Multi Target localization with Block Orthogonal Least Squares for Multistatic MIMO Radars

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***Abstract*—Recently, there has been a growing interest in multistatic radar configurations to improve the localization of multiple targets. Theoretically, the maximum likelihood (ML) approach enables to fuse the information provided by each radar pair to localize the different targets. However, it involves a multi-dimensional search process whose complexity exponentially grows with the number of targets. Consequently, heuristic methods, notably including the block orthogonal matching pursuit (BOMP), have been used in the multistatic radar context to approach the ML estimation greedily. Interestingly, the more accurate block orthogonal least squares (BOLS) method has not been studied in this context because the performance improvement is usually low in regard to the computational complexity. In this work, we investigate the application of BOLS to an angle-based localization of multiple targets using a multistatic multiple-input and multiple-output (MIMO) radar. Using Monte Carlo simulations, we demonstrate the significant advantage of an efficient implementation of BOLS over BOMP in this scenario featuring highly correlated signals. The impact of radar parameters on the localization root mean square error and on the computational complexity of both algorithms is also studied.**

***Index Terms*—Multistatic, Data Fusion, Maximum Likelihood, BOMP, BOLS**

I. INTRODUCTION

In recent years, multistatic radars have received considerable attention in the literature for their enhanced sensing capability compared to their mono-/bistatic counterparts [1]. While the latter are composed of one sensing transmitter (STx) and one sensing receiver (SRx), forming a radar pair (RP), a multistatic radar combines multiple RPs observing the same scene. The combined information provided by all the RPs is exploited either with a *two-step* fusion, aggregating local decisions about the target position, or with a *one-step* fusion, combining the raw signals to directly estimate the target position. One-step approaches typically provide more robust estimations while being more demanding in computation time and data transmission, as highlighted in [2], [3]

In this study, we focus on the localization of multiple targets using a one-step angle-based multistatic multiple-input and multiple-output (MIMO) radar system. More precisely, we focus on the one-step fusion rule derived from the maximum likelihood (ML) criterion applied to our sensing scenario. Given additive white Gaussian noise (AWGN) measurements, this approach is known to yield a non-convex least squares minimization problem, whose solving complexity scales exponentially with the number of targets. Low-complexity heuristics, such as MUSIC or ESPRIT, are commonly used in single-RP estimation scenarios [4], [5]. However, they do not extend directly to *one-step multistatic* scenarios [6].

Alternatively, greedy techniques like orthogonal matching pursuit (OMP) [7], [8] and orthogonal least squares (OLS) [9], [10], common for sparse decomposition of signals, can be used as fast approximate solvers for the above ML problem [11], [12]. Their formulation straightforwardly extends to the one-step multistatic context, as seen with the block OMP (BOMP) algorithm [13], which has been leveraged in many multistatic radar applications [14]–[21]. While OLS —and its extension: block OLS (BOLS) [22]–[24]— inherently offer more accurate approximations of ML and maximum a posteriori (MAP) solutions [12], [25], OMP-like approaches are preferred in radar applications for their simplicity, often providing a better performance/complexity trade-off. As a result, and to the best of the authors' knowledge, there is no work defining and applying a BOLS method tailored to multistatic radars.

The trade-off favorable to the OMP family is rooted in their equivalence to OLS when the signals from different targets are orthogonal. The advantage of OLS become evident in the context of highly correlated signals, as empirically observed in [26] and theoretically suggested in [27]. Unlike high-resolution delay-Doppler radar systems, the angle-based localization studied in this paper meets this context and motivates our implementation of a BOLS algorithm.

This leads to our contributions, outlined below:

- We propose a BOLS implementation for angle-based multi-target localization by a multistatic MIMO radar.
- We raise awareness on an efficient implementation of OLS-like methods seldom found in the literature. By leveraging an efficient projection matrix decomposition, we achieve a complexity competitive with BOMP.
- Numerical simulations validate the advantages of BOLS over BOMP in terms of localization root mean square error (RMSE) against the signal-to-noise ratio (SNR).
- The compararison between BOLS and BOMP is further investigated by studying the influence of the number of targets and antennas per node (through their impact on the resolution) on the respective performance.

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The rest of this paper is organized as follows. Section II describes the system model and Section III derives the ML fusion. Next, BOMP and our efficient implementation of BOLS are developed in Section IV. Finally, Section V presents simulation results and we conclude in Section VI.

In terms of notations, the vectors and matrices are defined as $\mathbf{a}$ and $\mathbf{A}$, respectively. The trace, the transpose, the Hermitian transpose and the Frobenius norm are denoted $\mathrm{Tr}\left\{\mathbf{A}\right\}, \mathbf{A}^{\mathrm{T}}$, $\mathbf{A}^{\mathrm{H}}$ and $\|\mathbf{A}\|_2$. The Moore-Penrose inverse is defined as $\mathbf{A}^{\dagger} = \left(\mathbf{A}^{\mathrm{H}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{H}}$. The Kronecker product is denoted $\otimes$. In the ML framework, the true, candidate and estimated values of a parameter are denoted θ, $\widetilde{\theta}$ and $\widehat{\theta}$, respectively.

II. SYSTEM MODEL

In this study, we investigate a multistatic MIMO radar configuration designed for angle-based target localization in the x-y plane. The configuration comprises multiple STx-SRx RPs, transmitting their processed data to a central processor. This data is then fused to determine the target positions. Details of the data transmission and the fusion rule used at the central processor are discussed in Section III and IV.

A. Scenario

In this scenario, we make the following assumptions:

Assumption 1: The MIMO waveforms sent by each STx are orthogonal, which is achieved through frequency or time division multiple access techniques. This orthogonality assumption allows each SRx to process the frame transmitted by its corresponding STx devoid of interference from other STxs.

Assumption 2: The direct signal between the STxs and the SRxs, along with clutter contributions, are effectively suppressed from the estimated channel. The processing steps to achieve this suppression are discussed in Section II-B.

Assumption 3: Only multipath signals with a single reflection on a target are significantly impacting the observed channel model. In this single bounce model, signals with multiple reflections are thus omitted [28]. This assumption is fundamental in the development of radar processing algorithms as the inclusion of second-order paths would significantly increase the complexity of the channel model, while still having a much lower contribution.

Assumption 4: The number of targets to localize is assumed to be known. Model selection methods for such problems are well defined in the literature [29], [30], and are therefore not the focus of this study.

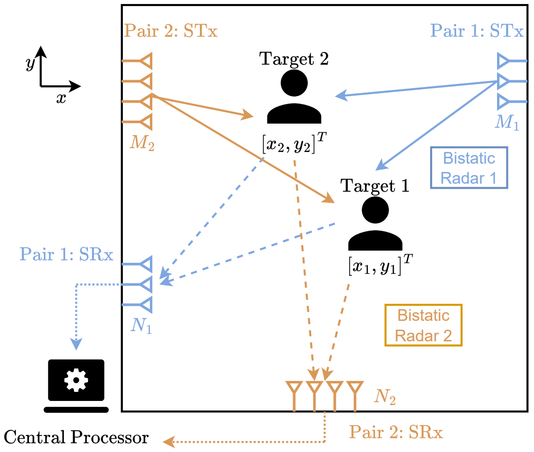

Fig. 1. Illustration of the scenario. The multistatic configuration comprises two bistatic radars and a central processor. In the illustration, solid lines represent the incident waveforms, dashed lines represent the reflected waveforms, and dotted lines depict the data transmitted to the central processor.

The scenario is depicted in Fig. 1, where the multistatic radar system aims at localizing K targets within its coverage area. The true position of the k^{th} target is defined by the vector $\mathbf{x}_k = [x_k, y_k]^{\mathrm{T}} \in \mathbb{R}^{2\times 1}$. The matrix $\mathbf{X} = [\mathbf{x}_1 \ \dots \ \mathbf{x}_K] \in \mathbb{R}^{2\times K}$ gathers the K true targets positions. The system comprises P RPs, whose p^{th} RP consists of an STx and an SRx equipped with uniform linear arrays, respectively composed of M_p transmitting and N_p receiving antennas. For simplicity, we assume the array has half-wavelength spacing and is oriented towards the coverage area. In the p^{th} radar pair, the angle-of-departure (AoD) and angle-of-arrival (AoA) for the k^{th} target are denoted by $\varphi_p(\mathbf{x}_k)$ and $\vartheta_p(\mathbf{x}_k)$, respectively. These angles are defined between the wavefront and the normal vector of the corresponding antenna array. The AoD $\mathbf{v}_p \in \mathbb{C}^{M_p\times 1}$, AoA $\mathbf{w}_p \in \mathbb{C}^{N_p\times 1}$ and joint AoD/AoA steering vectors $\mathbf{a}_p \in \mathbb{C}^{M_pN_p\times 1}$ are respectively defined as

$$\mathbf{v}_p(\mathbf{x}_k) = [1 \ e^{j\pi\sin(\varphi_p(\mathbf{x}_k))} \ \dots \ e^{j\pi(M_p-1)\sin(\varphi_p(\mathbf{x}_k))}]^{\mathrm{T}}, \quad (1)$$

$$\mathbf{w}_p(\mathbf{x}_k) = [1 \ e^{j\pi\sin(\vartheta_p(\mathbf{x}_k))} \ \dots \ e^{j\pi(N_p-1)\sin(\vartheta_p(\mathbf{x}_k))}]^{\mathrm{T}}, \quad (2)$$

$$\mathbf{a}_p(\mathbf{x}_k) = \mathbf{v}_p(\mathbf{x}_k) \otimes \mathbf{w}_p(\mathbf{x}_k) \quad (3)$$

B. Channel Model

In order to localize the targets, the p^{th} RP estimates the MIMO channel at Q_p different time instants. For each of these observations $q \in \{1, \dots, Q_p\}$, the STx transmits a spatially multiplexed signal enabling the estimation of the MIMO channel at its corresponding SRx. The model for these observations is given by

$$\widehat{\mathbf{h}}_{p,q} = \mathbf{h}_{p,q}(\mathbf{X}, \boldsymbol{\beta}_{p,q}) + \mathbf{n}_{p,q} = \mathbf{A}_p(\mathbf{X}) \ \boldsymbol{\beta}_{p,q} + \mathbf{n}_{p,q}, \quad (4)$$

- $\widehat{\mathbf{h}}_{p,q}, \mathbf{h}_{p,q} \in \mathbb{C}^{M_pN_p\times 1}$ denote the estimated and the true channel vectors of the p^{th} RP for observation q.
- $\mathbf{A}_p(\mathbf{X}) = [\mathbf{a}_p(\mathbf{x}_1) \ \dots \ \mathbf{a}_p(\mathbf{x}_K)] \in \mathbb{C}^{M_pN_p\times K}$ represents the joint AoD/AoA steering matrix.
- $\boldsymbol{\beta}_{p,q} = [\beta_{p,q,1} \ \dots \ \beta_{p,q,K}]^{\mathrm{T}} \in \mathbb{C}^{K\times 1}$ is the complex channel coefficient vector. Any stochastic model can be associated with the channel coefficients (e.g. the Swerling model [31]).
- $\mathbf{n}_{p,q} \in \mathbb{C}^{M_pN_p\times 1}$ denotes the estimation errors, which are assumed to be AWGN with variance σ_p^2.

Following *Assumption* 2, this formulation of the channel model assumes the existence of a perfect direct signal and clutter suppression. This can be achieved at the receiver by employing the widely utilized extensive cancellation algorithm (ECA) [32] or the computationally efficient average removal [33]. These algorithms exploit the null Doppler shift along the observations of the clutter and direct signal to effectively remove their contribution from the estimated channels.

III. MAXIMUM LIKELIHOOD ESTIMATOR

In this section, we formulate the ML combination rule for estimating the positions of the K targets based on the MIMO channel estimates acquired by each RP. The parameters to be estimated are defined by the vector
$\boldsymbol{\gamma} = [\mathbf{x}_1^{\mathrm{T}} \ \dots \ \mathbf{x}_K^{\mathrm{T}} \ \boldsymbol{\beta}_{1,1} \ \boldsymbol{\beta}_{1,2} \ \dots \ \boldsymbol{\beta}_{1,Q_1} \ \boldsymbol{\beta}_{2,1} \ \dots \ \boldsymbol{\beta}_{P,Q_P}]^{\mathrm{T}}$.

Considering independent noise contributions for the estimated channel vector from all radar pairs, the combined likelihood function is derived as the product of individual Gaussian density functions. After taking the natural logarithm of the combined likelihood function, the sum of the local log-likelihood functions has to be maximized,

$$\widehat{\boldsymbol{\gamma}} = \arg\min_{\widetilde{\boldsymbol{\gamma}}} \textstyle\sum_{p=1}^{P} \sum_{q=1}^{Q_p} \frac{1}{\sigma_p^2} \left\| \widehat{\mathbf{h}}_q - \mathbf{A}_p(\widetilde{\mathbf{X}}) \ \widetilde{\boldsymbol{\beta}}_{p,q} \right\|_2^2. \tag{5}$$

First, we maximize (5) with respect to the channel coefficients $\widetilde{\boldsymbol{\beta}}_{p,q}$ to obtain a closed-form expression as a function of $\widetilde{\mathbf{X}}$. Following the solution of the resulting linear least squares problem, the ML estimate of the channel coefficients for every observation q and radar pair p is expressed as

$$\widehat{\boldsymbol{\beta}}_{p,q}(\widetilde{\mathbf{X}}) = \mathbf{A}_p^{\dagger}(\widetilde{\mathbf{X}}) \ \widehat{\mathbf{h}}_{p,q}. \tag{6}$$

The projection matrix onto the subspace formed by the steering vectors corresponding to each candidate positions in $\widetilde{\mathbf{X}}$ is defined as

$$\mathbf{P}_p(\widetilde{\mathbf{X}}) = \mathbf{A}_p(\widetilde{\mathbf{X}}) \ \mathbf{A}_p^{\dagger}(\widetilde{\mathbf{X}}). \tag{7}$$

By substituting (6) and (7) back into (5), we obtain

$$\widehat{\mathbf{X}} = \arg\min\nolimits_{\widetilde{\mathbf{X}}} \textstyle\sum_{p=1}^{P} \sum_{q=1}^{Q_p} \frac{1}{\sigma_p^2} \left\| \widehat{\mathbf{h}}_{p,q} - \mathbf{P}_p(\widetilde{\mathbf{X}}) \ \widehat{\mathbf{h}}_{p,q} \right\|_2^2. \tag{8}$$

After some mathematical manipulations, we can show that this is equivalent to solving the following problems:

$$\widehat{\mathbf{X}} = \arg\max_{\widetilde{\mathbf{X}}} \textstyle\sum_{p=1}^{P} \sum_{q=1}^{Q_p} \frac{1}{\sigma_p^2} \left\| \mathbf{P}_p(\widetilde{\mathbf{X}}) \ \widehat{\mathbf{h}}_{p,q} \right\|_2^2 \tag{9}$$

$$= \arg\max_{\widetilde{\mathbf{X}}} \textstyle\sum_{p=1}^{P} \frac{Q_p}{\sigma_p^2} \operatorname{Tr}\left\{ \mathbf{P}_p(\widetilde{\mathbf{X}}) \ \widehat{\mathbf{R}}_p \right\} \tag{10}$$

in which the sample covariance matrix of the channel vector is defined as $\widehat{\mathbf{R}}_p = \frac{1}{Q_p} \sum_{q=1}^{Q_p} \widehat{\mathbf{h}}_{p,q} \ \widehat{\mathbf{h}}_{p,q}^{\mathrm{H}}$.

We seek the set of K positions $\widetilde{\mathbf{x}}$ that solves the optimization problem defined in (10). The decisions for all targets are not independent due to the presence of sidelobes in the ambiguity function. As a result, solving the exact ML problem is often impractical due to its computational complexity, which scales in the order $\mathcal{O}(n^{2K})$, where n is the number of grid points discretizing the search space for the variable $\widetilde{\mathbf{x}}$.

IV. BLOCK GREEDY ALGORITHMS

In order to approach the solution of the optimal MLE derived in Section III with a lower computational complexity, it is possible to apply greedy methods such as BOMP and BOLS. Both methods estimate the targets position one after the other, while taking into account the decisions taken during the previous steps. In this section, we first detail the steps of BOMP and BOLS. Then, we show how the computational complexity of BOLS can be reduced.

The successive step for a block greedy algorithm in this multistatic MIMO radar context are described in Algorithm 1.

Algorithm 1: Block Greedy Algorithms

```
Input: $\widehat{\mathbf{R}}_p \ \forall p \in \{1, \dots, P\}$
Output: $\widehat{\mathbf{X}}_K = [\widehat{\mathbf{x}}_1 \ \dots \ \widehat{\mathbf{x}}_K]$
begin
    Initialization: $\widehat{\mathbf{X}}_0 \leftarrow [\ ]$ ;
    for $k$=1 to $K$ do
        1. Selection Step: $\widehat{\mathbf{x}}_k$ given by (11) for BOMP,
           (12) for Slow-BOLS or (13) for Fast-BOLS;
        2. Update Step: $\widehat{\mathbf{X}}_k \leftarrow [\widehat{\mathbf{X}}_{k-1}, \widehat{\mathbf{x}}_k]$;
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The selection steps for the BOMP and the BOLS methods are respectively given by

$$\widehat{\mathbf{x}}_k = \arg\max_{\widetilde{\mathbf{x}}_k} \textstyle\sum_{p=1}^{P} \frac{Q_p}{\sigma_p^2} \mathbf{a}_p^{\mathrm{H}}\left(\widetilde{\mathbf{x}}_k\right) \ \widehat{\mathbf{R}}_p^{res} \ \mathbf{a}_p\left(\widetilde{\mathbf{x}}_k\right), \tag{11}$$

$$\widehat{\mathbf{x}}_k = \arg\max_{\widetilde{\mathbf{x}}_k} \textstyle\sum_{p=1}^{P} \frac{Q_p}{\sigma_p^2} \operatorname{Tr}\left\{ \mathbf{P}_p\left([\widehat{\mathbf{X}}_{k-1}, \widetilde{\mathbf{x}}_k] \right) \widehat{\mathbf{R}}_p \right\}, \tag{12}$$

where $\widehat{\mathbf{R}}_p^{res} = (\mathbf{I} - \mathbf{P}_p(\widehat{\mathbf{X}}_{k-1})) \ \widehat{\mathbf{R}}_p, \ \forall p = 1 \dots P$.

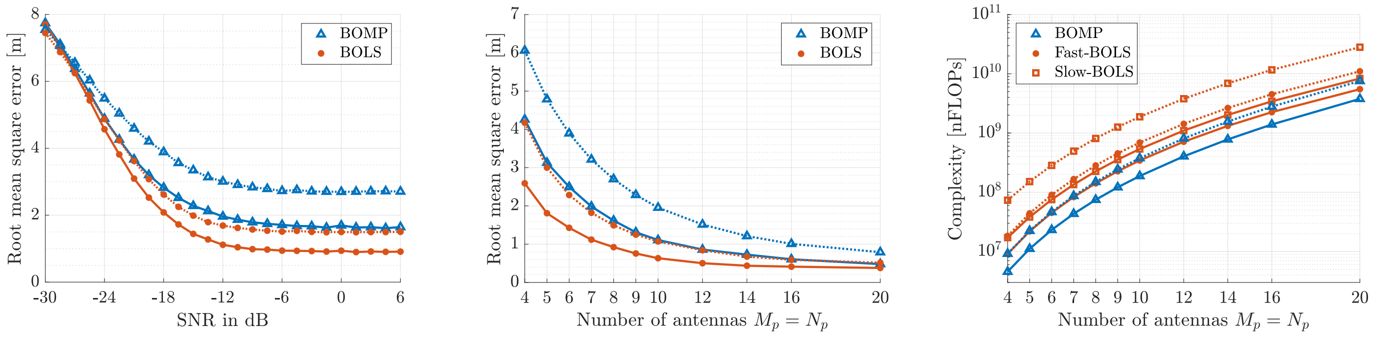
From (11) and (12), we can observe that BOMP and BOLS differ in their handling of previously selected positions during the iterative process. BOMP selects positions based on their projection with the residual sample covariance matrix $\widehat{\mathbf{R}}_p^{res}$, ensuring orthogonality by updating the residual after each iteration. In contrast, BOLS refines its selection by explicitly minimizing the residual error over all previously chosen positions, effectively re-optimizing their contributions at each step. This re-optimization grants BOLS higher recovery accuracy compared to BOMP but comes at the cost of increased computational complexity.

Using a projection matrix decomposition as defined in [34], we can reduce the natural computational complexity of the BOLS algorithm by avoiding the computation of the full projection matrix $\mathbf{P}_p\left([\widehat{\mathbf{X}}_{k-1}, \widetilde{\mathbf{x}}_k] \right)$ for each candidate position. The mathematical steps of this decomposition applied in this scenario are presented in the supplementary materials. As a consequence, the BOLS selection step defined in (12) can be equivalently reformulated in an efficient manner as

$$\widehat{\mathbf{x}}_k = \arg\max_{\widetilde{\mathbf{p}}_k} \sum_{p=1}^{P} \frac{Q_p}{\sigma_p^2} \frac{\mathbf{r}_p^{res}(\widetilde{\mathbf{x}}_k)^{\mathrm{H}} \ \widehat{\mathbf{R}}_p \ \mathbf{r}_p^{res}(\widetilde{\mathbf{x}}_k)}{\|\mathbf{r}_p^{res}(\widetilde{\mathbf{x}}_k)\|_2^2}, \tag{13}$$

where $\mathbf{r}_p^{res}(\widetilde{\mathbf{x}}_k) = (\mathbf{I} - \mathbf{P}_p(\widehat{\mathbf{X}}_{k-1})) \ \mathbf{a}_p(\widetilde{\mathbf{x}}_k)$. As for the computation of $\widehat{\mathbf{R}}_p^{res}$ in BOMP, the projection matrix $\mathbf{P}_p(\widehat{\mathbf{X}}_{k-1})$ in BOLS has to be computed only once at each step.

The 2 proposed greedy methods can be evaluated by the central processor solely using the sample covariance matrix $\widehat{\mathbf{R}}_p$ estimated for each $p \in \{1, \dots, P\}$. Therefore, the proposed multistatic fusion algorithms require each RP to transmit an $(M_p N_p \times M_p N_p)$ matrix to the central processor, instead of a $(M_p \times N_p \times Q_p)$ estimated channel tensor. This is more efficient since typically $Q_p > M_p N_p$ [35]. Besides, no time synchronization is required for this angle-based localization.



(a) (b) (c)

Fig. 2. Comparison of the performance of BOMP and BOLS for $Q_p = 512,\ \forall p$. (a) displays the evolution of the RMSE with respect to the SNR for $M_p = N_p = 8,\ \forall p$, (b) shows the RMSE as a function of the number of antennas in a noiseless scenario, and (c) represents in log scale the computational complexity as a function of the number of antennas. In all graphs, the solid and dotted lines depict a scenario with 4 and 8 targets, respectively.

V. SIMULATION RESULTS

This section compares the BOMP and BOLS algorithms outlined in Section IV for an angle-based multistatic MIMO radar system localizing targets within its coverage area. The performance of both algorithms is compared in terms of localization RMSE and computational complexity. The computational complexity is evaluated by counting the number of complex floating point operations. The RMSE is computed through $30,000$ Monte-Carlo simulations for which the error is defined between the true target positions and the corresponding detected positions. The association problem between the true and the detected positions is well-defined in the literature [36], and is here solved using the Hungarian algorithm [37].

For each simulation, we analyze a fixed setup with two RPs to locate multiple targets randomly positioned. A squared x-y coverage area is defined in $[-30, 30] \times [0, 60]$ meters. Both RPs share an STx located at $[0, 0]$ and have their SRx at $[-30, 30]$ and $[30, 30]$, respectively. In practice, each selection step in Algorithm 1 is performed by detecting the maximum value on a discrete search grid with a grid spacing of 2 meters, followed by a refinement step using a gradient descent to enable off-grid detections. The results can be observed on Fig. 2.

Fig. 2a depicts the RMSE obtained for both methods as a function of the mean SNR. The SNR is defined here as the mean of the quotient of the channel coefficients of each path by the noise variance. Two multitarget scenarios are compared with 4 and 8 targets, respectively. We can observe that, at low SNRs, both methods have similar RMSE as it corresponds to a noise-limited region. The improvement for BOLS compared to BOMP becomes more significant as the SNR increases. In such an interference-limited region, the impact of the correlation between the signals reflected by the targets dominates the noise, making the use of BOLS particularly interesting. We can also observe that in scenarios with more targets, the performance gap between BOLS and BOMP increases as the impact of the interference increases with the number of targets. Finally, the RMSE reaches a plateau at high SNRs due to interference among targets. This is a classical behaviour for heuristic methods and can be solved with additional refinement steps, for example, based on the alternating projections [38].

Fig. 2b and 2c illustrate, respectively, the RMSE and the computational complexity obtained for both methods as a function of the number of antennas per node. The considered scenario is noiseless to focus the discussion on the impact of the number of targets and antennas in the interference-limited region. The smaller the antenna arrays, the higher the benefit of BOLS over BOMP in terms of RMSE due to the increased correlation between the reflected signals from each target. It can also be observed that this gap increases with the number of targets as the impact of the interference increases too. We can notably see in this scenario that BOLS achieves a similar RMSE for detecting 8 targets as BOMP for detecting 4 targets. However, for very large antenna arrays (i.e. with more than 16-20 antennas), the localization RMSE, for all algorithms and number of targets, converge. Finally, BOMP always presents a smaller complexity compared to any BOLS implementation. However, this computational gap is much reduced using the efficient implementation of BOLS based on the projection matrix decomposition, especially as the number of targets increases. In summary, while BOMP may remain a better trade-off for systems with a high number of antennas, BOLS appears significantly more performant in scenarios with highly correlated signals.

VI. CONCLUSION

In this paper, we investigate the multitarget angle-based localization capabilities of a multistatic MIMO radar using block greedy methods. The performance of BOLS is compared to that of BOMP, commonly used in the literature, in terms of RMSE and computational complexity. Monte-Carlo simulations demonstrate the superiority of the proposed implementation of BOLS in highly correlated scenarios such as the one considered in this paper. The influence of the number of antennas per node and of the number of targets on the importance of this benefit is also discussed. We also raised awareness on an efficient implementation of BOLS, seldom found in the literature, based on a projection matrix decomposition. Finally, we have shown that both greedy methods can be implemented without time synchronization and with a limited data transfer. In future works, multistatic experimental measurements can further validate the benefit of BOLS, owing to its computationally efficient implementation.

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