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The risk premium in New Keynesian DSGE models: The cost of inflation channel ☆

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A B S T R A C T

We study the role of the cost of inflation channel in determining the risk premium in a (nonlinear) New Keynesian DSGE model. Relying on a Calvo (or Rotemberg) price setting, we show that while the cost of inflation channel generates the desired term premium moments, it suffers from nontrivial, counterintuitive approximation errors in the price dispersion function. In addition to documenting the issues, we propose ways to alleviate them, including a quasikinked demand function as a risk-generating mechanism.

1. Introduction

The government bond market is vital for modern economies. Countries issue bonds to finance their debt, investors buy government bonds to seek low-risk (or risk-free) returns, and central banks consider the yield curve (the function linking the bond's yields with their maturity) as the critical component of monetary policy transmission. Hence, understanding the underpinnings of government bonds' prices/yields is of pivotal importance. A key determinant of bonds' prices is the risk compensation investors demand for holding long(er) term maturity bonds, i.e., the risk premium. While an undisputed empirical finding in the finance literature is that risk premiums are substantial and time-varying (see Campbell and Shiller (1991), and Cochrane and Piazzesi (2005), among others), the economic forces behind this time variation remain unclear.

A popular approach to understand the determinant of bond risk premia relates to structural macroeconomic models, such as dynamic stochastic general equilibrium (DSGE) models (see Christiano et al. (2005), Smets and Wouters (2007)), whereby economic agents (households and firms) make optimal decisions and form rational expectations in the face of fundamental shocks. Within this class of models, the study of risk premia is usually done (i) by inducing Epstein and Zin (1989) generalized recursive preferences, which allow the researcher to disentangle the degree of intertemporal substitutability from risk attitudes and generate sizable risk

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premium variation, and (ii) by introducing nominal rigidities to jointly match consumption and inflation moments with yield curve moments (risk premia and yield curve slope) and to allow for the propagation of structural shocks. The presence of nominal price rigidities implies that inflation becomes disruptive and costly. This paper studies this cost of inflation channel and its importance for the term premium behavior in macrofinance structural macroeconomic models. Additionally, we highlight that standard New Keynesian DSGE models with nominal rigidities match risk premium dynamics but suffer from substantial approximation error in the cost of inflation channel.

The starting point of our analysis is a standard DSGE model, such as in Rudebusch and Swanson (2012), which allows for recursive preferences and nominal rigidities à la Calvo (1983) or Rotemberg (1982). Following Andreasen (2012), we produce a time-varying risk premium by focusing on the third-order approximation of the model around the steady state. A higher order approximation of the model generates additional effects typically not observed in the linear case, with the cost of inflation being the most important, especially for generating time-varying risk premium dynamics. For example, Swanson (2015) noted that price dispersion, the state variable accounting for the cost of inflation, generates conditional heteroskedasticity in the stochastic discount factor even with homoskedastic structural shocks. However, Swanson (2015) does not investigate the potential problem of approximation error. Maršál et al. (2019) analyze the approximation issue in a similar setup but with the trending inflation present and observe more puzzling results on top of the ones discussed in this paper.

We note that appealing results of Rudebusch and Swanson (2012) rest on extreme calibration settings and are linked to severe approximation errors that, in fact, undermine the internal consistency of the model. First, we document the role of the price dispersion mechanism as a main driving force of term premium variation in the model. Without the price dispersion feedback channel, the model loses a great portion of state-dependency, conditional heteroskedasticity of the stochastic discount factor, and finally, time-variation in the term premium. Second, we document a counterintuitive result linked to price dispersion (the inefficiency generated by staggered price setting). In a standard parametrization setting, we show that the third-order approximation of price dispersion implies that when inflation deviates from the steady-state value, price dispersion *decreases*, even below theoretical lower bound (of 1), due to the presence of substantial approximation error.

A first logical solution to these problems is the realistic calibration of the parameters such as markup, Calvo probability, and Cobb-Douglas labor output elasticity. Our results stress the fact that, in calibrating a DSGE model to fit asset pricing moments, the researcher should carefully investigate whether the calibration setting generates economically consistent transmission channels. As the second solution, we propose an alternative risk-generating mechanism that complements the price dispersion channel. Specifically, we introduce real rigidities in form of the Kimball (1995) aggregator. Its main advantage is that one can assume a lower degree of nominal rigidities (more frequent price changes) consistent with microeconomic literature (for example, Bils and Klenow (2004)) by applying a high degree of real rigidities. We draw inspiration from Harding et al. (2022) who use a non-linear macroeconomic model with real rigidities to resolve the missing deflation puzzle. In turn, in our setup, it serves as a remedy to the price dispersion problem and an attractive option for the asset-pricing general equilibrium model. We show that the Kimball specification maintains a similar level of state dependence and generates a comparable term premium while being free from the severe approximation error observed in the original case.

The remainder of the paper proceeds as follows. Section 2 introduces the benchmark structural model with the asset pricing block. Section 3 analyzes the role played by the price dispersion mechanism in reproducing risk premium moments and investigates the price approximation errors in the price dispersion function linked with benchmark calibration settings. In section 4, we propose the alternative specifications (Kimball real rigidities; Rotemberg nominal rigidities) and show that they can reproduce the desired risk premium moments while eliminating the internal inconsistencies brought about by the Calvo setting. In section 5, we demonstrate why the alternative frameworks are not a full remedy to the approximation error issue but are still preferable over the Calvo specification. Finally, section 6 concludes the paper.

2. A simple macroeconomic model with asset pricing block

The backbone of our model is similar to Rudebusch and Swanson (2012). In a nutshell (i) households have recursive preferences over state-contingent plans of labor and consumption, (ii) they offer labor input to monopolistically competitive firms facing inefficiencies in the price-setting mechanism, either via staggered price contracts (à la Calvo) or via price adjustment costs (à la Rotemberg), and finally, (iii) to transfer consumption over time households can use the bond market, whereby the one-period interest rate is set by the monetary authority.

2.1. Households

At any time t, the household obtains a utility flow:

$$u(c_t, l_t) = \frac{c_t^{1-\kappa}}{1-\kappa} + \eta \frac{(1-l_t)^{1-\chi}}{1-\chi}, \tag{1}$$

that is additively separable, strictly concave, increasing in consumption c_t, decreasing in labor l_t, and where κ, χ and η are positive model parameters.[1]

[1] Intertemporal elasticity of substitution is given by $IES = \frac{1}{\kappa}$, η is calibrated such that $l_{ss} = 1/3$, and $\chi/2$ is an inverse of Frisch elasticity.

Following the idea of Epstein and Zin (1989), we assume that the household's preferences are ordered using the recursive functional $\tilde{V}_t$:

$$\tilde{V}_t = u(c_t, l_t) + \beta \left(\mathbb{E}_t \tilde{V}_{t+1}^{1-\alpha}\right)^{\frac{1}{1-\alpha}},$$

where $\beta \in (0,1)$ is the household's discount factor, $\mathbb{E}_t$ is a conditional expectation given information in period t, and α is a positive curvature parameter that governs the relative risk aversion coefficient.

The value function of the household then satisfies the maximization problem (generalized Bellman equation)[2]

$$V_t = \max_{(c_t, l_t)} \left\{ u(c_t, l_t) + \beta \left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{\frac{1}{1-\alpha}} \right\},$$

subject to the household's flow budget constraint

$$a_{t+1} + P_t c_t = e^{i_t} a_t + w_t l_t + d_t,$$

were a_t is the nominal assets holdings, i_t is the (continuously-compounded) nominal interest rate, d_t is exogenous transfers to the household, P_t is the price of consumption good, and w_t is the nominal wage determined by the household's optimality condition:

$$\frac{w_t}{P_t} = \eta(1-l_t)^{-\chi} / c_t^{-\kappa}.$$

Finally, following the logic of Rudebusch and Swanson (2012), we obtain the model-implied real stochastic discount factor (SDF):

$$m_{t+1} = \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\kappa} \left(\frac{V_{t+1}}{\left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{1/(1-\alpha)}} \right)^{-\alpha}.$$

2.2. Production

Final output Y_t is produced by a perfectly competitive final good producer, which assembles intermediate goods $y_t(f)$ of type $f \in [0,1]$ using the following CES production function (Dixit-Stiglitz aggregator):

$$Y_t = \left(\int_0^1 y_t(f)^{1/\lambda} df \right)^{\lambda},$$

where Y_t denotes the amount of final good produced. The parameter $\lambda > 1$ governs the elasticity of demand with respect to price, $\lambda/(\lambda-1)$, in the resulting demand function for intermediate good of type f with price $p_t(f)$:

$$y_t(f) = \left(\frac{p_t(f)}{P_t} \right)^{-\lambda/(\lambda-1)} Y_t. \tag{2}$$

Monopolistically, infinitely-lived and competitive firms produce differentiated intermediate good $y_t(f)$ by hiring households, $L_t(f)$, and by using fixed capital k. All firms combine the production's inputs by identical Cobb-Douglas production functions:

$$y_t(f) = A_t k^{1-\theta} L_t(f)^{\theta}, \tag{3}$$

with $\theta \in (0,1)$ and technology A_t evolving according to the following AR(1) process:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_A \epsilon_t^A,$$

where $\rho_A \in [0,1]$, ϵ_t^A are the i.i.d. Gaussian productivity shock with mean zero and variance 1, and the standard deviation σ_A modulates the size of the shocks.

The last element of the production's part of the model relates to the price setting of the intermediate firms. Following the New-Keynesian macroeconomics' paradigm, we introduce nominal rigidities in the production sector, allowing for nonneutrality of short-term policy rate.

We follow Calvo (1983) and assume that the firm maximizes its profits subject to a stochastic staggered price setting mechanism. Each period, a firm can reset the price to an optimal level with probability $(1-\xi)$ while with probability ξ the firm will adjust its price $p_t(f)$ according to the following indexation rule (to steady-state inflation):

$$p_t(f) = p_{t-1}(f)\bar{\Pi},$$

[2] Similar to Rudebusch and Swanson (2012), we scale the value function V_{t+1} under the power operator by the steady state of the value function and then unscale it outside of it. This manipulation serves a numerical purpose and helps to avoid extremely low values of $\mathbb{E}_t V_{t+1}^{1-\alpha}$ given high relative risk aversion (hence α) without perturbing the results of the model; also see Swanson (2018).

where $\bar{\Pi}$ is a steady-state (gross) inflation rate. The optimal price of the re-optimizing firm is determined by the standard first-order condition:

$$p_t^*(f) = \lambda \frac{\sum_{j=0}^{\infty} \xi^j E_t \left\{ m_{t,t+j} \left(P_t / P_{t+j} \right) y_{t+j}(f) \mu_{t+j}(f) \right\}}{\sum_{j=0}^{\infty} E_t \left\{ m_{t,t+j} \left(P_t / P_{t+j} \right) \xi^j y_{t+j}(f) e^{j\pi} \right\}},$$

where $\mu_t(f)$ is the firm's marginal cost and $m_{t,t+j} = \prod_{k=1}^{j} m_{t+k}$ is the firm's (household's) real stochastic discount factor from period $t+1$ up to $t+j$.

The aggregate output, Y_t, depends on how the inputs are distributed among the various intermediate good producers. Given the frictions introduced by the staggered price mechanism, resources are not allocated optimally, with some production done by firms with nonoptimal price setting. The output lost due to this price distortion mechanism is captured by the measure of price dispersion, that we propose here in the following form:

$$\Delta_t^{1/\theta} = (1-\xi)^{1-\lambda/\theta} \left(1 - \xi X_t^{1/(\lambda-1)} \right)^{\lambda/\theta} + \xi X_t^{\lambda/((\lambda-1)\theta)} \Delta_{t-1}^{1/\theta}, \tag{4}$$

with

$$X_t = \Pi_t / \bar{\Pi}, \tag{5}$$

measuring the ratio of current (gross) inflation Π_t to the price adjustment coefficient for non-optimizing firms (indexation). Equation (4) indicates that the current level of price dispersion is linked to the price dispersion of the last period and contemporaneous inflation. If any shock operates through the aggregate rate of inflation, it has an immediate impact on the price dispersion measure.

Let L_t denote the aggregate labor demand. Labor market equilibrium requires that the labor demanded by firms is equal to the labor supplied by households or $L_t = l_t$. From the definition of L_t,

$$L_t = \int_0^1 L_t(f) df.$$

We find that L_t satisfies

$$Y_t = \Delta_t^{-1} A_t K^{1-\theta} L_t^{\theta}, \tag{6}$$

where $K = k$ is the aggregate capital stock. The inflation dynamics is given by:

$$X_t^{1/(1-\lambda)} = (1-\xi) \left(\frac{p_t^*}{P_t} \right)^{1/(1-\lambda)} X_t^{1/(1-\lambda)} + \xi. \tag{7}$$

Finally, the market clearing condition for the final good requires $Y_t = C_t + G + \bar{I}$, with constant government purchases as a share of steady-state output, $\gamma_G \bar{Y}$, constant investments $\bar{I} = \delta K$ that cover capital K depreciation with the rate δ, and $C_t = c_t$ is the aggregate consumption demanded by households.

2.3. Monetary policy rule

The monetary policy rule is assumed to be in the form of the Taylor rule with inertia. A central bank sets one-period nominal interest rate i_t,

$$i_t = \rho_i i_{t-1} + (1-\rho_i) \left(\bar{i} + \phi_\pi (\tilde{\pi}_t - \bar{\pi}) + \phi_y (y_t - \bar{y}) / \bar{y} \right).$$

Here ϕ_π and $\phi_y \in \mathbb{R}$ are the policy reaction coefficients, $\pi_t = \log \Pi_t$ is the inflation rate, $\bar{\pi} = \log \bar{\Pi}$ denotes the inflation target that coincides with the steady-state inflation, $y_t = \log Y_t$ is the output, $\bar{y}_t = \log \bar{Y}$ is the log of the steady-state output. Finally, $\tilde{\pi}_t$ stands for a geometric moving average of inflation:

$$\tilde{\pi}_t = \theta_\pi \tilde{\pi}_{t-1} + (1-\theta_\pi) \pi_t.$$

By setting $\theta_\pi = 0.7$ we assume an effective duration of about four quarters.[3]

2.3.1. Asset pricing block

Following Rudebusch and Swanson (2012) paper, this block uses m_t, the real SDF implemented in the model, to price nominal default-free government bonds.

By definition, in the model, a default-free zero-coupon nominal bond pays one nominal dollar at maturity. If $p_t^{(n)}$ denotes the nominal price of a such n-period bond, then $p_t^{(0)} = 1$ and for $n \geq 1$ in equilibrium we get

[3] The adoption of average inflation targeting does not significantly alter the nature of the price dispersion mechanism discussed in the paper. We have chosen to retain this feature in order to maintain close alignment with Rudebusch and Swanson (2012), considering it as a stylistic choice of the authors.

$$p_t^{(n)} = \mathbb{E}_t m_{t+1} e^{-\pi_{t+1}} p_{t+1}^{(n-1)}.$$

Furthermore, let $i_t^{(n)}$ be the yield on a nominal bond:

$$i_t^{(n)} = -\frac{1}{n} \log p_t^{(n)}.$$

The risk-neutral nominal price $\hat{p}_t^{(n)}$ of an n-period zero-coupon nominal bond is defined by

$$\hat{p}_t^{(n)} = e^{-i_t} \mathbb{E}_t \hat{p}_{t+1}^{(n-1)},$$

and n-period nominal term premium $\psi_t^{(n)}$ is given by

$$\psi_t^{(n)} = \frac{1}{n}(\log \hat{p}_t^{(n)} - \log p_t^{(n)}).$$

2.3.2. Model calibration

We calibrate our model similar to Rudebusch and Swanson (2012) best fit specification. The intertemporal elasticity of substitution is set to 0.11 which translates into $\kappa = 9.09$. Frisch elasticity is set to 0.28, which is equivalent to $\chi = 7.14$, while keeping the steady state-value of the labor equal to 1/3 by adjusting η constant. The Epstein-Zin parameter is set to imply the steady-state of relative risk aversion to 110. We do not assume any growth of productivity in the model and simply set household's discount factor $\beta = 0.99$.[4] The capital depreciation rate is set to $\delta = 0.02$, the steady-state capital-output ratio is set to 2.5. The steady-state government spending to output ratio is fixed to 0.17. On the firms' side, we set gross markup $\lambda = 1.2$, output elasticity with respect to labor $\theta = 0.6666$, and the Calvo parameter $\xi = 0.78$. The monetary policy rule is defined by $\psi_\pi = 0.53$, $\psi_y = 0.93$, $\rho_i = 0.73$ and zero inflation target that coincides with steady-state inflation $\bar{\pi} = 0$. Alternatively, any steady-state inflation value can be considered, as we assume steady-state indexation in the firms' optimization problem. Finally, our model uses only one fundamental shock, namely, productivity shock, and we set its persistence $\rho_a = 0.96$ and variance $\sigma_A = 0.005$.

2.3.3. Solution method

To solve our model, we use the perturbation method, one of the most popular ways to solve DSGE models with differentiable policy functions, see Schmitt-Grohe and Uribe (2004). The main advantage of the method is the computational efficiency (compared to alternatives) that allows for estimating nonlinear DSGE models. We specify nonlinear model and solve it up to the third order. It is acknowledged in the literature that such methods can exhibit two issues: (i) a high approximation error and (ii) explosive simulated paths, even when the linear model is stable, see Den Haan and De Wind (2012). To address this, we employ the popular pruning methodology introduced by Kim et al. (2008) to eliminate the explosiveness of the solution. Consequently, our model generates counterintuitive approximations, which are partially influenced by the pruning procedure. The specific impact of the pruning procedure cannot be isolated from the broader perturbation-related challenges, primarily because, in our calibration, non-pruned simulations consistently result in explosive dynamics. The third-order approximation is an essential step in generating time-varying risk within the model. Under this condition, the term premium becomes a linear function of states, see Andreasen (2012). We formulate and solve our model with the aforementioned methods using the Dynare package by Adjemian et al. (2011).[5] Additional information about the algorithms employed to produce our figures can be found in Appendix E.

3. Cost of inflation in the Calvo world

In New Keynesian models with the Calvo setting, price-setting frictions imply that some firms cannot optimally set prices, leading to inefficient input allocation and to lower aggregate output. These nonoptimizing firms find themselves further from the optimal price state, and as inflation increases, the overall distribution of prices changes and no longer corresponds to the existing marginal costs. In other words, the cost of inflation is driven by the price dispersion channel. In this section, we emphasize that the price dispersion mechanism, implied by price stickiness, is a key component of the nonlinear model. It accounts for the majority of the nonlinear effects and is pivotal in generating economically meaningful term premia. We proceed in two steps. First, we artificially shut down the impact of price dispersion on the aggregate production function to highlight the importance of price dispersion for the volatility of macro variables and the term premium. Subsequently, we analyze the relationship between the numerical errors in the price dispersion function's approximation and the term premium's volatility.

3.1. Price dispersion, volatility and term premium

To assess the role of price dispersion in term premium volatility, we compare three types of models: (i) the baseline macrofinance model developed in the previous section solved linearly; (ii) the same baseline model solved nonlinearly (3rd order); and (iii) a

[4] In Rudebusch and Swanson (2012), β is set to imply a value of $\tilde{\beta} = \beta\gamma^{-\kappa} = 0.99$, where $\gamma = 1.0025$ is a growth rate of productivity (1% annual growth). Given their values, the discount factor should be $\beta = \tilde{\beta}\gamma^{\kappa} = 0.99 \cdot 1.0025^{9.09} = 1.01$ that goes at odds with common intuition of $\beta < 1$. We avoid this problem by assuming no growth.

[5] During our unreported exercises, we cannot draw the conclusion that the 4th order solution performs better. In fact, the 4th order solution, without pruning, actually worsens the performance by reducing overall stability. Although pruning can be a beneficial extension to stabilize the dynamics, it is important to note that (i) it is currently not available in the Dynare package and (ii) it does not alleviate the computational complexity associated with the 4th order approximation.

Table 1
Model-based simulated standard deviations.

Variable (SD)	Linear	Nonlinear	Simplified
Consumption, %	0.75	1.13	0.79
Labor, %	1.92	2.43	1.88
Inflation, an.%	2.08	2.71	2.17
Nominal interest rate, an.%	2.37	3.05	2.47
Real interest rate, an.%	1.17	1.53	1.22
10Y nominal yield, an.%	1.42	2.30	1.52
10Y nominal TP, an.%	0.00	0.54	0.04

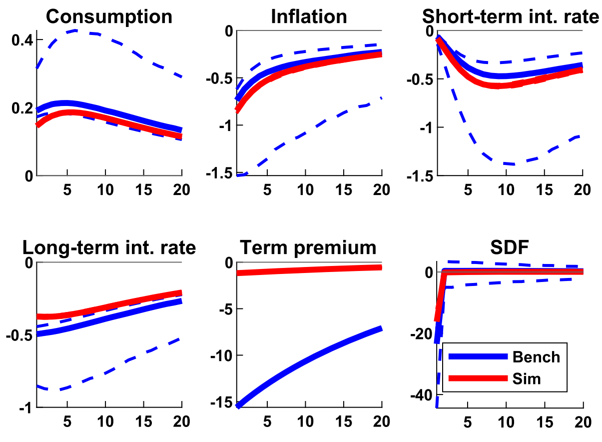

Fig. 1. Nonlinear impulse response functions to a one-standard-deviation positive productivity shock. The results are plotted for the Benchmark (Bench, blue) and Simplified (Sim, red) models. The solid line represents the median impulse response, while the dashed lines represent 5% and 95% bands. For the Simplified model, the median response almost coincides with the bands. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

simplified version of it where we switch off price dispersion's inefficiency in the aggregate production, i.e., in equation (6), we set $\Delta_t = 1$:

$$Y_t = A_t K^{1-\theta} L_t^{\theta}. \tag{8}$$

Intuitively, this modification allows for a nonlinear solution of the model, but the price dispersion mechanism is absent, analogous to the linear model. For each model, we use a calibration setting similar to Rudebusch and Swanson (2012), namely $\lambda = 1.2$, $\theta = 2/3$, $\xi = 0.78$, and $\psi_\pi = 0.53$. To gauge the importance of price dispersion on term premium volatility and other key variables, we compare the simulated standard deviations for the selected set of variables and the impulse response functions (IRFs) to productivity shocks.

We summarize model-implied standard deviations for the three models in Table 1. The set of selected variables is the following: consumption, C; labor, L; inflation, π; short-term nominal interest rate, i; short-term real interest rate, r; 10-year nominal bond yield, $i^{(40)}$; and 10-year nominal bond term premium, $\psi^{(40)}$. The moments in the first column correspond to the linearly solved baseline model. Consistent with theory, the term premium standard deviation is 0 in the linear setup. The second column reports values from the nonlinear benchmark model. The nonlinear effects in the model have a significant impact on the macroeconomic variables, increasing overall volatility. The model is able to generate sizable term premium volatility in agreement with Rudebusch and Swanson (2012). The third column of Table 1 reports the results from the simplified model without price dispersion's inefficiency. Two major points stand out. First, despite being solved nonlinearly, the simplified model generates macroeconomic moments much closer to those of the linear model rather than the nonlinear model. This prominent result suggests that the price dispersion mechanism is one of the fundamental nonlinear channels in the baseline model, and when removed, the model behaves very similarly to the linear version. Second, the term premium volatility is almost negligible compared to the nonlinear model. The main reason for this outcome is that the price dispersion mechanism creates the conditional heteroskedasticity of the stochastic discount factor that results in the time variation of the term premium.

To shed more light on our findings, we turn to the analysis of the IRFs. The blue (red) line in Fig. 1 depicts the IRFs to a one-standard-deviation positive productivity shock in the original (simplified) model for aggregate output (in %), the annualized inflation rate (in %), the annualized short-term interest rate (in %), the 10-year yield on nominal zero-coupon bonds (in %), the 10-year nominal term premium for such bonds (in ann.bp.), and the model-implied stochastic discount factor (in %).[6]

[6] The IRFs to a negative productivity shock can be found in Appendix A. Despite the fact that a third-order solved model produces non-symmetric results, the IRFs demonstrate similar patterns, and the same logic may be applied to explain the IRFs.

Focusing on the median response in both models, the responses of consumption, inflation, and short-term interest rates are comparable in size. Furthermore, they are in line with intuition: (i) inflation decreases as a fraction of firms reduces their prices in response to the decrease in firms' marginal costs; (ii) short-term interest rates decrease, following the central's bank monetary policy rule; and (iii) consumption demand increases as households become wealthier in present-value terms and in accordance with the increase in aggregate supply. However, the consumption path in the simplified model follows the lower band of the benchmark model. The major reason for that difference is the price dispersion channel, which creates two counteracting effects. First, due to high relative risk aversion and the precautionary motives that operate through the Euler equation, both models exhibit a negative bias in inflation and the nominal interest rate. In the benchmark case, this bias propagates through the increase in price dispersion, and decrease in aggregate output and consumption. Consequently, when a positive productivity shock hits the benchmark economy, inflation decreases and price dispersion increases even more, thus placing downward pressure on production and, therefore, consumption. Second, due to the presence of price dispersion, this productivity shock decreases the conditional volatility of the SDF, decreases the demand for precautionary savings, and creates upward pressure on consumption; see Swanson (2015) for details. The second effect dominates, and as a result, we observe an enhanced consumption response in the benchmark model compared to the simplified model, where the former mechanisms are muted.

The median responses of the models differ if we examine the stochastic discount factor, long-term interest rates, and term premium. The (instantaneous) median response of the (stochastic discount factor) long-term interest rates is more pronounced in the benchmark model, while the term premium response in the simplified model is almost absent. To understand the latter differences, we turn to the impact of price dispersion on the conditional heteroskedasticity of the IRFs. Focusing on the 5% and 95% mass bands of the IRFs, depicted by the dashed lines in Fig. 1, we observe that all the mass of the IRFs for the simplified model is concentrated in the median response. In other words, when we switch off the price dispersion mechanism, the conditional heteroskedasticity of the IRFs is lost, in line with the intuition given for the consumption path difference.

A final point to note is that in the simplified version of the model, following a positive technological shock, long-term bond interest rates decrease as the real value of long-term bond payments increases due to the persistent decrease in inflation. However, in the benchmark model, the decline in the conditional heteroskedasticity of the SDF results in a reduction in households' risk aversion and, consequently, in a decline in the term premium (see the bottom-middle panel of Fig. 1). The drop in households' risk aversion is reflected via the term premium and in the response in the interest rates of long-term bonds. In other words, when the response of the SDF is conditionally heteroskedastic, the decrease in long-term bond interest rates is more pronounced due to the contemporaneous decrease in risk aversion and the risk premium.

3.2. The price dispersion function

Given the central role that price dispersion plays in generating a time-varying term premium, in this section we delve further into the property of its third-order approximation. In our model, that price dispersion function is summarized by the following equation:

$$\Delta_t^{1/\theta} = (1-\xi)^{1-\lambda/\theta}\left(1-\xi X_t^{1/(\lambda-1)}\right)^{\lambda/\theta} + \xi X_t^{\lambda/((\lambda-1)\theta)}\Delta_{t-1}^{1/\theta}. \tag{9}$$

Equation (9) highlights that the price dispersion measure is a function of two sets of variables and parameters, two relationships that will be the focus of the remainder of this section. First, it hinges upon lagged price dispersion Δ_{t-1} and $X_t = \Pi_t/\bar{\Pi}$, i.e., the ratio of current (gross) inflation Π_t to the price adjustment coefficient for nonoptimizing firms (indexation).[7] Second, the price dispersion measure relates to three structural parameters: the monopolistic markup $\lambda \in (1;\infty)$, the Cobb-Douglas labor share $\theta \in [0;1]$, and the Calvo parameter $\xi \in (0;1)$, with $(1-\xi)$ being a firm's probability of resetting (optimizing) its price.[8]

Fig. 2 depicts the behavior of the price dispersion as function of the inflation deviation from the price index for nonoptimizing firms, $x_t = \log X_t$, and of the lagged dispersion Δ_{t-1}. In each subplot, the blue surfaces depict the theoretical value of the price dispersion function, while the red surfaces portray its perturbation approximation of order one (left panel), two (middle panel), or three (right panel). In the first-order approximation case, inflation deviations do not affect Δ_t, while the changes in lagged price dispersion are partially transmitted to its current value. Given that no shocks perturb the current and lagged price dispersion, the first-order setup results in a constant price dispersion over time ($\Delta_t = 1$). Further on, the second-order approximation follows a nonlinear pattern that satisfies Schmitt-Grohe and Uribe (2007)'s lower bound conditions but results in increasing approximation errors, with negative (positive) inflation deviations leading to an overestimation (underestimation) of the true price dispersion change. Finally, focusing on the third-order approximation, note that negative price deviations lead to the counterintuitive outcome that the approximated price dispersion is below unity. This result is in contrast with Schmitt-Grohe and Uribe (2007), who show that the theoretical lower bound for this function is 1, i.e., $\Delta_t \geq 1$. Intuitively, if $\Delta_t > 1$ the prices of intermediate goods are heterogeneous, and intermediate producers inefficiently allocate production inputs, leading to suboptimal final output. Price heterogeneity and production inefficiencies disappear when $\Delta_t = 1$, which corresponds to the state where aggregate output is at its optimal level.

[7] We have explored a potential approach to incorporate a non-zero steady-state inflation rate with either full or partial indexation. A full indexation with partial indexation to past inflation can be useful for fine-tuning the model but generally reduces risk. On the other hand, less than complete, or trending, inflation indexation tends to yield worse results overall.

[8] The condition that ensures that price dispersion is real-valued is $1-\xi X_t^{1/(\lambda-1)} \geq 0$ and simplifies to $X_t \leq \xi^{(1-\lambda)}$. Another condition ensures that the steady-state value of Δ is real-valued and simplifies to $X_t \leq \xi^{(1-\lambda)\theta/\lambda}$. We solve our model by a perturbation method (with pruning) that provides the Taylor series expansion around a steady state. Given that a solution exists, the constraint is not necessary because the Taylor polynomial exists even beyond the boundary.

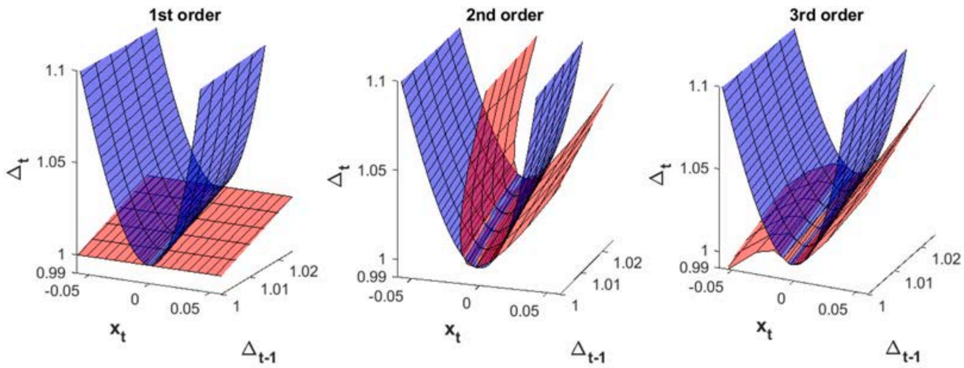

Fig. 2. Price dispersion given baseline calibration of $\lambda = 1.20$, $\theta = 0.65$, $\xi = 0.75$. The red surface denotes simulated surface from perturbation solution (without pruning) of different order. The blue surface denotes theoretical surface based on equation (9).

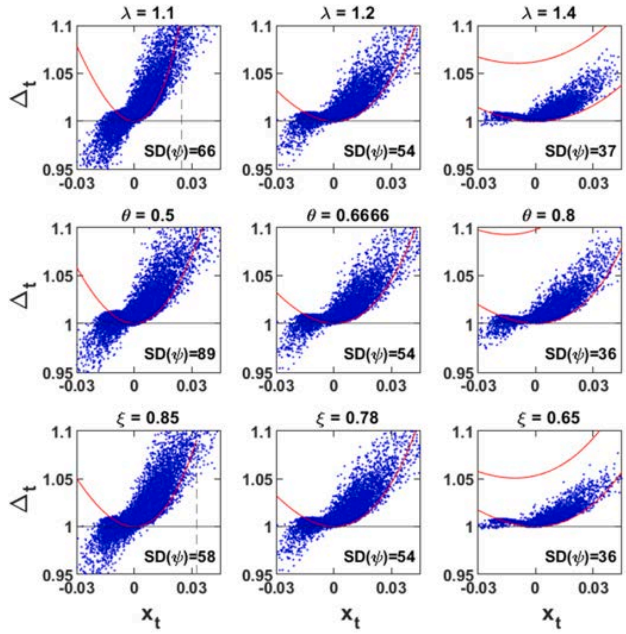

Fig. 3. Price dispersion as a function of x_t, changing markup (λ), share of labor in the production function (θ), and Calvo probability (ξ). The blue dots denote model-implied simulated points (50000 periods). The red lines represent lower and upper theoretical boundary given minimum (equal to 1) and maximum (sample maximum) previous-period price dispersion. Dashed lines represent the theoretical boundary on price dispersion. Baseline calibration (middle column): $\lambda = 1.2$, $\theta = 0.65$ and $\xi = 0.75$.

Following the third-order approximation, Δ_t can be smaller than 1, implying that in the presence of price dispersion, aggregate output can be higher than its optimal level (with no frictions).

To gain further insights into the properties of the third-order approximation of Δ_t (and its relationship with term premium volatility), Fig. 3 depicts its behavior for different calibration specifications, i.e., for different values of λ, θ, and ξ. The bottom-right number reported in each subplot is the standard deviation of the nominal risk premium implied by the parameter specification, while the red lines represent the lower (upper) theoretical boundary of the price dispersion measure for Δ_{t-1} equal to 1 (Δ_{t-1} equal to the sample maximum), and the blue dots are the model-implied simulated points. The subplots of the figure are organized as follows. The three (identical) subplots in the middle column refer to the baseline specification for λ, θ, and ξ, i.e., the Rudebusch and Swanson (2012) values.[9] The subplots in the left (right) column of Fig. 3 report the results for the models where the monopolistic markup (λ),

[9] To keep the simulations comparable, we adjust the model-implied inflation dynamics by assuming a strict monetary policy response to the inflation gap. For λ-plot $\psi_\pi = \{1.76, 1.53, 1.44\}$, for θ-plot $\psi_\pi = \{1.67, 1.53, 1.45\}$, and for ξ-plot $\psi_\pi = \{1.57, 1.53, 1.51\}$ from left to right, respectively. This manipulation helps to obtain the same standard deviation of inflation across the models.

top row, the Cobb-Douglas labor share (θ), middle row, and the Calvo parameter (ξ), bottom row, decrease (increase) with respect to the baseline specification.

A casual inspection of the theoretical boundaries in Fig. 3 reveals their curvatures and widths increase as we move from right-column subplots to left-column subplots, leading to an increase in the variability of price dispersion. These results are in line with intuition. First, a reduction in the monopolistic power of intermediate producers, i.e. lower markup λ, increases firms' heterogeneity because optimizing firms are prone to make higher prices changes to maximize profits. Second, higher price stickiness (higher ξ) implies that an average firm is less frequently given the opportunity to reoptimize its price. This higher reoptimization time leads to higher price heterogeneity due to (i) the increase in the price difference between the newest and oldest optimizing firms and (ii) the forward-looking behavior of optimizing firms that anticipate the higher reoptimization time by setting higher/lower prices. Finally, as θ decreases, the production of intermediates hinges more on firm-specific fixed capital, leading to a higher heterogeneity of marginal costs across firms and, consequently, to higher price dispersion.[10]

We now turn our attention to the performance of the third-order approximation of the model in capturing the theoretical behavior of price dispersion. Focusing on the blue dots in Fig. 3, two general patterns emerge across the subplots. First, the approximated price dispersion fails to capture the U-shaped pattern of the price dispersion function around the positive and negative changes in price deviations. Specifically, while for positive changes in price deviations, the approximated price dispersion follows an increasing path, for negative price deviations, the approximation cloud follows the opposite pattern as the theoretical function. Second, for negative price deviations, the approximated price dispersion tends to go below the theoretical lower bound of one.

4. Alternative specifications

Throughout the previous section, we have been relying on the model structure developed by Rudebusch and Swanson (2012). The model accounts for the patterns of output and inflation movements given the assumptions of (i) Calvo price setting and (ii) firm-specific capital. Altig et al. (2011) emphasize the importance of the latter assumption for generating data-consistent inflation persistence. From a structural perspective, firm-specific capital assumption allows fitting the appropriate New Keynesian Phillips curve slope. However, it comes at the cost of the poorly approximated price dispersion mechanism. In search of a possible resolution to the approximation problem, in this section, we analyze two alternative model specifications. First, we consider the Kimball aggregator, an alternative approach to the real rigidities as a substitute for firm-specific capital proposition. Second, we consider the Rotemberg model, the other workhorse option to introduce nominal rigidities instead of the Calvo assumption.

4.1. Kimball aggregator specification

The first specification that we consider is a Kimball aggregator model. The underlying motivation for this modification is driven by the desire to replace a potentially problematic fixed-capital assumption that, in fact, represents real rigidities in the benchmark model.

4.2. Problem with the fixed-capital assumption

To generate more intuition about the fixed-capital assumption, we consider the linear version of the New Keynesian Phillips Curve (NKPC). It is well known that the linearized NKPC is given by[11]:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \gamma_p \kappa_p \hat{\mu}_t, \tag{10}$$
$$\text{with } \kappa_p = \frac{(1-\xi)(1-\beta\xi)}{\xi} \text{ and } \gamma_p = \left(1 + \frac{1-\theta}{\theta}\frac{\lambda}{\lambda-1}\right)^{-1},$$

where $\hat{\mu}_t$ is the log-deviation of real marginal costs from their steady state, κ_p captures the degree of nominal rigidities in the model, while γ_p reflects the degree of strategic complementarities or the degree of real rigidities. Typically, the slope of the NKPC ($\gamma_p \kappa_p$) is estimated to be a relatively small number. For example, Smets and Wouters (2007), Altig et al. (2011), and Justiniano and Primiceri (2008) provide median estimates around $\gamma_p \kappa_p = 0.01$.

In our benchmark model, we follow Rudebusch and Swanson (2012) and set $\xi = 0.78$, which implies an average price contract duration of four quarters. The resulting degree of nominal rigidities is $\kappa_p = 0.085$. To match the desired value of the NKPC slope, we need to introduce some sort of real rigidities ($\gamma_p < 1$), i.e., firm-specific capital in the benchmark case. Under these considerations, γ_p is an increasing function of the labor share in the production function, θ, and monopolistic markup, λ; thus, we have the objective to lower these two parameters. In the benchmark calibration, we set $\theta = 2/3$ and $\lambda = 1.2$, which imply an NKPC slope of $\gamma_p \kappa_p = 0.016$, slightly higher than the median value in the literature.

Hence, in the fixed-capital specification, we have two contradicting motives. On the one hand, one wants to push these parameters further (decrease θ or λ and/or increase ξ) to lower the NKPC slope and match the inflation persistence observed in the data. Moreover, as θ or λ decreases, we generate higher term premium volatility in line with our objective; see Fig. 3 and discussion in the previous section. On the other hand, we have revealed that such calibrations begin to trigger problematic behavior of the price

10 It is important to note that the same conclusions hold for different levels of RRA.

11 For example, see Galí (2008).

dispersion function that feeds back into the rest of the economy. In fact, the fixed-capital assumption is a cornerstone issue, and an alternative approach to the real rigidities can serve as a potential remedy to the aforementioned problem.

4.2.1. Kimball aggregator

The main idea behind the real rigidities is to make firms averse to large changes in their prices when they are given a chance to reoptimize prices following a Calvo paradigm. A way to introduce such a concept into the model is to introduce a quasikinked demand function as a replacement for the Dixit-Stiglitz aggregator.[12] We allow for a Kimball (1995) type of aggregator, that is, the final goods producer utilizes intermediate goods using the production technology $\int_0^1 G_Y\left(\frac{y_t(f)}{Y_t}\right)df = 1$. We chose the functional form of G_Y to be as in Dotsey and King (2005):

$$G_Y\left(\frac{y_t(f)}{Y_t}\right) = \frac{\phi_p}{1+\psi_p}\left(\left(1+\psi_p\right)\frac{y_t(f)}{Y_t} - \psi_p\right)^{\frac{1}{\phi_p}} + 1 - \frac{\phi_p}{1+\psi_p}, \tag{11}$$

where $\phi_p = \frac{\lambda(1+\psi_p)}{1+\lambda\psi_p}$ and the parameter $\psi_p \leq 0$ determines the degree of curvature of the firm's demand curve. Specifically, when $\psi_p = 0$, we have the case of the Dixit-Stiglitz model as before. The nonlinear equations that follow from this assumption are discussed in Appendix B.

Following Levin et al. (2007), the real rigidities parameter, γ_p, is changed in the linearized NKPC of the model[13]:

$$\gamma_p = \frac{1}{1-\lambda\psi_p}. \tag{12}$$

The introduction of the real rigidities in the form of quasikinked demand lowers γ_p due to the additional term $-\lambda\psi_p$ and makes the NKPC flatter. Specifically, in the quasikinked demand case, the higher markup λ implies higher real rigidities (lower γ_p), and we have the opposite effect from the Dixit-Stiglitz case, i.e., a desire to increase λ to generate a flatter NKPC.

The intuition for the Kimball aggregator mechanism can be demonstrated in comparison to the Disxit-Stiglitz aggregator, a subcase of the Kimball aggregator with $\psi_p = 0$. In the Dixit-Stiglitz case, the demand curve is characterized by the constant demand elasticity formulation:

$$\frac{y_t(f)}{Y_t} = \left(\frac{p_t(f)}{P_t}\right)^{-\lambda/(\lambda-1)}. \tag{13}$$

That is, we have a standard log-linear relation between the relative demand, $y_t(f)/Y_t$, and the relative price, $p_t(f)/P_t$. A decrease or an increase in the (log-)relative price causes a symmetric response of the (log-) relative demand. An assumption about $\psi_p < 0$ brings about a quasikinked demand curve:

$$\frac{y_t(f)}{Y_t} = \frac{1}{1+\psi_p}\left(\frac{p_t(f)}{\vartheta_t P_t}\right)^{-\lambda/(\lambda-1)} + \frac{\psi_p}{1+\psi_p}, \tag{14}$$

where ϑ_t is a Lagrangian multiplier in the firm profit optimization problem; see Appendix B. In this case, a decline in the (log-) relative price induces a smaller increase in the (log-)relative demand, whereas an increase in the (log-)relative price delivers a stronger decline in the (log-)relative demand, namely, we have real rigidities on the demand side, compared to the Dixit-Stiglitz setup. Hence, a firm faces the quasikinked demand function.

4.2.2. Quasikinked demand: price dispersion approximation and the term premium

Using the results from the previous subsection, we make two changes to the benchmark model. First, we relax the fixed-capital assumption, allowing capital to be flexible across firms with the total capital stock fixed, i.e., we no longer have strategic complementarities in style of Altig et al. (2011). Flexible capital allows firms to set the same capital-to-labor ratio and encounter the same marginal costs. Hence, the marginal cost channel that influenced price dispersion via the parameter θ is absent. Second, we implement the quasikinked demand setting that substitutes from recently removed strategic complementarities and helps to maintain the same NKPC slope.

To obtain an impression of the price dispersion approximation in the new Kimball-type model, we compare it with the benchmark results. Fig. 4 portrays the price dispersion for two models, with blue dots representing the Kimball-type model and red dots depicting the benchmark model. In the bottom-right corner we display the standard deviation of the 10-year nominal term premium for both models (red for the baseline model and blue for the Kimball-type model). In the first line, we use the same nominal stickiness $\xi = 0.78$ for both models. The middle subplot refers to the baseline calibration of $\xi = 0.78$, $\theta = 0.6666$, and $\lambda = 1.2$, while in the Kimball-type

[12] Alternatively, one can assume sticky intermediate prices introduced by Basu (1995). We do not analyze this case in the paper.

[13] The term $\frac{1-\theta}{\theta}\frac{\lambda}{\lambda-1}$ in the (previous) definition of γ_p was due to the fixed factors (capital) at the firm level. We relax this assumption and make capital flexible across firms. The main reason that we do this is that under this assumption, it is impossible to find a closed-form solution for the price-setting problem given the arbitrary θ, the labor share in the production function.

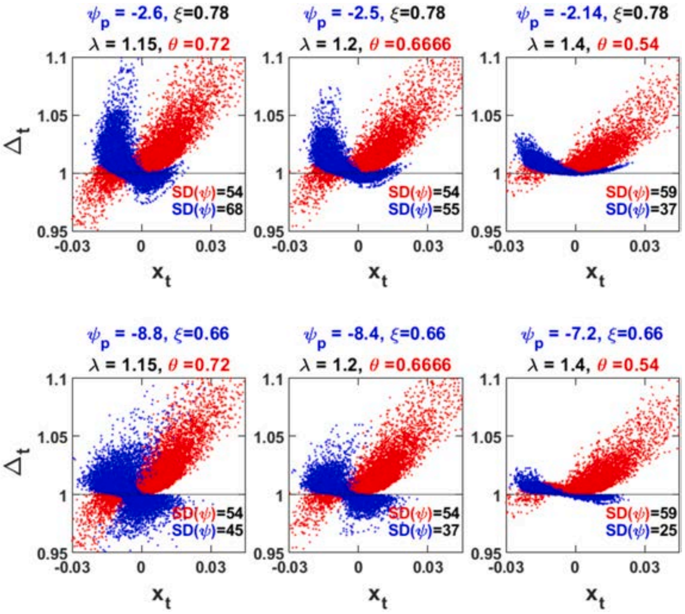

Fig. 4. Price dispersion as a function of x_t, a changing share of labor in the production function (θ) and Kimball curvature (ψ_p). The blue dots represent the Kimball-type model simulated points. The red dots denote the Dixit-Stiglitz-type model simulated points. Baseline calibration (middle column): $\lambda = 1.2$, $\theta = 0.65$, $\xi = 0.75$, and $\psi_p = -2.7$.

model we set $\psi_p = -2.5$ to match the linear NKPC slope.[14] In the left (right) subplot, we deviate from the initial calibration but maintain the same slope of the NKPC; thus, we allow λ and ψ_p to change in the Kimball-type model, and we allow λ and θ to change in the baseline model.

In the baseline calibration, the Kimball-type model demonstrates an improvement in price dispersion approximation compared to the original case. We still observe price dispersion realizations that fall below unity, but the overall dynamics are bounded from below compared to the benchmark model. Regarding the left subplot that has a higher degree of curvature of demand, namely, a more negative ψ_p, two comments can be made. First, we observe how the effect of the quasikinked demand restricts the negative deviation of inflation. Second, more severe violations of the constraint $\Delta_t \geq 1$ may arise. On the contrary, in the right subplot, we see a significant improvement over the benchmark model in terms of price dispersion approximation, but the model falls behind in term premium variability. The intuition behind this result is that we have decreased the nonlinearity of the price dispersion function by increasing the markup (λ) and decreasing the Kimball curvature (ψ_p). This modification decreases the variation in price dispersion, reduces the overall state-dependency, and consequently, decreases the implied term premium volatility.

The second line of Fig. 4 depicts the results for a different calibration. Given a chance to impose a higher degree of real rigidities, we end up lowering the degree of nominal rigidities and set the Calvo parameter $\xi = 0.66$ in agreement with Nakamura and Steinsson (2008). This modification requires the Kimball curvature parameter to be significantly lower than in the first calibration. However, we keep the nominal rigidities unchanged for the benchmark model (red dots). An inspection of the second line of subplots reveals the problematic behavior of the Kimball model when the Kimball curvature coefficient ψ_p is low. Inspecting the middle subplot, we note puzzling patterns in price dispersion realizations for the Kimball-type model, namely, many negative realizations in the right tail, a positive inflation deviation zone. However, in the right subplot we impose a slightly lower curvature of the Kimball aggregator and end up with much more plausible realizations. The other problem is the significant decline in term premium volatility that we experience once we switch to the low nominal rigidities calibration.

To assess the implications of quasikinked demand on the macro variables and term premium volatility, we compare the IRFs to a one-standard-deviation positive productivity shock for the benchmark model (Bench, red) and Kimball-type model (Kimb, blue). We use the baseline calibration $\xi = 0.78$, $\theta = 0.6666$, $\lambda = 1.2$, and $\psi_p = -2.5$. We concentrate on the main differences: consumption, SDF, and term premium responses. First, we know that the price dispersion channel is far less active in the Kimball-type model due to the absence of the fixed-capital assumption. This fact should imply a dramatic fall in IRF state-dependency. However, this is not entirely the case because another mechanism that operates through marginal costs (the quasikinked demand mechanism) is now active. Given the various states of the economy, marginal costs react (decrease) differently in response to the positive productivity

[14] Additionally, we decrease Frisch elasticity from 0.28 (in the baseline model) to 0.20 and increase the persistence of the shock from $\rho_A = 0.96$ (in the baseline model) to $\rho_A = 0.98$ in our Kimball-type model in every subplot. This adjustment generates comparable inflation variation and term premium variation in the Kimball-type model while maintaining the same quantity of risk (standard deviation of the shock) across the models.

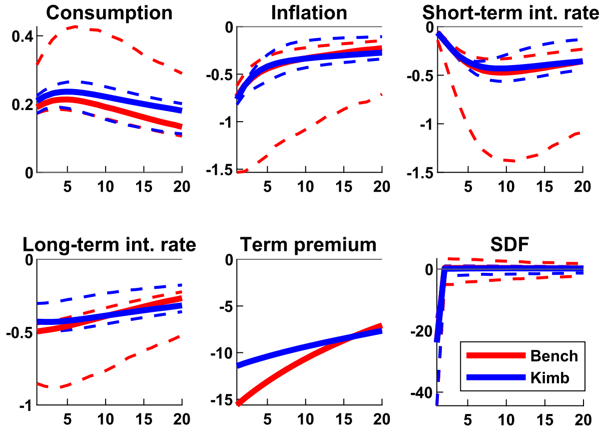

Fig. 5. Nonlinear impulse response functions to a one-standard-deviation positive productivity shock. The results are plotted for the benchmark (Bench, red) and Rotemberg (Rot, blue) models. The solid line represents the median impulse response, while the dashed lines represent 5% and 95% bands.

shock. Therefore, based on marginal costs and due to the quasikinked demand, the intermediate firm has distinct incentives to cut prices, which, in turn, creates optimal price and inflation state-dependency. The aforementioned mechanism complements the price dispersion mechanism, and its power increases as the Kimball curvature ψ_p decreases. Nevertheless, in the Kimball case, the conditional variance of the SDF falls slightly more on impact, and we observe an upward shift in the median consumption response. Finally, due to the lower state-dependency of the SDF, the response of the term premium is weakened.

4.3. Rotemberg specification

Our second alternative specification resides on the nominal rigidities proposed by Rotemberg. In the Rotemberg setup, we assume that a monopolistic firm faces a quadratic cost of adjusting nominal prices measured in terms of final output. Hence, unlike the Calvo model, shocks do not produce an inefficient distribution of prices but introduce an additional output-consumption wedge. However, the role of the wedge is similar to that of price dispersion, namely, an inefficient decrease in aggregate consumption as a cost of inflation. The output-consumption wedge, Δ^R, is summarized by the following equation:

$$\Delta_t^R = \frac{1}{1 - \frac{\phi}{2}(X_t - 1)^2} \geq 1. \tag{15}$$

Note that the wedge function is no longer a state variable (does not have a lagged component) but still a nonlinear function of steady-state-adjusted inflation, X_t. Furthermore, $\phi > 0$ determines the degree of nominal price rigidity and can be mapped to the Calvo stickiness parameter ξ. Specifically, Ascari and Rossi (2012) prove that Calvo and Rotemberg models have the same dynamics if (i) both models are log-linearized and (ii) trend inflation is zero. Under these conditions, the mapping is given by:

$$\phi = \frac{(\theta + \frac{(1-\theta)\lambda}{\lambda-1})(\frac{\lambda}{\lambda-1} - 1)\xi}{\theta(1-\xi)(1-\beta\xi)}. \tag{16}$$

Note that a decrease in markup, λ, or the Cobb-Douglas labor share, θ, increases ϕ. This is a direct link between the nonlinearity of the price dispersion, Δ, and the curvature of the output-consumption wedge, Δ^R, that is implied by the firm-specific capital assumption.

4.3.1. Output-consumption wedge and term premium

To further understand the differences generated by the Rotemberg assumption in the third-order approximation of Δ_t^R and its relationship with term premium volatility, Fig. 6 depicts price dispersion and output-consumption wedge behavior for different calibrations. Similar to the Kimball model section, we consider three alternative calibrations that keep the NKPC slope intact. In the benchmark model, we vary the combination of markup λ and labor share θ, whereas in the Rotemberg model, the degree of nominal price rigidity ϕ is also changed according to the mapping in equation (16). The rest of the parameters are identical in both models.

As Fig. 6 shows, the output-consumption wedge in the Rotemberg specification performs in line with intuition on the cost of inflation. First, unlike the Calvo case in the benchmark model, it follows a U-shaped pattern with the minimum in $x_t = 0$ an optimal state. Second, the specification mostly respects the lower bound of 1. Third, despite being a contemporaneous, non-state relation (see equation (15)), it demonstrates some fuzziness, especially on the edges, where the approximation is less precise. This evidence suggests that the Rotemberg specification is not free from approximation error in the left tail similar to the benchmark model, but the misapproximation itself implies less counterintuitive outcomes for the output-consumption wedge. Finally, in the bottom-right corner of each subplot, we can see that the Rotemberg specification does not hinder term premium volatility, although the volatility is somewhat lower than in the benchmark model.

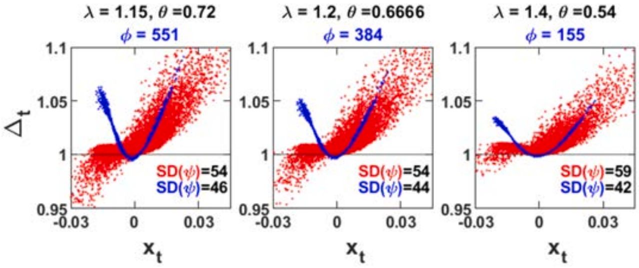

Fig. 6. Price dispersion or output-consumption wedge as a function of x_t, changing markup (λ) and the share of labor in the production function (θ). The blue dots represent the Rotemberg model simulated points. The red dots denote the benchmark model simulated points. Baseline calibration (middle column): $\lambda = 1.2$, $\theta = 0.6666$, $\xi = 0.78$, and $\phi = 384$.

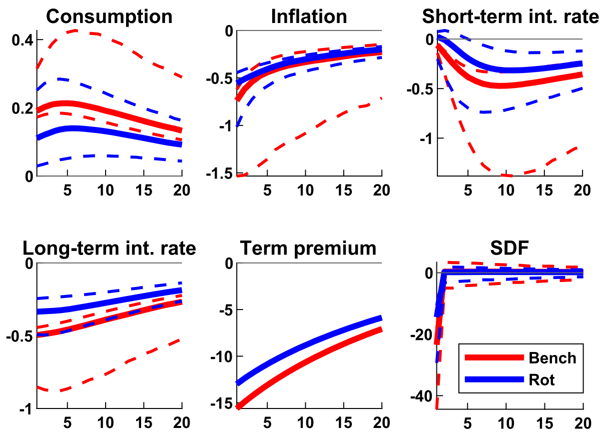

Fig. 7. Nonlinear impulse response functions to a one-standard-deviation positive productivity shock. The results are plotted for the benchmark (Bench, red) and Rotemberg (Rot, blue) models. The solid line represents the median impulse response, while the dashed lines represent 5% and 95% bands.

The lower standard deviation of the term premium is connected with the lower conditional heteroskedasticity in the Rotemberg case. To demonstrate this, in Fig. 7, we plot the IRFs to the one-standard-deviation positive productivity shock for both models. A general inspection of the mass bands suggests that state-dependency is lower in the Rotemberg case. Specifically, the lower state-dependency and median response of the SDF has two major implications. First, a lower decline in the conditional heteroskedasticity of the SDF compared to the benchmark model triggers a lower decrease in precautionary savings demand, and in turn, the median response of consumption is lower. Second, the same reduced decrease in conditional heteroskedasticity generates a lessened response by the term premium. The rest of the variables behave in line with common intuition: the inflation state-dependency and initial response is milder due to the more restricted nature of the output-consumption wedge compared to price dispersion; the short-term interest rate reaction follows the shortened response of output (consumption) and inflation according to the monetary policy rule.

5. Discussion

Thus far, we have shown that (i) the Calvo model with firm fixed capital can account for high risks but is flawed in terms of approximation; (ii) the Kimball-type model is a (potential) remedy in terms of approximation precision but is less successful in generating term premium dynamics; and (iii) the Rotemberg model can successfully account for the intuition on the cost of inflation channel while maintaining comparable term premium dynamics. Our ultimate goal is discriminated among these models and formulates practical advice for the future work. In this section, we advance in two ways. First, we use a measure of an approximation error that helps us to compare the models. Second, we offer a discussion on the general and practical implications of each mechanism and their relation to the term premium generating motives.

5.1. Approximation errors

We formulate the measure of approximation error as follows. First, we take the realization of the cost of inflation variable (price dispersion in the benchmark and Kimball models and the output-consumption wedge in Rotemberg model) implied by the perturbation solution. Second, we compute the equation-implied value for each case taking the inputs for the equation from the perturbation solution. See the details for each model in Appendix D. The difference between these two realizations yields intuition

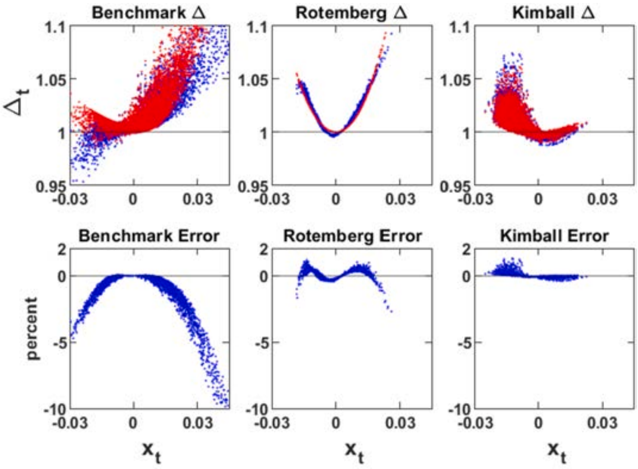

Fig. 8. Approximation error implied by the price dispersion equation.

about the one-period approximation error. When presented in percent, it roughly shows how the approximation underrepresents (overrepresents) the impact of the cost of inflation variable on final consumption.[15]

We report the results for each specification in Fig. 8. The first line of subplots depicts the model-implied values (blue dots) and equation-implied values (red dots) of price dispersion or the output-consumption wedge. The second line portrays the absolute errors in percent. Each model is calibrated according to the benchmark calibration of $\lambda = 1.2$, $\theta = 0.6666$, $\xi = 0.78$, $\phi = 384$, and $\psi_p = -2.5$.[16]

The left column shows results for the benchmark Calvo case, and we see that the error takes predominantly negative realizations as inflation (x_t) deviates from its steady state. This evidence suggests that we gravely underestimate the price dispersion mechanism and the decrease in output should be up to 10% higher.

The middle column presents results for the Rotemberg model. Two comments are in order. First, the red dots clearly show that equation-implied values are not state-dependent, while the perturbation-implied (blue dots) values are fuzzy. Second, although the output wedge plot (top middle) appears to be more in line with theory than the Calvo case, the error plot (bottom middle) reveals the same (mis-)approximation patterns for larger inflation deviations. Note that once brought to data, the Rotemberg model should experience a higher inflation standard deviation (so is x_t), and the impact of the tails should be more prominent. Consequently, the approximation of the Rotemberg-type model tends to underestimate the impact on output reduction in when facing large inflation deviations, whereas for moderate deviations, the impact is overestimated by a small margin (up to 1%).

Finally, the right column portrays the results for the Kimball model. The price dispersion plot reveals that the solution values (blue) are very close to the equation values (red). Looking at the bottom-right plot, we can see that the model delivers disproportionally lower approximation error. Specifically, on the positive deviation side, the model tends to consistently underrepresent the price dispersion channel by a small margin, while on the negative side, the price dispersion is overestimated. However, the top-right subplot reveals a possible caveat in the results, namely, the equation-implied values are also not consistent. The explanation for that fact is that the Kimball model offers much more complex nonlinear state-dependent mechanism, the approximation error accumulates more gradually, and the equation-implied results do not change dramatically.

Ultimately, the benchmark model with Calvo price setting has a flawed approximation that suggests price dispersion values lower than unity and with high approximation error. The model with Rotemberg price setting may appear a good alternative at first, as it does not suffer from lower than unity values and generally follows an economically motivated shape of the function. The analysis of the approximation error reveals that it can be subject to underestimation of the price dispersion mechanism in the tails similar to the Calvo model. Finally, the introduction of real rigidities in the form of the Kimball aggregator can help to overcome the problems of (i) an excessively active price dispersion channel and (ii) inaccurate price dispersion approximation while keeping competitive state-dependency in the model and delivering acceptable term premium volatility.

5.2. Further discussion

Here, we highlight the practical implications of every model. Because we solve the models up to the third order, it becomes crucial to maintain a limited number of state variables within the model. The issue becomes more relevant during an estimation process that requires multiple evaluations of the model. With this in mind, the Rotemberg model is the best candidate because it introduces

[15] This measure of the approximation error does not exactly give the approximation error value. For example, in benchmark case, Δ_t is a function of state variable Δ_{t-1} and contemporaneous inflation, π_t. Thus, when taking inflation as given, the feedback effects are neglected. There is no trivial way to express inflation as a function of state variables; therefore, we proceed with the former approach.

[16] For the Kimball model, we also adjust the Frisch elasticity and shock persistence in line with Subsection 4.2.2.

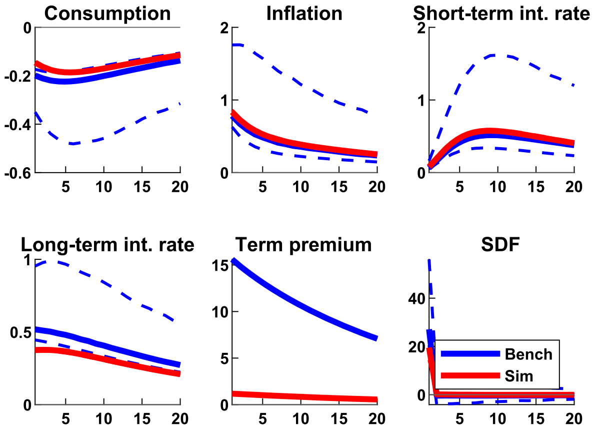

Fig. 9. Nonlinear impulse response functions to a one-standard-deviation negative productivity shock. The results are plotted for the Benchmark (Bench, blue) and Simplified (Sim, red) models. The solid line represents the median impulse response, while the dashed lines represent 5% and 95% bands. For the Simplified model, the median response almost coincides with the bands.

nominal rigidities in a parsimonious way without additional states. The Calvo model is ranked next and generates a new state – price dispersion. Finally, the Kimball-type model introduces at least four new states. Given that the solution time of the nonlinear model grows exponentially with every additional state variable, see Levintal (2017), we should keep the model as parsimonious as possible. Consequently, the Kimball-type model can be a viable option in small-scale models or when one has access to higher computational power.

Additionally, following our sensitivity exercise in Fig. 3, it is sensible to avoid setting the core parameters close to the extreme values. Such calibrations would imply a reduction in term premium volatility, and then complementary mechanisms should be introduced. Therefore, when deciding on the best approach, further modifications should be taken into account. For example, uncertainty shocks should be a viable mechanism to bolster the quantity of risk component and to have a significant impact on term premium variation. Consequently, in his paper, Oh (2020) analyzes the effects of uncertainty shocks in the Calvo and Rotemberg specifications and concludes that the Rotemberg model behaves more in line with empirical results. Given its superior approximation precision, practical plausibility, and empirical consistence with respect to uncertainty shocks, the Rotemberg specification should be preferred.

6. Conclusions

A New Keynesian DSGE model with staggered price setting can reproduce the observed moments of the term premium. However, this comes at a cost. The model becomes internally inconsistent by generating third-order approximations of the price dispersion function that are at odds with intuition and theoretical boundaries. We propose ways to alleviate these issues.

The first is a careful calibration. When the main parameters that define the crookedness of the price dispersion function are $\lambda > 1.2$ (markup), $\theta > 2/3$ (Cobb-Douglas labor share), and $\xi < 0.75$ (Calvo probability), we may minimize the consequences of the approximation errors. The state-dependency implied by the price dispersion function can generate heterogeneity in the SDF and translate it into term premium variability. In that sense, a model with volatile price dispersion is favorable to generate volatile term premiums but is prone to numerical issues.

Hence, in addition to a proper calibration, we propose a more structural remedy to the numerical issues and excess state-dependency due to price dispersion. First, we suggest enhancing the real rigidities channel. A quasikinked demand function in the form of the Kimball aggregator provides an appealing alternative to the price dispersion mechanism implied by the Dixit-Stiglitz aggregator. It keeps the term premium results intact while resolving the approximation error problem. Second, we make use of a Rotemberg specification for nominal rigidities and demonstrate that it mitigates a major part of approximation problem and is generally the preferred alternative over the Calvo model.

Appendix A. The IRFs to a negative productivity shock

The blue (red) line in Fig. 9 depicts the IRFs to a one-standard-deviation negative productivity shock in the original (simplified) model for aggregate output (in %), the annualized inflation rate (in %), the annualized short-term interest rate (in %), the 10-year yield on nominal zero-coupon bonds (in %), the 10-year nominal term premium for such bonds (in ann.bp.), and the model-implied stochastic discount factor (in %). Despite the fact that a third-order solved model produces non-symmetric results, the IRFs to a positive shock demonstrate similar patterns, and the same logic may be applied to explain the IRFs, see Fig. 1 in the main text.

Appendix B. Quasi-kinked demand: nonlinear equations

To obtain the model equations for the Kimball aggregator we majorly follow Harding et al. (2022). However, most of the derivations could be found in the Technical Appendix of Lindé and Trabandt (2018).

The price-setting problem of firms determines the optimal relative price p^* that is given by:

$$s_t = f_t p_t^* - a_t (p_t^*)^{1+(1+\theta_p)(1+\psi_p)/\theta_p},$$

where s_t, f_t and a_t are helping variables that evolve according to:

$$s_t = \frac{(1+\psi_p)(1+\theta_p)}{1+\psi_p+\theta_p\psi_p}\mu_t y_t \vartheta_t^{(1+\theta_p)(1+\psi_p)/\theta_p} + \beta\xi E_t (e^{-x_{t+1}})^{-(1+\theta_p)(1+\psi_p)/\theta_p} s_{t+1},$$

$$f_t = y_t \vartheta_t^{(1+\theta_p)(1+\psi_p)/\theta_p} + \beta\xi E_t (e^{-x_{t+1}})^{-(1+\psi_p+\psi_p\theta_p)/\theta_p} f_{t+1},$$

$$a_t = \frac{\psi_p\theta_p}{1+\psi_p+\theta_p\psi_p} y_t + \beta\xi E_t e^{-x_{t+1}} a_{t+1}.$$

Here $\theta_p = \lambda - 1$ is a net markup and $x_t = \pi_t - \bar{\pi}$ is similar to the basic Calvo model.

Zero profit condition and aggregate price index are given by:

$$\vartheta_t = 1 + \psi_p - \psi_p \Delta_{t,2},$$

$$\vartheta_t = \Delta_{t,3}.$$

Finally, different measures of price dispersion. The overall price dispersion used in aggregate production function Δ_t:

$$\Delta_t = \frac{\vartheta_t^{(1+\theta_p)(1+\psi_p)/\theta_p}}{1+\psi_p} \Delta_{t,1}^{-(1+\theta_p)(1+\psi_p)/\theta_p} + \frac{\psi_p}{1+\psi_p}.$$

The helping price dispersion $\Delta_{t,1}$, $\Delta_{t,2}$, and $\Delta_{t,3}$:

$$\Delta_{t,1}^{-(1+\theta_p)(1+\psi_p)/\theta_p} = (1-\xi_p)(p_t^*)^{-(1+\theta_p)(1+\psi_p)/\theta_p} + \xi \left(e^{-x_t}\Delta_{t-1,1}\right)^{-(1+\theta_p)(1+\psi_p)/\theta_p},$$

$$\Delta_{t,2} = (1-\xi_p) p_t^* + \xi e^{-x_t} \Delta_{t-1,2},$$

$$\Delta_{t,3}^{-(1+\psi_p+\psi_p\theta_p)/\theta_p} = (1-\xi_p)(p_t^*)^{-(1+\psi_p+\psi_p\theta_p)/\theta_p} + \xi \left(e^{-x_t}\Delta_{t-1,3}\right)^{-(1+\psi_p+\psi_p\theta_p)/\theta_p}.$$

Appendix C. Rotemberg price-setting

In the Rotemberg model, each firm faces an identical quadratic adjustment cost problem:

$$\max_{L_t(f),P_t(f)} \mathbb{E}_t \sum_{j=0}^{\infty} m_{t,t+j} \left[\frac{P_{t+j(f)}}{P_{t+j}} Y_{t+j}(f) - W_{t+j} L_{t+j}(f) - \frac{\phi}{2}\left(\frac{P_{t+j(f)}}{P_{t+j-1(f)}}\frac{1}{\bar{\Pi}} - 1\right)^2 Y_{t+j}\right],$$

subject to demand function, given by equation (2), and the production function, given by equation (3). The resulting nonlinear first-order condition can is

$$\phi(\frac{\Pi_t}{\bar{\Pi}} - 1)\frac{\Pi_t}{\bar{\Pi}} y_t = \frac{-1}{\lambda-1} Y_t + \frac{\lambda}{\lambda-1}\mu_t y_t + \phi \mathbb{E}_t m_{t+1} (\frac{\Pi_{t+1}}{\bar{\Pi}} - 1)\frac{\Pi_{t+1}}{\bar{\Pi}} y_{t+1}.$$

The final output costs paid by the optimizing firms have an impact on the market clearing condition that is given by:

$$\frac{Y_t}{\Delta_t^R} = C_t + G + \bar{I},$$

with the output consumption wedge, Δ_t^R, being:

$$\Delta_t^R = \frac{1}{1 - \frac{\phi}{2}(X_t - 1)^2} \geq 1.$$

Appendix D. Measure of approximation error

D.1. Benchamrk Calvo model

We start by computing the equation-implied price dispersion according to

$$\Delta_t^{eq} = \left((1-\xi)^{1-\lambda/\theta} \left(1 - \xi \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{1/(\lambda-1)}\right)^{\lambda/\theta} + \xi \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\lambda/((\lambda-1)\theta)} \Delta_{t-1}^{1/\theta} \right)^{\theta},$$

and treating Π_t and Δ_{t-1} as given in the simulated sample of the perturbation-solved model. Then we take the realization of the current period price dispersion Δ_t and compute the approximation error:

$$Err^{Bench} = 100(\Delta_t - \Delta_t^{eq}).$$

D.2. Rotemberg model

In a similar fashion, we compute the equation-implied output-consumption wedge:

$$\Delta_t^{R,eq} = \left[1 - \frac{\phi}{2}\left(\frac{\Pi_t}{\bar{\Pi}} - 1\right)^2\right]^{-1},$$

and treat Π_t as given. The resulting error is:

$$Err^{Rotemberg} = 100(\Delta_t^R - \Delta_t^{R,eq}).$$

D.3. Kimball model

In the Kimball model, we make use of the equation for the total price dispersion to compute the equation values:

$$\Delta_t^{eq} = \frac{(\Delta_{t,3}^{eq})^{(1+\theta_p)(1+\psi_p)/\theta_p}}{1+\psi_p}(\Delta_{t,1}^{eq})^{-(1+\theta_p)(1+\psi_p)/\theta_p} + \frac{\psi_p}{1+\psi_p},$$

where the helping price dispersion $\Delta_{t,1}^{eq}$ and $\Delta_{t,3}^{eq}$ are defined by:

$$(\Delta_{t,1}^{eq})^{-(1+\theta_p)(1+\psi_p)/\theta_p} = \left(1-\xi_p\right)(p_t^*)^{-(1+\theta_p)(1+\psi_p)/\theta_p} + \xi\left(e^{-x_t}\Delta_{t-1,1}\right)^{-(1+\theta_p)(1+\psi_p)/\theta_p},$$
$$(\Delta_{t,3}^{eq})^{-(1+\psi_p+\psi_p\theta_p)/\theta_p} = \left(1-\xi_p\right)(p_t^*)^{-(1+\psi_p+\psi_p\theta_p)/\theta_p} + \xi\left(e^{-x_t}\Delta_{t-1,3}\right)^{-(1+\psi_p+\psi_p\theta_p)/\theta_p},$$

while treating the optimal price, p_t^*, the steady-state adjusted inflation, $x_t = \pi_t - \bar{\pi}$, and lagged helping price dispersion variables, namely, $\Delta_{t-1,1}$ and $\Delta_{t-1,3}$, as given in the simulated sample. Finally, the error is defined by:

$$Err^{Kimball} = 100(\Delta_t - \Delta_t^{eq}).$$

Appendix E. Simulating the model and generating IRFs

For our simulations, we conducted model simulations for a total of 50100 periods and discarded the initial 100 periods as a standard warming period, following common practice in Dynare. We maintained the same random seed throughout, ensuring that shock paths remained consistent across models and calibrations. However, in unreported exercises, we verified that the results followed the same underlying logic even when different shock paths were utilized, provided that the simulation length was sufficiently long. This procedure was utilized to generate Fig. 3, Fig. 4, Fig. 6, and Table 1.

Our procedure for generating impulse response functions (IRFs) closely follows the procedure provided by the Dynare software. It can be summarized as follows:

1. Generate a random shock path of 120 periods.
2. Perform simulation of the model Y_1.
3. Add one standard deviation to the simulated series for productivity shock in period 101.
4. Perform simulation of the model Y_2.
5. The IRF is equal to $Y_2 - Y_1$.
6. Initial 100 periods are warming periods and we discard them.
7. In order to obtain a full distribution of the IRFs, we perform 5000 replications.
8. Dynare presents only the mean responses. Instead, we compute 5% and 95% percentiles responses along with the median response and report it in the plots.

We use this procedure to generate Fig. 1, Fig. 5, and Fig. 7.

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