LENIENCY IN ANTITRUST INVESTIGATIONS AS A COOPERATIVE GAME

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Leniency in antitrust investigations as a cooperative game

Pierre Dehez* and Samuel Ferey**

Abstract

Leniency programs in antitrust investigations exist in Europe since the late nineties. They cover secret agreements and concerted practices between companies, and provide total or partial immunity to companies reporting evidence. This raises the question of assessing correctly the contribution of each company that take part in a leniency program. This question is formalized within a cooperative game with transferable utility. The resulting game being convex, its core is nonempty and contains the Shapley value in its center. It defines a reference allocation that treats the participants symmetrically. In practice, companies report sequentially leading to allocations that are vertices of the core.

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1. Introduction

Leniency programs are designed to encourage companies suspected of being part of a cartel to cooperate with the authorities in antitrust investigations by granting reductions in fines to those who trigger the leniency program the earliest and provide the strongest evidence of illicit practices.

There are two important motivations behind leniency programs. On the one hand, leniency programs add more instability to the cartels and, on the other hand, they make it possible to reduce the cost of collecting evidence. The existing literature tends to focus on the first motivation, namely the non-cooperative elements that undermine the cooperative nature of a cartel (Volpin and Chokesuwattanaskul, 2023). The second motivation aroused little interest. It is the object of the present paper.

To benefit from immunity or a reduction in fines, participants in a leniency program are expected to cooperate fully with the Competition Authority and provide all the evidence of illegal practices that they possess. In many countries, in Europe in particular, leniency programs are open to companies as soon as they provide evidence of "significant added value".[1]

At the end of the investigation process, the set of evidence available to rule on the case and establish the fine reductions is vast and complex: the evidence is either provided by the companies or seized by the Competition Authority; some evidence is redundant while others are not; some evidence is solid, others must be corroborated; some pieces of evidence provided by a company incriminate themselves, others incriminate other members of the cartel, including other participants in the leniency program, etc.

The difficulty of calculating the contribution of each company to the establishment of evidence of unlawful behavior sometimes leads to conflicts between the Competition Authority and the participants in the leniency program. Indeed, the method used to calculate the contributions of each company to the establishment of evidence is often unclear. Therein lies one of the reasons why companies are reluctant today to trigger a leniency program.[2] And there seems to be no guarantee that the determination of the fine reduction on the basis of the evidentiary contribution of each party is consistent with the fact of favoring the companies that denounced the cartel the earliest.

The main issue that concerns us is the allocation to each participant in a leniency program of a share of the total value of the evidence submitted, taking into account the evidence provided by the other participants and the time at which the evidence is submitted. This information can then be used to establish the reductions in fines that could be applied once the total damage has been assessed and distributed among the members of the cartel in the form of fines. We model

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[1] "*In order to qualify, an undertaking must provide the Commission with evidence of the alleged infringement which represents significant added value with respect to the evidence already in the Commission's possession. [...] The concept of 'added value' refers to the extent to which the evidence provided strengthens, by its very nature and/or its level of detail, the Commission's ability to prove the alleged cartel*. " (EU, 2006, §25).

[2] A recent survey by Ysewyna and Kahman (2018, p.45) states that leniency programs are less attractive because of "increased exposure to civil damages claims (36%), […] perceived uncertainty in how authorities will grant leniency reductions (19%), perceived uncertainty in how authorities will calculate fines (14%), and perceived uncertainty in how authorities will deal with requests for access to file for leniency submissions (12%)".

this issue as a cooperative game based on the value of the evidence provided by each participant in the leniency program. Our concern is to focus on allocations of evidence that are fair while being free from objections that could be raised, either by an individual participant or by a group of participants. The fairness of an allocation is captured by the concept of *Shapley value* while the absence of objections is covered by the concept of *core*.

The definition of the (transferable utility) game is straightforward: it is defined by the function that assigns to each coalition of companies the value of the evidence they are *alone* to report. The Shapley value then provides a reference allocation that corresponds to a situation where companies report simultaneously. The game being convex, it defines an allocation that is centrally located within the core. However, companies generally report sequentially. It turns out that the vertex of the core corresponds to an allocation of evidence that results from a particular sequential participation of companies.

The paper is organized as follows. Section 2 covers transferable utility games and their properties, together with the notions of marginal contributions and dividend. Leniency games are introduced in Section 3. The core and the Shapley value are defined in Section 4. Section 5 deals with the determination of fines on the basis of illegal practices that each member of the cartel has committed, alone or with other companies, before the possible application of partial or total reductions in fines. Concluding remarks are offered in the last section.

2. Transferable utility games

This section covers transferable utility games, their properties and some of the concepts used later, in particular marginal contributions and dividends.

2.1 The characteristic function

A *transferable utility game* is a pair (N,v) where N is the set of *players* and v is a (characteristic) function that associates a real number to each coalition: $v(S)$ is the *worth* of coalition S. It may measure the payoff the coalition S should or could obtain if it forms. By convention, $v(\varnothing)=0$.

Notation: Finite sets are denoted by upper-case letters and lower-case letters are used to denote their cardinals: $t = |T|$, $s = |S|$, … By convention, a sum over an empty set is equal to 0. Set inclusion (non-strict) is denoted by $\subset$. Braces are sometimes omitted for coalitions, for instance, $v(i, j)$ replaces $v(\{i, j\})$ or $N \setminus i, j$ replaces $N \setminus \{i, j\}$. For a given vector x, we denote by $x(S)$ the sum of the components whose coordinates are included in the subset S and, by convention, we set $x(\varnothing)=0$.

It is usually assumed that disjoint coalitions never lose by getting together:

$$S \cap T = \varnothing \;\Rightarrow\; v(S \cup T) \geq v(S) + v(T).$$

This is *superadditivity*. Convexity is a more demanding property. A game is *convex* if

$$v(S \cup T) \geq v(S) + v(T) + v(S \cap T) \quad \text{for all } S,T \subset N.$$

Clearly, convexity implies superadditivity.[3]

[3] Convexity of transferable utility games has been introduced by Shapley (1971).

Cooperative game theory is mainly concerned by the allocation of the worth of the "grand coalition" N among the n players, knowing what each coalition could or should obtain by itself. An *allocation* is a vector $x=(x_1,...,x_n)$ such that $x_1+x_2+...+x_n=v(N).$

2.2 Marginal contributions

Players' contributions to coalitions play a central role in solutions to the allocation problem. The *marginal contribution* of player i to a coalition S is defined by $MC_i^v(S)=v(S)-v(S\setminus i).$ It is of course null if $i\notin S.$

Two players i and j are *substitutable* if they contribute identically for all coalitions containing them: $MC_i^v(S)=MC_j^v(S)$ for all S containing i and j. A player i is *null* if he never contributes: $MC_i^v(S)=0$ for all $S\subset N.$

Shapley (1971) proves that a game is convex if marginal contributions of any player never decrease with the size of the coalition $S\subset T \;\Rightarrow\; MC_i^v(S)\le MC_j^v(T).$ The converse is actually true as shown by Ichiishi (1981).

Marginal contributions vectors are associated to players' ordering. The vector $\mu^{\pi}(N,v)$ associated to the ordering $\pi=(i_1,i_2,...,i_n)$ is defined by:

$$\mu_{i_1}^{\pi}(N,v)=v(i_1),$$
$$\mu_{i_k}^{\pi}(N,v)=v(i_1,...,i_k)-v(i_1,...,i_{k-1}),\;\; k=2,...,n-1$$
$$\mu_{i_n}^{\pi}(N,v)=v(N)-v(i_1,...,i_{n-1}).$$

There are n! marginal contribution vectors, not necessarily distinct. Each vector $\mu^{\pi}(N,v)$ is an allocation of $v(N).$

2.3 Harsanyi dividends

Given a set of players N, the set $G(N)$ of all characteristic functions on N is equivalent to the specification of a vector in $\mathbb{R}^{2^n-1}.$ In his proof of the uniqueness of the value, Shapley (1953) uses the fact that the collection of *unanimity games* $(N,u_T)_{T\subset N}$ defined by:

$$\begin{aligned} u_T(S)&=1 \quad \text{if } T\subset S,\\ &=0 \quad \text{if } T\not\subset S, \end{aligned}$$

forms a basis of $G(N)$: for any given function $v\in G(N),$ there exists a unique (2^n-1)-dimensional vector $\alpha=(\alpha_T)$ such that:

$$v(S)=\sum_{T\subset N}\alpha_T\, u_T(S)=\sum_{T\subset S}\alpha_T \tag{1}$$

assuming $\alpha_{\varnothing}=0.$ *Positive games* are games whose dividends are nonnegative.

3. Leniency games

3.1 Evidence

The existence of a cartel is suspected by the Competition Authority (*Authority* for short) but it does not dispose of all the evidence necessary for a ruling. A leniency program is then proposed and some of the companies that are suspected decide to apply and disclose the evidence they hold against themselves *and* against other companies. Evidence are accepted only if they offer formal proofs of well-defined illegal practices.

We denote by $N = \{1,...,n\}$ the set of companies that are part of the leniency program. Among the companies that are suspected of being part of the cartel, some may not be included in the leniency program while those that are part of it may have evidence of their participation in the cartel. We group the outside companies into a single unit indexed "0" and we denote by N_0 the set of all members of the cartel. We denote by E_{ih} the evidence that is held by company $i \in N$ against company $h \in N_0$ and for which the Authority has no proof. In particular, E_{ii} is the set of evidence held by company *i* against itself.

We denote by E_i the set of evidence submitted by company *i* and by *E* the set of all evidence:

$$E_i = \bigcup_{h \in N_0} E_{ih} \text{ and } E = \bigcup_{i \in N} E_i .$$

In case of an agreement between two companies, say *i* and *j*, a same evidence $e \in E$ may appear in the evidence sets E_{ii} and E_{jj}, in which case we also have $e \in E_{ji} \cap E_{ij}$. However, we do not exclude the case where an agreement has taken place for which only one of the companies involved has provided the formal proof. More generally, we do *not* assume that $E_{hi} \subset E_{ii}$ for $h \neq i$.

We have to keep in mind that we are not interested in calculating penalties. Our objective is to correctly evaluate the evidence provided by each company participating in the leniency program. All we need to know are the individual evidence sets $E_1,...E_n$.

Example 1 Consider a leniency program involving three companies: $N_0 = \{0,1,2,3\}$. Table 1 lists the evidence submitted by the three companies where the row assigned to company *i* defines the evidence set E_i.

Proven…	against company 1	against company 2	against company 3	against other companies
by company 1	*a, b, c*	*a*	*f*	*c*
by company 2	*a*	*a, d, e*	*d*	*e*
by company 3	–	*e*	*f, g*	-

Table 1: Evidence sets

Table 1 reveals the existence of four agreements involving:

- company 1 and company 2 with a value *a*,
- company 2 and company 3 with a value *d*,
- company 1 and others with a value *c*,
- company 2 and others with a value *e*.

Notice that company 3 has not provided the proof of its agreement with company 2. The set of all evidence is given by $E = \{a,b,c,d,e,f,g\}$. In calculation, we will use the values (37, 53, 20, 23, 33, 19, 31), for a total equal to 216.

3.2 The value of evidence

Evidence reveal illegal practices to which monetary values are associated. Hence, the value of any subset of evidence is known and given by:

$$\Phi(A) = \sum_{e \in A} \Phi(e) \text{ for all } A \subset E$$

where $\Phi(e)$ is the value of evidence e.

3.3 Leniency games

Since companies also submit evidence that concerns other companies, the question arises of the correct evaluation of the evidence they submit. That information can then serve as the basis on which the Authority can decide upon the extent of the immunity to be granted to each company. Hence, the question: how to fairly allocate $\Phi(E)$ between the companies in a way that avoid objections from individual companies or group of companies?

In order to answer that question, we construct a transferable utility game (N, v) whose characteristic function *v* associates to each coalition $S \subset N$ the value of the evidence provided by the members of *S* and not provided by those outside *S*:

$$v(S) = \Phi\left(\bigcup_{i \in S} E_i \setminus \bigcup_{i \in N \setminus S} E_i\right).$$

The question then concerns the allocation of $v(N) = \Phi(E)$, the total value of the evidence reported by the *n* companies.[4]

We will simplify the characteristic function using a simple partitioning of the set of evidence that will reveal the Harsanyi dividends. For each coalition, we identify the evidence that its members have *in common and are the only ones* to report:

$$T \subset N \rightarrow \Omega(T) = \bigcap_{i \in T} E_i \setminus \bigcup_{i \in N \setminus T} E_i$$

with $\Omega(\emptyset) = 0$. We have:

$$\bigcup_{i \in S} E_i \setminus \bigcup_{i \in N \setminus S} E_i = \bigcup_{T \subset S} \Omega(T) \quad \text{for all } S \subset N.$$

By construction, the family of subsets $\left(\Omega(S) \mid S \subset N\right)$ is a *partition* of the set *E*. Therefore, by additivity, the characteristic function can be written as:

$$v(S) = \Phi\left(\bigcup_{T \subset S} \Omega(T))\right) = \sum_{T \subset S} \Phi\left(\Omega(T)\right) \quad \text{for all } S \subset N. \tag{2}$$

Referring to (1), we observe that the dividend of coalition *T* is precisely equals the value of $\Omega(T)$:

$$\alpha_T = \Phi\left(\Omega(T)\right) = \sum_{e \in \Omega(T)} \Phi(e) \quad \text{for all } T \subset N. \tag{3}$$

[4] This game corresponds to the 3-player "*helix game*" introduced by Mêgnigbêto (2018) for measuring the contributions to innovation of universities, the industry and the government. The extension to *n* players is investigated in Dehez and Mêgnigbêto (2024).

The "leniency games" defined by (2) have interesting properties. The following proposition is the immediate consequences of the nonnegativity of the dividends.

Proposition 1 Leniency games are *positive* and *convex* games.

Indeed, according to (1), $v(S)$ is a linear combination of unanimity games whose coefficients are the dividends. These coefficients being nonnegative, convexity follows from the convexity of unanimity games.

Example 1 (continued) Applied to the data given in Table 1, we have successively:

$$\begin{array}{lclcl}
\Omega(1)=\{b,c\}, & & \alpha_{\{1\}}=\Phi(b,c)=73, & & v(1)=73, \\
\Omega(2)=\{d\}, & & \alpha_{\{2\}}=\Phi(d)=23, & & v(2)=23, \\
\Omega(3)=\{g\}, & & \alpha_{\{3\}}=\Phi(g)=31, & & v(3)=31, \\
\Omega(1,2)=\{a\}, & \Rightarrow & \alpha_{\{1,2\}}=\Phi(a)=37, & \Rightarrow & v(1,2)=133, \\
\Omega(1,3)=\{f\}, & & \alpha_{\{1,3\}}=\Phi(f)=19, & & v(1,3)=123, \\
\Omega(2,3)=\{e\}, & & \alpha_{\{2,3\}}=\Phi(e)=33, & & v(2,3)=87, \\
\Omega(1,2,3)=\varnothing. & & \alpha_{\{1,2,3\}}=0. & & v(1,2,3)=216.
\end{array}$$

4. Allocation of evidence

4.1 Stability: the core

Consider an allocation $x=(x_1,...,x_n)$ of $\Phi(E)$. Because $v(i)$ is the value of the evidence that company i is alone to report, a minimal restriction is that each company should be allocated *at least the value of the evidence that it is alone to report*: $x_i \geq v(i)$ for all $i \in N$. This is a legal requirement. It defines individually rational allocations, known as *imputations*.

The *core* extends these inequalities from individual players to coalitions:

$$C(N,v)=\left\{x \in \mathbb{R}^n \,\middle|\, x(N)=v(N) \text{ and } x(S) \geq v(S) \text{ for all } S \subset N\right\}. \tag{4}$$

It is the set of allocations such that each coalition receives *at least the value of the evidence that it is alone to report*. This is a requirement of *stability*: faced with core allocations, no coalition of companies (nor individual company) is in a position to object.

In general, the core of a transferable utility game is a convex polyhedron, possibly empty. Convexity of leniency games ensures that their core is nonempty. Moreover, its vertices are precisely the marginal contribution vectors.

It is easily verified that core allocations defined by (4) satisfy the complementary inequalities requiring that the amount allocated to a coalition should not exceed its contribution to the grand coalition:

$$x(S) \leq v(N)-v(N \setminus S) \text{ for all } S \subset N. \tag{5}$$

In particular, the amount assigned to an individual company should not exceed its (marginal) contribution to the grand coalition: $x_i \leq v(N)-v(N \setminus i)$ for all $i \in N$. Applied to leniency games, a core allocation of the value of the set of all evidence is such that no coalition of companies is allocated less than the value of the evidence it is alone to report. Alternatively, following (5), no coalition of companies is allocated more than its overall contribution.

Example 1 (continued) The leniency game associated to Table 1 is given by (73, 23, 31 | 133, 123, 87 | 216). Its marginal contribution vectors are given by (73, 60, 83), (73, 93, 50), (110, 23, 83), (129, 23, 64), (92, 93, 31), (129, 56, 31), corresponding to the orderings 123, 132, 213, 231, 312 and 321. They happen to be distinct, but this is not always the case.[5]

Remark 1 Convexity implies that the core coincides with the set of *weighted Shapley values*.[6] Positivity implies that the core coincides with the *Harsanyi set*, the set of all distributions of the dividends.[7] The relations existing between these different solution concepts are reviewed by Dehez (2017).

What we need is an *allocation rule* that selects a particular core allocation. There are several rules among which the nucleolus (Schmeidler, 1969) and the Shapley value (Shapley, 1951 and 1953). Here, we retain the latter because it relies on simple and appealing axioms. Furthermore, leniency games being convex, the Shapley value, like the nucleolus, belongs to the core.

4.2 Fairness: the Shapley value

Given a transferable utility game (N,v), the Shapley value is a natural choice for a fair division of the overall gain given by $v(N)$. It is an allocation rule that is uniquely defined by a set of axioms (Shapley, 1953). There are several axiomatizations. Shapley's original axioms are:

- *efficiency* (the value of the game is exactly allocated),
- *symmetry* (equal players are remunerated equally),
- *null player* (players that do not affect coalitions' worth are not remunerated),
- *additivity* (the value of a sum of games on a common set of players is the sum of the values).

The first three axioms are appealing and unavoidable. This is not the case of additivity. Another axiomatization due Young (1985) replaces additivity by a more appealing axiom:

- *monotonicity* (if the marginal contribution of a player increases, other things being equal, his remuneration cannot decrease).

Hence, two major principles underlie the Shapley value: contributions and fairness. What a player obtains is linked to his contributions and players that contribute identically are remunerated equally.

There are several formulations. Here, we shall retain the formula based on Harsanyi dividends. Following Harsanyi (1959), α_T is the *dividend* accruing to coalition T once all sub-coalitions have obtained their dividends. By (1), the sum of all dividends is equal to $v(N)$. An allocation can then be obtained by distributing the dividends of each coalition among its members.

Harsanyi shows that the Shapley value gives to each player the sum of the *per capita* dividend of the coalitions of which he is a member:

$$SV_i(N,v) = \sum_{T \subset N: i \in T} \frac{1}{t}\alpha_T. \tag{6}$$

[5] Distinct marginal contribution vectors characterizes *strictly* convex games (Shapley, 1971).

[6] Weighted Shapley values have been introduced by Kalai and Samet (1987). See also Moderer and Shapley (1992).

[7] The Harsanyi set (also called selectope) was introduced as a solution concept independently by Hammer at al. (1977) and by Vasil'ev (1978).

Hence, combining (3) and (6), the Shapley value of a leniency game (N,v) is given by:

$$SV_i(N,v) = \sum_{T\subset N: i\in T} \frac{1}{t}\Phi\big(\Omega(T)\big) = \sum_{T\subset N: i\in T} \frac{1}{t}\sum_{e\in\Omega(T)} \Phi(e). \tag{7}$$

Example 1 (continued) The Shapley value of the leniency game (73, 23, 31 | 133, 123, 87 | 216) is given by (101, 58, 57). It is the "natural" allocation, given the evidence brought by each company: $E_1 = \{a,b,c,f\}$, $E_2 = \{a,d,e\}$ and $E_3 = \{e,f,g\}$. Indeed, we have:

$$SV_1 = \frac{1}{2}\Phi(a) + \Phi(b) + \Phi(c) + \frac{1}{2}\Phi(f) = 101,$$

$$SV_2 = \frac{1}{2}\Phi(a) + \Phi(d) + \frac{1}{2}\Phi(e) = 58,$$

$$SV_3 = \frac{1}{2}\Phi(e) + \frac{1}{2}\Phi(f) + \Phi(g) = 57.$$

The Shapley value can also be defined as the *average marginal contribution vector*. By convexity, the core of a leniency game is a convex polyhedron whose vertices are precisely the marginal contribution vectors. As a consequence, the Shapley value occupies a central position in the core.

The Shapley value is a reference: it proposes an allocation of the evidence reported by the participating companies in a situation where companies report simultaneously. The next subsection deals with situations where companies report sequentially.

4.3. Following the order of entry into the program

Leniency program offer different compensations to participating companies that may depend on the timing of their entry into the program. Being the first to access the program may even lead to a complete exoneration.

Assume that the companies report to the Authority sequentially, following the natural ordering: 1, 2, …, *n*. The actual value of the evidence reported by a company then depends upon the evidence that was not *previously* disclosed. The first discloses all the information it has i.e. the evidence against *all* companies including itself:

$$b_1(N,v) = \Phi\big(E_1\big) = \Phi\left(\bigcup_{T\subset N: 1\in T} \Omega(T)\right) = \sum_{T\subset N: 1\in T} \Phi\big(\Omega(T)\big).$$

The company entering the second discloses the evidence it has and was not revealed by the first:

$$b_2(N,v) = \Phi\Big(E_2 \setminus E_1\Big) = \Phi\Big(\bigcup_{\substack{T\subset N:\\ 1\notin T, 2\in T}} \Omega(T)\Big) = \sum_{\substack{T\subset N:\\ 1\notin T, 2\in T}} \Phi\big(\Omega(T)\big).$$

In general, for the k^{th} company, we have:

$$b_k(N,v) = \sum_{\substack{T\subset N:\\ 1,\dots,k-1\notin T, k\in T}} \Phi\big(\Omega(T)\big), \quad k = 2,\dots,n.$$

In particular, $b_n(N,v) = \Phi\big(\Omega(\{n\})\big)$ and, by construction, $\sum_{i\in N} b_i(N,v) = \sum_{T\subset N} \Phi\big(\Omega(T)\big) = v(N).$

Example 1 (continued) Given the leniency game (73, 23, 31 | 133, 123, 87 | 216), the three successive subsets are $\{a,b,c,f\}$, $\{d,e\}$ and $\{g\}$, resulting in the allocation (129, 56, 31).

Proposition 2 The vector $b=(b_1,...,b_n)$ is the marginal contribution vector associated to the *reverse order* $\pi=(n,...,1)$ where the company that enters first in the leniency program is in the *last position*.

Proof We indeed have successively:

$$
\begin{aligned}
&v(n)=\Phi\left(\Omega(n)\right)=b_n(N,v),\\
&v(k,...,n)-v(k+1,...,n) = \sum_{T\subset\{k,...,n\}}\Phi\left(\Omega(T)\right)-\sum_{T\subset\{k+1,...,n\}}\Phi\left(\Omega(T)\right)\\
&\qquad\qquad = \sum_{\substack{T\subset N:\\ 1,...,k-1\notin T,k\in T}}\Phi\left(\Omega(T)\right) \quad \text{for } k=n-1,...,2,\\
&v(N)-v(N\setminus 1)=\sum_{T\subset N:1\in T}\Phi\left(\Omega(T)\right)=b_1(N,v). \quad \blacklozenge
\end{aligned}
$$

By definition, $b(N,v)$ is one of the vertices of the core.

By positivity, the core coincides with the set of weighted Shapley values: there exist normalized weights such that the corresponding weighted Shapley value coincides with $b(N,v)$ The core also coincides with the Harsanyi set: there exist distributions of the dividends that coincide with $b(N,v)$.

In the case where the Authority considers that several companies have reported evidence simultaneously, they should be allocated the average of the marginal contribution vectors associated to the corresponding orderings.

Example 1 (continued) If companies 1 and 2 had entered the program simultaneously, they would be allocated the average of the vectors (92, 93, 31) and (129, 56, 31) corresponding to the orders 321 and 312, resulting in the allocation (110.5, 74.5, 31).

5. Determination of potential fines

Given the evidence collected by the Authority, before launching the leniency program and after, each company is eventually associated to specific illicit actions, whether committed alone or within agreements with other companies, knowing that a value has been associated to each illicit action. This information is then used to establish the fines that should, in principle, be paid by each member of the cartel.

The Authority may decide to exonerate, partly or fully, the members of the cartel that have taken part in the leniency program, depending on the timing of their entry in the leniency program and on the quality of the evidence they have provided, an information that corresponds to the outcome of the game constructed in Section 4. The determination of the potential fines gives rise to a similar game where the set of players includes all cartel members. We consider the game (N,v) whose characteristic function associates to each coalition *S* the value of the damage that the members of *S* have been alone to commit:

$$v(S)=\Phi\left(\bigcup_{i\in S}D_i\setminus\bigcup_{i\in N\setminus S}D_i\right)=v(S)=\sum_{T\subset S}\Phi\left(\Omega(T)\right) \tag{8}$$

where D_i is the set of damage that has resulted from the illicit actions of company *i*, $\Phi(d)$ is the value of damage *d* and $\{\Omega(T) \mid T \subset N\}$ is the partition of the set of all damage *D* defined by:

$$\Omega(T) = \bigcap_{i \in T} D_i \setminus \bigcup_{i \in N \setminus T} D_i.$$

$\Omega(T)$ is the set of damage resulting from the *joint* actions of the companies in *T* while not resulting from the actions of outside companies. It is the dividend associated with coalition *T*. As a result, the game defined by (8) is positive and convex, and the Shapley value is given by (7). It provides a fair division of the total economic damage created by the cartel. By convexity, it is a core allocation: each coalition of companies *S* must pay at least $v(S)$, the value of the damage that it has been alone to create. Alternatively, referring to (5), no coalition of companies is asked to pay more than $v(N) - v(N \setminus S)$, its contribution to the total economic damage.

Again, the Shapley value is a natural allocation that treats companies equally: in case of an illegal action that involves several companies, the corresponding damage is divided equally among them. Some companies may eventually be partly or fully exonerated, a decision that remains in the hands of the Competition Authority and depends on the contribution in terms of evidence of each company that has participated in the leniency program.

Example 2 Consider a cartel that includes four companies whose damage sets are given by $D_1 = \{a,b,c\}$, $D_2 = \{c,d,e\}$, $D_3 = \{c,e,f,g\}$ and $D_4 = \{c,d,e,f\}$. We observe that there are four agreements between companies, leading to damage *c*, *d*, *e* and *f*, while the other damage, *a*, *b* and *g*, were caused by individual companies. We assume that the damage values are given by (16, 34, 24, 14, 33, 40, 39). The *nonempty* elements of the partition of the set of damage $D = \{a,b,c,d,e,f,g\}$ and the associated dividends are given by:

$$\Omega(1) = \{a,b\} \to 50,$$
$$\Omega(3) = \{g\} \to 39,$$
$$\Omega(2,4) = \{d\} \to 14,$$
$$\Omega(3,4) = \{f\} \to 40,$$
$$\Omega(2,3,4) = \{e\} \to 33,$$
$$\Omega(1,2,3,4) = \{c\} \to 24.$$

The associated game is given by (50, 0, 39, 0 | 50, 89, 50, 39, 14, 79 | 89, 64, 129, 126 | 200) whose Shapley value is (56, 24, 76, 44), covering the total damage equal to 200.

6. Concluding remarks

We have taken a fresh look at the analysis of leniency programs. Unlike most publications which rely on a non-cooperative approach to solve the stability/instability problem, we have focused on the fairness of fine reductions based on evidence. By taking seriously into account the fact that the Competition Authority establish the reduction of fines by applying two principles, namely the order of entry and the quality of the evidence provided, we show that these two principles are compatible. However, our results are based on the assumption that each piece of evidence provided by the companies is strong enough to prove part of the illegal behavior. Additional research would be useful to deal with other evidence structures that might be encountered, such as, for example, complementary evidence or insufficient evidence.

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