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Efficiency in overlapping generations economies with longevity choices and fair annuities ☆

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ABSTRACT

When individuals can influence their life-expectancies and save in annuities, suboptimal savings result from the lack of incentives to choose the optimal longevity, even when annuity returns can be made contingent to longevity-related choices. Specifically, the golden rule steady state maximizing the representative agent utility cannot be attained as a competitive equilibrium under laissez-faire, even with actuarially fair annuities contingent to longevity-enhancing choices. In order to decentralize through markets the golden rule, longevity-enhancing expenditures need to be taxed if the steady state old-age consumption exceeds the annuitized capital return, and subsidized otherwise—the government budget being balanced through lump-sum transfers or taxes. Interestingly, with positive population growth the *expected* net contribution is negative when longevity-enhancing expenditures are taxed, and positive when subsidized.

1. Introduction

Individuals decide not only what and when to consume (given their resources and the available financial contracts), but also about how long to expect to live. Indeed, individuals do influence their life expectancy in various ways by undertaking or avoiding actions and behaviors that may increase or decrease it.[1] Off course, increasing one's life expectancy can typically be done only at a cost, either in terms of resources (e.g. healthcare undertaken at the expense of lost consumption), or in terms of disutility (e.g. forgone unhealthy pleasures or a constraining way of life).[2] If that was it, the problem would not be different from

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[1] For instance, agents can affect they life expectancy through their choice of preventive and palliative medical care, diet, lifestyle (see Balia and Jones, 2008). Poikolainen and Escola (1986) study in particular the impact of health expenditures on life expectancy. Several other studies document the impact of physical activity (Kaplan et al., 1987; Okamoto, 2006), overweight (see Solomon and Manson, 1997; Bender et al., 1998) and smoking (Doll and Hill, 1950) on life expectancy.

[2] While the most obvious way to increase life expectancy is to increase medical treatment, which requires the actual spending of income, individuals can also make behavioral choices to that end (e.g. exercising, abstaining from smoking, eating a healthy diet, driving safely) that do not necessarily require an additional spending, but may inflict nonetheless some disutility.

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any other implying a trade-off between competing goals, namely between quantity and quality of life here. Nevertheless, things are in this case (unsurprisingly) more complicated: our choice of an expected life-span is intermingled with the problem of how to deal with the risk of outliving our resources. In effect, for most people, most consumption is financed out of labor income, which stops at retirement for legal reasons, or earlier for physiological ones, so that the most straightforward way to guarantee an income at retirement is to save. Still, individual savings do not eliminate this risk entirely, since miscalculation may lead to either running short of savings before dying or to leaving accidental bequests.[3]

To address in particular the risk of individuals saving insufficiently for old age, mandatory pension schemes force the inter- and intra-generational transfers necessary to guarantee adequate old-age living standards. Moreover, these mandatory public pension schemes are increasingly supplemented by private market-based schemes, such as annuities providing a life-long income after retirement in exchange of previous contributions. Nevertheless, once insured against the risk of running out of income, anyone has obvious incentives to take measures to increase his or her life-expectancy.[4] Mandatory public pension schemes in which the pension received is not contingent to the individuals' life-enhancing choices are clearly unable to deal with this incentives problem, and are likely to induce excessive longevity, compared to the optimal one. Indeed, annuities that are independent of the individuals' choices make them invest in the quantity of life without internalizing the subsequent negative impact on its quality (through lower annuity returns). Instead, private annuity markets should have more chances of solving this incentives problem, by being able to make returns contingent to observable individual choices and behavior.[5] This conjecture is precisely what this paper explores. To do so, we use a Diamond (1965) overlapping-generations economy of agents living up to two periods in which longevity—i.e. the probability of survival into the second period—is assumed to be endogenous (in Section 6, we extend the analysis to an overlapping-generations model akin to a "perpetual youth" model à la Blanchard (1985) without constant survival probability: households can live any number of periods, but face at any time a probability of survival into the next period that depends on their longevity-enhancing expenditures).

Specifically, can perfect annuity markets—which can price longevity-enhancing observable behavior into annuity returns—overcome alone the problem of inefficient investment in longevity? We show here, first, that in a laissez-faire, dynamic setup the answer to this question is negative. More specifically, we establish that typically the golden rule steady state maximizing the expected utility of the representative agent is not an equilibrium outcome under laissez-faire, even with actuarially fair, longevity-contingent competitive annuity markets. Moreover, in order to lay bare the fact that longevity choices are the root of the problem, we show that for an exogenous survival probability the golden rule *is* attainable through annuities and money. We show next that, in this set-up, life-expectancy is actually not necessarily higher at the competitive equilibrium steady state than at the golden rule: there may as well be, at equilibrium, a problem of under-investment in longevity as one of over-investment. In other words, depending on their preferences, the agents may uncoordinatedly choose to live shorter and richer lives, as well as longer and poorer lives, than the optimal lives the resources allow for if they coordinated their choices. Finally, we identify a policy of taxes and transfers that decentralizes the golden rule as a competitive outcome.

This paper is related to the literature on endogenous longevity in overlapping generations setups. In Chakraborty (2004) and De la Croix and Ponthière (2010) survival chances are a function of a constant fraction of, respectively, the marginal productivity of labor and the output per worker. Chakraborty (2004) argue for the positive impact on growth of publicly funded health care through the subsequent increase in the returns to investment in human capital, while De la Croix and Ponthière (2010) study how the golden rule is modified when longevity depends on per capita income. At any rate, longevity is not in these two papers the result of any deliberate choice by the agents of setting aside resources devoted specifically to modify it.[6] To the contrary, in our paper, longevity is actually a direct private choice as it depends on the agent's private healthcare expenditures, like in Pestieau et al. (2008) and Jouvet et al. (2010).[7] Nevertheless, Jouvet et al. (2010) explicitly avoid addressing any risk aspect of the problem assuming deterministic life-spans that are function of health expenditures, while Pestieau et al. (2008) restrict the analysis to the case in which there exists a pay-as-you-go pension system with an exogenously given replacement ratio of benefits to wages.

The papers above focus on either the competitive equilibrium steady state, as in Chakraborty (2004), or on the golden rule steady state, as in De la Croix and Ponthière (2010), or on the solution of a planner constrained by an exogenously given replacement ratio of an existing pay-as-you-go pension scheme, as in Jouvet et al. (2010). We show here instead that the golden rule cannot typically be attained under laissez-faire as a competitive equilibrium with annuitized savings when the agents can influence their longevity, *even if the annuities contracts are devised to cope with the incentives to over-invest in longevity when insured*. Notwithstanding, we show that an active fiscal policy allows to implement the golden rule as an equilibrium outcome. Specifically, whether longevity-enhancing expenditures need to be taxed or

[3] Another primitive way to address this risk is to have children, although it is clearly not entirely free of risk either, in spite of the premeditate attempts by most cultures to instil in children devotion towards their parents.

[4] This point was developed in Davies and Kuhn (1992) and Philipson and Becker (1998).

[5] Philipson and Becker (1998) note that this externality can also arise with private annuity markets, if it is too costly to control longevity-enhancing behavior through insurance premia. Yet, the authors argue that public schemes are more subject to this moral-hazard problem, given their lack of a profit-maximization motive.

[6] It is indirectly a consequence of their saving decisions, but for that reason the agents cannot disentangle the two choices.

[7] Jouvet et al. (2010) also study the impact of a production externality (such as pollution) on longevity.

subsidized in order to implement the golden rule turns out to depend on whether the golden rule consumption of old agents exceeds the annuitized return to their capital savings or not. The policy is such that the government budget is balanced every period by period by means of lump-sum transfers to, or taxes from, the old depending on whether they got their longevity-enhancing expenditures taxed or subsidized respectively. Interestingly enough, and in stark contrast with the naive intuition that subsidizing longevity-enhancing expenditures (say, health expenditures) is good and taxing them is bad, it turns out that, whenever there is population growth, the expected net contribution of an agent from taxing them is negative when they need to be taxed to implement the golden rule, while his expected net contribution is positive when they need to be subsidized.[8]

This paper can also be related to the vast literature on the annuitization of savings. This branch of the literature has grown rapidly following the development of annuity markets.[9] The seminal contribution of Yaari (1965) assumes exogenous longevity, and shows that the optimal annuity pattern is independent of the individuals mortality profile. On the contrary, Bommier et al. (2011) study, in a partial equilibrium framework, the optimal taxation of annuities when individuals have different (exogenous) life-expectancies, and shows that optimal taxation differs across individuals (under some assumptions on individuals' preferences). Some other papers also consider the optimal fiscal policy when longevity is endogenous. For instance, in Leroux et al. (2011), individuals do not perfectly anticipate the consequences of longevity-enhancing efforts on the return of their annuitized savings—as in Philipson and Becker (1998)—and it shows that a tax on health expenditure is required to implement the optimum in a partial equilibrium setting. Nevertheless, to the best of our knowledge, it appears that studying the optimal taxation of annuities in a dynamic setup has not been paid much attention until now.

The paper is organized as follows. In Section 2, we present the model and show that with exogenous survival probability, annuities and money suffice to reach the golden rule. In Sections 3 and 4, we derive respectively the laissez-faire competitive equilibrium with actuarially fair annuities contingent to longevity choices as well as the golden rule steady state, and thus show that the latter fails to be an equilibrium outcome. In Section 5, we show how to decentralize the golden rule as a competitive equilibrium steady state. In Section 6, we discuss alternative assumptions regarding longevity. First, we show how our previous results are modified when longevity-enhancing behavior is costly in terms of utility rather than in terms of forgone consumption. Second, we extend the model to consider endogenous second-period labor supply which depends on the health expenditures made in the first period, and we show that all main results hold up. Third, we show that our results remain true when households live any number of periods with a positive survival probability, as long as longevity-enhancing expenditures are positive too. The last section concludes.

2. The model

2.1. Assumptions

At every date t, a continuum of identical agents are born. The representative agent born at t lives at least one period but at most two, conditional on survival with a probability $\pi(e^t)$ that he can influence through e^t, a fraction of first-period income devoted to health expenditures.

The survival probability $\pi(\cdot)$ is assumed to satisfy the following assumption.

Assumption 1.

$$\begin{aligned} &\pi'(e^t) > 0 \\ &\pi''(e^t) < 0 \\ &\lim_{e^t \to 0^+} \pi'(e^t) = +\infty \end{aligned} \tag{1}$$

Hence, the higher the longevity-enhancing expenditures e^t, the higher is the survival probability, but also the smaller is the increase in life-expectancy due to one additional unit of such expenditures. Longevity-enhancing expenditures are assumed to be observable and verifiable, so that they can be contracted upon.[10] Note that the probability of survival $\pi(e^t)$ is also the proportion of individuals born at t and choosing e^t that is expected to survive into the next period. For large enough cohorts of agents choosing an effort level e^t, the actual survival rate will be arbitrarily close to $\pi(e^t)$, according to the law of large numbers.

At the end of the first period—i.e. before the mortality shock realizes—each agent gives birth to n children, so that the size of each cohort of young agents is n times that of the previous one.

[8] At any rate, the golden rule gets implemented in both cases (and which of them realizes depends on the characteristics of the economy: preferences, technology, etc.), so that agents are not ex ante worse-off with a positive net contribution than with a negative one. The result highlights the importance of looking at the whole (general equilibrium) picture of consequences of taxes and transfers to assess a policy, instead of focusing on what gets *directly* taxed or subsidized.

[9] For a good review of the issues raised by the annuitization of savings, see Sheshinski (2007).

[10] Assuming imperfect observability would only add an additional source of inefficiency, on top of the one on which this paper focuses, namely the impossibility for the agents to internalize the effects of their longevity choices on their savings returns. Also, since we consider a representative agent, these expenditures, if not directly observable, could anyway be deduced from the individual's problem.

When young, the representative agent supplies inelastically one unit of labor for a real wage rate w_t that he splits between first-period consumption c_0^t, longevity-enhancing expenditures e^t, and savings.[11]

Saved income can be held in money or invested in annuities from competitive pension funds lending the contributions to the firms producing output. The returns to the investments in the pension funds are supposed to be made contractually contingent to the observable longevity-enhancing expenditures. Therefore, for actuarially fair annuities, whenever the representative agent makes a longevity effort e^t, the return to these annuities is the marginal productivity of capital over the corresponding survival rate, $r_{t+1}/\pi(e^t)$. As for the money savings,[12] note first that *money* stands here for an asset bubble whose returns cannot be contractually annuitized—not to mention made contingent to longevity-related choices—as it is indeed the case for cash holdings or bank account balances. Then the question of what happens to money savings of the deceased naturally arises. As a matter of fact, whether they are assumed to vanish as claims with their owners or taxed away and redistributed among survivors makes no difference to the equilibrium allocation. Only the rate at which prices decrease as population grows would change, depending on whether the amount of money remains constant or shrinks as the monetary savings of the deceased are redistributed or vanish respectively. Accordingly, for the sake of notational simplicity, we will write all expressions below as if the claims supported by money savings of the deceased vanished with them.[13]

Denoting c_0^t and c_1^t the first- and second-period consumptions of an agent born at time t, his problem consists therefore in maximizing the following expected utility

$$u(c_0^t) + \pi(e^t)v(c_1^t) \tag{2}$$

under the budget constraints[14]

$$\begin{aligned} c_0^t + k^t + \frac{1}{p_t}M^t + e^t &\leqslant w_t \\ c_1^t &\leqslant \frac{1}{\pi(e^t)}r_{t+1}k^t + \frac{1}{p_{t+1}}M^t \end{aligned} \tag{3}$$

where M^t represent savings held in money, k^t, the savings held in annuities, w_t the real wage rate, and p_t prices, at time t. According to the second-period budget constraint above, the agent knows that the annuity contract makes its return, $r_{t+1}/\pi(e^t)$, depend on his choice of observable longevity-enhancing expenditures.

Per period utility functions u and v are twice continuously differentiable, increasing, strictly concave functions satisfying the Inada condition, i.e. such that

Assumption 2.

$$\begin{aligned} &u'(c_0) > 0 < v'(c_1) \\ &u''(c_0) < 0 > v''(c_1) \\ &\lim_{c_0 \to 0^+} u'(c_0) = +\infty = \lim_{c_1 \to 0^+} v'(c_1) \end{aligned} \tag{4}$$

Finally, in order to guarantee the quasi-concavity of the representative agent's utility, as well as that the first-order conditions are not only necessary but also sufficient to characterize the agents' optimal choices, we also make throughout the paper the following assumption:

Assumption 3.

$$\pi(e)\pi''(e)v(c_1)v''(c_1) \geqslant (\pi'(e)v'(c_1))^2 \tag{5}$$

The above condition can as well be expressed (if v is assumed to take strictly positive values) as follows

[11] A longer life, especially a healthier one, is also likely to increase the labor force available for production at any time but, since in this model only young agents work and supply inelastically one unit of labor, an increased life expectancy has no impact on the labor supply. In Section 6.2, we extend our framework allowing for second-period labor supply to depend on the health expenditure made in the first period. We show that this does not change qualitatively the message of the paper and we are still able to identify the inefficiency created by the inability of agents to internalize the impact on saving returns of endogenous longevity.

[12] As in Diamond (1965), there is no hope that the golden rule be an equilibrium outcome in the absence of money (except for a knife-edge case). In Diamond (1965) the introduction of money (or national debt) fixes the problem (see Blanchard and Fisher, 1993, Chapter 4). Here, money alone will not be sufficient, but it will nonetheless be necessary. In effect, should there be no money to save in, but only annuities, then the competitive equilibrium steady state would necessarily differ from the golden rule but for a degenerate case (see Proposition 3).

[13] See Footnotes 14 and 20, as well as Sheshinski (2007) or Diamond (2005) for references. In order to keep the problem separate from intergenerational redistribution issues, we do not allow here for these claims to be redistributed among the newly born *young* agents (see Heijdra et al. (2014) for a comparison of these different options in an exogenous longevity setup).

[14] Should the deceased money savings be distributed among survivors the last term in the second period budget constraint would be $\frac{1}{p_{t+1}}\frac{M^t}{\pi(e^t)}$ instead. Note however that the representative agent would not in this case take into account the impact of his choice of e^t on this term, since (i) the return to money cannot be made contingent to longevity-related choices (as opposed to the return to each individual's annuities, which he duly takes into account when solving (2–3)), and (ii) the impact of each individual on the survival rate is negligible.

$$\frac{\pi''(e)}{\pi'(e)}e\cdot\frac{v''(c_1)}{v'(c_1)}c_1 \geqslant \frac{\pi'(e)}{\pi(e)}e\cdot\frac{v'(c_1)}{v(c_1)}c_1 \tag{6}$$

i.e. the product of the elasticities of π' and v' is bigger than the product of the elasticities of π and v. In other words, the (proportional) marginal increases in survival probability and second-period utility have to be *jointly* more sensitive to effort and consumption respectively than the probability and utility themselves, a sort of strong enough concavity of second-period expected utility.

At every period t, firms produce, out of aggregate capital $K_t = n^{t-1}k^{t-1}$ and labor $L_t = n^t$, an amount $F(K_t, L_t)$ of good—that can be consumed (by young and old), saved (to serve as next period's capital), or devoted to longevity-enhancing expenditures—according to a neoclassical production function satisfying the following standard assumption.[15]

Assumption 4. The production function $F(K_t, L_t)$ is increasing, concave, has constant returns to scale, and, for all $K_t, L_t \geqslant 0$,

$$\begin{aligned} &F(K_t,0) = 0 = F(0,L_t) \\ &\lim_{K\to 0^+} F_K(K,L_t) = +\infty = \lim_{L\to 0^+} F_L(K_t,L) \end{aligned} \tag{7}$$

Factor markets are perfectly competitive, so that at every period t the real wage rate is equal to the marginal productivity of labor and the marginal productivity of capital remunerates capital loans from the pension funds to firms,[16] i.e.

$$\begin{aligned} w_t &= F_L\left(\frac{k^{t-1}}{n},1\right) \\ r_{t+1} &= F_K\left(\frac{k^t}{n},1\right) \end{aligned} \tag{8}$$

Before going further, let us mention that in the main part of the manuscript, we will assume that longevity can only be increased using up resources. Only in Section 6.1, we relax this assumption to assume instead that longevity can be increased through efforts that are costly in terms of utility but not in terms of the budget constraint.

2.2. The benchmark with exogenous longevity

In order to lay bare how the agents' ability to influence their life expectancy generates the inefficiency pointed at in this paper, let us first assume that the survival probability is exogenous so that it does not depend on agents' health expenditures. In such a case, $\pi(e^t)$ then reduces to some constant probability π. As it will become apparent below, in this set up the golden rule will be reached if the agents can save in fair annuities and money.

The representative agent's problem is now

$$\begin{aligned} \max_{0\leqslant c_0^t, c_1^t, k^t, M^t} \quad & u(c_0^t) + \pi v(c_1^t) \\ \text{s.t.} \quad & c_0^t + k^t + \tfrac{1}{p_t}M^t \leqslant w_t \\ & c_1^t \leqslant \tfrac{1}{\pi} r_{t+1}k^t + \tfrac{1}{p_{t+1}}M^t \end{aligned} \tag{9}$$

given w_t, r_{t+1}, p_t, and p_{t+1}. The first-period budget constraint differs from (3) in that, under exogenous survival, the agent does not invest in e^t. The second-period budget constraint is also modified as the return from the annuity contract is in fixed proportion to the return to capital.

Assumption 2 guarantees that $c_0^t, c_1^t > 0$ as well as strictly positive savings, which implies that either $k^t > 0$ or $M^t > 0$ (or both, as long as the two assets have the same return). In order to see whether the golden rule can be an equilibrium outcome, we are nonetheless interested only in a competitive equilibrium steady state such that $k^t > 0$ and $M^t > 0$ for all t.[17]

Solving problem (9), the choice of an agent born at time t is a solution to

[15] For the sake of notational simplicity, capital is assumed to depreciate completely in one period.

[16] Competition among pension funds leads to the complete distribution as annuities of the return to the capital lent to firms among the proportion $\pi(e^t)$ of survivors.

[17] Any steady state with either $k^t = 0$ or $M^t = 0$ for all t is Pareto-dominated by an interior competitive equilibrium steady state. This is obvious if $k^t = 0$ for all t. And if $M^t = 0$ for all t, any such equilibrium steady state would be characterized by the system (14), except for the first line, which would be

$$\frac{u'(\tilde{c}_0)}{v'(\tilde{c}_1)} = F_K\left(\frac{\tilde{k}}{n},1\right)$$

so that the marginal productivity of capital is not guaranteed to coincide with the rate of growth of the population, and therefore the net output may not be maximized, as it is indeed at an interior competitive equilibrium steady state.

$$\begin{aligned} &\frac{1}{\pi}\frac{u'(c_0^t)}{v'(c_1^t)}=\frac{p_t}{p_{t+1}}=\frac{1}{\pi}r_{t+1}\\ &c_0^t+k^t+\frac{1}{p_t}M^t=w_t\\ &c_1^t=\frac{1}{\pi}r_{t+1}k^t+\frac{1}{p_{t+1}}M^t \end{aligned} \tag{10}$$

The first equation in the first line equates the marginal rate of substitution between first- and second-period consumptions to the rate at which income can be transferred from the first to the second period of life. It determines thus the agent's optimal level of savings. The second equation in the first line requires the absence of arbitrage between the two saving instruments (money and annuities) for the agent to be willing to hold them both. The second and third lines are the agent's budget constraints.

Note that at a competitive equilibrium, the real wage w_t and the rental rate of capital r_{t+1} are determined by the marginal productivities of labor and capital respectively as in conditions (8). Also, adding up the budget constraints of the agents alive at any given period t,

$$c_0^t+\frac{\pi}{n}c_1^{t-1}+k^t+\frac{1}{p_t}M^t=F_L\left(\frac{k^{t-1}}{n},1\right)+F_K\left(\frac{k^{t-1}}{n},1\right)\frac{k^{t-1}}{n}+\frac{\pi}{n}\frac{1}{p_t}M^{t-1} \tag{11}$$

it follows that the feasibility of the allocation is equivalent to

$$\frac{M^t}{M^{t+1}}=\frac{n}{\pi} \tag{12}$$

which at the steady state implies also that

$$\frac{p_t}{p_{t+1}}=\frac{n}{\pi} \tag{13}$$

Hence, the equilibrium return to money increases with the population growth factor but decreases with the survival chance.

A competitive equilibrium steady state consists, therefore, of a profile $(\tilde{c}_0,\tilde{c}_1,\tilde{k},\tilde{m})$ satisfying

$$\begin{aligned} &\frac{u'(\tilde{c}_0)}{v'(\tilde{c}_1)}=n=F_K\left(\frac{\tilde{k}}{n},1\right)\\ &\tilde{c}_0+\tilde{k}+\tilde{m}=F_L\left(\frac{\tilde{k}}{n},1\right)\\ &\frac{\pi}{n}\tilde{c}_1=F_K\left(\frac{\tilde{k}}{n},1\right)\frac{\tilde{k}}{n}+\tilde{m}. \end{aligned} \tag{14}$$

We now derive the golden rule steady state that maximizes the utility of the representative agent, under the feasibility constraint. For an economy with large enough generations, it is characterized by the solution to the following problem

$$\begin{aligned} \max_{0\leqslant c_0,c_1,k}\quad & u(c_0)+\pi v(c_1)\\ \text{s.t.}\quad & c_0+\tfrac{\pi}{n}c_1+k\leqslant F\left(\tfrac{k}{n},1\right) \end{aligned} \tag{15}$$

The resource constraint in the optimization problem above requires that the output per worker (net of capital replacement) allows at any time to satisfy the consumptions of young and old agents alive that period, the latter being only a proportion $\frac{1}{n}$ of the former because of the population growth, of which, moreover, only a fraction π would have survived, for large enough cohorts. The golden rule steady state (c_0^*,c_1^*,k^*) is then a solution to the optimization problem above and hence necessarily satisfies the following first-order conditions

$$\begin{aligned} &\frac{u'(c_0^*)}{v'(c_1^*)}=n=F_K\left(\frac{k^*}{n},1\right)\\ &c_0^*+\frac{\pi}{n}c_1^*+k^*=F\left(\frac{k^*}{n},1\right) \end{aligned} \tag{16}$$

The first line follows on the one hand, from equating the marginal rate of substitution $\frac{1}{\pi}\frac{u'(c_0^*)}{v'(c_1^*)}$ to the rate at which consumption can be transferred between contemporaneous young and old agents $\frac{n}{\pi}$, and, on the other hand, from the condition for net output maximization which results in equating the marginal productivity of capital to the growth factor of the population. The second line is the feasibility of the allocation of resources so that output net of capital replacement must equal at any period the consumption of young and surviving old agents.

Under the assumptions made, the golden rule is defined by the first-order conditions in (16) above and it is unique (see Proposition A1 in the appendix).

Comparing the systems (14) and (16) characterizing, respectively, the competitive equilibrium steady state and the golden rule, it is immediate to see that the golden rule solves the competitive equilibrium steady state equations, for some positive amount of money. Thus, the introduction of an exogenous survival probability does not change the well-known result in a Diamond (1965) model that a bubble asset—in this case supplemented by annuities—allows to reach the golden rule as a competitive equilibrium steady state, as the next proposition states.

Proposition 1. *Consider a Diamond (1965) overlapping generations production economy with money and an exogenously growing population with a representative agent facing an exogenous probability of survival, and saving in actuarially fair annuities. The golden rule is a competitive equilibrium steady state of the economy.*

So, as long as the survival probability is exogenous, fair annuities and money allow to reach the golden rule. We will now show that Proposition 1 does not hold anymore when longevity depends on individual choices, even if the annuity returns can be made contingent to those choices.

3. Competitive equilibria with endogenous longevity

The representative agent's problem under perfect competition and endogenous longevity is

$$\begin{aligned} \max_{0\leqslant c_0^t, c_1^t, k^t, M^t, e^t} \quad & u(c_0^t) + \pi(e^t)v(c_1^t) \\ \text{s.t.} \quad & c_0^t + k^t + \tfrac{1}{p_t}M^t + e^t \leqslant w_t \\ & c_1^t \leqslant \tfrac{1}{\pi(e^t)} r_{t+1}k^t + \tfrac{1}{p_{t+1}}M^t \end{aligned} \tag{17}$$

given w_t, r_{t+1}, p_t, and p_{t+1}. Thus, the individual has not only to decide how much to save—as well as how to allocate his savings between capital and money—but also the amount e^t of his income to put aside so as to increase his life expectancy, taking into account its impact on the return of annuities.

Indeed, in the second-period budget constraint, the representative agent is aware that the annuity return contractually depends on his choice of e^t, according to the assumption that pension funds can make the individual annuity contract contingent to his longevity-enhancing observable choices. Note also that Assumptions 1 and 2 guarantee that $c_0^t, c_1^t, e^t > 0$ as well as strictly positive savings, which implies that either $k^t > 0$ or $M^t > 0$ (or both, as long as the two assets have the same return). For the same reason as before, we focus nonetheless on the characterization of interior competitive equilibria such that $k^t > 0$ and $M^t > 0$ for all t.[18] The agent's choice in such equilibria is then necessarily characterized by the first-order conditions[19] or, equivalently,

$$\begin{aligned} & \frac{1}{\pi(e^t)} \frac{u'(c_0^t)}{v'(c_1^t)} = \frac{p_t}{p_{t+1}} = \frac{1}{\pi(e^t)} r_{t+1} \\ & c_0^t + k^t + \frac{1}{p_t}M^t + e^t = w_t \\ & c_1^t = \frac{1}{\pi(e^t)} r_{t+1}k^t + \frac{1}{p_{t+1}}M^t \\ & \pi'(e^t)v(c_1^t) = \left[1 + \frac{\pi'(e^t)}{\pi(e^t)}k^t\right] v'(c_1^t) r_{t+1}. \end{aligned} \tag{18}$$

The first equality in the first line equates the marginal rate of substitution between first- and second-period consumptions to the rate at which income can be transferred from the first to the second period of life. It determines thus the agent's optimal level of savings. The second equality in the first line requires the absence of arbitrage between the two saving instruments (money and annuities) for the agent to be willing to hold them both. Note that it depends on health expenditures e_t. The second and third lines are the agent's budget constraints. The last equation states that the agent's optimal level of longevity-enhancing expenditure e^t should be such that the marginal gain $\pi'(e^t)v(c_1^t)$ of increasing it equalizes its marginal cost, which is twofold: the part following from a smaller first-period or second-period consumption, $u'(c_0^t) = v'(c_1^t)r_{t+1}$, and the part following from a reduced return from annuitized savings, that is to say $\pi(e^t)v'(c_1^t)\frac{\pi'(e^t)}{\pi(e^t)^2} r_{t+1}k^t$.

Using the equilibrium factor prices in (8), we obtain from the addition of the budget constraints of the agents alive at any given period t,

[18] See Footnote 17.

[19] The first-order conditions are not sufficient (although the objective function is quasi-concave under Assumption 3, the constrained set is not necessarily convex). Nonetheless, they are still necessary, so that for the purpose of showing that the golden rule is not a market outcome (see Proposition 2) not being a sufficient condition is not essential.

$$c_0^t + \frac{\pi(e^{t-1})}{n} c_1^{t-1} + k^t + \frac{1}{p_t} M^t + e^t = F_L\left(\frac{k^{t-1}}{n}, 1\right) + F_K\left(\frac{k^{t-1}}{n}, 1\right)\frac{k^{t-1}}{n} + \frac{\pi(e^{t-1})}{n}\frac{1}{p_t} M^{t-1} \tag{19}$$

so that the feasibility of the allocation is equivalent to

$$\frac{M^t}{M^{t+1}} = \frac{n}{\pi(e^t)} \tag{20}$$

which at the steady state implies also

$$\frac{p_t}{p_{t+1}} = \frac{n}{\pi(e)}. \tag{21}$$

The equilibrium return to money now depends, indirectly, on the equilibrium choice of effort.[20] The link between effort and the return to money is however not internalized by the agents: while each agent's annuities return is contractually contingent to his own individual effort (which he duly takes into account), the return to money is the result of demographics and hence, depends on the average effort (on which each agent has a negligible impact and, therefore, duly disregards). Of course, given our representative agent model, average and individual efforts coincide, but they affect differently the agents' behavior.

A competitive equilibrium steady state consists, therefore, of a profile $(\tilde{c}_0, \tilde{c}_1, \tilde{e}, \tilde{k}, \tilde{m})$ satisfying

$$\begin{aligned}
&\frac{u'(\tilde{c}_0)}{v'(\tilde{c}_1)} = n = F_K\left(\frac{\tilde{k}}{n}, 1\right)\\
&\tilde{c}_0 + \tilde{k} + \tilde{m} + \tilde{e} = F_L\left(\frac{\tilde{k}}{n}, 1\right)\\
&\frac{\pi(\tilde{e})}{n}\tilde{c}_1 = F_K\left(\frac{\tilde{k}}{n}, 1\right)\frac{\tilde{k}}{n} + \tilde{m}\\
&\pi'(\tilde{e})v(\tilde{c}_1) = \left[1 + \frac{\pi'(\tilde{e})}{\pi(\tilde{e})}\tilde{k}\right] v'(\tilde{c}_1)n,
\end{aligned} \tag{22}$$

which we compare to the golden rule steady state in the next sections.

4. The golden rule with endogenous longevity

For an economy with large enough generations the golden rule is characterized by the unique solution to the problem maximizing the steady state expected utility of the representative agent under the feasibility constraint (written next in per young terms)[21]

$$\begin{aligned}
\max_{0 \leqslant c_0, c_1, k, e} \quad & u(c_0) + \pi(e)v(c_1)\\
\text{s.t.} \quad & c_0 + \tfrac{\pi(e)}{n} c_1 + k + e \leqslant F\left(\tfrac{k}{n}, 1\right)
\end{aligned} \tag{23}$$

The resource constraint above differs from the exogenous survival probability case in that we now need to take into account that longevity can be increased devoting resources to health expenditures e. Accordingly, the fraction of survivors depends on the investment in longevity-enhancing expenditures.

The solution to the optimization problem above is characterized by the first-order conditions,[22] or equivalently

[20] If the monetary savings of the deceased were distributed among the survivors, the second period budget constraint faced by the representative agent would be

$$c_1^t \leqslant \frac{r_{t+1}}{\pi(e^t)} k^t + \frac{1}{\pi(e^t)} \frac{M^t}{p_{t+1}}$$

which implies at equilibrium (for the allocation of resources to be feasible) that $\frac{M^t}{M^{t+1}} = n$, instead of $\frac{M^t}{M^{t+1}} = \frac{n}{\pi(e^t)}$, so that at a competitive equilibrium steady state prices $\frac{p_t}{p_{t+1}}$ should evolve according to n as well, instead of $\frac{n}{\pi(e^t)}$. Nonetheless the competitive equilibrium steady state allocation would still be given by the system (22), as long as the representative agent is not able to internalize the impact the survival probability on the actual return to money, which is not n but $n/\pi(e)$. If he was able to do so, then the competitive equilibrium steady state would implement the golden rule but, as discussed above, he is prevented from doing so by (i) the impossibility to annuitize the returns of money holdings and bank accounts balances (not to mention to make them contingent to longevity-related choices), and (ii) each individual's negligible impact on the survival rate.

[21] The steady state expected utility of the representative agent is the right welfare criterion in this setup. In particular, since population grows each period *exogenously* by a factor n (reproduction is not a choice variable for the agents), welfare criteria used in endogenous fertility setups—see Golosov et al. (2007)—are of no application here.

[22] It should be noted that, although the objective function in (23) is nicely quasi-concave under Assumption 3, the constrained set is not necessarily convex, so that the planner's problem fails to be convex. Nevertheless, the existence (from the compactness of the constrained set) and interiority (by Assumptions 1 and 2) of the solution to (23) makes possible to characterize it as the only solution (see Proposition A2 in the appendix) to the first-order conditions in (24).

$$\begin{aligned}
&\frac{u'(c_0^*)}{v'(c_1^*)} = n = F_K\left(\frac{k^*}{n}, 1\right)\\
&c_0^* + \frac{\pi(e^*)}{n}c_1^* + k^* + e^* = F\left(\frac{k^*}{n}, 1\right)\\
&\pi'(e^*)v(c_1^*) = \left[1 + \frac{\pi'(e^*)}{n}c_1^*\right]v'(c_1^*)n
\end{aligned} \tag{24}$$

The first line follows on the one hand, from equating the marginal rate of substitution $\frac{1}{\pi(e^*)}\frac{u'(c_0^*)}{v'(c_1^*)}$ to the rate at which consumption can be transferred between contemporaneous young and old agents $\frac{n}{\pi(e^*)}$, and, on the other hand, from the condition for the net output maximization which leads to equating the marginal productivity of capital to the growth factor of the population. The second line is the feasibility of the allocation of resources so that output net of capital replacement must equal at any period the consumption of the young and of the contemporaneous surviving old agents plus the longevity-enhancing expenditures. Finally, the third line requires that, at the golden rule, the marginal benefit of increasing the life-expectancy, $\pi'(e)v(c_1)$, exactly matches its marginal cost, which consists of (i) the direct impact that an increase in health expenditures has on first-period consumption or second-period consumption, $u'(c_0) = v'(c_1)n$, and (ii) the indirect cost in terms of the additional pressure put on resources following from bigger cohorts of survivors, $\pi'(e)v'(c_1)c_1$.

Note that the last equation in the system (24) differs from the last equation in the competitive system (22): the factor $\frac{\tilde{k}}{\pi(\tilde{e})}$ within brackets in the competitive equilibrium steady state is now replaced by $\frac{c_1^*}{n}$ in the golden rule. As a consequence, in the absence of intervention, no competitive equilibrium steady state solution to Eqs. (22) is the golden rule (except in the knife-edge case in which the equilibrium demand for money happens to be zero) as established in the following proposition.

Proposition 2. *Consider a Diamond (1965) overlapping generations production economy with money and an exogenously growing population with a representative agent who can increase his survival probability at a real cost and can save in actuarially fair annuities contractually contingent to such choices. Under laissez-faire, no competitive equilibrium steady state of the economy with a positive demand for money implements the golden rule steady state.*

Proof. There is no need to consider possible non-interior competitive equilibrium steady states, since they are Pareto-dominated by any (interior) solution to (22) and therefore could not coincide with the golden rule.[23] Thus, letting $(\tilde{c}_0, \tilde{c}_1, \tilde{k}, \tilde{m}, \tilde{e})$ be a competitive equilibrium steady state solution to (22) and (c_0^*, c_1^*, k^*, e^*) be the golden rule—i.e. the solution to (24)-– it follows trivially from the last equation in each of the systems that, should the two steady states coincide, then necessarily

$$\frac{1}{\pi(\tilde{e})}\tilde{k} = \frac{1}{n}\tilde{c}_1 \tag{25}$$

which from the second-period budget constraint in (22) implies $\tilde{m} = 0$. In other words, no competitive equilibrium steady state with $\tilde{m} \neq 0$ implements the golden rule. □

Therefore, in the absence of intervention, the demand for money should be zero for a competitive equilibrium steady state to implement the golden rule. Nevertheless, typically money holdings will be nonzero at equilibrium, since money is what allows the agents to disentangle the optimal level of capital from the optimal level of savings (which needs not be the one that, if held as capital, maximizes net output).[24] Accordingly, Section 5, provides a policy that decentralizes the golden rule with positive money demand.[25]

Nonetheless, before moving to the decentralization of the golden rule, note that Proposition 2 does *not* say that, without intervention, the presence of money prevents attaining the golden rule, but rather that only if money is not demanded (even if present in the model) at a competitive equilibrium steady state can the latter decentralize the golden rule. To make this point clearer, Proposition 3 shows that *in the absence of money* the competitive steady state and the best steady state generically differ.

Proposition 3. *Consider a Diamond (1965) overlapping generations production economy* **without money** *and an exogenously growing population with a representative agent who can increase his survival probability at a real cost and can save in actuarially fair annuities contractually contingent to such choices. Then no competitive equilibrium steady state of the economy implements the golden rule.*

[23] See Footnote 17.

[24] In general, without the possibility of such a separation there is no hope of attaining the golden rule, as in the original Diamond (1965) setup.

[25] Alternatively, if also the return to money could be made contingent to the longevity choices, then the golden rule could be attained as a competitive outcome without intervention but, as discussed above, the impossibility to contractually annuitize the returns to cash and bank accounts and the negligible impact of each individual on the survival rate prevent this possibility.

Proof. If money is removed from the model it is straightforward to see that a competitive steady state (c_0, c_1, k, e) coincides with the best steady state only if

$$n = F_K\left(\frac{k}{n}, 1\right) \tag{26}$$

is satisfied, i.e. only if the competitive steady state return to capital equals the population growth rate, but nothing in the competitive steady state equations guarantees that this is the case. As a matter of fact, typically this will *not* be the case: a competitive steady state is characterized by four variables (c_0, c_1, k, e) satisfying the following four equations

$$\begin{aligned} &\frac{u'(c_0)}{v'(c_1)} = F_K\left(\frac{k}{n}, 1\right) \\ &c_0 + k + e = F_L\left(\frac{k}{n}, 1\right) \\ &\frac{\pi(e)}{n} c_1 = F_K\left(\frac{k}{n}, 1\right)\frac{k}{n} \\ &\pi'(e)v(c_1) = \left[1 + \frac{\pi'(e)}{\pi(e)}k\right]v'(c_1)n. \end{aligned} \tag{27}$$

Condition (26) will therefore not be satisfied by a competitive steady state, generically in the space of the fundamentals of the economy (preferences and technology). □

Given that the competitive equilibrium steady state typically differs from the golden rule, how do they compare? Does the market lead to over-saving or under-saving? Does it lead to over- or under-investment in life-enhancing expenditures? As the next proposition shows, the level of capital (and hence of net output) at the golden rule and at the competitive equilibrium steady state actually coincide since in both cases, the marginal productivity of capital is equalized to the rate of growth of the population. However, the life-expectancy and hence the size of the population are different: at a monetary competitive equilibrium steady state, people live either longer but poorer lives, or shorter but richer lives, than those corresponding to the best trade-off between "quantity" and "quality" of life.

Proposition 4. *Consider a Diamond (1965) overlapping generations production economy with money and an exogenously growing population with a representative agent who can increase his survival probability at a real cost and can save in actuarially fair annuities contractually contingent to such choices. At every competitive equilibrium steady state* $(\tilde{c}_0, \tilde{c}_1, \tilde{k}, \tilde{m}, \tilde{e})$ *the agent consumes either too little (in both periods) and devotes too much effort to increase his life-expectancy compared to the golden rule* (c_0^*, c_1^*, k^*, e^*)*, or consumes too much (in both periods) and devotes too little effort. More specifically,*

$$\text{either} \quad \begin{aligned} &c_0^* < \tilde{c}_0 \\ &c_1^* < \tilde{c}_1 \\ &k^* = \tilde{k} \\ &e^* > \tilde{e} \end{aligned} \quad \text{or} \quad \begin{aligned} &c_0^* > \tilde{c}_0 \\ &c_1^* > \tilde{c}_1 \\ &k^* = \tilde{k} \\ &e^* < \tilde{e} \end{aligned} \tag{28}$$

Proof. See Appendix A. □

The above proposition thus establishes that, in the overlapping generations setup considered here, there may be over- as well as under-investment in longevity, even with a perfectly competitive market of annuities contingent to the individuals' longevity-enhancing behavior.[26] Whether agents under- or over-invest in their longevity at the steady state competitive equilibrium depends on their relative preferences for consumption and longevity and thus on the concavity of the functions $u(\cdot)$, $v(\cdot)$ and $\pi(\cdot)$. In any case, a golden rule investment in longevity that is higher (resp. lower) than at the competitive equilibrium leads to less (resp. more) consumption in *both* periods.

Having shown that, without intervention, no competitive equilibrium steady state can be the golden rule, we show, in the following section, how to decentralize the latter as a competitive outcome under the adequate fiscal policy.

5. Policy implementing the golden rule steady state as a competitive equilibrium steady state

Assume the government taxes longevity-enhancing expenditures e^t of agents born at time t at a rate σ^t, while it hands the proceeds of the tax at $t+1$ through a lump-sum transfer T^t. The problem of an agent born at t becomes

$$\begin{aligned} &\max_{0 \leqslant c_0^t, c_1^t, k^t, e^t, M^t} && u(c_0^t) + \pi(e^t)v(c_1^t) \\ &\text{s.t.} && c_0^t + k^t + \tfrac{1}{p_t}M^t + (1+\sigma^t)e^t \leqslant w_t \\ & && c_1^t \leqslant \tfrac{1}{\pi(e^t)} r_{t+1}k^t + \tfrac{1}{p_{t+1}}M^t + T^t \end{aligned} \tag{29}$$

[26] Philipson and Becker (1998), as a result of addressing this issue in an essentially static setup, find that agents always have incentives to over-invest in their longevity.

Note that Assumptions 1 and 2 guarantee again that $c_0^t, c_1^t, e^t > 0$ but not necessarily strictly positive savings, i.e. $k^t > 0$ or $M^t > 0$ (or both, as long as the two assets have the same return), nonetheless we can focus on interior competitive equilibria such that $k^t, M^t > 0$. [27] The agent's choice in such equilibria is then necessarily characterized by the first-order conditions or, equivalently, by

$$\begin{aligned}
&\frac{1}{\pi(e^t)}\frac{u'(c_0^t)}{v'(c_1^t)} = \frac{p_t}{p_{t+1}} = \frac{1}{\pi(e^t)} r_{t+1}\\
&c_0^t + k^t + \frac{1}{p_t}M^t + (1+\sigma^t)e^t = w_t\\
&c_1^t = \frac{1}{\pi(e^t)} r_{t+1}k^t + \frac{1}{p_{t+1}}M^t + T^t\\
&\pi'(e^t)v(c_1^t) = \left[(1+\sigma^t) + \frac{\pi'(e^t)}{\pi(e^t)}k^t\right] v'(c_1^t) r_{t+1}
\end{aligned} \tag{30}$$

At a competitive equilibrium, the conditions requiring the wage and rental rate of capital to be the marginal productivities still hold. We require also that the government runs a balanced budget at every period so that, in every period t,

$$\sigma^t e^t = \frac{\pi(e^{t-1})}{n} T^{t-1} \tag{31}$$

holds, where the amount raised by taxes on health expenditures on the left-hand side matches at every period the lump-sum amount handed out to the survivors of the previous generation, on the right-hand side. Finally, adding up the budget constraints of the agents alive at any given period,

$$c_0^t + \frac{\pi(e^{t-1})}{n} c_1^{t-1} + k^t + \frac{1}{p_t}M^t + (1+\sigma^t)e^t = F_L\left(\frac{k^{t-1}}{n}, 1\right) + F_K\left(\frac{k^{t-1}}{n}, 1\right)\frac{k^{t-1}}{n} + \frac{\pi(e^{t-1})}{n}\frac{1}{p_t}M^{t-1} + \frac{\pi(e^{t-1})}{n}T^{t-1} \tag{32}$$

It follows that the feasibility condition is again equivalent to (20) which implies (21) at the steady state.

Therefore, the competitive equilibrium steady state is characterized now by a profile (c_0, c_1, e, k, m) satisfying

$$\begin{aligned}
&\frac{u'(c_0)}{v'(c_1)} = n = F_K\left(\frac{k}{n}, 1\right)\\
&c_0 + k + m + (1+\sigma)e = F_L\left(\frac{k}{n}, 1\right)\\
&c_1 = \frac{1}{\pi(e)} F_K\left(\frac{k}{n}, 1\right)k + \frac{n}{\pi(e)}m + T\\
&\pi'(e)v(c_1) = \left[(1+\sigma) + \frac{\pi'(e)}{\pi(e)}k\right] v'(c_1)n
\end{aligned} \tag{33}$$

with

$$\sigma e = \frac{\pi(e)}{n} T \tag{34}$$

for the government budget to be balanced. Comparing these conditions with those characterizing the golden rule in (24), it is straightforward that the golden rule is a competitive equilibrium steady state under the tax and transfer policy defined above if, and only if, the tax rate is

$$\sigma = \pi'(e^*)\left[\frac{c_1^*}{n} - \frac{k^*}{\pi(e^*)}\right] \tag{35}$$

where c_1^*, k^*, and e^* are the golden rule values. In particular, if the golden rule second-period consumption exceeds the return from annuities, i.e.

$$c_1^* > \frac{n}{\pi(e^*)} k^* = \frac{1}{\pi(e^*)} F_K\left(\frac{k^*}{n}, 1\right) k^* \tag{36}$$

then it can be decentralized as a competitive equilibrium taxing first-period longevity-enhancing expenditures and making positive second-period transfers, and conversely in the opposite case.

The intuition is the following. Whenever the golden rule second-period consumption level exceeds the return from capital—which, from Proposition 4, is equal to the competitive equilibrium steady state level of capital—it is because the return from annuities is too low. This means that at the competitive equilibrium steady state the agent invests too much in his

[27] The same comment as in Footnote 17 applies here. Since we focus on the steady state, any steady state with $k^t = 0$ or $M^t = 0$ is Pareto-dominated by any interior steady state.

longevity. The only way to reduce $\tilde{e}$ toward e^* consists then in taxing longevity-enhancing expenditures. A symmetric reasoning applies when the golden rule consumption is smaller than the return from capital and health expenditures need to be subsidized.

Specifically, in order to implement the golden rule, it suffices to announce at the beginning of each period t (i) a longevity-enhancing expenditure tax (or subsidy, depending on its sign) rate equal to

$$\sigma^t = \pi'(e^{t-1})\left[\frac{c_1^{t-1}}{n} - \frac{k^{t-1}}{\pi(e^{t-1})}\right] \tag{37}$$

and (ii) a lump-sum transfer to old agents at $t+1$ of an amount[28]

$$T^t = \frac{n}{\pi(e^{t-1})} e^{t-1} \sigma^t. \tag{38}$$

Substituting these two expressions into conditions (30) characterizing now the competitive equilibrium, it is straightforward to check that, at the steady state, the government budget is balanced in every period and that the conditions coincide with those of the golden rule in (24). Therefore, such a tax-and-transfer scheme decentralizes the golden rule. This result is summarized in the next proposition.

Proposition 5. *Consider a Diamond (1965) overlapping generations production economy with money and an exogenously growing population with a representative agent who can increase his survival probability at a real cost and can save in actuarially fair annuities contractually contingent to such choices. The golden rule steady state is a competitive equilibrium outcome if each generation t faces a tax (or a subsidy) on its longevity-enhancing expenditures, whose expression is given by Eq. (37) and receives (or is subject to) a second-period lump-sum transfer (or tax) in $t+1$ of an amount defined by Eq. (38).*

In case it seems counterintuitive that taxing longevity-enhancing expenditures may ever be welfare-improving, let us consider, in order to assess the true impact of this fiscal policy, the *expected* per capita net taxes paid by any given generation t, i.e. $\tau^t = \sigma^t e^t - \pi(e^t)T^t$. First, note that the transfer T^t is conditional on the individual's survival, while the contribution $\sigma^t e^t$ is paid in first period, with certainty. Second, substituting the expressions for σ^t and T^t, it follows that

$$\tau^t = \pi'(e^{t-1})\left[c_1^{t-1} - \frac{n}{\pi(e^{t-1})}k^{t-1}\right]\left[\frac{e^t}{n} - \frac{\pi(e^t)e^{t-1}}{\pi(e^{t-1})}\right] \tag{39}$$

which at the steady state becomes

$$\tau = \pi'(e)\left[c_1 - \frac{n}{\pi(e)}k\right]\left[\frac{1}{n} - 1\right]e. \tag{40}$$

When population growth is positive, i.e. $n > 1$, the second term in brackets is negative and the net contribution is different from zero as soon as second-period consumption differs from the return from annuities. Specifically, if the golden rule second-period consumption exceeds the annuitized return to capital—so that the implementation of the golden rule requires *taxing* health expenditures—then the expected net contribution of the representative agent is actually *negative*. In this case, he pays a tax in the first period that is more than offset (in expectation) by a second-period lump-sum transfer that is made significantly big thanks to the taxes paid by the next generation of a growing population. On the contrary, if the golden rule second-period consumption is smaller than the annuitized return to capital—so that the implementation of the golden rule requires *subsidizing* instead health expenditures—then the expected net contribution of the agent is actually *positive*, as he receives a subsidy when young that he has to pay back through a lump-sum tax when old to subsidize the health expenditures of a growing population.[29] This result is summarized in the next proposition.

Proposition 6. *Consider a Diamond (1965) overlapping generations production economy with money and an exogenously growing population with a representative agent who can increase his survival probability at a real cost and can save in actuarially fair annuities contractually contingent to such choices. When the golden rule steady state is implemented through the taxation (resp. subsidization) of longevity-enhancing expenditures, the agents expected net contribution is actually negative (resp. positive).*

Finally, since at the heart of the problem there is the ability of the planner (but not of the agents) to internalize the externality of longevity-enhancing expenditures on the return to money in Eq. (21), it may seem that a policy that manipulates the return to money could also succeed in implementing the golden rule steady state, without having to go through a Pigouvian tax. This is unfortunately not the case, since not only distorting the return to money is needed, but also distorting the marginal cost of increasing longevity-enhancing expenditures, which the above Pigouvian tax achieves—through the presence of σ^t in the first-order condition with respect to e^t in (30). To the opposite, a distortion in the return to money would only show up in the first-order condition with respect to M^t, with no impact on the marginal condition on e^t and hence on its level.

28 Note that these expressions depend only on past variables so that they cannot be manipulated by individuals born in period t.

29 In the absence of population growth (i.e. $n = 1$), the net contribution of the individual *in the steady state* is $\tau = 0$.

6. Discussion

Before concluding, let us consider to other extensions. First, we assume that agents can influence their life-expectancy by means of efforts that are no longer costly in terms of resources but in terms of utility. Second, we assume that agents also have the possibility to supply labor in the second period of their life and that the amount supplied depends on the health expenditures made in the first period. Third, we assume that agents may live more than just two periods and consider a model akin to a "perpetual youth model" à la Blanchard (1985) but with endogenous longevity. In the following, we show how these alternative assumptions may modify the results obtained in the previous sections.

6.1. Efforts in terms of utility but not of resources

Individuals can certainly influence their life expectancy not only through health expenditures but also through behavioral choices (such as a constraining way of life, healthy diets, no smoking, and exercising) that may not have an impact on their budget constraints, but have nonetheless an impact on the agent's utility. One may wonder whether our previous results still hold and, whether something can be done about it, given that there is no use of resources (i.e. health spending) to tax now.

In essence, all our previous results carry over. Specifically, the golden rule is not a competitive equilibrium outcome, but it can still be made to become one.

Defining γe as the utility cost of longevity-enhancing behavior and γ being the intensity of effort disutility, it is straightforward to check that the golden rule solution to

$$\begin{aligned} \max_{0\leqslant c_0,c_1,k,e} \quad & u(c_0)+\pi(e)v(c_1)-\gamma e \\ \text{s.t.} \quad & c_0+\tfrac{\pi(e)}{n}c_1+k\leqslant F\left(\tfrac{k}{n},1\right) \end{aligned} \tag{41}$$

is now characterized by

$$\begin{aligned} & \frac{u'(c_0^*)}{v'(c_1^*)}=n=F_K\left(\frac{k^*}{n},1\right) \\ & c_0^*+\frac{\pi(e^*)}{n}c_1^*+k^*=F\left(\frac{k^*}{n},1\right) \\ & \pi'(e)v(c_1^*)=\gamma+\pi'(e^*)v'(c_1^*)c_1^* \end{aligned} \tag{42}$$

It is easy to check that it is not a competitive equilibrium steady state using the same arguments as before.[30] Its decentralization is nevertheless less straightforward, since the standard Pigouvian tax which was used in the simple case of health expenditure does not work here. Indeed, the healthy behaviors we consider in this setting do not translate into expenditures that enter the agents' budget constraints and that could be taxed directly. Moreover, taxing them indirectly, by taxing, for example, saving returns (held in either capital or money) in order to worsen the prospect of a high life-expectancy, would at the same time distort the consumption-saving decision, making it impossible to attain the golden rule.[31] Therefore, an alternative intervention is needed.

Consider instead the following policy. Announce to the agent born at t that, in the second period $t+1$, a lump-sum tax or subsidy of an amount $T^t=\frac{1}{p_t}M^{t-1}\ln\frac{\pi(e^t)}{\pi(e^{t-1})}$ will be raised from or transferred to him, depending on its sign. Note that while M^{t-1} and $\pi(e^{t-1})$—the survival rate at t—are known at time t, the effort e^t in the survival probability $\pi(e^t)$ of an agent born at time t is not known at the time the announcement is made. Nonetheless, it will be known by the government at the time the policy is implemented in $t+1$. This tax can thus be manipulated by the individual, so that choosing a given level of effort in t influences the level of tax or subsidy he faces in period $t+1$.

The representative agent's problem then becomes

$$\begin{aligned} \max_{0\leqslant c_0^t,c_1^t,k_1^t,e^t,M^t} \quad & u(c_0^t)+\pi(e^t)v(c_1^t)-\gamma e^t \\ \text{s.t.} \quad & c_0^t+k^t+\tfrac{1}{p_t}M^t\leqslant w_t \\ & c_1^t\leqslant \tfrac{1}{\pi(e^t)}r_{t+1}k^t+\tfrac{1}{p_{t+1}}M^t-\tfrac{1}{p_t}M^{t-1}\ln\tfrac{\pi(e^t)}{\pi(e^{t-1})} \end{aligned} \tag{43}$$

and it is not difficult to check that the resulting competitive equilibrium steady state equations are exactly those of the golden rule in (42).

[30] The competitive equilibrium steady state condition for effort is in this case

$$\pi'(\tilde{e})v(\tilde{c}_1)=\gamma+\frac{\pi'(\tilde{e})}{\pi(\tilde{e})}v'(\tilde{c}_1)\tilde{k}n.$$

[31] This works instead in a static partial equilibrium framework, as in Leroux (2011), where the best allocation is restored through a tax on savings or, equivalently, on second-period consumption. In this case, the individual has less incentives to invest in a higher life expectancy as his second-period consumption is lower.

Note that the value of $T^t = \frac{1}{p_t} M^{t-1} \ln \frac{\pi(e^t)}{\pi(e^{t-1})}$ is zero at the steady state, so that no tax or subsidy is actually raised or handed out when the golden rule is implemented, keeping the government budget trivially balanced. As a matter of fact, the mere announcement of such a policy makes the agents modify their choices in such a way that the golden rule is attained in a decentralized way. Individuals are then given the opportunity to manipulate the tax or the subsidy in their interest and, in order to do that, they will change their behavior. Interestingly enough, it turns out that, trying to manipulate the tax, it is actually them who are manipulated to implement the golden rule.

6.2. Extension to second period endogenous labor supply

It is worth noticing that on top of the effect of health expenditures on the agents' survival probability, the model can easily be extended so that health expenditures also have a direct productivity effect along the lines of Grossman (1972). It suffices to assume that agents work in both periods, supplying inelastically one unit of labor when young (as before), while supplying an amount of labor when old that effectively depends on their first-period health expenditures.

Specifically, the agent born at time t now solves

$$\begin{aligned}
&\max_{0 \leqslant c_0^t, c_1^t, k^t, M^t, e^t} && u(c_0^t) + \pi(e^t) v(c_1^t) \\
& && c_0^t + k^t + \tfrac{1}{p_t} M^t + e^t \leqslant w_t \\
& && c_1^t \leqslant \tfrac{1}{\pi(e^t)} r_{t+1} k^t + \tfrac{1}{p_{t+1}} M^t + w_{t+1} l(e^t)
\end{aligned} \tag{44}$$

where $l(e^t)$ in the second budget constraint is his second period labor supply, satisfying $l(e^t) \in [0,1], l''(e^t) < 0 < l'(e^t)$ for all $e^t > 0, \lim_{e^t \to 0^+} l'(e^t) = +\infty$, and $\lim_{e^t \to +\infty} l(e^t) = 1$. [32]

Accordingly, firms employ at equilibrium an amount of labor equal to $1 + \frac{\pi(e^t)}{n} l(e^t)$ to produce at $t+1$, so that factors are remunerated at rates

$$\begin{aligned}
w_t &= F_L\left(\frac{k^{t-1}}{n}, 1 + \frac{\pi(e^{t-1})}{n} l(e^{t-1})\right) \\
r_{t+1} &= F_K\left(\frac{k^t}{n}, 1 + \frac{\pi(e^t)}{n} l(e^t)\right)
\end{aligned} \tag{45}$$

The agent's choice in such equilibria is then necessarily characterized by

$$\begin{aligned}
&\frac{1}{\pi(e^t)} \frac{u'(c_0^t)}{v'(c_1^t)} = \frac{p_t}{p_{t+1}} = \frac{1}{\pi(e^t)} r_{t+1} \\
&c_0^t + k^t + \frac{1}{p_t} M^t + e^t = w_t \\
&c_1^t = \frac{1}{\pi(e^t)} r_{t+1} k^t + \frac{1}{p_{t+1}} M^t + w_{t+1} l(e^t) \\
&\pi'(e^t) v(c_1^t) = \left[1 + \frac{\pi'(e^t)}{\pi(e^t)} k^t\right] v'(c_1^t) r_{t+1} - \pi(e^t) w_{t+1} l'(e^t) v'(c_1^t).
\end{aligned} \tag{46}$$

Only the third and fourth lines—the second period budget constraint and the optimality condition for e, respectively—differ from those of the case without second period labor supply. In the fourth line, $\pi(e^t) w_{t+1} l'(e^t) v'(c_1^t)$ represents the marginal second-period utility benefit of an increased labor supply, through the increase of health expenditure.

The feasibility of the allocation is still equivalent to Eq. (20)—i.e. (21) at the steady state—so that a monetary competitive equilibrium steady state consists of a profile satisfying

$$\begin{aligned}
&\frac{u'(\tilde{c}_0)}{v'(\tilde{c}_1)} = n = F_K\left(\frac{\tilde{k}}{n}, 1 + \frac{\pi(\tilde{e})}{n} l(\tilde{e})\right) \\
&\tilde{c}_0 + \tilde{k} + \tilde{m} + \tilde{e} = F_L\left(\frac{\tilde{k}}{n}, 1 + \frac{\pi(\tilde{e})}{n} l(\tilde{e})\right) \\
&\frac{\pi(\tilde{e})}{n} c_1 = F_K\left(\frac{\tilde{k}}{n}, 1 + \frac{\pi(\tilde{e})}{n} l(\tilde{e})\right) \frac{\tilde{k}}{n} + \tilde{m} + F_L\left(\frac{\tilde{k}}{n}, 1 + \frac{\pi(\tilde{e})}{n} l(\tilde{e})\right) \frac{\pi(\tilde{e}) l(\tilde{e})}{n} \\
&\pi'(\tilde{e}) v(\tilde{c}_1) = \left[1 + \frac{\pi'(\tilde{e})}{\pi(\tilde{e})} \tilde{k} - F_L\left(\frac{\tilde{k}}{n}, 1 + \frac{\pi(\tilde{e})}{n} l(\tilde{e})\right) \frac{\pi(\tilde{e})}{n} l'(\tilde{e})\right] v'(\tilde{c}_1) n
\end{aligned} \tag{47}$$

[32] Note that second period work duration cannot exceed second period life duration, i.e. $l(e^t) \leqslant 1$. As before (see Footnote 19), the constrained set is not guaranteed to be convex but, as in the previous case, the necessity of the FOC's suffices to establish the main result, namely that the planner's steady state is not a laissez-faire competitive equilibrium steady state.

Note that, in comparison to the competitive equilibrium steady state with no second period labor supply (see Eq. (20)), there is an additional term within the brackets on the right-hand side of the last equation. This term directly follows from the endogeneity of second-period labor supply: a marginal increase in health expenditures e^t in the first period now leads to a marginal increase in second-period income through increased labor supply.

On the other hand, the golden rule is again characterized by the unique solution to the problem maximizing the steady state expected utility of the representative agent under the feasibility constraint (written next in per young terms)

$$\begin{aligned} \max_{0\leqslant c_0,c_1,k,e} \quad & u(c_0)+\pi(e)v(c_1) \\ \text{s.t.} \quad & c_0+\tfrac{\pi(e)}{n}c_1+k+e\leqslant F\left(\tfrac{k}{n},1+\tfrac{\pi(e)}{n}l(e)\right) \end{aligned} \tag{48}$$

The solution to this problem is characterized by the equations[33]

$$\begin{aligned} & \frac{u'(c_0^*)}{v'(c_1^*)}=n=F_K\left(\frac{k^*}{n},1+\frac{\pi(e^*)}{n}l(e^*)\right) \\ & c_0^*+\frac{\pi(e^*)}{n}c_1^*+k^*+e^*=F\left(\frac{k^*}{n},1+\frac{\pi(e^*)}{n}l(e^*)\right) \\ & \pi'(e^*)v(c_1^*)=\left[1+\frac{\pi'(e^*)}{n}c_1^*-F_L\left(\frac{k^*}{n},1+\frac{\pi(e^*)}{n}l(e^*)\right)\left(\frac{\pi(e^*)}{n}l'(e^*)+\frac{\pi'(e^*)}{n}l(e^*)\right)\right]v'(c_1^*)n \end{aligned} \tag{49}$$

Let us compare the competitive equilibrium steady state defined by (47) and the golden rule defined by (49) under endogenous second-period labor supply with our baseline case where second-period labor supply was nil. Under endogenous labor supply, we still obtain that the competitive equilibrium steady state and the golden rule levels of e differ since the factor $\frac{\tilde{k}}{\pi(\tilde{e})}$ within brackets in the competitive equilibrium steady state equations is now replaced by $\frac{c_1^*}{n}$ in the golden rule equations. However, unlike the zero second-period labor supply case, there is now at the golden rule, a term, $F_L\left(\frac{k^*}{n},1+\frac{\pi(e^*)}{n}l(e^*)\right)\pi'(e^*)l(e^*)v'(c_1^*)$, which is absent from the competitive equilibrium equation. This is a direct consequence from the fact that, at the competitive equilibrium, the agent does not take into account the impact of the increase in survival (equivalently the increase in number of survivors) on the return from labor.

Like in the case with no second-period labor supply, the tax-and-transfer scheme described in Section 5 allows to decentralize the golden rule as a competitive equilibrium (only the rates would differ). The agent's choice in such equilibria is then necessarily characterized by the first-order conditions or, equivalently, by

$$\begin{aligned} & \frac{1}{\pi(e^t)}\frac{u'(c_0^t)}{v'(c_1^t)}=\frac{p_t}{p_{t+1}}=\frac{1}{\pi(e^t)}r_{t+1} \\ & c_0^t+k^t+\frac{1}{p_t}M^t+(1+\sigma^t)e^t=w_t \\ & c_1^t=w_{t+1}l(e^t)+\frac{1}{\pi(e^t)}r_{t+1}k^t+\frac{1}{p_{t+1}}M^t+T^t \\ & \pi'(e^t)v(c_1^t)=\left[(1+\sigma^t)+\frac{\pi'(e^t)}{\pi(e^t)}k^t\right]v'(c_1^t)r_{t+1}-w_{t+1}\pi(e^t)l'(e^t)v'(c_1^t) \end{aligned} \tag{50}$$

In equilibrium, the wage and the rental rate of capital are still equal to marginal productivities. The government must also run a balanced budget at every period so that, in every period t, Eq. (31) still holds. Finally, the budget constraints of the agents alive at any given period, still imply the feasibility condition to be equal to (20), and hence (21) at the steady state. Therefore, the competitive equilibrium steady state is now characterized by a profile satisfying

$$\begin{aligned} & \frac{u'(c_0)}{v'(c_1)}=n=F_K\left(\frac{k}{n},1+\frac{\pi(e)}{n}l(e)\right) \\ & c_0+k+m+(1+\sigma)e=F_L\left(\frac{k}{n},1+\frac{\pi(e)}{n}l(e)\right) \\ & c_1=F_L\left(\frac{k}{n},1+\frac{\pi(e)}{n}l(e)\right)l(e)+\frac{1}{\pi(e)}F_K\left(\frac{k}{n},1+\frac{\pi(e)}{n}l(e)\right)k+\frac{n}{\pi(e)}m+T \\ & \pi'(e)v(c_1)=\left[(1+\sigma)+\frac{\pi'(e)}{\pi(e)}k-F_L\left(\frac{k}{n},1+\frac{\pi(e)}{n}l(e)\right)\pi(e)l'(e)\right]v'(c_1)n \end{aligned} \tag{51}$$

[33] As in the case with no second-period labor supply, the planner's problem is not convex (see Footnote 22). Nonetheless, the generic regularity of any solution to any system along an homotopy between the system (49) above with second period labor supply and the system without it in (24)—along with the uniqueness of the solution to the system in the latter case—guarantees the characterization of the planner's choice by Eqs. (49) above (see Proposition A3 in the appendix).

with

$$\sigma e = \frac{\pi(e)}{n} T. \tag{52}$$

Comparing these conditions with those characterizing the golden rule in (49), it follows that the golden rule is a competitive equilibrium steady state under the tax-and-transfer policy described above if, and only if, the tax rate is equal to

$$\sigma = \pi'(e^*)\left[\frac{c_1^*}{n} - \frac{k^*}{\pi(e^*)}\right] - F_L\left(\frac{k^*}{n}, 1 + \frac{\pi(e^*)}{n} l(e^*)\right)\frac{\pi'(e^*)}{n} l(e^*) \tag{53}$$

where c_1^*, k^*, and e^* are the golden rule values. In particular, if the golden rule second-period consumption exceeds second-period income from savings in capital *and from labor*—i.e. the return from annuities *plus second-period labor income*—it can then be decentralized as a competitive equilibrium taxing first-period longevity-enhancing expenditures and making second-period transfers, and conversely in the opposite case.

Note finally that the level of σ is here distorted downward as compared to our initial framework (Eq. (35)) since the second term on the right-hand side of (53) is negative. The tax rate is thus smaller because of the positive impact of the increase in the number of the survivors on the marginal productivity of labor (and thus, on the real wage).

6.3. A "perpetual youth" model with endogenous and varying survival probability

We finally consider an extension of the model that allows agents to survive *a priori* any number of periods, in the spirit of Blanchard (1985), but with an endogenous survival probability that depends on some health expenditures. Specifically, we assume that agents face at every period a survival probability $\pi(e^{t-1})$ that depends on the health expenditures they made in the previous period, e^{t-1}.

An agent born at t solves, therefore,

$$\begin{array}{ll} \max\limits_{c_j^t, k_j^t, m_j^t, e_j^t} & \sum\limits_{j=0}^{+\infty} \beta^j \pi(e_{j-1}^t) u(c_j^t) \\ \text{s.t.} & c_j^t + k_j^t + m_j^t + e_j^t \leqslant w_{t+j} + \frac{r_{t+j}}{\pi(e_{j-1}^t)} k_{j-1}^t + m_{j-1}^t \frac{p_{t+j-1}}{p_{t+j}} \quad \forall j = 0, 1 \ldots \end{array} \tag{54}$$

where t stands for the birth date of the agent and $j = 0, 1, \ldots$ for his age (so the c_j^t stands for the consumption at $t+j$ of an agent born at t) with $\pi(e_{-1}^t) = 1$ and $k_{-1}^t = 0 = m_{-1}^t$ trivially. The parameter β is the discount factor of the agent. The solution to the problem faced by an agent born at t is then characterized by the following first order conditions (with respect to $c_j^t, k_j^t, m_j^t, e_j^t$)

$$\begin{pmatrix} \beta^j \pi(e_{j-1}^t) u'(c_j^t) \\ 0 \\ 0 \\ \beta^{j+1} \pi'(e_j^t) u(c_{j+1}^t) \end{pmatrix} = \lambda_j^t \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \lambda_{j+1}^t \begin{pmatrix} 0 \\ -\frac{1}{\pi(e_j^t)} r_{t+j+1} \\ -\frac{p_{t+j}}{p_{t+j+1}} \\ \frac{\pi'(e_j^t)}{\pi(e_j^t)^2} r_{t+j+1} k_j^t \end{pmatrix} \tag{55}$$

for all $j = 0, 1, \ldots$, where λ_j^t and λ_{j+1}^t are the Lagrange multipliers associated with the budget constraints at ages j and $j+1$, along with the budget constraints themselves binding. Rearranging these terms, we obtain the following condition on effort:

$$\pi'(e_j^t) u(c_{j+1}^t) = \left[1 + \frac{\pi'(e_j^t)}{\pi(e_j^t)} k_j^t\right] u'(c_{j+1}^t) r_{t+j+1} \tag{56}$$

for all t integer and $j = 0, 1, \ldots$ with

$$r_t = F_K\left(\frac{K_t}{N_t}, \frac{L_t}{N_t}\right) \tag{57}$$

where factors are expressed in per 0-year old agents,[34] and

$$\begin{aligned} \frac{L_t}{N_t} &= \sum_{j=0}^{+\infty} \left[\frac{1}{n^j} \prod_{h=0}^{j} \pi(e_{h-1}^{t-j})\right] \\ \frac{K_t}{N_t} &= \sum_{j=0}^{+\infty} \left[\frac{1}{n^j} \prod_{h=0}^{j} \pi(e_{h-1}^{t-j-1}) \frac{k_j^{t-j-1}}{n}\right] \end{aligned} \tag{58}$$

[34] Output is linearly homogeneous, so that marginal productivities are homogeneous of degree 0.

for all integer t, where $N_t = nN_{t-1}$ and $N_0 = 1$. As in the setup considered in the rest of the paper, condition (56) requires that the return to an increase in the longevity-enhancing expenditures, at any age $j = 0, 1, \ldots$ for any generation t, matches exactly the direct and indirect costs of doing so. More specifically, at a competitive equilibrium steady state,

$$\pi'(e_j)u(c_{j+1}) = \left[1 + \frac{\pi'(e_j)}{\pi(e_j)}k_j\right]u'(c_{j+1})r \tag{59}$$

must hold for all $j = 0, 1, \ldots$ with

$$r = F_K\left(\frac{K_t}{N_t}, \frac{L_t}{N_t}\right), \tag{60}$$

and

$$\begin{aligned} \frac{L_t}{N_t} &= \sum_{j=0}^{+\infty}\left[\frac{1}{n^j}\prod_{h=0}^{j}\pi(e_{h-1})\right] \\ \frac{K_t}{N_t} &= \sum_{j=0}^{+\infty}\left[\frac{1}{n^j}\prod_{h=0}^{j}\pi(e_{h-1})\frac{k_j}{n}\right]. \end{aligned} \tag{61}$$

At the steady state, K_t and L_t grow at the same rate as N_t, i.e. by the same factor n each period, while the per 0-year old capital and labor, K_t/N_t and L_t/N_t respectively stay constant at the levels in (61).

A planner, on the contrary, solves the following problem for the representative agent at the steady state

$$\begin{aligned} &\max_{c_j,k_j,e_j} \quad \sum_{j=0}^{+\infty}\beta^j\pi(e_{j-1})u(c_j) \\ &\text{s.t.} \quad \sum_{j=0}^{+\infty}\frac{\prod_{h=0}^{j}\pi(e_{h-1})}{n^j}\left[c_j + k_j + e_j\right] \leqslant F\left(\sum_{j=0}^{+\infty}\frac{\prod_{h=0}^{j}\pi(e_{h-1})}{n^j}\frac{k_j}{n}, \frac{\prod_{h=0}^{j}\pi(e_{h-1})}{n^j}\right) \end{aligned} \tag{62}$$

with $\pi(e_{-1}) = 1$. First-order conditions with respect to c_j, k_j, e_j of this problem are

$$\begin{pmatrix} \beta^j\pi(e_{j-1})u'(c_j) \\ 0 \\ \beta^{j+1}\pi'(e_j)u(c_{j+1}) \end{pmatrix} = \lambda\begin{pmatrix} \frac{1}{n^j}\prod_{h=0}^{j}\pi(e_{h-1}) \\ \frac{1}{n^j}\prod_{h=0}^{j}\pi(e_{h-1})(1-\frac{1}{n}F_K) \\ \frac{1}{n^j}\prod_{h=0}^{j}\pi(e_{h-1}) + \left(\sum_{l=j+1}^{+\infty}\frac{1}{n^l}\prod_{h=0}^{l}\pi(e_{h-1})\left[c_l + k_l + e_l - F_K\cdot\frac{k_{l-1}}{n} - F_L\right]\right)\frac{\pi'(e_j)}{\pi(e_j)} \end{pmatrix} \tag{63}$$

for all $j = 0, 1, \ldots$ and some Lagrange multiplier $\lambda > 0$. This yields the following rearranged necessary condition for effort e_j at each age j

$$\pi'(e_j)u(c_{j+1}) = \left[1 + \left(\sum_{l=j+1}^{+\infty}\frac{1}{n^{l-j}}\prod_{h=j+1}^{l}\pi(e_{h-1})\left[c_l + k_l + e_l - F_K\cdot\frac{k_{l-1}}{n} - F_L\right]\right)\frac{\pi'(e_j)}{\pi(e_j)}\right]u'(c_{j+1})F_K. \tag{64}$$

Note that, in the two-period case (*i.e.* when $\pi(e_1) = 0$ for any e_1), necessarily $k_j = e_j = 0$ for all $j \geqslant 1$ and this necessary condition becomes

$$\pi'(e_0)u(c_1) = \left[1 + \frac{\pi'(e_0)}{n}c_1\right]u'(c_1)n. \tag{65}$$

This is equivalent to the last line in (24) which describes the golden rule condition for health expenditure (where v in (24) stands for βu).

From conditions (59) and (64), it follows that actuarially fair annuities alone cannot implement the planner's steady state. The reason is that the agents fail to internalize the impact on marginal return to money of the longevity-enhancing expenditures *at all possible ages they may attain*. The results obtained in the two-period set up then carry over to this extended perpetual youth model.

7. Concluding remarks

This paper addresses the question of whether an actuarially fair annuity market, which makes the return of annuities contingent to longevity-enhancing choices, is able to cope with the agents' incentives to extend their lives inefficiently beyond optimal levels, once insured against the risk of outliving their savings. Davies and Kuhn (1992) and Philipson and Becker (1998) addressed a related question by showing, in a static set-up, that public pension systems, by insuring individuals against the risk of running out of income, lead them to invest too much in their longevity. The reason was that agents, having access to annuity contracts that are not contingent on their behavior, do not take into account that increased longevity will, in the aggregate, decrease the return of their own annuities. As a consequence, individuals actually evaluate incorrectly the trade-off between "quantity" and "quality" of life. The fact that public pensions are not contingent to the individuals' choices clearly prevents them to price-in this externality. Private annuity markets instead should in principle be—by making annuities contingent to the agents' longevity choices—better able to restore the right incentives. Nevertheless, we showed here in an overlapping generations production economy à la Diamond (1965) with money and an actuarially fair competitive annuity market, that the competitive equilibrium steady state still differs from the best possible steady state, the golden rule steady state. However, we also showed that when population growth is positive, the golden rule steady state can be decentralized as a competitive equilibrium by means of a tax-and-transfer policy that is balanced at the steady state.

We also derived the model under alternative assumptions such as assuming second-period endogenous labor supply, a health investment which is costly in terms of utility but not in terms of resources, and a more than two periods model. We showed that given the nature of the mechanism driving the results in our baseline case, the results of these extended setups did not differ greatly from those delivered by our stripped down model.

Of course, our model relies on other simplifying assumptions. One of these is the representative agent assumption. Allowing for different types may create moral hazard problems and the intervention of the government may certainly be complicated by the necessity to introduce incentive constraints. Such an extension has been dealt with in Leroux (2011) when agents have different disutility of effort and in Leroux et al. (2011), when agents differ in productivity and in their genetic backgrounds. These models are derived in a static equilibrium framework. Introducing differences in agents' characteristics affecting longevity in an overlapping generations model would certainly be interesting and is on our research agenda.

Appendix A

Proposition A1. *Consider a Diamond (1965) overlapping generations production economy with money and an exogenously growing population with a representative agent facing an exogenous probability of survival, and saving in actuarially fair annuities. The golden rule is well defined as the unique steady state maximizing the representative agent's utility under the feasibility constraint.*

Proof. Assume both (c_0, c_1, k) and (c_0', c_1', k') satisfy the equations in (16) characterizing any solution to the problem (15), so that

$$F\left(\frac{k}{n},1\right) = n = F\left(\frac{k'}{n},1\right) \tag{66}$$

from which $k = k^* = k'$ for some k^*, and

$$\frac{u'(c_0)}{v'(c_1)} = n = \frac{u'(c_0')}{v'(c_1')} \tag{67}$$

$$c_0 + \frac{\pi}{n}c_1 = F\left(\frac{k^*}{n},1\right) - k^* = c_0' + \frac{\pi}{n}c_1' \tag{68}$$

Assume without loss of generality that $c_0 < c_0'$. Since $u'', v'' < 0$, the first line above implies that $c_1 < c_1'$ as well. There is a contradiction with the second line. Hence $c_0 = c_0'$ and therefore $c_1 = c_1'$ as well. □

Proposition A2. *In a Diamond (1965) overlapping generations production economy with a representative agent who can increase his life expectancy at a cost in terms of resources, the golden rule is well defined as the unique steady state maximizing the representative agent's utility under the feasibility constraint.*

Proof. Assume both (c_0, c_1, k, e) and (c_0', c_1', k', e') satisfy Eqs. (24) characterizing any solution to the problem (23), so that

$$F\left(\frac{k}{n},1\right) = n = F\left(\frac{k'}{n},1\right) \tag{69}$$

from which $k = k^* = k'$ for some k^*, and

$$\frac{u'(c_0)}{v'(c_1)} = n = \frac{u'(c'_0)}{v'(c'_1)} \tag{70}$$

$$c_0 + \frac{\pi(e)}{n} c_1 + e = F\left(\frac{k^*}{n}, 1\right) - k^* = c'_0 + \frac{\pi(e')}{n} c'_1 + e' \tag{71}$$

Assume without loss of generality that $c_0 < c'_0$. Since $u'', v'' < 0$, the first line above implies that $c_1 < c'_1$ as well. Hence from the second line, $e > e'$ since $\pi' > 0$. But the last line of (24) requires

$$\pi'(e) \frac{v(c_1) - v'(c_1)c_1}{v'(c_1)} = n \tag{72}$$

to be satisfied by both (e, c_1) and (e', c'_1). This cannot be since, on the left-hand side, the function $(v(c) - v'(c)c)/v'(c)$ is strictly increasing in c (as $v'' < 0$), and $\pi'(e)$ is decreasing in e (as $\pi'' < 0$). Hence $c_0 = c'_0$. Similarly, $c_1 = c'_1$, and hence $e = e'$ also, since the left-hand side of the feasibility condition in (16) is monotone in e. □

Proof of Proposition 4. Firstly, $\tilde{k} = k^*$ follows trivially from (14) and (16):

$$F_K\left(\frac{\tilde{k}}{n}, 1\right) = n = F_K\left(\frac{k^*}{n}, 1\right). \tag{73}$$

(1) Assume $e^* > \tilde{e}$ and $c_1^* \geqslant \tilde{c}_1$. Then

$$\frac{\pi(e^*)}{n} c_1^* > \frac{\pi(\tilde{e})}{n} \tilde{c}_1 \tag{74}$$

and hence $c_0^* < \tilde{c}_0$ from the feasibility condition

$$c_0^* + \frac{\pi(e^*)}{n} c_1^* + e^* = F\left(\frac{k^*}{n}, 1\right) - k^* = F\left(\frac{\tilde{k}}{n}, 1\right) - \tilde{k} = \tilde{c}_0 + \frac{\pi(\tilde{e})}{n} \tilde{c}_1 + \tilde{e} \tag{75}$$

so that

$$u'(c_0^*) > u'(\tilde{c}_0). \tag{76}$$

Moreover, since $c_1^* \geqslant \tilde{c}_1$, then

$$\frac{1}{v'(c_1^*)} \geqslant \frac{1}{v'(\tilde{c}_1)}. \tag{77}$$

Therefore,

$$\frac{u'(c_0^*)}{v'(c_1^*)} \geqslant \frac{u'(c_0^*)}{v'(\tilde{c}_1)} > \frac{u'(\tilde{c}_0)}{v'(\tilde{c}_1)} \tag{78}$$

which cannot be since at the competitive equilibrium steady state and at the golden rule steady state, the marginal rates of substitution are both equal to the rate of growth of the population, n.

Therefore, either $e^* \leqslant \tilde{e}$ or $c_1^* < \tilde{c}_1$ or both hold.

(2) Assume that both $e^* \leqslant \tilde{e}$ and $c_1^* < \tilde{c}_1$ hold. Then

$$\frac{\pi(e^*)}{n} c_1^* < \frac{\pi(\tilde{e})}{n} \tilde{c}_1 \tag{79}$$

and hence $c_0^* > \tilde{c}_0$ again by the feasibility condition, from which

$$u'(c_0^*) < u'(\tilde{c}_0). \tag{80}$$

Moreover, since $c_1^* < \tilde{c}_1$, then

$$\frac{1}{v'(c_1^*)} < \frac{1}{v'(\tilde{c}_1)}. \tag{81}$$

Therefore,

$$\frac{u'(c_0^*)}{v'(c_1^*)} < \frac{u'(c_0^*)}{v'(\tilde{c}_1)} < \frac{u'(\tilde{c}_0)}{v'(\tilde{c}_1)} \tag{82}$$

which again cannot be since both the LHS and the RHS are equal to n.
Therefore, either $e^* \leqslant \tilde{e}$ and $c_1^* \geqslant \tilde{c}_1$, or $e^* > \tilde{e}$ and $c_1^* < \tilde{c}_1$.
Now let us see that $e^* \leqslant \tilde{e}$ and $c_1^* \geqslant \tilde{c}_1$ can only hold with both inequalities being strict.

(3) Assume that $e^* = \tilde{e}$ and $c_1^* > \tilde{c}_1$. Then

$$\frac{\pi(e^*)}{n} c_1^* > \frac{\pi(\tilde{e})}{n} \tilde{c}_1 \tag{83}$$

and hence $c_0^* < \tilde{c}_0$ by the feasibility condition, from which

$$u'(c_0^*) > u'(\tilde{c}_0). \tag{84}$$

Moreover, since $c_1^* > \tilde{c}_1$, then

$$\frac{1}{v'(c_1^*)} > \frac{1}{v'(\tilde{c}_1)}. \tag{85}$$

Therefore,

$$\frac{u'(c_0^*)}{v'(c_1^*)} > \frac{u'(c_0^*)}{v'(\tilde{c}_1)} > \frac{u'(\tilde{c}_0)}{v'(\tilde{c}_1)} \tag{86}$$

which again cannot be since both at the competitive equilibrium steady state and the golden rule steady state, these marginal rates of substitution are equal to the growth factor of the population n.

(4) Assume that $e^* < \tilde{e}$ and $c_1^* = \tilde{c}_1$. Then

$$\frac{\pi(e^*)}{n} c_1^* < \frac{\pi(\tilde{e})}{n} \tilde{c}_1 \tag{87}$$

and hence $c_0^* > \tilde{c}_0$ by the feasibility condition, from which

$$u'(c_0^*) < u'(\tilde{c}_0). \tag{88}$$

Moreover, since $c_1^* = \tilde{c}_1$, then

$$\frac{1}{v'(c_1^*)} = \frac{1}{v'(\tilde{c}_1)}. \tag{89}$$

Therefore,

$$\frac{u'(c_0^*)}{v'(c_1^*)} = \frac{u'(c_0^*)}{v'(\tilde{c}_1)} < \frac{u'(\tilde{c}_0)}{v'(\tilde{c}_1)} \tag{90}$$

which again cannot be since both the LHS and the RHS are equal to the growth factor of the population n.

(5) Assume $e^* = \tilde{e}$ and $c_1^* = \tilde{c}_1$. Then

$$\frac{\pi(e^*)}{n} c_1^* = \frac{\pi(\tilde{e})}{n} \tilde{c}_1 \tag{91}$$

and hence $c_0^* = \tilde{c}_0$, i.e. $(c_0^*, c_1^*, e^*) = (\tilde{c}_0, \tilde{c}_1, \tilde{e})$ which cannot be by Proposition 2.
Therefore, either $e^* > \tilde{e}$ and $c_1^* < \tilde{c}_1$, or $e^* < \tilde{e}$ and $c_1^* > \tilde{c}_1$.

(6) Finally, since both at the golden rule steady state and the competitive equilibrium steady state, it holds

$$\frac{u'(c_0^*)}{v'(c_1^*)} = n = \frac{u'(\tilde{c}_0)}{v'(\tilde{c}_1)} \tag{92}$$

either $e^* < \tilde{e}, c_1^* > \tilde{c}_1$ and $c_0^* > \tilde{c}_0$, or $e^* > \tilde{e}, c_1^* < \tilde{c}_1$ and $c_0^* < \tilde{c}_0$. □

Proposition A3. *The planner's steady state in the second period labor supply case is generically—in the C^1-uniform convergence topology for u, v, π, l—characterized as the unique solution to the system (49).*

Proof. The planer's steady state equations in (49) for the second-period labor supply case can be rewritten as follows (where obvious arguments have been dropped for the sake of readability)

$$\begin{aligned} &u' - nv' = 0 \\ &F_K - n = 0 \\ &c_0 + \frac{\pi}{n} c_1 + k + e - F = 0 \\ &\pi' v - \left[1 + \frac{\pi'}{n} c_1\right] v' n + F_L [l' \pi + l \pi'] v' = 0 \end{aligned} \tag{94}$$

whose Jacobian of the left-hand side is

$$\begin{pmatrix} u'' & -nv'' & 0 & 0 \\ 0 & 0 & F_{KK}\frac{1}{n} & F_{KL}A\frac{1}{n} \\ 1 & -\frac{\pi}{n} & 0 & \left(\left[1+\frac{\pi'}{n}c_1\right]n - F_L A\right)\frac{1}{n} \\ 0 & -\left(\left[1+\frac{\pi'}{n}c_1\right]n - F_L A\right)v'' & F_{LK}\frac{1}{n}Av' & \pi''[v - v'c_1] + F_{LL}A^2 v' + F_L B v' \end{pmatrix} \tag{95}$$

where $A = l'\pi + l\pi'$ and $B = l''\pi + 2l'\pi' + l\pi''$.

Note that the planer's steady state equations do not depend on second derivatives of u, v, π, l so that, should the Jacobian at a solution to the system be singular, there exists an arbitrarily close[35] economy differing from the given one just in the second order derivatives of some of these functions (but not in their values or first order derivatives, so that the solution still satisfies the system) such that the Jacobian is regular. In other words, the solution is generically—in the C^1-uniform topology—a regular zero of the system, i.e. locally unique.

Since it is moreover globally unique when $l_0(e) = 0$ for all e, then it is globally unique as well for any $l(e)$ as long as it stays locally unique along the following homotopy parametrized by $\alpha \in [0, 1]$

$$\begin{aligned} & u'(c_0) - nv'(c_1) = 0 \\ & F_K\left(\frac{k}{n}, 1 + \frac{\pi(e)}{n}[\alpha l(e) + (1-\alpha)l_0(e)]\right) - n = 0 \\ & c_0 + \frac{\pi(e)}{n}c_1 + k + e - F\left(\frac{k}{n}, 1 + \frac{\pi(e)}{n}[\alpha l(e) + (1-\alpha)l_0(e)]\right) = 0 \\ & \pi'(e)v(c_1) - \left[1 + \frac{\pi'(e)}{n}c_1\right]v'(c_1)n + F_L\left(\frac{k}{n}, 1 + \frac{\pi(e)}{n}[\alpha l(e) + (1-\alpha)l_0(e)]\right) \\ & \cdot \left[[\alpha l'(e) + (1-\alpha)l_0'(e)]\pi(e) + [\alpha l(e) + (1-\alpha)l_0(e)]\pi'(e)\right]v'(c_1) = 0 \end{aligned} \tag{96}$$

connecting continuously the system with second period labor $l(e)$ for $\alpha = 1$ to the system (24) without second period labor, i.e. $l_0(e) = 0$ for all e, for $\alpha = 0$. Note that indeed the solution stays locally unique as α runs $[0, 1]$ since, for all α, the Jacobian is generically regular for the same reason as above. □

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[35] With respect to the metric $\|f - g\|_{C^1} = \sup\|f - g\| + \sup\|f' - g'\|$ requiring, for f to be close to g, to be so uniformly in derivatives as well as in values, i.e. to be close in the C^1-uniform topology.