

Simulation of a Radio Frequency Quadrupole with the Method of Moments

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Abstract—The Method of Moments (MoM) is widely used for radiation and scattering simulations. LINear ACcelerator (LINAC) such as Radio Frequency Quadrupole (RFQ) can also be simulated with the MoM. Present solvers for linear accelerators are quite intensive. Faster solvers would allow better analysis and possible optimisation of LINACs. In this abstract, the link between the MoM and the electromagnetic fields simulation for RFQ is developed. First some generalities about LINAC and RFQ are presented. Then, the application of the MoM for RFQ simulations is detailed. In particular, a method based on the combination of Macro Basis Functions (MBFs) and the loop-tree or loop-star decompositions is proposed to solve the problem of low-frequency breakdown due to the unavoidable very fine mesh used for the simulation of a RFQ. Finally, preliminary results obtain with the MoM with Rao-Wilton-Glisson (RWG) basis functions are presented.

I. RADIO FREQUENCY QUADRUPOLE

An Accelerator Driven System (ADS) [1] is a sub-critical nuclear reactor fed by the neutrons from a spallation target. The spallation target must be fed with a constant high proton beam in order to produce neutrons. LINACs [2], [3] are used to accelerate the protons in view of their ability of delivering high beam current. Myrrha [4] is a project of the Belgian Nuclear Research Centre (SCK-CEN) that aims to build an ADS. The purpose of Myrrha is, among others, the transmutation of nuclear waste. The examples in this paper are based on the Myrrha accelerator. A RFQ [5],[6] is a critical element of a LINAC. It is situated at the very beginning of the accelerator and has three functions: bunching, focusing and acceleration. Bunching consists of the creation of packets of particles from a continuous current distribution. The term focusing in this context means keeping the particles inside the accelerator without changing the transversal velocity distribution. The focusing is then indirectly obtained by the longitudinal acceleration of the beam. At the end of the RFQ the transversal velocity distribution of the particles remains the same as at the entrance, but the longitudinal velocity is much higher. The RFQ is based on the well known electrostatic quadrupole. However, an RFQ is supplied by a harmonic voltage source for two reasons: bunching and acceleration. Due to the inversion of polarity, the accelerator is alternatively focusing and defocusing the beam, as shown Fig. 1.

The transversal motion of the particles in such a device is described by the Hill's equations [3]. It is known that under some hypotheses, stable (bounded) solutions exist for these

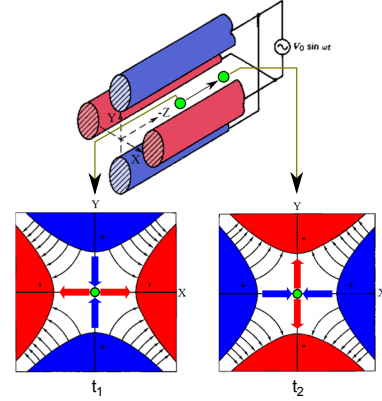


Fig. 1. RFQ without the acceleration function. Picture taken and modified from [2]

equations. Furthermore, it can be shown that the velocity distribution of the particles at the entrance and at the exit of the accelerator remains nearly the same. Therefore, the focusing as defined above can be achieved even with an alternatively focusing and defocusing electric field. Fig. 2 shows the undulation of the electrodes in the direction of the beam and the electric-field decomposition at the boundary of an electrode at two different times. In order to accelerate the bunch, the RF period must be synchronized with it. On Fig. 2 the swap of polarity is represented. At time t_0 the bunch of particles experiences an accelerating field; at time t_1 , thanks to the movement of the particles and the swap of polarity, the bunch again experiences an accelerating field.

Locally, the fields can be assumed quasi-static but globally the RFQ can be seen as a transmission line. For instance, the Myrrha RFQ length is four meters long and the resonance frequency is around 176 MHz. The electrodes are made of four rods supported by several stems connected to ground. Such a RFQ is called a four-rods and is a resonator. The supports holding the rods and called stems close the resonant circuit. Two consecutive stems are connected to two opposite voltage rods. Hence, a current loop is created and an inductive effect appears as shown Fig. 3.

Fig. 4 shows the current flow along two rods of same polarity of a section of the Myrrha RFQ. The two other rods have been omitted for clarity of the picture. By tuning the number of stems and their spacing, one can adjust the

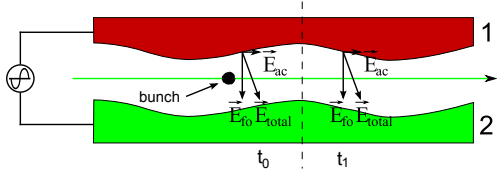
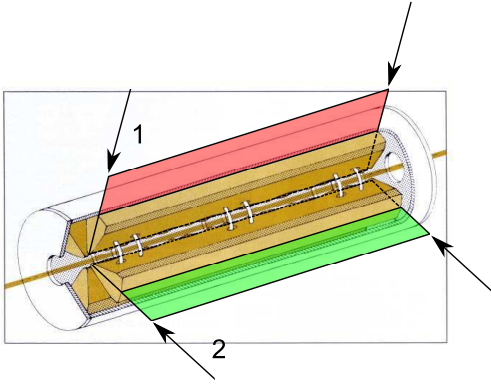


Fig. 2. RFQ with the acceleration function. Top picture taken from [7]

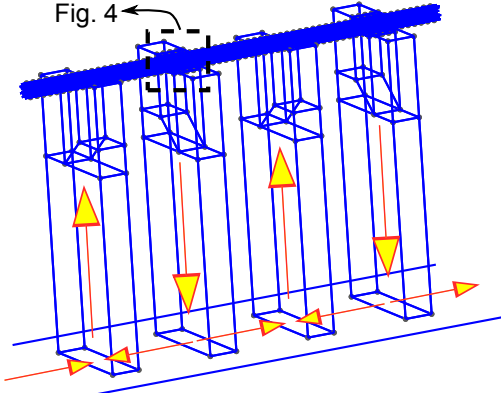


Fig. 3. Current flow in the stems of a section of the Myrrha RFQ

inductance. This is used to adjust the resonance frequency of the RFQ.

At the present time, numerical solvers for linear accelerators require an important computation time, which limits the possibilities of numerical optimization. For instance, Toutatis [8] is a solver for RFQ electromagnetic fields simulations and is based on the Poisson equation solved in a finite difference scheme. Our aim is to improve the accuracy and computation time of current solvers with the help of the Method of Moments (MoM). The analysis and optimization of the Myrrha RFQ will be carried out with MoM solvers.

II. A SIMPLE METHOD BASED ON THE MOM FOR ELECTROMAGNETIC SIMULATIONS OF RFQ

The fields between the rods must be calculated very accurately; therefore the mesh must fit very well the

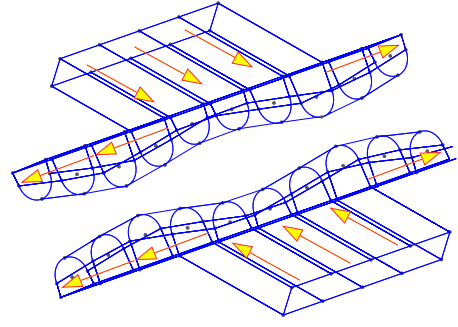


Fig. 4. Current flow in the rods of a section of the Myrrha RFQ

undulations of the electrodes (see Fig. 2). In terms of stability of the MoM system of equations, since the basis functions are several orders of magnitude smaller than the wavelength, the system is very ill conditioned. This problem is known as the low-frequency breakdown [9], [10]. A stable system is obtained by taking basis functions with dimensions of the order of a tenth of wavelength. In order to gain stability, one simple idea is to use a subspace of current distributions. The new basis functions are commonly called Macro Basis Functions (MBFs) [11],[12] or Characteristic basis functions (CBFs) [13], [14], [15]. The generation of MBFs is a vast subject but a critical aspect well developed in the literature. In order to generate the MBFs, one must solve a smaller but still ill conditioned system of equations. A wide literature basis can be found about the low-frequency breakdown. In particular, the loop-tree and the loop-star decompositions [9], [10], [16], [17] have been widely used to solve the problem and are still reference methods. The idea is to use another set of basis functions namely the loop-tree basis or the loop star basis. The system of equations obtained using such a basis is more stable but cannot yet be used with iterative solvers. In order to gain more stability, a normalization procedure and a permutation of the loop-tree or loop-star basis functions by a connection matrix allows one to use iterative solvers. This method is explained in details in [16]. Combining the MBFs with this method should lead to an other level a stability and this combination constitutes our objective.

III. PRELIMINARY RESULTS

In this section, one can find some preliminary simulations obtained with the MoM. The mesh has been generated with the help of GMSH [18]. The simulations are performed with the RWG basis functions. The goal is to analyse the stability of the MoM system of equations. Since the complexity of the whole accelerator is around 350000 basis functions, one only simulates a slice of the accelerator, as shown Fig. 5. Fig. 6 shows the front and top view of the RFQ slice.

The slice of the RFQ is made of 5802 RWG basis functions. An additional RWG placed on the top of the RFQ (as shown Fig. 5) is used as the source. A delta-gap of 1000 V centered on this basis function generates a current that radiates the

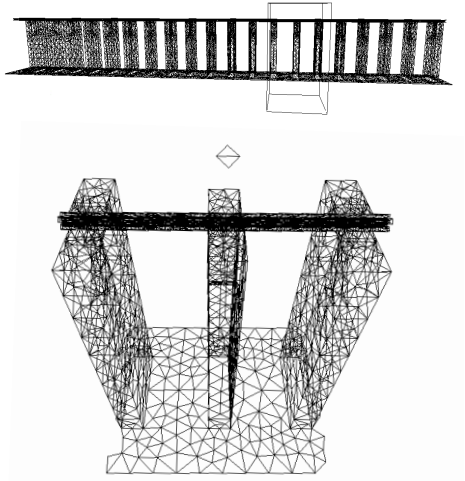


Fig. 5. Definition of the slice for the MoM simulation

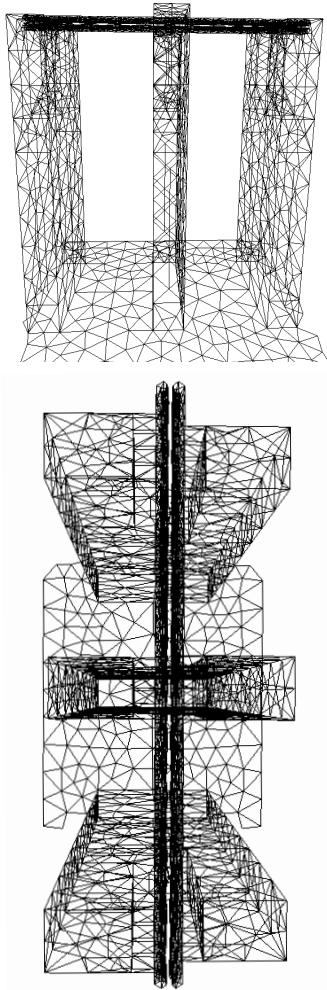


Fig. 6. Front and top view of the slice

excitation field. The chosen frequency is 175 MHz, which

corresponds to the working frequency of the accelerator. Unexpectedly, the condition number of the impedance matrix is equal to $3.9024 \cdot 10^5$. The residual error ($r = Zx - e$, where Z is the impedance matrix, x the coefficients of the RWG basis functions and e the excitation) is equal to $2.4606 \cdot 10^{-14}$. The inversion was carried out with the `mldivide` function of Matlab. GMRES has been used as well but, has shown, a very slow convergence has been obtained; 280 iterations are required to reach a residual error of 10^{-2} and 720 to reach 0.005. The norm of the current density distribution for a zero phase $e^{j\omega t} = 1$ is shown Fig. 7. The norm of the current density is calculated at each node of each triangle. The norm is then linearly re-scaled between -1 and 1. A linear color scale between red and green is then created (red for -1 and green for +1).

The black color corresponds to the basis functions for which the current is lower than a specific threshold value. These basis functions are omitted in order to keep a reasonable contrast of colors. One can notice a high current density between the rods as shown Fig. 7.

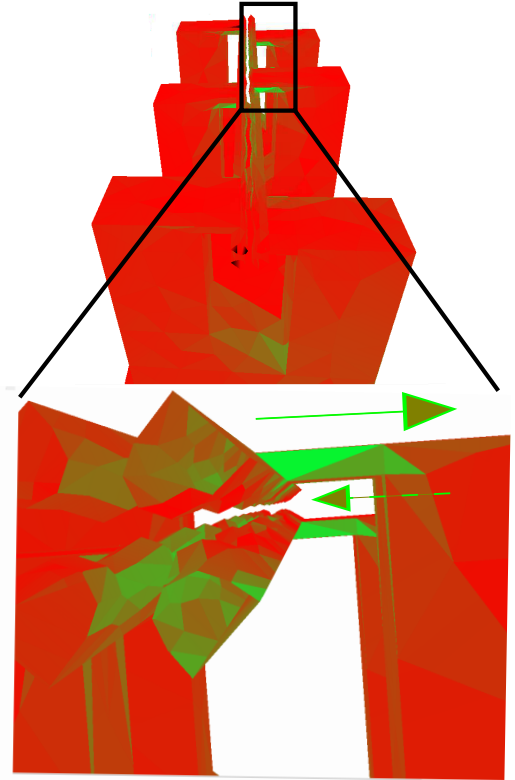


Fig. 7. Zoom on the current density distribution in the rods.

The concentration of the current on the surface of the rods is due to the capacity between them. As shown on the picture, two rods angularly separated by 90 degrees are connected to different stems while two rods angularly separated of 180 degrees are connected to the same ones. This due to the fact that the polarity of two opposite rods must be equal and the

polarity of two rods angularly spaced of 90 degrees must be identical, as shown in Fig. 1. As shown on Fig. 7, the current density is high at the connections between the rods and the stems. This is due to the fact that the current flows from one rod to the others through the stems and the ground plane. Two arrows underline the current flow going through the stems in Fig. 7. The small thickness of the connections causes a high current density. Since the length of the slice is 50 cm and the wavelength is 170 cm the current distribution doesn't vary much in the longitudinal direction.

Fig. 8 and Fig. 9 show the current density distribution but for colors mapped with a logarithm function instead of the linear function.

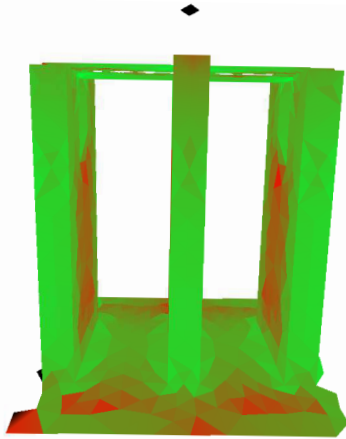


Fig. 8. Norm of the current density distribution calculated via a logarithmic scale. Red color corresponds to the lowest current density and the green color to the highest one.

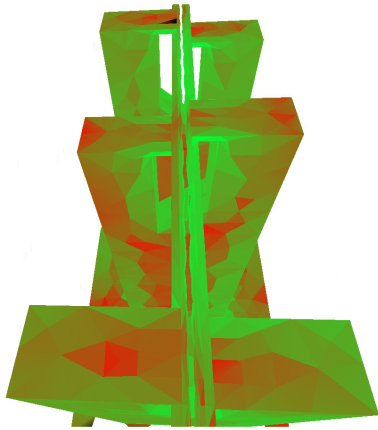


Fig. 9. Top view of the current density distribution for the logarithm mapping.

IV. CONCLUSION

The MoM is likely to improve the computation speed of the present numerical solvers for linear accelerators. However,

as one pointed it out, the very fine mesh may lead to the well known low-frequency breakdown. Coupling the MBF concept with the loop-tree or loop-star basis functions should, on one hand, solve the stability issue and, on the second hand, improve the computation speed. The simulations carried out with the RWG basis functions for a relatively small problem has demonstrated the relevance of the MoM for the electromagnetic simulation of RFQs. Unexpectedly, the MoM system of equations was still sufficiently well conditioned to allow an accurate resolution. However, and as expected, GMRES turned out to be inefficient.

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