

Constraint-Based Graphic Statics: new paradigms of computer-aided structural equilibrium design

Corentin Fivet¹, Denis Zastavni²

¹ Arch.Eng. PhD candidate, LOCI, UCLouvain, Louvain-la-Neuve, Belgium, corentin.fivet@uclouvain.be

² Prof. Dr. Arch.Eng., LOCI, UCLouvain, Louvain-la-Neuve, Belgium, denis.zastavni@uclouvain.be

Summary: This paper presents a graphical, constraint-based system aimed at assisting designers during the earliest stages of the structural design process. While taking full advantage of well-tried methods of graphic statics, this system introduces labour-saving devices for the computer-aided shaping of structures. As a result, it is argued that this system supports new paradigms of structural equilibrium design.

Keywords: *structural design; static equilibrium; graphic statics; constraint-based geometric solver; computer-aided design*

ABSTRACT

Structural morphogenesis can be defined as the preliminary definition of a structure's geometry (and hence its stresses) before any detailed analysis is undertaken. If looking for structural efficiency and robustness, this morphogenesis is of crucial importance because the structural behaviour it determines will not generally be modified after the subsequent structural analysis.

When looking at current structural morphogenesis tools, form-finding algorithms and manual methods both reflect a relative lack of design efficiency. On the one hand, form-finding results are difficult for the designer to amend; on the other, computers do not help with manual methods. Against this backdrop, this paper discusses the features and the effects of a newly developed system aimed at interactively supporting designers in their search for static equilibrium.

This system uses a strut-and-tie abstraction which allows the stability of a very large range of structures to be modeled: isostatic or hyperstatic reticular structures, mechanisms, irregular beams subjected to bending, compression-only structures, discontinuous stress fields etc. In order to manage all the variables in an entirely graphical way, the strut-and-tie model is described in full by the two reciprocal diagrams of graphic statics: a form diagram whose points control the geometries and a force diagram whose points control the stress intensities.

Starting from scratch, the designer has a set of instructions available that allow any strut-and-tie model to be built and modified stage by stage. The geometric properties of these instructions guarantee that the strut-and-tie model stays in static equilibrium whatever the stage. This feature prevents the user from ascertaining the actual static equilibrium manually and hence allows a narrower focus on each structural choice.

Three kinds of instructions can be identified: the first applies a geometric constraint to a point; the second resolves forces or transforms rods into two opposite forces and vice versa; and the third is a procedure comprising any other instructions, allowing for instance the combination or the refinement of existing sub-systems of equilibrium.

When applied in the form diagram, a geometric constraint can, for example, compel a rod to stay within a certain area or to have a certain

orientation. When applied in the force diagram, a geometric constraint can, for example, compel a rod to be under a certain maximum stress.

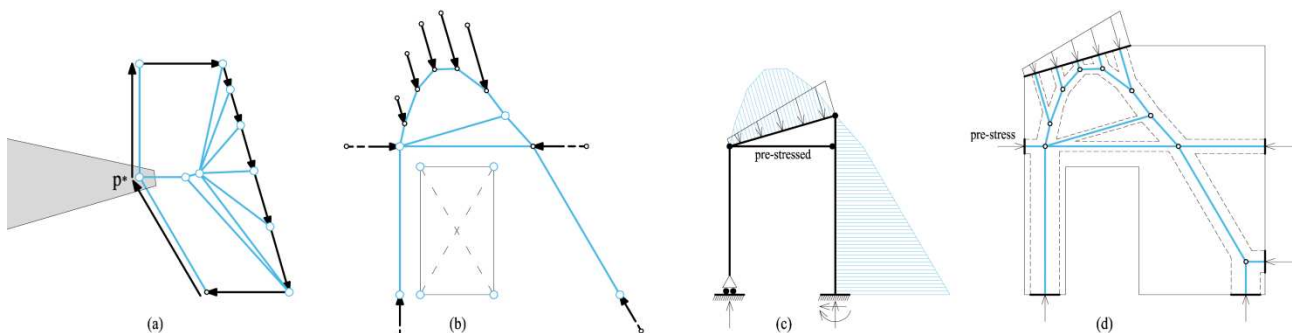
If planar, those geometric constraints are any Boolean combination of three fundamental shapes: a half-plane, the interior of a disc and the exterior of a disc. These three constraints are sufficient for extending the usual ruler-and-compass constructions considerably. Indeed, their definition allows the user to compute inequalities graphically and apply geometric constraints that vary exactly like dynamic conditional statements. On the other hand, interdependencies of constraints allow the user to constrain a point on any non-standard curve or shape.

The sum of the constraints applied on a given point is simply displayed as a geometric shape inside which the point has to stay. If those shapes are propagated on every child point, it is impossible for a point to have no position in which to stay. Each shape therefore visually synthesizes all the solutions of the geometric problem.

Since these geometric constraints are based on a very small set of primitive constraints and only take points as parameters, the system allows constraints to be permuted on demand and is therefore completely free of the hierarchy inherent in classical parametric modelers. The consequence of this feature is that the user does not have to predict the parametric hierarchy in advance or rebuild the model whenever the hierarchy is inverted.

When creating forces or rods, new geometric constraints are automatically produced. It follows that the graphical domain of each point displays all the design freedoms available to the user so that the structure stays in equilibrium. The user is then able to achieve manually and graphically customized topological optimizations.

As a result, the system removes all the usual precepts of a hierarchical and chronological design process since it independently and simultaneously deals with the geometries of the structure, its inner stresses, its hyperstaticity, its boundary conditions and its material strengths. Furthermore all these design freedoms are integrated in a unified interactive graphical interface. Through various applications, this paper shows how these features reveal the way ahead towards new paradigms of structural equilibrium design.



The hatched area in the force diagram (a) corresponds to the geometric domain of point p^* , where no rod from the form diagram (b) crosses the rectangle; (c) a resulting reticular frame with its moment diagram; (d) a resulting concrete shear-wall.