Artificial Intelligence Alter Egos: Who benefits from Robo-investing?*

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July 6, 2019

Abstract

Artificial intelligence, or AI, enhancements are increasingly shaping our daily lives. Financial decision-making is no exception to this. We introduce the notion of AI Alter Egos, which are shadow robo-investors, and use a unique data set covering brokerage accounts for a large cross-section of investors over a sample from January 2003 to March 2012, which includes the 2008 financial crisis, to assess the benefits of robo-investing. We have detailed investor characteristics and records of all trades. Our data set consists of investors typically targeted for robo-advising. We explore robo-investing strategies commonly used in the industry, including some involving advanced machine learning methods. The man versus machine comparison allows us to shed light on potential benefits the emerging robo-advising industry may provide to certain segments of the population, such as low income and/or high risk averse investors.

*The authors are grateful to the online brokerage house for providing the data. Any errors are the full responsibility of the authors. We thank conference participants at the Chinese Economists Society, Dalian, the 2019 SoFiE Annual Meetings, Shanghai, and seminar participants at HEC Liège, the Toulouse School of Economics, Georgia State University, UC San Diego, University of Chicago, and the University of Southern California for helpful comments.

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1 Introduction

To assess the benefits of robo-investing we use a unique data set covering brokerage accounts for a large cross-section of 22,972 individual investors covering a sample from January 2003 to March 2012, and therefore includes the 2008 financial crisis. We have records of all trades, and in addition have detailed information about each individual investor's characteristics such as age, gender, education, annual net income, and most importantly, risk aversion assessed on the basis of responses to survey questions. Although we work with Belgian individual investors, most of their trading activities pertain to foreign stocks (86% are non-Belgian and roughly a quarter are US). Hence, our analysis pertains to international portfolio selection of stocks and ETFs.

To the best of our knowledge there has not been any assessment of the potential benefits of robo-investing over a long period of time for a heterogeneous panel of individual investors. We explore robo-investing strategies commonly used in the industry, including some involving advanced machine learning methods. The man versus machine comparison allows us to shed light on potential benefits the emerging robo-advizing industry may provide to certain targeted segments of the population, such as low income and/or investors with relatively little financial literacy.[1]

Our sample has a number of appealing features to study robo-investing. Many investment brokerage firms are now targeting individuals with modest savings as it is generally believed that smaller investors don't get the investment advice they need. In fact, 71% or almost 90 million American families have investment account balances worth less than $100,000. The growth of automated investment advisory services is filling a need for such investors.

[1] In the US, robo-advisor start-ups saw an eight-fold increase in their AUM in recent years on the back of some retirement savings shifting to robo-advisor accounts. Cost advantages have been creating significant momentum for the industry. In addition, the success of passive investment strategies in recent years has also been beneficial. It is therefore fair to say that robo-advisors are posing a challenge to traditional financial advisory services. One expects that some robo-advisory start-ups will probably end up in partnerships or be the subject of takeovers by established asset management firms or banks in the coming years. Moreover, the traditional asset managers themselves are also adopting robo-investing strategies. In that respect, robo-advising will become more mainstream.

Our data set consists of individual investors typically targeted by robo-advising. In terms of annual net income, approximately 70% of the investors in our sample declare an income between 20,000 and 75,000 euros. The mean portfolio value in our sample is 29,244 euros and the average investor is about 48 years old.

Note that our paper does not directly address the effect on wealth management of adopting robo-advising, as studied by for example D'Acunto, Prabhala, and Rossi (2019). On the one hand, our data is richer in terms of details regarding the characteristics - such as income, education, gender, risk aversion, trading habits - for each individual investor. On the other hand, we study a sample where robo-advising was not adopted by the brokerage firm whose trading data we examine. Instead, we introduce the idea of shadow robo-investors to assess the potential benefits of robo-advising. Namely, we study various robo-investors that shadow the individuals in our data set and the novelty of our approach is that we know what the investors have done in reality versus what a robo-investor would have done instead. In that sense our analysis is a real-time experiment with real data.

Robo-investors are limited to the set of stocks and ETFs in each individual investor's history of trading - using a rolling 2-year sample.[2] This constraint ties each robot to a specific investor in our sample via their trading history. Note that the robo-investors use *all* the stocks/ETFs individual investor i held in the past two years, but may have sold in the meantime. Hence, the rationale is that the investor knows about the stocks/ETFs held by the shadow robo-investor. We call these shadow robo-investors Artificial Intelligence Alter Egos, or AI Alter Egos.[3]

The notion of AI Alter Egos is not unique to finance, although we might be the first to coin the term. To illustrate, let's look at machine learning (ML) advances in other fields, such as literature and music. Today, a ML text mining algorithm can analyze the writings of a famous author and create entirely new literature in the style of the writer it was exposed to

[2]The majority of trading occurs in either equity or ETFs as described in detail in the Appendix.

[3]Since the robo-investor schemes go beyond machine learning, as they involve portfolio allocation rules, we use the more general term of artificial intelligence. In our case the AI pertains a set of computer-driven self-learning rules which determine portfolio allocations.

and trained on. The same can be done with music. For example, Franz Schubert started his Unfinished Symphony in B minor in 1882, but wrote only two complete movements, though he lived another six years. Now, deep-learning ML has produced a completed version of the entire symphony. We can characterize this as Schubert's AI Alter Ego composing a new score. Would Schubert have done better than his AI Alter Ego? We prefer to leave that debate to the musicologists, but it's fair to say it would probably be hard to address the question. Fortunately, it's much easier to apply the notion of AI Alter Egos in a setting where comparing the outcomes of human and AI alternatives is more straightforward – such as in financial investments.

We consider three investment strategies. Two are based on a Markowitz (1952) mean-variance (MV) scheme and a third is based on DeMiguel, Garlappi, and Uppal (2007) involving the $1/N_{it}$ scheme where N_{it} is the number of stocks held by investor i over a 2-year trailing sample up to time t. The two MV strategies differ in terms of the sophistication regarding the conditional mean and variance estimates. The first involves two-year rolling sample estimates for both the mean and variance. For the second we rev up the robot engines and replace the rolling sample estimators by respectively expected return predictions using machine learning algorithms and sophisticated conditional covariance estimators. More specifically for the conditional mean we use Elastic-Net, Random Forest, Neural Network, and model ensemble estimators. For the conditional covariance matrix - looking at a total of 683 stocks and 393 ETFs - we use the Engle, Ledoit, and Wolf (2019) nonlinear shrinkage method derived from random matrix theory to correct in-sample biases of sample eigenvalues. Finally, it is important to note that robo-investors have the option to hold cash, i.e. decide to avoid market risk exposure. No short selling is allowed, however.

We study three rebalancing schemes: once a year, quarterly and monthly. In the main body of the paper we focus exclusively on the quarterly rebalancing scheme. Note that robo-investors buy and hold at fixed sampling frequencies - end of quarter in the lead example. This is in contrast to the individual investors in our sample who execute their trades at any point in time.

Overall our findings are as follows. The AI Alter Ego robo-investors involving equal weighting or rolling sample mean and variance estimates perform poorly and are of little value to any of our investors.[4] In contrast the machine learning MV AI Alter Egos result in significant investment portfolio performance improvements for certain types of investors. In particular, those featuring high risk aversion benefit greatly from following the robo-investor strategies. Low income (low education) investors typically also gain from the AI advise. These results confirm the claims made by practitioners in the industry regarding the promises the use of AI hold for the future of the FinTech industry. More intriguing, and somewhat unexpected are our results pertaining to the performance during the financial crisis. Robo-investors outperform a large swath of investors. In fact, the median robo-investor moves into cash (because of negative expected returns using AI) whereas individuals feature behavioral biases, such as the disposition effect (cfr. Odean (1998)) with unfortunate consequences during the onset of the financial crisis.

As a by-product of our analysis, we also identify which machine learning methods perform well. While deep learning is often the best across a large cross-section of stocks, a close second-best is a much simpler linear prediction model with elastic net penalty based on the same set of predictor, namely those suggested by Welch and Goyal (2007), which consist of a mixture of firm-specific and macroeconomic covariates. Put differently, the gains from non-linear models is marginal at best.

The paper is organized as follows. In section 2 we describe the brokerage data, with some of the details appearing in Appendix A.[5] Section 3 describes the various robo-investor schemes. Section 4 reports the empirical results. Section 5 concludes the paper.

[4]Note that among the existing robo-investor practices there are number which proclaim using MV allocations and most likely use some type of rolling sample scheme - although most white papers are rather vague on the actual implementation.

[5]Further details also appear in the Online Appendix.

2 A Large Panel of Individual Brokerage Accounts

Our primary data set comes from a large Belgian online brokerage firm and consists of the trading accounts of 22,972 individual investors. This unique data spans about 10 years from January 2003 to March 2012, and therefore includes the 2008 financial crisis. We have detailed information about each trade, such as the instrument, the time-stamp, the trade direction, the executed quantity, the trade price, and explicit transaction costs. The details of the data are described in Appendix A. We focus on common stock investments as well as ETFs and exclude other financial instruments.[6] Trading of ETFs, mutual funds, options and warrants is more prevalent with high income/education investors. Trading of bonds is overall insignificant. Because we examine robo-advisors which are mean-variance investors we focus exclusively on stocks and ETFs which best fit the portfolio allocation model. For high income/education investors in particular this means we leave out to a certain degree other assets which we have available. After applying some filters described in the Appendix, we end up with a sample of 1,590,199 (stocks) + 60,344 (ETFs) = 1,650,543 trades (and more than 13 billion euros traded in stocks and close to 1 billion euros in ETFs) over the 111-month period covering 683 stocks and 393 ETFs or 70% of all the investors' trading activity.

Using the trading data we build end-of-month portfolios for each investor and use historical market data to compute monthly portfolio market values. We also compute both monthly and daily returns. Combining end-of-month portfolio market values with the corresponding monthly aggregate cash-flows, we calculate for each investor 110 (i.e. from February 2003 to March 2012) monthly portfolio gross and net returns (the latter net of transaction costs). Investors are included as robo-investor candidates if they satisfy the return criteria - sufficient time periods and returns with no extraordinary outliers - and minimum trade restrictions (see Appendix A for further details). In particular, we drop investors with more

[6]In Appendix A we document that 6,741 investors also traded options and warrants with an aggregate number of 602,833 trades and 6,665 investors traded mutual funds with an aggregate number of 260,120 trades. Only a few investors (i.e. 1,813) traded bonds with an aggregate number of 5,999 trades.

than 106 missing values in their return series (i.e. at least 4 months of returns are needed to keep an investor) and drop outliers as well. This decreases the sample to 20,622 investors (down from 22,972). To simplify the analysis we do not take into account transaction costs (which are available for each trade) neither for the individual investment accounts nor for the robo-investor ones. Since robo-investors trade less than the average/median investor, namely only once a quarter in the lead example, this should yield conservative estimates of the robo-investing gains.

Our data set also includes an extensive set of individual investor characteristics, such as age, gender, education, annual net income and a risk aversion measure based on surveys (described in Appendix section A.1). Although we work with Belgian individual investors, most of their trading activities involve foreign stocks (mainly the US and bordering countries France and Germany - see Appendix Table A.3 for details). The majority of stocks pertain to the technology sector (16.93%), financials (15.91%), and industrials (14.01%).

As noted in the Introduction, our data set consists of individual investors typically targeted by robo-advising. We have about 70% of the investors in our sample who declare an annual net income between 20,000 and 75,000 euros. Only a minority (3.36%) earns more than 150,000 euros per year.[7] The average investor is about 48 years old and executes monthly 2.76 trades across 2.05 different stocks for a volume of 18,237 euros. Consistent with the literature, investors in our sample are under-diversified; the average (median) investor holds a five-stock (three-stock) portfolio. The average end-of-month portfolio value is about 28,003 euros (with a median value of about 7,552 euros). As for risk aversion, the majority of investors seem to be risk tolerant since 65.33% of them declare a medium risk aversion and 27.88% of them even a low risk aversion.

In terms of performance, our investors earn an average monthly gross return of 0.42% on stocks and ETFs (median return of 0.13% - see Table A.9 in the Appendix), with a volatility

[7] The income measure reported in our data is recorded once, when the investor completed the MiFID tests. The classification may therefore be noisy over the 10 year sample period, particularly for the early entries.

of 10.04%. This high average volatility of individual portfolio gross returns is not surprising given our sample period includes turbulent market conditions.[8]

3 Robo-Investors

The robo-investors are limited to the set of stocks and ETFs in each individual investor's history of trading - using a rolling 2-year sample. This constraint ties each robot to a specific investor in our sample via their trading history. We call these shadow robo-investors Artificial Intelligence Alter Egos. Robo-investors have the option to hold cash, i.e. decide to avoid market risk exposure, but no short-selling occurs in our sample nor is it allowed for in the design of the robots. The two-year window is arguably somewhat arbitrary. Our results hold for longer windows. Shorter windows are less appealing given the trading frequency of many investors, with only a median of 2 trades per month (see Table A.9 in the Appendix). The portfolio allocations of robo-investors occur at fixed intervals, either monthly, quarterly or annually. In the main body of the paper we focus exclusively on the quarterly results.[9]

3.1 AI Alter Egos

We construct three types of AI Alter Ego robo-investors. As we already noted, each setting only uses stocks and ETFs held by an individual investor over the past two years, not the entire universe of stocks. The table below provides two illustrative examples. When we refer to t, we mean end of the year, or quarter or month, depending on the case being considered.

[8]As detailed in the Appendix, to calculate portfolio returns, we opt for an approximation of the Modified Dietz Method, aiming at delivering a return close to the money-weighted rate of return (e.g., Shestopaloff and Shestopaloff (2007)).

[9]In the Online Appendix, the monthly and detailed quarterly results are reported. The annual results are available on request. In the computations of returns we ignore transaction costs. Since our focus is quarterly trading frequencies this is a reasonable abstraction. The monthly robo-investor results are arguably more suspect of being overstated because transaction costs are not accounted for.

Initial holdings	Trading t - 1	Trading t	Investor holdings	Robo-investor potential holdings
Stocks 1 & 2	Sells all of 2	Buys stock 3	Stocks 1 & 3	Stocks 1, 2 & 3
Stock 1	Sells all of 1	Buys ETF 4	ETF 4	Stock 1 & ETF 4

The first line portrays an investor holding two stocks - say 1 and 2 - at time t - 2 (column called Initial holdings). At the end of t - 1, the investors sells all holdings of stock 2 and at the end of the subsequent period t buys stock 3. Hence, at the end of t she/he holds stocks 1 and 3. The robo-investor has stocks 1, 2 and 3 to form a portfolio. The second case is similar, but the investor only holds stock 1, sells all of it in t - 1 and buys ETF 4 in t. The robo-investors has two assets to select from. It is important to stress that the robo-investor may hold cash, i.e. decide not to put all the money in the stock market. This will be important as will become clear when discussing the empirical results.

To proceed, we need to introduce some notation. Let S_{it} be the set of stocks/ETFs investor i held over a two-year period up to time t. The above illustrative examples clarified that this does not mean that the investor holds these stocks/ETFs at the end of year/quarter/month t. It only means that the investor held these stocks/ETFs in the recent two-year history. We denote by T_i the duration of time (months/quarters/years whichever applies) investor i appears in the sample. Moreover, we denote by $N_{it} = \#S_{it}$, the number of stocks/ETFs in the set. We only consider investors with $N_{it} \geq 2 \ \forall \ t = 1, \ldots, T_i$. This ensures that the investment opportunity set contains a minimally sufficient set of stocks/ETFs for the robo-investors. This leaves us with 20,622 investors who satisfy this criteria and are included in our analysis.

The robo-investors buy at the end of t and hold until end of $t + 1$, i.e. for a month, quarter or full year.[10] We then compute holding period returns for the robo-investor, $r^{ae}_{i,t+1}$,

[10]In between rebalancing periods, the portfolio weights adjust according to the performance of an individual

and compute the alter-ego-less-investor's realized return spread as $r^s_{i,t} = r^{ae}_{i,t} - r_{i,t}$.

The first type shadows each individual investor in our sample using the DeMiguel, Garlappi, and Uppal (2007) equal weighting rule, the second and third rely on a mean-variance Markowitz (1952) strategy with a short-sale constraint. The difference between the second and third variations is the sophistication of expected return and risk estimators. In the second approach a simple rolling sample estimator is involved for expected returns and the linear shrinkage estimator of Ledoit and Wolf (2004) for second conditional moments. In the third case, machine learning and conditional covariance estimators are used. More specifically for the conditional mean we use Elastic-Net, Random Forest, Neural Network, and model ensemble estimators. For the conditional covariance matrix we use the Engle, Ledoit, and Wolf (2019) nonlinear shrinkage method derived from random matrix theory to correct in-sample biases of sample eigenvalues.

Equal Weights We endow the robo-investor with a DeMiguel, Garlappi, and Uppal (2007) $1/N_{i,t}$ strategy. In particular, for each individual i, the Alter Ego buys and holds at time t all the stocks in the set S_{it} with equal allocations $1/N_{it}$. Henceforth we will refer to this as the EW portfolio rule.

Rolling Sample Markowitz The mean-variance optimal portfolio is constructed as the maximum Sharpe ratio subject to the short-sale constraint and the individual's investment opportunity set. Investor i selects from the set S_{it} of stocks. Critical to the optimal portfolios are estimates of conditional expected returns (μ^i_t) and the conditional covariance matrix of returns (Σ^i_t) for the stocks in the set S_{it}. The robo-investor solves for $\hat{w}_{i,t}$ selecting among

asset relative to the performance of the portfolio as a whole. In particular when $t+1$ is not a rebalancing period, $w_{i,t+1} = w_{i,t}(1 + r_{i,t+1})/[\sum_{i=1}^{N} w_{i,t}(1 + r_{i,t+1})]$

these stocks according to:

$$\max_{w_{i,t}} w'_{i,t}\mu_t^i - \frac{\gamma}{2} w'_{i,t}\Sigma_t^i w_{i,t} \tag{1}$$

$$\text{s.t. } w_{i,t} \geq 0, \tag{2}$$

where γ is often interpreted as a risk aversion parameter which we set equal to one as it maximizes the Sharpe ratio. We estimate μ_t^i with two-year rolling-window historical averages, $\hat{\mu}_t^i = \frac{1}{k}\sum_{j=0}^{k-1} r_{t-j}^d$, where r_t^d is an $N_{it} \times 1$ vector of daily returns and k is the number of days in the two-year historical sample. For covariance, we also use rolling sample estimator with linear shrinkage as in Ledoit and Wolf (2004) based on daily returns over the same time span. These estimates form our naive benchmark.

Machine learning and Shrinkage Continuing with the Markowitz allocation scheme, we explore whether increasing the complexity of the rolling sample estimators translates into improved robo-investor performance. We assume that each investor's Alter Ego robo-investor has access to a common set of models that replace the rolling sample schemes. For expected return predictions we use machine learning algorithms applied to each of the 1076 assets (683 stocks and 393 ETFs) and the Alter Ego robo-investor picks the prediction pertaining to the stocks in the sets S_{it}. More specifically for the conditional mean estimates we use Elastic-Net (Zou and Hastie (2005)), Random Forest (Breiman (2001)), Neural Network (Friedman, Hastie, and Tibshirani, 2016, Chap. 11), and model ensemble estimators (Friedman, Hastie, and Tibshirani, 2016, Chap. 16). For the conditional covariance matrix - looking at a total of 1076 assets - we use the Engle, Ledoit, and Wolf (2019) nonlinear shrinkage method derived from random matrix theory to correct in-sample biases of sample eigenvalues.

3.2 Machine Learning Expected Returns

What we have in mind is a situation where the robo-investors rely on a modeling department within the brokerage house to provide them with estimates of conditional means and conditional covariances for the entire universe of stocks/ETFs and supplying the Alter Ego investor associated with each individual investor with the estimates μ_t^i and Σ_t^i for the stocks in the set S_{it}. The modelers estimate a wide class of models and use out-of-sample performance metrics to determine the most appropriate panel of conditional means and conditional covariances to supply to the robo-investors. Our goal here is to provide a simple approximation to the comprehensive conditional modeling process that such a brokerage research group would undertake. In terms of expected returns models our analysis shares some of the methods also considered by Gu, Kelly, and Xiu (2018).

For the purpose of our analysis, let $r_{i,t-} = (r_{i,t}, \cdots, r_{i,t-k+1})'$ be the $k \times 1$ vector of own-lagged stock returns for stock i. We have $N = 1076$ stocks/ETFs to consider and $T = 110$ monthly periods. We use 70% of the data for training, 20% of the data as a validation sample (for hyperparameter tuning), and 10% of the sample for testing out-of-sample performance. To maximize the use of our unique data set, we start building our models using returns data from January 1993 to December 2002 - namely a 10-year sample prior to the start of our individual investor data.

We augment the panel of monthly stock/ETF returns with the five Fama-French monthly factors (Mkt, SMB, HML, RMW, CMA) as well as their momentum factor (see Ken French website for definitions), and Welch and Goyal (2007) predictors: div. price ratio, div. yield, earnings price ratio, div. payout ratio, stock variance, BM DJ stocks, net equity expansion, TBill, long-term yield, term spread, default yield spread, inflation (see their paper for definitions). We understand that a true data engineering group would likely create a much larger and more robust set of data sources. Our goal is not to replicate the true data-source generating process, but to provide a simple approximation to the set of all useful signals for prediction. Let x_t represent an $M \times 1$ vector of these predictors.

In each model class we estimate individual models for each stock/ETF separately, rather than pooling across stocks/ETFs, in order to allow as much heterogeneity as possible in model parameter estimates. The common modeling objective is to estimate $E_t[r_{i,t+1}]$, where $E_t[r_{i,t+1}] = f_i(z_{i,t})$. The modelers therefore employ different approaches to estimate $f_i()$, and also work to curate the best possible set of covariates $z_{i,t}$.

We separate our conditional mean models into (1) linear and (2) nonlinear model sets. Within the linear models, we consider OLS and elastic-net models. For nonlinear models we consider random forests of regression trees and shallow feed-forward neural networks. Hence, we consider two popular nonparametric and parametric machine learning models designed to introduce nonlinear interactions between covariates: random forests of regression trees (nonparametric) and artificial neural networks (parametric). Finally we consider a simple model ensemble across all models.

Linear Models The linear models we estimate for each stock i across time periods $t = k, \ldots, T-1$ are of the form:

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,r} r_{i,t} + \beta_{i,x} x_t + \epsilon_{i,t+1} \tag{3}$$

In addition to estimating this model with OLS, we fit sets of linear models per stock i using Elastic Net involving two tuning parameters (α, λ) that we optimize over the validation sample.

$$\mathcal{L}_i(\theta) = \frac{1}{T-k} \sum_{t=1}^{T} \epsilon_{i,t+1}^2 + \alpha\lambda \sum_{m \in \#\beta} |\beta_m| + \frac{1}{2}(1-\alpha)\lambda \sum_{m \in \#\beta} \beta_m^2 \tag{4}$$

where $\beta = (\beta_{i,0}\beta_{i,r}\beta_{i,x})'$

Nonlinear Models We consider two popular nonparametric and parametric machine learning models designed to introduce nonlinear interactions between covariates: random forests of regression trees (nonparametric) and artificial neural networks (parametric). We employ the algorithm of (Breiman (2001)) to estimate random forest models and we use stochastic gradient descent to minimize an ℓ_2 objective function with regularization terms in order to train the neural networks. In both cases our estimation techniques are standard. Again we estimate the model on the training data and optimize all respective tuning parameters on the validation set.

Random Forest A random forest is a combination of individual regression trees. It is a bootstrapping method that seeks to avoid both overfitting and decrease correlation among trees by using random subsets of predictors at each branch of a given tree. Each tree can be classified as having K terminal nodes (called "leaves") with a depth of L. The prediction of a given tree then can be stated as:

$$h(z_{i,t}; \beta, K, L) = \sum_{k=1}^{K} \beta_k 1\{z_{i,t} \in P_k(L)\} \tag{5}$$

where $P_k(L)$ is the k-th partition that has at most L different branches that it considers. A set of branches for a given partition can be represented as a product of indicators for sequential branches. For a given partition, then $\hat{\beta}_k$ is the average of the returns for all members of that given partition. A standard greedy search algorithm is used to maximize the information gained at each split. The recursive binary splitting algorithm continues until a set of stopping criterion are met, which typically rely on the maximal additional information gained from a split being less than a threshold, or a max number of leaves and/or depth of a tree being reached.

For the random forest models, the key tuning parameters are the number of bootstraped trees, the depth of each tree, and the random subset of predictors that are considered at each potential split within a tree. The random forest prediction is then the bootstrapped average at any prediction point across trees.

Neural Network Our neural network architecture is two hidden layers with 10 neurons per layer, sigmoid transfer functions in the input and hidden layers, and a linear transfer function in the output layer. We use stochastic gradient descent to minimize an ℓ^2 objective function with regularization terms in order to train the neural networks. In both cases our estimation techniques are standard. Again we estimate the model on the training data and optimize all respective tuning parameters on the validation set.

Finally we consider a model ensemble of the above linear and nonlinear estimators, restricting ourselves to an equal-weighting scheme across predicted expected returns as to limit introducing additional estimation uncertainty.

Figure 1 displays a set of 10 bar plot clusters. Each displays end-of-year (last quarter) snapshots of forecasting performance. The 10 rolling samples displayed, each pertaining to a 10-year sample of return data to estimate, validate and forecast returns. For each of the 10 rolling samples the relative performance of the competing models (only looking at equities) is displayed. The out-of-sample performance is measured in terms of MSE and the height of each bar represents the percentage a particular model has the lowest MSE in predicting the cross-section of returns for all the stocks in the sample. For each cluster the height of the bars add up to 100% and each represents the fraction a particular class of models provides the best return prediction for the 683 in the cross-section. We note that neural network models represent the most successful class of models, typically being the best for between 40 and 50 percent of the assets in the cross-section. Often a close second is the class of Elastic Net models. All other methods are less successful, although there is quite some variation across time.

The results displayed in Figure 1 may leave the impression that neural network models are dominant. Let us turn our attention to Figure 2 which sheds perhaps a different light on this result. It provides a collection of four histograms documenting for a particular 10-year rolling sample, the 1994-2004 period, the cross-section of out-of-sample MSE's. The models are OLS, Elastic Net (EN), Random Forest (RF), and Neural Net (NN). The out-of-sample (OOS) performance is measured in terms of MSE. Eyeballing the four histograms we see that OLS is clearly worse than EN (Elastic Net), RF (random forest) and NN (neural network), but the differences among the three ML methods is not as clear. The cross-section of MSE's appears indeed similar. Further evidence of this appears in Table 1 which reports the average MSE and MAE of out of sample forecasts across all assets and rolling sample schemes. It shows that the elastic-net and neural net models deliver the lowest out-of-sample MSE when aggregating performance across stocks/ETFs. However, the differences between EN and NN are very small, indicating that while NN perhaps provides the best predictions, EN is typically a close second and arguably much easier to implement. Moreover, the EN is a linear model, whereas the NN is nonlinear. The presence of nonlinearities does not seem to substantially pay off.

All the models/estimators have dimensions on where they could be refined, but ultimately the modeling group delivers a set of conditional mean estimates by stock/ETFsto the robo-investors. Each of these chosen conditional mean estimates come from the model with the lowest out-of-sample MSE. A common model need not be chosen across stocks/ETFs, and indeed we can see that even in a few cases, OLS with all covariates included is the model with the best out-of-sample performance. The final panel of $(\hat{E}_t[r_{i,t+1}])_{i,t}$ is used in the robo-investors' optimal portfolio problems.

4 Empirical Results

The empirical results focus on answering a number of questions: (a) are more sophisticated models better, (b) who gains from robo-advice, (c) how does robo-investing perform during a major financial crisis, (d) how do AI Alter Egos compare to passive investment schemes and (e) are spreads due to behavioral biases? A subsection is devoted to answering each of these questions. In the main body of the paper we report a summary set of results pertaining to quarterly rebalancing. In the Online Appendix, the monthly and detailed quarterly results are reported.

4.1 Are more sophisticated models better?

In Table 2 we report for all investors in our sample the median, first (Q1) and third quartiles (Q3) of the cross-sectional distribution of return spreads $r^s_{i,t} = r^{ae}_{i,t}$ - $r_{i,t}$, considering only equity holdings (left panel) or the entire universe of 683 stocks and 393 ETFs (right panel). The AI Alter Ego schemes are: (a) MV with Rolling Mean/Rolling Variance, (b) Machine Learning (ML) Mean/Rolling Variance - using the methods displayed in Table 1, (c) ML Mean/Nonlinear Smoothed Variance, and finally the equally weighted (EW) portfolio scheme. Neither rolling sample mean nor equally weighted portfolios have positive median spreads. Hence, the median shadow robo-investor performs worse than the humans. The highest median spread is obtained from the ML Mean/Rolling Variance, namely 2.93% per year (equities only) and 3.37% for the universe of stocks and ETFs. Using the nonlinear smoothing approach to covariance estimation slightly reduces the median return by 15 basis points or even 41 basis points when ETFs are included. While there is a large cross-sectional heterogeneity, judging by the inter-quartile range, we also observe a right shift in the entire distribution. The first quartiles for MV Rolling Mean and EW are 3 percent lower, whereas Q3 is 5 percent lower compared to either type of MV ML. Figure 3 provides further evidence of our findings. The six plots display 'man versus machine' scatter plots of returns (top three plots) and portfolio risk measured via realized variances (lower three plots). The x-axis is the average returns of individual investors over the life-span of their portfolio holdings in our sample, and the y-axis is the corresponding shadow AI performance. For the top three plots, all dots above the 45 degree line correspond to robo-investor improvements. The gray shaded area represents robo-investor improvements *and* overall positive returns. For the lower three plots, all dots below the 45 degree line represent less risk taking by the robo-investor. For the top three subplots (a) - (c) we see a clear upward shift in the cloud of dots as we move from (a) to (b) and very little movement from (b) to (c). This corresponds to the better performance of ML/Rolling Variance robo-investor schemes highlighted in Table 2. When we examine the lower set of plots we see that most robo-investors feature lower portfolio return fluctuations, since the majority of dots lie below the 45 degree line. Contrary to the return results, we see that the ML based robo-investors feature more variance risk. All the results reported so far pertain to quarterly portfolio rebalancing. In the Online Appendix, we provide detailed evidence showing that the findings extend to monthly rebalancing. The annual rebalancing yield qualitatively the same findings as well.

Overall, the results clearly show that the ML expected return scheme is superior to any of the two relatively naive and simple robo-investing schemes. Hence, the answer is clearly that more sophisticated models are better. In the remainder of this section we will therefore focus exclusively on the MV ML/Rolling Variance robo-investor AI Alter Egos.

4.2 Who gains from robo-advise?

Continuing with quarterly rebalancing and MV ML/Rolling Variance robo-investors, in Table 4 we report the median, Q1, Q3 as well as confidence interval for the median of the cross-sectional distributions of the spreads between AI Alter Ego and individual investor returns, considering the entire universe of 683 stocks and 393 ETFs.[11] Summary statistics are computed for separate samples with low/high education, low/high risk aversion and low/high income classification for investors.

Let us start with high and low risk aversion, which is the panel in the middle of the Table. High risk averse median individual investors stand to gain 5.14 percent from robo-investor shadow Alter Egos. Their low risk aversion counterparts only gain 3.29. Both clearly benefit,

[11] To construct confidence intervals for aggregate summary statistics we do the following. We first randomly sample individuals according to an individual bootstrap method whereby each investor is assumed independent of each other investor, and sample the entire time-series path of each investor to maintain the dependence structure. For each bootstrap repetition we compute the relevant statistic per individual and aggregate the per-individual statistics over all of the sampled investors. Let $\{\tilde{\theta}_r\}_{r=1}^R$ be the constructed statistic over R bootstrap repetition, and let $\hat{\theta}$ be the point estimate of interest. Let $\tilde{\theta}_{(\alpha/2)}$ and $\tilde{\theta}_{(1-\alpha/2)}$ represent the $\alpha/2$ and $1-\alpha/2$ percentiles of the bootstrap statistic. We then construct pivotal $1-\alpha$ confidence intervals according to $[2\hat{\theta} - \tilde{\theta}_{(1-\alpha/2)}, 2\hat{\theta} - \tilde{\theta}_{(\alpha/2)}]$.

since the confidence intervals for either type of investor indicates that the median spreads are significantly different from zero. In addition, the 95% confidence interval for the difference in medians is $[0.5546, 3.1111]$, and therefore excludes zero. Hence, the median high risk averse investor gains statistically significantly more from robo-investing than the median low risk averse investor does. A similar pattern emerges for high/low income, with the median low income investor gaining roughly two-thirds more (4.13 percent versus 2.76) than the high income median investor. Low and high education differences are not as pronounced, with a wedge of 63 basis points. The inference indicates, however, that the high/low median spread for income and education are not statistically significant. Needless to say that a spread between median 2.76 (high income) and 4.13 (low income) percent return per year is economically quite substantial.

4.3 How does robo-advising perform during major financial crisis?

In Table 5 we report the Alter Ego return spreads for stocks/ETFs as they relate to the financial crisis and Great Recession financial. The subsamples are benchmarked using the NBER chronology identifying the crisis period as 12/2007 - 6/2009.[12] The focus is again on the MV ML/Rolling Variance AI Alter Ego scheme. For each of the subsamples we compute the median, Q1 and Q3 realized returns along with the same statistics for the AI Alter Ego returns. Note that, since the median of a spread is not the difference in median returns, we are not inferring something directly related to the spreads reported in prior tables. We focus on the returns instead in order to highlight a very important finding. Prior to the crisis we note that the median investor had an annual return of 9.26%, almost double the return of the median AI Alter Ego (4.17%). We also note though that the inter-quartile spread for investors is twice as large as the same statistic for robo-investors using ML. For individual investors the Q1-Q3 spans from -4.70 to 20.98 percent, whereas the AI Alter Egos feature a better Q1 of minus two percent, and a lower Q3 of almost eleven percent.

[12]We also examined the more specifically targeted Belgian crisis dates related to the severe difficulties of the country's financial sector. The results are broadly speaking similar and not reported here.

During the crisis things take a dramatic turn. The median robo-investor has zero return - meaning the median AI Alter Ego holds cash. In contrast, for individual investors the median is a 29 percent loss and even the Q3 investor still has a -3.59% negative annual return. Compare that with 23.81 percent return for Q3 of the AI Alter Egos.

After the crisis, things reverse to the pattern observed prior to the crisis - namely the median investor does better than the median AI Alter Ego, with again a much wider inter-quartile range for individual investors, even more than double the dispersion among robo-investors.

A more striking picture emerges when we turn our attention to Figure 4. The five lines correspond to (a) realized return of median investor, (b) median AI Alter Ego returns using MV Rolling Mean/Rolling Variance scheme (c) median AI Alter Ego returns using MV ML/Rolling Variance scheme (d) median AI Alter Ego returns using MV ML/Nonlinear Variance and finally (e) the median EW robo-investor. One word of caution: these medians do not represent the same investor or AI Alter Ego through time, so this is not the performance of a specific individual or robot. Each line starts out with one unit of investment at the beginning of the sample and the median returns are compounded subsequently. Prior to the crisis, the median investor reaches roughly 2.3. This means that the initial capital is doubled over a five year span from 2002 until 2007. By the time the devastation of the crisis took its toll, the median investor is under water by 20 percent and finally ends up with a meager 20 percent return over a 10-year period. It is remarkable that even the EW robo-investor, whom we know from prior analysis is neither sophisticated nor particularly successful, achieves a higher return at the end of the sample. The best overall performance is obtained from the MV ML/Rolling Variance median robo-investor (again not shadowing always the same investor across time) with a 60 percent overall return. This median robo-investor has a relatively slow start and under-performs prior to the crisis, but features small losses during the tumultuous market conditions. Note also that the MV ML/Nonlinear Variance AI Alter Ego is almost identical to the ML/Rolling Variance scheme. Finally, the

MV Rolling Mean/Rolling Variance scheme tracks the ML performance very closely until the financial crisis.

To shed further light on this we turn our attention to Table 3 displaying the ranking of the regressors based on their ℓ_2 contribution across stocks for the Elastic Net regressions defined in equations (3) - (4). We focus on the EN regressions as they provide a fairly simple regression-based interpretation. In addition, it is often the best or nearly the best prediction model. The ranks are computed for 10-year rolling samples starting with 93-03 and ending with 02-12. Of particular interest is the crisis period spanning across the 96-06 through 99-09 samples. The top ranked predictor in all but the last of rolling samples is dfy namely the default yield spread. Another top-ranked series during the crisis is lty or the long term yield. Looking across all samples we also see $svar$ stock variance, $ntis$ net equity expansion and $infl$ inflation. Interestingly, the usual Fama-French regressors rarely appear among the top-ranked regressors. This should not perhaps come as a surprise, since the Fama-French factors are meant to price the cross-section of returns.

Finally, in Figure 5 we provide a time series plot of the fraction with negative expected returns among the cross-section of stocks, according to the best machine learning model. Early in the sample we see that typically between 20 and 30% of the stocks featured negative expected returns. The fraction shoots up above 50% in 2008 and goes as high as 60%. As a result, the majority of stocks featured negative expected returns, which explains why the AI Alter Egos have a propensity to move out of the market.

4.4 AI Alter Egos versus Passive Investments

How do AI Alter Egos measure up against passive investment strategies, in particular buying and holding a market-wide ETF? To address this question we turn our attention to Table 6. We report summary statistics for spreads with respect to two ETFs. One tracks the S&P 500 index and the other is the iShares MSCI Belgium ETF. Neither is ideal, but we did not find an index available throughout the entire sample period that mimics the basket of stocks held by the investors in the brokerage data set.[13] Unfortunately, the results reported in Table 6 depend on which ETF is selected. In the right panel displays the results for stocks+ETFs returns minus the benchmark ETF spreads, either S&P 500 or Belgian and in the left panel AI Alter Ego MV/ML/Nonlinear against the same benchmarks. Each panel contains the median, first and third quartile of the spreads. The full sample results appear in the top part of Table 6. Subsamples stratified according to NBER crisis dates appear in the lower part. The median investor has a spread of -8.50% against the S&P 500, meaning the median investor vastly under-performs the benchmark. For the Belgian ETF the results are not as dramatic, since the median investor does better with a positive spread of 1.37%. There is wide cross-sectional variation, although the third quartile for the US market index is only 2% (while 12% for the Belgian index). The AI Alter Ego spreads are better in both cases, although the US benchmark still yields a negative spread of -6.18%. Against the Belgian ETF, the AI Alter Ego has a positive median spread of almost 4 percent.

When we look at the pre-crisis sample we note that the median investor and AI Alter Ego have returns below the two benchmarks, more so for the Belgian ETF than its US counterpart. It is also worthwhile noting that the median AI Alter Ego performs worse. The crisis period is a totally different story. The AI Alter Egos median investors vastly outperform the benchmark by respectively 18.32% (SPDR) and 48.31%. Moreover, the median investor does better than the Belgian ETF by a substantial margin of 21.24% but is 8.78% below the S&P 500 ETF. In both cases we see significant improvements from the Alter Ego schemes. Post-crisis things return back to the pre-crisis situation.

4.5 Are the spreads due to market or behavioral factors?

Are the sharp findings regarding the crisis related to well documented behavioral biases? In the Appendix section A.4 we report that the investors in our sample feature the behavioral biases studied in the literature. Regarding the crisis results, we would like to focus on two

[13]According to the results reported in Table A.3 investors hold 26% of US stocks and 14% of Belgian stocks.

key ones: (1) the disposition effect (DE) - selling winners too soon, holding on to losers too long - as in Odean (1998) for each investor and (2) trading frequency.

In Table 7 we document the results of cross-sectional median regressions, where the AI Alter Ego spreads from the MV ML/Rolling Variance robo-investors are a function of DE (left column) as well as DE combined with trading frequencies. We report the AI Alter Ego return spreads as well as the spreads vis-à-vis the S&P 500 ETF. The DE has a positive impact on the spreads, albeit not always statistically significant when combined with trading frequency indicator regressors. When we add dummies for the 2nd through 4th quartile of trading frequency we note that the DE spreads are affected in a statistically significant way, monotonically deteriorating for spreads and increasing for spreads vis-à-vis the S&P 500 ETF. This means that investors who trade a lot tend to have larger AI spreads against the ETF benchmark (in unreported results it is also the case for the Belgian ETF). Conversely, frequent traders tend to have less benefits from AI Alter Egos, and DE is insignificant when combined with trading frequency. Overall, the results indicate that behavioral biases explain to a certain degree the cross-section of AI Alter Ego spreads. Particularly, spreads against a passive investment strategy increase with trading frequency and disposition effect. Additional results involving controls for investor characteristics appear in Tables OA.21 and OA.22 of the Online Appendix. Besides adding controls, we also consider quantile regressions for 5%, 10% 20%, 80%, 90% and 95%. Overall the findings remain, particularly for the right tail of the distribution. The DE is significant for the median and extreme right tail when looking at AI Alter Ego spreads with respect to the benchmark ETF.

5 Conclusions

Artificial intelligence enhancements are increasingly shaping our daily lives. Financial decision-making is no exception to this. We introduce the notion of AI Alter Egos, machine-driven decision makers which shadow a particular individual, and apply it in the area of robo-investing using a brokerage accounts data set rich in both cross-sectional and time series features.

The purpose of our analysis is to assess the highly touted benefits of robo-advising. Through the AI Alter Ego scheme we address a number of questions: (a) are more sophisticated models better, (b) who gains from robo-advise, (c) how does robo-investing perform during a major financial crisis, (d) how do AI Alter Egos compare to passive investment schemes and (e) are spreads due to behavioral biases? Overall, we find that investors displaying certain characteristics - in particular high risk averse and low income - stand to gain significantly. In particular: high risk-aversion, low income investors. Moreover, machine learning methods provide important portfolio return improvements. AI Alter Ego spreads are related to behavioral biases - in particular the disposition effect and trading frequency. During the financial crisis, robo-investors have a greater propensity to cash out of the market, which contributes to their overall return superiority. Finally, compared to passive ETF investment, we find that the evidence is mixed, although during the financial crisis AI Alter Egos were vastly better than the passive strategy.

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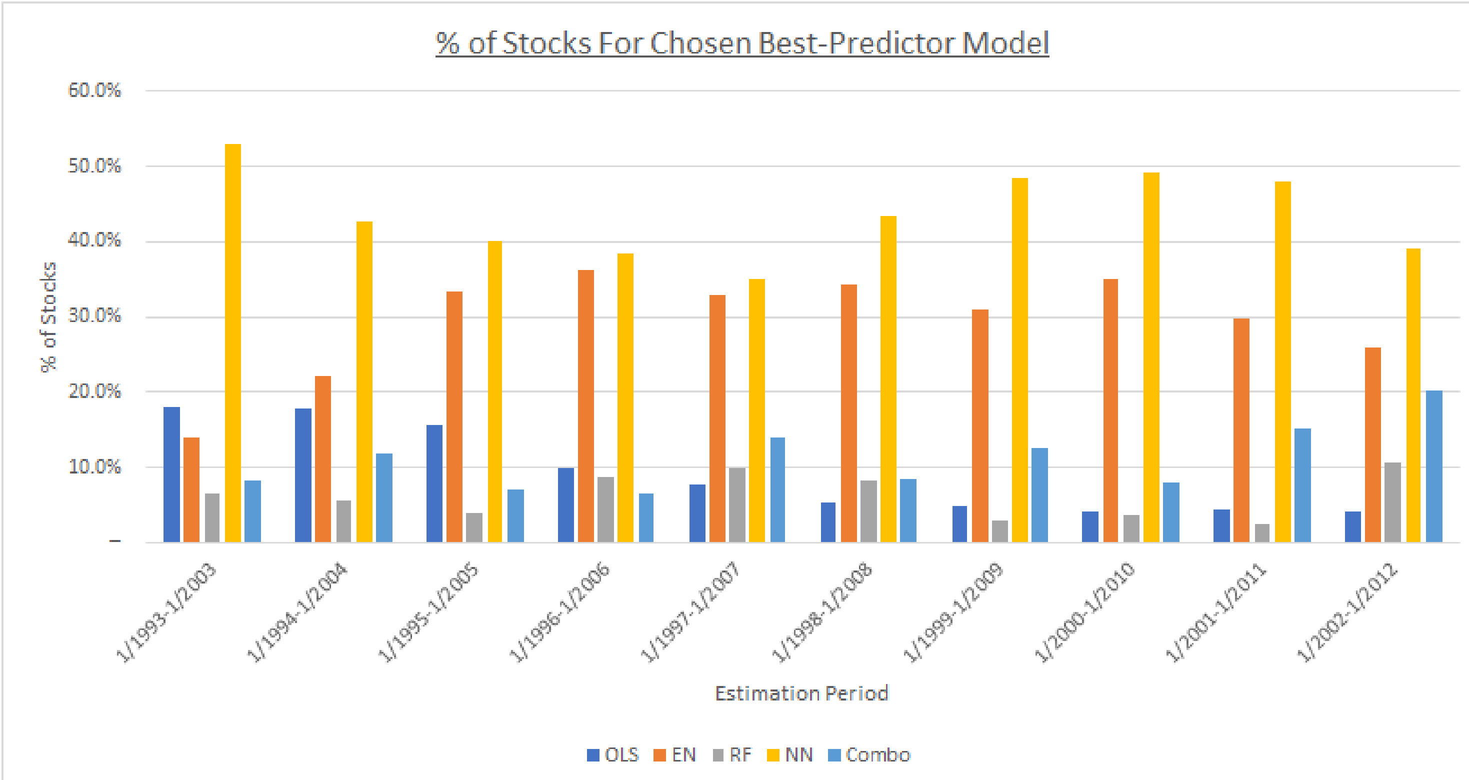


Fig. 1. Bar charts for 10-year rolling samples are displayed, where we only display yearly snapshots. The first covers the sample Jan 1993 - Jan 2003 and the last Jan 2002 - Jan 2012. For each of the 10 rolling samples the relative performance of the competing models (only looking at equities) is displayed. The bars add up to 100% for each of the 10 rolling samples. The out-of-sample (OOS) performance is measured in terms of MSE and the height of each bar represents the percentage a particular model has the lowest MSE in predicting the cross-section of returns for all the stocks in the sample. The models are OLS, Elastic Net (EN), Random Forest (RF), Neural Net (NN) and Ensemble (Comb). We use 70% of the data for training, 20% of the data as a validation sample (for hyperparameter tuning), and 10% of the sample for testing OOS performance. The bar charts pertain to the OOS performance.

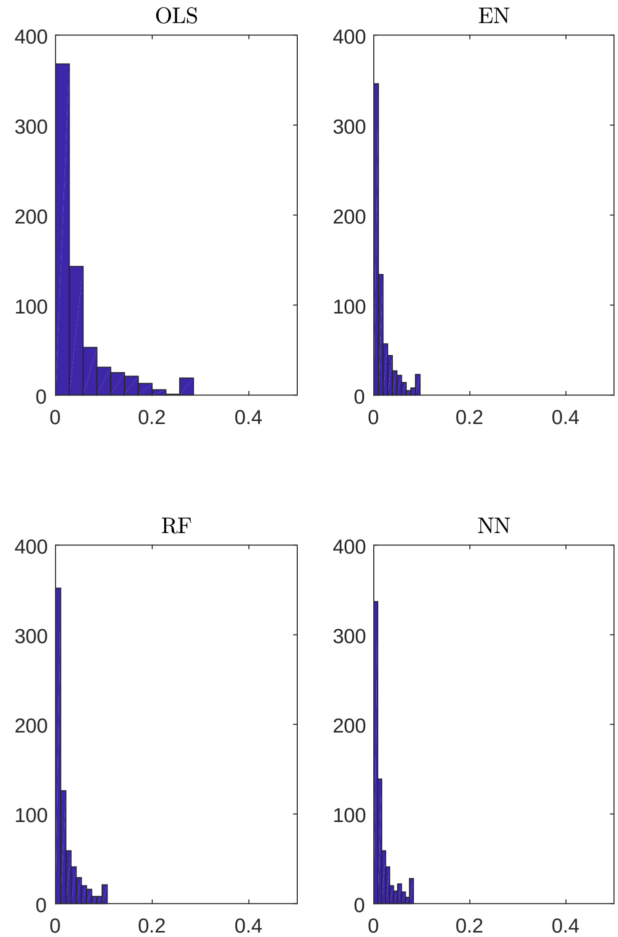
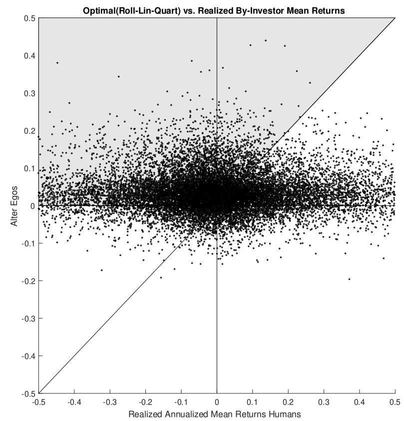
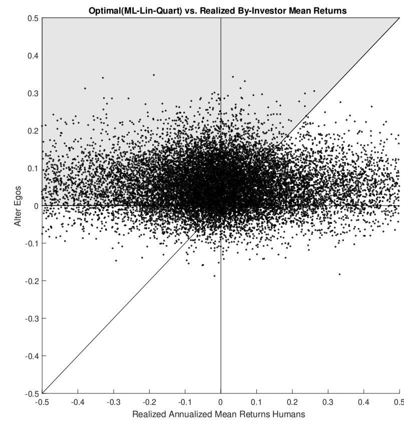


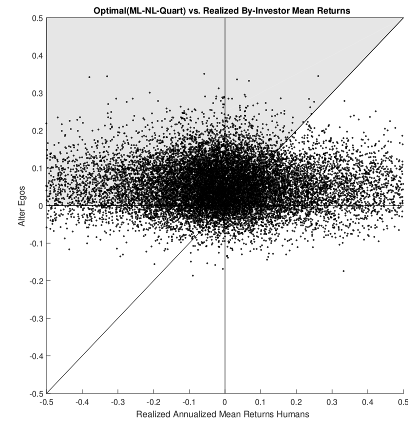
Fig. 2. The models are OLS, Elastic Net (EN), Random Forest (RF), and Neural Net (NN). The out-of-sample (OOS) performance is measured in terms of MSE. Histograms pertain to MSE for 1994-2004 subsample pertaining to the cross-section of returns. We use 70% of the data for training, 20% of the data as a validation sample (for hyperparameter tuning), and 10% of the sample for testing OOS performance. The histograms pertain to the cross-section of OOS MSE performance.



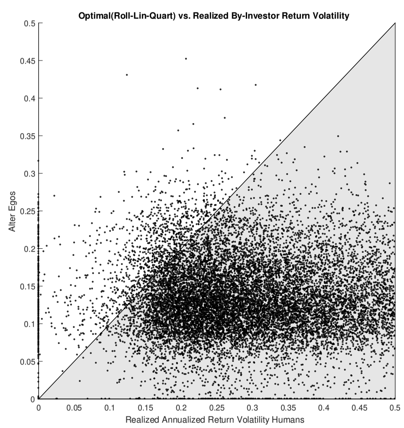
(a) **Rolling M/V**



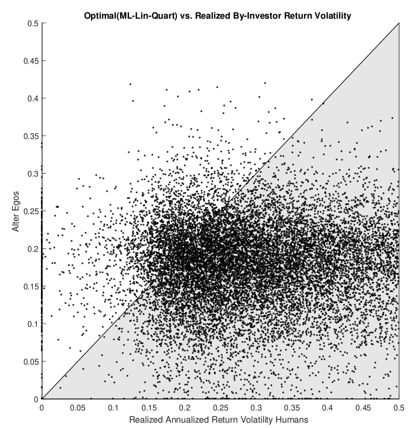
(b) **ML/Rolling V**



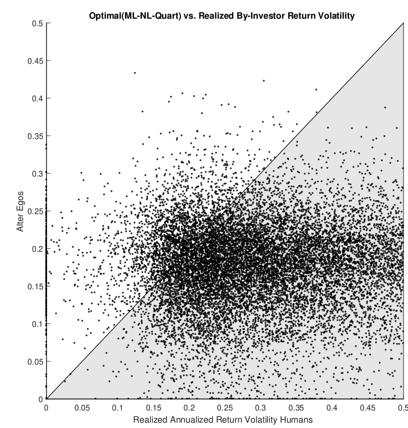
(c) **ML/Nonlinear**



(d) **Rolling M/V**



(e) **ML/Rolling V**



(f) **ML/Nonlinear**

Fig. 3. Top panel: scatter plots of investor realized returns (X-axis) versus corresponding AI Alter Egos realized returns (Y-axis). The gray cones correspond to AI improvements, above 45 degree line and fourth quadrant. Bottom panel: scatter plots of investor return variances (X-axis) versus corresponding AI Alter Egos variance (Y-axis). The gray cones corresponds to AI improvements, below 45 degree line.

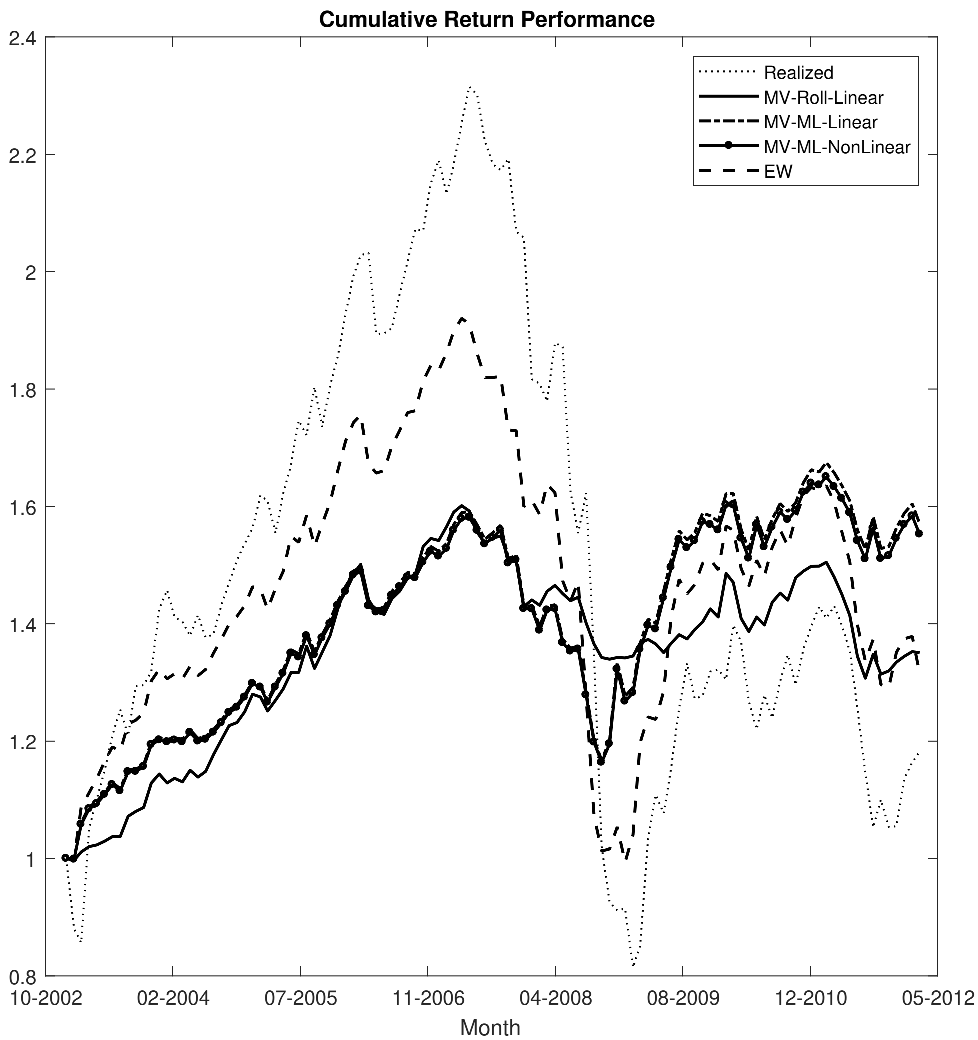


Fig. 4. The lines correspond to (a) realized return of median investor, (b) median AI Alter Ego returns using MV Rolling Mean/Rolling Variance scheme (c) median AI Alter Ego returns using MV ML/Rolling Variance scheme (d) median AI Alter Ego returns using MV ML/Nonlinear Variance and finally (e) the median EW robo-investor. All start out with one unit of investment at the beginning of the sample and median returns are compounded.

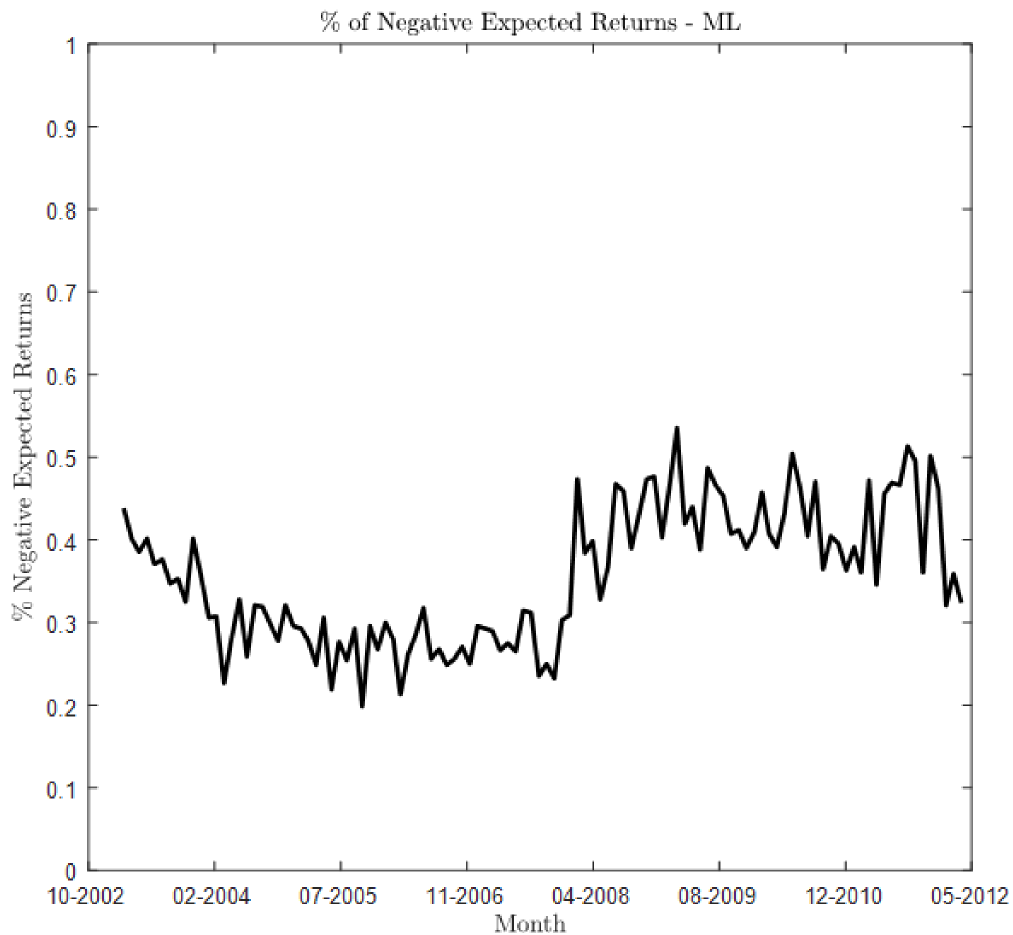


Fig. 5. Time series plot of the fraction among the cross-section of stocks with negative expected returns, according to the best machine learning model - see Figure 1 for details.

Table 1: Out-of-Sample MSE Across Stocks

	Cross-Sectional MSE				
	OLS	EN	RF	NN	Comb
Mean	0.0207	0.0104	0.0114	0.0100	0.0101
Median	0.0147	0.0072	0.0081	0.0066	0.0070

Notes: Cross-sectional average and median MSE's on the out-of-sample testing data for: OLS, Elastic-Net (EN), Random Forest (RF), Neural Network (NN), and ensemble (Comb).

Table 2: AI Alter Ego Return Spreads - All Investors

	Equities only			Equities + ETF		
	Median	Q1	Q3	Median	Q1	Q3
			Mean Variance			
Rolling Mean/Rolling Variance	-0.08	-13.60	12.84	0.58	-12.72	13.16
ML Mean/Rolling Variance	2.93	-10.66	17.41	3.37	-10.19	17.20
ML Mean/Nonlinear Smoothed Variance	2.78	-10.72	17.13	2.96	-10.49	17.09
			Equally Weighted			
	-0.67	-13.88	11.96	-0.54	-13.68	11.98

Notes: Entries are median, first (Q1) and third (Q3) quartiles of the cross-sectional distributions of the spreads between AI Alter Ego and individual investor returns, $r^s_{i,t} = r^{ae}_{i,t}$ - $r_{i,t}$, for three Mean Variance (MV) types of robo-investors and one equally weighted (EW) considering only equity holdings (left panel) or the entire universe of 683 stocks and 393 ETFs (right panel). The AI Alter Ego schemes are: (a) MV with Rolling Mean/Rolling Variance, (b) Machine Learning (ML) Mean/Rolling Variance - using the methods displayed in Table 1, (c) ML Mean/Nonlinear Smoothed Variance, and finally the equally weighted portfolio scheme. The spreads are in percentage per year.

Table 3: Ranked Variables Based on Relative ℓ_2 Contribution Across Stocks

Rank	93-03	94-04	95-05	96-06	97-07	98-08	99-09	00-10	01-11	02-12
1	SMB	svar	ntis	dfy	dfy	dfy	lty	Mkt-RF	ntis	svar
2	dfy	lty	lty	lty	svar	ntis	dfy	lty	svar	dfy
3	lty	infl	infl	svar	SPvwx	lty	svar	svar	Mkt-RF	lty
4	RMW	dfy	tbl	infl	infl	svar	tbl	dfy	infl	infl
5	infl	Mkt-RF	svar	HML	Mkt-RF	tbl	Mkt-RF	SPvw	SPvw	ntis
6	ntis	SMB	HML	tbl	dy	Mkt-RF	infl	ntis	CMA	RMW
7	HML	RMW	RMW	Mkt-RF	lty	infl	ntis	tbl	bm	tbl
8	Mkt-RF	ntis	dy	bm	RMW	dy	SPvw	CMA	lty	SPvwx
9	svar	HML	bm	SMB	ntis	SPvwx	RF	bm	ltr	SPvw
10	SPvw	bm	dfy	SPvw	dp	dp	SPvwx	RMW	SMB	Mkt-RF
11	Mom	Mom	Mkt-RF	RF	SMB	SPvw	HML	SMB	SPvwx	CMA
12	tbl	ltr	ep	ntis	tbl	bm	CMA	HML	dfy	HML
13	ltr	dy	RF	dy	bm	SMB	RMW	SPvwx	HML	SMB
14	de	SPvw	SMB	CMA	HML	RMW	bm	dy	dy	ltr
15	bm	CMA	dp	RMW	RF	HML	SMB	Mom	Mom	dp
16	SPvwx	de	CMA	SPvwx	SPvw	CMA	ltr	dp	dp	RF
17	RF	SPvwx	SPvw	Mom	Mom	RF	dy	ltr	tbl	Mom
18	CMA	dp	Mom	ltr	de	de	dp	infl	RMW	bm
19	dy	tbl	ltr	ep	ep	ep	Mom	ep	ep	dy
20	ep	RF	de	dp	CMA	ltr	de	de	RF	ep
21	dp	ep	SPvwx	de	ltr	Mom	ep	RF	de	de

Notes: Elastic Net regressions defined in equations (3)-(4) involve the following set of regressors: *dp* Dividend/Price, *dy* Dividend Yield, *ep* Earnings/Price, *de* Dividend Payout, *svar* Stock Variance, *bm* Book-to-Market, *ntis* Net Equity Expansion, *tbl* T-Bill Rate, *lty* Long Term Yield, *ltr* Long Term Return, *dfy* Default Yield Spread, *infl* Inflation, *SPvw* S&P 500, *SPvwx* S&P 500 (excl. dividends), the following Fama French factors $Mkt - RF$ Market, SMB, HML, RMW, CMA, RF, and Mom Momentum (see Ken French website for definitions), and Welch and Goyal (2007) for definitions.

Table 4: AI Alter Egos Return Spreads - Education, Risk Aversion and Income

	Return Spreads			CI Median	
	Median	Q1	Q3	C(2.5%)	C(97.5%)
Education					
Low	2.83	-10.47	16.79	1.18	4.22
High	3.46	-10.15	17.26	3.01	3.85
Confidence interval difference in medians: [-0.5989, 2.1723]					
Risk Aversion					
Low	3.29	-10.86	17.43	2.39	4.38
High	5.14	-9.12	17.28	3.63	6.36
Confidence interval difference in medians: [0.5546, 3.1111]					
Income					
Low	4.13	-10.00	17.57	3.01	5.20
High	2.76	-11.70	15.01	0.82	4.91
Confidence interval difference in medians: [-3.1016, 0.6014]					

Notes: Entries are median, first and third quartiles, as well as confidence interval for the median of the cross-sectional distributions of the spreads between AI Alter Ego and individual investor returns, considering the entire universe of 683 stocks and 393 ETFs. The AI Alter Ego scheme is Mean Variance (MV) with Machine Learning (ML) Mean/Rolling Variance - using the methods displayed in Table 1. Summary statistics are computed for separate samples with low/high education, low/high risk aversion and low/high income classification for investors. 95% confidence intervals for differences in medians are computed as described in footnote 11. The spreads are in percentages per year.

Table 5: Returns Pre-Crisis, Crisis and Post-Crisis

	Median	Q1	Q3
Pre			
Realized	9.26	-4.70	20.98
MV ML	4.17	-2.00	10.70
During			
Realized	-29.04	-45.87	-3.59
MV ML	0.00	-13.81	23.81
Post			
Realized	5.64	-6.47	15.30
MV ML	2.05	-1.76	8.42

Notes: The subsamples are benchmarked based on the NBER Crisis Time Period 12/2007 - 6/2009. The Pre-crisis sample starts in 2002 and ends 11/2007, the post-crisis sample covers 7/2008 until end of sample, 2012. MV ML refers to the AI Alter Ego scheme is Mean Variance with Machine Learning Mean/Rolling Variance - using the methods displayed in Table 1. The spreads are in percentages per year.

Table 6: AI Alter Ego Return Spreads vis-à-vis benchmark ETFs

	Realized Stocks+ETFs minus ETF			AI Alter Ego minus ETF		
	Median	25q	75q	Median	25q	75q
			Full sample			
S&P 500 ETF	-8.50	-21.05	2.00	-6.18	-13.99	1.32
Belgian ETF	1.37	-9.90	12.00	3.93	-4.89	13.10
			Pre-Crisis			
S&P 500 ETF	-2.22	-16.02	9.18	-7.20	-13.88	-0.41
Belgian ETF	-9.57	-21.28	1.45	-14.58	-21.33	-7.02
			During Crisis			
S&P 500 ETF	-8.78	-27.02	12.48	18.32	-1.75	39.32
Belgian ETF	21.24	1.23	40.35	48.31	25.60	71.02
			Post-Crisis			
S&P 500 ETF	-11.37	-23.16	-1.78	-14.97	-19.62	-8.43
Belgian ETF	-1.48	-12.59	8.26	-4.82	-9.85	2.36

Notes: The AI Alter Ego scheme is the ML Mean/Nonlinear Smoothed Variance - using the methods displayed in Table 1. The spreads are in percentage per year. The subsamples are benchmarked based on the NBER Crisis Time Period 12/2007 - 6/2009. The Pre-crisis sample starts in 2002 and ends 11/2007, the post-crisis sample covers 7/2008 until end of sample, 2012. The benchmark ETFs are the SPDR ETF tracking the S&P 500 index and the iShares MSCI Belgium ETF.

Table 7: Disposition Effect and AI Alter Ego Return Spreads - Median regression

	Spreads		Spreads vis-à-vis S&P 500 ETF	
	Model DE Only	Model DE + Trading Freq.	Model DE Only	Model DE + Trading Freq.
Trading frequency				
2nd quartile		-1.361*		0.782***
3rd quartile		-1.944***		1.168***
4th quartile		-2.850***		1.175***
Disposition Effect	0.014*	0.011	0.004*	0.007***

Notes: Cross-sectional Median Regression Alter Ego Spreads with MV ML/Rolling Variance, * $p<0.10$ ** $p<0.05$, *** $p<0.01$. Number of observation $N = 19118$. Detailed parameter estimates for the controls (gender, education, risk aversion, income, funds invested, ETF use) appear in the Online Appendix Tables OA.21 and OA.22.

Appendix

A Data - details

Our primary data set comes from a large Belgian online brokerage firm and consists of the trading accounts of 22,972 individual investors. This unique data spans about 10 years from January 2003 to March 2012, which includes the 2008 financial crisis. We have detailed information about each trade, i.e. the ISIN code of the instrument, the time-stamp, the trade direction, the executed quantity, the trade price, and the explicit transaction costs.[14] For the purpose of this research, we focus on common stocks and ETFs investments and exclude other financial instruments. Over the sample period, 6,741 investors also traded options and warrants for an aggregate number of 602,833 trades and 6,665 investors traded mutual funds for an aggregate number of 260,120 trades. Only a few investors (i.e. 1,813) traded bonds for an aggregate number of 5,999 trades. Overall, the individual investors trade across 12,818 different stocks but we count for most of the stocks only a few trades. We therefore filter the trades and keep the 683 stocks that are most traded over the sample period. This universe covers 71.5% of all the investors' trading activity on stocks. We end up with a sample of 1,590,199 trades on stocks (and 13,015,509,557 traded volume in euros) over the 111-month period. In addition to stocks, 4,693 investors (i.e. 20.45%) also trade ETF. Over the whole sample period, those ETF investments correspond to 60,344 trades on 393 different ETFs for a total monetary volume of 898,496,821 in euros.

Types of Assets in the Data Base and Prevalence of Trading

	Income		**Risk aversion**		**Education**	
	High	Low	High	Low	High	Low
%investors						
Stocks	100%	100%	100%	100%	100%	100%
ETFs	30%	18%	21%	22%	22%	14%
Mutual funds	37%	26%	28%	29%	31%	18%
Options	13%	8%	9%	13%	11%	4%
Warrants	27%	18%	20%	22%	20%	13%
Bonds	9%	7%	8%	7%	9%	4%

TABLE A.1. The table shows the types of assets in our database as well as the proportion of active investors - low/high income, risk aversion and education.

Table A.1 shows the types of assets in our database as well as the proportion of active investors per type of investors - low/high income, risk aversion and education. We note that all investors trade equities. Trading of ETFs, mutual funds, options and warrants is more

[14] We also know the currency in which the trade is executed, which allows us to compute monetary volumes in euros using historical exchange rates from the European Central Bank and Bloomberg.

prevalent with high income/education investors. Trading of bonds is overall insignificant. Because we examine robo-advisors which are mean-variance investors we focus exclusively on stocks and ETFs which best fit the portfolio allocation model. For high income/education investors in particular this means we leave out to a certain degree other assets which we have available.

A.1 Portfolios and Individual Investor Characteristics

Using the trades data, we build end-of-month portfolios for each investor and use historical market data to compute monthly portfolio market values. We also compute both monthly and daily asset returns.[15] Combining end-of-month portfolio market values with the corresponding monthly aggregate cash-flows, we calculate for each investor 110 monthly portfolio gross and net returns, i.e. from February 2003 to March 2012. To calculate portfolio returns, we opt for an approximation of the Modified Dietz Method, aiming at delivering a return close to the money-weighted rate of return (e.g., Shestopaloff and Shestopaloff (2007)). Specifically, we compute portfolio returns per investor and month assuming that all the purchases (sales) executed in a given month take place on the first (last) day of the month. Mathematically, it gives: $R_t = (EMV_t - EMV_{t-1} - P_t + S_t)/(EMV_{t-1} + P_t)$ where R_t is the portfolio gross return for month t, EMV_t is the end-of-month portfolio value at month t, EMV_{t-1} is the end-of-month portfolio value at month $t-1$, P_t and S_t are the aggregate monetary value of all purchases and sales executed during month t. When calculating net returns, we subtract from the numerator the aggregate monetary value paid on transaction costs during month t.

Robo-investors can hold cash. It is therefore of interest to comment on how investor cash holdings - which we do not observe - affect our return calculations. To that end, let us consider an illustrative example. Suppose an investor has 10 euros (units don't matter) at end of t - 1 with 5 invested in stock 1 and 5 in stock 2. To further simplify the example assume that this investor did not hold any other stocks in the past. The robo-investor decides the allocation between stock 1 and 2 - buying and holding the shares. Furthermore, the investor cashes out 5 euros in stock 1 shortly thereafter (assuming stock 1 does not appreciate). At time t investor only has stock 2 and holds a portfolio worth 5 euros. The robo-investor can buy stock 1 and 2 for a value of portfolio up to 5 euros but suppose expected returns are negative for stock 1 so only holds stock 2. In the end investor and shadow Alter Ego both hold the same portfolio. In this example, the time $t + 1$ returns from the robo-investor's strategy are going to be independent of the actions that the investor takes with respect to their non-negative (could be 0) allocation between stocks 1 and 2 at time t. The robo-investor only sees that the investment opportunity set has remained the same. Table A.2 provides a few scenarios - including the aforementioned one - which show that the return calculations are relatively robust to various cash inflow and outflow calculations.

Two final additional filters were applied on the resulting portfolio gross returns (prior to transaction costs). First, we drop any investors if they had more than 106 missing values in their return series (i.e. at least 4 months of returns are needed to keep an investor). Second, because of some outliers, we disregard investors for whom we observe at least one month

[15] Historical price data come from both Eurofidai and Bloomberg.

	t-1	Trading	**t**	**t-1**	Trading	**t**	**t-1**	Trading	**t**
Stock 1	5	-5	0	5	-5	0	5	0	5
Stock 2	5	0	5	5	5	10	5	5	10
EMV	10		5	10		10	10		15
Cash-in		0			5			5	
Cash-out		5			5				
Return			0%			0%			0%

Table A.2: Illustrative Examples of Cash flows and Return Computations

where the absolute value of their return is greater than 100. These two filters decrease the sample from 22,972 to 20,622 investors.

In addition to the above information about trading activity, we have an extensive set of individual investor characteristics, including age, gender, education, but also risk aversion, annual net income, and amount invested in financial markets. Some of the individual characteristics are based on surveys and pertain to gauging attitudes towards risk, i.e. measure risk aversion. These survey-based individual investor measures of risk aversion are collected by the brokerage firm within the context of the MiFID regulation for all EU member states that came into effect in November 2007.[16] European regulation has made it compulsory for brokerage firms to collect specific information about their retail clients' needs and preferences. Accordingly, investment firms operating in the EU are obliged to submit questionnaires (referred to as "MiFID tests") to their clients in order to determine their level of knowledge and experience, their investment objectives as well as their financial literacy. MiFID tests can be viewed as regulated Investment Policy Statements (IPS) required when any retail investor asks for financial advice and/or portfolio management services. It is worth noting that the MiFID regulation does not impose standardized questionnaires. Each brokerage firm is free to devise and organize its own questionnaire(s) provided it abides by some general guidelines.[17]

A.2 Sample of stocks and ETFs

Table A.3 shows that the 683 stocks retained in our sample are international assets. Although we work with Belgian individual investors, most of their trading activities concern foreign stocks (mainly US and neighboring countries) despite the well-known home bias.[18] This feature of our data is mainly due to the small size of the Belgian stock market. About 150 stocks are listed on the Euronext Brussels Stock Exchange in comparison to respectively, 1,791 and 2,545 domestic companies listed on the New York Stock Exchange and the Nasdaq

[16]MiFID stands for the Markets in Financial Instruments Directive. MiFID I (2004/39/EC) is known as the first version of this Directive while a review of it was recently implemented in January 2018 (known as MiFID II (2014/65/UE)). For more details, please visit the European Commission website (`http://ec.eurosopa.eu/internal_market/securities/isd/mifid2/index_en.htm`).

[17]For more details on the MiFID tests, please refer to Bellofatto, D'Hondt, and De Winne (2018).

[18]Investors' nationality is not available in the data set but we can reasonably assume that most of the investors are Belgian or based in Belgium.

(WFE Annual Statistics Guide 2017). Industry sectors are listed in Table A.4. The top 3 consist of technology (16.93%), financial (15.91%), and industrial stocks (14.01%).

International coverage of stocks

Countries	Stocks (%)
USA	26 %
France	22 %
Germany	18 %
Belgium	14 %
Netherlands	10 %
Others	10 %

TABLE A.3. Trades cover 12,818 different stocks. Those with few trades are discarded and we keep 683 stocks, which cover 71.5% of all the investors' trading activity on stocks. The table shows the main countries where the 683 stocks in our sample originate.

Stock distribution across industry sectors

Sectors	Stocks (%)
Technology	16.93
Financials	15.91
Industrials	14.01
Consumer Goods	10.07
Health Care	9.78
Consumer Services	9.63
Basic Materials	9.05
Oil and Gas	8.46
Utilities	3.50
Telecommunications	1.89

TABLE A.4. Trades cover 12,818 different stocks. Those with few trades are discarded and we keep 683 stocks, which cover 71.5% of all the investors' trading activity on stocks. The table reports the proportion of stocks across sectors based on the Industry Classification Benchmark (ICB) industry classification taxonomy. This information is available for 680 stocks in our sample.

Table A.5 provides summary information about the ETFs in our sample. Most of them are equities or commodities-linked. Although the majority of ETFs are passive (344 out of 393, i.e. 87%), some use a multiplier different from one. In particular, such leveraged and/or inverse ETFs display the highest number of trades (i.e. 24,624) in the equities category. This is consistent with the top 5 provided in Table A.6. Among the five most traded ETF, four are equities-linked, and, more importantly, three of them are either leveraged, inverse or even leveraged-inverse. Our sample period, which covers the 2008 financial crisis, probably explains why such non-passive ETFs attracted some investors. Among the 4,693 investors who traded ETF, 3,225 focused only on passive ETF; the others (i.e. 30%) traded non-passive ETFs (most of the time, in addition to passive ETF). Table A.6 also reveals that the

Statistics about ETFs underlying asset (or basket of assets)

Underlying	Multiplier=1	Multiplier $\neq$ 1
Panel A : # ETF		
Equities	227	29
Commodities	76	19
Fixed-income	31	0
Currencies	4	1
Real Estate	6	0
Panel B : # trades		
Equities	23,053	24,624
Commodities	11,501	436
Fixed-income	512	0
Currencies	11	2
Real Estate	205	0

TABLE A.5. Our sample is made of 393 different ETFs for a total number of 60,344 trades. The table reports aggregate statistics per type of underlying asset (or basket of assets). For each category, we distinguish ETFs based on their multiplier. When the multiplier is equal to one, the ETF simply tracks the underlying asset (or basket of assets). When the multiplier differs from one, the ETF can be leveraged, inverse, or even leveraged-inverse depending on the multiplier value. Panel A gives the number of ETFs while the corresponding number of trades are provided in Panel B.

trading activity on ETFs is mainly concentrated on some ETF, since the top 5 accounts for about 39% of the whole activity on ETFs (in number of trades or in euros).

ETFs Top 5

Name	# trades		euros	
Lyxor CAC 40 Daily (-2x) Inverse	10,098	(17%)	177,819,698	(20%)
Gold Bullion Securities ETC	4,080	(7%)	62,324,680	(7%)
Lyxor CAC 40 Daily (2x) Leveraged	3,583	(6%)	61,342,462	(7%)
Lyxor Euro Stoxx 50 Daily (-1x) Inverse	2,987	(5%)	28,499,711	(3%)
Lyxor BEL 20 ETFs	2,866	(5%)	17,453,004	(2%)

TABLE A.6. The table lists the five most traded ETFs in our sample. For each one, we report the aggregate number of trades and the monetary value in euros. Corresponding percentages as a fraction of the whole trading activity on the 393 ETFs are provided in brackets.

Table A.7 provides cross-sectional statistics about asset prices, monthly returns and risk factors. The latter are estimated only for stocks using daily prices and Fama-French data available online. Prices and market cap are in euros whereas returns are monthly. On average, the stocks in our sample have a one percent monthly return and 11 percent volatility. They are slightly right skewed and not surprisingly feature fat tails. For ETF, the average monthly return is 3.23% with an average volatility of 37.84%.

Cross-sectional statistics for asset prices, monthly returns and risk factors

	Mean	Q3	Median	Q1
Panel A : Stocks				
Price (euros)	39.74	34.41	19.49	9.92
Market Cap (euros)	14,485.74	13,112.77	2,601.70	591.39
Return (%)	1.01	1.68	0.88	0.32
Volatility (%)	11.02	13.52	10.52	7.79
Skewness	0.1956	0.5368	0.1348	-0.2125
Kurtosis	2.3082	2.9150	1.3910	0.6234
Market	0.3923	0.5817	0.4183	0.2083
SMB	-0.3823	0.1498	-0.3426	-0.8722
HML	0.2931	0.5454	0.2690	-0.0138
RMW	-0.0569	0.1744	-0.1136	-0.5022
CMA	-0.3884	-0.1029	-0.3365	-0.6760
Momentum	0.1449	0.3312	0.1243	-0.0724
Panel B : ETF				
Price (euros)	74.57	88.82	35.75	19.25
Return (%)	3.23	0.95	0.35	-0.08
Volatility (%)	37.84	8.73	6.26	4.83
Skewness	-0.1216	0.1963	-0.1839	-0.4933
Kurtosis	1.5899	1.6593	0.6270	0.0904

TABLE A.7. The table reports the cross-sectional mean, median, lower and upper quartiles for asset prices and monthly returns. Panel A refers to stocks while Panel B refers to ETF. *Price* is expressed in euros. *Market Cap* is expressed in millions of euros and is based on the monthly number of shares outstanding available for the stocks in our sample. *Return* is the monthly price return expressed in %. *Volatility* is the standard deviation of monthly returns expressed in %. *Skewness* and *Kurtosis* computed on monthly returns are also provided. *Market*, *SMB*, *HML*, *RMW* and *CMA* refer to the estimated Fama-French 5 factors for stocks. These factors summarize the excess return on the market (*Market*), the performance of small stocks relative to big stocks (*SMB*, Small Minus Big), the performance of value stocks relative to growth stocks (*HML*, High Minus Low), the performance of robust stocks relative to weak stocks (*RMW*, Robust Minus Weak), and the performance of conservative stocks relative to aggressive stocks (*CMA*, Conservative Minus Aggressive). *Momentum*, which is the average return on the two high prior return portfolios minus the average return on the two low prior return portfolios, is also estimated using Fama-French available data.

A.3 Sample of investors

Table A.8 shows that 10.11% of investors are female. Moreover, our sample is mainly composed of highly educated investors since 73.59% of them have a university degree or equivalent. As for risk aversion, the majority of investors seem to be risk tolerant since 65.33% of them are classified as medium risk averse and 27.88% of them even featuring low risk aversion. When considering the annual net income over the entire sample, we observe that about 70% of the investors declare an income between 20,000 and 75,000 euros. Only a minority (3.36%) earn more than 150,000 euros per year. The income measure reported in our data is recorded once, when the investor completed the MiFID tests. Hence, the classification may therefore be noisy over the 10-year sample period, particularly for the early entries. However, when looking at risk aversion and income together, the income range 20,000-75,000 euros features the most investors with a low or medium aversion while high risk averse investors tend to have lower incomes.

Investor Characteristics Joint Percentages

	Gender	Female			Male			Total		
RA	**Income // Educ**	No	HS	Univ	No	HS	Univ	Total		
Low	<20,000	0.05	0.19	0.16	0.19	0.82	1.49	2.90		13.86
	20,000-40,000	0.08	0.22	0.54	0.54	2.40	6.06	9.85		36.89
	40,000-75,000	0.06	0.16	0.62	0.45	1.56	6.42	9.28		33.02
	75,000-150,000	0.01	0.04	0.32	0.13	0.45	3.54	4.49		12.89
	>150,000	0.02	0.02	0.12	0.07	0.14	1.01	1.37	27.88	3.36
Mid	<20,000	0.14	0.41	0.66	0.74	2.06	4.01	8.01		
	20,000-40,000	0.22	0.74	1.72	1.56	5.10	15.66	25.01		
	40,000-75,000	0.14	0.30	1.45	1.20	2.90	16.53	22.52		
	75,000-150,000	0.07	0.07	0.54	0.34	0.45	6.45	7.94		
	>150,000	0.01	0.01	0.21	0.05	0.11	1.46	1.85	65.33	
High	<20,000	0.05	0.12	0.21	0.32	0.63	1.62	2.95		
	20,000-40,000	0.05	0.05	0.12	0.17	0.45	1.18	2.03		
	40,000-75,000	0.01	0.03	0.09	0.06	0.14	0.90	1.23		
	75,000-150,000	0.00	0.01	0.01	0.02	0.05	0.38	0.47		
	>150,000	0.00	0.01	0.01	0.01	0.01	0.09	0.13	6.80	
	Total	0.92	2.39	6.81	5.84	17.27	66.79			
Total				10.11			89.90			
		6.76	19.66	73.59						

TABLE A.8. The table reports investor characteristics joint percentages computed on survey data. For education (*Educ*), 'No' refers to investors without any degree, 'HS' to investors with a secondary/high school degree, and 'Univ' to investors with an university degree or equivalent. For risk aversion (*RA*), 'High' refers to investors who state a high risk aversion (i.e. they do not want to take any risks and prefer safe investments), 'Mid' refers to investors who state a medium risk aversion, and 'Low' to investors who state a low risk aversion (i.e. they would invest even more when facing a sharp loss of 20% to reduce their average purchase prices). For annual net income (*Income*), five categories are available: lower than 20,000 euros, 20,000-40,000 euros, 40,000-75,000 euros, 75,000-150,000 euros, and larger than 150,000 euros.

Table A.9 provides cross-sectional statistics for investors' age, monthly trading activity and portfolio. The average investor is about 48 years old. When focusing on stocks only (Panel

A), the average investor executes monthly 2.76 trades across 2.05 different stocks for a volume of 18,237 euros. Consistent with the literature, investors in our sample are under-diversified: the average (median) investor holds a five-stock (three-stock) portfolio. The average end-of-month portfolio value is about 28,003 euros (with a median value of about 7,552 euros). For comparison, Kumar and Lee (2006) and Korniotis and Kumar (2013) document that a typical investor at a major U.S. discount brokerage house over the period 1991-1996 holds a four-stock portfolio (with a median of three) with an average size of 35,629 USD (with a median of 13,869 USD). Using a more recent sample, Leal, Loureiro, and Armada (2017) find that a typical investor at a Portuguese brokerage holds 2.34 different stocks over the period 2003-2007. As a proxy of experience, we look at the investor stock portfolio holding

Cross-sectional statistics for investors' age, monthly trading activity and portfolio

	Mean	Q3	Median	Q1
Age	48.57	58	48	38
Panel A : Stocks only				
# trades	2.76	3	2	1.4
Volume (euros)	18,237.29	13,278.06	5,584.25	2,488.75
# ISIN	2.05	2.36	1.68	1.25
# assets	4.43	5.66	2.75	1.27
Portfolio value (euros)	28,003.08	22,350.32	7,552.07	2,484.01
# months	54.12	81	50	27
Return (%)	0.28	0.79	0.03	-0.70
Volatility (%)	10.63	11.66	8.02	5.89
Panel B : Stocks and ETF				
# trades	3.88	4.22	2.66	1.88
Volume (euros)	29,211.35	20,936.43	9,115.29	4,430.90
# ISIN	2.73	3.15	2.18	1.6
# assets	6.68	9.17	4.5	2
Portfolio value (euros)	49,961.58	41,461.24	14,696.78	4,925.21
# months	57.47	88	59	29
Return (%)	0.42	0.72	0.13	-0.42
Volatility (%)	10.04	9.38	6.72	5.35

TABLE A.9. The table reports the cross-sectional mean, median, lower and upper quartiles for investors' age as well as trade-based and portfolio-based variables. Panel A refers to stocks only and Panel B refers to both stocks and ETF. Consequently, statistics in Panel A are computed across the entire sample (i.e. 22,972 investors) while those reported in Panel B are computed across the subsample of investors who traded ETFs in addition to stocks (i.e. 4,693 investors). *Age* is computed as the difference between 2012 and the investor's year of birth. *# trades* is the monthly number of trades executed. *Volume* is the corresponding monthly monetary volume in euros. *# ISIN* is the monthly number of different assets traded. *# assets* is the monthly average number of assets held portfolio computed over the holding period. *Portfolio value* is the corresponding monthly average market portfolio value in euros. *# months* refers to the portfolio holding period. *Return* is the monthly average gross portfolio return expressed in %. *Volatility* is the standard deviation of monthly gross portfolio returns expressed in %.

period (*# months*): our average investor holds stocks for 54.12 months out of 111. However, this average hides different holding patterns: 21,865 investors (i.e. 95%) hold stocks at some point over the sample period. In particular, 1.9% of them hold throughout, 1.5% hold then leave at some point, 81.5% enter and hold throughout and 15.1% enter and exit.

In terms of performance, our average investor earns a monthly gross return of 0.28% on stocks, with a volatility of 10.63%. This relatively high average volatility of individual stock portfolio gross returns is not surprising given our sample period that includes market ups and downs.

Statistics for ETFs usage

	ETFs users	non-ETFs users
Age	51.40***	47.85
% Females	9.14	9.91
Education (%)		
No	4.54***	7.39
HS	16.51***	20.60
Univ	78.95***	72.01
Risk aversion (%)		
Low	30.79***	27.27
Mid	62.20***	65.81
High	7.01	6.92
Income (%)		
Low	12.21***	14.62
Mid	82.87	82.43
High	4.92***	2.94
Funds invested (%)		
Low	35.86***	53.20
Mid	34.88***	29.77
High	29.26***	17.03

TABLE A.10. The table reports the mean age and characteristics percentages computed on survey data for investors who traded ETFs ('ETF users') versus those who did not ('non-ETF users'). 'Age' is defined as the difference between 2012 and the investor's year of birth. For the level of education ('Education'), 'No' refers to investors without any degree, 'HS' to investors with a secondary/high school degree, and 'Univ' to investors with an university degree or equivalent. For the level of risk aversion ('Risk aversion'), 'High' refers to investors who state a high risk aversion (i.e. they do not want to take any risks and prefer safe investments), 'Mid' refers to investors who state a medium risk aversion, and 'Low' to investors who state a low risk aversion (i.e. they would invest even more when facing a sharp loss of 20% to reduce their average purchase prices). For annual net income ('Income'), 'High' refers to investors who report they earn more than 150,000 euros, 'Mid' to investors who earn between 20,000-150,000 euros, and 'Low' to investors who state earning less than 20,000 euros. For the amount invested in financial markets ('Amount invested'), 'High' refers to investors who report they invest more than 250,000 euros, 'Mid' to investors who invest between 50,000-250,000 euros, and 'Low' to investors who invest less than 50,000 euros. *, **, *** indicate whether percentages (or means) differ between ETF users and non-ETF users at the level of 10%, 5%, and 1%, respectively.

Panel B of Table A.9 reports similar cross-sectional statistics for the subsample of investors who traded both stocks and ETF. Not surprisingly, these investors exhibit a higher monthly trading activity, either in number of trades or in euros. Combining stocks and ETF, these investors hold more diversified and wealthy portfolios: the typical investor holds a six-asset portfolio (with a median of 4.5) with an average size of 49,961 euros (with a median of 14,696 euros). This better diversification appears profitable since the average investor earns a monthly gross return of 0.42%, with a volatility of 10.04%.

We should point out that almost all the variables in Table A.9 are positively skewed and display a large heterogeneity. This is consistent with usual tremendous variations in behavior and in outcomes across individual investors (Barber and Odean (2013)). Finally, Table A.10 provides statistics on ETF usage in our sample. They show that ETF users are somewhat older and that they are more educated, report lower risk aversion, have higher income, invest more money in financial markets. In addition, they are more active investors on stocks (based on both the number of trades or the monetary volume traded).

Table A.11: Summary Statistics Disposition Effect

	Mean	SD	Median	Obs
Global	8.60	23.83	3.35	20,622
Education				
Low	7.36	15.76	2.81	1,374
High	8.49	25.60	3.24	15,149
Low-High pval	*0.0467*			
Risk Aversion				
Low	10.33	20.75	4.22	5,858
High	10.07	22.60	3.78	1,423
Low-High pval	*0.7210*			
Income				
Low	8.30	18.01	3.14	2,849
High	13.94	47.50	5.24	696
Low-High pval	*0.0021*			

Notes: Summary statistics for the disposition effect – selling winners too soon, holding losers too long – as in Odean (1998) for each investor. Entries to the table are the average disposition effect, their standard deviations, median and sample sizes. The p-values (pval) of the differences in mean between high and low tests within education, risk aversion and income investors are reported.

A.4 Behavioral Biases

The investors in our sample feature the typical behavioral biases. We start with disposition effect (DE) - selling winners too soon, holding on to losers too long - as in Odean (1998) for each investor. The DE is a difference of proportions: Proportion of Gains Realized (PGR) minus Proportion of Losses Realized (PLR), where PGR = 100*Number of realized gains / (Number of realized gains + Number of Paper Gains), the same reasoning for losses.

Some summary statistics of the disposition effect as observed for different sub-populations appear in Table A.11. Entries to the table are the average disposition effect, their standard deviations, median and sample sizes. Higher values imply large DE, meaning investors sell winners too soon and hold on to loosers too long. We also report significance between means tests for the various sub-populations. For all classifications there are important differences. Low minus high level of education result in significant large DE for the former. Same for low minus high risk aversion and for low minus high incomes. The results in Table A.11 indicate that less educated investors feature large DE effects compared to their highly educated peers. Similarly, low income investors have large DE effects than high income ones. Finally, high risk aversion also results in large DE compared to the less averse investors.

Table A.12: Summary Statistics Monthly Trading Frequency Stocks and ETFs

	Mean	SD	Median	Obs
Global	2.80	3.22	2	22,972
Education				
Low	2.80	3.25	2	1,563
High	2.76	3.28	2	16,868
Low-High pval	*0.5760*			
Risk Aversion				
Low	3.13	3.33	2.23	6,430
High	3.18	3.43	2.16	1,594
Low-High pval	*0.6060*			
Income				
Low	2.79	2.83	2	3,246
High	3.52	5.49	2.35	769
Low-High pval	*0.0004*			

Notes: Summary statistics for trading frequency, defined as the average number of monthly trades, for each investor. Entries to the table are the averages, standard deviations, median and sample sizes. The p-values (pval) of the differences in mean between high and low tests within education, risk aversion and income investors are reported.

Table A.12 tells us something about trading frequencies. The median investor trades two assets a month, with a slightly higher mean of 2.8. There is very little variation in the median across the various cross-sectional sample splits. The means of trading frequencies are also relatively flat across the various sub-populations, with the exception perhaps for income, where high income investors trade more often.

Online Appendix

Table OA.1: Returns Monthly Rebalancing

	Equities				*Equities & ETFs*			
	Mean	Median	25%	75%	Mean	Median	25%	75%
	MV - Rolling Means/Linear Covariances							
Mean	-0.46	0.00	-4.71	3.44	0.17	0.00	-4.05	4.00
SD	12.52	12.63	8.45	16.51	12.09	12.07	8.47	15.78
Sharpe Ratio	-0.03	-0.05	-0.43	0.33	0.01	0.01	-0.38	0.38
Max Draw		0.2105	0.0838	0.3315		0.2016	0.0841	0.3131
	MV - ML Means/Linear Covariances							
Mean	-0.18	0.00	-5.90	5.48	0.60	0.32	-4.80	6.06
SD	18.11	18.31	12.60	23.58	17.72	17.79	12.57	22.72
Sharpe Ratio	-0.01	0.00	-0.35	0.35	0.07	0.05	-0.30	0.39
Max Draw		0.2698	0.1120	0.4248		0.2686	0.1120	0.4156
	MV - ML Means/Non-Linear Covariances							
Mean	0.12	0.00	-5.52	5.57	0.19	0.02	-5.29	5.76
SD	17.79	18.05	12.24	23.27	17.41	17.42	12.04	22.66
Sharpe Ratio	0.02	0.02	-0.35	0.36	0.01	0.03	-0.33	0.38
Max Draw		0.2643	0.1060	0.4196		0.2610	0.1081	0.4143
	Equal-Weighted							
Mean	-2.03	-1.32	-8.49	4.37	-1.79	-1.10	-8.04	4.60
SD	19.59	19.41	14.17	24.87	19.38	19.15	14.05	24.46
Sharpe Ratio	-0.01	-0.09	-0.41	0.29	0.02	-0.08	-0.39	0.30
Max Draw		0.3591	0.1439	0.5735		0.3605	0.1473	0.5699

Notes: Entries are mean, median, first (Q1) and third (Q3) quartiles of the cross-sectional distributions of AI Alter Ego returns for three Mean Variance (MV) types of robo-investors and one equally weighted (EW) considering only equity holdings (left panel) or the entire universe of 683 stocks and 393 ETFs (right panel). The AI Alter Ego schemes are: (a) MV with Rolling Mean/Rolling Variance, (b) Machine Learning (ML) Mean/Rolling Variance - using the methods displayed in Table **??**, (c) ML Mean/Nonlinear Smoothed Variance, and finally the equally weighted portfolio scheme. The spreads are in percentage per year.

Table OA.2: Returns Monthly Rebalancing - Education-Sorted

	Low				High			
	Mean	Median	25q	75q	Mean	Median	25q	75q
	MV - Rolling Means/Linear Covariances							
Mean	-27.60	-0.04	-12.39	12.79	62.60	-0.63	-12.42	12.00
SD	344.64	34.26	23.50	52.98	540.79	32.81	22.98	51.95
Sharpe Ratio	0.03	-0.01	-0.34	0.37	0.06	-0.02	-0.35	0.36
Max Draw		0.56	0.33	0.76		0.57	0.32	0.77
	MV - ML/Linear Covariances							
Mean	-27.60	-0.04	-12.39	12.79	62.60	-0.63	-12.42	12.00
SD	344.64	34.26	23.50	52.98	540.79	32.81	22.98	51.95
Sharpe Ratio	0.03	-0.01	-0.34	0.37	0.06	-0.02	-0.35	0.36
Max Draw		0.56	0.33	0.76		0.57	0.32	0.77
	MV - ML/Non-Linear Covariances							
Mean	-27.60	-0.04	-12.39	12.79	62.60	-0.63	-12.42	12.00
SD	344.64	34.26	23.50	52.98	540.79	32.81	22.98	51.95
Sharpe Ratio	0.03	-0.01	-0.34	0.37	0.06	-0.02	-0.35	0.36
Max Draw		0.56	0.33	0.76		0.57	0.32	0.77
	Equal-Weighted							
Mean	-27.60	-0.04	-12.39	12.79	62.60	-0.63	-12.42	12.00
SD	344.64	34.26	23.50	52.98	540.79	32.81	22.98	51.95
Sharpe Ratio	0.03	-0.01	-0.34	0.37	0.06	-0.02	-0.35	0.36
Max Draw		0.56	0.33	0.76		0.57	0.32	0.77

Table OA.3: Returns Monthly Rebalancing - Risk Aversion-Sorted

	Low				High			
	Mean	Median	25q	75q	Mean	Median	25q	75q
				MV - Rolling Means/Linear Covariances				
Mean	-26.13	-0.42	-12.49	12.67	164.92	-1.94	-12.38	10.59
SD	341.43	32.51	23.04	51.29	1,094.82	32.71	22.99	49.50
Sharpe Ratio	0.09	-0.02	-0.34	0.38	-0.08	-0.06	-0.36	0.30
Max Draw		0.56	0.31	0.75		0.58	0.35	0.78
				MV - ML/Linear Covariances				
Mean	-26.13	-0.42	-12.49	12.67	164.92	-1.94	-12.38	10.59
SD	341.43	32.51	23.04	51.29	1,094.82	32.71	22.99	49.50
Sharpe Ratio	0.09	-0.02	-0.34	0.38	-0.08	-0.06	-0.36	0.30
Max Draw		0.56	0.31	0.75		0.58	0.35	0.78
				MV - ML/Non-Linear Covariances				
Risk-Aversion								
Mean	-26.13	-0.42	-12.49	12.67	164.92	-1.94	-12.38	10.59
SD	341.43	32.51	23.04	51.29	1,094.82	32.71	22.99	49.50
Sharpe Ratio	0.09	-0.02	-0.34	0.38	-0.08	-0.06	-0.36	0.30
Max Draw		0.56	0.31	0.75		0.58	0.35	0.78
				Equal-Weighted				
Mean	-26.13	-0.42	-12.49	12.67	164.92	-1.94	-12.38	10.59
SD	341.43	32.51	23.04	51.29	1,094.82	32.71	22.99	49.50
Sharpe Ratio	0.09	-0.02	-0.34	0.38	-0.08	-0.06	-0.36	0.30
Max Draw		0.56	0.31	0.75		0.58	0.35	0.78

Table OA.4: Returns Monthly Rebalancing - Income-Sorted

	Low					High			
	Mean	Median	25q	75q		Mean	Median	25q	75q
				MV - Rolling Means/Linear Covariances					
Mean	-147.04	-0.94	-12.36	11.92		-75.31	0.48	-9.74	13.55
SD	540.00	32.47	23.21	50.05		217.85	33.39	23.85	51.71
Sharpe Ratio	0.00	-0.04	-0.36	0.35		0.10	0.01	-0.27	0.38
Max Draw		0.55	0.31	0.76			0.61	0.35	0.78
				MV - ML/Linear Covariances					
Mean	-147.04	-0.94	-12.36	11.92		-75.31	0.48	-9.74	13.55
SD	540.00	32.47	23.21	50.05		217.85	33.39	23.85	51.71
Sharpe Ratio	0.00	-0.04	-0.36	0.35		0.10	0.01	-0.27	0.38
Max Draw		0.55	0.31	0.76			0.61	0.35	0.78
				MV - ML/Non-Linear Covariances					
Mean	-147.04	-0.94	-12.36	11.92		-75.31	0.48	-9.74	13.55
SD	540.00	32.47	23.21	50.05	217.85		33.39	23.85	51.71
Sharpe Ratio	0.00	-0.04	-0.36	0.35		0.10	0.01	-0.27	0.38
Max Draw		0.55	0.31	0.76			0.61	0.35	0.78
				Equal-Weighted					
Mean	-147.04	-0.94	-12.36	11.92		-75.31	0.48	-9.74	13.55
SD	540.00	32.47	23.21	50.05		217.85	33.39	23.85	51.71
Sharpe Ratio	0.00	-0.04	-0.36	0.35		0.10	0.01	-0.27	0.38
Max Draw		0.55	0.31	0.76			0.61	0.35	0.78

Table OA.5: AI Spreads Monthly Rebalancing

	Equities				*Equities & ETFs*			
	Mean	Median	25%	75%	Mean	Median	25%	75%
	MV - Rolling Means/Linear Covariances							
Mean	-43.44	-0.20	-13.73	12.67	-40.27	0.40	-13.01	12.86
SD	476.02	35.29	26.25	53.25	485.10	35.12	26.00	53.20
Sharpe Ratio	-0.06	-0.00	-0.37	0.33	-0.05	0.01	-0.35	0.34
Max Draw		0.5357	0.3201	0.7452		0.5333	0.3165	0.7429
	MV - ML/Linear Covariances							
Mean	-43.17	0.01	-14.07	13.37	-39.83	0.64	-12.87	13.73
SD	477.63	37.39	28.44	54.34	486.62	37.17	28.12	54.10
Sharpe Ratio	-0.08	0.00	-0.36	0.33	-0.05	0.02	-0.33	0.34
Max Draw		0.5263	0.3184	0.7365		0.5182	0.3173	0.7295
	MV - Rolling Means/Non-Linear Covariances							
Mean	-42.86	0.23	-13.78	13.44	-40.24	0.20	-13.47	13.41
SD	477.51	37.21	28.30	54.12	486.63	37.19	28.11	54.14
Sharpe Ratio	-0.05	0.01	-0.35	0.34	-0.07	0.01	-0.35	0.34
Max Draw		0.5269	0.3171	0.7358		0.5300	0.3191	0.7401
	Equal-Weighted							
Mean	-45.02	-1.88	-15.22	10.70	-42.23	-1.75	-15.03	10.71
SD	477.13	37.00	28.15	53.61	486.29	36.91	28.07	53.71
Sharpe Ratio	-0.14	-0.05	-0.39	0.27	-0.12	-0.05	-0.38	0.27
Max Draw		0.5140	0.3097	0.7231		0.5162	0.3106	0.7295

Notes: Entries are mean, median, first (Q1) and third (Q3) quartiles of the cross-sectional distributions of the spreads between AI Alter Ego and individual investor returns, $r^s_{i,t} = r^{ae}_{i,t}$ - $r_{i,t}$, for three Mean Variance (MV) types of robo-investors and one equally weighted (EW) considering only equity holdings (left panel) or the entire universe of 683 stocks and 393 ETFs (right panel). The AI Alter Ego schemes are: (a) MV with Rolling Mean/Rolling Variance, (b) Machine Learning (ML) Mean/Rolling Variance - using the methods displayed in Table **??**, (c) ML Mean/Nonlinear Smoothed Variance, and finally the equally weighted portfolio scheme. The spreads are in percentage per year.

Table OA.6: AI Spreads Equities & ETFs Monthly Rebalancing - Education-Sorted

	Low				High				Low		High	
	Spreads								Median 95 % CI			
	Mean	Median	25q	75q	Mean	Median	25q	75q	2.5%	97.5%	2.5%	97.5%
					MV - Rolling Means/Linear Covariances							
Mean	32.82	0.20	-13.83	13.35	-59.73	0.50	-12.91	12.84	-1.15	1.69	0.13	0.91
SD	353.37	36.01	25.96	53.97	547.00	35.19	26.12	53.60	34.88	37.46	34.86	35.50
Sharpe Ratio	-0.03	0.01	-0.38	0.34	-0.04	0.02	-0.35	0.34	-0.02	0.05	0.01	0.03
Max Draw		0.53	0.31	0.75		0.54	0.32	0.75	0.52	0.55	0.53	0.54
					MV - ML/Linear Covariances							
Mean	32.90	-0.14	-14.37	13.17	-59.21	0.81	-12.75	13.85	-1.58	1.51	0.42	1.26
SD	355.09	37.83	28.81	55.24	548.46	37.19	28.16	54.45	37.03	38.73	36.80	37.55
Sharpe Ratio	-0.08	-0.00	-0.36	0.32	-0.04	0.03	-0.33	0.35	-0.05	0.05	0.01	0.04
Max Draw		0.52	0.32	0.73		0.52	0.32	0.73	0.51	0.54	0.52	0.53
					MV - ML/Non-Linear Covariances							
Mean	32.67	-0.35	-14.67	13.06	-59.66	0.29	-13.37	13.46	-1.84	1.00	-0.07	0.79
SD	355.13	37.94	28.96	55.24	548.45	37.28	28.14	54.43	36.87	38.96	37.00	37.66
Sharpe Ratio	-0.06	-0.01	-0.37	0.34	-0.07	0.01	-0.34	0.34	-0.05	0.03	-0.00	0.02
Max Draw		0.53	0.32	0.74		0.53	0.32	0.74	0.51	0.54	0.53	0.54
					Equal-Weighted							
Mean	30.72	-2.43	-15.03	10.79	-61.65	-1.65	-14.95	10.70	-3.65	-0.61	-2.05	-1.30
SD	354.66	37.81	28.55	54.07	548.14	36.96	28.16	54.01	36.93	38.92	36.62	37.27
Sharpe Ratio	-0.12	-0.06	-0.40	0.26	-0.12	-0.05	-0.38	0.27	-0.10	-0.01	-0.06	-0.04
Max Draw		0.52	0.31	0.73		0.52	0.31	0.73	0.50	0.53	0.52	0.52

Table OA.7: AI Spreads Equities & ETFs Monthly Rebalancing - Risk Aversion-Sorted

	Low				High				Low		High	
	Spreads								Median 95 % CI			
	Mean	Median	25q	75q	Mean	Median	25q	75q	2.5%	97.5%	2.5%	97.5%
MV - Rolling Means/Linear Covariances												
Mean	23.23	0.33	-13.57	13.14	-182.26	1.37	-11.19	13.68	-0.46	0.98	-0.04	2.77
SD	348.67	34.81	26.03	53.08	1,080.05	35.44	26.00	51.14	34.39	35.42	34.19	36.60
Sharpe Ratio	-0.08	0.01	-0.37	0.34	0.09	0.04	-0.31	0.34	-0.01	0.03	-0.01	0.08
Max Draw		0.53	0.32	0.74		0.54	0.33	0.75	0.52	0.54	0.52	0.56
MV - ML/Linear Covariances												
Mean	23.74	0.48	-13.07	14.19	-181.85	1.72	-11.76	14.39	-0.34	1.14	0.24	3.02
SD	350.19	36.91	28.03	54.00	1,081.45	37.36	28.00	52.73	36.44	37.30	36.19	38.55
Sharpe Ratio	-0.05	0.02	-0.34	0.35	0.03	0.06	-0.30	0.36	-0.01	0.04	0.02	0.10
Max Draw		0.51	0.31	0.73		0.52	0.32	0.74	0.50	0.52	0.50	0.54
MV - ML/Non-Linear Covariances												
Mean	23.32	0.06	-13.73	13.55	-182.18	1.57	-12.52	14.67	-0.67	0.77	0.31	3.09
SD	350.24	36.74	28.13	53.99	1,081.43	37.50	28.09	52.52	36.26	37.21	36.54	38.91
Sharpe Ratio	-0.05	0.00	-0.36	0.33	-0.08	0.05	-0.31	0.37	-0.02	0.02	0.01	0.09
Max Draw		0.52	0.32	0.74		0.52	0.31	0.75	0.52	0.53	0.50	0.54
Equal-Weighted												
Mean	21.26	-2.02	-15.35	10.48	-184.70	-0.89	-14.35	10.69	-2.80	-1.25	-2.05	0.16
SD	349.91	36.69	28.07	53.48	1,081.14	37.10	28.28	51.68	36.11	37.34	36.06	38.12
Sharpe Ratio	-0.16	-0.06	-0.40	0.27	-0.08	-0.02	-0.36	0.29	-0.08	-0.03	-0.05	0.02
Max Draw		0.51	0.31	0.73		0.52	0.32	0.74	0.50	0.52	0.51	0.54

Table OA.8: AI Spreads Equities & ETFs Monthly Rebalancing - Income-Sorted

	Low				High				Low		High	
	Spreads								Median 95 % CI			
	Mean	Median	25q	75q	Mean	Median	25q	75q	2.5%	97.5%	2.5%	97.5%
					MV - Rolling Means/Linear Covariances							
Mean	130.96	0.68	-12.24	12.76	65.56	-0.25	-14.25	10.26	-0.30	1.38	-2.51	1.71
SD	548.38	35.08	26.00	51.48	222.69	35.80	26.51	53.30	34.25	35.76	34.19	37.74
Sharpe Ratio	-0.01	0.02	-0.34	0.34	-0.09	-0.00	-0.34	0.26	-0.01	0.05	-0.08	0.06
Max Draw		0.52	0.30	0.73		0.56	0.35	0.77	0.51	0.53	0.54	0.60
					MV - ML/Linear Covariances							
Mean	131.53	0.98	-12.72	14.79	66.11	0.26	-12.94	10.85	0.06	2.10	-1.54	2.67
SD	550.15	37.13	28.52	53.61	224.33	37.19	28.49	54.96	36.40	37.88	35.61	38.74
Sharpe Ratio	-0.02	0.03	-0.33	0.35	-0.05	0.01	-0.33	0.29	-0.00	0.06	-0.04	0.07
Max Draw		0.50	0.29	0.71		0.55	0.35	0.75	0.49	0.51	0.51	0.57
					MV - ML/Non-Linear Covariances							
Mean	131.04	0.32	-13.00	14.19	65.69	-0.15	-13.76	10.65	-0.52	1.46	-2.19	1.51
SD	550.01	37.03	28.32	53.08	224.39	37.73	28.55	54.92	36.32	37.69	36.17	39.35
Sharpe Ratio	-0.01	0.01	-0.35	0.35	-0.05	-0.00	-0.34	0.28	-0.01	0.04	-0.06	0.05
Max Draw		0.51	0.30	0.73		0.57	0.36	0.76	0.50	0.53	0.53	0.60
					Equal-Weighted							
Mean	129.15	-1.18	-14.60	10.81	64.01	-1.95	-15.14	9.68	-2.20	-0.29	-4.04	0.17
SD	549.67	36.57	28.14	52.22	223.71	37.33	28.22	54.00	35.83	37.21	35.69	38.84
Sharpe Ratio	-0.05	-0.03	-0.38	0.28	-0.15	-0.05	-0.37	0.23	-0.07	-0.01	-0.12	0.01
Max Draw		0.50	0.30	0.71		0.56	0.35	0.74	0.49	0.51	0.53	0.59

Table OA.9: Returns Quarterly Rebalancing

	Equities				*Equities & ETFs*			
	Mean	Median	25%	75%	Mean	Median	25%	75%
				MV - Rolling Means/Linear Covariances				
Mean	-0.25	0.00	-4.31	3.53	0.38	0.00	-3.43	4.16
SD	12.62	12.73	8.61	16.61	12.29	12.28	8.67	15.93
Sharpe Ratio	0.05	-0.03	-0.39	0.34	-0.18	0.03	-0.33	0.39
Max Draw		0.2022	0.0848	0.3259		0.1964	0.0840	0.3116
				MV - ML Means/Linear Covariances				
Mean	3.42	1.94	-2.92	9.14	3.84	2.56	-2.26	9.17
SD	17.80	17.96	11.80	23.58	17.22	17.32	11.77	22.47
Sharpe Ratio	0.15	0.17	-0.21	0.52	0.22	0.20	-0.16	0.54
Max Draw		0.2239	0.0916	0.3734		0.2248	0.0912	0.3725
				MV - ML Means/Non-Linear Covariances				
Mean	3.31	1.75	-2.85	8.89	3.62	2.11	-2.58	9.11
SD	17.55	17.75	11.49	23.39	17.13	17.01	11.32	22.66
Sharpe Ratio	0.07	0.16	-0.21	0.51	0.20	0.19	-0.19	0.54
Max Draw		0.2213	0.0891	0.3716		0.2192	0.0880	0.3675
				Equal-Weighted				
Mean	-0.74	-0.34	-6.83	5.23	-0.51	-0.11	-6.40	5.45
SD	19.30	19.11	14.04	24.34	19.09	18.90	13.91	23.96
Sharpe Ratio	-0.01	-0.04	-0.35	0.33	0.01	-0.02	-0.33	0.35
Max Draw		0.3449	0.1411	0.5387		0.3437	0.1440	0.5338

Notes: Entries are mean, median, first (Q1) and third (Q3) quartiles of the cross-sectional distributions of AI Alter Ego returns for three Mean Variance (MV) types of robo-investors and one equally weighted (EW) considering only equity holdings (left panel) or the entire universe of 683 stocks and 393 ETFs (right panel). The AI Alter Ego schemes are: (a) MV with Rolling Mean/Rolling Variance, (b) Machine Learning (ML) Mean/Rolling Variance - using the methods displayed in Table **??**, (c) ML Mean/Nonlinear Smoothed Variance, and finally the equally weighted portfolio scheme. The spreads are in percentage per year.

Table OA.10: AI Spreads Quarterly Rebalancing

	Equities				*Equities & ETFs*			
	Mean	Median	25%	75%	Mean	Median	25%	75%
				MV - Rolling Means/Linear Covariances				
Mean	-43.23	-0.08	-13.60	12.84	-40.06	0.58	-12.72	13.16
SD	476.19	35.53	26.42	53.34	485.30	35.38	26.25	53.34
Sharpe Ratio	-0.02	-0.00	-0.37	0.33	-0.03	0.02	-0.34	0.34
Max Draw		0.5363	0.3167	0.7476		0.5344	0.3150	0.7461
				MV - ML/Linear Covariances				
Mean	-39.57	2.93	-10.66	17.41	-36.60	3.37	-10.19	17.20
SD	477.89	37.77	28.56	54.90	486.80	37.41	28.16	54.65
Sharpe Ratio	0.02	0.08	-0.28	0.41	0.05	0.10	-0.27	0.41
Max Draw		0.5029	0.3017	0.7224		0.4994	0.3001	0.7213
				MV - Rolling Means/Non-Linear Covariances				
Mean	-39.67	2.78	-10.72	17.13	-36.82	2.96	-10.49	17.09
SD	477.80	37.60	28.43	54.91	486.88	37.52	28.25	54.89
Sharpe Ratio	0.05	0.08	-0.28	0.41	0.01	0.08	-0.28	0.41
Max Draw		0.5041	0.3022	0.7220		0.5080	0.3036	0.7279
				Equal-Weighted				
Mean	-43.73	-0.67	-13.88	11.96	-40.95	-0.54	-13.68	11.98
SD	477.24	37.03	28.11	53.76	486.39	36.94	28.04	53.81
Sharpe Ratio	-0.13	-0.02	-0.36	0.30	-0.21	-0.01	-0.35	0.30
Max Draw		0.5003	0.3063	0.7073		0.5032	0.3065	0.7143

Notes: Entries are mean, median, first (Q1) and third (Q3) quartiles of the cross-sectional distributions of the spreads between AI Alter Ego and individual investor returns, $r^s_{i,t} = r^{ae}_{i,t}$ - $r_{i,t}$, for three Mean Variance (MV) types of robo-investors and one equally weighted (EW) considering only equity holdings (left panel) or the entire universe of 683 stocks and 393 ETFs (right panel). The AI Alter Ego schemes are: (a) MV with Rolling Mean/Rolling Variance, (b) Machine Learning (ML) Mean/Rolling Variance - using the methods displayed in Table **??**, (c) ML Mean/Nonlinear Smoothed Variance, and finally the equally weighted portfolio scheme. The spreads are in percentage per year.

Table OA.11: AI Spreads Equities & ETFs Quarterly Rebalancing - Education-Sorted

	Low				High				Low		High	
	Spreads								Median 95 % CI			
	Mean	Median	25q	75q	Mean	Median	25q	75q	2.5%	97.5%	2.5%	97.5%
MV - Rolling Means/Linear Covariances												
Mean	33.13	0.33	-13.78	12.76	-59.53	0.67	-12.62	13.30	-1.30	1.56	0.24	1.02
SD	353.61	36.04	26.47	54.34	547.19	35.46	26.37	53.76	34.96	36.98	35.09	35.86
Sharpe Ratio	-0.02	0.01	-0.36	0.34	-0.02	0.02	-0.34	0.35	-0.03	0.05	0.01	0.03
Max Draw		0.53	0.31	0.75		0.54	0.32	0.75	0.51	0.54	0.53	0.54
MV - ML/Linear Covariances												
Mean	36.33	2.83	-10.47	16.79	-55.95	3.46	-10.15	17.26	1.18	4.22	3.01	3.85
SD	354.97	38.19	28.82	55.48	548.69	37.53	28.30	55.10	37.12	39.11	37.21	37.91
Sharpe Ratio	0.13	0.08	-0.27	0.42	0.05	0.10	-0.27	0.42	0.03	0.12	0.09	0.11
Max Draw		0.50	0.30	0.73		0.50	0.30	0.73	0.48	0.51	0.50	0.51
MV - ML/Non-Linear Covariances												
Mean	36.21	2.62	-10.83	17.17	-56.17	3.08	-10.46	17.17	1.03	4.36	2.62	3.49
SD	3355.10	38.13	28.57	55.95	548.78	37.62	28.40	55.26	36.77	39.32	37.24	37.97
Sharpe Ratio	0.12	0.07	-0.28	0.40	0.01	0.09	-0.28	0.42	0.03	0.12	0.07	0.10
Max Draw		0.51	0.30	0.74		0.51	0.31	0.73	0.49	0.52	0.51	0.52
Equal-Weighted												
Mean	32.21	-1.21	-13.78	12.13	-60.39	-0.52	-13.64	12.08	-2.63	0.36	-0.87	-0.20
SD	354.72	37.86	28.41	54.17	548.24	37.00	28.13	54.16	36.87	39.18	36.66	37.35
Sharpe Ratio	-0.06	-0.03	-0.36	0.30	-0.25	-0.01	-0.35	0.30	-0.07	0.01	-0.02	-0.00
Max Draw		0.51	0.31	0.71		0.51	0.31	0.72	0.49	0.52	0.50	0.51

Table OA.12: AI Spreads Equities & ETFs Quarterly Rebalancing - Risk Aversion-Sorted

	Low				High				Low		High	
	Spreads								Median 95 % CI			
	Mean	Median	25q	75q	Mean	Median	25q	75q	2.5%	97.5%	2.5%	97.5%
					MV - Rolling Means/Linear Covariances							
Mean	23.39	0.31	-13.39	13.63	-181.67	1.79	-10.57	13.56	-0.29	0.85	0.51	3.12
SD	348.92	35.18	26.31	53.17	1,080.30	35.43	26.30	51.53	34.67	35.69	34.20	36.61
Sharpe Ratio	-0.07	0.01	-0.37	0.34	0.10	0.05	-0.30	0.36	-0.01	0.03	0.01	0.10
Max Draw		0.53	0.31	0.74		0.54	0.32	0.76	0.52	0.54	0.52	0.55
					MV - ML/Linear Covariances							
Mean	26.75	3.29	-10.86	17.43	-178.62	5.14	-9.12	17.28	2.39	4.28	3.63	6.36
SD	350.37	37.34	28.10	54.61	1,081.66	37.75	28.30	52.77	36.92	37.80	36.70	38.88
Sharpe Ratio	0.03	0.10	-0.30	0.42	0.08	0.15	-0.25	0.42	0.08	0.12	0.11	0.18
Max Draw		0.50	0.30	0.72		0.50	0.30	0.74	0.49	0.50	0.49	0.52
					MV - ML/Non-Linear Covariances							
Mean	26.45	2.75	-11.00	17.09	-178.79	4.50	-9.58	17.86	2.02	3.35	3.17	6.04
SD	350.46	37.47	28.12	54.79	1,081.72	37.71	28.36	52.83	36.93	37.99	36.41	38.89
Sharpe Ratio	-0.07	0.08	-0.30	0.41	0.08	0.14	-0.26	0.43	0.06	0.09	0.09	0.17
Max Draw		0.50	0.30	0.72		0.51	0.31	0.74	0.50	0.51	0.50	0.53
					Equal-Weighted							
Mean	22.52	-0.94	-14.05	11.95	-183.51	0.42	-12.82	11.98	-1.73	-0.29	-0.80	1.68
SD	350.00	36.69	28.11	53.48	1,081.22	37.16	28.11	51.55	36.06	37.30	35.73	38.32
Sharpe Ratio	-0.12	-0.03	-0.37	0.30	-0.04	0.02	-0.34	0.32	-0.05	-0.01	-0.02	0.05
Max Draw		0.50	0.30	0.71		0.51	0.31	0.72	0.49	0.51	0.49	0.53

Table OA.13: AI Spreads Equities & ETFs Quarterly Rebalancing - Income-Sorted

	Low				High				Low		High	
	Spreads								Median 95 % CI			
	Mean	Median	25q	75q	Mean	Median	25q	75q	2.5%	97.5%	2.5%	97.5%
					MV - Rolling Means/Linear Covariances							
Mean	131.22	1.10	-12.78	13.50	65.78	0.23	-13.79	10.00	0.15	2.02	-1.90	1.96
SD	548.60	35.05	26.26	51.85	222.89	36.48	26.88	53.79	34.36	35.68	34.89	38.28
Sharpe Ratio	0.00	0.04	-0.34	0.36	-0.08	0.01	-0.31	0.28	0.01	0.07	-0.07	0.06
Max Draw		0.52	0.30	0.73		0.55	0.35	0.78	0.51	0.53	0.52	0.58
					MV - ML/Linear Covariances							
Mean	134.54	4.13	-10.00	17.57	69.48	2.76	-11.70	15.01	3.01	5.20	0.82	4.91
SD	550.12	37.04	28.28	53.74	224.66	37.72	29.22	55.80	36.27	37.67	36.12	39.11
Sharpe Ratio	0.06	0.11	-0.27	0.43	0.04	0.07	-0.29	0.38	0.08	0.14	0.02	0.13
Max Draw		0.48	0.28	0.70		0.54	0.33	0.75	0.47	0.49	0.52	0.57
					MV - ML/Non-Linear Covariances							
Mean	134.37	3.84	-10.45	17.89	68.78	1.79	-11.84	15.03	2.84	4.98	-0.35	4.01
SD	550.17	37.22	28.24	54.40	224.72	37.90	29.14	55.99	36.50	37.90	36.48	39.41
Sharpe Ratio	0.07	0.11	-0.27	0.43	0.02	0.05	-0.29	0.39	0.07	0.15	-0.00	0.10
Max Draw		0.49	0.29	0.71		0.55	0.34	0.76	0.48	0.50	0.52	0.59
					Equal-Weighted							
Mean	130.47	-0.12	-13.62	12.04	65.47	-0.86	-13.88	11.20	-1.07	0.81	-2.38	1.32
SD	549.74	36.55	28.04	52.85	223.77	37.33	28.42	53.56	35.61	37.35	35.75	38.74
Sharpe Ratio	-0.04	-0.00	-0.35	0.31	-0.11	-0.02	-0.33	0.28	-0.02	0.03	-0.07	0.04
Max Draw		0.48	0.29	0.69		0.54	0.34	0.74	0.47	0.49	0.52	0.57

Table OA.14: Summary Statistics Returns - Equities & ETFs - Pre NBER Crisis

	Monthly Rebalancing				Quarterly Rebalancing				Annual Rebalancing			
	Mean	Median	25%	75%	Mean	Median	25%	75%	Mean	Median	25%	75%
					Mean Variance - Rolling Means/Linear Covariances							
Mean	3.36	3.77	-2.61	10.78	3.63	3.97	-2.37	10.75	3.89	4.07	-2.40	10.98
SD	11.90	11.14	8.23	15.07	11.67	10.94	8.00	14.71	11.57	10.84	8.06	14.59
Sharpe Ratio	0.17	0.40	-0.28	0.94	0.19	0.42	-0.26	0.96	0.28	0.44	-0.25	0.98
Max Draw		0.0000	0.0000	0.0975		0.0000	0.0000	0.0942		0.0000	0.0000	0.0944
					Mean Variance - ML Means/Linear Covariances							
Mean	4.19	3.81	-2.09	10.54	4.12	4.17	-2.00	10.70	2.20	2.65	-3.45	9.07
SD	11.72	10.91	7.99	14.91	11.92	11.25	8.09	15.27	11.53	10.97	7.99	14.67
Sharpe Ratio	0.25	0.40	-0.22	0.93	0.33	0.43	-0.21	0.93	0.18	0.30	-0.35	0.83
Max Draw		0.0000	0.0000	0.0911		0.00000	0.0000	0.0937		0.0000	0.0000	0.1002
					Mean Variance - ML Means/Non-Linear Covariances							
Mean	4.27	3.79	-1.99	10.54	4.17	4.08	-2.01	10.64	2.37	2.68	-3.32	9.15
SD	11.59	10.78	7.86	14.72	11.83	11.13	7.89	15.18	11.44	10.89	7.74	14.64
Sharpe Ratio	0.29	0.41	-0.22	0.94	0.29	0.43	-0.22	0.94	0.16	0.31	-0.34	0.85
Max Draw		0.0000	0.0000	0.0888		0.0000	0.0000	0.0925		0.0000	0.0000	0.0966
					Equal-Weighted							
Mean	2.66	3.96	-3.15	10.50	2.82	4.11	-3.11	10.63	2.84	4.11	-2.93	10.65
SD	11.49	11.07	8.82	13.70	11.50	11.07	8.84	13.76	11.45	11.04	8.83	13.72
Sharpe Ratio	0.25	0.41	-0.29	0.94	0.31	0.42	-0.28	0.95	0.32	0.42	-0.27	0.95
Max Draw		0.0000	0.0000	0.1065		0.0000	0.0000	0.1057		0.0000	0.0000	0.1062

Table OA.15: AI Spreads - Equities & ETFs - Pre NBER Crisis

	Monthly Rebalancing				Quarterly Rebalancing				Annual Rebalancing			
	Mean	Median	25%	75%	Mean	Median	25%	75%	Mean	Median	25%	75%
					Mean Variance - Rolling Means/Linear Covariances							
Mean	-103.79	-4.90	-19.03	10.17	-103.52	-4.71	-18.71	10.30	-103.26	-4.58	-18.61	10.35
SD	283.63	24.41	18.77	33.88	283.51	24.32	18.67	33.84	283.46	24.21	18.66	33.73
Sharpe Ratio	-0.06	-0.21	-0.73	0.37	-0.12	-0.21	-0.72	0.37	-0.11	-0.20	-0.71	0.39
Max Draw		0.0000	0.2951	0.0000		0.0000	0.2896	0.0000		0.0000	0.0000	0.2873
					Mean Variance - ML Means/Linear Covariances							
Mean	-102.96	-4.56	-18.32	10.15	-103.03	-4.83	-18.81	10.19	-104.95	-6.36	-19.92	8.63
SD	283.63	24.53	18.72	33.80	283.71	24.49	18.78	34.13	283.28	23.98	18.39	33.48
Sharpe Ratio	-0.14	-0.20	-0.71	0.37	-0.10	-0.21	-0.72	0.37	-0.17	-0.28	-0.77	0.31
Max Draw		0.0000	0.2945	0.0000		0.0000	0.2909	0.0000		0.0000	0.0000	0.2963
					Mean Variance - ML Means/Non-Linear Covariances							
Mean	-102.88	-4.62	-18.20	10.07	-102.98	-4.80	-18.76	10.28	-104.78	-6.03	-19.82	8.84
SD	283.59	24.44	18.67	33.72	283.69	24.45	18.73	34.10	283.29	24.02	18.42	33.56
Sharpe Ratio	-0.12	-0.20	-0.71	0.37	-0.38	-0.20	-0.72	0.38	-0.14	-0.26	-0.76	0.32
Max Draw		0.0000	0.0000	0.2935		0.0000	0.0000	0.2911		0.0000	0.0000	0.2969
					Equal-Weighted							
Mean	-104.50	-5.02	-18.70	8.61	-104.33	-4.93	-18.52	8.81	-104.31	-4.97	-18.48	8.86
SD	283.24	23.71	18.34	33.42	283.25	23.74	18.35	33.36	283.22	23.67	18.31	33.27
Sharpe Ratio	-0.12	-0.23	-0.72	0.32	-0.12	-0.22	-0.71	0.33	0.08	-0.22	-0.71	0.33
Max Draw		0.0000	0.0000	0.2791		0.0000	0.0000	0.2782		0.0000	0.0000	0.2782

Table OA.16: Summary Statistics Returns - Equities & ETFs - During NBER Crisis

	Monthly Rebalancing				Quarterly Rebalancing				Annual Rebalancing			
	Mean	Median	25%	75%	Mean	Median	25%	75%	Mean	Median	25%	75%
					Mean Variance - Rolling Means/Linear Covariances							
Mean	-4.94	0.00	-8.93	0.00	-5.95	-0.28	-9.29	0.00	-5.93	-1.06	-10.23	0.00
SD	9.93	5.84	0.00	16.08	9.50	5.52	0.00	15.13	9.45	5.92	0.00	15.05
Sharpe Ratio	-0.55	-0.71	-1.05	0.08	-0.62	-0.79	-1.10	-0.09	-0.73	-0.79	-1.14	-0.15
Max Draw		0.000	0.000	0.1097		0.000	0.000	0.1115		0.000	0.000	0.1110
					Mean Variance - ML Means/Linear Covariances							
Mean	-0.31	-2.25	-16.80	10.63	7.37	0.00	-13.81	23.81	4.43	0.00	-15.49	18.67
SD	24.64	25.56	15.01	33.45	25.07	24.86	13.58	36.33	26.05	25.34	14.09	38.21
Sharpe Ratio	0.00	-0.20	-0.77	0.47	0.18	0.02	-0.75	0.84	0.06	-0.06	-0.81	0.62
Max Draw		0.1024	0.0000	0.3451		0.0550	0.0000	0.2923		0.0819	0.0000	0.3188
					Mean Variance - ML Means/Non-Linear Covariances							
Mean	-1.90	-2.70	-18.26	9.55	6.65	0.00	-14.19	23.34	3.77	0.00	-16.30	18.87
SD	24.07	24.94	13.95	33.11	24.80	23.81	13.17	36.48	25.73	24.38	13.61	38.16
Sharpe Ratio	-0.03	-0.23	-0.85	0.44	0.11	0.00	-0.80	0.83	-0.02	-0.08	-0.87	0.64
Max Draw		0.0974	0.0000	0.3334		0.0561	0.0000	0.2762		0.0795	0.0000	0.3060
					Equal-Weighted							
Mean	-8.06	-16.15	-28.03	0.00	-2.25	-11.30	-21.39	4.80	-2.02	-11.21	-21.25	5.04
SD	29.77	30.48	20.76	39.25	28.90	29.62	20.41	37.94	28.80	29.55	20.42	37.73
Sharpe Ratio	-0.05	-0.59	-0.85	-0.04	-0.04	-0.42	-0.72	0.15	0.05	-0.42	-0.72	0.16
Max Draw		0.1645	0.0000	0.5114		0.1457	0.0000	0.4728		0.1452	0.0000	0.4703

Table OA.17: AI Spreads - Equities & ETFs - During NBER Crisis

	Monthly Rebalancing				Quarterly Rebalancing				Annual Rebalancing			
	Mean	Median	25%	75%	Mean	Median	25%	75%	Mean	Median	25%	75%
					Mean Variance - Rolling Means/Linear Covariances							
Mean	-362.76	23.20	-6.13	42.74	-363.77	22.67	-6.62	41.86	-363.75	22.45	-6.46	41.77
SD	824.99	48.73	33.53	81.94	825.03	48.52	33.52	82.28	824.89	48.31	33.26	82.04
Sharpe Ratio	0.22	0.51	-0.10	0.90	0.22	0.49	-0.11	0.88	0.23	0.49	-0.11	0.88
Max Draw		0.2424	0.0000	0.4546		0.2423	0.0000	0.4556		0.2393	0.0000	0.4541
					Mean Variance - ML Means/Linear Covariances							
Mean	-358.13	22.16	-6.37	46.73	-350.45	27.31	-3.48	59.14	-353.39	24.93	-4.98	54.00
SD	827.30	51.33	37.60	82.61	829.02	54.07	38.74	84.88	829.62	55.20	39.33	85.55
Sharpe Ratio	-0.09	0.43	-0.11	0.87	0.17	0.53	-0.05	0.99	0.19	0.48	-0.08	0.90
Max Draw		0.2500	0.0000	0.4597		0.2434	0.0000	0.4494		0.2578	0.00000	0.4576
					Mean Variance - ML Means/Non-Linear Covariances							
Mean	-359.72	21.27	-8.76	46.29	-351.17	26.86	-4.78	59.15	-354.05	24.79	-7.44	54.65
SD	827.35	51.35	37.62	83.00	829.15	54.34	38.71	84.97	829.76	55.43	39.26	85.76
Sharpe Ratio	0.28	0.41	-0.16	0.86	0.21	0.52	-0.08	0.99	0.16	0.47	-0.12	0.90
Max Draw		0.2531	0.0000	0.4667		0.2465	0.0000	0.4574		0.2588	0.0000	0.4661
					Equal-Weighted							
Mean	-365.88	10.37	-14.74	33.65	-360.07	15.97	-7.95	38.95	-359.83	16.37	-7.56	39.20
SD	828.28	53.33	39.74	81.99	828.55	53.55	39.78	82.76	828.45	53.39	39.67	82.55
Sharpe Ratio	0.46	0.19	-0.26	0.61	0.32	0.30	-0.14	0.70	0.27	0.31	-0.13	0.71
Max Draw		0.2849	0.0000	0.4984		0.2723	0.0000	0.4724		0.2704	0.0000	0.4705

Table OA.18: Summary Statistics Returns - Equities & ETFs - Post NBER Crisis

	Monthly Rebalancing				Quarterly Rebalancing				Annual Rebalancing			
	Mean	Median	25%	75%	Mean	Median	25%	75%	Mean	Median	25%	75%
					Mean Variance - Rolling Means/Linear Covariances							
Mean	0.58	0.00	-4.36	4.84	1.06	0.00	-3.39	5.44	0.56	0.00	-4.07	5.01
SD	11.57	12.17	8.27	15.38	12.01	12.47	8.62	15.88	12.01	12.52	8.68	15.85
Sharpe Ratio	0.07	0.03	-0.43	0.50	-0.09	0.07	-0.34	0.52	0.17	0.04	-0.40	0.49
Max Draw		0.1493	0.0357	0.2343		0.1450	0.0385	0.2303		0.1553	0.0385	0.2360
					Mean Variance - ML Means/Linear Covariances							
Mean	1.09	0.00	-4.94	6.71	3.58	2.05	-1.76	8.42	3.32	2.06	-1.96	8.26
SD	15.30	15.35	10.22	20.41	14.11	14.25	9.51	18.95	13.96	14.10	9.56	18.62
Sharpe Ratio	0.10	0.07	-0.38	0.51	0.30	0.23	-0.17	0.64	0.29	0.23	-0.19	0.63
Max Draw		0.1560	0.0405	0.2606		0.1347	0.0390	0.2225		0.1368	0.0377	0.2223
					Mean Variance - ML Means/Non-Linear Covariances							
Mean	0.70	0.00	-5.31	6.44	3.24	1.41	-2.11	7.84	2.90	1.41	-2.34	7.69
SD	14.95	14.90	9.24	20.56	13.99	13.87	8.59	19.32	13.67	13.71	8.64	18.82
Sharpe Ratio	0.05	0.06	-0.42	0.52	0.33	0.19	-0.20	0.62	0.26	0.20	-0.23	0.62
Max Draw		0.1484	0.0322	0.2578		0.1299	0.0317	0.2229		0.1306	0.0315	0.2197
					Equal-Weighted							
Mean	1.14	0.91	-4.96	7.47	1.26	0.93	-4.94	7.42	1.82	1.29	-4.58	7.83
SD	15.92	16.21	11.43	20.46	15.95	16.13	11.43	20.42	15.95	16.07	11.41	20.41
Sharpe Ratio	0.17	0.12	-0.35	0.53	0.47	0.12	-0.35	0.52	0.22	0.14	-0.33	0.54
Max Draw		0.2057	0.0570	0.3090		0.2050	0.0593	0.3046		0.2008	0.0598	0.2989

Table OA.19: AI Spreads - Equities & ETFs - Post NBER Crisis

	Monthly Rebalancing				Quarterly Rebalancing				Annual Rebalancing			
	Mean	Median	25%	75%	Mean	Median	25%	75%	Mean	Median	25%	75%
					Mean Variance - Rolling Means/Linear Covariances							
Mean	30.84	-5.01	-17.00	7.86	31.32	-4.50	-16.70	8.62	30.82	-4.97	-17.31	8.11
SD	246.03	28.28	21.55	39.61	246.40	28.81	21.92	39.97	246.40	28.86	21.90	40.02
Sharpe Ratio	-0.25	-0.19	-0.62	0.25	-0.23	-0.16	-0.59	0.27	-0.25	-0.18	-0.61	0.26
Max Draw		0.3232	0.2105	0.4523		0.3201	0.2086	0.4479		0.3245	0.2111	0.4565
					Mean Variance - ML Means/Linear Covariances							
Mean	31.35	-4.41	-17.51	9.31	33.84	-2.33	-14.28	10.98	33.58	-2.43	-14.50	10.77
SD	247.86	30.74	23.64	41.33	247.21	29.92	22.89	40.71	247.12	29.75	22.84	40.55
Sharpe Ratio	-0.26	-0.15	-0.59	0.28	-0.13	-0.08	-0.50	0.34	-0.17	-0.08	-0.51	0.34
Max Draw		0.3274	0.2124	0.4585		0.3106	0.2027	0.4356		0.3120	0.2045	0.4357
					Mean Variance - ML Means/Non-Linear Covariances							
Mean	30.96	-4.66	-17.89	9.00	33.50	-2.72	-14.56	10.82	33.16	-2.78	-15.16	10.55
SD	247.82	30.90	23.48	41.21	247.28	30.14	22.82	40.76	247.12	29.90	22.64	40.51
Sharpe Ratio	-0.27	-0.16	-0.59	0.28	-0.19	-0.09	-0.51	0.34	-0.39	-0.10	-0.53	0.33
Max Draw		0.3325	0.2163	0.4671		0.3156	0.2067	0.4404		0.3177	0.2086	0.4434
					Equal-Weighted							
Mean	31.40	-4.20	-17.21	8.84	31.52	-4.15	-17.16	8.89	32.08	-3.70	-16.67	9.38
SD	247.17	29.74	22.97	40.44	247.16	29.77	22.95	40.46	247.14	29.80	22.91	40.45
Sharpe Ratio	-0.31	-0.15	-0.59	0.28	-0.45	-0.15	-0.58	0.28	-0.49	-0.13	-0.57	0.30
Max Draw		0.3113	0.2008	0.4473		0.3115	0.2006	0.4461		0.3092	0.1990	0.4421

Table OA.20: Hypothesis Tests Median

		Monthly Rebalancing High minus Low		Quarterly Rebalancing High minus Low		Annual Rebalancing High minus Low	
Model	Characteristic	CI(2.5%)	CI(97.5%)	CI(2.5%)	CI(97.5%)	CI(2.5%)	CI(97.5%)
MV Roll-Linear	Education	-0.9790	1.8562	-0.9162	1.4445	-0.6900	1.9954
	Risk-Aversion	-0.1839	2.3496	0.1594	2.7474	0.5449	2.5652
	Income	-1.9732	1.1685	-2.6676	0.5298	-2.2432	0.9252
MV ML-Linear	Education	0.1849	2.4513	-0.5989	2.1723	-0.8826	2.0941
	Risk-Aversion	-0.0772	2.5332	0.5546	3.1111	0.0296	2.5896
	Income	-2.4204	0.5635	-3.1016	0.6014	-2.6429	1.4669
MV ML-Nonlinear	Education	-0.9857	2.3394	-0.7765	1.8614	-0.9314	1.9420
	Risk-Aversion	0.3590	2.5165	0.1305	3.6186	0.0270	2.8486
	Income	-1.8915	1.1449	-3.3510	-0.1937	-3.0906	0.8836
EW	Education	-0.2899	2.2258	-0.3346	1.7664	-0.6122	2.1451
	Risk-Aversion	0.1334	2.8904	0.3316	2.6218	0.2814	2.5593
	Income	-2.3182	0.6158	-1.9809	1.2162	-2.5144	1.0012

Table OA.21: Disposition Effect and Alter Ego Spreads - Quantile Regressions

	5%	10%	20%	50%	80%	90%	95%
Disposition Effect	0.001	-0.001	0.003	0.011	0.023	-0.002	-0.089
Trading freq. 2nd quartile	-0.386**	-0.367	-0.839**	-1.167*	-2.620	-7.759	-10.388
Trading freq. 3rd quartile	-0.276*	-0.432	-1.021***	-1.760***	-5.180***	-10.000*	-8.060
Trading freq. 4th quartile	-0.408**	-0.535	-0.836**	-2.597***	-4.411**	-5.201	8.533
Male	-0.090	-0.326	-0.023	1.493***	3.397*	7.608	21.233**
Educ. HS	-0.078	0.160	-0.251	-0.767	-0.410	8.156	-15.957
Educ. University	0.019	0.348	-0.073	-0.917	-1.827	-0.182	-26.987
Aversion - Medium	0.164	-0.023	0.138	-0.161	1.345	6.485**	9.052
Aversion - High	0.022	-0.263	-0.319	-0.569	3.010	11.655	33.829
Income: 20k-40k	0.119	0.017	-0.191	0.666	0.540	-3.548	19.814
Income: 40k-75k	0.405**	0.304	0.266	0.652	1.344	1.427	21.032
Income: 75k-150k	0.172	0.129	0.453	2.265***	1.030	-7.159	9.198
Income: > 150k	0.454**	-0.074	0.656	3.986**	11.372**	16.875	73.250
Funds invested: 20k-50k	0.082	0.141	-0.022	-0.990*	-2.422	0.058	7.729
Funds invested: 50k-250k	0.096	0.071	-0.141	-1.212**	-2.912*	-5.473	-7.889
Funds invested: 250k-1MM	-0.082	-0.101	-0.276	-1.914***	-3.900**	-1.577	-4.796
Funds invested: > 1 MM	0.189	0.844	-0.137	-2.645***	-5.885*	-2.518	-21.711
ETFs	0.042	0.084	0.177	0.599	2.300	5.698	8.129
Levered ETFs	0.076	-0.094	-0.692	-0.867	-4.786	-13.864*	-4.731
Constant	1.394***	2.783***	6.068***	15.659***	37.703***	61.315***	104.718**

Notes: Cross-sectional Regressions Alter Ego Spreads on DE with MV ML/Rolling, * p<0.10 ** p<0.05 and *** p<0.01.

TABLE OA.22. Disposition Effect and Alter Ego Spreads vis-à-vis S&P 500 ETF - Quantile Regressions

	5%	10%	20%	50%	80%	90%	95%
Disposition Effect	-0.002	-0.001	0.002	0.005**	0.002	-0.002	-0.011**
Trading freq. 2nd quartile	-0.185	0.198	0.482***	0.770***	0.639***	0.628**	0.358
Trading freq. 3rd quartile	-0.164	0.219	0.716***	1.245***	1.254***	1.140***	1.182**
Trading freq. 4th quartile	-0.436*	-0.114	0.649***	1.321***	1.402***	1.478***	1.226**
Male	-0.234	-0.163	-0.168	0.089	0.226	0.054	0.526*
Educ. HS	-0.070	-0.415	-0.268	-0.061	0.237	0.564**	0.740**
Educ. University	0.198	-0.355	0.001	0.198	0.211	0.575**	0.832***
Aversion - Medium	0.198	0.146	0.226**	0.085	-0.040	-0.034	-0.028
Aversion - High	0.361	0.555**	0.286	-0.316	-0.449*	-0.713**	-0.799*
Income: 20k-40k	0.003	-0.017	0.039	-0.039	0.089	-0.006	-0.114
Income: 40k-75k	-0.025	0.178	0.150	-0.083	0.148	0.072	0.122
Income: 75k-150k	0.630**	0.723***	0.584***	0.190	0.381	0.197	0.674
Income: $>$ 150k	0.334	0.885**	0.519	0.561	0.443	1.385*	1.068
Funds invested: 20k-50k	0.246	0.072	0.084	-0.347**	-0.259	-0.149	0.020
Funds invested: 50k-250k	-0.232	-0.339*	-0.170	-0.305**	-0.339**	0.061	0.275
Funds invested: 250k-1MM	-0.100	-0.454**	-0.205	-0.626***	-0.619***	-0.040	-0.220
Funds invested: $>$ 1 MM	-0.204	-0.676**	-0.695**	-0.382	-0.806***	-0.698	0.570
ETFs	-0.150	-0.029	0.036	-0.167	-0.226	-0.241	-0.269
Levered ETFs	-0.089	0.069	-0.378	-0.305	-0.170	-0.073	-0.872**
Constant	-11.909***	-9.812***	-8.439***	-4.112***	1.053**	3.631***	5.740***

Notes: Cross-sectional Regressions Alter Ego Spreads on DE with MV ML/Rolling, * $p<0.10$ ** $p<0.05$ and *** $p<0.01$.

Table OA.23: Individual Investors Benefiting from Robo-Investing

	Gray Area			*Above 45 degree Line*			*Gray Area*			*Above 45 degree Line*		
	Total	High	Low	Total	High	Low	Total	High	Low	Total	High	Low
	Mean Variance - Rolling Means/Linear Covariances						*Mean Variance - ML Means/Non-Linear Covariances*					
Monthly												
Education	45.6%	46.1%	43.3%	56.5%	56.5%	55.4%	43.5%	43.8%	42.1%	55.7%	55.6%	55.0%
Aversion	45.6%	46.6%	45.3%	56.5%	57.4%	56.4%	43.5%	44.5%	43.5%	55.7%	56.3%	56.0%
Income	45.6%	47.8%	45.3%	56.5%	56.8%	57.4%	43.5%	46.6%	44.3%	55.7%	56.3%	57.1%
ETF holder	45.6%	46.8%	45.3%	56.5%	56.9%	56.4%	43.5%	43.1%	43.7%	55.7%	55.5%	55.8%
Trading freq	45.6%	44.3%	44.4%	56.5%	55.9%	55.7%	43.5%	42.2%	43.2%	55.7%	54.9%	54.9%
Quarterly												
Education	46.4%	46.8%	45.1%	56.8%	56.7%	55.7%	51.4%	51.6%	50.7%	60.0%	59.9%	58.9%
Aversion	46.4%	46.9%	45.9%	56.8%	57.3%	56.6%	51.4%	52.0%	51.8%	60.0%	60.9%	60.6%
Income	46.4%	49.0%	46.3%	56.8%	57.6%	57.1%	51.4%	51.9%	51.7%	60.0%	59.6%	60.8%
ETF holder	46.4%	46.9%	46.3%	56.8%	57.2%	56.7%	r 51.4%	51.7%	51.4%	60.0%	60.4%	59.8%
Trading freq	46.4%	44.9%	45.9%	56.8%	55.8%	56.5%	51.4%	52.1%	47.0%	60.0%	60.2%	57.6%
	Mean Variance - ML Means/Linear Covariances						*Equal-Weighted*					
Monthly												
Education	43.2%	43.5%	41.2%	55.7%	55.6%	54.4%	43.0%	43.8%	38.5%	55.7%	55.9%	53.8%
Aversion	43.2%	44.7%	42.9%	55.7%	56.9%	55.7%	43.0%	45.0%	42.5%	55.7%	57.3%	55.6%
Income	43.2%	45.7%	43.7%	55.7%	56.3%	57.0%	43.0%	48.6%	41.5%	55.7%	57.9%	56.1%
ETF holder	43.2%	43.0%	43.2%	55.7%	55.3%	55.8%	43.0%	47.5%	41.8%	55.7%	57.3%	55.3%
Trading freq	43.2%	42.3%	41.7%	55.7%	55.0%	54.4%	43.0%	47.5%	37.7%	55.7%	57.1%	52.7%
Quarterly												
Education	52.0%	52.2%	51.3%	60.4%	60.3%	59.1%	43.0%	43.8%	38.5%	55.7%	55.9%	53.8%
Aversion	52.0%	53.4%	52.3%	60.4%	61.1%	60.9%	43.0%	45.0%	42.5%	55.7%	57.3%	55.6%
Income	52.0%	52.7%	52.4%	60.4%	60.3%	61.2%	43.0%	48.6%	41.5%	55.7%	57.9%	56.1%
ETF holder	52.0%	52.4%	52.0%	60.4%	60.8%	60.3%	43.0%	47.5%	41.8%	55.7%	57.3%	55.3%
Trading freq	52.0%	53.5%	47.3%	60.4%	60.9%	58.0%	43.0%	47.5%	37.7%	55.7%	57.1%	52.7%

Notes: Entries to the table are percentage of the population of individual investors who benefit from robo-investing. The left set of entries pertain to observations in the shaded gray areas appearing in the top plots of Figure **??** corresponding to improvements *and* positive returns. The right set of entries pertain to observations above the 45 degree line, implying improvements but not necessarily resulting in positive returns for robo-investors.

Table OA.24: Monthly trading activity and portfolio depending on investor income

	High	Mid	Low
Panel A : Stocks only			
# trades	2.3	2	2
Volume (euros)	14,495.05	6,283.50	4,828.43
# ISIN	2	1.71	1.66
# assets	3.66	2.92	2.55
Portfolio value (euros)	24,166.58	9,197.40	5,948.34
# months	62	53	46
Return (%)	0.16	0.08	-0.02
Volatility (%)	7.14	7.83	8.28
Panel B : Stocks and ETF			
# trades	3.27	2.67	2.62
Volume (euros)	21,717.59	9,756.52	7,829.03
# ISIN	2.56	2.19	2.12
# assets	6.96	4.89	4.08
Portfolio value (euros)	43,983.19	17,435.81	11,276.56
# months	69	62	55
Return (%)	0.22	0.16	0.08
Volatility (%)	6.08	6.68	6.89

Notes: The table reports the cross-sectional median for trade-based and portfolio-based variables depending on investor income. Panel A refers to stocks only and Panel B refers to both stocks and ETF. Consequently, statistics in Panel A are computed across the entire sample (i.e. 22,972 investors) while those reported in Panel B are computed across the subsample of investors who traded ETF in addition to stocks (i.e. 4,693 investors). *# trades* is the monthly number of trades executed. *Volume* is the corresponding monthly monetary volume in euros. *# ISIN* is the monthly number of different assets traded. *# assets* is the monthly average number of assets held portfolio computed over the holding period. *Portfolio value* is the corresponding monthly average market portfolio value in euros. *# months* refers to the portfolio holding period. *Return* is the monthly average gross portfolio return expressed in %. *Volatility* is the standard deviation of monthly gross portfolio returns expressed in %. For annual net income ('income'), 'High' refers to investors who report they earn more than 150,000 euros, 'Mid' to investors who earn between 40,000-150,000 euros, and 'Low' to investors who state earning less than 40,000 euros.

Table OA.25: Monthly trading activity and portfolio depending on investor risk aversion

	High	Mid	Low
Panel A : Stocks only			
# trades	2.14	1.92	2.2
Volume (euros)	6,341.79	5,142.82	6,711.00
# ISIN	1.75	1.65	1.79
# assets	2.19	2.75	2.87
Portfolio value (euros)	6,208.07	7,150.70	9,263.56
# months	49	50	50
Return (%)	0.03	0.03	0.04
Volatility (%)	7.93	7.92	8.30
Panel B : Stocks and ETF			
# trades	2.99	2.5	2.91
Volume (euros)	11,189.66	8,558.09	9,928.17
# ISIN	2.29	2.1	2.3
# assets	3.12	4.48	4.98
Portfolio value (euros)	10,128.04	14,261.04	16,703.92
# months	56	59	60
Return (%)	0.14	0.12	0.16
Volatility (%)	6.82	6.56	7.15

Notes: The table reports the cross-sectional median for trade-based and portfolio-based variables depending on investor risk aversion. Panel A refers to stocks only and Panel B refers to both stocks and ETF. Consequently, statistics in Panel A are computed across the entire sample (i.e. 22,972 investors) while those reported in Panel B are computed across the subsample of investors who traded ETF in addition to stocks (i.e. 4,693 investors). # *trades* is the monthly number of trades executed. *Volume* is the corresponding monthly monetary volume in euros. # *ISIN* is the monthly number of different assets traded. # *assets* is the monthly average number of assets held portfolio computed over the holding period. *Portfolio value* is the corresponding monthly average market portfolio value in euros. # *months* refers to the portfolio holding period. *Return* is the monthly average gross portfolio return expressed in %. *Volatility* is the standard deviation of monthly gross portfolio returns expressed in %. For the level of risk aversion ('risk aversion'), 'High' refers to investors who state a high risk aversion (i.e. they do not want to take any risks and prefer safe investments), 'Mid' refers to investors who state a medium risk aversion, and 'Low' to investors who state a low risk aversion (i.e. they would invest even more when facing a sharp loss of 20% to reduce their average purchase prices).

Table OA.26: Monthly trading activity and portfolio depending on investor education

	Univ	HS	No
Panel A : Stocks only			
# trades	2	2.12	2
Volume (euros)	5,525.23	6,048.64	5,208.12
# ISIN	1.67	1.75	1.67
# assets	2.84	2.59	2.39
Portfolio value (euros)	7,760.13	7,369.06	6,145.48
# months	51	51	44
Return (%)	0.06	-0.06	-0.04
Volatility (%)	7.85	8.42	8.76
Panel B : Stocks and ETF			
# trades	2.61	2.90	2.70
Volume (euros)	8,815.49	10,182.63	9,582.28
# ISIN	2.15	2.34	2.24
# assets	4.54	4.43	3.58
Portfolio value (euros)	15,033.30	14,801.56	9,906.08
# months	59	61.5	53
Return (%)	0.14	0.07	0.00
Volatility (%)	6.67	6.86	7.37

Notes: The table reports the cross-sectional median for trade-based and portfolio-based variables depending on investor education. Panel A refers to stocks only and Panel B refers to both stocks and ETF. Consequently, statistics in Panel A are computed across the entire sample (i.e. 22,972 investors) while those reported in Panel B are computed across the subsample of investors who traded ETF in addition to stocks (i.e. 4,693 investors). *# trades* is the monthly number of trades executed. *Volume* is the corresponding monthly monetary volume in euros. *# ISIN* is the monthly number of different assets traded. *# assets* is the monthly average number of assets held portfolio computed over the holding period. *Portfolio value* is the corresponding monthly average market portfolio value in euros. *# months* refers to the portfolio holding period. *Return* is the monthly average gross portfolio return expressed in %. *Volatility* is the standard deviation of monthly gross portfolio returns expressed in %. For the level of education ('education'), 'No' refers to investors without any degree, 'HS' to investors with a secondary/high school degree, and 'Univ' to investors with an university degree or equivalent.