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Bearing capacity of surface foundations resting on Hoek – Brown materials using equivalent Mohr – Coulomb parameters

Capacité portante des fondations superficielles reposant sur un materiau Hoek – Brown en utilisant des paramètres Mohr – Coulomb équivalents

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ABSTRACT: Estimating the bearing capacity of surface foundations resting on rock masses is a necessary first step in the design of critical infrastructures. Although it is known that the rock masses behave nonlinearly according to the Hoek – Brown (H-B) failure criterion, there has long been a tendency to model rock failure as a linear Mohr – Coulomb (M-C) law for which extensive literature is available in terms of the foundation ultimate bearing response. This has provided an impetus for practicing engineers to find equivalent friction angle and cohesion so that the classical bearing capacity relationships can be used. This paper provides specific guidance for selecting equivalent M-C parameters to compute bearing capacity of surface foundation resting on rock masses. Numerical models validate that the suggested equivalent M-C parameters values result in bearing capacity values practically equal to those of the H-B material.

RÉSUMÉ: L'estimation de la capacité portante des fondations superficielles reposant sur un massif rocheux est une étape primordiale dans la conception d'infrastructures. Bien que l'on sache que la roche atteint la rupture selon le critère non linéaire de Hoek – Brown (H-B), il existe depuis longtemps une tendance à la modélisation des massifs rocheux en tant que matériau linéaire Mohr – Coulomb (M-C) pour lequel une littérature abondante est disponible. Cela a incité les ingénieurs praticiens à trouver un angle de frottement et une cohésion équivalents, de sorte que les relations de capacité portante classiques puissent être utilisées. Ce document fournit des instructions spécifiques pour la sélection de paramètres M-C équivalents pour le calcul de la capacité portante des fondations superficielles reposant sur un massif rocheux. Des modèles numériques attestent que les valeurs des parameteres M-C équivalents ainsi suggérées produisent des valeurs de capacité portante pratiquement égales à celles résultant du critère H-B.

Keywords: Bearing capacity, Surface foundations, Hoek-Brown criterion, Rock mass, Equivalent Mohr-Coulomb parameters.

1 INTRODUCTION

Since the introduction of the Hoek – Brown (H-B) criterion (Hoek, 2002) in 1980, several authors have used it in order to investigate the bearing capacity of foundations on rock masses. These authors have employed various methods. Among them one can cite the characteristic lines theory (Serrano, 2000), the upper bound – lower bound theorems of limit analysis (Merifield, 2006) and displacement finite element method (Clausen, 2013). It appears that the published

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solutions for strip footing resting on weightless rock (Serrano, 2000; Merifield, 2006) are in excellent agreement with each other. These solutions are presented in the form of charts and tables for discrete values of H-B input parameters. Thus, graphical reading and interpolation are required, which is not practical and can lead to imprecise results.

Nevertheless, the industry has a tendency to use constant Mohr – Coulomb (M-C) parameters to characterize the rock masses. This is indeed, due to the fact that the physical interpretation of these parameters is much easier and that classical bearing capacity equations can be used. One of such equation is written in the following form:

$$q_{ult} = cN_c + qN_q + \frac{1}{2}\gamma B N_\gamma \tag{1}$$

Here, the non-dimensional parameters N_c, N_q and N_γ are bearing capacity factors, which are functions of the friction angle. They are related, respectively, to the cohesion c, the surcharge load q and the unit weight γ of the soil.

The processes of finding equivalent M-C parameters boil down to finding an appropriate upper limit of confining stress $\sigma'_{3\max}$ over which the equivalence between the H-B and the M-C criteria is considered. The issue of determining $\sigma'_{3\max}$ depends on the specific application. There are no theoretically established methods for choosing this stress range. More specific guidance has been provided for selecting appropriate values of $\sigma'_{3\max}$ for tunnels and slopes (Hoek, 2002). However, no guidance has been provided for the case of surface foundations. To the authors' knowledge, there are no published solutions to estimate the ultimate bearing capacity values of a rock mass using equivalent M-C parameters. Most of previous published works (Serrano, 2000; Merifield, 2006; Clausen, 2013) indicate that using equivalent M-C parameters significantly overestimates the actual bearing capacity values. The primary focus of this paper is to provide specific guidance for selecting appropriate equivalent M-C parameters, which results in acceptable bearing capacity values and failure mechanism when compared to the original H-B failure criterion.

The paper is organized as follows. First, a bried introduction of the H-B criterion is provided. Then, the problem statement and the proposed method are respectively presented. Finally, a numerical study is conducted to compare numerical results to results obtained using the proposed method.

2 H-B FAILURE CRITERION

The H-B failure criterion (Hoek, 2002) is an empirical criterion developed through curve-fitting of triaxial test data. This criterion assumes isotropic rock and should only be applied to rock masses in which there is a sufficient number of closely spaced discontinuities. In other words, the H-B failure criterion is valid for intact rocks or heavily jointed rock masses (i.e. sufficient dense and randomly distributed joints).

The latest version of the H-B criterion (Hoek, 2002) is defined by:

$$\sigma'_1 = \sigma'_3 + \sigma_{ci}\left(m_b \frac{\sigma'_3}{\sigma_{ci}} + s\right)^a \tag{2}$$

Here σ'_1 and σ'_3 denote, respectively, the major and the minor principal stresses at failure. σ_{ci} is the uniaxial compressive strength of the intact rock. The strength parameters m_b, s and a describe the rock mass strength characteristics and depend on the Geotechnical Strength Index GSI, the disturbance factor D and the intact frictional strength component m_i. They are calculated as:

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right), \tag{3}$$

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right), \tag{4}$$

$$a = \frac{1}{2} + \frac{1}{6}\left(\exp\left(-\frac{GSI}{15}\right) - \exp\left(-\frac{20}{3}\right)\right), \quad (5)$$

On the other hand, the Uniaxial Compressive Strength (UCS) and the unaxial tensile strength are, respectively, obtained by setting $\sigma_3' = 0$ and $\sigma'_1 = 0$ with $\sigma_3' = \sigma_t$ in Eq. (2) giving:

$$UCS = \sigma_c = \sigma_{ci} s^a \quad (6)$$

$$\frac{\sigma_t'}{\sigma_{ci}} = -\left(m_b \frac{\sigma_t'}{\sigma_{ci}} + s\right)^a \quad (7)$$

Moreover, the deformation modulus E_{rm} can be expressed as follows (Hoek, 2002):

$$E_{rm}\,[GPa] = \begin{cases} \left(1-\frac{D}{2}\right)\sqrt{\frac{\sigma_{ci}}{100}}\,10^{\left(\frac{GSI-10}{40}\right)} & ;\sigma_{ci} \le 100\ MPa \\ \left(1-\frac{D}{2}\right)10^{\left(\frac{GSI-10}{40}\right)} & ;\sigma_{ci} > 100\ MPa \end{cases} \quad (8)$$

3 PROBLEM STATEMENT

In order to estimate equivalent M-C parameters, a direct comparison can be made by fitting an average linear relationship to the curve generated by plotting Eq. (2) for a range of the minor principal stress $\sigma_3' \in [\sigma_t, \sigma_{3\max}']$. A fitting process (Hoek, 2002) involves balancing the area above and below the M-C plot in the $\sigma_1' - \sigma_3'$ plane within the $[\sigma_t, \sigma_{3\max}']$ range (cf. Figure 1). This results in the following equations for the friction angle ϕ and cohesion c:

$$(a)\ \sin(\phi) = \frac{6am_b(s + m_b\sigma_{3n}')^{a-1}}{2(1+a)(2+a) + 6am_b(s + m_b\sigma_{3n}')^{a-1}}$$

$$(b)\ \frac{c}{\sigma_{ci}} = \frac{[(1+2a)s + (1-a)m_b\sigma_{3n}']\,(s + m_b\sigma_{3n}')^{a-1}}{(1+a)(2+a)\sqrt{1 + \frac{6am_b(s + m_b\sigma_{3n}')^{a-1}}{(1+a)(2+a)}}} \quad (9)$$

where $\sigma_{3n}' = \sigma_{3\max}'/\sigma_{ci}$

The issue of determining the appropriate value of $\sigma_{3\max}'$ for use in Eq. (9) depends on the specific application. There are no theoretically established methods for choosing this range. From experience and by trial and error, the following value is in general suggested (Hoek, 2002):

$$\sigma_{3max}' = 0.25\sigma_{ci} \quad (10)$$

More specific guidance was provided for selecting appropriate values of $\sigma_{3\max}'$ for tunnels and slopes (Hoek, 2002). However, no guidance has been provided for the case of surface foundations.

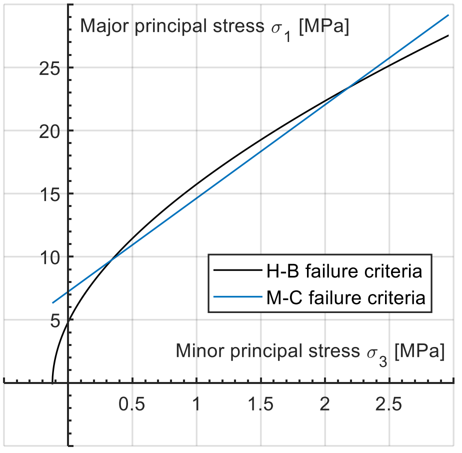

Figure 1: Mohr Coulomb and Hoek Brown failure envelope.

4 PROPOSED METHOD

The proposed method is initially established for a strip footing resting on weightless rock mass ($\gamma = 0$) and in the absence of surcharge load ($q = 0$). Then, the method is extended to include rock self weight for strip footing and circular footings, respectively

To properly conduct the study, it was felt necessary to use H-B core parameters ranges that cover practically all rocks. We have considered the following range for the strength parameters: $GSI \in [10,100]$, $m_i = \{7,10,15,17,25\}$ while D has been set to zero.

4.1 Strip footing (weightless rock)

In case of a strip footing resting on weightless rock with no surcharge load, Eq (1) reduces to

$$q_{ult}^{strip} = cN_c = c\,\frac{N_\phi e^{\pi \tan(\phi)} - 1}{\tan(\phi)} \qquad (11)$$

where $N_\phi = \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$.

Eq. (11) can further be written in non dimensional form, as:

$$\left(\frac{q_{ult}^{Strip}}{\sigma_{ci}}\right)_{\gamma=0} = \frac{c}{\sigma_{ci}}\frac{N_\phi e^{\pi.\tan(\phi)} - 1}{\tan(\phi)} \qquad (12)$$

Here, the $\phi - c$ values are calculated per Eq. (9) for a specific σ'_{3max} over which the equivalence between M-C and H-B criteria is considered.

Since the exact bearing capacity values for different H-B core parameters can be found in the literature (Merifield, 2006), Eq (12) may be solved for σ'_{3max}. Surprinsigly, no such solution exists. This may be due to the fact that the minor principal stress field beneath the foundation has stress values starting from zero and that the linear fit proposed by Eq. (9) overestimates the H-B failure envelope close to $\sigma_3 = 0$. To tackle this problem, we proceeded as follows. Firstly, the σ'_{3max} has been chosen such that the difference is minimized between bearing capacity values evaluted per Eq. (12) and those published in litterature (Merifield, 2006). It was found that $\sigma'_{3\max}$ varies as a function of UCS (Eq. (6)), GSI and m_i as follows:

$$\sigma'_{3max} = \frac{\sigma_c}{100}(1.83m_i + 34.02) \qquad (13)$$

Secondly, the friction angle evaluated per Eq. (9) was downgraded such that the bearing capacity would match published solutions within 1% (Merifield, 2006). The factor adjusting the friction angle is defined as $f_\phi = \phi_{adj}/\phi$ (< 1). It is a function of GSI and m_i:

$$f_\phi = exp\left[\frac{1}{100}.\left\{b_1.exp\left(b_2.\frac{GSI}{100}\right) - b_3.exp\left(b_4.\frac{GSI}{100}\right)\right\}\right] \qquad (14)$$

Here b_j; j=1,2,3,4 are parameters correlated to m_i using a power relation of the form $b_j = p_j.m_i^{\frac{q_j}{100}} + t_j$; j=1,2,3,4. Herein p_j, q_j and t_j are constants given in Table 1.

To sum up, the following steps should be followed to assess the bearing capacity of a strip footing resting on a weighless rock masses:
(1) Choose the H-B imput parameters.
(2) Calculate σ'_{3max} per Eq.(13)
(3) Calculate M-C parameteres per Eq.(9)
(4) Calculate the adjusted friction angle ϕ_{adj} per Eq.(14)
(5) Evaluate the bearing capacity per Eq.(12)

The calculation process can be easily programmed in an Excel spreadsheet.

Table 1: Power relation constants

	$j = 1$	$j = 2$	$j = 3$	$j = 4$
p_j	818.40	7.59	29.80	−17.50
q_j	−0.76	−162	−70.60	−53.90
t_j	−826	−0.52	−18.40	−1.59

4.2 Strip footing – Extension of the method to include rock self-weight

In this section, the previous case is extended to include the effect of rock self weight on bearing capacity values. In this case, the following bearing capacity equation (normalized) should be used instead of Eq. (12):

$$\left(\frac{q_{ult}^{Strip}}{\sigma_{ci}}\right) = \left(\frac{q_{ult}^{Strip}}{\sigma_{ci}}\right)_{\gamma=0} + \frac{1}{2}\frac{\gamma B}{\sigma_{ci}} N_\gamma \quad (15)$$

While the expression of N_γ remains unknown, the values for N_c as given per Eq.(11) is generally agreed upon. It is found that the bearing capacity factor N_γ varies widely between authors and N_γ published in (Baars, 2015) which is derived from FEM analysis for rough and smooth footings, gives more conservative values and will be adopted, in this paper. It is expressed as follows:

$$N_\gamma = 2e^{\pi \tan(\phi_{adj})} \tan(\phi_{adj}) \quad (16)$$

To study the effect of rock self-weight on bearing capacity values, the non dimentional factor $\frac{\sigma_{ci}}{\gamma . B}$ belonging to the following range [125, 250, 500, 1000, 2000, 5000, 10000, ∞] is used. A value of $\frac{\sigma_{ci}}{\gamma . B} = \infty$ means that the rock is weightless. For example, the normalized bearing capacity values evaluated per Eq. (15) are depicted in Figure 2 for $GSI = 30$ and 60. A general observation is that the evolution of $\left(\frac{q_{ult}^{Strip.f}}{\sigma_{ci}}\right)$ is almost linearly dependent on m_i and becomes more and more independent of the rock self-weight when GSI increases. Indeed, for $GSI > 60$, the ratio $\frac{\sigma_{ci}}{\gamma B}$ has almost no effect on bearing capacity values.

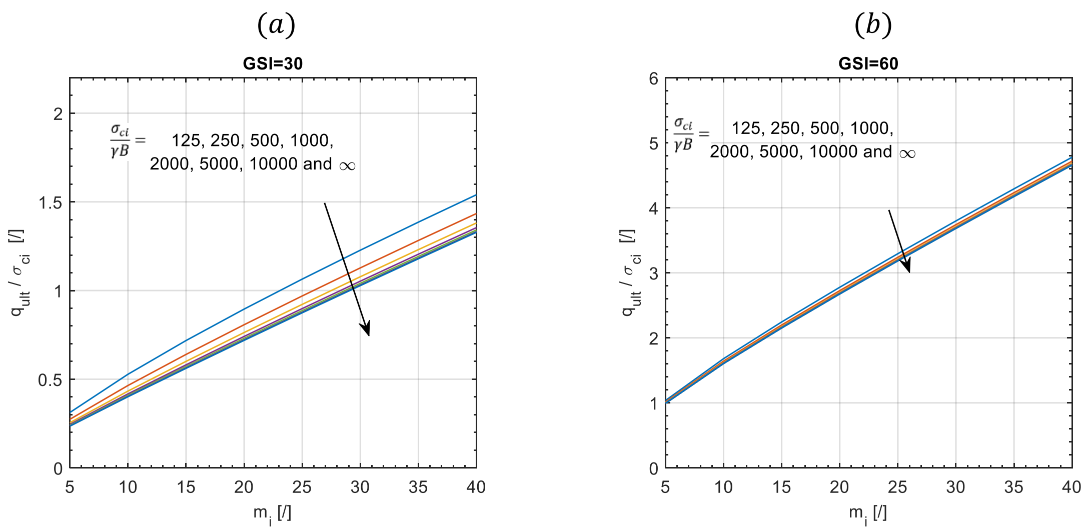

Figure 2: Normalized bearing capacity as a function of mi for: (a) GSI =30 / (b) GSI=60

4.3 Extension of the method to include circular footing

A shape factor denoted s_c can be introduced to take into account the circular shape of the footing. Thus, the bearing capacity expression is formulated, in non dimensional form, as follows:

$$\left(\frac{q_{ult}^{Circular}}{\sigma_{ci}}\right)_{\gamma=0} = s_c \left(\frac{q_{ult}^{strip}}{\sigma_{ci}}\right)_{\gamma=0} \quad (17)$$

A series of analyses were performed by comparing bearing capacity values evaluated per Eq. (17) using $\phi_{adj} - c$ pair and those published in litterature (Clausen, 2013). This analysis has led to the following shape factor formula:

$$s_c = 1 + \left[0.22 \tan\left(\frac{\pi}{4} + \frac{\phi_{adj}}{2}\right)^{1.4}\right] \frac{B}{L} \quad (18)$$

where B and L are, respectively, the foundation width and the foundation length. This factor goes to 1 for $L \rightarrow \infty$ (case of strip footing). In case of circular footing, $B = L$ and Eq. (18) is simplified to:

$$s_c = 1 + \left[0.22 \tan\left(\frac{\pi}{4} + \frac{\phi_{adj}}{2}\right)^{1.4}\right] \quad (19)$$

5 EXEMPLES AND COMPARISON

In order to validate the proposed method, three different quality rock masses; namely, poor rock, medium rock and very good rock are considered. Table 2 summarizes the H-B input parameters as well as the equivalent M-C parameters of each rock mass.

Table 2: Different quality rock masses –M-C parameters

Rock quality	H-B input parameters (D= 0)		M-C equivalent parameters per Eq. (9) and (14)	
	GSI	m_i	$\frac{c}{\sigma_{ci}}$ ($\times 10^3$)	Φ_{adj} [°]
Very poor	30	7	3.80	40.51
Medium	50	15	16.59	41.16
Very good	80	25	105.46	39.00

5.1 *Strip footing (weightless rock)*

The bearing capacity values of a strip footing resting on the three different quality rock masses (weightless) are calculated per Eq. (12). Results are summarized in Table 3. It can be noted that these values are in excellent agreement compared to published results (Merifield, 2006).

5.2 *Strip footing (self weight rock)*

When considering the rock self weight and by comparing charts of the present work (cf. Figure 2) to published charts (Merifield, 2006), it was concluded that curves presented the same tendency but bearing capacity values for $GSI < 30$, $\frac{\sigma_{ci}}{\gamma.B} < 500$ and $m_i > 10$ remain 10% below values graphically read from published charts (Merifield, 2006) i.e.,

$$0.9 q_{ult}^{(\text{Merifield,2006})} < q_{ult}^{Eq.(15)} < q_{ult}^{(\text{Merifield,2006})}$$

This is essentially due to the conservative choice of bearing capacity factor N_γ (Eq. (16)).

Table 3: Bearing capacity – Strip foundation (weightless rock).

Rock Quality	q_{ult}/σ_{ci}		
	Eq.(12)	(Merifield, 2006)	Error (%)
Very Poor	0.303	0.300	+1.0
Average	1.415	1.400	+1.0
Very Good	7.142	7.150	−0.1

5.3 *Circular footing (weightless rock)*

The bearing capacity of a circular footing resting on the three different quality rock masses are calculated per Eq. (17). Results are summarized in Table 4. It can be seen that results are in excellent agreement compared to published results (Clausen, 2013).

Table 4: Bearing capacity – Circular foundation (weightless rock).

Rock Quality	q_{ult}/σ_{ci} [/]		
	Eq. (17)	(Clausen, 2013)	Relative difference (%)
Very Poor	0.500	0.495	+0.9
Medium	2.354	2.319	+1.5
Very Good	11.568	11.612	+0.4

6 NUMERICAL VALIDATION

In this section, bearing capacity of surface foundations resting on the three different quality rock masses (cf. Table 2) are numerically investigated using the geotechnical analysis software RS2. Recalling that published solutions (Serrano, 2000; Merifield, 2006; Clausen, 2013) are based on the assumptions of elastic perfectly plastic material behavior and associated flow rule, the same assumptions were assumed in the numerical model.

6.1 Strip footing (weightless rock)

The soil (rock layer) was first modeled using the H-B model. The model uses six parameters, which consist of elastic parameters (E, ν) and the H-B core parameters $(\sigma_{ci}, m_i, GSI\ and\ D = 0)$. Then, the mechanical response of the soil was modelled according to the M-C model and the following peak parameters were defined: cohesion (c), friction angle (ϕ) and tensile strength (σ_t). It should be noted that ν was fixed to 0.2 and E is calculted per Eq.(8).
The geometry details of the model are as shown in Figure 3. A graded mesh of 2380 triangular elements was used. It should be noted that the mesh was refined beneath the foundation.

The materials properties for the three differents quality rock masses are as given in Table 2.
Load-displacement curves for poor, average and hard rock are depicted in Figure 4. It can be noted that a good agreement between the two model solutions are demonstrated and peak stresses almost equal to values summarized in Table 3 were reached.

6.2 Circular footing (weightless rock)

The numerical model type is set as an axisymmetric. Load-displacement curves for poor, average and hard rock are depicted in Figure 5. Peak stresses almost equal to the values summarized in Table 4 were reached which confirms the accuracy of the proposed method.

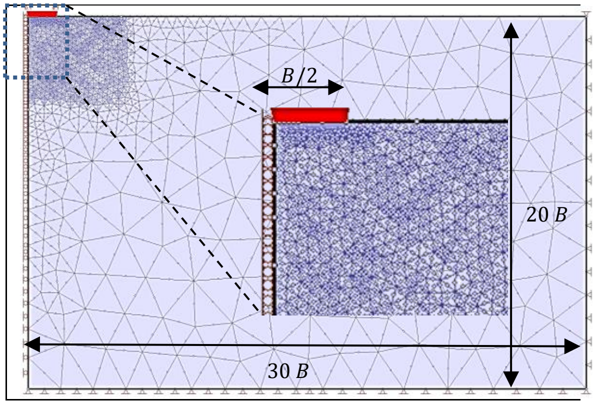

Figure 3: Boundary conditions, mesh and geometry details of the numerical model.

7 CONCLUSION

The bearing capacity of surface foundations resting on H-B material has been investigated. For a specific upper limit of confining stress over which the correspondence between the H-B and the M-C criteria has been considered, equivalent M-C parameters have been suggested using an adjustment friction angle factor. The following conclusions can be made:

(1) Results obtained from the proposed approach fit within 1% published solutions in case of a strip footing resting on weightless rock masses.
(2) Ignoring rock self-weight leads to a potentially very conservative estimate of the ultimate bearing capacity. This is especially the case for poor rock where the underestimation can reach 60%. This is particularly true for poor rock type with GSI < 30.
(3) The proposed method is also valid for estimating the bearing capacity of circular footing resting on weightless rock via shape factor. Bearing capacity values are in good agreement compared to published results (Clausen, 2013) within 1.5 %.
(4) The comparison of proposed approach with results from finite element limit analysis validates the accuracy of the proposed method.

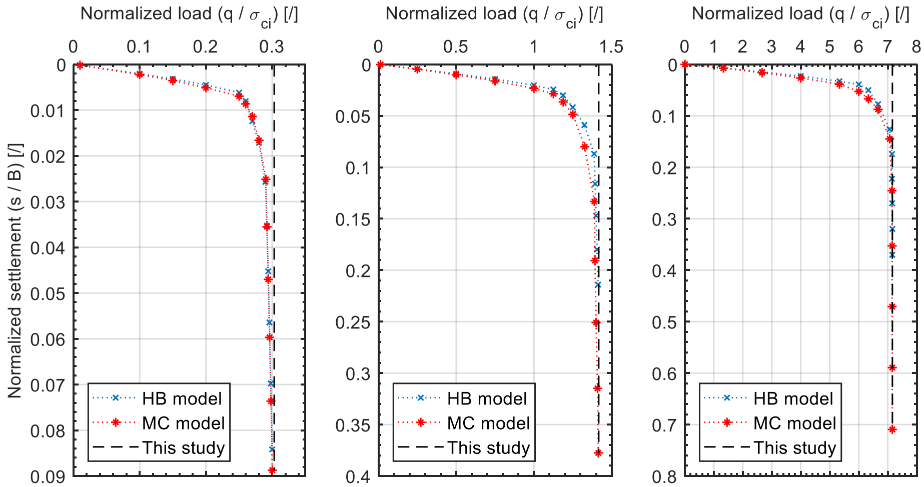

Figure 4: Load-displacement curves – Strip footing resting on poor, average and hard weightless rock.

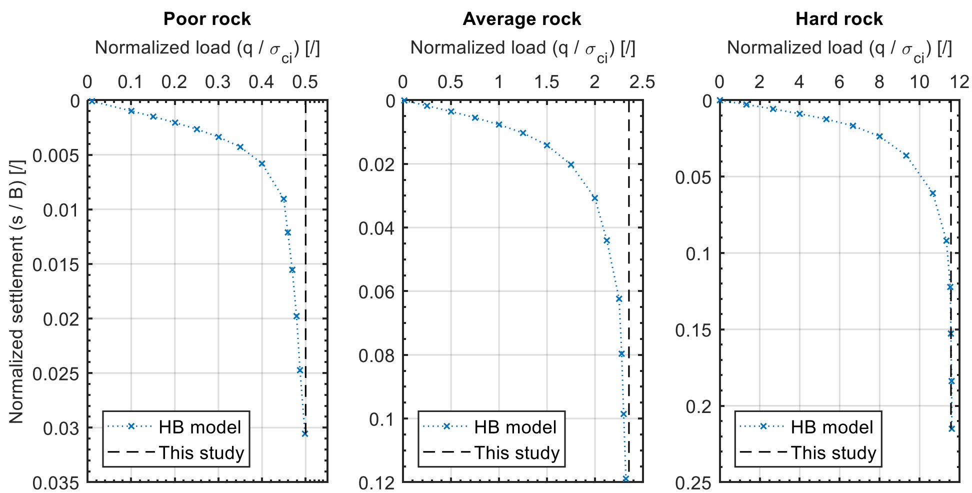

Figure 5: Load-displacement curves – Circular footing resting on poor, average and hard weightless rock

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