application to antenna arrays and metasurfaces,” Ph.D. dissertation, Inst. Inf. Commun. Technol., Electron. Appl. Math. (ICTEAM), UCLouvain, Ottignies-Louvain-la-Neuve, Belgium, 2021.

[13] Computer Simulation Technology (CST). [Online] Available: https://www.3ds.com/fr/products/simulia/cst-studio-suite