To what extent is resampling useful in portfolio management?

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Abstract

We take a new look at the resampled efficiency™ technique developed by Michaud (1998) and compare it with the Markowitz mean-variance portfolio construction technique by assessing the performance of three representative portfolios, i.e. the global minimum variance portfolio, the intermediate return portfolio, and the maximum return portfolio. We show that resampling leads to more stable and more diversified portfolios. However, the out-of-sample analysis shows that resampling does not systematically increase (decrease) the risk-adjusted performance (turnover) of the portfolios.

Keywords: sampling error, resampling, Markowitz, Michaud.

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1. Introduction

The mean-variance optimization technique has been shown to significantly depend on the values of a set of inputs, i.e. the means and the variance-covariance matrix. Since the means and the (co)variances are typically estimated from historical samples, these estimates are likely to be affected by sampling error which in turn bears on the composition of the portfolio. In order to improve the Markowitz optimization technique and address estimation risk, Michaud (1998) proposes the *resampled efficiency* method which is a statistical resampling procedure based on the well-known bootstrapping procedure:[3]

The crucial aspect to understand is that the resampling procedure consists in computing "statistically equivalent" efficient frontiers: All the simulated efficient frontiers are associated with the original set of data inputs, used in the simulation process. As such, the statistical equivalence region displays the instability and ambiguity of the traditional mean-variance optimization procedure. A practical implication is that a portfolio lying in the statistical equivalence region would not require any rebalancing, allowing investors to avoid ineffective transaction costs.

In this paper, we compare the resampling method (based on the parametric bootstrap) with the Markowitz mean-variance portfolio construction technique by assessing the performance of three representative portfolios, i.e. the global minimum variance portfolio, the intermediate return portfolio, and the maximum return portfolio. The sample period ranges from January 1980 to December 2008, yielding a total of 348 observations. For each of the 9 asset classes given in Table 1, we compute monthly log returns as well as the variance-covariance matrix. The in-sample and out-of-sample analyses indicate that the resampling technique enhances the allocation but not the risk-adjusted performance and turnover of the portfolios.

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[3] Resampled Efficiency optimization was co-invented by Richard Michaud and Robert Michaud, U.S. patent 6,003,018. New Frontier Advisors, LLC (NFA) is an exclusive worldwide licensee.

Table 1

Asset class for empirical analysis	
Asset class	
CASH - EURO	BONDS - EURO GOVT
EQUITIES - US	BONDS - US GOVT
EQUITIES - US Small Cap	BONDS - US CREDIT Inv Grade
EQUITIES - EUROPE	REAL ESTATE - US
EQUITIES - JAPAN	

Source: Datastream.

2. In-sample analysis

We start our in-sample analysis by generating mean-variance and resampled portfolios with respect to the full 348 observations i.e., T=348. The resampled portfolios are obtained by averaging on 500 scenarios. So, in our case, S=500.

2.1. Statistical equivalence region

The statistical equivalence region is plotted in the following way. First, we compute the mean-variance efficient frontier (in red) from the original set of inputs i.e. the sample historical means (μ^h) and covariances ($\sigma_{ij}{}^h$). So, only the weights computed with the Markowitz equations are optimal regarding this original set of inputs. Second, we apply the resampling algorithm. In this case, each new weight can be interpreted as a set of statistically equivalent weights regarding the mean-variance efficient weights. In combination with the original set of inputs, all resampled portfolio weights (in blue) will form frontiers below the mean-variance efficient frontier. Hence,

Figure 2 shows the mean-variance efficient frontier together with the statistical equivalence region, the shape of which looks like a "comet". It -gives us an immediate idea on how sampling errors can affect the determination of an efficient frontier. This figure demonstrates that even small changes in the sample data can cause significant changes in the mean-variance efficient curve.

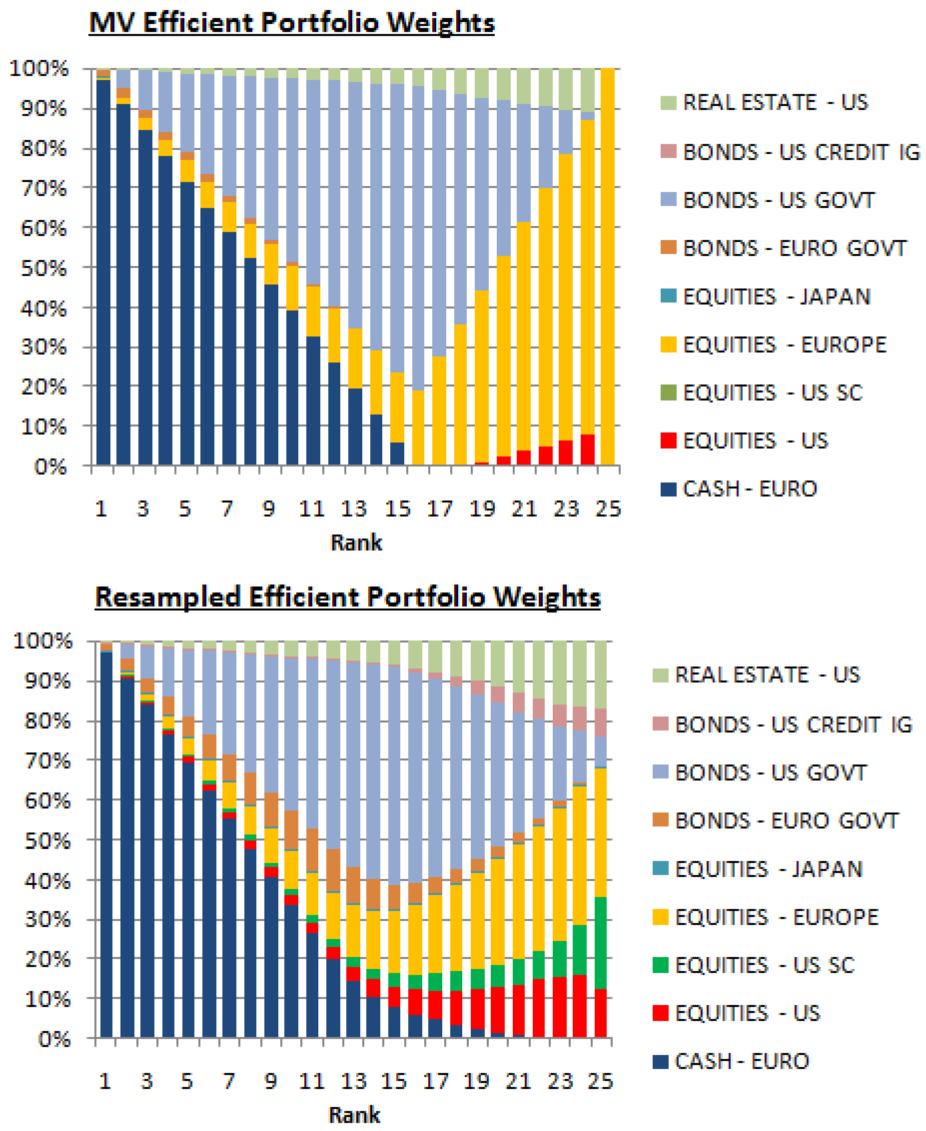
Figure 2

We notice that the dispersion increases as we move from the lower to the upper part of the efficient frontier. The distance between portfolios on the mean-variance frontier and their resampled analogues is indeed increasing as we move to the right in the risk–return space. This result is consistent with the higher level of estimation error that characterizes intermediate and extreme portfolios in comparison with low-risk portfolios.

2.2. Allocation

Figure 3 shows the mean-variance efficient allocation together with the resampled efficient allocation.

Figure 3



The first striking result is that resampled portfolios exhibit greater diversification than the mean-variance portfolios. According to the mean-variance procedure, an intermediate portfolio (rank 13) includes four assets, while nine asset classes are included in the resampling procedure.

The second result is that resampled portfolios show smooth transitions (i.e. less-sudden shifts) in allocation along the resampled frontier. For the mean-variance procedure, notice the undesirable difference in weights between portfolios of rank 24 and rank 25: at least 20% of portfolio composition is changed. In addition, as we move from portfolios of rank 1 to portfolios of rank 25, the mean-variance approach provides an extreme portfolio fully invested in European equities. A portfolio with these characteristics is likely to maximize sampling errors and exhibit poor out-of-sample performance. In contrast to the mean-variance allocation, diversification is preserved in the resampling procedure where resampled portfolios show a tendency towards the 'equities-Europe' asset class.

These results show that resampled portfolios have desirable characteristics for investors. The low degree of diversification and the sudden shifts in the allocation along portfolios are undesirable characteristics of the mean-variance portfolios.

2.3. Statistical analysis

Table 3 focuses on the weights of three specific portfolios: the Global Minimum Variance portfolio (GMV); which ranks as the 1st portfolio, the Intermediate Return portfolio (I); which ranks as the 13th portfolio and the Maximum Return portfolio (M); which ranks as the 25th portfolio.

The first column of this table displays the weights to invest in each asset according to the resampling method. The second and third column shows the extreme weights we get among the 500 scenarios of the resampling procedure. Finally, the last column displays the weights to invest in each asset according to the classical mean-variance method.

Table 3

Statistical Analysis: Global Minimum Variance Portfolio				
Asset Name	Resampled Efficient Portfolio Weights	5th percentile	95th percentile	MV Efficient Portfolio Weights
CASH - EURO	97,48%	96,17%	98,75%	97,60%
EQUITIES - US	0,06%	0,00%	0,39%	0,01%
EQUITIES - US SC	0,02%	0,00%	0,16%	0,00%
EQUITIES - EUROPE	0,06%	0,00%	0,34%	0,05%
EQUITIES - JAPAN	0,23%	0,00%	0,52%	0,27%
BONDS - EURO GOVT	1,93%	0,71%	3,16%	2,01%
BONDS - US GOVT	0,02%	0,00%	0,09%	0,00%
BONDS - US CREDIT IG	0,04%	0,00%	0,33%	0,00%
REAL ESTATE - US	0,15%	0,00%	0,54%	0,06%
Statistical Analysis: Intermediate Return Portfolio				
Asset Name	Resampled Efficient Portfolio Weights	5th percentile	95th percentile	MV Efficient Portfolio Weights
CASH - EURO	14,55%	0,00%	44,09%	19,48%
EQUITIES - US	3,55%	0,00%	17,39%	0,00%
EQUITIES - US SC	2,32%	0,00%	15,31%	0,00%
EQUITIES - EUROPE	13,16%	0,00%	33,47%	15,16%
EQUITIES - JAPAN	0,38%	0,00%	2,89%	0,00%
BONDS - EURO GOVT	9,37%	0,00%	40,51%	0,00%
BONDS - US GOVT	51,47%	22,93%	70,46%	61,85%
BONDS - US CREDIT IG	0,32%	0,00%	0,00%	0,00%
REAL ESTATE - US	4,88%	0,00%	20,30%	3,51%
Statistical Analysis: Maximum Return Portfolio				
Asset Name	Resampled Efficient Portfolio Weights	5th percentile	95th percentile	MV Efficient Portfolio Weights
CASH - EURO	0,00%	0,00%	0,00%	0,00%
EQUITIES - US	12,40%	0,00%	100,00%	0,00%
EQUITIES - US SC	23,40%	0,00%	100,00%	0,00%
EQUITIES - EUROPE	32,40%	0,00%	100,00%	100,00%
EQUITIES - JAPAN	0,40%	0,00%	0,00%	0,00%
BONDS - EURO GOVT	0,00%	0,00%	0,00%	0,00%
BONDS - US GOVT	7,20%	0,00%	100,00%	0,00%
BONDS - US CREDIT IG	7,20%	0,00%	100,00%	0,00%
REAL ESTATE - US	17,00%	0,00%	100,00%	0,00%

Both methods give for the GMV portfolio almost the same composition. A majority has to be invested in the 'Cash-Euro' index (around 97,5%). This is a traditional result. The 'Cash-Euro' index has the lowest variance and is the least correlated. The narrowness of the intervals between the 5th percentile and the

95th percentile show the high level of confidence we can have in the weights. The GMV portfolio is likely to be less affected by sampling errors.

Compared with Portfolio GMV, Portfolio I is invested in more risky assets. The 'Bonds - Us Govt' asset class receives the highest weight in both methods. Nevertheless, the composition of portfolio I differs significantly between the two methodologies. The magnitude of the intervals between the 5th percentile and the 95th percentile increases. Since these assets have higher historical variance, they are more likely to be far from their historical values among all different scenarios in the simulation process. In other words, the riskier is the portfolio, the higher is the estimation risk.

Regarding portfolio M, the composition is totally different between the two methods. The mean-variance optimization gives a one-asset portfolio composed by the 'Equities-Europe' asset class, while the resampling optimization provides a portfolio composed by 7 asset classes. Again, the statistical analysis proves that estimation risk affects riskier portfolios to a larger extent as the magnitude of the intervals between the 5th percentile and the 95th percentile vary from 0% to 100%.

3. Out-of-sample analysis

We now perform a rolling out-of-sample simulation to compare the performance of the two asset allocation strategies. Two lengths for the in-sample estimation window are considered: $T = 30$ and $T = 90$. Simulations are performed for the global minimum variance portfolio, the intermediate return portfolio and the maximum return portfolio. Optimal portfolios are computed for each period, then the sample period is moved forward one month and optimization process is repeated. Subsequently, we compute the realized returns generated by the optimal portfolios. Finally, we compute the average realized returns and the average risk of the portfolios across the out of sample period. In that manner, we are able to compute an average realized Sharpe ratio that enables us to compare the performance and turnover of the two allocation strategies.

3.1. Performance

The out-of-sample performance measured by the average Sharpe ratios is summarized in Table 4.

Table 4

	Mean-variance			Resampling		
	Return	Risk	Sharpe Ratio	Return	Risk	Sharpe Ratio
T=30						
GMV	0,556%	0,124%	4,465	0,567%	0,124%	**4,561**
I	0,833%	2,106%	0,396	0,894%	1,827%	**0,489**
M	0,912%	4,996%	0,183	0,918%	4,114%	**0,223**
T=90						
GMV	0,548%	0,170%	3,222	0,551%	0,169%	**3,264**
I	0,806%	1,734%	**0,465**	0,784%	1,711%	0,458
M	0,959%	4,155%	**0,231**	0,914%	4,040%	0,226

For an in-sample period of T = 30, the resampling strategy performs better than mean-variance strategy in all portfolios. In this case, the estimation period is quite short and it is therefore reasonable to expect that the effect of estimation risk will be more significant, penalizing mean-variance and favoring resampling methods.

When the in-sample period is set to T=90, we are not able to conclude that the resampling procedure outperforms the mean-variance procedure. The Sharpe ratio of the resampling procedure is indeed higher only for the global minimum variance portfolio in T=90.

Hence, the larger the sample size, the better the performance of the mean-variance portfolios with respect to the resampling procedure. Of course, this result could depend on the sample period considered. For example, taking extreme positions (as the Markowitz procedure often leads to) can be highly rewarding in trending markets.

3.2. Turnover

Finally, we evaluate the turnover of each portfolio. The average turnover measures the rate of trading activity across portfolio assets. As such, it represents the percentage of portfolio that is bought and sold in exchange for other assets. Turnover is computed as the sum of the absolute values of purchases and sales during a pre-set time period, divided by 2. The portfolio turnover from time $t-1$ to time t , $TO(t-1, t)$, is defined as:

$$\mathrm{TO(t-1,t)} = \frac{\sum_{i=1}^{K} |w_i(t-1) - w_i(t)|}{2}$$

where K is the number of assets in portfolio, w_i (t) and w_i (t +1) indicate the weight of the generic asset class i at time t −1 and t respectively.

Table 5 shows that portfolio turnover is higher when the sampling length is equal to T=30. It proves that the size of the sampling length influences the stability of the outputs. Intuitively, a short sampling length will have a tendency to be influenced by large variations in returns while a long sampling length will smooth these large variations. In our case, the portfolio weights are influenced by the large variations in the inputs (mean and covariance) when the sampling length is shorter, causing a higher turnover in the portfolios.

Table 5

	Mean-variance turnover	Resampling turnover
T=30		
GMV	0,466%	0,484%
I	16,081%	12,142%
M	21,000%	17,010%
T=90		
GMV	0,242%	0,365%
I	7,982%	8,358%
M	12,121%	12,172%

Looking at Table 5, we cannot conclude that the resampling procedure leads to lower turnover with respect to the mean-variance procedure. Active portfolio management would not necessarily benefit from using the resampling procedure since transaction costs (directly related to turnover) are not reduced in every case.

4. Conclusion

Portfolio resampling offers an intuitive way to deal with sampling error in portfolio optimization. Resampling addresses estimation risk with simulations. These simulated return and risk measures help to quantify the effect on the optimization process of uncertainty inherent in the investment world.

In terms of allocation, the comparison between the mean-variance optimization and the resampled optimization shows that the resampled optimization strategy leads to more stable and more diversified portfolios.

In terms of risk-adjusted performance, we were not able to prove that resampled optimization outperforms the mean-variance optimization. When small samples are relied upon, estimates are more affected by sampling errors. In that case, the resampled optimization works better than the mean-variance optimization for all portfolios. Similar results are obtained when turnover is used to compare the two approaches.

All in all, the resampled portfolios are undeniably better diversified than their mean-variance analogues. However, it is far from clear why averaging over resampled portfolio weights should necessarily represent an *optimal* portfolio construction solution to deal with sampling error. Our empirical analysis reinforces this skeptical view.

Reference

MICHAUD R. (1998), *Efficient Portfolio Asset Management*, Boston, Harvard Business School Press.