Cooperative game-theoretic features of cost sharing in location-routing*

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Abstract

This article studies several variants of the location-routing problem using a cooperative game-theoretic framework. The authors derive characteristics in terms of subadditivity, convexity, and non-emptiness of the core. Moreover, for some of the game variants, it is shown that for facility opening costs substantially larger than the costs associated with routing, the core is always non-empty. The theoretical results are supported by numerical experiments aimed at illustrating the properties and deriving insights. Among others, it is observed that, while in general it is not possible to guarantee core allocations, in a huge majority of cases the core is non-empty.

Keywords: Collaborative logistics; Location-routing; Cooperative game theory; Cost allocation

1 Introduction

Recent literature surveys by Guajardo and Rönnqvist (2016), Gansterer and Hartl (2018), and Cleophas et al. (2018) reveal a considerable growth in the number of articles and applications of collaborative transportation. This literature positions collaborative transportation as an important mechanism to reduce cost and negative environmental effects, and to increase profits and service levels. In parallel, the literature on location-routing problems has also grown considerably. The central problem in this literature is to locate facilities while simultaneously finding routes to serve customers from these facilities. It has early been recognized that tackling these decision problems separately may lead to suboptimal solutions (Perl, 1987; Salhi and Rand, 1989). Reviews by Prodhon and Prins (2014), Drexl and Schneider (2015), and Schneider and Drexl (2017) give account of a broad range of applications as well as a considerable progress in solution methods for location-routing problems.

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Recent emergence of dynamic on-demand warehousing (Sinha, 2016) emphasizes the potential in applicability even further as it introduces facility location decisions of an operational (short-term) nature.

Cleophas et al. (2018) list the cooperative planning within location-routing problems as one of future challenges in operations research. Despite the growing interest in both collaborative transportation and location-routing, the integration of these two areas remains fairly unattended by researchers. The only exceptions, to our knowledge, are van Oost (2016), Quintero-Araujo et al. (2017), and Ouhader and Kyal (2017), which present promising studies on the benefits of collaboration in location-routing. Besides the academic interest, the intersection of these two areas motivates from practical situations, such as the installation of urban consolidation centres for city logistics and the formation of strategic alliances. For example, Paddeu (2017) recounts the case of the Bristol-Bath freight consolidation centre from the perspective of its users and points to pricing and cost coverage as important factors for success or failure. If a consolidation centre is located in the periphery of a city to serve users in the city centre by shared routes, the users will naturally be concerned about the cost of the service and how this compares to the non-shared solution. The location of the centre may, therefore, play an important role in the willingness of users to adopt the shared solution. Another example in practice is given by the alliance between Colgate-Palmolive, GlaxoSmithKline, Henkel, and Sara Lee in France (Eyers, 2010). The alliance started in 2005 with cooperation on routing only, but subsequently led to a decrease in the number of facilities of the alliance from four to only one. With these sharing practices emerging as important mechanisms to improve operations, it turns important for firms to understand the economic foundations for cooperation in location-routing.

In this article, we study several variants of the location-routing problem using a cooperative game-theoretic framework. By defining a transferable utility game for each of these variants, we are able to derive essential properties that characterize the behaviour of cooperation in location-routing from a cost sharing perspective. In particular, this framework is useful to answer whether incentives exist for all firms to align under a collaborative location-routing approach.

The rest of the article is organized as follows. In Section 2, we briefly overview related literature. Section 3 describes the main variants of the location-routing problem and defines the corresponding cooperative games. In Section 4, we derive theoretical results for these games. Section 5 summarizes numerical results obtained from experiments. Section 6 presents our concluding remarks.

2 Literature review

Horizontal cooperation in transportation and logistics has been recognized as an important instrument to reduce costs, increase productivity, improve customer service, stabilize or im-

prove the market position, among other benefits (Cruijssen et al., 2007a,b). Recent literature surveys by Guajardo and Rönnqvist (2016) and Gansterer and Hartl (2018) reveal a considerable growth in the number of articles and applications of collaborative transportation.

The vehicle routing problem tackles a question of how to perform tours to visit a group of customers from one or more facilities using one or more vehicles. For a current state of this literature, see for example Adewumi and Adeleke (2018). In the vehicle routing problems, the horizontal cooperation can be found on several different levels in form of cooperation of customers, carriers or shippers. For the customer level, the traveling salesman game and the basic vehicle routing game are among the most studied. For their definitions, see for example studies by Potters et al. (1992) and Göthe-Lundgren et al. (1996). These problems focus on allocating costs of realized tours among the visited customers. When it comes to the shippers and carriers, the collaborative vehicle routing problem has been introduced. In the collaborative vehicle routing problem, the cooperating shippers and carriers pool their customers and allow for visiting customers of different shippers within the same tour. Gansterer and Hartl (2018) present a survey on this literature and acknowledge a usual lack of distinction between a cooperation of shippers and a cooperation of carriers. They suggest that it might be sometimes important to recognize a difference in the information they possess. However, since the focus of this article is on centralized collaborative planning, we assume that the shippers and carriers possess the same information and make decisions jointly. Thus, their distinction is not needed and we refer to them simply as shippers.

For situations where the shippers already know where their potential customers would be, but they do not have a facility from which to serve them, the facility location problem becomes useful. The facility location problem aims to find the optimal location of facilities such that each customer gets assigned to a facility. The optimality lies in minimizing the sum of facility opening costs and connection costs. Computation of the connection costs differs among various formulations of the facility location problem. For an overview of several variants of the problem, see Laporte et al. (2015). Goemans and Skutella (2004) introduced the cooperative facility location game to deal with a cooperative formulation of the problem in which several shippers aim to get their customers assigned to a facility. By allowing the customers to be connected to facilities of different shippers, this problem might allow for substantial savings.

Perl (1987) and Salhi and Rand (1989) pointed out that tackling the decisions on facility location and vehicle routing separately may lead to suboptimal solutions. With the aim to connect these problems, the location-routing problem was addressed by a large stream of literature since then, as documented in surveys by Nagy and Salhi (2007), Prodhon and Prins (2014), Drexl and Schneider (2015), and Schneider and Drexl (2017). Among others, these surveys discuss different variants of the location-routing problem such as, for example, the standard location-routing problem (LRP), the capacitated location-routing problem (LRP-C), and the location-routing problem with a limited number of facilities (LRP-L). The

LRP is used in numerous applications as documented, for example, by Watson-Gandy and Dohrn (1973), Or and Pierskalla (1979), Bruns et al. (2000), and Ambrosino et al. (2009) for the cases of food and drink distribution, blood bank location, parcel delivery, and food distribution, respectively. As claimed by Prodhon and Prins (2014), the LRP-C is nowadays addressed to a larger extent than the LRP. For example, Nambiar et al. (1981, 1989), Gunnarsson et al. (2006), and Marinakis and Marinaki (2008) utilize the LRP-C in rubber plant location, shipping industry, and wood distribution, respectively. The LRP-L arises when some types of facilities cause nuisance and social rejection. A city taking this into account could decide to impose a limit on the number of a certain type of facilities. For instance, Caballero et al. (2007) deal with one such problem by using the LRP-L for location of incineration plants for disposal of solid animal waste.

While a large number of articles in this literature stream have been devoted to development of solution methods, very few have studied collaboration in location-routing. Among the exceptions, Quintero-Araujo et al. (2017) use numerical experiments to compare the non-cooperative and cooperative scenarios in the location-routing problem in terms of the total cost and the total traveling-related CO_2 emissions. For the location-routing problem variant with two-echelons, Ouhader and Kyal (2017) analyze the cooperation based on three different objectives, minimizing the total cost, minimizing the total amount of traveling-related CO_2 emissions, and maximizing the number of created job opportunities (social impact). Through numerical experiments, they observe how each objective separately affects the other measures. In literature on collaborative transportation problems, cooperative game theory is commonly used to derive theoretical properties and investigate implications of the collaboration. To our knowledge, our article is the first one studying cooperation in location-routing problems from a cooperative game-theoretic perspective.

3 Location-routing problems and game definitions

In this section, we briefly present some of the main location-routing problem variants and then formally introduce the definition of transferable utility games for these variants.

3.1 Location-routing problem variants

3.1.1 Standard location-routing problem

Our departing point is the standard location-routing problem (LRP). Its definition, as described for example by Prodhon and Prins (2014), assumes a set of potential facility locations and a set of customers (and their corresponding demands) to be given. The LRP then aims at finding locations and routes that minimize the total routing costs, costs of using vehicles and costs of opening facilities while assuring that all customers are visited and their demand is satisfied.

Let G be the set of feasible sites of candidate facilities, I the set of customers to be served and $V = G \cup I$ the set of all such nodes. Let K be a set of all vehicles available for routing from the facilities (no facility has a specific fleet). Let c_{ij} be the cost of traveling from node $i \in V$ to node $j \in V$, a the cost of acquiring a vehicle, and f_g the cost of establishing and operating a facility at site $g \in G$. The number of units demanded by customer $i \in I$ is d_i. The capacity of one vehicle is q. To make the values comparable, they need to be normalized with respect to a certain time period. This depends on a particular application in question. Sometimes, for example, an annual average may serve the purpose.

The LRP can be formulated as an integer linear programming model. Let X_{ijk} be a binary decision variable which takes value 1 if vehicle k travels directly from node i to node j, and zero otherwise ($k \in K$, $i \in V$, $j \in V$, such that $i \neq j$ and at least one of these two nodes belongs to I). Then, the model is formulated below.

$$\min_{X_{ijk}, Z_g} \left(\sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} X_{ijk} + \sum_{k \in K} \left(a \sum_{g \in G} \sum_{j \in I} X_{gjk} \right) + \sum_{g \in G} f_g Z_g \right) \tag{1}$$

$$\text{s.t.} \quad \sum_{k \in K} \sum_{i \in V} X_{ijk} = 1 \qquad \forall j \in I, \tag{2}$$

$$\sum_{j \in I} \sum_{i \in V} d_j X_{ijk} \leq q \qquad \forall k \in K, \tag{3}$$

$$\sum_{i \in V} X_{ipk} - \sum_{j \in V} X_{pjk} = 0 \qquad \forall p \in V, \forall k \in K, \tag{4}$$

$$\sum_{g \in G} \sum_{j \in I} X_{gjk} \leq 1 \qquad \forall k \in K, \tag{5}$$

$$\sum_{i \in S} \sum_{j \in S} X_{ijk} \leq |S| - 1 \qquad \forall S \subseteq I : S \neq \varnothing, \forall k \in K, \tag{6}$$

$$\sum_{j \in I} X_{gjk} - Z_g \leq 0 \qquad \forall k \in K, \forall g \in G, \tag{7}$$

$$X_{ijk} \in \{0, 1\} \qquad \forall k \in K, \forall i, j \in V : i \neq j, \{i, j\} \cap I \neq \varnothing, \tag{8}$$

$$Z_g \in \{0, 1\} \qquad \forall g \in G. \tag{9}$$

Objective function (1) minimizes the sum of routing costs, the costs of using vehicles and the costs of operating facilities. Constraints (2) state that each customer must be served by one vehicle. Constraints (3) state that the capacity of vehicles must be respected. Constraints (4) are flow conservation constraints. Constraints (5) express that no vehicle can depart from more than one facility. Constraints (4) and (5) impose that every used vehicle has to come back to the same facility it departed from. Constraints (6) eliminate sub-tours. Constraints (7) ensure that a facility can be used if and only if this facility is open. Constraints (8) and (9) state the binary nature of the variables.

Note that in this formulation, constraints (2) and (3) imply that each customer needs to be served by a single vehicle. In combination with constraints (7) this also means that the whole demand of a customer needs to be satisfied from only one facility. Consequently, the

problem might become infeasible if there is not enough vehicles to satisfy the total demand or if a customer demands more than the capacity of the vehicles.

3.1.2 Capacitated location-routing problem

The capacitated location-routing problem (LRP-C) introduces an upper bound in the supply available at the facilities. The limited capacity of the facilities leads to the following modifications of the model (1)–(9).

Parameter w_g is added and defined as the capacity of facility $g \in G$. New binary decision variable F_{ig} takes value 1 if customer i is assigned to facility g, and 0 otherwise ($i \in I$, $g \in G$). In addition to constraints (2)–(9), constraints

$$\sum_{u \in V} X_{guk} + \sum_{u \in V} X_{uik} \leq 1 + F_{ig} \qquad \forall g \in G, \forall i \in I, \forall k \in K \tag{10}$$

need to hold. To make constraints (7) account for the limited capacity of the candidate facilities, they are replaced by

$$\sum_{i \in I} d_i F_{ig} \leq w_g Z_g \qquad \forall g \in G. \tag{11}$$

Constraints (10) state that each customer served by a vehicle departing from a certain facility must be assigned to this facility. The term $\sum_{u \in V} X_{guk}$ is equal to 1 if the vehicle k departs from the facility g while $\sum_{u \in V} X_{uik}$ is equal to 1 if this vehicle supplies customer i. If both terms are equal to 1, it implies F_{ig} to equal 1. Constraints (11) ensure that the capacities of the facilities must be respected. At the same time, constraints (11) forbid customers to be assigned to a facility which is not open.

To avoid infeasibility of the LRP-C, there need to be enough facility candidates with large enough capacities such that each customer can be assigned to and fully supplied by only one facility. Besides, as for the LRP, there need to be enough vehicles to satisfy the total demand and customers cannot demand more than the capacity of the vehicles.

3.1.3 Location-routing problem with a limited number of facilities

The location-routing problem with a limited number of facilities (LRP-L) introduces an upper bound in the maximum number of facilities that can be opened. If a condition that only l facilities can be used takes place, the model (1)-(9) needs to be modified by introducing the parameter l and adding the constraint

$$\sum_{g \in G} Z_g \leq l \tag{12}$$

which ensures that the total number of opened facilities is less or equal to the limit l.

The feasibility of the LRP-L is subject to the same conditions as in the case of the LRP, that is, there need to be enough vehicles to satisfy the total demand and customers cannot demand more than the capacity of the vehicles.

3.2 Location-routing games

Our interest lies in a collaborative version of the location-routing problem. When shippers collaborate, the overall problem opens opportunities to combine their customers within the same tours and serve their demands from shared facilities. To model this situation, we use a cooperative game-theoretic framework.

3.2.1 Standard location-routing game

The standard location-routing game (LRG) is defined as a transferable utility game by the tuple (N, C) where $N = \{1, \ldots, n\}$ denotes the set of all players (shippers), $\mathscr{S}$ the set of all subsets of N, and $C \colon \mathscr{S} \to \mathbb{R}$ the characteristic function. The characteristic function C assigns to each coalition $S \in \mathscr{S}$ its optimal cost, that is, the optimal objective value of the corresponding LRP (by convention, $C(\varnothing) = 0$). Let I_n be the set of customers originally to be served by a shipper $n \in N$. The corresponding LRP for computing $C(S)$ is then the LRP in which the set of customers to be served is $I = \bigcup_{n \in S} I_n$.

3.2.2 Location-routing game extensions

Similarly, for the two other variants of the location-routing problem, the LRP-C and the LRP-L, we define the capacitated location-routing game (LRG-C1) and the location-routing game with a limited number of facilities (LRG-L1), respectively. The LRG-C1 introduces a problem where each coalition solves a location-routing problem with capacitated facilities. The LRG-L1 extends the standard location-routing game by assuming a limit on the number of facilities that can be located by each coalition. These capacities and limits are independent of the coalitions' size or members and remain constant.

Unlike in other collaborative transportation problems (such as the collaborative vehicle routing problem or the facility location game), in both the LRG-C1 and the LRG-L1, when cooperation takes place, the original strategies before such cooperation are not necessarily feasible. When two shippers use the same facility and operate on its full capacity in their stand-alone strategies, this strategy is not possible once they cooperate. The same problem occurs for the LRG-L1 if the shippers already use the maximum allowed number of facilities in their stand-alone strategies. Therefore, we also formulate alternative models for the capacitated location-routing game and the location-routing game with a limited number of facilities as LRG-C2 and LRG-L2, respectively.

Let the LRG-C2 be defined as a transferable utility game in a similar way as the LRG-C1, that is, for computation of each characteristic function value, the LRP-C is solved. Now, however, these LRP-C's differ not only in the sets of customers $I = \bigcup_{n \in S} I_n$, but also in the capacities $w_g = \sum_{n \in S} w_{gn}$ where w_{gn} denotes a partial capacity available at site $g \in G$ if shipper $n \in N$ is in the coalition.

Similarly, let the LRG-L2 be defined as a transferable utility game in which the LRP-L

is solved for each characteristic function value computation. As opposed to the LRG-L1, however, these LRP-L's differ not only in the sets of customers $I = \bigcup_{n \in S} I_n$, but also in the limits on the number of facilities being computed as $l = \sum_{n \in S} l_n$ where l_n stands for a partial limit on the number of facilities.

As opposed to the LRG-C1 and the LRG-L1, in the LRG-C2 and the LRG-L2, the capacities and limits are dependent on the coalitions' size and members. The shippers are associated with partial capacities or limits which may be utilized in any coalition by adding up the partial capacities or limits of all its members.

All the formulations have their place in practice. The LRG-C1 can be used, for example, if shippers may for some reason be associated only with facilities up to a certain capacity per location. For situations where several facilities can be opened at each location, the LRG-C1 is an appropriate model too. This reflects a case in which facilities consist of blocks of a certain capacity and each shipper or coalition of shippers determines how many blocks to open. On the other hand, in a single-commodity situation, if shippers for example have a maximum supply available for each candidate facility, its value stands for the partial capacity of this facility in the LRG-C2 formulation. If the facilities cause social rejection and shippers do not want to be associated with more than a certain number of them, cooperation might not increase the number and the LRG-L1 becomes the appropriate model. If, on the other hand, there is an enforcement preventing a shipper from operating more than a certain number of facilities, cooperation might allow the total limit to be a sum of the limits of all involved shippers. Such situation requires the use of the LRG-L2.

4 Theoretical results

Cooperation does not necessarily guarantee beneficial outcomes for all parties. To assess such issues, properties of subadditivity and convexity of a cooperative game come in useful.

Cooperative game (N, C) is considered *subadditive* if its characteristic function is subadditive, i.e.,

$$C(S \cup T) \leq C(S) + C(T) \qquad \forall S, T \subseteq N \colon S \cap T = \varnothing. \tag{13}$$

If the subadditivity holds, no coalition is less profitable than some of its partitions. That is, there is no loss involved in merging coalitions with respect to the total costs incurred.

The *convexity* of a cooperative game is a stronger property which requires submodularity of its characteristic function, i.e.,

$$C(S \cup T) + C(S \cap T) \leq C(S) + C(T) \qquad \forall S, T \subseteq N, \tag{14}$$

or equivalently (Schrijver, 2003),

$$C(S \cup \{i\}) - C(S) \geq C(T \cup \{i\}) - C(T) \qquad \forall i \in N, \forall S, T \subseteq N \colon S \subseteq T \subseteq N \setminus \{i\}. \tag{15}$$

Studies of transferable utility games naturally lead to studies of cost allocations which prescribe the costs to be paid by particular players within the cooperation. For a cooperative

game of n players, the cost allocation is a vector $(\pi_1, \ldots, \pi_n) \in \mathbb{R}^n$. Conditions of efficiency,

$$\sum_{j \in N} \pi_j = C(N), \tag{16}$$

and rationality,

$$\sum_{j \in S} \pi_j \leq C(S) \qquad \forall S \subseteq N, \tag{17}$$

define a set of cost allocations known as the *core* as first introduced by Shapley (1955). A vector in the core proposes a redistribution of total costs of the grand coalition N and is said to be *stable*, as there are no incentives for any subset of players to deviate from the collaboration. In the literature, this has been recognized as an essential condition to sustain the cooperation in practice. It is hence desirable for a cooperative game to have a non-empty core and it turns interesting to study whether a game admits or not allocations in the core.

4.1 Game-theoretic properties of the standard location-routing game

Theorem 1. *The standard location-routing game is subadditive.*

Proof. For the LRG, it is easy to see that for any disjoint sets S and T, the solution of a LRP where customers of shippers in S and T are served using the same facilities, routes and vehicles as in the separate problems for S and T is a feasible solution of the LRP for set $S \cup T$. The value of $C(S) + C(T)$ is therefore an upper bound on the optimal objective value $C(S \cup T)$ of the problem for $S \cup T$ and the LRG is hence subadditive. □

Theorem 2. *The core of the standard location-routing game can be empty.*

Proof. Göthe-Lundgren et al. (1996) present an example to prove that the core of the basic vehicle routing game can be empty. By adapting this example, we can prove that the core of the LRG can be empty as well. Figure 1 illustrates the location of three customers (1, 2 and 3) and feasible sites of candidate facilities (A, B and C). Each customer is a client of a different shipper which can hence be referred to by 1, 2 and 3 as well. The figure also contains the transportation costs. The costs of establishing and operating a facility are equal to one unit for each of the sites. The demand of each customer is of one unit. The capacity of each vehicle is of two units. The cost of using a vehicle is set to zero. Calculating the characteristic cost function for the singletons, we obtain $C(\{1\}) = C(\{2\}) = C(\{3\}) = 3$. The routing cost is equal to 2 and the facility opening cost to 1. The facility opened is one of the two adjacent to the customer. For the two-player coalitions, $C(\{1,2\}) = C(\{1,3\}) = C(\{2,3\}) = 4.7$ (routing cost of 3.7 and facility opening cost of 1). For the three-player coalition, $C(\{1,2,3\}) = 7.7$ (routing cost of 5.7 and facility opening cost of 2). From the costs, we can notice there is an incentive for a two-player coalition. It is clearly more beneficial than a non-cooperative state. In case of the three-player coalition, whichever the cost allocation is, there will always be two shippers that could get better off by excluding the third one. Hence, the core is empty. □

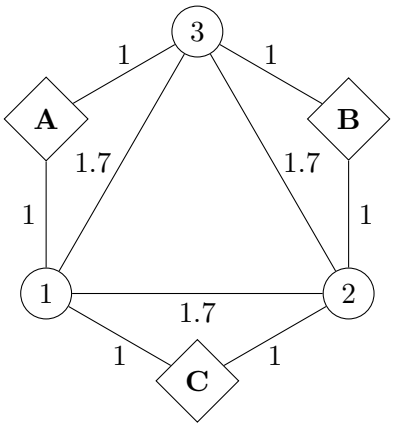


Figure 1: A standard collaborative location-routing problem with an empty core (the circles and diamonds represent customers and candidate facilities, respectively, and the arcs indicate the traveling costs)

Corollary 3. *The standard location-routing game is not necessarily convex.*

Proof. According to Shapley (1971), the core of a convex game is not empty. Equivalently, by contraposition, if the core of a game is empty, the game must be non-convex. Since the core of the LRG can be empty, it follows that this game cannot be convex in general.

Moreover, in the problem of Figure 1, taking coalitions $S = \{1\}$, $T = \{1, 2\}$ and $i = 3$, the left-hand side of inequality (15) is $4.7 - 3 = 1.7$ while the right-hand side is $7.7 - 4.7 = 3$, thus the inequality is violated and this is an example of a non-convex game. □

4.2 Game-theoretic properties of the location-routing game extensions

Theorem 4. *The LRG-C1 is not necessarily subadditive or convex.*

Proof. A counter-example is illustrated in Figure 2. Again, we consider each customer being assigned to a different shipper. The costs of establishing and operating facilities are equal

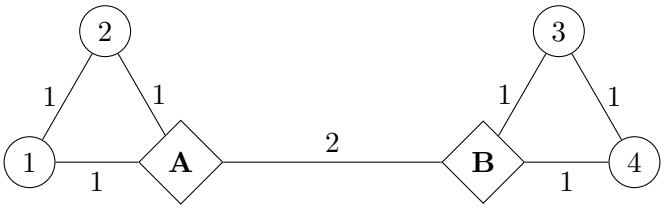


Figure 2: A non-subadditive capacitated location-routing game (the circles and diamonds represent customers and candidate facilities, respectively, and the arcs indicate the traveling costs)

to one unit for site A and six units for site B. Both candidate facilities have a maximum capacity equal to one unit. The demand for each customer is of one unit. The capacity of each vehicle is of two units. The cost of using a vehicle is set to zero. For coalition $S = \{1, 2\}$,

facility A is opened and $C(S) = 4$. For coalition $T = \{3, 4\}$, facility A is opened as well and $C(T) = 8$. However, for $S \cup T$, both facilities are opened and $C(S \cup T) = 13$ which causes that $C(S \cup T) \nleq C(S) + C(T)$ and the subadditivity (as well as convexity) does not hold. □

Theorem 5. *The LRG-L1 is not necessarily subadditive or convex.*

Proof. Considering the same example (Figure 2) for the location-routing game with a limited number of facilities, but now with omitting the maximum capacities, introducing a limit of $l = 1$ on the number of facilities and having the costs of establishing and operating facilities equal to 1 for both A and B, leads to $C(S) = 4$, $C(T) = 4$ and $C(S \cup T) = 11$. Hence, again $C(S \cup T) \nleq C(S) + C(T)$ and the subadditivity and convexity properties do not hold. □

Theorem 6. *The core of the LRG-C1 and the LRG-L1 can be empty.*

Proof. Clearly, if each facility offered a maximum capacity that could accommodate all customers in the game, the LRG-C1 would reduce to the LRG. Similarly, if the limit on the number of facilities l was equal or higher than the number of all potential facility sites, the LRG-L1 would reduce to the LRG. Therefore, the example in Figure 1 could be used to show a possibility of an empty core in the LRG-C1 and the LRG-L1 as well as for the case of the LRG. □

Theorem 7. *The LRG-C2 and the LRG-L2 are subadditive.*

Proof. For both the LRG-C2 and the LRG-L2, it is easy to see that for any disjoint sets S and T, the solution where customers of shippers in S and T are served using the same facilities, routes and vehicles as for the separate problems for S and T is a feasible solution for set $S \cup T$. The value of $C(S) + C(T)$ is therefore an upper bound on the optimal objective value $C(S \cup T)$ and subadditivity is hence satisfied. □

Theorem 8. *The core of the LRG-C2 and the LRG-L2 can be empty.*

Proof. For illustration of the possibility of an empty core, the same reasoning as in the case of the LRG-C1 and the LRG-L1 could be used. That is, large enough capacities and large enough limits on the number of facilities would reduce the LRG-C2 and the LRG-L2 to the LRG which, as shown in the example of Figure 1, can have an empty core. □

Corollary 9. *The LRG-C2 and the LRG-L2 are not necessarily convex.*

Proof. As in the proof of Corollary 3, the posible emptiness of the core consequently leads to non-convexity. □

All the aforementioned results are summarized in Table 1 along with the property satisfaction of some of the collaborative transportation problems outlined in Section 2, namely the collaborative vehicle routing problem (CoopVRP) and the facility location game (FLG).

In both the CoopVRP and the FLG, as defined for example by Zibaei et al. (2016) and Goemans and Skutella (2004), respectively, it is easy to see that the subadditivity is generally satisfied. Clearly, when the facilities' capacities are unconstrained, a combination of any non-cooperative solutions forms a feasible solution in the cooperative formulation. On the other hand, Goemans and Skutella (2004) show for the case of the FLG that the convexity is not generally satisfied and the core can be empty. For the case of the CoopVRP, the traveling salesman game can be reformulated as a special case of the CoopVRP with vehicles of a large capacity and shippers each serving one customer and possessing one facility located in the same position for all shippers. Then, Potters et al. (1992) show that the convexity is not generally satisfied and the core can be empty. In Table 1, checkmarks show that the respective property holds for any instance of the problem. If a checkmark is missing, this property might be satisfied for particular instances, but does not hold in general. The

Table 1: Properties of different variants of the location-routing games, the collaborative vehicle routing problem, the facility location game, and the transportation game

	subadditive	**convex**	**non-empty core**
LRG	✓	-	-
LRG-C1	-	-	-
LRG-L1	-	-	-
LRG-C2	✓	-	-
LRG-L2	✓	-	-
CoopVRP	✓	-	-
FLG	✓	-	-
TG	✓	-	✓

results might suggest that all collaborative transportation problems allow for an empty core. To avoid such misinterpretation, without its definition, we also include the transportation game (TG) introduced by Samet et al. (1984). It is an example of a game which is always subadditive and has a non-empty core, but is not necessarily convex (Sánchez-Soriano et al., 2001).

4.3 Impact of the facility costs

The facility opening costs, f_g in the model (1)–(9), play an important role in the location-routing games. They stand for all costs necessary to establish and operate a facility. In what follows, we will show that, if the facility opening costs are substantially larger than the traveling costs and the costs of using vehicles, the core of the LRG is guaranteed to be non-empty.

Theorem 10. *For any traveling and vehicle-related costs, there exists $K \in \mathbb{R}$ such that, if $f_g \geq K$ for all $g \in G$, the core of the respective standard location-routing game is non-empty.*

Proof. Solving the model (1)–(9) for the grand coalition N results in an optimal objective value of $C(N)$. This cost could be allocated to the shippers for example such that each of them is allocated the same cost, that is,

$$\pi_j = \frac{C(N)}{|N|} \qquad \forall j \in N. \tag{18}$$

This allocation clearly satisfies the efficiency condition (16).

It would be feasible to open only the least costly facility and serve all customers from this facility. Denoting the opening cost of this facility by $f_{g^*} = \min_{g \in G} f_g$ and any (not necessarily minimal) routing costs and costs of using vehicles needed to serve all customers by utilizing only this facility by VRC_{g^*}, the total cost would be at least as large as $C(N)$,

$$C(N) \leq f_{g^*} + VRC_{g^*}, \tag{19}$$

and hence

$$\pi_j \leq \frac{1}{|N|} \left(f_{g^*} + VRC_{g^*}\right) \qquad \forall j \in N. \tag{20}$$

Then, for any proper subset $S \subset N$,

$$\sum_{j \in S} \pi_j \leq \frac{|S|}{|N|} \left(f_{g^*} + VRC_{g^*}\right) \leq \frac{|N|-1}{|N|} \left(f_{g^*} + VRC_{g^*}\right). \tag{21}$$

If the facility costs are substantially larger than the traveling costs and the costs of using vehicles, for example

$$VRC_{g^*} \leq \frac{f_{g^*}}{|N|-1}, \tag{22}$$

(21) then implies

$$\sum_{j \in S} \pi_j \leq f_{g^*} \qquad \forall S \subset N. \tag{23}$$

Since each coalition $S \subset N$ needs to locate at least one facility and f_{g^*} is the minimal cost associated with this, then

$$\sum_{j \in S} \pi_j \leq C(S) \qquad \forall S \subset N. \tag{24}$$

In combination with satisfaction of the efficiency condition, this means that all rationality conditions (17) are satisfied and $(\pi_1, \ldots, \pi_{|N|})$ belongs to the core. Hence, the core is non-empty.

VRC_{g^*} is not dependent on the value of f_{g^*}. It is then easy to see that any K such that

$$VRC_g \leq \frac{K}{|N|-1} \qquad \forall g \in G \tag{25}$$

guarantees a non-empty core and proves the Theorem 10. □

This result for the LRG can be generalized to the LRG-L1 and the LRG-L2 as well. However, in the case of the LRG-C1 or the LRG-C2, due to their capacities, it is not always feasible to locate only one facility and the same argument cannot be used.

5 Numerical results

To illustrate the theoretical results and explore the satisfaction of the properties that do not hold in general, we have conducted a numerical experiment. We also address here the problem of how to split the savings by applying cost allocation methods frequently used in the literature.

All presented models and methods are implemented and solved using AMPL/Gurobi 7.5.0.

5.1 Experiment design

We have generated 10,000 instances, all of them containing nine sites of candidate facilities and three shippers, each of them having two or three customers.

Given a square 100×100, the coordinates x and y of the customers and facility candidates follow a uniform distribution between 0 and 100. The transportation cost c_{ij} between two nodes is the Euclidean distance between the two nodes. Each shipper is randomly assigned two or three customers. The demand d_j for each customer follows a uniform distribution between 10 and 100 and the vehicle capacity q follows a uniform distribution between 100 and 200. The fleet is homogeneous. The cost of using a vehicle a is the same for all vehicles and ranges between 10 and 200 and each facility opening cost f_g follows a uniform distribution between 100 and 300.

In the case of the LRG-L1, the limit on number of facilities l takes value 1, 2 or 3. The partial limits on number of facilities l_n in the LRG-L2 equal either 1 or 2. For the LRG-C1, the facility capacities w_g range from 100 to 500, whereas for the LRG-C2, the partial facility capacities w_{gn} range from 35 to 200. Generation of the values of w_{gn} in this way implies that for some coalitions there might be customers whose demand cannot be satisfied from only one facility. In such case, the LRG-C2 would not yield a feasible solution. We observe only two instances in which this happens. For further analysis, we exclude those and take into account only the instances with feasible solutions.

In the non-cooperative case, the facility opening costs range from 12 % to 67 % of the total costs, the costs of using a vehicle from 3 % to 64 % and the routing costs from 12 % to 65 %. Respective histograms are shown in Figure 3. Overall, the experiment covers a wide range of how the costs are distributed.

5.2 Game properties

Regarding the properties of subadditivity, convexity and core-emptiness, results of the experiment are shown in Table 2. All instances of the LRG, LRG-C2 and LRG-L2 are subadditive which confirms the theoretical results from Section 4. There is only a slight change in the results when it comes to the LRG-C1 and the LRG-L1 in which only 2.7 % and 0.2 % of the instances respectively end up as non-subadditive. We observe a huge majority of instances

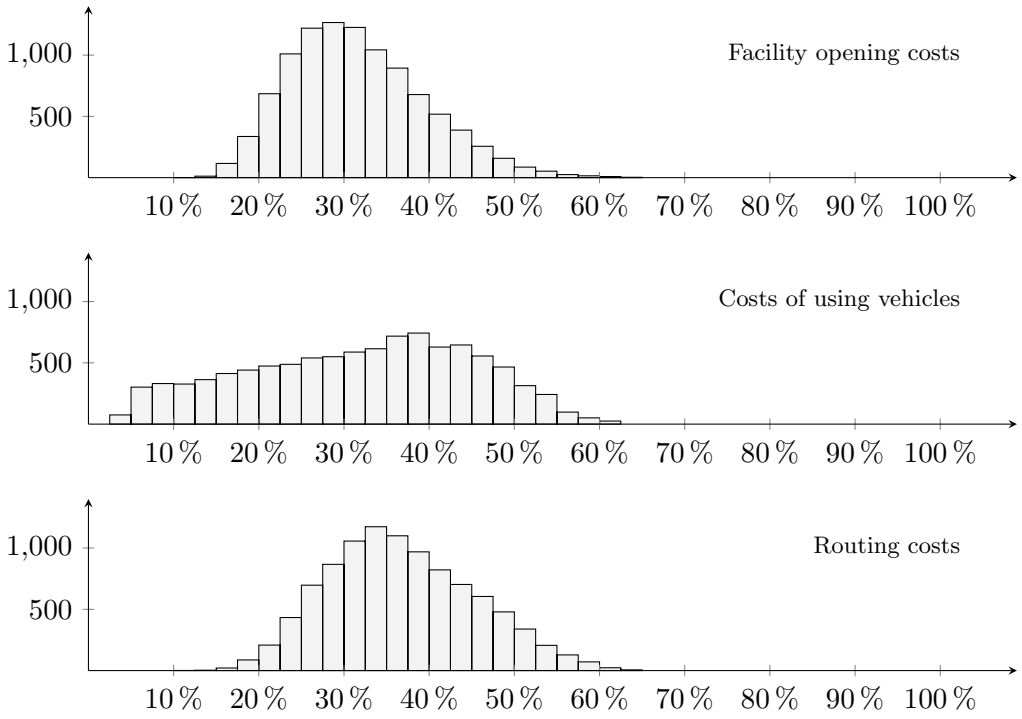


Figure 3: Histograms of facility opening costs, costs of using vehicles, and routing costs as a percentage of the total costs incurred in a case of no cooperation taking place

having a non-empty core. Nevertheless, only less than a third of the instances end up being convex. There are no substantial differences among the models. Only, in the case of the LRG-C1, satisfaction of the properties is generally slightly lower than in the other models.

Table 2: Satisfaction of properties in location-routing games

	subadditive	**convex**	**non-empty core**
LRG	100 %	30.5 %	99.3 %
LRG-C1	97.3 %	22.7 %	92.6 %
LRG-L1	99.8 %	30.3 %	99.1 %
LRG-C2	100 %	26.1 %	99.7 %
LRG-L2	100 %	30.4 %	99.3 %

By definition, in the instances with an empty core, no cost allocation can satisfy the efficiency condition (16) and the rationality conditions (17) at the same time. By requiring the efficiency, it can be measured what violation of the rationality constraints is inevitable. For this, the strong ε-core can be utilized. The *strong ε-core*, as introduced by Shapley and Shubik (1966), is a set of all optimal solutions of $(\pi_1, \ldots, \pi_{|N|})$ to problem

$$\min_{\pi_j, \varepsilon} \varepsilon \tag{26}$$

$$\text{s.t.} \quad \sum_{j \in N} \pi_j = C(N), \tag{27}$$

$$\sum_{j \in S} \pi_j \leq C(S) + \varepsilon \qquad \forall S \subseteq N, \tag{28}$$

$$\pi_j \in \mathbb{R} \qquad \forall j \in N, \tag{29}$$

$$\varepsilon \in \mathbb{R}, \tag{30}$$

where ε represents the maximal violation of the rationality constraints. For all variants of the location-routing game, the average maximal violation is rather low and ranges from 0.91 % to 1.70 % of the respective $C(N)$.

5.3 Impact of the facility costs

As discussed in Section 4.3, for the LRG, LRG-L1, and LRG-L2, the core is non-empty when the facility costs reach a certain size. We investigate this by generating new sets of instances with the only change being in the facility cost values. In the new instances we generate the facility opening costs as multiples of the original facility opening costs, that is, for a multiplier of 0, all facility opening costs equal zero, for a multiplier of 1, the results are the same as in Table 2, for a multiplier of 2, all facility opening costs are doubled, and so on. The results for the LRG are reported in Table 3. We can see that the results confirm

Table 3: Impact of the facility costs on properties of the LRG

	facility cost multiplier	**subadditive**	**convex**	**non-empty core**
LRG	0	100 %	41.4 %	97.1 %
LRG	1	100 %	30.5 %	99.3 %
LRG	2	100 %	32.0 %	99.9 %
LRG	3	100 %	32.9 %	100 %
LRG	4	100 %	33.5 %	100 %
LRG	5	100 %	33.9 %	100 %

the theoretical findings and the core becomes non-empty already for a multiplier of 3. As expected, the results are very similar for the case of the LRG-L1 and the LRG-L2. However, we do not observe similar behavior in the LRG-C1 and the LRG-C2.

5.4 Impact of the vehicle cost

The cost of using a vehicle, a in model (1)–(9), stands for not only the usage of the vehicle as it might suggest. All kinds of costs regarding the tours, but not dependent on the distance traveled, should be included. This covers loading and unloading costs, driver-related costs,

vehicle maintenance costs, and so on. The cost of using a vehicle might hence differ substantially across applications. Therefore, it is worthwhile to investigate how the properties of the location-routing games are affected by the magnitude of this cost.

For the LRG, when we compare instances resulting with an empty core with those resulting with a non-empty core, we can see a substantial difference between the average costs of using a vehicle. This average is 164 for the empty-core instances and 104 for the non-empty-core instances. We investigate this issue further by generating new sets of instances in the same way as for the facility costs analysis. Now, the new sets of instances differ only in the cost of using a vehicle. The results are provided in Table 4. We can notice that

Table 4: Impact of the vehicle cost on properties of the LRG

	vehicle cost multiplier	**subadditive**	**convex**	**non-empty core**
LRG	0	100 %	28.1 %	100 %
LRG	1	100 %	30.5 %	99.3 %
LRG	2	100 %	30.7 %	97.3 %
LRG	3	100 %	30.8 %	96.4 %
LRG	4	100 %	30.7 %	95.8 %
LRG	5	100 %	30.7 %	95.7 %

the proportion of instances with a non-empty core decreases as the cost of using a vehicle increases. This indeed supports the observation of different average costs of using a vehicle. Similar effects can be observed in the case of the LRG-C1, LRG-L1, LRG-C2, and LRG-L2.

5.5 Cost allocations

Regardless of the core emptiness or non-emptiness, in practice, a unique cost allocation often needs to be specified to assess the contribution of different cooperating parties. With a focus on collaborative transportation, Guajardo and Rönnqvist (2016) recognized the Shapley value (Shapley, 1953), the nucleolus (Schmeidler, 1969) and proportional methods to be some of the most preferred allocation methods used in literature.

It turns interesting to investigate whether the allocation methods result in allocations that are rational, that is, satisfy constraints (17). This relates to suitability of different allocation methods for the location-routing game. Besides the Shapley value and the nucleolus, we investigate the lexicographic equal profit method known as EPML (Frisk et al., 2010; Dahlberg et al., 2017), and two proportional methods, the first of them proportional to the stand-alone costs of each shipper and the second one to the total demand of each shipper's customers. All these allocation methods by definition satisfy the efficiency condition (16). Additional satisfaction of the rationality conditions hence means that the respective cost

allocation belongs to the core. Such an analysis is therefore meaningless for the instances with an empty core in which all allocations would end up as non-rational. We investigate how often the particular allocation methods satisfy the rationality conditions in the instances with a non-empty core. The results are presented in Table 5.

Table 5: Rationality satisfaction by various cost allocation methods

	Shapley value	**Nucleolus**	**EPML**	**Cost prop. method**	**Demand prop. method**
LRG	97.0 %	100 %	100 %	79.7 %	67.7 %
LRG-C1	84.4 %	100 %	100 %	60.5 %	53.1 %
LRG-L1	96.9 %	100 %	100 %	79.3 %	67.4 %
LRG-C2	95.9 %	100 %	100 %	83.7 %	80.3 %
LRG-L2	97.0 %	100 %	100 %	79.7 %	67.7 %

The nucleolus and the EPML excel in the rationality satisfaction, which is not surprising as they are defined to belong to the core if it is not empty. In a huge majority of the instances, the rationality is also satisfied by the Shapley value. The proportional methods do not perform that well. Nevertheless, in the computation of the Shapley value, the nucleolus, and the EPML, the cost associated with each possible coalition needs to be determined. This becomes an obstacle when the number of shippers increases as the number of coalitions grows exponentially. The proportional methods, on the other hand, do not face this problem and might therefore be a preferred option. In case of seeking a proportional method, from Table 5 we can suggest the allocation proportional to the stand-alone costs to be the one to choose. In fact, we observe that the cost proportional method performs better in all the location-routing game variants.

As already mentioned, in the instances with an empty core, no allocation method can satisfy all the rationality conditions. To propose a similar allocation quality measure, a modification of the strong ε-core model (26)–(30) becomes useful. If, for a given allocation $(\pi_1, \ldots, \pi_{|N|})$, the model is formulated as

$$\min_{\varepsilon} \varepsilon \tag{31}$$

$$\text{s.t.} \quad \sum_{j \in S} \pi_j \leq C(S) + \varepsilon \qquad \forall S \subseteq N, \tag{32}$$

$$\varepsilon \in \mathbb{R}, \tag{33}$$

the optimal solution of ε represents the maximal violation of the rationality constraints (17). Table 6 shows the average maximal violation as a percentage of the respective $C(N)$ for various cost allocation methods (the EPML is not included because it is defined only for games with a non-empty core). The results support the previous findings with the

Table 6: Average maximal violation of rationality constraints by various cost allocation methods in instances with an empty core

	Shapley value	**Nucleolus**	**Cost proportional method**	**Demand proportional method**
LRG	2.5 %	1.1 %	5.3 %	7.3 %
LRG-C1	3.5 %	1.7 %	5.5 %	7.7 %
LRG-L1	2.7 %	1.2 %	5.6 %	7.3 %
LRG-C2	3.2 %	0.9 %	5.3 %	7.3 %
LRG-L2	2.5 %	1.1 %	5.3 %	7.3 %

nucleolus having the lowest average maximal violation of the rationality constraints. This, ranging from 0.9 % to 1.7 %, outperforms the Shapley value, the cost proportional method, and the demand proportional method in all the location-routing game variants. Among the proportional methods, the cost proportional method again performs better in all the variants.

5.6 Savings

From the results of the LRG, it can be seen that the savings of the shippers can be substantial, as shown in Figure 4. The histogram shows percentage savings when the grand coalition is

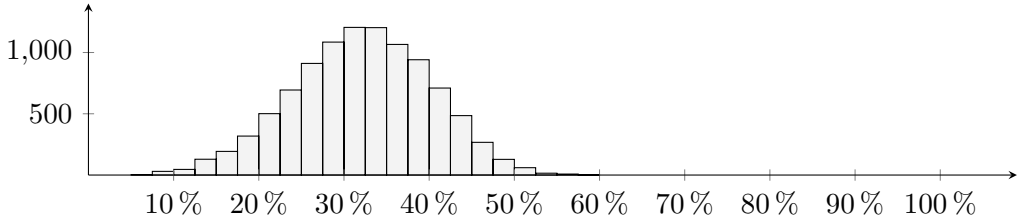


Figure 4: Histogram of percentage savings of the grand coalition with respect to the sum of the stand-alone costs

formed as opposed to a non-cooperative case represented as a sum of the stand-alone costs. These cost savings range from 6 % to 62 % with an average of 32 %. The main source of the savings is that less facilities are often needed when the grand coalition forms. The facility opening costs are on average reduced by 63 %.

Although it might seem counter-intuitive, we observe 14 % of instances where the total routing cost increases in comparison to the non-cooperative solution as seen in Figure 5. The intuition for this is that, since collaboration aims at reducing the overall costs (which include routing costs, facility opening costs and costs of using vehicles), it is possible that in some situations the selection of a reduced number of facilities might offset the increase in costs due to larger distance traveled. On average, however, the routing costs are reduced by

16 %.

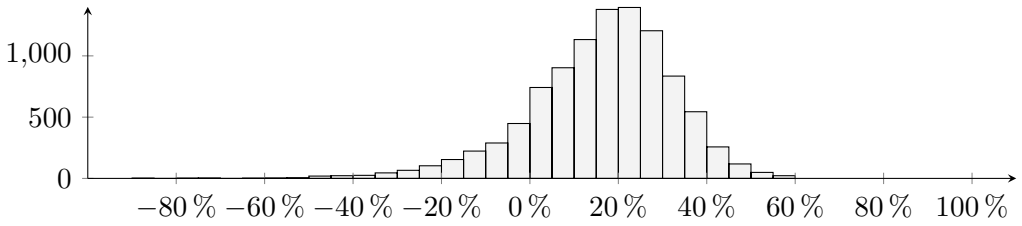


Figure 5: Histogram of percentage difference of the grand coalition's routing cost with respect to the sum of the stand-alone routing costs

The transportation is commonly recognized as one of the main contributors of CO_2 emissions (Ballot and Fontane, 2010). The increase in the traveled distance might thus lead to negative environmental effects. As this is accompanied by a change in facility selection, it is difficult to draw general conclusions. However, shippers pursuing collaboration should be aware of this possible side effect.

6 Concluding remarks

Horizontal cooperation is receiving more and more attention across transportation and logistics processes. The location-routing problem is no exception with companies cooperating on both locating their facilities and serving their customers. While there exists evidence of successful cooperation in practice (Eyers, 2010; Paddeu, 2017), the literature lacks general assessment of benefits coming from horizontal cooperation in location-routing problems.

In this article, we have introduced the standard location-routing game, a collaborative formulation of the standard location-routing problem. For both the capacitated location-routing problem and the location-routing problem with limited number of facilities, we have defined two alternative formulations of their collaborative versions. In three of these problems, we have shown the subadditivity property to hold in general. However, in the other two, the subadditivity is not always satisfied. Moreover, none of the problems guarantees the convexity or a non-empty core. Nevertheless, for the standard location-routing game and the location-routing game with limited number of facilities, we have shown that, when the facility opening costs are substantially larger than the traveling costs and the costs of using vehicles, the core is guaranteed to be non-empty.

Although it is not possible to guarantee core allocations in general, with a numerical experiment, we have shown that the core allocations exist in a huge majority of our instances. The results have also supported the findings of the effect of facility opening costs on the core emptiness. As the facility opening costs increase, the likelihood of a non-empty core increases. On the other hand, with the costs of using vehicles, we have observed the opposite effect. As the vehicle costs increase, the likelihood of a non-empty core decreases. It is important to note that the core emptiness does not necessarily outrule the cooperation. Often, regardless

of the emptiness or non-emptiness of the core, it is preferred to pursue the cooperation and choose a unique cost allocation. We have tested the performance of various cost allocation methods. With respect to the stability, the results have shown dominance of the nucleolus and the lexicographical equal profit method. The latter is however not defined for the cases with an empty core and cannot thus be used under any circumstances. The Shapley value has shown a fairly good performance as well, followed by the cost proportional method and the demand proportional method.

The focus of this article was on exploration of the properties of location-routing games. While the numerical experiment was conducted in small instances that can be solved to optimality, an interesting avenue for future research is to explore location-routing games where the approximate solutions, instead of the optimal ones, are taken into account. In fact, large-scale instances often occurring in practice are commonly solved with heuristic approaches (Schneider and Drexl, 2017). It is worthwhile to investigate whether different solution approaches preserve the same properties.

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