Prandtl second optimal wing: model design and testing with near wake PIV, and 3-D simulation

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Prandtl first optimal wing

- consider a **family of wings with fixed span and fixed lift**
- wing with **lowest induced drag** = **elliptical circulation distribution**
- hence the near wake rollup displays a strong « tip vortex »
- the downwash velocity (induced by the wake) is uniform across the span
- **wing without twist**: elliptical chord distribution, hence all profiles have the same effective AoA and thus operate at the same C_l



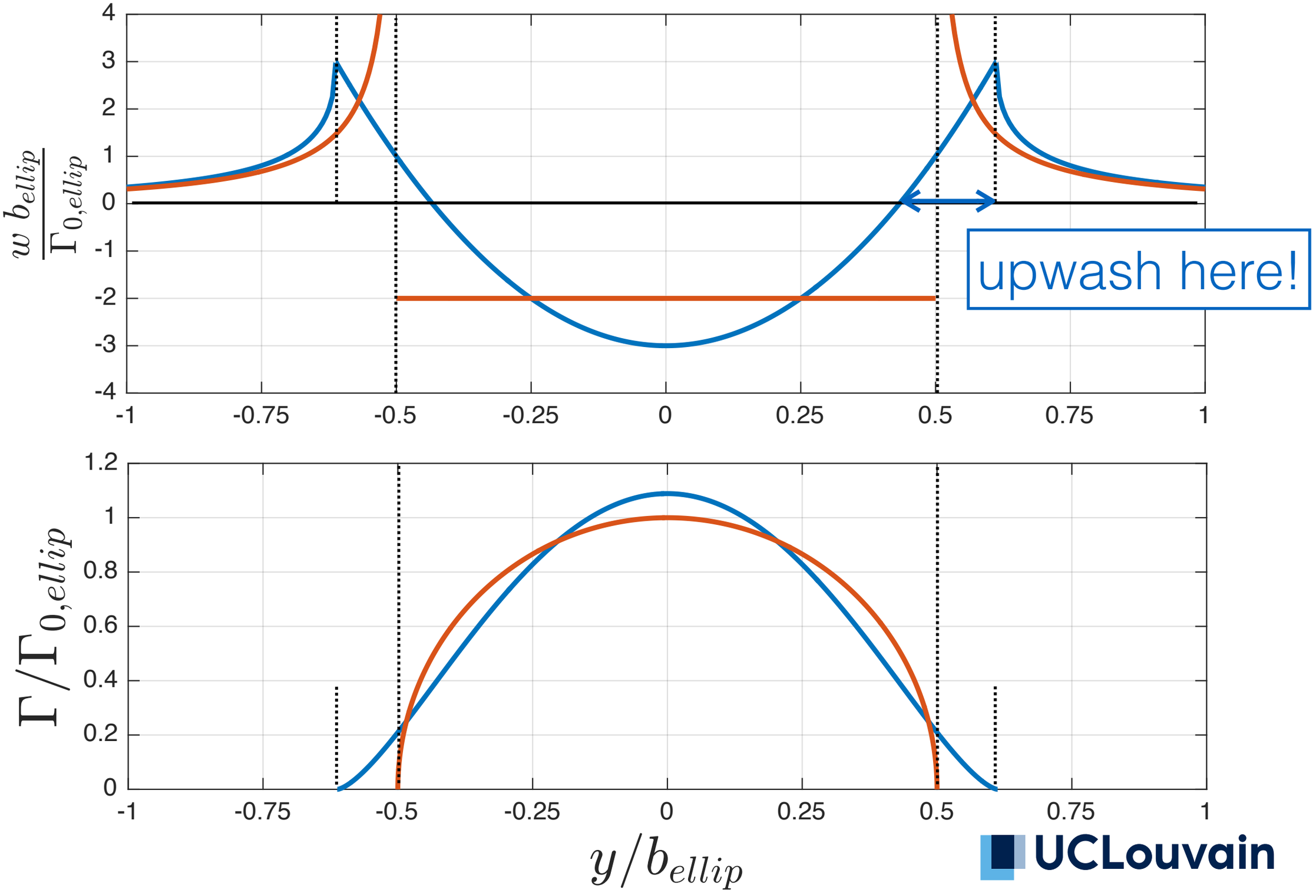
Prandtl second optimal wing

- short (6 pages) and little noticed article: L. Prandtl, Über Tragflügel kleinsten induzierten Widerstandes, Zeitschrift für Flugtechnik und Motorluftschiffahrt. 24 Jg. (1933) S. 305–306
- consider a **family of wings with fixed lift and fixed structural weight** (itself is proportional to the integral of the bending moment distribution)
- wing with **lowest induced drag** = **bell-shaped circulation distribution**
- hence the near wake rollup does not display a tip vortex
- the downwash velocity is not uniform across the span; some is upwash
- closer to **bird's wings**; allows efficient maneuvers without vertical tail





Comparison


$\frac{w\ b_{ellip}}{\Gamma_{0,ellip}}$
upwash here!
$\Gamma/\Gamma_{0,ellip}$
y/b_{ellip}

Comparison

- same:

$$L = \rho\, U_\infty \int_{-b/2}^{b/2} \Gamma(y)\, dy$$

$$L\, r^2 = \int_{-b/2}^{b/2} M(y)\, dy = \ldots = \rho\, U_\infty \int_{-b/2}^{b/2} \Gamma(y)\, \frac{y^2}{2}\, dy$$

- obtain:

$$b_{\text{ellip}} = 4\, r$$

$$b_{\text{bell}} = 4\sqrt{\frac{3}{2}}\, r \simeq 1.225\; b_{\text{ellip}}$$

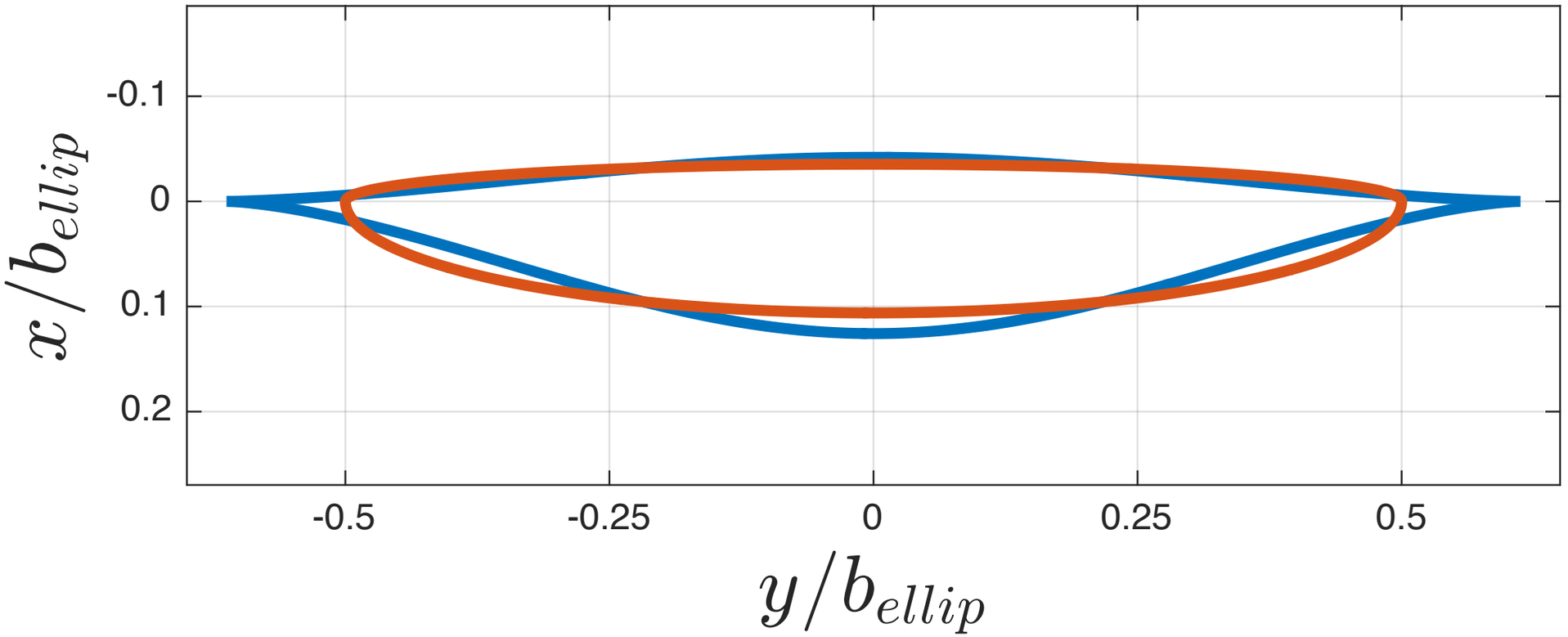
$$D_{i,\text{ellip}} = \frac{1}{8\pi}\, \frac{L^2}{\rho U_\infty^2 r^2}$$

$$D_{i,\text{bell}} = \frac{1}{9\pi}\, \frac{L^2}{\rho U_\infty^2 r^2} \simeq 0.889\, D_{i,\text{ellip}}$$

22.5% larger wingspan
11.1% smaller induced drag

Comparison: case without twist

- the wing without twist is too pointed near the wing tip: it cannot be built!

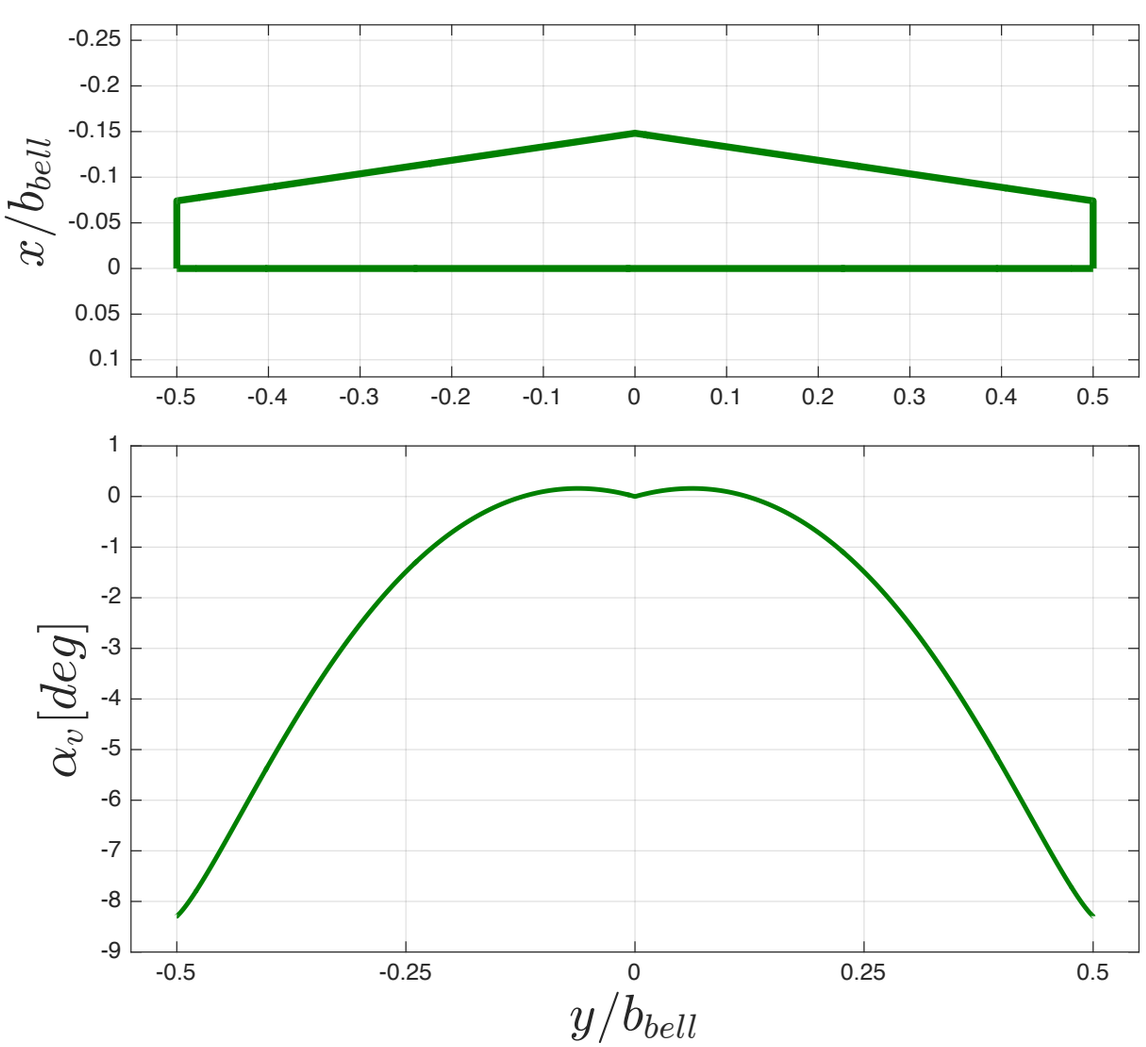


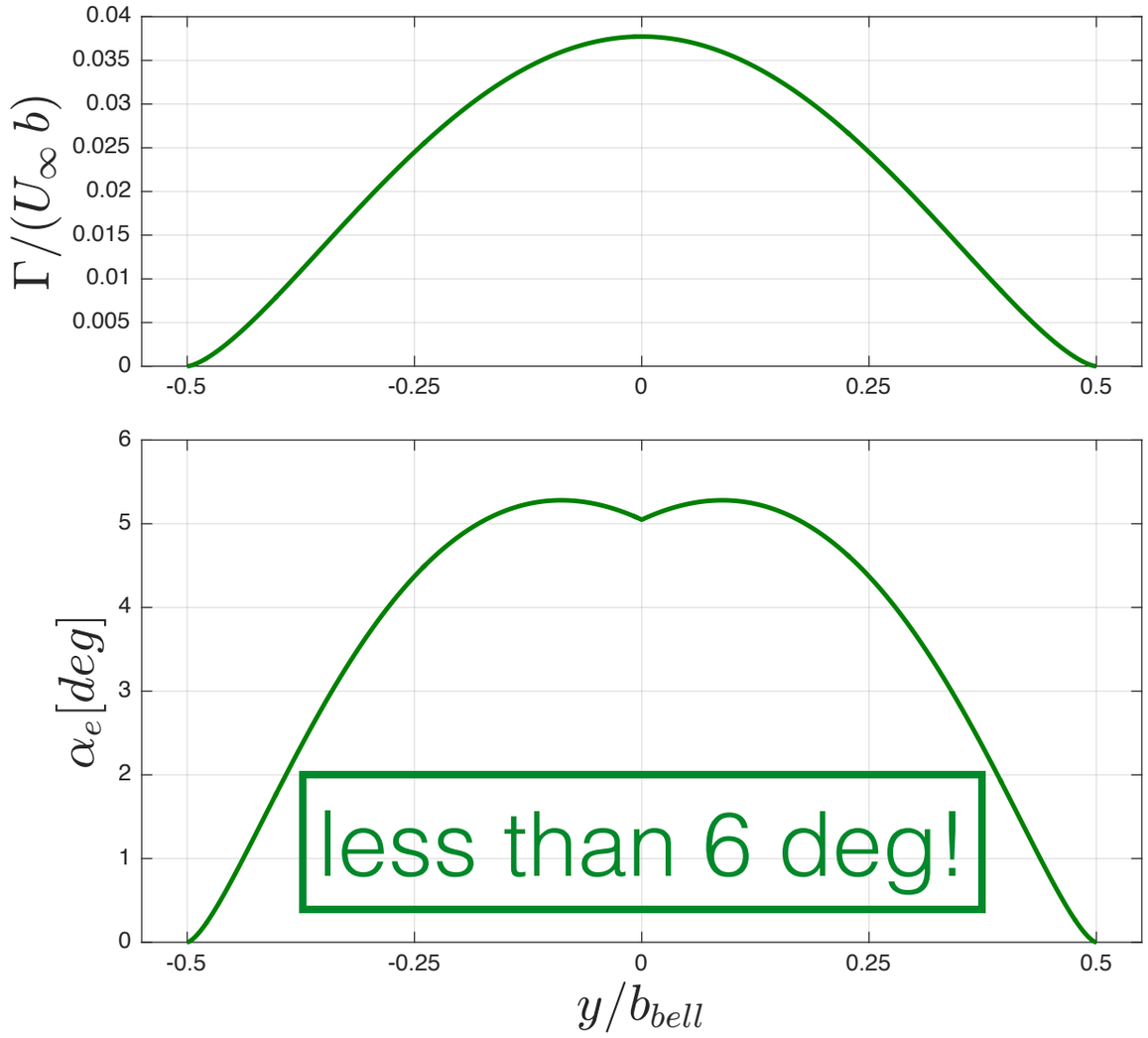
- **Solution**: *choose a wing shape*, and then find the twist angle distribution such that, at the design C_L , the loading is bell-shaped

Our wing

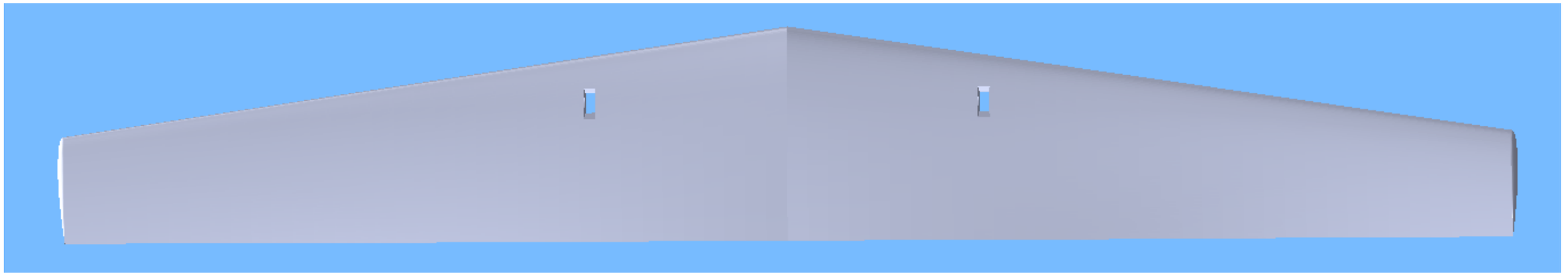
- linearly tapered wing with straight TE (needed for near wake PIV!)
- $C_{L,\mathrm{design}} = 0.40$, aerodynamic profile = NACA0012

$$b_{\mathrm{bell}} = 36.75\ \mathrm{cm}, c_r = 5.44\ \mathrm{cm}, c_t = \frac{c_r}{2} \Rightarrow S = 150\ \mathrm{cm}^2, AR = 9.0$$

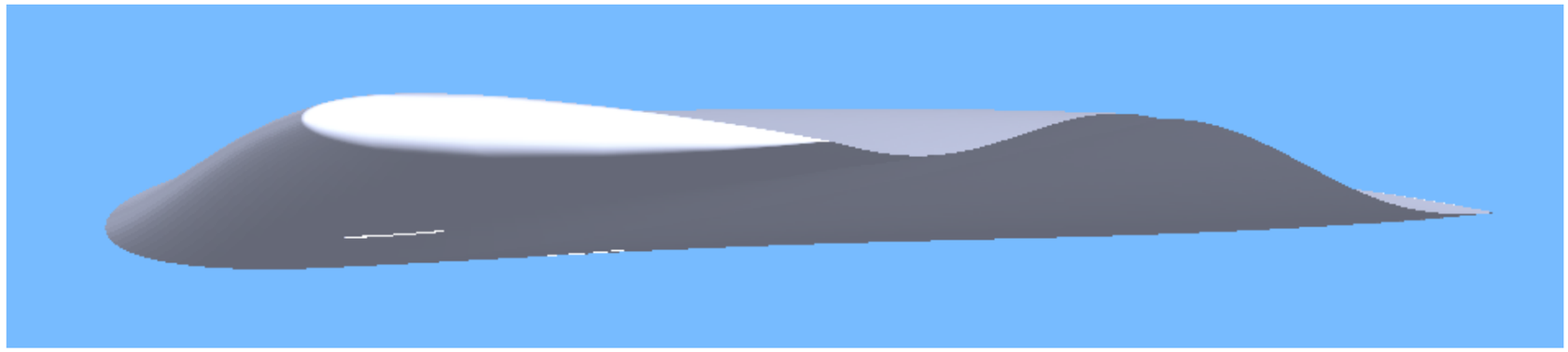


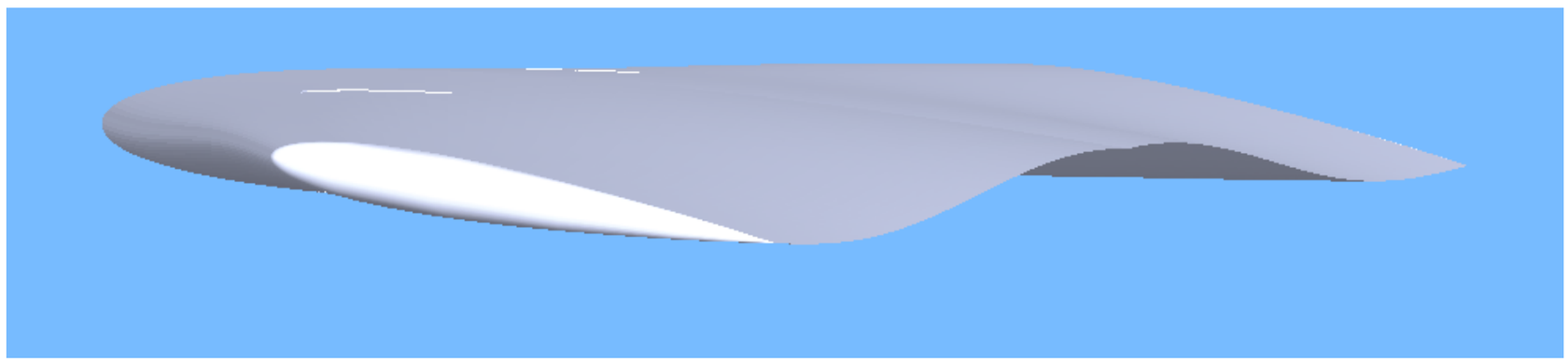


Our wing

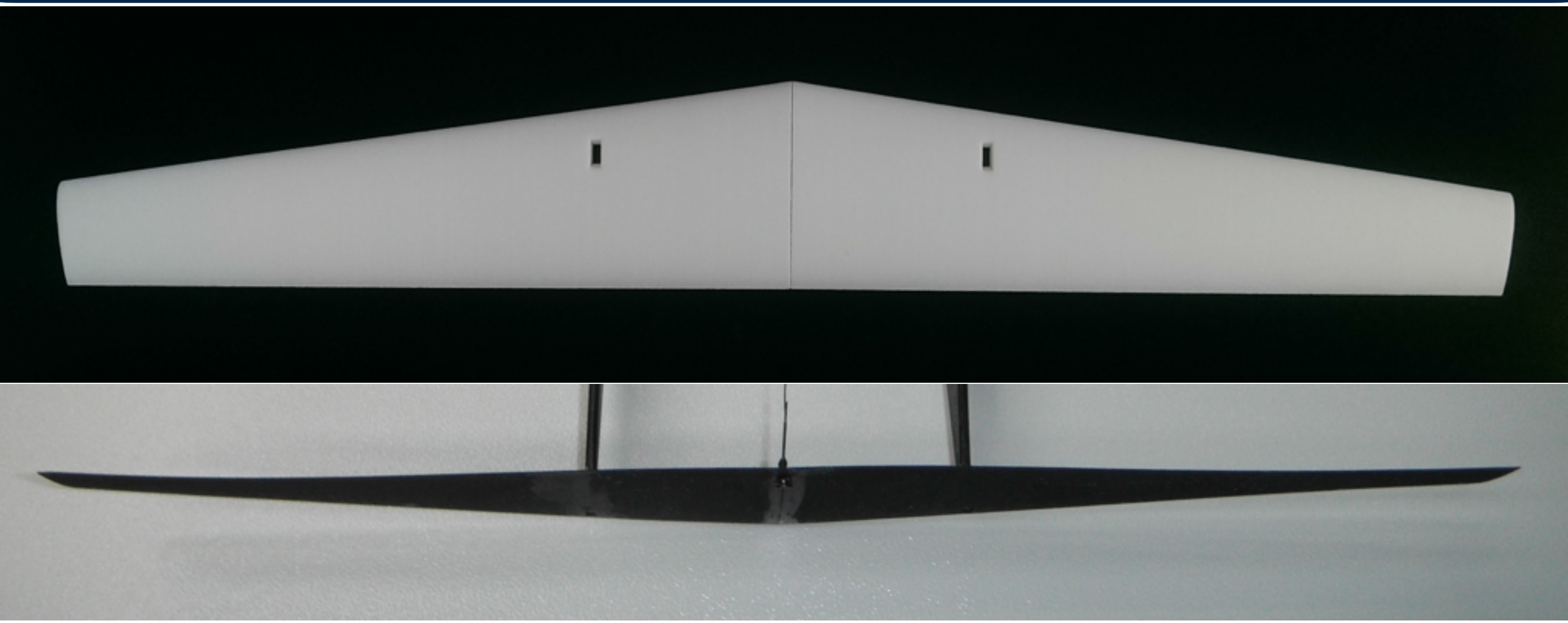








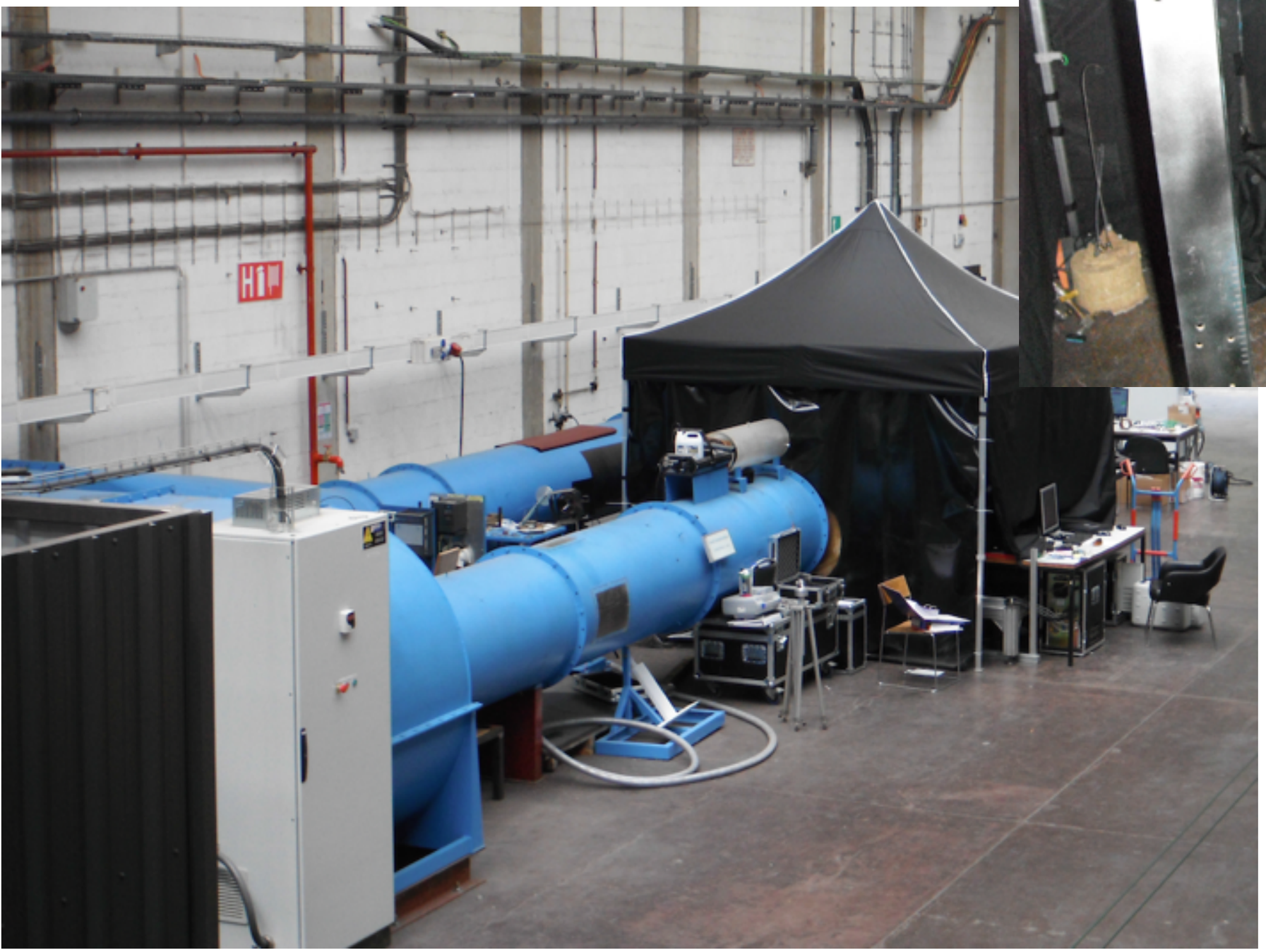
Our wing





Our open section wind tunnel



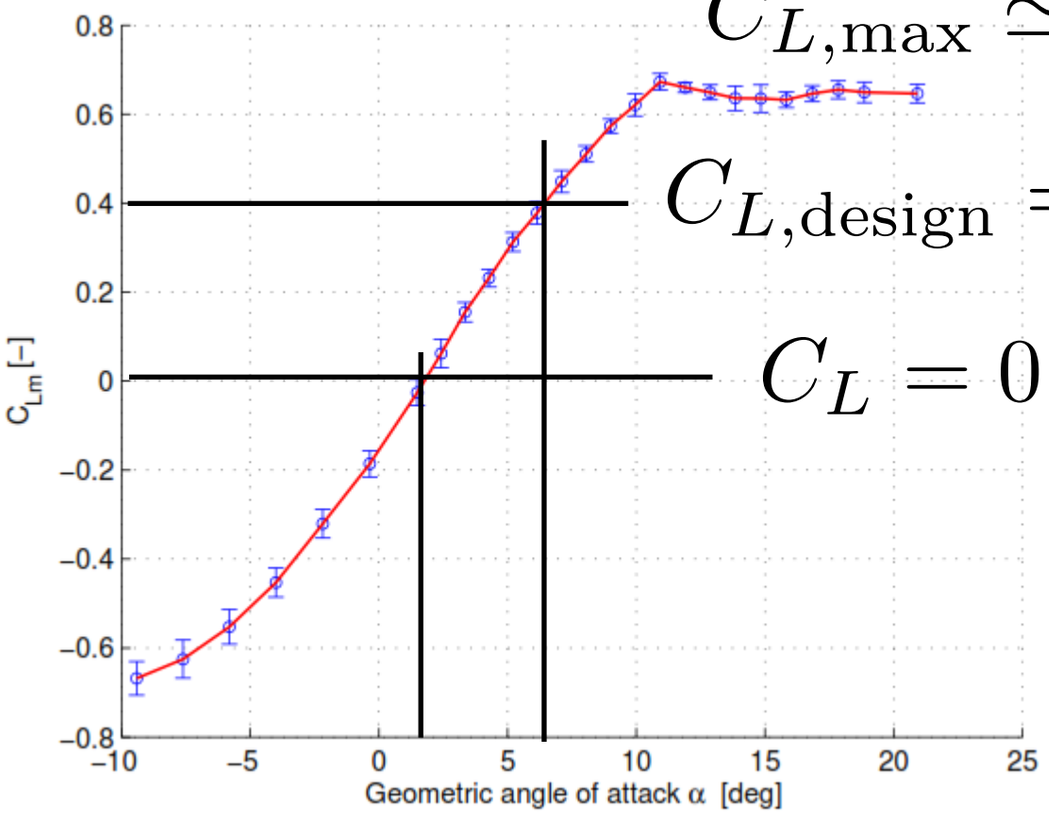


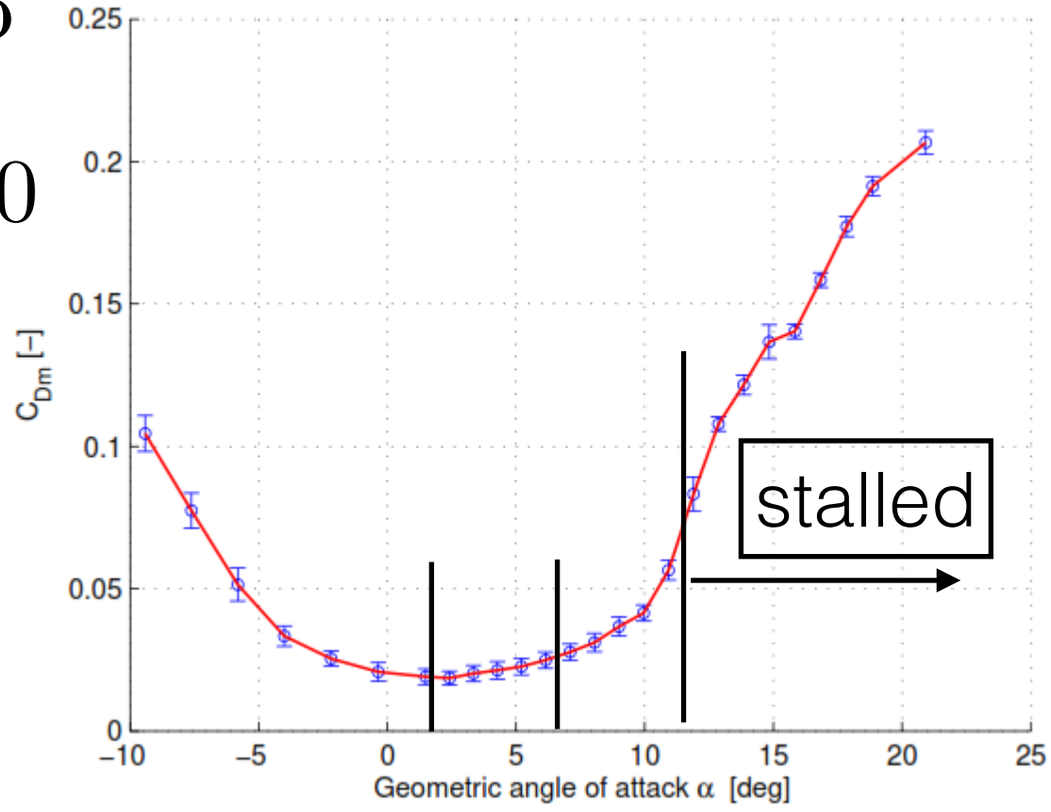
Lift and drag results

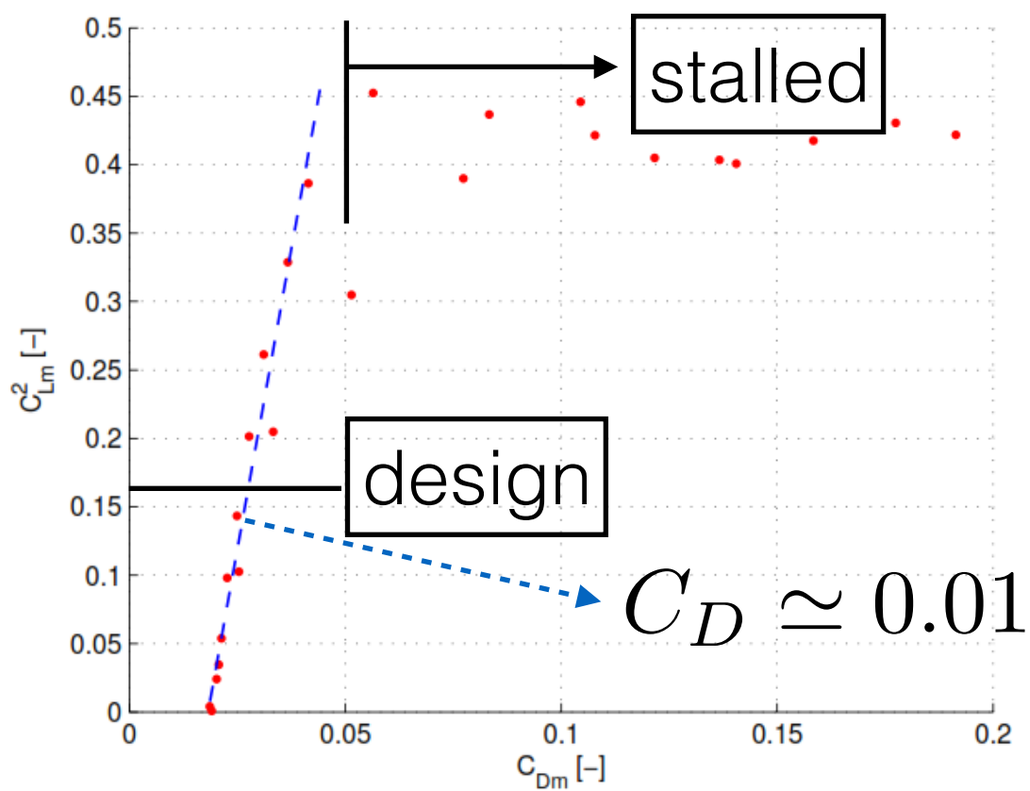
$$C_{L,\max} \simeq 0.65$$

$$C_{L,\text{design}} = 0.40$$

$$C_L = 0$$







$$C_D \simeq 0.019 + 0.049\ C_L^2$$

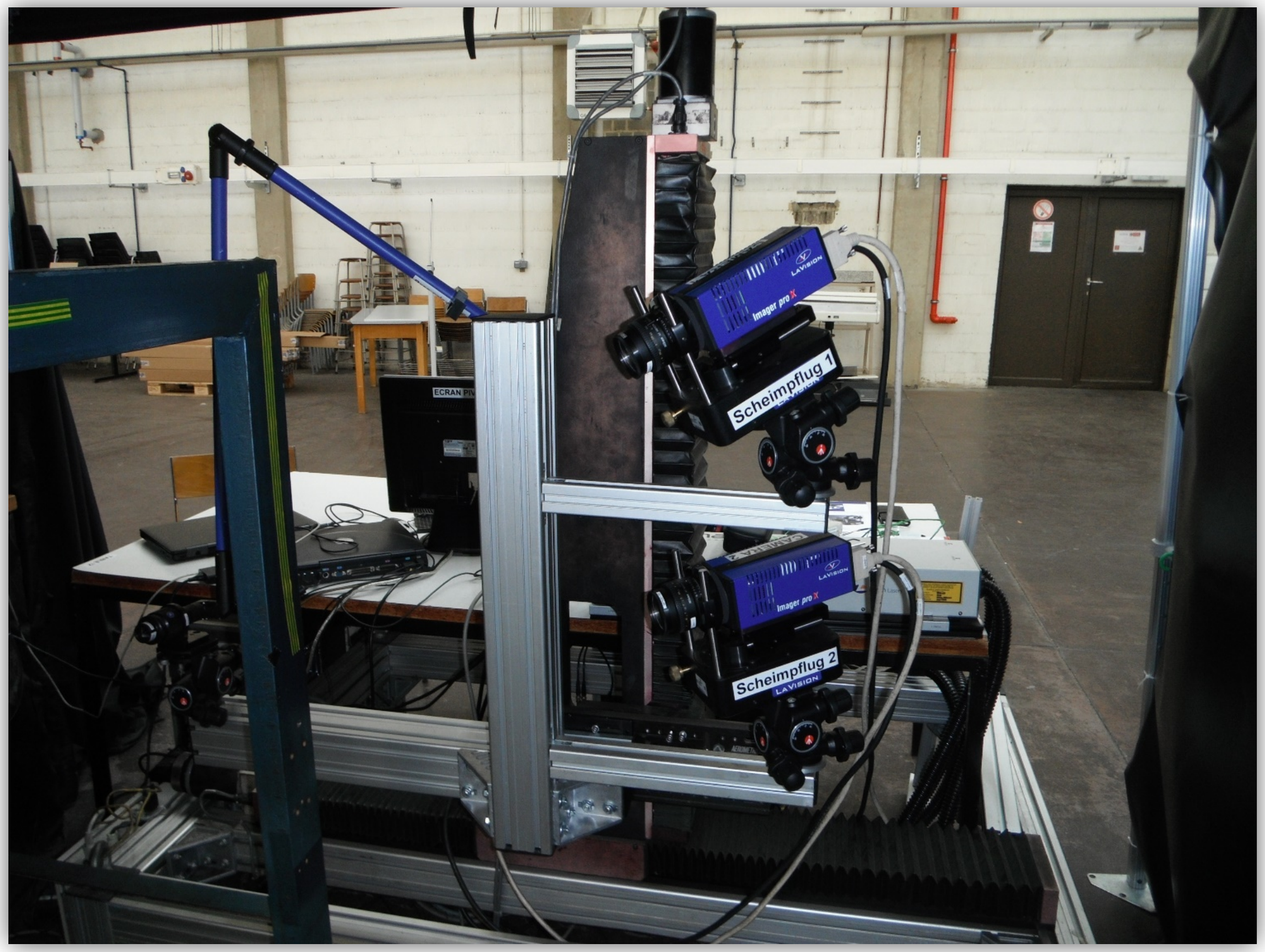
$$U_\infty = 30\ \text{m/s}$$

$$Re = \frac{U_\infty\, c_r}{\nu} \simeq 1.1\ 10^5$$

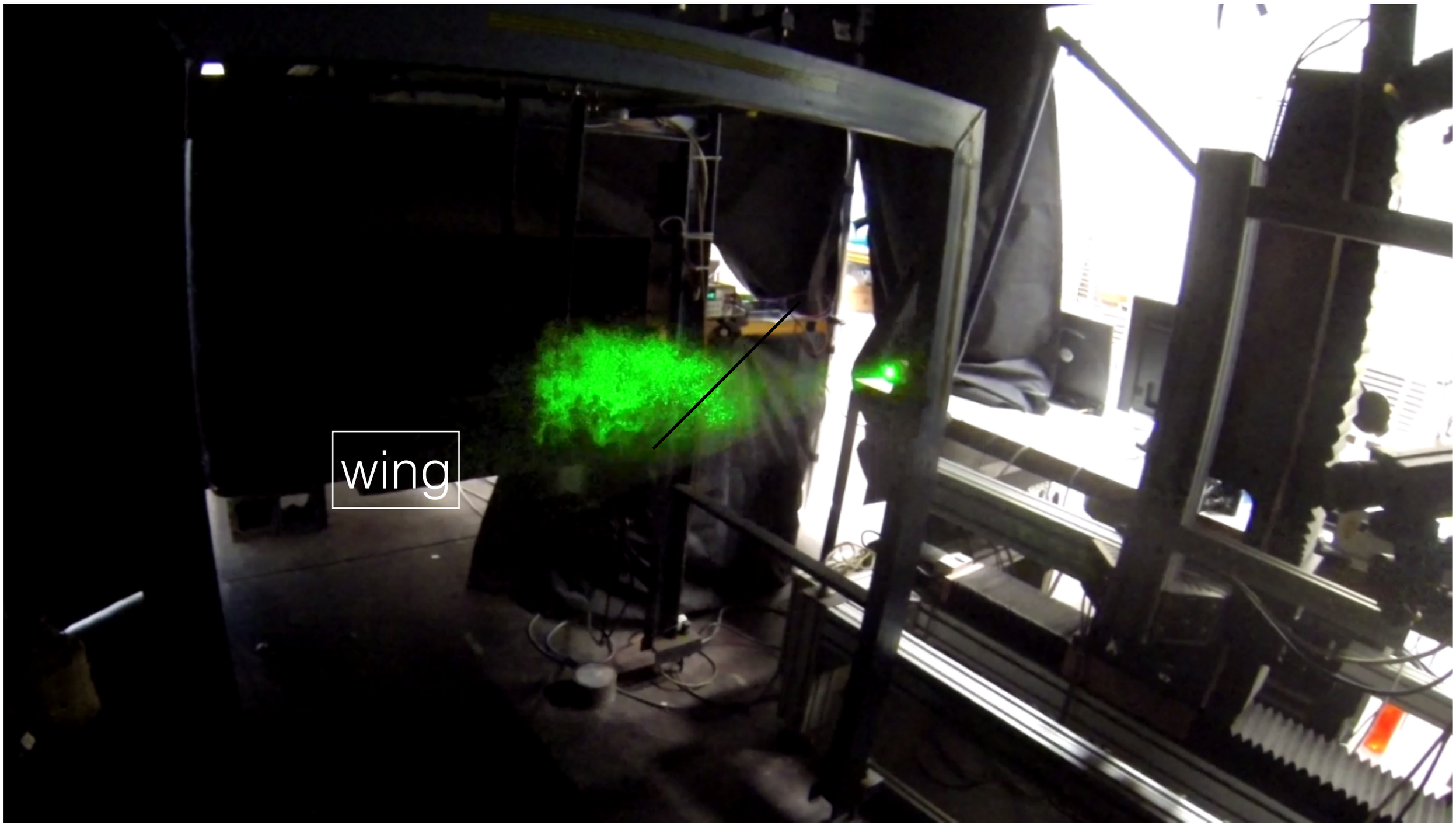
Lift and drag results

- At design:
  - Prandtl theory: $C_{D,i} = \frac{4}{3} \frac{1}{\pi\, AR} C_L^2 = 0.0472\, C_L^2$
  - Caution: Prandtl theory only captures the drag induced by the wake!
  - there is also a variation of the aerodynamic profile drag: $C_d = C_{d,0} + \beta\, C_l^2$

Stereo-PIV setup

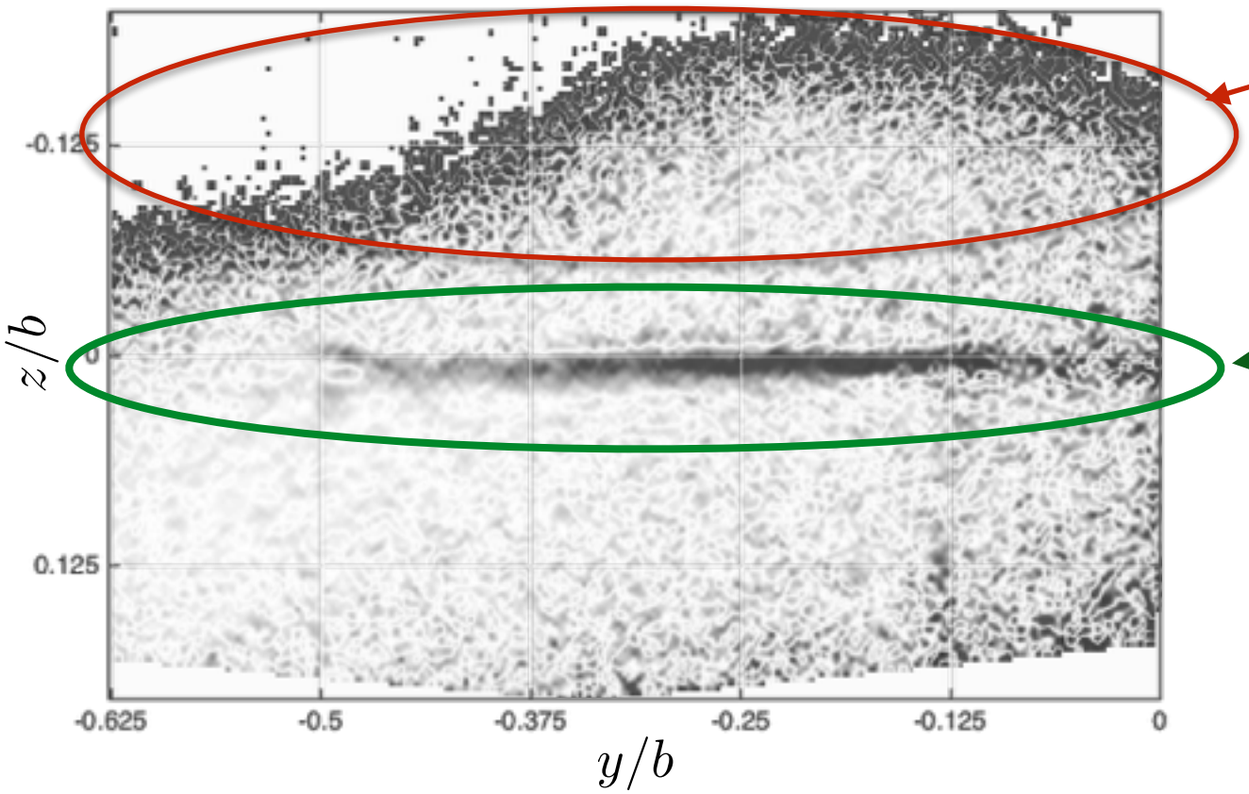


stereo-PIV setup and wind tunnel



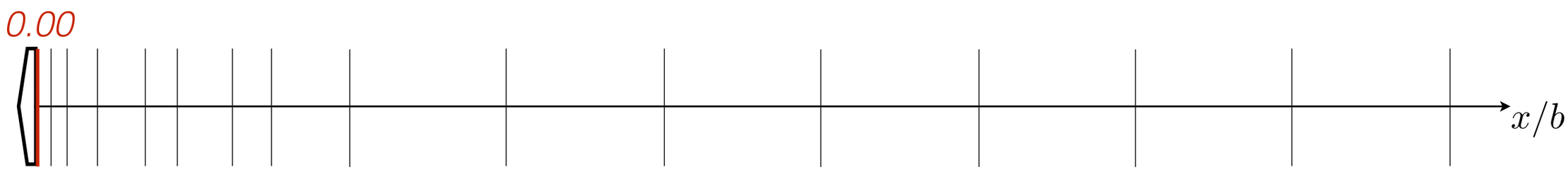
x-vorticity: PIV

Only the left part of the wake was measured

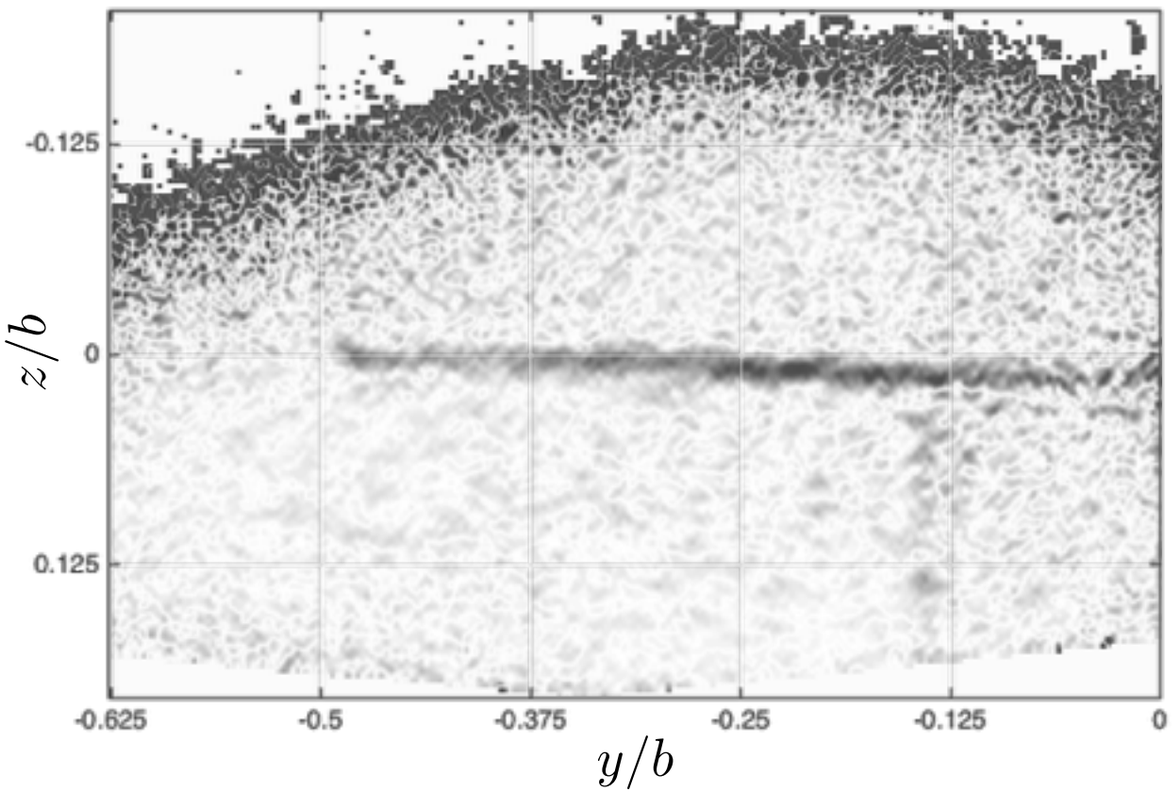


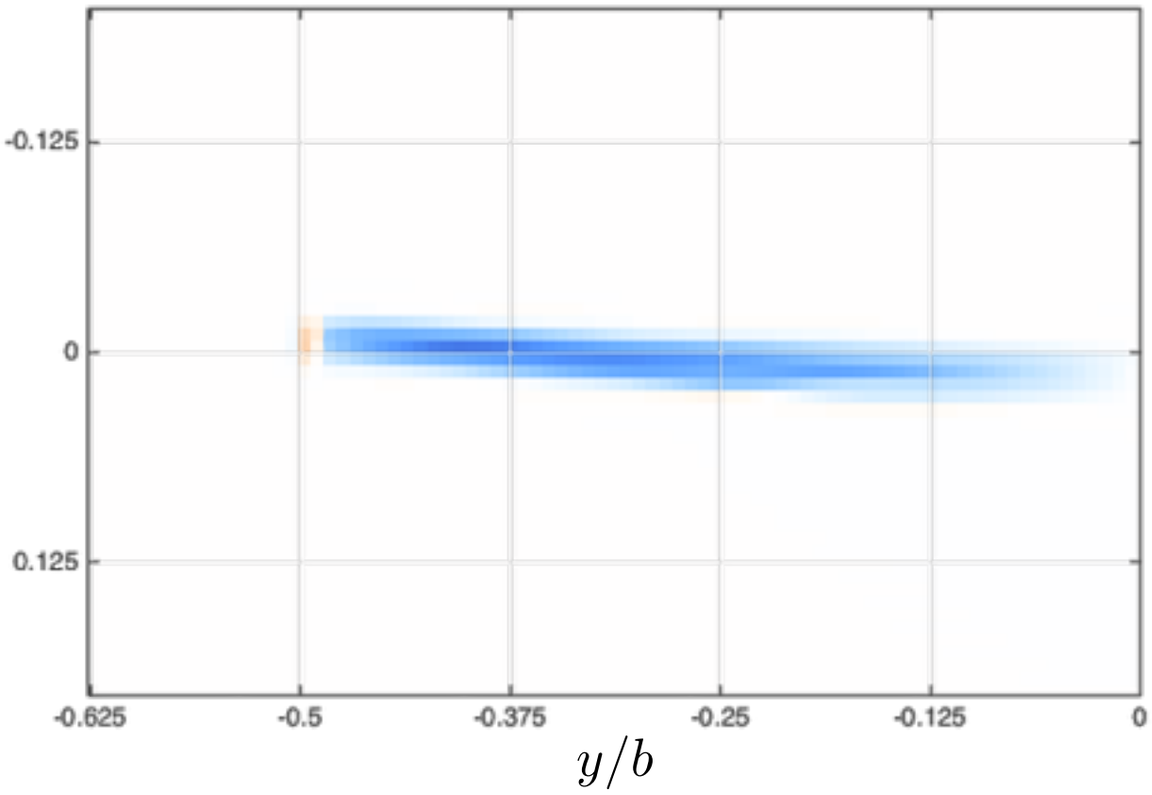
noise! = edge of PIV data

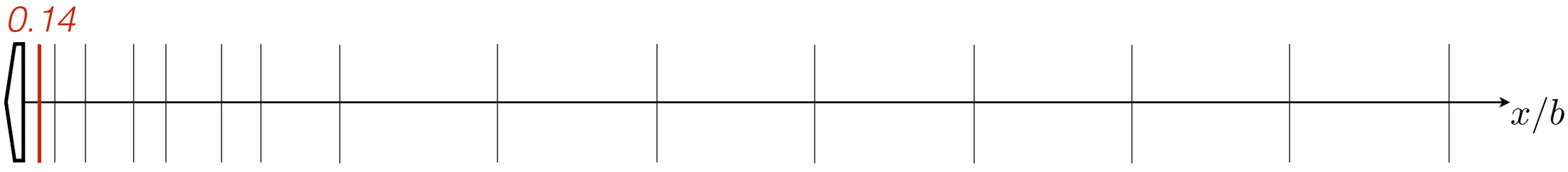
look only at this! = wake vorticity field



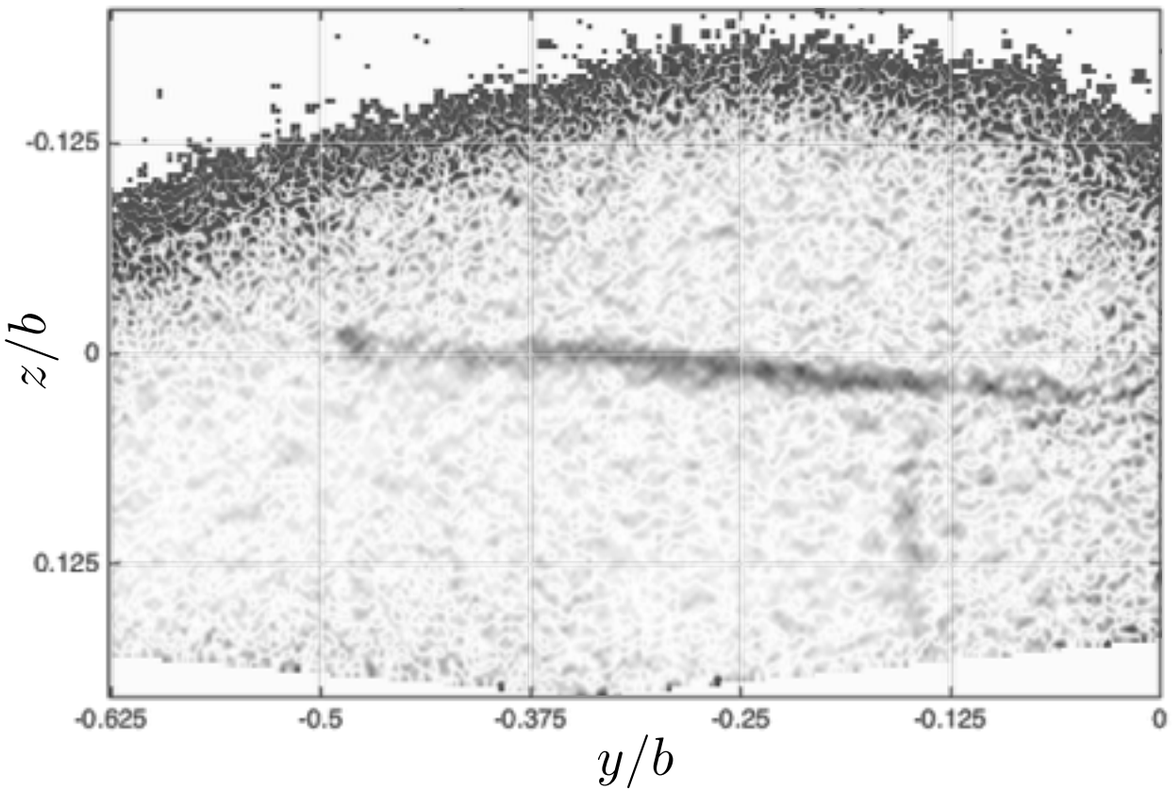
x-vorticity: PIV versus Simulation

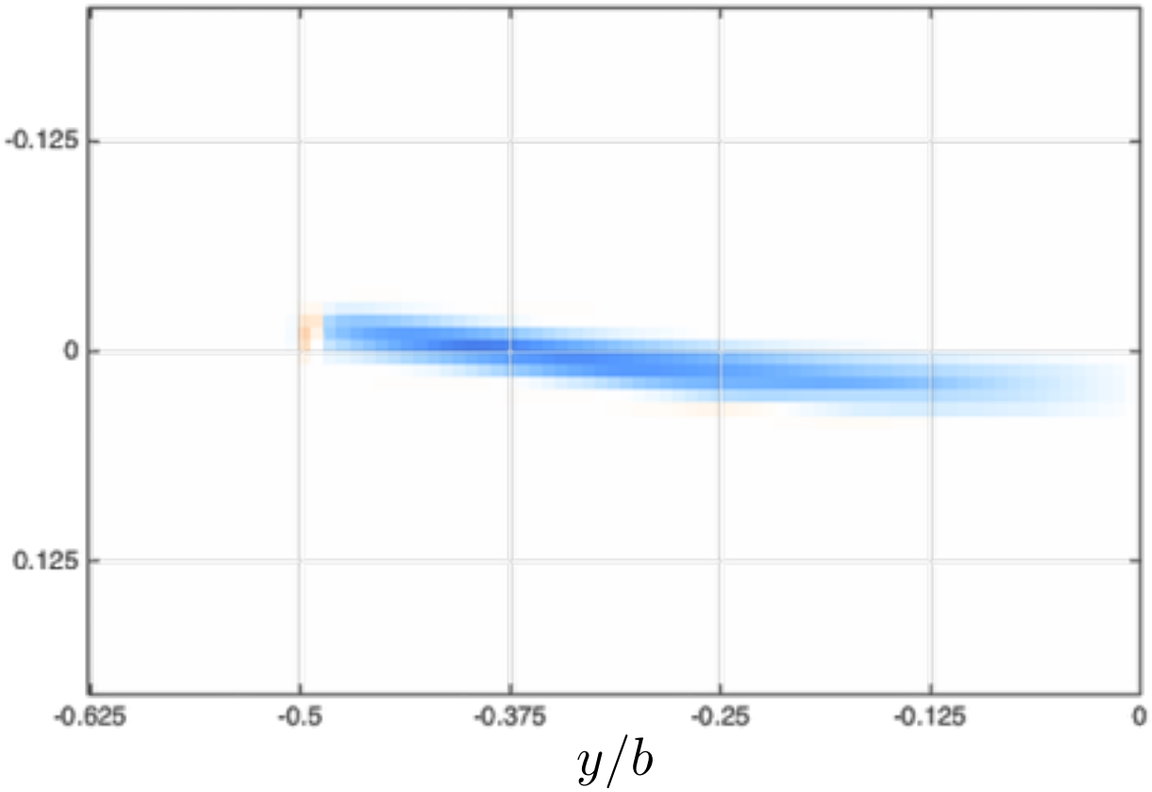


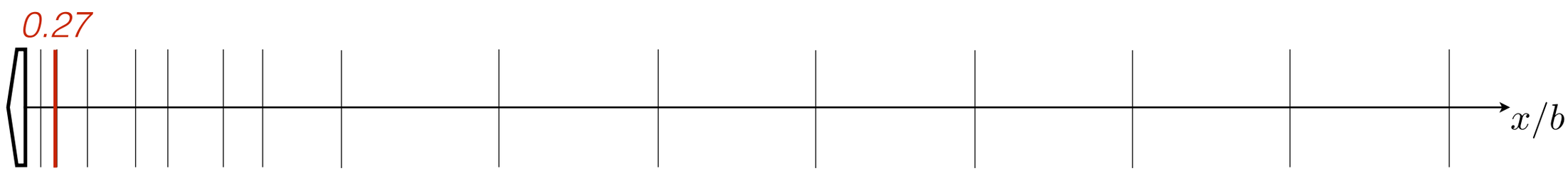




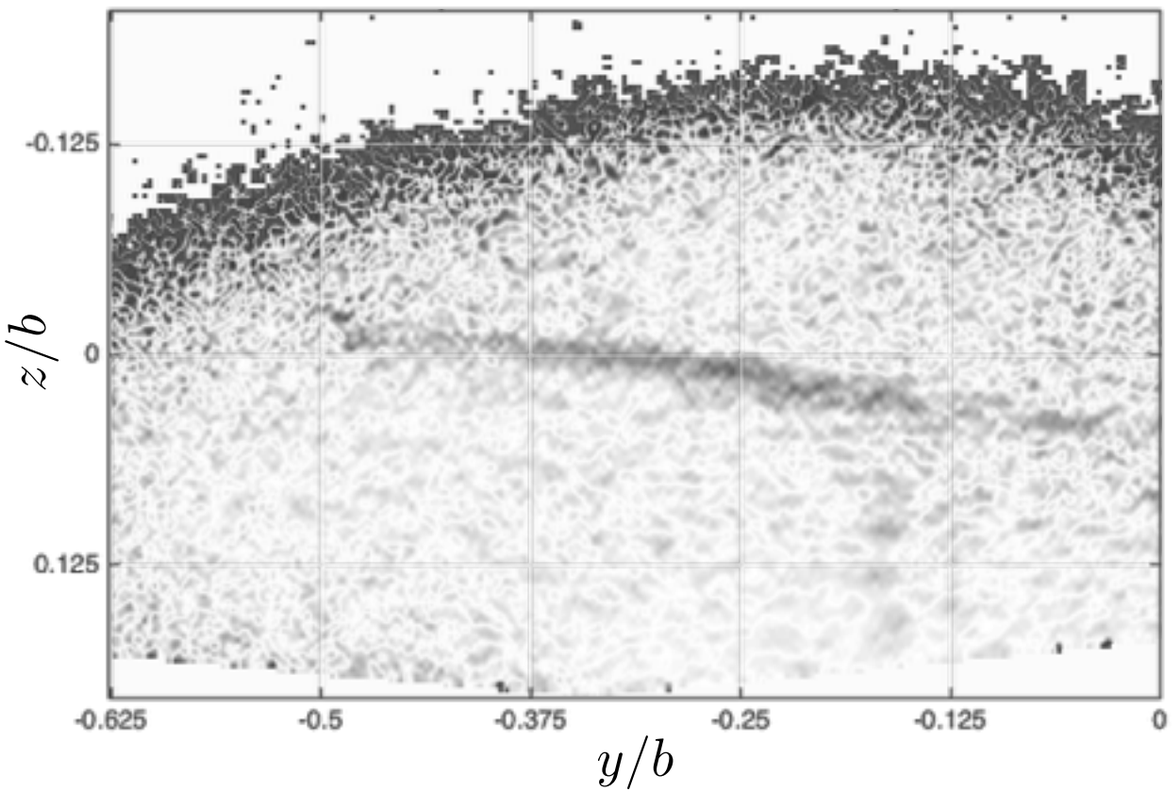
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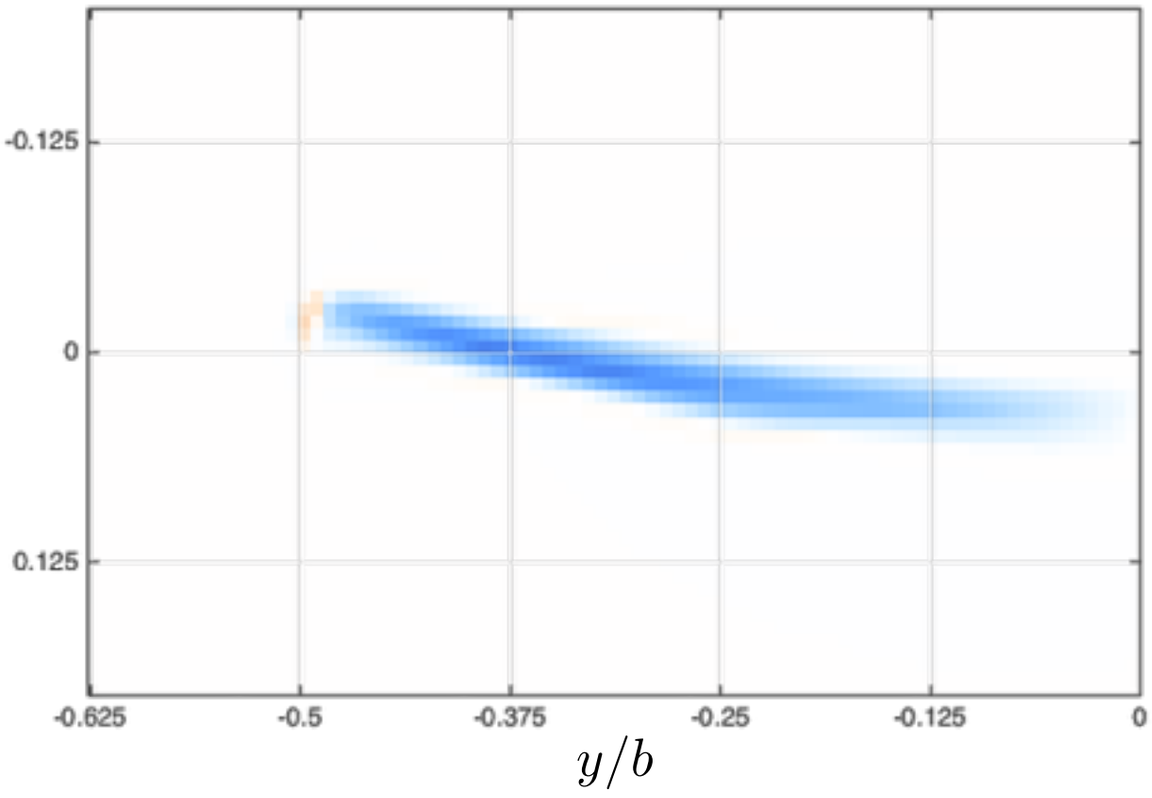


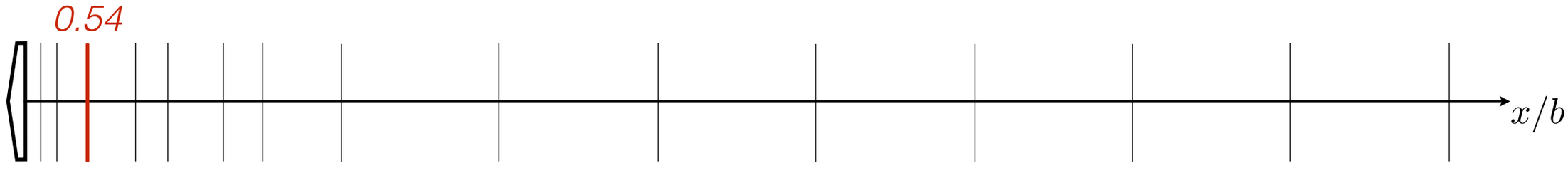




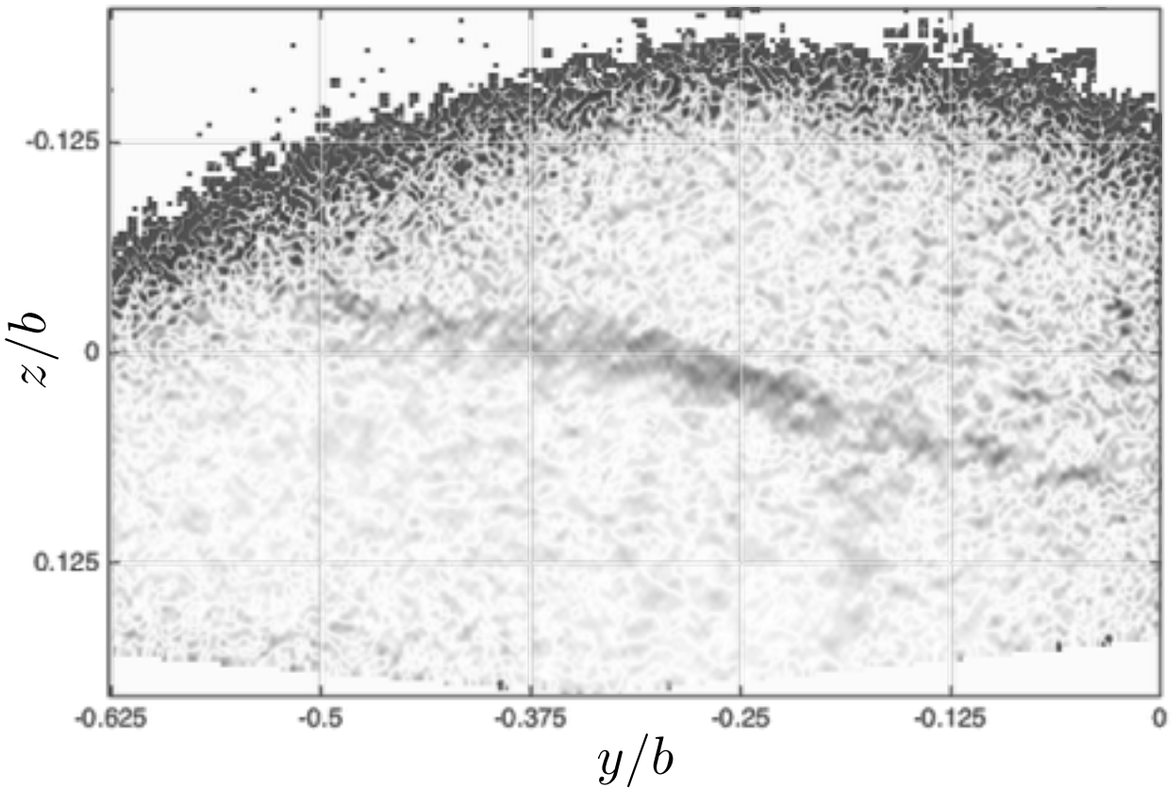
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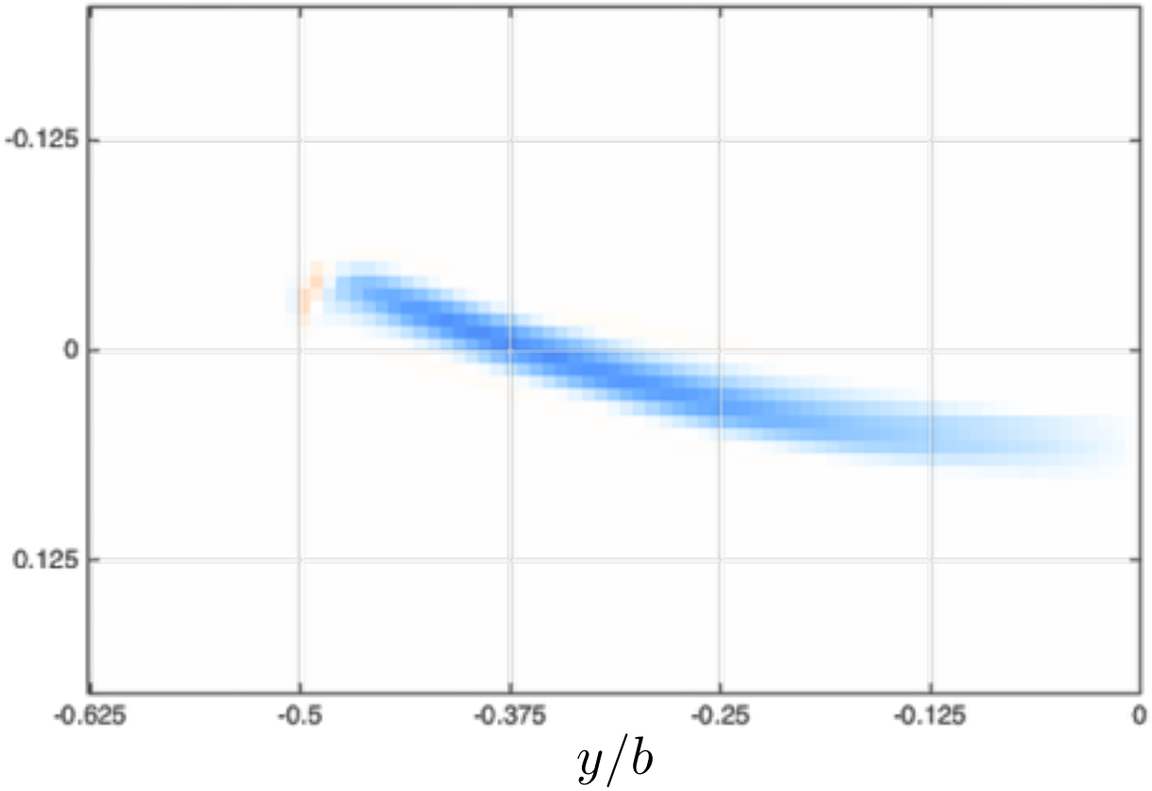


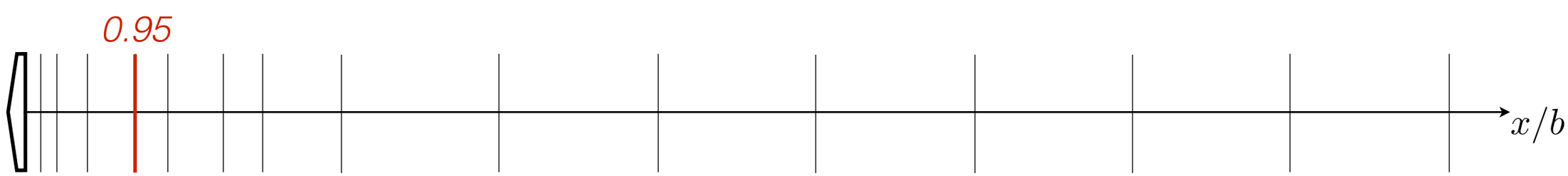




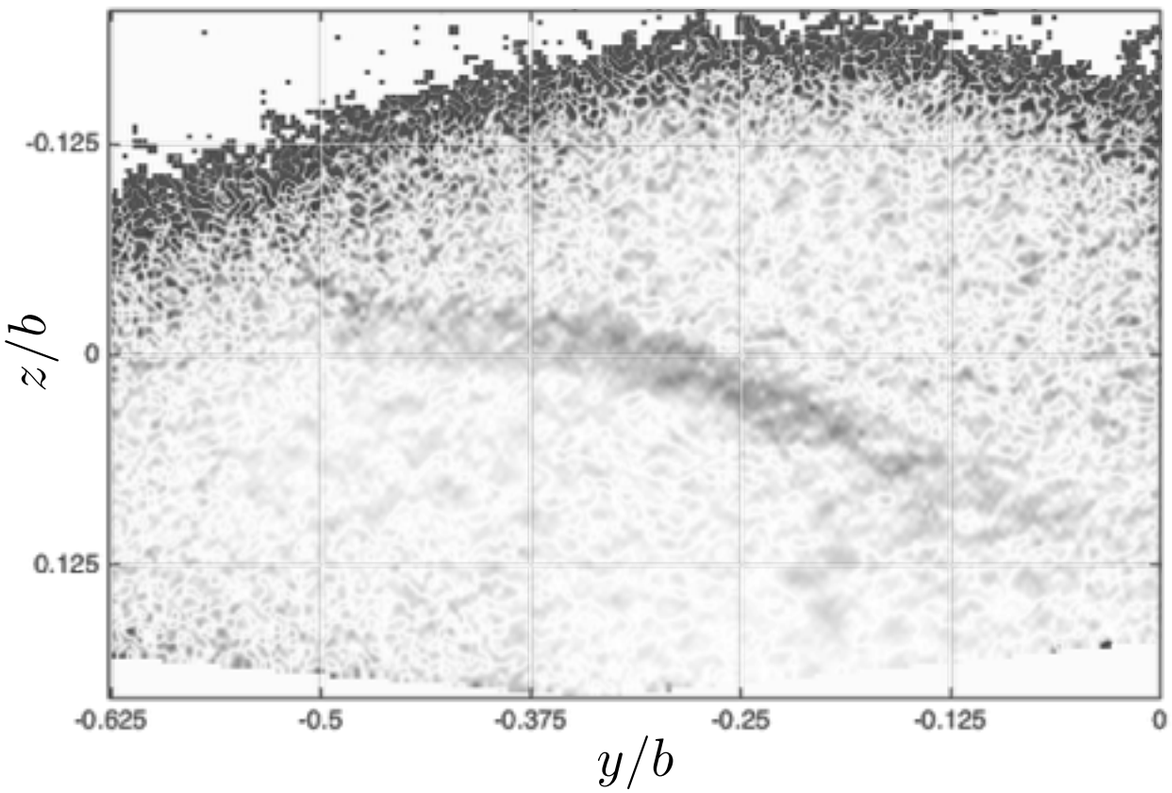
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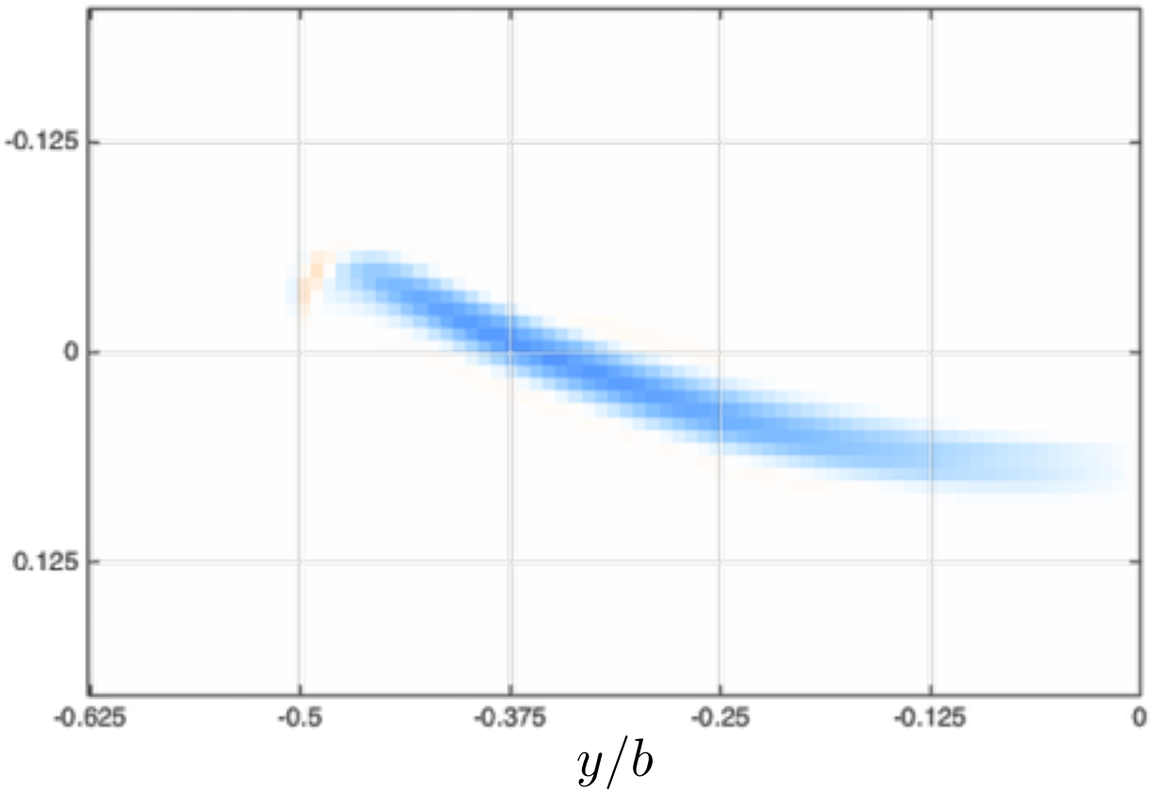


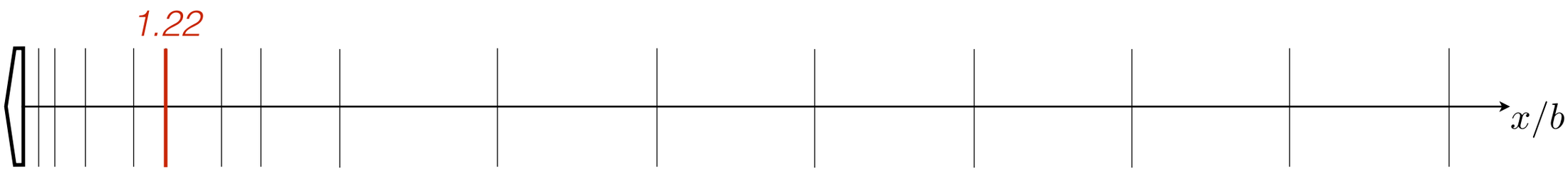




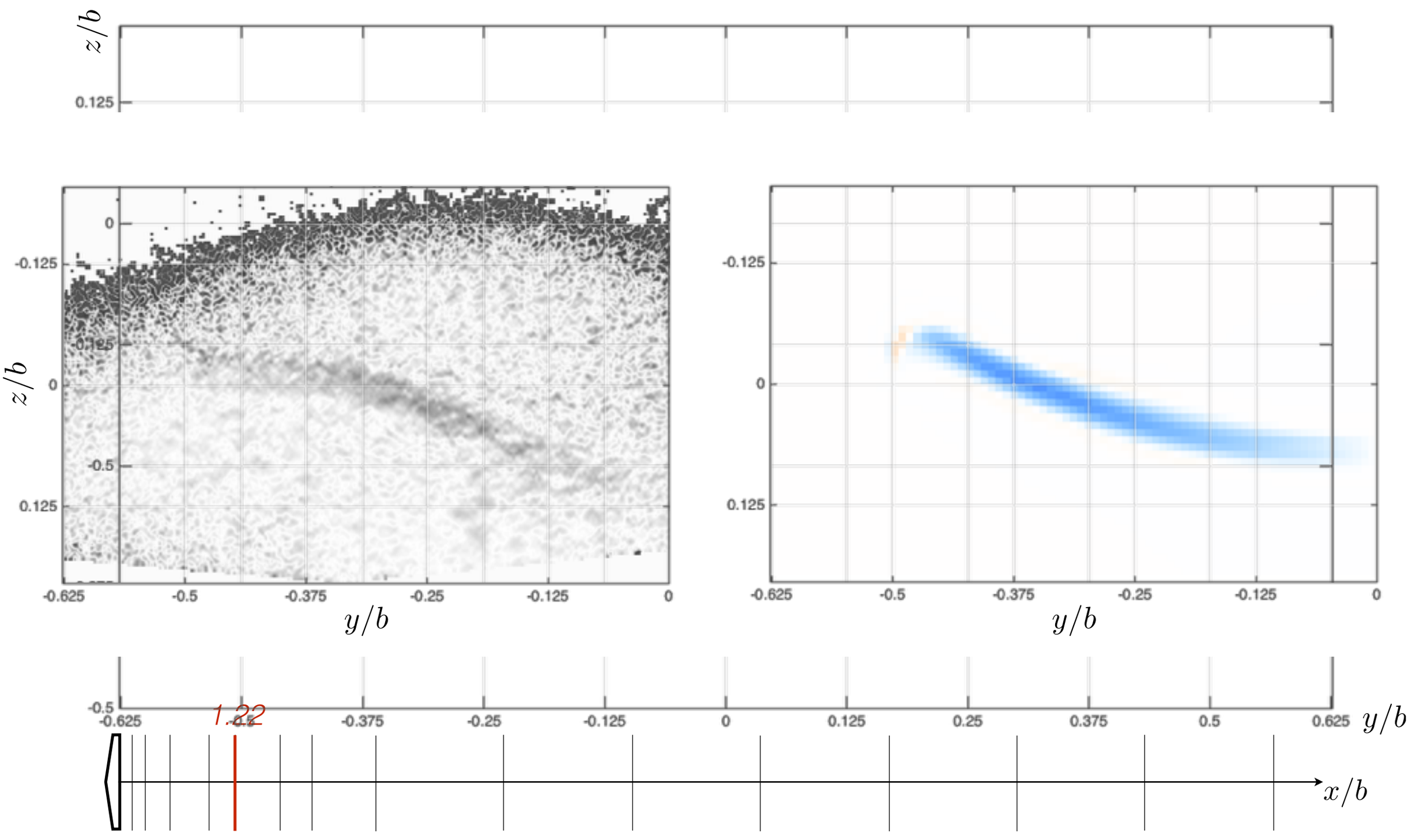
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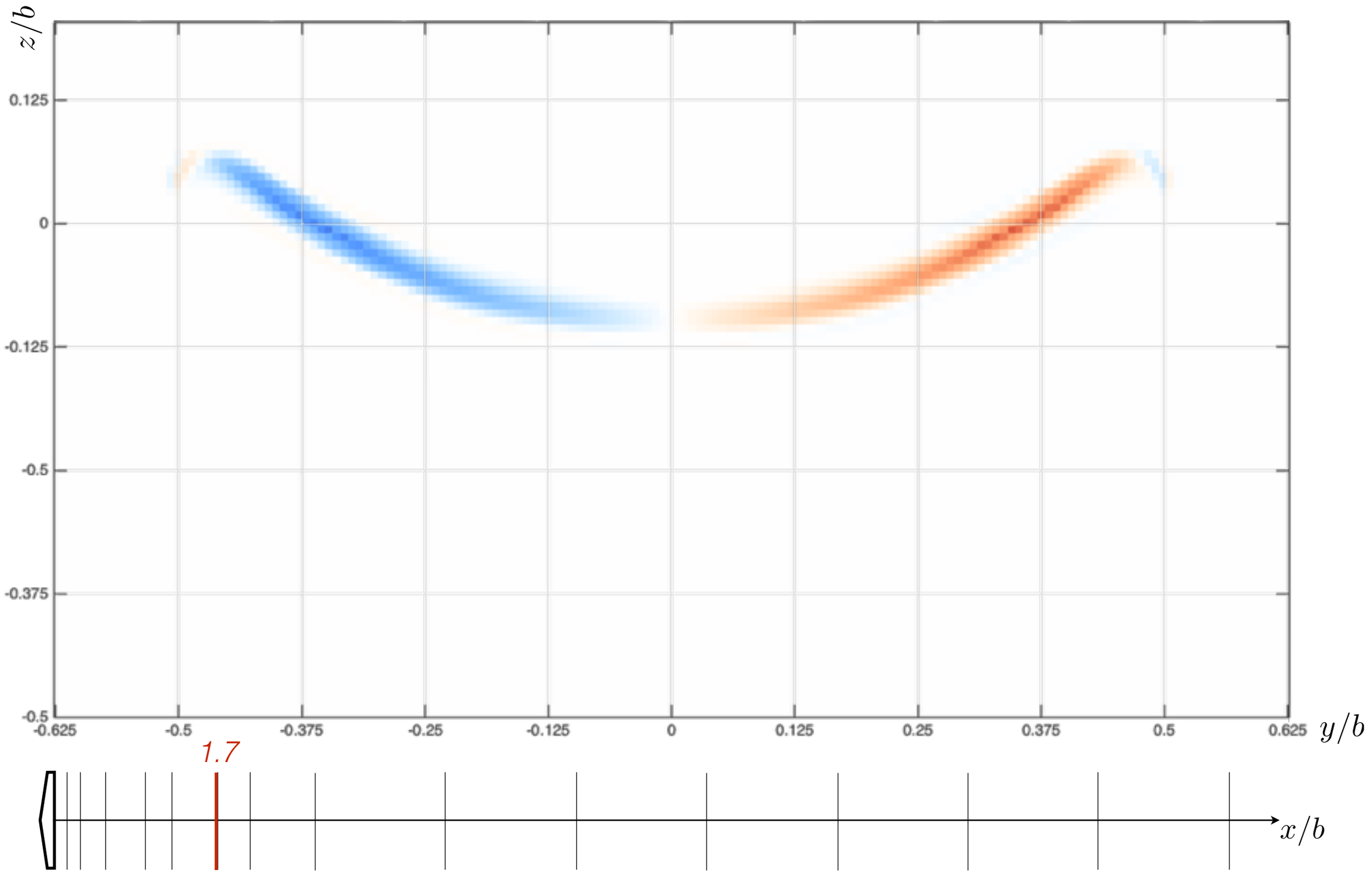




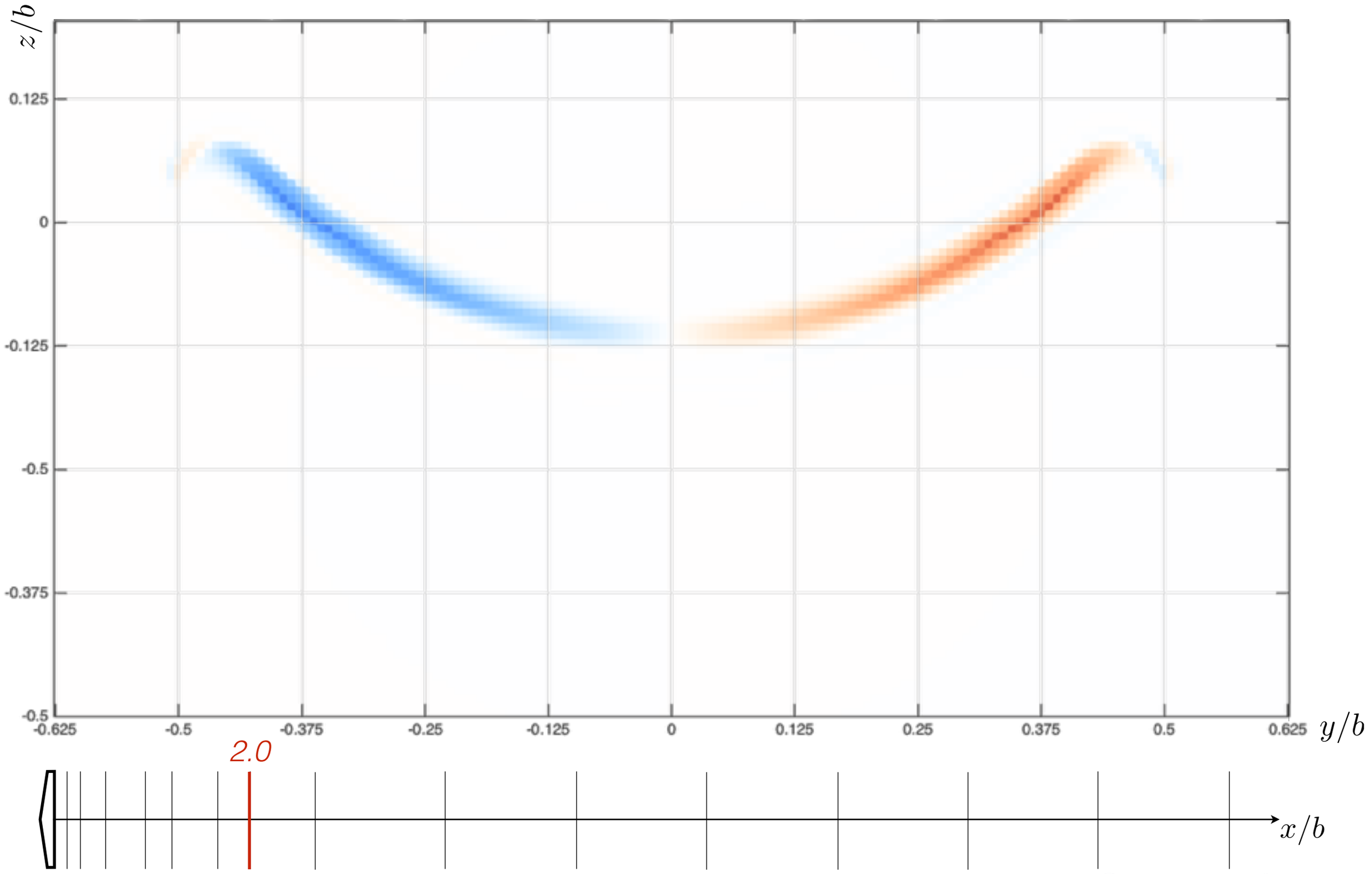
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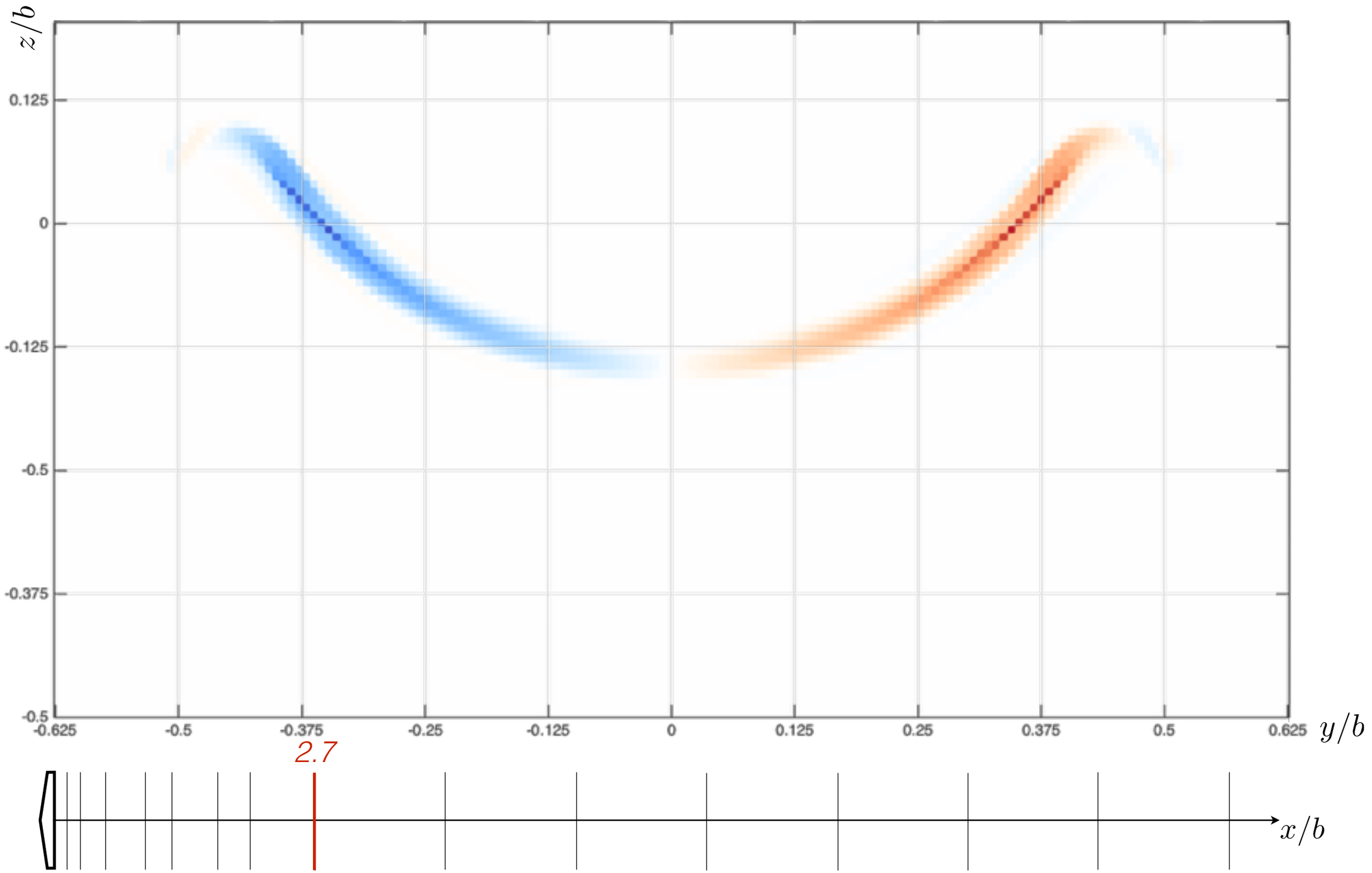
x-vorticity: Simulation



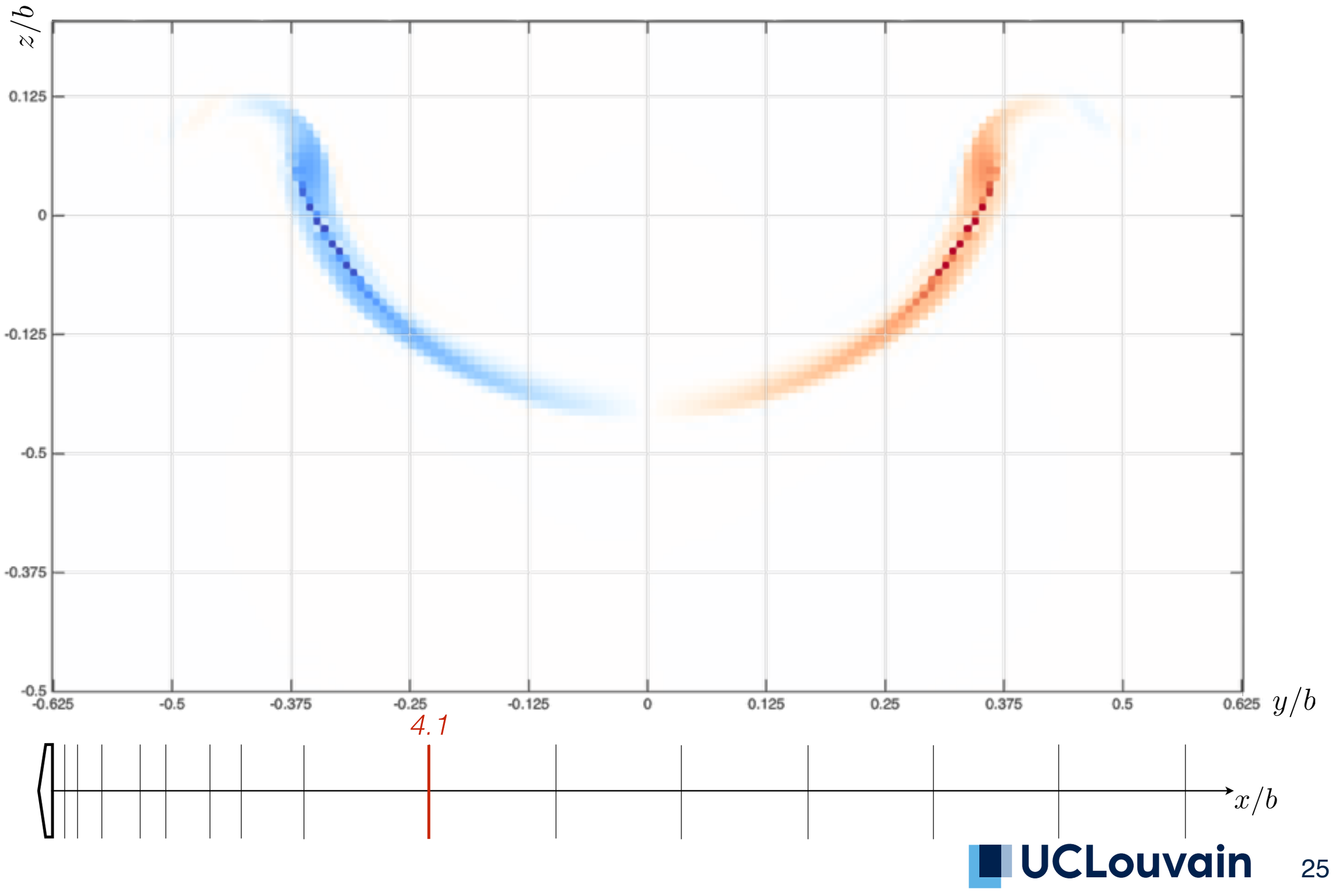
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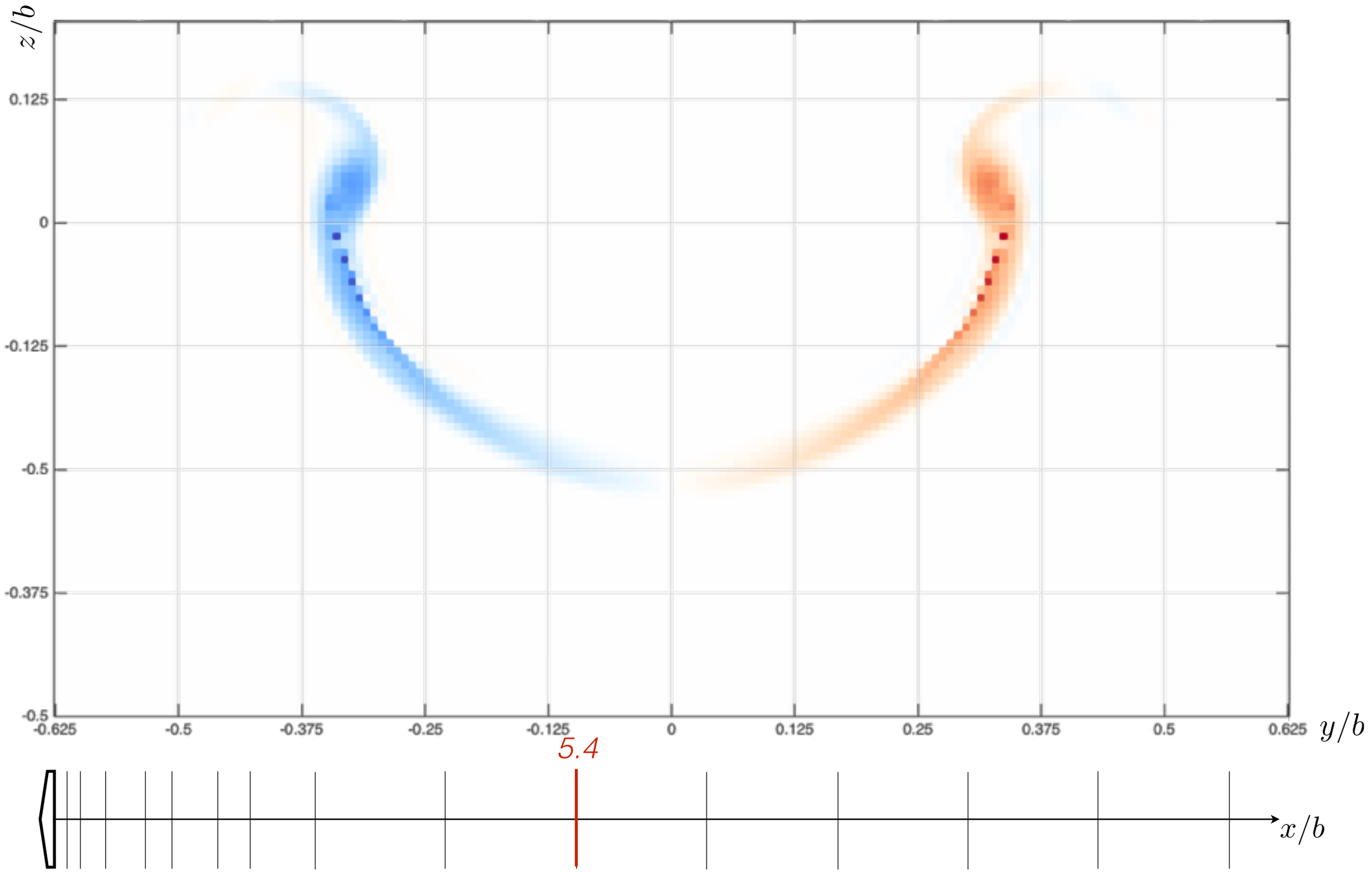
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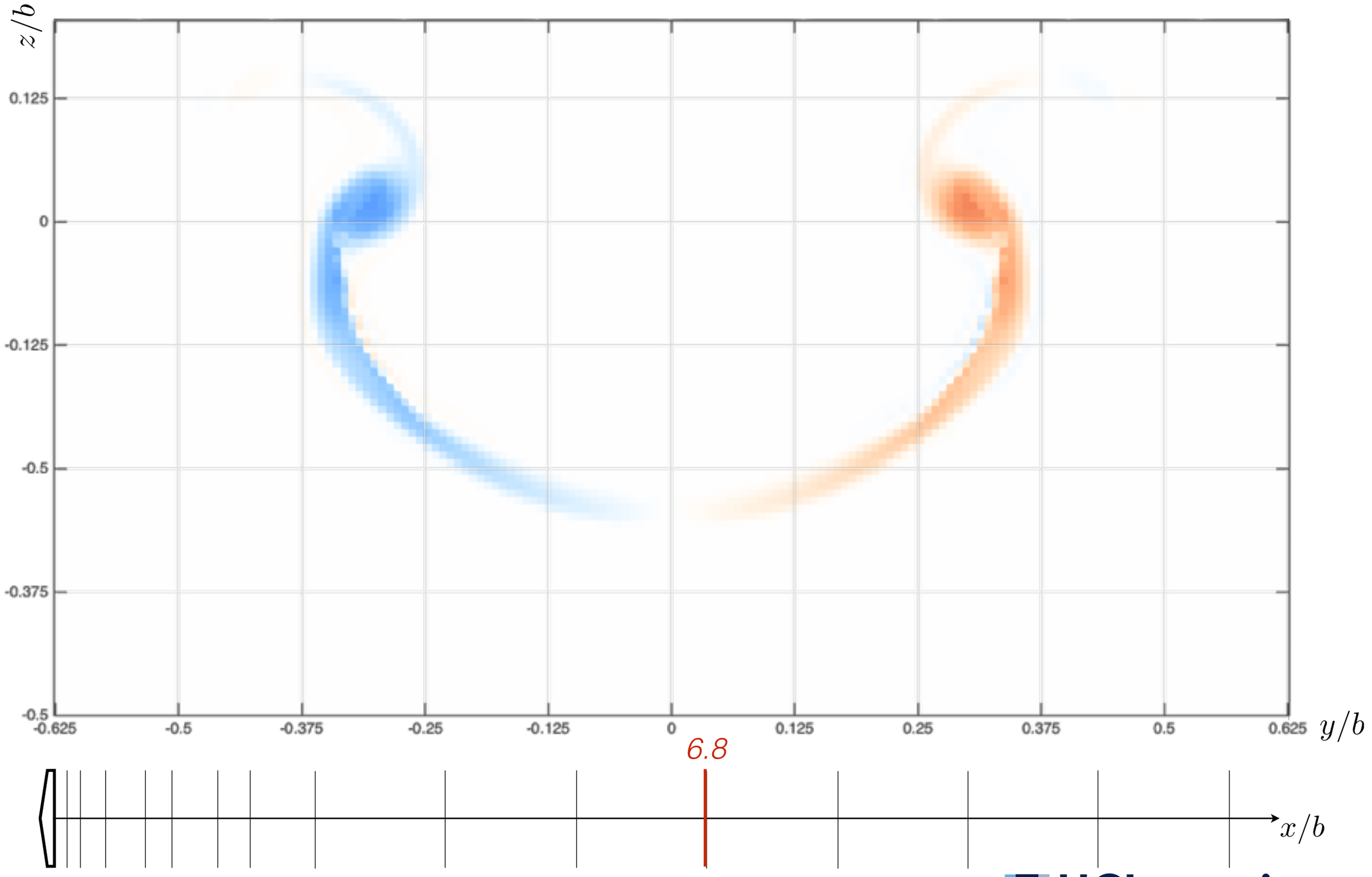
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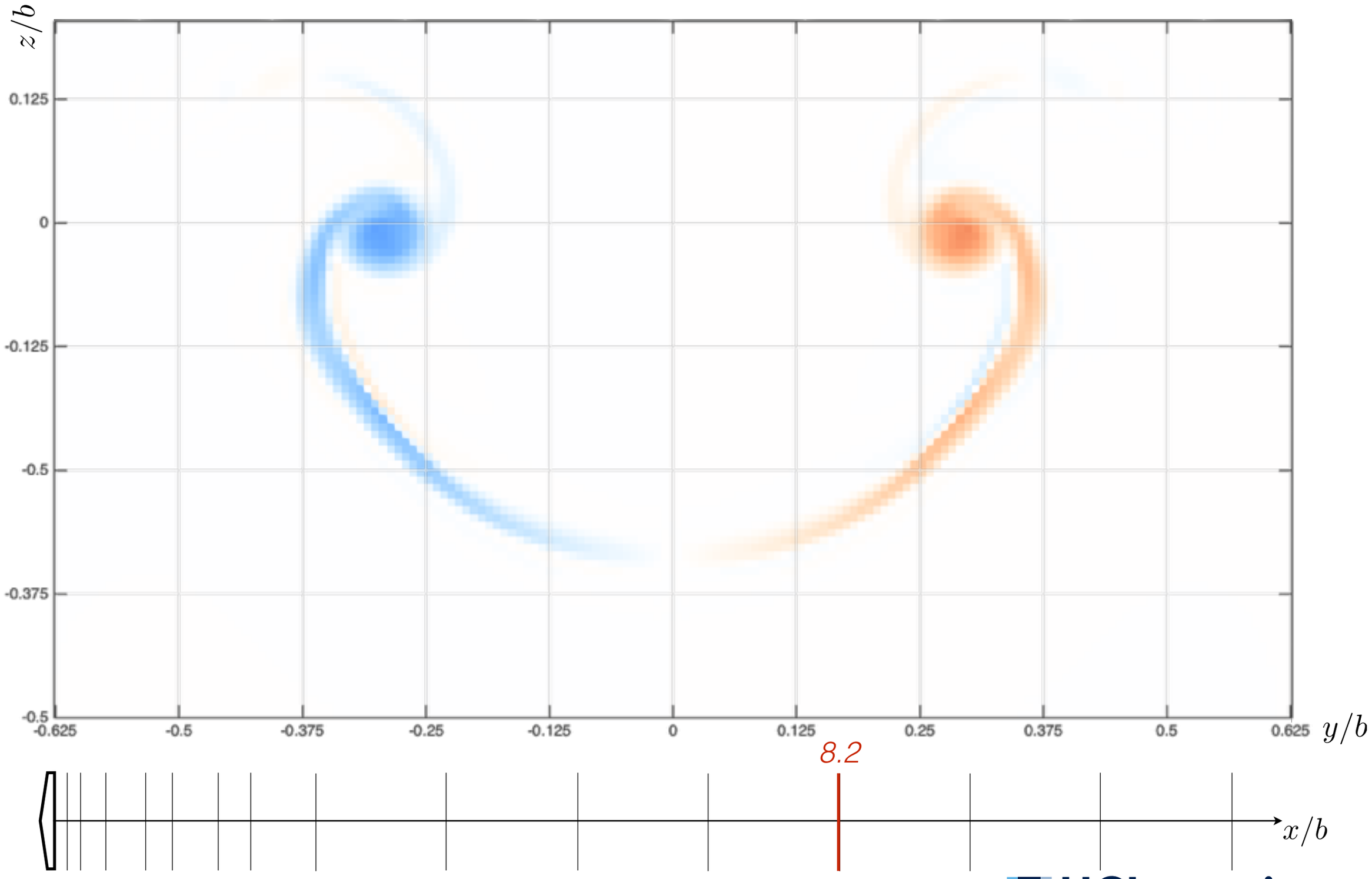
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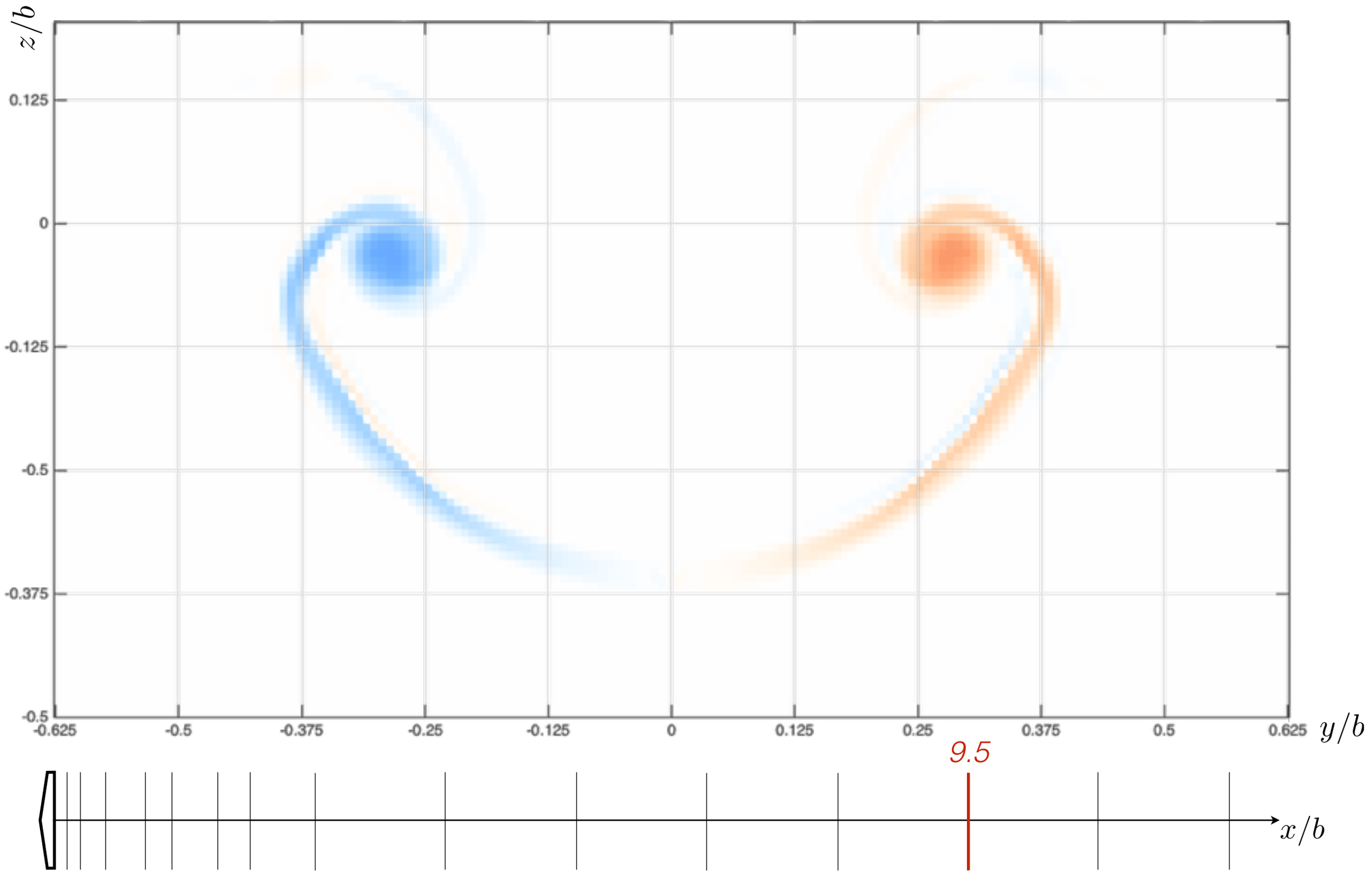
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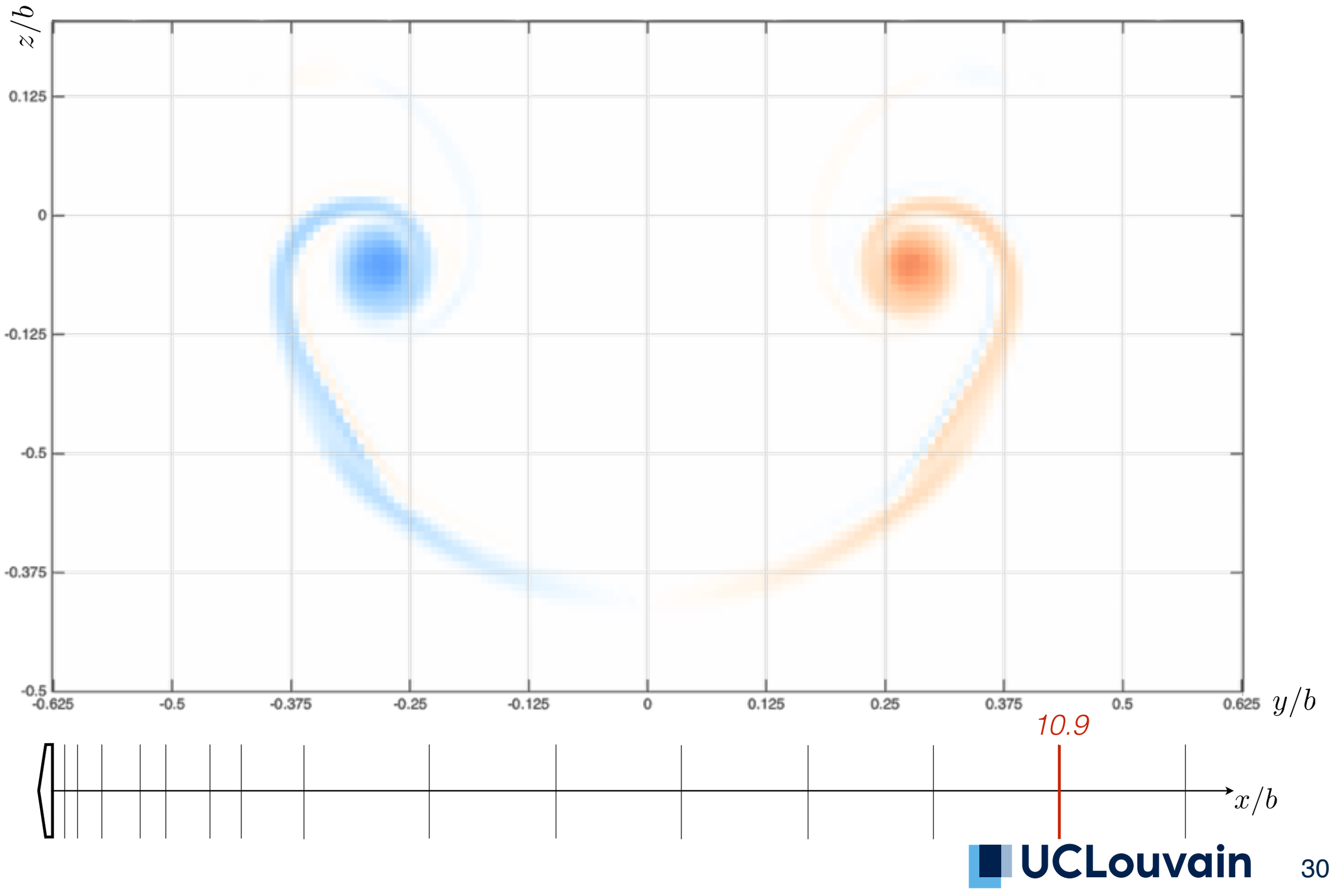
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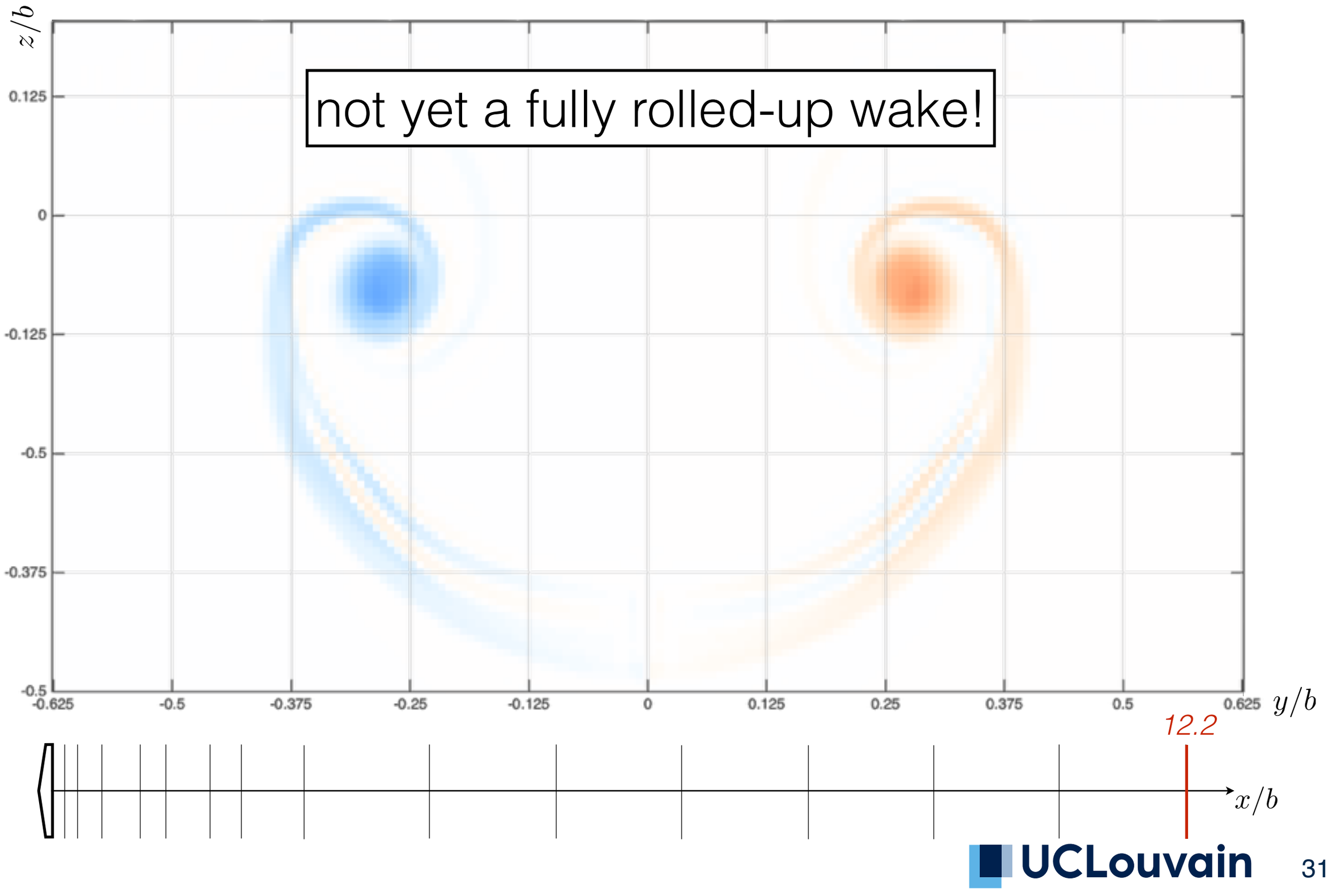
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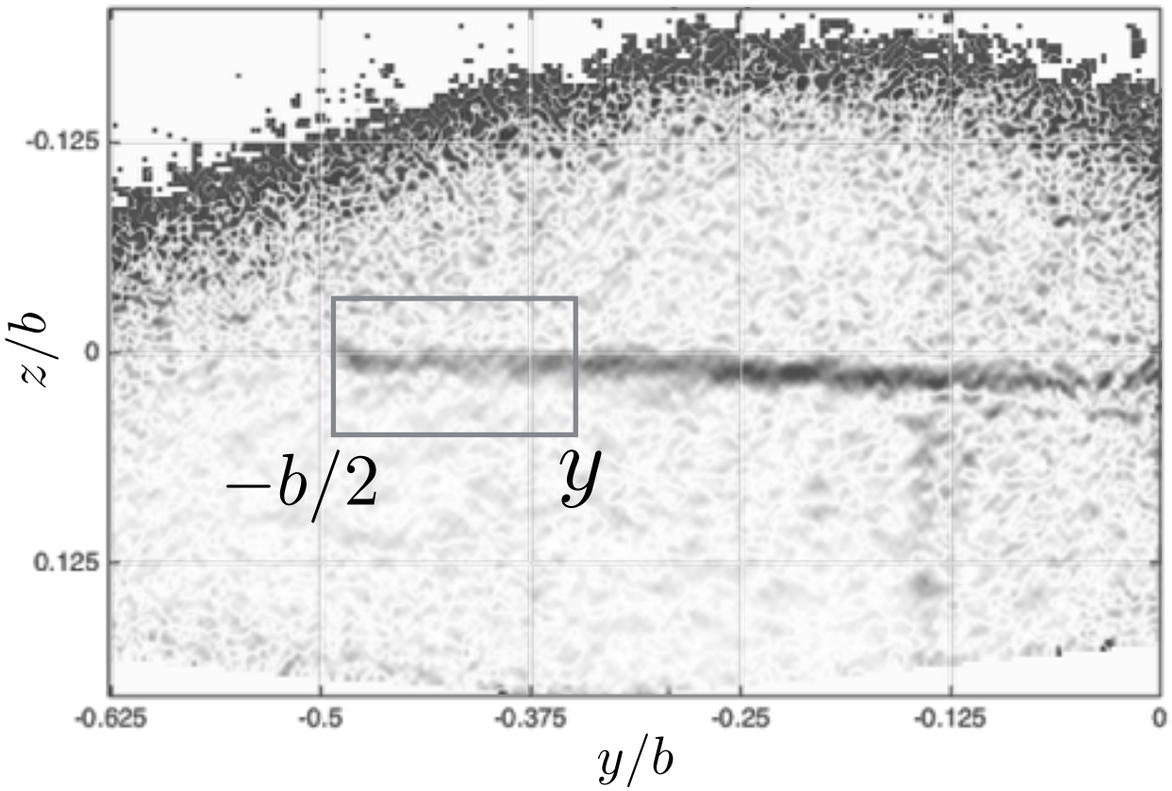


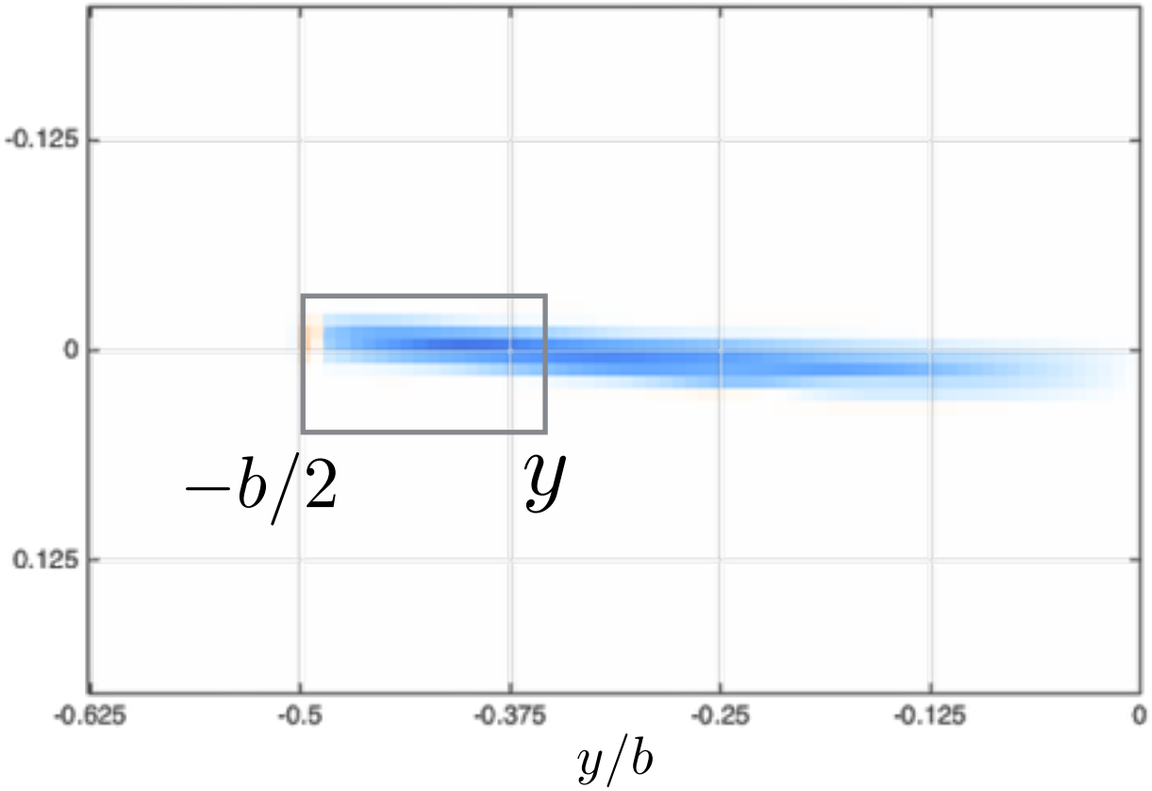
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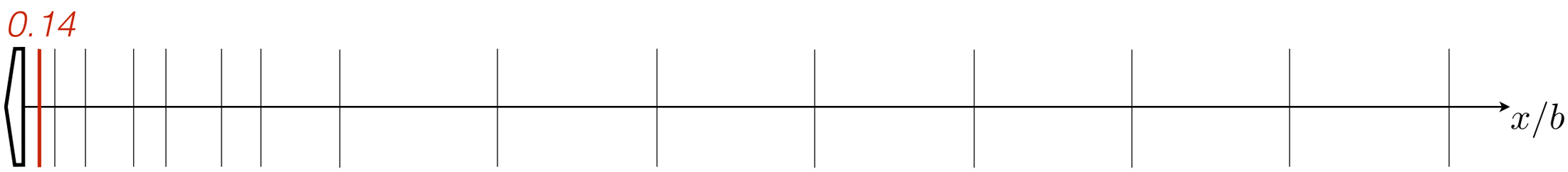


Wing loading recovered from near wake x-vorticity

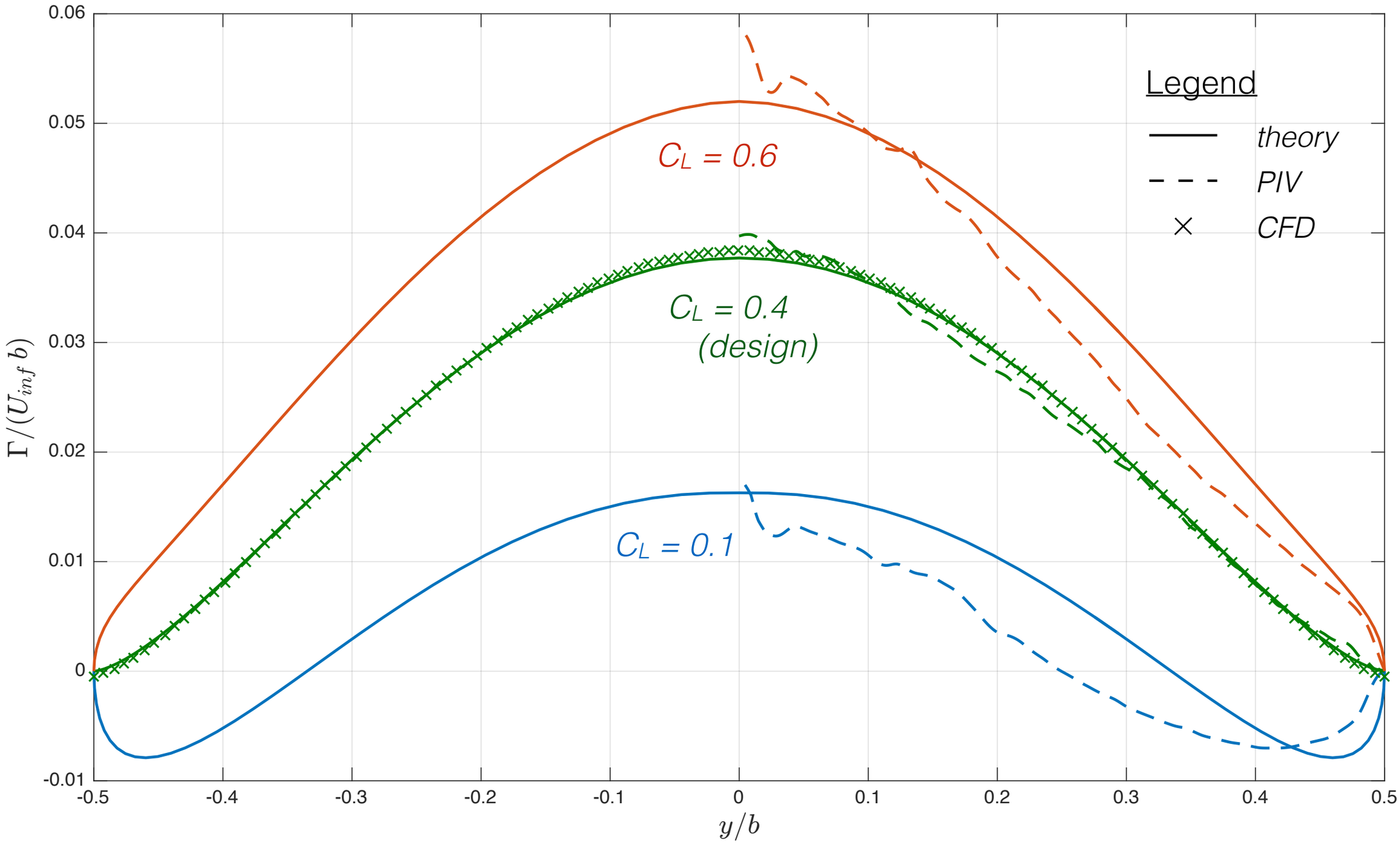
$$\Gamma(y) = -\int_{-b/2}^{y}\int_{-l/2}^{l/2} \omega_x(y', z')\, dy'\, dz'$$







Wing loading recovered from near wake x-vorticity



some conclusions

- At design $C_L = 0.4$:
  - good agreement between the theory and the experiment:
    - on the induced drag $k\,C_L^2$ (obtained from the polar)
    - on the span loading $\Gamma(y)$ (recovered from the near wake PIV)
  - no tip vortex, hence no significant rollup of the near wake
  - good agreement between the theory and the simulation (= ongoing work)
  - of course, there is rollup in the mid wake: so as to end as a two-vortex system in the far wake
- Off design:
  - good agreement between the theory and the experiment on the induced drag
  - fair agreement on the span loading: the shape is OK

NASA Prandtl-D flying wings

- Much regained interest by NASA:
  - A.H. Bowers, O.J. Murillo, R.R. Jensen, B. Eslinger, C. Gelzer, On wings of the minimum induced drag: spanload implications for aircraft and birds, NASA/TP—2016–219072
  - **Prandtl-D** = Preliminary Research Aerodynamic Design to Lower Drag. Test of various Prandtl-D flying wings since 2015







Thank you!

Any question?

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