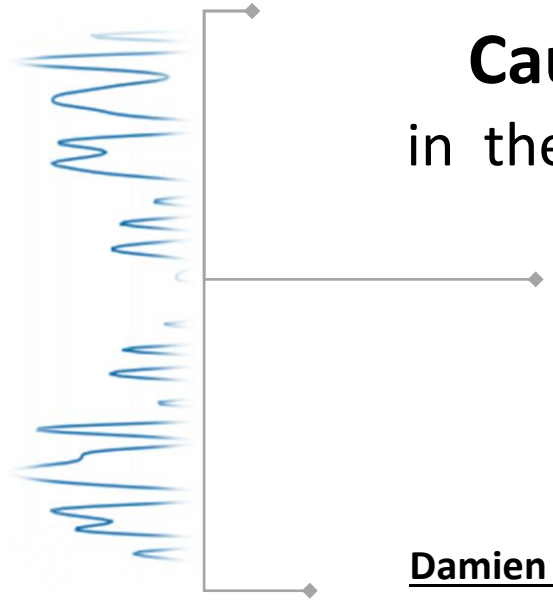
UCLouvain
Earth and Life Institute (ELI)

Hydrological connectivity
From
Causal analysis of time series
in the Lhomme Karst System (Belgium)



8th of April, 2019
EGU General Assembly, Vienna

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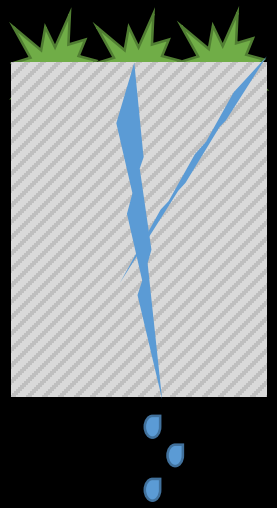
[1] Université catholique de Louvain (UCLouvain), [2] Royal Observatory of Belgium (ROB), [3] University of Namur (UNamur), [4]University of Mons (UMons)

Photography encouraged

Hydrological connectivity ?

STRUCTURAL APPROACH

Karst structure → connections



"it is difficult, even impossible, to infer the karst structure and its role only from the geological framework and fracturing distribution"
Bakalowicz, 2005

DYNAMICAL APPROACH

Using Time Series Analysis
And Causal inference

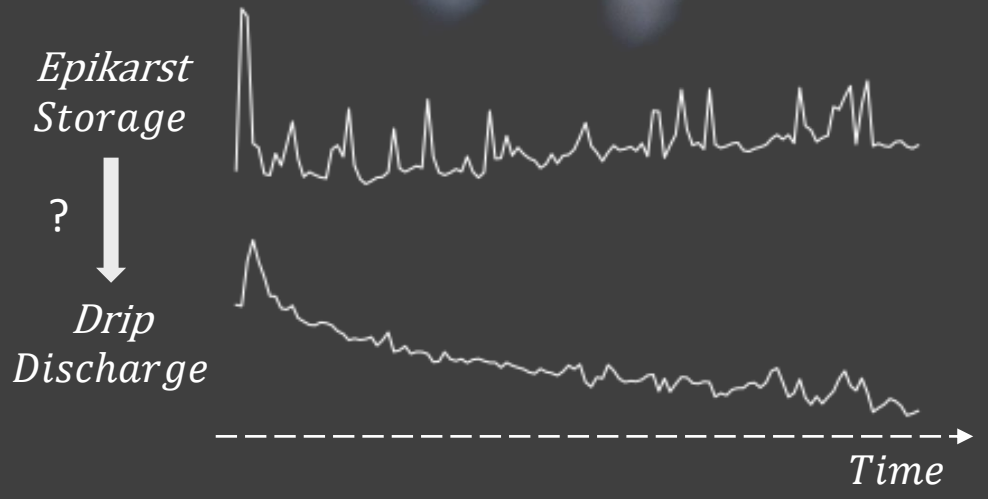
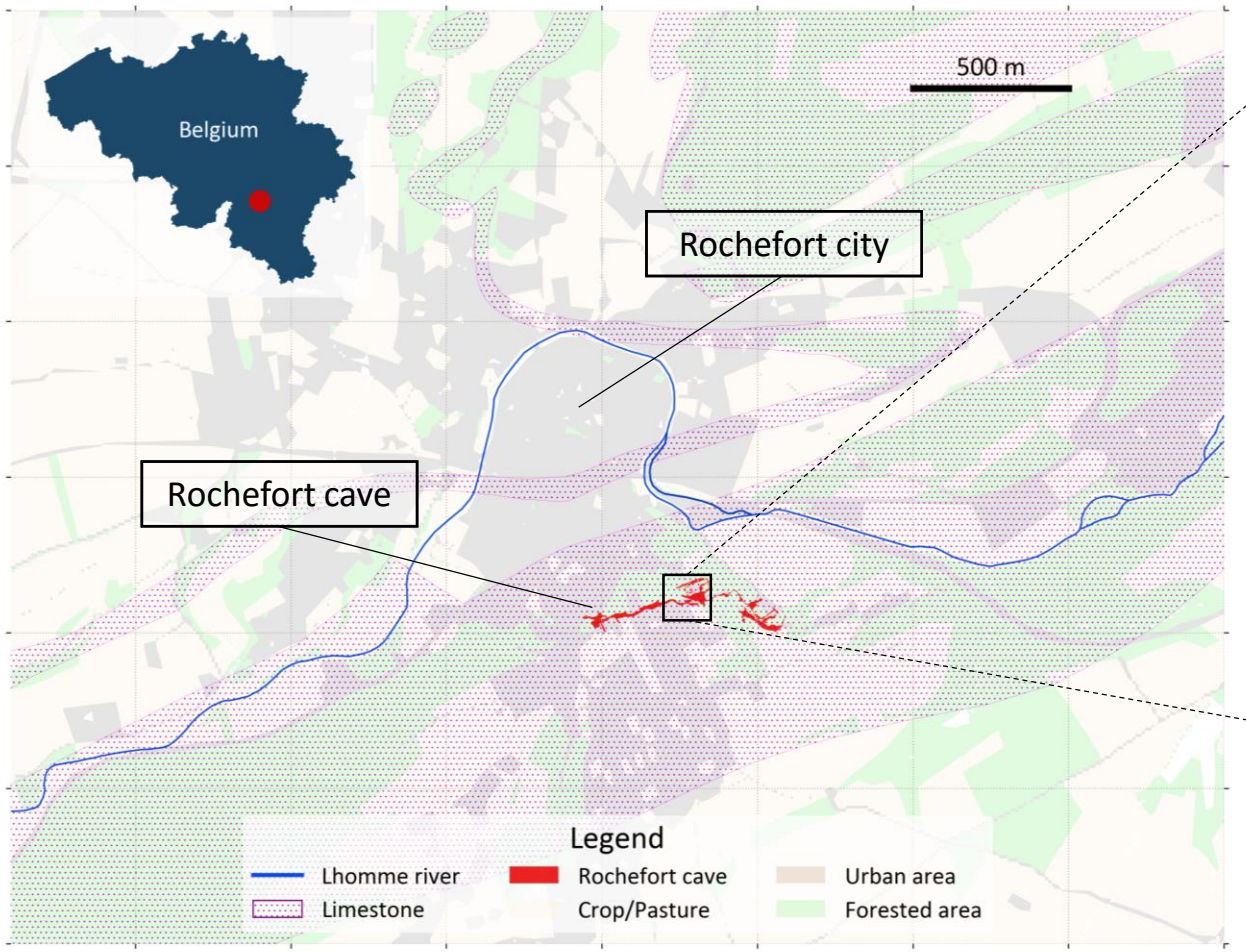
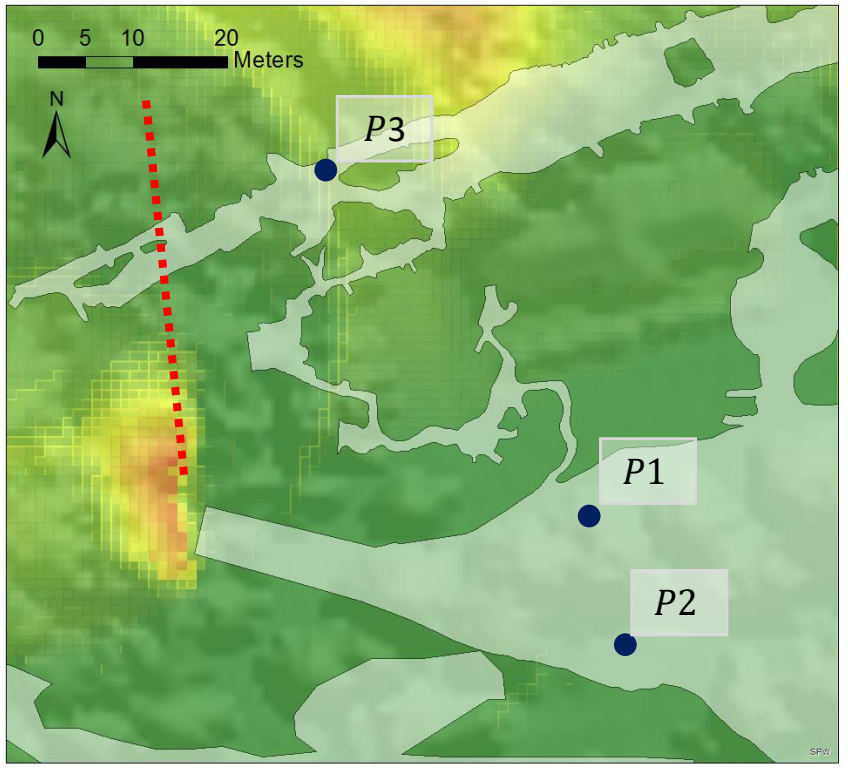


Image: Public domain C.C.0 1.0, www.maxpixel.net

Lhomme Karst System and Rochefort cave laboratory



Data source SPW, EPSG: 31370



DEM/DSM source: SPW
Adapted from A. Watlet

Related papers:
Watlet *et al*, 2018
Poulain *et al*, 2018

KARAG: Karst Aquifer ReseArch by Geophysics (2013-2017)

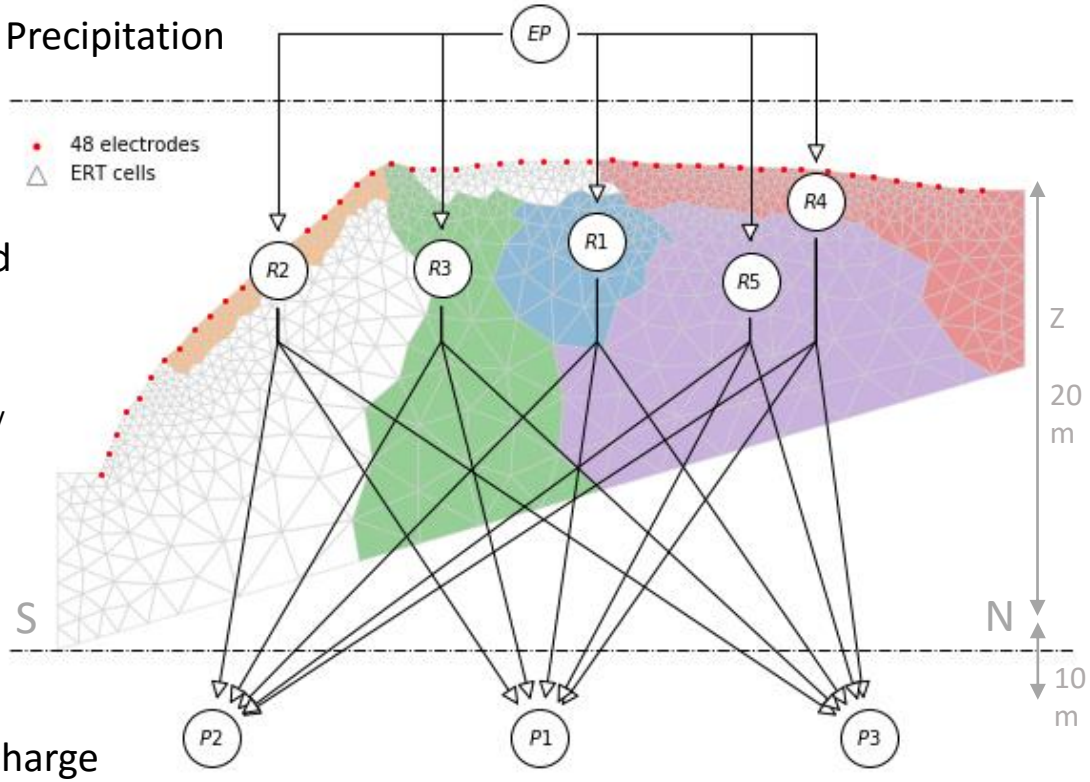
Study case: infiltration in the cave (quickflow)

Investigated causal links (over 3 days)

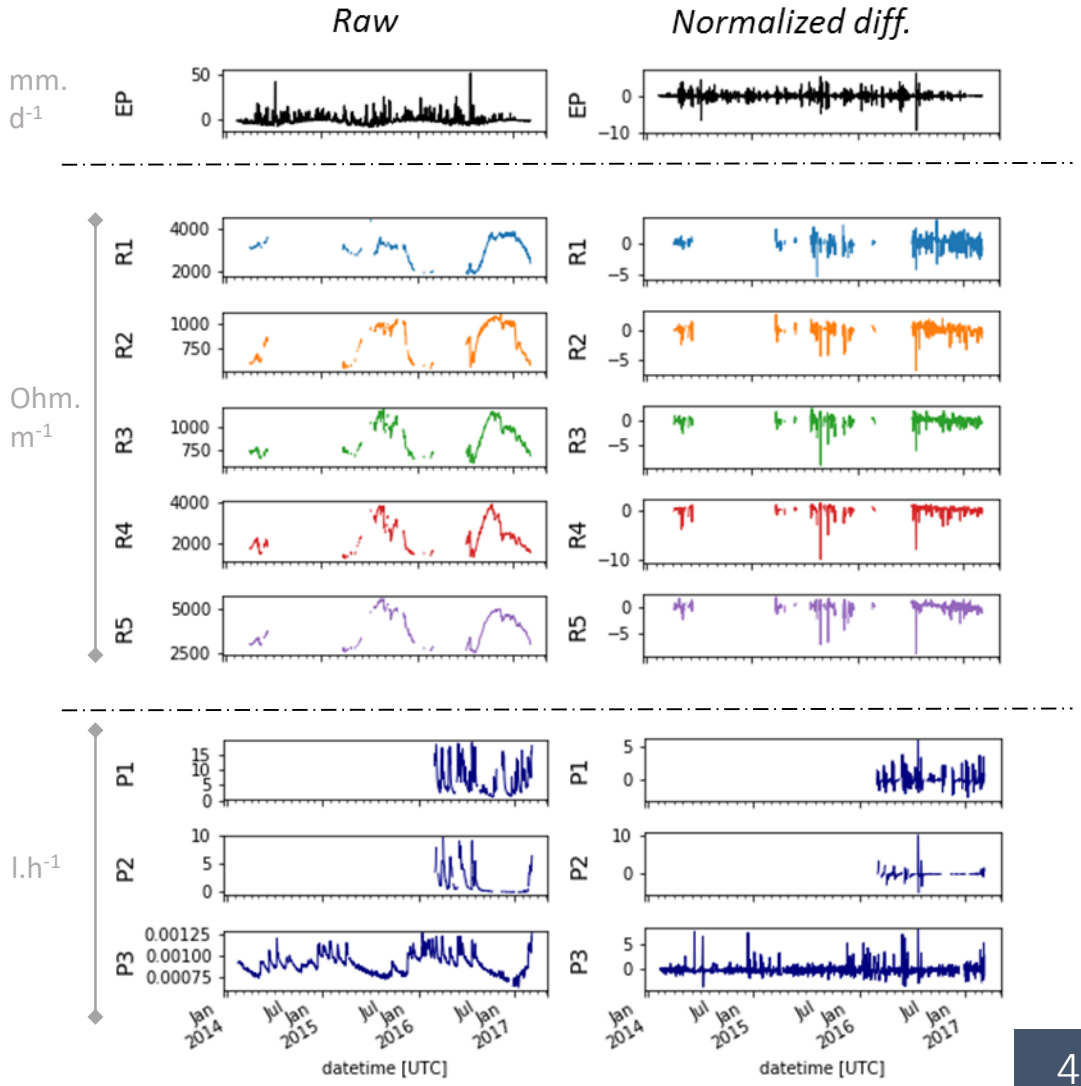
Effective Precipitation

Clustered Electrical Resistivity Tomography Data

Cave Drip Discharge



Time series (1121 daily values)



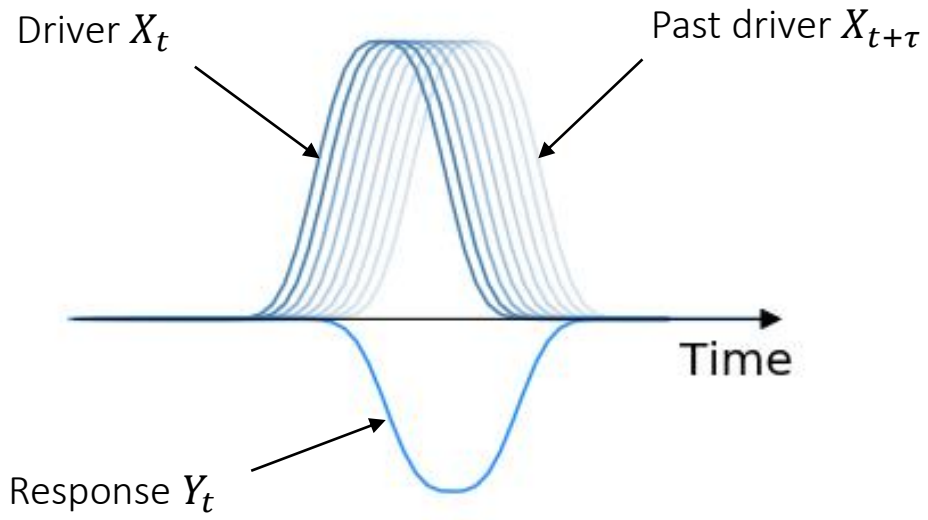
Causal inference methods

1. Cross-correlation
2. Pairwise Granger causality
3. Convergent Cross Mapping

Method 1: Cross-correlation function (CCF)

The past of a driver $X_{t+\tau}$ is linearly correlated with the present response Y_t

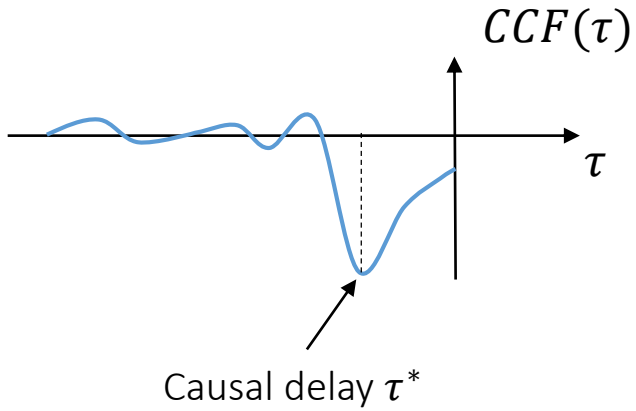
Principle: Correlation of successive lags



$$CCF(\tau) = \rho(X_{t+\tau}, Y_t), \qquad \tau \leq 0$$

With the Pearson's correlation $\rho(X,Y) = \frac{\widehat{\sigma_{XY}}}{\widehat{\sigma_X}\,\widehat{\sigma_Y}} \in [-1,1]$

Output: CCF-Correlogram



Optional constraint

$CCF(\tau^*) > 0$ or

$CCF(\tau^*) < 0$

Icon: Idea by Adrien Coquet from the Noun Project

Method 2: Pairwise Granger Causality (PGC)

Past information about a driver $X_{t+\tau}$ improves linear prediction of the response Y_t

Principle: test if better fitting by including the driver $\boldsymbol{X_t}$

AR process of order p

without driver X_t

$$Y_t = c + \beta_{Y,1}Y_{t-1} + \cdots + \beta_{Y,p}Y_{t-p} + \varepsilon_t$$

VAR process of order p

with driver X_t

$$Y_t = c + \beta_{Y,1}Y_{t-1} + \cdots + \beta_{Y,p}Y_{t-p} + \beta_{X,1}X_{t-1} + \cdots + \beta_{X,p}X_{t-p} + \varepsilon_t$$

Fit

$$\hat{\varepsilon}_{t,\mathrm{AR}} = Y_t - \hat{Y}_{\mathrm{t,AR}}$$

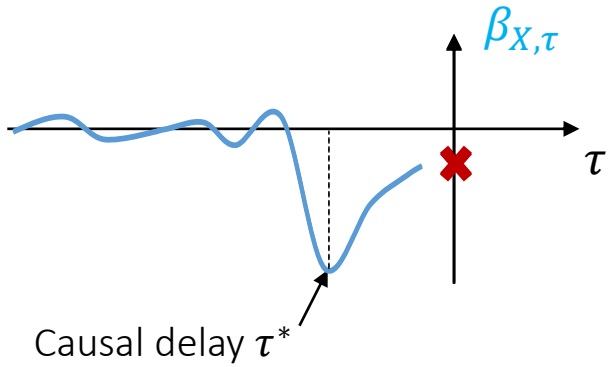
$$\sigma^2_{\hat{\varepsilon},\mathrm{AR}} = \frac{\sum(\hat{\varepsilon}^2_{t,\mathrm{AR}})}{N}$$

$$>$$

$$\hat{\varepsilon}_{t,\mathrm{VAR}} = Y_t - \hat{Y}_{\mathrm{t,VAR}}$$

$$\sigma^2_{\hat{\varepsilon},\mathrm{VAR}} = \frac{\sum(\hat{\varepsilon}^2_{t,\mathrm{VAR}})}{N}$$

Output: PGC-Correlogram



Optional constraint

$\beta_{X,\tau^*} > 0$ or

$\beta_{X,\tau^*} < 0$

Granger, 1969, doi:10.2307/1912791

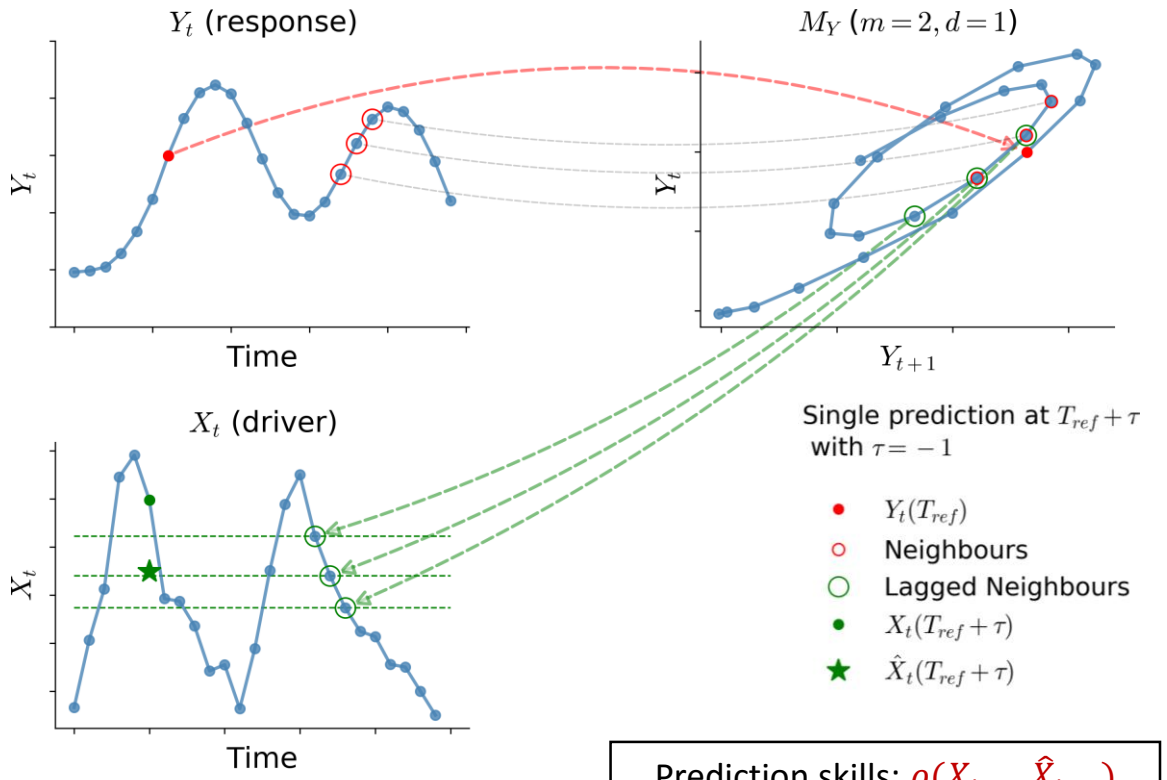
Icon: Idea by Adrien Coquet from the Noun Project

Method 3: Convergent Cross Mapping (CCM)

Similar states of a response Y_t are able to predict the past of a driver $X_{t+\tau}$ consistently

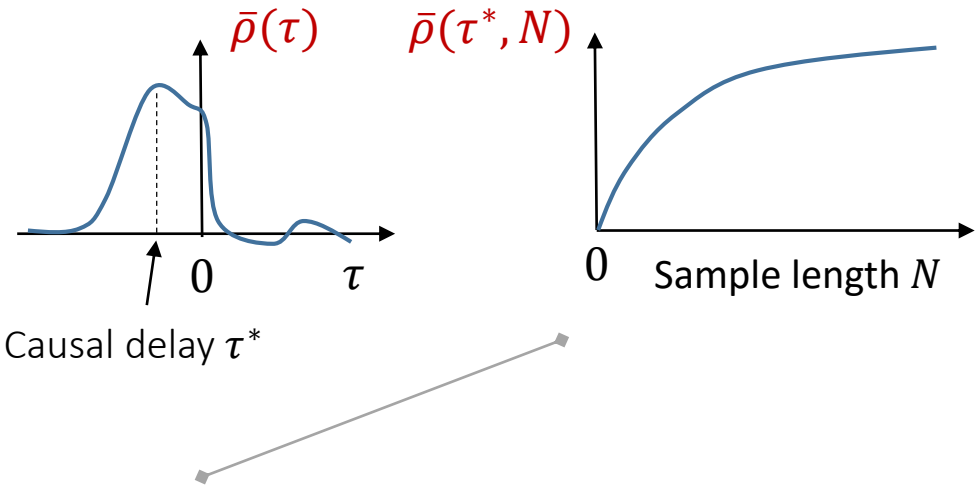
Principle: Nearest Neighbours cross-predictions

Based on Takens' embedding: $M_Y(m,d) = \{Y_t, Y_{t+d}, \dots, Y_{t+(m-1)d}\}$



Prediction skills: $\rho(X_{t+\tau}, \hat{X}_{t+\tau})$

Output: CCM-Correlogram and **CCM-Convergence**



Convergence $\stackrel{\text{def}}{=}$

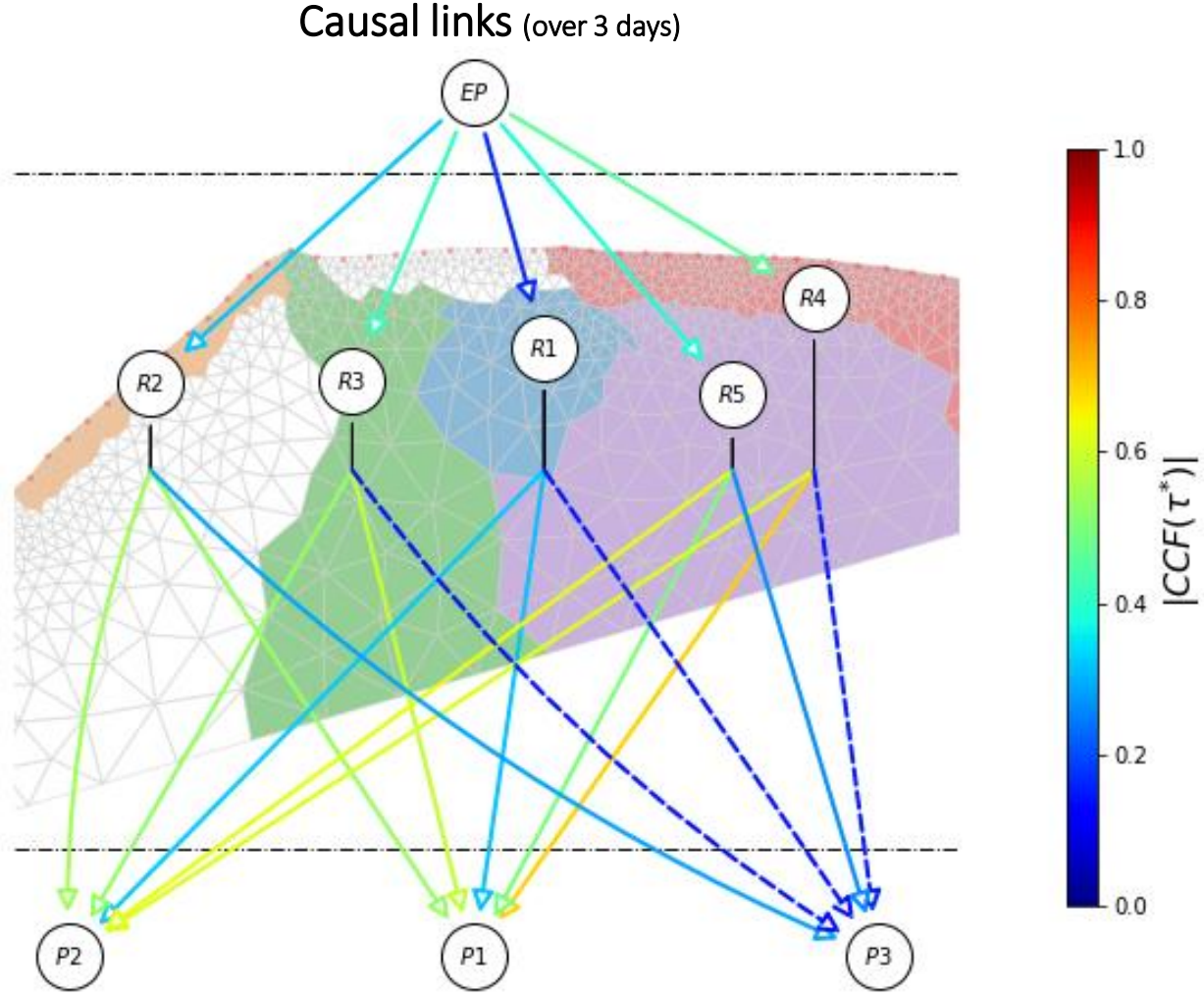
more similar states → more predictive power

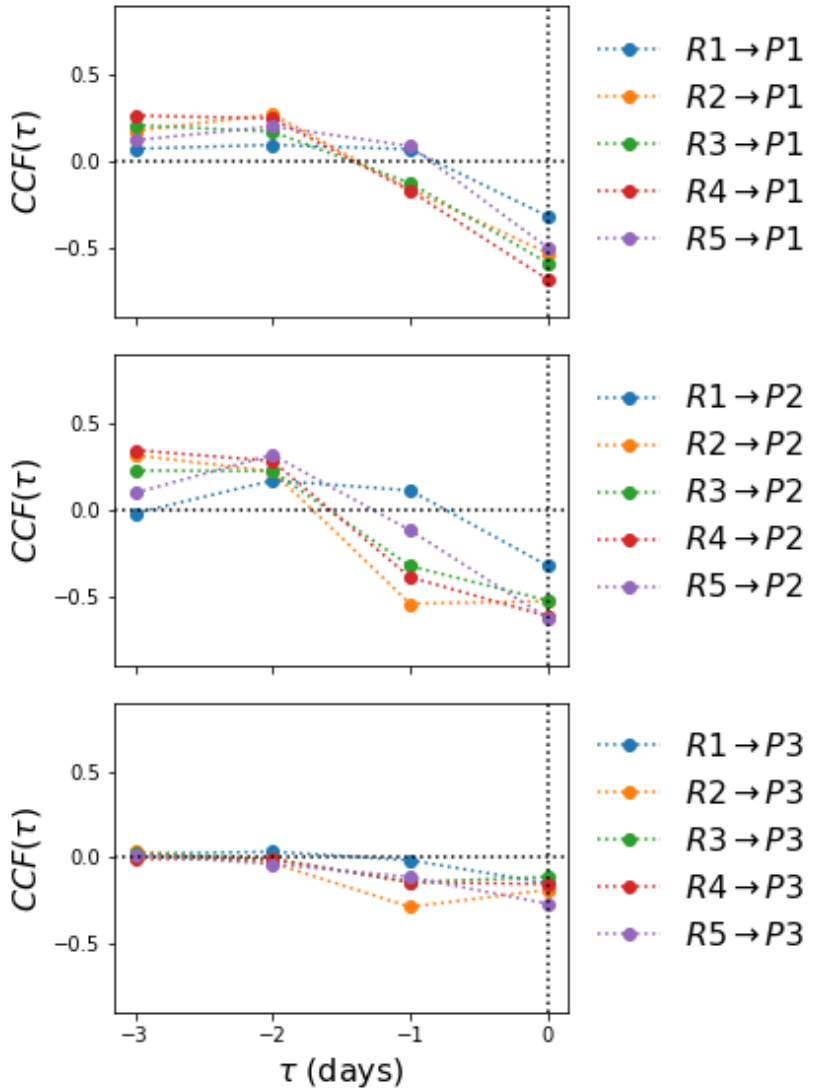
Sugihara *et al.*, 2012: doi:10.1126/science.1227079

Ye *et al.*, 2015: doi:10.1038/srep14750

Icon: Idea by Adrien Coquet from the Noun Project

Result 1: Cross-correlation function



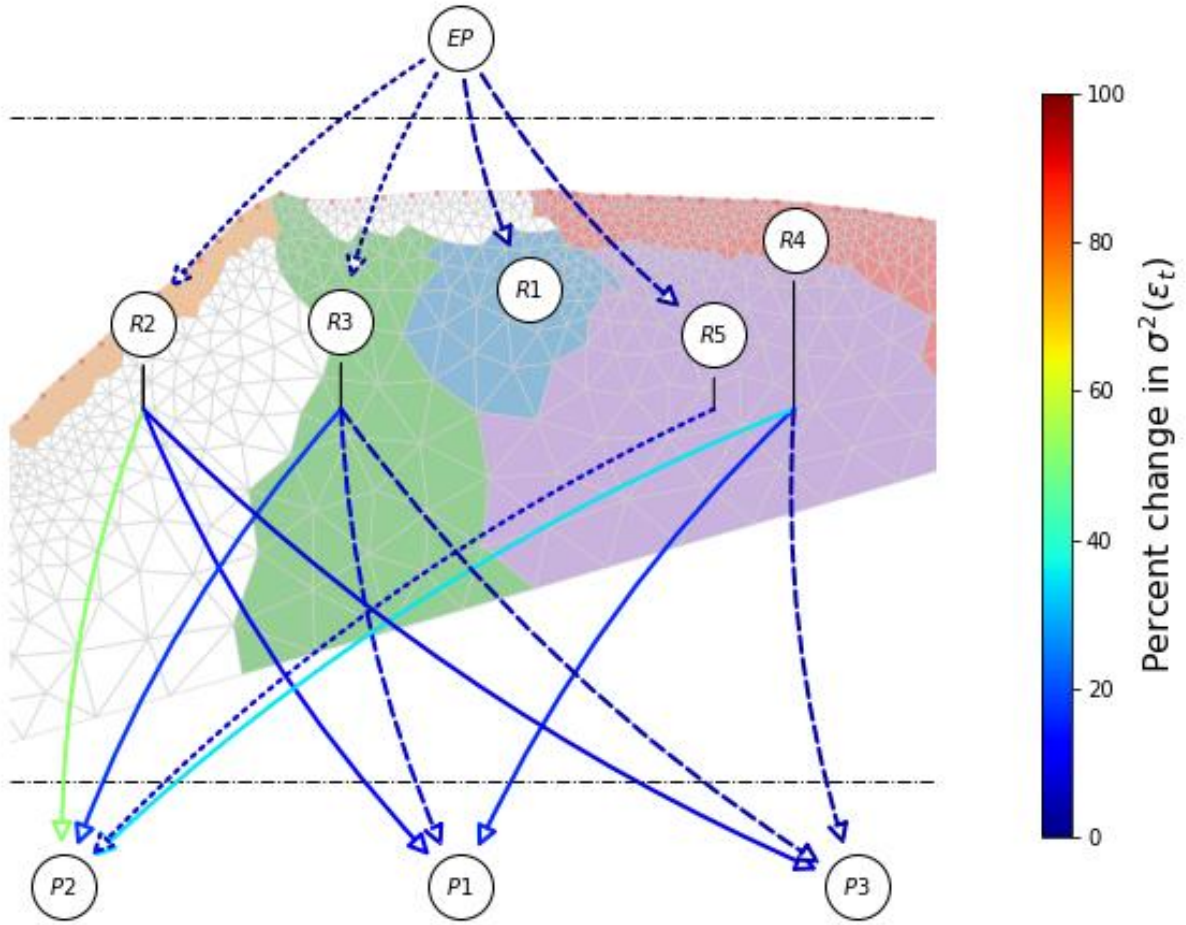


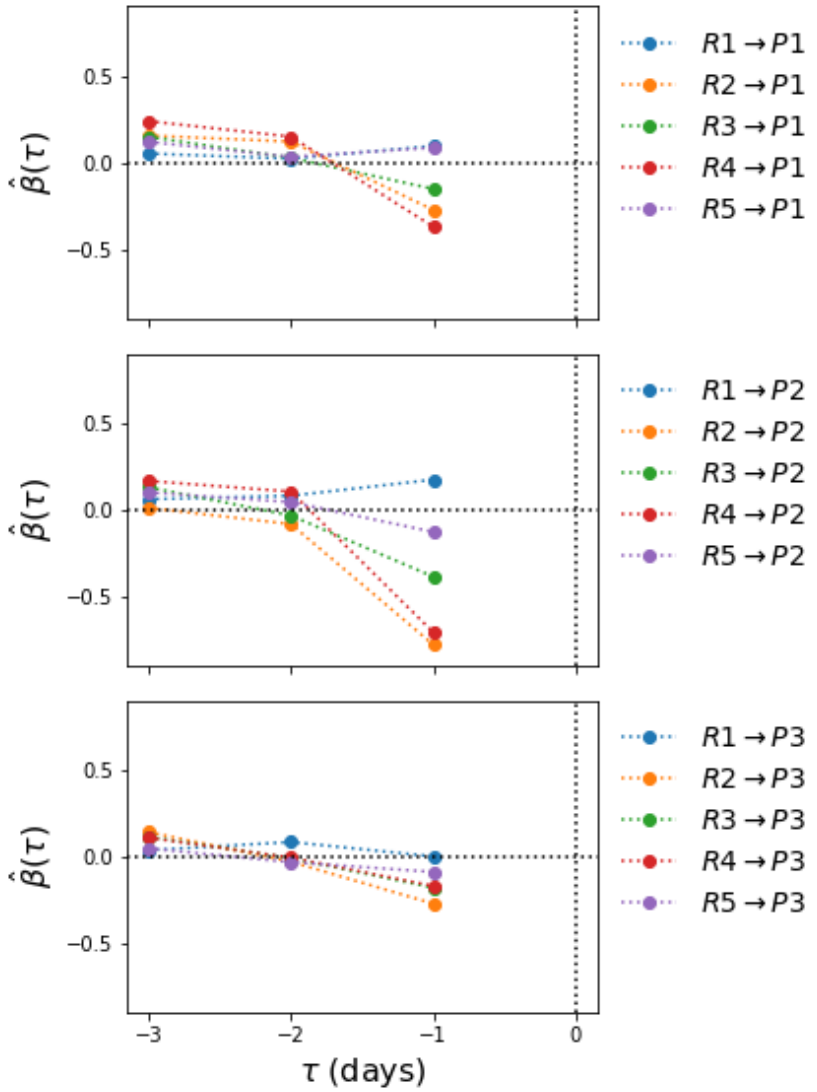
Signif. level: —— < 0.001 – – < 0.01 ····· < 0.05

Straight line = instantaneous, Curved = delayed ($\tau^* < 0$)

Result 2: Pairwise Granger Causality

Causal links (over 3 days)



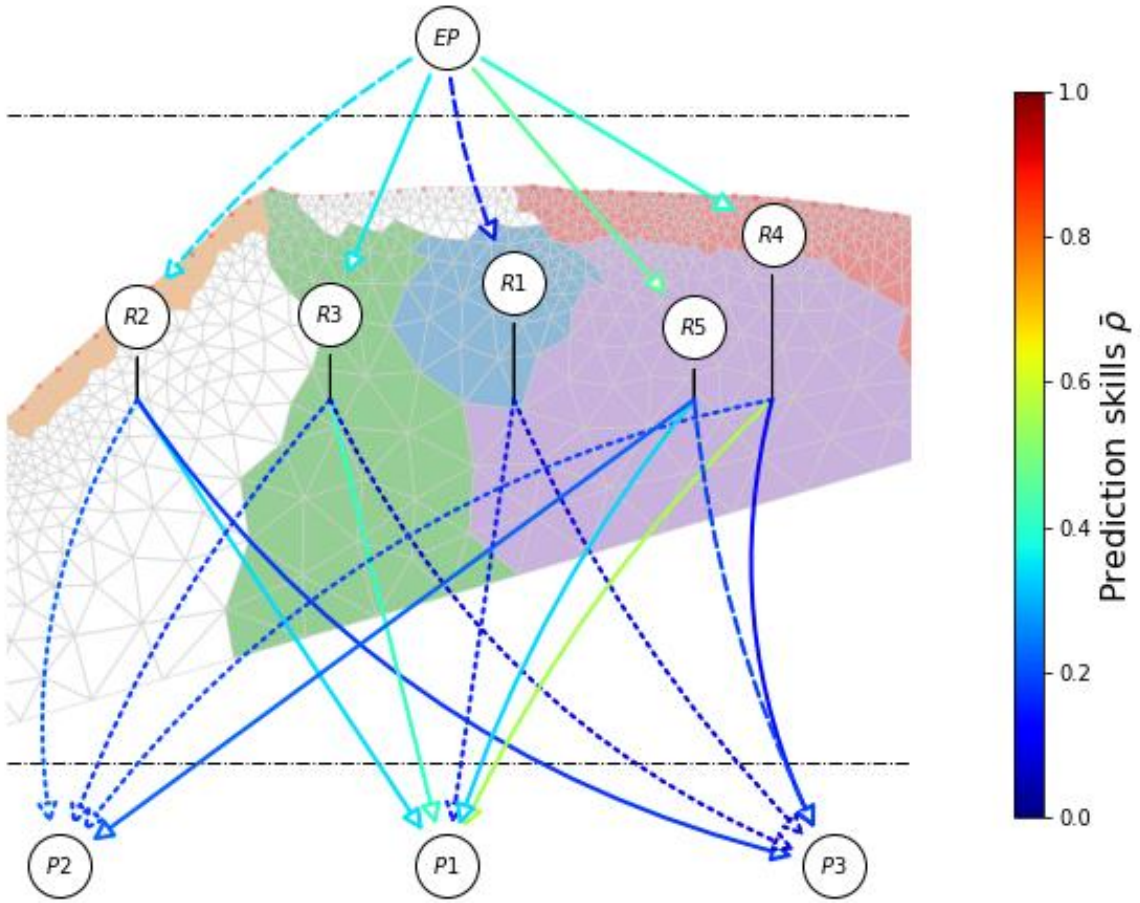


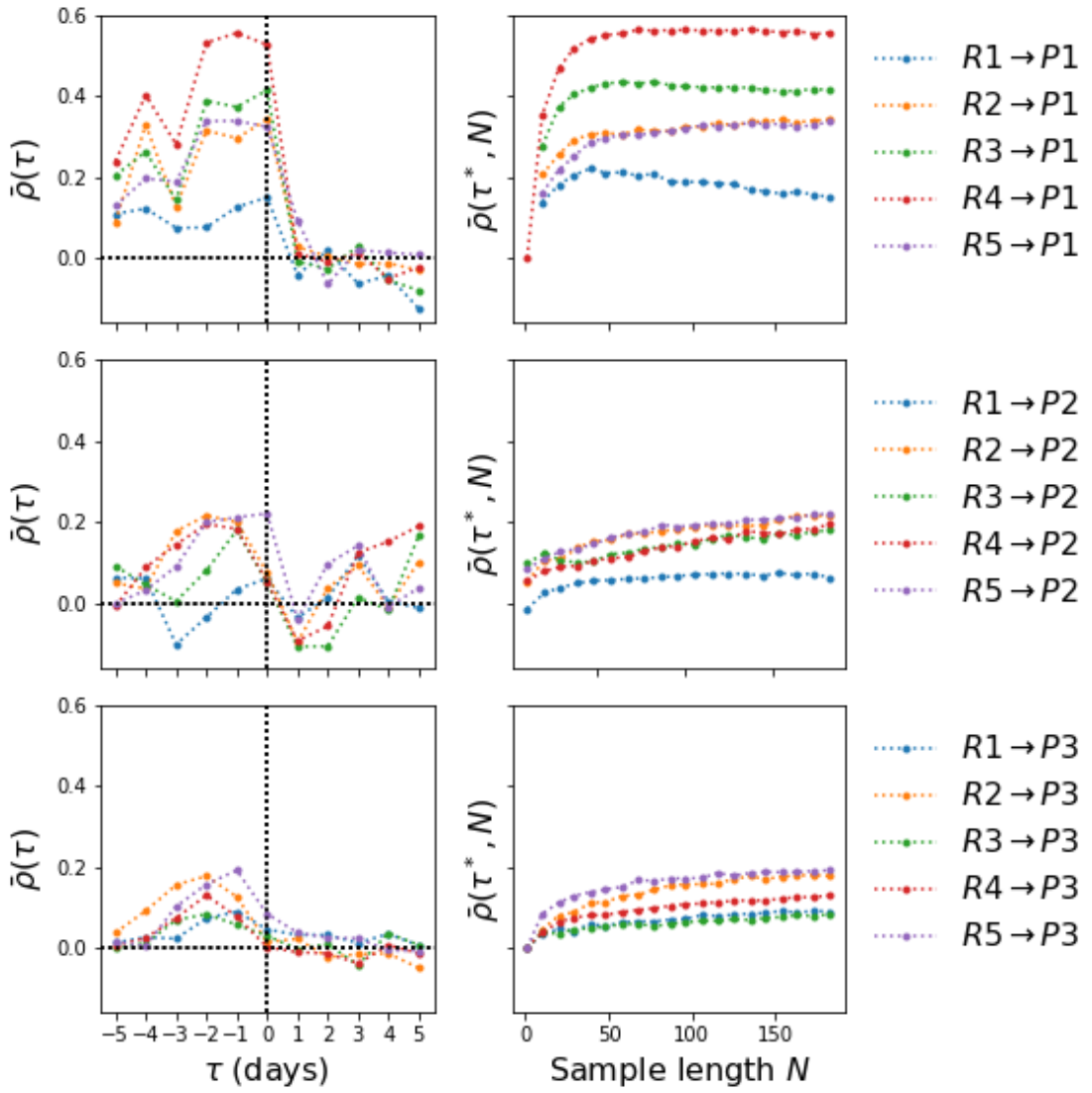
Signif. level: —— < 0.001 – – < 0.01 ····· < 0.05

Straight line = instantaneous, Curved = delayed ($\tau^* < 0$)

Result 3: Convergence Cross Mapping

Causal links (over 5 days, $\boldsymbol{m = 3}$, $\boldsymbol{d = 1}$)

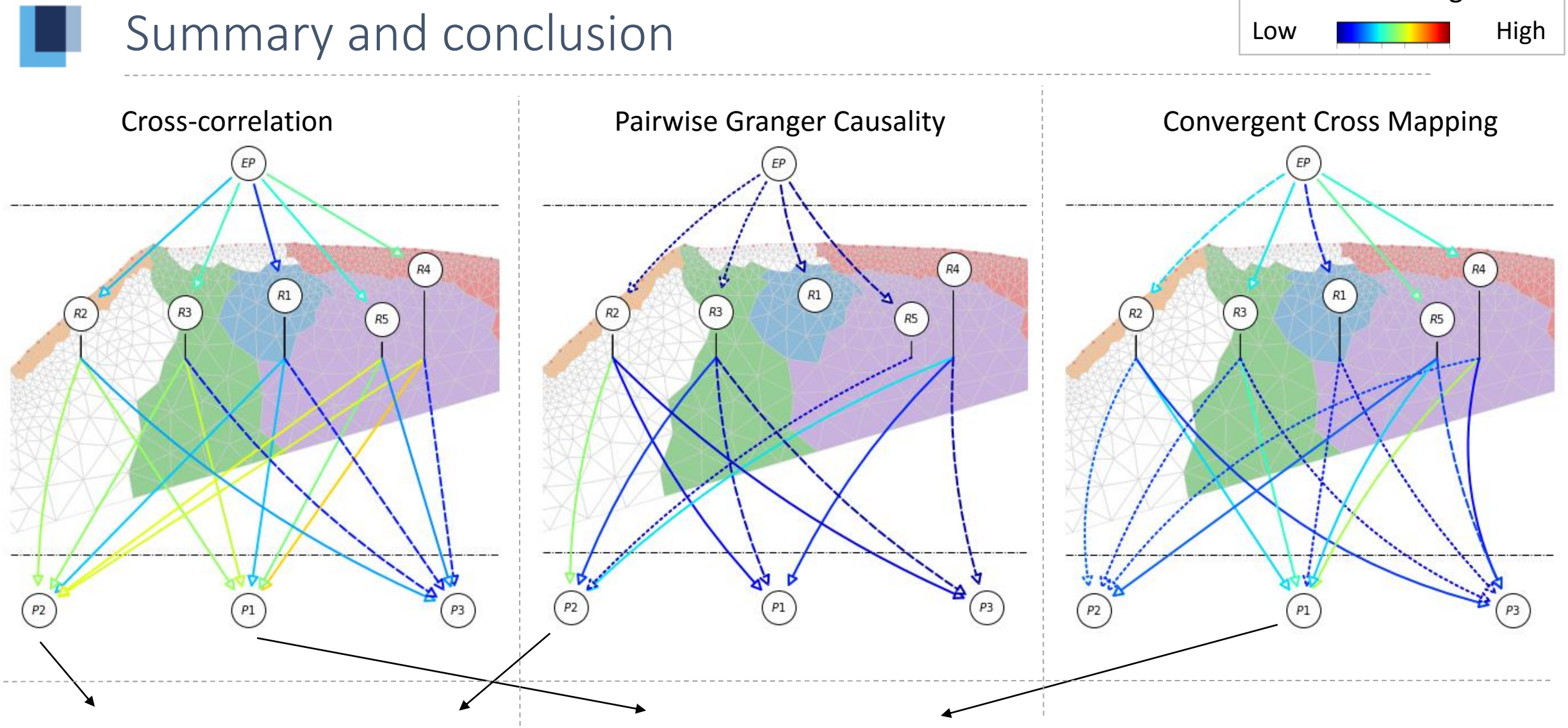




Signif. level: —— < 0.001 – – < 0.01 ····· < 0.05

Straight line = instantaneous, Curved = delayed ($\tau^* < 0$)

Summary and conclusion



P2 - Multilinear vector autoregressive

P1 - Multilinear deterministic - nonlinear?

P3 – Unidentified quickflow pattern