

Predictability of numerical models for reciprocal structures

Martin Steinmetz^{a,*}, Pierre Latteur^b, Luca Sgambi^a

^a Louvain Research Institute for Landscape, Architecture, Built Environment, UCLouvain, Louvain-la-Neuve 1348, Belgium

^b Institute of Mechanics, Materials and Civil Engineering, UCLouvain, Louvain-la-Neuve 1348, Belgium

ARTICLE INFO

Keywords:

Reciprocal structures
Motion capture system
Steel structure
Predictability
Hinged connection
Connection characterisation
Experimental study
Numerical simulation

ABSTRACT

Scientific literature shows that the actual behaviour of metallic reciprocal structures can differ significantly from that predicted by numerical models. This discrepancy is mainly due to the connections, whose complex designs generally exhibit nonlinear behaviour, including friction, locking effects, and semi-rigid responses. When the connections are perfectly rigid, as in welded beams, the structure is referred to as a gridshell, which is not the subject of this paper. Conversely, a reciprocal structure is theoretically composed of stacked beams and therefore features perfectly hinged (pinned) connections. In this article, we demonstrate that hinged connections enable a reliable numerical model. We develop a connection consisting of a spherical hinge assembled to rods attached to the ends of the bars, and we propose a methodology for integrating this connection into a numerical model to obtain reliable computational results. To achieve this, large-scale steel structural prototypes equipped with such a connection were constructed and tested. The results show a strong similarity between the numerical and experimental values of displacements and stresses. The experimental measurements were performed using a redundant measurement system, notably a motion capture system, which is highly effective for measuring three-dimensional displacements and had never previously been used for reciprocal structures.

1. Introduction and objectives

Reciprocal structures (RS) are planar structures (denoted RS^{2D}) or spatial structures (denoted RS^{3D}), composed of beams that mutually support one another without any apparent hierarchy (Fig. 1). An RS consists of at least one single module (SM RS) composed of three beams [1]. Each beam is supported at its ends by another beam or by a support, such that the connections link only two beams at a time. By assembling single modules, it is theoretically possible to cover any surface, thereby forming a multi-module reciprocal structure (MM RS). In this way, reciprocal structures can span large distances using stacked beams of relatively small dimensions, while producing an unusual architectural appearance. Despite these two advantages, very few MM RS^{3D} have been constructed worldwide for real projects [2], for three reasons:

- **Iterative geometric design:** the position of each element influences that of all the others, and any local adjustment propagates to the global geometry. However, the eccentricity between the beams (denoted e in Fig. 1) should ideally be identical to ensure uniform connections. Therefore, iterative numerical methods must be used to converge toward the desired geometry, for example, using genetic

algorithms [3], dynamic relaxation [4], or form-finding algorithms [5].

- **Limited reliability in predicting structural behaviour:** connections often exhibit uncertain stiffness parameters and nonlinear behaviour, which strongly influences the global response of the structure. In addition, reciprocity implies that the members are subjected to all combinations of internal forces and undergo three-dimensional displacements, even under vertical loading [6]. Moreover, local and global geometrical stability is sometimes difficult to guarantee [7], [8].
- **Assembly/disassembly complexity:** the type of connection plays a crucial role in the construction sequence. The more a connection restricts movement, the less the structure can adapt to geometric deviations during construction. In the extreme case where beams are welded together, as in the structure of the Rokko Mountain Observatory, any error during assembly makes it impossible to install the subsequent beams [9]. Moreover, since the beams support one another, temporary supports or formwork are required during assembly [10].

These three factors impose significant constraints on the design of

* Corresponding author.

E-mail address: martin.steinmetz@uclouvain.be (M. Steinmetz).

<https://doi.org/10.1016/j.engstruct.2026.123249>

Received 11 March 2026; Received in revised form 23 May 2026; Accepted 17 June 2026

Available online 23 June 2026

0141-0296/© 2026 Elsevier Ltd. All rights are reserved, including those for text and data mining, AI training, and similar technologies.

reciprocal structures, particularly their connections, which must allow easy beam assembly, tolerate fabrication or assembly inaccuracies, and ensure sufficient strength. Given their number and visual prominence, connections must also be aesthetically pleasing and cost-effective.

A previous study by the authors [7] highlights the wide variety of possible RS connections while emphasising that designers of RS^{3D} generally select connections that restrain one or more rotational degrees of freedom between beams, in addition to the three translational degrees of freedom. These rotational constraints promote the stability of RS^{3D}, but they also complicate construction and, above all, contradict the definition of reciprocal structures, which are supposed to be composed of beams simply resting on one another [11]. Selecting completely hinged (pinned) connections and supports that restrain only the three translational degrees of freedom is therefore recommended. In this case, members hinged at their ends remain geometrically stable in rotation about their longitudinal axis only due to the eccentricities between beams' axes [7]. It should be noted that some hinged RS^{3D} geometries are not stable, which requires careful consideration of the geometry during the design process [7].

The objective of this article is to demonstrate that hinged connections, whose behaviour has been experimentally characterised, allow a reliable numerical model to be obtained for reciprocal structures. To this end:

- We develop a connection comprising a spherical hinge linked by rods to the ends of the members, which makes it possible to construct reciprocal structures more easily and inspire real-world projects.
- We propose a methodology to reliably integrate this type of connection into a numerical model.
- We assess the assembly difficulties of a large-scale MM RS^{3D} prototype and of the modules composing it. This structure is shown in Fig. 2 and presented in Section 4.
- We validate this hinged connection by comparing numerical and experimental results in terms of nodal displacements and stresses in the members.

2. Experimental testing on RS connections: state of the art

Steinmetz et al. describe in [9] the state of the art regarding experimental campaigns aimed at validating the behaviour of RS connections. Fig. 3 presents the connections used in the six most comprehensive studies: three RS^{2D} connections (Fig. 3a, b and c) and three RS^{3D} connections (Fig. 3d, e and f).

- In the study by Pizzigoni [12], RS^{2D} prototypes made of fibre-reinforced concrete were developed to assess the feasibility of demountable, reusable reciprocal structures. The double S-shaped beams are connected by a metallic sphere, allowing rotation while maintaining a constant and strong contact point (Fig. 3a). Experimental measurements on a three-beam structure show good agreement with the numerical results obtained using solid finite-element modelling presented by Garavaglia et al. [13].
- Gustafsson [14] designed an MM RS^{2D} structure made of squared timber using a half-lap joint to ensure that the beam axes remain in the same plane (Fig. 3b). The numerical model idealises the connection behaviour using six springs, whose stiffnesses are determined analytically. Experimental validation of this model was affected by the intrinsic variability of the timber material and by modelling simplifications, notably the absence of local section reduction and the use of linear analysis. For these reasons, the prototype deformed under a relatively low load (approximately 3 times the predicted value) and failed at 71% of the estimated maximum load. Despite this modelling difficulty, the connection enables rapid assembly and disassembly, which facilitates structural reuse.
- Steinmetz et al. [15] proposed an experimental methodology applied to an SM RS^{2D} composed of three laterally connected steel tubes (Fig. 3c). They highlight the benefit of using cable potentiometers to measure a single component of nodal displacement. In addition, determining internal forces using strain gauges enables a clearer discussion of load transfer within the structure. For this simple structure, the numerical model is slightly stiffer (up to 2.1%) and accurately predicts von Mises stresses (<0.5%).
- For an MM RS^{3D} structure made of circular steel tubes, Rizzuto and Hulse [16] proposed a connection using a through-bolt and saddle washers to facilitate load transfer (Fig. 3d). Rizzuto [17] compared experimental measurements with numerical results while considering various assumptions regarding connection behaviour. None of the models provided satisfactory agreement between calculations and measurements because the real behaviour of the connections is difficult to determine accurately.
- Godina Rodríguez [18] proposed a connection between two bamboo poles made of three wooden blocks machined to fit the shape of the poles and then clamped with rods and steel plates (Fig. 3e). This connection is modelled as fixed because it is assumed to be much stiffer than the members. Tests on an SM RS^{3D} with three beams showed good agreement with numerical models, but this correlation deteriorated for an MM RS^{3D}; numerical models tended to over-estimate stiffness. The difficulties mainly stemmed from uncertainty

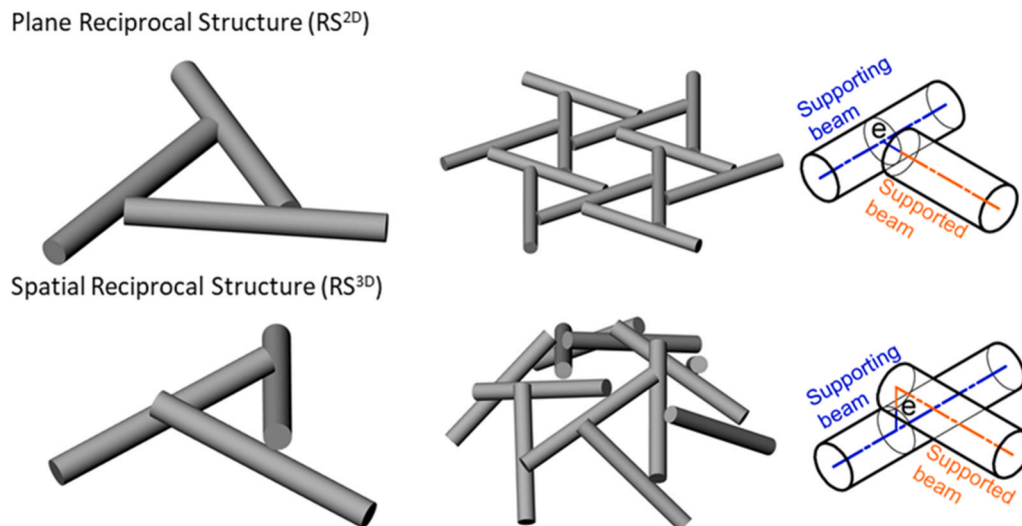


Fig. 1. Planar and spatial reciprocal structures (left: single modules SM RS, centre: multi-module structures MM RS, right: connection type).



Fig. 2. The RS-24B prototype, built outdoors: a) General view and b) Central rings.

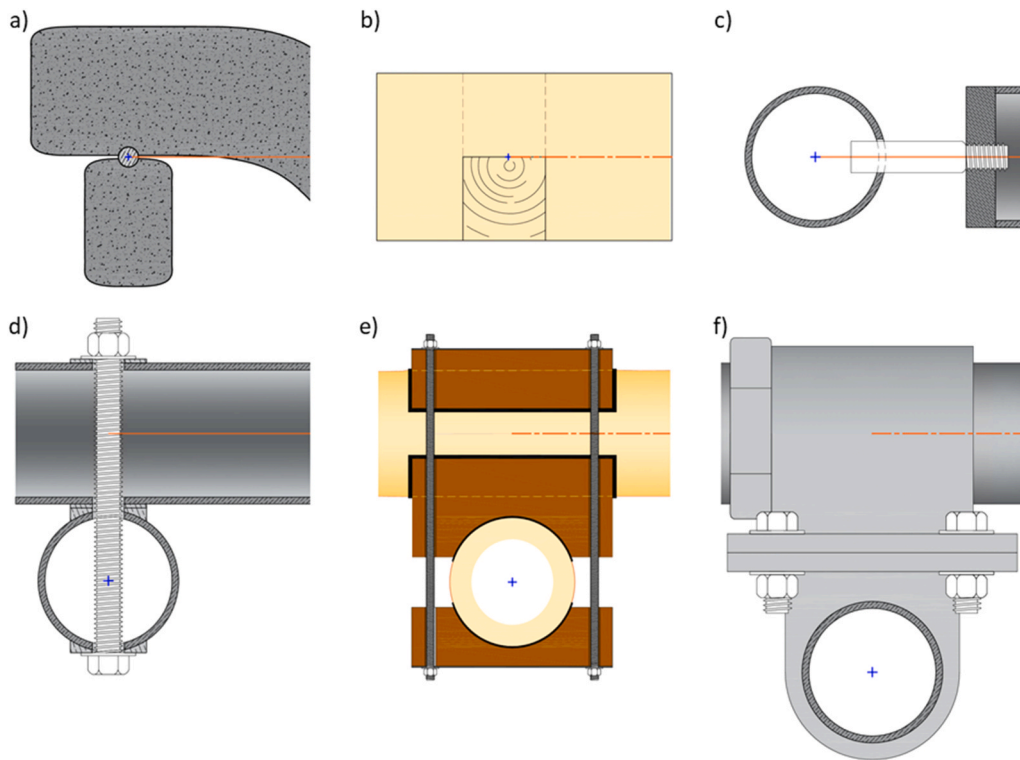


Fig. 3. Connections tested for RS^{2D} in cases a), b) and c), and for RS^{3D} in cases d), e) and f).

in bamboo's mechanical properties and geometric discrepancies between the prototypes and the models, which limited the scope of validation.

- Qi et al. [19] developed a rigid connection between steel tubes, which corresponds more closely to the definition of a gridshell than to that of a reciprocal structure. This connection consists of two identical parts assembled with four bolts according to the orientation of the two tubes (Fig. 3f). The results of a test performed on an SM RS^{3D} with six beams showed good overall agreement with the numerical model, with average deviations below 7.5%.

Overall, across these studies, the reliability of numerical analysis, the accuracy of measurements, and the understanding of connection behaviour remain major challenges.

3. Development of a reliable hinged connection for RS^{3D}

The developed connection was the subject of several successive variants tested on an SM RS^{3D} composed of three tubes (S235). Nu-

merical results were compared with experimental measurements in terms of nodal displacements and stresses in the tubes:

- The first tested connection is inspired by the one proposed by Rizzuto [17], presented in Fig. 3d. To limit rotational constraints, the M12 threaded rod (8.8 class) passes only through the supporting tube (Fig. 4a). A neoprene layer is added between the tubes (1 mm) and at the interface between the bolt head and the inside of the supported tube (2 mm). This connection is then modelled by a hinge at the contact point between the two tubes, connected to the tube axes by two infinitely rigid elements whose lengths equal the tubes' radii. The tests showed that this type of connection does not behave linearly, with significant discrepancies between numerical results and measurements: about 10% for nodal displacements and 20% for bending stresses. This behavioural difference is due to the supported tube being restrained in rotation about its axis by the bolt head, despite the spacing provided by the neoprene, with no bolt preload.
- In the second proposed solution (Fig. 4b), the top of the supported tube is perforated and placed on a sphere so that it can rotate freely on it in the three directions, thanks to a sufficiently large opening on

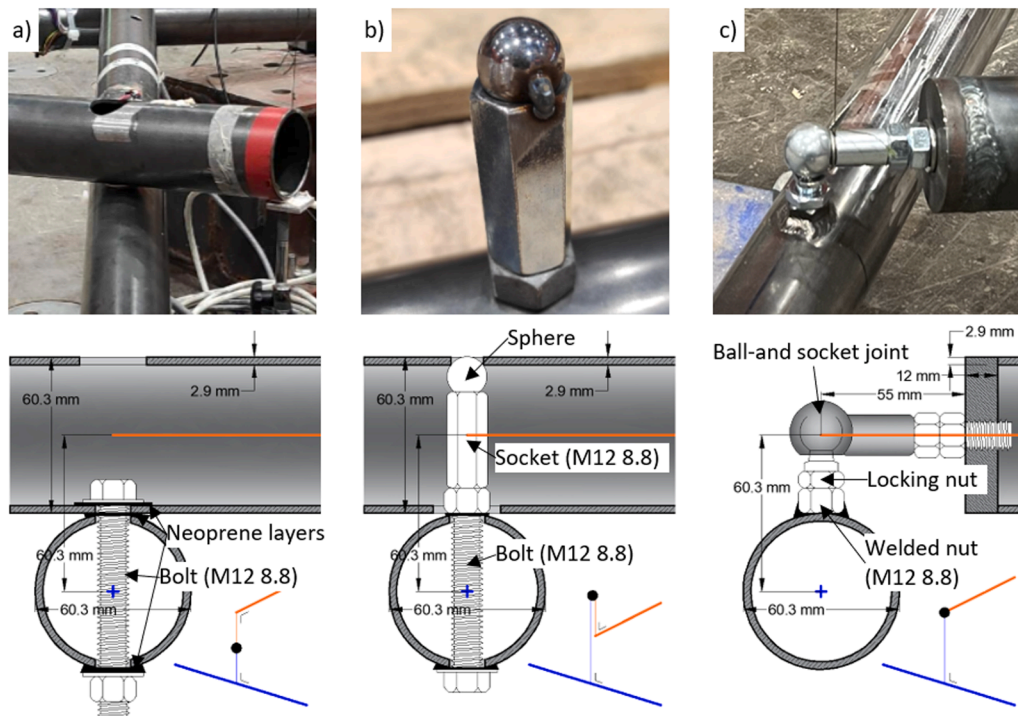


Fig. 4. Three proposed steel tube connections and their numerical modelling.

the opposite lower face. The sphere is welded to a hexagonal socket nut, which is connected to the supporting tube by an M12 steel bolt passing through the nut. An additional nut between the socket nut and the supporting tube is necessary to ensure that the eccentricity between the tubes equals the sum of the radii of both tubes. On average, the numerical results at the measurement points correspond well with the experimental measurements, with deviations of only a few per cent. Although this test confirms that perfect hinges ensure reliable numerical results, this connection is not very relevant because it does not prevent uplift or disengagement.

- The third connection is illustrated in Fig. 4c. The supported member ends with a flat plate that has a threaded hole. During assembly, the threaded rod is inserted into this hole, two M12 nuts are tightened, and the female part of a commercial ball-and-socket M12 joint is connected. At the level of the supporting tube, the male part is screwed into a M12 nut welded to the tube and secured with a locking nut. The successive metallic parts of the connection are modelled by short solid cylindrical elements whose stiffness is determined experimentally (this procedure is detailed later in this

section). The numerical results show good agreement between experimental tests and numerical calculations, with differences of about 1% for nodal displacements and 4% for bending stresses. However, this connection was not adopted because inserting the male part of the joint into the supporting tube nut is difficult, as perfect alignment with the welded nut is required for screwing. In addition, the welded nut engages only the upper part of the tube, which may cause local ovalisation.

- The fourth solution, an improvement of the previous one, is presented in Fig. 5 and was selected for the construction of the prototypes. It ensures ease of assembly while preserving the hinge between the two beams and improving force transmission. At the supporting tube, the male part of the ball-and-socket joint M16 is screwed into a sleeve ensuring continuity with the threaded rod, which is itself inserted into a tubular reinforcement welded inside the supporting tube. A locking nut placed on the opposite side of the reinforcing tube secures the connection in case of uplift. The thickness of this tubular

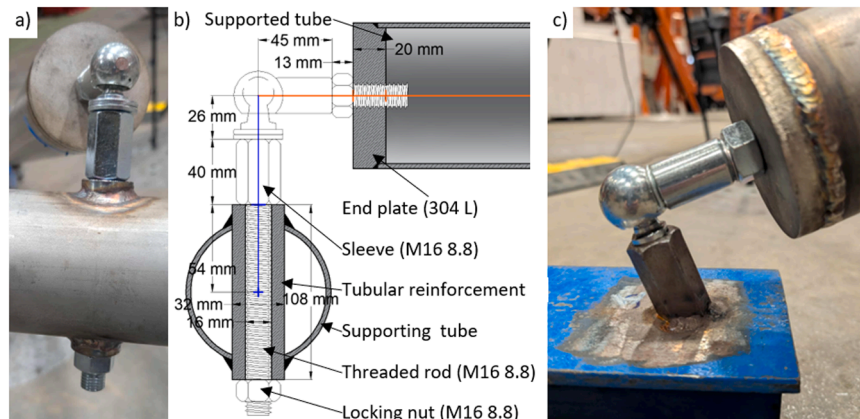


Fig. 5. Final connection proposal: a) connection between members, b) descriptive diagram, c) support connection.

reinforcement is relatively large due to welding constraints. This connection can also be used to create hinged supports (Fig. 5c).

The connection shown in Fig. 5 consists of a succession of elements with specific mechanical properties. In the numerical model, we propose representing them by a hinge connected by two cylinders, whose fictitious diameters are determined from experimental tests on the connection. Fig. 6 illustrates how these tests were performed. The actuator force is applied progressively (Fig. 6a): three loading–unloading cycles up to 2 kN, followed by three cycles up to 3.5 kN, three cycles up to 5 kN, and finally a monotonic loading up to failure.

This loading protocol is applied to the connection placed in two configurations (Figs. 6b and 6c). For the part located in the extension of the supported beam (denoted *p*, Configuration 1), elastic behaviour is observed up to at least 5 kN, followed by a slightly plastic phase before failure of the rod at the flat plate. For the eccentricity part (denoted *e*, Configuration 2), the initial stiffness is higher, but a clear elastic limit appears between the cyclic loads of 2 kN and 3.5 kN, before reaching a ductile failure by thread stripping near the interface between the reinforcement and the sleeve. The loading protocol applied to the prototypes tested in the laboratory (Section 4) must therefore ensure that these forces (Configuration 1: 5 kN and Configuration 2: 2 kN) are not exceeded at any connection.

The equivalent diameters of the two cylinders (ϕ_p et ϕ_e) are determined assuming fixed-end conditions. It is then possible to relate the stiffness obtained from the test (δ/F) to the geometric parameters (κ , l , A , and I) and material properties (E and G) of each cylinder, considering only bending effects (bending moment and associated shear force):

$$\frac{\delta}{F} = \frac{l^3}{3EI} + \frac{l}{G\kappa A} \quad (1)$$

Eq. 1 makes it possible to determine the diameter of the cylinders:

$$\phi = \sqrt{\frac{2}{\pi \kappa G \delta} + \sqrt{\left(\frac{2}{\pi \kappa G \delta}\right)^2 + \frac{64}{3} \frac{F l^3}{\pi E \delta}}} \quad (2)$$

The stiffness is determined during the last cycle, reaching 5 kN and 2 kN, respectively, for configurations 1 and 2, using linear regression over the entire loading phase of the last cycle. Assuming an isotropic material ($E_p = E_e = 200000 \text{ MPa}$ and $G_p = G_e = 77000 \text{ MPa}$), and knowing the equivalent beam lengths ($l_p = 58 \text{ mm}$ and $l_e = 66 \text{ mm}$) indicated in Fig. 6d, the equivalent diameters are computed using Eq. 2: $\phi_p = 12.8 \text{ mm}$ et $\phi_e = 17.1 \text{ mm}$. These informations are recalled in Table 1.

It is important to emphasise that, in the context of a real project involving connections of the same type but different sizes, laboratory tests will also be required to introduce the correct connection data (the two-cylinder diameters) into the numerical model.

4. Description of the tests performed on the prototypes

The proposed connection is used for connecting the 24 stainless steel 304 L tubes (88.9×3.05) of a large-scale MM RS^{3D} prototype that covers an area of 9.4 m in diameter and reaches a maximum height of 3.5 m. This structure, denoted RS–24B, is composed of four rings (denoted A, B, C and D), each consisting of six beams (Fig. 7a). The six beams of the rings A, B and C are identical and measure respectively 161 mm, 221 mm, and 251 mm, which simplified fabrication. The lengths of the supporting beams in ring D vary from 330 to 571 mm to ensure the desired inclination (Fig. 7b). This structure could only be tested outdoors at its final site due to its large size (see Section 6).

4.1. Tested prototypes

Before testing the RS–24B, prototypes with fewer bars are tested in

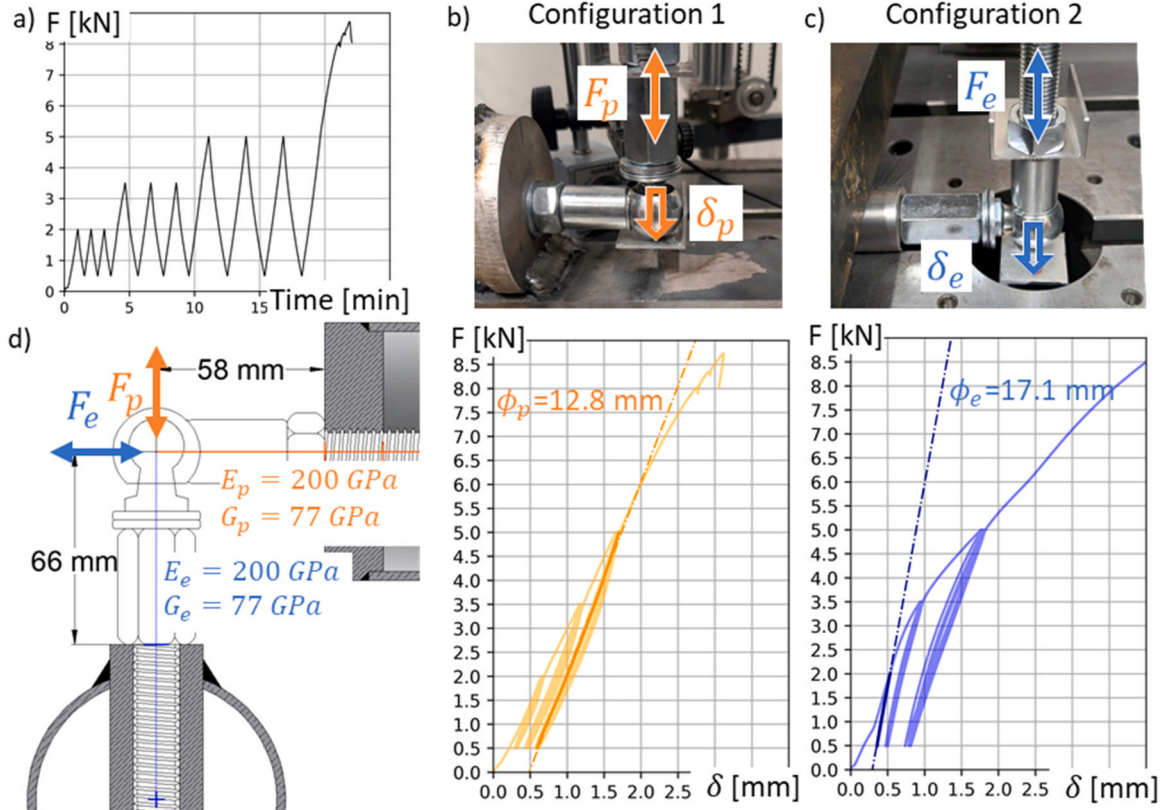
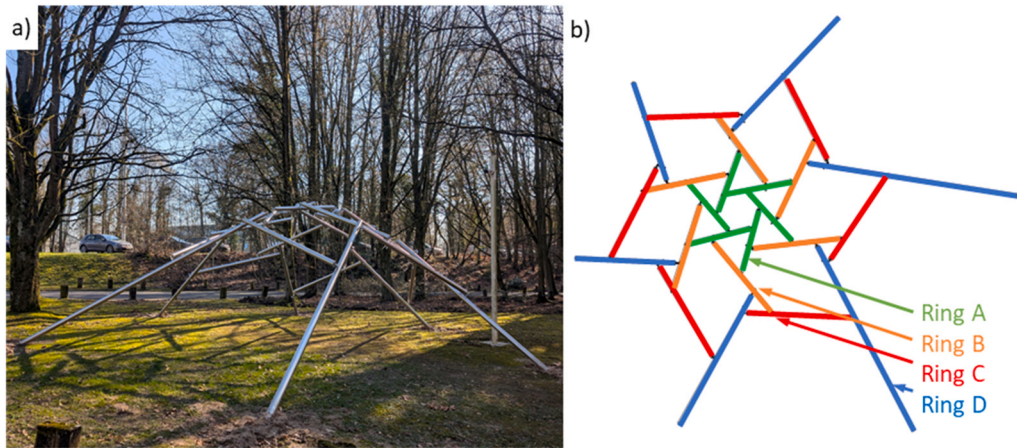


Fig. 6. Connection characterisation: a) loading protocol, b) Configuration 1 with the load applied to the supporting part. c) Configuration 2 with the load applied to the eccentricity part. d) Scheme of the connection.

Table 1

Material and geometric properties for the modelled beam elements. Values with asterisk (*) are measured. NR = Not Relevant.

Property	Symbol	Unit	Values		
			Tube 88.9 × 3.05	Equi. cylinder Config 1	Equi. cylinder Config 2
Material	Young's Modulus	E	193000*	200000	200000
	Shear's Modulus	G	74230	77000	77000
	Yield strength at 0.2%	f_y	215*	NR	NR
	Poisson's Ratio	ν	0.3	0.3	0.3
Geometric	Outer Radius	R	44.45	6.4	8.55
	Inner Radius	r	41.40	0	0
	Area	$A = \pi(R^2 - r^2)$	822	129	230
	Moment of Inertia	$I = \frac{\pi}{4}(R^4 - r^4)$	758802	1318	4197
	Polar Moment of Inertia	$I_p = \frac{\pi}{2}(R^4 - r^4)$	1517604	2635	8394
	Radius Ratio	$m = r/R$	0.931	0	0
	Shear Coefficient [20]	$\kappa = \frac{6(1+\nu)(1+m^2)^2}{(7+6\nu)(1+m^2)^2 + (20+12\nu)m^2}$	0.532	0.886	0.886

**Fig. 7.** a) RS-24B constructed on site. b) Scheme of RS-24B with ring indication.

the laboratory to validate the numerical modelling and to understand the effect of modelling uncertainties related to connections on the structure's response. Specifically, the following structures were tested in the laboratory (Fig. 8):

- A single three-beam module RS-3B, composed of two beams (denoted A1 and A2) belonging to ring A and one beam (denoted B1) belonging to ring B.
- Ring A, denoted RS-6B (six beams).
- Rings A and B together, denoted RS-12B (12 beams).

The stainless-steel tubes (grade 304 L) have an outer diameter of 88.9mm with a thickness of 3.05mm whose material and geometric properties are presented in Table 1.

As explained in the introduction, erecting a reciprocal structure is complex because the structure remains unstable until all beams are assembled. For the RS prototypes tested in the laboratory, the supports were first fixed to the test floor with a positioning accuracy ensured by the Motion Capture System (MCS) described in Section 4.2. A marker is placed on top of the sleeve, allowing the support to be positioned with millimetre accuracy using live tracking (Fig. 9a).

The assembly of RS-3B and RS-6B is relatively simple because they consist of a single module and require only one temporary support to hold the first beam. The assembly of RS-12B and RS-24B is more complex. The erection of RS-12B follows the six assembly steps shown in Fig. 9b, which require pre-assembling the three-beam modules and using three temporary supports. The assembly of RS-24B is described in

Section 6.1.

4.2. Measurement instrumentation

The instrumentation must be redundant and account for structural points moving in all directions and for most members being subjected to six internal forces. The measurement locations are shown in Fig. 8. Measurements were performed using the following instruments (Fig. 10):

- The cable potentiometer (Fig. 10a) measures the vertical displacement of the connection point expected to experience the largest displacement. Its advantage is that it is only slightly affected by horizontal displacement components if placed sufficiently far from the cable attachment point.
- The Motion Capture System (MCS) measures three-dimensional displacements of reflective markers (small white plastic spheres) using eight optical cameras (Fig. 10b). This method allows a large number of measurement points, provided each marker is observed by at least two cameras [21]. The measurement accuracy is approximately $\pm 0.05\text{mm}$. Markers are placed at the beam ends, beneath the connection rods, and on the load cell.
- Six strain gauges (SG) are glued around the perimeter of the same cross-section to determine the internal forces in the beam's local coordinate system (defined in Fig. 10c according to their positive direction) and the maximum von Mises stress ($\sigma_{VM,Max}$). This cross-section is located midway between the two central connections of

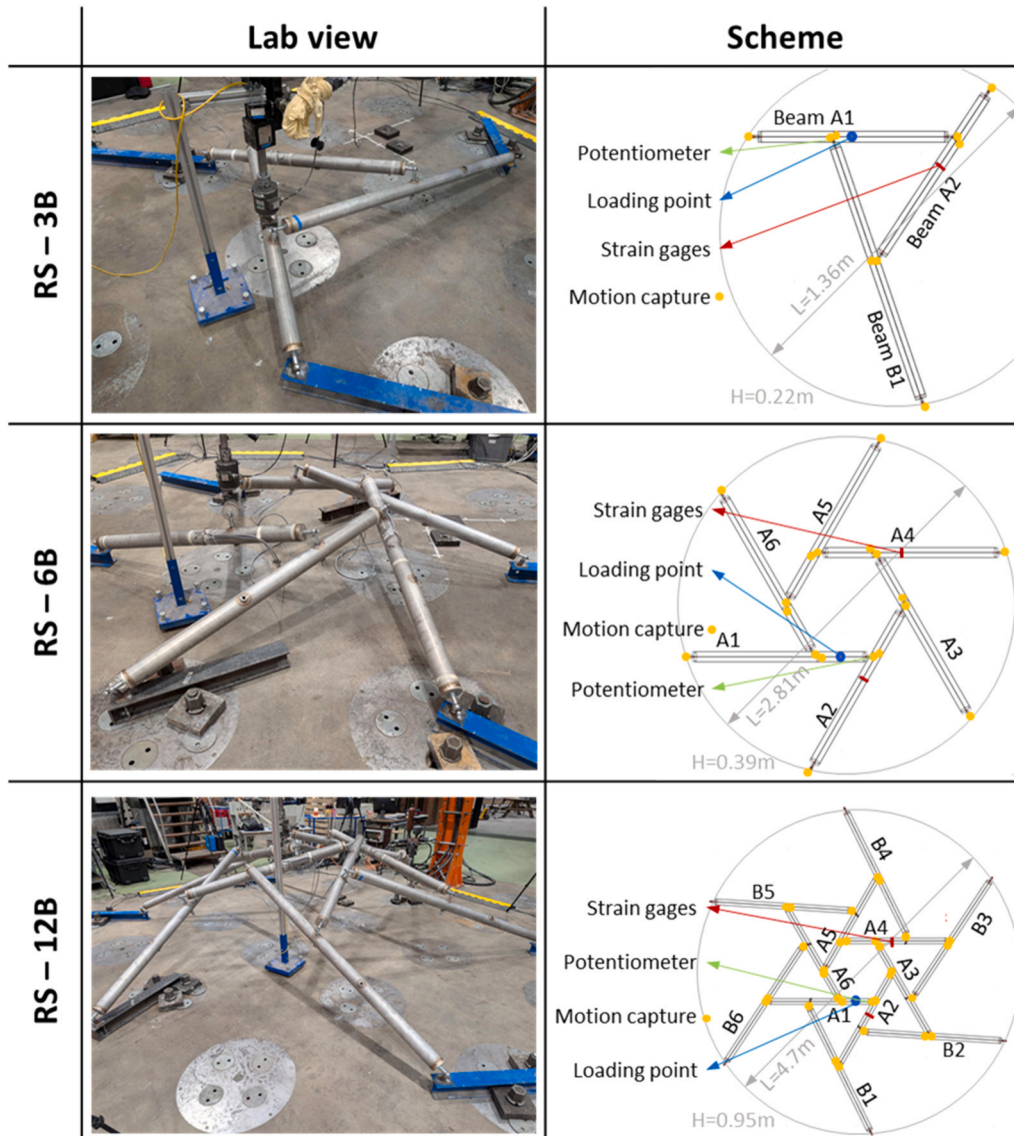


Fig. 8. Structures RS-3B, RS-6B and RS-12B. L is the diameter of the circle passing through the support points, and H is the total height.

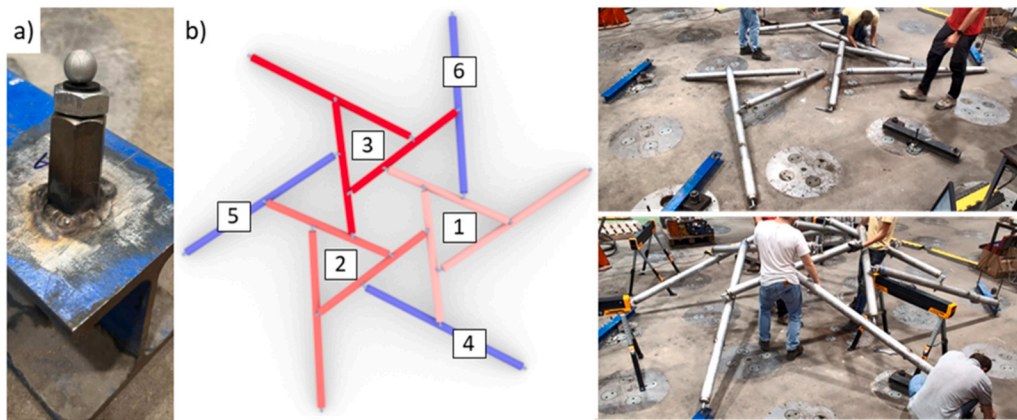


Fig. 9. a) Positioning of support using the MCS. b) Six assembly steps (shown above) of RS-12B in the laboratory, with photographs of the pre-assembly and assembly step 5.

Beam A2 and, for RS-6B and RS-12B, also between those of Beam A4. This location corresponds to a highly stressed region. Still, it is

located $239\text{mm} (\geq 4R)$ from each connection, ensuring the validity of Saint-Venant's principle. The methodology used to compute

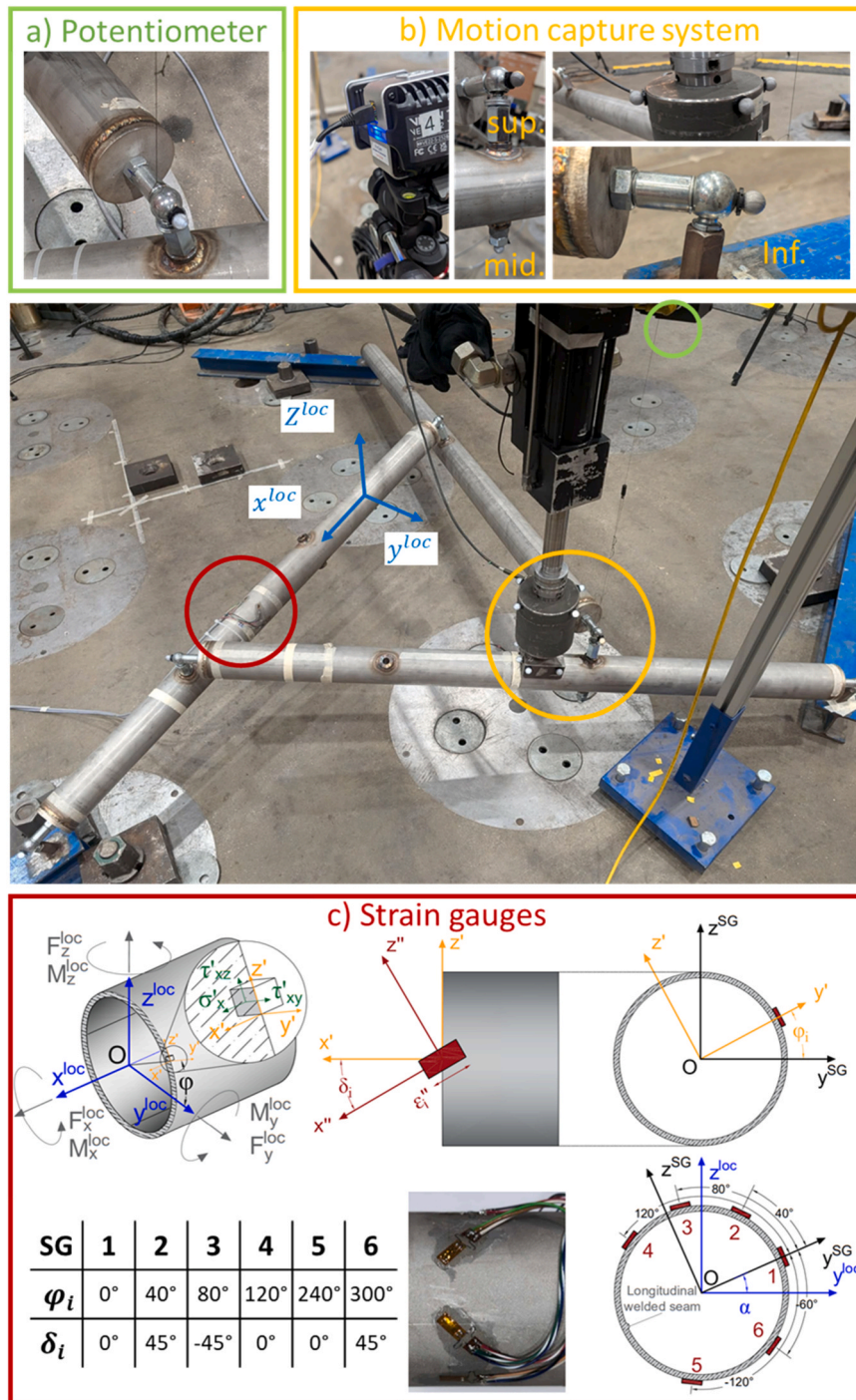


Fig. 10. Instrumentation details on RS-3B (similar for the other structures tested): a) cable potentiometer, b) motion capture system, and c) strain gauges.

internal forces and von Mises stresses from strain gauge measurements was described by Steinmetz et al. [15] and is not repeated here.

4.3. Loading protocol

The loading protocol is identical for the three prototypes (Fig. 11):

- The actuator force F is applied to Beam A1 through a steel plate to avoid local stress concentrations. The loading rate is slow (0.025 mm/s) to obtain a sufficient number of measurement points.
- Eight loading–unloading cycles are first applied progressively between $F_{\min} = 0.25\text{kN}$ and $F_{\max}$ equal respectively to 6.25kN, 4.75kN and 3.25kN for RS-3B, RS-6B and RS-12B.
- Five additional loading cycles (cycles 9–13) are then applied up to $F_{\max}$. Only these cycles are discussed in the results, which are presented relative to the beginning of cycle 9.

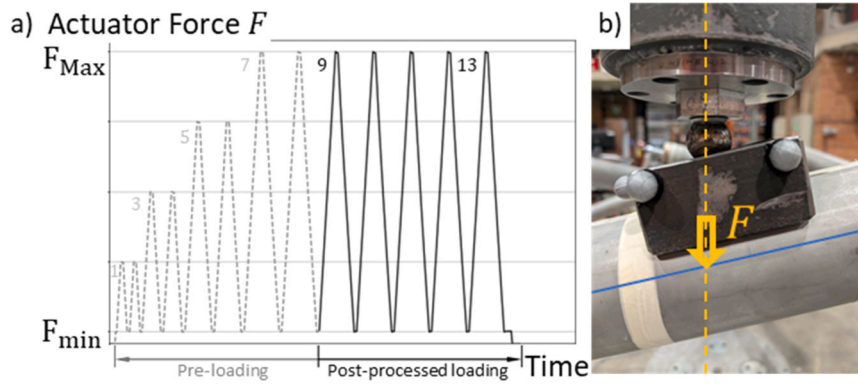


Fig. 11. a) Loading protocol for RS-3B, RS-6B and RS-12B tested in the laboratory. b) Application of the actuator force on Beam A1.

5. Comparison between experimental measurements and numerical results

5.1. Numerical model assumptions

The structures were analysed using the commercial software SCIA, with the following assumptions:

- Each tube is modelled by its centroidal axis. The local coordinate system used to determine internal forces is shown in Fig. 10.
- Each connection and support (Fig. 5) is modelled as a hinge connected to the tubes by two cylindrical elements (Section 3) whose mechanical characteristics are presented in Table 1 and Fig. 6.
- The actuator force is applied on the centroidal axis of Beam A1 at the intersection between the vertical actuator axis and the tube axis (Fig. 11b).
- Nodes corresponding to the positions of the MCS markers are connected to the model by massless beams.
- A second-order analysis is performed to account for geometric nonlinearities, since some prototypes, such as RS-12B or RS-24B, are relatively flexible. Material nonlinearity is not considered because the loading is intentionally limited to remain within the elastic range of both tubes and connections.

The numerical modelling and stability checks were performed before testing the four structures. Fig. 12 shows the RS-12B model with bending moment response (Fig. 12a), as well as the modelling details of the connection (Fig. 12b) and support (Fig. 12c).

In the following, the difference between a numerical result (denoted *Num.*) and an experimental measurement during the loading phase of the last cycle (cycle 13, denoted *Exp.*) is expressed as an “error” defined

by: $100 \times (\text{Num.} - \text{Exp.}) / \text{Exp.} [\%]$.

5.2. Test results

5.2.1. Measurements using the potentiometer

Fig. 13 compares the cable potentiometer measurements for the last five cycles (superimposed) with the numerical displacements at the same point. The errors are respectively -6.6% for RS-3B, $+2.4\%$ for RS-6B, and $+7.2\%$ for RS-12B. Although the potentiometer measures only a single point of the structure, the agreement between experimental and numerical results is very good for the three prototypes.

5.2.2. Measurements using the motion capture system

Fig. 10b defines the notation: for a given beam A_1 , the marker located at its highest end is denoted A_1^{sup} , at its lowest end A_1^{inf} , and the intermediate one A_1^{mid} . For example, the connection between A_1 and A_2 has two markers: above, the marker of the supported beam (A_1^{sup}) and below, the marker of the supporting beam (A_2^{mid}).

Fig. 14 presents, for the three structures:

- Histograms of the vertical displacement error during the last loading cycle (except for support markers, whose vertical displacement is nearly zero).
- A plan view showing the measured horizontal displacement vectors of the markers (coloured) and the numerically calculated displacement (darkened).

For RS-3B, the marker closest to the potentiometer measurement (B_1^{sup}) shows an error of -8.7% , similar to that measured by the potentiometer (-6.6%). However, larger errors occur at some measurement

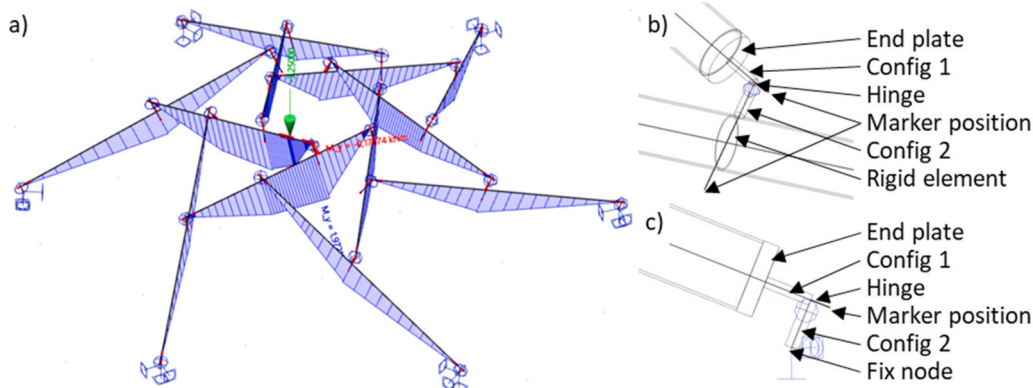


Fig. 12. a) Numerical model of RS-12B, with bending moment for maximal actuator force. b) Detailed modelling of the connection. c) Detailed modelling of the support.

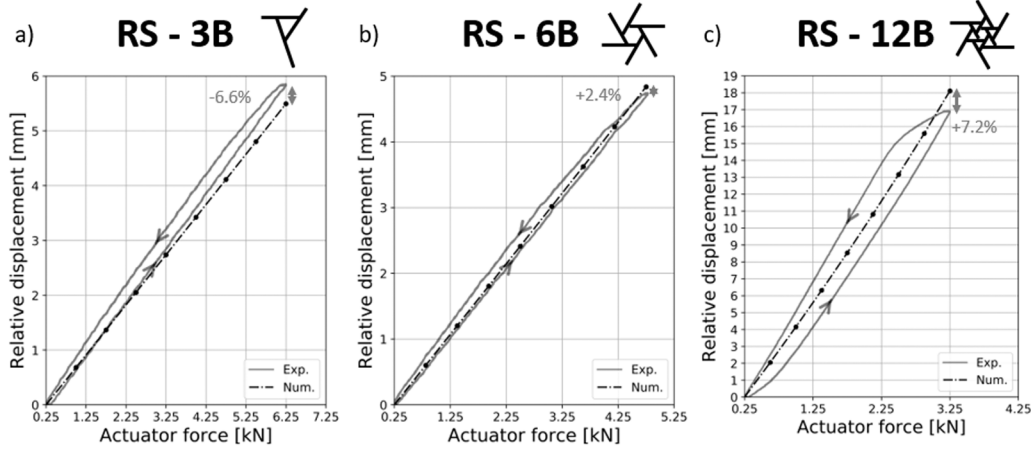


Fig. 13. Comparison between the displacements (relative to the beginning of cycle 9) measured by the potentiometer and those predicted by the numerical model for: a) RS-3B, b) RS-6B, and c) RS-12B.

points, reaching up to +36.7% (Fig. 14a). These discrepancies are attributed to the horizontal components of the support reactions (Fig. 8), which induce unexpected elastic bending of the support beams. In particular, the marker A_1^{inf} presents a total horizontal displacement of 1.87 mm, compared to the computed 0.14 mm.

For RS-6B, the marker closest to the potentiometer measurement (A_1^{sup}) shows an error of -3.6%, close to the potentiometer error (+2.4%). The model predicts vertical displacements very well, with errors between -3.6% and +6.1%, even though some supports moved horizontally due to the flexion of the support beams (A_1^{inf} and A_4^{inf}).

For RS-12B, the marker closest to the potentiometer measurement (A_1^{sup}) shows an error of +3.4%, consistent with the potentiometer (+7.2%). The model predicts vertical displacements very accurately for the 18 central markers (A^{sup} , A^{mid} and B^{sup}), with errors ranging from +0.4% to +5.5%.

For five of the twelve peripheral markers (B^{mid} and A^{inf}), the errors are larger, reaching -6.9% and +20.2%. Regarding the horizontal behaviour shown in Fig. 14c, the model correctly predicts the “spinning-top” effect, with a slight overestimation for all markers. Unlike the other tests, the support nodes were not instrumented to prevent the cameras from being placed too far from the central markers. It is therefore not possible to determine whether some support points moved.

5.2.3. Measurements using strain gauges

Beams A2 and A4 are equipped with strain gauges enabling the determination of internal forces and maximum stresses. Fig. 15 presents

only the values of the total bending moment $M_{y,z,\text{tot}} =$

$\left((M_y^{\text{loc}})^2 + (M_z^{\text{loc}})^2 \right)^{1/2}$ and the von Mises stress σ_{vM} during the last five loading cycles, which are perfectly superimposed. The other internal forces were also measured but are negligible, generating less than 1.5% of the von Mises stress.

For RS-3B, only Beam A2 is instrumented. Fig. 15a shows that the maximum bending moment is very close to the maximum numerical value, with an error of -2.5%. The maximum von Mises stress is also very close to the numerical value, differing by only +0.1%.

For RS-6B (Fig. 15b), the errors are also very small. Beams A2 and A4 exhibit maximum deviations of -1.5% and -2.3%, respectively, for the maximum bending moment, and -0.8% and -0.9% for the maximum von Mises stresses.

For RS-12B (Fig. 15c), Beam A4 shows an error of -3.4% for the bending moments and -4.2% for the von Mises stresses. However, these errors are larger for Beam A2, reaching +10.9% and +10.6%,

respectively.

5.3. Discussion

5.3.1. Validation of the numerical model and relevance of the hinged connection

With a few exceptions, which can be explained by more flexible support than expected, the numerical model's predictions are very good for the three structures. The redundancy of the experimental measurements, obtained with three different instruments, confirms that:

- The choice of a hinged connection is appropriate for reciprocal structures.
- The connection modelling approach described in Section 3, including experimental characterisation according to the defined protocol, is relevant and reproducible for other types of reciprocal structures and for real projects.

It should be noted that a clear hysteresis is visible for RS-12B in both displacements and stresses. This behaviour can be explained by friction at the ball-and-socket joints, as observed during local tests on the connection (Fig. 6). Since these joints were reused across different tests and assemblies, it would have been advisable to apply additional lubrication in addition to the factory-applied grease. Other sources of hysteresis can be excluded as the structures are tested in their elastic range, and the loading rate is very slow.

5.3.2. Ease of assembly

The proposed connection (Fig. 5) enables rapid assembly and disassembly of the structures: RS-12B can be assembled in less than 30 min by four people (excluding the time required to anchor the supports to the test floor). The proposed connection allows two assembly methods:

- **Method A** (Fig. 16a): the threaded rod is first fixed to the sleeve, then inserted into the tubular reinforcement (Phase 1). The connection is then secured by the locking nut (Phase 2). This method is very fast but requires sufficient separation between the two tubes during assembly. If this is not possible—for example, when the supported tube is already connected to other tubes—Method B must be used.
- **Method B** (Fig. 16b): the sleeve attached to the joint is first aligned with the tubular reinforcement, after which the threaded rod is inserted into the tubular reinforcement (Phase 1). In Phase 2, the threaded rod is screwed into the sleeve.

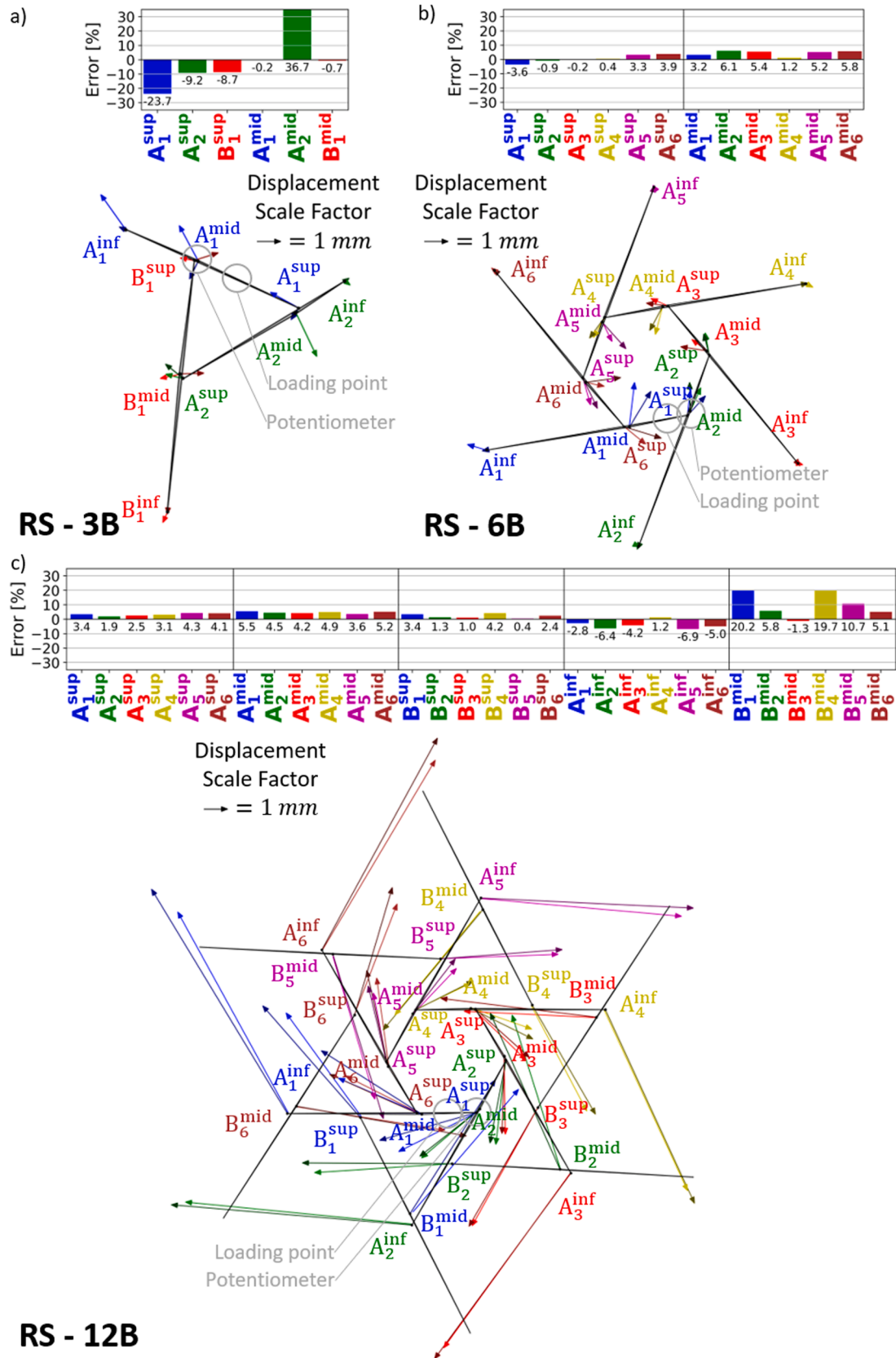


Fig. 14. For the three prototypes, error value on vertical displacement at the markers, and plan view of the structure's behaviour with measured displacement vectors (in colour) and numerically calculated displacement vectors (in dark).

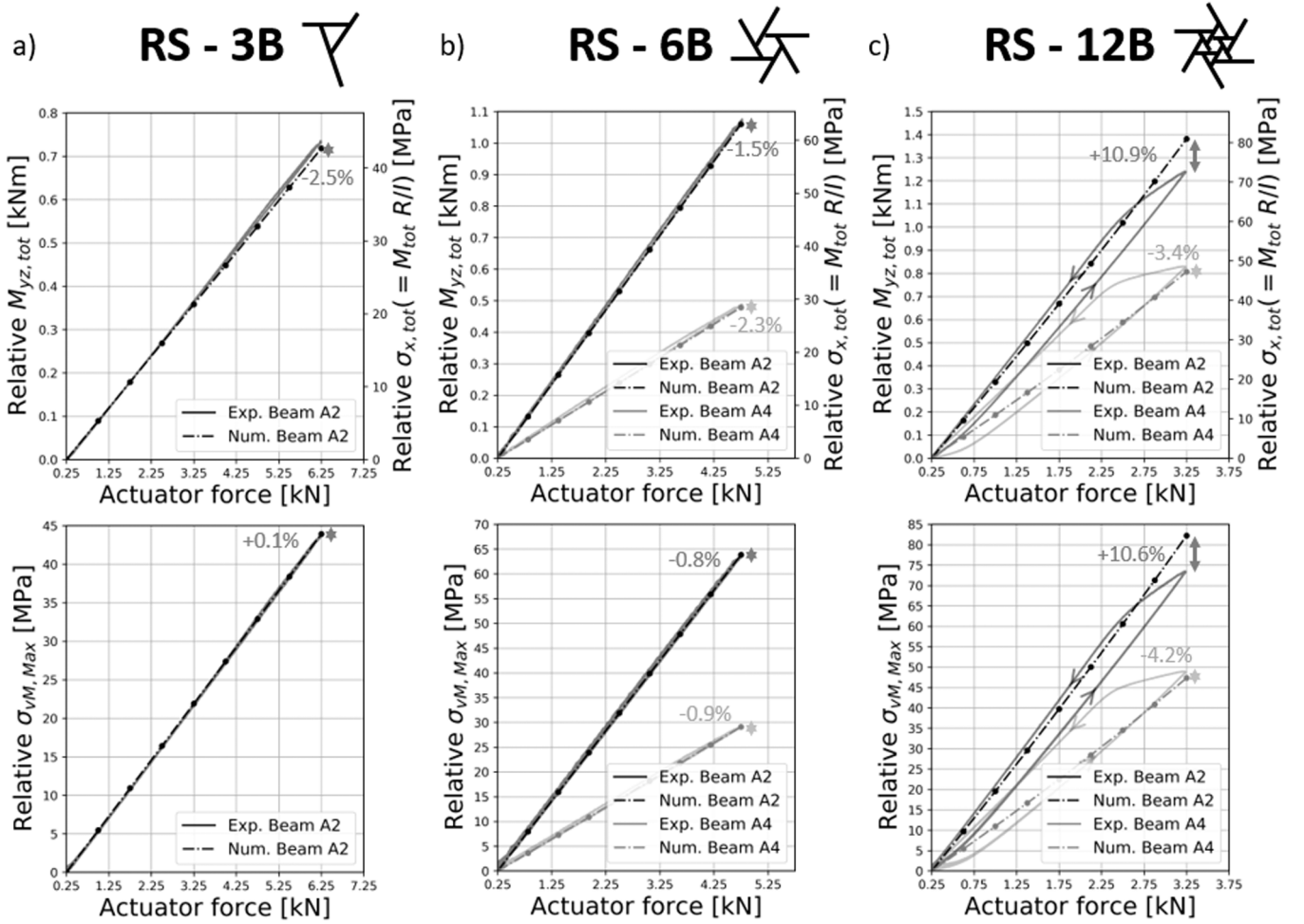


Fig. 15. Comparison between the total bending moment and von Mises stresses (relative to the beginning of cycle 9) and those predicted by the numerical model for: a) RS-3B, b) RS-6B, and c) RS-12B.

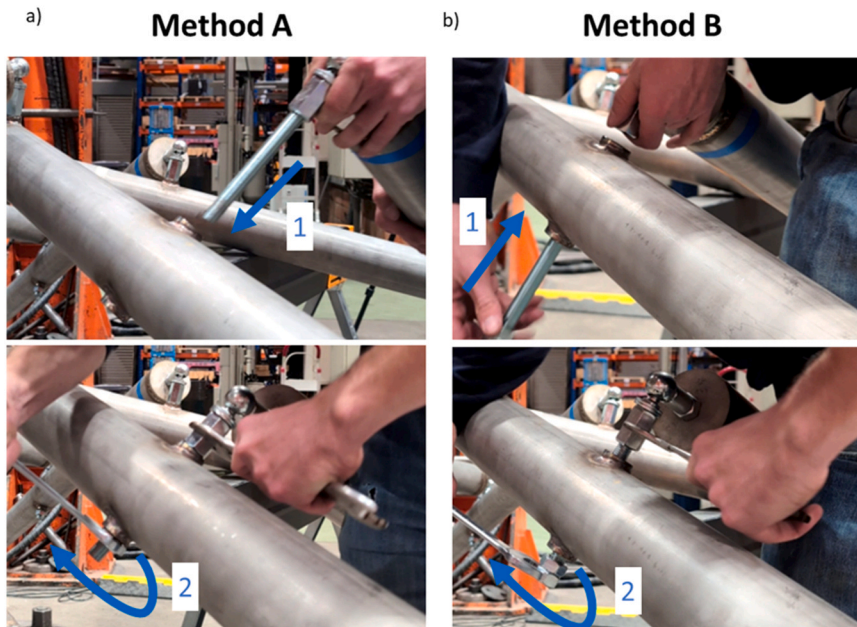


Fig. 16. Assembly stages for Methods A and B.

5.3.3. Choice of instrumentation

The Motion Capture System (MCS) is highly effective because it enables measurement of displacements at many points and facilitates the assembly and positioning of the supports. In addition, it enables observation of the spinning-top effect associated with the nodes' horizontal displacements. However, the markers must be positioned as accurately as possible at locations that are easily identifiable in the numerical model, to facilitate their correspondence with the model nodes. Strain gauges are also advantageous because they are less sensitive to parasitic effects from support displacements, as the structures are isostatic. Finally, the cable potentiometer provides a more traditional measurement, enabling, at least, local validation of the results obtained with the MCS.

6. Assembly and testing of the full structure RS–24B

The RS–24B structure was constructed directly on an outdoor site because of its large size (9.4 m in diameter with a height of 3.5 m, as shown in Fig. 2 and Fig. 7).

6.1. Assembly

The assembly was carried out in four stages, illustrated in Fig. 17:

- **Preparation (Fig. 17a):** concrete anchoring slabs are placed in holes excavated to a depth of at least 25 cm. A temporary platform is built at the centre, with temporary supports equipped with threaded pads allowing vertical adjustment of a few centimetres.



Fig. 17. a) Site preparation. b) The lifting equipment is placing the RS–12B on the temporary supports. c) Tubes of rings C and D are being installed sequentially. d) View of the central area. e) Tube supports are anchored in the concrete slabs.

- **Pre-assembly, transport and placement of RS–12B (Fig. 17b):** the RS–12B, weighing at least 180 kg, is first assembled in the laboratory and braced with cables to preserve its geometry during transportation. It is then placed on the temporary platform by lifting equipment. The tubes of ring B rest on pads of different heights to ensure the desired inclination.
- **Assembly of the peripheral tubes (Figs. 17c, 17d and 17e):** the tubes of the two lower rings are installed one after another. Once the structure is complete, the supports are fixed to the slabs using chemical anchors.
- **Finishing:** the foundations are covered with soil, and the temporary support system is removed.

This assembly sequence is inspired by that used at the Centre Masureel, where the inner modules were first assembled and temporarily propped at the desired height, before the supporting beams were installed. Nevertheless, other assembly strategies were also considered:

- **Installing and fixing the lower rings before bringing the pre-assembled RS–12B.** This option avoids constructing the temporary platform but carries a higher risk. In addition, the required geometric tolerance is larger because the supports' positions relative to the foundations cannot be adjusted.
- **Assembling the structure tube by tube, starting from the lower rings.** Despite the large number and size of temporary supports required, this solution would have been adopted if the lifting equipment had not been able to access the site.

6.2. Experimental setup

Fig. 16a shows a plan view of RS–24B, depicting the entire experimental setup. Because the test was carried out *in situ*, this setup differs from that used for the three other RS prototypes:

- The load is applied by suspending masses from a lifting ring attached beneath the connection linking Beams A6 and A1 (Fig. 18b).
- The loading protocol consists of two cycles from 0 to 2354 N, with 196 N increments applied with masses of 20 kg.
- The vertical displacement is measured using mechanical dial comparators placed at four points. To limit the influence of the spinning-top effect, each dial comparator is mounted on a ballasted plate and attached to the structure, as with the loading device (Fig. 18c).
- Strain gauge measurements could also be performed because the gauges had been protected before assembly.

6.3. Test results

As for the three other RS prototypes, the experimental results are compared with numerical predictions. The geometry of the numerical model was obtained from a LiDAR survey of the constructed structure. The resulting point cloud enables the determination of the tubes' centroidal axes and hinge positions. The supports, modelled as perfectly fixed, are located at the tube ends at the interface with the concrete slab. Their depth was measured before being covered.

The vertical displacements measured at four points of the structure are denoted A to D (Fig. 18a). The first cycle shows a significant settlement effect, whereas the second does not. Consequently, the numerical results are compared with the measurements during the loading phase of the second cycle. The displacement measurements show good agreement with the numerical model (Fig. 19). The errors are –2.4% at Point A, –9.2% at Point B, +1.7% at Point C, and +4.2% at Point D.

Fig. 20 presents the bending moments and stresses calculated from the strain-gauge measurements on Beams A2 and A4. The bending moments are very similar to the numerical results, with an error of –3.9% for Beam A2 and +2.8% for Beam A4. Regarding the von Mises stresses, the error is very small for Beam A2 (–1.4%), but larger for Beam A4 (+15.0%). This difference can be explained by a greater discrepancy

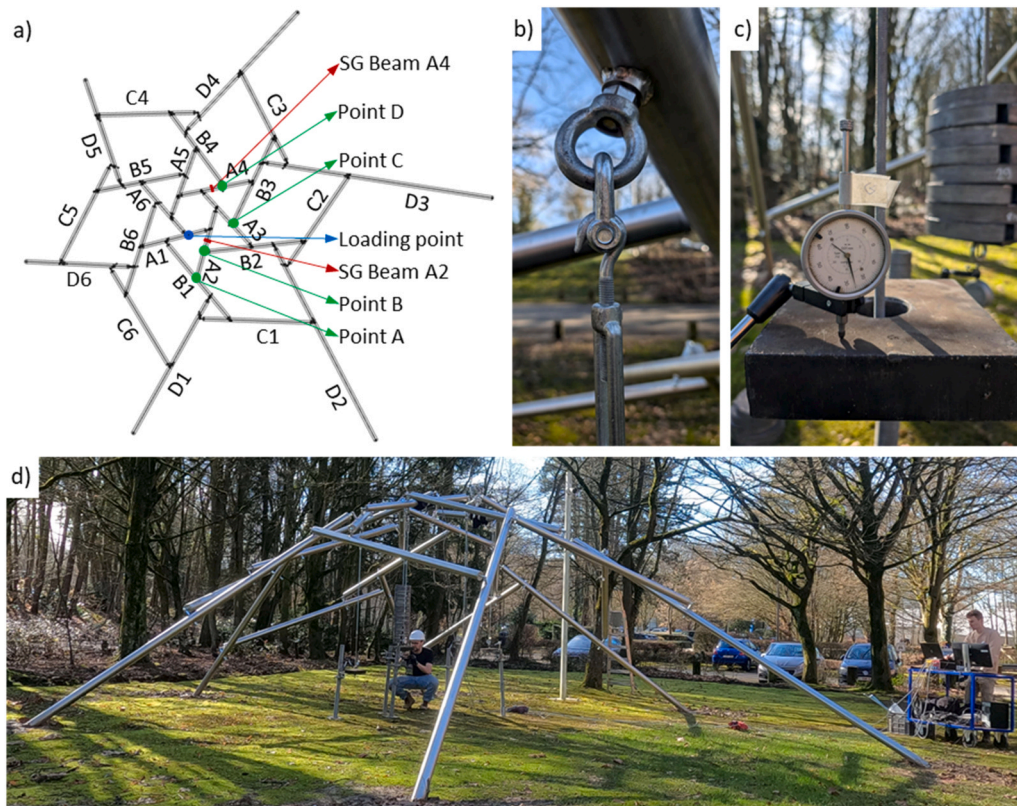


Fig. 18. a) Plan view of RS–24B with the locations of measurement and loading points. b) Loading attachment and displacement measurement device. c) Displacement measurement device. d) Overall view of the test site.

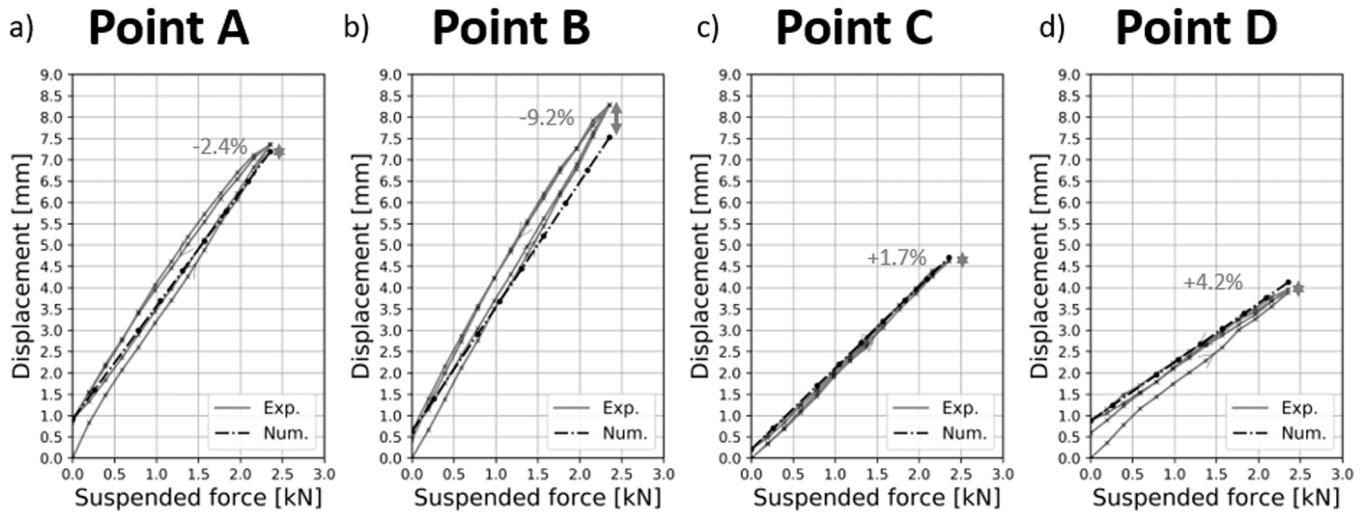


Fig. 19. Vertical displacement at four points of RS-24B: a) Point A, b) Point B, c) Point C, and d) Point D.

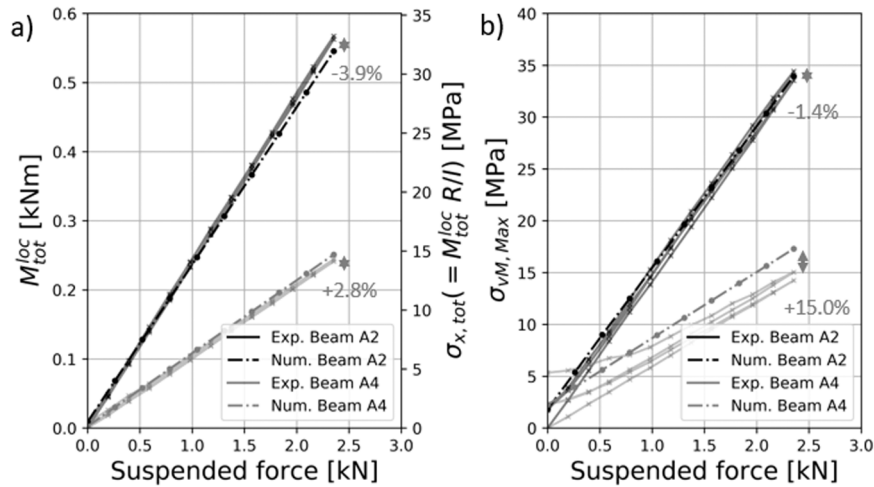


Fig. 20. Bending moment and von Mises stress for Beams A2 and A4 of RS-24B.

between the numerical and experimental shear force results.

7. Conclusion

This article demonstrates that real metallic reciprocal structures can be designed using reliable numerical modelling, thereby increasing structural safety. This is achieved by implementing hinged connections, which must first be tested in the laboratory according to a precise protocol.

To achieve this objective, the article proposes a hinged connection and a characterisation protocol. This protocol is validated through tests on several large-scale prototypes: RS-3B, RS-6B and RS-12B (tested in the laboratory), and RS-24B (tested in situ). The results show excellent agreement between numerical and experimental results, with discrepancies generally below a few per cent. The occasional deviations observed can mainly be explained by secondary effects, notably the supports' unexpected flexibility or geometric imperfections.

The redundancy of the measurement systems (potentiometer, Motion Capture System and strain gauges) strengthens the reliability of the conclusions and enables a detailed analysis of the three-dimensional structural behaviour, including the horizontal "spinning-top" effects characteristic of RS^{3D}.

In addition to the mechanical validation, this work highlights the

constructive advantages of the proposed connection. As the number of beams increases, the assembly sequence becomes more complex and the required geometric tolerances become more stringent. The developed connection facilitates these operations thanks to its simplicity of implementation and the ability to adopt different assembly strategies.

CRedit authorship contribution statement

Luca Sgambi: Writing – original draft, Supervision, Project administration, Methodology, Funding acquisition, Conceptualization. **Pierre Latteur:** Writing – original draft, Supervision, Project administration, Methodology, Funding acquisition, Conceptualization. **Martin Steinmetz:** Writing – original draft, Visualization, Validation, Methodology, Investigation, Formal analysis, Data curation, Conceptualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The authors would like to thank Rodin Bricoult and Jean Goux for

their assistance in constructing the principal structure, as well as the technicians of the laboratory LEMSC for their support during the testing process (<https://uclouvain.be/en/research-institutes/immc/lemsc>).

Appendices

A. List of symbols

Symbol	Unit	Property
A	mm^2	Area of the beam
E	N/mm^2	Elastic modulus
e	mm	Eccentricity
F_{max}	N	Maximum actuator force applied to the structure
F_{min}	N	Minimum actuator force applied to the structure
G	N/mm^2	Shear modulus
H	mm	Height of the structure
I	mm^4	moment of inertia
I_p	mm^3	Polar moment of inertia
L	mm	Circle diameter passing through all the supports
$M_y^{loc}, M_z^{loc}, M_{yz}^{loc}$	Nmm	Moment computed around the y local axis, Moment computed around the z local axis, combined moment
MCS		Motion Capture System
MM RS		Multiple module reciprocal structure
N^{loc}	N	Normal force
R	mm	Outer radius of the tube
r	mm	Inner radius of the tube
RS, RS^{2D}, RS^{3D}		Reciprocal structure: plane and spatial
SM RS		Single module reciprocal structure
SG		Strain gauge
V_y^{loc}, V_z^{loc}	N	Shear forces computed along the y and z local axes,
$x^{loc}, y^{loc}, z^{loc}$		Local coordinate system of a beam
$\delta, \delta_e, \delta_p$	mm	Displacement of a hinge during the local test
κ	–	shear coefficient
σ_{vM}	N/mm^2	von Mises stress
σ_x	N/mm^2	Normal stress
τ_{xz}	N/mm^2	Shear stress
ν	–	Poisson ratio
ϕ	mm	Diameter of cylinder

Data availability

Data will be made available on request.

References

- [1] Latteur P, Geno J, Vandamme M. Design rules for timber log reciprocal floors, based on mass optimization under stiffness constraint. *Int J Sp Struct* 2021;36(3): 217–40. <https://doi.org/10.1177/09560599211041699>.
- [2] Xu P, Zhao P, Liu Y, Liu N. Reciprocal frame (RF) architecture design based on statistical analysis of geometrical form parameters from built cases and parametric simulations. *Constr Build Mater* 2024;447(March). <https://doi.org/10.1016/j.conbuildmat.2024.138082>.
- [3] Baverel O. Nexorades: A family of interwoven space structures. *Univ Surrey* 2000; (December):1–323.
- [4] Song P, -Wing Fu C, Goswami P, Zheng J, Mitra NJ, Cohen-Or D. Reciprocal frame structures made easy. *ACM Trans Graph* 2013;32(4). <https://doi.org/10.1145/2461912.2461915>.
- [5] Su Y, Wu Y, Qiao G, Shen S. Self-adaptive form generation method for reciprocal grid structures. *Comput Civ Infrastruct Eng* 2019;34(5):444–54. <https://doi.org/10.1111/mice.12423>.
- [6] Douthe C, Baverel O. Design of nexorades or reciprocal frame systems with the dynamic relaxation method. *Comput Struct* 2009;87(21–22):1296–307. <https://doi.org/10.1016/j.compstruc.2009.06.011>.
- [7] Steinmetz M, Sgambi L, Latteur P. Geometric stability of reciprocal structures 2026;152(3):1–20. <https://doi.org/10.1061/JSENDH.STENG-15466>.
- [8] Xia YQ, Xiao N, Chen HP, Qian XQ. Determination of static and kinematic determinacy of pin-jointed assemblies using rigid-body displacements as primary unknown variables. *Eng Struct* 2019;181(June 2018):643–52. <https://doi.org/10.1016/j.engstruct.2018.12.037>.
- [9] Goto K, Kidokoro R, Matsuo T. A mountain observatory. *Arup J* 2011;26(2):208. <https://doi.org/10.2307/1776215>.
- [10] Anastas Y, Rhode-Barbarigos L, Adriaenssens S. Design-to-construction workflow for cell-based pattern reciprocal free-form structures. *J Int Assoc Shell Spat Struct* 2016;57(2):159–76. <https://doi.org/10.20898/j.iaass.2016.188.737>.
- [11] Brocato M. Reciprocal frames: Kinematical determinacy and limit analysis. *Int J Sp Struct* 2011;26(4):343–58. <https://doi.org/10.1260/0266-3511.26.4.343>.
- [12] A. Pizzigoni, “A high fiber reinforced concrete prototype for reciprocal structures of demountable building A High Fiber Reinforced Concrete Prototype for reciprocal structures of demountable building.”, no. May, 2009.
- [13] Garavaglia E, Pizzigoni A, Sgambi L, Basso N. Collapse behaviour in reciprocal frame structures. *Struct Eng Mech* 2013;46(4):533–47. <https://doi.org/10.12989/sem.2013.46.4.533>.
- [14] J. Gustafsson, “Connections in Timber Reciprocal Frames,” 2016.
- [15] M. Steinmetz, L. Sgambi, and P. Latteur, “Reliable experimental testing methodology for reciprocal structures,” 2024.
- [16] Rizzuto JP, Hulse R. Dodecahedral mutually supported element space structure: Experimental investigation. *Int J Sp Struct* 2007;22(2):107–21. <https://doi.org/10.1260/026635107781482640>.
- [17] Rizzuto JP. Experimental investigation of reciprocally supported element (RSE) lattice honeycomb domes structural behaviour. *Eng Struct* 2018;166(February): 496–510. <https://doi.org/10.1016/j.engstruct.2018.03.094>.
- [18] Rodríguez M.C.G. August, 2019. Design framework for bamboo culms - A study of bamboo reciprocal structures [Ph.D. thesis in Civil Engineering].
- [19] Qi L, Li Z, Pan H, Li L, Huang X. Study on adjustable connector of reciprocal frame. *Stavební Obz - Civ Eng J* 2021;30(2):378–92. <https://doi.org/10.14311/cej.2021.02.0028>.
- [20] Cowper GR. The shear coefficient in timoshenko's beam theory. *J Appl Mech Trans ASME* 1964;33(2):335–40. <https://doi.org/10.1115/1.3625046>.
- [21] Park HS, Park K, Kim Y, Choi SW. Deformation monitoring of a building structure using a motion capture system. *IEEE/ASME Trans Mechatron* 2015;20(5):2276–84. <https://doi.org/10.1109/TMECH.2014.2374219>.