Joint Spectral Characteristics: A Tale of Three Disciplines*

Raphaël M. Jungers

ICTEAM Institute, Université catholique de Louvain and FNRS
raphael.jungers@uclouvain.be

Joint spectral characteristics describe the stationary behavior of a discrete time linear switching system. Well, that's what an electrical engineer would say. A mathematician would say that they characterize the asymptotic behavior of a semigroup of matrices, and a computer scientist would perhaps see them as describing languages generated by automata.

Because of their connections with these wide research topics, joint spectral characteristics have been at the center of rich and diverse research efforts in recent years (see [7,11,15] for general surveys). They are notoriously very hard to compute (NP-hardness, Undecidability, etc. are the rule rather than the exception [16,10]), but it turns out that one can often get around these difficulties, and modern optimization techniques seem particularly useful for studying them [14,13].

I will survey and connect several powerful and interesting results, emphasizing the role of optimization methods. I will present applications, ranging from wireless control protocols to viral diseases treatment, malicious agents tracking, etc... I will emphasize the numerous connections with language and automata theory. In particular, I will cover the asymptotics of repetition-free languages [8,4,9,5], the capacity of codes avoiding forbidden differences [12,6,3], and recent results linking automata theory with the generation of Linear Matrix Inequalities for computing the joint spectral radius [1,2]. I will present several open problems and promising research directions.

References

1. Ahmadi, A.A., Jungers, R.M., Parrilo, P., Roozbehani, M.: Analysis of the joint spectral radius via Lyapunov functions on path-complete graphs. In: Proceedings of: Hybrid Systems: Computation and Control (HSCC 2011), Chicago (2011)
2. Ahmadi, A.A., Jungers, R.M., Parrilo, P., Roozbehani, M.: When is a set of LMIs a sufficient condition for stability? In: Proceedings of: ROCOND 2012, Aalborg (2012)
3. Asarin, E., Dima, C.: On the computation of covert channel capacity. RAIRO - Theoretical Informatics and Applications 44(1), 37–58 (2010)
4. Berstel, J.: Growth of repetition-free words–a review. Theoretical Computer Science 340(2), 280–290 (2005)

* Work supported by the Communauté française de Belgique - Actions de Recherche Concertées, and by the Belgian Programme on Interuniversity Attraction Poles initiated by the Belgian Federal Science Policy Office.

M.-P. Béal and O. Carton (Eds.): DLT 2013, LNCS 7907, pp. 27–28, 2013.

5. Blondel, V.D., Cassaigne, J., Jungers, R.M.: On the number of α-power-free words for $2 < \alpha \leq 7/3$. Theoretical Computer Science 410, 2823–2833 (2009)
6. Blondel, V.D., Jungers, R., Protasov, V.Y.: On the complexity of computing the capacity of codes that avoid forbidden difference patterns. IEEE Transactions on Information Theory 52(11), 5122–5127 (2006)
7. Jungers, R.M.: The joint spectral radius, theory and applications. LNCIS, vol. 385. Springer, Heidelberg (2009)
8. Jungers, R.M., Protasov, V.Y., Blondel, V.D.: Overlap-free words and spectra of matrices. Theoretical Computer Science 410, 3670–3684 (2009)
9. Karhumäki, J., Shallit, J.: Polynomial versus exponential growth in repetition-free binary words. Journal of Combinatorial Theory Series A 105(2), 335–347 (2004)
10. Kozyakin, V.A.: Algebraic unsolvability of problem of absolute stability of desynchronized systems. Automation and Remote Control 51, 754–759 (1990)
11. Liberzon, D.: Switching in Systems and Control. Birkhäuser, Boston (2003)
12. Moision, B.E., Orlitsky, A., Siegel, P.H.: On codes that avoid specified differences. IEEE Transactions on Information Theory 47, 433–442 (2001)
13. Parrilo, P.A., Jadbabaie, A.: Approximation of the joint spectral radius of a set of matrices using sum of squares. In: Bemporad, A., Bicchi, A., Buttazzo, G. (eds.) HSCC 2007. LNCS, vol. 4416, pp. 444–458. Springer, Heidelberg (2007)
14. Protasov, V.Y., Jungers, R.M., Blondel, V.D.: Joint spectral characteristics of matrices: a conic programming approach. SIAM Journal on Matrix Analysis and Applications 31(4), 2146–2162 (2010)
15. Shorten, R., Wirth, F., Mason, O., Wulff, K., King, C.: Stability criteria for switched and hybrid systems. SIAM Review 49(4), 545–592 (2007)
16. Tsitsiklis, J.N., Blondel, V.D.: The Lyapunov exponent and joint spectral radius of pairs of matrices are hard- when not impossible- to compute and to approximate. Mathematics of Control, Signals, and Systems 10, 31–40 (1997)