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Magnitude Representation in Children

Its Development and Dysfunction

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INTRODUCTION

The semantics of a given number is rich. For instance, the semantics of nine could include information such as “is larger than eight,” “is close to ten,” “is the square of three,” “is an odd number,” and “is the age of my elder son.” Among these, the magnitude is extremely important. Miller and Gelman (1983) showed that when kindergartners and third graders are presented with triads of single digits and asked to select the two that are the more closely related to one another and the two that are the least, they base their judgments on magnitude information only. Later on, in sixth grade, magnitude is still the more used dimension, although other dimensions such as parity are used, also.

Magnitude of numbers refers to the cases in which numbers are used in a cardinal context, that is, when they denote the “manyness” of a set (e.g., five is the number of fingers I have on one hand; see Fuson, 1992). In reference to this cardinal concept, learning takes place along two directions. One of them is the distinction between the transformations that do or do not modify the cardinal of a set (e.g., spatial transformations, such as spreading or grouping the items, do not modify the cardinal, whereas addition or subtraction of items does). The other is the comparison between the cardinal of different sets (e.g., set A’s cardinal could be smaller, larger, or equal to set B’s). The child first learns these notions through his/her experience with sets of real objects. But later on, the child will learn to reason with symbols, such as words or Arabic numerals.

This chapter focuses on the magnitude comparison of numbers. The first part of the chapter will briefly present the effects that have been encountered in magnitude comparison tasks with adults as well as the models that have been proposed to account for these data. The second part of the chapter will review studies that have traced the development of these effects. The third part will aim at identifying the age at which number magnitude starts to be automatically activated in children. Finally, we will consider the hypothesis that developmental dyscalculia could be due to a basic dysfunction in representing number magnitude.

CLASSICAL EFFECTS REPORTED IN ADULT STUDIES AND EXPLANATORY MODELS

Adult studies of number magnitude comparison have identified several robust effects. First, response times (RTs) decrease monotonously as the numerical distance between the two numbers increases (e.g., 7–8 versus 7–2). This *distance effect* was observed first by Moyer and Landauer (1967) but has been replicated frequently since (e.g., Banks, Fujii, & Kayra-Stuart, 1976). Second, for equal numerical distance, RTs increase with the size of the numbers (e.g., –3 versus 7–8). This *size effect* has also been reported many times (e.g., Antell & Keating, 1983; Strauss & Curtis, 1981). Third, the *semantic congruity effect* (first observed by Banks, Fujii, & Kayra-Stuart, 1976) refers to the fact that large numbers are more quickly compared under the instructions to "choose the larger" as opposed to "choose the smaller," whereas small numbers are more quickly compared under the instructions to "choose the smaller." Finally, a *spatial congruency effect* has also been reported: subjects who respond "larger" with their right hand are faster than subjects who answer "larger" with their left hand (Dehaene, Dupoux, & Mehler, 1990).

Several models have been proposed to account for these effects, but, broadly speaking, two theoretical types of models can be distinguished: the symbolic and the analogue models of magnitude comparison.

Banks's Semantic Coding Model

The semantic coding model proposed by Banks and colleagues (Banks, Clark, & Lucy, 1975; Banks, Fujii, & Kayra-Stuart, 1976; Banks, 1977) belongs to the first type of models. It assumes that comparison is held on discrete codes and follows two processing steps: an encoding and a comparison stage. In the encoding stage, a very crude semantic description of the numbers is generated: each of them is independently coded as larger (L+) or smaller (S+) than a cutoff criterion. The value of the cutoff point would vary from trial to trial and present a skewed distribution that places its mean below the arithmetic midpoint of the digits used in the experiment. This is due to the compressive mapping of digits onto subjective magnitude. Consequently, smaller numbers are spaced farther apart on the subjective continuum than larger numbers, and the cutoff is more likely to fall between two small numbers than between two large ones. When considering a pair of numbers to compare, two possibilities can thus be encountered at this first stage: either the codes attributed to the two numbers are different (L+/S+) or they are identical (L+/L+ or S+/S+). In the latter case, more precise coding of the two numbers is needed to determine which of the two large digits is the larger one (e.g., L and L+) or which of the two small digits is the smaller one (e.g., S and S+).

When *discriminable* codes are finally obtained, the comparison stage can take place. At this stage, a correct response is computed by matching the instructional codes (e.g., "choose the larger") with the codes generated by the encoding stage. In the case of an L+/S+ pair, the response is easily computed. However, in the case of an L+/L pair, it is easier to provide a correct response under the instruction "choose the larger" than under the instruction "choose the smaller." Indeed, in the latter situation, a transformation of the initial labels is needed to match the instructions: the L+/L labels will have to be recoded into S/S+. The same reasoning can be held for S/S+ pairs under the instructions "choose the larger."

In this model, the distance effect is easily explained: digits that are farther apart are likely to be coded differently right away, whereas digits that are close are more likely to share the same primary coding and thus to need further semantic elaboration before comparison. The size effect comes about because the cutoff point is usually placed among the smaller numbers. Consequently, large digits are more likely to fall beyond the cutoff point and to share the same initial coding as the small digits. They will thus need further elaboration of the initial coding, which will slow down the encoding stage. Finally, the semantic congruity effect emerges because the codes of the digits depend on the overall size of the pair, and the comparison stage is faster when the codes of the digits match the form of the instructional codes. Otherwise, the initial labels have to be transformed to match the instructions, and this supplementary processing step slows down the response. However, this model cannot account for the spatial congruity effect.

Banks's semantic coding model thus postulates that the comparison takes place on discrete or categorical information, although an analogue representation of numbers is assumed in memory. On the contrary, analogue models assume that comparison takes place directly on this analogue medium.

Analogue Models

The assumption of a comparison process acting directly on analogue representations of number magnitude can be traced back to rather old papers, such as in Moyer and Landauer (1967) or Restle (1970). This type of model automatically accounts for the distance effect since numerically close numbers are also close on the analogue medium and thus more difficult to disambiguate than more distant numbers. Yet, two possible explanations have been proposed to account for the size effect: one in the context of the accumulator model and the other under the number line model.

The accumulator model was first proposed by Mech and Church (1983) and elaborated by Gallistel and Gelman (1992). According to this model, when faced with a collection of objects, the individual's accumulator fills up proportionally to the number of items to enumerate; for each counted entity, a fixed quantity of energy is delivered into the accumulator. The final state of the accumulator then represents the numerosity of the array. Yet, as the quantity of energy entering the accumulator may vary slightly from one trial to the next, the state of the accumulator for a given numerosity presents a variability that increases with the size of that numerosity. In other words, magnitude representation is approximate and the variability of the mental magnitude distribution for a given numerosity increases proportionally with the numerosity (i.e., scalar variability; see Cordes, Gelman, Gallistel, & Whalen, 2001, for recent data on this point). Such an assumption can account for the size effect, as the representations of the magnitude of two large numbers are noisier and more difficult to compare than those of two small numbers.

The other way to explain the size effect has been to assume that magnitude representations are quite precise but that the mapping on the analogue medium is not linear. According to Dehaene, for instance (Dehaene, Dupoux, & Mehler, 1990; Dehaene, 1992), the mental number line would be compressed in such a way that for equal numerical distance, the subjective distance between the corresponding mental representations would be smaller as the size of the numbers increases. This relationship would follow a logarithmic curve such as defined by the Weber–Fechner law (i.e., the subjective distance between two numbers would be a function of the difference between their logarithms). Such an assumption makes it possible to account for the size effect, as small numbers are farther apart on the number line than large numbers.

If the analogue models easily account for the distance and size effects, they are less suited to explain the semantic congruity effect. Yet, they are perfectly able to account for the spatial congruency effect: subjects who respond "larger" with their right hand are faster than subjects who answer "larger" with their left hand because the number line is spatially oriented with the small numbers being coded on the left and large numbers on the right. Congruency between the spatial orientation of the number line and the orientation of the answer key allows faster responses.

Yet, the origin of this effect is still a matter of debate. This spatial bias or SNARC effect (Spatial–Numerical Association Response Code) was first reported in a parity judgment task (saying if a number is odd or even; Dehaene, Bossini, & Giraux, 1993). But Dehaene et al. (1993) showed that this spatial orientation was inverted for Iranian subjects and might, thus, be related to cultural habits such as the direction of reading. Other authors have also shown that it can be manipulated by experimental instructions (Bächtold, Baumüller, & Brugger, 1998). Furthermore, one may wonder whether this SNARC effect is really linked to the magnitude information of numbers or simply to their ordinal characteristics. Indeed, a SNARC effect has also been obtained with non-numerical ordinal series such as the letters of the alphabet or the months of the year (see Gevers, Reynvoet, & Fias, 2003, for an in-depth discussion of this point, and see chapter 3 in this volume).

APPEARANCE AND MODULATION OF THESE EFFECTS THROUGHOUT CHILDHOOD

In this second section, we will trace the appearance of the effects reported in the comparison literature and describe their developmental modulation throughout childhood. The first part of the section will deal with perceptual comparison in infants and preschoolers, whereas the second part will cover symbolic comparison in school-aged children.

Perceptual Comparison in Infants and Preschoolers

The Facts

To test discrimination capacities in babies, much research has used the habituation or the preferential-looking paradigms. In both cases, the child is presented with a series of stimuli that share a common property (e.g., a series of pictures of animals differing in color, size, or spatial location, but always containing the same number of stimuli). After several presentations of this type, the child gets habituated to the stimuli and his looking time in response to the presentation of the display decreases. At this point, the experimenter introduces a novel stimulus (e.g., a picture of six animals) simultaneously or successively with an old one (e.g., another picture of three animals). If the child perceives the difference between the old and the new material, he will look longer or more preferentially at the new stimuli than at the old one (i.e., he will dishabituate). If this happens, the crucial question then is to determine the nature of the change between the novel and the old stimulus that was detected by the child.

Using this method, many experiments have provided evidence that babies, even neonates, are able to discriminate collections of varying numerosities. The first studies of this type tried mainly to identify the border at which the child could discriminate a collection of *n versus n+1* items. These studies found that although infants could detect a change in small collections (e.g., one versus two or two versus three), they would fail when the size of the set exceeded three or four elements (e.g., four versus five; Starkey & Cooper, 1980; Antell & Keating, 1983; Strauss & Curtis, 1981; van Loosbroek & Smistman, 1990).

The Interpretations

The Numerical Hypothesis

Two main hypotheses were first proposed to account for these observations. Several authors, such as Gallistel and Gelman (1992), Wynn (1998), and Dehaene, Dehane-Lambertz, and Cohen (1998), assumed that human infants are born with a specialized mental mechanism for processing numerosities. In their view, this mechanism is preverbal and is shared by certain animal species. For instance, the accumulator model proposed by Gallistel and Gelman (1992) can account for the limit of the precise number discrimination observed in infants, as it assumes an increasing imprecision of the magnitude representation as numerosities get bigger.

The General Non-Numerical Capacities Hypothesis

Others authors, however (Simon, 1997; Uller, Carey, Huntley-Fenner, & Klatt, 1999), have proposed that infants' behavior can be explained by cognitive capacities that are general and not specifically numerical. For instance, Simon (1997) proposed that infants use a mental token to represent each item to be enumerated. These tokens derive from a pre-attentive mechanism dedicated to the representation and tracking of individual objects within a visual scene called the "the object-files." According to this proposal, infants' numerical discrimination can be explained as follows: when infants see a collection, each entity is mentally represented by a mental token. These mental tokens can be stored in memory when the set disappears. The presentation of another set gives rise to the same process of mental token elaboration. These two sets of mental tokens are then compared by a process of one-to-one correspondence. The perception of a mismatch between models induces a reaction of surprise in babies. Such an explanation perfectly predicts the size effect observed in infant studies since the number of items that can be represented and stored in memory at once is supposed to be limited to three or four.

One way to oppose the numerical and object file models is to consider large numerosities. According to the numerical model, large numerosities could be discriminated, provided the distance between the two numerosities considered is great enough. On the contrary, the object file model assumes that large numerosities could not be discriminated at all. Recently, several studies have used large collections of items and have manipulated the ratio of their numerosities. Remember that according to the compressed number line model, for instance, the difference between the magnitude of two and four is subjectively bigger than the difference between the magnitude of eight and ten. This is because the subjective distance between two magnitudes is a function of the difference between the logarithm of each number. Number pairs which present the same difference between the logarithm of their constituting numbers also share the same ratio (e.g., the pairs two–four or five–ten enter in the ratio of 1/2 and $\log(4) - \log(2) = \log(10) - \log(5)$). Furthermore, the ratio combines the size and the distance effect given that it approaches one when the distance between two numerosities decreases (2/10, 2/8, 2/6 . . .) or when the size of the set increases (4/8, 8/12, 12/16 . . .).

Using large collections with a manipulation of the ratio, Xu and Spelke (2000) showed that 6-month-old infants could discriminate large visual arrays entering in a 1/2 ratio (8 vs. 16 elements) but not those entering in a 2/3 ratio (8 vs. 12).[1] Similarly, Lipton and Spelke (2003) reported that 6-month-old infants could discriminate sequences of sounds entering in a 1/2 ratio (e.g., 8 vs. 16 sounds) but not sequences entering in a 2/3 ratio (8 vs. 12). At 9 months of age, however, infants are able to discriminate sequences entering in a 2/3 ratio but not those in a 4/5 ratio, which suggests that the precision of numerical discrimination increases over the infancy period. In preschoolers, Huntley-Fenner and Cannon (2000) also reported a ratio effect with the performance of 3- to 5-year-old children being more accurate in magnitude comparison for a ratio of 1/2 than for a ratio of 2/3. These results are thus incompatible with the object file model but support the numerical models.

The Perceptual Hypothesis

A third interpretation has recently been proposed. According to Mix and colleagues (Mix, Huttenlocher, & Levine, 2002; Clearfield & Mix, 2001), infants discriminate and quantify sets by using perceptual non-numerical cues that naturally covary with number such as the volume, the area, and the length. According to them, infants are initially unable to represent discrete number properties independently of their correlated perceptual variables and instead represent discrete and continuous quantities in terms of overall amount. In this case, an analogue medium would probably serve to represent the amount since perceptual representations are inherently imprecise. This analogue representation of amount would explain why infants' quantitative performance is subject to Weber's law and would account for the size, the distance, and the ratio effects. Such a proposal is supported by recent data showing that when perceptual cues such as surface area or contour length are pitted against number, 6- to 7-month-old infants dishabituate to a change in the continuous variables but not to a change in number (Feigenson, Carey, & Spelke, 2002; Clearfield & Mix, 1999). Similar results were also obtained by Rousselle, Palmers, and Noël (2004) with 3-year-old children: they were able to select the larger of two collections of dots if number and surface area were confounded but performed randomly when surface area was equated in the two collections.

Concluding Remarks

Currently, there is a lack of strong empirical evidences to rule out the perceptual account. Indeed, as outlined by Mix et al. (2002), the main problem is the simultaneous control of all perceptual variables that naturally covary with number. However, even if we only consider the studies which took great care in the control of perceptual variables, a heterogeneous picture still emerges: children seem to be able to discriminate large collections entering in a 1/2 ratio (e.g., Xu & Spelke, 2000; Lipton & Spelke, 2003) but fail to do so for small collections such as one vs. two (e.g., Feigenson, Carey, & Spelke, 2002; Clearfield & Mix, 1999). Such an opposition has been clearly underlined by Xu (2003), who showed that 6-month-old infants succeeded in discriminating four from eight visual elements but failed to discriminate two from four elements. Faced with these surprising results, one may consider like Feigenson, Carey, and Spelke (2002) or Xu (2003) that children would have two available mechanisms: an analogue numerical magnitude process and an object file model. But several questions remain open under this hypothesis. For instance, one may wonder why the analogue magnitude process does not code numerosities when small collections are presented. One possibility would be to assume that the object file process is faster than the analogue process, meaning that the former would always be activated first except in cases in which it is overwhelmed (i.e., when faced with large collections). Second, if one assumes that the object file mechanism is used for small collections, it remains to be explained why infants are unable to discriminate them when continuous perceptive variables are controlled for. Feigenson, Carey, and Spelke (2002) propose that these object files may also code objects' properties such as their size so that computations on these object files could be establishing one-to-one correspondence but could also include comparison of overall continuous extent. However, even this modified view of the object files could not explain a child's failure to discriminate small sets controlled for perceptual variables, as the possibility of one-to-one correspondence still predicts discrimination capacities in such situations.

In summary, the issue of number magnitude representation in babies has yielded much information over the past decades. Many facts have been collected and several interesting models have been proposed. Yet future research will be needed to come up with a satisfactory explanation. A possible way to clarify this problem could be to use cross-modal or cross-format paradigms, such as those used by Barth, Kanwisher, and Spelke (2003) with adults. The presence of a similar ability to compare numerosities across formats (e.g., simultaneous and sequential sets) or modalities (e.g., visual and auditory stimulus sets) in young children would certainly constitute strong evidence in favor of the hypothesis of a real processing of numerosities. Attempts in this direction were made by Starkey, Spelke, and Gelman (1990) but not replicated by Moore, Benenson, Reznick, Peterson, and Kagan (1987) or by Mix, Levine, and Huttenlocher (1997).

Many of the problems raised in perceptual comparison of numerosities disappear, however, with symbolic comparison. In this case, the physical properties of the stimuli are completely unrelated to the numerical ones. This type of material has been used in studies with older children that will be reviewed in the following section.

SYMBOLIC COMPARISON IN SCHOOL-AGED CHILDREN

Basic Results

Two studies have tested children of different ages in symbolic number comparison tasks. Both aimed to trace the appearance of the classic effects as well as to describe and interpret their developmental changes.

The first of these studies was run by Sekuler and Mierkiewicz (1977). They tested kindergarten and first-, fourth-, and seventh-grade children as well as university students with pairs of single-digit Arabic numerals. They observed a global decrease in response time with increasing age. Moreover, a distance effect was observed in each group, the slope of which grew steeper in the younger groups.

These results were replicated by Duncan and McFarland (1980) who tested kindergarten and first-, third-, and fifth-grade children as well as university students with single-digit pairs. A distance effect was present in each group with a decreasing size as participants were older. These authors also considered the semantic congruity effect and observed it in each age group, with its size being greater in the younger groups.

Thus, effects of the numerical distance and of the semantic congruity in magnitude comparison tasks are already present from kindergarten with single-Arabic digits. However, the magnitude of these effects decreases across development. Duncan and McFarland (1980) tried to identify the processing stage at which these effects occur. They considered two possibilities: either these effects and their changes across development take place at the encoding stage or they occur at the comparison stage. If these effects are linked to the encoding stage, a manipulation of the quality of the stimuli might modify their size; if not, this manipulation would just add a constant to the response time. Based on this reasoning, they contrasted a magnitude comparison task of Arabic digits with or without a line grid being superimposed over the stimuli. The results were clear: while the distance effect was not affected by the manipulation, the semantic congruity effect was stronger in degraded than in nondegraded stimuli. These results thus suggest that the semantic congruity effect lies at the encoding level, whereas the distance effect comes from the comparison stage. As the size of both these effects decreases across development, Duncan and McFarland concluded that "both encoding and comparison improve with age" (p. 621).

Explanations for the Developmental Changes in Number Comparison

Given that comparison improves with age, one may wonder about the reasons underlying those changes. According to Sekuler et al. (1977), three types of modifications of magnitude representation could be evoked:

1. The average distance between analogue representations of the digits could be reduced in younger subjects because their analogue number line could be more compressed than in adults.
2. The discriminal dispersions around each mean representation could be larger in younger subjects.
3. Both explanations could be correct; that is, younger subjects' number magnitude representation could be both more compressed than the adults' and also present a higher dispersion.

These propositions actually correspond to different theoretical conceptions of the analogue magnitude representation of numbers: the first proposition is more compatible with the logarithmically compressed number line, whereas the second is more compatible with the scalar variability of the accumulator.

Siegler and Opfer (2003) contrasted the predictions of these two models in two estimation tasks. In the number-to-position task, participants were shown a one- to three-digit number and asked to estimate its position on a drawn number line, with the left end labeled "0" and the right end labeled "100" or "1000." In the position-to-number task, they were shown a position on a number line and asked to estimate the corresponding number. Second, fourth, and sixth graders, along with undergraduates, were presented with the two tasks. If their magnitude representation follows a logarithmic compression, as in the number line model, their mean estimates should increase logarithmically with numerical magnitude in the number-to-position task and exponentially with the number in the position-to-number task. The variability of these estimates should be constant across the number range. If, on the contrary, their magnitude representation is best modeled by the accumulator model, the mean estimates should increase linearly with the size of the number, and the variability of these estimates should increase with the size of the numbers on both tasks.

Results supported the logarithmic model in the 0–1000 number line in grades two and four. Yet, none of these above-mentioned models could account for the 0–100 number line in grades two and four nor for the 0–100 and 0–1000 number lines in grade six and in adults. Indeed, in the latter cases, mean estimates increased linearly while variability remained constant across numerosity. Siegler and Opfer referred to this profile as the linear-ruler model.

One could, however, wonder whether estimates on a physical number line are a perfect modelization of the magnitude representation on an internal number line. Indeed, the physical number line has attributes (such as the two ends marked, for instance) that allow the child to build specific strategies to deal with the task. For instance, the large numbers that are near the right end of the physical number line are consequently better estimated than smaller numbers that do not benefit from these external marks. The material used in the task thus induces biases that are not supposed to be present in the magnitude representation. Consequently, we fear that the use of such a paradigm may lead to erroneous conclusions.

In this sense, Huntley-Fenner's (2001) task might be considered the more appropriate. Collections of 5 to 11 black squares were flashed for 250 ms, and children (5–7 years old) were asked to estimate their cardinal and select the corresponding Arabic numeral between 1 and 20. Results are largely compatible with the accumulator model: the means of the estimates increased with the presented numerosity, and there was a scalar variability as the SD/mean estimate ratio was constant among the stimuli set. Moreover, the authors reported a slight developmental trend in which this SD/mean estimate ratio grew smaller in older children. This would suggest a growing precision of the magnitude representation with age. Based on these observations, one may speculate that this increasing precision due to a reduced dispersion around each mean magnitude representation might account for the decrease in the distance effect observed in magnitude comparison tasks.

Two Last Issues

Two other questions have been addressed in the literature. First, according to Dehaene, (1992) number magnitude representation is spatially oriented (see chapter 3 for a deep review of this topic). Accordingly, one may wonder whether this spatial orientation of the number line (SNARC effect) is precocious or appears late in development. Second, the magnitude representation is supposed to recruit the bilateral inferior parietal areas (Dehaene & Cohen, 1995). One could thus wonder whether the same brain areas are activated during magnitude processing in young children and in adults. The few studies that have addressed these two issues will be considered successively.

The question of the onset of a spatial bias in number magnitude processing in children is interesting because predictions in this matter differ according to the various interpretations that have been proposed to account for its observation in adults. At least three possibilities can be considered. First, one could suppose that the spatial orientation of the number line is an intrinsic property of number magnitude representation. In this case, the SNARC effect should be present simultaneously with the other effects, such as the distance and size effects. Second, one could argue that this spatial orientation is related to cultural habits such as the direction of reading. It should then appear only when children become used to that direction of reading. Third, this SNARC effect could be associated with the ordinal nature of numbers rather than with their cardinal meaning. Consequently, the SNARC effect should appear when children are familiar with the order of the numbers (i.e., when they are able to recite the number sequence). To our knowledge, only one study has addressed this issue. Berch, Foley, Hill, and Ryan's (1999) study asked children (grades two, three, four, six, and eight) to make an odd/even judgment on the 0–9 Arabic numerals. A significant SNARC effect appeared from grades three and up. However, the failure to obtain any SNARC effect in grade two was attributed to the high error rate and large RT variance in that age group. Further research will be needed, and, preferably, with simpler tasks as well as with tasks requiring a processing of number magnitude per se, to identify the time at which this spatial association of numbers appears.

The second question relates to the developmental changes in the brain regions supporting number magnitude representation and has only been addressed by one study. In this research project, Temple and Posner (1998) tested 5-year-old children in a magnitude comparison task of dots and Arabic-digit stimuli, using an event-related potentials paradigm. They observed a distance effect that appeared on the same locations of electrodes (in the bilateral parietal regions) as in adults and that was delayed by only 50 ms.

Concluding Remarks

In summary, studies on the development of Arabic number magnitude representation have put forward more similarities between a 5-year-old child and an adult than differences. Indeed, at this young age, children already show the distance and semantic congruity effects in magnitude comparison tasks with single-Arabic digits. The brain areas activated during this task appear to be remarkably similar to those of adults. The main developmental changes observed are quantitative rather than qualitative; over time, children's global RTs decrease, as does the size of the distance and semantic congruity effects. This is probably due to the general speed-up phenomenon that is observed across development in all speeded tasks (see Kail, 1993). Yet the reduction in the size of the distance effect probably also results from an increasing precision of the number magnitude representation. Even so, alongside these quantitative changes, it is possible that some qualitative changes may have gone unnoticed. For instance, it is possible that the spatial orientation of the number line could only develop later, around grade three, but more empirical evidence on that topic would be needed.

AUTOMATIC ACTIVATION OF NUMBER MAGNITUDE

Introduction

All the studies reported in the previous section used tasks that explicitly required the processing of number magnitude, and the obtained results have underlined the large commonalties between children and adults. Yet, adults have also been shown to process number magnitude in tasks for which this dimension is not relevant. For instance, when they are presented with pairs of digits and asked to judge if the two stimuli are physically identical or not, adults are slower to say "different" when the two numbers are numerically close than when they are numerically distant from one another (Dehaene & Akhavein, 1995). Similarly, if they are presented with a sequence of digits and asked to count the symbols (e.g., 3333, answer "four"), they are slower when the number of items differs from the identity of the items (555 vs. 333), and this interference is larger when the number of symbols is close to the identity of the digits (e.g., stronger in the case 444 than in the case 888; see Pavese & Umiltà, 1998). These interferences of the numerical dimension in these tasks, where it is irrelevant, argue for an automatic and unintentional activation of number magnitude. In this section, we will report on studies that have aimed to determine the age at which this automatic activation of number magnitude appears in children.

The Data

The first evidence in this domain comes from the study of Duncan and McFarland (1980). These authors presented pairs of single-digit Arabic numbers to participants of different ages (from kindergarten to university students) who had to decide whether the two symbols were physically identical or not. When different, the digits composing the pairs were numerically close (distance of 1, 2, or 3) or distant (i.e., distance of 6, 7, or 8). Both adults and children (even those from kindergarten) showed a distance effect. Thus, even kindergartners processed the digits up along the semantic stage in this simple physical judgment task.

Girelli, Lucangeli, and Butterworth (2000) drew a quite different conclusion. These authors used a Stroop paradigm: pairs of Arabic digits were presented in a numerical and in a physical comparison in which subjects had, respectively, to select the stimulus larger in magnitude or in physical size. In both tasks, congruency between the magnitude and the physical dimensions was manipulated: in congruent trials the digit that was physically bigger was also the numerically larger (e.g., 2–6); in incongruent pairs, the digit that was physically smaller was numerically larger (e.g., 2–6); and in neutral pairs, the digits were displayed with the same physical size (2-6). In these conditions, the presence of a (numerical) size-congruity effect in the physical comparison indicates that the irrelevant number magnitude information has been automatically processed. Further, pairs with a numerical distance of 1 or 5 were contrasted, as the presence of a distance effect in the physical judgment is another indicator of an automatic activation of number magnitude. Results indicated the presence of a size congruity and a distance effect in adults' physical judgment as well as in that of fifth-grade children. Third graders, however, only showed a size congruity effect, while none of these effects were significant in the first grade. The authors thus concluded that automatization is achieved gradually as younger children do not show any signs of an automatic activation of the magnitude information.

Recently, a very similar study was undertaken by Rubinsten and colleagues (2002). Five groups of students were tested (beginning of first grade, end of first grade, third grade, fifth grade, and university students), and three numerical distances between the single-Arabic digits of the pairs (1, 2, and 4) were considered. In the physical comparison task, the size congruity effect was absent in the beginning of the first grade but was significant by the end of that school year and later on.[2] Thus, by introducing this group of "end first graders," the authors were able to show much earlier signs of automatic activation of number magnitude than reported by Girelli et al. However, and contrary to Girelli et al., they failed to find any significant distance effect in the physical comparison (although it was present in the magnitude comparison condition).

In relation to these Stroop studies, two questions will be addressed. First, how can we account for the fact that the size congruity and distance effects sometimes dissociate in the physical tasks? Second, how can the failure to obtain any of these effects in certain age groups be interpreted?

Dissociations between Size-Congruity and Distance Effects

If both the size-congruity and the distance effects (observed in the physical task) are indicative of an automatic activation of number magnitude representation, then how is it possible to observe one of these effects without the other? Although intriguing, such a result profile is not rare. In Girelli et al.'s (2000) third graders and in all the groups in Rubinsten et al.'s study (2002), a significant size-congruity effect was observed without any distance effect. Tzelgov, Meyer, and Henik (1992) have proposed a possible interpretation of this dissociation. According to these authors, the distance and the size-congruity effects do not reflect the same process of number magnitude. In their view, the distance effect results from an algorithmic process that codes both numbers on the number line and enables their comparison, whereas the size-congruity effect reflects a different process. Indeed, as we accumulate experience of Arabic digit comparison, we store instances in which digits are judged as small or large quantities, and it is the retrieval from memory of those instances that is reflected by the size-congruity effect. In intentional conditions, that is, in number magnitude comparison tasks, the distance effect always appears and signs the use of the algorithmic process. However, in nonintentional conditions (e.g., in the physical comparison task), both mechanisms take place but, as memory retrieval is faster than the algorithmic process, only a size-congruity effect appears, even in adults.

Rubinsten et al. (2002) follow this reasoning to interpret their data. First, as the physical comparison task does not require an intentional access to number magnitude, the memory-based mechanism is dominant, which explains the absence of any significant distance effect in all the groups considered. Second, as the size-congruity effect (which is the signature of this memory-based mechanism) only appears when the child has accumulated enough experience with magnitude comparison of Arabic digits, it failed to appear in young and inexperienced beginners of first grade but was present in older groups.

Finally, Rubinsten et al. try to reinterpret the data obtained by Girelli et al. (2000). Remember that in this experiment, fifth graders and adults showed both a size-congruity and a distance effect. As magnitude information was not intentionally processed, the presence of this last effect is surprising under Tzelgov et al.'s interpretations. However, Rubinsten et al. noticed that in this experiment, numerical distance was totally confounded with the type of pairs in terms of "memory labels": stimuli with a small numerical distance of one were made of two numbers sharing the same labels (either two small or two large numbers), whereas the large-distance pairs were all made of one small and one large number. In these conditions, the algorithmic and the memory processes are impossible to disentangle: the distance effect might just reveal the autonomous activation of memory labels rather than of magnitude per se.

Although appealing, this interpretation does not seem able to account for two facts of Rubinsten et al.'s study. First, if the memory-based process dominates the nonintentional processing of magnitude, then in the physical task, same-label pairs should be processed faster than different-label pairs. Second, the size-congruity effect should be stronger in the different-label pairs than in the same-label pairs. The type of label of the pairs was not introduced as such in the study, but it was partially confounded with the numerical distance: pairs with a distance of 1 and 2 were all same-label pairs, whereas pairs with a distance of 4 were all different-label pairs. However, no distance effect emerged in the physical task, even between the distance 1–2 (same-label pairs) and 4 (different-label pairs). Furthermore, there is no mention of an interaction between size congruity and numerical distance.

A Relative-Speed Account

The second issue that needs to be addressed regards the interpretation of the null effect (i.e., the failure to measure any significant size-congruity or distance effect in certain age groups). Most of the time, the conclusion has simply been that number magnitude representation was not automatically activated in these young children who failed to show any of those significant effects. Alternatively, one may consider the possibility that this failure may be due to the large discrepancy between processing time for the physical and the magnitude dimensions: number magnitude could be activated in the physical comparison task, whereas a decision for the physical dimension could be reached before the magnitude information has had the time to interfere with the response. This possibility is mentioned briefly by Girelli et al. (2000) and by Rubinsten et al. (2002).

Recently, Schwarz and Ischebeck (2003) have proposed a relative speed account of the number-size interference in Stroop tasks. According to their model, decisions are not based on information conveyed in an all-or-none fashion but, rather, on noisy partial information that continuously accumulates until the decision is reached. As both the processing of the relevant and of the irrelevant dimensions is influenced by the distance (i.e., the physical and the numerical distance), decreasing the relevant distance or increasing the irrelevant distance should lengthen the period of time during which the irrelevant information has an opportunity to influence the processing of the relevant attribute. In other words, to favor the emergence of a numerical interference on the physical judgment, one should increase the difficulty of the physical comparison (i.e., presenting pairs of digits with close physical size) or increase the ease of the numerical comparison (i.e., presenting pairs of digits with large numerical distance).

In Girelli et al. (2000), the physical size distance was so important (24 points vs. 48 points) that in first grade, the physical judgment took about half the time required for the numerical judgment (in the neutral conditions, physical comparison took 1,188 ms, whereas numerical comparison took 2017 for a distance of 5 and 2,130 for a distance of 1). Later on, RTs decreased overall but much more significantly for the numerical task than for the physical task, which may account for the appearance of a number distance effect in the physical task for the older groups. For Rubinsten, the differences in physical size were much smaller (either 6 vs. 7 mm height or 7 vs. 9 mm height). The difference between the physical and the numerical judgments were thus smaller as well (mean difference of 257 over all the groups) and decreased with age (this difference was 128 ms greater in the beginning of first grade than by the end of first grade, but only 39 ms greater at the end of first grade compared with the third grade). These differences in ease of physical comparison may account for the more precocious observation of a size-congruity effect in Rubinsten et al.'s than in Girelli et al.'s studies.

Recently, Mussolin (2002) has tried to palliate this problem of processing time difference by balancing the speed of the physical and the numerical processing. A Stroop paradigm was used, but in contrast to the classic version of the task, the number pairs did not appear directly with a physical size difference but were first presented in a same intermediate physical size (e.g., 3–6). Only after a certain delay did their physical size differences emerge with one digit's physical size increasing and the other one's decreasing (e.g., 3–6). The delay before the appearance of the physical size difference was calculated for each participant and corresponded to his/her difference in processing speed between a physical comparison task on similar digits (e.g., 3–3) and a magnitude comparison of digits presented with the same physical size (e.g., 3–6). In this way, we could assume that the number magnitude information and the physical size information were reached simultaneously, which provides good conditions for the irrelevant number magnitude information to alter the physical comparison task.

The other aim of this study was to examine whether the age at which the numerical magnitude is automatically activated does or does not depend on the size of the numbers considered. Therefore, three numerical sizes of number pairs were used: pairs of single digits (e.g., 2–4), pairs of two-digit numbers smaller than 50 (e.g., 23–46), and pairs of two-digit numbers with at least one bigger than 50 (e.g., 34–68[3]). For each of them, the ratio was manipulated with half of the pairs having a ratio of 1/2 (e.g., 23–46) and the other having a ratio of 2/3 (e.g., 32–48).

Second, third, and fourth graders were presented with these number pairs and asked to select the one that was physically bigger. Besides the usual decrease in RTs with age, a highly significant size-congruity effect and an interaction between this effect and the ratio were observed. Congruent trials were processed faster than incongruent ones, but this was true for the ratio of 1/2 only. In other words, number magnitude interfered with the physical judgment when the two numbers of the pair were highly distant numerically, as predicted by Schwartz and Ischebeck's (2003) model. Another important finding was the failure to obtain any size effect, which means that this automatic activation of number magnitude was observed for both one- and two-digit numbers. Yet, analyses run separately in each group indicated that for second graders, only the single-digit and the small two-digit numbers gave rise to a size-congruity effect, which indicates that only those smaller numbers led to an automatic activation of their corresponding magnitude representation.

Summary

In conclusion, using a single-digit matching task, Duncan and McFarland (1980) reported a distance effect that suggested an automatic activation of number magnitude from as early as the age of 6 years. Converging evidence was also obtained in a Stroop paradigm by Rubinsten et al. (2002), who reported a size congruity effect from the end of first grade. The failure to observe similar results in Girelli et al. (2000) may be accounted for by the relative-speed model (Schwartz et al., 2003). Finally, the results of Mussolin (2002) indicate that second graders are able to automatically activate not only the magnitude of single-digit numbers but also that of two-digit numbers, provided the task at hand has been slowed down sufficiently to allow numerical information to interfere with it.

A BASIC DYSFUNCTION OF NUMBER MAGNITUDE REPRESENTATION IN DYSCALCULIC CHILDREN?

As we have underlined in the beginning of this chapter, magnitude is a key dimension to the semantic of numbers. The results reported in the two previous sections have shown that children starting elementary school show the same basic effects in Arabic number magnitude comparison as those observed in adults and that they automatically activate the magnitude of Arabic numerals, even when this dimension is not relevant for the task. About 6% of school-aged children have major difficulties in mathematics (Gross-Tsur, Manor, & Shalev, 1996). As number magnitude seems to be one of the roots of this learning, Butterworth (1999) has proposed that a dysfunction of that representation might well be one of the possible causes of mathematical disabilities. In support of this hypothesis, he reported the case of Charles, who had difficulty with mathematics for many years. At the age of 31, he still counted on his fingers to solve simple additions. A deep investigation of his number-processing capacities showed several abnormalities in his processing of number magnitude. Thus, when required to select the larger of two Arabic digits, he relied on counting, which led him to be four times slower than control subjects and to present a reverse distance effect (he was thus faster when digits were numerically close than when they were numerically distant). Finally, in a Stroop task such as reported in Girelli et al. (2000), he failed to show any distance effect in the physical comparison condition. This profile was interpreted by Butterworth as a basic dysfunction of Charles's number magnitude representation, which accounted for his mathematical difficulties.

Other pieces of evidence for this hypothesis can be found in the recent study of Isaacs, Edmonds, Lucas, and Gadian (2001). These authors compared teenagers who were born prematurely and who had normal versus weak calculation abilities with structural fMRI and observed more gray matter in the former group in the left intraparietal sulcus only. This result is of particular interest because this specific brain area is probably a major structure for supporting the representation of number magnitude. Indeed, it has been shown to be activated in Arabic magnitude comparison (Pesenti, Thioux, Seron, & De Volder, 2000) and to be sensitive to the numerical distance of numbers to be compared, independent of their presentation format (Arabic digits or words; Pinel, Dehaene, Rivière, & LeBihan (2001).

Neuropsychological data also provide converging evidence for a role of the parietal lobe in number magnitude representation and in dyscalculia. In acquired disorders, it is well known that a lesion of the parietal lobe of the dominant hemisphere may lead to acalculia, which often appears in association with finger agnosia, left–right disorientation, and dysgraphia (i.e., the Gerstmann syndrome). For instance, the patient MAR (Dehaene & Cohen, 1997), who suffered from an infarct of the parietal lobe of his dominant hemisphere, exhibited a Gerstmann syndrome and was mostly impaired in numerical tasks that required the manipulation of number magnitude.

The existence of the Gerstmann syndrome has inspired the study of Fayol, Barrouillet, and Marinthe (1998). These authors tested normal 5-year-old children's digital agnosia and found that these measures predicted arithmetic performance 1 year later. Following this research, Noël (2002) wondered whether these predictions concerned numerical performance in general or, more specifically, performance in tasks requiring the processing of number magnitude. Indeed, according to Dehaene's (1992) triple-code model, one could suppose that a parietal weakness would be associated with low performances in tasks based on the magnitude representation (e.g., number comparison or subitizing[4]) but might preserve number processing that could be realized without appealing to this representation (e.g., number transcoding or simple arithmetic). Digital agnosia was tested in first-grade children (at the beginning of the school year) and their numerical abilities one year later. The global results were not in favor of a specific relationship between digital agnosia and number magnitude processing. Yet children were doing quite well with the test of digital agnosia, and only two of them showed a real deficit at that level. One of them, Simon, had a very interesting profile. Although scoring very poorly on the digital agnosia test (Z score of –2.38), he did not exhibit a complete Gerstmann syndrome, as he scored normally on a left–right orientation test (Z score = 0.68). One year later, his numerical performance was tested. In a magnitude comparison of dot collections, he performed slowly (Z score of 1.71) and produced numerous errors (Z score = 2.59). He was also extremely slow (Z score of 2.78), although accurate (Z score of 0.02) when asked to select the bigger of two single-digit Arabic numbers. This slowness was not generalized, as his RTs were normal in a physical matching task of the same Arabic digits (Z = 0.47). Furthermore, he could only subitize two dots, which is very few (Z score = –2.00), and he made too many errors when asked to say how many fingers were raised (Z score = 2.36). However, his difficulties with numbers were not generalized: he scored above the mean for fast single-digit additions (Z=0.94) and number transcoding (Z score = 0.73). Finally, his deficits in numerical tasks were not accompanied by a reading disability as he scored above the mean for reading words (Z score = 1.18). The profile exhibited by Simon thus suggests that digital agnosia measured at entry level in elementary school may predict disabilities in learning mathematics 1 year later. Furthermore, these learning problems might specifically affect number magnitude processing, leaving intact more verbal numerical abilities (such as arithmetical facts and transcoding).

At this point, evidence supporting Butterworth's hypothesis of a basic number magnitude dysfunction in developmental dyscalculia is scarce and heterogeneous. Yet, these results are encouraging. It is hoped that further research will enable a deeper understanding of developmental dyscalculia in the case of otherwise normal children as well as in the case of genetic disorders (Ansari & Karmiloff-Smith, 2002).

GENERAL CONCLUSION

Magnitude is a key dimension of numbers' semantic. Its representation is usually evaluated through number comparison tasks. In such situations, children's performance is sensitive to the same variables as adults' (i.e., the size of the numbers and the distance between them). Furthermore, from the age of 6 or 7, the presentation of a one- or a two-digit number automatically activates its corresponding magnitude, even when irrelevant for the task. Given the importance of this dimension and its degree of development before any explicit mathematical instruction, we have considered Butterworth's hypothesis of a possible link between a dysfunction of the number magnitude representation and mathematics learning disabilities. Although strong evidence is currently not available, several pieces of information support the plausibility of this hypothesis. Much work remains to be done and several questions need to be addressed. First, what precisely is the dysfunction seen in the magnitude representation in dyscalculic children? Second, does this dysfunction correspond to structural or functional peculiarities of certain brain areas? Third, are all developmental dyscalculia linked to a dysfunction of the magnitude representation? Finally, in case of abnormalities in the magnitude representation, is all number processing impaired or is some preserved? We are confident that future research will provide answers to these questions.

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NOTES

1. However, inconsistent observations were reported by Feigenson and colleagues (2002) with 10- to 12-months- old infants failing to compare arrays that differed by a ratio of 1/2 (two vs. four, three vs. six).
2. The interference effect obtained at the end of the first grade was accompanied by a facilitation effect in the third grade and up.
3. All of the two-digit pairs were compatible; i.e., the comparison of the decade digits and of the unit digits led to the same result. Nuerk et al. (2001) have indeed underlined the impact of decade–unit compatibility on magnitude representation of two-digit Arabic numerals.
4. Subitizing refers to the fast and precise apprehension of small numerosities up to four or five.

REFERENCES

Ansari, D., & Karmiloff-Smith, A; (2002). Atypical trajectories of number development: A neuroconstructivist perspective. *Trends in Cognitive Science, 6*(12), 511–516.

Antell, S. E., & Keating, D P. (1983). Perception numerical invariance in neonates. *Child Development, 54*, 695–701.

Bächtold, D., Baumüller, M., & Brugger, P. (1998). Stimulus–response compatibility in the representational space. *Neuropsychologia, 36*(8), 731–735.

Banks, W. P. (1977). Encoding and processing of symbolic information in comparative judgments. In G. H. Bower (Ed.), *The psychology of learning and motivation* (Vol. 11, pp. 101–159). New York: Academic Press

Banks, W. P., Clark, H. H., & Lucy, P. (1975). The locus of the semantic congruity effect in comparative judgments. *Journal of Experimental Psychology: Human Perception and Performance, 1*, 35–47.

Banks, W. P., Fujii, M., & Kayra-Stuart, F. (1976). Semantic congruity effects in comparative judgments of magnitudes of digits. *Journal of Experimental Psychology: Human Perception and Performance, 2*, 435–447.

Barth, H., Knawisher, N., & Spelke, E. (2003). The construction of large number representations in adults. *Cognition, 86*, 201–221.

Berch, D. B., Foley, E. J., Hill, R. J., & McDonough Ryan, P. (1999). Extracting parity and magnitude from Arabic numerals: Developmental changes in number processing and mental representation. *Journal of Experimental Child Psychology, 74*, 286–308.

Butterworth, B. (1999). *The mathematical brain*. London: Macmillan.

Clearfield, M. W., & Mix, K. S. (1999). Number versus contour length in infants' discrimination of small visual sets. *Psychological Science, 10*, 408–411.

Clearfield, M. W., & Mix, K. S. (2001). Amount versus number: Infants' use of area and contour length to discriminate small sets. *Journal of Cognition and Development, 2*(3), 243–260.

Cordes, S., Gelman, R., Gallistel, C. R., & Whalen, J. (2001). Variability signatures distinguish verbal from nonverbal counting for both large and small numbers. *Psychonomic Bulletin & Review, 8*(4), 698–707.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition, 44*, 1–42.

Dehaene, S., & Akhavein, R. (1995). Attention, automaticity and levels of representation in number processing. *Journal of Experimental Psychology: Learning, Memory and Cognition, 21*, 314–326.

Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition, 1*, 83–120.

Dehaene, S., & Cohen, L. (1997). Cerebral pathways for calculation: double dissociation between rote verbal and quantitative knowledge of arithmetic. *Cortex, 33*, 219–250.

Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General, 122*, 371–396.

Dehaene, S., Dehaene-Lambertz, G., & Cohen, L. (1998). Abstract representations of numbers in the animal and human brain. *Trends In NeuroSciences, 21*(8), 355–361.

Dehaene, S., Dupoux, E., & Mehler, J. (1990). Is numerical comparison digital? Analogical and symbolic effects in two-digit number comparison. *Journal of Experimental Psychology: Human Perception and Performance, 16*, 626–641.

Duncan, E. M., & McFarland, C. E. (1980). Isolating the effects of symbolic distance and semantic congruity in comparative judgments: An additive-factors analysis. *Memory & Cognition, 8*(6), 612–622.

Fayol, M., Barrouillet, P., & Marinthe, C. (1998). Predicting arithmetical achievement from neuropsychological performance: A longitudinal study. *Cognition, 68*, B63–B70.

Feigenson, L., Carey, S., & Hauser, M (2002). The representation underlying infants' choice of more: Object files vs. analog magnitudes. *Psychological Science, 13*(2), 150–156.

Feigenson, L., Carey, S., & Spelke, E. (2002). Infants' discrimination of number vs. continuous extent. *Cognitive Psychology, 44*, 33–66.

Fuson, K. C. (1992). Relationships between counting and cardinality from age 2 to age 8. In J. Bideaud, C. Meljac, & J.-P. Fisher (Eds.), *Pathways to number. Children's developing numerical abilities* (pp. 127–149). Hillsdale, NJ: Erlbaum.

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition, 44*, 43–74.

Gevers, W., Reynvoet, B., & Fias, W. (2003). The mental representation of ordinal sequences is spatially organized. *Cognition, 87*, B87–B95.

Girelli, L., Lucangeli, D., & Butterworth, B. (2000). The development of automaticity in accessing number magnitude. *Journal of Experimental Child Psychology, 76*, 104–122.

Gross-Tsur, V., Manor, O., & Shalev, R. S. (1996). Developmental dyscalculia: Prevalence and demographic features. *Developmental Medicine and Child Neurology, 38*, 25–33.

Huntley-Fenner, G. (2001). Children's understanding of number is similar to adults' and rats': Numerical estimation by 5–7-year-olds. *Cognition, 78*(1), B27–B40.

Huntley-Fenner, G., & Cannon, E. (2000). Preschoolers' magnitude comparisons are mediated by a preverbal analog mechanism. *Psychological Science, 11*(2), 147–152.

Isaacs, E. B., Edmonds, C. J., Lucas, A., & Gadian, D. G. (2001). Calculation difficulties in children of very low birthweight. A neural correlate. *Brain, 124*, 1701–1707.

Kail, R. (1993). Processing time decreases globally at an exponential rate during childhood and adolescence. *Journal of Experimental Child Psychology, 56*, 254–265.

Lipton, J. S., & Spelke, E. S. (2003). Origins of number sense: Large-number discrimination in human infants. *Psychological Science, 14*(5), 396–401.

Mech, W. H., & Church, R. M. (1983). A mode control model of counting and timing processes. *Journal of Experimental Psychology: Animal Behavior Processes, 9*, 320–334.

Miller, K., & Gelman, R. (1983). The child's representation of number: A multidimensional scaling analysis. *Child Development, 54*(6), 1470–1479.

Mix, K. S., Levine, S. C., & Huttenlocher, J. (1997). Numerical abstraction in infants: Another look. *Developmental Psychology, 33*, 423–428.

Mix, K. S., Huttenlocher, J., & Levine, S. C. (2002). Multiple cues for quantification in infancy: Is number one of them? *Psychological Bulletin, 128*(2), 278–294.

Moore, D., Benenson, J., Reznick, J. S., Peterson, M., & Kagan, J. (1987). Effect of auditory numerical information on infants' looking behavior: Contradictory evidence. *Developmental Psychology, 23*, 665–670.

Moyer, R. S., & Landauer, T. K. (1967). Time required for judgments of numerical inequality. *Nature, 215*, 1519–1520.

Mussolin, C. (2002). *L'automaticité de l'accès à la représentation numérique chez l'enfant.* Unpublished Master's thesis, Université Catholique de Louvain, Belgium.

Noël, M.-P. (2002). *Les signes du syndrome de Gerstmann: Une aide à la compréhension des bases neuroanatomiques du calcul et de certaines dyscalculies développementales?* Presentation at the Troisièmes journées "troubles de l'apprentissage du langage oral et écrit." Marseille-Aix, 15–16 November.

Pavese, A., & Umiltà, C. (1998). Symbolic distance between numerosity and identity modulates Stroop interference. *Journal of Experimental Psychology, Human Perception & Performance, 24*, 1535–1545.

Pesenti, M., Thioux, M., Seron, X., & De Volder, A. (2000). Neuroanatomical substrates of Arabic number processing, numerical comparison and simple addition: A PET study. *Journal of Cognitive Neuroscience, 12*(3), 461–479.

Pinel, P., Dehaene, S., Rivière, D., & LeBihan, D. (2001).

Modulation of parietal activation by semantic distance in a number comparison task. *NeuroImage, 14,* 1013–1026.
Restle, F. (1970). Speed of adding and comparing numbers. *Journal of Experimental Psychology, 83,* 274–278.
Rousselle, L., Palmers, E., & Noël, M.-P. (2004). Magnitude comparison in preschoolers: What counts? Influence of perceptual variables. *Journal of Experimental Child Psychology, 87,* 57–84.
Rubinsten, O., Henik, A., Berger, A., & Shahar-Shalev, S. (2002). The development of internal representations of magnitude and their association with Arabic numerals. *Journal of Experimental Child Psychology, 81,* 74–92.
Schwarz, W., & Ischebeck, A. (2003). On the relative speed account of the number-size interference in comparative judgments of numerals. *Journal of Experimental Psychology: Human Perception and Performance, 29*(3), 507–522.
Sekuler, R., & Mierkiewicz, D. (1977). Children's judgments of numerical inequalities. *Child Development, 48,* 630–633.
Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological Science, 14*(3), 237–243.
Simon, T. J. (1997). Reconceptualizing the origins of number knowledge: A "non-numerical" account. *Cognitive Development, 12,* 349–372.
Starkey, P., & Cooper, R. G. (1980). Perception of numbers by human infants. *Science, 210,* 1033–1035.
Starkey, P., Spelke, E. S., & Gelman, R. (1990). Numerical abstraction by human infants. *Cognition, 36,* 97–127.
Strauss, M. S., & Curtis, L. E. (1981). Infant perception of numerosity. *Child Development, 52,* 1146–1152.
Temple, E., & Posner, M. I. (1998). Brain mechanisms of quantity are similar in 5-year-old children and adults. *Proceedings of the National Academy of Science, 95,* 7836–7841.
Tzelgov, J., Meyer, J., & Henik, A. (1992). Automatic and intentional processing of numerical information. *Journal of Experimental Psychology: Learning, Memory and Cognition, 18*(1), 166–179.
Uller, C., Carey, S., Huntley-Fenner, G., & Klatt, L. (1999). What representations might underlie infant numerical knowledge? *Cognitive Development, 14,* 1–36.
van Loosbroek, E., & Smitsman, W. (1990). Visual perception of numerosity in infancy. *Developmental Psychology, 26*(6), 916–922.
Wynn, K. (1998). Psychological foundations of number: Numerical competence in human infants. *Trends in Cognitive Science, 2*(8), 296–303.
Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition, 74,* B1–B11.
Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representations. *Cognition, 89,* B15–B25.