

UCLouvain, Faculté des sciences
Earth and Life Institute

Tipping cascades, future Earth system trajectories, and the prospect of a hothouse : a study with a reduced-complexity model

Victor COUPLET

Thesis submitted in partial fulfillment
of the requirements for the degree of
Docteur en Sciences

Thesis committee:

Prof. Michel CRUCIFIX (<i>Supervisor</i>)	UCLouvain
Prof. Qiuzhen YIN (<i>President</i>)	UCLouvain
Prof. François MASSONNET (<i>Secretary</i>)	UCLouvain
Prof. Marisa MONTAYA REDONDO	Universidad Complutense de Madrid
Prof. Wim THIERY	Vrije Universiteit Brussel
Prof. Anna VON DER HEYDT	Utrecht University

Version of March 2025

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This work was supported by the Fonds de la Recherche Scientifique - FNRS
under the Aspirant Fellowship FC 38941 (4 years) and by UCLouvain under an
extension grant (6 months).

Abstract

The potential for the Earth system to transition irreversibly into a "Hothouse Earth" has generated significant scientific debate and media attention but remains underexplored in modelling studies. To address this, we develop SURFER, a reduced-complexity Earth system model integrating a dynamic carbon cycle and a network of interconnected tipping and nonlinear elements. SURFER's computational efficiency allows us to conduct a large ensemble of multi-millennial simulations, accounting for the uncertainties associated with tipping elements and exploring the Earth system's response across various time scales and a wide range of forcing scenarios. Our results provide no evidence of a planetary threshold beyond which the Earth system evolves irreversibly into a Hothouse state. However, we found that tipping cascades triggered under low-emission scenarios can drive the Earth system toward a long-term "Wethouse" state characterised by substantial sea-level rise. Such cascades lead to diverging Earth system trajectories where stabilisation of sea level at intermediate values becomes impossible.

Acknowledgments - Remerciements

First and foremost, I would like to warmly thank my thesis supervisor, Michel Crucifix, for his kind guidance and advice. I am especially grateful for the many opportunities he provided, from proposing an exciting research topic to facilitating my involvement with several European research projects such as TiPES and CriticalEarth. While not directly funded by these projects, I had the privilege of participating in many of their conferences, where I met a fantastic community of people and had many enriching experiences.

I would also like to thank the members of my thesis committee, François Massonnet and Wim Thiery, as well as the other members of the jury, Quizhen Yin, Marisa Montoya Redondo, and Anna von der Heydt for taking the time to read and review this thesis. I truly appreciated our discussions, and their thoughtful comments have significantly improved the quality of this work.

I am also grateful to all the researchers who shared their codes and data, saving me valuable time. More broadly, I want to thank everyone who contributes to a more open and collaborative scientific community.

Merci à Dominique, Alice, Livia et Pierre-Yves pour leur support indispensable. Merci à mon équipe de recherche, Marina, Lilian, Justin, Loïc, Oisín, et à tous les collègues du Mercator pour nos nombreux échanges, scientifiques ou pas, et surtout pour la camaraderie et la bonne ambiance qui ont toujours régné au boulot.

Enfin, merci à ma famille et amis pour leur soutien sans faille. Un merci tout particulier à mes colocataires, Edouard, Sébastien, Léo, Camille et Laura, avec qui j'ai partagé cette longue aventure. Pour finir, merci à Anne d'avoir su habilement me rappeler les deadlines quand il le fallait, sans jamais arrêter de croire en moi.

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List of abbreviations

Tipping and nonlinear elements

TEs	Tipping Elements
NEs	Nonlinear Elements
GRIS	Greenland Ice Sheet
AIS	Antarctic Ice Sheet
WAIS	West Antarctic Ice Sheet
EAIS	East Antarctic Ice Sheet
EASB	East Antarctic Subglacial Basins
AMOC	Atlantic Meridional Overturning Circulation
AMAZ	Amazon Rainforest
PERM	Permafrost
ASI	Arctic Sea Ice
GLCR	Mountain Glaciers

Model hierarchy

ESM	Earth System Model
GCM	Global Climate Model
EMIC	Earth system Model of Intermediate Complexity

Projects

IPCC	Intergovernmental Panel on Climate Change
CMIP6	Coupled Model Inter-comparison Project phase 6
LTMIP	Long Tail Model Inter-comparison Project
ZECMIP	Zero Emissions Commitment MIP

★ | List of abbreviations

TIPMIP Tipping Point Model Inter-comparison Project

Others

GMT Global Mean (surface) Temperature
GCB Global Carbon Budget
SLR Sea Level Rise
PI Pre-Industrial (period)
ECS Equilibrium climate sensitivity
D-O Dansgaard-Oeschger (event or cycle)
SSP Shared Socioeconomic Pathways (scenarios)

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Introduction

As humans, we are profoundly altering the environment, primarily through the massive release of greenhouse gases into the atmosphere (IPCC, 2021a). The consequences of these emissions are increasingly evident, with 2023 and 2024 marking the warmest years on record (WMO, 2025). The frequency and intensity of extreme weather events such as heatwaves, droughts, and heavy precipitations have increased and are projected to increase further under global warming (IPCC, 2021b). While these short-term impacts are well understood and documented, for example in IPCC reports, the multi-centennial to millennial consequences of anthropogenic emissions remain uncertain. For instance, while it is very likely that melting polar ice caps will contribute to sea level rise for centuries and even millennia after emissions of greenhouse gases are reduced or stopped (Breedam et al., 2020; Clark et al., 2016), there is only a medium agreement between model projections. There is also growing concern over climate tipping points, where potentially abrupt and irreversible changes could occur in climate subsystems beyond certain warming thresholds (Armstrong McKay et al., 2022; Lenton et al., 2008; Lenton et al., 2019).

To gain insight into potential changes over millennial timescales, we can turn to paleoclimate data, which reveal that Earth has experienced a wide range of climate states throughout its history (Judd et al., 2024; Westerhold et al., 2020). These past states range from entirely ice-covered conditions (Hoffman, 1998) to ice-free periods with rainforests extending to the poles (Klages et al., 2020), offering possible analogues for future climates, even if exact comparisons remain challenging. For example, intermediate emission

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scenarios might lead to CO₂ levels and warming last seen three million years ago during the mid-Pliocene period, as shown in figure 1.1 (Burke et al., 2018; IPCC, 2021a). Higher-emission scenarios could push Earth's climate toward conditions resembling the Mid-Miocene epoch (~15-17 Ma), characterized by mean temperatures 4–5°C warmer than pre-industrial levels and sea levels 10–60 meters higher (IPCC, 2021a; Kidder and Worsley, 2012; Steffen et al., 2018; von der Heydt, 2022). Under the most extreme emission scenarios, the Earth system might approach early Eocene (~50 Ma) ice-free conditions, with temperatures exceeding pre-industrial levels by more than 10°C (Burke et al., 2018; IPCC, 2021a).

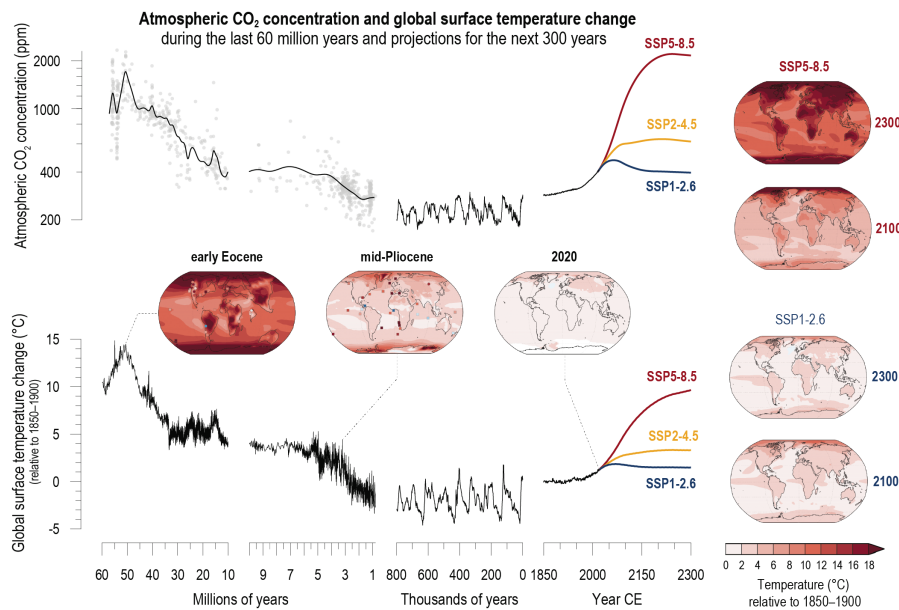


Fig. 1.1 Changes in atmospheric CO₂ and global surface temperature (relative to 1850–1900) from the deep past to the next 300 years. This figure is a reproduction of Figure TS.1 in IPCC (2021a) : Technical summary. It shows that intermediate-to-high emission scenarios could, within a few centuries, lead to conditions comparable to those last experienced millions of years ago.

Paleoclimate data also reveal that transitions between climate states have sometimes been abrupt on geological timescales. Notable examples include the Eocene–Oligocene transition (~34 Ma) and shifts from glacial to interglacial periods during the last three million years (Boers et al., 2022, and references therein). Additionally, studies across the model hierarchy suggest that certain climate states may coexist under the same boundary conditions, defined by

factors such as the astronomical configuration, the palaeogeography, or the atmospheric CO₂ concentration (Brunetti et al., 2019; Budyko, 1969; Ferreira et al., 2011; Kilic et al., 2017; Lucarini and Bódai, 2017; Popp et al., 2016; Sellers, 1969; von der Heydt and Ashwin, 2016). In this context, multistability provides a useful framework for understanding how the Earth system can undergo large shifts triggered by relatively minor changes in external conditions. It also raises a concerning possibility: we may be nearing a planetary tipping point, where even modest additional emissions could propel the Earth system into a fundamentally different state.

Abrupt shifts and tipping points are well-documented at local and regional scales in both paleoclimate records (Bagniewski et al., 2023; Boers et al., 2022; Brovkin et al., 2021) and climate model simulations (e.g. Drijfhout et al., 2015). However, whether these regional shifts could combine in a planetary-scale tipping point remains uncertain (Hughes et al., 2013; Lenton and Williams, 2013). One possible mechanism for global-scale changes is tipping cascades, where a transition in one climate subsystem initiates further transitions in interconnected subsystems (Klose et al., 2020; Klose et al., 2021; Wunderling et al., 2024). Steffen et al. (2018) suggest that tipping cascades involving elements that impact planetary albedo (e.g., sea ice and ice sheets) or release large amounts of greenhouse gases (e.g., Amazon rainforest dieback, permafrost thaw, methane hydrate destabilization) could lead to substantial warming and sea-level rise, even under low- and medium-emission scenarios. Such cascades would effectively eliminate "middle-of-the-road" climate trajectories, leaving humanity with a binary choice depicted in Figure 1.2: either significantly reduce emissions to stabilize the Earth system at a relatively cool, manageable state or risk crossing a planetary threshold beyond which a so-called "Hothouse Earth" state becomes inevitable.

While perspective pieces have highlighted the risks of a potential Hothouse Earth scenario and generated substantial scientific and media interest (Kemp et al., 2022; Lenton et al., 2019; Steffen et al., 2018), these discussions largely lack support from detailed model simulations, which are essential for more robust, evidence-based conclusions. Figure 1.3 shows that over centennial timescales, current models predict a linear relationship between global mean temperature and cumulative CO₂ emissions, indicating a minimal risk of abrupt Hothouse Earth transitions. In contrast, experiments with offline ice sheet models show that we may be committed to substantial sea level rise over the next millennia, even under relatively low emission scenarios (Breedam et al., 2020; Clark et al., 2016). These experiments suggest a risk of reaching a "Wethouse" state,

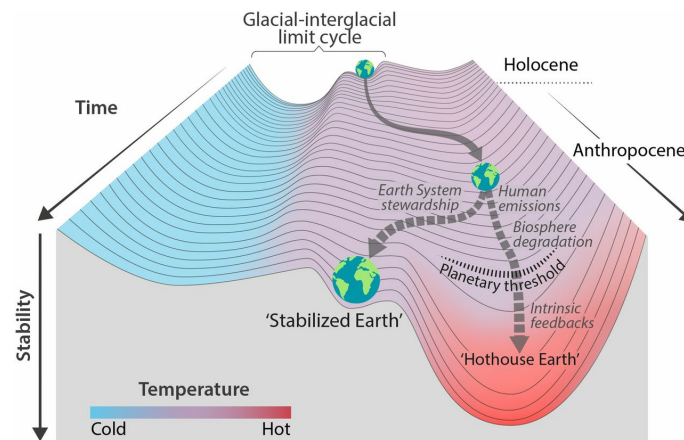


Fig. 1.2 Stability landscape and trajectories of the Earth system. This figure is a reproduction of Figure 2 in Steffen *et al.* (2018), licensed under CC-BY-NC-ND 4.0. The illustrated hypothesis suggests a planetary threshold beyond which biogeophysical feedbacks and tipping cascades lock the Earth system on a pathway towards a Hothouse state.

characterised by elevated sea levels rather than high temperatures (Lenton *et al.*, 2023). However, multi-millennial emission-driven simulations of the future that include an interactive carbon cycle and dynamic ice sheets are still lacking. This is primarily due to computational limitations but also because it is challenging to couple all these elements in complex models successfully.

In this context, the present thesis aims to investigate tipping cascades and Earth system trajectories, and assess the potential for a Hothouse Earth scenario using a reduced-complexity model. Section 1.1 introduces the concepts of tipping elements and cascades, while section 1.2 motivates using dynamical systems theory and simple dynamical systems to investigate such concepts. Section 1.3 clarifies the research questions and provides an overview of the methodological approach employed to address them.

1.1 Tipping elements and tipping cascades

1.1.1 Tipping elements

A tipping element is an Earth system component that can undergo a self-perpetuated qualitative state change once a critical forcing threshold, or tipping point, is crossed. This transition, driven by one or more amplifying

Every tonne of CO₂ emissions adds to global warming

Global surface temperature increase since 1850–1900 (°C) as a function of cumulative CO₂ emissions (GtCO₂)

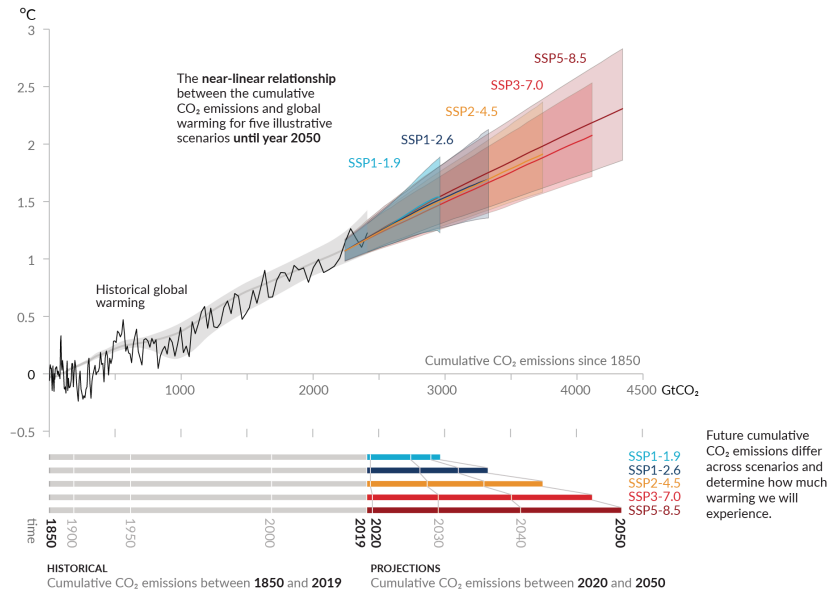


Fig. 1.3 Near-linear relationship between cumulative CO₂ emissions and the increase in global surface temperature. This figure is a reproduction of Figure SPM.10 in IPCC (2021a) : Summary for Policymakers

feedback loops, may be abrupt and/or irreversible (Armstrong McKay et al., 2022; Lenton et al., 2008; Lenton et al., 2023). A frequently cited example is the Atlantic Meridional Overturning Circulation (AMOC) (Rahmstorf, 2024, for an accessible review). In the Atlantic Ocean, warm and salty surface waters flow northward, cool at high latitudes, become denser and sink, forming deep currents that flow back southward. This circulation is crucial in transporting heat northward, particularly to regions like Great Britain and Scandinavia, which enjoy relatively mild climates compared to other areas at similar latitudes, such as northern Canada.

It has long been hypothesized that a gradual increase in freshwater input at high latitudes, resulting from enhanced precipitation or meltwater from Greenland, could disrupt this circulation (Stommel, 1961). This possibility arises from the salt-advection feedback: as high-latitude waters become less salty, their density decreases, reducing sinking and slowing the circulation. A weakened circulation further reduces the northward transport of salt, amplifying the initial disturbance and potentially triggering a complete shutdown of

the AMOC. Such transitions have been simulated with EMICs (Lohmann et al., 2023; Rahmstorf et al., 2005) and recently with a comprehensive global climate model (GCM) (van Westen et al., 2024a).

The Greenland ice sheet (GRIS) is another example of a suspected tipping element (Armstrong McKay et al., 2022). The primary mechanism that could drive a complete collapse of the ice sheet past a warming threshold is the melt-elevation feedback (Levermann and Winkelmann, 2016). As the ice sheet melts, its surface elevation decreases and is exposed to higher temperatures, similar to what a person would feel hiking down a mountain. These higher temperatures can then drive further melting, which lowers elevation even more, creating a self-reinforcing loop. However, even in such cases, the transition to a wholly melted ice sheet would unfold over millennia, illustrating that tipping processes are not always abrupt. Similar self-amplifying feedbacks have been identified in a range of climate subsystems. Candidate tipping elements include the Amazon Rainforest (AMAZ), the West and East Antarctic Ice Sheets (WAIS and EAIS), the East Antarctic subglacial basins (EASB), monsoon systems, and others. Comprehensive reviews, such as those by Armstrong McKay et al. (2022), Wang et al. (2023), and Lenton et al. (2023), provide an exhaustive list of these systems along with the associated feedback mechanisms.

There is evidence that tipping events have happened in the past (Boers et al., 2022; Brovkin et al., 2021), and increasing concern that some may occur in the future as a result of global warming (Armstrong McKay et al., 2022; IPCC, 2021a; Lenton et al., 2023; McNeall et al., 2011; Wang et al., 2023). For instance, parts of the Amazon rainforest, the Greenland ice sheet, and the AMOC are already exhibiting critical slowing down, a sign of declining stability (Boers, 2021; Boers and Rypdal, 2021; Boulton et al., 2022; Dakos et al., 2008; Ditlevsen and Ditlevsen, 2023; Scheffer et al., 2009). Recent studies have even projected a 50% or higher chance of AMOC collapse within this century (Ditlevsen and Ditlevsen, 2023; van Westen et al., 2024b), though such predictions are debated (Ben-Yami et al., 2024b).

Drivers of tipping elements can vary in nature; for instance, freshwater influx in the North Atlantic can destabilise the AMOC, while local temperatures play a critical role in the stability of the Greenland Ice Sheet. However, these drivers are often closely linked to global mean temperature, which is a useful proxy for the state of the entire climate system. For this reason, in reviews of tipping elements, their critical thresholds are typically expressed as a function of global temperature rise. Figure 1.4, reproduced from (Armstrong McKay et al., 2022), illustrates the estimated ranges for the critical thresholds of suspected

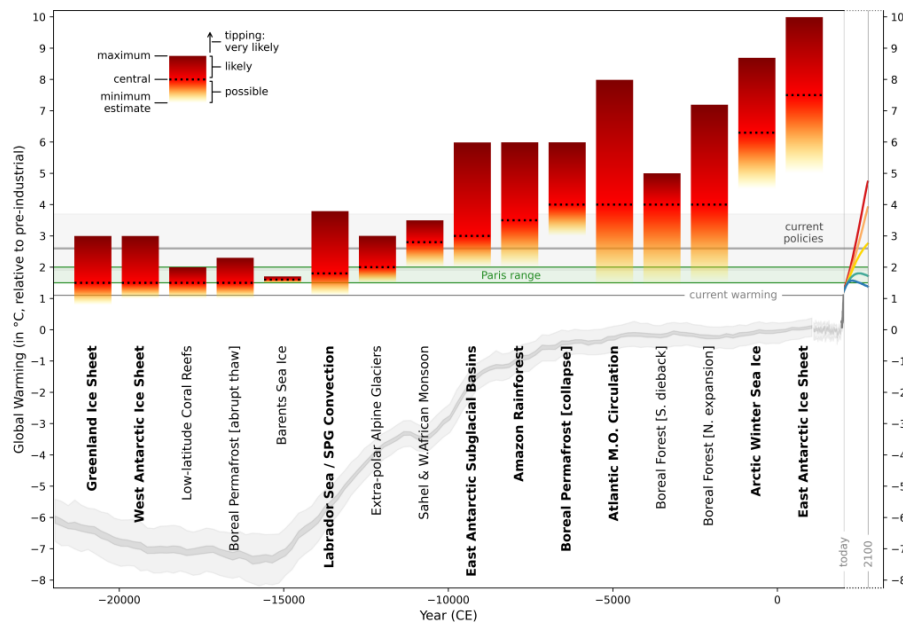


Fig. 1.4 Global warming threshold estimates for climate tipping elements. This figure is an adaptation of Figure 2a in Armstrong McKay et al. (2022), and was reproduced from Armstrong McKay (2022).

tipping elements, showing that several systems, including the Greenland and West Antarctic ice sheets, are already at risk of tipping within the 1.5–2°C warming range targeted by the Paris Agreement.

1.1.2 Tipping cascades

Tipping elements, as parts of the Earth system, are linked together through oceanic and atmospheric circulation systems and can interact across scales in time and space, as shown in Figure 1.5 (Kriegler et al., 2009; Rocha et al., 2018; Wunderling et al., 2021). A recent review assessed all known interactions between tipping elements and found that most of them are destabilising (Wunderling et al., 2024). This suggests that the tipping of one element could facilitate or even cause the tipping of other elements in what is called a tipping cascade. Additionally, some elements can influence global mean temperatures through albedo changes or by releasing greenhouse gases, which would also act to destabilise other elements further. There is evidence that such cascading events may have happened in the past, for example during the Eocene-Oligocene transition (Tigheelaar et al., 2011), the Paleocene-Eocene

thermal maximum (Kender et al., 2021) and during the last interglacial period where interactions and cascading impacts in the Arctic may explain the high polar temperatures (+8°C compared to today) and sea-level rise (+6-9m compared to today) (Thomas et al., 2020).

Dansgaard-Oeschger (D-O) events or cycles may also be interpreted as an example of past tipping cascades. D-O events are a series of abrupt warming and cooling episodes that occurred during the last glacial period and likely throughout all Quaternary glacial periods (Held et al., 2024; Kindler et al., 2014). These events are thought to result from rapid transitions in North Atlantic sea-ice cover and the AMOC, leading to global changes, including in the hydroclimate and monsoon systems (Malmierca-Vallet et al., 2023). Additionally, it has been hypothesized that during the cold phases of D-O cycles, a weakened AMOC and expanded sea-ice cover facilitated the subsurface accumulation of heat in the North Atlantic, potentially triggering large releases of icebergs from the Laurentide Ice Sheet—a phenomenon known as Heinrich events (Alvarez-Solas et al., 2010; Heinrich, 1988). Thus, D-O events are associated with abrupt transitions across multiple interconnected systems and are often cited as an example of a tipping cascade (Brovkin et al., 2021; Wunderling et al., 2024). However, inferring the causal chain of events from proxy records remains a challenge, and it remains unclear whether these records reflect a series of coupled tipping events across different systems or multiple systems responding linearly to a single tipping event.

Some interactions between fast-responding tipping elements have been investigated using CMIP6 models and observational data. Such interactions include the impact of an AMOC shutdown on the Amazon rainforest and monsoons (Ben-Yami et al., 2024a; Nian et al., 2023), as well as teleconnections between the Amazon rainforest, the West Antarctic ice sheet, and the snow cover of the Tibetan plateau (Liu et al., 2023). Cascades involved in oscillations similar to Dansgaard-Oeschger events have also been studied in several global climate models (Klockmann et al., 2018; Kuniyoshi et al., 2022; Malmierca-Vallet et al., 2023; Malmierca-Vallet et al., 2024; Vettoretti et al., 2022). Due to computational constraints, cascades involving ice-sheets have been studied only in conceptual models (Klose et al., 2024; Wunderling et al., 2021; Wunderling et al., 2022). We will follow a similar approach to study long-term earth system trajectories, as explained in sections 1.2 and 1.3.

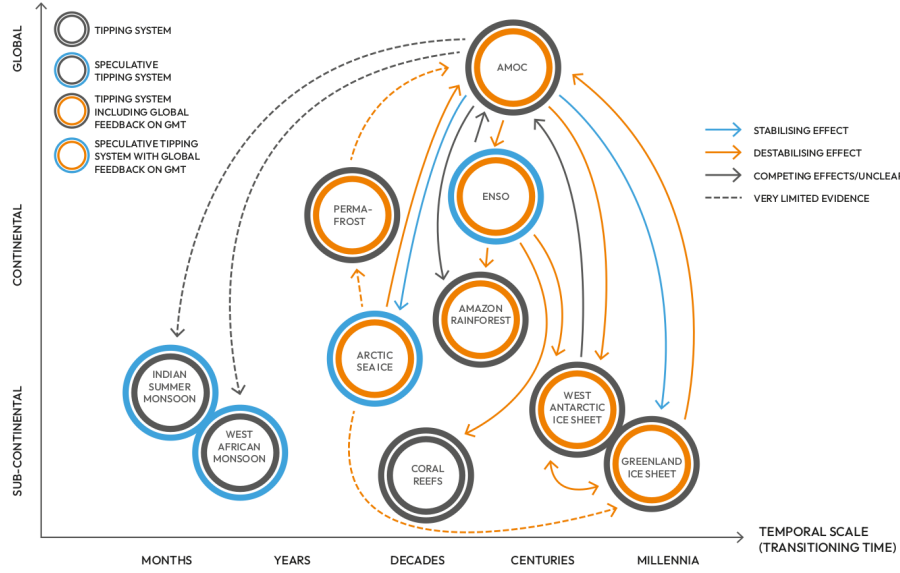


Fig. 1.5 Tipping elements interact across scales in space and time. This figure is an adaptation from Figure 2 in Wunderling et al. (2024), and was reproduced from Figure 1.5.2 in Lenton et al. (2023), licensed under CC-BY-NC-ND 4.0.

1.2 Dynamical systems theory

Dynamical systems theory provides a set of powerful tools for studying systems that change over time. It has been widely used in the climate community to understand the physical processes and phenomena that control the evolution of the Earth system. In particular, concepts such as *stability*, and *bifurcations* are particularly useful to understand and model the behaviour of tipping elements, as described in the following.

Let us consider a system described by the state variable x and whose time evolution is given by

$$\frac{dx}{dt} = f(x, q) = \left(-x^3 + x - \sqrt{\frac{4}{27}} \frac{q}{q_{crit}} \right) \frac{1}{\tau} \quad (1.1)$$

where q is a control parameter. Points x^* satisfying $f(x^*, q) = 0$ are referred to as equilibrium points because, at these points, the system state remains constant (i.e., the derivative is zero). The stability of an equilibrium point depends on the eigenvalues of the linearized system around it: if all eigenvalues have negative real parts, the equilibrium is stable; if all have positive real parts,

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the equilibrium is unstable (Strogatz, 2015). Intuitively, trajectories of the system converge towards stable equilibria and diverge from unstable ones.

For the system described by equation 1.1, since $f(x, q)$ is a cubic polynomial in x , there can be one, two, or three equilibria depending on the parameter q . Figure 1.6 illustrates these equilibria as a function of q , forming what is known as a bifurcation diagram. This diagram reveals three equilibrium branches: two stable branches, $x_+^s(q)$ and $x_-^s(q)$, and one unstable branch, $x^u(q)$. These branches meet at critical points (q_{crit}, x_+) and $(-q_{crit}, x_-)$, known as fold bifurcations. For $-q_{crit} < q < q_{crit}$, the system is bi-stable. For $-q_{crit} < q < q_{crit}$, the system is bi-stable.

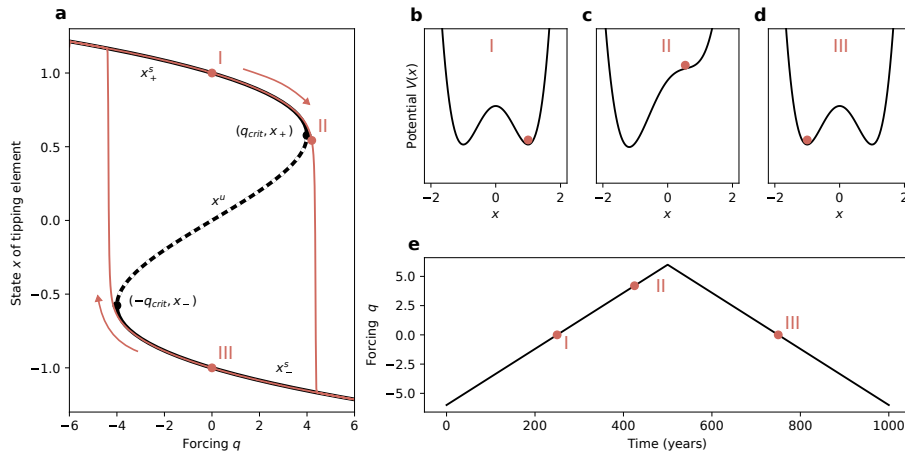


Fig. 1.6 An example of bifurcation tipping. *a)* Bifurcation diagram of equation 1.1, with $q_{crit} = 4$ and $\tau = 1$ yr. The evolution of the tipping element under the time varying forcing $q(t)$ is plotted in red. It traces an hysteresis curve. *b-d)* Potential $V(x)$, defined by equation 1.2, for three different times. *e)* Time evolution of the forcing.

Suppose now that the system lies on the upper equilibrium branch $x_+^s(q)$ and that the forcing q slowly increases, as depicted in Figure 1.6a and 1.6e. As q changes, the equilibrium position shifts, but because it is stable, the system will follow it. However, if the forcing increases beyond the critical value q_{crit} , the upper equilibrium disappears, and strong positive feedback drives the system to the lower equilibrium branch. This phenomenon is known as bifurcation tipping, the most studied tipping mechanism.

The transition between the two equilibrium branches can be abrupt if the system's timescale τ is small compared to the timescale of the forcing changes. If the forcing is reversed, the system remains on the lower branch until the other bifurcation point is reached. Consequently, for a range of forcing values, the transition is irreversible. This also implies that the equilibrium state in

which the system lies for a given forcing value depends on the system's history, a phenomenon known as hysteresis.

Rewriting the right-hand side of equation 1.1 as the derivative of a potential offers a complementary view of the system dynamics. We have

$$\frac{dx}{dt} = -\frac{\partial V}{\partial x}, \quad \text{with } V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2} + \sqrt{\frac{4}{27}} \frac{qx}{q_{crit}} \right) \frac{1}{\tau}. \quad (1.2)$$

The potential for different values of q is shown in Figure 1.6b-d. First, consider the potential in figure 1.6b. The system's dynamics can be intuitively understood by imagining it as a small ball moving within this potential. If the ball is placed at any point, it will naturally roll toward one of the two local minima, which represent the stable equilibria. If the ball is placed precisely at the local maximum, it will remain there; however, even the slightest perturbation will cause it to roll toward one of the local minima. The local maximum thus corresponds to the unstable equilibrium. As q varies, the shape of the potential changes and one of the local minima may disappear (Figure 1.6c). In this case, the ball will move toward the remaining minimum and settle there, even if the original local minimum reappears as q is reversed (Figure 1.6d).

Potentials can always be computed in one-dimensional systems; however, this is often not the case for higher-dimensional systems. Nevertheless, the concept is frequently used metaphorically, under various names, to represent system dynamics. For instance, Waddington (1957) introduced the epigenetic landscape to describe cell differentiation. Similarly, Steffen et al. (2018) hypothesise the shape of a stability landscape, as reproduced in Figure 1.2, to illustrate potential future trajectories of the Earth system.

Fold bifurcations and bi-stability have been identified in simple models designed to study the Atlantic Meridional Overturning Circulation (Stommel, 1961, see also Figure 1.7), the atmosphere-vegetation system in the Sahara (Brovkin et al., 1998), the Greenland ice sheet (Levermann and Winkelmann, 2016), and many other systems (Bathiany et al., 2016). While tracing the bifurcation diagram of more complex models is challenging, insights into their equilibrium structure can be gained through hysteresis experiments, where the forcing is slowly increased and then decreased, as illustrated in Figure 1.6. If the forcing changes slowly enough relative to the system's intrinsic dynamics, the system is expected to remain near its equilibrium state, and hysteresis behavior would indicate multi-stability. Such behavior has been observed in the AMOC flow using Earth system models of intermediate complexity

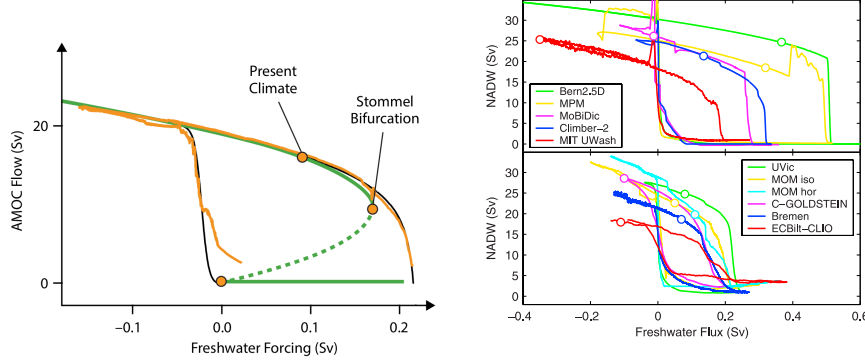


Fig. 1.7 Fold bifurcations and hysteresis behaviour have been observed for the AMOC across the model hierarchy. Left: In green, bifurcation diagram from a conceptual box model. In black, hysteresis curve traced with the same box model. In orange, the hysteresis curve found in a global ocean circulation model. Figure reproduced from Rahmstorf (2024), and originally from Rahmstorf (1996). Right: AMOC hysteresis curves for various models of intermediate complexity. Figure reproduced from Rahmstorf et al. (2005).

(Rahmstorf et al., 2005, see also Figure 1.7), and state-of-the-art global climate models (van Westen and Dijkstra, 2023). It has also been found in offline models of the Greenland and Antarctic ice sheets (Garbe et al., 2020; Robinson et al., 2012), further supporting the possibility that all these systems may be bi-stable or multi-stable.

These observations across the model hierarchy motivate the use of dynamical systems theory, which provides a unifying interpretative framework. This research line turned out to be quite fertile and, among others, yielded the idea of detecting possible "early warning signals", which would announce the proximity of bifurcations (Dakos et al., 2008). Equations such as (1.1), and the associated double-fold bi-stable structure shown in Figure 1.6, are widely used in conceptual studies to capture the first-order dynamics of tipping elements (e.g. Dekker et al., 2018; Klose et al., 2020; Möller et al., 2024; Wunderling et al., 2021). Similar representations will be employed throughout this thesis.

However, it should be noted that bifurcation tipping alone is insufficient to capture the full diversity of tipping phenomena observed or anticipated in the climate system. For instance, tipping can also be triggered by threshold effects associated with the rate of change of the forcing. It has long been suggested that a potential collapse of the AMOC could depend on the rate of increase in greenhouse gas concentrations (Stocker and Schmittner, 1997). Here again, dynamical systems theory provides a valuable framework to analyze, explain,

and generalize such phenomena, known as rate-induced tipping (Ashwin et al., 2012; Feudel, 2023).

1.3 Thesis objectives, methods, and outline

1.3.1 Objectives and methods

The reports of the intergovernmental panel on climate change provide ample evidence that every degree of temperature increases the risk and amplitude of potentially disruptive effects on the ecosystem and societies. However, whether a "cascade" of tipping is a possibility and whether there is a planetary tipping point leaving us with a binary choice between a "governed Earth" and a "hothouse" with no midway alternative is speculative and has not been demonstrated in comprehensive Earth system models. In this context, the general objective of the present thesis is to use the available knowledge about the stability of the Earth system components and processes to investigate this possibility more rigorously. Specifically, we aim to answer the following questions :

**To what extent do tipping events and other nonlinear feedbacks increase the risk of reaching a Hothouse state? A Wethouse state?
Could this outcome occur under low-emission scenarios?**

To what extent do tipping events and other nonlinear feedbacks create a divergence in Earth system trajectories? In other words, is there a threshold that prevents stabilisation at intermediate temperatures or sea-level rise?

To address these questions, we need a model that meets the following criteria:

- It must include a reasonable number of interconnected tipping elements to explore the possibility of tipping cascades.
- It should incorporate a dynamic carbon cycle to account for feedback mechanisms arising from tipping elements such as the Amazon rainforest or permafrost, which could release substantial amounts of greenhouse gases into the atmosphere.

1 | Introduction

- It must be capable of running simulations over several thousand years to capture the response of slow components, such as ice sheets.
- It should be computationally efficient, enabling a large number of simulations. This is essential as we need to run the model across a range of emission scenarios and repeat these runs for different parameter variations to account for uncertainties in the behaviour of tipping elements.

The third and fourth requirements exclude using Earth system models and comprehensive GCMs. EMICs offer an appealing trade-off between the inclusion of physical processes and computational efficiency, but conducting hundreds or thousands of multimillennial experiments with these models remains very expensive. As a result, we have developed our own reduced-complexity model, aiming to realistically capture the Earth system's response timescales and sensitivities with a parsimonious set of assumptions. The resulting dynamical system consists of 25 differential equations, but the following three equations summarise its core structure:

$$\frac{dC}{dt} = E + \sum_i E_{\text{TE},i}(x_i) - g(C, \delta T, x_i) \quad (1.3)$$

$$\frac{d\delta T}{dt} = \left(F_{2\times} \log_2 \left(\frac{C(t)}{280} \right) - \beta \delta T - H + \sum_i F_{\text{TE},i}(x_i) \right) \frac{1}{c_{vol} h_U} \quad (1.4)$$

$$\frac{dx_i}{dt} = \left(-x_i^3 + a_{2,i}x_i^2 + a_{1,i}x_i + c_{0,i} + c_{1,i}\delta q_i(\delta T, x_1, x_2, \dots, x_n) \right) \frac{1}{\tau_i} \quad (1.5)$$

The first equation represents the carbon cycle component. Atmospheric CO₂ concentration (C) increases due to anthropogenic emissions from burning fossil fuels and deforestation (E). This additional carbon is then progressively absorbed from the atmosphere by various natural carbon sinks ($g(C, \Delta T, x_i)$), which operate over a range of timescales. Tipping elements (x_i) may release greenhouse gases in the atmosphere ($E_{\text{TE},i}(x_i)$) and affect the carbon sinks. The second equation represents the climate component. The increased atmospheric CO₂ concentration leads to a stronger greenhouse effect ($F_{2\times} \log_2 (C(t)/280)$). The resulting warming of Earth's surface, modelled as the top $h_U = 150$ m of the ocean with volumetric heat capacity c_{vol} , is controlled by the climate feedback parameter β and heat exchanges with the deep ocean, denoted by H . The climate feedback parameter accounts for

physical feedbacks—such as the Planck response, water vapor and lapse rate feedbacks, surface-albedo feedbacks from sea ice and snow cover changes, and cloud feedbacks—as well as biogeophysical and non-CO₂ biogeochemical feedbacks (Forster et al., 2021). The slower albedo feedback from melting ice sheets—included as tipping elements in our model—is accounted for by the term $\sum_i F_{\text{TE},i}(x_i)$. The CO₂ biogeochemical feedbacks, or carbon cycle feedbacks, are explicitly modelled in the carbon cycle component (Eq. 1.3), with carbon sinks depending on the state of the the Earth system, and with the possibility of greenhouse gas emissions from tipping elements. The last equation represents a network of interconnected tipping elements, modelled using a more general version of equation 1.1. Each tipping element is influenced by the global mean surface temperature δT and by the state of other elements (x_i , $i \in 1, 2, \dots, n$), allowing for coupling and cascades.

Our general strategy for investigating tipping cascades and future Earth system trajectories involves forcing our model with a family of 100 emission scenarios. These scenarios are defined by the total amount of carbon released into the atmosphere after 2015 AD, with scenario j corresponding to a $j \cdot 50$ PgC release of CO₂. A conservative scenario, with immediate and drastic emission cuts, is obtained with $j = 1$, while "business as usual" is obtained with $j = 100$ (Figure 1.8, left-hand-side). The climate model maps this family of scenarios onto a family of responses.

We systematically compare the model's response to successive scenarios, each separated by 50 PgC of emissions, equivalent to approximately five years of emissions at current rates. Comparing the tipping elements' responses allows us to define tipping cascades, while comparing the temperature and sea level rise responses enables us to investigate the impact of these cascades on Earth system trajectories. In particular, we assess whether tipping cascades could push the Earth system into a hothouse state. Additionally, a large difference in the model's response between two successive emission scenarios would suggest that tipping cascades prevent the stabilisation of the Earth system at intermediate states (Figure 1.8, right-hand-side).

1.3.2 Outline

The thesis is divided into three main chapters.

- In **Chapter 2**, we construct the carbon cycle and climate components of our model (equations 1.3 and 1.4), building upon SURFER v2.0, a reduced complexity model designed to estimate the impact of CO₂ emissions

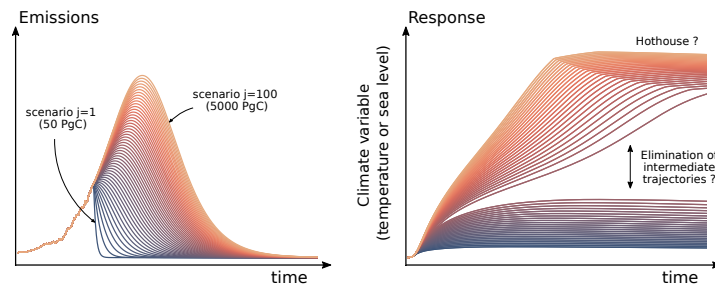


Fig. 1.8 We study the response of our reduced complexity model to a family of emission scenarios, with the total amount of carbon released after 2015 AD ranging from 50 to 5000 PgC. We explore which scenarios may lead to a Hothouse state and how tipping cascades influence this outcome. Additionally, we investigate whether tipping cascades can lead to divergent trajectories, preventing the stabilisation of the Earth system at intermediate temperatures or sea levels.

and solar radiation modification on global mean temperatures, sea-level rise, and ocean pH. However, SURFER v2.0 is unsuitable for simulations beyond a few thousand years because it lacks the relevant carbon cycle processes. To address this, we extend the model by incorporating a dynamic alkalinity cycling in the ocean, a carbonate sediments reservoir, and weathering fluxes. We show that this updated version, SURFER v3.0, reproduces results from a large class of models, ranging from centennial CMIP6 projections to 1 Myr runs performed with the cGENIE model of intermediate complexity. These results demonstrate that SURFER v3.0 can accurately simulate the climate, the carbon cycle, and sea level rise on time scales ranging from decades to millions of years.

- In **Chapter 3**, we extend the model by incorporating a network of interconnected tipping elements (equation 1.5). Using this framework, we conduct simulations across a range of emission scenarios, as outlined in section 1.3. To address uncertainties in key parameters related to tipping elements, we perform these simulations with an ensemble of parameter sets constructed from the most recent literature. Our analysis focuses on the dynamics of tipping cascades, while their broader impacts are examined in Chapter 4. Specifically, we investigate how interactions between tipping elements and their feedback on global temperature influence the likelihood of tipping events and cascading effects. Particular attention is given to cascades that may occur under low-emission scenarios, offering insights into risks for scenarios aligned with current climate targets.

Additionally, we identify the tipping elements most likely to initiate cascades, highlighting their critical role in Earth system stability.

- In **Chapter 4**, we analyse the simulations performed in Chapter 3, focusing on the impacts of tipping cascades. We define a Hothouse state as an Earth system condition with global temperatures exceeding 5°C and sea levels rising more than 10 meters above pre-industrial levels. If only the sea-level criterion is met, we refer to it as a "Wethouse" state. By examining our model's temperature and sea-level rise responses, we explicitly test the Hothouse hypothesis proposed by Steffen et al. (2018). We assess the risk of reaching a Hothouse or Wethouse state as a function of both emission scenarios and time, investigating how these risks are influenced by tipping cascades. Additionally, as outlined in section 1.3, we examine whether cascades lead to a divergence in Earth system trajectories, potentially preventing stabilisation of temperature or sea-level rise at intermediate levels.

We conclude this thesis in Chapter 5 by providing a summary of the research outcomes and some perspectives.

Note : Chapter 2 is based on Couplet et al. (2024b), accepted in *Geoscientific Model Development*. Chapters 3 and 4 are based on paper drafts that still need to be submitted. The overall structure of these papers has been adapted to avoid some redundancies.

2

The SURFER model

2.1 Introduction

Making informed decisions about climate change necessitates a comprehensive examination across multiple temporal scales, to balance the immediate needs of current populations with the imperative of safeguarding the planet's habitability for future generations (Raworth, 2012). To this end, climate models are indispensable for scientists and policymakers. They come in different sizes and complexities (see section 3.2 in Goosse, 2015). On the one side of the complexity spectrum, state-of-the-art Earth system models include detailed representation of physical and biogeochemical processes. However, due to their size and complexity, they are hard to analyse and computationally expensive to run, with most simulations ending in 2100 (for example, this is the case in most CMIP6 experiments). On the other side of the spectrum, conceptual box models trade complexity and spatial resolution for speed and simplicity. They provide valuable insights into fundamental climate processes and feedbacks, facilitating transparent and intuitive assessments of long-term climate trends. Due to their low computational cost, they can be run many times, with different choices of parameters and for different forcing scenarios. This allows an extensive exploration of mitigation and adaptation strategies, such as to take into account possible errors caused by simplifications and other lacks of knowledge.

This is the spirit in which SURFER was designed (Martínez Montero et al., 2022). SURFER v2.0 is a model based on nine ordinary differential equations

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designed to estimate global mean temperature increase, sea level rise and ocean acidification in response to CO₂ emissions and aerosols injections. It is fast, easy to understand and modify, making it appropriate for use in policy assessments. For example, in Montero et al. (2023), it helped assess the long-term sustainability of short term climate policies based on a novel commitment metric. Yet, SURFER v2.0 lacks some processes in its carbon cycle implementation. It can only simulate quantities reliably for up to two or three millennia, and only accounts for carbon dioxide emissions in the carbon cycle. Here, we introduce an extended version of the model with new processes, SURFER v3.0. Among other things, we have added: a representation of atmospheric methane, a distinction between land-use and fossil emissions, an additional oceanic layer, a dependence of the solubility and dissociation constants on temperature and pressure, a dynamic representation of alkalinity, a sediments box, and weathering processes.

This chapter is structured as follow. We explain in detail the new additions of SURFER v3.0 in section 2.2. In section 2.3, we compare the results of SURFER v3.0 to other models on time scales ranging from centuries to a million year. We first show that SURFER v3.0 can reproduce the historical record. Then, we show that SURFER v3.0 is in the range of IPCC class models for different quantities modelled up to 2100 and 2300 AD. We next compare the CO₂ draw-down computed by SURFER over 10000 years to models that participated in the LTMIP project. Lastly, we compare SURFER v3.0 outputs to 1 Myr runs performed with the cGENIE climate model of intermediate complexity. In section 2.4, we show that including new processes in the carbon cycle reduces the committed sea level rise (SLR) estimations on millennial time scales compared to SURFER v2.0. In section 2.5 we discuss the advantages and limitations of our model. Finally, in section 2.6, we conclude and provide some perspectives on future research.

2.2 Model specification

The nine ordinary differential equations of SURFER v2.0 describe the exchanges of carbon between four different reservoirs (atmosphere, upper ocean layer, deeper ocean layer and land), the evolution of temperature anomalies of two different reservoirs (upper ocean layer and deeper ocean layer), the volumes of Greenland and Antarctic ice sheets, and the sea level rise related to glaciers (Martínez Montero et al., 2022). In version v3.0, we have added :

- an ocean layer of intermediate depth,
- a sediment box with CaCO_3 accumulation and burial fluxes,
- dynamic alkalinity cycling between the 3 ocean layers,
- an explicit description of the soft-tissue and carbonate pumps in the ocean,
- volcanic outgassing and weathering fluxes,
- an equation for the methane evolution in the atmosphere,
- a temperature and depth (pressure) dependence of the solubility and dissociation constants,
- a second equation for the land reservoir that allows for a better distinction between land-use and fossil greenhouse gas emissions.

SURFER v3.0 now consists of 17 coupled ordinary differential equations, while keeping its original architecture. It contains three main components, for, respectively, the carbon cycle, climate, and the sea-level. The model is forced by land-use and fossil emissions of CO_2 and CH_4 , and by aerosols injections. State variables defining the model components, carbon and energy fluxes between those components, and forcings are schematically summarised in Figure 3.1.

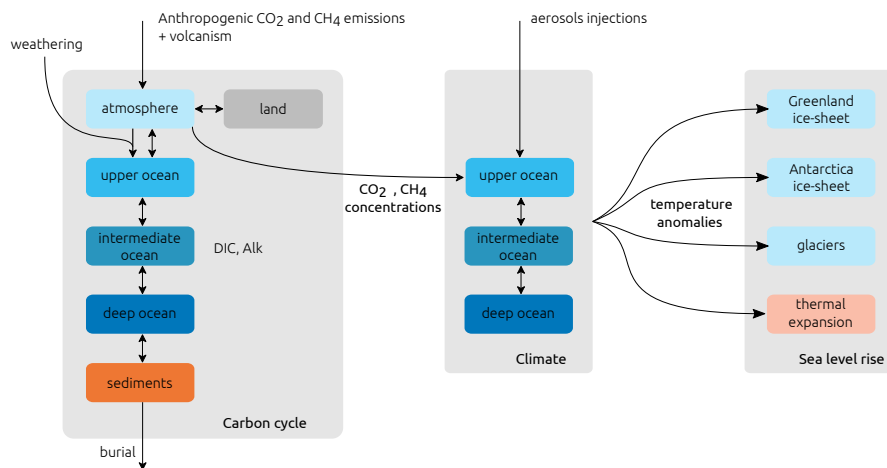


Fig. 2.1 Conceptual diagram of SURFER v3.0.

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In this section, we explain in detail the model implementation. Sub-sections 2.2.1, 2.2.2, and 2.2.3 focus respectively on the implementation of the the carbon, climate and sea-level rise components. In sub-section 2.2.4 we show how the model was calibrated and how the initial conditions are chosen. Sub-section 2.2.5 discusses the algorithmic implementation and speed.

2.2.1 Carbon cycle component

The equations for the carbon cycle component of SURFER v3.0 are given by

$$\frac{dM_A}{dt} = V + E_{\text{fossil}}^{\text{CO}_2} + E_{\text{land-use}}^{\text{CO}_2} - F_{A \rightarrow U} - F_{A \rightarrow L} + F_{\text{CH}_4, \text{ox}} - E_{\text{natural}}^{\text{CH}_4} - F_{\text{weathering}}, \quad (2.1)$$

$$\frac{dM_A^{\text{CH}_4}}{dt} = E_{\text{fossil}}^{\text{CH}_4} + E_{\text{land-use}}^{\text{CH}_4} + E_{\text{natural}}^{\text{CH}_4} - F_{\text{CH}_4, \text{ox}}, \quad (2.2)$$

$$\frac{dM_L}{dt} = F_{A \rightarrow L} - E_{\text{land-use}}^{\text{CO}_2} - E_{\text{land-use}}^{\text{CH}_4}, \quad (2.3)$$

$$\frac{dM_U}{dt} = F_{A \rightarrow U} - F_{U \rightarrow I} + F_{\text{river}}, \quad (2.4)$$

$$\frac{dM_I}{dt} = F_{U \rightarrow I} - F_{I \rightarrow D}, \quad (2.5)$$

$$\frac{dM_D}{dt} = F_{I \rightarrow D} - F_{\text{acc}}, \quad (2.6)$$

$$\frac{d\tilde{Q}_U}{dt} = -\tilde{F}_{U \rightarrow I} + \tilde{F}_{\text{river}}, \quad (2.7)$$

$$\frac{d\tilde{Q}_I}{dt} = \tilde{F}_{U \rightarrow I} - \tilde{F}_{I \rightarrow D}, \quad (2.8)$$

$$\frac{d\tilde{Q}_D}{dt} = \tilde{F}_{I \rightarrow D} - \tilde{F}_{\text{acc}}, \quad (2.9)$$

$$\frac{dM_S}{dt} = F_{\text{acc}} - F_{\text{burial}}. \quad (2.10)$$

They describe the fluxes of carbon between six reservoirs, the atmosphere (A), the land (L), the upper (U), intermediate (I), and deep (D) ocean layers, and the deep sea sediments (S). M_A and $M_A^{\text{CH}_4}$ are the masses of carbon in the atmosphere in CO_2 and CH_4 forms respectively. M_L is the mass of carbon on land (in vegetation and soils) and M_S is the mass of carbon in the erodible CaCO_3 deep sea sediments. M_U , M_I and M_D are the masses of dissolved inorganic carbon in the ocean layers, while $\tilde{Q}_U$, $\tilde{Q}_I$, $\tilde{Q}_D$ are total alkalinities.

All these quantities are expressed in PgC, and the fluxes in PgC yr⁻¹ (equation 2.14 explains what this means for alkalinity).

Variable names are chosen to be self-explanatory, and symbols with tildes indicate quantities and fluxes related to alkalinity. Volcanism (V) and anthropogenic CO₂ emissions ($E_{\text{fossil}}^{\text{CO}_2}$, $E_{\text{land-use}}^{\text{CO}_2}$) increase the CO₂ content of the atmosphere. Methane anthropogenic emissions ($E_{\text{fossil}}^{\text{CH}_4}$, $E_{\text{land-use}}^{\text{CH}_4}$) as well as natural emissions ($E_{\text{natural}}^{\text{CH}_4}$) increase the CH₄ content in the atmosphere. Methane is rapidly oxidised into CO₂ ($F_{\text{CH}_4, \text{ox}}$). Atmosphere and land reservoirs exchange CO₂ through photosynthesis and respiration (combined in $F_{\text{A} \rightarrow \text{L}}$). Weathering ($F_{\text{weathering}}$) removes carbon dioxide from the atmosphere through chemical reactions with rocks and minerals. The products of these reactions are then transported to the ocean by the rivers (F_{river} , $\tilde{F}_{\text{river}}$). Carbon is exchanged between the different layers of the ocean by mixing and the different carbon pumps ($F_{\text{U} \rightarrow \text{I}}$, $F_{\text{I} \rightarrow \text{D}}$, $\tilde{F}_{\text{U} \rightarrow \text{I}}$, $\tilde{F}_{\text{I} \rightarrow \text{D}}$). A fraction of the carbon accumulates as sediments on the ocean floor (F_{acc} , $\tilde{F}_{\text{acc}}$) where it can be permanently buried (F_{burial}) and removed from the system. Ultimately, plate tectonics movements transport this carbon in the mantle of the Earth (not explicitly modelled in SURFER) where it will be transformed back to CO₂ and eventually emitted in the atmosphere through volcanism, closing the cycle (for a detailed description of the full carbon cycle, see, e.g. Archer, 2010).

In the following, we detail the implementation of the above fluxes. Before that, we motivate the addition of a new oceanic layer and discuss the treatment of alkalinity and of the solubility and dissociation constants.

Additional ocean layer

SURFER v2.0 used two ocean layers: an upper layer (U) of 150 m depth and a deep layer (D) of 3000 m depth. In SURFER v3.0 we use three ocean layers: an upper layer (U) of 150 m depth, an intermediate layer (I) of 500 m depth, and a deep layer (D) of 3150 m depth. Overall, the total ocean depth in SURFER v3.0 is greater than in SURFER v2.0 and closer to the global mean estimate (~3700 m). The new intermediate layer allows a smoother transition between the upper and deeper layers which have different properties (temperature, salinity, pH, ...), see Figure 2.3. It also allows a faster carbon transport out of the upper layer, because the exchange with the new intermediate layer is faster than with the old deep layer. This modification partly fixes two problems we had in SURFER v2.0, and which are visible in Figure 2.7: CO₂ uptake by the ocean was too slow, and the surface pH decreased too fast (surface ocean acidified too quickly, see Martínez Montero et al., 2022). Indeed, since the dissolved CO₂

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now leaves the upper layer quicker, the surface ocean can absorb CO_2 from the atmosphere at faster rates. Moreover, with reduced "stagnation" of dissolved CO_2 in the upper layer, surface acidity increases slower (pH decreases slower). Differences in atmosphere-to-ocean carbon flux and surface ocean pH between SURFER v2.0 and SURFER v3.0 are shown in Figure 2.7.

Total alkalinity

Dissolved inorganic carbon (DIC) is defined as the sum of the carbonate species: carbonate ions CO_3^{2-} , bicarbonate ions HCO_3^- and H_2CO_3^* which represents a mix of aqueous carbon dioxide, $\text{CO}_2(\text{aqueous})$ and carbonic acid H_2CO_3 . For concentrations, we have

$$\text{DIC} = [\text{H}_2\text{CO}_3^*] + [\text{HCO}_3^-] + [\text{CO}_3^{2-}] . \quad (2.11)$$

Alkalinity, on the other hand, is defined as the excess of bases over acids in water (Middelburg et al., 2020) :

$$\text{Alk} = [\text{HCO}_3^-] + 2 [\text{CO}_3^{2-}] + [\text{OH}^-] - [\text{H}^+] + [\text{B}(\text{OH})_4^-] + \text{minor bases} . \quad (2.12)$$

Intuitively, it measures the ability of a water mass to resist changes in pH when an acid is added. This happens for example when excess CO_2 in the atmosphere dissolves in seawater (see eqs 2.17-2.19). Alkalinity thus plays a crucial role in buffering ocean acidification, which is important for many marine organisms who depend on stable pH levels for their survival and growth (Ross et al., 2011). Being related to the concentrations of dissolved inorganic carbon species (CO_3^{2-} and HCO_3^-), alkalinity also has a role in regulating the oceanic uptake of atmospheric CO_2 . In SURFER v2.0, alkalinity is considered constant and is furthermore approximated by carbonate alkalinity $\text{Alk} \approx \text{Alk}_C = [\text{HCO}_3^-] + 2 [\text{CO}_3^{2-}]$. In SURFER v3.0, we include CaCO_3 sediment dissolution, and this requires having variable alkalinity (equations 2.7-2.9). We estimate alkalinity based on the carbonate, borate and water self-ionisation alkalinity (CBW), which includes the contributions of the hydroxide ions $[\text{OH}^-]$, free protons $[\text{H}^+]$ and borate ions $[\text{B}(\text{OH})_4^-]$:

$$\text{Alk} \approx \text{Alk}_{CBW} = [\text{HCO}_3^-] + 2 [\text{CO}_3^{2-}] + [\text{OH}^-] - [\text{H}^+] + [\text{B}(\text{OH})_4^-] . \quad (2.13)$$

This treatment produces more accurate computations of the concentration of chemical species than SURFER v2.0, specifically $[\text{H}^+]$ and thus pH, but comes at a greater computational cost. See appendix A.4 for more details.

We use units of PgC for the variables representing alkalinity $\tilde{Q}_U$, $\tilde{Q}_I$, and $\tilde{Q}_D$ even though our working approximation now include terms that do not contain carbon. We do this for purely practical reasons, as all other variables of the carbon cycle component are also in PgC. The alkalinity concentration Alk_i for the layer $i \in \{U, I, D\}$ in $\mu\text{mol kg}^{-1}$, a more common unit, is simply given by

$$\text{Alk}_i = \frac{\tilde{Q}_i}{W_i \bar{m}_C} \times 10^{18}, \quad (2.14)$$

where W_i is the weight in kg of ocean layer i , and $\bar{m}_C$ is the molar mass of carbon in mol kg^{-1} . This is the same equation as for the conversion from dissolved inorganic carbon mass in PgC to concentration in $\mu\text{mol kg}^{-1}$

$$\text{DIC}_i = \frac{M_i}{W_i \bar{m}_C} \times 10^{18}. \quad (2.15)$$

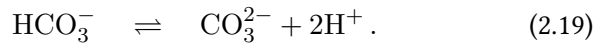
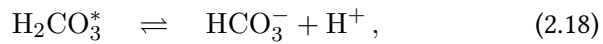
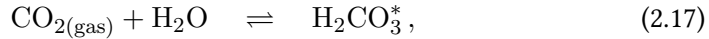
The weight of layer i is given by

$$W_i = \frac{h_i \bar{m}_w m_O}{h_U + h_I + h_D}, \quad (2.16)$$

with $\bar{m}_w$ the molar mass of water and m_O the number of moles in the ocean.

Solubility and dissociation constants

When atmospheric CO_2 enters the ocean, it undergoes a series of chemical reactions



These reactions are fast and we assume that they are in equilibrium (Sarmiento and Gruber, 2006). The relationships between the equilibrium concentrations of the different chemical species are determined by the dissociation constants :

$$K_1 = \frac{[\text{H}^+][\text{HCO}_3^-]}{[\text{H}_2\text{CO}_3^*]}, \quad (2.20)$$

$$K_2 = \frac{[\text{H}^+][\text{CO}_3^{2-}]}{[\text{HCO}_3^-]}. \quad (2.21)$$

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Similarly, we have for the equilibrium concentrations of OH^- and $\text{B}(\text{OH})_4^-$

$$K_w = [\text{H}^+] [\text{OH}^-] , \quad (2.22)$$

$$K_b = \frac{[\text{H}^+] [\text{B}(\text{OH})_4^-]}{[\text{H}_3\text{BO}_3]} , \quad (2.23)$$

where we additionally assume that the total equilibrium boron concentration is proportional to salinity (Uppström, 1974) :

$$\text{TB} = [\text{B}(\text{OH})_4^-] + [\text{H}_3\text{BO}_3] = c_b \cdot S, \quad (2.24)$$

with $c_b = 11.88 \mu\text{mol kg}^{-1} \text{psu}^{-1}$. The solubility of CO_2 in seawater, K_0 , relates the concentration of H_2CO_3^* and the partial pressure of dissolved CO_2 , p_{CO_2} :

$$K_0 = \frac{[\text{H}_2\text{CO}_3^*]}{p_{\text{CO}_2}} . \quad (2.25)$$

In SURFER v2.0, K_0 , K_1 and K_2 were constant. In SURFER v3.0, K_0 , K_1 , K_2 , K_b , and K_w depend on temperature and salinity based on Dickson and Millero (1987), Dickson (1990), Mehrbach et al. (1973), Millero (1995), and Weiss (1974). Salinity is assumed to be invariant on time, so we effectively only have a dependence on temperature . We also introduce a pressure (depth) dependence for K_1 , K_2 , K_w , and K_b , based on Millero (1995). This allows a better characterisation of pH in the deep ocean layer. Details on the parametrisations are in appendix A.3.

$F_{\text{A} \rightarrow \text{U}}$

We derive an expression for the atmosphere-to-ocean flux $F_{\text{A} \rightarrow \text{U}}$ in a slightly different way than in SURFER v2.0. The sea-air CO_2 exchange is proportional to the difference in CO_2 partial pressure between the atmosphere and the surface ocean layer (Goosse, 2015). We have

$$F_{\text{A} \rightarrow \text{U}} = \bar{m}_C A_O \rho k K_0 (p_{\text{CO}_2}^{\text{A}} - p_{\text{CO}_2}^{\text{U}}) , \quad (2.26)$$

where A_O is the ocean surface area (in m^2), ρ is the density of sea-water (in kg m^3), $\bar{m}_C$ is the molar mass of carbon (in kg mol^{-1}), k is the gas transfer velocity (in m yr^{-1}) and K_0 is the solubility constant of CO_2 (in $\text{mol kg}^{-1} \text{atm}^{-1}$). This same expression is used by Lenton (2000) and Zeebe (2012), for carbon cycles models of similar complexity, with a multiplication by the molar mass of carbon $\bar{m}_C$ added here to obtain a flux in kg yr^{-1} instead of mol yr^{-1} . We

can express $p_{\text{CO}_2}^{\text{A}}$ and $p_{\text{CO}_2}^{\text{U}}$ in terms of model variables M_{A} and M_{U}

$$F_{\text{A} \rightarrow \text{U}} = \bar{m}_{\text{C}} A_{\text{O}} \rho k K_0 (p_{\text{CO}_2}^{\text{A}} - p_{\text{CO}_2}^{\text{U}}), \quad (2.27)$$

$$= \bar{m}_{\text{C}} A_{\text{O}} \rho k K_0 \left(\frac{M_{\text{A}}}{m_{\text{A}} \bar{m}_{\text{C}}} \cdot 1 \text{ atm} - \frac{M'_{\text{U}}}{W_{\text{U}} \bar{m}_{\text{C}} K_0} \right), \quad (2.28)$$

$$= \frac{A_{\text{O}} \rho k}{m_{\text{A}}} K_0 \left(M_{\text{A}} \cdot 1 \text{ atm} - \frac{m_{\text{A}}}{W_{\text{U}} K_0} \frac{M'_{\text{U}}}{M_{\text{U}}} M_{\text{U}} \right), \quad (2.29)$$

$$= \bar{k}_{\text{A} \rightarrow \text{U}} K_0 \left(M_{\text{A}} \cdot 1 \text{ atm} - \frac{m_{\text{A}}}{W_{\text{U}} K_0} B_{\text{U}}(M_{\text{U}}, \tilde{Q}_{\text{U}}, T_{\text{U}}) M_{\text{U}} \right). \quad (2.30)$$

Here, M'_{U} represents the mass of H_2CO_3^* in the upper ocean layer. The factor 1 atm is introduced for unit consistency and if M_{A} and M_{U} are expressed in PgC, then the flux will be in PgC yr^{-1} . We have defined $\bar{k}_{\text{A} \rightarrow \text{U}} = \frac{A_{\text{O}} \rho k}{m_{\text{A}}}$. We would recover the equation of SURFER v2.0 with $k_{\text{A} \rightarrow \text{U}} = \bar{k}_{\text{A} \rightarrow \text{U}} K_0$, but K_0 now depends on temperature. There are two advantages of proceeding as we did here compared to SURFER v2.0. First the coefficient $\bar{k}_{\text{A} \rightarrow \text{U}}$ is a function of well-identified physical quantities. Second, we have not used the equilibrium condition for $F_{\text{A} \rightarrow \text{U}}(t_{\text{PI}})$ for the derivation of our expression, meaning that we can use it to constrain other quantities. Indeed, once $\bar{k}_{\text{A} \rightarrow \text{U}}$ and M_{A} are set, the equilibrium condition $F_{\text{A} \rightarrow \text{U}}(t_{\text{PI}}) = -(F_{\text{CaCO}_3,0} + F_{\text{CaSiO}_3,0})$ will determine M'_{U} (see section about initial conditions).

The term B_{U} is a factor tracking the ocean's buffer capacity. In SURFER v2.0, it was a function of M_{U} only. Now, the buffer factor also depends on the variable alkalinity ($\tilde{Q}_{\text{U}}$) and on temperature through the dissociation constants:

$$B_{\text{U}}(M_{\text{U}}, \tilde{Q}_{\text{U}}, T_{\text{U}}) \equiv \frac{M'_{\text{U}}}{M_{\text{U}}} = \frac{[\text{H}_2\text{CO}_3^*]_{\text{U}}}{\text{DIC}_{\text{U}}} = \left(1 + \frac{K_1}{[\text{H}^+]_{\text{U}}} + \frac{K_1 K_2}{[\text{H}^+]_{\text{U}}^2} \right)^{-1}. \quad (2.31)$$

To obtain this equation, we have used equations 2.20 and 2.21 to write DIC (eq 2.11) in terms of $[\text{H}_2\text{CO}_3^*]_{\text{U}}$ and $[\text{H}^+]$. In SURFER v2.0, we could compute analytically $[\text{H}^+]$ and B_{U} as a function of M_{U} and $\tilde{Q}_{\text{U}}$. We cannot do that anymore because we use a more complete approximation for alkalinity. We still compute $[\text{H}^+]$ as a function of M_{U} and $\tilde{Q}_{\text{U}}$ (and the dissociation constants), but we have to numerically solve a degree 5 equation in $[\text{H}^+]$. Appendix A.4 explains how we proceed.

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$F_{U \rightarrow I}$, $F_{I \rightarrow D}$, $\tilde{F}_{U \rightarrow I}$, and $\tilde{F}_{I \rightarrow D}$

We now focus on carbon transport within the ocean. The processes and fluxes included in SURFER v3.0 are schematised in Figure 2.2. Carbon enters the upper ocean layer (U) through CO_2 exchanges with the atmosphere and through the riverine influx of weathering products. CO_2 intake from the atmosphere increases DIC but doesn't change alkalinity (see equations 2.17-2.19). This is why we have a $F_{A \rightarrow U}$ DIC flux, but no $\tilde{F}_{A \rightarrow U}$ alkalinity flux. The riverine influx of weathering products increases DIC and alkalinity in the same amount (1:1 ratio); see section 2.2.1. We divide the exchange of carbon between the ocean layers into three parts which we briefly explain below: the carbonate pump, the soft-tissue pump, and residual processes. For a comprehensive treatment of these processes, see Sarmiento and Gruber (2006).

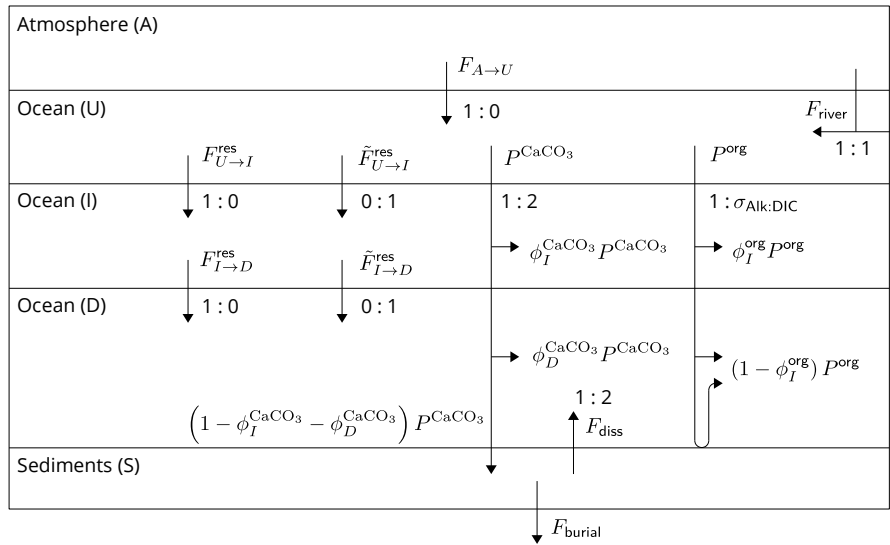
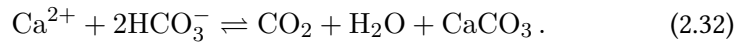


Fig. 2.2 Schematic diagram of the DIC and Alk fluxes in SURFER v3.0. The ratios are ratios of DIC to alkalinity changes, i.e a : b ratio indicates that for a DIC change of a moles, there is an associated alkalinity change of b moles.

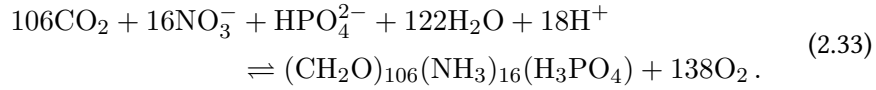
In the surface layer, calcifying organisms take up bicarbonate ions to form their calcium carbonate (CaCO_3) shells (forward reaction 2.32).



These organisms eventually die and sink to the bottom ocean. On the way down, some of the CaCO_3 is dissolved (reverse reaction 2.32), resulting in a

downward transport of DIC and alkalinity at a 1:2 ratio. This is the carbonate pump. We represent the export of CaCO_3 at a depth of 150 m (so from layer U to I) by P^{CaCO_3} . A fraction $\phi_I^{\text{CaCO}_3}$ of this export simultaneously dissolves in the intermediate layer (I) and a fraction $\phi_D^{\text{CaCO}_3}$ simultaneously dissolves in the deep layer (D), which leaves a fraction $1 - \phi_I^{\text{CaCO}_3} - \phi_D^{\text{CaCO}_3}$ reaching the sediments. Of that CaCO_3 that reaches the bottom of the ocean, some dissolves and some is permanently buried (more details on this below).

In the surface layer, organisms also take up carbon through photosynthesis (primary production, forward reaction 2.33).



The CO_2 is transformed into organic carbon that will eventually sink to the bottom ocean. On the way down, some of the organic carbon is remineralised (reverse reaction 2.33), resulting in a downward transport of DIC. This is the soft-tissue pump (Volk and Hoffert, 1985). It also acts on alkalinity, mainly through the uptake and release of H^+ needed for the transformation of inorganic nitrate (NO_3^-) into organic nitrogen. Primary production of organic matter increases alkalinity, while remineralisation decreases alkalinity. We represent the ratio of alkalinity to DIC change for primary production and remineralisation with the parameter $\sigma_{\text{Alk:DIC}}$. We represent the export of organic carbon at a depth of 150 m (so from layer U to I) by P^{org} . We consider that all this organic carbon is simultaneously remineralised in the water column, with a fraction ϕ_I^{org} remineralised in the intermediate layer (I), and a fraction $\phi_D^{\text{org}} = 1 - \phi_I^{\text{org}}$ remineralised in the deep layer (D). In reality, some organic carbon accumulates on the sea-floor sediments where a large part is remineralised and only a small amount is permanently buried (Sarmiento and Gruber, 2006). Here, by setting $\phi_D^{\text{org}} = 1 - \phi_I^{\text{org}}$, we neglect this burial and effectively consider that all organic carbon that falls on sediments is remineralised.

Apart from the carbonate and soft-tissue pumps, forming the biological pumps, other processes are responsible for the transport of carbon between the ocean layers. Ocean circulation and mixing will propagate variations in DIC caused by spatial and temporal variations of air-sea gas exchanges. This is termed the gas exchange pump by Sarmiento and Gruber (2006). For example, a component of the gas exchange pump is the solubility pump (Volk and Hoffert, 1985): cold waters have a higher solubility and are thus enriched in CO_2 , they are also denser and will sink in the high latitudes, resulting in a

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downward transport of DIC. Besides, upwelling is responsible for an upward transport of DIC and alkalinity, necessary to counteract the carbonate and soft-tissue pumps (Goosse, 2015). We gather all the processes related to ocean circulation and mixing in the residual terms $F_{U \rightarrow I}^{\text{res}}$ and $F_{I \rightarrow D}^{\text{res}}$, $\tilde{F}_{U \rightarrow I}^{\text{res}}$ and $\tilde{F}_{I \rightarrow D}^{\text{res}}$. We consider the residual fluxes of DIC and alkalinity to be independent because some processes such as the solubility pump only act on DIC.

Based on the above considerations, the masses of dissolved inorganic carbon and the mass of carbon in the sediments evolve according to the following equations:

$$\frac{dM_U}{dt} = F_{A \rightarrow U} - P^{\text{CaCO}_3} - P^{\text{org}} - F_{U \rightarrow I}^{\text{res}} + F_{\text{river}}, \quad (2.34)$$

$$\frac{dM_I}{dt} = \phi_I^{\text{CaCO}_3} P^{\text{CaCO}_3} + \phi_I^{\text{org}} P^{\text{org}} + F_{U \rightarrow I}^{\text{res}} - F_{I \rightarrow D}^{\text{res}}, \quad (2.35)$$

$$\frac{dM_D}{dt} = \phi_D^{\text{CaCO}_3} P^{\text{CaCO}_3} + (1 - \phi_I^{\text{org}}) P^{\text{org}} + F_{I \rightarrow D}^{\text{res}} + F_{\text{diss}}, \quad (2.36)$$

$$\frac{dM_S}{dt} = (1 - \phi_I^{\text{CaCO}_3} - \phi_D^{\text{CaCO}_3}) P^{\text{CaCO}_3} - F_{\text{diss}} - F_{\text{burial}}. \quad (2.37)$$

We recover equations 3.25-2.10 by setting

$$F_{U \rightarrow I} = F_{U \rightarrow I}^{\text{CaCO}_3} + F_{U \rightarrow I}^{\text{org}} + F_{U \rightarrow I}^{\text{res}}, \quad (2.38)$$

$$F_{I \rightarrow D} = F_{I \rightarrow D}^{\text{CaCO}_3} + F_{I \rightarrow D}^{\text{org}} + F_{I \rightarrow D}^{\text{res}}, \quad (2.39)$$

$$F_{\text{acc}} = (1 - \phi_I^{\text{CaCO}_3} - \phi_D^{\text{CaCO}_3}) P^{\text{CaCO}_3} - F_{\text{diss}}, \quad (2.40)$$

with the fluxes associated to the carbonate pump defined as

$$F_{U \rightarrow I}^{\text{CaCO}_3} = P^{\text{CaCO}_3}, \quad (2.41)$$

$$F_{I \rightarrow D}^{\text{CaCO}_3} = (1 - \phi_I^{\text{CaCO}_3}) P^{\text{CaCO}_3}, \quad (2.42)$$

and the fluxes associated to the soft-tissue pump defined as

$$F_{U \rightarrow I}^{\text{org}} = P^{\text{org}}, \quad (2.43)$$

$$F_{I \rightarrow D}^{\text{org}} = (1 - \phi_I^{\text{org}}) P^{\text{org}}. \quad (2.44)$$

Consequently, for the carbonate alkalinity fluxes, we have

$$\tilde{F}_{U \rightarrow I} = 2 \times F_{U \rightarrow I}^{\text{CaCO}_3} + \sigma_{\text{Alk:DIC}} \times F_{U \rightarrow I}^{\text{org}} + \tilde{F}_{U \rightarrow I}^{\text{res}}, \quad (2.45)$$

$$\tilde{F}_{I \rightarrow D} = 2 \times F_{I \rightarrow D}^{\text{CaCO}_3} + \sigma_{\text{Alk:DIC}} \times F_{I \rightarrow D}^{\text{org}} + \tilde{F}_{I \rightarrow D}^{\text{res}}, \quad (2.46)$$

$$\tilde{F}_{\text{acc}} = 2 \times F_{\text{acc}}. \quad (2.47)$$

In the experiments presented here (sections 2.3 and 2.4), we keep P^{CaCO_3} , $\phi_{\text{I}}^{\text{CaCO}_3}$, $\phi_{\text{D}}^{\text{CaCO}_3}$, P^{org} , and $\phi_{\text{I}}^{\text{org}}$ constant. This is an idealisation. Production and export of CaCO_3 and organic matter are biological processes and, in reality, depend on the temperature, pH, salinity, nutrient concentration and other properties of the ocean. For example, changes in primary production (and thus export of organic matter) may have contributed to the CO_2 changes during the glacial-interglacial cycles (Kohfeld and Ridgwell, 2009). In the future, changes in the biological pumps are also possible and might lead to additional feedbacks in the carbon cycle (Henson et al., 2022; Planchat et al., 2023).

For the residual exchange terms, we adopt a linear formalism with exchanges coefficients, similarly as in SURFER v2.0:

$$F_{\text{U} \rightarrow \text{I}}^{\text{res}} = k_{\text{U} \rightarrow \text{I}} M_{\text{U}} - k_{\text{I} \rightarrow \text{U}} M_{\text{I}}, \quad (2.48)$$

$$F_{\text{I} \rightarrow \text{D}}^{\text{res}} = k_{\text{I} \rightarrow \text{D}} M_{\text{I}} - k_{\text{D} \rightarrow \text{I}} M_{\text{D}}, \quad (2.49)$$

$$\tilde{F}_{\text{U} \rightarrow \text{I}}^{\text{res}} = \tilde{k}_{\text{U} \rightarrow \text{I}} \tilde{Q}_{\text{U}} - \tilde{k}_{\text{I} \rightarrow \text{U}} \tilde{Q}_{\text{I}}, \quad (2.50)$$

$$\tilde{F}_{\text{I} \rightarrow \text{D}}^{\text{res}} = \tilde{k}_{\text{I} \rightarrow \text{D}} \tilde{Q}_{\text{I}} - \tilde{k}_{\text{D} \rightarrow \text{I}} \tilde{Q}_{\text{D}}. \quad (2.51)$$

F_{acc} , $\tilde{F}_{\text{acc}}$, and F_{burial}

The CaCO_3 raining on the ocean floor either accumulates and gets buried in sediments, or dissolves, depending on the saturation state of the ocean waters with respect to CO_3^{2-} (Archer et al., 1998; Zeebe and Westbroek, 2003). Typically, the upper ocean is supersaturated in CO_3^{2-} while the deeper ocean is under-saturated, mainly due to the pressure dependence of CaCO_3 solubility (Sarmiento and Gruber, 2006). This means that most of the accumulation in sediments will happen in a region above where most of the dissolution happens. A transition zone separates the accumulation and dissolution regions. The top boundary of the transition zone is the lysocline, defined as the depth where the calcium carbonate content of sediments starts to decrease sharply, or in other words, the depth below which the rate of dissolution of CaCO_3 starts to increase significantly. The bottom boundary of the transition zone is called the carbonate (or calcite) compensation depth (CCD) and is the depth at which the rate of CaCO_3 dissolution is equal to the rate of supply through the CaCO_3 rain. Below this depth, no CaCO_3 is preserved in the sediments. The depth of the transition zone varies between places and ocean basins but is generally between 3000 and 5000 m deep (Sarmiento and Gruber, 2006). We may thus safely consider in our model that both dissolution and accumulation happen in

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the deep ocean layer (D), and that is why we group both processes in a single term, F_{acc} which can be positive or negative, negative values indicating net dissolution.

Estimates indicate that carbonate accumulation in shallow waters may be comparable in magnitude to that occurring in open-ocean deep sediments (Milliman and Droxler, 1996; Milliman, 1993; Vecsei, 2004). However, shallow waters are characterised by significantly higher sedimentation rates, necessitating distinct models or parametrisations (Ridgwell and Hargreaves, 2007). Moreover, factors such as carbonate fixation by corals and algae must also be taken into account. Given the significant uncertainty surrounding these processes and the lack of reliable data to quantify them, we excluded them from our model. This approach is equivalent to assuming that CaCO_3 accumulation on shelves and the weathering flux required to balance it remain constant throughout the simulations (Archer et al., 1998; Ridgwell and Hargreaves, 2007), despite potential influences from human activities (Andersson and Mackenzie, 2004).

In the open-ocean deep sediments, the local dissolution rate depends on the concentration of CaCO_3 in the sediments and the saturation state of pore water around the sediments with respect to carbonate (Sarmiento and Gruber, 2006). We thus assume that the globally integrated dissolution flux is a function of two variables only: the deep ocean mean concentration of CO_3^{2-} , and the total mass of erodible CaCO_3 sediments. We use the following parametrisation

$$F_{\text{diss}} = \begin{cases} R^{\text{CaCO}_3} & \text{if } M_S = 0 \text{ and } D > R^{\text{CaCO}_3}, \\ D & \text{otherwise,} \end{cases} \quad (2.52)$$

with

$$R^{\text{CaCO}_3} = (1 - \phi_1^{\text{CaCO}_3} - \phi_D^{\text{CaCO}_3}) P^{\text{CaCO}_3}, \quad (2.53)$$

and

$$D = F_{\text{diss},0} + \alpha_{\text{diss}} ([\text{CO}_3^{2-}]_D - [\text{CO}_3^{2-}]_D(t_{\text{PI}})) + \beta_{\text{diss}} (M_S - M_S(t_{\text{PI}})) + \gamma_{\text{diss}} ([\text{CO}_3^{2-}]_D - [\text{CO}_3^{2-}]_D(t_{\text{PI}})) (M_S - M_S(t_{\text{PI}})). \quad (2.54)$$

The first case in equation 2.52 indicates that if the erodible sediments reservoir is empty, the dissolution flux can't exceed the CaCO_3 rain flux R^{CaCO_3} , regardless of the saturation state of deep waters. The accumulation fluxes of DIC and alkalinity, F_{acc} and $\tilde{F}_{\text{acc}}$, are then given by equations 2.40 and 2.47. A

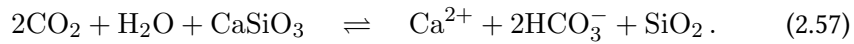
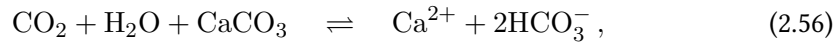
similar parametrisation is proposed and tested in Archer et al. (1998), but they use the CO_3^{2-} concentration at a particular point in the deep pacific and the mass of CaCO_3 in the bioturbated layer of the sediments as their two variables, instead of the mean deep ocean CO_3^{2-} and the mass of erodible CaCO_3 . They show that the parametrised version of their model is comparable to the non-parametrised one, except for a dissolution spike in the first 1000-2000 years following the fossil fuel emissions and invasion in the ocean. We choose the coefficient of our parametrisation based on theirs. Details can be found in section 2.2.4.

In the sediments, the bioturbated layer is the layer where sediments are mixed by biological activity (bioturbation). Dissolution only occurs in the top few centimetres near the sediment-ocean interface but can effectively reach deeper because of bioturbation (Sarmiento and Gruber, 2006). If the dissolution rate is greater than the rain rate of CaCO_3 on the sea floor, the sediments will lose CaCO_3 until the bioturbated layer becomes saturated with non-erodible material. At this point, dissolution stops and deeper carbonates are isolated from the carbon cycle. This explains why the effective reservoir of sediment carbonates to be accounted for here (M_S), has a limited size, which is estimated to be around 1600 PgC as of today (Archer et al., 1998). The erodible depth is defined as the lower boundary of the erodible sediments inventory (Archer et al., 1998). By definition, CaCO_3 sediments that move below the erodible depth will never be dissolved and we say that they are buried. Locally, the burial rate depends on the rain rate of non-erodible material and the concentration of CaCO_3 in sediments at the erodible depth. We assume that the former is constant and that the total burial flux only depends on the total mass of erodible CaCO_3 . We use the following linear parametrisation

$$F_{\text{burial}} = \alpha_{\text{burial}} M_S . \quad (2.55)$$

$V, F_{\text{weathering}}, F_{\text{river influx}}, \tilde{F}_{\text{river influx}}$

Continental weathering of carbonate and silicate rocks can absorb CO_2 out of the atmosphere through the following reactions (Berner et al., 1983; Walker et al., 1981) :



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Let us consider F_{CaCO_3} and F_{CaSiO_3} the fluxes of Ca^{2+} produced by these two processes. For carbonate weathering (equation 2.56), the production of one mole of Ca^{2+} consumes one mole of carbon (CO_2) from the atmosphere and produces two moles of DIC and alkalinity that are transported by the rivers to the ocean. For the silicate weathering (equation 2.57), the production of one mole of Ca^{2+} consumes two moles of carbon (CO_2) from the atmosphere and produces two moles of DIC and alkalinity that are transported by the rivers to the ocean. Hence, we set

$$F_{\text{weathering}} = F_{\text{CaCO}_3} + 2F_{\text{CaSiO}_3} \quad (2.58)$$

$$F_{\text{river influx}} = 2F_{\text{CaCO}_3} + 2F_{\text{CaSiO}_3} \quad (2.59)$$

$$F_{\text{river influx}}^{\text{alk}} = 2F_{\text{CaCO}_3} + 2F_{\text{CaSiO}_3} \quad (2.60)$$

Like with alkalinity, we use PgC yr^{-1} for F_{CaCO_3} and F_{CaSiO_3} , even though they are defined as fluxes of Ca^{2+} . To go from Tmol yr^{-1} to PgC yr^{-1} , one just has to divide by the molar mass of carbon $12 \cdot 10^{-3} \text{ mol kg}^{-1}$.

For every two moles of DIC produced by carbonate weathering, one mole of carbon is taken from the atmosphere reservoir and one mole of carbon comes from carbonate rocks on land, which are not explicitly described as a reservoir in our model. This extra mole is thus treated as an external source of carbon, like volcanism. The carbon entering the ocean from weathering fluxes will eventually precipitate back as CaCO_3 in the sediments, which releases CO_2 . Thus, the net effect of carbonate weathering is to transfer CaCO_3 from rocks on land to sediments in the ocean, but with no long-term net effect on the atmospheric CO_2 . On the other hand, since silicate weathering consumes one more mole of CO_2 from the atmosphere for the same Ca^{2+} flux, its net effect is to remove carbon from the atmosphere. At equilibrium, this net removal is compensated by volcanic outgassing.

The F_{CaCO_3} and F_{CaSiO_3} fluxes aren't constant and depend on many factors (for a review, see Kump et al., 2000). For example, temperature affects the rates of reactions 2.56 and 2.57, water run-off on land influences how much under-saturated water will come in contact with rocks for weathering, and vegetation may affect the acidity of soils and thus the rates of dissolution. Colbourn et al. (2013) provide parametrisations of all these processes for the model of terrestrial rock weathering RockGEM, which was included in the GENIE Earth system modelling framework. They showed that the temperature feedback was the dominant factor, and we choose to consider this one only, especially given that SURFER lacks a representation of the hydrological cycle.

Following their parametrisation, we set

$$F_{\text{CaCO}_3} = F_{\text{CaCO}_3,0}(1 + k_{\text{Ca}}\delta T_{\text{U}}), \quad (2.61)$$

$$F_{\text{CaSiO}_3} = F_{\text{CaSiO}_3,0}e^{k_{\text{T}}\delta T_{\text{U}}}, \quad (2.62)$$

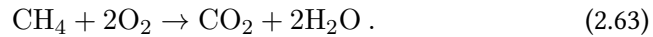
where k_{Ca} and k_{T} are constants and δT_{U} is the temperature anomaly of the upper ocean (and atmosphere) modelled by SURFER. This is also the parametrisation used by Lord et al. (2016) in their cGENIE simulations, which we will use as a basis for comparison with SURFER (see section 2.3.4).

Methane

The evolution of the methane concentration in the atmosphere is mainly controlled by 3 processes: natural and anthropogenic emissions increase the CH_4 concentration whereas oxidation into CO_2 decreases it (Saunois et al., 2020).

Anthropogenic emissions can be divided in two sources, fossil emissions ($E_{\text{fossil}}^{\text{CH}_4}$) and land-use emissions ($E_{\text{land-use}}^{\text{CH}_4}$). Fossil emissions come from the industry sector and the use and exploitation of fossil fuels. Land-use emissions result from agriculture (rice production, cattle,...), agricultural waste burning, and burning of biomass such as forests, grasslands and peat. Natural emissions ($E_{\text{natural}}^{\text{CH}_4}$) mainly come from anaerobic decomposition of organic matter in wetlands, but also from freshwater systems, termites, and geological sources such as volcanoes, permafrost and methane hydrates. For a detailed treatment of the different methane emissions see Saunois et al. (2020). The rate of natural emissions may depend on temperature, and if they increase upon global warming, this could lead to positive feedbacks and eventually tipping points (Archer et al., 2009b; Fewster et al., 2022; Nisbet et al., 2023). For simplicity, we assume constant natural emissions. To ensure the conservation of carbon, land-use CH_4 emissions are taken from the land reservoir, while natural CH_4 emissions are taken directly from the CO_2 atmospheric reservoir of carbon. The reason for this difference is explained in the next section.

Oxidation of methane ($F_{\text{CH}_4,\text{ox}}$, equation 2.63) is the main sink of methane out of the atmosphere and ultimately leads to the release of CO_2 and H_2O :



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We describe this a simple decay process :

$$F_{\text{CH}_4, \text{ox}} = -\frac{M_{\text{A}}^{\text{CH}_4}}{\tau_{\text{CH}_4}}, \quad (2.64)$$

where τ_{CH_4} is the atmospheric CH_4 lifetime. In principle, it may vary depending on temperature and on the availability of the hydroxyl radical OH, which is necessary for the intermediate steps of reaction 2.63, and which itself depends on the concentrations of CH_4 , N_2O , CO and other trace gases (Skeie et al., 2023). However, for simplicity, we choose to keep τ_{CH_4} constant and set its value to 9.5 yr. We add the product of oxidation to M_{A} .

Land reservoir and land-use emissions

In SURFER v3.0, we distinguish greenhouse gas emissions caused by fossil fuel use from those caused by land use. While fossil CO_2 and CH_4 emissions are directly added to the atmosphere, land-use CO_2 and CH_4 emissions ($E_{\text{land-use}}^{\text{CO}_2}$ and $E_{\text{land-use}}^{\text{CH}_4}$) must be taken from the land reservoir (eq 3.27). In SURFER v2.0, based on the outputs of the ZECMIP experiments (Jones et al., 2019; MacDougall et al., 2020), we parametrised the carbon flux from the land to the atmosphere in the following way

$$F_{\text{A} \rightarrow \text{L}} = k_{\text{A} \rightarrow \text{L}} \left(\beta_{\text{L}} M_{\text{A}}(t_{\text{PI}}) \left(1 - \frac{M_{\text{A}}(t_{\text{PI}})}{M_{\text{A}}} \right) - (M_{\text{L}} - M_{\text{L}}(t_{\text{PI}})) \right). \quad (2.65)$$

This is equivalent to saying that M_{L} relaxes to an equilibrium mass $M_{\text{L}}^{\text{eq}}(M_{\text{A}})$ equal to

$$M_{\text{L}}^{\text{eq}}(M_{\text{A}}) = \beta_{\text{L}} M_{\text{A}}(t_{\text{PI}}) \left(1 - \frac{M_{\text{A}}(t_{\text{PI}})}{M_{\text{A}}} \right) + M_{\text{L}}(t_{\text{PI}}), \quad (2.66)$$

with a timescale $1/k_{\text{A} \rightarrow \text{L}}$.

Now suppose that we have a certain amount of land-use emissions; we transfer some carbon out of the land reservoir into the atmosphere reservoir, and let the model evolve without changing anything else. Then the $F_{\text{A} \rightarrow \text{L}}$ flux, as it is, will increase and land will absorb the carbon lost until initial equilibrium is reached again. Physically, this means that the forest that was replaced by grassland or crop fields has grown back to its original size. In the real world, this may happen, but if the land is managed (e.g. for agriculture), the forest does not regrow. To take this into account, we subtract the cumulative

CO₂ land-use emissions from the equilibrium mass to which M_L relaxes

$$M_L^{\text{eq}}(M_A, t) = \beta_L M_{A(t_{\text{PI}})} \left(1 - \frac{M_A(t_{\text{PI}})}{M_A} \right) + M_L(t_{\text{PI}}) - \int_{t_0}^t E_{\text{land-use}}^{\text{CO}_2}(t) \, dt. \quad (2.67)$$

This is the same procedure as in the model from Lenton (2000), except that they subtract only a fraction of the cumulative land-use emissions, allowing for some forest regrowth. Here we subtract all land-use emissions because, in principle, the negative emissions coming from forest regrowth should already be accounted for in the net reported land-use emissions (IPCC, 2021a, p.688). We can rewrite the new $F_{A \rightarrow L}$ flux as

$$F_{A \rightarrow L} = k_{A \rightarrow L} \left(\beta_L M_{A(t_{\text{PI}})} \left(1 - \frac{M_A(t_{\text{PI}})}{M_A} \right) - (M_L - M_L^*) \right), \quad (2.68)$$

where M_L^* is a new model variable, the evolution of which is described by

$$\frac{dM_L^*}{dt} = -E_{\text{land-use}}^{\text{CO}_2}(t), \quad (2.69)$$

with $M_L^*(t_{\text{PI}}) = M_L(t_{\text{PI}})$.

We haven't included methane land-use emissions in equations 2.67-3.28. This means that the carbon mass in CH₄ form taken from the land reservoir is reabsorbed by land once methane is oxidized back to CO₂. In other words, methane land-use emissions do not cause a net addition of CO₂ in the atmosphere through oxidation. This makes sense if you consider methane emissions coming from the anaerobic decomposition of organic matter in rice cultures or cattle stomachs. Indeed, the decomposed organic matter was formed not too long before by absorbing CO₂ from the atmosphere through photosynthesis, so when the methane oxidises into CO₂, it closes the loop and there is no net effect on atmospheric CO₂.

The same reasoning applies to most natural emissions of methane which arise from wetlands: they do not cause a net increase of atmospheric CO₂ concentrations. In principle, these natural methane emissions should be subtracted from the land carbon reservoir, as anthropogenic land-use emissions, and there should be a non-zero atmosphere-to-land equilibrium flux that compensates the CO₂ created from oxidation. However, this is impossible with our parametrisation of the atmosphere-to-land flux, which is zero at pre-industrial times by construction. To avoid introducing yet another parametrisation or a more detailed representation of the carbon on land and to maintain carbon

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conservation, we merely subtract the natural emissions directly from the CO₂ mass of carbon in the atmosphere. This approach is justified by our assumption that the natural methane production-oxidation cycle is in equilibrium.

Natural CH₄ emissions coming from geological processes such as volcanism or natural gas leaks would have a net impact on CO₂ levels but they are negligible compared to emissions coming from wetlands (Saunois et al., 2020). Natural CH₄ emissions coming from permafrost would also have a lasting effect on CO₂ concentrations because the organic matter from which they originate was formed thousands of years before. Accounting for emissions from permafrost would require yet another parametrisation, and we rather neglected them in SURFER v3.0.

2.2.2 Climate component

The equations for the climate component are essentially the same as in SURFER v2.0, with the addition of an oceanic box and the radiative forcing due to methane. The atmosphere is considered to be in thermal equilibrium with the surface ocean layer (U). The evolutions of temperature anomalies for the three oceanic layers are dictated by

$$c_{\text{vol}} h_U \frac{d\delta T_U}{dt} = F(M_A, M_A^{\text{CH}_4}, I) - \beta \delta T_U - \gamma_{U \rightarrow I} (\delta T_U - \delta T_I), \quad (2.70)$$

$$c_{\text{vol}} h_I \frac{d\delta T_I}{dt} = \gamma_{U \rightarrow I} (\delta T_U - \delta T_I) - \gamma_{I \rightarrow D} (\delta T_I - \delta T_D), \quad (2.71)$$

$$c_{\text{vol}} h_D \frac{d\delta T_D}{dt} = \gamma_{I \rightarrow D} (\delta T_I - \delta T_D), \quad (2.72)$$

where $F(M_A, M_A^{\text{CH}_4}, I)$ is the anthropogenic radiative forcing. Its expression is given by

$$F(M_A, M_A^{\text{CH}_4}, I) = F_{2\times} \log_2 \left(\frac{M_A}{M_A(t_{\text{PI}})} \right) + \alpha_{\text{CH}_4} \sqrt{M_A^{\text{CH}_4} - M_A^{\text{CH}_4}(t_{\text{PI}})} - \alpha_{\text{SO}_2} \exp \left(- (\beta_{\text{SO}_2}/I)^{\gamma_{\text{SO}_2}} \right). \quad (2.73)$$

The first two terms describe the contribution of CO₂ and methane to an increased greenhouse effect. The third term corresponds to solar radiation modification in the form of SO₂ injections (Martínez Montero et al., 2022).

2.2.3 Sea level component

Sea level rise is estimated as the sum of four contributions: thermal expansion, and melt from the mountain glaciers, the Greenland ice sheet, and the Antarctic ice sheet:

$$S_{\text{tot}} = S_{\text{th}} + S_{\text{gl}} + S_{\text{GIS}} + S_{\text{AIS}}. \quad (2.74)$$

Compared to SURFER v2.0, we only change the parametrisation of thermal expansion, where we need to add a term to take into account the new intermediate layer.

$$S_{\text{th}} = \alpha_{\text{U}} h_{\text{U}} \delta T_{\text{U}} + \alpha_{\text{I}} h_{\text{I}} \delta T_{\text{I}} + \alpha_{\text{D}} h_{\text{D}} \delta T_{\text{D}}. \quad (2.75)$$

Here α_i is the thermal expansion coefficient corresponding to layer $i \in \{\text{U}, \text{I}, \text{D}\}$. The other contributions are the same as in Martínez Montero et al. (2022). We recall them here, and more details are provided in the original publication.

The evolution of the sea-level rise contribution from glaciers is given by the equation

$$\frac{dS_{\text{gl}}}{dt} = \frac{1}{\tau_{\text{gl}}} (S_{\text{gleq}}(\delta T_{\text{U}}) - S_{\text{gl}}), \quad (2.76)$$

with

$$S_{\text{gleq}}(\delta T_{\text{U}}) = S_{\text{gl pot}} \tanh\left(\frac{\delta T_{\text{U}}}{\zeta}\right). \quad (2.77)$$

Here τ_{gl} is a relaxation timescale, $S_{\text{gl pot}}$ is the potential sea level rise due to mountain glaciers and ζ is a sensitivity coefficient.

The contributions from Greenland and Antarctica are given by

$$S_{\text{GIS}} = S_{\text{GIS}} (1 - V_{\text{GIS}}(t)), \quad S_{\text{AIS}} = S_{\text{AIS}} (1 - V_{\text{AIS}}(t)), \quad (2.78)$$

where S_{GIS} and S_{AIS} are the sea-level rise potential of Greenland's and Antarctic ice sheet and V_{GIS} , V_{AIS} are the volume fractions of the ice sheets with respect to their pre-industrial volume. The evolution of ice sheets volumes are described with the equation

$$\frac{dV}{dt} = \mu(V, \delta T_{\text{U}}) \underbrace{(-V^3 + a_2 V^2 + a_1 V + c_1 \delta T_{\text{U}} + c_0)}_H, \quad (2.79)$$

with

$$\mu(V, \delta T_{\text{U}}) = \begin{cases} 1/\tau & \text{if } H > 0 \text{ or } (H < 0 \text{ and } V > 0), \\ 0 & \text{if } H < 0 \text{ and } V = 0, \end{cases} \quad (2.80)$$

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and

$$\tau = \tau_- + \frac{\tau_+ - \tau_-}{2} \left(1 + \tanh \left(\frac{H}{k_\tau} \right) \right). \quad (2.81)$$

The time scales τ_+ and τ_- are associated with the asymmetric processes of freezing and melting. The first case in equation 2.80 was separated in two cases in SURFER v2.0, depending on the sign of H . In SURFER v3.0, we introduce a smooth transition between τ_+ and τ_- when H changes sign. This formulation effectively prevents small fluctuations around the equilibrium having different timescales (when H is close to zero). The parameter k_τ controls the smoothness of the transition, SURFER v2.0 being recovered with $k_\tau = \infty$. The constant parameters (a_2, a_1, c_1, c_0) are given in terms of $(T_+, V_+), (T_-, V_-)$, which are the bifurcation points of the steady state structure induced by equation 2.79:

$$a_2 = \frac{3(V_- + V_+)}{2}, \quad (2.82)$$

$$a_1 = -3V_-V_+, \quad (2.83)$$

$$c_1 = -\frac{(V_+ - V_-)^3}{2(T_+ - T_-)}, \quad (2.84)$$

$$c_0 = +\frac{T_+V_-^2(V_- - 3V_+) - T_-V_+^2(V_+ - 3V_-)}{2(T_- - T_+)}. \quad (2.85)$$

These relationships allow SURFER's ice sheets to be easily calibrated on the steady-state structure of more complex ice-sheet models.

2.2.4 Calibration and initial conditions

We calibrate the parameters and initial conditions of the model using known physics, observations, model results, and the hypothesis that the carbon cycle was at equilibrium during pre-industrial times. This follows standard practice, even though processes involving longer time scales active at glacial-interglacial time scales are not necessarily in balance (Brovkin et al., 2016). We assume

$$0 = V - F_{A \rightarrow U}(t_{PI}) - F_{A \rightarrow L}(t_{PI}) + F_{CH_4,ox}(t_{PI}) - E_{nat}^{CH_4} - F_{weathering}(t_{PI}), \quad (2.86)$$

$$0 = E_{nat}^{CH_4} - F_{CH_4,ox}(t_{PI}), \quad (2.87)$$

$$0 = F_{A \rightarrow L}(t_{PI}), \quad (2.88)$$

$$0 = F_{A \rightarrow U}(t_{PI}) - F_{U \rightarrow I}(t_{PI}) + F_{river}(t_{PI}), \quad (2.89)$$

$$0 = F_{U \rightarrow I}(t_{PI}) - F_{I \rightarrow D}(t_{PI}), \quad (2.90)$$

$$0 = F_{I \rightarrow D}(t_{PI}) - F_{acc}(t_{PI}), \quad (2.91)$$

$$0 = -\tilde{F}_{U \rightarrow I}(t_{PI}) + \tilde{F}_{river}(t_{PI}), \quad (2.92)$$

$$0 = \tilde{F}_{U \rightarrow I}(t_{PI}) - \tilde{F}_{I \rightarrow D}(t_{PI}), \quad (2.93)$$

$$0 = \tilde{F}_{I \rightarrow D}(t_{PI}) - \tilde{F}_{acc}(t_{PI}), \quad (2.94)$$

$$0 = F_{acc}(t_{PI}) - F_{burial}(t_{PI}). \quad (2.95)$$

From equations 2.92-2.94 we get that $\tilde{F}_{acc}(t_{PI}) = \tilde{F}_{I \rightarrow D}(t_{PI}) = \tilde{F}_{U \rightarrow I}(t_{PI}) = \tilde{F}_{river}(t_{PI}) = 2F_{CaCO_3,0} + 2F_{CaSiO_3,0}$. The dissolution/precipitation of $CaCO_3$ produces/consumes moles of DIC and alkalinity at a 1:2 ratio (see section 2.2.1), hence we have $F_{acc}(t_{PI}) = \frac{1}{2}\tilde{F}_{acc}(t_{PI}) = F_{CaCO_3,0} + F_{CaSiO_3,0}$. We then get from equations 2.90, 2.91 and 2.95 that $F_{U \rightarrow I}(t_{PI}) = F_{I \rightarrow D}(t_{PI}) = F_{burial}(t_{PI}) = F_{acc}(t_{PI}) = F_{CaCO_3,0} + F_{CaSiO_3,0}$. Equation 2.89 gives us $F_{A \rightarrow L}(t_{PI}) = -(F_{CaCO_3,0} + F_{CaSiO_3,0})$, equation 2.87 tells us that $E_{nat}^{CH_4} = F_{CH_4,ox}(t_{PI})$, and finally, from equation 2.86 we find $V = F_{CaSiO_3,0}$. Equation 2.88 provides no extra information since $F_{A \rightarrow L}(t_{PI}) = 0$ by construction. Developing the expressions of the carbon fluxes, we get the following system of equations :

$$V = F_{CaSiO_3,0}, \quad (2.96)$$

$$E_{nat}^{CH_4} = \frac{M_A^{CH_4}(t_{PI})}{\tau_{CH_4}}, \quad (2.97)$$

$$0 = 0, \quad (2.98)$$

$$\begin{aligned} \bar{k}_{A \rightarrow U} K_0 \left(M_A(t_{PI}) - \frac{m_A}{W_U K_0} M'_U(t_{PI}) \right) \\ = -(F_{CaCO_3,0} + F_{CaSiO_3,0}), \end{aligned} \quad (2.99)$$

$$\begin{aligned} P^{CaCO_3} + P^{org} + k_{U \rightarrow I} M_U(t_{PI}) - k_{I \rightarrow U} M_I(t_{PI}) \\ = F_{CaCO_3,0} + F_{CaSiO_3,0}, \end{aligned} \quad (2.100)$$

$$\begin{aligned} (1 - \phi_I^{CaCO_3}) P^{CaCO_3} + (1 - \phi_I^{org}) P^{org} + k_{I \rightarrow D} M_I(t_{PI}) \\ - k_{D \rightarrow I} M_D(t_{PI}) = F_{CaCO_3,0} + F_{CaSiO_3,0}, \end{aligned} \quad (2.101)$$

$$\begin{aligned} 2P^{CaCO_3} + \sigma_{Alk:DIC} P^{org} + \tilde{k}_{U \rightarrow I} \tilde{Q}_U(t_{PI}) - \tilde{k}_{I \rightarrow U} \tilde{Q}_I(t_{PI}) \\ = 2(F_{CaCO_3,0} + F_{CaSiO_3,0}), \end{aligned} \quad (2.102)$$

$$\begin{aligned} 2(1 - \phi_I^{CaCO_3}) P^{CaCO_3} + \sigma_{Alk:DIC} (1 - \phi_I^{org}) P^{org} \\ + \tilde{k}_{I \rightarrow D} \tilde{Q}_I(t_{PI}) - \tilde{k}_{D \rightarrow I} \tilde{Q}_D(t_{PI}) = 2(F_{CaCO_3,0} + F_{CaSiO_3,0}), \end{aligned} \quad (2.103)$$

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$$(1 - \phi_I^{\text{CaCO}_3} - \phi_D^{\text{CaCO}_3})P^{\text{CaCO}_3} - F_{\text{diss},0} = F_{\text{CaCO}_3,0} + F_{\text{CaSiO}_3,0}, \quad (2.104)$$

$$\alpha_{\text{burial}}M_S(t_{\text{PI}}) = F_{\text{CaCO}_3,0} + F_{\text{CaSiO}_3,0}. \quad (2.105)$$

The equilibrium hypothesis effectively provides us with constraints linking some parameters and initial conditions. In section 2.2.4 we discuss the choice and calibration of the model parameters. A sensitivity analysis for most parameters is presented in appendix A.5. In section 2.2.4, we provide a set of initial conditions for the model.

Parameters

We start with the parameters of the carbon cycle component. The parameters that control the CO_2 uptake by vegetation (β_L , $k_{A \rightarrow L}$) and the CO_2 uptake by the ocean on short time scales ($k_{U \rightarrow I}$ and $\tilde{k}_{U \rightarrow I}$) are tuned to reproduce historical observations of CO_2 concentrations (Figure 2.4), historical estimations of land and ocean sinks (Figure 2.6) as well as historical and SSP runs of CMIP6 (Figures 2.7 and 2.8). The parameters $k_{I \rightarrow D}$ and $\tilde{k}_{I \rightarrow D}$, which control the ocean carbon uptake on multi-centennial to multi-millennial time scales, are chosen to produce a reasonable fit with the 1 Myr runs of cGENIE (see section 2.3.4). Overall, the calibration to other models is performed qualitatively, without optimising well-defined metrics.

The parameter $\bar{k}_{A \rightarrow U}$ is defined as $\bar{k}_{A \rightarrow U} = A_O \rho k / m_A$. With ocean area $A_O = 361 \times 10^{12} \text{ m}^2$, mean sea water density $\rho = 1026 \text{ kg m}^{-3}$, gas transfer velocity $20 \text{ cm h}^{-1} = 0.2 \cdot 365 \cdot 24 \text{ m yr}^{-1}$ (Yang et al., 2022), and number of moles in the atmosphere $m_A = 1.727 \times 10^{20} \text{ mol}$, we obtain $\bar{k}_{A \rightarrow U} = 4.7 \text{ kg}(\text{mol yr})^{-1}$. The parameter β_L , which controls the amount of CO_2 uptake by vegetation (see eq 2.68), is set to 1.7. This is the same value as for SURFER v2.0, which was calibrated on the outputs of the ZECMIP experiments (Jones et al., 2019). The parameter $k_{A \rightarrow L}$ is set to 0.044, which is an increase by a factor 1.75 compared to SURFER v2.0. This is done to have a better match with historical CO_2 concentrations.

The parameters that dictate the DIC and alkalinity exchanges between the ocean layers are physically determined by the oceanic circulation, hence we expect them to be similar ($k_{i \rightarrow j} \approx \tilde{k}_{i \rightarrow j}$ for $i, j \in \{U, I, D\}$), but we do not require them to be equal because some processes such as the solubility pump can impact DIC and alkalinity independently. Yet, for simplicity, we set both $k_{U \rightarrow I}$ and $\tilde{k}_{U \rightarrow I}$ to 0.13 yr^{-1} , and both $k_{I \rightarrow D}$ and $\tilde{k}_{I \rightarrow D}$ to 0.009 yr^{-1} . Then, $k_{I \rightarrow U}$ and $\tilde{k}_{I \rightarrow U}$, $k_{D \rightarrow I}$ and $\tilde{k}_{D \rightarrow I}$ are computed from equations 2.100-2.103.

We have

$$k_{I \rightarrow U} = \left(P^{\text{CaCO}_3} + P^{\text{org}} - (F_{\text{CaCO}_3,0} + F_{\text{CaSiO}_3,0}) + k_{U \rightarrow I} M_U(t_{\text{PI}}) \right) \frac{1}{M_I(t_{\text{PI}})}, \quad (2.106)$$

$$k_{D \rightarrow I} = \left((1 - \phi_I^{\text{CaCO}_3}) P^{\text{CaCO}_3} + (1 - \phi_I^{\text{org}}) P^{\text{org}} - (F_{\text{CaCO}_3,0} + F_{\text{CaSiO}_3,0}) + k_{I \rightarrow D} M_I(t_{\text{PI}}) \right) \frac{1}{M_D(t_{\text{PI}})}, \quad (2.107)$$

$$\tilde{k}_{I \rightarrow U} = \left(2P^{\text{CaCO}_3} + \sigma_{\text{Alk:DIC}} P^{\text{org}} - 2(F_{\text{CaCO}_3,0} + F_{\text{CaSiO}_3,0}) + \tilde{k}_{U \rightarrow I} \tilde{Q}_U(t_{\text{PI}}) \right) \frac{1}{\tilde{Q}_I(t_{\text{PI}})}, \quad (2.108)$$

$$\tilde{k}_{D \rightarrow I} = \left(2(1 - \phi_I^{\text{CaCO}_3}) P^{\text{CaCO}_3} + \sigma_{\text{Alk:DIC}} (1 - \phi_I^{\text{org}}) P^{\text{org}} - 2(F_{\text{CaCO}_3,0} + F_{\text{CaSiO}_3,0}) + \tilde{k}_{I \rightarrow D} \tilde{Q}_I(t_{\text{PI}}) \right) \frac{1}{\tilde{Q}_D(t_{\text{PI}})}. \quad (2.109)$$

With the choices for the other parameters described hereafter, we obtain $k_{I \rightarrow U} = 0.0383 \text{ yr}^{-1}$, $\tilde{k}_{I \rightarrow U} = 0.0392 \text{ yr}^{-1}$, $k_{D \rightarrow I} = 1.44 \times 10^{-3} \text{ yr}^{-1}$, and $\tilde{k}_{D \rightarrow I} = 1.43 \times 10^{-3} \text{ yr}^{-1}$. Overall, this corresponds to a time scale range of 7.7 - 26.1 yr ($1/k_{U \rightarrow I} - 1/k_{I \rightarrow U}$) for the oceanic carbon exchanges between surface and intermediate layers, and a time scale range of 111.1 - 693.8 yr ($1/k_{I \rightarrow D} - 1/k_{D \rightarrow I}$) for the oceanic carbon exchanges between intermediate and deep layers. This also gives pre-industrial DIC fluxes of 175 PgC yr^{-1} for subduction ($k_{U \rightarrow I} M_U(t_{\text{PI}})$) and 183 PgC yr^{-1} for obduction ($k_{I \rightarrow U} M_I(t_{\text{PI}})$). Although these carbon fluxes are an order of magnitude greater than the fluxes from the biological pumps, they are much less studied, and estimates in the literature are rare. IPCC AR5 gave estimates of 90 PgC yr^{-1} and 101 PgC yr^{-1} for subduction and obduction respectively (Stocker et al., 2013), while IPCC AR6 gave estimates of 264 PgC yr^{-1} and 275 PgC yr^{-1} (Canadell et al., 2021b; Levy et al., 2013).

The CaCO_3 export out of the ocean surface has been estimated between 0.6 and 1.8 PgC yr^{-1} (see supplementary material in Sulpis et al. (2021) and references therein). Sulpis et al. (2021) provide a tighter range of 0.77 to 1.06 PgC yr^{-1} , of which 0.34 to 0.53 PgC yr^{-1} are dissolved in the water column before reaching the sediments. We set P^{CaCO_3} to 1 PgC yr^{-1} , which is also

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the value given in Sarmiento and Gruber (2006) for the open-ocean export at a 100 m depth. We set $\phi_I^{\text{CaCO}_3} = 0.15$ and $\phi_D^{\text{CaCO}_3} = 0.39$. This gives us a total of 0.54 PgC yr^{-1} dissolved in the water column and 0.46 PgC yr^{-1} that rains on the sediments, which is close to estimates from Sulpis et al. (2021) and Sarmiento and Gruber (2006, see Figure 9.1.1).

Estimates of the export of organic carbon out of the euphotic zone range from 4 to 12 PgC yr^{-1} (DeVries and Weber (2017), and references therein). The euphotic zone is the uppermost layer of the ocean that receives sunlight and where photosynthesis can happen. Since the remineralisation of organic matter is quite fast in the water column, estimates of carbon export vary greatly depending on the specific definition of the euphotic zone and its depth. Based on a data-assimilated model, DeVries and Weber (2017) give an estimate of $9.1 \pm 0.2 \text{ PgC yr}^{-1}$ for the organic carbon export out of the euphotic zone and an estimate of 6.7 PgC yr^{-1} for the organic carbon export at 100 m depth. In our model, we set the organic carbon export P^{org} at a 150 m depth to 7 PgC yr^{-1} , which corresponds to the estimate given for the open ocean in Sarmiento and Gruber (2006), and we set $\phi_I^{\text{org}} = 0.72$. Equation 2.33 suggests that for a 106-mole decrease in DIC due to organic matter production, alkalinity will increase by 17 moles (the uptake of 18 moles of H^+ increases alkalinity by 18 moles, while the uptake of HPO_4^{2-} , which is one of the minor bases included in the full definition of alkalinity (eq 2.12), decreases alkalinity by one mole). Here, we follow Sarmiento and Gruber (2006) and set $\sigma_{\text{Alk:DIC}}$ to $-16/117$, meaning that for 1 mole uptake of DIC in organic matter production, alkalinity is increased by $16/117 = 0.14$ moles. Setting $\sigma_{\text{Alk:DIC}} = -16/117$ instead of $\sigma_{\text{Alk:DIC}} = -17/106$ doesn't result in much change, as can be seen from the sensitivity analysis in appendix A.5.

For our parametrisation of CaCO_3 dissolution, we need to calibrate 4 parameters. The value of $F_{\text{diss},0}$ is computed using equilibrium conditions. From equation 2.104, we obtain

$$F_{\text{diss},0} = (1 - \phi_I^{\text{CaCO}_3} - \phi_D^{\text{CaCO}_3})P^{\text{CaCO}_3} - (F_{\text{CaCO}_3,0} + F_{\text{CaSiO}_3,0}) \quad (2.110)$$

With the choice of P^{CaCO_3} , $\phi_I^{\text{CaCO}_3}$, $\phi_D^{\text{CaCO}_3}$ described above, and the choice for $F_{\text{CaCO}_3,0}$, $F_{\text{CaSiO}_3,0}$ described below, we get $F_{\text{diss},0} = 0.33 \text{ PgC yr}^{-1}$. The parameters α_{diss} , β_{diss} and γ_{diss} are obtained based on a parametrisation provided in Archer et al. (1998) for accumulation (rain minus dissolution). The values we use are given in table A.2.

We split the total weathering flux evenly between silicate and carbonate weathering ($F_{\text{CaCO}_3,0} = F_{\text{CaSiO}_3,0}$) and we set them such as to obtain a pre-industrial burial flux of 0.13 PgC yr^{-1} . This is the estimate given in Sarmiento and Gruber (2006). For comparison, the IPCC AR6 WG1 gives an estimate of 0.2 PgC yr^{-1} . From equation 2.98, we have $F_{\text{CaCO}_3,0} + F_{\text{CaSiO}_3,0} = \alpha_{\text{burial}} M_S(t_{\text{PI}}) = F_{\text{burial}}(t_{\text{PI}}) = 0.13 \text{ PgC yr}^{-1}$, which gives us $F_{\text{CaCO}_3,0} = F_{\text{CaSiO}_3,0} = 0.065 \text{ PgC yr}^{-1}$ or $5.42 \text{ Tmol yr}^{-1}$. This implies pre-industrial CO_2 consumption fluxes of $5.42 \text{ Tmol yr}^{-1}$ for carbonate weathering ($F_{\text{CaCO}_3,0}$) and $10.84 \text{ Tmol yr}^{-1}$ for silicate weathering ($2 \times F_{\text{CaSiO}_3,0}$). These values are relatively low compared to literature estimates of $8.6 - 12.3 \text{ Tmol yr}^{-1}$ and $11.7 - 17.9 \text{ Tmol yr}^{-1}$, respectively (Colbourn et al., 2015, and references therein). Nevertheless, they are close to the values from Lord et al. (2016), whose 1 Myr cGENIE runs we use as a calibration target for SURFER v3.0. This being said, setting higher pre-industrial weathering fluxes, and particularly higher carbonate weathering fluxes, has a minimal impact on simulated atmospheric CO_2 concentrations, as demonstrated in Appendix A.5

From these choices, we get $\alpha_{\text{burial}} = (F_{\text{CaCO}_3,0} + F_{\text{CaSiO}_3,0}) / M_S(t_{\text{PI}}) = 0.13 / M_S(t_{\text{PI}})$. Volcanic outgassing is set to $V = F_{\text{CaSiO}_3,0} = 0.065 \text{ PgC yr}^{-1}$ as per equation 2.96. The IPCC estimate is 0.1 PgC yr^{-1} . Following Colbourn et al. (2013), we set k_{Ca} to 0.049 K^{-1} and k_{T} to

$$k_{\text{T}} = \frac{1000 E_a}{RT_0^2}, \quad (2.111)$$

where R is the gas constant (in $\text{J K}^{-1} \text{mol}^{-1}$), T_0 is the global mean pre-industrial temperature (in K) and E_a is the activation energy for dissolution (in kJ mol^{-1}). West et al. (2005) provide an estimate for the activation energy: $E_a = 74 \pm 29 \text{ kJ mol}^{-1}$. With $T_0 = T_{\text{U}}(t_{\text{PI}}) = 288.38 \text{ K}$ (as set in section 2.2.4), this gives a range for k_{T} between 0.065 K^{-1} and 0.149 K^{-1} . We set $k_{\text{T}} = 0.095 \text{ K}^{-1}$. Together with the other parameter choices, this leads to long-term CO_2 atmospheric which fit well the ones simulated by Lord et al. (2016) with cGENIE (see Section 2.3.4). We note that more recent work provided updated estimates for the action energy E_a as low as 22 kJ mol^{-1} , which would imply lower values for k_{T} than the ones we are using (Brantley et al., 2023). On the other hand, using a higher value may help compensate for the absence of a simulated hydrological cycle, which is critical for representing silicate and carbonate weathering fluxes but whose response to anthropogenic emissions is less certain than that of temperature (Kukla et al., 2023; Maher and Chamberlain, 2014).

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Natural methane emissions are set to $E_{\text{nat}}^{\text{CH}_4} = M_{\text{A}}^{\text{CH}_4}(t_{\text{PI}})/\tau_{\text{CH}_4}$. With $M_{\text{A}}^{\text{CH}_4}(t_{\text{PI}})$ chosen as in section 2.2.4 and $\tau_{\text{CH}_4} = 9.5 \text{ yr}$, we get natural emissions of $0.157 \text{ PgC yr}^{-1}$ or $209 \text{ TgCH}_4 \text{ yr}^{-1}$. This is in the range of the top-down estimate of the IPCC ($176\text{--}243 \text{ TgCH}_4 \text{ yr}^{-1}$) and a bit below the bottom-up estimate range ($245\text{--}484 \text{ TgCH}_4 \text{ yr}^{-1}$) (Canadell et al., 2021b).

We now discuss the parameters for the climate component. As in SURFER v2.0, $F_{2\times}$ is set to 3.9 W m^{-2} and β to $1.1143 \text{ W m}^{-2} \text{ }^\circ\text{C}^{-1}$ such that we have an equilibrium climate sensitivity (ECS) equal to $F_{2\times}/\beta = 3.5 \text{ }^\circ\text{C}$. For the parametrisation of the heat exchange between the ocean layers, we first distinguished $\gamma_{\text{U}\rightarrow\text{I}}$ and $\gamma_{\text{I}\rightarrow\text{D}}$, and ended up setting $\gamma_{\text{U}\rightarrow\text{I}} = \gamma_{\text{I}\rightarrow\text{D}} = 0.8357 \text{ W m}^{-2} \text{ }^\circ\text{C}^{-1}$. This is the value chosen for the unique γ in SURFER v2.0. This gives us a transient climate response (TCR) of $1.9 \text{ }^\circ\text{C}$, well in the likely range of $1.4\text{--}2.2 \text{ }^\circ\text{C}$ given by the IPCC (Forster et al., 2021). In section 2.4 we show that this choice gives a good fit to the estimated heat uptake by the deep ocean in the period 1971-2018.

For the contribution of methane to the radiative forcing F_{CH_4} (in W m^{-2}), we use a common parametrisation (Myhre et al., 1998)

$$F_{\text{CH}_4} = 0.036 \sqrt{C_{\text{CH}_4} - C_{\text{CH}_4}(t_{\text{PI}})}, \quad (2.112)$$

where C_{CH_4} is the methane atmospheric concentration in ppb. We have

$$C_{\text{CH}_4} = \frac{M_{\text{A}}^{\text{CH}_4}}{\bar{m}_{\text{C}} m_{\text{A}}} 10^{21}, \quad (2.113)$$

where M^{CH_4} is expressed in PgC , and thus

$$\alpha_{\text{CH}_4} = 0.036 \sqrt{\frac{10^{21}}{\bar{m}_{\text{C}} m_{\text{A}}}}. \quad (2.114)$$

We end the discussion of parameters with the sea level rise component of SURFER. The thermal expansion coefficient (for the density) of a water parcel is defined as $\alpha = \frac{1}{\rho} \frac{\partial \rho}{\partial t} |_{P,S}$, where ρ , P , and S are the density, the pressure and salinity of that water parcel. To obtain the averaged thermal expansion coefficients for each ocean layer α_{U} , α_{I} and α_{D} , we proceed in three steps, as in Williams et al. (2012). First, we use the GLODAPv2.2016b mapped climatology (Lauvset et al., 2016) to compute the thermal expansion coefficient at each ocean point. To do this, we use the International Thermodynamic Equation Of Seawater - 2010 (TEOS-10) and the Python implementation of the GSW

Oceanographic toolbox of TEOS-10 (McDougall and Barker, 2011). Second, we average over each horizontal level of the GLODAPv2.2016b climatology to obtain a vertical profile of the thermal expansion coefficient. Third, we average over each of our defined ocean layers, using the areas of each horizontal level as weights for the horizontally averaged values of the thermal expansion coefficient. We obtain $\alpha_U = 2.20 \times 10^{-4} \text{ K}^{-1}$, $\alpha_I = 1.61 \times 10^{-4} \text{ K}^{-1}$ and $\alpha_D = 1.39 \times 10^{-4} \text{ K}^{-1}$. This is close to the values used in SURFER v2.0 ($\alpha_U = 2.3 \times 10^{-4} \text{ K}^{-1}$ and $\alpha_D = 1.3 \times 10^{-4} \text{ K}^{-1}$). In section 2.4, we show that these expansion coefficients give a good fit to the thermosteric sea level rise on multi-millennial time scales as simulated by the Earth System Model of Intermediate Complexity UVic 2.8. All other parameters of the sea level rise component are as in SURFER v2.0 and are recapped in tables C.1 and A.5.

Initial conditions

As in SURFER v2.0, the initial mass of carbon in atmospheric CO_2 , $M_A(t_{\text{PI}})$, is set such as to have a pre-industrial atmospheric CO_2 concentration of 280 ppm. We have

$$M_A(t_{\text{PI}}) = 280 \times 10^{-18} m_A \bar{m}_C = 580.27 \text{ PgC}. \quad (2.115)$$

The initial mass of carbon in atmospheric CH_4 , $M_A^{\text{CH}_4}(t_{\text{PI}})$, is set such as to have a pre-industrial atmospheric CH_4 concentration of 720 ppb. We have

$$M_A^{\text{CH}_4}(t_{\text{PI}}) = 720 \times 10^{-21} m_A \bar{m}_C = 1.49 \text{ PgC}. \quad (2.116)$$

The initial mass of carbon in land soils and vegetation $M_L(t_{\text{PI}})$ is set to 2200 PgC, as in SURFER v2.0. Hence, we have $M_L^*(t_{\text{PI}}) = M_L(t_{\text{PI}}) = 2200 \text{ PgC}$. The initial mass of carbon in erodible CaCO_3 sediments, $M_S(t_{\text{PI}})$, is set to 1600 PgC, following Archer et al. (1998).

For each ocean layer, we have 17 quantities (T , S , K_0 , K_1 , K_2 , K_w , K_b , $[\text{H}^+]$, $[\text{H}_2\text{CO}_3^*]$, $[\text{HCO}_3^-]$, $[\text{CO}_3^{2-}]$, $[\text{OH}^-]$, $[\text{H}_3\text{BO}_3]$, $[\text{H}(\text{BO})_4^-]$, DIC, Alk, p_{CO_2}) that are linked by a nonlinear system of 13 equations (equations 2.11, 2.13, 2.20-2.25, A.1, A.2-A.5 (+A.7)). Hence, only $17 - 13 = 4$ of these quantities may be set independently. Equation A.7 for the pressure dependence of the dissociation constants isn't counted because it can be combined with equations A.2-A.5. For each ocean layer, we will set initial temperature, salinity, alkalinity and DIC, except for the surface layer where we set $[\text{H}_2\text{CO}_3^*]$ instead of DIC. This is because equilibrium conditions give a constraint on the H_2CO_3^* mass

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(and thus $[\text{H}_2\text{CO}_3^*]$) in the upper layer. Equation 2.99 gives us

$$M'_U(t_{\text{PI}}) = \frac{W_U K_0}{m_A} M_A(t_{\text{PI}}) + \frac{W_U}{k_{A \rightarrow U} m_A} (F_{\text{CaSiO}_3,0} + F_{\text{CaCO}_3,0}). \quad (2.117)$$

We obtain $M'_U(t_{\text{PI}}) = 6.94 \text{ PgC}$ and $[\text{H}_2\text{CO}_3^*]_U(t_{\text{PI}}) = M'_U(t_{\text{PI}})/(W_U \bar{m}_C) \times 10^{18} = 10.43 \text{ } \mu\text{mol kg}^{-1}$. To set the other quantities, we use the GLODAPv2.2016b mapped climatologies (Lauvset et al., 2016), which include climatologies for temperature, salinity, alkalinity, dissolved inorganic carbon, pre-industrial dissolved inorganic carbon and pH, among other biogeochemical variables, which were computed based on data gathered between 1972 and 2013. The dissolved inorganic carbon data is normalized to 2002, and the pH is computed based on temperature, alkalinity and the normalised DIC. We compute the global averages of these data fields over our defined ocean layers by the same averaging method as used for computing the thermal expansion coefficients (see section 2.2.4). The values obtained are in table A.6.

We set the initial (and constant) salinities S_U , S_I , S_D of our ocean layers to the computed averages from the GLODAP data. The temperatures of the ocean layers are defined as

$$T_i(t) = T_{i,0} + \delta T_i(t) \quad \text{for } i \in \{U, I, D\}. \quad (2.118)$$

By definition, the initial conditions for the temperature anomalies δT_i are zero. We set $T_{i,0}$ such that $T_i(t = 2002)$, obtained from experimental runs, is approximately equal to the temperature average computed from the GLODAP data. We set $\tilde{Q}_U(t_{\text{PI}})$, $\tilde{Q}_I(t_{\text{PI}})$, $\tilde{Q}_D(t_{\text{PI}})$, $M_I(t_{\text{PI}})$, and $M_D(t_{\text{PI}})$ based on the computed averages for DIC and Alk, converted to carbon masses with equations 2.14 and 2.15. $M_U(t_{\text{PI}})$ is computed from the fixed $[\text{H}_2\text{CO}_3^*]_U(t_{\text{PI}})$, S_U , $T_{U,0}$, $\text{Alk}_U(t_{\text{PI}})$. Details for the computation are provided in appendix A.4. We obtain $M_U(t_{\text{PI}}) = 1344.78 \text{ PgC}$, which corresponds to $\text{DIC}_U(t_{\text{PI}}) = 2022.08 \text{ } \mu\text{mol kg}^{-1}$. This is only 0.22% off compared to the averaged value for the upper layer obtained from GLODAP. The total dissolved inorganic carbon in the ocean is 37772 PgC, which is close to the 38000 PgC estimate from the IPCC (Canadell et al., 2021b).

For the sea level rise components, as in SURFER v2.0, we set $S_{\text{gl}}(t_{\text{PI}}) = 0$, $V_{\text{GIS}}(t_{\text{PI}}) = 1$, and $V_{\text{AIS}}(t_{\text{PI}}) = 1$. All initial conditions are recapped in table A.7.

In Figure 2.3, we compare the horizontally averaged vertical depth profiles of GLODAP to the vertical profiles of SURFER v3.0 for different model quantities.

The vertical profiles of SURFER are computed by running the model from 1750 to 2002, forced with historical CH_4 and CO_2 emissions, and starting from the initial conditions described above. We observe that the chosen initial conditions produce a model state in 2002 that matches the GLODAP data.

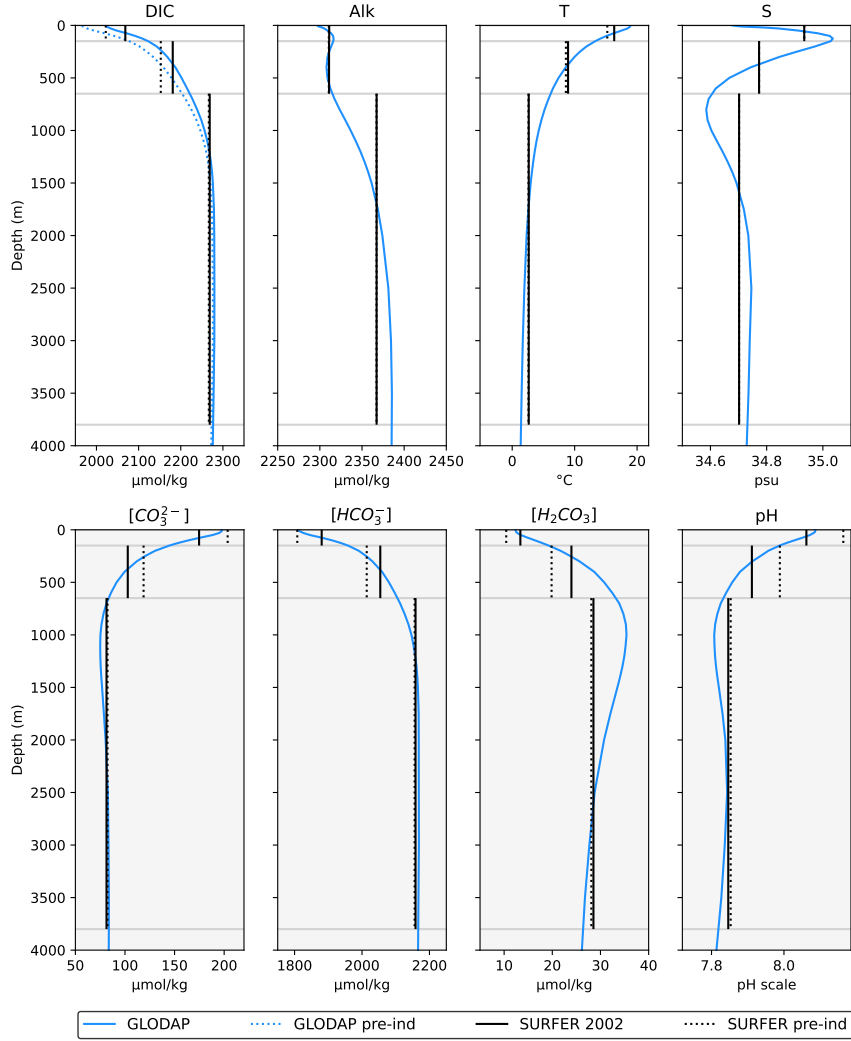


Fig. 2.3 Horizontally averaged quantities from the GLODAPv2.2016b mapped climatologies (Lauvset et al., 2016) compared to initial and simulated values by SURFER v3.0. The carbonate species are not provided in the GLODAP climatologies. We computed their values at each ocean point based on the climatologies of DIC, Alk, temperature and salinity, and then averaged them horizontally. Details for the computation of the carbonates species are in appendix A.4.

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2.2.5 Numerics

The model is implemented in Python 3.0. using the library `solve_ivp` with the integration method LSODA. The LSODA method has an automatic stiffness detection and switches accordingly between an Adams and BDF method (Petzold, 1983). The local error estimates are kept below $atol + rtol \times abs(y)$ where $atol$ and $rtol$ are parameters that control the relative and absolute accuracy and where y is a model variable. By default, we set $atol$ to 10^{-3} for the variables $M_A^{CH_4}$, M_S , δT_U , δT_L , δT_D , S_{gl} , V_{GIS} , V_{AIS} , and we set $atol$ to 10^{-6} for the other variables. The reason for this difference is that the variables in the first group can have small or near zero values, meaning that $atol$ will dominate the local error estimate. If it is too small, the solver takes too many steps and is slow. We set $rtol$ to 10^{-6} for all variables. The code is compiled with Numba and the model runs fast. When forced with CO_2 and CH_4 emissions of a given SSP scenario, runs of 10^3 to 10^6 years take typically around 60 milliseconds on a laptop with processor Intel® Core™ i5-10210U CPU @ 1.60GHz $\times$ 8. The run time is not a linear function of simulated time because the LSODA method uses an adaptive time step.

The exact version of SURFER used to produce the results showed in this chapter is archived on Zenodo (<https://zenodo.org/records/12774163>, Couplet et al., 2024a), as is the input data to run the model and most of the data to produce the plots. The code of SURFER is licensed under MIT license.

2.3 Numerical results and comparisons

In this following section, we test SURFER v3.0 and show that it is an adequate representation of the real climate system. We show that it reproduces well-known dynamics of the carbon cycle and we compare it with outputs of other models over a large range of time scales.

2.3.1 Historical period

We show here that SURFER v3.0 is able to reproduce the measured historical CO_2 and CH_4 concentrations, and the estimated land and ocean carbon sinks. We perform a historical run by starting SURFER in 1750 with the parameters and initial conditions described in section 2.2.4. We force the model with fossil

and land-use emissions of CO_2 and CH_4 . Emissions from other greenhouse gases such as nitrous oxide (N_2O), ozone (O_3), halogenated gases and aerosols are not taken into account. Figures 2.4 and 2.5 show the historical CO_2 and CH_4 concentrations as simulated by SURFER v3.0 compared to the measurements from Köhler et al. (2017). SURFER v3.0 is in good agreement with the historical CO_2 observations, with a difference of at most ~ 6 ppm, which is better than SURFER v2.0. For methane concentrations, SURFER v3.0 is in relatively good agreement with the historical observations, although it doesn't capture well the apparent stabilisation in the 2000s. The cause for this stabilisation is not totally clear (Turner et al., 2019). Main hypotheses include a decline in fossil emissions (Chandra et al., 2024), and a shortening of the lifetime of atmospheric CH_4 , due to increasing concentrations of the hydroxyl radical caused by changes in emissions of other gases such as N_2O and carbon monoxide (CO) (Skeie et al., 2023). These processes are not modelled in SURFER v3.0, where the atmospheric lifetime of CH_4 is kept constant.

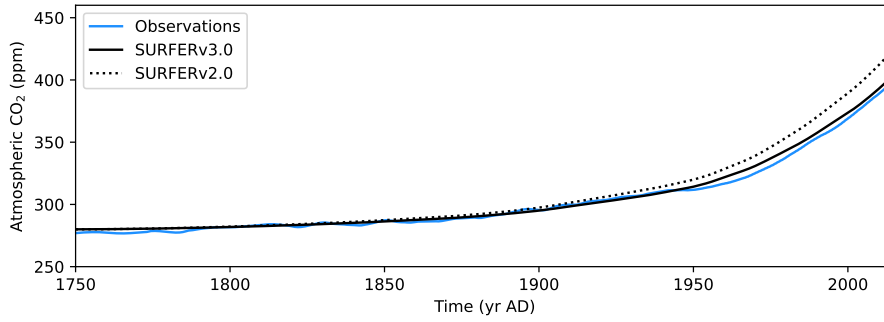


Fig. 2.4 *Historical atmospheric CO_2 concentrations. Comparison between (smoothed) observations (Köhler et al., 2017) and outputs from SURFER v2.0 and SURFER v3.0 when forced with historical emissions.*

In Figure 2.6, we compare the partitioning of CO_2 emissions in the atmosphere, land and ocean reservoirs with the estimates from the Global Carbon Budget (GCB) (Friedlingstein et al., 2022). Fossil and land-use CO_2 emissions used in SURFER v3.0 up to the year 1990 are the estimates provided by the GCB, and after that, we start using the emission values provided for the SSP scenarios. These are slightly different than the estimates from the GCB (see appendix A.2), which explains the small mismatch visible in Figure 2.6. In SURFER, we compute the ocean sink as $S_{\text{ocean}} = F_{\text{A} \rightarrow \text{U}} - F_{\text{A} \rightarrow \text{U}}(t_{\text{PI}})$, following the definition of Hauck et al. (2020), the land sink as $S_{\text{land}} = F_{\text{A} \rightarrow \text{L}} - F_{\text{A} \rightarrow \text{L}}(t_{\text{PI}}) = F_{\text{A} \rightarrow \text{L}}$, and the atmospheric

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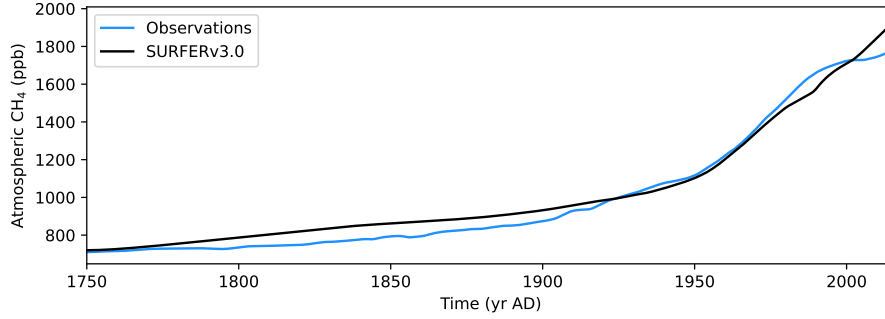


Fig. 2.5 Historical atmospheric CH_4 concentrations. Comparison between (smoothed) observations (Köhler et al., 2017) and outputs from SURFER v3.0 when forced with historical emissions.

growth as $\frac{dM_A}{dt}$. These quantities simulated by SURFER v3.0 for the historical period are very close to the estimates from the GCB. For the years 2000 to 2010, the GCB gives a mean estimate of $2.3 \pm 0.4 \text{ PgC yr}^{-1}$ for the ocean sink, $2.7 \pm 0.5 \text{ PgC yr}^{-1}$ for the land sink, and $4 \pm 0.02 \text{ PgC yr}^{-1}$ of atmospheric growth. Values simulated in SURFER v3.0 are respectively 2.16 PgC yr^{-1} , 3.05 PgC yr^{-1} and 3.96 PgC yr^{-1} , with only the atmospheric growth being just below the GCB estimated range. The cumulative budgets are also very similar: of the total amount of emissions in the period 1850-2014, the GCB estimates that around $26 \pm 5\%$ are absorbed by the ocean, $31 \pm 7\%$ are absorbed by the land and $40 \pm 1\%$ stay in the atmosphere, while for SURFER v3.0, those numbers are respectively 24%, 37% and 41%. In the GCB, there is a cumulative budget imbalance of 15 PgC for the years 1850-2014, which arises from errors in independent estimates of emissions and sinks, as well as from missing terms in the budget computation. In SURFER v3.0, however, carbon is explicitly conserved and the budget imbalance (B_{im}) only results from the definition of the sinks, which don't capture processes such as methane oxidation or changes in carbonate and silicate weathering fluxes. Indeed, we have

$$B_{\text{im}} = E_{\text{fossil}}^{\text{CO}_2} + E_{\text{land-use}}^{\text{CO}_2} - \left(\frac{dM_A}{dt} + S_{\text{ocean}} + S_{\text{land}} \right) \quad (2.119)$$

$$= -V + F_{A \rightarrow U}(t_{\text{PI}}) + F_{\text{weathering}} - \left(F_{\text{CH}_4, \text{ox}} - E_{\text{natural}}^{\text{CH}_4} \right) \quad (2.120)$$

$$= \left(F_{\text{weathering}} - F_{\text{weathering}}(t_{\text{PI}}) \right) - \left(F_{\text{CH}_4, \text{ox}} - E_{\text{natural}}^{\text{CH}_4} \right), \quad (2.121)$$

and the cumulative budget imbalance for the years 1850-2014 is -15 PgC, with a contribution of +1 PgC from increased weathering fluxes and -16 PgC from methane oxidation.

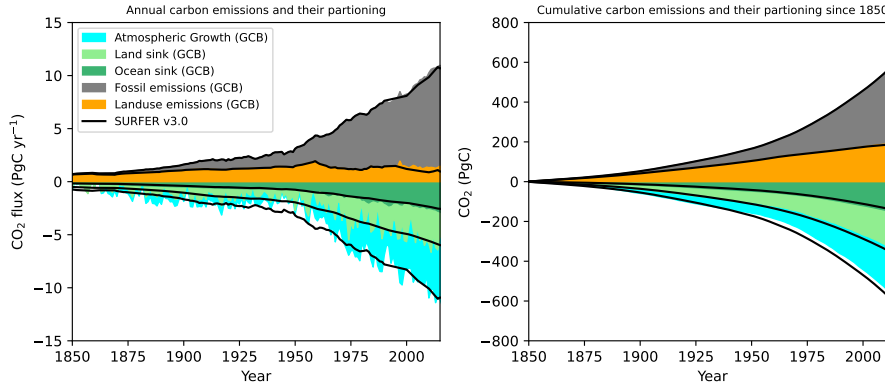


Fig. 2.6 Partitioning of fossil and land-use CO₂ emissions in the atmosphere, ocean and land in SURFER v3.0 (black lines) compared to estimates from the Global Carbon Budget (in colour) (GCB, Friedlingstein et al., 2022).

2.3.2 CMIP6 projections

We now compare SURFER v3.0 to the CMIP6 ensemble for the SSP1-2.6 and SSP3-7.0 scenarios. As for the historical runs in section 2.3.1, SURFER v3.0 is forced with CO₂ and CH₄ fossil and land-use emissions, but no other greenhouse gases nor aerosols. Runs are started in 1750 and results for atmospheric CO₂, temperature, surface ocean pH, ocean carbon uptake and land carbon uptake are plotted in Figure 2.7. Additionally, we compare with outputs from SURFER v2.0, forced with the total CO₂ emissions and run with the parameters and initial conditions described in Martínez Montero et al. (2022).

As already shown in Figure 2.4, SURFER v3.0 reproduces well the historical CO₂, and for the SPP scenario projections, falls within the lower range of the CMIP6 model ensemble. SURFER v3.0 can simulate a global mean temperature anomaly that generally remains within the CMIP6 range, considering only the effects of CO₂ and methane. This is because the contributions from the other major drivers of temperature changes such as nitrous oxide (N₂O), ozone (O₃), halogenated gases and aerosols approximately cancel each other (Forster et al., 2021, see Figure 7.8). However, between 1960 and 2015, the cooling effect of aerosols from anthropogenic and volcanic sources was likely more significant,

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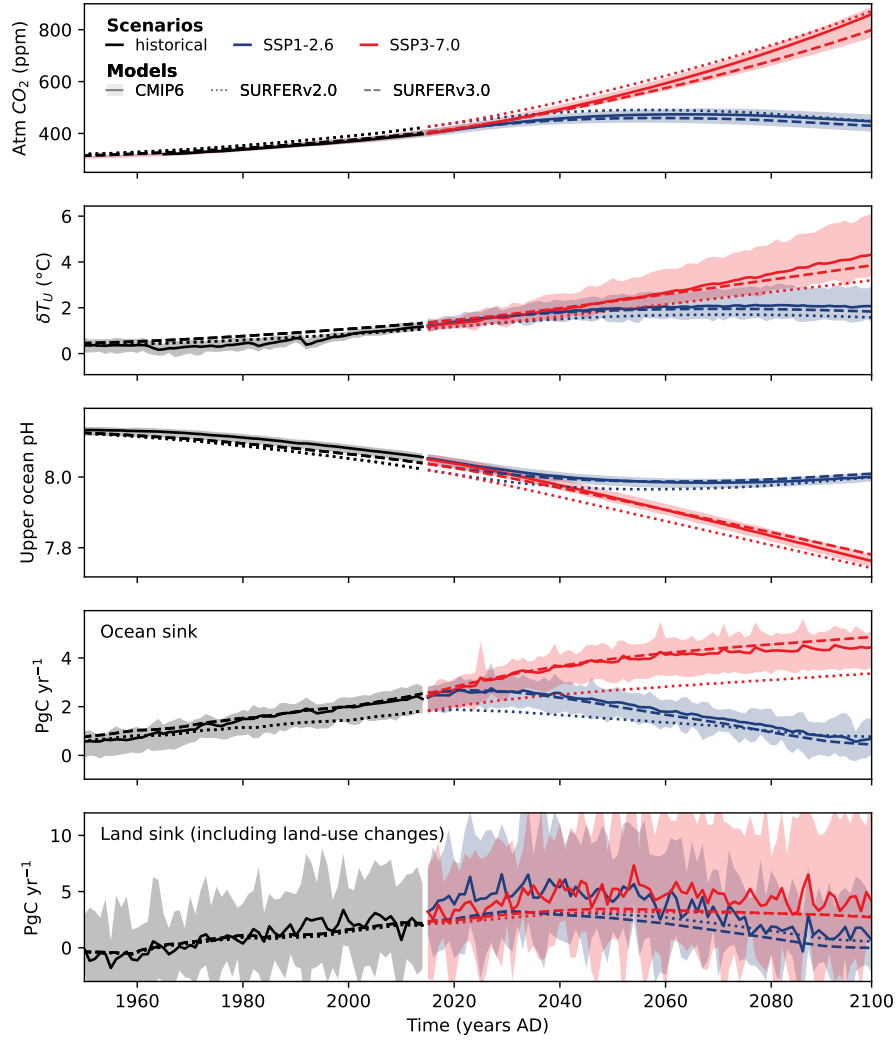


Fig. 2.7 Comparison between SURFER v3.0, SURFER v2.0 and the CMIP6 model ensemble mean for the historical period (1750-2014) and the near future (2015-2100) under SSP1-2.6 and SSP3-7.0 scenarios. The CMIP6 data is from concentration driven runs, except the atmospheric CO_2 which comes from emission driven runs. The ocean sink is computed in SURFER as $F_{A \rightarrow U} - F_{A \rightarrow U}(t_{PI})$. The land sink is taken here as the Net Biome Productivity (NBP) which includes land-use fluxes. In SURFER, it is computed as $F_{A \rightarrow U} - E_{land-use}^{\text{CO}_2} - E_{land-use}^{\text{CH}_4}$.

and without accounting for this, SURFER v3.0 simulates temperatures slightly above the CMIP6 range.

Surface pH as simulated by SURFER v3.0 generally aligns with the CMIP6 range for both the historical period and SSP projections, an improvement over

SURFER v2.0 which showed too rapid ocean acidification. This improvement is primarily due to the addition of a new intermediate layer in SURFER v3.0, which facilitates faster carbon transfer out of the upper ocean layer, thereby slowing surface acidification. This enhanced carbon transfer to intermediate and deep ocean layers also allows the ocean to absorb CO_2 more efficiently. As a result, the ocean carbon uptake in SURFER v3.0 now falls within the CMIP6 model range. The land carbon uptake in SURFER v3.0 and v2.0 are very similar, and both are in the range of CMIP6 models, which is quite large and demonstrates a higher uncertainty.

In Figure 2.8, we compare the land and ocean sinks of SURFER to four CMIP6 models and one EMIC that have been run to the year 2300 under the SSP1-2.6, SSP3-4.3, and SSP5-8.5 scenarios. We observe that SURFERv3.0 remains within the range of CMIP6-class models even for these longer time scales. For all three scenarios, the land sink is expected to become negative at some point, indicating that the land reservoir will release some of the carbon it had previously absorbed (Canadell et al., 2021b; Tokarska et al., 2016; Zickfeld et al., 2013). For the SSP1-2.6 and SSP-3.4 scenarios, this negative land sink in CMIP6 models is attributed to the land carbon-concentration feedback: as CO_2 concentrations decrease after strong negative emissions, vegetation releases carbon. For the SSP5-8.5 scenario, the negative land sink is rather due to a stronger land carbon-climate feedback, where warming leads to a release CO_2 from the land reservoir, for example through increased decomposition rates (Tokarska et al., 2016). In SURFER, the parametrisation of the atmosphere-to-land flux, ($F_{A \rightarrow L}$) depends on the atmospheric CO_2 concentration (M_A) but not on temperature, effectively including only a carbon-concentration feedback. This explains why, for the SSP5-8.5 scenario, the land sink in SURFER only becomes slightly negative around 2250, when the atmospheric CO_2 concentrations begin to decline. Despite this, the land sink from SURFER remains mostly in the range of the other models, which is quite large and reflects the large uncertainty in processes related to the terrestrial biosphere.

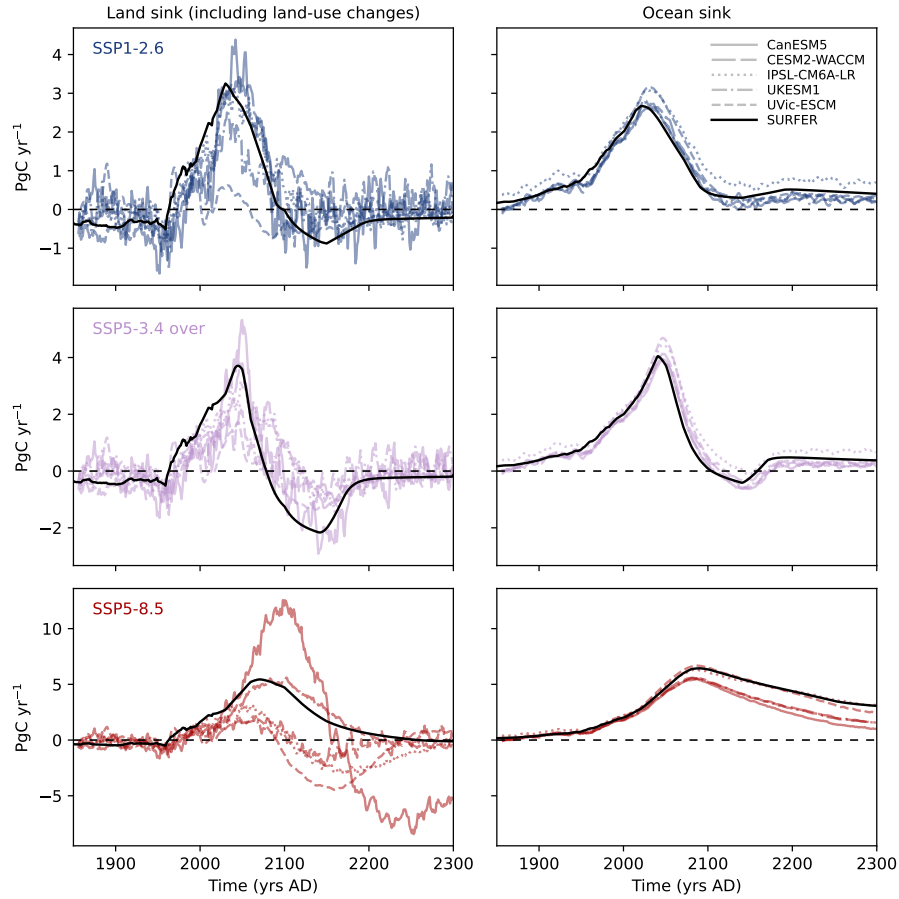


Fig. 2.8 Comparison of atmosphere-to-ocean and atmosphere-to-land carbon fluxes as simulated by SURFER v3.0 and CMIP6 models for the historical period (1750-2014) and the future (2015-2300) under SSP1-2.6, SSP5-3.4 over, and SSP5-8.5 scenarios. The ocean sink is computed in SURFER as $F_{A \rightarrow U} - F_{A \rightarrow U}(t_{PI})$. The land sink is taken here as the Net Biome Productivity (NBP) which includes land-use fluxes. In SURFER, it is computed as $F_{A \rightarrow U} - E_{land-use}^{CO_2} - E_{land-use}^{CH_4}$.

2.3.3 Long Tail Model Inter-comparison Project (LTMIP)

In the previous sections, we have seen that SURFER v3.0 can reproduce the historical record and outputs from CMIP6-class models for projections up to 2300. Here, we focus on longer time scales and compare SURFER v3.0 with results from the LTMIP (Long Tail Model Inter-comparison Project, Archer et al., 2009b). In these experiments, several models were used to assess the CO₂ draw-down from the atmosphere for 10000 years, after emissions pulses of 1000 PgC and 5000 PgC. For each emission pulse, 5 experiments are performed with different physical processes progressively included to assess their impact on atmospheric CO₂ uptake. These experiences are named with a combination of letters indicating the processes included: climate feedbacks (C), sediments (S), weathering (W), and vegetation (V). We reproduce these experiments with SURFER v3.0 by successively reducing the number of active processes. For, the CSWV experiment, we use the standard version of SURFER v3.0. For the CSW experiment, we set $k_{A \rightarrow L} = 0$, so that vegetation is kept constant and has no influence on carbon uptake ($F_{A \rightarrow L} = 0$). For the CS experiment, we additionally keep the weathering fluxes F_{CaCO_3} and F_{CaSiO_3} constant and equal to their pre-industrial values, thus eliminating weathering feedbacks. For the C experiment, we further keep the accumulation and burial fluxes constant and equal to their pre-industrial values, thus effectively eliminating interactions with the sediments. Finally for the baseline experiment, on top of all the modifications described above, we keep the solubility and dissociation constants constant. In this last case, SURFER v3.0 is very similar to SURFER v2.0.

Results for these five experiments are visible in Figure 2.9 for the 1000 PgC pulse, and in Figure 2.10 for the 5000 PgC pulse. Overall, SURFER v3.0 falls within the range of other models, except in the following cases: the 5000 PgC baseline experiment after 1000 years, the 5000 PgC C experiment between years 1000 and 5000, and the CSWV experiments after year 1000, where SURFER v3.0 simulates slightly lower atmospheric CO₂ levels than the other models. For the baseline experiment, SURFER v2.0 doesn't absorb CO₂ from the atmosphere fast enough in the first thousand years after the emission pulse. As already mentioned, this is improved in SURFER v3.0 thanks to the addition of a third oceanic layer at intermediate depth.

We can define and quantify the climate, sediment, weathering, and vegetation feedbacks by taking the difference in simulated atmospheric CO₂ between consecutive experiments (C-baseline, CS-C, CSW-CS, and CSWV-CSW). Re-

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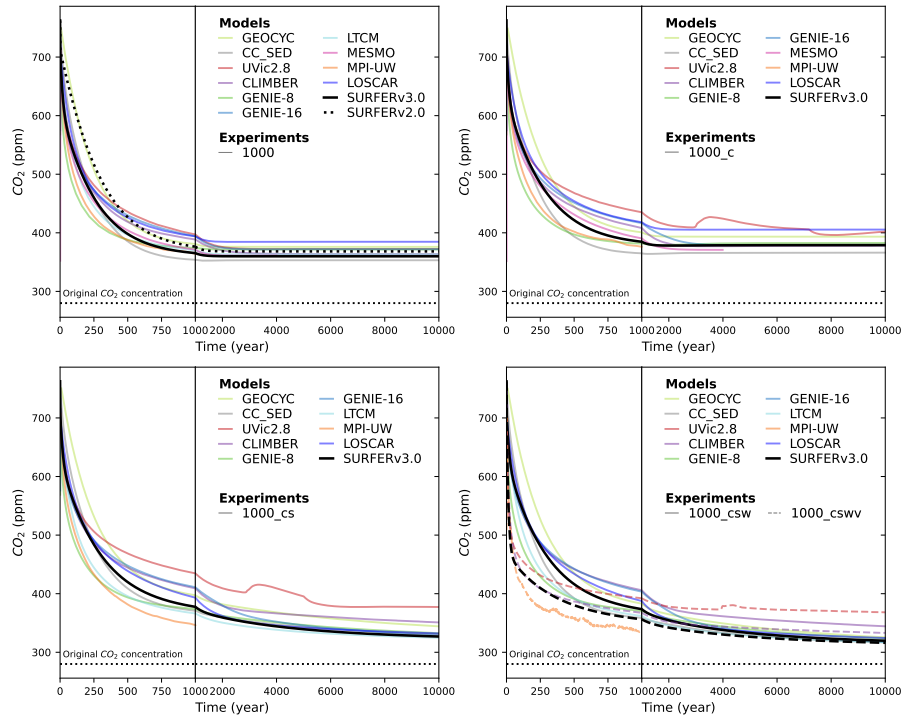


Fig. 2.9 Atmospheric CO_2 simulated by different models and SURFER v3.0 after a 1000 PgC emission pulse for the LTMIP experiments. The five experiments are the baseline (ocean only) experiment, the climate (C), the climate plus sediments (CS), the climate plus sediments plus weathering (CSW) and the climate plus sediments plus weathering plus vegetation (CSWV) experiments. We have added the results of the LOSCAR model (Zeebe, 2012) which wasn't part of the original LTMIP publication (Archer et al., 2009b).

sults are plotted in Figure 2.11 for the 1000 PgC pulse, and in Figure 2.12 for the 5000 PgC pulse. Not all experiments were performed for each model so feedbacks can't always be computed. All experiments are only available for CLIMBER and SURFER v3.0. Overall, the feedbacks in SURFER v3.0 fall within the range of the other models, demonstrating that the associated processes are reasonably well simulated by SURFER. For 5000 PgC pulse experiments, the climate feedback in SURFER v3.0 is very similar to the LOSCAR and GEOCYC models but quite different to the other models. This is probably explained by SURFER V3.0, as well as LOSCAR and GEOCYC, all missing a dynamic ocean circulation and hence feedbacks associated with temperature-induced circulation changes. The sediment feedback in SURFER v3.0 for the 1000 PgC is in the higher range (more negative) of the other models, which is consistent with the dissolution flux being generally larger (accumulation more negative)

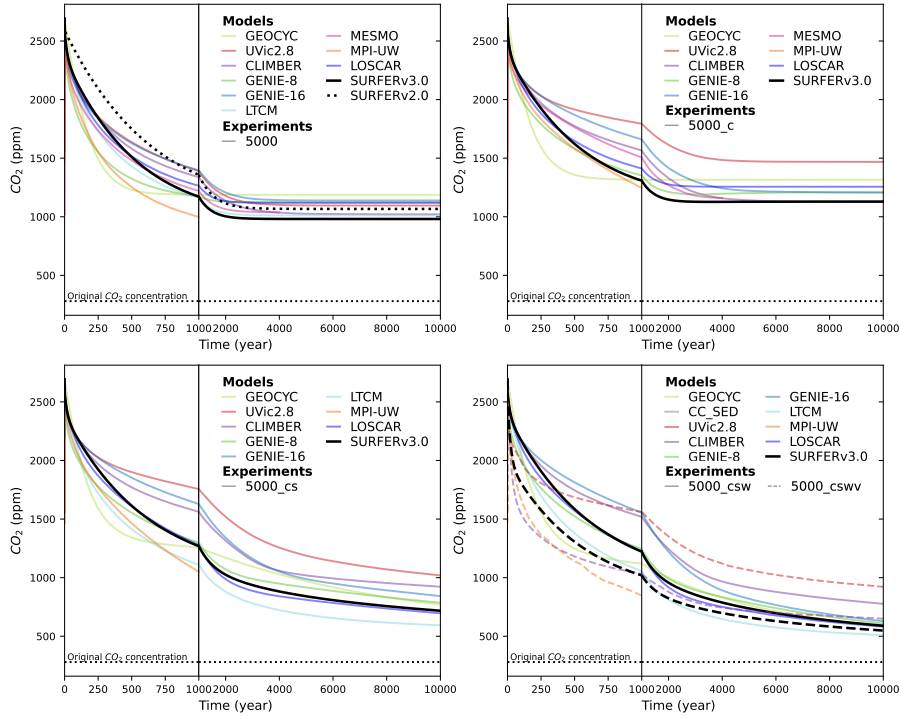


Fig. 2.10 Atmospheric CO_2 simulated by different models and SURFER v3.0 after a 5000 PgC emission pulse for the LTMIP experiments. The five experiments are the baseline (ocean only) experiment, the climate (C), the climate plus sediments (CS), the climate plus sediments plus weathering (CSW) and the climate plus sediments plus weathering plus vegetation (CSWV) experiments. We have added the results of the LOSCAR model (Zeebe, 2012) which wasn't part of the original LTMIP publication (Archer et al., 2009b).

than in the other models (see Figure 2.13). For the 5000 PgC, the sediment feedback in SURFER v3.0 is in the mid-to-lower range of the other models, despite the dissolution flux still being in the higher range. In general, other than oceanic invasion, vegetation has the biggest impact on CO_2 uptake before the year 1000, while sediments have the biggest impact between the year 1000 and 10000.

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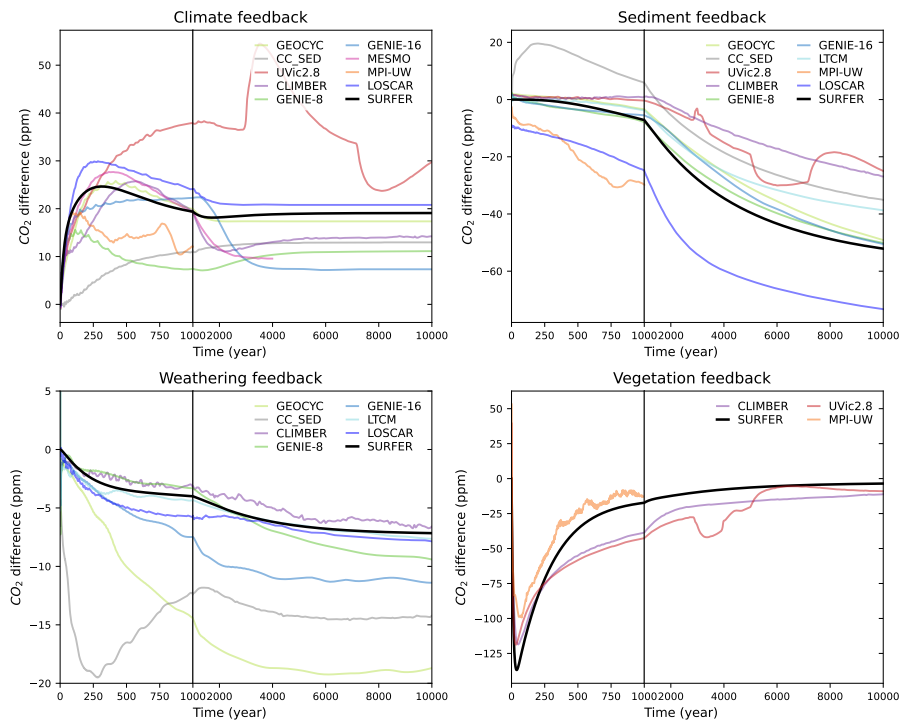


Fig. 2.11 Impacts of the climate, sediments, weathering, and vegetation feedbacks on the atmospheric CO₂ concentration, after a 1000 PgC emission pulse. Here, a feedback is defined as the difference in CO₂ concentration resulting from the addition of the associated process.

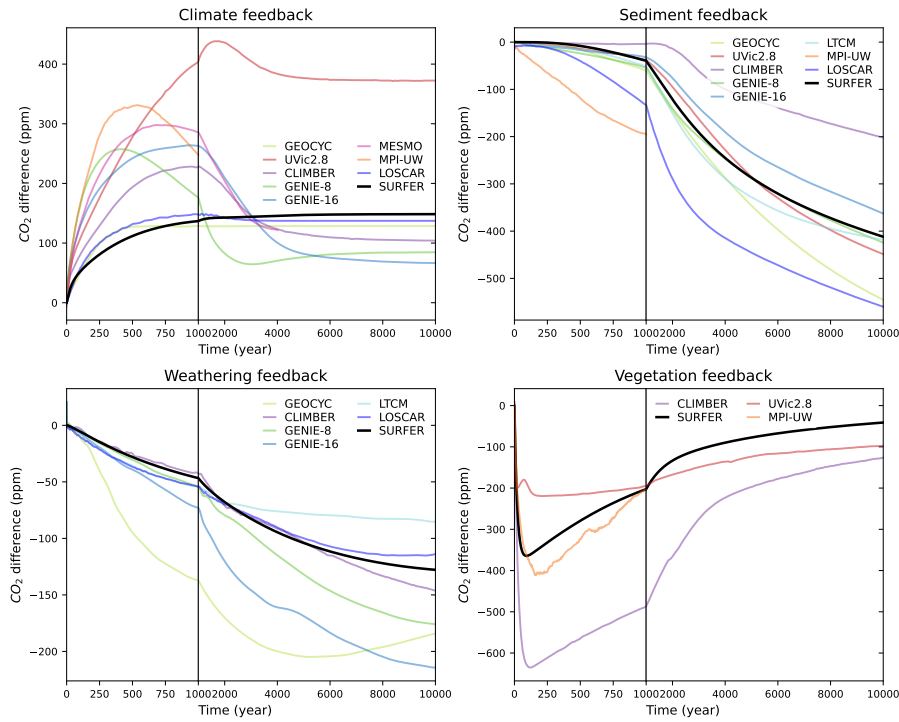


Fig. 2.12 Impacts of the climate, sediments, weathering, and vegetation feedbacks on the atmospheric CO_2 concentration, after a 5000 PgC emission pulse. Here, a feedback is defined as the difference in CO_2 concentration resulting from the addition of the associated process.

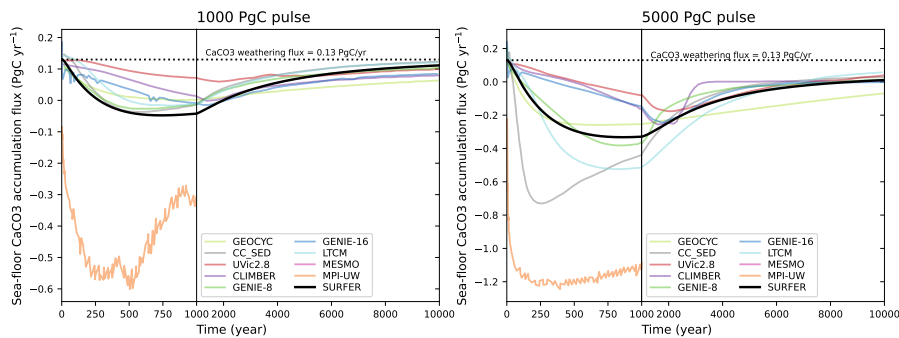


Fig. 2.13 CaCO_3 accumulation fluxes in sediments for the different models. Negative values indicate net dissolution of CaCO_3 .

2.3.4 cGENIE

Only a few models of intermediate complexity have been run for 100 kyr or more to investigate the carbon cycle's response to (anthropogenic) CO₂ emissions. Some examples include cGENIE (Colbourn et al., 2013; Colbourn et al., 2015; Lord et al., 2016), and CLIMBER-X (Kaufhold et al., 2024a). Here, we compare SURFER v3.0 with the 1 Myr runs performed with the cGENIE model of intermediate complexity (Lord et al., 2016). This model comprises a 2-D Energy-Moisture Balance atmosphere, a 3-D frictional geostrophic ocean circulation model, and a representation of the global carbon cycle, with ocean cycling of DIC, alkalinity, and a nutrient (PO₄), CaCO₃ marine sediments, and terrestrial weathering (Colbourn et al., 2013; Edwards and Marsh, 2005; Ridgwell et al., 2007; Ridgwell and Hargreaves, 2007). For the runs presented here (Lord et al., 2016), cGENIE was used without the terrestrial biosphere module and its associated carbon fluxes. The model had 8 ocean levels, with the surface layer being 175 meters deep, comparable to the surface layer in SURFER v3.0.

We perform equivalent runs in SURFER v3.0 with $k_{A \rightarrow L} = 0$ to neglect the role of vegetation. Results are plotted in Figures 2.14 and 2.15 for atmospheric CO₂, global mean temperature, ocean surface pH, ocean surface calcite saturation state and CaCO₃ content in sediments. Ocean surface calcite saturation state, Ω_U , is defined as

$$\Omega_U = \frac{[\text{CO}_3^{2-}]_U [\text{Ca}^{2+}]_U}{K_{\text{sp}}^{\text{CaCO}_3}}. \quad (2.122)$$

The solubility product $K_{\text{sp}}^{\text{CaCO}_3} = [\text{CO}_3^{2-}]_{\text{sat}} [\text{Ca}^{2+}]_{\text{sat}}$ depends on salinity, temperature and pressure (Millero, 1995; Mucci, 1983). For the computation of Ω_U , we use parametrisations of $K_{\text{sp}}^{\text{CaCO}_3}$ described in appendix A.3, and we assume that $[\text{Ca}^{2+}]_U$ is constant and equal to 0.01028 mol kg⁻¹ (Sarmiento and Gruber, 2006).

Overall, SURFER v3.0 reproduces well the behaviour of cGENIE. For all emission pulses, the relative difference in simulated atmospheric CO₂ with cGENIE doesn't exceed 18%, is lower than 8% after 1000 yrs and below 5% after 50 kyr. These are smaller differences than those between models for the LTMIP experiments (see Figures 2.9 and 2.10). This good agreement for millennial and longer time scales was expected, as SURFER v3.0 was qualitatively tuned to cGENIE's long-term atmospheric CO₂ output. The agreement in ocean surface pH is also very strong, with absolute differences below 0.06 pH units

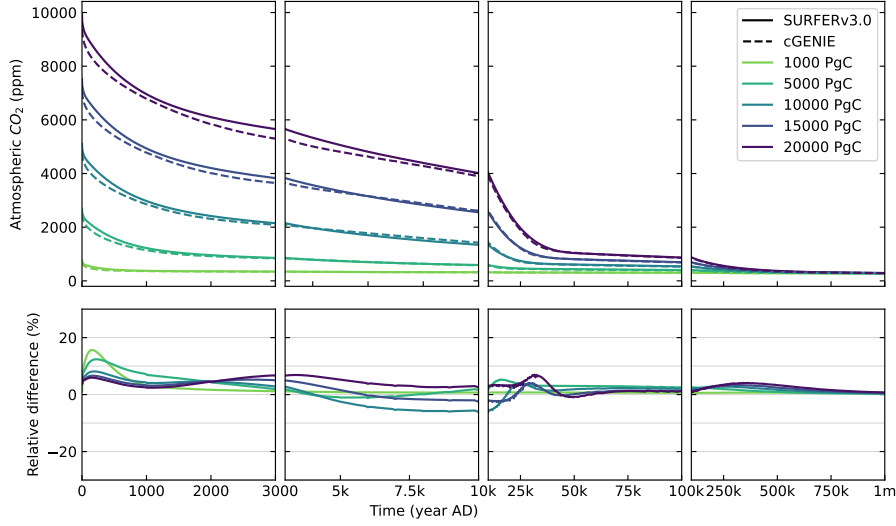


Fig. 2.14 Atmospheric CO₂ concentrations simulated by cGENIE and SURFER v3.0 after emissions pulses of different sizes. The bottom plot shows the relative difference between SURFER v3.0 and cGENIE.

after 5 years, below 0.04 pH units after 1000 years, and below 0.02 pH units after 50 kyr, corresponding to relative difference below 1% after 5 years for all emissions pulses. Because the carbon exchanges between the atmosphere and the surface ocean reach equilibrium relatively fast, the good agreement for ocean surface pH directly results from the good agreement for atmospheric CO₂ concentrations. Regarding temperatures, peak warming occurs later in SURFER v3.0 than in cGENIE, primarily because SURFER models only ocean temperatures, leading to slower global warming compared to cGENIE, which also accounts for the thermal balance of the continents.

After 1 Myr, the state is almost back to equilibrium in SURFER v3.0, with atmospheric CO₂ concentrations ranging from 280.68 ppm for the 1000 PgC emission pulse, to 292.08 ppm for the 20000 PgC pulse. Carbon is removed from the atmosphere through a range of processes. First, atmospheric CO₂ dissolves in the upper ocean layer following the reaction



which is equivalent to reactions 2.17-2.19. This causes a decrease in ocean surface pH (acidification) and consumes CO_3^{2-} , which results in a decrease of the surface calcite saturation state. Both these effects are observable in

2 | The SURFER model

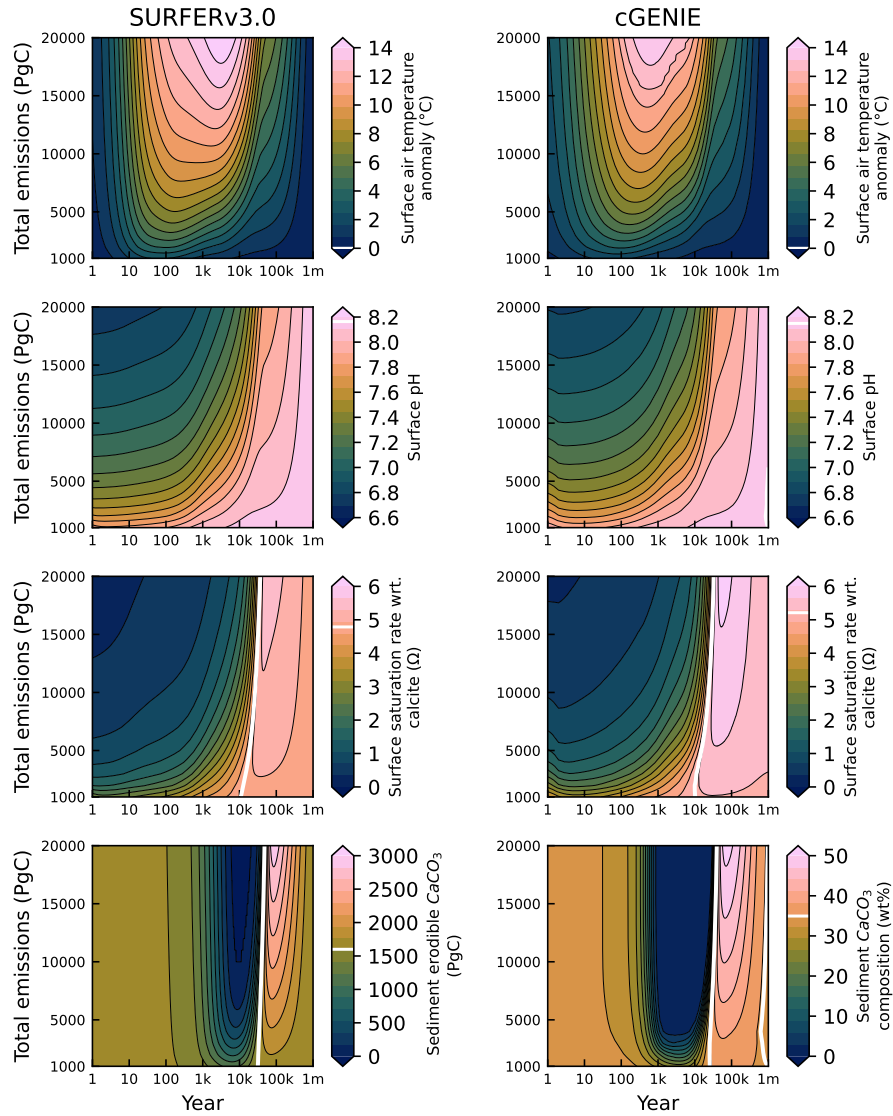


Fig. 2.15 Global mean temperature anomaly, surface ocean pH, surface ocean saturation state with respect to calcite, and CaCO_3 sediments content in SURFER v3.0 and cGENIE after emissions pulses ranging from 1000 PgC to 20000 PgC. White lines indicate pre-industrial values used in each model. Note : SURFER v3.0 and cGENIE use different units for the CaCO_3 sediments content. SURFER uses the total erodible CaCO_3 mass, while cGENIE uses the mean dry weight fraction (mass of CaCO_3 divided by the mass of CaCO_3 and non erodible material in sediments). Although these two quantities are strongly correlated, they do not necessarily depend linearly on one another, complicating direct comparisons.

SURFER v3.0 and cGENIE. On centennial to millennial timescales, the CO_3^{2-} anomaly mixes into the ocean interior and deep waters become less saturated, causing an increase in the dissolution of deep-sea CaCO_3 sediments and a release of carbonate ions. Some of these carbonate ions can then react with CO_2 (reaction 2.123), leading to further oceanic CO_2 uptake. This process is called sea-floor neutralisation, as it is the dissolution of previously deposited deep-sea sediments that allows the neutralisation of atmospheric CO_2 (Archer et al., 1997; Archer et al., 1998). Moreover, the increased dissolution of CaCO_3 sediments compared to the pre-industrial state creates an imbalance between the alkalinity input to the ocean by weathering and the alkalinity output by accumulation. This replenishes the ocean CO_3^{2-} concentration and the erodible CaCO_3 sediments stock while leading to a further uptake of atmospheric CO_2 . This second process is called terrestrial neutralisation, as it is the (imbalanced) dissolution of carbonate and silicate rocks on land that neutralise the atmospheric CO_2 (Archer et al., 1997; Archer et al., 1998; Ridgwell and Hargreaves, 2007). We observe an overshoot in surface calcite saturation state and deep sea ocean sediment content compared to the pre-industrial situation in both SURFER v3.0 and cGENIE because of increased weathering rates due to warming. The extra CaCO_3 in the sediments will eventually be buried, leading to a permanent transfer of carbon to the geological reservoir.

2.4 Sea level rise and importance of long time scale processes

So far, we have focused primarily on the carbon cycle, as the additions to SURFER v3.0 are related to it. In this section, we examine sea-level rise (SLR), which is computed as the sum of four contributions: thermosteric (thermal expansion), glaciers, Greenland and Antarctica. The parametrisation for thermosteric sea level rise in SURFER v3.0 is essentially the same as in SURFER v2.0, with the key differences being the use of three 3 ocean layers instead of two, and the use of new thermal expansion coefficients. In subsection 2.4.1, we verify that these changes still provide a reasonable approximation of thermosteric SLR. The parametrisations for glaciers, Greenland and Antarctica remain unchanged between SURFER v2.0 and SURFER v3.0, so any difference in SLR contributions under a given forcing scenario results only from differences in simulated temperatures. We investigate the SLR contribution from the ice sheets in subsection 2.4.2.

2.4.1 Thermosteric sea level rise and ocean heat content

Heat transfer in SURFER v3.0's ocean is controlled by the parameters $\gamma_{U \rightarrow I}$ and $\gamma_{I \rightarrow D}$. The heat accumulated in ocean layers, along with the corresponding temperature increases, determine thermosteric sea level rise. To ensure that our chosen values for $\gamma_{U \rightarrow I}$, $\gamma_{I \rightarrow D}$, and for the thermal expansion coefficients are reasonable, we compare the ocean heat content and thermosteric sea level rise in SURFER v3.0 with IPCC estimates for 1971-2018 (Figure 2.16).

SURFER v3.0 simulates significantly higher ocean heat content above 700 m (layers U+I) than the IPCC estimates, resulting in a larger thermosteric sea level rise. This discrepancy arises for two reasons. First, in SURFER, all energy imbalance is absorbed by the ocean, with no energy allocated to warming the land and atmosphere or for melting glaciers and ice sheets. Consequently, the ocean warms more than it should. Second, and more importantly, SURFER v3.0 doesn't account for faster and larger land temperature increases, which causes the global mean temperature rise to be higher when averaged over land and oceans. In other words, SURFER assumes that the global mean temperature is equivalent to the ocean's mean surface temperature. As a result, SURFER needs more energy to reproduce observed global mean temperatures, and overestimates surface ocean temperatures. For the ocean below 700 m (layer D in SURFER), the ocean heat content and the thermosteric sea level

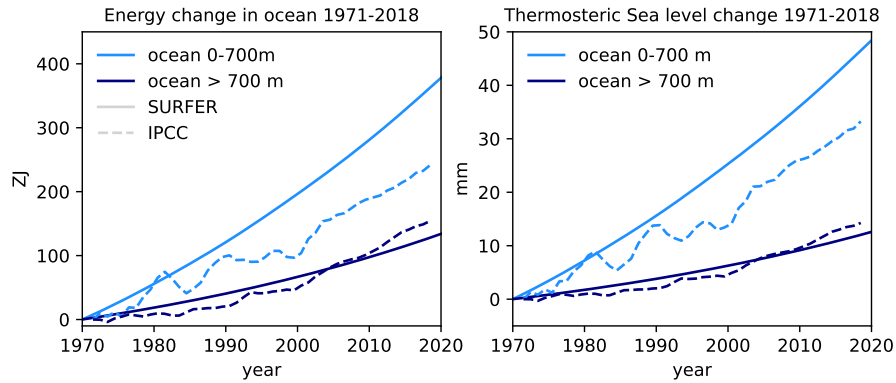


Fig. 2.16 Ocean heat content and thermosteric sea level changes between the years 1971 and 2018 as estimated by the IPCC (Fox-Kemper et al., 2021), and simulated by SURFER v3.0.

rise simulated by SURFER v3.0 matches the IPCC estimates quite well. This indicates that the oceanic heat transport at depth is a little too slow, which compensates for the ocean receiving more energy than it should.

We also check that our parametrisation for thermosteric sea level rise is valid on longer time scales. In Figure 2.17, we compare outputs from the intermediate complexity model UVic, versions 2.8 and 2.9 (Clark et al., 2016; Eby et al., 2009), to outputs from SURFER v2.0 and SURFER v3.0. The UVic 2.8 and UVic 2.9 models both include a sediment module and have equilibrium climate sensitivities around 3.5°C , the same as in SURFER. Emission scenarios used to force the models follow historical estimates of CO_2 emissions up to the year 2000. Following this, cumulative emissions of either 1280 or 3840 PgC are added between the years 2000 and 2300. For more details on the experimental setup, see Clark et al. (2016).

SURFER v3.0 has a faster and larger atmospheric CO_2 uptake than both UVic 2.8 and UVic 2.9. This is consistent with the LTMIP experiments (figures 2.9 and 2.10) where UVic 2.8 was already the model with the smallest CO_2 uptake after 10000 years for the CSWV experiments. UVic 2.9 has an even slower CO_2 uptake than UVic 2.8 due to the difference in the sediments representation (Clark et al., 2016). For SURFER v2.0, the atmospheric CO_2 concentration reaches equilibrium after ~ 4000 years since it only takes into account the process of ocean CO_2 invasion. These differences in atmospheric CO_2 concentrations lead to relatively large differences in global mean surface temperatures. Nevertheless, the thermosteric SLR is comparable in all models,

2 | The SURFER model

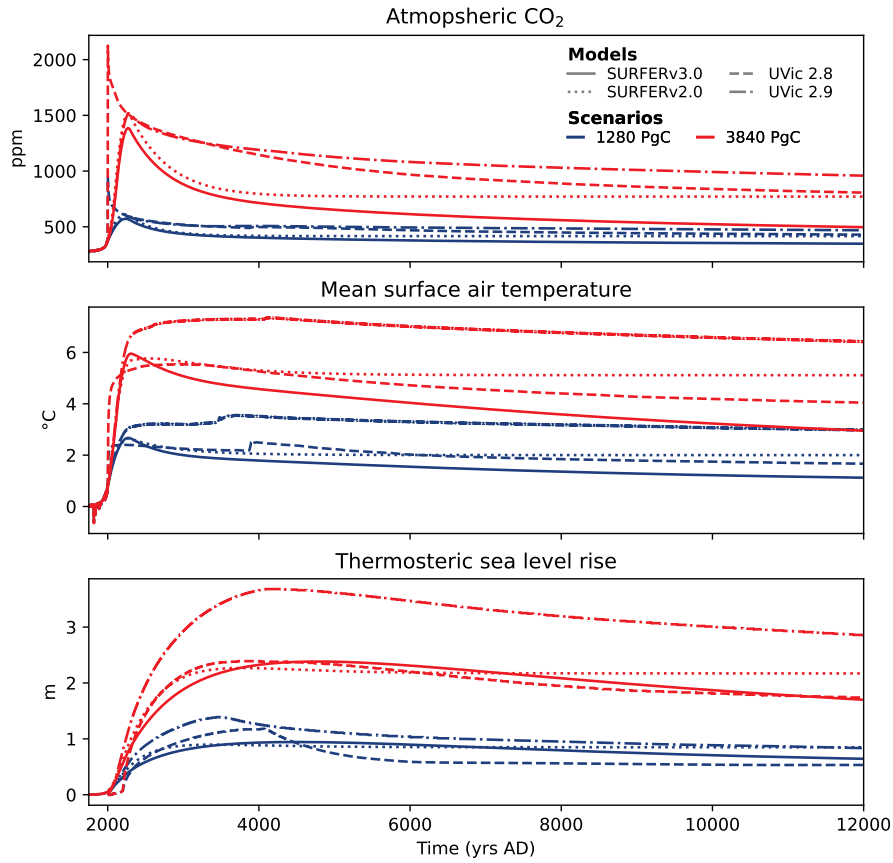


Fig. 2.17 Atmospheric CO₂, surface temperature, and thermosteric sea level rise simulated by SURFER v3.0, SURFER v2.0, and two versions of the UVic model of intermediate complexity for two emission scenarios : 1280 PgC (blue) and 3840 PgC (red).

except for UVic 2.9 under the 3840 PgC scenario, where it is larger than for the other models. The thermosteric SLR from SURFER v3.0 is close to the one from UVic 2.8 for both scenarios, even though the simulated temperature is lower in SURFER v3.0. This is again a consequence of SURFER not simulating land temperatures. The global mean ocean temperature increase in UVic 2.8 is smaller than its global mean temperature increase, and probably comparable to the mean ocean temperature observed in SURFER v3.0. SURFER v3.0 and SURFER v2.0 also have comparable thermosteric SLR despite SURFER v2.0 simulating a larger temperature increase. This is because the ocean in SURFER v3.0 is deeper than in SURFER v2.0 (3800 m deep vs 3150 m) and has thus more potential for expansion.

2.4.2 Ice sheets

In SURFER v2.0, atmospheric CO₂ concentrations, and hence temperatures, stabilise a few thousand years after the end of emissions. In SURFER v3.0, thanks to new processes added in the carbon cycle, CO₂ drawdown from the atmosphere continues until return to pre-industrial conditions, leading to lower temperatures than in SURFER v2.0 on millennial and longer time scales. Since ice sheets respond on the millennial time scale, we expect these differences to have a significant impact on their melting. To test this, we force both SURFER v3.0 and SURFER v2.0 with the CO₂ emissions from five SSP scenarios (SSP1-2.6, SSP2-4.5, SSP4-6.0, SSP3-7.0, SSP5-8.5) which cover a range of possible futures. Simulations last for 500 kyr and results are plotted in Figure 2.18.

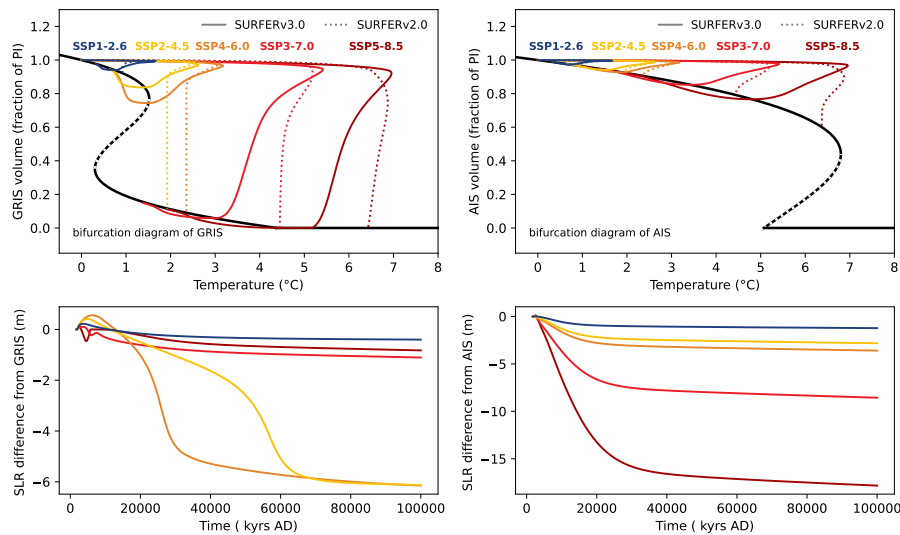


Fig. 2.18 Comparison of Greenland and Antarctic sea level rise contributions in SURFER v3.0 and SURFER v2.0 for SSP emission scenarios. Top plots: Volume fractions of Greenland and Antarctica ice sheets as a function of temperature increase. Equilibrium values are shown in black, with stable equilibria represented by solid lines and unstable equilibria by dashed lines. Bottom plots: difference in sea level rise contributions from Greenland and Antarctica in SURFER v3.0 and SURFER v2.0. Negatives values indicate a reduced sea level rise contribution in SURFER v3.0 compared to SURFER v2.0.

In SURFER, the ice sheets are designed as tipping elements (see section 2.2.3). For both SURFER v2.0 and SURFER v3.0, and under all scenarios, the simulated temperature increase overshoots the critical warming threshold of the Greenland ice sheet. However, the Greenland ice sheet doesn't always

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collapse. Indeed, in general, tipping can be avoided if the overshoot duration is short relative to the effective time scale of the tipping element (Ritchie et al., 2019; Ritchie et al., 2021). For SURFER v2.0 this happens for the SSP1-2.6 scenario. For all other scenarios, the temperature stabilises past the critical threshold and thus Greenland eventually transitions to a completely melted state. In SURFER v3.0, thanks to a greater decrease in temperature after reaching a maximum, Greenland overshoots safely its critical threshold also under SSP2-4.5 and SSP4-6.0 scenarios. This happens even though peak warming reaches 2.62 °C and 3.18 °C respectively, well above the Greenland ice sheet's critical threshold of 1.52 °C (as set in SURFER). For these scenarios, This leads to a ~ 6 m reduction in the long-term sea level rise contribution simulated by SURFER v3.0 compared to SURFER v2.0.

Meanwhile, The Antarctic Ice Sheet doesn't tip in our simulations, regardless of using SURFER v2.0 or SURFER v3.0. This is because the critical warming threshold for the Antarctic ice sheet, set to 6.8 °C in SURFER, is much higher than that for Greenland and seldom reached, even for the SSP5-8.5 scenario. However, despite no changes in the tipping behaviour, differences in simulated temperatures still give rise to large differences in SLR contribution, particularly for high-emission scenarios. The long-term sea level rise contributions from Antarctica for the SSP3-7.0 and SSP5-8.5 scenarios are respectively reduced by ~ 8 m and ~ 18 m in SURFER v3.0 compared to SURFERv2.0.

2.5 Discussion

Our primary goal with SURFER v3.0 was to improve the representation of carbon cycle dynamics to enable simulations of the Earth system on multi-millennial time scales. To this end, we have added to SURFER v2.0 a dynamic and more precise representation of alkalinity, an explicit representation of the carbonate and soft-tissue pumps, a sediments reservoir with associated accumulation and burial fluxes, and weathering as well as volcanic out-gassing fluxes. We have shown that these additions allow for an accurate simulation of carbon cycle dynamics on multi-millennial time scales by comparing SURFER v3.0 to the outputs from the LTMIP experiments, and to the outputs of cGENIE for 1 Myr runs. Furthermore, we showed that the stabilisation of atmospheric CO₂ and temperature at lower levels in SURFER v3.0 than in SURFER v2.0 leads to a significant reduction in simulated sea level rise. While this outcome is robust, the tipping behaviour of the Greenland ice sheet in response to specific

emissions scenarios is sensitive to the choice of model parameters. For the parameter set used in this study (see Appendix A.1), we found that, in contrast to SURFER v2.0, Greenland avoids tipping in SURFER v3.0 under intermediate emissions scenarios SSP2-4.5 and SSP4-6.0.

A secondary goal with SURFER v3.0 was to improve on SURFER v2.0 for the decadal to centennial time scales. This was done by adding an ocean layer of intermediate depth, a temperature and pressure dependence of the solubility and dissociation constants, a representation of atmospheric methane, and by carefully setting the initial conditions of the oceanic variables based on the GLODAP dataset. We have shown that SURFER v3.0 successfully reproduces the historical CO₂ and CH₄ concentrations, the estimated historical land and ocean sinks, the CMIP6 ensemble mean for the evolution of different quantities under the SSP1-2.6 and SSP3-7.0 scenarios, and that in all these tasks it performs equally well or better than SURFER v2.0.

In summary, SURFER v3.0 outperforms SURFER v2.0 on all time scales, and despite having doubled the number of differential equations, it stays fast, transparent and easy to modify. This manuscript contains all the equations for the model, all the parameter values, and the initial conditions for all the variables. SURFER v3.0 is coded in Python, which is one of the most widely used open-source programming languages. The code of the model, as well as the code for all the figures, is available online in a Jupyter notebook, providing already a wide range of example use cases.

Of course, many coupled atmosphere-ocean box models of the carbon cycle already exist, some of them including carbonate sediments and weathering processes (Keir, 1988; Köhler and Munhoven, 2020; Lenton and Britton, 2006; Munhoven and François, 1996; Zeebe, 2012). These models, often primarily focused on the ocean, offer a more complex representation of the carbon cycle than SURFER v3.0. For instance, BICYCLE-SE (Köhler and Munhoven, 2020) includes 1 atmospheric box, 10 ocean boxes (5 surface boxes), ocean cycling of DIC, alkalinity, dissolved oxygen, and phosphate, sediment columns in each ocean basin and at various depths, and 7 terrestrial biosphere boxes. Still, despite being simpler, SURFER v3.0 effectively captures the essential dynamics of the carbon cycle, as demonstrated in the precedent sections.

Moreover, SURFER includes a dynamic representation of temperature and sea level rise, which is often lacking in other models where those quantities are constant or prescribed. Among existing models, the one from Lenton and Britton (2006) is the closest to SURFER v3.0, with 4 ocean boxes, 10 sediment boxes, 2 terrestrial biosphere boxes, and a weathering flux parametrisation. As

2 | The SURFER model

in SURFER v3.0, its carbon cycle is coupled with an energy balance representation of global mean temperature and both models can be classified as "simple Earth system models". While it is possible to integrate SURFER's temperature equations and sea level rise parametrisation into other models, we believe the unique combination of included processes, simplicity, transparency, and speed makes SURFER v3.0 a valuable addition to the literature, on top of being a worthwhile update to SURFER v2.0.

Although we are quite satisfied with SURFER's capabilities, we are certainly not claiming it is *the one model to rule them all*. Indeed, being that simple comes with several limitations that we discuss here.

First, SURFER v3.0 lacks horizontal resolution. In particular, SURFER v3.0 has no high latitude ocean surface box, which is a feature of most ocean carbon cycle box models and could help with simulating more realistically the atmosphere-to-ocean CO₂ flux. The lack of spatial resolution also implies that SURFER v3.0 doesn't represent land temperatures, hence equating global mean temperatures with ocean mean surface temperatures. As shown in section 2.4.1, this leads to an overestimation of ocean surface temperature and thermosteric SLR for historical and SSP forced runs.

Additionally, several oceanic and land carbon-climate feedbacks are missing in SURFER v3.0. For example, there is no dynamic ocean circulation and as such, changes in atmosphere-to-ocean CO₂ fluxes resulting from climate-induced changes in the oceanic circulation are not represented. This was suggested in section 2.3.3 by comparing the "climate feedback" of SURFER v3.0 with other models for the 5000 PgC pulse LTMIP experiment. A second example is the organic matter and CaCO₃ productions in the upper ocean layer that are kept constant, neglecting eventual changes in marine ecosystem production and associated carbon uptake. Last but not least, is our parametrisation of atmosphere-to-land carbon flux, which depends on the atmospheric CO₂ concentration, but not on temperature. As explained in section 2.3.2, this is probably the reason why SURFER's land sink stays positive longer than other models for high-emission scenarios. Furthermore, this parametrisation doesn't account for hypothesised tipping elements such as the Amazon and boreal forests or the permafrost, which may release important amounts of greenhouse gases past critical warming thresholds (Armstrong McKay et al., 2022).

Moreover, SURFER lacks a detailed representation of land surface-vegetation-albedo feedbacks, which can influence temperatures and hydro-climate on decadal to multi-centennial timescales through changes in vegetation cover (Feng et al., 2022; Forster et al., 2021; Willeit et al., 2014). In the current model,

these feedbacks are indirectly represented via the climate feedback parameter β (see *Biogeophysical feedbacks* in IPCC (2021a), p 976). However they are not explicitly linked to the land carbon reservoir (M_L), the variable most closely associated with vegetation cover. This limitation, combined with the absence of a hydrological cycle and horizontal spatial resolution, constrains SURFER's ability to capture the full consequences of vegetation feedbacks. For example, SURFER may fail to account for their contribution to polar amplification (Swann et al., 2010; Willeit et al., 2014) and their subsequent impact on sea level changes. Simulations of the last glacial inception by Willeit et al. (2023) demonstrated that dynamic vegetation responses led to an additional 15-meter decrease in sea level, highlighting the critical role of these feedbacks in shaping long-term Earth system trajectories.

For the sea level rise module, the contribution of the Antarctic ice sheet should probably be split into several components. Indeed, it has been shown that the West Antarctic ice sheet, as well as some East Antarctic subglacial basins, could collapse for lower warming thresholds than the East Antarctic ice sheet and contribute to several meters of sea level rise already by 2300 for high emission scenarios (Alley et al., 2015; Coulon et al., 2024; Garbe et al., 2020).

Finally, in its current state, SURFFER v3.0 accounts for only one process influencing the carbon cycle on the 100 kyr timescale: the weathering of silicate rocks. Other potentially important processes acting on these time scales, such as orbital forcing variations, are currently neglected. These variations modulate the seasonal and spatial repartition of incoming solar energy and are known to drive or act as a pacemaker of the Quaternary glacial cycles (Hays et al., 1976).

Also neglected in SURER, organic carbon burial may influence the climate system through its role in long-term carbon sequestration (Archer, 2010; Berner, 2004). For instance, during past oceanic anoxic events, reduced oxygen levels inhibited remineralisation processes, resulting in increased organic carbon preservation and burial in sediments and decreased atmospheric CO_2 levels (Barclay et al., 2010; Schlanger and Jenkyns, 1976). As warming is suspected to lead to oceanic deoxygenation due to reduced oxygen solubility and enhanced weathering fluxes (Ruvalcaba Baroni et al., 2020; Schmittner et al., 2008), organic carbon burial may act as a negative feedback mechanism that mitigates warming.

In addition, organic carbon plays a critical role in carbonate sedimentation. In oxic pore waters, the respiration of organic matter creates acidic micro-environments that enhance the dissolution of CaCO_3 sediments (Sarmiento

and Gruber, 2006). Incorporating organic carbon burial into SURFER would require adding a sediment reservoir for organic carbon, along with accumulation and burial fluxes, similar to the implementation of CaCO_3 burial. Ideally, the model should also account for the flux of organic carbon from land to the ocean through river input and exchange fluxes between ocean layers due to oceanic circulation. While these modifications are conceptually straightforward, parametrising the fluxes would require incorporating oxygen and nutrient cycling, which is essential to determine the rates of organic carbon production and remineralisation. Implementing these features would demand additional effort but offer a more comprehensive representation of the long-term carbon cycle and enable the study of oceanic anoxic events.

2.6 Conclusion

We have presented SURFER v3.0, a simple Earth system model that includes a dynamic carbon cycle and simulates various important quantities such as atmospheric CO_2 and CH_4 concentrations, temperature anomalies, ocean surface pH, and sea-level rise in response to anthropogenic greenhouse gases emissions. SURFER v3.0 extends SURFER v2.0 by incorporating dynamic alkalinity cycling, CaCO_3 sediments, and weathering processes. These additions enable SURFER v3.0 to accurately simulate the dynamics of the coupled carbon-climate system over timescales ranging from decades to millions of years. We have validated this by comparing SURFER v3.0 to historical data and outputs from GCMs, EMICs, and other box models.

We have also demonstrated that SURFER v3.0 can simulate thermosteric sea-level rise reasonably well on millennial timescales, though it tends to overestimate it for shorter timescales. Furthermore, we have shown that incorporating long-term carbon cycle processes in SURFER v3.0 leads to a significant reduction in simulated contributions of Greenland and Antarctica to sea-level rise, compared to SURFER v2.0. These results highlight the critical importance of considering these processes to better predict committed sea-level changes.

SURFER v3.0 is fast, transparent, easy to modify and use, and hence an ideal tool for policy assessments that wish to take into account centennial to multi-millennial time scales. In the future, we plan to add to SURFER a representation of glacial cycle dynamics and include several tipping elements

to investigate the stability of the Earth system and the impact of anthropogenic emissions on its future long-term trajectories.

3

Tipping interactions and cascades

3.1 Introduction

To this day, interactions between tipping elements and cascading dynamics have been studied mainly with conceptual models. These models have been applied to both generic tipping elements (Dekker et al., 2018; Klose et al., 2020; Klose et al., 2021) and specific cases, such as AMOC and ENSO (Dekker et al., 2018), sea ice and AMOC (Lohmann et al., 2021), or ice sheets and AMOC (Klose et al., 2024; Sinet et al., 2023). Wunderling et al., 2021 developed a conceptual model of four tipping elements (GRIS, WAIS, AMOC, AMAZ) based on the expert elicitation of Kriegler et al., 2009 and a boolean model of tipping interactions from Gaucherel and Moron, 2016. They found that interactions tend to destabilize the entire system, increasing the risk of observing tipping events and cascades, even for a warming below 2 °C. They also show that the ice sheets often initiate tipping cascades while AMOC acts as a mediator.

These studies use overly simplistic forcing scenarios. For instance, Wunderling et al., 2021 use constant temperature scenarios, placing a strong emphasis on the critical temperatures at which tipping elements might tip and potentially overlooking the time scale of their internal dynamics. These timescales can play an important role in determining tipping behaviour in case of non-stationary forcing. Specifically, tipping can be avoided if the overshoot duration of the forcing over the critical threshold is short relative to the effective time scale of

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the tipping element (Ritchie et al., 2019; Ritchie et al., 2021). In a follow-up study, Wunderling and co-authors applied idealized non-stationary temperature forcings to the same 4-element model mentioned above (Wunderling et al., 2022). They showed that the risk of tipping increases with peak temperature, convergence temperature, and convergence time. Using the same model but focusing on scenarios that remain within or temporarily exceed the Paris limit of 2 °C, Möller et al., 2024 obtained similar results. They also showed that the risk of observing tipping events by 2300 presents a nonlinear increase around 2 °C of peak warming, primarily driven by the increased risk of tipping the Amazon, a fast element.

Tipping elements may also affect other elements because of their impact on globally averaged temperatures. For example, Deutloff et al., 2023 study the impact of additional CO₂ emissions resulting from Amazon rainforest and permafrost collapse on the probability of crossing other tipping points by 2500, without considering any other tipping interactions. They found a positive but small impact compared to the scenario dependence.

In summary, existing studies often address different aspects of tipping dynamics, but none have fully incorporated interactions between elements and their feedbacks on global temperature in a single framework, while performing multi-millennial simulations under realistic emission scenarios. In this chapter, we present such a study using SURFER, a reduced complexity model with a process-based carbon cycle described in chapter 2 (Couplet et al., 2024b; Martínez Montero et al., 2022). We have added to this model several tipping elements, based on the most recent knowledge available. With it, we aim to answer the following questions :

- Considering realistic emission scenarios resulting in non-stationary temperatures, how do the intrinsic time scales of tipping elements affect they tipping behaviour?
- To what extent do interactions and feedbacks on global mean temperature increase the risk of observing tipping events ?
- What are the cascades that could happen in the future and particularly for scenarios that stay within or close to the 2 °C Paris range ?
- Can we identify specific roles for the elements parts of a tipping cascade. Specifically, are the Greenland and West Antarctic ice sheets really the main initiators of cascades, despite having a slow response time scale?

The chapter is structured as follows. In section 3.2 we describe our model, how we implement the tipping elements, and the overall experimental setup. Section 3.3 describes the results of our experiments. We focus on tipping elements individually in section 3.3.1, and their increased risk of tipping when interactions are taken into account. We then focus on tipping cascades in section 3.3.2. In section 3.4, we summarise our results compare them to other studies, highlight advantages and limitations of our approach, and suggest further research directions

3.2 Methods

3.2.1 The SURFER model of reduced complexity

We use SURFER v3.0, a simple Earth system model that includes a dynamic carbon cycle (Couplet et al., 2024b, also presented in Chapter 2 of this thesis). It reliably estimates CO₂ concentrations, global mean temperatures, sea-level rise, and many ocean acidification metrics in response to anthropogenic greenhouse gas emissions on timescales from decades to millions of years. Version 3.0 consists of 17 differential equations which describe the exchanges of carbon between six different reservoirs: the atmosphere, land, upper, intermediate, and deep ocean layers, and deep sea sediments; temperature anomalies in the three ocean layers; volumes of Greenland and Antarctic ice sheets; and the sea level rise related to glaciers.

In the extended version used here, we keep the Greenland ice sheet (GRIS), and we split the Antarctic ice sheet into the West Antarctic ice sheet (WAIS), the East Antarctic subglacial basins (EASB), and the remainder of the East Antarctic ice sheet (EAIS). We also add parametrisations for the Atlantic meridional overturning circulation (AMOC), the Amazon rainforest (AMAZ), boreal permafrost (PERM) and Arctic sea-ice (ASI). All these systems, important for climate, were classified as global core tipping elements in Armstrong McKay et al., 2022.

Mountain glaciers (GLCR), which are also included in this extended version, are classified as regional tipping elements in Armstrong McKay et al., 2022. However, when aggregated globally, they tend to respond fairly linearly to temperature changes (Rounce et al., 2023; Zekollari et al., 2024). The classification of Arctic sea-ice and boreal permafrost as tipping elements is debated. In

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the recent Global Report on Tipping points, they are not classified as tipping systems (Lenton et al., 2023).

Summer Arctic sea-ice is projected to respond linearly to temperature changes. This is because strong negative feedbacks in winter associated with ocean heat loss overcompensate the positive ice-albedo feedback (Notz and Community, 2020; Notz and Stroeve, 2016). Winter sea ice may respond nonlinearly to changes in temperature, but only because of the geometry of the Arctic Ocean. There is no self-amplifying feedback phenomenon (Lenton et al., 2023). Arctic sea-ice loss is thus reversible upon a decrease in temperatures. Our model does not feature an explicit seasonal cycle. We therefore introduced a parametrisation for mean annual arctic sea-ice, not distinguishing between winter sea ice and summer sea ice. It is modelled as a system with a unique equilibrium state that depends nonlinearly on temperature.

Permafrost thaw processes are typically classified as either gradual or abrupt (Armstrong McKay et al., 2022; Lenton et al., 2023). While abrupt thaw processes can lead to regional tipping points, permafrost thaw aggregated globally is generally considered to scale roughly linearly with temperature (Nitzbon et al., 2024; Schuur et al., 2015). Some degree of nonlinearity may be expected if large permafrost regions become susceptible to abrupt thaw processes simultaneously. For example, this could be the case for the deep and carbon-rich Yedoma permafrost deposits (Armstrong McKay et al., 2022; Strauss et al., 2017). In this study, we consider permafrost as a whole, and as for Arctic sea-ice, we model it as an element with a unique equilibrium state that is slightly nonlinear dependent on temperature.

Although not considered as tipping elements in themselves, we include Arctic sea ice and permafrost in this study because of their potential to produce strong feedbacks on temperature, and because of their close coupling with other tipping elements (Wunderling et al., 2024). Since neither exhibit signs of bi-stability, we refer to them here as *monostable elements* (MEs). By contrast, we refer to GRIS, WAIS, EASB, EAIS, AMOC, and AMAZ as proper *tipping elements*. When discussing all elements collectively, we use the term *nonlinear elements* (NEs) or simply *elements*. Any reference to an element "tipping" implicitly refers specifically to a tipping element.

The following sections describe the parametrisations we use for the elements and their interactions, and the implementation of the elements feedbacks on temperature, either through albedo changes or additional greenhouse gas. The different components of the model and the interactions between those components are schematically summarised in Figure 3.1.

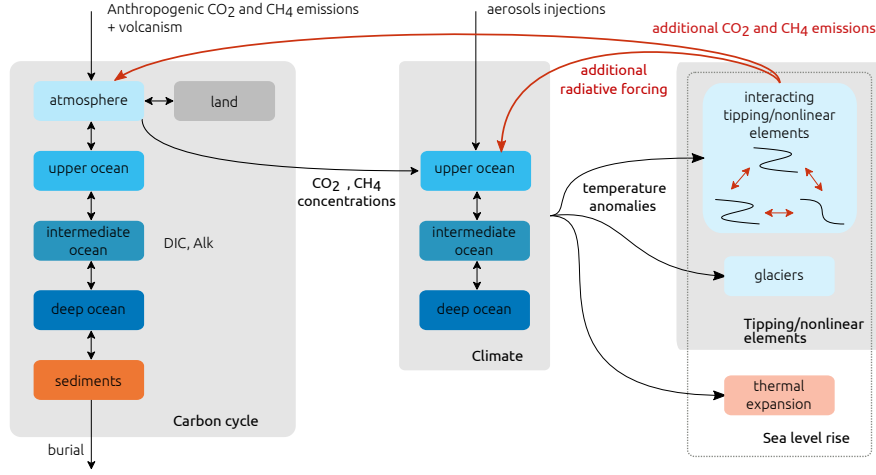


Fig. 3.1 Conceptual diagram of SURFER with the inclusion of interacting tipping elements and their feedbacks on climate.

3.2.2 Representation of tipping elements and other nonlinear components

The evolution of the sea-level rise contribution from glaciers is the same as in previous SURFER versions and is given by the equation

$$\frac{dS_{gl}}{dt} = \frac{1}{\tau_{gl}} (S_{gleq}(\delta T_U) - S_{gl}) , \quad (3.1)$$

with

$$S_{gleq}(\delta T_U) = S_{gl\ pot} \tanh\left(\frac{\delta T_U}{\zeta}\right) . \quad (3.2)$$

Here τ_{gl} is a relaxation timescale, $S_{gl\ pot}$ is the potential sea level rise due to mountain glaciers and ζ is a sensitivity coefficient.

The state evolution of the other tipping elements and nonlinear components is modelled with

$$\frac{dx_i}{dt} = f(x_i, q_i) = \left[\underbrace{-x_i^3 + a_{2,i}x_i^2 + a_{1,i}x_i + c_{0,i} + c_{1,i}\delta q_i}_{H_i} \right] \mu(x_i, H_i) , \quad (3.3)$$

with $i \in \{\text{GRIS, WAIS, EASB, EAIS, ASI, AMOC, AMAZ, PERM}\}$. More specifically, x_{GRIS} , x_{WAIS} , x_{EASB} , and x_{EAIS} represent the fraction of ice volumes compared to pre-industrial levels ; x_{ASI} represent the fraction of mean annual

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Arctic sea-ice surface compared to pre-industrial values ; x_{AMOC} represents the fraction of AMOC strength compared to pre-industrial values ; and x_{AMAZ} and x_{PERM} represent the fraction of carbon stored in the Amazon Rainforest and permafrost respectively, compared to pre-industrial levels. The first group of terms in equation 3.3 represents the internal dynamics of the element. Since it is a cubic polynomial of the variable x_i , it allows the elements to have 1, 2 or 3 stable states depending on δq_i , which represents the forcing anomaly relative to pre-industrial times ($\delta q_i(t_{\text{PI}}) = 0$). The factor $\mu(x_i, H_i)$ is an inverse time scale and is given by

$$\mu(x_i, H_i) = \begin{cases} 1/\tau_i^+ & \text{if } H_i > 0 \text{ and } 0 < x_i < 1, \\ 1/\tau_i^- & \text{if } H_i < 0 \text{ and } 0 < x_i < 1, \\ 0 & \text{if } x_i \leq 0 \text{ or } x_i \geq 1. \end{cases} \quad (3.4)$$

The dependence of μ on the sign of H allows the representation of the asymmetric character of processes driving the evolution of some elements. For instance, ice sheets tend to melt faster than they form. The third case in 3.4 constrains x_i to remain between 0 and 1, with $x_i = 0$ indicating a completely collapsed state and $x_i = 1$ indicating the pre-industrial state. While the first constraint is physical, the second one is introduced here to facilitate the analysis and the parametrisation of the interactions (see section 3.2.4). Furthermore, in the numerical implementation of the model, the function μ is smoothed out such that the right-hand side of eq 3.3 is differentiable everywhere.

The stability structure induced by eq. 3.3 depends on the constant parameters (a_2, a_1, c_1, c_0) . When $a_2^2 + 3a_1 > 0$, $f(x_i, q_i)$ has 1, 2 or 3 real roots depending on q_i , and we have a double-fold structure : there are three equilibrium branches, two stable, $x_+^s(\delta q)$ and $x_-^s(\delta q)$, and one unstable $x^u(\delta q)$, that meet at critical points (q_+, x_+) and (q_-, x_-) in two fold bifurcations (see figure 3.2). For $q_- < \delta q < q_+$, the system is thus bi-stable. This structure, along with the bi-stability condition, corresponds to the classical representation of tipping points in conceptual studies, for example, in Wunderling et al., 2021. We use it for representing our tipping elements GRIS, WAIS, EASB, EAIS, AMOC and AMAZ. The parameters (a_2, a_1, c_1, c_0) can be obtained from the coordinates of the bifurcations points (q_+, x_+) and (q_-, x_-) :

$$a_2 = \frac{3(x_- + x_+)}{2}, \quad (3.5)$$

$$a_1 = -3x_-x_+, \quad (3.6)$$

$$c_1 = -\frac{(x_+ - x_-)^3}{2(q_+ - q_-)}, \quad (3.7)$$

$$c_0 = +\frac{q_+x_-^2(x_- - 3x_+) - q_-x_+^2(x_+ - 3x_-)}{2(q_- - q_+)}. \quad (3.8)$$

These relationships allow to easily calibrate SURFER such as to match the steady-state structure of more complex models. We further impose the pre-industrial condition $x_+^s(\delta q = 0) = 1$ such that have only to set 3 of the 4 parameters independently. We choose to set q_+ , x_+ , and q_- , then x_- is given by

$$x_- = \frac{-2 + x_+(1 + G^{1/3} + G^{-1/3})}{-1 + G^{1/3} + G^{-1/3}}, \quad (3.9)$$

with

$$G = \frac{q_+ + q_- + 2\sqrt{q_+q_-}}{q_+ - q_-}. \quad (3.10)$$

When $a_2^2 + 3a_1 < 0$, $f(x_i, q_i)$ has a unique real root for any value of q_i , and so the stability structure induced by equation 3.3 has a unique stable equilibrium branch $x^s(\delta q)$ that has an inflexion point (q_I, x_I) (see figure 3.2). The parameters (a_2, a_1, c_1, c_0) can be obtained from the coordinates of the inflexion point (q_I, x_I) , the derivative at the inflexion point, $d_I = \frac{dx^s(q_I)}{d\delta q}$ and the pre-industrial condition $x^s(\delta q = 0) = 1$:

$$a_2 = 3x_I, \quad (3.11)$$

$$a_1 = M_I - 3x_I^2, \quad (3.12)$$

$$c_1 = -d_I M_I, \quad (3.13)$$

$$c_0 = 1 - 3x_I + 3x_I^2 - M_I, \quad (3.14)$$

with

$$M_I = \frac{(x_I - 1)^3}{x_I - q_I d_I - 1} \quad (3.15)$$

In the limit case $a_2^2 + 3a_1 = 0$, the two bifurcation points (q_+, x_+) and (q_-, x_-) merge in (q_I, x_I) and the slope of the unique stable branch at this point is infinite ($\frac{dx^s(q_I)}{d\delta q} = -\infty$).

The forcings are given by

$$\delta q_i = \delta T_U + d \cdot \sum_j \epsilon_{ij} \delta L_j(x_j, \dot{x}_j) q_{*,i}, \quad (3.16)$$

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with

$$q_{*,i} = \begin{cases} q_{+,i} & \text{for } i \in \text{GRIS, WAIS, EASB, EAIS, AMOC, AMAZ,} \\ q_{I,i} & \text{for } i \in \text{AIS, PERM.} \end{cases} \quad (3.17)$$

Each element i is forced by the upper ocean temperature δT_U , which is equivalent to the global mean surface temperature in SURFER (see section 2.4.1). This global mean temperature should be understood as a proxy for the corresponding spatially inhomogeneous distributions of temperatures and precipitations to which the tipping and nonlinear elements respond in reality. The main advantage of this formulation is that it allows us the direct use of reviews on tipping elements such as Armstrong McKay et al. (2022), which typically provide critical thresholds for tipping elements as a function of global mean surface temperature. However, our formulation does not take into account that some elements may also respond to the intermediate and deep ocean temperatures (δT_I and δT_D), which evolve on slower time scales than the surface temperature. For instance, this is the case for the AMOC, and experiments not discussed here have shown that including additional terms in the forcing proportional to δT_I or δT_D leads to phenomena such as rate-induced tipping (Alkhayuon et al., 2019; Stocker and Schmittner, 1997).

Each element i can also be coupled to other elements j through the quantities δL_j . As further developed in section 3.2.4, interactions between elements are modelled by taking δL_j as a linear function of element x_j or its time derivative $\dot{x}_j$. The latter case is used, for example, to model the impact of Greenland melt ($\propto \dot{x}_{\text{GRIS}}$) on the AMOC. The forcing δq_i is expressed in units of temperature ($^{\circ}\text{C}$) even though the quantities δL_j are not necessarily linked to temperature. The coupling coefficients ϵ_{ij} give the strength and the sign of interactions, and have the inverse units of δL_j . The adimensional parameter d is introduced for practical purposes and allows to scale the strength of all interactions simultaneously. We set $d = 1$ to take interactions into account, $d = 0$ to suppress coupling. In the supplementary material, we perform some experiments with $d = 2$ to study the impact of overall stronger interactions between the elements.

For uncoupled tipping elements ($d = 0$), we have $\delta q = \delta T_U$, and we can set parameters a_2 , a_1 , c_1 , and c_0 as a function of $T_+ = q_+$, $T_- = q_-$, x_+ , and x_- following equations 3.5-3.8. Similarly, for uncoupled monostable elements, we can set parameters a_2 , a_1 , c_1 , and c_0 as a function of $T_I = q_I$, x_I , and d_I , following equations 3.11-3.14. This is helpful since most studies report the

critical thresholds of tipping and other nonlinear elements as temperature thresholds, without taking into account interactions. We use this to our advantage and fix the values of a_2 , a_1 , c_1 , and c_0 for all elements to match the temperature threshold ranges given in Armstrong McKay et al., 2022. This is also the reference we use to calibrate the timescales τ_- and τ_+ , as well as the feedbacks of the elements on global mean temperature. Finally, we calibrate the coupling coefficients ϵ_{ij} based a recent review of tipping interactions from Wunderling et al., 2024. More details on the calibration process are in section 3.2.4.

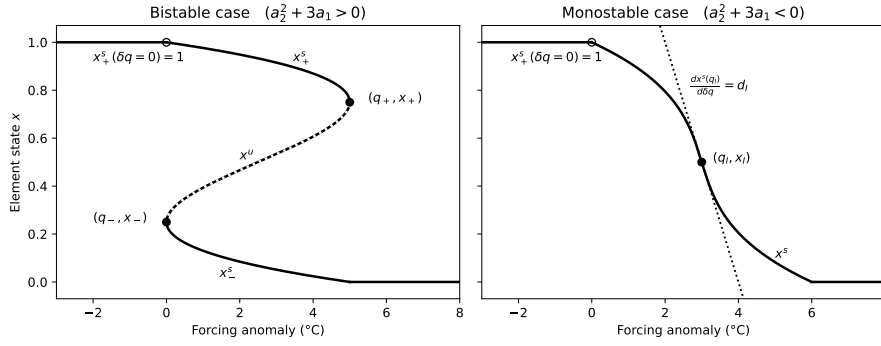


Fig. 3.2 Examples of steady state structures produced by equation 3.3 for the elements. For $a_2^2 + 3a_1 > 0$, there is a bistable double-fold stability structure. Parameters a_2 , a_1 , c_1 , c_0 can be expressed as the coordinates of the two bifurcation points (q_+, x_+) and (q_-, x_-) . When $a_2^2 + 3a_1 < 0$, there is a monostable stability structure. Parameters a_2 , a_1 , c_1 , c_0 can be expressed as the coordinates of the inflexion points (q_I, x_I) and the slope d_I of the stable branch at this point.

3.2.3 Monte-Carlo ensemble and Experimental setup

To deal with the uncertainty surrounding tipping and other elements, we create an ensemble of $N_{\text{params}} = 100$ different parameters sets. The values for T_+ or T_I , τ_- and the coupling coefficients ϵ_{ij} are sampled uniformly in ranges based on the literature, using latin hypercube sampling (specifically, we use the LatinHypercube class from the Quasi-Monte Carlo submodule of SciPy, with the random-cd optimisation scheme). This sampling method ensures a more representative sampling across the entire high dimensional parameter space ($n_{\text{dim}} = 31$) than simple random sampling. The other parameters for the elements, T_- and x_- or x_I and d_I , and τ_+ are fixed. This is also the case for the parameters $\bar{F}$, $\bar{E}^{\text{CO}_2}$, and $\bar{E}^{\text{CH}_4}$ that regulate the feedbacks of the

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Tipping elements	T_+ (°C)	T_- (°C)	x_+ adim	x_- adim	τ_- (years)	τ_+ (years)	$\bar{F}$ (W m ⁻²)	$\bar{E}^{\text{CO}_2}$ (PgC)	$\bar{E}^{\text{CH}_4}$ (PgC)
GRIS	[0.8, 3.0]	0.0	0.75	0.25	[305, 1978]	5500	0.145	0	0
WAIS	[1.0, 3.0]	0.0	0.75	0.25	[153, 1714]	5500	0.056	0	0
EASB	[2.0, 6.0]	0.0	0.75	0.25	[88, 847]	5500	0.056	0	0
EAIS	[5.0, 10.0]	0.0	0.75	0.25	[469, 741]	5500	0.669	0	0
AMOC	[1.4, 8.0]	0.0	0.75	0.25	[2.1, 21]	$= \tau_-$	0	0	0
AMAZ	[2.0, 6.0]	0.0	0.75	0.25	[7.9, 15]	$= \tau_-$	0	75	0
Other elements	T_I (°C)	x_I adim	d_I (°C ⁻¹)						
ASI	[4.5, 8.7]	0.5	-0.50		[1.9, 8.4]	$= \tau_-$	0	0	0
PERM	[3.0, 6.0]	0.5	-0.25		[5.3, 76]	3000	0	300	7.5

Table 3.1 *Parameters for the tipping elements and other nonlinear monostable elements. For T_+ and τ_- , parameters are sampled uniformly in the ranges based on the literature (see section 3.2.4). Other parameters are fixed. For the AMOC, the AMAZ and ASI, τ_+ is set equal to the sampled value for τ_- . For the arctic sea ice (ASI) temperature threshold T_I , we use the range given in Armstrong McKay et al., 2022 for the winter sea ice, as it is responsible for the nonlinear behaviour. Similarly, for our permafrost temperature threshold T_I , we take the values given in Armstrong McKay et al., 2022 for the boreal permafrost collapse (PFTP).*

elements on the temperature (see section 3.2.4). Table 3.1 summarises the values given to the parameters for the elements, and more details are provided in section 3.2.4.

For each parameter set of our ensemble, we run the extended SURFER model for a family of $N_{\text{scenarios}} = 100$ emission scenarios that we describe below. This forms one single experiment with $N_{\text{params}} \times N_{\text{scenarios}} = 10000$ model runs. The emission scenarios start in 1850 AD and follow historical CO₂ emissions until 2015 AD. After that, between 50 and 5000 PgC of CO₂ are released in the atmosphere following a logistic equation, with each successive scenario adding 50 PgC. Methane emissions are the same in all scenarios: they follow historical emissions from 1850 AD until 1995 AD, then follow the SSP1-2.6 scenario. More details about the construction of these scenarios are in appendix B.1. For the same parameters and without including the feedbacks of the elements on temperature, the temperature difference in our model between two successive scenarios at any given time is lower than 0.1°C (for ECS = 3.5°C). Throughout this chapter, we will refer to scenarios by their CO₂ cumulative emissions value after 2015, rather than their total cumulative emission value since 1850.

We perform several experiments for different model settings. The two main experiments are the *baseline* and *fully coupled* experiments. For the baseline experiment, we switch off the interactions between the elements ($d = 0$) and their feedbacks on temperature ($\bar{F}_i = 0$, $\bar{E}_i^{\text{CO}_2} = 0$, $\bar{E}_i^{\text{CH}_4} = 0$). In the fully coupled experiment, they are taken into account. We also perform an *interactions only* experiment where feedbacks of elements on temperature are turned off, and a *feedback only* experiment where interactions between elements are turned off. In appendix B.3, we also provide some results for experiments with a higher climate sensitivity, with stronger interactions between tipping elements, and without considering the nonlinear monostable elements ASI and PERM. Overall we performed $N_{\text{params}} \times N_{\text{scenarios}} \times 8 = 80000$ model runs, cumulating a total of around 8 billion years of simulation.

We now define the *tipping risk* or the *risk of a tipping event* in a given experiment, and for a given scenario, as the fraction of runs with different parameter sets featuring this tipping event. We emphasize that the tipping risks defined here are illustrative rather than predictive. By using uniform distributions to sample key parameters, we are not representing the actual likelihood of any specific value occurring, so our tipping risks should not be interpreted as probabilistic projections of events. Instead, by assigning equal probability to all possible parameter values, we perform a maximum uncertainty assessment. This approach follows previous exploratory studies (e.g., Wunderling et al., 2021; Wunderling et al., 2022) and provides insights into robust patterns that could emerge under a variety of conditions.

3.2.4 Calibration

Critical temperatures

For each tipping element, the temperature threshold T_+ is sampled within the intervals provided in Table 3.2, based on the review by Armstrong McKay et al., 2022. These intervals align closely with values in the more recent Global Tipping Points Report (Lenton et al., 2023). Generally, in assessments of tipping elements—and in the broader literature—there is limited information on x_+ and the "back-tipping" point (T_-, x_-) . Therefore, to streamline our analysis, we set $x_+ = 0.75$ and $T_- = 0$ for all tipping elements. The corresponding value of x_- is then determined by equations 3.9-3.10, resulting in $x_- = 0.25$ across elements, independent of the sampled values of T_+ .

The choices of $x_+ = 0.75$ and $x_- = 0.25$ provide a reasonable, physically plausible estimate. However, setting $T_- = 0$ may be less realistic for certain

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Elements	Critical temperature (°C)			Transition timescale (years)			Max impact
	Min	Max	Best	Min	Max	Best	
GRIS	0.8	3.0	1.5	1k	15k	10k	0.13°C
WAIS	1.0	3.0	1.5	500	13k	2k	0.05°C
EASB	2.0	6.0	3.0	500	10k	2k	0.05°C
EAIS	5.0	10.0	7.5	10k*	?	?	0.60°C
AMOC	1.4	8.0	4.0	15	300	50	-0.60°C
AMAZ	2.0	6.0	3.5	50	200	100	75 PgC
AWSI	4.5	8.7	6.3	10	100	20	0.60°C
PERM	3.0	6.0	4.0	10	300	50	?*

Table 3.2 Literature-based temperature thresholds and transition time scales for tipping elements, as given in Armstrong McKay et al., 2022. *For our calibration procedure of τ_- (see section 3.2.4), we use minimum estimate of 5 kyr instead of 10 kyr for the transition time scale of the East Antarctic ice sheet (EAIS), and a maximum estimate of 15 kyr. This provides a better estimation of committed sea level rise (see Appendix C.1). **Armstrong McKay et al., 2022 divides the permafrost into three categories and doesn't provide a single number for the maximum impact of permafrost as a whole.

tipping elements, such as the East Antarctic Ice Sheet (EAIS), where hysteresis experiments like that of Garbe et al., 2020 suggest a significantly higher back-tipping threshold ($T_{-,EAIS} > 5^\circ\text{C}$). Nonetheless, as our focus here is primarily on the "forward tipping points", i.e., transitions from the baseline upper branch regime to the tipped lower branch regime (see figure 3.2), we retain $T_- = 0^\circ\text{C}$ for all tipping elements to maintain mathematical simplicity.

For Arctic sea ice (ASI) and permafrost (PERM), which are here considered as nonlinear but monostable elements ($a_2^2 + 3a_1 > 0$), we fix $x_I = 0.5$ and we sample T_I within the intervals provided in Table 3.1, based on the review by Armstrong McKay et al., 2022, as for the tipping elements. For Arctic sea ice, we set $d_I = -0.5^\circ\text{C}^{-1}$, and for permafrost we set $d_I = -0.25^\circ\text{C}^{-1}$, which, together with the set T_I range, results in a slightly nonlinear dependence of its equilibrium state on temperature, as discussed in section 3.2.1.

Transition timescales

The parameters τ_- and τ_+ in equations 3.3-3.4 fix the time scales of the elements intrinsic dynamics. Specifically, τ_- has an important role in governing the transition time of an element from its baseline state to its collapsed state. Armstrong McKay et al., 2022 provide range estimates for these transition time scales, reproduced in Table 3.2. Our calibration strategy is to sample $\tau_{-,i}$ in a

range $[\tau_{-,i}^{\min}, \tau_{-,i}^{\max}]$ chosen such that if the element i tips, its transition time falls within (or close to) the assessed range. To find the adequate $\tau_{-,i}^{\min}$ and $\tau_{-,i}^{\max}$, we first define the *transition time scale* of a tipping element i as the time it takes to go from $x_i = x_+$ to $x_i = x_-$ when tipping. We then set $\tau_{-,i}^{\min}$ by simulating element i under a constant temperature forcing $T = T_{+,i} + 3^\circ\text{C}$ with no couplings, and finding the value of τ_- such that the computed transition time scale is equal to the lower bound provided in Table 3.2. For the critical temperature threshold $T_{+,i}$, we choose the best estimate given in Armstrong McKay et al., 2022, also listed in Table 3.2. The temperature forcing with $T = T_{+,i} + 3^\circ\text{C}$ ensures that the element tips relatively quickly. To fix $\tau_{-,i}^{\max}$, we proceed similarly, except that we use a constant temperature forcing $T = T_{+,i} + 1^\circ\text{C}$ and that we search for the value of τ_- such that the computed transition time scale is equal to the upper bound provided in Table 3.2. This lower temperature forcing still leads to tipping while ensuring a slower transition between the states. The resulting ranges $[\tau_{-,i}^{\min}, \tau_{-,i}^{\max}]$ are presented in Table 3.1. In our experiments, the effective transition time scale will not only depend on the choice of τ_- but also on the choices for T_+ , the couplings, and the forcing scenario. Figure B.2 shows that for the majority of cases where elements tip, the transition timescale falls within the range assessed by Armstrong McKay et al., 2022, indicating that our calibration procedure is effective.

For the nonlinear monostable elements (arctic sea ice and permafrost), we define the transition time scale as the time it takes to go from $x_i = 1$ to $x_i = x_I$, and then follow the same calibration procedure for τ_- as described above.

For the AMOC, Amazon rainforest (AMAZ), and Arctic sea ice (ASI), we set τ_+ equal to the sampled value for τ_- , indicating a symmetric timescale. For the ice sheets (GRIS, WAIS, EASB, and EAIS), we set $\tau_+ = 5500$ years, a much higher value than τ_- , to capture the asymmetry between the faster melting and slower freezing processes. For permafrost, while the frozen soil area is considered reversible with decreasing temperatures, the burial of organic carbon is irreversible on short timescales (Lenton et al., 2023). This asymmetry between rapid carbon release upon thaw and the much slower carbon capture when soil refreezes is represented in our model by defining x_{PERM} as the releasable carbon content in permafrost (rather than permafrost area) and setting $\tau_+ = 3000$ years, a value much higher than τ_- (see Table 3.1).

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Interactions between the elements

For a time evolution described by equation 3.3, an element will tip from the upper branch $x_s^+(\delta q)$ to the lower branch $x_s^-(\delta q)$ if the forcing $q = \delta T_U + d \cdot \sum_j \epsilon_{ij} \delta L_j q_+$ exceeds the critical value $q_+ = T_+$ for a long enough period. If $\delta T_U = 0$ and $\delta L_{k \neq j} = 0$, we have $q = \epsilon_{ij} \delta L_j q_+$ and element i will tip for $\epsilon_{ij} \delta L_j > 1$. In other words, the critical value L_j^+ of the forcing is given by

$$L_j^+ = \frac{1}{\epsilon_{ij}}, \quad (3.18)$$

implying that if we can derive it from observations or from appropriate model experiments, we can set $\epsilon_{ij} = \frac{1}{L_j^+}$.

For example let's consider the AMOC. As temperatures increase, this changes the evaporation-precipitation budget over the Atlantic region, which leads to changes in salinity of surface waters. This may lead to a slowdown of the AMOC, and eventually a collapse (Rahmstorf, 2024). This is what is represented by the forcing term $c_1 \delta T_U$. Recent assessments locate the critical temperature for AMOC tipping between 1.4°C and 8°C of warming (Armstrong McKay et al., 2022). This knowledge allows to set $q_+ = T_+$. But the AMOC may also be impacted by the freshwater flux F_{GRIS} coming from the melting Greenland ice sheet. This flux can be expressed in sverdrups as

$$F_{\text{GRIS}} = \frac{S_{\text{pot,GRIS}} A_O}{3600 \cdot 24 \cdot 365 \cdot 10^6} \cdot \dot{x}_{\text{GRIS}} \quad (3.19)$$

and the impact on the AMOC can be included in a coupling term $\epsilon_{\text{AMOC,GRIS}} F_{\text{GRIS}}$. How much change in forcing, and whether meltwater fluxes from Greenland could be sufficient to tip AMOC, is not well known (Weijer et al., 2019). According to an early EMIC inter-comparison project, AMOC could tip with a hosing flux from 0.1 sv to 0.5 sv (Rahmstorf et al., 2005). Recently, van Westen et al., 2024a found a tipping point at 0.5-0.6 sv in CESM but the model has salinity biases and the tipping point may occur for a smaller forcing. Here, based on Rahmstorf et al. (2005), we suppose that the critical meltwater flux F_{GRIS}^+ lies in the range $[0.1, 0.5]$ sv and we can thus sample $\epsilon_{\text{AMOC,GRIS}}$ in $[\frac{1}{0.5}, \frac{1}{0.1}] = [2, 10]$.

Meltwater from the west-Antarctic ice-sheet (WAIS) could also impact the AMOC and we model it through the coupling term $\epsilon_{\text{AMOC,WAIS}} F_{\text{WAIS}}$, with

$$F_{\text{WAIS}} = \frac{S_{\text{pot,WAIS}} A_O}{3600 \cdot 24 \cdot 365 \cdot 10^6} \cdot \dot{x}_{\text{WAIS}} \quad (3.20)$$

In this case it is less clear what the critical flux F_{WAIS}^+ should be. Wunderling et al. (2024) consider the WAIS $\rightarrow$ AMOC interaction as weak to moderate, while it considers the GRIS $\rightarrow$ AMOC interaction strong. Based on this, we decide that the maximum value that $\epsilon_{\text{AMOC,WAIS}}$ can take is a fraction $2/3$ of the maximum value that $\epsilon_{\text{AMOC,GRIS}}$ can take. In this way, for the same meltwater flux, the maximal impact of WAIS on AMOC is smaller than the maximal impact of GRIS, which reflects the assessment from Wunderling et al. (2024). Besides, the sign of the WAIS $\rightarrow$ AMOC interactions is not clear. There are competing effects at play and a melting WAIS could both act to stabilise and destabilise the AMOC. For this reason, we sample $\epsilon_{\text{AMOC,WAIS}}$ in the range $[-\frac{2}{3} \cdot 10, \frac{2}{3} \cdot 10]$. A negative $\epsilon_{\text{AMOC,WAIS}}$ generates a stabilising interaction.

The other interactions are less studied in model experiments. Hence, choosing appropriate δL_j and L_j^+ is harder. As a consequence we choose the simplest case of linear couplings, as in Wunderling et al., 2021. We set $\delta L_j = (1 - x_j)$ such that the forcing from element j on element i is zero at pre-industrial times and maximal when j has completely collapsed ($x_j = 0$). Then, for all element pairs (i, j) except (AMOC,GRIS) and (AMOC,WAIS), we sample ϵ_{ij} in intervals based on the assessed strength and signs of the interactions by Wunderling et al., 2024, following the rule given in figure 3.3.

Here are some examples to clarify our rationale. For a weak destabilizing interaction, such as WAIS $\rightarrow$ GRIS, we sample ϵ_{ij} in the range $]0, 1/3]$, meaning that WAIS alone cannot trigger a collapse of GRIS without additional warming or interactions. In other words, we set $L_j^+ = 1/\epsilon_{ij} > 1$, which exceeds the maximum possible value for δL_j ($\delta L_{j,\text{max}} = 1$). For a moderate destabilizing interaction, such as GRIS $\rightarrow$ WAIS, ϵ_{ij} is sampled in the range $]1/3, 2/3]$, indicating that GRIS alone cannot collapse WAIS, but could do so if warming reaches at least $\delta T_U = \frac{2}{3} \cdot T_{+, \text{WAIS}}$, or even $\delta T_U = \frac{1}{3} \cdot T_{+, \text{WAIS}}$ in the worst cases. For stabilizing interactions, ϵ_{ij} is sampled within negative ranges. If no interaction is identified from element j to i , we set $\epsilon_{ij} = 0$.

For a monostable element i and any influencing element j , a value of $\epsilon_{ij} \geq 1$ implies that element j on its own can push element i past its inflexion point, the stage at which i exhibits the greatest sensitivity to changes in external forcing.

Feedbacks on climate

Tipping elements, or other nonlinear elements may impact the global mean temperature. This can happen for example through additional greenhouse gas emissions for the biospheric elements (AMAZ, PERM) or through changes

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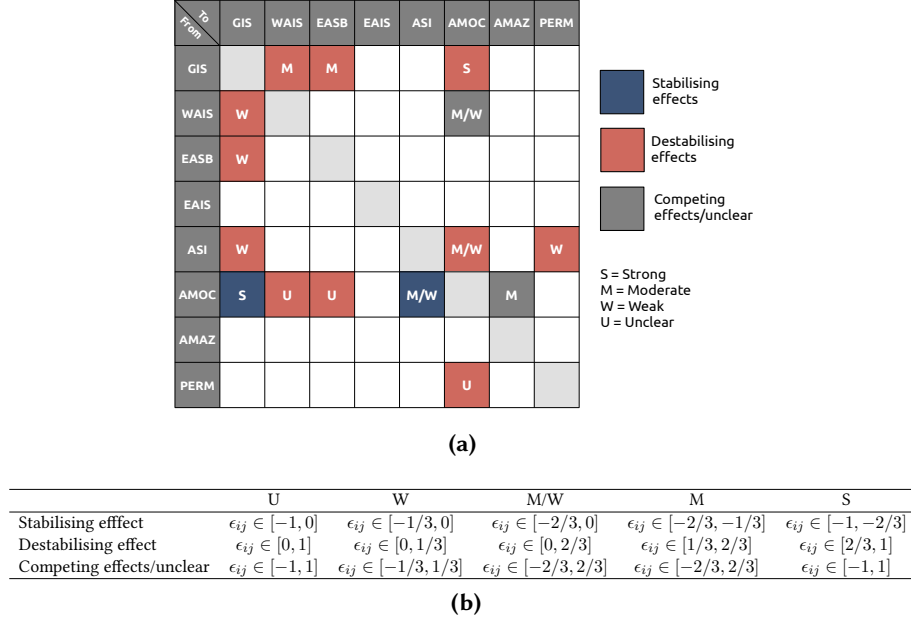


Fig. 3.3 Assessment and parametrisation of the interactions between the elements. *a)* Interactions between tipping elements and other nonlinear components. Colours give the sign of the interaction (Stabilising, Destabilising, Competing effects or unclear), while the letter gives the assessed strength (Strong, Moderate, Weak, or unclear). Adapted from the review on climate tipping point interactions from Wunderling et al., 2024, and more details on specific interactions and their physical mechanisms can be found therein. We have added EASB and we consider it likely to play a similar role in tipping interactions as WAIS, so we have based its interactions with other elements on those involving WAIS. The exception is the EASB→AMOC interaction, which we set to zero, unlike the WAIS→AMOC interaction. This choice reflects the high uncertainty in both the sign and strength of the WAIS-AMOC interaction, making an EASB-AMOC link even less certain. *b)* sampling ranges for the parameters ϵ_{ij} that define interactions in the model, as a function of the assessed sign and strength of the interactions by Wunderling et al., 2024. For the interaction from GRIS to AMOC and from WAIS to AMOC, ϵ_{ij} is sampled in different ranges than the ones provided here, respectively $[2, 10]$ and $[-20/3, 20/3]$.

in albedo for the cryospheric elements (ASI, GRIS, WAIS, EASB, EAIS, and mountain glaciers). We modify the climate and carbon components of SURFER v2.0 and v3.0 to include these processes. The evolution of temperature anomaly in the surface ocean layer, δT_U , which is considered to be in thermal equilibrium

with the atmosphere, is dictated by

$$c_{\text{vol}} h_U \frac{d\delta T_U}{dt} = F(M_A, M_A^{\text{CH}_4}, I) - \beta \delta T_U + \sum_i F_{\text{NE},i}(x_i) - \gamma_{U \rightarrow I} (\delta T_U - \delta T_I). \quad (3.21)$$

The term $F(M_A, M_A^{\text{CH}_4}, I)$ is the radiative forcing due to greenhouse gases and aerosols. Its expression is given by

$$F(M_A, M_A^{\text{CH}_4}, I) = F_{2\times} \log_2 \left(\frac{M_A}{M_A(t_{\text{PI}})} \right) + \alpha_{\text{CH}_4} \sqrt{M_A^{\text{CH}_4} - M_A^{\text{CH}_4}(t_{\text{PI}})} - \alpha_{\text{SO}_2} \exp \left(- (\beta_{\text{SO}_2}/I)^{\gamma_{\text{SO}_2}} \right). \quad (3.22)$$

The first two terms describe the contribution of CO_2 and methane to an increased greenhouse effect. The third term corresponds to solar radiation modification in the form of SO_2 injections. In this chapter, they are always set to zero. The term $\gamma_{U \rightarrow I} (\delta T_U - \delta T_I)$ models heat exchanges with the intermediate ocean layer. The climate feedback parameter β represents the feedbacks of fast processes (centennial time scale) and is related to the equilibrium climate sensitivity (in $^\circ\text{C}$)

$$ECS = \frac{F_{2\times}}{\beta} \quad (3.23)$$

with $F_{2\times}$ the radiative forcing corresponding to a doubling of CO_2 . As in previous SURFER versions, we set $F_{2\times} = 3.9 \text{ W m}^{-2}$ and $\beta = 1.1143 \text{ W m}^{-2} \text{ } ^\circ\text{C}^{-1}$, such that we have an ECS equal to 3.5°C .

The term $\sum_i F_{\text{NE},i}(x_i)$ represents the slower feedbacks from the tipping and other nonlinear elements. It is the only new term that has been added in equation 3.21 compared to SURFER v3.0. For GRIS, WAIS, EASB, and EAIS, we set

$$F_{\text{NE},i}(x_i) = \bar{F}_i (1 - x_i^{4/5}) \quad (3.24)$$

where $\bar{F}_i$ is the top of atmosphere radiative anomaly resulting from the albedo change following the complete removal element i . The $4/5$ exponent converts ice sheet volume to ice sheet surface, based on the volume-area scaling law for ice-caps (Bahr et al., 2015). The parameters $\bar{F}_i$ are calibrated such that the additional warming from a complete removal of the elements in SURFER matches the values $\Delta T_{\text{ref},i}$ given in Wunderling et al., 2020, would SURFER have an ECS of 3°C . In other words, we set $\bar{F}_i = \beta_{\text{ref}} \Delta T_{\text{ref},i}$ with $\beta_{\text{ref}} = \frac{F_{2\times}}{3^\circ\text{C}}$. This is done to account for the fact that Wunderling et al. (2020) obtained mean

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values for the $\Delta T_{\text{ref},i}$ using a model ensemble with a climate sensitivity range of 2.0-3.75°C. With the parameters $\bar{F}_i$ fixed, the effective temperature impacts of elements removal in SURFER will scale with the climate sensitivity of the model, which is set to 3.5°C (in appendixes B and C, we perform experiments with an ECS equal to 5°C). The $\Delta T_{\text{ref},i}$ values from Wunderling et al. (2020) are the ones given in Armstrong McKay et al. (2022) for the maximum impact of ice-sheet tipping elements, and are reproduced in table 3.2.

For ASI, AMAZ and PERM, because they evolve on faster timescales, we consider that their feedback on global mean temperatures through albedo are already included in the climate feedback parameter β . This is also the case for the impacts of AMAZ on temperatures through evapotranspiration and cloud formation.

AMAZ and PERM can additionally impact global mean temperatures through release of CO₂ and CH₄ in the atmosphere. The evolution of the atmospheric CO₂ and CH₄ content (M_A , $M_A^{\text{CH}_4}$) is described by the following equations

$$\frac{dM_A}{dt} = V + E_{\text{fossil}}^{\text{CO}_2} + E_{\text{land-use}}^{\text{CO}_2} + E_{\text{NE}}^{\text{CO}_2} - F_{A \rightarrow U} - F_{A \rightarrow L} + F_{\text{CH}_4, \text{ox}} - E_{\text{natural}}^{\text{CH}_4} - F_{\text{weathering}}, \quad (3.25)$$

$$\frac{dM_A^{\text{CH}_4}}{dt} = E_{\text{fossil}}^{\text{CH}_4} + E_{\text{land-use}}^{\text{CH}_4} + E_{\text{NE}}^{\text{CH}_4} + E_{\text{natural}}^{\text{CH}_4} - F_{\text{CH}_4, \text{ox}}, \quad (3.26)$$

The precise meaning and expression of each term is detailed in Chapter 2. Here, we have added CO₂ and CH₄ emissions ($E_{\text{NE}}^{\text{CO}_2}$ and $E_{\text{NE}}^{\text{CH}_4}$) from nonlinear elements. They are treated as land-use emissions, and are thus taken from the land reservoir (M_L) :

$$\frac{dM_L}{dt} = F_{A \rightarrow L} - E_{\text{land-use}}^{\text{CO}_2} - E_{\text{land-use}}^{\text{CH}_4} - E_{\text{NE}}^{\text{CO}_2} - E_{\text{NE}}^{\text{CH}_4}, \quad (3.27)$$

$$\frac{dM_L^*}{dt} = -E_{\text{land-use}}^{\text{CO}_2}(t) - E_{\text{NE}}^{\text{CO}_2} - E_{\text{NE}}^{\text{CH}_4}, \quad (3.28)$$

The variable M_L^* is an ad-hoc variable that is used in the definition of the atmosphere to land carbon flux $F_{A \rightarrow L}$ and which can be thought of as the amount of carbon that the land can hold for $\delta T_U = 0$. Including $E_{\text{NE}}^{\text{CO}_2}$ and $E_{\text{NE}}^{\text{CH}_4}$ in equation 3.28 means that the carbon loss is irreversible until the permafrost and the Amazon rainforest regrow.

Emissions from nonlinear elements are given by

$$E_{\text{NE}}^{\text{CO}_2} = E_{\text{AMAZ}}^{\text{CO}_2} + E_{\text{PERM}}^{\text{CO}_2}, \quad (3.29)$$

$$E_{\text{NE}}^{\text{CH}_4} = E_{\text{PERM}}^{\text{CH}_4}, \quad (3.30)$$

with

$$E_{\text{AMAZ}}^{\text{CO}_2} = -C_{\text{AMAZ}}^{\text{CO}_2} \cdot \dot{x}_{\text{AMAZ}}, \quad (3.31)$$

and

$$E_{\text{PERM}}^{\text{CO}_2} = \begin{cases} -C_{\text{PERM}}^{\text{CO}_2} \cdot \dot{x}_{\text{PERM}} & \text{if } \dot{x}_{\text{PERM}} < 0, \\ -\left(C_{\text{PERM}}^{\text{CO}_2} + C_{\text{PERM}}^{\text{CH}_4}\right) \cdot \dot{x}_{\text{PERM}} & \text{if } \dot{x}_{\text{PERM}} > 0, \end{cases} \quad (3.32)$$

$$E_{\text{PERM}}^{\text{CH}_4} = \begin{cases} -C_{\text{PERM}}^{\text{CH}_4} \cdot \dot{x}_{\text{PERM}} & \text{if } \dot{x}_{\text{PERM}} < 0, \\ 0 & \text{if } \dot{x}_{\text{PERM}} > 0. \end{cases} \quad (3.33)$$

For permafrost emissions, we differentiate explicitly thawing ($\dot{x}_{\text{PERM}} < 0$) formation ($\dot{x}_{\text{PERM}} > 0$). Indeed, when it thaws, the organic carbon that it contains is released either as CO_2 or as CH_4 , but the organic carbon that slowly accumulates in the active layer during the warm seasons, and that is permanently captured as the soil refreezes, comes from photosynthesis and thus atmospheric CO_2 .

The coefficients $C_i^{\text{CO}_2/\text{CH}_4}$ represent the maximal amount of carbon releasable by element i as CO_2 or CH_4 . Indeed, we have

$$\int_{x_i=1}^{x_i=0} E_i^{\text{CO}_2/\text{CH}_4} dx_i = -C_i^{\text{CO}_2/\text{CH}_4} \int_{x_i=1}^{x_i=0} \dot{x}_i dx_i = C_i^{\text{CO}_2/\text{CH}_4}. \quad (3.34)$$

For AMAZ, we set $C_{\text{AMAZ}}^{\text{CO}_2} = 75 \text{ PgC}$ based on Armstrong McKay et al., 2022. For PERM, estimates of CO_2 and CH_4 emissions are not well constrained. It is generally considered that permafrost soils hold around 1035 PgC of carbon (Schuur et al., 2015) but not all this carbon is necessarily released to the atmosphere upon complete thaw. Here we set $C_{\text{PERM}}^{\text{CO}_2} = 300 \text{ PgC}$ and $C_{\text{PERM}}^{\text{CH}_4} = 7.5 \text{ PgC}$ meaning that around 2.4% of the carbon is emitted as CH_4 . This fraction has been estimated between 2.3% and 2.7% from expert reviews (Schuur et al., 2013; Schuur et al., 2015) but could range from 1 to 12% Schuur et al., 2022, and overall there is low confidence in the relative roles of CO_2 and CH_4 (Canadell et al., 2021a). The total amount of carbon releasable is 307.5 PgC, or 1370 PgCO_2eq (using a 100 year global warming potential for CH_4 relative to CO_2

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equal to 27). This is comparable to the value of 1330 PgCO₂eq cited in the Global Tipping Points report for permafrost carbon release at 10°C warming (Lenton et al., 2023), at which point we can assume that all the carbon has been released.

For the AMOC, changes in the oceanic circulation and exchanges between the surface and deep waters could impact the overall heat and carbon exchanges with the atmosphere. For example, Boot et al., 2024, found in CESM2 and for the SSP5-8.5 scenario an atmospheric pCO₂ decrease of 1.3 ppm per sverdrup decrease in AMOC strength. Furthermore, since AMOC causes a large inter-hemispheric transport of heat, a collapse would cool the North Atlantic and Northern hemisphere, and warm the Southern hemisphere. These initial cooling and warming responses are then amplified asymmetrically by sea-ice albedo feedbacks and other atmospheric feedbacks, which result in a overall decrease in global mean temperature in the first few centuries after the collapse (Drijfhout, 2015b; Laurian et al., 2009). This cooling could be compensated by simultaneous global warming (Drijfhout, 2015a; van Westen et al., 2024b). What happens on longer time scale is less certain. Drijfhout, 2015b show that a reorganisation of radiative transfers between the ocean and atmosphere reduce the cooling and even lead to a slight warming, before returning back to cooling around 700 years after the collapse. Capturing such a complex response is beyond the capabilities of SURFER v3.0, which only includes a 3-layer ocean model, does not include high latitude boxes, nor any horizontal resolution, and does not represent moisture and water vapor in the atmosphere. Consequently, we choose to neglect the global pCO₂ and temperature response of an AMOC collapse. Nevertheless, the local temperature effects of AMOC on other elements (ASI, PERM, GRIS, ...), as well as other climate impacts such as changes in precipitation patterns over the Amazon rainforest are taken into account through the coupling terms in equation 3.3.

3.3 Results

We first assign, for every tipping element in every model run, a *tipping status* which is either *tipped* or *not tipped*. An element i is assigned the status *tipped* if there exists a time t ($1850 < t < 100000$) such that $x_i(t) < x_- = 0.25$; otherwise it is assigned the status *not tipped*. This definition thus considers an element as *tipped* if it transitioned at least once to its collapsed state (lower equilibrium branch) during the model run, irrespective of whether the element

transitioned back to its original state due to decreasing forcings. This is avoided in the baseline experiment because even after 100 kyr and even under low emissions scenarios, temperatures are still above the back tipping threshold $T_- = 0$, due to the long residence time of CO_2 in the atmosphere. However, when including interactions between elements, some might transition back to their original state due to the combined effect of decreasing temperatures and stabilising interactions. In our experiments, this only happens for the Amazon rainforest, which sometimes tips back thanks to a stabilising action of the AMOC.

We perform two types of analysis. In section 3.3.1 we compare the tipping status of elements between different experiments. This allows to investigate how much interactions between elements or their feedbacks on temperature impact the tipping risk. In section 3.3.2, we compare the elements tipping status in successive scenarios of a given experiment, allowing us to define and investigate tipping cascades.

3.3.1 Individual tipping elements

We begin by examining the factors that govern the tipping behaviour of an element in the baseline experiment, providing a basis for comparison when investigating the roles of interactions and feedbacks. With T_- and x_+ fixed to the same value for all tipping elements, we will focus on their critical temperature T_+ and their internal timescales τ_- (and τ_+).

In the baseline experiment, each element is forced by temperature only ($\delta q = \delta T_U$), which is determined by the emission scenario. Under a constant temperature forcing, an element will tip if $\delta q > q_+$ or $\delta T_U > T_+$. In other words, the tipping status is, in this case, determined only by T_+ . However, when the temperature varies in time, as for our emissions scenarios, tipping can be avoided even if the temperature exceeds the critical threshold T_+ , provided that the overshoot time is short compared to the intrinsic timescale of the element (Ritchie et al., 2019; Ritchie et al., 2021). An example of this behaviour is shown in Figure 3.4, which depicts the evolution of AMOC and GRIS under the 2000 PgC emission scenario, with both elements having the same critical temperature threshold, $T_+ = 2^\circ\text{C}$. Initially, the temperature rises quickly due to anthropogenic emissions and peaks at 4.07°C , well above the critical threshold for AMOC and GRIS, then gradually declines due to natural CO_2 sinks. AMOC tips, as the overshoot duration is long relative to its internal

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timescale, whereas GRIS remains stable due to its slower dynamics. In such cases, the tipping status of an element depends both on T_+ and τ_- .

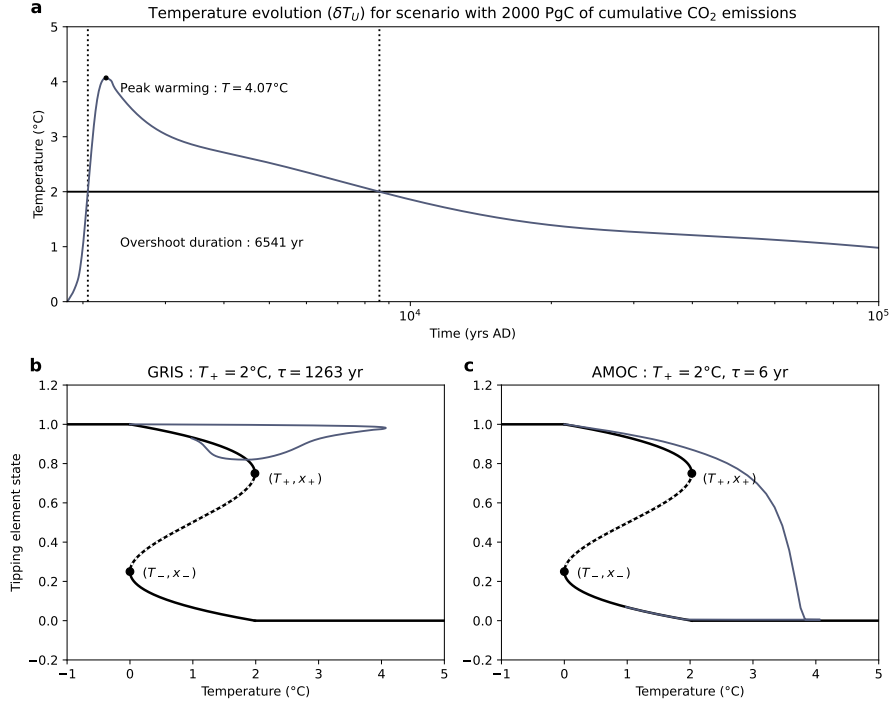


Fig. 3.4 a) Temperature evolution (δT_U) for the scenario with 2000 PgC of cumulative CO₂ emissions. The x-axis has a logarithmic scale. b-c) Evolution of two tipping elements, GRIS and AMOC, under the same scenario, with same critical thresholds but different intrinsic timescales. Both elements overshoot their critical threshold, but only AMOC tips, because it is a fast tipping element. These examples are from the baseline experiment (i.e. no interactions between elements and no feedbacks from elements on temperature).

In overshoot scenarios, Ritchie et al., 2019 showed that an element will avoid tipping if

$$4a_0^2\kappa(q_{\max} - q_+)t_{\text{over}}^2 < 16 \quad (3.35)$$

where $q_{\max}$ is the maximal value of the forcing, t_{over} is the overshoot duration and $4a_0^2\kappa$ is element dependent quantity which can be approximated from observational data or obtained from the equations that describe the evolution of the element, as in our case :

$$a_0 = \left. \frac{\partial f(x, \delta q)}{\partial \delta q} \right|_{(x_+, q_+)} \quad (3.36)$$

$$\kappa = \frac{1}{2a_0} \frac{\partial^2 f(x, \delta q)}{\partial x^2} \Big|_{(x_+, q_+)} \quad (3.37)$$

For the baseline experiment ($\delta q = \delta T_U$), and replacing x_+ and x_- by their fixed values, this gives us

$$\frac{3}{16} \left(\frac{T_{\max} - T_+}{T_+} \right) \left(\frac{t_{\text{over}}}{\tau_-} \right)^2 < 16 \quad (3.38)$$

This criterion is only valid when the forcing change is slow enough compared to the dynamics of the element. In other words, it will be most accurate for fast elements such as the AMOC or the Amazon rainforest.

Figure 3.5a illustrates the theoretical "Ritchie" tipping thresholds (dotted lines) derived from inequality 3.38 for different emission scenarios. These thresholds delineate regions in the (τ_-, T_+) plane, separating parameter pairs (τ_-, T_+) that satisfy inequality 3.38 from those that do not. The thresholds depend not only on τ_- and T_+ but also on the emission scenario through $T_{\max}$ and t_{over} . In principle, they should also depend on T_- and x_+ , but, in this study, we have set these parameters to the same values for all tipping elements. For a given scenario, parameter pairs (τ_-, T_+) below the corresponding threshold satisfy inequality 3.38, and theory predicts that tipping elements with such parameter combinations will tip under that scenario. Conversely, parameter pairs above the threshold indicate that tipping elements will not tip under the associated emission scenario.

As an example, we plotted the parameter combinations of GRIS and AMOC from Figure 3.4. In this case, the pair $(\tau_{-, \text{GRIS}}, T_{+, \text{GRIS}})$ lies above the theoretical threshold for the 2000 PgC emission scenario, and GRIS does not tip under this scenario. Conversely, the pair $(\tau_{-, \text{AMOC}}, T_{+, \text{AMOC}})$ lies below the same threshold, and AMOC tips under that scenario. In this instance, the theoretical prediction aligns with the observed tipping outcomes.

To assess whether this agreement holds more generally, we compare the theoretical thresholds with the empirical tipping thresholds (solid lines in Figure 3.5a), which were determined using a bisection algorithm. Overall, for the scenarios considered, the theoretical thresholds align well with the observed tipping thresholds. This alignment is particularly strong for tipping elements with timescales shorter than or comparable to the timescale of global warming (~ 100 years), where the assumptions made to derive inequalities 3.35 and 3.38 hold true (Ritchie et al., 2019).

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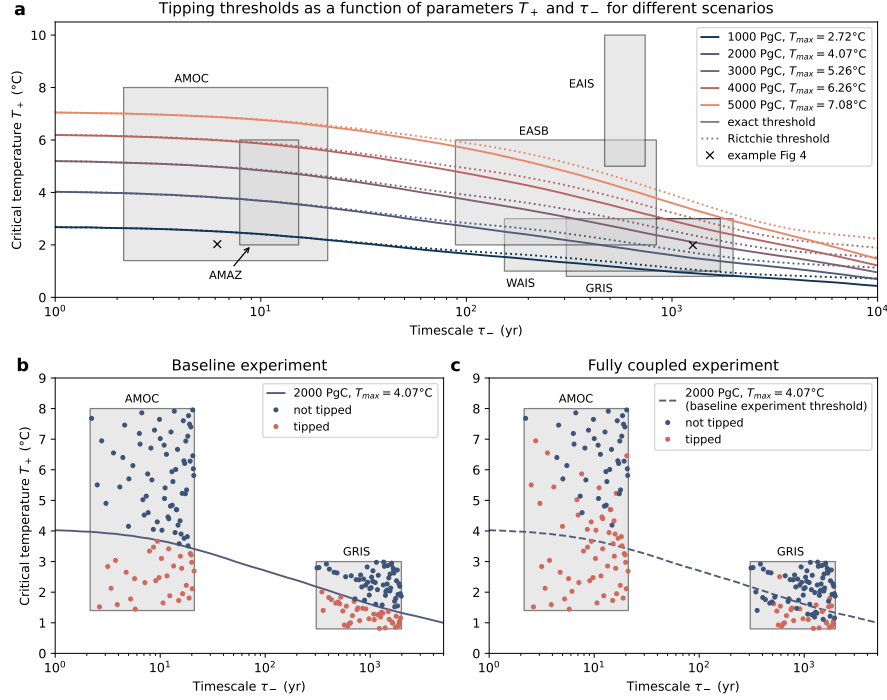


Fig. 3.5 a) Tipping thresholds for the tipping elements, as a function of their critical temperature T_+ and their intrinsic time scale τ_- . The gray rectangles indicate the range of possible parameter values for each element. The plain coloured lines represent the tipping thresholds for different scenarios in the baseline experiment (interactions between tipping elements and their feedback on temperature are not included). An element with a (T_+, τ_-) combination that is below a given threshold will tip for this scenario. The coloured dotted lines are the theoretical predictions for the tipping thresholds based on the criterion from Ritchie et al., 2019, see equation 3.38. b-c) Zoom on the tipping threshold for the scenario with 2000 PgC of cumulative emissions, and for the AMOC and GRIS. The dots represent parameter a hundred combinations from our ensemble of parameter sets, and their colour represent the element tipping status. Panel (b) is for the baseline experiment, while panel (c) is for the fully coupled experiment. In the latter case, the tipping threshold, which was computed for the baseline case, doesn't neatly separate tipping and non tipping behaviours. This indicates that including interactions between elements and their feedbacks on temperatures modifies the effective tipping threshold.

Based on the tipping thresholds, we observe that AMOC and AMAZ are likely to tip if peak warming exceeds their critical temperature threshold. As they are fast-tipping elements (with low τ_-), even small and brief overshoots may lead to tipping. This contrasts with slower elements. For example WAIS and GRIS are unlikely to tip under the 1000 PgC scenario, where temperatures peak at 2.7°C . Even in the 4000 PgC scenario, where peak warming reaches

6.26°C, well above the highest possible threshold for GRIS and WAIS ($T_+ = 3^\circ\text{C}$), they may still avoid tipping. For GRIS, this stability for high warming scenario has been confirmed in state-of-the-art ice sheet models, provided that temperatures decrease sufficiently fast after peaking (Bochow et al., 2023; Höning et al., 2024).

Figures 3.5b and 3.5c present sampled combinations of (T_+, τ_-) from our parameter ensemble for AMOC and GRIS, indicating whether these elements tip under the 2000 PgC emission scenario for both baseline and fully coupled experiments. In the baseline experiment the computed tipping threshold effectively separates between tipped and non tipped elements, by construction. In this case, the tipping risk for an element under a given scenario—defined as the percentage of sampled parameter combinations that lead to tipping—approximates the fraction of the element’s possible parameter space (represented by the gray rectangle) that falls below the scenario’s tipping threshold.

In the fully coupled experiment, Figure 3.5c shows that interactions between elements and their feedbacks on temperature causes to AMOC and GRIS tipping in cases where they would otherwise not tip. For GRIS, we also observe the opposite effect, with cases where it tips in the baseline experiment but doesn’t in the fully coupled one, indicating that interactions may both destabilise and stabilise elements. Identifying a clear tipping threshold in (T_+, τ_-) space is no longer possible as the tipping behaviour now depends on additional parameters such as the coupling coefficients ϵ_{ij} . Nevertheless, we can still compute the tipping risk.

Figure 3.6 shows the tipping risk of each element as a function of cumulative emissions across four experiment setups: baseline, interactions only, feedbacks only, and fully coupled. By comparing the fully coupled experiment to the feedbacks only, and the interactions-only experiment to the baseline, we observe that interactions consistently increase the tipping risk for WAIS, EASB, and AMOC across all emission scenarios, with the most pronounced effects at intermediate emission levels (2000–3000 PgC). In low-emission scenarios (<1000 PgC), tipping events are rare, so interactions have minimal impact. Under high emission scenarios (>4000 PgC), elements have high chances to tip regardless of whether we consider interactions. For GRIS, interactions reduce the tipping, due to the strong stabilising influence of AMOC. For the AMAZ, the risk differs little whether interactions are considered or not, as AMOC is the only element affecting it, and its impact can be stabilising or destabilising. This means in this case that while interactions significantly affect individual simulations, they have a minimal effect on aggregated risk. In

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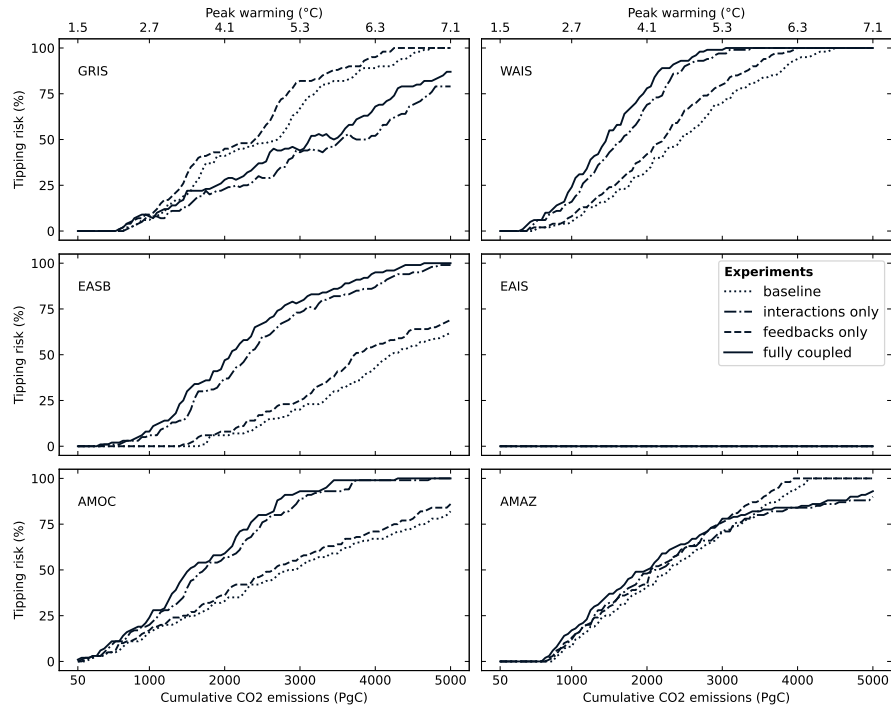


Fig. 3.6 Tipping risk for the different tipping elements, as a function of cumulative emissions and for different experiments (baseline, fully coupled, interactions only, feedbacks only). For a given element and scenario, the tipping risk is defined as the percentage of ensemble simulations generated with different parameter sets in which this element tips. For reference, the peak warming corresponding to a given level of emissions in the baseline experiment is indicated on the upper axis of the plots.

high-emission scenarios (>4000 PgC), interactions slightly diminish the AMAZ tipping risk, likely because AMAZ tipping becomes nearly inevitable without AMOC influence, so the latter only impacts the tipping behaviour when it is stabilising, reducing the aggregated tipping risk. In these experiments, EAIS never tips, thanks to its combination of high temperature threshold and slow dynamics (see Figure 3.5), and because in this model, it isn't impacted by other elements, except indirectly through global mean temperature.

Comparing the fully coupled experiment to the interactions only, and the feedbacks-only experiment to the baseline, shows that including feedbacks of elements on global mean temperature always increases the tipping risk. Indeed, all else being equal, increasing temperatures has a destabilising impact on elements in our model ($c_1 < 0$ by design, see equations 3.3, 3.7, and 3.13). While the impact of feedbacks on tipping risk is smaller than that of direct interactions,

it can still be substantial for a given scenario and element, increasing tipping risk by up to 10%. In appendix B.3, we further test tipping risk sensitivity to an increased ECS, doubled interaction strength, and the exclusion of monostable elements ASI and PERM.

In figure 3.7, we present the tipping risk for each element in both the baseline and fully coupled experiments using "burning embers" diagrams, and compare with the tipping risks that would result from constant temperature scenarios. Under these constant temperature scenarios, the risk only depends on the warming and the critical temperature threshold T_+ of the elements, which is often the assumption made, explicitly or implicitly, when such "burning embers" plots appear in the literature and media (see for example: Armstrong McKay et al., 2022; Schellnhuber et al., 2016; Wunderling et al., 2021). This communicates risk simply and efficiently, but may misrepresent the risk for the slower elements. Indeed, as shown in Figure 3.7, the tipping risk in the baseline experiment for GRIS, WAIS, EASB, and EAIS is significantly lower than that predicted by a constant temperature scenario with equivalent peak warming. In the fully coupled experiment, the risk further changes as described above, highlighting the importance of considering both non-stationary temperature dynamics and interactions between the elements.

3.3.2 Tipping cascades

In this section we compare the tipping outcomes in all pairs of successive scenarios, i.e., scenarios that are separated by only 50 PgC of CO₂ emissions. This allows us to define *tipping cascades*. We order scenarios by increasing emissions and index them such that scenario j corresponds to the scenario where $j \cdot 50$ PgC of CO₂ emissions are added after 2015. In a given experiment and for a given set of parameters, we consider that there is a *tipping event* for scenario j if at least one tipping element has changed status in scenario j compared to in scenario $j - 1$. If only one element has changed status, we classify the event as a *single tipping event*. If more than one element has changed status, we call it a *tipping cascade*. The number of elements whose status have changed is the *size of the cascade*. If all the elements whose status have changed went from "not tipped" for scenario $j - 1$ to "tipped" for scenario j , we call it a *destabilising cascade*. If at least one element went from "tipped" to "not tipped", we call it a *stabilizing cascade*, regardless of the number of elements that were destabilised.

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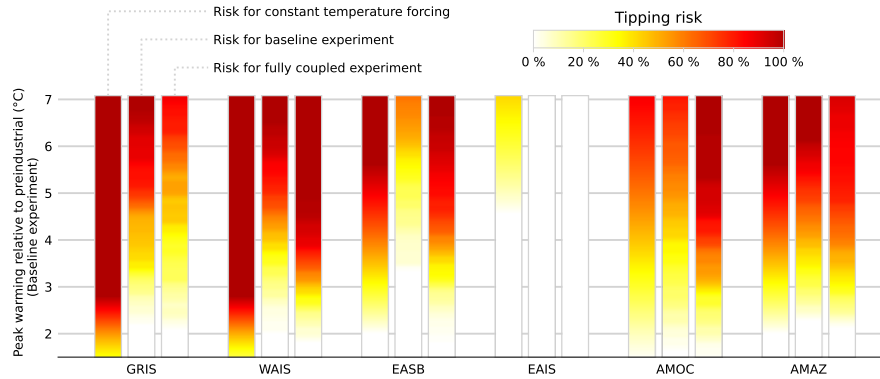


Fig. 3.7 Tipping risk as a function of peak warming for the baseline and fully coupled experiments, compared to the tipping risk that would result from constant temperature scenarios. In the latter case, the constant temperature is also the peak temperature, and the tipping risk is : 0% if the temperature T is lower than the minimum possible temperature threshold for the element $T_{+,i}^{min}$; 100% if the temperature T is above the maximum possible temperature threshold $T_{+,i}^{max}$; and equal to $100 \cdot (T - T_{+,i}^{min}) / (T_{+,i}^{max} - T_{+,i}^{min})$ if $T_{+,i}^{min} \leq T \leq T_{+,i}^{max}$. In the baseline and fully coupled experiments, the tipping elements experience a time varying temperature and the relationship between peak warming and tipping risk is more complex. In the fully coupled experiment, peak warming furthermore depends on the state of the elements and may not always be equal for the same emission scenario. To simplify the analysis, we have plotted for both experiments the risk as a function of cumulative emissions, and indicated on the y-axis the corresponding peak warming from the baseline experiment, where warming does not depend on the state of the tipping elements. Note that, as demonstrated in Chapter 4, the difference in peak warming between the baseline and fully coupled setups for a given model run is rather small and stays below 0.5°C (see Figure 4.2).

What cascades do we observe ?

Figure 3.8 shows an example of a specific cascade, while Figure 3.9 shows all the observed tipping events in the fully coupled experiment. Single tipping events are the most common, accounting for 77 % of all events, while the remaining 23% are tipping cascades. We observe no cascades of size 5 or more. The most frequent cascades involve either the AMOC and the ice sheets or only the ice sheets. Stabilizing cascades make up approximately 24% of all cascades, primarily driven by the stabilization of GRIS due to AMOC's influence. Notably, we don't observe cascades where the AMAZ is stabilised by AMOC. This is because AMAZ is a fast tipping element, and in a scenario where AMOC changes tipping status, the latter generally tips too late to prevent AMAZ from tipping. For similar reasons, we do not observe cascades where GRIS tipping leads to the AMOC tipping, despite it being often mentioned as a

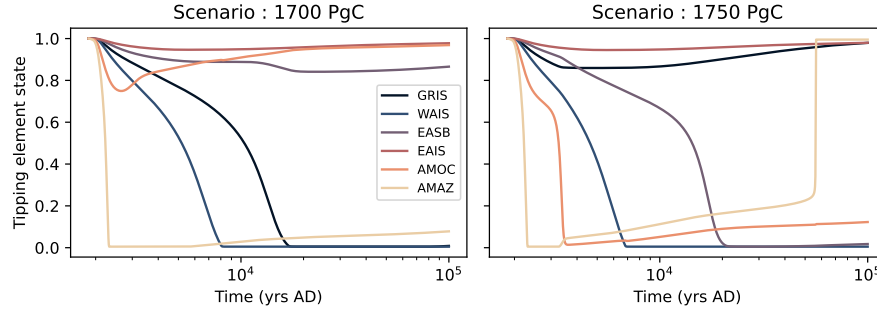


Fig. 3.8 Example of a tipping cascade happening in the fully coupled experiment for a given set of parameters. In the 1700 PgC scenario, GRIS, WAIS, and AMAZ are tipped. When 50 PgC of emissions are added, AMOC and EASB additionally tip, while GRIS does not tip anymore due to the influence of the AMOC. In this case, we have a size-3 stabilising cascade for the 1750 PgC emission scenario. In the 1750 PgC scenario, AMAZ returns to its baseline state after 50 kyrs because of the stabilisation effect from AMOC and the temperatures slowly decreasing due to the natural uptake of CO₂ by carbon sinks. Nevertheless, AMAZ is counted as "has tipped" since it reached its collapsed state for some time.

typical example of tipping cascade (Lenton et al., 2019; Steffen et al., 2018). In our model, this absence results from our definition of tipping cascade, which requires changes between two very close scenarios. If GRIS doesn't tip in scenario $j - 1$, but tips in scenario j due to the slight increase in temperature, its trajectory will pass very close to the bifurcation point, where the dynamics are slow. This has two consequences : (1) the meltwater flux produced ($\propto x_{\text{GRIS}}$) is small, with a limited impact on AMOC; (2) if the AMOC does tip, it will do so on a shorter time scale than GRIS, and its stabilising influence will be enough to revert GRIS's tipping, excluding GRIS from the cascade count. However, not observing a destabilising GRIS-AMOC cascade does not mean that GRIS has no influence on AMOC. As shown in Figure 3.6, interactions with other elements, including GRIS, increase the tipping risk for AMOC. Overall, these specific examples emphasise once more the critical role of time scales when considering realistic scenarios with non-stationary temperatures.

At which levels of emissions do we observe most cascades?

Figure 3.10 shows the distribution of tipping events among the scenarios in the fully coupled experiment. Figure 3.10a focuses specifically on cascades. We observe that the majority (76%) of cascades happen for scenarios with between 1000 PgC and 3000 PgC of added cumulative emissions, or with between $\sim 2.7^\circ\text{C}$

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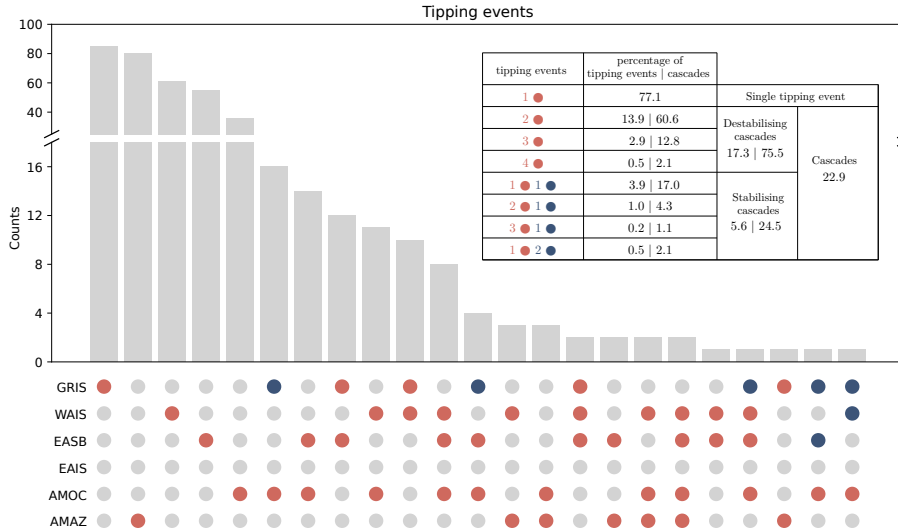


Fig. 3.9 *Tipping events frequency for the fully coupled experiment. The dots represent the tipping status change of each element. For a tipping event happening for scenario j , a red dot indicates that the element does not tip in scenario $j - 1$ but tips in scenario j (destabilisation) ; a blue dot indicates that the element tips in scenario $j - 1$ but does not tip in scenario j (stabilisation) ; and a grey dot indicates that the element doesn't change tipping status between two successive scenarios. The inset table shows the fraction of events that are classified as single tipping events, cascades, stabilising cascades or destabilising cascades. When two numbers are given, the first one indicates the percentage over all tipping events, and the second one indicates the percentage over all tipping cascades.*

and $\sim 5.3^{\circ}\text{C}$ of peak warming. This range is a bit higher than the $1\text{-}3^{\circ}\text{C}$ range observed for the bulk of cascades in Wunderling et al., 2021, because we use more realistic non-stationary scenarios, rather than constant temperature scenarios. About 16% of observed cascades occur in scenarios with emissions up to 1000 PgC . This is equivalent in our model to a peak warming of up to 2.7°C , which is the temperature anomaly we are expected to reach in 2100 under current policies (Climate Action Tracker, 2023). We observe very few cascades happening for high emissions scenarios, as it is very probable that all tipping elements have already tipped by then.

There is a peak in the frequency of size-2 destabilising cascades for scenarios with cumulative emissions between 1250 and 1500 PgC. About half of this peak (7 out of 16 cascades) is due to the destabilisation of the East Antarctic subglacial basins (EASB) by the AMOC. While this cascade is the most frequent overall (16 occurrences, see Figure 3.9), it remains unclear why

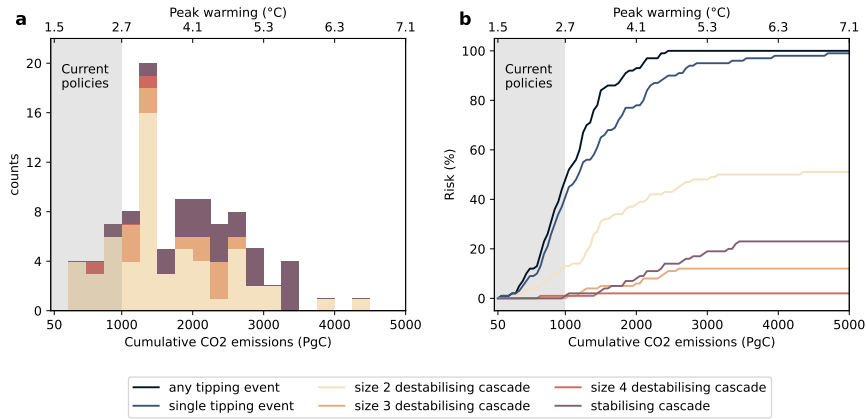


Fig. 3.10 *Timing of tipping events. A tipping event is considered to happen for scenario j if some tipping elements change status compared to scenario $j - 1$. a) distribution of tipping cascades as a function of cumulative emissions in the fully coupled experiment. b) Risk of observing a given tipping event before a given amount of cumulative emissions. For reference, the peak warming corresponding to a given level of emissions in the baseline experiment is indicated on the upper axis of the plots. The shaded areas correspond to scenarios with cumulative emissions lower or equal to 1000 PgC. In our model, these scenarios lead to a peak warming lower or equal to 2.7°C which is the warming expected by 2100 under current policies.*

it is concentrated within this emission range. One possible explanation is that, for these scenarios, reaching the EASB tipping point becomes almost possible from the temperature rise alone (baseline case, see Figures 3.5 and 3.6), and AMOC, whose tipping risk is increasing rapidly, provides the additional push needed. However, more model runs are needed to determine if this is the correct explanation and if the frequency peak is a robust feature or a statistical anomaly.

Figure 3.10b shows the risk of various tipping events occurring for scenarios with cumulative emissions lower or equal to a given value, in the fully coupled experiment. For a given event type and emissions level X , the risk is here defined as the percentage of parameter sets which lead to at least one such tipping event in any scenario with cumulative emissions less than or equal to X . Figure 3.10b is similar but not equivalent to the "integration" of Figure 3.10a, as for a given parameter set the same tipping events may occur in multiple scenarios. We find that the risk of observing tipping events occurring in scenarios with cumulative emissions of up to 1000 PgC is 48%, mainly due to single tipping events. The risks of observing a size 2, 3, or 4 destabilising

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cascade is 13%, 0%, and 1%, respectively, while the risk of a stabilizing cascade remains very low at 1%. In appendix B.3, we examine how these numbers change for higher climate sensitivity, stronger interactions, and with the removal of ASI and PERM. The risk of observing tipping events increases rapidly with emissions : for scenarios up to 2000 PgC (corresponding to peak warming up to 4.1°C), the risk of observing tipping event increases to 93%, while the risk of observing a size 2 destabilising cascade increases to 38%.

What are the roles of elements in a cascade ?

Understanding the roles of tipping elements within a cascade, especially those that initiate it, is instructive. It is valuable from a theoretical perspective, to clarify the dynamics of cascades, but also from a very practical standpoint, as these elements should be carefully monitored in priority, and if possible managed.

Wunderling et al., 2021, define the initiator of a tipping cascade as the element of the cascade with the critical temperature threshold closest to the constant temperature forcing for which the cascade occurred. With this definition, they find that GRIS and WAIS are often-times the initiators of cascades, which results from them having the lowest critical temperatures. Using the same model as Wunderling et al., 2021, but different analysis techniques, Rosser et al., 2024 reach the similar conclusion that GRIS and WAIS are the most decisive elements influencing tipping risks and cascades. However both studies use constant temperature forcings, under-representing the influence of elements' timescales, as shown in section 3.3.1. To address this, we take a different approach, outlined below.

We define an element i as an initiator of a tipping cascade occurring under scenario j if it meets the following criteria: (1) element i changes tipping status solely due to the increase in anthropogenic emissions from scenario $j - 1$ to j ; and (2) this change in i triggers the tipping of another element $k \neq i$. Condition (1) can be verified by rerunning the model with all other elements $k \neq i$ held at their values from scenario $j - 1$. Condition (2), however, is more challenging to test, as it requires a substantial number of targeted experiments that selectively enable and disable specific interactions and feedbacks, a level of analysis not conducted in this study. We therefore focus on condition (1) alone and define elements that meet it as *potential initiators*.

A potential initiator does not tip due to changes in other elements, but may not necessarily be the primary driver of a cascade. The cascade could also be driven by other elements that change tipping status under the same

scenario, or even by elements not counted as part of the cascade, including sea ice and permafrost, that have been sufficiently altered by the emissions increase to influence the tipping status of others. While this approach has limitations, it still provides valuable insights into cascade dynamics, as shown in Figure 3.11.

We observe that AMOC is the element that appears the most in cascades, followed by GRIS, EASB and WAIS. This is not surprising as these elements are tightly coupled (see figure 3.3a). We find that AMOC is a potential initiator in most of the cascades it appears in, and in more than 50% of all cascades, which makes it the most frequent potential initiator overall. The next most frequent potential initiator of cascades is GRIS, which is a potential initiator in about 20% of all cascades, and in a bit less than half of the cascades it appears in. AMAZ appears in fewer than 15% of cascades but serves as a potential initiator in most of them, despite only affecting other elements through a relatively small feedback on global mean temperature. A deeper analysis shows that in most cascades where AMAZ is a potential initiator, another element also acts as a potential initiator, illustrating the limitations of our approach in determining the precise causal sequence of events within a cascade.

We obtain some further insights by separating the analysis between stabilising and destabilising cascades. AMOC and GRIS appear in all stabilising cascades. Since GRIS is stabilised, it cannot by definition be a potential initiator in these cascades. In contrast, it is a potential initiator of most destabilising cascades it appears in. On the other hand, AMOC is a frequent potential initiator in both stabilising and destabilising cascades, underlining its significant role in tipping dynamics.

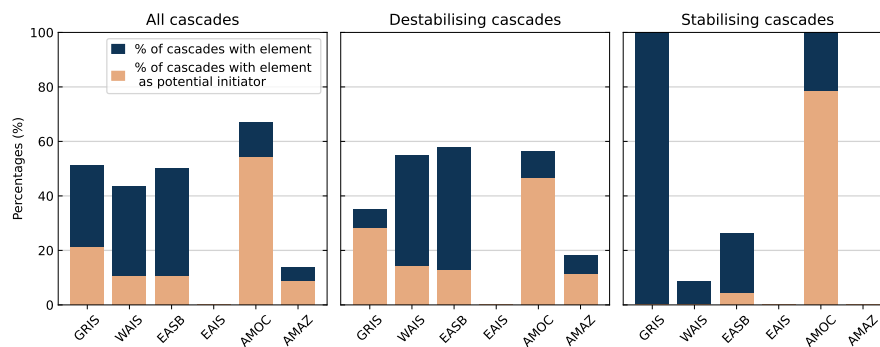


Fig. 3.11 Occurrence and roles of the tipping elements in cascades for the fully coupled experiment. Left plot is for all cascades. Middle and right plots consider the specific cases of destabilising and stabilising cascades.

3.4 Discussion and conclusion

Our objective was to explore tipping cascades using a simple modelling framework that integrates essential physics and latest knowledge on tipping elements and their interactions. The extended version of SURFER presented here reflects that approach, providing a fast model with a dynamic carbon cycle, enabling us to conduct multi-millennial, emission-based simulations. The parametrization of nonlinear elements is flexible, accommodating 6 bi-stable tipping elements (GRIS, WAIS, EASB, EAIS, AMOC and AMAZ), as well as 2 mono-stable nonlinear elements (ASI and PERM). This is, in total, twice as much elements as existing similar studies (Möller et al., 2024; Rosser et al., 2024; Wunderling et al., 2021; Wunderling et al., 2022). Key parameters—such as critical temperature thresholds, timescales, interaction strengths, and feedback effects on global mean temperature—are calibrated based on the most recent literature. Uncertainties in these parameters are taken into account using a large Monte-Carlo ensemble.

With this framework, we explored tipping dynamics and cascades across a range of emission scenarios. Our findings highlight that, in addition to critical temperature thresholds, the intrinsic timescales of individual elements play a substantial role in determining tipping risk. Slow-tipping elements, in particular, can avoid tipping even under substantial and prolonged overshoots of their critical temperature thresholds. While these findings align with well-established theory (Ritchie et al., 2019) and observed behaviour in both conceptual models (Wunderling et al., 2022) and complex model studies (Bochow et al., 2023; Höning et al., 2024), we note that this aspect is often overlooked in risk assessments, which tend to focus mainly on temperature thresholds and the likelihood of crossing them under projected warming.

Our results also show that including interactions between elements increases the tipping risk for WAIS, EASB, and AMOC and reduce tipping risk for GRIS. These findings support previous work by Wunderling et al. (2021). Additionally, including feedbacks from tipping elements on global mean temperatures raises the tipping risk across all elements, though this effect remains secondary to the influence of interactions. This aligns with other studies suggesting that feedbacks from tipping elements on global temperature have a limited effect on increasing the chances of crossing tipping points (Deutloff et al., 2023). Overall, we found a 48% risk of observing a tipping event for a scenario with peak warming less than or equal to 2.7°C, which is the warming

expected in 2100 under current policies. This risk is similar to the one given in other studies that consider interactions only (Möller et al., 2024; Wunderling et al., 2022). In our model, it results mainly from the risk of observing single tipping events. Nevertheless, we observe that cascades can occur within 2.7°C of peak warming, albeit with a reduced risk compared to Wunderling et al. (2021), who found the majority of cascades to occur between 1°C and 3°C of warming. This discrepancy stems from our use of realistic, non-stationary scenarios instead of constant-temperature scenarios.

We found AMOC to be the main potential imitator of cascades, which is concerning given that some studies project its tipping within this century with a 50% likelihood. This is in contrast with the results from Wunderling et al. (2021), who identified GRIS and WAIS as the main initiators of cascades. One of the reasons for this difference is that we do not observe cascades where GRIS destabilises the AMOC. These cascades, frequent in Wunderling et al. (2021), would provide numerous instances where GRIS acts as an initiator while AMOC does not. It is probable that Wunderling et al. (2021) observes a lot of these cascades because they model the impact of GRIS on AMOC as proportional to x_{GRIS} , whereas we model it as proportional to $\dot{x}_{\text{GRIS}}$.

While our definition of a tipping cascade is similar to the one used by Wunderling et al. (2021), it differs slightly from the term's broader use in the literature. By comparing tipping status across closely spaced emission scenarios, our approach highlights how even minimal changes in forcing can result in distinct Earth system states, where one, two, three, or more elements have tipped. We use scenarios separated by 50 PgC of emissions, which corresponds to only five years of emissions at today's rates. This type of cascade aligns with definitions of joint or domino cascades by Klose et al. (2021). However, if only one element changes status for a given scenario in the fully coupled experiment but would not have tipped under the same scenario in the baseline experiment, we do not count it as a cascade, despite the element tipping with the aid of interactions. This would align with the definition of a two-phase cascade by Klose et al. (2021), where the tipping of one element brings another one closer to its bifurcation point, but additional forcing is needed to trigger its tipping.

While the results of this study provide insights into tipping dynamics and cascades, they inevitably rest upon certain assumptions and simplifications. Using a unique equation to represent all elements allows a streamlined analysis and easy comparison between elements, which differ by only a few parameters. However, this equation does not fully capture the complex dynamics of real

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worlds tipping elements. For example it only allows for bifurcation-tipping, i.e., the forcings needs to go past the bifurcation point to trigger tipping. For some elements like the AMOC, the rate of forcing change may also have an effect on whether the element tips or not, which may impact the cascading dynamics with the ice sheets (Klose et al., 2024; Sinet et al., 2023). Tipping elements may also have a multitude of intermediate states (Bastiaansen et al., 2022). This was shown in some models for the GRIS (Höning et al., 2023) and the AMOC (Lohmann et al., 2023), or in vegetation systems (Rietkerk et al., 2021). Additionally, real-world interactions between elements are certainly more complicated than simple linear dependencies.

The parameter calibration could also be improved. Due to the lack of information available, we have fixed arbitrarily the values of T_- , x_- , and x_+ . A better calibration of the "back tipping" points (T_- , x_-) would enable the study of the long term reversibility of cascades. For these matters, we encourage the community to report more on these quantities, for example in future literature reviews and assessments. Furthermore, the calibration of the coupling coefficients based on the assessments of interactions strengths from Wunderling et al. (2024) is somewhat arbitrary. As explained in section 3.2.4, targeted experiments in more complex models with isolated forcings could help to provide better calibration. Future experiments made in the contexts of projects such as TipESM or TIPMIP may also provide further constraints on the distributions of the critical thresholds T_+ and timescales τ_- . Using formal distributions for the parameters would allow to interpret our tipping risks as probabilistic risk assessments.

Finally, it should be noted that the impacts of tipping elements may take several centuries to unfold, or even millennia. In this study we only looked at whether the elements tipped or not during the 100 000 year span of the model runs, but didn't focus specifically on the tipping times nor impacts. When we talk about cascade that occur for scenario with peak warming below of 2.7°C, it doesn't mean that all the elements will have transitioned to their collapsed state by the time the temperature reaches 2.7°C. We'll examine the impact of cascades in the next chapter, specifically their impact on global mean temperatures and sea level rise, to examine if they could lock us on a pathway towards a hothouse Earth.

4

Future Earth system trajectories

4.1 Introduction

In Chapter 2, we introduced SURFER v3.0, a reduced complexity model which includes a process-based carbon cycle and reliably estimates CO₂ concentrations, global mean temperature, sea-level rise, and many ocean acidification metrics over timescales from decades to millions of years (Couplet et al., 2024b; Martínez Montero et al., 2022). In Chapter 3, we have incorporated a network of interconnected tipping elements that influence the climate through albedo changes and additional greenhouse gas emissions into the model. This has allowed the investigation of tipping cascades. We are now ready to explore the future Earth system trajectories and the prospect of a hothouse state.

We use the same experimental setup as in Chapter 3, recapped in Section 4.2, but focus on the impacts of tipping events and cascades on temperature and sea-level rise. In this Chapter, we specifically aim to address our main research questions (see Section 1.3), which drive our investigation and shape our findings in Section 4.3:

- To what extent do tipping events and other nonlinear feedbacks increase the risk of reaching a Hothouse state? A Wethouse state? Could this outcome occur under low-emission scenarios? (Section 4.3.1)

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- To what extent do tipping events and other nonlinear feedbacks create a divergence in Earth system trajectories? In other words, is there a threshold that prevents stabilisation at intermediate temperature or sea-level rise? (Section 4.3.2)

In Section 4.4, we discuss our results and our the limitations of our approach.

Before proceeding, we emphasize that this Chapter does not address runaway greenhouse scenarios, sometimes mistakenly confused with Hothouse Earth scenarios in science communication. Runaway greenhouse warming occurs when increasing radiative forcing causes a planet's moist atmosphere to reach a theoretical limit on the long-wave radiation it can emit. Beyond this limit, achieving radiative balance requires the planet's surface to warm by several hundred degrees, allowing it to emit radiation at shorter wavelengths in the near-infrared, where water vapour is less effective as a greenhouse gas (Goldblatt and Watson, 2012). This process would cause the oceans to evaporate entirely, leaving behind an atmosphere akin to that of Venus: hot, dense, and hostile to life. Such an event has not occurred in Earth's history, even during periods of significantly elevated CO₂ levels, or we wouldn't be here to discuss it. Furthermore, it is considered highly unlikely to occur in the future under anthropogenic forcing at current solar luminosity (Goldblatt and Watson, 2012; IPCC, 2021a).

4.2 Methods

SURFER is a reduced-complexity box model describing the exchanges of carbon and heat between the land, the atmosphere, and the ocean. We use version 3.0 of SURFER, which includes dynamic alkalinity cycling, CaCO₃ sediments, and weathering processes, enabling simulations of the coupled carbon-climate system over multi-millennial timescales (Couplet et al., 2024b). We have incorporated in the model parametrisations for six tipping elements: the Greenland ice sheet, the West and East Antarctic ice sheets, the East Antarctic subglacial basins, the AMOC and the Amazon rainforest; as well as three threshold-free nonlinear elements: the mountain glaciers, Arctic sea-ice, and boreal permafrost. The state evolution of these elements is modelled using the following equations :

$$\frac{dx_i}{dt} = \left[\underbrace{-x_i^3 + a_{2,i}x_i^2 + a_{1,i}x_i + c_{0,i} + c_{1,i}}_{H_i} \left(\delta T_U + d \cdot \sum_j \epsilon_{ij} \delta L_j q_{*,i} \right) \right] \mu(x_i, H_i), \quad (4.1)$$

with $i \in \{\text{GRIS, WAIS, EASB, EAIS, ASI, AMOC, AMAZ, PERM}\}$. The adimensional variables x_i represent the states of the elements and evolve between 0 and 1, a value of 1 indicating the pre-industrial state and 0 the collapsed state. The evolution of mountain glaciers is described by a different parametrisation, which originates from previous versions of the model (see appendix C.1).

The parameters (a_2, a_1, c_1, c_0) in Eq 4.1 are related to key quantities, such as the coordinates of the bifurcation points in the case of tipping elements, and are calibrated based on the review by Armstrong McKay et al. (2022). The factor μ is an inverse timescale and is calibrated to reproduce the transition time scales also provided in Armstrong McKay et al., 2022. Each element i is forced by temperature, and it can also be forced with any other quantity δL_j simulated by SURFER, allowing for numerous coupling possibilities between the different elements. In particular, the parameters ϵ_{ij} control the sign and strength of interactions. They are calibrated based on the recent review of tipping interactions by Wunderling et al. (2024).

We include the impacts of the ice sheets (GRIS, WAIS, EASB, EAIS) on global mean temperatures through albedo changes based on Wunderling et al. (2020). In our model, the maximum impact they can have on mean temperatures for an ECS of 3.5°C is 0.15°C, 0.06°, 0.06°C, and 0.7°C, respectively. We also consider the additional greenhouse gas emissions from the Amazon rainforest dieback and boreal permafrost thaw. In the model, AMAZ can release a maximum of 75 PgC of CO₂, while permafrost can release a maximum of 300 PgC of CO₂ and 7.5 PgC of methane. Other feedbacks from the elements on temperature, such as the albedo changes resulting from Arctic sea ice loss or Amazon dieback, are considered to be already included in the climate feedback parameter β . This parameter is fixed in SURFER to provide an equilibrium climate sensitivity of 3.5°C. As in Chapter 3, we neglect the impacts of an AMOC collapse on global mean temperature and oceanic carbon release. However, the local effects on other elements (ASI, PERM, GRIS, AMAZ...) are taken into account through the coupling terms in equation 4.1. Sea level rise is computed as the sum of

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6 distinct contributions: oceanic expansion (thermosteric sea level rise), and meltwater from mountain glaciers, Greenland ice sheet, West Antarctic ice sheet, East Antarctic subglacial basins, and East Antarctic ice sheet. More details are provided in appendix C.1.

In general, large uncertainties surround the critical threshold of tipping elements and the time scales over which they may collapse. Arguably, there are even more uncertainties surrounding the interactions between the elements. To deal with the uncertainty, we sample relevant parameters in uniform distributions and create an ensemble of $N_{\text{params}} = 100$ different parameter sets using Latin hypercube sampling. For each parameter set in our ensemble, we run the model from 1850 AD to 100000 AD for a family of $N_{\text{scenarios}} = 100$ emission scenarios. This constitutes a single experiment with $N_{\text{params}} \times N_{\text{scenarios}} = 10000$ model runs. We perform several such experiments for different model settings. The two main experiments discussed in this chapter are the *baseline* and *fully coupled* experiments. In the baseline experiment, interactions between the elements and their feedback on temperature are switched off, while in the fully coupled experiment, they are taken into account. In appendix C.2, we also provide some results for experiments with a higher climate sensitivity and with overall stronger interactions between the nonlinear elements. We emphasize that by sampling the parameters of tipping elements and their interactions from uniform distributions, we aim to explore the range of possible Earth system trajectories rather than offer a strictly quantitative probabilistic projection of the future.

We use emission scenarios that follow historical CO₂ emissions from 1850 to 2015 AD. After that, between 50 and 5000 PgC of CO₂ are released following a logistic equation. Methane emissions are the same for all scenarios, following historical emissions from 1850 AD until 1995 AD, then following the SSP1-2.6 scenario. We order scenarios by increasing cumulative emissions and index them such that scenario j corresponds to $j \cdot 50$ PgC of CO₂ emissions produced after 2015. Note that throughout this chapter, we will refer to scenarios by their CO₂ cumulative emissions after 2015 rather than their total cumulative emission value since 1850.

Let us now describe our definition of a tipping cascade. In every model run, we assign to the six tipping elements a *tipping status*, which is either *tipped* or *not tipped*. We assign to an element i the status *tipped* if there exists a time t ($1850 < t < 100000$) such that $x_i(t) < x_- = 0.25$; otherwise, we assign the status *not tipped*. In a given experiment and for a given set of parameters, we say there is a *tipping event* for scenario j if at least one tipping element has

changed status in scenario j compared to scenario $j - 1$. If only one element has changed status, we classify the event as a *single tipping event*. If $n > 1$ elements have changed status, we call it a *tipping cascade* of size n .

The term *hothouse* generally refers to a planet several degrees warmer with wholly or partially melted ice sheets, but it needs to be precisely defined. It is often used in the literature to depict different Earth system states. For instance, when Steffen et al. (2018) refers to hothouse earth, they are talking about a state akin to the Mid-Pliocene epoch, where temperatures were 4-5°C warmer than today, and sea levels 10-60 m higher. However, authors that consider Earth's history on a larger time scale would still view it as a cool greenhouse (e.g. Kidder and Worsley, 2012) or even a coolhouse (e.g. Westerhold et al., 2020), reserving the term hothouse for Eocene-like climates, with 10+°C temperatures and ice-free conditions. Here, we stick to Steffen et al. (2018) framing and define a hothouse as an Earth system state with temperatures higher than 5°C and sea level higher than 10 m above pre-industrial levels. If only the sea level condition is met, we will call it a *Wethouse* state.

4.3 Results

Figure 4.1 provides an illustrative example of a model run, highlighting the key quantities analyzed in this study. Section 4.3.1 focuses on temperature and sea-level rise, examining how they are influenced by interactions between nonlinear elements and their feedback on the climate. We also evaluate the conditions under which the Earth system might reach the defined Hothouse or Wethouse states. Section 4.3.2 examines the impacts of tipping cascades by comparing successive scenarios. We explicitly test the hypothesis proposed by Steffen et al. (2018), assessing whether tipping cascades could drive the Earth system into a Hothouse state.

4.3.1 Impacts of tipping elements on temperature and sea level rise

Figure 4.2 compares SURFER's temperature and sea level rise response in the baseline and fully coupled experiment. These quantities are displayed as a function of cumulative emissions and at times $t = 2300$, $t = 5000$, and $t = 50000$, to contrast the different response time scales.

Including the feedbacks of tipping elements on global mean temperature in the fully coupled experiment leads to an increase in temperatures compared

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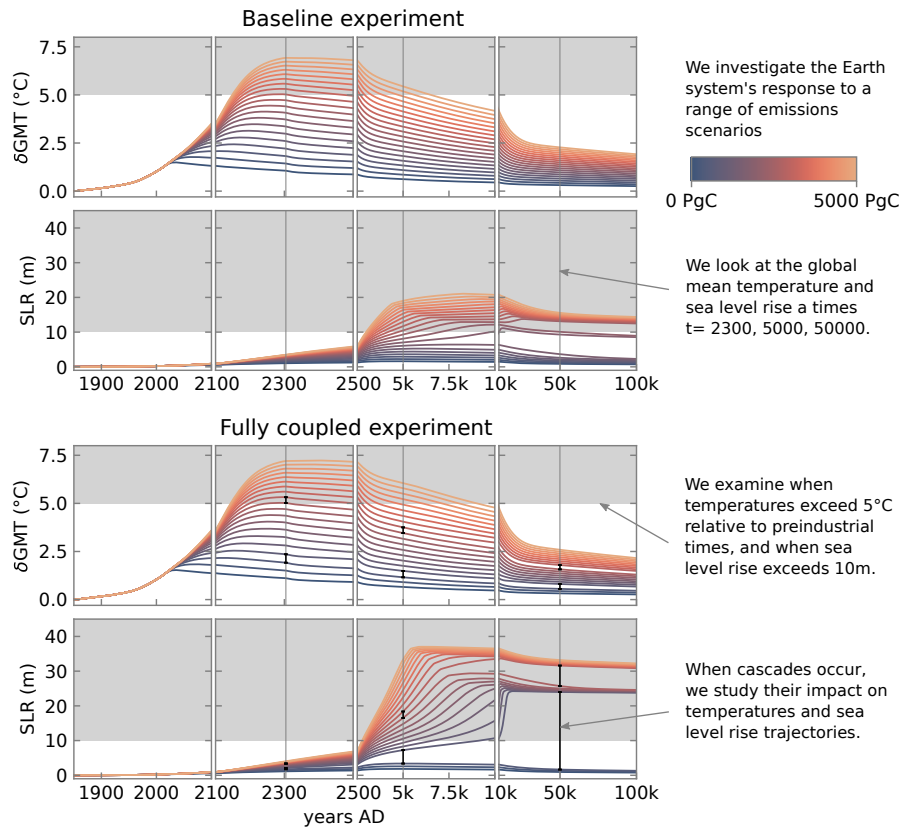


Fig. 4.1 *Experimental setup and example output of temperature and sea level rise from SURFER. We conduct two experiments: a baseline experiment, where interactions between elements and their feedback on temperature are disabled, and a fully coupled experiment, where these interactions and feedbacks are included. Each experiment involves running the model for 100 parameter sets and over 100 emission scenarios, with cumulative CO_2 emissions since 2015 AD ranging from 50 to 5000 PgC. This figure shows temperature and sea level rise outputs for both experiments under a single parameter set, with results displayed for 20 of the 100 scenarios for clarity.*

to the baseline experiment. The difference generally grows with cumulative emissions and is the largest for intermediate time scales, with a difference of about $0.4\text{--}0.7^\circ\text{C}$ at time $t = 5000$ for 5000 PgC of emissions. However, by this time, natural carbon sinks have begun to reduce temperatures, meaning that the impact of tipping elements on peak warming is, in the end, relatively small. The near-linear relationship between global mean temperature and greenhouse gas emissions, as predicted by GCMs for the next century, persists even over longer time scales. In the long term ($t \geq 5000$), tipping elements

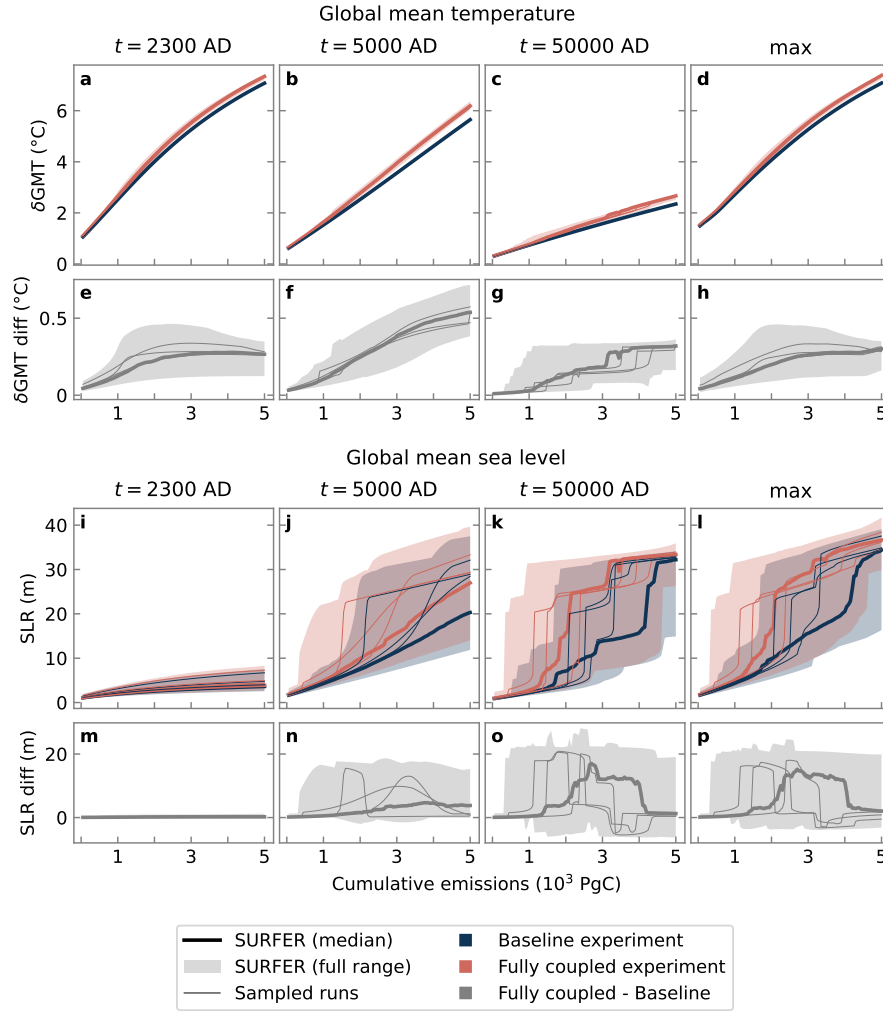


Fig. 4.2 Temperature (a–h) and sea level rise (i–p) as a function of cumulative emissions for the baseline and fully coupled experiments. Panels e–h and m–p display the differences between fully coupled and baseline experiments. Thick solid lines represent median values, while shaded areas show the full range across all simulations. Thinner solid lines indicate example values for three specific parameter sets. The first three columns represent results at different time points, and the fourth column shows the maximum values reached during the simulations. Note that the timing of maximum values may differ between the baseline and fully coupled experiments for a given parameter set and emission scenario.

may introduce small nonlinearities, but these are secondary in magnitude compared to the predominant linear behaviour.

Due to the slow response of ice sheets, significant differences in sea-level rise between the experiments emerge only on intermediate time scales. For a given emissions scenario, sea-level rise is generally larger in the fully coupled experiment. This is primarily because interactions between tipping elements increase the tipping risk for the West Antarctic Ice Sheet and the East Antarctic Subglacial Basin (see Chapter 2). Notably, this may lead to differences of up to 20m by 5000 AD, even for low emissions scenarios (<1000 PgC). In some cases, however, sea-level rise is smaller in the fully coupled experiment due to the stabilizing influence of AMOC. On longer time scales ($t=50000$) and under high-emission scenarios, differences between experiments diminish as ice sheets are tipped, regardless of whether interactions are included. Our results indicate a nonlinear long-term response of sea-level rise to cumulative emissions, with potentially significant differences even between similar emission scenarios.

Following the approach from Chapter 3, we define the risk of a specific event or Earth system state as the percentage of ensemble simulations (with varying parameter sets) in which the event or state occurs. Figure 4.3 shows the risk of exceeding 5°C of warming, 10 meters of sea-level rise (Wethouse), or both (hothouse) as a function of cumulative emissions and time. Our results indicate that, in the baseline experiment, the risk of reaching a hothouse state begins for cumulative emissions exceeding 3000 PgC and increases only moderately in the fully coupled experiment. Importantly, we find no risk of reaching a hothouse state under low-emission scenarios (<1000 PgC), generally considered compatible with staying within or slightly overshooting the 2°C Paris limit.

Our results also indicate that a hothouse state is reversible on millennial timescales, persisting between a few centuries and up to 8,000 years for emissions scenarios ranging from 3000 to 5000 PgC. Inspecting separately the two conditions that define our hothouse state ($T > 5^{\circ}\text{C}$ and $\text{SLR} > 10\text{ m}$) reveals that sea-level rise, with its slower response, determines the entry time into the hothouse state, while naturally decreasing temperatures limit its duration (see also Figure 4.1).

Focusing solely on the 10 m sea-level rise condition, we find a serious risk of entering a far less reversible Wethouse state, even under low-emission scenarios. In the fully coupled experiment, this could occur by the year 5000 for scenarios with less than 600 PgC of emissions. For higher-emission scenarios, a Wethouse state could occur as early as the year 2500, with smaller differences in risk between the baseline and fully coupled experiments, consistent with the results in Figure 4.2. Once established, a Wethouse state can persist more

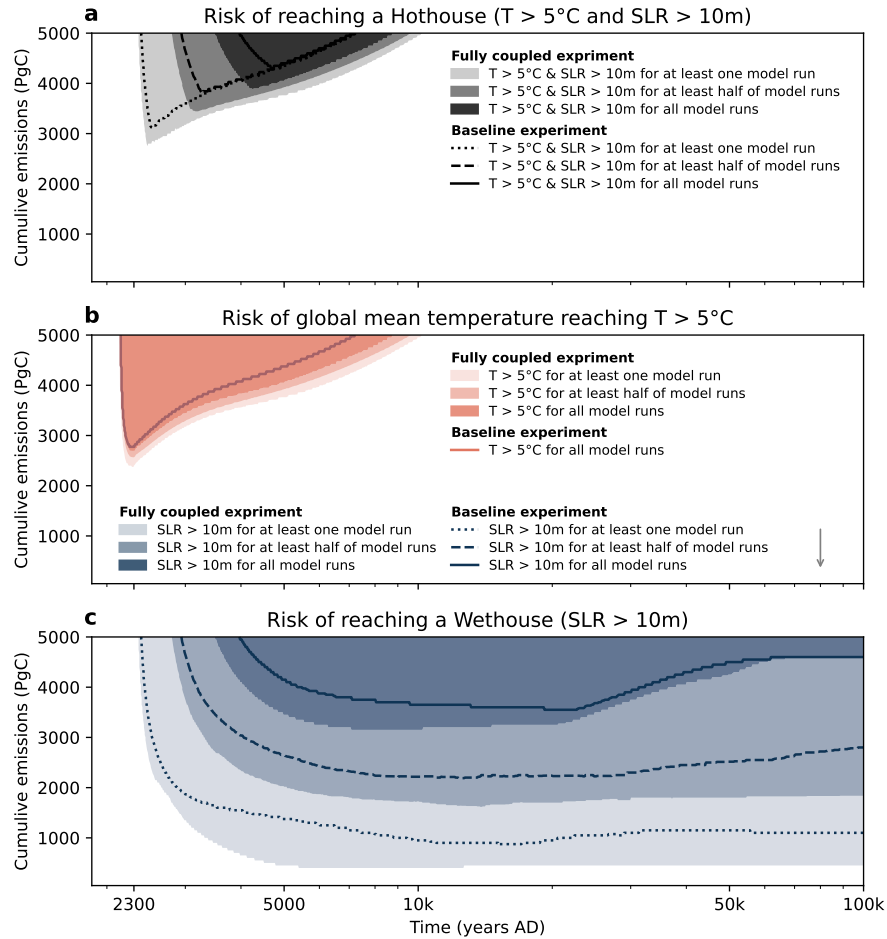


Fig. 4.3 Risk of warming exceeding 5°C and sea level rise exceeding 10 m as a function of time and cumulative emissions for the baseline and fully coupled experiments. For each time t and emission scenario, the risk is defined as the percentage of ensemble simulations, with varying parameter sets, that exceed the reference thresholds: $T > 5^{\circ}\text{C}$ and $\text{SLR} > 10\text{m}$ for the Hothouse (panel a), $T > 5^{\circ}\text{C}$ for warming only (panel b), and $\text{SLR} > 10\text{m}$ for sea level rise only (Wethouse, panel c). For the fully coupled experiment, shaded areas represent emission scenarios and times where risk is greater than 0%, greater than 50%, or equal to 100%. For the baseline experiment, these areas are outlined by dotted, dashed, and solid lines.

than 100,000 years due to the slow recovery rate of ice sheets and the extended time required for temperatures to decrease enough to allow the regrowth of tipped ice sheets.

4.3.2 Tipping cascades and diverging Earth system trajectories

The previous Section discussed the temperature and sea-level rise responses to cumulative emissions. We evaluated how interactions between nonlinear elements and their feedbacks on temperature influence these responses overall. Here, we focus on the impact of isolated tipping events, assessing whether they can lead to divergent Earth system trajectories.

To achieve this, we compare temperatures and sea-level rise from each model run where a tipping event occurs to the corresponding run with 50 PgC fewer emissions. For example, if a tipping event occurs in a model run using parameter set i under scenario j , we compare it to the model run with the same parameter set i under scenario $j - 1$. Figure 4.4 examines the impact of tipping events on maximum temperatures and sea level rise, while Figures C.4 and C.5 focus on impacts at specific times ($t = 2300$, $t = 5000$, and $t = 50000$). As a reference, we also provide the mean differences in temperature and sea level rise across all parameter sets and successive scenarios without tipping events. Differences significantly exceeding these reference values indicate a divergence of trajectories.

In the baseline experiment, the reference difference in peak warming between two scenarios separated by 50 PgC of CO₂ emissions is 0.056°C. In the coupled experiment, where tipping elements can influence temperature even without tipping, this reference value is slightly higher at 0.059°C. Our results show that tipping events have a limited impact on peak warming, with a maximum effect of ~0.06°C in our experiments. There are two primary reasons for this. First, most tipping elements included in our model, such as the Greenland Ice sheet, The West Antarctic ice sheet, the East Antarctic subglacial basins, and the Amazon rainforest, do not produce substantial changes in radiative forcing. The East Antarctic Ice Sheet has the potential to contribute up to ~0.7°C of warming but never tips in our scenarios. Permafrost could, in principle, release significant amounts of CO₂ and CH₄ in the atmosphere. However, it responds mostly linearly to temperature increases and is not directly coupled to other tipping elements, limiting the potential for abrupt changes in thaw emissions between successive scenarios. Second, as shown in Figure C.4, the influence of tipping events on temperature becomes noticeable only over longer time scales, well after peak warming. Even then, the maximum impact observed in our experiments is 0.4°C (at $t = 50000$). In summary, our findings suggest that tipping cascades are unlikely to drive the Earth system

into a hothouse state and to prevent the stabilization of global temperatures at intermediate levels.

The analysis of maximum sea-level rise tells a different story, with tipping events playing a more significant role. In the baseline experiment, the reference difference in maximum sea-level rise between two scenarios separated by 50 PgC of emissions is 0.19 m. In the fully coupled experiment, this value is slightly lower, at 0.16 m, due to the stabilizing influence of AMOC on the Greenland Ice Sheet (GRIS). In both experiments, single tipping events can already result in substantial increases in maximum sea level rise, reaching around 5 m for the tipping of GRIS or the West Antarctic Ice Sheet (WAIS), and up to 10 m for the East Antarctic Subglacial Basins (EASB). Since ice sheets are often partially melted before collapsing, the computed impacts of tipping events are generally smaller than their theoretical maximum sea-level rise potentials.

Including interactions between elements in the fully coupled experiment significantly increases the occurrence of cascades, which can result in substantial sea-level rise when they involve multiple cryospheric elements. Some of these cascades occur even for low-emission scenarios, such as the size-4 cascade highlighted in Figure 4.1. In that example, a cascade involving the AMAZ, the AMOC, the WAIS, and the EASB occurs in the scenario with 650 PgC of emissions, leading to a max SLR of 24 m, which is 20.4 m more than the maximum 3.6 m SLR observed in the scenario with $650 - 50 = 600$ PgC emissions. Overall, 13% of our parameter sets generate fully coupled runs where tipping events or cascades occur under low emission scenarios (<1000 PgC) and result in a max sea level rise exceeding 10 m, with a difference of at least 5 m with the max sea level rise in the scenario with 50 PgC less emissions. Most of these events involve tipping the Greenland ice sheet, but the ones leading to the largest divergence in trajectories involve tipping the EASB, as in the example from Figure 4.1. Figure C.5 shows that this divergence in sea level rise due to tipping events becomes evident over intermediate to long time scales ($t \geq 5000$). In conclusion, our findings suggest that tipping cascades may lock the Earth system on a pathway towards a Wethouse, preventing the stabilisation of long-term sea level rise at intermediate values.

We combine all our results and provide a visual overview of possible Earth system trajectories in Figure 4.5, which shows the time evolution of temperature and sea-level rise distributions across all model runs and scenarios. These distributions offer insights into the stability landscape of the Earth system, with high values indicating states that remain stable in SURFER across

multiple emissions scenarios. In this way, the figure can be viewed as a model-derived counterpart to the conceptual stability landscape proposed by (Steffen et al., 2018) (see their Figure 2), where 'valleys' and 'mountains' are inverted. It demonstrates that there is no clear divergence in Earth system temperature trajectories. In other words, a mostly continuous range of temperatures can be reached at any given time, depending on anthropogenic emissions. While the distributions shown here may smooth out and differ from those generated with a given set of parameters from our ensemble, our previous analysis indicates that temperature divergence due to specific cascades remains small. Temperature differences are generally less than 0.27°C at any given time between two scenarios that differ by only 50 PgC of emissions.

By contrast, we observe clear divergences in sea level rise trajectories. On centennial time scales, the distribution of sea-level rise is unimodal, with a high sea-level rise tail gradually emerging. After several thousand years, multiple modes develop, corresponding to different combinations of tipped elements, as shown in figure 4.6. This is a robust behaviour across our ensemble of runs, although specific choices of parameters may result in different distributions with only a subset of modes present. Our results show that long-term sea level rise may depend more on whether ice sheets tip than the precise choice of emission scenario. We observe a divergence of trajectories in both the baseline and fully coupled experiments, with some differences in the long-term distributions. Specifically, interactions between tipping and other nonlinear elements decrease the tipping risk of GRIS, suppressing the long-term SLR modes at ~ 10 m and ~ 15 m while increasing the tipping risk for EASB, thereby amplifying the long-term SLR modes at ~ 25 m and ~ 30 m. Finally, it is important to note that when we refer to a 'divergence in trajectories', we imply that intermediate trajectories are unlikely rather than strictly impossible. In fact, by considering a continuous family of forcing scenarios, we can always identify one where a given (intermediate) sea level rise occurs at a given finite time t .

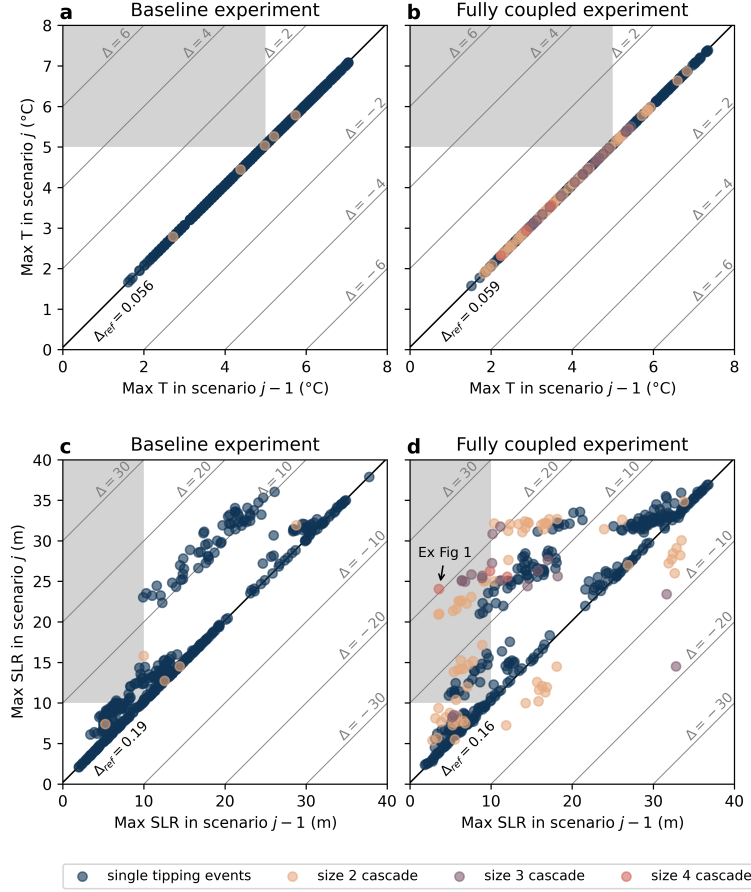


Fig. 4.4 Impact of tipping events on maximum temperature (a,b) and sea level rise (c,d) for the baseline (a,c) and fully coupled (b,d) experiments. Each tipping event observed in these experiments is represented by a point, with colour indicating the event type. The impact of a tipping event is assessed by comparing the y and x coordinates of the corresponding point, which represent, respectively, the maximum temperature or sea level rise in the scenario j where the event occurred and in scenario $j-1$ (which assumes 50 PgC lower cumulative emissions). The thick black diagonal in each plot has equation $y = x + \Delta_{ref}$, where Δ_{ref} is the mean difference in maximum temperature or sea level rise between two scenarios j and $j-1$ for cases where no tipping event occurred. Thus, tipping events on this line have a zero or negligible impact on maximum temperature or sea level rise. The farther away the points are from this diagonal, the greater the impact of the associated event. The thinner grey $y = x + \Delta$ diagonals represent lines of constant impact and serve as visual guides. Shaded regions highlight events that cause maximum temperature to exceed 5°C (a,b) or maximum sea level rise to exceed 10m (c,d) above pre-industrial levels. Note that in the baseline experiment (a,c), the interactions between elements are turned 'off', which means that size-2 cascades are due to elements tipping for the same scenario by coincidence.

4 | Future Earth system trajectories

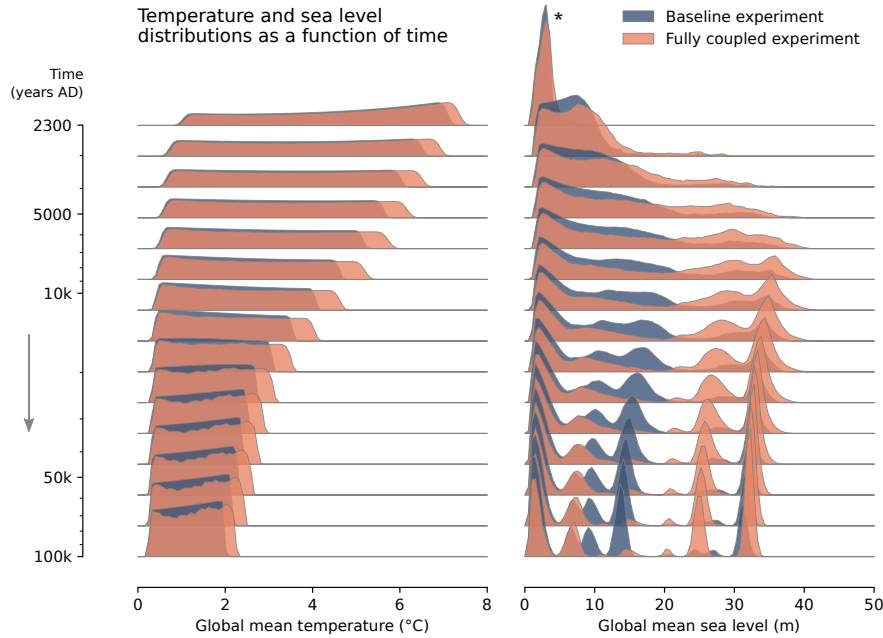


Fig. 4.5 Distribution of temperatures and sea level rise across all scenarios and model runs as a function of time for the baseline and fully coupled experiments. The time axis is on a logarithmic scale. The sea level rise distributions for $t = 2300$ have been scaled down to fit the figure.

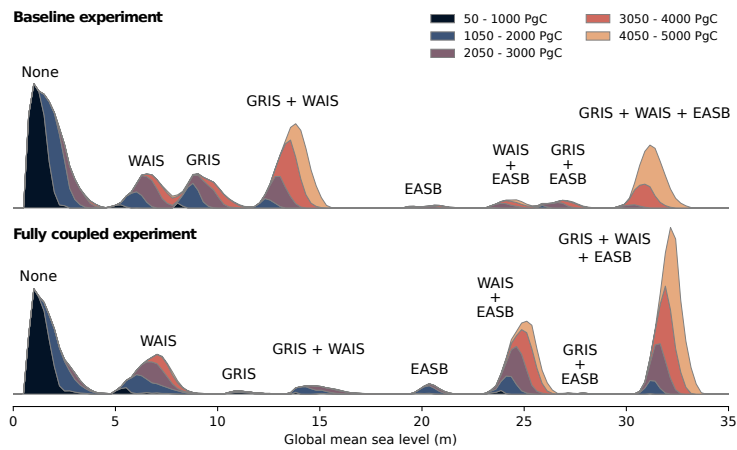


Fig. 4.6 Distributions of sea level rise at time $t = 100$ kyr across all scenarios and model runs for both the baseline and fully coupled experiments. Peaks in the distributions correspond to different combinations of tipped elements contributing to sea level rise, indicated by black labels.

4.4 Discussion and conclusion

Overall, our results suggest that global mean temperature responds nearly linearly to cumulative emissions, even over the long term, with the impact of tipping and other nonlinear elements remaining relatively small compared to the choice of emission scenario. This aligns with findings from the sixth IPCC assessment report and studies by Wang et al. (2023) and Deutloff et al. (2023), which modelled the effects of various tipping elements on temperature trajectories up to 2300 and 2500 using the FaIR climate emulator (Smith et al., 2018). With a mid-range climate sensitivity of 3.5°C, our model predicts a hothouse state only under high-emission scenarios (> 3000 PgC). Even in these cases, the hothouse state is reversible, ending after several thousand years thanks to naturally active carbon sinks. While tipping cascades can occur within 1000 PgC of emissions, we found no evidence for a planetary threshold beyond which the Earth would be locked in a pathway towards a hothouse state, with warming exceeding 5°C and sea-level rise surpassing 10 meters compared to pre-industrial levels, as hypothesized by Steffen et al. (2018).

In contrast, tipping elements have a more pronounced impact on sea level, with ice sheet dynamics rather than the choice of emission pathway being the dominant factor in determining long-term sea-level rise. Including interactions between elements increases the tipping risk of WAIS and EASB, which increases the risk of reaching a Wethouse state under low-emission scenarios. In that case, the sea level would rise over several thousand years. For high-emission scenarios, we found that a Wethouse state could develop on centennial timescales, consistent with IPCC projections, which do not rule out up to 15 meters of sea-level rise by 2300 under SSP5-8.5 (IPCC, 2021a).

Importantly, we demonstrated that tipping events and cascades create divergent sea level rise trajectories, preventing long-term stabilisation at intermediate values. In other words, there may be a planetary threshold beyond which tipping cascades lock the Earth system in a pathway towards a Wethouse state. Where such a threshold lies depends on the behaviour of the ice sheets, but our results indicate that it could be reached in scenarios with less than 1000 PgC of CO₂ emissions released after 2015. Our results also suggest that, regardless of emission levels, a Wethouse state will likely persist for tens of thousands of years. It should be noted that our parametrisations for ice sheet regrowth are very simplified. For instance, we set the critical temperatures for tipping back to the original state to $T_- = 0^\circ\text{C}$ for all ice sheets, and we set

the time scales for growth τ_{+} to larger arbitrary values than the melting time scales τ_{-} (see Table 3.1). While these assumptions are physically motivated, they may lead to erroneous estimations of the Wethouse duration. At last, regrowth dynamics will be affected, on these time scales, by changes in Earth's orbital elements and obliquity. However, these considerations do not affect the general message, that the Wethouse state is effectively irreversible within any reasonable foreseeable horizon for humans.

Naturally, our results depend on the specific tipping elements and nonlinear processes included in the model. In the following paragraphs, we address processes not incorporated into this framework but that could significantly influence long-term Earth system trajectories, particularly those related to carbon-climate and cloud feedback.

The total carbon-climate feedback in SURFER v3.0 is relatively weak, meaning temperature changes have only a modest effect on the carbon cycle (Couplet et al., 2024b, also in Chapter 2). In this study, the feedback is enhanced by including emissions from Amazon dieback and permafrost thaw, but additional processes could amplify it further. For example, making oceanic carbon pumps sensitive to variables like temperature and pH could introduce new feedback mechanisms (Henson et al., 2022; Planchat et al., 2023). In a conceptual model study, Rothman (2019) demonstrated that such mechanisms could disrupt the carbon cycle, leading to transient spikes in atmospheric CO_2 . Methane hydrates present another potential feedback mechanism that could amplify warming (Archer et al., 2009a; Renssen et al., 2004). These ice-like structures, located beneath the seabed, are estimated to store approximately 1,000–2,000 PgC of methane, a fraction of which could be released upon destabilization. However, most of these emissions are either retained in sediments beneath the seabed (Minshull et al., 2016) or dissolved in seawater and oxidized into CO_2 before reaching the atmosphere (Ruppel and Kessler, 2017). As a result, the long-term feedback from methane hydrates may be comparable in magnitude to that from permafrost thaw.

Cloud feedbacks remain one of the largest sources of uncertainty in climate prediction, with the potential to drive high-end climate sensitivities of 5°C or more (Bjordal et al., 2020). In SURFER, climate sensitivity is set at 3.5°C . Increasing this value heightens the likelihood of reaching a Hothouse state under lower emissions scenarios (see appendix C.2). Notably, the interplay between high climate sensitivity and strong carbon-climate feedbacks could further exacerbate the risk of observing large temperatures, even under low-emission scenarios (Kaufhold et al., 2024b). Cloud feedbacks may also be

nonlinear. In large eddy simulations, Schneider et al. (2019) found an instability for CO₂ levels above 1200 ppm where subtropical stratocumulus clouds break up, substantially lowering the planetary albedo, and leading to a global surface warming of 8°C. In SURFER, 1200 ppm are first reached in the baseline experiment for the scenarios with 3300 PgC of emissions, which has a peak warming of 5.6°C. This process would thus happen in a already warm Earth system state but could prevent mid-term stabilisation of temperatures between ~ 6 and $\sim 6 + 8 = 14^\circ\text{C}$.

A range of additional positive feedbacks could amplify warming (Ripple et al., 2023), many of which are not implemented in SURFER. Some of these feedbacks are poorly represented or entirely absent even in state-of-the-art models. This is suggested by most models' difficulty in accurately simulating reconstructed past warm climates (Kopp et al., 2017), although recent progress is encouraging (Zhu et al., 2024). Moreover, certain feedback mechanisms may have not yet been identified or fully understood. These uncertainties highlight the need for further research to constrain future Earth system trajectories better.

5

Conclusions

5.1 Research outcomes

Climate tipping points and cascades are widely recognised as potential sources of abrupt and irreversible change in the Earth's climate and environment. In a perspective paper, Steffen et al. (2018) evoked the possibility of crossing a planetary threshold beyond which tipping cascades and biogeophysical feedbacks lock the Earth system in a pathway towards an irreversible hothouse state. This hypothesis has generated significant scientific debate and media attention but remains underexplored in modelling studies. To address this, we have developed SURFER, a reduced-complexity Earth system model integrating a dynamic carbon cycle and a network of interconnected tipping and nonlinear elements. Through targeted experiments, we answered the overarching questions presented in the introduction of this thesis:

**To what extent do tipping events and other nonlinear feedbacks increase the risk of reaching a Hothouse state? A Wethouse state?
Could this outcome occur under low-emission scenarios?**

To what extent do tipping events and other nonlinear feedbacks create a divergence in Earth system trajectories? In other words, is there a threshold that prevents stabilisation at intermediate temperatures or sea-level rise?

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We summarise our main achievements and findings below, chapter by chapter.

In Chapter 2, we presented SURFER v3.0, the reduced complexity model we used throughout this thesis. It is an extension of SURFER v2.0 that incorporates a dynamic cycling of alkalinity in the ocean, a carbonate sediments reservoir, and weathering fluxes. We have demonstrated that with these additions, SURFER can accurately simulate the climate, the carbon cycle, and sea level rise on time scales ranging from decades to millions of years. We show that compared to SURFER v2.0, including long-term carbon-cycle processes in SURFER v3.0 leads to a stabilisation of the Greenland ice sheet for the middle of the road emission scenarios and a significant reduction in sea level rise contribution from Antarctica for high emissions scenarios.

In Chapter 3, we coupled SURFER v3.0 to a network of interacting tipping and other nonlinear elements. These included the Greenland Ice sheet, the West and East Antarctic ice sheets, the East Antarctic subglacial basins, the Amazon rainforest, the AMOC, the Arctic sea ice, and the boreal permafrost. With this new model, we have explored tipping events and cascades. Our results have highlighted the important role of the elements' intrinsic time scales and the need for realistic, non-stationary scenarios in assessing tipping risk. Notably, slow tipping elements such as the ice sheets may remain stable even under relatively large and prolonged overshoots of their critical temperature thresholds. This confirms early results from Chapter 2 and suggests that assessing the tipping risk of a slow element solely based on whether warming crosses its critical temperature threshold may lead to an overestimation of the risk.

On the other hand, we found that interactions among elements generally increase tipping risks. Feedbacks from tipping elements on global mean temperature also raises overall tipping risk, although to a lesser extent than direct interactions. We found that most cascades happen for scenarios with peak warming between 2.7°C and 5.3°C, with some occurring for lower peak warming. This suggests that tipping cascades may occur due to the projected warming under current policies, underscoring - if further evidence were truly still necessary - the need for urgent action to reduce our emissions. In contrast to previous studies, we identified the AMOC as the primary potential initiator of cascades, which is concerning given that some studies have shown it could tip within this century, even under intermediate warming scenarios (Ditlevsen and Ditlevsen, 2023; van Westen et al., 2024b).

In Chapter 4, we investigated the Earth system's response to anthropogenic emissions using the experiments performed in Chapter 3. This allowed us to answer our main research questions: we found no evidence of a planetary threshold beyond which the Earth system evolves irreversibly into a Hothouse state, as hypothesised by Steffen et al. (2018). Instead, we observed a mostly linear temperature response with cumulative emissions at all time scales. This suggests that future warming largely depends on our CO₂ and methane emissions. Although this may appear good news in the context of this thesis, we emphasise that even steady, incremental changes, if unmitigated, can pose significant challenges to human societies.

Besides, we demonstrated that tipping dynamics in the ice sheets lead to a nonlinear response of sea level to cumulative emissions in the long term. A difference in emissions of 50 PgC, five years' worth of emissions at today's rate, could trigger cascades that drive ~20 m of sea level rise on multi-millennial time scales, preventing stabilisation at intermediate values. In other words, we found that tipping cascades could lock the Earth system on a pathway towards a Wethouse state. Crucially, this could occur under low emission scenarios that stay within or slightly overshoot the Paris Agreement 2°C limit.

5.2 Some reflections on SURFER's development

Although the conclusions presented above naturally focus on the results obtained, a significant portion of this thesis was dedicated to the development and calibration of SURFER, which itself represents a key outcome of this work. This is reflected in the length of the method sections in Chapters 2 and 3. During the development process, we encountered several challenges, which we reflect upon here.

A primary challenge was to develop the carbon cycle component based on SURFERv2.0 (Chapter 2). This was a lengthy endeavour, not least because the author knew next to nothing about oceanic geochemistry. In hindsight, time could have been saved by incorporating parametrisations for tipping elements and sea level into pre-existing box models of the carbon cycle, such as LOSCAR (Zeebe, 2012) or the model proposed by Lenton and Britton (2006). Nevertheless, the experience was highly valuable, as it provided a better comprehension of the Earth system's dynamics and allowed full control over the model's development.

A second challenge was incorporating tipping elements into the model (Chapter 3). Initially, we planned to couple pre-existing process-based models of tipping elements, reserving the "double-fold parametrisation" (Eq. 1.5) only for elements for which we could not find or easily design a model. By "process-based model," we refer to conceptual models explicitly grounded in physical processes, as opposed to parametrisations, which are designed to replicate observed outcomes without necessarily reflecting the underlying physics. This initial plan had the advantage of physically constraining most parameters, reducing the need for arbitrary choices, particularly regarding interaction strengths between elements. However, this approach had drawbacks: it resulted in a somewhat cumbersome model, harder to analyse mathematically, and less flexible for modifications. For these reasons – and given that the first-order dynamics of most elements can be approximated with a double-fold parametrisation (see section 1.2) – we opted to represent all elements using the same equation, varying only the parameters. This choice resulted in a flexible and easily expandable model. It also enabled a streamlined analysis and a more systematic approach to handling parameter uncertainties for the tipping elements. Nevertheless, these simplifications inherently limit SURFER's ability to fully capture the complexity of real-world tipping dynamics. Consequently, our results should be regarded as illustrative rather than strictly predictive.

A third challenge, perhaps more philosophical in nature, was to convince ourselves that we are gaining insights from the model that go beyond the assumptions we initially built into it. The results obtained in Chapter 3 help address this concern, as not all outcomes could have been predicted before running the model experiments. However, some of the main results in Chapter 4 seem more directly tied to our initial implementation choices. For example, the absence of cascades driving the Earth system into a Hothouse state ultimately reflects the limited impact on temperature assigned to the tipping elements included in the model. Similarly, the observed bifurcation in sea-level rise trajectories directly results from the decision to include ice sheets as tipping elements, each with two distinct stable states.

We argue that this sense of predictability stems from the framing of the research question. If the question is, "Could tipping cascades drive the Earth system into a Hothouse or Wethouse state?" the answer inevitably depends on which tipping elements are included in the model. However, by narrowing the scope slightly to, "Given current knowledge about this specific set of eight tipping and nonlinear elements, their interactions, and their effects on temperature, what are the chances they could push the Earth into a Hothouse

or Wethouse state?", then SURFER provides a clear and systematic answer, as demonstrated in Chapter 4. This highlights the model's true purpose: to serve as a simple, flexible, and transparent framework for testing hypotheses, such as the Hothouse hypothesis proposed by Steffen et al. (2018), in a structured and systematic way.

5.3 Perspectives

In the end, SURFER has proven to be a valuable tool for exploring future Earth system trajectories. Nevertheless, it is essential to recognise that there is still a significant amount of uncertainty surrounding tipping elements and amplifying feedbacks. While we have integrated some of these into our modelling framework, our results must be confirmed in more comprehensive models. Such models could also offer critical insights into the regional impacts of tipping events, which SURFER cannot address due to its lack of spatial resolution.

Currently, comprehensive models are rarely used to run long-term emission-driven simulations, which are essential to better constrain future Earth system trajectories. However, this may change in the near future. Recent calls have been made to prioritize emission-driven simulations in CMIP7 (Sanderson et al., 2024) and to extend the time horizon of these simulations beyond 2100 and 2300 (Lyon et al., 2021). Besides, several Earth system models of intermediate complexity can already conduct multi-millennial simulations with interactive carbon cycles and ice sheets. However, most currently fail to reproduce past glacial cycles in this setup, limiting confidence in their future projections. Nevertheless, this is an active area of research, and CLIMBER-2 has successfully simulated four glacial cycles, demonstrating the feasibility of such simulations (Ganopolski and Brovkin, 2017).

In parallel, SURFER can be used for other exploratory studies. As for EMICs, a promising research line consists of including in SURFER processes that enable it to reproduce the Quaternary glacial cycles. This would improve its representation of carbon-climate feedbacks and provide a more comprehensive understanding of the Earth system's stability landscape. In particular, it would allow us to explore the divergence of Earth system trajectories from glacial-interglacial cycles, the other "fork in the road" discussed by (Steffen et al., 2018, see figure 1.2). Indeed, studies suggest that anthropogenic emissions will likely

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cause the Earth system to skip one or more glacial cycles (Ganopolski et al., 2016; Loutre and Berger, 2000; Talento and Ganopolski, 2021).

SURFER could also be used to investigate the possibilities for Earth system management. When we say that tipping cascades may *lock* the Earth system on a pathway towards a Wethouse, we mean that it is the long-term committed state based on a given emission scenario and without any further human intervention. However, beyond emissions mitigation, humanity could actively influence the climate through methods such as carbon dioxide removal (CDR) and solar radiation modification (SRM), despite these approaches coming with significant challenges and risks. For example, CDR may require vast amounts of land (Dooley et al., 2024), and SRM could have unintended consequences on ecosystems (Zarnetske et al., 2021). Whether these strategies could avert a Hothouse or Wethouse outcome under specific emissions scenarios, potentially creating the Stabilized Earth trajectory described by (Steffen et al., 2018), could be systematically investigated using SURFER, building on the methodology developed in Martínez Montero et al. (2024).

A

Supplement to chapter 2

A.1 SURFER parameters and initial conditions

Symbol	Comment	Value
m_A	number of moles in the atmosphere	1.727×10^{20} mol
m_O	number of moles in the ocean	7.8×10^{22} mol
$\bar{m}_C$	carbon molar mass	12×10^{-3} kg mol ⁻¹
$\bar{m}_w$	water molar mass	18×10^{-3} kg mol ⁻¹
c_{vol}	sea water volumetric heat capacity	0.13 W yr m ⁻³
R	gas constant	8.314 J mol ⁻¹ K ⁻¹
h_U	upper layer depth	150 m
h_I	intermediate layer depth	500 m
h_D	deep layer depth	3150 m
W_U	upper layer weight	$h_U \bar{m}_w m_O / (h_U + h_I + h_D)$
W_I	intermediate layer weight	$h_I \bar{m}_w m_O / (h_U + h_I + h_D)$
W_D	deep layer weight	$h_D \bar{m}_w m_O / (h_U + h_I + h_D)$
z_U	upper layer depth mid-depth point	75 m
z_I	intermediate layer mid-depth point	400 m
z_D	deep layer mid-depth point	2225 m

Table A.1 *Physical parameters and geometry.*

Symbol	Comment	Value
$F_{\text{CaCO}_3,0}$	PI weathering of carbonate rocks	$0.065 \text{ PgC yr}^{-1}$
$F_{\text{CaSiO}_3,0}$	PI weathering of silicate rocks	$0.065 \text{ PgC yr}^{-1}$
k_{Ca}	parametrisation of carbonate weathering flux	0.049 K^{-1}
k_{T}	parametrisation of silicate weathering flux	0.095 K^{-1}
$k_{\text{A} \rightarrow \text{L}}$	controls rate of carbon uptake by vegetation	0.044 yr^{-1}
β_{L}	controls amount of carbon uptake by vegetation	1.7
$k_{\text{A} \rightarrow \text{U}}$	controls rate of air-sea CO_2 exchanges	$4.7 \text{ kg (mol yr)}^{-1}$
$k_{\text{U} \rightarrow \text{I}}$	controls rate of DIC transfer via ocean mixing from layer U to I	0.13 yr^{-1}
$k_{\text{I} \rightarrow \text{D}}$	controls rate of DIC transfer via ocean mixing from layer I to D	0.009 yr^{-1}
$\tilde{k}_{\text{U} \rightarrow \text{I}}$	controls rate of alkalinity transfer via ocean mixing from layer U to I	0.13 yr^{-1}
$\tilde{k}_{\text{I} \rightarrow \text{D}}$	controls rate of alkalinity transfer via ocean mixing from layer I to D	0.009 yr^{-1}
P_{org}	organic matter export at 150 m	7 PgC yr^{-1}
P_{CaCO_3}	CaCO_3 export at 150 m	1 PgC yr^{-1}
$\phi_{\text{I}}^{\text{org}}$	fraction of organic matter rain that remineralises in layer I	0.72
$\phi_{\text{I}}^{\text{CaCO}_3}$	fraction of CaCO_3 rain that dissolves in layer I	0.15
$\phi_{\text{D}}^{\text{CaCO}_3}$	fraction of CaCO_3 rain that dissolves in layer D	0.39
$\sigma_{\text{Alk:DIC}}$	Alk to DIC changes in organic matter production	$-16/117$
α_{diss}	parametrisation of CaCO_3 sediments dissolution	$-1.07 \times 10^{-2} \text{ PgC yr}^{-1} (\mu\text{mol kg}^{-1})^{-1}$
β_{diss}	parametrisation of CaCO_3 sediments dissolution	$1.82 \times 10^{-5} \text{ yr}^{-1}$
γ_{diss}	parametrisation of CaCO_3 sediments dissolution	$-4.53 \times 10^{-6} \text{ yr}^{-1} (\mu\text{mol kg}^{-1})^{-1}$
τ_{CH_4}	atmospheric lifetime of methane	9.5 yr

Table A.2 Parameters for the carbon cycle component (the table continues on the next page).

Symbol	Comment	Value
V	volcanic outgassing	$F_{\text{CaSiO}_3,0}$
$k_{\text{I} \rightarrow \text{U}}$	controls rate of DIC transfer via ocean mixing from layer I to U	eq 2.106
$k_{\text{D} \rightarrow \text{I}}$	controls rate of DIC transfer via ocean mixing from layer D to I	eq 2.107
$\tilde{k}_{\text{I} \rightarrow \text{U}}$	controls rate of alkalinity transfer via ocean mixing from layer I to U	eq 2.108
$\tilde{k}_{\text{D} \rightarrow \text{I}}$	controls rate of alkalinity transfer via ocean mixing from layer D to I	eq 2.109
$F_{\text{diss},0}$	parametrisation of CaCO_3 sediments dissolution	eq 2.110
α_{burial}	parametrisation of CaCO_3 sediments burial	$(F_{\text{CaCO}_3,0} + F_{\text{CaSiO}_3,0})/M_{\text{S}}(t_{\text{PI}})$
$E_{\text{nat}}^{\text{CH}_4}$	natural methane emissions	$M_{\text{CH}_4}(t_{\text{PI}})/\tau_{\text{CH}_4}$

Table A.2 (continued) Parameters for the carbon cycle component. This part of the table contains parameters that are computed from equilibrium conditions (equations 2.96-2.105).

Symbol	Comment	Value
$F_{2\times}$	extra radiative forcing due to a doubling of atmospheric CO ₂	3.9 W m ⁻²
β	climate feedback parameter	1.1143 W m ⁻² °C ⁻¹
$\gamma_{U\rightarrow I}$	parametrisation of heat exchange between ocean layers	0.8357 W m ⁻² °C ⁻¹
$\gamma_{I\rightarrow D}$	parametrisation of heat exchange between ocean layers	0.8357 W m ⁻² °C ⁻¹
α_{CH_4}	parametrisation of radiative forcing of methane	0.791 W m ⁻² PgC ^{-1/2}
α_{SO_2}	parametrisation of radiative forcing of SO ₂	65 W m ⁻²
β_{SO_2}	parametrisation of radiative forcing of SO ₂	2246 TgS yr ⁻¹
γ_{SO_2}	parametrisation of radiative forcing of SO ₂	0.23

Table A.3 Parameters for the climate component.

Parameter	Comment	Value
$S_{\text{gl pot}}$	SLR potential from mountain glaciers	0.5 m
ξ	sensitivity coeff. for glacier parametrisation	2°C
τ_{gl}	timescale for glacier melt	200 yr
α_{U}	thermal expansion coeff. for layer U	$2.20 \times 10^{-4} \text{ K}^{-1}$
α_{I}	thermal expansion coeff. for layer I	$1.61 \times 10^{-4} \text{ K}^{-1}$
α_{D}	thermal expansion coeff. for layer D	$1.39 \times 10^{-4} \text{ K}^{-1}$

Table A.4 *Parameter values for the sea level rise component.*

Parameter	Greenland's value	Antarctica's value
T_{+}	1.52 °C	6.8 °C
T_{-}	0.3 °C	4.0 °C
V_{+}	0.77	0.44
V_{-}	0.3527	-0.3200
τ_{+}	5500 yr	5500 yr
τ_{-}	470 yr	3000 yr
k_{τ}	0.05	0.05
S_{pot}	7.4 m	55 m

Table A.5 *Parameter values used for Greenland and Antarctic ice sheets.*

	Upper layer (0-150 m)	Intermediate layer (150-650 m)	Deep layer (650-3800 m)
Pre-industrial DIC ($\mu\text{mol kg}^{-1}$)	2017.65	2152.62	2266.57
Total alkalinity ($\mu\text{mol kg}^{-1}$)	2310.61	2310.60	2367.21
Temperature ($^{\circ}\text{C}/^{\circ}\text{K}$)	16.34 / 289.49	8.95 / 282.10	2.65 / 275.80
Salinity (psu)	34.93	34.77	34.70

Table A.6 *GLODAPv2.2016b quantities averaged over ocean layers equivalent to those of SURFER.*

Variable	Comment	Initial (PI) Value
M_A	mass of carbon in atmospheric CO_2	580.27 PgC
$M_A^{CH_4}$	mass of carbon in atmospheric CH_4	1.49 PgC
M_L	mass of carbon on land (soils+vegetation)	2200 PgC
M_L^*	additional land variable that integrates land-use emission	2200 PgC
M_U	dissolved inorganic carbon mass in ocean layer U	1344.78 PgC
M_I	dissolved inorganic carbon mass in ocean layer I	4772.02 PgC
M_D	dissolved inorganic carbon mass in ocean layer D	31655.16 PgC
Q_U	alkalinity mass in ocean layer U	1536.67 PgC
Q_I	alkalinity mass in ocean layer I	5122.24 PgC
Q_D	alkalinity mass in ocean layer D	33060.77 PgC
M_S	erodible $CaCO_3$ sediments mass	1600 PgC
δT_U	temperature anomaly in ocean layer U	0 K
δT_I	temperature anomaly in ocean layer I	0 K
δT_D	temperature anomaly in ocean layer D	0 K
S_{gl}	sea level rise contribution from mountain glaciers	0 msle
V_{GIS}	volume fraction of Greenland ice sheet with respect to pre-industrial value	1
V_{AIS}	volume fraction of Antarctic ice sheet with respect to pre-industrial value	1

Table A.7 Initial conditions for the 17 model variables of SURFER v3.0 (the table continues on the next page).

Variable	Comment	Initial (PI) Value
S_U	salinity of ocean layer U (constant)	34.93 psu
S_I	salinity of ocean layer I (constant)	34.77 psu
S_D	salinity of ocean layer D (constant)	34.70 psu
$T_{U,0}$	pre-industrial temperature of ocean layer U (constant)	288.38 K
$T_{I,0}$	pre-industrial temperature of ocean layer I (constant)	281.75 K
$T_{D,0}$	pre-industrial temperature of ocean layer D (constant)	275.76 K

Table A.7 (continued) Initial conditions for SURFER v3.0. This part of the table gives the fixed values for salinity and the pre-industrial temperatures, which are necessary to compute the dissociation constants and the solubility constant of CO₂.

A.2 Emission scenarios

Historical and SSP runs (Figures 2.3-2.8, 2.16, 2.18, A.1-A.8)

For CO₂ emissions up to 1989 (included), we use the data from the Global Carbon Budget Friedlingstein et al., 2022. Fossil emissions start in 1750 and we include the cement carbonation sink in them. Land-use emissions are only provided from the year 1850 and so we assume that they have grown linearly from zero since 1750. This adds around 33 PgC of emissions compared to a scenario where land-use emissions are considered zero before 1850.

For CH₄ emissions up to 1989 (included), we use the data from Jones et al., 2023. Both fossil and land-use emissions start in 1830 so, similarly to the CO₂ land-use emissions, we assume that they increased linearly from 0 since 1750. This adds a total of ~1147 MtCH₄ of emissions compared to a scenario where land-use and fossil CH₄ emissions are considered zero before 1830.

For all emissions (CO₂ and CH₄, fossil and land-use) from 1990 to 2100, we use the values provided in the SSP database (<https://tntcat.iiasa.ac.at/SspDb/dsd>, Gidden et al., 2019; Riahi et al., 2017). All SSP scenarios have the same emissions between 1990 and 2015, so we perform our historical runs (1750-2014) with the values from any SSP scenarios. The CO₂ and CH₄ emissions provided in 1990 by Friedlingstein et al., 2022 and Jones et al., 2023 are not equal to the values provided in 1990 in the SSP database, which causes small jumps in our emissions. For emissions between 2100 and 2300, we use the SSP extensions as described in Meinshausen et al., 2020. Before the year 1750, after 2300, all anthropogenic emissions are set to zero.

Pulse experiments (Figures 2.9-2.15)

For these experiments, the model is started with an additional amount of carbon x in the atmospheric reservoir, depending on the emission pulse : $M_A(t_{PI}) = 580.3 + x$ PgC. No other emissions are used.

Others (Figure 2.17)

For the experiments in section 2.4.1 where we compare SURFER to UVic, we use the CO₂ emissions provided in the supplementary material of Clark et al., 2016, while CH₄ emissions are set to zero.

A.3 Solubility and dissociation constants

For the temperature (and salinity) dependence of K_0 , K_1 , K_2 , K_b , K_w , and $K_{sp}^{CaCO_3}$, we use the equations from Sarmiento and Gruber, 2006, and originally from Dickson and Millero, 1987; Dickson, 1990; Mehrbach et al., 1973; Millero, 1995; Mucci, 1983; Weiss, 1974:

$$\ln K_0 = -60.2409 + 93.4517 \left(\frac{100}{T} \right) + 23.3585 \cdot \ln \left(\frac{T}{100} \right) + S \left(0.023517 - 0.023656 \left(\frac{T}{100} \right) + 0.0047036 \left(\frac{T}{100} \right)^2 \right), \quad (A.1)$$

$$-\log K_1^0 = -62.008 + \frac{3670.7}{T} + 9.7944 \cdot \ln T - 0.0118 \cdot S + 0.000116 \cdot S^2, \quad (A.2)$$

$$-\log K_2^0 = +4.777 + \frac{1394.7}{T} - 0.0184 \cdot S + 0.000118 \cdot S^2, \quad (A.3)$$

$$\ln K_b^0 = \frac{1}{T} \left(-8966.9 - 2890.53 \cdot S^{1/2} - 77.942 \cdot S + 1.728 \cdot S^{3/2} - 0.0996 \cdot S^2 \right) + 148.0248 + 137.1942 \cdot S^{1/2} + 1.62142 \cdot S + 0.053105 \cdot S^{1/2} T + \ln T (-24.4344 - 25.085 \cdot S^{1/2} - 0.2474 \cdot S), \quad (A.4)$$

$$\ln K_w^0 = 148.96502 + \frac{-13847.26}{T} - 23.6521 \cdot \ln T + S^{1/2} \left(-5.977 + \frac{118.67}{T} + 1.0495 \cdot \ln T \right) - 0.01615 \cdot S, \quad (A.5)$$

$$\ln K_{sp}^{CaCO_3,0} = -395.8293 + \frac{6537.773}{T} + 71.595 \cdot \ln T - 0.17959 \cdot T + \left(-1.78938 + \frac{410.64}{T} + 0.0065453 \cdot T \right) S^{1/2} - 0.17755 \cdot S + 0.0094979 \cdot S^{2/3}. \quad (A.6)$$

Here, K_0 , K_1^0 , K_2^0 , K_w^0 , K_b^0 , and $K_{sp}^{CaCO_3,0}$ are the values of the constants at atmospheric pressure, given in $\text{mol}(\text{kg atm})^{-1}$ for K_0 , in mol kg^{-1} for K_1^0 , K_2^0 , K_b^0 , and in $(\text{mol kg}^{-1})^2$ for K_w^0 and $K_{sp}^{CaCO_3,0}$, T is the temperature in Kelvin and S is the salinity on the practical salinity scale. In SURFER, only temperature anomalies are computed, meaning that to compute the absolute temperature and then the dissociation constants, we need to provide an initial temperature for each layer. This is done in section 2.2.4.

	a_0	a_1	a_2	b_0	b_1
K_1	-25.50	0.1271	0	-3.08	0.0877
K_2	-15.82	-0.0219	0	1.13	-0.1475
K_b	-29.48	0.1622	-0.002608	-2.84	0
K_w	-25.60	0.2324	-0.0036246	-5.13	0.0794
$K_{sp}^{CaCO_3}$	-48.76	-0.5304	0	-11.76	0.3692

Table A.8 Parametrisation for the effect of pressure on dissociation constants ; coefficients for equations A.8 and A.9. The values are taken from Millero (1995). We have flipped the signs of a_1 and a_2 for K_b , and b_1 for $K_{sp}^{CaCO_3}$ to match the values given in Millero (1979).

The pressure (depth) dependence for K_1 , K_2 , K_w , K_b , and $K_{sp}^{CaCO_3}$ is from Millero, 1995,

$$\ln (K_i^p/K_i^0) = -\frac{\Delta V_i}{RT}P + \frac{0.5\Delta K_i}{RT}P^2, \quad (A.7)$$

where K_i^p is the value of the dissociation constant at pressure P (in bar), K_i^0 is the value of the dissociation constant at atmospheric pressure (1.01325 bar or 101325 Pa), T is the temperature in Kelvin and R is the gas constant in $\text{bar cm}^3 \text{mol}^{-1} \text{K}^{-1}$ (10 times the value in $\text{J mol}^{-1} \text{K}^{-1}$). The quantities ΔV_i and ΔK_i are changes in molal volume and compressibility and are estimated following :

$$\Delta V_i = a_{0,i} + a_{1,i}(T - 273.15) + a_{2,i}(T - 273.15)^2, \quad (A.8)$$

$$1000\Delta K_i = b_{0,i} + b_{1,i}(T - 273.15). \quad (A.9)$$

Values for the coefficients are in table A.8.

Hydrostatic balance provides the pressure at a given depth, thus, pressure (in bar) is given by $P = \rho g z / 100000$ where $\rho = 1026 \text{ kg/m}^3$ is sea water mean density, $g = 9.81 \text{ kg m s}^{-2}$ is the gravitational acceleration and z is the depth (in m). For the computation of mean quantities in the different ocean levels U, I and D, we use the depths $z_U = 75 \text{ m}$, $z_I = 400 \text{ m}$ and $z_D = 2225 \text{ m}$ which corresponds to the mid-depth points of the layers.

A.4 Alkalinity and solving the carbonate system

In SURFER v2.0, we approximated alkalinity by carbonate alkalinity

$$\text{Alk} \approx \text{Alk}_C = [\text{HCO}_3^-] + 2 [\text{CO}_3^{2-}] . \quad (\text{A.10})$$

To compute $[\text{H}^+]$ and the other carbonate species, we had to rewrite eq A.10 as a function of the four known quantities DIC, Alk, T , and S , and the unknown quantity $[\text{H}^+]$. Using the definition of DIC (eq 2.11) and of the dissociation constants K_1 and K_2 (eq 2.20 and eq 2.21) we can express DIC as a function of $[\text{HCO}_3^-]$ and $[\text{H}^+]$:

$$\text{DIC} = \left(1 + \frac{K_2}{[\text{H}^+]} + \frac{[\text{H}^+]}{K_1} \right) [\text{HCO}_3^-] , \quad (\text{A.11})$$

or equivalently, we can express $[\text{HCO}_3^-]$ as a function of DIC and $[\text{H}^+]$

$$[\text{HCO}_3^-] = \frac{\text{DIC} \cdot K_1 [\text{H}^+]}{K_1 K_2 + K_1 [\text{H}^+] + [\text{H}^+]^2} . \quad (\text{A.12})$$

Using equation 2.21, we can then express $[\text{CO}_3^{2-}]$ as a function of DIC and $[\text{H}^+]$

$$\begin{aligned} [\text{CO}_3^{2-}] &= \frac{[\text{HCO}_3^-] K_2}{[\text{H}^+]} \\ &= \frac{\text{DIC} \cdot K_1 K_2}{K_1 K_2 + K_1 [\text{H}^+] + [\text{H}^+]^2} . \end{aligned} \quad (\text{A.13})$$

Inserting eqs A.12 and A.13 in A.10, we get

$$\text{Alk}_C = \text{DIC} \cdot \frac{K_1 [\text{H}^+] + K_1 K_2}{K_1 K_2 + K_1 [\text{H}^+] + [\text{H}^+]^2} , \quad (\text{A.14})$$

which we can solve for $[\text{H}^+]$, given Alk, DIC and the dissociations constants K_1 and K_2 (which depend on T and S). To do so, we write eq A.14 as a degree 2 polynomial equation

$$P_C([\text{H}^+]) \equiv [\text{H}^+]^2 + a_1 [\text{H}^+] + a_0 , \quad (\text{A.15})$$

with

$$a_1 = K_1 \left(1 - \frac{\text{DIC}}{\text{Alk}} \right), \quad (\text{A.16})$$

$$a_0 = K_1 K_2 \left(1 - \frac{2 \cdot \text{DIC}}{\text{Alk}} \right). \quad (\text{A.17})$$

The positive root is given by

$$[\text{H}^+] = \frac{K_1}{2 \cdot \text{Alk}} \left(\sqrt{(\text{DIC} - \text{Alk})^2 - 4 \frac{K_2}{K_1} \text{Alk} \cdot (\text{Alk} - 2 \cdot \text{DIC})} + (\text{DIC} - \text{Alk}) \right). \quad (\text{A.18})$$

Then, $[\text{HCO}_3^-]$ can be computed from equation A.12, $[\text{CO}_3^{2-}]$ can be computed from equation A.13, and finally, $[\text{H}_2\text{CO}_3^*]$ can be computed from equation 2.20.

In SURFER v3.0, we approximate alkalinity by the carbonate, borate and water self-ionisation alkalinity

$$\text{Alk} \approx \text{Alk}_{\text{CBW}} = [\text{HCO}_3^-] + 2 [\text{CO}_3^{2-}] + [\text{OH}^-] - [\text{H}^+] + [\text{B}(\text{OH})_4^-] \quad (\text{A.19})$$

As before, to compute $[\text{H}^+]$ and the other carbonate species, we have to rewrite eq A.19 as a function of the 4 know quantities DIC, Alk, T , and S and the unknown quantity $[\text{H}^+]$. We already know how express $[\text{HCO}_3^-]$ and $[\text{CO}_3^{2-}]$ as a function of DIC and $[\text{H}^+]$ (eqs A.12 and A.13). Equation 2.22 further gives us

$$[\text{OH}^-] = \frac{K_w}{[\text{H}^+]}, \quad (\text{A.20})$$

and we can use equations 2.23 and 2.24 to obtain

$$[\text{B}(\text{OH})_4^-] = \frac{c_b \cdot S \cdot K_b}{[\text{H}^+] + K_b}. \quad (\text{A.21})$$

Inserting these results in equation A.19, we get

$$\begin{aligned} \text{Alk} \approx \text{Alk}_{\text{CBW}} = \text{DIC} \cdot \frac{K_1 [\text{H}^+] + K_1 K_2}{K_1 K_2 + K_1 [\text{H}^+] + [\text{H}^+]^2} \\ + \frac{K_w}{[\text{H}^+]} - [\text{H}^+] + \frac{c_b \cdot S \cdot K_b}{[\text{H}^+] + K_b} \end{aligned} \quad (\text{A.22})$$

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which we can solve for $[H^+]$, given Alk, DIC and the dissociations constants K_1 , K_2 , K_w and K_b (which depend on T and S). To do this, we follow Munhoven, 2013 and we write eq A.22 as a polynomial equation that is now of degree 5

$$P_{CBW}([H^+]) \equiv [H^+]^5 + q_4[H^+]^4 + q_3[H^+]^3 + q_2[H^+]^2 + q_1[H^+] + q_0, \quad (A.23)$$

with

$$q_4 = \text{Alk} + K_1 + K_b, \quad (A.24)$$

$$q_3 = (\text{Alk} - \text{DIC} + K_b)K_1 + (\text{Alk} - c_b \cdot S)K_b + K_1K_2 - K_w, \quad (A.25)$$

$$q_2 = (\text{Alk} - 2 \cdot \text{DIC} + K_b)K_1K_2 + (\text{Alk} - \text{DIC} - c_b \cdot S)K_1K_b - K_1K_w - K_bK_w, \quad (A.26)$$

$$q_1 = (\text{Alk} - 2 \cdot \text{DIC} - c_b \cdot S)K_1K_2K_b - K_1K_2K_w - K_1K_bK_w, \quad (A.27)$$

$$q_0 = -K_1K_2K_bK_w. \quad (A.28)$$

We solve this equation using the Newton-Raphson method. To ensure quick convergence, we need a good initial guess that is not too far from the real solution. We adopt the following procedure from Munhoven, 2013 and Humphreys et al., 2022. We define the following coefficients

$$c_2 = K_b \left(1 - \frac{c_b \cdot S}{\text{Alk}} \right) + K_1 \left(1 - \frac{\text{DIC}}{\text{Alk}} \right), \quad (A.29)$$

$$c_1 = K_1K_b \left(1 - \frac{c_b \cdot S}{\text{Alk}} - \frac{\text{DIC}}{\text{Alk}} \right) + K_1K_2 \left(1 - 2 \frac{\text{DIC}}{\text{Alk}} \right), \quad (A.30)$$

$$c_0 = K_1K_2K_b \left(1 - \frac{2 \cdot \text{DIC} + c_b \cdot S}{\text{Alk}} \right), \quad (A.31)$$

from which we construct the quantities

$$H_{\min} = \frac{-c_2 + \sqrt{c_2^2 - 3c_1}}{3}, \quad (A.32)$$

and

$$H_0 = H_{\min} + \sqrt{-\frac{H_{\min}^3 + c_2H_{\min}^2 + c_1H_{\min} + c_0}{\sqrt{c_2^2 - 3c_1}}}. \quad (A.33)$$

The initial guess for the Newton-Raphson method is taken as

$$[\text{H}^+]_0 = \begin{cases} 10^{-3} & \text{if } \text{Alk} \leq 0, \\ H_0 & \text{if } 0 < \text{Alk} < 2 \cdot \text{DIC} + c_b \cdot S \text{ and } c_2^2 - 3c_1 > 0, \\ 10^{-7} & \text{if } 0 < \text{Alk} < 2 \cdot \text{DIC} + c_b \cdot S \text{ and } c_2^2 - 3c_1 < 0, \\ 10^{-10} & \text{if } \text{Alk} \geq 2 \cdot \text{DIC} + c_b \cdot S. \end{cases} \quad (\text{A.34})$$

The rationale behind this choice is given in Munhoven, 2013 and Humphreys et al., 2022. With it, we only need 5 Newton-Raphson iterations to obtain an accurate value for $[\text{H}^+]$ and pH, which is defined as

$$\text{pH} = -\log_{10}([\text{H}^+]). \quad (\text{A.35})$$

Once $[\text{H}^+]$ is computed, $[\text{HCO}_3^-]$ and $[\text{CO}_3^{2-}]$ can be computed from equations A.12 and A.13, $[\text{H}_2\text{CO}_3^*]$ can be computed from equation 2.20, $[\text{OH}^-]$ can be computed from equation A.20, and $[\text{B}(\text{OH})_4^-]$ can be computed from equation A.21. Equation A.19 is solved following this procedure at every time step of the numerical integration of the model. This is because we need $[\text{H}^+]_U$ for the computation of B_U in the atmosphere to upper ocean carbon flux $F_{A \rightarrow U}$ (see equations 2.30 and 2.31), and we need $[\text{CO}_3^{2-}]_D$ for the computation of the dissolution flux F_{diss} (see eq 2.54). This is also the procedure used to obtain climatologies (and vertical profiles) of the carbonate species from the GLODAP climatologies of DIC, Alk, temperature and salinity (see Figure 2.3). In this case, we computed the carbonate species at each ocean point where DIC, Alk, temperature and salinity were all given.

To determine the initial condition for the dissolved inorganic carbon mass in the upper layer $M_U(t_{PI})$, we need to compute $\text{DIC}_U(t_{PI})$ as a function of $\text{Alk}_U(t_{PI})$, $T_U(t_{PI})$, S_U , and $[\text{H}_2\text{CO}_3^*]_U(t_{PI})$, which is fixed by equilibrium conditions (eq 2.117). Now DIC is an unknown and we can't use the procedure described above. Instead, we write Alk as a function of $[\text{H}_2\text{CO}_3^*]$ and $[\text{H}^+]$ (and the dissociation). Using equations 2.20 and 2.21 we get

$$[\text{HCO}_3^-] = \frac{K_1[\text{H}_2\text{CO}_3^*]}{[\text{H}^+]}, \quad (\text{A.36})$$

$$[\text{CO}_3^{2-}] = \frac{K_1 K_2 [\text{H}_2\text{CO}_3^*]}{[\text{H}^+]^2}, \quad (\text{A.37})$$

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and thus

$$Alk \approx Alk_{CBW} = \frac{K_1[H_2CO_3^*]}{[H^+]} + 2\frac{K_1K_2[H_2CO_3^*]}{[H^+]^2} + \frac{K_w}{[H^+]} - [H^+] + \frac{c_b \cdot S \cdot K_b}{[H^+] + K_b} . \quad (A.38)$$

This equation can be solved numerically for $[H^+]$ using any algorithm. The speed of the algorithm is not of great importance here since we only perform the computation for the setting of the initial conditions, and not at each time step of the numerical integration. Once $[H^+]$ is obtained, $[HCO_3^-]$ and $[CO_3^{2-}]$ are recovered with equations 2.20 and 2.21 and DIC can then be computed as the sum of $[HCO_3^-]$, $[CO_3^{2-}]$ and $[H_2CO_3^*]$.

A.5 Sensitivity analysis

We present here a sensitivity analysis of the model in response to changes in specific parameters. While the primary goal is to assess how these sometimes arbitrary parameter choices affect model behaviour, an added benefit is the ability to investigate the timescales on which the associated processes are significant.

We first test how atmospheric CO₂ levels simulated for the SSP3-7.0 scenario from 1750 AD to 1 000 000 AD respond to changes in 20 parameters associated with the carbon cycle component of SURFER v3.0 (all parameters from the first part of table A.2, except τ_{CH_4}). Most parameters are adjusted one at a time, with interdependent parameters and $M_U(t_{\text{PI}})$ being modified accordingly to obey the pre-industrial equilibrium condition (see second part of table A.2). Each parameter is varied within a range of 0.25 to 1.75 times its default value. Note that these ranges do not necessarily reflect physically plausible values.

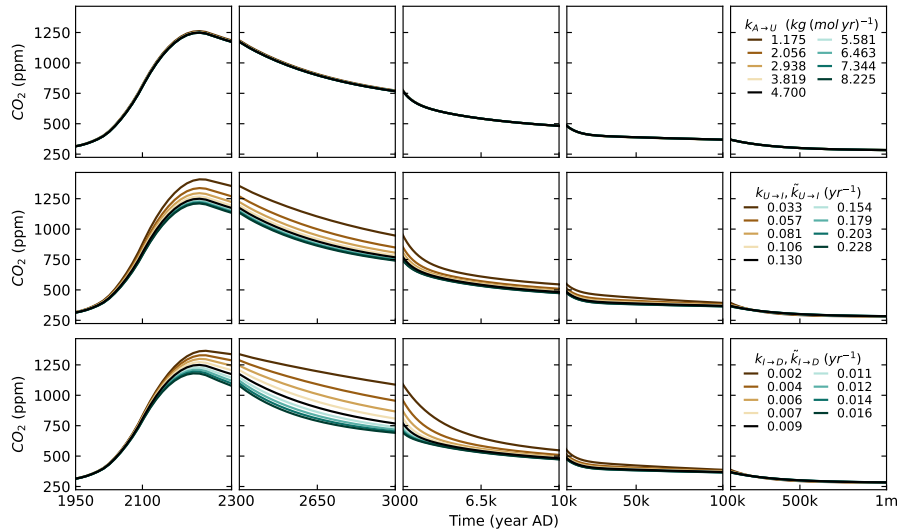


Fig. A.1 Atmospheric CO₂ concentration simulated by SURFER v3.0 under the SSP3-7.0 scenario for different values of $k_{A \rightarrow U}$, $k_{U \rightarrow I}$, and $k_{I \rightarrow D}$ (and $\tilde{k}_{U \rightarrow I}$, and $\tilde{k}_{I \rightarrow D}$). Black lines indicate the atmospheric CO₂ concentration simulated for the default parameter values, as described in section 2.2.4.

Changing $k_{A \rightarrow U}$ in the range described above has a negligible impact on the atmospheric CO₂ draw-down (figure A.1). This is because regardless of

the values set for $k_{A \rightarrow U}$, the transfer rate of CO_2 between the atmosphere and the ocean surface layer is very fast compared to the subsequent transport of dissolved carbon at depth, which therefore is the real limiting factor of for oceanic CO_2 uptake. Indeed, we observe that increasing $k_{U \rightarrow I}$ or $k_{I \rightarrow D}$ leads to an increased CO_2 uptake for up to 100 kyr. After that, the ocean, atmosphere and sediments reservoir are in equilibrium, and atmospheric CO_2 is regulated by the imbalance between volcanism and weathering. Changing $k_{I \rightarrow D}$ only has a very little effect before 2100 AD, reflecting the longer time scales associated with the deeper ocean. Note that we have varied $\tilde{k}_{U \rightarrow I}$ and $\tilde{k}_{I \rightarrow D}$ simultaneously as $k_{U \rightarrow I}$ and $k_{I \rightarrow D}$ because these parameters are all linked to the implicit ocean circulation.

Figure A.2 shows the sensitivity of atmospheric CO_2 to changes in parameters associated with the carbonate and soft-tissue pumps. Overall, varying these parameters has a small or negligible impact. This is due to the pre-industrial equilibrium condition that we impose, which creates a compensating effect. For example, if we have stronger pumps initially, we need stronger upwelling fluxes at equilibrium and so higher $k_{I \rightarrow U}$ and $k_{D \rightarrow I}$ (and $\tilde{k}_{I \rightarrow U}$ and $\tilde{k}_{D \rightarrow I}$). This leads to weaker $F_{U \rightarrow I}^{\text{res}}$ and $F_{I \rightarrow D}^{\text{res}}$ (and $\tilde{F}_{U \rightarrow I}^{\text{res}}$ and $\tilde{F}_{I \rightarrow D}^{\text{res}}$) fluxes which mostly compensates the effects of stronger pumps in transient runs (see equations 2.48-2.51). This compensation effect is not exact and we can observe a small impact for changes in P^{org} and ϕ_I^{org} . In comparison, the impact for changes in P^{CaCO_3} , $\phi_I^{\text{CaCO}_3}$, and $\sigma_{\text{Alk:DIC}}$ are almost non-existent because these parameters are associated to smaller DIC and alkalinity fluxes. Changing $\phi_D^{\text{CaCO}_3}$ has no impact on model results because we keep P^{CaCO_3} constant. In this case the terms in F_{acc} involving $\phi_D^{\text{CaCO}_3}$ cancel out (see equations 2.40, 2.54, and 2.110). Still, we have kept the parameter to facilitate further model updates. Increasing P^{org} implies that we need stronger upwelling fluxes to respect the pre-industrial condition, and so overall the ocean is better ventilated and it is harder to store carbon in the deep ocean, slowing down atmospheric CO_2 uptake. Increasing ϕ_I^{org} has the opposite effect: it leads to a weaker soft-tissue pump, which implies weaker upwelling fluxes and thus a faster CO_2 uptake.

Figures A.3 and A.4 show the sensitivity of atmospheric CO_2 to changes in parameters associated with the dissolution of CaCO_3 sediments and with weathering fluxes. Atmospheric CO_2 is sensitive to changes in α_{diss} and γ_{diss} , but barely to changes in β_{diss} , meaning that the deep CO_3^{2-} concentration rather than the mass of erodible CaCO_3 sediments, M_S , is the dominant driver of dissolution in our parametrisation. With the biggest changes in atmospheric

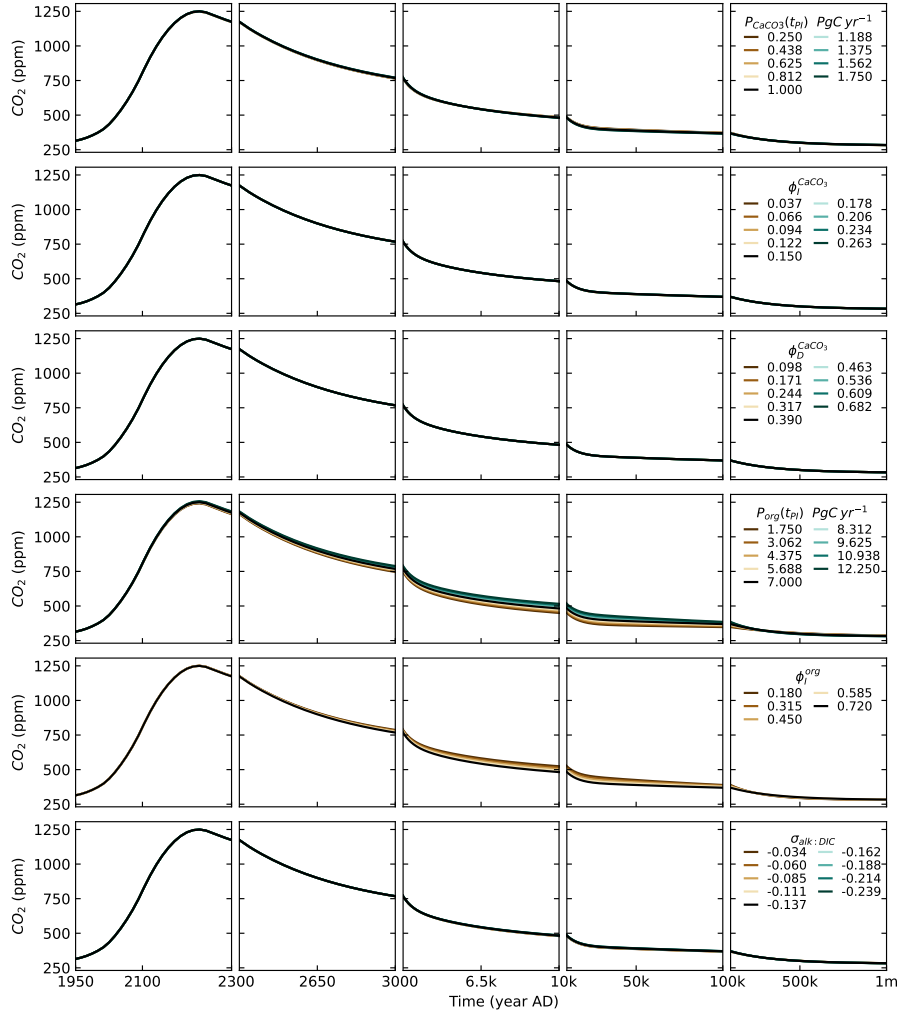


Fig. A.2 Atmospheric CO_2 concentration simulated by SURFER v3.0 under the SSP3-7.0 scenario for different values of P_{CaCO_3} , ϕ_{CaCO_3} , $\phi_{\text{D}}^{\text{CaCO}_3}$, P_{org} , $\phi_{\text{I}}^{\text{org}}$, and $\sigma_{\text{alk-DIC}}$. Black lines indicate the atmospheric CO_2 concentration simulated for the default parameter values, as described in section 2.2.4.

CO_2 concentration observed between 3000 yrs AD and 50 kyr AD, these experiments showcase nicely the timescales associated with sediment processes. Changes in $F_{\text{CaCO}_3,0}$ and k_{Ca} , associated to carbonate weathering have an negligible impact on CO_2 levels compared to changes in $F_{\text{CaSiO}_3,0}$ and k_{T} , associated to silicate weathering. This results from silicate weathering rather than carbonate weathering being the process that is ultimately responsible for the draw-down of excess atmospheric CO_2 , at least in our model. We observe

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that a larger initial silicate weathering flux ($F_{\text{CaSiO}_3,0}$), or a larger response of silicate weathering to warming (k_T) leads to a faster uptake of CO_2 , mostly noticeable after 100 kyr .

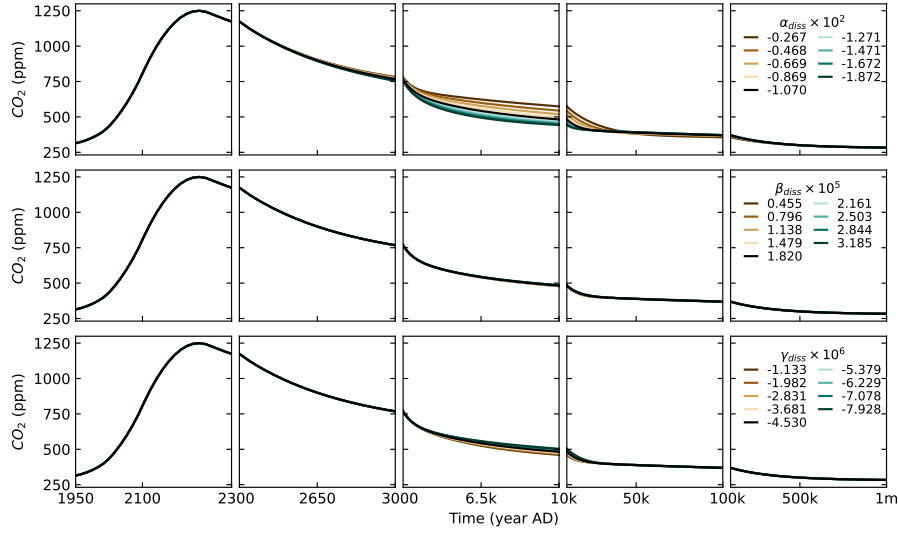


Fig. A.3 Atmospheric CO_2 concentration simulated by SURFER v3.0 under the SSP3-7.0 scenario for different values of α_{diss} , β_{diss} , and γ_{diss} . Black lines indicate the atmospheric CO_2 concentration simulated for the default parameter values, as described in section 2.2.4.

Focusing on vegetation in Figure A.5, we observe that the impact of varying $k_{\text{A} \rightarrow \text{L}}$ is small and limited to years before 2300 AD. This is because the exchanges of carbon between the atmosphere and the land are relatively fast and so the limiting factor to land uptake is rather the amount of carbon that land can store, which is controlled by β_{L} . A larger β_{L} means that more carbon can be stored in the land reservoir for a given atmospheric concentration, thus increasing the land sink. Together with $k_{\text{U} \rightarrow \text{I}}$ and $k_{\text{I} \rightarrow \text{D}}$ (and $\tilde{k}_{\text{U} \rightarrow \text{I}}$ and $\tilde{k}_{\text{I} \rightarrow \text{D}}$), β_{L} is the parameter that has the biggest influence on CO_2 uptake up to the year ~ 6500 AD, showing that the fertilisation effect and oceanic invasion of CO_2 are the main processes driving atmospheric CO_2 draw-down on these timescales in SURFER v3.0.

We also test how simulated atmospheric CO_2 levels, temperatures and sea level rise respond to changes in $\gamma_{\text{U} \rightarrow \text{I}}$, $\gamma_{\text{I} \rightarrow \text{D}}$, and β , which are parameters linked to the climate module of SURFER v3.0. The experimental setup is the same as the one described above.

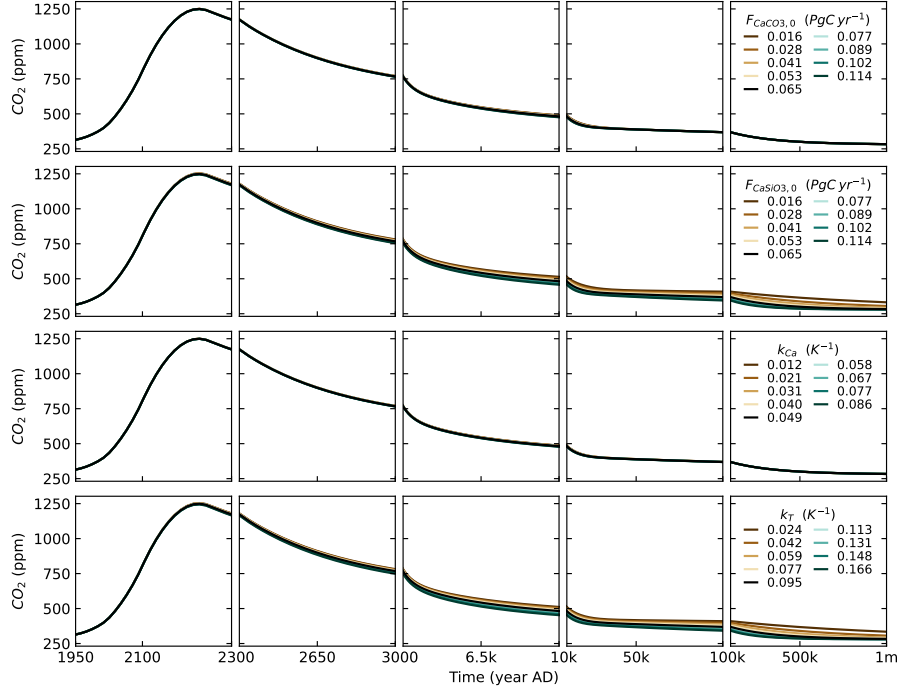


Fig. A.4 Atmospheric CO_2 concentration simulated by SURFER v3.0 under the SSP3-7.0 scenario for different values of $F_{\text{CaCO}_3,0}$, $F_{\text{CaSiO}_3,0}$, k_{Ca} , and k_{T} . Black lines indicate the atmospheric CO_2 concentration simulated for the default parameter values, as described in section 2.2.4.

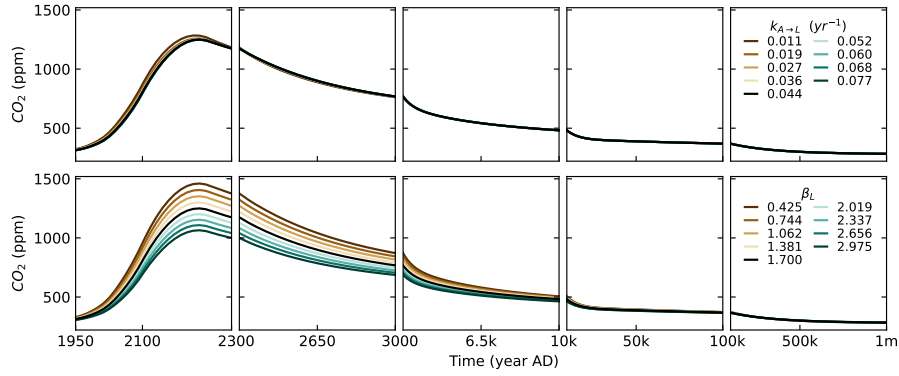


Fig. A.5 Atmospheric CO_2 concentration simulated by SURFER v3.0 under the SSP3-7.0 scenario for different values of $k_{\text{A} \rightarrow \text{L}}$, and β_{L} . Black lines indicate the atmospheric CO_2 concentration simulated for the default parameter values, as described in section 2.2.4.

Figures A.6 and A.7 show the sensitivity analysis for $\gamma_{\text{U} \rightarrow \text{I}}$ and $\gamma_{\text{I} \rightarrow \text{D}}$. Increasing $\gamma_{\text{U} \rightarrow \text{I}}$ decreases the heat accumulation and thus temperatures in

the surface layer, but increases temperature in the intermediate and deep layers. On the other hand, increasing $\gamma_{I \rightarrow D}$ decreases the heat accumulation and temperature in the surface and intermediate layers, but increases the temperature in the deep layer. For both parameters, an increase leads to an overall increase in thermosteric sea level rise because the deep layer dominates the thermal expansion. Although both parameters impact surface temperature, they have almost no impact on atmospheric CO_2 because SURFER v3.0 has a very weak carbon-climate feedback (see discussion, section 2.5). We also observe little to no impact on temperatures after 10 000 kyr, suggesting that by that time, the ocean has reached thermal equilibrium.

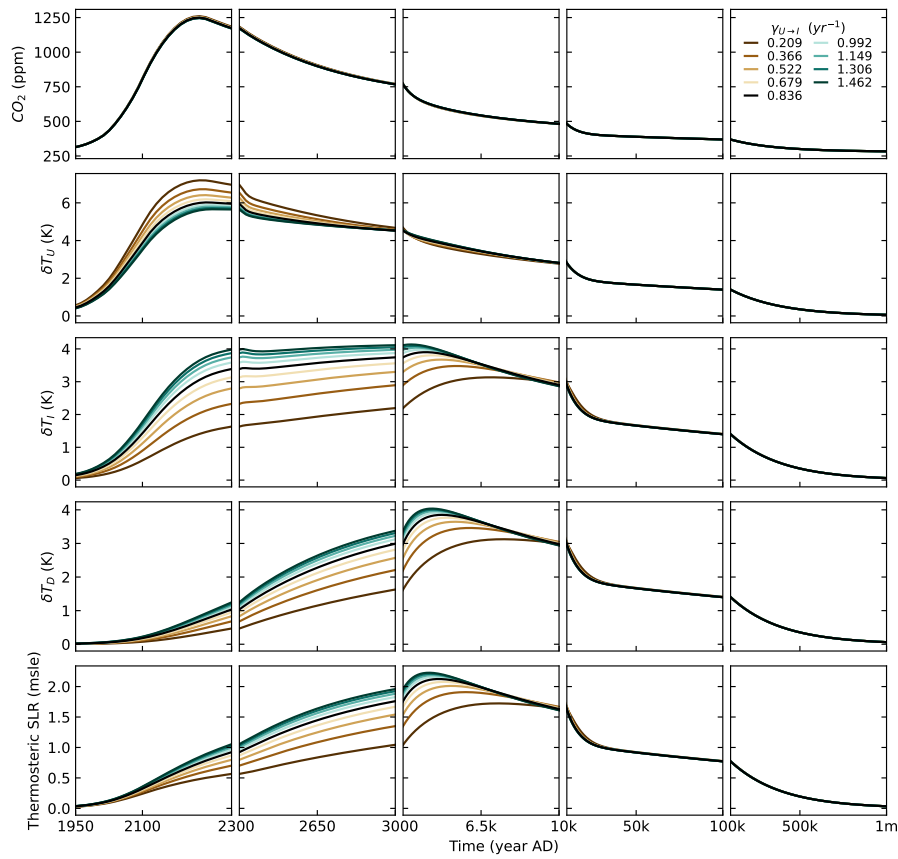


Fig. A.6 Atmospheric CO_2 concentration, temperatures, and thermosteric sea level rise simulated by SURFER v3.0 under the SSP3-7.0 scenario for different values of $\gamma_{U \rightarrow I}$. Black lines indicate outputs simulated for the default parameter values, as described in section 2.2.4.

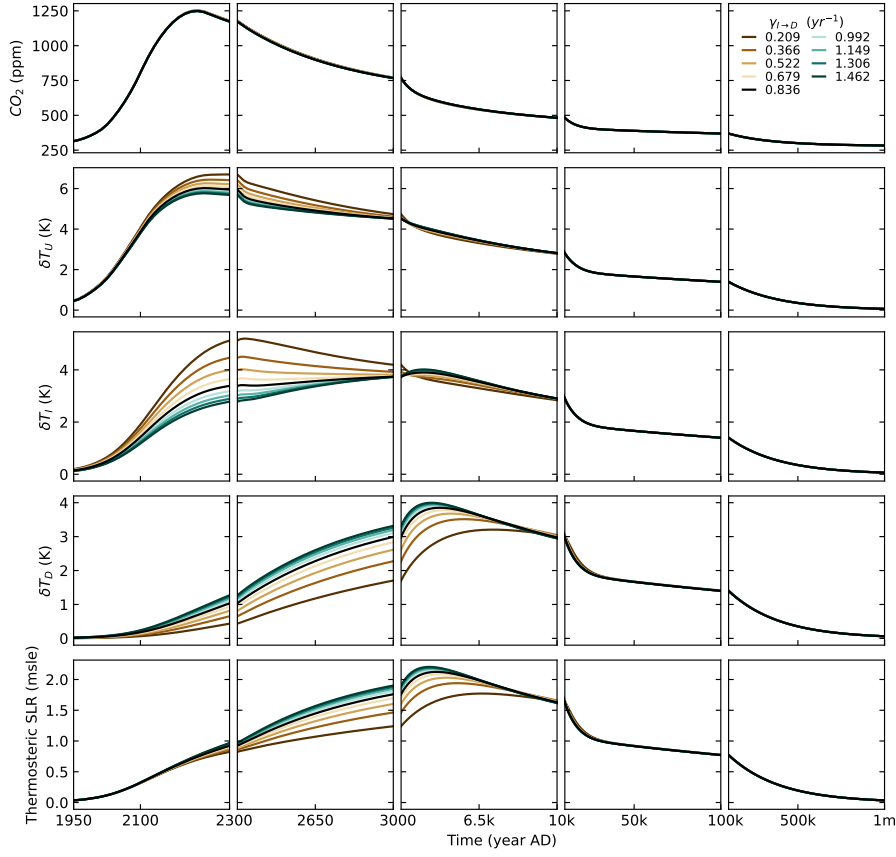


Fig. A.7 Atmospheric CO_2 concentration, temperatures, and thermosteric sea level rise simulated by SURFER v3.0 under the SSP3-7.0 scenario for different values of $\gamma_{I \rightarrow D}$. Black lines indicate outputs simulated for the default parameter values, as described in section 2.2.4.

In Figure A.8, we investigate SURFER v3.0's response to changes in β , the climate feedback parameter. Decreasing β while keeping $F_{2\times}$ constant increases the equilibrium climate sensitivity. In this case, we vary β in the range $0.780\text{--}1.671 \text{ W m}^{-2} \text{ } ^\circ\text{C}^{-1}$ giving an ECS range of $2.33\text{--}5^\circ\text{C}$ which is within the estimated very likely $2\text{--}5^\circ\text{C}$ range given by the IPCC Smith et al., 2021. Lower values for β , and thus higher values of climate sensitivity naturally lead to more warming and consequently a higher sea level rise. Big differences in projected sea level rise after 10 kyr AD, even for close β values, are the consequence of tipping the Greenland ice sheet. As for $\gamma_{U \rightarrow I}$, and $\gamma_{I \rightarrow D}$, changes in β do not impact CO_2 levels that much since the total carbon-climate feedback in SURFER v3.0 is weak. For higher values of climate sensitivity, the

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positive climate-carbon feedback resulting from the temperature-dependent solubility and dissociation constants leads to higher CO₂ levels up to around 10 kyr AD. Conversely, the negative carbon-climate feedback from silicate weathering, acting on longer timescales, results in lower CO₂ values after 10 kyr AD, and especially after 100 000 kyr.

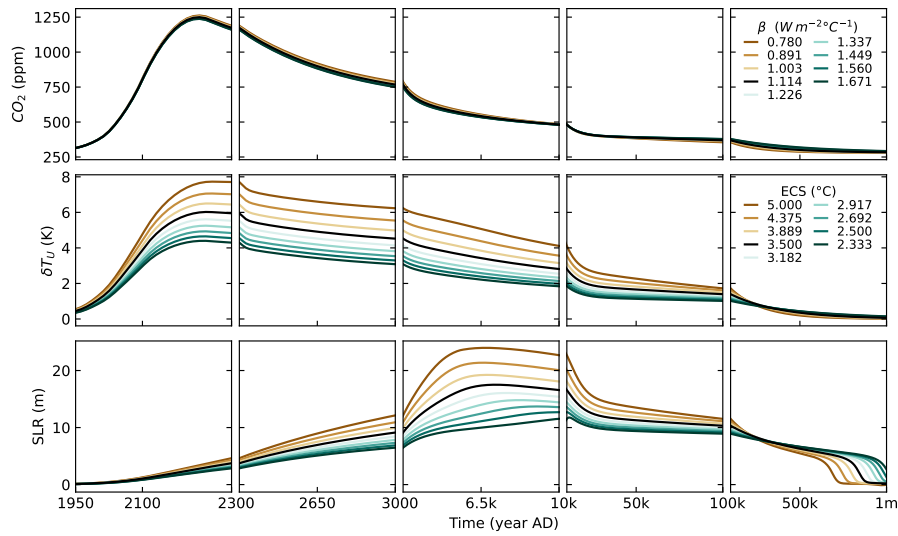


Fig. A.8 Atmospheric CO₂ concentration, surface temperature, and total sea level rise simulated by SURFER v3.0 under the SSP3-7.0 scenario for different values of β . The equilibrium climate sensitivity of SURFER v3.0 is related to β through $ECS = F_{2\times} / \beta$. Black lines indicate outputs simulated for the default parameter values, as described in section 2.2.4.

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B.1 Emission scenarios

We use CO₂ emissions data from the Global Carbon Budget (Friedlingstein et al., 2022) for the period 1850 to 2015, incorporating fossil fuel emissions, land-use change emissions, and the cement carbonation sink. For simplicity, we treat land-use emissions as fossil emissions, although SURFER allows for distinctions between emission types. Cumulative CO₂ emissions from 1850 to 2015, including land-use emissions, amount to 639 PgC. From 2015 to 3000 AD, emissions $E(t)$ follow the logistic equation

$$E(t) = -\frac{dF(t)}{dt} = -aF(t) (F(1850) - F(t)) \quad (\text{B.1})$$

similarly as in Winkelmann et al., 2015 or Lord et al., 2016. Emissions before 1850 and after 3000 AD are set to zero. Here, $F(t)$ represents the total emissions remaining to be released at time t , and a is a parameter defined by

$$a = \frac{-E(2015)}{F(1850)(F(1850) - F(2015))} , \quad (\text{B.2})$$

where $E(2015)$ are the historical CO₂ emissions in 2015 and $F(2015)$ is the amount of total emissions still to be released after 2015.

In this study, we explore a range of 100 emissions scenarios, with $F(2015)$ varying between 50 PgC and 5000 PgC. Indexing each scenario by j , we define

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$F_j(2015) = j \cdot 50$ PgC, for $j \in 1, 2, \dots, 100$. Solving equation B.1 for F , we find

$$F(t) = \frac{F(t^*)F(1850)e^{aF(1850)(t-2015)}}{F(2015)e^{aF(1850)(t-2015)} + F(1850) - F(2015)} \quad (\text{B.3})$$

where the total cumulative emissions are given by $F(1850) = 639$ PgC + $F(2015)$. Throughout Chapters 3 and 4, we often refer to scenarios by their cumulative emissions value after 2015, meaning that when we mention a scenario with cumulative emissions X , we refer to the scenario j with $F_j(2015) = X$ PgC. Our scenarios cover a similar range to that of the SSP scenarios used in IPCC reports, ensuring their policy relevance (see Figure B.1).

We use the same CH_4 emissions for all scenarios. For emissions from 1850 up to 1989 (included), we use the data from Jones et al. (2023) and include fossil and land-use emissions. From 1990 to 2100, use the values for the SSP1-2.6 scenario as provided in the SSP database (<https://tntcat.iiasa.ac.at/SspDb/dsdGidden> et al. (2019) and Riahi et al. (2017)). From 2100 to 2300 we use the values for the SSP1-2.6 scenario extension, as described in Meinshausen et al. (2020). Outside of this range, CH_4 emissions are set to zero. This approach results in a total cumulative CH_4 emission of 20 PgC.

B.2 Transition time scales

We sample the timescales $\tau_{-,i}$ from uniform distributions with bounds selected to ensure that the effective transition timescales of the tipping elements fall within or close to the assessed range reported in the literature. Figure B.2 shows the distribution of effective transition timescales for tipping elements in our baseline experiment, for three different scenarios. For each element and scenario, transition timescales are influenced not only by τ_- but also by the value of T_+ , resulting in a non-uniform distribution. In the majority of cases, the transition timescales fall within the range assessed by Armstrong McKay et al., 2022, indicating that our calibration procedure effectively aligns with established estimates.

It is important to note that while some observed transition times are computed as ~ 700 years for GRIS, ~ 200 years for WAIS, and ~ 400 years for EASB, this does not imply that these elements are fully collapsed by 2550 AD, 2050 AD, or 2250 AD. This is because our definition of transition time for an element i only takes into account the duration for the element to evolve from $x_i = x_+ = 0.75$ to $x_i = x_- = 0.25$. In our simulations for the 5000 PgC

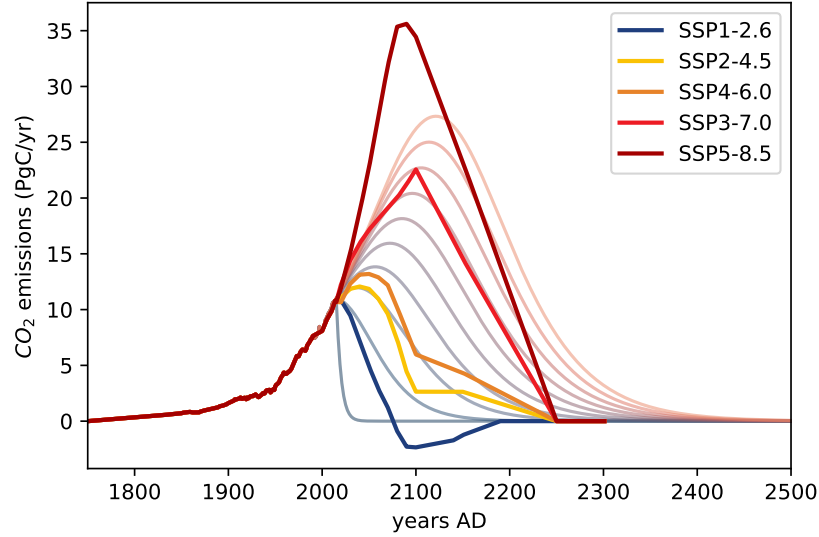


Fig. B.1 Comparison between CO_2 emissions from SSP scenarios (land-use + fossil) with scenarios used in Chapters 3 and 4. In the experiments we perform, we force the model with 10 emissions scenarios, starting with 50 PgC of total cumulative emissions after 2015 AD, adding 50 PgC of CO_2 emissions for each successive scenario until we reach 500 PgC of cumulative emissions. In this figure, only one in ten scenarios is plotted.

scenario, the scenario with the fastest transitions, we find that GRIS, WAIS, and EASB reach complete collapse ($x_i = 0$) at the earliest by ~ 3400 AD, ~ 2500 AD, and ~ 2700 AD, respectively.

In our experiments with an equilibrium climate sensitivity (ECS) of 3.5°C , the East Antarctic Ice Sheet (EAIS) does not exhibit tipping, making it difficult to evaluate the effectiveness of our calibration procedure for this element. Additionally, the assessed range for its transition timescale is poorly constrained. Armstrong McKay et al., 2022 give a minimum estimate of 10,000 years but do not provide a maximum estimate. For our calibration, we selected minimum and maximum transition timescales of 5,000 years and 15,000 years, respectively. This choice results in a sampling range for $\tau_{-, \text{EAIS}}$ of [469, 741] years, aimed at improving the simulation of the total sea level rise contribution from the Antarctic Ice Sheet, which is addressed in Appendix C.1. This situation also highlights that, since the effective transition timescale depends on T_+ , a lower sampling range for τ_- does not necessarily yield shorter transition times when comparing different elements.

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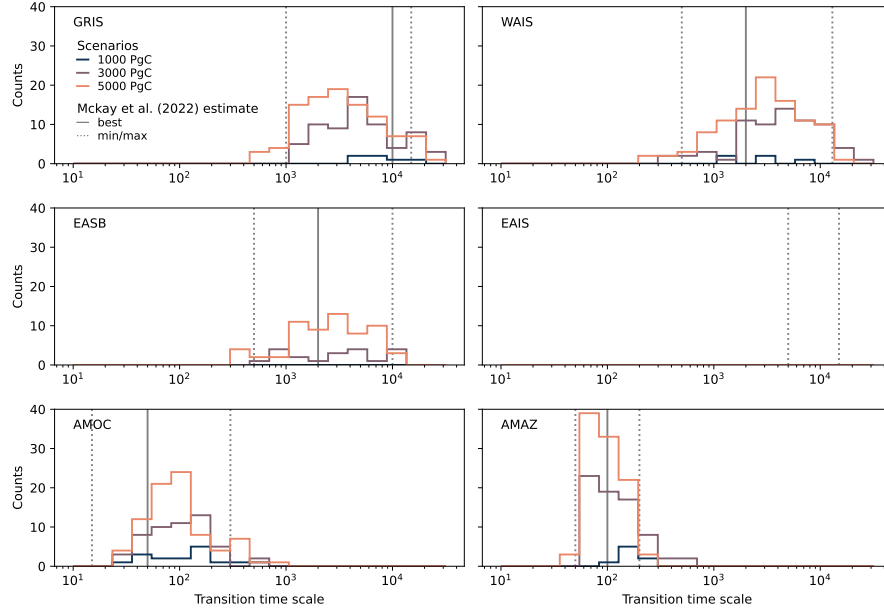


Fig. B.2 Distribution of effective transition time scales for three different scenarios (1000 PgC, 3000 PgC, and 5000 PgC) in the fully coupled experiment. The effective transition timescale is defined for an element that tips as the time it takes to transition from $x = x_+ = 0.75$ to $x = x_- = 0.25$. The EAIS never tips in the fully coupled experiment, which is why no distributions are plotted for this element. Vertical lines indicate the minimum, maximum and best estimate from Armstrong McKay et al., 2022, also given in table 3.2.

B.3 Sensitivity analysis

In the main text, we have discussed the results of 4 experiments : the baseline, interactions only, feedbacks only, and the fully coupled experiments. These experiments correspond to enabling or not the interactions between the elements and their feedback on temperature. In the following section, we test the sensitivity of our results to several other model settings : an increased climate sensitivity, increased interactions strengths , an a removal of Arctic sea-ice and permafrost.

IPCC assessed the very likely equilibrium climate sensitivity range to be between 2°C and 5°C . SURFER's climate sensitivity is determined by the climate feedback parameter (see eq 3.23), which we have chosen such that $\text{ECS} = 3.5^\circ\text{C}$, a value in the middle of the assessed range. Here, we repeat

the baseline and fully coupled experiment but with $ECS = 5.0^{\circ}\text{C}$, a value at the high end of the very likely range. To do this we set $\beta = 0.78\text{W m}^2\text{C}^{-1}$, while all other parameters are kept the same. Figure B.3 shows the tipping risk for each element in these new experiments, compared to the tipping risk for the same experiments with $ECS = 3.5^{\circ}\text{C}$. Unsurprisingly, we observe that for a given scenario, the tipping risk is higher for a higher climate sensitivity. This is because the a higher climate sensitivity leads to higher warming, and higher temperatures always increase the tipping risk in our model, all other things being equal. For AMOC and AMAZ, the tipping risk as a function of peak warming is almost identical for equivalent experiments with different ECS. This is because for these fast elements, their tipping status is mainly determined by short term temperatures and whether warming exceeds their critical temperature thresholds (see section 3.3.1). For GRIS, WAIS, and EASB, the tipping risk as a function of peak warming is similar but slightly larger for the experiments with higher climate sensitivity. For those slow tipping elements, peak temperatures matter, but also how fast will the temperature decrease after peaking (see section 3.3.1), and it turns out that in scenarios with different climate sensitivities but same peak warming, the temperatures for the scenario with $ECS = 5^{\circ}\text{C}$ decrease more slowly on the millennial time scale, as can be seen in Figure B.4

Overall it shows, that while the relationship between cumulative emissions and tipping risk may depend on climate sensitivity, the relationship between peak warming and tipping risk is more robust. This is the reason why we have always plotted tipping risk as a function of CO_2 emissions and peak warming in Chapter 3. Nevertheless, it should be noted that the relationship between peak warming and tipping risk can be impacted by anything that could change temperatures quite rapidly, such as methane emissions, strong negative CO_2 emissions or aerosols emissions.

Interactions between nonlinear elements are very poorly constrained, and admittedly, the rule we have chosen to assign values for the coupling coefficients as a function of the assessed interaction strengths and signs by Wunderling et al., 2024 is rather arbitrary (see Figure 3.3). In Wunderling et al., 2021, who use a similar experimental setup as us and also have a parameter d_W that scales all interactions, the value of the forcing induced by the tipping of element i on element j for strong interactions can reach up to 5 times the critical forcing value of j , and can trigger the tipping of j even for most of interactions which are considered as weak. A $d = 1$ value in our model (see eq 3.16) approximately corresponds to a value $d_W \sim 0.2 - 0.4$ in Wunderling

et al., 2021. In Figure B.5, we show the tipping risk for all elements in the fully coupled experiment but with $d = 2$. In this case, the tipping of an element i is always sufficient on its own to trigger the tipping of element j in case of strong interactions, and may trigger tipping even in case of moderate strength interactions. This corresponds to a value of about $d_W \sim 0.4 - 0.8$ in Wunderling et al., 2021. We observe that doubling all interactions strengths amplifies the effects of interactions already described in section 3.3.1. Compared to the baseline experiment, the AMOC, WAIS and EASB tipping risk increases more for $d = 2$ than for $d = 1$, the GRIS tipping risk decreases more and the AMAZ tipping risk doesn't change much, except for high emissions scenarios. The change in tipping risk resulting from interactions does not depend linearly on d , i.e., the difference in tipping risk for a given element and scenario between the baseline and fully coupled experiments for $d = 2$ is not twice the difference for $d = 1$.

We also examine tipping risk in the fully coupled experiment when Arctic sea ice (ASI) and permafrost (PERM) are excluded from the model. Figure B.5 shows that this exclusion leads to a significant reduction in the tipping risk for the AMOC, due to the destabilizing effects that both ASI and PERM have on the AMOC when included. This reduction in AMOC tipping risk subsequently propagates to other elements based on their dependence on AMOC: the tipping risk for WAIS and EASB decreases due to reduced destabilization from the AMOC, while the tipping risk for GRIS increases in its absence.

Interestingly, we observe that AMOC tipping risk is now only slightly higher than in the baseline case, suggesting that in our model, the AMOC is only moderately impacted by GRIS and WAIS. It is expected that WAIS has a negligible net effect on AMOC tipping risk due to its potential for both stabilizing and destabilizing interactions. GRIS, on the other hand, exerts only a limited destabilizing influence on AMOC, as its meltwater flux F_{GRIS} is in general small compared to the chosen critical threshold range ($F_{\text{GRIS}}^+ \in [0.1, 05]$ sv), as shown in Figure B.6.

Overall, these findings demonstrate the importance of including ASI and PERM in tipping cascade studies, even if they are not classified as tipping elements themselves. What matters most is their potentially nonlinear behaviour and, more critically, their strong coupling with other elements, rather than whether they strictly meet the definition of a tipping element.

Table B.1 presents the risk of observing any tipping event for a scenario with cumulative emissions of 1000 PgC or less, across all experiments performed. We find that the experiments with the highest tipping risk are the ones with an

ECS of 5.0°C , with a 86% risk for the fully coupled experiment and 76% for the baseline experiment. On the other hand, the likelihood of observing cascades of size 3 or 4 is the greatest in the $d = 2$ case, even surpassing that of the fully coupled experiment with $\text{ECS} = 5.0^{\circ}\text{C}$. This aligns with findings from Wunderling et al., 2021, who observed that interaction strength influences cascade distribution and size.

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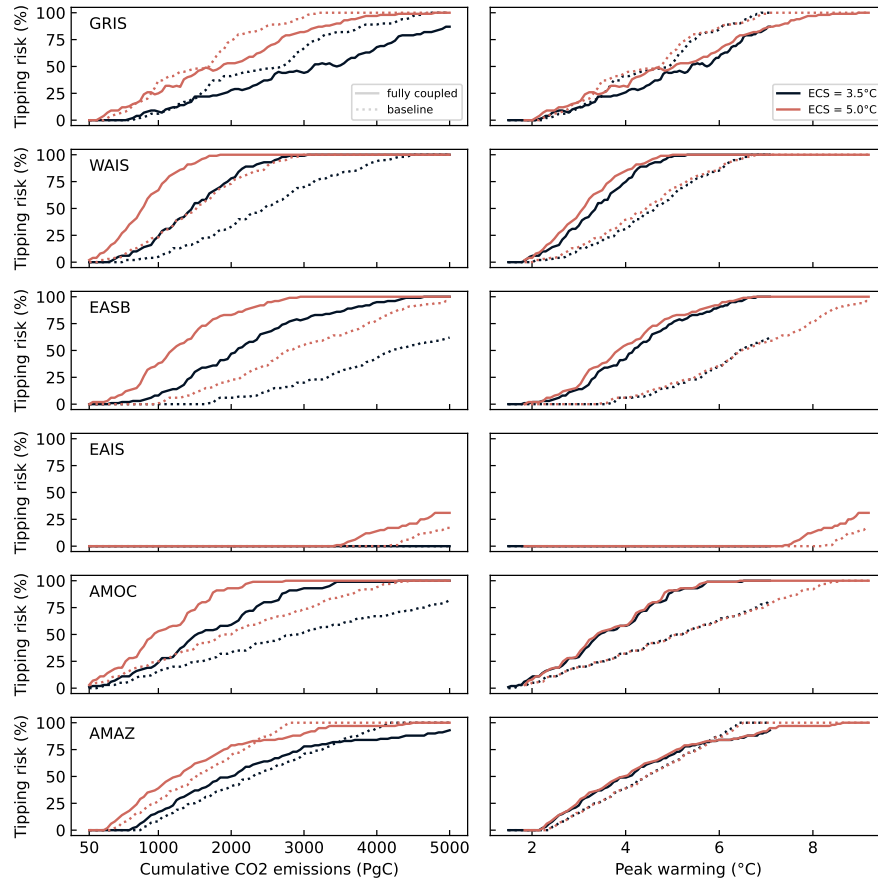


Fig. B.3 Tipping risk of the different tipping elements, for the baseline and fully coupled experiments, with ECS = 3.5°C or ECS = 5°C. For a given element and scenario, the tipping risk is defined as the percentage of ensemble simulations, generated with different parameter sets, in which the elements tips. In the left panels, the risk is given as a function of cumulative emissions. In the right panels, the risk is given as a function of peak warming in the baseline experiments, where warming does not depend on the state of the tipping elements.

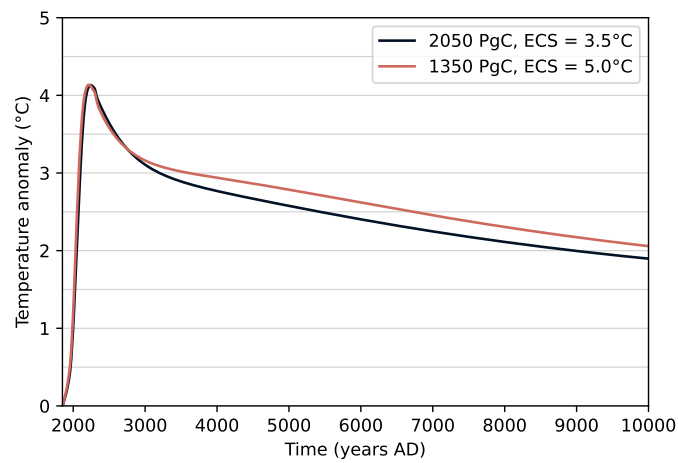


Fig. B.4 Temperature evolution in the baseline experiment settings (interactions between tipping elements and their feedback on temperature are not included) for two different scenarios and ECS. Both scenarios result in the same peak warming and short term evolution, but the multi-millennial evolution is different, which result in a different tipping risk for the slow tipping elements (see top 3 right panels of figure B.3).

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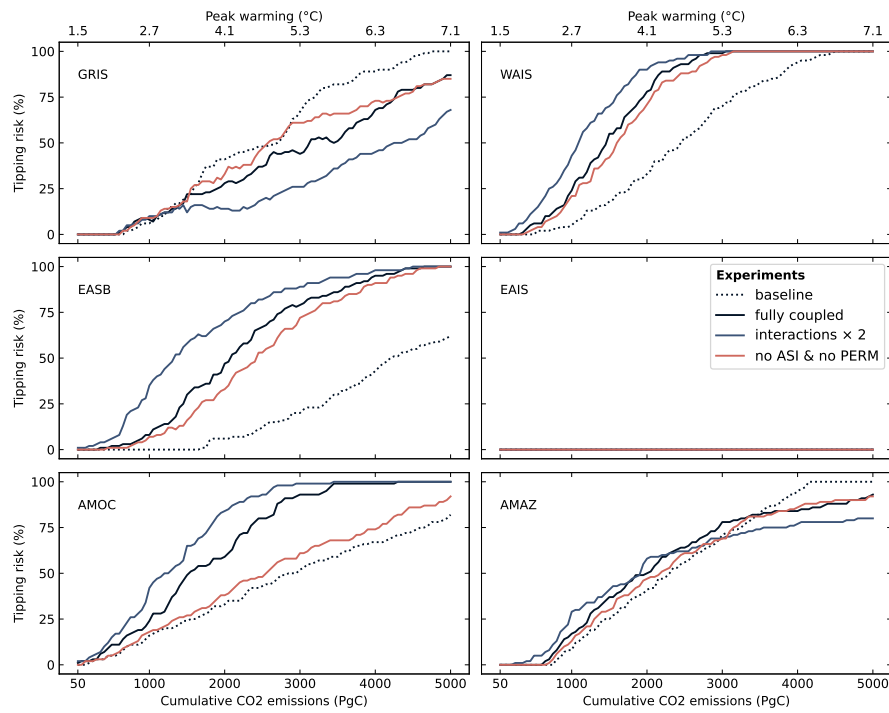


Fig. B.5 Tipping risk of the different tipping elements, for different experiments : the fully coupled experiment with $ECS = 3.5^{\circ}C$ (black), the fully coupled experiment with $ECS = 3.5^{\circ}C$ and interaction strengths doubled (blue), and the fully coupled experiment with $ECS = 3.5^{\circ}C$ but excluding ASI and PERM (red). For a given element and scenario, the tipping risk is defined as the percentage of ensemble simulations, generated with different parameter sets, in which the elements tips. For reference, the peak warming corresponding to a given level of emissions in the baseline experiment is indicated on the upper axis of the plots.

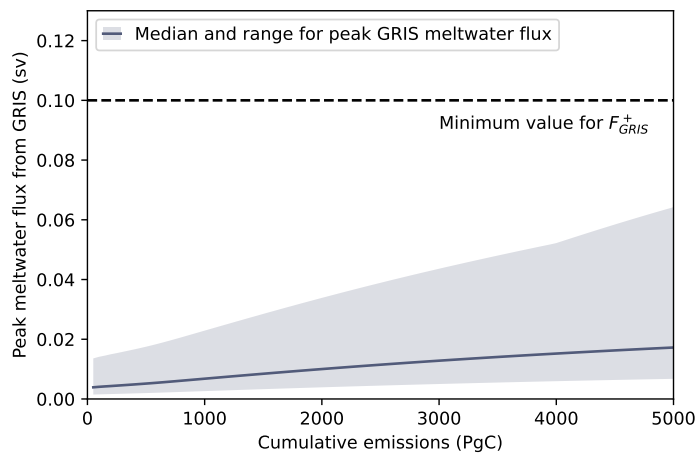


Fig. B.6 Peak meltwater flux from Greenland as a function of cumulative emissions in the fully coupled experiment. How fast Greenland melts under a given scenario depends on the values of $T_{+,GRIS}$ and $\tau_{-,GRIS}$, but also on the values of the coupling coefficients and the evolution of other elements, resulting in a large possible range for peak meltwater flux. Overall, the peak meltwater flux is quite small compared even to the smallest possible value for F_{GRIS}^+ , resulting in a limited impact on AMOC.

ECS	Experiments			any tipping event		single tipping event		destablilising cascade					stablilising cascade
	interacting TEs	feedbacks on GMT	ASI & PERM	size 2	size 3	size 4	size 5	size 6					
3.5 °C	no	no	yes	31	31	0	0	0	0	0	0		
3.5 °C	yes	yes	yes	48	41	13	0	1	0	0	1		
3.5 °C	yes	no	yes	40	36	8	0	1	0	0	0		
3.5 °C	no	yes	yes	39	39	0	0	0	0	0	0		
5.0 °C	no	no	yes	76	71	5	0	0	0	0	0		
5.0 °C	yes	yes	yes	86	70	32	8	2	0	0	7		
3.5 °C	yes (×2)	yes	yes	64	33	20	21	3	0	0	2		
3.5 °C	yes	yes	no	40	33	10	1	1	0	0	0		

Table B.1 Risk of observing tipping events for emissions scenarios with cumulative emissions below or equal to 1000 PgC, for different experiment settings. These experiments differ in several key aspects: the ECS used, whether interaction between tipping elements or other elements are included, whether feedbacks of these elements on temperature are considered, and whether ASI and PERM are included. For each experiment and a given tipping event, the risk is computed here as the percentage of ensemble members with different parameters for which the tipping event occurs at least once in a scenario with cumulative emissions below or equal to 1000 PgC.

B.4 Increase in tipping risk as function of emissions

In Figure B.7, we examine how the increase in risk per additional 50 PgC of emissions varies as a function of cumulative emissions. In other words, we estimate the derivative of curves shown in Figure 3.6. For low emissions scenarios (<1000 PgC) in the fully coupled experiment, we find that the AMOC initially experiences the highest increase in tipping risk, with an increase of approximately 1%-2% per 50 PgC. We observe a large acceleration of the tipping risk (increase of increase) within 1000 PgC of cumulative emissions for AMAZ and WAIS, but only the acceleration for AMAZ would be noticeable if we evaluated the tipping risk on short time scales. Comparison with the baseline experiment shows that for AMAZ, the tipping risk acceleration for low emission scenarios primarily results from AMAZ starting to tip under these scenarios for some combinations of (T_+, τ_-) , regardless of the interactions between elements and their feedbacks on temperature (see Figure 3.5). In contrast, the magnitude of the tipping risk acceleration for WAIS under low emissions scenario seems to be largely driven by interactions and feedbacks on temperature. An acceleration of the tipping risk within 1000 PgC is also observed for AMOC, EASB and GRIS, although with smaller magnitude than for WAIS and AMAZ. These findings are consistent with the results from Möller et al., 2024.

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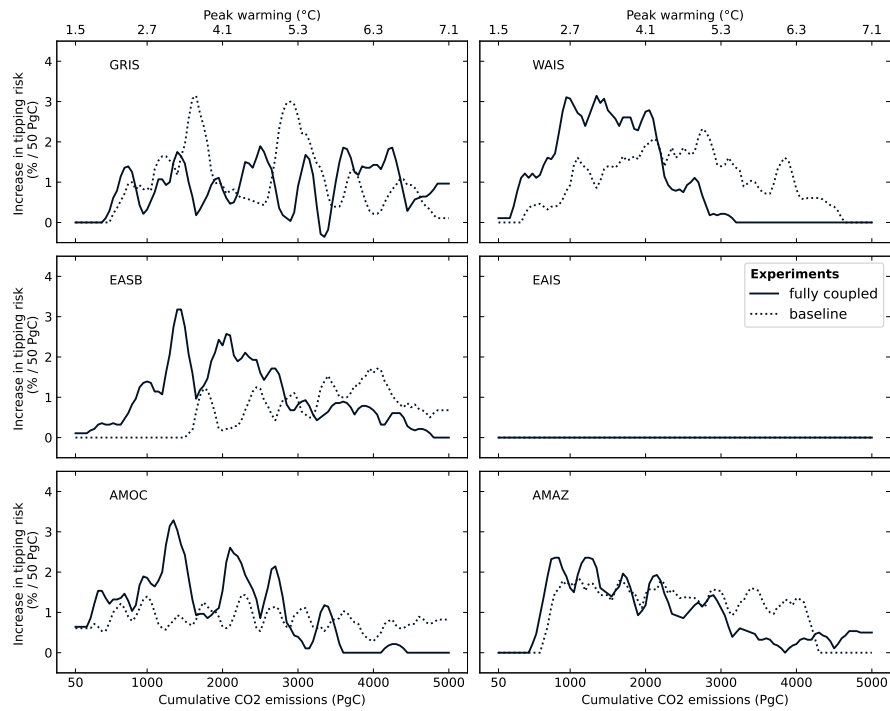


Fig. B.7 Increase in tipping risk of the different tipping elements, per additional 50 PgC of cumulative emissions, and as a function of cumulative emissions. In other words, we plot the derivative of the tipping risk curves shown in figure 3.6. The derivatives are estimated using a Savitzky-Golay filter with polynomial order 3 and a window length of 7 points. For reference, the peak warming corresponding to a given level of emissions in the baseline experiment is indicated on the upper axis of the plots.

B.5 TIPMIP protocol

TIPMIP is an international, ongoing model inter-comparison project aimed at studying tipping dynamics in various Earth system components and assessing the uncertainties involved. Results from TIPMIP would greatly benefit SURFER by providing constrained parameter ranges and a stronger basis for model calibration. With such inputs, SURFER could then serve as an emulator to conduct probabilistic risk assessments for specific scenarios. In this case, information flows downward within the model hierarchy, from complex, state-of-the-art models to reduced-complexity models.

However, the information can also flow upwards. For instance, SURFER can be used as an exploratory tool to highlight important concepts and uncover robust phenomena, which could then be further investigated with more complex models, as demonstrated in the main text of Chapter 3. Here, we showcase another possible use by running SURFER under the Tier 1 ESM TIPMIP protocol, providing a low-cost test of the protocol's feasibility, relevance, and potential challenges. This exercise offers preliminary insights into expected outputs and supplies a framework for interpreting them, serving as a proof of concept for TIPMIP.

The TIPMIP protocol consists of a control run and 5 emission based scenarios. CO₂ is emitted at a constant rate of $X \text{ PgC/yr}$ such as to produce a warming of around 0.2°C per decade until warming reaches 2°C or 4°C , at which point emissions are zero for 50 years. Then either emissions are kept to zero until the end of the simulation, either negatives emissions are introduced at the same rate of $X \text{ PgC/yr}$ until warming is back to zero. For the 4°C warming case, there is also an additional scenario where negative emissions are introduced until temperatures fall back to 2°C , then kept zero afterwards. Here we run SURFER under each of these scenarios and for each parameter set of our monte-carlo ensemble. We repeat this for 2 different model settings : the baseline, and fully coupled settings (see section 3.2.3). This constitutes 2 experiments, each consisting of 5×100 model runs. The baseline experiment is equivalent to studying each tipping element separately in off-line models.

Figure B.8 shows the evolution of the global mean temperature in SURFER across the five TIPMIP scenarios. In our model, a constant CO₂ emission rate of $\sim 13,3 \text{ PgC yr}^{-1}$ was required to obtain a warming rate of around 0.2°C per decade. It should be noted that the warming is not perfectly linear : reaching 2°C from 0°C requires about 90 years of emissions, while reaching 4°C

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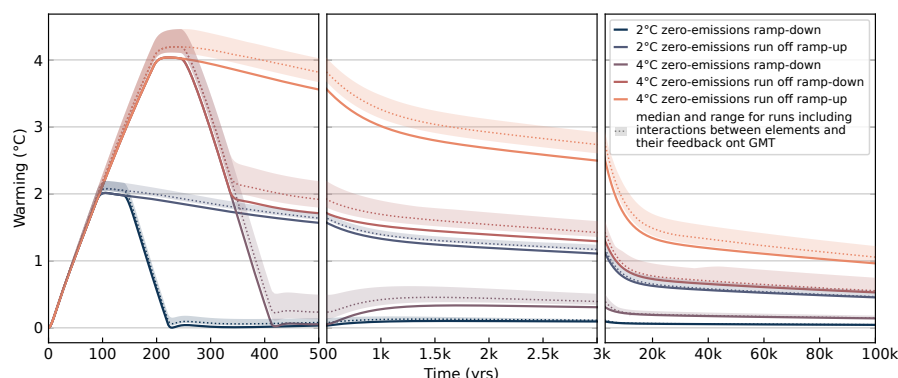


Fig. B.8 Temperature evolution for the different scenarios of the TIPMIP Tier 1 ESM protocol. Plain lines indicate the temperature evolution in our baseline experiment settings, that is not including the interactions between tipping elements and their feedback on temperature. The dotted lines and shaded areas indicate respectively the median and maximum range of temperatures in the fully coupled experiment settings.

from 2°C takes approximately 105 years. Additionally, temperature decreases during negative emissions are not symmetrical with the warming. For instance, while it takes 195 years of positive emissions to increase from 0°C to 4°C, only 167 years of negative emissions are needed to reverse this warming. This asymmetry is driven by the continuous action of natural land and ocean carbon sinks, and results in net cumulative emissions for the two scenarios with full temperature reversals: 120 PgC for the 2°C peak warming scenario and 373 PgC for the 4°C scenario. This carbon is accumulated in the ocean and land during the warming and cooling phases, and then partly released back to the atmosphere as the carbon reservoirs equilibrate after emissions are set to zero. This process explains the small rebound in atmospheric CO₂ and temperatures observed in these scenario.

Once emissions are set to zero, temperatures remain relatively stable, with changes not exceeding 0.5°C until around year 500. Beyond this point, temperatures decrease over multi-millennial timescales as atmospheric CO₂ gradually declines. In the fully coupled experiment, temperatures are up to 0.45°C higher than in the baseline experiment due to feedbacks from tipping and other nonlinear elements. These feedbacks may affect tipping risks and should be considered when defining scenarios. Specifically, it is important to specify whether the feedbacks of studied elements on temperature, especially faster ones, are included when determining the constant emission rate that results in a $\sim 0.2^\circ\text{C}$ warming per decade.

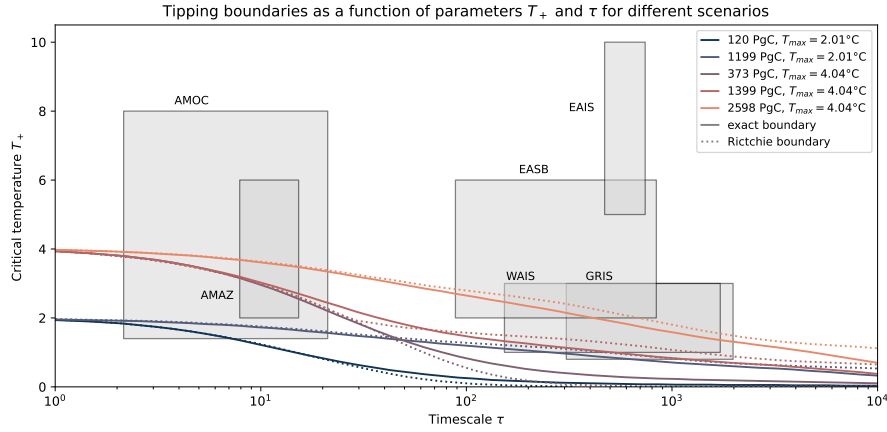


Fig. B.9 Tipping behaviour thresholds for the tipping elements, as a function of their critical temperature T_+ and their intrinsic time scale τ_- . The gray rectangles indicate the range of possible parameter values for each element. The plain coloured lines represent the tipping thresholds for the different scenarios of the TIPMIP Tier 1 ESM protocol, and in our baseline experiment (interactions between tipping elements and their feedback on temperature are not included). An element with a (T_+, τ_-) combination that is below a given threshold will tip for this scenario. The coloured dotted lines are the theoretical predictions for the tipping thresholds based on the criterion from Ritchie et al., 2019, see equation 3.38.

Figure B.9 shows the baseline tipping thresholds for the 5 TIPMIP scenarios in the (T_+, τ_-) space, similar to Figure 3.5 in the main text. The tipping risk for each element is shown in Figure B.10 for both the baseline and coupled experiments. To account for time-limited simulations from some Earth system models used in the TIPMIP project, we also present tipping risks based on only the first 500 years of the model runs, identifying whether an element is at risk of tipping within this initial period.

For AMAZ and AMOC, tipping risk remains relatively consistent whether we consider only the first 500 years of the simulations or the full run, as these are fast elements. Additionally, scenarios with a peak temperature of 4°C consistently show higher tipping risk than those with a 2°C peak. Among scenarios with the same peak temperature, those with higher cumulative emissions (i.e., less negative emissions) present an increased tipping risk.

For GRIS, WAIS, and EASB, the tipping risk is zero when considering only the first 500 yrs of the simulations, reflecting their nature as slow-tipping elements that require longer timescales for meaningful study (addressed in TIPMIP). The tipping risk increases across the scenarios with cumulative

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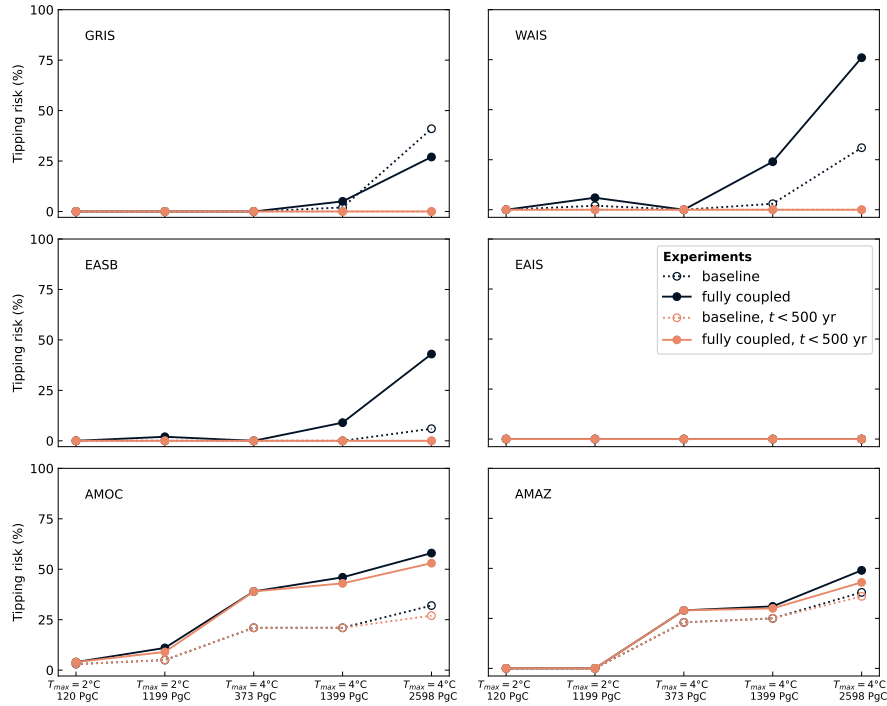


Fig. B.10 Tipping risk of the tipping elements, for the different scenarios of the TIPMIP Tier 1 ESM protocol, and for different experiment settings (baseline, fully coupled, only interactions, only feedbacks). For a given element and scenario, the tipping risk is defined as the percentage of ensemble simulations, generated with different parameter sets, in which the elements tips.

emissions, regardless of the peak temperature reached. This is because, for slow-tipping ice sheets, long-term temperatures, driven primarily by cumulative emissions, are more influential than short-term temperature fluctuations. Long-term temperature evolution also depend on each model's representation of the carbon cycle, which can yield significantly different long-term CO₂ concentrations even for identical emission scenarios (Archer et al., 2009b; Kaufhold et al., 2024a). Consequently, in the emissions-based TIPMIP protocol, whether the ice sheets ultimately tip in a given model may depend more on its simulation of long-term CO₂ draw down than on its specific representation of the ice sheets themselves (Couplet et al., 2024b).

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C.1 Sea-level rise in SURFER

Sea level rise is estimated as the sum of 6 contributions: thermal expansion, and melt from the mountain glaciers, the Greenland ice sheet, and the different parts of the Antarctic ice sheet:

$$S_{\text{tot}} = S_{\text{th}} + S_{\text{gl}} + S_{\text{GRIS}} + S_{\text{WAIS}} + S_{\text{EASB}} + S_{\text{EAIS}} . \quad (\text{C.1})$$

Thermal expansion is given by

$$S_{\text{th}} = \alpha_{\text{U}} h_{\text{U}} \delta T_{\text{U}} + \alpha_{\text{I}} h_{\text{I}} \delta T_{\text{I}} + \alpha_{\text{D}} h_{\text{D}} \delta T_{\text{D}} . \quad (\text{C.2})$$

where α_i is the thermal expansion coefficient corresponding to ocean layer $i \in \{\text{U}, \text{I}, \text{D}\}$.

The evolution of the sea-level rise contribution from glaciers is the same as in previous SURFER versions and is given by the equation

$$\frac{dS_{\text{gl}}}{dt} = \frac{1}{\tau_{\text{gl}}} (S_{\text{gleq}}(\delta T_{\text{U}}) - S_{\text{gl}}) , \quad (\text{C.3})$$

with

$$S_{\text{gleq}}(\delta T_{\text{U}}) = S_{\text{gl pot}} \tanh\left(\frac{\delta T_{\text{U}}}{\zeta}\right) . \quad (\text{C.4})$$

Parameter	Comment	Value
h_U	upper layer depth	150 m
h_I	intermediate layer depth	500 m
h_D	deep layer depth	3150 m
α_U	thermal expansion coefficient for layer U	$2.20 \times 10^{-4} \text{ K}^{-1}$
α_I	thermal expansion coefficient for layer I	$1.61 \times 10^{-4} \text{ K}^{-1}$
α_D	thermal expansion coefficient for layer D	$1.39 \times 10^{-4} \text{ K}^{-1}$
$S_{\text{gl pot}}$	sea level rise potential from mountain glaciers	0.5 m
ξ	sensitivity coefficient for glacier parametrisation	2°C
τ_{gl}	timescale for glacier melt	200 yr
$S_{\text{GRIS}}^{\text{pot}}$	sea level rise potential from Greenland ice-sheet	7 m
$S_{\text{WAIS}}^{\text{pot}}$	sea level rise potential from West Antarctic ice-sheet	4 m
$S_{\text{EASB}}^{\text{pot}}$	sea level rise potential from East Antarctic subglacial basins	19 m
$S_{\text{EAIS}}^{\text{pot}}$	sea level rise potential from East Antarctic ice-sheet	34 m

Table C.1 *Parameter values for SURFER’s sea level rise component.*

Here τ_{gl} is a relaxation timescale, $S_{\text{gl pot}}$ is the potential sea level rise due to mountain glaciers and ξ is a sensitivity coefficient.

The contributions from the ice sheets are given by

$$S_i = S_i^{\text{pot}} (1 - x_i(t)), \quad i \in \{\text{GRIS, WAIS, EASB, EAIS}\}, \quad (\text{C.5})$$

where S_i is the sea-level rise potential of the ice sheet. The value of the parameters from the equations above are given in table C.1.

Projections of sea-level rise from earlier SURFER versions were well-calibrated against higher complexity models. SURFER v2.0, which excludes long-term carbon cycle processes such as sedimentation and weathering, simulated short-term sea-level rise within the range of IPCC projections (Martínez Montero et al., 2022). In Couplet et al. (2024b), we demonstrated that SURFER v3.0 produced multi-millennial thermosteric sea-level rise comparable to other EMICs and showed that incorporating long-term carbon cycle processes reduces the contributions from the Greenland and Antarctic ice sheets. This appendix demonstrates that splitting the Antarctic ice sheet into the WAIS, EASB, and EAIS components still results in reasonable sea level rise projections.

Figure C.1 compares SURFER’s simulated sea-level rise under SSP1-2.6 and SSP5-8.5 emission scenarios with IPCC projections. For this comparison, we exclude interactions between tipping elements and their feedbacks on temperature. However, we account for the uncertainty associated with tipping elements by running the model across our ensemble of parameter sets (baseline experiment settings). SURFER’s projections align well with the IPCC’s for

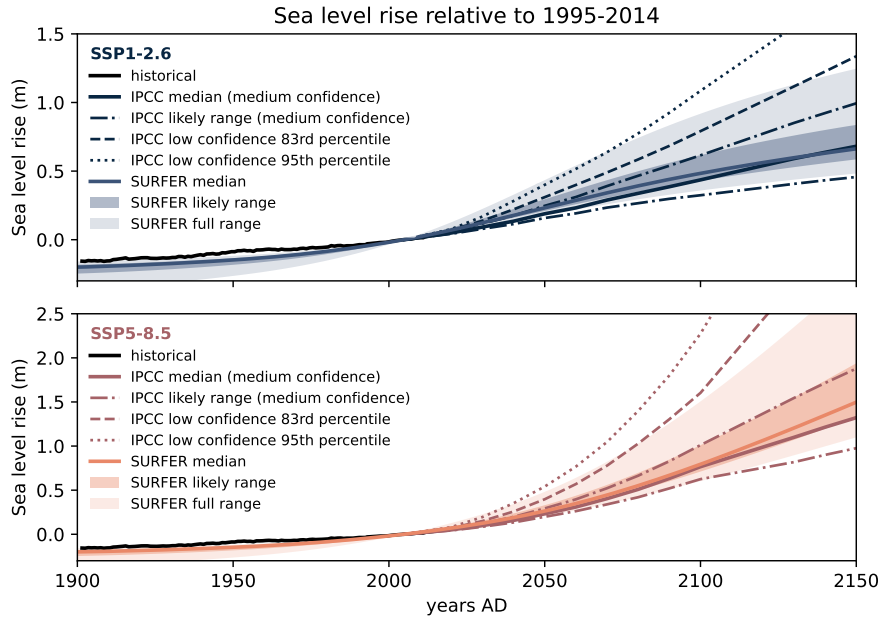


Fig. C.1 Sea level rise projections from the IPCC, compared with projections from SURFER for the SSP1-2.6 and SSP5-8.5 scenarios. The likely ranges, indicated by dash-dotted lines for the IPCC and shaded regions for SURFER, correspond to the 17th-83rd percentile ranges. SURFER's results are shown for the baseline experiment, where uncertainty arises solely from parameter choices in calibrating the ice sheets, with climate sensitivity fixed at 3.5°C. This figure is inspired by figure 9.27 of the IPCC AR6 WG1 report (IPCC, 2021a).

both scenarios: the median values are comparable, and SURFER's likely range generally falls within the IPCC's medium-confidence likely range. Beyond 2070, even under worst-case scenarios, SURFER does not simulate sea-level rise as high as the IPCC's low-confidence 95th percentile. It is important to note that the IPCC's uncertainty range reflects a combination of uncertainties in both climate sensitivity (ECS) and ice sheet dynamics. In contrast, for this figure, SURFER's ECS is fixed at 3.5°C, and its uncertainty range reflects only uncertain ice sheet dynamics. This explains why SURFER's range is narrower than that of the IPCC.

In Figure C.2, we compare SURFER's 10,000-year sea-level rise commitment with projections from Clark et al. (2016) and Breedam et al. (2020). We examine sea-level rise as a function of both peak global mean temperature, following the approach in the sixth IPCC report (IPCC, 2021a, see Fig. 9.30), and long-term warming. SURFER simulates lower sea-level rise contributions from ice sheets

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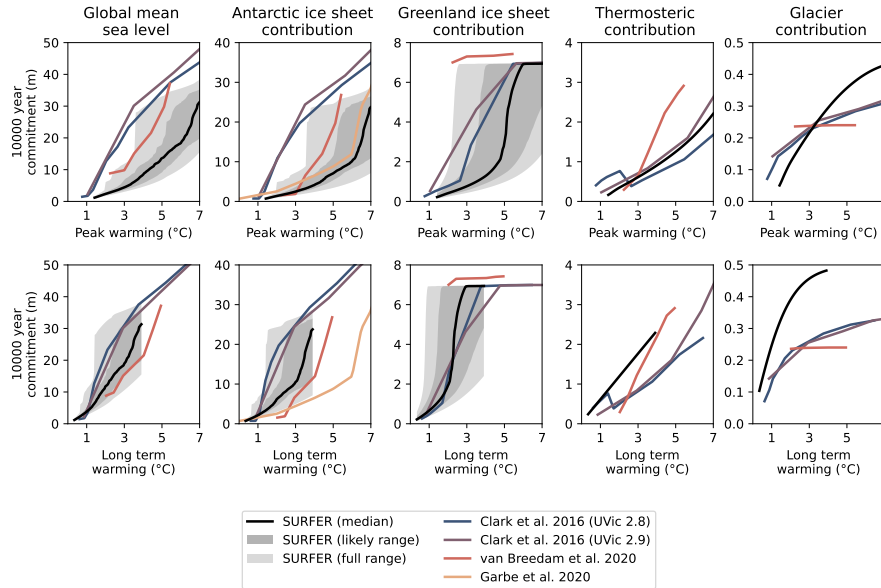


Fig. C.2 10,000-year sea-level rise commitment relative to the year 2000 as a function of peak global mean temperature and long-term temperature. In the bottom plots, long-term warming refers to the global mean temperature in 12,000 AD. The leftmost column shows the total global mean sea-level rise, while the other columns break down contributions from individual sources. Coloured lines represent projections from various models (Breedam et al., 2020; Clark et al., 2016; Garbe et al., 2020), while the black line represents SURFER's median outcome for the baseline experiment. Shaded regions denote SURFER's likely range (13th to 83rd percentile) and full range. The Antarctic ice sheet contribution in Garbe et al. (2020) is computed from quasi-equilibrium runs. This figure is inspired by Figure 9.30 of the IPCC AR6 WG1 report (IPCC, 2021a).

for the same peak temperature than the other models, resulting in an overall lower sea-level rise commitment. This difference arises because, in SURFER, temperatures decrease more rapidly after peaking than in UVic and LOVECLIM, the EMICs used in the studies referenced. However, when we evaluate sea-level commitment as a function of long-term warming, SURFER aligns with the other models, suggesting that the ice sheets are reasonably well-calibrated. This also highlights that a significant reduction in CO₂ concentrations over millennia, whether natural or through carbon dioxide removal, could help reduce committed sea-level rise, as demonstrated in Couplet et al. (2024b) (see also chapter 2).

C.2 Sensitivity analysis

In the main text, we presented two experiments: the baseline experiment, which excludes interactions between elements and their feedback on temperature, and the fully coupled experiment, which incorporates both. We conducted both experiments using an equilibrium climate sensitivity of 3.5°C. This section examines how our results change under a higher climate sensitivity and stronger interaction strengths, focusing specifically on the risk of reaching a Hothouse or Wethouse state, as illustrated in Figure C.3.

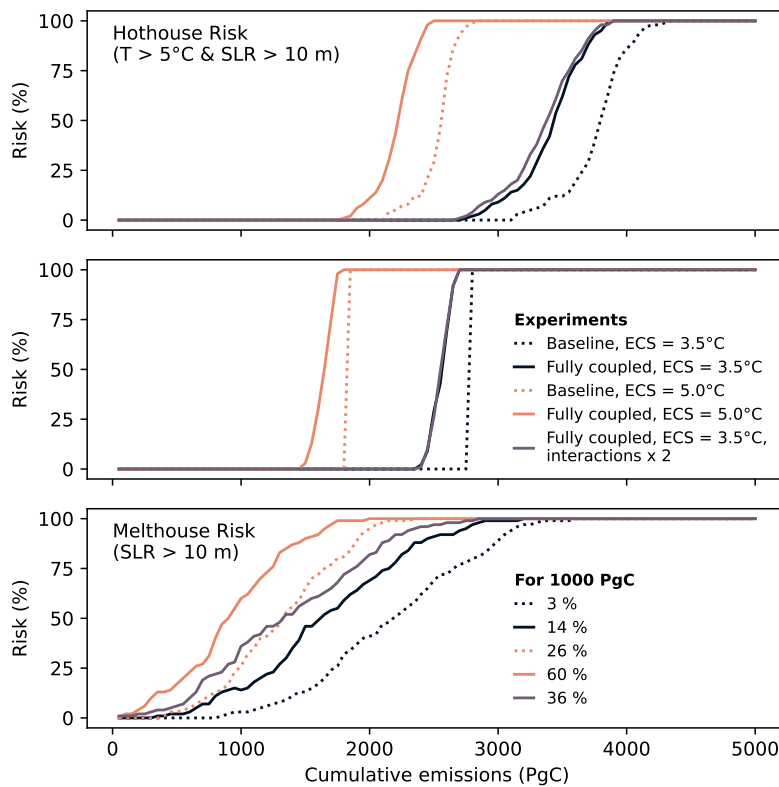


Fig. C.3 Risk of observing a hothouse as a function of cumulative emissions for different experiments. Cumulative emissions refer to the total amount of CO₂ emitted after 2015. For comparison, we have put the amounts corresponding to the SSP scenarios. It should be noted that SSP scenarios differ in terms of their methane and other greenhouse gas emissions, as opposed to the scenarios we use. The risk is defined as the percentage of ensemble simulations, with varying parameter sets, that exceed the reference thresholds ($T > 5^{\circ}\text{C}$ and $\text{SLR} > 10\text{m}$) at any given time.

The risk of reaching a Hothouse state is higher for a high-end equilibrium climate sensitivity (ECS) of 5°C compared to an ECS of 3.5°C and becomes non-negligible (~10%) for cumulative emissions around 2000 PgC. This results from a heightened risk of reaching both 5°C of warming and 10 m of sea level rise. Notably, with an ECS of 5°C, the risk of entering a Wethouse state (defined as 10 m of sea level rise) becomes significant even with 1000 PgC of emissions, rising to 60%, compared to just 14% for an ECS of 3.5°C.

Doubling all interaction strengths between tipping and other nonlinear elements while maintaining an ECS of 3.5°C results in a negligible increase in the risk of reaching a Hothouse state compared to the standard fully coupled experiment. Although more elements tip due to stronger interactions, their influence on maximum temperatures remains minimal. However, these elements can significantly affect sea level rise, increasing the risk of entering a Wethouse state.

These findings, along with those presented in section 4.3, suggest that emission pathways and the climate response primarily dictate short-term and peak warming and, consequently, the risk of reaching a Hothouse state. In comparison, the influence of nonlinear elements and cascading dynamics appears relatively minor. Long-term temperatures are additionally shaped by millennial-scale carbon cycle processes such as sedimentation and weathering.

Short-term sea level rise is primarily driven by near-term temperatures and their associated factors, as well as the characteristics of individual ice sheets. In contrast, long-term and peak sea level rise—which determine the risk of reaching a Wethouse state—are influenced by long-term temperatures and the properties of ice sheets, including their interactions with other elements of the Earth system.

It is important to note that the temperature response for a given CO₂ emissions scenario—and thus the risk of reaching a Hothouse or Wethouse state — also depends on other greenhouse gases, such as methane, and forcing agents like aerosols. In this study, methane emissions are set to the levels of the extended SSP1-2.6 scenario for all CO₂ emissions pathways (Meinshausen et al., 2020), while aerosol forcing is assumed to be zero throughout.

C.3 Additional figures

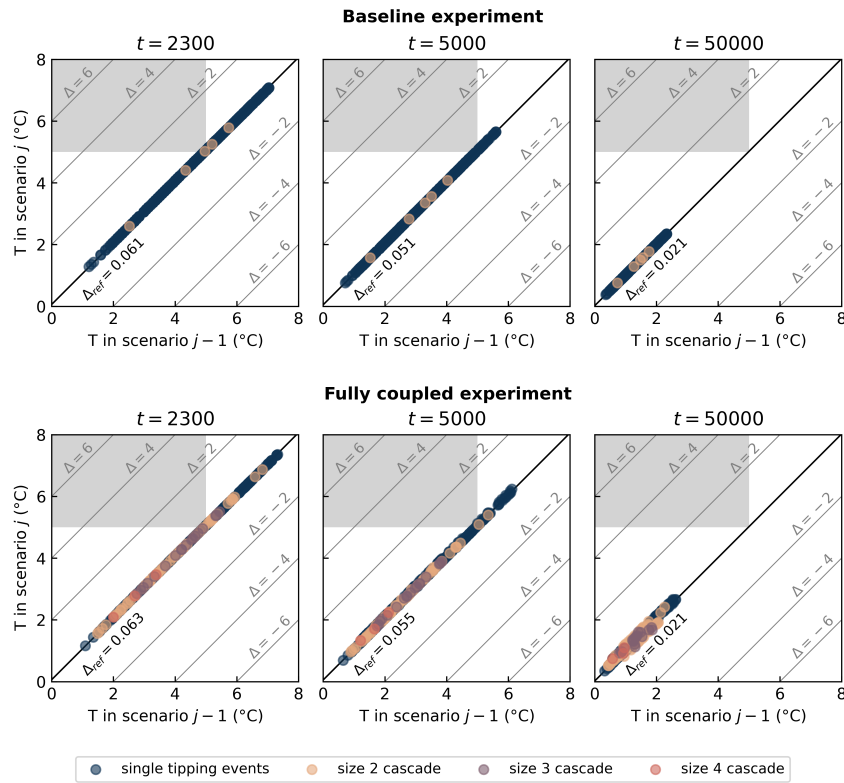


Fig. C.4 Impact of tipping events on temperature in 2300 AD, 5000 AD, and 50000 AD for the baseline and fully coupled experiments. Each tipping event observed in these experiments is represented by a point, with colour indicating the event type. The impact of a tipping event is assessed at a given time by comparing the y and x coordinates of the corresponding point, which represent, respectively, the temperature in the scenario j where the event occurred and in scenario $j - 1$ (which assumes 50 PgC lower cumulative emissions). The thick black diagonal in each plot has equation $y = x + \Delta_{ref}$, where Δ_{ref} is the mean difference in temperature at a given time between two scenarios j and $j - 1$ for cases where no tipping event occurred. Thus, tipping events on this line have a zero or negligible impact on temperature. The farther away the points are from this diagonal, the greater the impact of the associated event. The thinner grey $y = x + \Delta$ diagonals represent lines of constant impact and serve as visual guides. Shaded regions highlight events that cause temperature to exceed 5°C above pre-industrial levels. Note that in the baseline experiment, the interactions between elements are turned 'off', which means that size-2 cascades are due to elements tipping for the same scenario by coincidence.

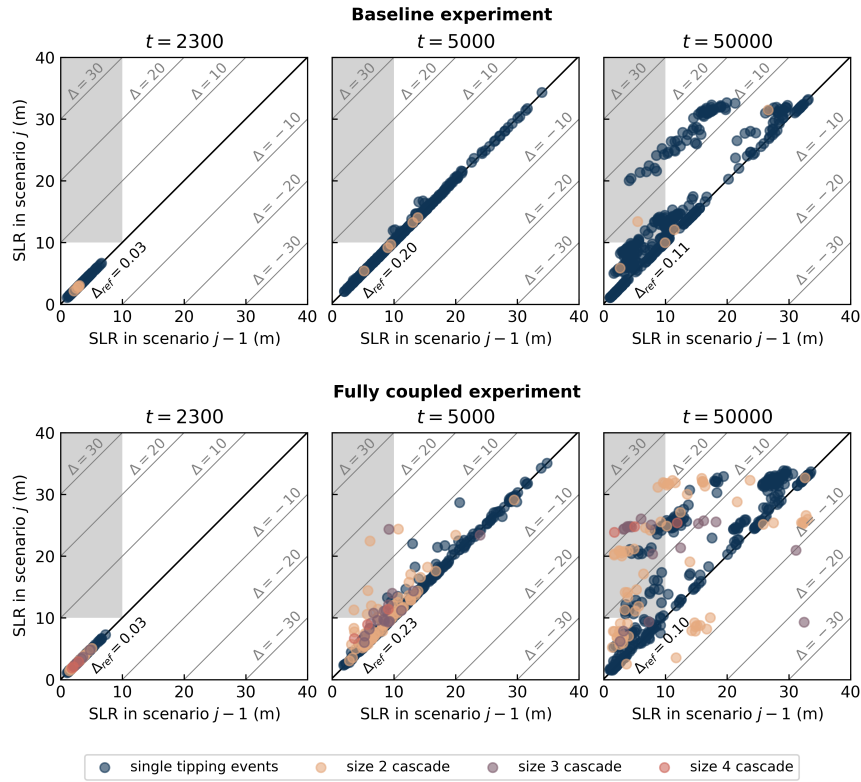


Fig. C.5 Impact of tipping events on sea level rise in 2300 AD, 5000 AD, and 50000 AD for the baseline and fully coupled experiments. Each tipping event observed in these experiments is represented by a point, with colour indicating the event type. The impact of a tipping event is assessed at a given time by comparing the y and x coordinates of the corresponding point, which represent, respectively, the sea level rise in the scenario j where the event occurred and in scenario $j - 1$ (which assumes 50 PgC lower cumulative emissions). The thick black diagonal in each plot has equation $y = x + \Delta_{ref}$, where Δ_{ref} is the mean difference in sea level rise at a given time between two scenarios j and $j - 1$ for cases where no tipping event occurred. Thus, tipping events on this line have a zero or negligible impact on sea level rise. The farther away the points are from this diagonal, the greater the impact of the associated event. The thinner grey $y = x + \Delta$ diagonals represent lines of constant impact and serve as visual guides. Shaded regions highlight events that cause sea level rise to exceed 10 m above pre-industrial levels. Note that in the baseline experiment, the interactions between elements are turned 'off', which means that size-2 cascades are due to elements tipping for the same scenario by coincidence.



Data availability

Data from other references used throughout this thesis for emission scenarios are as follows :

- Historical CO₂ emissions are from Friedlingstein et al. (2022) and are available at <https://doi.org/10.18160/GCP-2022>.
- Historical CH₄ emissions are from Jones et al. (2023) and are available at <https://zenodo.org/records/10839859>.
- CO₂ and CH₄ emissions for the SSP scenarios are available in the SSP database hosted by the IIASA Energy Program at <https://tntcat.iiasa.ac.at/SspDb>.

Data from other references used in Chapter 2 for comparison with SURFER's output are as follows :

- Figure 2.3 : GLODAPv2.2016b mapped climatologies (Lauvset et al., 2016) are available at <https://www.ncei.noaa.gov/access/metadata/landing-page/bin/iso?id=gov.noaa.nodc:0286118>.
- Figures 2.4 and 2.5 : data from Köhler et al. (2017) available at <https://doi.org/10.1594/PANGAEA.871273>.
- Figure 2.6 : data from Friedlingstein et al. (2022) available at <https://doi.org/10.18160/GCP-2022>.

D | Data availability

- Figure 2.7 : CMIP6 data can be accessed through the Earth System Grid Federation (ESGF) nodes (e.g. <https://esgf-index1.ceda.ac.uk/projects/cmip6-ceda/>).
- Figure 2.8 : data is from Figure 5.30 of IPCC AR6 WG1 (Canadell et al., 2021b). CMIP6 data can be accessed through the Earth System Grid Federation (ESGF) nodes (e.g. <https://esgf-index1.ceda.ac.uk/projects/cmip6-ceda/>). The code to process the data necessary for this specific IPCC figure is available online in a Jupyter notebook at https://github.com/IPCC-WG1/Chapter-5_Fig30/blob/main/longterm_carboncycle_withssp126.ipynb.
- Figures 2.9-2.13 : LTMIP data is available at <https://terra.seos.uvic.ca/LTMIP/>. The LOSCAR data is from Zeebe (2012) and available through personal correspondence with author.
- Figures 2.14 and 2.15 : cGENIE data is from Lord et al. (2016) and available through personal correspondence with author.
- Figure 2.16 : data is from Figure 1 in IPCC AR6 WG1 Cross-Chapter Box 9.1 (IPCC, 2021a). The code to produce the IPCC figure and the associated analysis is freely available online in a jupyter notebook: https://github.com/BrodiePearson/IPCC_AR6_Chapter9_Figures/blob/main/Plotting_code_and_data/Cross_Chapter_Box9_1/Plot_Figure/plot_AR6_CCBBox9_1_FGD.ipynb.
- Figure 2.17 : data from Clark et al. (2016) available at <https://www.nature.com/articles/nclimate2923>.

Data from other references used in the supplementary material of Chapter 4 (Appendix C) for comparison with SURFER's output are as follows :

- Figure C.1 : data is from Figure 9.27 of IPCC AR6 WG1 (Canadell et al., 2021b). CMIP6 data can be accessed through the Earth System Grid Federation (ESGF) nodes (e.g. <https://esgf-index1.ceda.ac.uk/projects/cmip6-ceda/>). The code to process the data necessary for this specific IPCC figure is available online in a Jupyter notebook at https://github.com/BrodiePearson/IPCC_AR6_Chapter9_Figures.
- Figures C.2 : data from Clark et al. (2016) available at <https://www.nature.com/articles/nclimate2923> ; data from Breedam et al. (2020)

available through personal correspondence with author ; data from Garbe et al. (2020) available through personal correspondence with author. Note that this figure is inspired from Figure 9.30 of IPCC AR6 WG1 (Canadell et al., 2021b), for which the data and processing code is available online in a jupyter notebook at https://github.com/BrodiePearson/IPCC_AR6_Chapter9_Figures.

The scientific colormap *batlow* is used in Figure 2.15 and the scientific colormap *lipari* is used for figures throughout the thesis (Crameri, 2023).

E

List of publications related to the thesis



First author

- Couplet, V., M. Martínez Montero, and M. Crucifix (2024). **SURFER v3.0: a fast model with ice sheet tipping points and carbon cycle feedbacks for short and long-term climate scenarios**, Geoscientific Model Development (*accepted*). (Chapter 2)



Co-author

- Martínez Montero, M., M. Crucifix, V. Couplet, N. Brede, and N. Botta (2022). **SURFER v2.0: a flexible and simple model linking anthropogenic CO₂ emissions and solar radiation modification to ocean acidification and sea level rise**, Geoscientific Model Development, vol. 15, no. 21.
- Martínez Montero, M., N. Brede, V. Couplet, M. Crucifix, N. Botta, and C. Wieners (2024). **Lost options commitment: how short-term policies affect long-term scope of action**, Oxford Open Climate Change, vol. 4, no. 1.

E | List of publications related to the thesis

- Wunderling, N., A. S. von der Heydt, Y. Aksenov, S. Barker, R. Bastiaansen, V. Brovkin, M. Brunetti, V. Couplet, T. Kleinen, C. H. Lear, J. Lohmann, R. M. Roman-Cuesta, S. Sinet, D. Swingedouw, R. Winkelmann, P. Anand, J. Barichivich, S. Bathiany, M. Baudena, J. T. Bruun, C. M. Chiessi, H. K. Coxall, D. Docquier, J. F. Donges, S. K. J. Falkena, A. K. Klose, D. Obura, J. Rocha, S. Rynders, N. J. Steinert, and M. Willeit (2024). **Climate tipping point interactions and cascades: a review**, *Earth System Dynamics*, vol. 15, no. 1, pp. 41–74.
- Laridon, A., Couplet, V., Gérard, J., Thiery, W., and Crucifix, M. (2025). **Connecting complex and simplified models of tipping elements: a nonlinear two-forcing emulator for the Atlantic meridional overturning circulation**, *Open Research Europe* (*submitted*).

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Code

- Couplet, V., M. Crucifix, and M. Martínez Montero (2024), **SURFERv3.0 code**, Zenodo (<https://zenodo.org/records/12774162>).

Bibliography

- Alkhayoun, H., P. Ashwin, L. C. Jackson, C. Quinn, and R. A. Wood (2019), Basin bifurcations, oscillatory instability and rate-induced thresholds for atlantic meridional overturning circulation in a global oceanic box model, *Proc. R. Soc. A.*, vol. 475, no. 2225, p. 20190051.
- Alley, R. B., S. Anandakrishnan, K. Christianson, H. J. Horgan, A. Muto, B. R. Parizek, D. Pollard, and R. T. Walker (2015), Oceanic forcing of ice-sheet retreat: west antarctica and more, *Annual Review of Earth and Planetary Sciences*, vol. 43 (Volume 43, 2015), pp. 207–231.
- Alvarez-Solas, J., S. Charbit, C. Ritz, D. Paillard, G. Ramstein, and C. Dumas (2010), Links between ocean temperature and iceberg discharge during heinrich events, *Nature Geosci.*, vol. 3, no. 2, pp. 122–126.
- Andersson, A. J. and F. T. Mackenzie (2004), Shallow-water oceans: a source or sink of atmospheric CO₂?, *Frontiers in Ecology and the Environment*, vol. 2, no. 7, pp. 348–353.
- Archer, D., B. Buffett, and V. Brovkin (2009a), Ocean methane hydrates as a slow tipping point in the global carbon cycle, *Proceedings of the National Academy of Sciences*, vol. 106, no. 49, pp. 20596–20601.
- Archer, D. (2010), The global carbon cycle, Princeton University Press.
- Archer, D., M. Eby, V. Brovkin, A. Ridgwell, L. Cao, U. Mikolajewicz, K. Caldeira, K. Matsumoto, G. Munhoven, A. Montenegro, and K. Tokos (2009b), Atmospheric lifetime of fossil fuel carbon dioxide, *Annu. Rev. Earth Planet. Sci.*, vol. 37, no. 1, pp. 117–134.
- Archer, D., H. Kheshgi, and E. Maier-Reimer (1997), Multiple timescales for neutralization of fossil fuel CO₂, *Geophys. Res. Lett.*, vol. 24, no. 4, pp. 405–408.
- (1998), Dynamics of fossil fuel CO₂ neutralization by marine CaCO₃, *Global Biogeochem. Cycles*, vol. 12, no. 2, pp. 259–276.

★ | Bibliography

- Armstrong McKay, D. (2022), *Exceeding 1.5°C global warming could trigger multiple climate tipping points – paper explainer*, climatetippingpoints.info, URL: <https://climatetippingpoints.info/2022/09/09/climate-tipping-points-reassessment-explainer/> (visited on 12/18/2024).
- Armstrong McKay, D. I., A. Staal, J. F. Abrams, R. Winkelmann, B. Sakschewski, S. Loriani, I. Fetzer, S. E. Cornell, J. Rockström, and T. M. Lenton (2022), Exceeding 1.5°C global warming could trigger multiple climate tipping points, *Science*, vol. 377, no. 6611, eabn7950.
- Ashwin, P., S. Wieczorek, R. Vitolo, and P. Cox (2012), Tipping points in open systems: bifurcation, noise-induced and rate-dependent examples in the climate system, *Phil. Trans. R. Soc. A.*, vol. 370, no. 1962, pp. 1166–1184.
- Bagniewski, W., D.-D. Rousseau, and M. Ghil (2023), The PaleoJump database for abrupt transitions in past climates, *Sci Rep*, vol. 13, no. 1, p. 4472.
- Bahr, D. B., W. T. Pfeffer, and G. Kaser (2015), A review of volume-area scaling of glaciers, *Rev Geophys*, vol. 53, no. 1, pp. 95–140.
- Barclay, R. S., J. C. McElwain, and B. B. Sageman (2010), Carbon sequestration activated by a volcanic CO₂ pulse during ocean anoxic event 2, *Nature Geosci*, vol. 3, no. 3, pp. 205–208.
- Bastiaansen, R., H. A. Dijkstra, and A. S. v. d. Heydt (2022), Fragmented tipping in a spatially heterogeneous world, *Environ. Res. Lett.*, vol. 17, no. 4, p. 045006.
- Bathiany, S., H. Dijkstra, M. Crucifix, V. Dakos, V. Brovkin, M. S. Williamson, T. M. Lenton, and M. Scheffer (2016), Beyond bifurcation: using complex models to understand and predict abrupt climate change, *Dyn Stat Clim Syst*, vol. 1, no. 1.
- Ben-Yami, M., P. Good, L. C. Jackson, M. Crucifix, A. Hu, O. Saenko, D. Swingedouw, and N. Boers (2024a), Impacts of AMOC collapse on monsoon rainfall: a multi-model comparison, *Earth’s Future*, vol. 12, no. 9.
- Ben-Yami, M., A. Morr, S. Bathiany, and N. Boers (2024b), “Uncertainties too large to predict tipping times of major earth system components from historical data”, *Science Advances*.
- Berner, R. A. (2004), “The phanerozoic carbon cycle: CO₂ and O₂”, Oxford, New York: Oxford University Press, 158 pp.
- Berner, R. A., A. C. Lasaga, and R. M. Garrels (1983), The carbonate-silicate geochemical cycle and its effect on atmospheric carbon dioxide over the past 100 million years, *American Journal of Science*, vol. 283, no. 7, pp. 641–683.

- Bjordal, J., T. Storelvmo, K. Alterskjær, and T. Carlsen (2020), Equilibrium climate sensitivity above 5 °C plausible due to state-dependent cloud feedback, *Nat. Geosci.*, vol. 13, no. 11, pp. 718–721.
- Bochow, N., A. Poltronieri, A. Robinson, M. Montoya, M. Rypdal, and N. Boers (2023), Overshooting the critical threshold for the greenland ice sheet, *Nature*, vol. 622, no. 7983, pp. 528–536.
- Boers, N. (2021), Observation-based early-warning signals for a collapse of the atlantic meridional overturning circulation, *Nat. Clim. Chang.*, vol. 11, no. 8, pp. 680–688.
- Boers, N., M. Ghil, and T. F. Stocker (2022), Theoretical and paleoclimatic evidence for abrupt transitions in the earth system, *Environ. Res. Lett.*, vol. 17, no. 9, p. 093006.
- Boers, N. and M. Rypdal (2021), Critical slowing down suggests that the western greenland ice sheet is close to a tipping point, *Proceedings of the National Academy of Sciences*, vol. 118, no. 21, e2024192118.
- Boot, A. A., A. S. von der Heydt, and H. A. Dijkstra (2024), Response of atmospheric pCO₂ to a strong AMOC weakening under low and high emission scenarios, *Clim Dyn*, vol. 62, no. 8, pp. 7559–7574.
- Boulton, C. A., T. M. Lenton, and N. Boers (2022), Pronounced loss of amazon rainforest resilience since the early 2000s, *Nat. Clim. Chang.*, vol. 12, no. 3, pp. 271–278.
- Brantley, S. L., A. Shaughnessy, M. I. Lebedeva, and V. N. Balashov (2023), How temperature-dependent silicate weathering acts as earth’s geological thermostat, *Science*, vol. 379, no. 6630, pp. 382–389.
- Breedam, J. V., H. Goelzer, and P. Huybrechts (2020), “Semi-equilibrated global sea-level change projections for the next 10 000 years”.
- Brovkin, V., E. Brook, J. W. Williams, S. Bathiany, T. M. Lenton, M. Barton, R. M. DeConto, J. F. Donges, A. Ganopolski, J. McManus, S. Praetorius, A. de Vernal, A. Abe-Ouchi, H. Cheng, M. Claussen, M. Crucifix, G. Gallopín, V. Iglesias, D. S. Kaufman, T. Kleinen, F. Lambert, S. van der Leeuw, H. Liddy, M.-F. Loutre, D. McGee, K. Rehfeld, R. Rhodes, A. W. R. Seddon, M. H. Trauth, L. Vanderveken, and Z. Yu (2021), Past abrupt changes, tipping points and cascading impacts in the earth system, *Nat. Geosci.*, vol. 14, no. 8, pp. 550–558.
- Brovkin, V., T. Brücher, T. Kleinen, S. Zaehle, F. Joos, R. Roth, R. Spahni, J. Schmitt, H. Fischer, M. Leuenberger, E. J. Stone, A. Ridgwell, J. Chappellaz, N. Kehrwald, C. Barbante, T. Blunier, and D. Dahl Jensen

- (2016), Comparative carbon cycle dynamics of the present and last interglacial, *Quaternary Science Reviews*, vol. 137, pp. 15–32.
- Brovkin, V., M. Claussen, V. Petoukhov, and A. Ganopolski (1998), On the stability of the atmosphere-vegetation system in the sahara/sahel region, *Journal of Geophysical Research: Atmospheres*, vol. 103 (D24), pp. 31613–31624.
- Brunetti, M., J. Kasparian, and C. V  rard (2019), Co-existing climate attractors in a coupled aquaplanet, *Clim Dyn*, vol. 53, no. 9, pp. 6293–6308.
- Budyko, M. I. (1969), The effect of solar radiation variations on the climate of the earth, *Tellus A: Dynamic Meteorology and Oceanography*, vol. 21, no. 5.
- Burke, K. D., J. W. Williams, M. A. Chandler, A. M. Haywood, D. J. Lunt, and B. L. Otto-Bliesner (2018), Pliocene and eocene provide best analogs for near-future climates, *Proc Natl Acad Sci USA*, vol. 115, no. 52, pp. 13288–13293.
- Canadell, J., P. Monteiro, M. Costa, L. Cotrim da Cunha, P. Cox, A. Eliseev, S. Henson, M. Ishii, S. Jaccard, C. Koven, A. Lohila, P. Patra, S. Piao, J. Rogelj, S. Syampungani, S. Zaehle, and K. Zickfeld (2021a), Global carbon and other biogeochemical cycles and feedbacks, in: *Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*, ed. by V. Masson-Delmotte, P. Zhai, A. Pirani, S. Connors, C. P  an, S. Berger, N. Caud, Y. Chen, L. Goldfarb, M. Gomis, M. Huang, K. Leitzell, E. Lonnoy, J. Matthews, T. Maycock, T. Waterfield, O. Yelek  i, R. Yu, and B. Zhou, Cambridge, United Kingdom and New York, NY, USA: Cambridge University Press, pp. 673–816.
- (2021b), Global carbon and other biogeochemical cycles and feedbacks supplementary material, in: *Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*, ed. by V. Masson-Delmotte, P. Zhai, A. Pirani, S. Connors, C. P  an, S. Berger, N. Caud, Y. Chen, L. Goldfarb, M. Gomis, M. Huang, K. Leitzell, E. Lonnoy, J. Matthews, T. Maycock, T. Waterfield, O. Yelek  i, R. Yu, and B. Zhou.
- Chandra, N., P. K. Patra, R. Fujita, L. H  oglund-Isaksson, T. Umezawa, D. Goto, S. Morimoto, B. H. Vaughn, and T. R  ockmann (2024), Methane emissions decreased in fossil fuel exploitation and sustainably increased in microbial source sectors during 1990–2020, *Commun Earth Environ*, vol. 5, no. 1, pp. 1–15.

- Clark, P. U., J. D. Shakun, S. A. Marcott, A. C. Mix, M. Eby, S. Kulp, A. Levermann, G. A. Milne, P. L. Pfister, B. D. Santer, D. P. Schrag, S. Solomon, T. F. Stocker, B. H. Strauss, A. J. Weaver, R. Winkelmann, D. Archer, E. Bard, A. Goldner, K. Lambeck, R. T. Pierrehumbert, and G.-K. Plattner (2016), Consequences of twenty-first-century policy for multi-millennial climate and sea-level change, *Nature Clim Change*, vol. 6, no. 4, pp. 360–369.
- Climate Action Tracker (2023), *The CAT thermometer*, URL: <https://climateactiontracker.org/global/cat-thermometer/>.
- Colbourn, G., A. Ridgwell, and T. M. Lenton (2013), The rock geochemical model (RokGeM) v0.9, *Geosci. Model Dev.*, vol. 6, no. 5, pp. 1543–1573.
- (2015), The time scale of the silicate weathering negative feedback on atmospheric CO₂, *Global Biogeochemical Cycles*, vol. 29, no. 5, pp. 583–596.
- Coulon, V., A. K. Klose, C. Kittel, T. Edwards, F. Turner, R. Winkelmann, and F. Pattyn (2024), “Disentangling the drivers of future antarctic ice loss with a historically calibrated ice-sheet model”, *The Cryosphere*.
- Couplet, V., M. Crucifix, and M. Martínez Montero (2024a), SURFERv3.0 code.
- Couplet, V., M. Martínez Montero, and M. Crucifix (2024b), SURFER v3.0: a fast model with ice sheet tipping points and carbon cycle feedbacks for short and long-term climate scenarios, *EGUsphere*, pp. 1–77.
- Cramer, F. (2023), *Scientific colour maps*, version 8.0.1.
- Dakos, V., M. Scheffer, E. H. van Nes, V. Brovkin, V. Petoukhov, and H. Held (2008), “Slowing down as an early warning signal for abrupt climate change”, p. 5.
- Dekker, M. M., A. S. von der Heydt, and H. A. Dijkstra (2018), Cascading transitions in the climate system, *Earth Syst. Dynam.*, vol. 9, no. 4, pp. 1243–1260.
- Deutloff, J. E., H. Held, and T. M. Lenton (2023), *The risky middle of the road – probabilities of triggering climate tipping points and how they increase due to tipping points within the earth’s carbon cycle*, preprint.
- DeVries, T. and T. Weber (2017), The export and fate of organic matter in the ocean: new constraints from combining satellite and oceanographic tracer observations, *Global Biogeochemical Cycles*, vol. 31, no. 3, pp. 535–555.
- Dickson, A. G. and F. J. Millero (1987), “A comparison of the equilibrium constants for the dissociation of carbonic acid in seawater media”, p. 11.
- Dickson, A. G. (1990), Thermodynamics of the dissociation of boric acid in synthetic seawater from 273.15 to 318.15 K, *Deep Sea Research Part A. Oceanographic Research Papers*, vol. 37, no. 5, pp. 755–766.

- Ditlevsen, P. and S. Ditlevsen (2023), Warning of a forthcoming collapse of the atlantic meridional overturning circulation, *Nat Commun*, vol. 14, no. 1, p. 4254.
- Dooley, K., K. L. Christiansen, J. F. Lund, W. Carton, and A. Self (2024), Over-reliance on land for carbon dioxide removal in net-zero climate pledges, *Nat Commun*, vol. 15, no. 1, p. 9118.
- Drijfhout, S. (2015a), Competition between global warming and an abrupt collapse of the AMOC in earth's energy imbalance, *Sci Rep*, vol. 5, no. 1, p. 14877.
- Drijfhout, S., S. Bathiany, C. Beaulieu, V. Brovkin, M. Claussen, C. Huntingford, M. Scheffer, G. Sgubin, and D. Swingedouw (2015), Catalogue of abrupt shifts in intergovernmental panel on climate change climate models, *Proceedings of the National Academy of Sciences*, vol. 112, no. 43, E5777–E5786.
- Drijfhout, S. S. (2015b), Global radiative adjustment after a collapse of the atlantic meridional overturning circulation, *Clim Dyn*, vol. 45, no. 7, pp. 1789–1799.
- Eby, M., K. Zickfeld, A. Montenegro, D. Archer, K. J. Meissner, and A. J. Weaver (2009), Lifetime of anthropogenic climate change: millennial time scales of potential CO₂ and surface temperature perturbations, *Journal of Climate*, vol. 22, no. 10, pp. 2501–2511.
- Edwards, N. R. and R. Marsh (2005), Uncertainties due to transport-parameter sensitivity in an efficient 3-d ocean-climate model, *Clim Dyn*, vol. 24, no. 4, pp. 415–433.
- Feng, R., T. Bhattacharya, B. L. Otto-Bliesner, E. C. Brady, A. M. Haywood, J. C. Tindall, S. J. Hunter, A. Abe-Ouchi, W.-L. Chan, M. Kageyama, C. Contoux, C. Guo, X. Li, G. Lohmann, C. Stepanek, N. Tan, Q. Zhang, Z. Zhang, Z. Han, C. J. R. Williams, D. J. Lunt, H. J. Dowsett, D. Chandan, and W. R. Peltier (2022), Past terrestrial hydroclimate sensitivity controlled by earth system feedbacks, *Nat Commun*, vol. 13, no. 1, pp. 1–11.
- Ferreira, D., J. Marshall, and B. Rose (2011), Climate determinism revisited: multiple equilibria in a complex climate model, *Journal of Climate*, vol. 24, no. 4, pp. 992–1012.
- Feudel, U. (2023), Rate-induced tipping in ecosystems and climate: the role of unstable states, basin boundaries and transient dynamics, *Nonlinear Processes in Geophysics*, vol. 30, no. 4, pp. 481–502.
- Fewster, R. E., P. J. Morris, R. F. Ivanovic, G. T. Swindles, A. M. Peregon, and C. J. Smith (2022), Imminent loss of climate space for permafrost peatlands

- in europe and western siberia, *Nat. Clim. Chang.*, vol. 12, no. 4, pp. 373–379.
- Forster, P., T. Storelvmo, K. Armour, W. Collins, J.-L. Dufresne, D. Frame, D. Lunt, T. Mauritsen, M. Palmer, M. Watanabe, M. Wild, and H. Zhang (2021), The earth’s energy budget, climate feedbacks, and climate sensitivity, in: *Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*, ed. by V. Masson-Delmotte, P. Zhai, A. Pirani, S. Connors, C. Péan, S. Berger, N. Caud, Y. Chen, L. Goldfarb, M. Gomis, M. Huang, K. Leitzell, E. Lonnoy, J. Matthews, T. Maycock, T. Waterfield, O. Yelekçi, R. Yu, and B. Zhou, Cambridge, United Kingdom and New York, NY, USA: Cambridge University Press, pp. 923–1054.
- Fox-Kemper, B., H. Hewitt, C. Xiao, G. Aðalgeirsdóttir, S. Drijfhout, T. Edwards, N. Golledge, M. Hemer, R. Kopp, G. Krinner, A. Mix, D. Notz, S. Nowicki, I. Nurhati, L. Ruiz, J.-B. Sallée, A. Slangen, and Y. Yu (2021), Ocean, cryosphere and sea level change supplementary material, in: *Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*, ed. by V. Masson-Delmotte, P. Zhai, A. Pirani, S. Connors, C. Péan, S. Berger, N. Caud, Y. Chen, L. Goldfarb, M. Gomis, M. Huang, K. Leitzell, E. Lonnoy, J. Matthews, T. Maycock, T. Waterfield, O. Yelekçi, R. Yu, and B. Zhou.
- Friedlingstein, P. et al. (2022), Global carbon budget 2022, *Earth Syst. Sci. Data*, vol. 14, no. 11, pp. 4811–4900.
- Ganopolski, A., R. Winkelmann, and H. J. Schellnhuber (2016), Critical insolation–CO₂ relation for diagnosing past and future glacial inception, *Nature*, vol. 529, no. 7585, pp. 200–203.
- Ganopolski, A. and V. Brovkin (2017), Simulation of climate, ice sheets and CO₂ evolution during the last four glacial cycles with an earth system model of intermediate complexity, *Clim. Past*, vol. 13, no. 12, pp. 1695–1716.
- Garbe, J., T. Albrecht, A. Levermann, J. F. Donges, and R. Winkelmann (2020), The hysteresis of the antarctic ice sheet, *Nature*, vol. 585, no. 7826, pp. 538–544.
- Gaucherel, C. and V. Moron (2016), “Potential stabilizing points to mitigate tipping point interactions in earth’s climate”, *Int. J. Climatol.* P. 10.

- Gidden, M. J., K. Riahi, S. J. Smith, S. Fujimori, G. Luderer, E. Kriegler, D. P. van Vuuren, C. Doelman, S. Frank, O. Fricko, M. Harmsen, T. Hasegawa, P. Havlik, J. Hilaire, R. Hoesly, J. Horing, A. Popp, E. Stehfest, and K. Takahashi (2019), “Global emissions pathways under different socioeconomic scenarios for use in CMIP6: a dataset of harmonized emissions trajectories through the end of the century”, *Geosci. Model Dev.*
- Goldblatt, C. and A. J. Watson (2012), The runaway greenhouse: implications for future climate change, geoengineering and planetary atmospheres, *Phil. Trans. R. Soc. A.*, vol. 370, no. 1974, pp. 4197–4216.
- Goosse, H. (2015), *Climate system dynamics and modelling*, Cambridge: Cambridge University Press.
- Hauck, J., M. Zeising, C. Le Quéré, N. Gruber, D. C. E. Bakker, L. Bopp, T. T. T. Chau, Ö. Gürses, T. Ilyina, P. Landschützer, A. Lenton, L. Resplandy, C. Rödenbeck, J. Schwinger, and R. Séférian (2020), Consistency and challenges in the ocean carbon sink estimate for the global carbon budget, *Front. Mar. Sci.*, vol. 7.
- Hays, J. D., J. Imbrie, and N. J. Shackleton (1976), “Variations in the earth’s orbit: pacemaker of the ice ages”, vol. 194, p. 12.
- Heinrich, H. (1988), Origin and consequences of cyclic ice rafting in the northeast atlantic ocean during the past 130,000 years, *Quat. res.*, vol. 29, no. 2, pp. 142–152.
- Held, F., H. Cheng, R. L. Edwards, O. Tüysüz, K. Koç, and D. Fleitmann (2024), Dansgaard-oeschger cycles of the penultimate and last glacial period recorded in stalagmites from türkiye, *Nat Commun*, vol. 15, no. 1, p. 1183.
- Henson, S. A., C. Laufkötter, S. Leung, S. L. C. Giering, H. I. Palevsky, and E. L. Cavan (2022), Uncertain response of ocean biological carbon export in a changing world, *Nat. Geosci.*, vol. 15, no. 4, pp. 248–254.
- Hoffman, P. F. (1998), A neoproterozoic snowball earth, *Science*, vol. 281, no. 5381, pp. 1342–1346.
- Höning, D., M. Willeit, R. Calov, V. Klemann, M. Bagge, and A. Ganopolski (2023), Multistability and transient response of the greenland ice sheet to anthropogenic CO₂ emissions, *Geophysical Research Letters*, vol. 50, no. 6, e2022GL101827.
- Höning, D., M. Willeit, and A. Ganopolski (2024), “Reversibility of greenland ice sheet mass loss under artificial carbon dioxide removal scenarios”, *Environ. Res. Lett.*

- Hughes, T. P., S. Carpenter, J. Rockström, M. Scheffer, and B. Walker (2013), Multiscale regime shifts and planetary boundaries, *Trends in Ecology & Evolution*, vol. 28, no. 7, pp. 389–395.
- Humphreys, M. P., E. R. Lewis, J. D. Sharp, and D. Pierrot (2022), PyCO2sys v1.8: marine carbonate system calculations in python, *Geoscientific Model Development*, vol. 15, no. 1, pp. 15–43.
- IPCC (2021a), Climate change 2021: the physical science basis. contribution of working group i to the sixth assessment report of the intergovernmental panel on climate change, vol. In Press, Cambridge, United Kingdom and New York, NY, USA: Cambridge University Press.
- (2021b), Summary for policymakers, in: *Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*, ed. by V. Masson-Delmotte, P. Zhai, A. Pirani, S. Connors, C. Péan, S. Berger, N. Caud, Y. Chen, L. Goldfarb, M. Gomis, M. Huang, K. Leitzell, E. Lonnoy, J. Matthews, T. Maycock, T. Waterfield, O. Yelekçi, R. Yu, and B. Zhou, Cambridge, United Kingdom and New York, NY, USA: Cambridge University Press, 332.
- Jones, C. D., T. L. Frölicher, C. Koven, A. H. MacDougall, H. D. Matthews, K. Zickfeld, J. Rogelj, K. B. Tokarska, N. P. Gillett, T. Ilyina, M. Meinshausen, N. Mengis, R. Séférian, M. Eby, and F. A. Burger (2019), The zero emissions commitment model intercomparison project (ZECMIP) contribution to c4mip: quantifying committed climate changes following zero carbon emissions, *Geoscientific Model Development*, vol. 12, no. 10, pp. 4375–4385.
- Jones, M. W., G. P. Peters, T. Gasser, R. M. Andrew, C. Schwingshackl, J. Gütschow, R. A. Houghton, P. Friedlingstein, J. Pongratz, and C. Le Quéré (2023), National contributions to climate change due to historical emissions of carbon dioxide, methane, and nitrous oxide since 1850, *Sci Data*, vol. 10, no. 1, p. 155.
- Judd, E. J., J. E. Tierney, D. J. Lunt, I. P. Montañez, B. T. Huber, S. L. Wing, and P. J. Valdes (2024), A 485-million-year history of earth’s surface temperature, *Science*, vol. 385, no. 6715, eadk3705.
- Kaufhold, C., M. Willeit, B. Liu, and A. Ganopolski (2024a), “Assessing the lifetime of anthropogenic CO₂ and its sensitivity to different carbon cycle processes”, *Biogeosciences*.

- Kaufhold, C., M. Willeit, S. Talento, and A. Ganopolski (2024b), *Uncertainties in climate sensitivity and residual carbon emissions permit for a hothouse climate ahead*, Copernicus Meetings.
- Keir, R. S. (1988), On the late pleistocene ocean geochemistry and circulation, *Paleoceanography*, vol. 3, no. 4, pp. 413–445.
- Kemp, L., C. Xu, J. Depledge, K. L. Ebi, G. Gibbins, T. A. Kohler, J. Rockström, M. Scheffer, H. J. Schellnhuber, W. Steffen, and T. M. Lenton (2022), Climate endgame: exploring catastrophic climate change scenarios, *Proc. Natl. Acad. Sci. U.S.A.*, vol. 119, no. 34, e2108146119.
- Kender, S., K. Bogus, G. K. Pedersen, K. Dybkjær, T. A. Mather, E. Mariani, A. Ridgwell, J. B. Riding, T. Wagner, S. P. Hesselbo, and M. J. Leng (2021), Paleocene/eocene carbon feedbacks triggered by volcanic activity, *Nat Commun*, vol. 12, no. 1, p. 5186.
- Kidder, D. L. and T. R. Worsley (2012), A human-induced hothouse climate?, *GSAT*, pp. 4–11.
- Kilic, C., C. C. Raible, and T. F. Stocker (2017), Multiple climate states of habitable exoplanets: the role of obliquity and irradiance, *ApJ*, vol. 844, no. 2, p. 147.
- Kindler, P., M. Guillevic, M. Baumgartner, J. Schwander, A. Landais, and M. Leuenberger (2014), Temperature reconstruction from 10 to 120 kyr b2k from the NGRIP ice core, *Clim. Past*, vol. 10, no. 2, pp. 887–902.
- Klages, J. P., U. Salzmann, T. Bickert, C.-D. Hillenbrand, K. Gohl, G. Kuhn, S. M. Bohaty, J. Titschack, J. Müller, T. Frederichs, T. Bauersachs, W. Ehrmann, T. van de Flierdt, P. S. Pereira, R. D. Larter, G. Lohmann, I. Niezgodzki, G. Uenzelmann-Neben, M. Zundel, C. Spiegel, C. Mark, D. Chew, J. E. Francis, G. Nehrke, F. Schwarz, J. A. Smith, T. Freudenthal, O. Esper, H. Pälike, T. A. Ronge, and R. Dziadek (2020), Temperate rainforests near the south pole during peak cretaceous warmth, *Nature*, vol. 580, no. 7801, pp. 81–86.
- Klockmann, M., U. Mikolajewicz, and J. Marotzke (2018), Two AMOC states in response to decreasing greenhouse gas concentrations in the coupled climate model MPI-ESM, *Journal of Climate*, vol. 31, no. 19, pp. 7969–7984.
- Klose, A. K., J. F. Donges, U. Feudel, and R. Winkelmann (2024), “Rate-induced tipping cascades arising from interactions between the greenland ice sheet and the atlantic meridional overturning circulation”.
- Klose, A. K., V. Karle, R. Winkelmann, and J. F. Donges (2020), Emergence of cascading dynamics in interacting tipping elements of ecology and climate, *R. Soc. open sci.*, vol. 7, no. 6, p. 200599.

- Klose, A. K., N. Wunderling, R. Winkelmann, and J. F. Donges (2021), What do we mean, ‘tipping cascade’?, *Environ. Res. Lett.*, vol. 16, no. 12, p. 125011.
- Kohfeld, K. E. and A. Ridgwell (2009), Glacial-interglacial variability in atmospheric CO₂, in: *Geophysical Monograph Series*, ed. by C. Le Quéré and E. S. Saltzman, vol. 187, Washington, D. C.: American Geophysical Union, pp. 251–286.
- Köhler, P. and G. Munhoven (2020), Late pleistocene carbon cycle revisited by considering solid earth processes, *Paleoceanography and Paleoclimatology*, vol. 35, no. 12.
- Köhler, P., C. Nehrbass-Ahles, J. Schmitt, T. F. Stocker, and H. Fischer (2017), A 156 kyr smoothed history of the atmospheric greenhouse gases CO₂, CH₄, and n₂O and their radiative forcing, *Earth System Science Data*, vol. 9, no. 1, pp. 363–387.
- Kopp, R., D. Easterling, T. Hall, K. Hayhoe, R. Horton, K. Kunkel, and A. LeGrande (2017), *Ch. 15: potential surprises: compound extremes and tipping elements. climate science special report: fourth national climate assessment, volume i*, U.S. Global Change Research Program.
- Kriegler, E., J. W. Hall, H. Held, R. Dawson, and H. J. Schellnhuber (2009), Imprecise probability assessment of tipping points in the climate system, *Proceedings of the National Academy of Sciences*, vol. 106, no. 13, pp. 5041–5046.
- Kukla, T., D. E. Ibarra, K. V. Lau, and J. K. C. Rugenstein (2023), All aboard! earth system investigations with the CH₂O-CHOO TRAIN v1.0, *Geoscientific Model Development*, vol. 16, no. 19, pp. 5515–5538.
- Kump, L. R., S. L. Brantley, and M. A. Arthur (2000), Chemical weathering, atmospheric CO₂, and climate, *Annual Review of Earth and Planetary Sciences*, vol. 28 (Volume 28, 2000), pp. 611–667.
- Kuniyoshi, Y., A. Abe-Ouchi, S. Sherriff-Tadano, W.-L. Chan, and F. Saito (2022), Effect of climatic precession on dansgaard-oeschger-like oscillations, *Geophysical Research Letters*, vol. 49, no. 6, e2021GL095695.
- Laurian, A., S. S. Drijfhout, W. Hazeleger, and R. van Dorland (2009), Global surface cooling: the atmospheric fast feedback response to a collapse of the thermohaline circulation, *Geophysical Research Letters*, vol. 36, no. 20.
- Lauvset, S. K., R. M. Key, A. Olsen, S. van Heuven, A. Velo, X. Lin, C. Schirnick, A. Kozyr, T. Tanhua, M. Hoppema, S. Jutterström, R. Steinfeldt, E. Jeansson, M. Ishii, F. F. Perez, T. Suzuki, and S. Watelet (2016), “A new global interior ocean mapped climatology: the 1° × 1° GLODAP version 2”.

★ | Bibliography

- Lenton, T. M., H. Held, E. Kriegler, J. W. Hall, W. Lucht, S. Rahmstorf, and H. J. Schellnhuber (2008), Tipping elements in the earth's climate system, *Proceedings of the National Academy of Sciences*, vol. 105, no. 6, pp. 1786–1793.
- Lenton, T. M. (2000), Land and ocean carbon cycle feedback effects on global warming in a simple earth system model, *Tellus B*, vol. 52, no. 5, pp. 1159–1188.
- Lenton, T. M., D. I. Armstrong McKay, S. Loriani, J. F. Abrams, S. J. Lade, J. F. Donges, M. Milkoreit, T. Powell, S. R. Smith, C. Zimm, J. E. Buxton, E. Bailey, L. Laybourn, A. Ghadiali, and J. G. Dyke, eds. (2023), *The global tipping points report 2023*, Exeter, UK: University of Exeter.
- Lenton, T. M. and C. Britton (2006), Enhanced carbonate and silicate weathering accelerates recovery from fossil fuel CO₂ perturbations, *Global Biogeochemical Cycles*, vol. 20, no. 3.
- Lenton, T. M., J. Rockström, O. Gaffney, S. Rahmstorf, K. Richardson, W. Steffen, and H. J. Schellnhuber (2019), Climate tipping points — too risky to bet against, *Nature*, vol. 575, no. 7784, pp. 592–595.
- Lenton, T. M. and H. T. Williams (2013), On the origin of planetary-scale tipping points, *Trends in Ecology & Evolution*, vol. 28, no. 7, pp. 380–382.
- Levermann, A. and R. Winkelmann (2016), “A simple equation for the melt elevation feedback of ice sheets”, *The Cryosphere*.
- Levy, M., L. Bopp, P. Karleskind, L. Resplandy, C. Etche, and F. Pinsard (2013), Physical pathways for carbon transfers between the surface mixed layer and the ocean interior, *Global Biogeochemical Cycles*, vol. 27, no. 4, pp. 1001–1012.
- Liu, T., D. Chen, L. Yang, J. Meng, Z. Wang, J. Ludescher, J. Fan, S. Yang, D. Chen, J. Kurths, X. Chen, S. Havlin, and H. J. Schellnhuber (2023), Teleconnections among tipping elements in the earth system, *Nat. Clim. Chang.*, vol. 13, no. 1, pp. 67–74.
- Lohmann, J., D. Castellana, P. D. Ditlevsen, and H. A. Dijkstra (2021), Abrupt climate change as a rate-dependent cascading tipping point, *Earth System Dynamics*, vol. 12, no. 3, pp. 819–835.
- Lohmann, J., H. A. Dijkstra, M. Jochum, V. Lucarini, and P. D. Ditlevsen (2023), *Multistability and intermediate tipping of the atlantic ocean circulation*, arXiv:2304.05664, arXiv.
- Lord, N. S., A. Ridgwell, M. C. Thorne, and D. J. Lunt (2016), An impulse response function for the “long tail” of excess atmospheric CO₂ in an earth system model, *Global Biogeochem. Cycles*, vol. 30, no. 1, pp. 2–17.

- Loutre, M. F. and A. Berger (2000), Future climatic changes: are we entering an exceptionally long interglacial?, *Climatic Change*, vol. 46, no. 1, pp. 61–90.
- Lucarini, V. and T. Bódai (2017), Edge states in the climate system: exploring global instabilities and critical transitions, *Nonlinearity*, vol. 30, no. 7, R32–R66.
- Lyon, C., E. E. Saupe, C. J. Smith, D. J. Hill, A. P. Beckerman, L. C. Stringer, R. Marchant, J. McKay, A. Burke, P. O’Higgins, A. M. Dunhill, B. J. Allen, J. Riel-Salvatore, and T. Aze (2021), Climate change research and action must look beyond 2100, *Glob Change Biol*, gcb.15871.
- MacDougall, A. H., T. L. Frölicher, C. D. Jones, J. Rogelj, H. D. Matthews, K. Zickfeld, V. K. Arora, N. J. Barrett, V. Brovkin, F. A. Burger, M. Eby, A. V. Eliseev, T. Hajima, P. B. Holden, A. Jeltsch-Thömmes, C. Koven, N. Mengis, L. Menviel, M. Michou, I. I. Mokhov, A. Oka, J. Schwinger, R. Séférian, G. Shaffer, A. Sokolov, K. Tachiiri, J. Tjiputra, A. Wiltshire, and T. Ziehn (2020), “Is there warming in the pipeline? a multi-model analysis of the zero emissions commitment from CO₂”.
- Maher, K. and C. P. Chamberlain (2014), Hydrologic regulation of chemical weathering and the geologic carbon cycle, *Science*, vol. 343, no. 6178, pp. 1502–1504.
- Malmierca-Vallet, I., L. C. Sime, and the D–O community members (2023), Dansgaard–oeschger events in climate models: review and baseline marine isotope stage 3 (MIS3) protocol, *Clim. Past*, vol. 19, no. 5, pp. 915–942.
- Malmierca-Vallet, I., L. C. Sime, P. J. Valdes, M. Klockmann, G. Vettoretti, and J. Slattery (2024), The impact of CO₂ and climate state on whether dansgaard–oeschger type oscillations occur in climate models, *Geophysical Research Letters*, vol. 51, no. 13, e2024GL110068.
- Martínez Montero, M., N. Brede, V. Couplet, M. Crucifix, N. Botta, and C. Wieners (2024), Lost options commitment: how short-term policies affect long-term scope of action, *Oxford Open Climate Change*, vol. 4, no. 1, kgae004.
- Martínez Montero, M., M. Crucifix, V. Couplet, N. Brede, and N. Botta (2022), SURFER v2.0: a flexible and simple model linking anthropogenic CO₂ emissions and solar radiation modification to ocean acidification and sea level rise, *Geoscientific Model Development*, vol. 15, no. 21, pp. 8059–8084.
- McDougall, T. J. and P. M. Barker (2011), “Getting started with TEOS-10 and the gibbs seawater (GSW) oceanographic toolbox”, SCOR/IAPSO WG127, 28 pp.

- McNeall, D., P. R. Halloran, P. Good, and R. A. Betts (2011), “Analyzing abrupt and nonlinear climate changes and their impacts”, *Vo lu me*.
- Mehrbach, C., C. H. Culberson, J. E. Hawley, and R. M. Pytkowicz (1973), Measurement of the apparent dissociation constants of carbonic acid in seawater at atmospheric pressure, *Limnol. Oceanogr.*, vol. 18, no. 6, pp. 897–907.
- Meinshausen, M., Z. R. J. Nicholls, J. Lewis, M. J. Gidden, E. Vogel, M. Freund, U. Beyerle, C. Gessner, A. Nauels, N. Bauer, J. G. Canadell, J. S. Daniel, A. John, P. B. Krummel, G. Luderer, N. Meinshausen, S. A. Montzka, P. J. Rayner, S. Reimann, S. J. Smith, M. van den Berg, G. J. M. Velders, M. K. Vollmer, and R. H. J. Wang (2020), The shared socio-economic pathway (SSP) greenhouse gas concentrations and their extensions to 2500, *Geosci. Model Dev.*, vol. 13, no. 8, pp. 3571–3605.
- Middelburg, J. J., K. Soetaert, and M. Hagens (2020), Ocean alkalinity, buffering and biogeochemical processes, *Reviews of Geophysics*, vol. 58, no. 3, e2019RG000681.
- Millero, F. J. (1979), The thermodynamics of the carbonate system in seawater, *Geochimica et Cosmochimica Acta*, vol. 43, no. 10, pp. 1651–1661.
- (1995), Thermodynamics of the carbon dioxide system in the oceans, *Geochimica et Cosmochimica Acta*, vol. 59, no. 4, pp. 661–677.
- Milliman, J. D. and A. W. Droxler (1996), Neritic and pelagic carbonate sedimentation in the marine environment: ignorance is not bliss, *Geol Rundsch*, vol. 85, no. 3, pp. 496–504.
- Milliman, J. D. (1993), Production and accumulation of calcium carbonate in the ocean: budget of a nonsteady state, *Global Biogeochemical Cycles*, vol. 7, no. 4, pp. 927–957.
- Minshull, T. A., H. Marín-Moreno, D. I. Armstrong McKay, and P. A. Wilson (2016), Mechanistic insights into a hydrate contribution to the paleocene-eocene carbon cycle perturbation from coupled thermohydraulic simulations, *Geophysical Research Letters*, vol. 43, no. 16, pp. 8637–8644.
- Möller, T., A. E. Högnér, C.-F. Schleussner, S. Bien, N. H. Kitzmann, R. D. Lamboll, J. Rogelj, J. F. Donges, J. Rockström, and N. Wunderling (2024), Achieving net zero greenhouse gas emissions critical to limit climate tipping risks, *Nat Commun*, vol. 15, no. 1, p. 6192.
- Montero, M. M., N. Brede, V. Couplet, M. Crucifix, N. Botta, and C. Wieners (2023), *Lost options commitment: how short-term policies affect long-term scope of action*, arXiv:2309.07743, arXiv.

- Mucci, A. (1983), The solubility of calcite and aragonite in seawater at various salinities, temperatures, and one atmosphere total pressure, *American Journal of Science*, vol. 283, no. 7, pp. 780–799.
- Munhoven, G. (2013), Mathematics of the total alkalinity–pH equation – pathway to robust and universal solution algorithms: the SolveSAPHE package v1.0.1, *Geoscientific Model Development*, vol. 6, no. 4, pp. 1367–1388.
- Munhoven, G. and L. M. François (1996), Glacial-interglacial variability of atmospheric CO₂ due to changing continental silicate rock weathering: a model study, *Journal of Geophysical Research: Atmospheres*, vol. 101 (D16), pp. 21423–21437.
- Myhre, G., E. J. Highwood, K. P. Shine, and F. Stordal (1998), New estimates of radiative forcing due to well mixed greenhouse gases, *Geophys. Res. Lett.*, vol. 25, no. 14, pp. 2715–2718.
- Nian, D., S. Bathiany, M. Ben-Yami, L. L. Blaschke, M. Hirota, R. R. Rodrigues, and N. Boers (2023), A potential collapse of the atlantic meridional overturning circulation may stabilise eastern amazonian rainforests, *Commun Earth Environ*, vol. 4, no. 1, pp. 1–11.
- Nisbet, E. G., M. R. Manning, E. J. Dlugokencky, S. E. Michel, X. Lan, T. Röckmann, H. A. C. Denier van der Gon, J. Schmitt, P. I. Palmer, M. N. Dyonisius, Y. Oh, R. E. Fisher, D. Lowry, J. L. France, J. W. C. White, G. Brailsford, and T. Bromley (2023), Atmospheric methane: comparison between methane’s record in 2006–2022 and during glacial terminations, *Global Biogeochemical Cycles*, vol. 37, no. 8, e2023GB007875.
- Nitzbon, J., T. Schneider von Deimling, M. Aliyeva, S. E. Chadburn, G. Grosse, S. Laboor, H. Lee, G. Lohmann, N. J. Steinert, S. M. Stuenzi, M. Werner, S. Westermann, and M. Langer (2024), No respite from permafrost-thaw impacts in the absence of a global tipping point, *Nat. Clim. Chang.*, vol. 14, no. 6, pp. 573–585.
- Notz, D. and S. Community (2020), Arctic sea ice in CMIP6, *Geophysical Research Letters*, vol. 47, no. 10, e2019GL086749.
- Notz, D. and J. Stroeve (2016), Observed arctic sea-ice loss directly follows anthropogenic CO₂ emission, *Science*, vol. 354, no. 6313, pp. 747–750.
- Petzold, L. (1983), Automatic selection of methods for solving stiff and nonstiff systems of ordinary differential equations, *SIAM J. Sci. and Stat. Comput.*, vol. 4, no. 1, pp. 136–148.

- Planchat, A., L. Bopp, L. Kwiatkowski, and O. Torres (2023), “Carbonate pump feedbacks on alkalinity and the carbon cycle in the 21st century and beyond”.
- Popp, M., H. Schmidt, and J. Marotzke (2016), Transition to a moist greenhouse with CO₂ and solar forcing, *Nat Commun*, vol. 7, no. 1, p. 10627.
- Rahmstorf, S. (1996), On the freshwater forcing and transport of the atlantic thermohaline circulation, *Climate Dynamics*, vol. 12, no. 12, pp. 799–811.
- Rahmstorf, S. (2024), Is the atlantic overturning circulation approaching a tipping point?, *Oceanography*, vol. 37, no. 3, pp. 16–29.
- Rahmstorf, S., M. Crucifix, A. Ganopolski, H. Goosse, I. Kamenkovich, R. Knutti, G. Lohmann, R. Marsh, L. A. Mysak, Z. Wang, and A. J. Weaver (2005), Thermohaline circulation hysteresis: a model intercomparison, *Geophys. Res. Lett.*, vol. 32, no. 23, p. L23605.
- Raworth, K. (2012), “A safe and just space for humanity: can we live within the doughnut?”
- Renssen, H., C. J. Beets, T. Fichefet, H. Goosse, and D. Kroon (2004), Modeling the climate response to a massive methane release from gas hydrates, *Paleoceanography*, vol. 19, no. 2.
- Riahi, K., D. P. van Vuuren, E. Kriegler, J. Edmonds, B. C. O’Neill, S. Fujimori, N. Bauer, K. Calvin, R. Dellink, O. Fricko, W. Lutz, A. Popp, J. C. Cuaresma, S. Kc, M. Leimbach, L. Jiang, T. Kram, S. Rao, J. Emmerling, K. Ebi, T. Hasegawa, P. Havlik, F. Humpenöder, L. A. Da Silva, S. Smith, E. Stehfest, V. Bosetti, J. Eom, D. Gernaat, T. Masui, J. Rogelj, J. Strefler, L. Drouet, V. Krey, G. Luderer, M. Harmsen, K. Takahashi, L. Baumstark, J. C. Doelman, M. Kainuma, Z. Klimont, G. Marangoni, H. Lotze-Campen, M. Obersteiner, A. Tabeau, and M. Tavoni (2017), The shared socioeconomic pathways and their energy, land use, and greenhouse gas emissions implications: an overview, *Global Environmental Change*, vol. 42, pp. 153–168.
- Ridgwell, A., J. C. Hargreaves, N. R. Edwards, J. D. Annan, T. M. Lenton, R. Marsh, A. Yool, and A. Watson (2007), Marine geochemical data assimilation in an efficient earth system model of global biogeochemical cycling, *Biogeosciences*, vol. 4, no. 1, pp. 87–104.
- Ridgwell, A. and J. C. Hargreaves (2007), Regulation of atmospheric CO₂ by deep-sea sediments in an earth system model: REGULATION OF CO₂ BY DEEP-SEA SEDIMENTS, *Global Biogeochem. Cycles*, vol. 21, no. 2, n/a–n/a.

- Rietkerk, M., R. Bastiaansen, S. Banerjee, J. van de Koppel, M. Baudena, and A. Doelman (2021), Evasion of tipping in complex systems through spatial pattern formation, *Science*, vol. 374, no. 6564, eabj0359.
- Ripple, W. J., C. Wolf, T. M. Lenton, J. W. Gregg, S. M. Natali, P. B. Duffy, J. Rockström, and H. J. Schellnhuber (2023), Many risky feedback loops amplify the need for climate action, *One Earth*, vol. 6, no. 2, pp. 86–91.
- Ritchie, P., Ö. Karabacak, and J. Sieber (2019), Inverse-square law between time and amplitude for crossing tipping thresholds, *Proc. R. Soc. A.*, vol. 475, no. 2222, p. 20180504.
- Ritchie, P. D. L., J. J. Clarke, P. M. Cox, and C. Huntingford (2021), Overshooting tipping point thresholds in a changing climate, *Nature*, vol. 592, no. 7855, pp. 517–523.
- Robinson, A., R. Calov, and A. Ganopolski (2012), Multistability and critical thresholds of the greenland ice sheet, *Nature Clim Change*, vol. 2, no. 6, pp. 429–432.
- Rocha, J. C., G. Peterson, Ö. Bodin, and S. Levin (2018), “Cascading regime shifts within and across scales”, p. 6.
- Ross, P. M., L. Parker, W. A. O’Connor, and E. A. Bailey (2011), The impact of ocean acidification on reproduction, early development and settlement of marine organisms, *Water*, vol. 3, no. 4, pp. 1005–1030.
- Rosser, J., R. Winkelmann, N. Wunderling, et al. (2024), Cryosphere tipping elements decisive for tipping risks and cascading effects in the earth system, *Preprint at Research Square*.
- Rothman, D. H. (2019), Characteristic disruptions of an excitable carbon cycle, *Proc Natl Acad Sci USA*, vol. 116, no. 30, pp. 14813–14822.
- Rounce, D. R., R. Hock, F. Maussion, R. Hugonnet, W. Kochtitzky, M. Huss, E. Berthier, D. Brinkerhoff, L. Compagno, L. Copland, D. Farinotti, B. Menounos, and R. W. McNabb (2023), Global glacier change in the 21st century: every increase in temperature matters, *Science*, vol. 379, no. 6627, pp. 78–83.
- Ruppel, C. D. and J. D. Kessler (2017), The interaction of climate change and methane hydrates, *Reviews of Geophysics*, vol. 55, no. 1, pp. 126–168.
- Ruvalcaba Baroni, I., V. Palastanga, and C. P. Slomp (2020), Enhanced organic carbon burial in sediments of oxygen minimum zones upon ocean deoxygenation, *Front. Mar. Sci.*, vol. 6.
- Sanderson, B. M., B. B. Booth, J. Dunne, V. Eyring, R. A. Fisher, P. Friedlingstein, M. J. Gidden, T. Hajima, C. D. Jones, C. G. Jones, A. King, C. D. Koven, D. M. Lawrence, J. Lowe, N. Mengis, G. P. Peters, J. Rogelj,

- C. Smith, A. C. Snyder, I. R. Simpson, A. L. S. Swann, C. Tebaldi, T. Ilyina, C.-F. Schleussner, R. Séférian, B. H. Samset, D. van Vuuren, and S. Zaehle (2024), The need for carbon-emissions-driven climate projections in CMIP7, *Geoscientific Model Development*, vol. 17, no. 22, pp. 8141–8172.
- Sarmiento, J. L. and N. Gruber (2006), “Ocean biogeochemical dynamics”, Princeton: Princeton University Press, 503 pp.
- Saunois, M., A. R. Stavert, B. Poulter, P. Bousquet, J. G. Canadell, R. B. Jackson, P. A. Raymond, E. J. Dlugokencky, S. Houweling, P. K. Patra, P. Ciais, V. K. Arora, D. Bastviken, P. Bergamaschi, D. R. Blake, G. Brailsford, L. Bruhwiler, K. M. Carlson, M. Carrol, S. Castaldi, N. Chandra, C. Crevoisier, P. M. Crill, K. Covey, C. L. Curry, G. Etiope, C. Frankenberg, N. Gedney, M. I. Hegglin, L. Höglund-Isaksson, G. Hugelius, M. Ishizawa, A. Ito, G. Janssens-Maenhout, K. M. Jensen, F. Joos, T. Kleinen, P. B. Krummel, R. L. Langenfelds, G. G. Laruelle, L. Liu, T. Machida, S. Maksyutov, K. C. McDonald, J. McNorton, P. A. Miller, J. R. Melton, I. Morino, J. Müller, F. Murguía-Flores, V. Naik, Y. Niwa, S. Noce, S. O’Doherty, R. J. Parker, C. Peng, S. Peng, G. P. Peters, C. Prigent, R. Prinn, M. Ramonet, P. Regnier, W. J. Riley, J. A. Rosentreter, A. Segers, I. J. Simpson, H. Shi, S. J. Smith, L. P. Steele, B. F. Thornton, H. Tian, Y. Tohjima, F. N. Tubiello, A. Tsuruta, N. Viovy, A. Voulgarakis, T. S. Weber, M. van Weele, Y. Yin, Y. Yoshida, W. Zhang, Z. Zhang, Y. Zhao, B. Zheng, Q. Zhu, Q. Zhu, and Q. Zhuang (2020), “The global methane budget 2000–2017”.
- Scheffer, M., J. Bascompte, W. A. Brock, V. Brovkin, S. R. Carpenter, V. Dakos, H. Held, E. H. van Nes, M. Rietkerk, and G. Sugihara (2009), Early-warning signals for critical transitions, *Nature*, vol. 461, no. 7260, pp. 53–59.
- Schellnhuber, H. J., S. Rahmstorf, and R. Winkelmann (2016), Why the right climate target was agreed in paris, *Nature Clim Change*, vol. 6, no. 7, pp. 649–653.
- Schlanger, S. and H. Jenkyns (1976), “Cretaceous oceanic anoxic events: causes and consequences”, *Geologie en mijnbouw*, vol. 55, no. 3.
- Schmittner, A., A. Oschlies, H. D. Matthews, and E. D. Galbraith (2008), Future changes in climate, ocean circulation, ecosystems, and biogeochemical cycling simulated for a business-as-usual CO₂ emission scenario until year 4000 AD, *Global Biogeochemical Cycles*, vol. 22, no. 1.
- Schneider, T., C. M. Kaul, and K. G. Pressel (2019), Possible climate transitions from breakup of stratocumulus decks under greenhouse warming, *Nat. Geosci.*, vol. 12, no. 3, pp. 163–167.

- Schuur, E. A. G., B. W. Abbott, W. B. Bowden, V. Brovkin, P. Camill, J. G. Canadell, J. P. Chanton, F. S. Chapin, T. R. Christensen, P. Ciais, B. T. Crosby, C. I. Czimczik, G. Grosse, J. Harden, D. J. Hayes, G. Hugelius, J. D. Jastrow, J. B. Jones, T. Kleinen, C. D. Koven, G. Krinner, P. Kuhry, D. M. Lawrence, A. D. McGuire, S. M. Natali, J. A. O'Donnell, C. L. Ping, W. J. Riley, A. Rinke, V. E. Romanovsky, A. B. K. Sannel, C. Schädel, K. Schaefer, J. Sky, Z. M. Subin, C. Tarnocai, M. R. Turetsky, M. P. Waldrop, K. M. Walter Anthony, K. P. Wickland, C. J. Wilson, and S. A. Zimov (2013), Expert assessment of vulnerability of permafrost carbon to climate change, *Climatic Change*, vol. 119, no. 2, pp. 359–374.
- Schuur, E. a. G., A. D. McGuire, C. Schädel, G. Grosse, J. W. Harden, D. J. Hayes, G. Hugelius, C. D. Koven, P. Kuhry, D. M. Lawrence, S. M. Natali, D. Olefeldt, V. E. Romanovsky, K. Schaefer, M. R. Turetsky, C. C. Treat, and J. E. Vonk (2015), Climate change and the permafrost carbon feedback, *Nature*, vol. 520, no. 7546, pp. 171–179.
- Schuur, E. A. G., B. W. Abbott, R. Commane, J. Ernakovich, E. Euskirchen, G. Hugelius, G. Grosse, M. Jones, C. Koven, V. Leshyk, D. Lawrence, M. M. Lorant, M. Mauritz, D. Olefeldt, S. Natali, H. Rodenhizer, V. Salmon, C. Schädel, J. Strauss, C. Treat, and M. Turetsky (2022), Permafrost and climate change: carbon cycle feedbacks from the warming arctic, *Annual Review of Environment and Resources*, vol. 47 (Volume 47, 2022), pp. 343–371.
- Sellers, W. D. (1969), A global climatic model based on the energy balance of the earth-atmosphere system, *Journal of Applied Meteorology and Climatology*, vol. 8, no. 3, pp. 392–400.
- Sinet, S., A. S. von der Heydt, and H. A. Dijkstra (2023), AMOC stabilization under the interaction with tipping polar ice sheets, *Geophysical Research Letters*, vol. 50, no. 2.
- Skeie, R. B., Ø. Hodnebrog, and G. Myhre (2023), Trends in atmospheric methane concentrations since 1990 were driven and modified by anthropogenic emissions, *Commun Earth Environ*, vol. 4, no. 1, pp. 1–14.
- Smith, C., Z. Nicholls, K. Armour, W. Collins, P. Forster, M. Meinshausen, M. Palmer, and M. Watanabe (2021), The earth's energy budget, climate feedbacks, and climate sensitivity supplementary material, in: *Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*, ed. by V. Masson-Delmotte, P. Zhai, A. Pirani, S. Connors, C. Péan, S. Berger, N. Caud, Y. Chen, L. Goldfarb, M. Gomis, M. Huang,

- K. Leitzell, E. Lonnoy, J. Matthews, T. Maycock, T. Waterfield, O. Yelekçi, R. Yu, and B. Zhou.
- Smith, C. J., P. M. Forster, M. Allen, N. Leach, R. J. Millar, G. A. Passerello, and L. A. Regayre (2018), FAIR v1.3: a simple emissions-based impulse response and carbon cycle model, *Geosci. Model Dev.*, vol. 11, no. 6, pp. 2273–2297.
- Steffen, W., J. Rockström, K. Richardson, T. M. Lenton, C. Folke, D. Liverman, C. P. Summerhayes, A. D. Barnosky, S. E. Cornell, M. Crucifix, J. F. Donges, I. Fetzer, S. J. Lade, M. Scheffer, R. Winkelmann, and H. J. Schellnhuber (2018), Trajectories of the earth system in the anthropocene, *Proc Natl Acad Sci USA*, vol. 115, no. 33, pp. 8252–8259.
- Stocker, T. F., D. Qin, G.-K. Plattner, M. Tignor, S. K. Allen, J. Boschung, A. Nauels, Y. Xia, V. Bex, and P. M. Midgley (2013), “Climate change 2013: the physical science basis”, *Contribution of working group I to the fifth assessment report of the intergovernmental panel on climate change*, vol. 1535.
- Stocker, T. F. and A. Schmittner (1997), “Influence of CO₂ emission rates on the stability of the thermohaline circulation”, vol. 388, p. 4.
- Stommel, H. (1961), Thermohaline convection with two stable regimes of flow, *Tellus*, vol. 13, no. 2, pp. 224–230.
- Strauss, J., L. Schirrmeister, G. Grosse, D. Fortier, G. Hugelius, C. Knoblauch, V. Romanovsky, C. Schädel, T. Schneider von Deimling, E. A. G. Schuur, D. Shmelev, M. Ulrich, and A. Veremeeva (2017), Deep yedoma permafrost: a synthesis of depositional characteristics and carbon vulnerability, *Earth-Science Reviews*, vol. 172, pp. 75–86.
- Strogatz, S. H. (2015), “Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering”, Second edition, Boulder, CO: Westview Press, a member of the Perseus Books Group, 513 pp.
- Sulpis, O., E. Jeansson, A. Dinauer, S. K. Lauvset, and J. J. Middelburg (2021), Calcium carbonate dissolution patterns in the ocean, *Nat. Geosci.*, vol. 14, no. 6, pp. 423–428.
- Swann, A. L., I. Y. Fung, S. Levis, G. B. Bonan, and S. C. Doney (2010), Changes in arctic vegetation amplify high-latitude warming through the greenhouse effect, *Proceedings of the National Academy of Sciences*, vol. 107, no. 4, pp. 1295–1300.
- Talento, S. and A. Ganopolski (2021), Reduced-complexity model for the impact of anthropogenic CO₂ emissions on future glacial cycles, *Earth Syst. Dynam.*, vol. 12, no. 4, pp. 1275–1293.

- Thomas, Z. A., R. T. Jones, C. S. Turney, N. Golledge, C. Fogwill, C. J. Bradshaw, L. Menviel, N. P. McKay, M. Bird, J. Palmer, P. Kershaw, J. Wilmshurst, and R. Muscheler (2020), Tipping elements and amplified polar warming during the last interglacial, *Quaternary Science Reviews*, vol. 233, p. 106222.
- Tigchelaar, M., A. S. von der Heydt, and H. A. Dijkstra (2011), A new mechanism for the two-step 18 o signal at the eocene-oligocene boundary, *Clim. Past*, vol. 7, no. 1, pp. 235–247.
- Tokarska, K. B., N. P. Gillett, A. J. Weaver, V. K. Arora, and M. Eby (2016), The climate response to five trillion tonnes of carbon, *Nature Clim Change*, vol. 6, no. 9, pp. 851–855.
- Turner, A. J., C. Frankenberg, and E. A. Kort (2019), Interpreting contemporary trends in atmospheric methane, *Proceedings of the National Academy of Sciences*, vol. 116, no. 8, pp. 2805–2813.
- Uppström, L. R. (1974), The boron/chlorinity ratio of deep-sea water from the pacific ocean, *Deep Sea Research and Oceanographic Abstracts*, vol. 21, no. 2, pp. 161–162.
- Van Westen, R. M. and H. A. Dijkstra (2023), Asymmetry of AMOC hysteresis in a state-of-the-art global climate model, *Geophysical Research Letters*, vol. 50, no. 22, e2023GL106088.
- Van Westen, R. M., M. Kliphuis, and H. A. Dijkstra (2024a), Physics-based early warning signal shows that AMOC is on tipping course, *Science Advances*, vol. 10, no. 6, eadk1189.
- Van Westen, R. M., E. Y. P. Vanderborght, M. Kliphuis, and H. A. Dijkstra (2024b), *Substantial risk of 21st century AMOC tipping even under moderate climate change*, arXiv:2407.19909, arXiv.
- Vecsei, A. (2004), A new estimate of global reefal carbonate production including the fore-reefs, *Global and Planetary Change*, vol. 43, no. 1, pp. 1–18.
- Vettoretti, G., P. Ditlevsen, M. Jochum, and S. O. Rasmussen (2022), Atmospheric CO₂ control of spontaneous millennial-scale ice age climate oscillations, *Nat. Geosci.*, vol. 15, no. 4, pp. 300–306.
- Volk, T. and M. I. Hoffert (1985), Ocean carbon pumps: analysis of relative strengths and efficiencies in ocean-driven atmospheric CO₂ changes, in: *The Carbon Cycle and Atmospheric CO₂: Natural Variations Archean to Present*, American Geophysical Union (AGU), pp. 99–110.

★ | Bibliography

- Von der Heydt, A. S. and P. Ashwin (2016), State dependence of climate sensitivity: attractor constraints and palaeoclimate regimes, *Dynamics and Statistics of the Climate System*, vol. 1, no. 1.
- Von der Heydt, A. S. (2022), Can the miocene climate inform the future?, *Science*, vol. 377, no. 6601, pp. 26–27.
- Waddington, C. (1957), *The strategy of the genes*.
- Walker, J. C. G., P. B. Hays, and J. F. Kasting (1981), A negative feedback mechanism for the long-term stabilization of earth's surface temperature, *Journal of Geophysical Research: Oceans*, vol. 86 (C10), pp. 9776–9782.
- Wang, S., A. Foster, E. A. Lenz, J. D. Kessler, J. C. Stroeve, L. O. Anderson, M. Turetsky, R. Betts, S. Zou, W. Liu, W. R. Boos, and Z. Hausfather (2023), Mechanisms and impacts of earth system tipping elements, *Reviews of Geophysics*, vol. 61, no. 1.
- Weijer, W., W. Cheng, S. S. Drijfhout, A. V. Fedorov, A. Hu, L. C. Jackson, W. Liu, E. L. McDonagh, J. V. Mecking, and J. Zhang (2019), Stability of the atlantic meridional overturning circulation: a review and synthesis, *Journal of Geophysical Research: Oceans*, vol. 124, no. 8, pp. 5336–5375.
- Weiss, R. (1974), Carbon dioxide in water and seawater: the solubility of a non-ideal gas, *Marine Chemistry*, vol. 2, no. 3, pp. 203–215.
- West, A. J., A. Galy, and M. Bickle (2005), Tectonic and climatic controls on silicate weathering, *Earth and Planetary Science Letters*, vol. 235, no. 1, pp. 211–228.
- Westerhold, T., N. Marwan, A. J. Drury, D. Liebrand, C. Agnini, E. Anagnostou, J. S. K. Barnet, S. M. Bohaty, D. De Vleeschouwer, F. Florindo, T. Frederichs, D. A. Hodell, A. E. Holbourn, D. Kroon, V. Lauretano, K. Littler, L. J. Lourens, M. Lyle, H. Pälike, U. Röhl, J. Tian, R. H. Wilkens, P. A. Wilson, and J. C. Zachos (2020), An astronomically dated record of earth's climate and its predictability over the last 66 million years, *Science*, vol. 369, no. 6509, pp. 1383–1387.
- Willeit, M., A. Ganopolski, and G. Feulner (2014), “Asymmetry and uncertainties in biogeophysical climate–vegetation feedback over a range of CO₂ forcings”.
- Willeit, M., R. Calov, S. Talento, R. Greve, J. Bernales, V. Klemann, M. Bagge, and A. Ganopolski (2023), “Glacial inception through rapid ice area increase driven by albedo and vegetation feedbacks”.
- Williams, R. G., P. Goodwin, A. Ridgwell, and P. L. Woodworth (2012), How warming and steric sea level rise relate to cumulative carbon emissions, *Geophysical Research Letters*, vol. 39, no. 19.

- Winkelmann, R., A. Levermann, A. Ridgwell, and K. Caldeira (2015), Combustion of available fossil fuel resources sufficient to eliminate the antarctic ice sheet, *Sci. Adv.*, vol. 1, no. 8, e1500589.
- WMO (2025), *WMO confirms 2024 as warmest year on record at about 1.55°C above pre-industrial level*, World Meteorological Organization, URL: <https://wmo.int/media/news/wmo-confirms-2024-warmest-year-record-about-155degc-above-pre-industrial-level> (visited on 01/21/2025).
- Wunderling, N., J. F. Donges, J. Kurths, and R. Winkelmann (2021), Interacting tipping elements increase risk of climate domino effects under global warming, *Earth Syst. Dynam.*, vol. 12, no. 2, pp. 601–619.
- Wunderling, N., A. S. von der Heydt, Y. Aksenov, S. Barker, R. Bastiaansen, V. Brovkin, M. Brunetti, V. Couplet, T. Kleinen, C. H. Lear, J. Lohmann, R. M. Roman-Cuesta, S. Sinet, D. Swingedouw, R. Winkelmann, P. Anand, J. Barichivich, S. Bathiany, M. Baudena, J. T. Bruun, C. M. Chiessi, H. K. Coxall, D. Docquier, J. F. Donges, S. K. J. Falkena, A. K. Klose, D. Obura, J. Rocha, S. Rynders, N. J. Steinert, and M. Willeit (2024), Climate tipping point interactions and cascades: a review, *Earth System Dynamics*, vol. 15, no. 1, pp. 41–74.
- Wunderling, N., M. Willeit, J. F. Donges, and R. Winkelmann (2020), Global warming due to loss of large ice masses and arctic summer sea ice, *Nat Commun*, vol. 11, no. 1, p. 5177.
- Wunderling, N., R. Winkelmann, J. Rockström, S. Loriani, D. I. Armstrong McKay, P. D. L. Ritchie, B. Sakschewski, and J. F. Donges (2022), Global warming overshoots increase risks of climate tipping cascades in a network model, *Nat. Clim. Chang.*
- Yang, M., T. G. Bell, J.-R. Bidlot, B. W. Blomquist, B. J. Butterworth, Y. Dong, C. W. Fairall, S. Landwehr, C. A. Marandino, S. D. Miller, E. S. Saltzman, and A. Zavarisky (2022), Global synthesis of air-sea CO₂ transfer velocity estimates from ship-based eddy covariance measurements, *Frontiers in Marine Science*, vol. 9.
- Zarnetske, P. L., J. Gurevitch, J. Franklin, P. M. Groffman, C. S. Harrison, J. J. Hellmann, F. M. Hoffman, S. Kothari, A. Robock, S. Tilmes, D. Visioni, J. Wu, L. Xia, and C.-E. Yang (2021), Potential ecological impacts of climate intervention by reflecting sunlight to cool earth, *Proceedings of the National Academy of Sciences*, vol. 118, no. 15, e1921854118.
- Zeebe, R. E. (2012), “LOSCAR: long-term ocean-atmosphere-sediment Carbon cycle reservoir model v2.0.4”, *Geosci. Model Dev.*

★ | Bibliography

- Zeebe, R. E. and P. Westbroek (2003), A simple model for the CaCO_3 saturation state of the ocean: the “strangelove,” the “neritan,” and the “cretan” ocean, *Geochemistry, Geophysics, Geosystems*, vol. 4, no. 12.
- Zekollari, H., M. Huss, L. Schuster, F. Maussion, D. R. Rounce, N. Champollion, L. Compagno, R. Hugonnet, B. Marzeion, S. Mojtavavi, and D. Farinotti (2024), “21st century global glacier evolution under CMIP6 scenarios and the role of glacier-specific observations”.
- Zhu, J., C. J. Poulsen, and B. L. Otto-Bliesner (2024), Modeling past hothouse climates as a means for assessing earth system models and improving the understanding of warm climates, *Annual Review of Earth and Planetary Sciences*, vol. 52 (Volume 52, 2024), pp. 351–78.
- Zickfeld, K., M. Eby, A. J. Weaver, K. Alexander, E. Crespin, N. R. Edwards, A. V. Eliseev, G. Feulner, T. Fichefet, C. E. Forest, P. Friedlingstein, H. Goosse, P. B. Holden, F. Joos, M. Kawamiya, D. Kicklighter, H. Kienert, K. Matsumoto, I. I. Mokhov, E. Monier, S. M. Olsen, J. O. P. Pedersen, M. Perrette, G. Philippon-Berthier, A. Ridgwell, A. Schlosser, T. S. V. Deimling, G. Shaffer, A. Sokolov, R. Spahni, M. Steinacher, K. Tachiiri, K. S. Tokos, M. Yoshimori, N. Zeng, and F. Zhao (2013), Long-term climate change commitment and reversibility: an EMIC intercomparison, *Journal of Climate*, vol. 26, no. 16, pp. 5782–5809.