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# Finite element modeling of spherical induction actuator

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# ABSTRACT

The thesis deals with finite element method simulations of the two-degree of freedom spherical induction actuator performed using the 2D and 3D models. In some cases non-linear magnetization curves, rotor movement and existence of higher harmonics are taken into account. The evolution of the model leading to its simplification is presented.

Several rotor structures are tested, namely the one-layer, two-layers and two-layers-with-teeth rotor. The study of some rotor parameters, i.e. teeth number and size, external and internal layer thickness, in order to improve the actuator electromechanical conversion and reduce the torque ripple is performed. The influence of the rotor teeth geometrical form on the electromechanical conversion is shown. General actuator parameters such as the airgap, source current and parameters of the magnetic material are verified as well.

Finally the superiority of the two-layers-with-teeth rotor structure over the one-layer and two-layers structures is confirmed.

## Keywords

spherical actuator; rotor with teeth; teeth number; torque oscillations; finite element method

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# List of Symbols

## VECTORS

|                                            |                                                    |
|--------------------------------------------|----------------------------------------------------|
| $\mathbf{A}$                               | magnetic vector potential                          |
| $\mathbf{B}$                               | magnetic flux density                              |
| $\mathbf{B}_c$                             | magnetic flux density surrounding conductor        |
| $\mathbf{C}$                               | coefficients matrix (FEM)                          |
| $\mathbf{E}$                               | electric field                                     |
| $\mathbf{F}$                               | force acting on conductor                          |
| $\mathbf{F}_s$                             | force vector (FEM)                                 |
| $\mathbf{H}$                               | magnetic field                                     |
| $\mathbf{H}_s$                             | magnetic field component created by source current |
| $\mathbf{I}_a, \mathbf{I}_b, \mathbf{I}_c$ | $a, b, c$ phase current                            |
| $\mathbf{I}_r$                             | rotor current                                      |
| $\mathbf{I}_r'$                            | rotor current (brought to stator side)             |
| $\mathbf{I}_s$                             | stator supply current                              |
| $\mathbf{I}_\mu$                           | magnetizing current                                |
| $\mathbf{J}$                               | current density                                    |
| $\mathbf{J}_s$                             | surface supply current                             |

---

|                            |                                                                     |
|----------------------------|---------------------------------------------------------------------|
| $\mathbf{J}_{ext}$         | source (external) current density                                   |
| $\mathbf{K}$               | stiffness matrix (FEM)                                              |
| $\mathbf{M}_{sr}$          | matrix of mutual inductance between stator and rotor windings       |
| $\mathbf{N}$               | vector of shape functions of one element (FEM)                      |
| $\mathbf{T}$               | electric vector potential                                           |
| $\mathbf{V}_s$             | stator voltage                                                      |
| $l_c$                      | conductor length                                                    |
| $\mathbf{r}_c$             | vector from the conductor element to the field point                |
|                            |                                                                     |
| $\varphi$                  | vector of searched function all nodal values (FEM)                  |
| $\varphi^{(e)}$            | vector of searched function nodal values of ( $e$ )-element (FEM)   |
| $\varphi_u, \varphi_{u-1}$ | vectors of nodal values at time steps ( $u$ ) and ( $u - 1$ ) (FEM) |

### NON-VECTORS

|          |                                                                                                                                                                                                |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $B$      | magnetic flux density                                                                                                                                                                          |
| $B_r$    | mean value of flux density radial component calculated along support line placed in the middle of 1/24 part of rotor external layer / in the middle of tooth (for two-layers-with-teeth rotor) |
| $B_s$    | saturation flux density                                                                                                                                                                        |
| $C_c$    | Carter coefficient                                                                                                                                                                             |
| $F_{mm}$ | magnetomotive force                                                                                                                                                                            |
| $H$      | magnetic field                                                                                                                                                                                 |
| $I$      | effective stator supply current density                                                                                                                                                        |

---

|                           |                                                                                                                |
|---------------------------|----------------------------------------------------------------------------------------------------------------|
| $I(\varphi)$              | functional of searched function (FEM)                                                                          |
| $I(\varphi)^{(e)}$        | functional of searched function for ( $e$ )-element (FEM)                                                      |
| $I_c$                     | current circulating in conductor                                                                               |
| $I_r$                     | rotor current                                                                                                  |
| $I_s$                     | effective stator supply current                                                                                |
| $I_{REL}$                 | total current induced in 1/24 part of rotor external layer /<br>in 1/22 part (for two-layers-with-teeth rotor) |
| $J_s$                     | magnitude of surface supply current                                                                            |
| $L_\mu$                   | magnetizing inductance                                                                                         |
| $M_{sr}$                  | mutual inductance between stator and rotor windings                                                            |
| $N_i, N_j, N_k, N_n, N_q$ | shape functions (FEM)                                                                                          |
| $P_{Joule\ rotor}$        | Joule losses power in rotor                                                                                    |
| $P_{mechanic}$            | outgoing mechanic power of rotor                                                                               |
| $P_{s \rightarrow r}$     | power transferred from stator to rotor                                                                         |
| $R_r$                     | rotor resistance                                                                                               |
| $R_r'$                    | rotor resistance brought to stator side                                                                        |
| $R_s$                     | stator resistance                                                                                              |
| $R_{Cu}$                  | resistance of 1/22 part of rotor external layer (two-layers-with-teeth rotor)                                  |
| $Rel$                     | magnetic circuit reluctance                                                                                    |
| $Rel_{airgap}$            | airgap reluctance                                                                                              |
| $Rel_{rotor}$             | rotor reluctance                                                                                               |
| $Rel_{stator}$            | stator reluctance                                                                                              |
| $Rel_{surface}$           | surface reluctance                                                                                             |

---

|                                                              |                                            |
|--------------------------------------------------------------|--------------------------------------------|
| $Rel_{tooth}$                                                | tooth reluctance                           |
| $Rel_{total}$                                                | magnetic circuit total reluctance          |
| $S$                                                          | surface of stator slot                     |
| $S_{ec}$                                                     | electric circuit section                   |
| $S_{mc}$                                                     | magnetic circuit section                   |
| $T$                                                          | actuator torque                            |
| $\Delta T$                                                   | actuator torque ripple amplitude           |
| $T_{em}$                                                     | electromagnetic torque                     |
| $T_{max}$                                                    | actuator maximal torque                    |
| $T_{mean}$                                                   | actuator torque mean value                 |
| $T_{op}$                                                     | actuator torque at operating point         |
| $V$                                                          | electric scalar potential                  |
| $V_p$                                                        | problem volume (FEM)                       |
| $W$                                                          | energy                                     |
| $W_e$                                                        | electric field energy                      |
| $W_m$                                                        | magnetic field energy                      |
| $a_i, a_j, a_k, a_n, b_i, b_j, b_k, b_n, c_i, c_j, c_k, c_n$ | coefficients (FEM)                         |
| $d$                                                          | distance between two neighbouring windings |
| $e$                                                          | airgap                                     |
| $e_{eq}$                                                     | equivalent airgap                          |
| $el$                                                         | rotor external layer thickness             |
| $el_{opt}$                                                   | rotor external layer optimal thickness     |
| $f$                                                          | stator supply current frequency            |

---

|                 |                                                                                                          |
|-----------------|----------------------------------------------------------------------------------------------------------|
| $f_r$           | rotor current frequency                                                                                  |
| $f_s$           | function describing distribution of sources inducing field                                               |
| $g$             | function defining nodal values at moment $t = 0$ (FEM)                                                   |
| $h_1, h_2, h_3$ | function defined in points at domain edge $\Gamma$ (FEM)                                                 |
| $il$            | rotor internal layer thickness                                                                           |
| $j$             | imaginary unit                                                                                           |
| $k$             | current ratio                                                                                            |
| $k_1, k_2$      | constant depending on stator and rotor turn number and on winding distribution in stator and rotor slots |
| $l$             | stator / rotor length                                                                                    |
| $l_r$           | rotor inductance                                                                                         |
| $l_r'$          | rotor inductance (brought to stator side)                                                                |
| $l_s$           | stator inductance                                                                                        |
| $l_{mc}$        | magnetic circuit length                                                                                  |
| $m$             | constant depending on material parameters                                                                |
| $n$             | total number of nodal values (FEM)                                                                       |
| $n_r$           | rotor rotating speed [ $rpm$ ]                                                                           |
| $n_s$           | rotor synchronous rotating speed [ $rpm$ ]                                                               |
| $ns$            | number of stator slots                                                                                   |
| $nt$            | number of rotor teeth                                                                                    |
| $o$             | diffusion (real number) coefficient                                                                      |
| $p$             | number of pole pairs                                                                                     |
| $q$             | number of nodes needed for chosen approximation of one element (FEM)                                     |

---

|                             |                                                                                               |
|-----------------------------|-----------------------------------------------------------------------------------------------|
| $r$                         | rotor radius                                                                                  |
| $r_c$                       | distance from the conductor element to the field point                                        |
| $s$                         | slot width                                                                                    |
| $sr$                        | steel ratio (of the rotor surface)                                                            |
| $t$                         | time                                                                                          |
| $\Delta t$                  | value of time step (FEM)                                                                      |
| $ts$                        | rotor tooth size                                                                              |
| $v_p$                       | rotating magnetic field propagation speed                                                     |
|                             |                                                                                               |
| $\Gamma$                    | problem domain edge (FEM)                                                                     |
| $\Phi$                      | magnetic scalar potential                                                                     |
| $\Phi_{red}$                | reduced magnetic scalar potential                                                             |
| $\Omega$                    | problem domain (FEM)                                                                          |
|                             |                                                                                               |
| $\alpha$                    | parameter describing section of slot width $s$ (Carter coefficient)                           |
| $\beta_1, \beta_2, \beta_3$ | constants describing solution surface of a triangular element with linear approximation (FEM) |
|                             |                                                                                               |
| $\theta_{em}$               | relative position of rotor equivalent windings with reference to stator windings              |
|                             |                                                                                               |
| $\gamma$                    | slip between rotor speed $\omega_m$ and synchronous speed $\omega_s/p$                        |
| $\gamma_{max}$              | slip at maximal torque                                                                        |
|                             |                                                                                               |
| $\varepsilon$               | electric permittivity                                                                         |
|                             |                                                                                               |
| $\eta_{s \rightarrow r}$    | efficiency of conversion between $P_{s \rightarrow r}$ and $P_{mechanic}$                     |
|                             |                                                                                               |
| $\kappa^2$                  | real, imaginary or complex coefficient                                                        |

---

|                                                                                                 |                                                                                  |
|-------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------|
| $\varkappa$                                                                                     | function defined in points at domain edge $\Gamma$ (FEM)                         |
| $\lambda$                                                                                       | slot pitch                                                                       |
| $\mu$                                                                                           | magnetic permeability                                                            |
| $\mu_0$                                                                                         | vacuum magnetic permeability                                                     |
| $\mu_r$                                                                                         | magnetic relative permeability                                                   |
| $\rho$                                                                                          | electric resistivity                                                             |
| $\varrho$                                                                                       | free electric charge density                                                     |
| $\sigma$                                                                                        | electric conductivity                                                            |
| $\tau$                                                                                          | electromechanical conversion parameter                                           |
| $\tau_{opt}$                                                                                    | electromechanical conversion parameter optimal value                             |
| $\phi$                                                                                          | magnetic flux                                                                    |
| $\varphi$                                                                                       | searched function describing field variable (FEM)                                |
| $\varphi^{(e)}$                                                                                 | searched function in one element (FEM)                                           |
| $\varphi_i^{(e)}, \varphi_j^{(e)}, \varphi_k^{(e)}, \varphi_i, \varphi_j, \varphi_k, \varphi_q$ | searched function nodal values of $(e)$ -element in its nodes $i, j, k, q$ (FEM) |
| $\omega$                                                                                        | sources pulsation                                                                |
| $\omega_m$                                                                                      | rotor rotating speed (in radians/s)                                              |
| $\omega_{m \max}$                                                                               | rotor rotating speed at maximal torque (in radians/s)                            |
| $\omega_r$                                                                                      | rotor current pulsation                                                          |
| $\omega_{r \max}$                                                                               | rotor current pulsation at maximal torque                                        |
| $\omega_s$                                                                                      | stator supply current pulsation                                                  |
| $\omega_s/p$                                                                                    | rotor synchronous rotating speed (in radians/s)                                  |
| $\omega_{op}$                                                                                   | rotor rotating speed at operating point (in radians/s)                           |

|          |                                     |
|----------|-------------------------------------|
| $\Delta$ | Laplace operator                    |
| $\nabla$ | nabla operator                      |
| $\Delta$ | surface of triangular element (FEM) |
| MD       | magneto-dynamic formulation         |
| TM       | transient magnetic formulation      |

# CHAPTER 1

## INTRODUCTION

In the field of robotic and automation the need of actuation with multiple degrees of freedom is obvious. Generally multi-degree of freedom motions are achieved by combination of several one-degree of freedom actuators, what requires complex transmission systems. The idea of creation of a two-degree of freedom spherical actuator is based on the presumption that such construction should guarantee higher level of precision, dynamic performance and lower friction comparing to a two-degree of freedom structure created using two independent one-degree of freedom actuators.

This thesis was developed at the same time as the work of Dr Bruno Dehez [5]. In his study he presented the evolution of the spherical actuator concept. He started with the solid rotor made of iron that was replaced by the two-layers rotor. In that structure the inner layer was made of iron and the external layer was made of copper. Finally the third rotor structure with the so-called rotor teeth made of iron and crossing the external layer, between the inner layer and airgap, was proposed.

Dr Dehez studied as well the bearings keeping the rotor in the position. His study showed the interest of using the aerostatic suspension allowing the decrease of the airgap and assuring the additional cooling effect. However such a choice of the rotor suspension imposed, due to the mechanic constraints, the solid structure of the stator. That structure will be not an object of my investigation because it goes beyond my competence of an electrical engineer.

At the same time Dr Dehez proposed the induction (asynchronous) actuation of the machine as the best adapted to the spherical actuator.

The reason of it is that in an induction machine the torque depends only on the speed difference between the rotor and the rotating magnetic field generated by the stator, and because of that there is no need to install any sensors measuring the rotor position in order to pilot the actuator supply.

The original aim of this thesis is to optimize the rotor of a spherical induction actuator. Its performance is expressed using the electromechanical conversion parameter. In order to do that three different rotor structures will be compared, namely one-layer, two-layers and two-layers-with-teeth structure. After that some rotor crucial parameters will be tested to further improve the chosen structure.

During my study I created an impressive number of torque-speed characteristics in order to evaluate the electromechanical conversion parameter  $\tau$ . These characteristics were obtained using two different methods. First of them used in the majority of cases was to change the supply current frequency  $f$  and, what follows, its pulsation  $\omega$ . Because the rotor was locked so the rotor pulsation  $\omega_r$  was equal to the stator pulsation. That is how the characteristics of torque in function of rotor speed (or pulsation) were obtained. Because the simulations were performed always for only some chosen values of the frequency  $f$  the resulting torque-speed characteristics are the approximate ones. The second method demanding much more time and computational power was used only in a few cases especially to confirm the other results. It consists in the no-load start-up of the actuator for friction defined to be equal to zero and only pure inertia imposed. In such a case the complete characteristic torque-speed is obtained. Because the second method requires the use of the transient magnetic formulation (TM), it takes into account as well the rotor movement and the possibility of the non-sinusoidal coupling between the stator and the rotor, comparing to the first method used in my study along with the magnetodynamic formulation (MD) neglecting these two phenomena.

In the area of my study many rules have already been known empirically, as for example the rotor teeth number versus the stator slot number [7], and have been widely used in practice for the design of electrical machines. However in my work I go often deeper inside the problem and try to present its more complex analysis.

In the second chapter of my thesis I present the Finite Element Method (FEM) which allows the study of very complex 2-dimensional

as well as 3-dimensional problems. The professional software I am using in my simulations, namely Flux2D and Flux3D [4], is based on this method.

The third chapter presents the criterion of evaluation which was created to compare all the obtained results. This criterion, which value is quite easy to determine, represents in some way the efficiency of the electromechanical energy conversion between the stator and the rotor in the actuator.

In the following fourth chapter the stator imposed structure and its different models are presented as well as the current supply of the actuator.

The fifth chapter compares the one-layer solid rotor with the two-layers one and shows the superiority of this second structure. Some other parameters, as for example the airgap, rotor external layer thickness versus airgap and the supply currents, are tested as well.

Finally the sixth chapter introduces the two-layers-with-teeth rotor structure and after some analysis shows its superiority over the two previously presented ones. An initial optimization consisting in the study of rotor parameters such as the external layer thickness, size and number of teeth in order to increase the actuator performance and reduce the torque oscillations is presented. Apart from this the influence of the magnetic material parameters on the electromechanical conversion is presented. Some ways of its improvement and the mechanical constraints are discussed. The influence of geometrical form of teeth is shown as well and it will require the deeper analysis in the future as an important parameter. Additionally the non-sinusoidal coupling phenomenon is briefly discussed, however in that case the further study are not needed because for the rotor with teeth such a phenomenon can be neglected.

As we can see the sixth chapter presents the main results of my thesis [9, 10]. The interest of increasing the teeth number in order to improve actuator performance and reduce torque oscillations is presented as the original result. Such a conclusion does not correspond with the empirical rules to be found in the literature. However, it must be verified and needs further confirmation because of the appearance of the high frequency losses. As for the optimal teeth number the obtained 252 teeth correspond to the chosen level of accepted torque oscillations and can be reduced if we allow higher torque ripple. Concerning the rotor structure

optimization the external layer of 11 *mm* thickness and the steel ratio parameter of value between 30 and 40 % were found as assuring the best actuator performance regardless the chosen teeth number.

The last, seventh chapter presents the concise summary of my dissertation with all the conclusions and essential comments.

## CHAPTER 2

# FINITE ELEMENT METHOD IN TWO AND THREE DIMENSIONS

In this chapter I will present the calculation method, which will be used later on in my work. The Finite Element Method (FEM) is the numerical method for solving partial differential equations, which is widely used in many engineering fields. Its main advantages are, as follows:

- the possibility to solve the problem of the complicated geometry either in two or three dimensions
- easier, comparing to the other methods, determination of the limit conditions
- taking into account the non-linearity within the problem domain

As the inconveniences, I could mention:

- complexity of the utilisation
- need of supplementary, comparing to the other methods, time and hardware memory to perform the calculations

### 2.1 Finite element

The FEM basic idea is to divide the problem domain  $\Omega$  into the finite number of sub-domains - elements (Figure 2.1). The curved edges of the domain are approximated with the broken line or flat segments (in 3D).

Each element possesses nodes, which the searched variable values are associated with. These nodes are to be found at the edges and at the corners of the elements, in such a way, that each node and its variable values are common for two or more adjoining elements. As the result, the problem domain is discrete and represented as mesh of elements. This mesh is, by definition, continuous and their elements does not cover each other.

The mesh of elements should be denser (elements should be smaller) in the regions where the fast changes of the approximated function gradient are expected [1]. It should be also more elements at the irregular edges comparing to the smooth ones. The elements should be proportional in shape, for example close to equilateral in case of triangles. The long and narrow elements must be avoided. Solution precision in such the elements may be no more credible. Generally the more elements ('good' ones), the better precision. However the cost in time and hardware needed will increase as well, that is why normally a compromise is required.

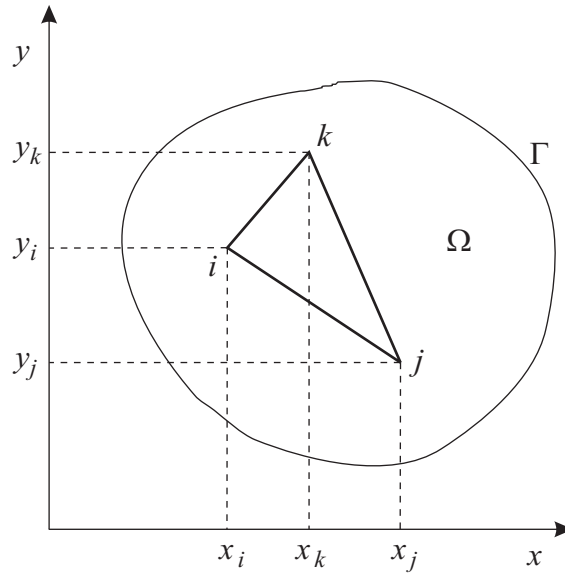


Figure 2.1 *Finite element in the two-dimensional domain  $\Omega$*

In electrotechnics the finite element represents the part of the problem domain, where the potential, as the searched field value, exists. Therefore the nodes of the elements are the points in the problem domain, where the potential, or another field value, and its derivatives, are known or searched.

## 2.2 Example of FEM approximation

In finite element method after the meshing of the domain the characteristics of the searched variable values over each element are approximated using the continuous functions. Equations of these functions depend on the chosen approximation and the variable values (and sometimes their derivatives as well) in the nodes. These functions defined over each finite element are called ‘shape functions’ [19]. The set of shape functions for the whole problem domain  $\Omega$  gives the approximation of the concerned variable.

Let consider a two-dimensional variable (function)  $\varphi(x, y)$  [1]. I will show that the nodal values  $\boldsymbol{\varphi}$  (vector of all nodal values) can define  $\varphi(x, y)$  in the whole problem domain  $\Omega$  (on the  $x$  0  $y$  surface) unambiguously and in continuous way.

Having the domain  $\Omega$  (with the edge  $\Gamma$ ), as presented on the figure 2.1, we create in it the triangular elements with the nodes in the corners of each element. It leads to the linear relation  $\varphi(x, y)$  in each element (Figure 2.2).

The surface crossing the three nodal values of the  $(e)$ -element (solution surface) is described with the equation:

$$\varphi^{(e)} = \beta_1 + \beta_2 x + \beta_3 y \quad (2.1)$$

The constants  $\beta_1$ ,  $\beta_2$  and  $\beta_3$  can be expressed using the  $(e)$ -element nodes coordinates and the corresponding nodal values  $\boldsymbol{\varphi}$  in the following way:

$$\left. \begin{aligned} \varphi_i^{(e)} &= \beta_1 + \beta_2 x_i + \beta_3 y_i \\ \varphi_j^{(e)} &= \beta_1 + \beta_2 x_j + \beta_3 y_j \\ \varphi_k^{(e)} &= \beta_1 + \beta_2 x_k + \beta_3 y_k \end{aligned} \right\} \quad (2.2)$$

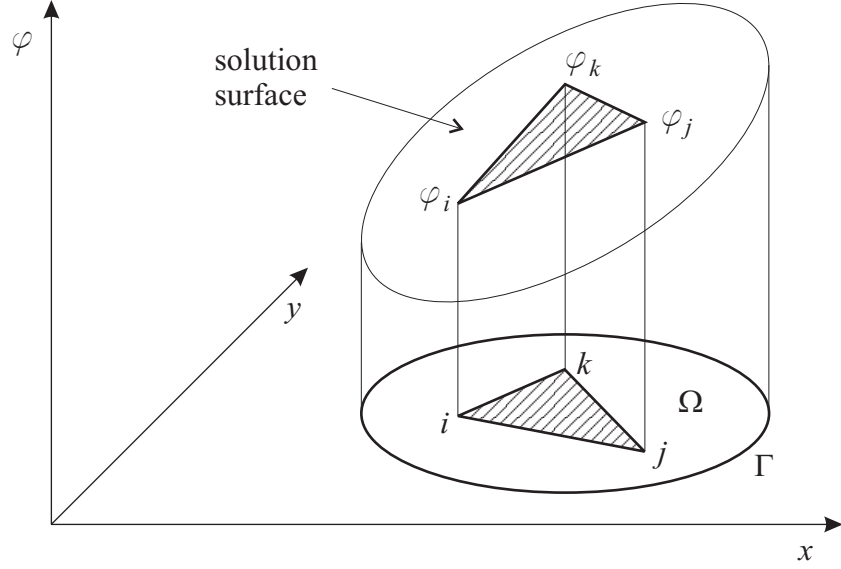


Figure 2.2 *Finite element and its solution surface crossing three nodal values  $\varphi_i, \varphi_j, \varphi_k$*

As the solution of equation 2.2 we obtain:

$$\left. \begin{aligned} \beta_1 &= \frac{\varphi_i (x_j y_k - x_k y_j) + \varphi_j (x_k y_i - x_i y_k) + \varphi_k (x_i y_j - x_j y_i)}{2 \Delta} \\ \beta_2 &= \frac{\varphi_i (y_j - y_k) + \varphi_j (y_k - y_i) + \varphi_k (y_i - y_j)}{2 \Delta} \\ \beta_3 &= \frac{\varphi_i (x_k - x_j) + \varphi_j (x_i - x_k) + \varphi_k (x_j - x_i)}{2 \Delta} \end{aligned} \right\} \quad (2.3)$$

where:

$$2 \Delta = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} \quad (2.4)$$

and:

$\Delta$  – surface of triangular element with nodes  $i, j, k$

Combining equations 2.3 and 2.1 we obtain:

$$\varphi^{(e)} = \frac{a_i + b_i x + c_i y}{2 \Delta} \varphi_i + \frac{a_j + b_j x + c_j y}{2 \Delta} \varphi_j + \frac{a_k + b_k x + c_k y}{2 \Delta} \varphi_k \quad (2.5)$$

where:

$$\left. \begin{aligned} a_i &= x_j y_k - x_k y_j \\ b_i &= y_j - y_k \\ c_i &= x_k - x_j \end{aligned} \right\} \quad (2.6)$$

The remaining coefficients  $a_j, b_j, c_j, a_k, b_k$  and  $c_k$  can be obtained by the cyclic replacing of the indexes  $i, j$  and  $k$ . To simplify nomenclature the upper index ( $e$ ) was omitted in nodal values  $\varphi_i, \varphi_j$  and  $\varphi_k$ .

We define:

$$N_n = \frac{a_n + b_n x + c_n y}{2 \Delta} \quad n = i, j, k \quad (2.7)$$

and:

$$\boldsymbol{\varphi}^{(e)} = \begin{bmatrix} \varphi_i \\ \varphi_j \\ \varphi_k \end{bmatrix} \quad (2.8)$$

$$\mathbf{N} = \begin{bmatrix} N_i \\ N_j \\ N_k \end{bmatrix} \quad (2.9)$$

The  $\mathbf{N}$  functions are called the, mentioned already, shape functions and they play important role in the finite element method.

In this case the  $\mathbf{N}$  functions are linear for the triangular element with three nodes. In the matrix notation we can rewrite equation 2.5 as follows:

$$\varphi^{(e)} = \mathbf{N}^T \boldsymbol{\varphi}^{(e)} = N_i \varphi_i + N_j \varphi_j + N_k \varphi_k \quad (2.10)$$

For the domain  $\Omega$  containing  $M$  elements in total, the representation of the variable  $\varphi(x, y)$  in the whole  $\Omega$  can be described in the following way:

$$\varphi(x, y) = \sum_{e=1}^M \varphi^{(e)} = \sum_{e=1}^M \mathbf{N}^T \boldsymbol{\varphi}^{(e)} \quad (2.11)$$

Looking at the equation 2.11 we can see clearly that knowing the nodal values  $\boldsymbol{\varphi}$  the searched variable  $\varphi(x, y)$  can be easily defined in the whole problem domain  $\Omega$ . The solution is the set of connected to each other triangular surfaces which linearly describe the variation of  $\varphi(x, y)$  in the  $\Omega$  (Figure 2.3).

Equations 2.10 and 2.11 are defined for the linear shape function and the triangular element with three nodes. We can imagine however more complicated elements and shape functions. In such a case the form of the mentioned equations remains unchanged and for one element looks

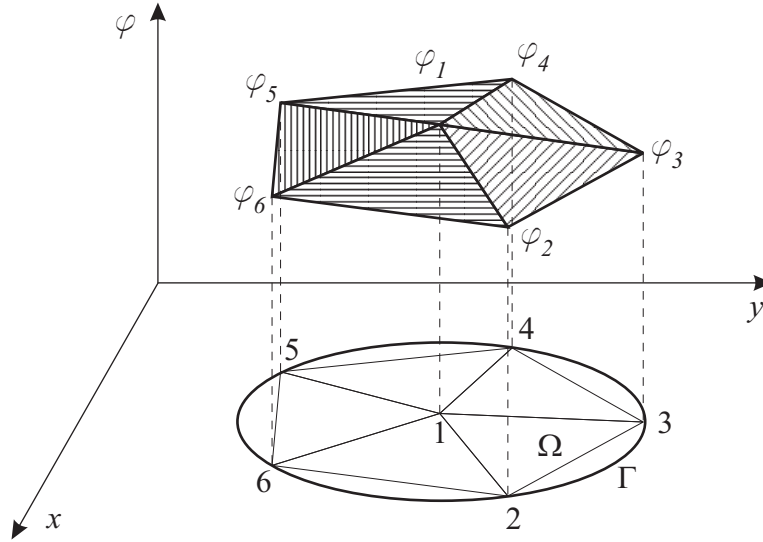


Figure 2.3 Linear approximation of  $\varphi(x, y)$

as follows [1]:

$$\varphi^{(e)} = \mathbf{N}^T \boldsymbol{\varphi}^{(e)} = [N_1 \ N_2 \ \dots \ N_q] \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_q \end{bmatrix} \quad (2.12)$$

where:

$q$  – number of nodes needed for approximation  
(number depending on the shape functions order)

## 2.3 Variational formulation of FEM equations

In this method after the meshing of the problem domain and choice of the shape functions, when the nodal values  $\boldsymbol{\varphi}$  are not known, the next step is to create the functional  $I(\boldsymbol{\varphi})$ . The solution of the problem using FEM consists in finding such a nodal values  $\boldsymbol{\varphi}$  which correspond to the minimum of the functional  $I(\boldsymbol{\varphi})$  [1].

### 2.3.1 Creation of functional $I(\boldsymbol{\varphi})$

#### Partial differential equations describing the problem

In electromagnetism theory the stationary sinusoidal field induced by the source  $f_s$  is described using the elliptic partial differential equation (Helmholtz' equation) of the following form [17]:

$$\nabla^2 \varphi + \kappa^2 \varphi = -f_s \quad (2.13)$$

where:

$\nabla$  – nabla operator  
 $\kappa^2$  – real, imaginary or complex coefficient  
 $\varphi$  – searched function describing field variable  
 $f_s$  – function describing spatial distribution of sources inducing the field

The corresponding boundary conditions are described generally as Hankel condition using the following expression:

$$\frac{\partial \varphi}{\partial n} = \varkappa \varphi + h_1 \quad (2.14)$$

where:

$\frac{\partial \varphi}{\partial n}$  – normal derivative of  $\varphi$  function in points at the domain edge  $\Gamma$

$\varkappa, h_1$  – functions defined in points at the domain edge  $\Gamma$

The slow-changing transient field induced by the source  $f_s$  is described by the parabolic partial differential equation (diffusion, Fourier equation) [1]:

$$\nabla^2 \varphi - \frac{1}{o} \frac{\partial \varphi}{\partial t} = -f_s \quad (2.15)$$

where:

$o$  – diffusion (real number) coefficient

The corresponding boundary conditions are either Neumann conditions:

$$\frac{\partial \varphi}{\partial n} = h_2 \quad (2.16)$$

or Dirichlet conditions:

$$\varphi = h_3 \quad (2.17)$$

where:

$h_2, h_3$  – functions defined in points at the domain edge  $\Gamma$

For the equation 2.15 we need to define also the initial condition:

$$\varphi(t=0) = g \quad (2.18)$$

where:

$g$  – function defining nodal values at the moment  $t = 0$

### Functional $I(\varphi)$ representing energy accumulated in problem domain

One of the way of solving the elliptic partial differential equation 2.13 is, as I already said, the minimizing of the functional expression  $I(\varphi)$  which represents the energy or power accumulated in the problem domain. It is the variational formulation of the problem, which however can not be applied to the parabolic partial differential equation 2.15. This equation describing transient field requires the use of, for example, Galerkin method. This method will not be presented in my dissertation where I will concentrate on the variational formulation.

The electric field energy  $W_e$  collected in the problem volume  $V_p$  are represented by the equation:

$$W_e = \int_{V_p} \frac{\varepsilon \mathbf{E}^2}{2} dv \quad (2.19)$$

where:

$\varepsilon$  – electric permittivity  
 $\mathbf{E}$  – electric field

Defining the electric scalar potential  $V$ :

$$\mathbf{E} = -\text{grad } V \quad (2.20)$$

we obtain:

$$W_e = \int_{V_p} \frac{\varepsilon}{2} (\text{grad } V)^2 dv \quad (2.21)$$

Similarly we can present the magnetic field energy  $W_m$ :

$$W_m = \int_{V_p} \frac{\mathbf{B}^2}{2\mu} dv \quad (2.22)$$

where:

$\mu$  – magnetic permeability  
 $\mathbf{B}$  – magnetic field flux density

Defining the magnetic vector potential  $\mathbf{A}$ :

$$\mathbf{B} = \text{rot } \mathbf{A} \quad (2.23)$$

we obtain:

$$W_m = \int_{V_p} \frac{1}{2\mu} (\text{rot } \mathbf{A})^2 dv \quad (2.24)$$

Equations 2.24 and 2.21 have a different form but in a particular case of the magnetic field when  $\mathbf{A} = A\mathbf{1}_z$  the expression  $(\text{rot } \mathbf{A})^2$  can be replaced by  $(\text{grad } A)^2$ . Now in the equations of the both energies we have an expression of the form  $(\text{grad } V)^2$ . In such a case we can generalize the equations 2.24 and 2.21 using Cartesian coordinates in isotropic, homogeneous and linear domain  $\Omega$ :

$$W = \int_{V_p} m \left[ \left( \frac{\partial V}{\partial x} \right)^2 + \left( \frac{\partial V}{\partial y} \right)^2 + \left( \frac{\partial V}{\partial z} \right)^2 \right] dx dy dz \quad (2.25)$$

where:

$m$  – constant depending on the material parameters

Concluding, for the problems described by the equations 2.13 and 2.14 in the two-dimensional domain  $\Omega$  and at its edge  $\Gamma$  we can create the following functional:

$$\begin{aligned} I(\varphi) = \int_{\Omega} \left[ m \left\{ \left( \frac{\partial \varphi}{\partial x} \right)^2 + \left( \frac{\partial \varphi}{\partial y} \right)^2 \right\} - \kappa^2 \varphi^2 - 2 f_s \varphi \right] dx dy + \\ - \int_{\Gamma} \left[ \varkappa \varphi^2 + 2 h_1 \varphi \right] d\Gamma \end{aligned} \quad (2.26)$$

In case of the boundary conditions described by the Neumann condition (Equation 2.16) the second element in the equation 2.26 will have the following form:

$$-2 \int_{\Gamma} h_2 \varphi \, d\Gamma \quad (2.27)$$

Finally for the Dirichlet boundary condition (Equation 2.17) the second element in the equation 2.26 will be equal to zero and disappear.

### 2.3.2 Minimization of functional $I(\varphi)$

As I said already before in order to solve the partial differential equation describing our problem we need to minimize the functional  $I(\varphi)$ . The necessary condition of existence of such a minimum is:

$$\frac{\partial I(\varphi)}{\partial \varphi_i} = 0 \quad i = 1, 2, \dots, n \quad (2.28)$$

where:

$n$  – total number of discrete nodal values  $\varphi_i$  in domain  $\Omega$

Functional  $I(\varphi)$  defined for the whole domain can be replaced by the sum of local functionals  $I(\varphi)^{(e)}$  defined for each of the  $M$  elements separately:

$$I(\varphi) = \sum_{e=1}^M I(\varphi)^{(e)} \quad (2.29)$$

We can rewrite now the equation 2.28 as:

$$\frac{\partial I(\varphi)}{\partial \varphi_i} = \sum_{e=1}^M \frac{\partial I(\varphi)^{(e)}}{\partial \varphi_i} = 0 \quad i = 1, 2, \dots, n \quad (2.30)$$

what represents a system of  $n$  equations containing  $n$  nodal values  $\varphi_i$ , which has a unique precise solution (if only one or several nodal values satisfy a Dirichlet condition).

If functional  $I(\varphi)$  is a function of second order (Equation 2.26) and with respect of equation 2.11 we can write generally:

$$\frac{\partial I(\varphi)}{\partial \varphi} = \mathbf{K}\varphi - \mathbf{F}_s = \mathbf{0} \quad (2.31)$$

or:

$$\mathbf{K}\varphi = \mathbf{F}_s \quad (2.32)$$

where:

- $\mathbf{K}$  – stiffness matrix (reluctances; impedances)
- $\mathbf{F}_s$  – force vector (sources; loads)
- $\varphi$  – searched vector of nodal values

In system of equations 2.32 the boundary conditions of Hankel or Neumann are already took into consideration. The Dirichlet boundary conditions must be considered only after the minimization of the functional  $I(\varphi)$  resulting in the creation of the system of equations 2.32.

Here I want to add only, without any analysis of the Galerkin method, that for the parabolic partial differential equation 2.15 describing the transient fields the final matrix equation will have the following form:

$$(\Delta t \mathbf{K} + \mathbf{C}) \varphi_u = \mathbf{C} \varphi_{u-1} - \Delta t \mathbf{F}_s \quad (2.33)$$

where:

- $\mathbf{K}, \mathbf{F}_s$  – stiffness matrix and force vector of the same form as in the stationary field problem
- $\Delta t$  – value of time step
- $\varphi_u, \varphi_{u-1}$  – nodal values at the time step ( $u$ ) and the preceding step ( $u - 1$ )

and the elements of the matrix  $\mathbf{C}$  are described as:

$$C_{ij} = \int_{\Omega} \frac{1}{o} N_i N_j d\Omega \quad (2.34)$$

In the first step of iteration (Equation 2.33) we compute the nodal values at the time  $u = \Delta t$ . As the nodal values corresponding to the preceding step ( $u - 1$ ) = 0 we use the initial conditions (Equation 2.18).

## 2.4 Magnetic field equations

Equations describing the slow-changing electromagnetic fields can be expressed using one of two different notations [1, 15, 16, 17]. The problem domain is isotropic, non-homogeneous (there are a few sub-domains which are homogeneous) and can be non-linear<sup>1</sup>.

I present here the notations I used in my study. All these notations are available in the Flux2D / Flux3D software [4].

### 2.4.1 The $\mathbf{AV}$ notation

It is used in the two-dimensional modeling, which is generally less complex, as the more precise and general notation demanding however more time and computational power. It is using a pair of presented already potentials  $\mathbf{AV}$ , where  $\mathbf{A}$  is the magnetic vector potential defined as follows:

$$\mathbf{B} = \text{rot } \mathbf{A} \quad (\text{for: } \text{div } \mathbf{B} = 0) \quad (2.35)$$

and  $V$  is the electric scalar potential defined as:

$$\mathbf{E} = -\text{grad } V - \frac{\partial \mathbf{A}}{\partial t} \quad \left( \text{for: } \text{rot } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \right) \quad (2.36)$$

Taking the first Maxwell equation (Equation C.1):

$$\text{rot } \mathbf{H} = \mathbf{J}_{ext} + \sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \quad (2.37)$$

where:

$\mathbf{H}$  – magnetic field

$\mathbf{J}_{ext}$  – source (external) current density

$\sigma$  – electric conductivity

and knowing that the displacement current can be ignored (for relatively low frequencies) we can write:

$$\text{rot } \mathbf{H} = \mathbf{J}_{ext} + \sigma \mathbf{E} = \mathbf{J}_{ext} + \sigma \left( -\text{grad } V - \frac{\partial \mathbf{A}}{\partial t} \right) \quad (2.38)$$

---

<sup>1</sup>Using the FEM tools we have, it was not possible to perform 3D simulation with the non-linear materials

For:

$$\mathbf{B} = \mu \mathbf{H} \quad (2.39)$$

and basing on the equation 2.35 we develop the equation 2.38 to the following form:

$$\text{rot} \left( \frac{1}{\mu} \text{rot} \mathbf{A} \right) = \mathbf{J}_{ext} - \sigma \text{grad} V - \sigma \frac{\partial \mathbf{A}}{\partial t} \quad (2.40)$$

where:

$$\text{grad} V = 0 \quad (2.41)$$

because the external, conducting layer is short-circuited.

Knowing that:

$$\text{rot} (\text{rot} \mathbf{A}) = \text{grad} (\text{div} \mathbf{A}) - \text{div} (\text{grad} \mathbf{A}) \quad (2.42)$$

we can write:

$$\text{grad} \left( \frac{1}{\mu} \text{div} \mathbf{A} \right) - \text{div} \left( \frac{1}{\mu} \text{grad} \mathbf{A} \right) = \mathbf{J}_{ext} - \sigma \frac{\partial \mathbf{A}}{\partial t} \quad (2.43)$$

Finally using the Coulomb gauge [1]:

$$\text{div} \mathbf{A} = 0 \quad (2.44)$$

we obtain the general equation describing the magnetic field as a function of the magnetic vector potential  $\mathbf{A}$ :

$$\text{div} \left( \frac{1}{\mu} \text{grad} \mathbf{A} \right) - \sigma \frac{\partial \mathbf{A}}{\partial t} = -\mathbf{J}_{ext} \quad (2.45)$$

As we can see the equation 2.45 is a vector form of the parabolic partial differential equation 2.15.

### 2.4.2 The $\mathbf{T}\Phi\text{-}\Phi\text{-}\Phi_{red}$ notation

Although the  $\mathbf{AV}$  notation is more precise, in the generally more complex three-dimensional modeling the less demanding in time and hardware needed the  $\mathbf{T}\Phi\text{-}\Phi\text{-}\Phi_{red}$  notation is used.

#### Conducting region without source

In this notation  $\mathbf{T}$  is the electric vector potential defined, for the conducting region without source, as follows:

$$\mathbf{J} = \sigma \mathbf{E} = \text{rot } \mathbf{T} \quad (\text{for: } \text{div } \mathbf{J} = 0) \quad (2.46)$$

and  $\Phi$  is the magnetic scalar potential defined for the same region as:

$$\mathbf{H} = \mathbf{T} - \text{grad } \Phi \quad (\text{for: } \text{rot } \mathbf{H} = \sigma \mathbf{E}) \quad (2.47)$$

Starting with the forth Maxwell equation:

$$\text{div } \mathbf{B} = 0 \quad (2.48)$$

we obtain:

$$\text{div} \left( \mu (\mathbf{T} - \text{grad } \Phi) \right) = 0 \quad (2.49)$$

Equations 2.46 and 2.47 correspond to the  $\mathbf{T}\Phi$  sub-notation describing in 3D the conducting region without source. Knowing that this region is not only isotropic but also homogeneous (it is a sub-domain of the whole problem domain) and linear (Flux3D constraint) we obtain:

$$\text{div} (\mathbf{T} - \text{grad } \Phi) = 0 \quad (2.50)$$

Using the Lorentz gauge [18]:

$$\text{div } \mathbf{T} = \mu \sigma \frac{\partial \Phi}{\partial t} \quad (2.51)$$

leads to the following equation:

$$\text{div} (\text{grad } \Phi) - \mu \sigma \frac{\partial \Phi}{\partial t} = 0 \quad (2.52)$$

Because in 3D we are limited to analyse only the sinusoidal signals (first harmonic component) we obtain finally:

$$\operatorname{div}(\operatorname{grad} \Phi) - j \omega \mu \sigma \Phi = 0 \quad (2.53)$$

where:

$$\begin{aligned} j &- \text{imaginary unit } (j^2 = -1) \\ \omega &- \text{sources pulsation} \end{aligned}$$

We can see that the equation 2.53 is a Helmholtz equation.

#### Non-conducting region without source

The magnetic field in the non-conducting region without source is described using the magnetic scalar potential  $\Phi$  defined as follows:

$$\mathbf{H} = -\operatorname{grad} \Phi \quad (\text{for: } \operatorname{rot} \mathbf{H} = 0) \quad (2.54)$$

Starting with the equation 2.48 for the isotropic, homogeneous and linear region we obtain finally:

$$\operatorname{div}(\operatorname{grad} \Phi) = 0 \quad (2.55)$$

Equation 2.55 is a Laplace equation.

#### Non-conducting region with source

The magnetic field in the non-conducting region with source is described using the reduced magnetic scalar potential  $\Phi_{red}$  defined as follows:

$$\mathbf{H} = \mathbf{H}_s - \operatorname{grad} \Phi_{red} \quad (\text{for: } \operatorname{rot} \mathbf{H} = \mathbf{J}_{ext}) \quad (2.56)$$

where:

$$\begin{aligned} \mathbf{H}_s &- \text{magnetic field component created by the source current} \\ &(\text{to be found using the Biot-Savart law - Equation C.5}) \end{aligned}$$

Starting with the equation 2.48 for the isotropic, homogeneous and linear region we obtain finally:

$$\text{div}(\text{grad } \Phi_{red}) = 0 \quad (2.57)$$

Equation 2.57 is a Laplace equation.

#### 2.4.3 Transient magnetic TM formulation

‘Transient magnetic’ or TM formulation delivers the step-by-step resolution taking into account the rotating movement of the rotor and the existence of the slot effect. It takes into account also the non-sinusoidal coupling between the stator and the rotor which is a result of the higher harmonics existence. In the case of the software which I used this formulation is available only in 2D simulations and it uses the equation 2.45 to describe the magnetic field in the problem domain.

#### 2.4.4 Magnetodynamic MD formulation

In magnetodynamic formulation the assumption is made that only the sinusoidal first harmonic component is present. 2D simulations performed using this formulation describe the magnetic field using the modified equation 2.45 which gets the following form:

$$\text{div} \left( \frac{1}{\mu} \text{grad } \mathbf{A} \right) - j \omega \sigma \mathbf{A} = -\mathbf{J}_{ext} \quad (2.58)$$

Equation 2.58 is a Helmholtz equation.

In case of the 3D simulations, where only MD formulation is available, the equations 2.53, 2.55 and 2.57 are used separately in each problem sub-domain, according to its characteristics.



## CHAPTER 3

# CRITERION OF EVALUATION

In order to evaluate and compare the different rotor structures and parameters sets, we have decided to use the electromechanical conversion efficiency criterion. Because the model we used in our simulations did not let us calculate the exact value of the efficiency of the conversion between the power transferred from the stator to the rotor and the outgoing mechanic power, the chosen criterion is assumed to be close to this efficiency and its value can be estimated on a basis of the actuator torque-speed characteristics. In such a way the electromechanical conversion parameter  $\tau$  was created [5]. As for the choice of efficiency as the basic parameter to evaluate we must say that it is an important parameter in the mobile robotics, for which the spherical actuator had been prepared.

Considering the characteristics torque-speed of a classic induction actuator and a load (Figure 3.1) we define the operating point of actuator-load system in the intersection point of the both given characteristics.

At that point, the outgoing mechanic power of the rotor  $P_{mechanic}$  is given by the equation:

$$P_{mechanic} = \omega_{op} T_{op} \quad (3.1)$$

where:

$\omega_{op}$  – rotor rotating speed at the operating point (in radians/s)

$T_{op}$  – torque generated at the operating point

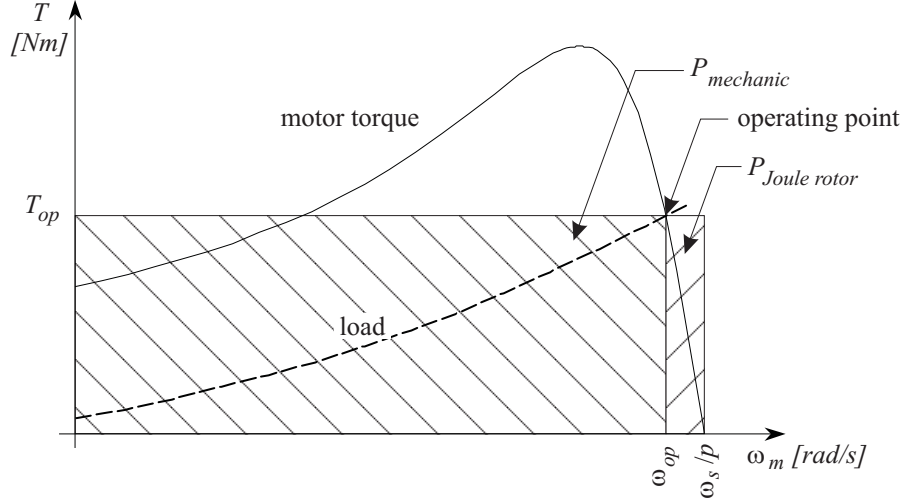


Figure 3.1 Operating point, delivered mechanic power and Joule losses in rotor, presented on a basis of the torque-speed characteristic

At the same time Joule losses power in the rotor  $P_{Joule rotor}$  is equal to:

$$P_{Joule rotor} = (\omega_s/p - \omega_{op}) T_{op} \quad (3.2)$$

where:

$\omega_s$  – stator supply current pulsation

$p$  – pole pairs number

$\omega_s/p$  – rotor synchronous rotating speed (in radians/s)

Knowing these two parameters we can calculate the efficiency  $\eta_{s \rightarrow r}$  of the conversion between the power transferred from the stator to the rotor  $P_{s \rightarrow r}$  and the outgoing mechanic power:

$$\eta_{s \rightarrow r} = \frac{P_{mechanic}}{P_{s \rightarrow r}} = \frac{P_{mechanic}}{P_{mechanic} + P_{Joule rotor}} \quad (3.3)$$

Taking into account equations 3.1 and 3.2 we can rewrite equation 3.3 and obtain:

$$\eta_{s \rightarrow r} = 1 - \gamma_{op} \quad (3.4)$$

where  $\gamma$  is the slip between the rotor rotating speed  $\omega_m$  and the synchronous speed, defined as follows:

$$\gamma = \frac{\omega_s - p \omega_m}{\omega_s} = \frac{\omega_s/p - \omega_m}{\omega_s/p} \quad (3.5)$$

Looking at equation 3.4 we realize, that the lower the slip, the higher electromechanical conversion efficiency we have. However, the slip depends not only on the load but as well on the motor characteristic and especially on its form around the synchronous speed (Figure 3.2). In this area the torque-speed characteristic is quasi-linear and generally can be replaced with a straight line. The more this line inclined, the lower the slip corresponding to the operating point and the higher the  $\eta_{s \rightarrow r}$  conversion.

Finally, in a first approximation, it seems possible to compare the  $\eta_{s \rightarrow r}$  conversions of the two different actuators on a basis of the torque-speed characteristic slope around the synchronous speed. This angular

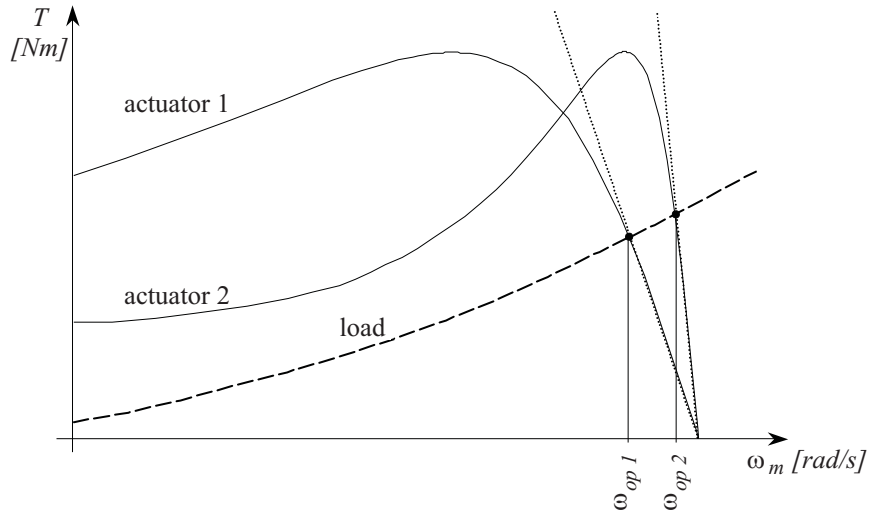


Figure 3.2 Operating point dependence on the actuator torque-speed characteristic

coefficient can be approximated by the following rapport:

$$\frac{T_{max}}{\omega_s/p - \omega_{m \max}} \quad (3.6)$$

where:

$T_{max}$  – actuator maximal torque  
 $(\omega_s/p - \omega_{m \max})$  – difference between synchronous speed  $\omega_s/p$   
and speed at the maximal torque  $\omega_{m \max}$   
(Figure 3.3)

Using the rotor current pulsation  $\omega_r$  that is proportional to the difference between the synchronous and rotor speed:

$$\omega_r = \omega_s - p \omega_m \quad (3.7)$$

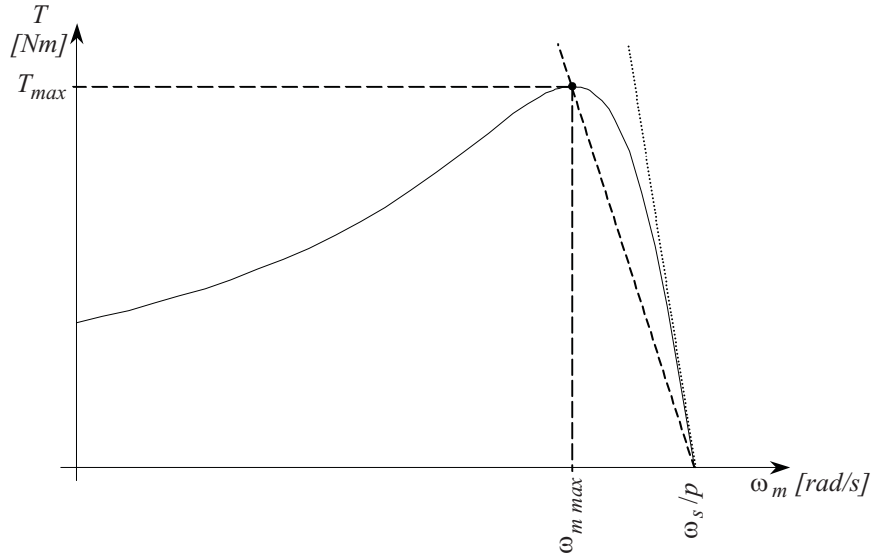


Figure 3.3 Approximation of the torque-speed characteristic slope around the operating point

we can finally replace equation 3.6 by the rapport:

$$\frac{T_{max}}{\omega_{r\ max}} = \tau \quad (3.8)$$

where  $\omega_{r\ max}$  is the rotor current pulsation at the maximal torque and  $\tau$  is the newly defined electromechanical conversion parameter, which I will use in my study.



## CHAPTER 4

# STATOR MODEL

Having presented the calculation method and the evaluation criterion, I will now move to the stator of the spherical actuator to study. I will start by introduction of the geometry, continue with the material and finish with the supply currents, chosen to be used in my simulations as the basic parameters.

### 4.1 Geometry

#### 4.1.1 Prototype stator

I want to present here the stator of the existing actuator prototype [5]. It consists of five separated inductors (Figure 4.1). Supplied by three-phase current systems, four one-degree of freedom inductors (Figure 4.2) are able to generate sliding magnetic fields in the direction of the motorized degree of freedom. The speeds of these sliding fields depend on the frequency of the three-phase supply. The two-degree of freedom inductor (Figure 4.2) is able to generate a sliding magnetic field in any of both directions perpendicular to the slots, or in any direction between these two axes, if both windings are supplied. In that case the direction of the sliding magnetic field depends on the frequency ratio of the both three-phase supplies.

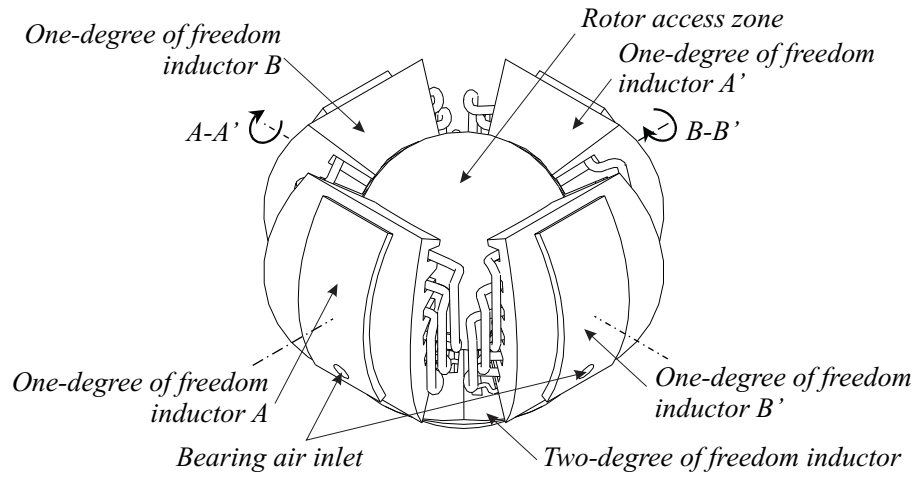


Figure 4.1 Two-degree of freedom actuator

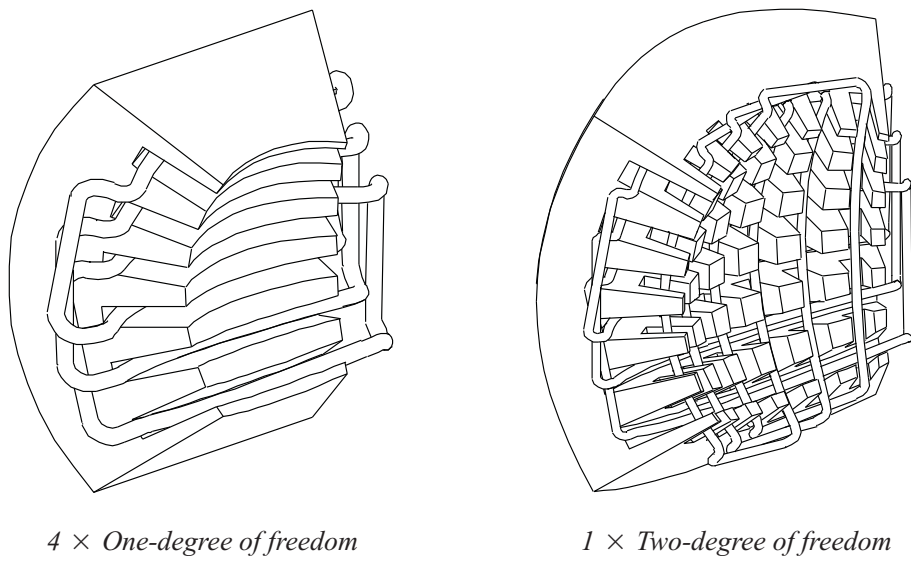


Figure 4.2 Inductors

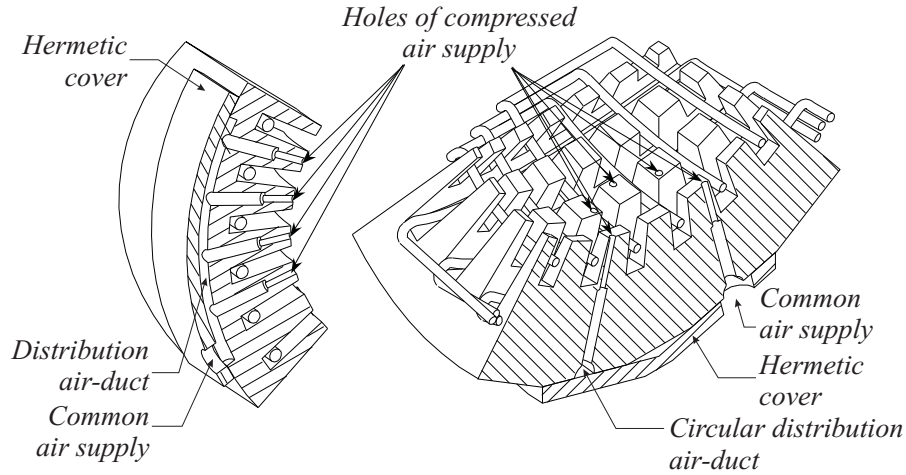


Figure 4.3 Aerostatic suspension

### Aerostatic suspension

There is a need to mention, that in order to be able to integrate the actuation and support functions in the same area and hence maximize the inductor surface, the aerostatic suspension of the rotor was used [5] (Figure 4.3). Besides, such a solution allows us to minimize the airgap (5.2.1), on which depends directly the maximal reachable torque value, and friction disappears as well. Furthermore, the positive side effect is cooling of the actuator.

#### 4.1.2 Prototype stator 3D simplified model - 'A-model'

In order to use the 2- and 3-dimensional finite element method calculation software (Flux2D and Flux3D) I need to simplify the stator structure.

The 3-dimensional simulations are very demanding in time and hardware needed that is why in my case they will be limited to the study of the actuator section containing only one one-degree of freedom inductor contributing to only one axe of rotation (Figure 4.4).

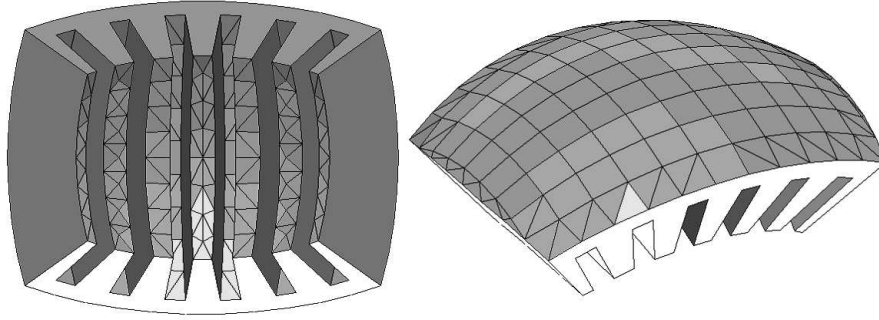


Figure 4.4 3D simplified stator structure - 'A-model'

#### 4.1.3 Prototype stator 2D model - 'B-model'

In first approximation stator will consist of three one-degree of freedom inductors, corresponding to the vertical cross-section of the prototype stator, as presented on the figure 4.5. This model takes into account the edge effects.

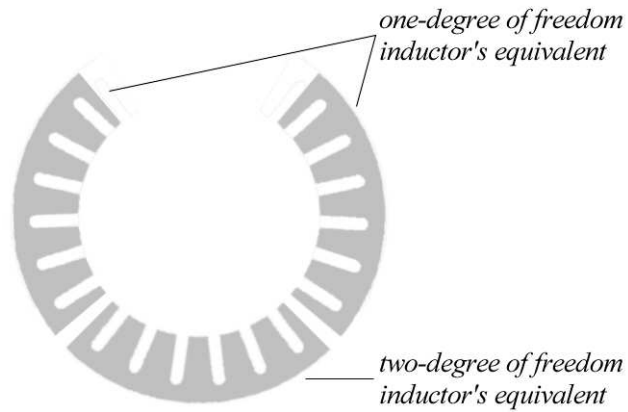


Figure 4.5 2D equivalent stator structure - 'B-model'

#### 4.1.4 Prototype stator 2D simplified model - 'C - model'

The edge effects in the three-inductors stator can be neglected, what illustrates figure 4.6 (difference in torque per pair of poles about 10 %), and that is why I performed final simplification and obtained the classic structure with twenty four slots (not taking into account the edge effects in the real actuator), which I will use to represent the stator in further 2D simulations. Figure 4.7 shows its geometry and the precise dimensions are to be found in the table 4.1. The actual torque of the spherical actuator will be than the 3/4 of the calculated torque value.

I need to mention, that shape of the stator slot is very basic, straight without any reduction of the slot opening, what can result in amplification of the slot effect. However, the influence of the stator on the spherical actuator performance is not the object of my research, so the stator will not change during my simulations.

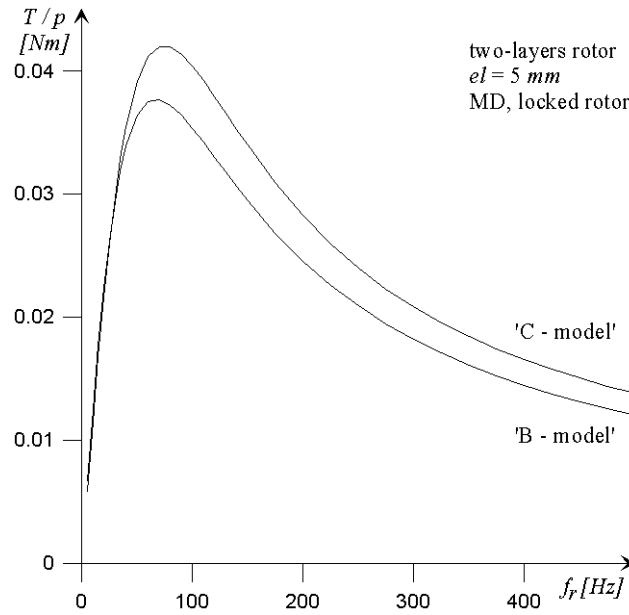


Figure 4.6 Comparison of 'B-model' and 'C-model' in term of edge effects

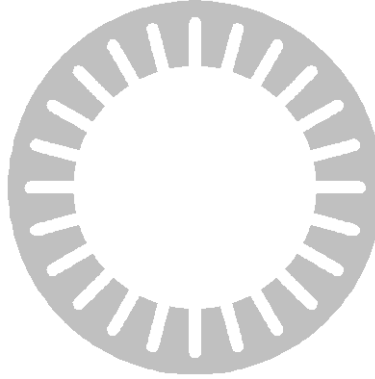


Figure 4.7 *Final simplified 2D stator structure - 'C-model'*

#### 4.1.5 Prototype stator analytical model - 'D-model'

Some results presented in my thesis were obtained formerly [5] using the analytical model based on the following hypothesis:

##### First hypothesis

There is no electromagnetic coupling between inductors. The study can be limited to one inductor supposing that the effects of the others are the same.

##### Second hypothesis

Supposing the relatively simple rotor structure (one or two layers) the electromagnetic phenomena generated by the inductors are concentrated on the rotor surface. That is why the curve of the rotor can be neglected and the spherical problem can be replaced by the plane one (Figure 4.8).

##### Third hypothesis

The currents circulating in the inductors are replaced by the surface currents circulating in the interface surface between the stator and the airgap. The surface currents are sinusoidal (see: the sixth hypothesis).

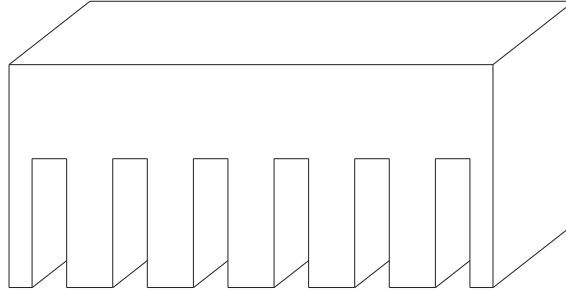


Figure 4.8 *Plane stator - second analytical hypothesis*

#### Fourth hypothesis

The stator structure is infinite in the plane  $x$ - $y$  what allows the neglect of the edge effects around the stator (Figure 4.9).

All the precedent hypothesis allow the replacement of the initial 3D problem by the 2D one.

#### Fifth hypothesis

The magnetic permeability of the stator is supposed to be infinite. That is why the magnetic field in the stator and below it is equal to zero.

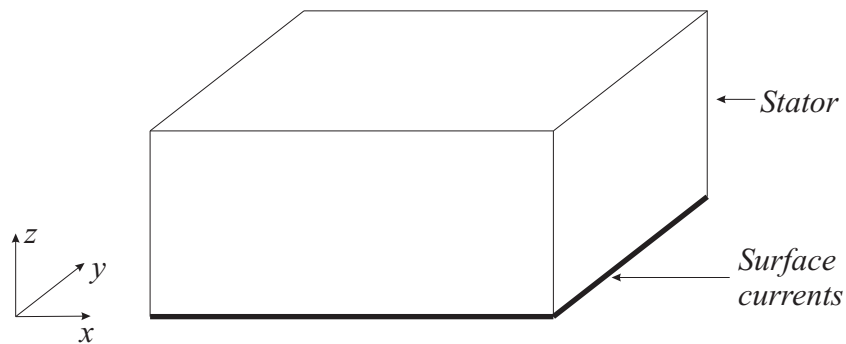


Figure 4.9 *Plane infinite stator with surface currents - third and fourth analytical hypothesis*

## Sixth hypothesis

For the sinusoidal sources and the linear, isotropic and homogeneous materials all the variables are supposed to change in a sinusoidal way, as well.

Table 4.1 *Stator parameters*

| Parameters                        |                                           | Values |                          |
|-----------------------------------|-------------------------------------------|--------|--------------------------|
| Dimensions                        |                                           |        |                          |
|                                   | Stator inner diameter                     | 51     | <i>mm</i>                |
|                                   | Stator outer diameter                     | 78.6   | <i>mm</i>                |
|                                   | Stator length $l$                         | 50     | <i>mm</i>                |
|                                   | Slot depth                                | 10     | <i>mm</i>                |
|                                   | Slot width $s$                            | 2.6    | <i>mm</i>                |
|                                   | Slot surface $S$                          | 26     | <i>mm</i> <sup>2</sup>   |
|                                   | Slots number                              | 24     |                          |
|                                   | Slot pitch $\lambda$                      | 6.68   | <i>mm</i>                |
| Material                          |                                           |        |                          |
| Carpenter Stainless<br>Type 430FR | Magnetic relative<br>permeability $\mu_r$ | 500    |                          |
| Solenoid Quality steel            | Electric resistivity $\rho$               | 76     | $\mu\Omega cm$           |
|                                   | Saturation flux density $B_s$             | 1.35   | <i>Tesla</i>             |
| Current Supply                    |                                           |        |                          |
|                                   | Effective current density $I$             | 10     | <i>A/mm</i> <sup>2</sup> |
|                                   | Current frequency $f$                     | 50     | <i>Hz</i>                |
|                                   | Slots per pole pitch                      | 3      |                          |
|                                   | Winding pitch                             | 3      |                          |
|                                   | Pole pairs number $p$                     | 4      |                          |

## 4.2 Materials

For manufacturing constraints, the stator has been built of Carpenter Stainless Type 430FR Solenoid Quality, which is a ferritic chromium-iron stainless alloy. It is the widely used alloy steel with good magnetic relative permeability and average electric conductivity. I will define it here as a material with linear magnetization curve, however later in my dissertation I will discuss as well the use of non-linear characteristic of magnetization (5.1.1). Materials parameters are presented in the table 4.1.

This material shows obviously important iron losses and is probably not the best one to be used, as the first experimental results confirmed [5].

## 4.3 Current supply

Figures 4.10 and 4.11 show the layout of the supply currents in the stator slots, that is currents of the phases  $a$ ,  $b$  and  $c$  :

$$I_a = \sqrt{2} I_s \cos \left[ 2 \pi f t \right] \quad (4.1)$$

$$I_b = \sqrt{2} I_s \cos \left[ 2 \pi f t + \frac{2 \pi}{3} \right] \quad (4.2)$$

$$I_c = \sqrt{2} I_s \cos \left[ 2 \pi f t - \frac{2 \pi}{3} \right] \quad (4.3)$$

where :

$$I_s = I S \quad (4.4)$$

and:

- $I_s$  – effective stator supply current
- $f$  – stator supply current frequency
- $I$  – effective stator supply current density
- $S$  – surface of stator slot

Such a layout of the currents in the twenty four slots corresponds to four pairs of poles  $p$ . For a not full stator (Figure 4.5), one of these four pole pairs is not supplied.

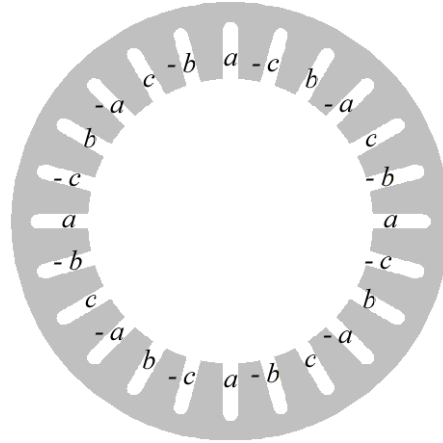


Figure 4.10 *Supply currents phases layout in the stator slots*

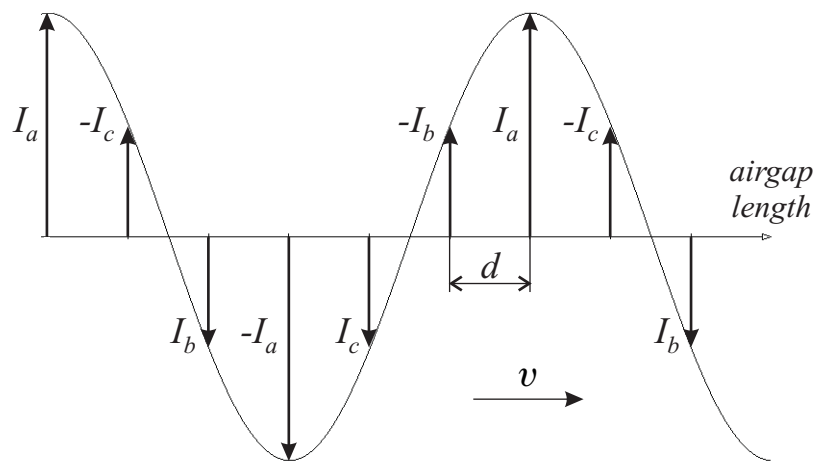


Figure 4.11 *Linear representation of the supply currents layout in the slots of the stator along the airgap*

Looking at the figure 4.11 we can see directly, that the chosen supply currents layout will result in the rotating magnetic field of the sinusoidal form, propagating itself with the speed of:

$$v_p = 6 d f \quad (4.5)$$

where:

$d$  – distance between two neighbouring windings  
(here: equal to distance between two neighbouring slots)

In the analytical model ('D-model') the surface supply currents  $\mathbf{J}_s$  are sinusoidal as well and described by the following equation:

$$\mathbf{J}_s = J_s \cos \left[ 2 \pi f t - \frac{p}{r} x \right] \mathbf{1}_y \quad (4.6)$$

where:

$J_s$  – magnitude of the surface supply currents  
 $x$  –  $x$  coordinate (according to figure 4.9)

and:

$$J_s = \frac{3 \sqrt{2} p I_s}{\pi r} \quad (4.7)$$

The value of the effective current density  $I$  was chosen as the relatively high one,  $10 \text{ A/mm}^2$ , which however does not cause the saturation of the magnetic material, computed for the case of a solid sphere rotor made of the same steel as the stator. Looking closer at the figure 4.12 we can observe, that for the chosen supply current density only a few areas in the actuator (represented in white) have the magnetic flux density higher than the saturation flux density  $B_s$  equal to  $1.35 \text{ Tesla}$  and that the actuator is still not saturated, what supports the choice of the supply current value.

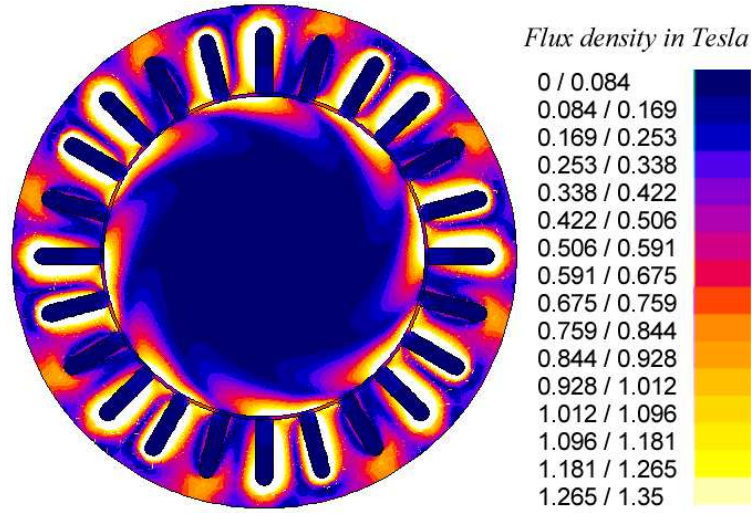


Figure 4.12 *Flux density in the actuator*

Considering the cooling effect of the aerostatic suspension (4.1.1), the copper losses, even if higher (4 times) than the ones commonly admissible in classical motor windings, remains acceptable. The closer analysis of the losses in the spherical actuator will not be, however, the subject of my study.

## CHAPTER 5

# TWO-LAYERS ROTOR

In this chapter I will introduce the structure of the rotor, its initial evolution and some parameters, which influence on the actuator performance will be verified as well.

### 5.1 Rotor structure initial evolution

The aim of the rotor structure changes presented in this section is to improve the spherical actuator performance.

#### 5.1.1 Solid rotor

The simplest structure of the rotor to consider in my simulations is the solid sphere, presented on the figure 5.1. The diameter of the rotor is given in the table 5.1.

This basic geometry of the rotor will be the subject of further improvements, which utility will be confirmed.

The solid rotor is made of the same material as the stator, namely Carpenter Stainless Type 430FR Solenoid Quality. For the simplicity of computations I will consider it as a material with the linear magnetization curve (Table 5.1), as the saturation effect can be neglected in the first study of influence of some parameters. However, in some other cases the validation computations using the non-linear characteristic of magnetization will be performed (Figure 5.2).

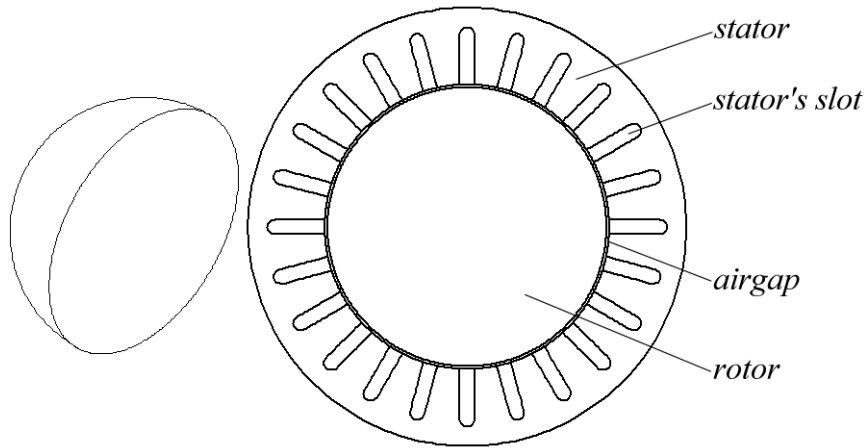


Figure 5.1 Solid rotor and its 2D model

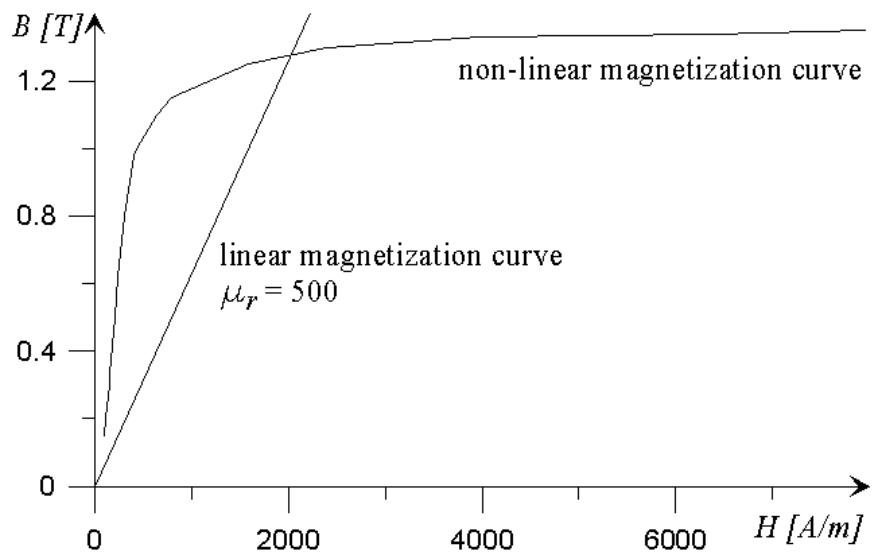


Figure 5.2 Magnetization curve of Carpenter Stainless Type 430FR Solenoid Quality steel and its simplified linear characteristic

Table 5.1 *Rotor parameters*

| Parameters                        |                                           | Values              |
|-----------------------------------|-------------------------------------------|---------------------|
| Dimensions                        |                                           |                     |
|                                   | Rotor radius $r$                          | 25 <i>mm</i>        |
|                                   | Rotor length $l$                          | 50 <i>mm</i>        |
|                                   | Airgap $e$                                | 0.5 <i>mm</i>       |
| Materials                         |                                           |                     |
| Carpenter Stainless<br>Type 430FR | Magnetic relative<br>permeability $\mu_r$ | 500                 |
| Solenoid Quality steel            | Electric resistivity $\rho$               | 76 $\mu\Omega cm$   |
|                                   | Saturation flux density $B_s$             | 1.35 <i>Tesla</i>   |
| Copper                            | Magnetic relative<br>permeability $\mu_r$ | 1                   |
|                                   | Electric resistivity $\rho$               | 1.72 $\mu\Omega cm$ |

### 5.1.2 Two-layers rotor

As the first improvement of the rotor structure, the external layer made of copper will be added (Figure 5.3). The rotor core, so called inner layer, will stay composed of the magnetic material, as presented in section 4.2.

The high, comparing to the magnetic material, electric conductivity of copper (Table 5.1) should make the circulation of the inductive currents in the rotor much easier, it means that the inductive currents should achieve higher density, than in the case of the solid one-layer rotor, and as the result, the generated torque should be higher. The separation of the conducting and magnetic layers leads as well to limitation of the negative skin effect reducing the penetration of electromagnetic field inside the material [5].

On the other hand, the low magnetic permeability of copper leads to a higher equivalent airgap and hence to a reduction of the magnetic field. There will be then a compromise value of the copper layer thickness to find.

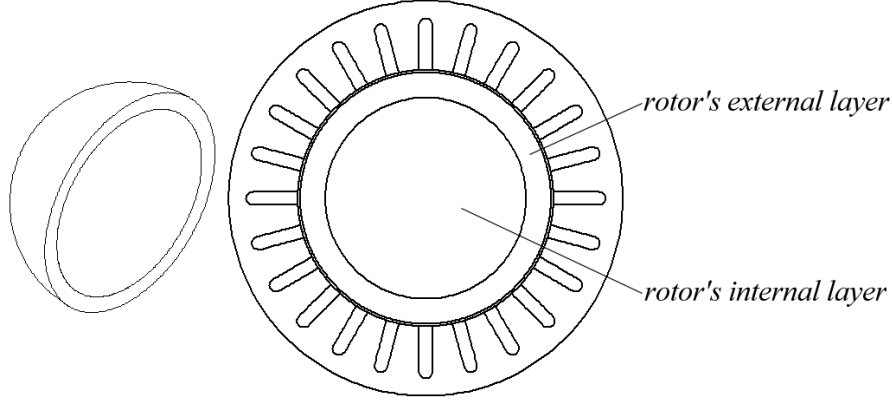


Figure 5.3 Two-layers rotor and its 2D model

### 5.1.3 Comparison between solid and two-layers rotor

For some values of the rotor external layer thickness  $el$ , as well as for the solid rotor, the characteristics torque-speed, where speed is represented as a function of the rotor current frequency  $f_r$ , were computed;

$$f_r = f - p \frac{\omega_m}{2\pi} \quad (5.1)$$

Figure 5.4 presents the results for the values of the external layer thickness between 0 (solid rotor) and 5 mm. We can see directly, that the actuator with two-layers rotor produces generally more torque than the actuator with the solid rotor. Looking at the evolution of the curves we can observe, that it exists certain  $el$  value (0.2 mm), which corresponds to the highest value of the maximal torque. Increasing the thickness of the external layer over 0.2 mm, decreases the torque maximal value. We can see, that for the  $el$  of 25 mm, what corresponds to the solid rotor made of copper, the characteristic torque-speed is placed below the characteristic of the solid rotor made of steel. This can be caused by the lower magnetic permeability of copper comparing to steel. On the other hand, decreasing the external layer thickness below 0.2 mm means, that the rotor tends to become the solid steel rotor, and the characteristic torque-speed for  $el$  of 0.1 mm shows this tendency.

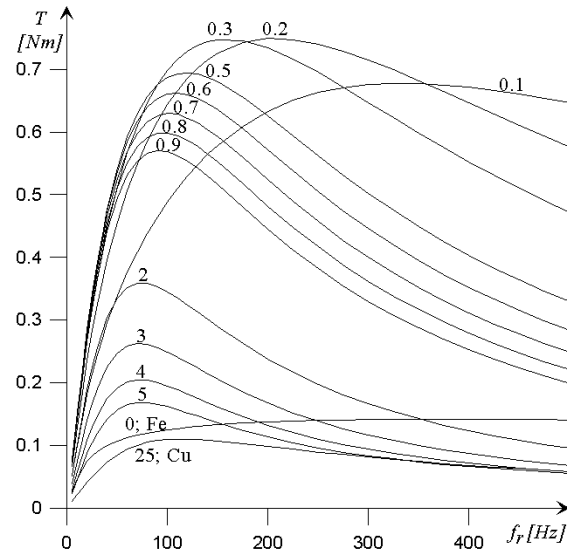


Figure 5.4 Characteristics torque-speed for solid (Fe or Cu) and two-layers rotor for different rotor external layer thickness  $el$

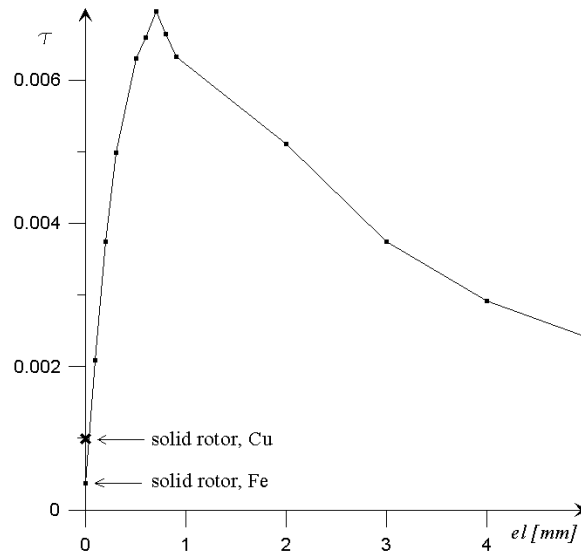


Figure 5.5 Electromechanical conversion  $\tau$  as a function of rotor external layer thickness  $el$

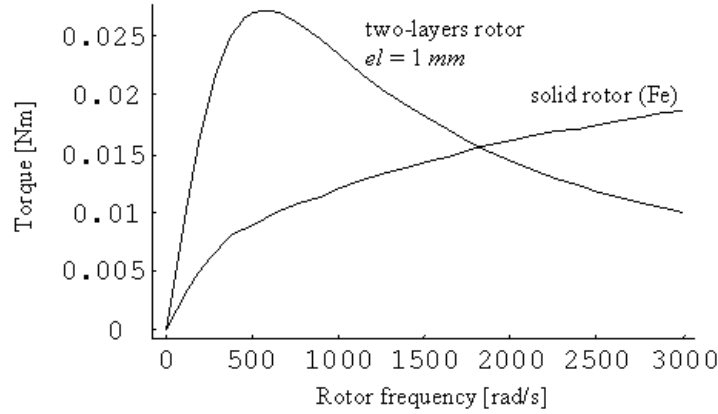


Figure 5.6 *Characteristics torque-speed for solid and two-layers rotor - analytical results [6]*

Coming to my main criterion of evaluation, for each curve from the figure 5.4 value of the  $\tau$  parameter was calculated and presented on the figure 5.5. As we can see, for each considered two-layers rotor the  $\tau$  value is higher, comparing to the solid rotors, and the optimum corresponds to the  $el$  of  $0.7\text{ mm}$ . So, we do not only have the higher value of the maximal torque produced by the actuator, but as well the better electromechanical conversion. Comparing the  $\tau$  value for both solid rotors reveals that, despite the lower maximal torque, rotor made entirely of copper ensures better electromechanical conversion. All of this confirms the previous analytical results [6] presented on the figure 5.6 and shows the advantages of replacing the solid rotor with the two-layers rotor.

#### Solid and two-layers rotors – results explanation

In the subsection 5.1.2 I introduced the theory supporting the use of the two-layers rotor, namely the possibility to induce higher currents inside such a structure thanks to high conductivity of copper, the inner steel rotor core allowing to maintain the flux at the high level. Now I will present some of the results, which show the validity of my reasoning. To obtain these results I first divided the two-layers rotor of my model into twenty four equal parts, what corresponds to the number of the stator slots, and for one of this parts, as shown on the figure 5.7,

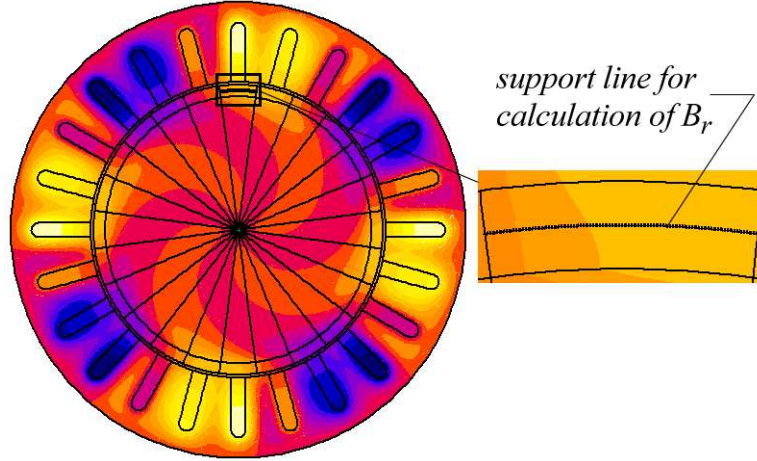


Figure 5.7 Rotor divided in 24 parts and calculation support line.  
Magnetic flux distribution

I calculated the total current  $I_{REL}$  induced in the rotor external layer, as a function of this layer thickness  $el$  (Figure 5.8). After that, for each  $el$  considered, I found the mean value of the flux density radial component  $B_r$  calculated along the support line placed in the middle of the rotor external layer, as presented on the figure 5.7.

Support line was placed in the middle of the rotor external layer (so it is changing its position as the  $el$  value changes) with the assumption, that using it in the calculation will give me results from which I will be able to approximately conclude, how the calculated physical parameter behave in the whole area of the external layer. That is why I will treat the  $B_r$  mean value calculated on the basis of the support line as the  $B_r$  mean value for the whole considered external layer part.

As we can observe on the figure 5.8, increasing the rotor external layer thickness  $el$  results in the higher total current  $I_{REL}$  induced in this layer, however, the more we increase the  $el$  value, the slower the current increases and it achieves its maximum value for the  $el$  of 4 mm. After that the current value starts to decrease. The initial increase of the current can be explained, as I proposed before, by the decrease of the external layer resistance. At the same time the flux density radial component  $B_r$  value (responsible for the induction of current in this

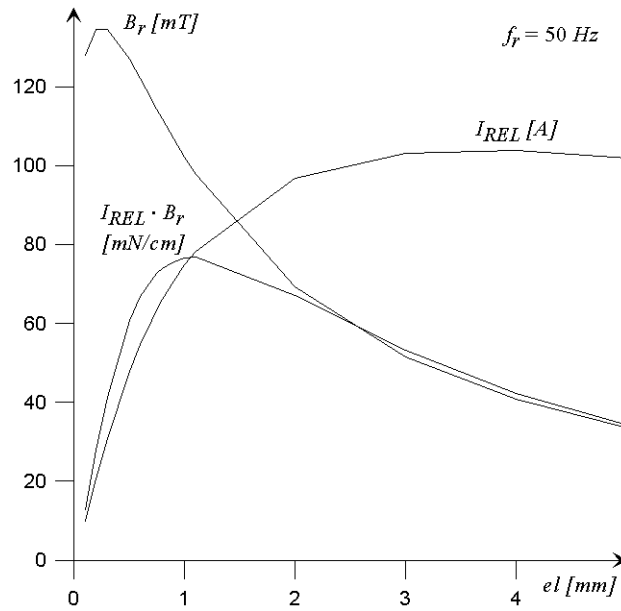


Figure 5.8 Current  $I_{REL}$ , flux density radial component  $B_r$  and  $I_{REL} \cdot B_r$  in 1/24 part of the rotor

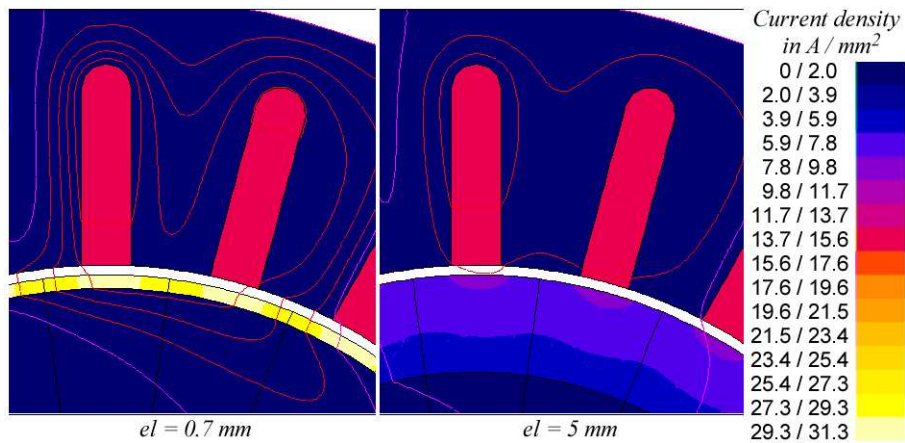


Figure 5.9 Induction currents and equiflux lines for different  $el$  values

particular part of the rotor) is decreasing with the increase of the equivalent airgap composed of the rotor external layer and the airgap itself ( $el + e$ ). We can speak here about equivalent airgap because from the magnetic point of view these two just mentioned regions are not different at all and they participate equally and strongly in the changes of the magnetic circuit reluctance. Coming back to the decreasing  $B_r$  value, it results in the stabilization of the current value. Finally  $B_r$  is not high enough to generate more current in the external layer despite the reduction of its resistivity, so the  $I_{REL}$  current starts to decrease as well.

Looking at the figure 5.9 we can notice the differences, in both the induction current and magnetic flux, caused by the changing value of the external layer thickness. For the  $el$  value of  $0.7\text{ mm}$  we can see, that the current density are elevated and they do not change at the different 'depth' of the external layer. There are as well some equiflux lines indicating the magnetic flux circulation through the external layer. However for the  $el$  value of  $5\text{ mm}$  the current density are lower comparing to the previous case and, what is more important, they are diversified according to the position in the external layer. The closer to the rotor surface the higher the current density. That is caused by the magnetic flux leakage increasing with the increase of external layer thickness. It is also visible, that with the same scale of representation as used for the equiflux lines in the previous case, there are few lines crossing the  $5\text{ mm}$  'deep' external layer. Of course it does not mean that there is not a flux at all, but that the flux circulation is highly reduced, what confirms the results discussed earlier and presented on the figure 5.8.

Considering the theory of magnetic circuits and the results presented on the figure 5.9 it is quite obvious, that the total magnetic flux passing through the rotor external layer decreases from the very first moment when the  $el$  value begins to growth. Increasing the thickness of the external layer composed of the non-magnetic material increases the magnetic circuit reluctance and if we assume the constant magnetomotive force we can only obtain one result - the decrease of the magnetic flux in the circuit.

The initial unexpected increase of the  $B_r$  value can be explained by the method of its calculation along the support line placed in the middle of the external layer. Since I calculated the mean value along the support line, what I judged sufficient for this study, and not the

mean value for the chosen part whole external layer area, my results can show some small inaccuracy.

Finally I will discuss the  $I_{REL} \cdot B_r$  result characteristic presented on the figure 5.8. The utility of this parameter comes out from the Laplace equation:

$$\mathbf{F} = I_c (\mathbf{l}_c \times \mathbf{B}_c) \quad (5.2)$$

where:

$\mathbf{F}$  – force acting on the conductor

$I_c$  – current circulating in the conductor

$\mathbf{l}_c$  – conductor length

$\mathbf{B}_c$  – magnetic field flux density surrounding the conductor

For the considered rotor part (Figure 5.7) the  $I_{REL} \cdot B_r$  result characteristic shows us the form of the characteristic of the force acting on this part and responsible for the rotating movement of the rotor. As we can see, this force achieves its maximum for the  $el$  of 1 *mm*, or, considering the inaccuracy in  $B_r$  estimation, for the  $el$  value close to it. Such a characteristic corresponds a lot with the  $\tau$  parameter characteristic presented on the figure 5.5.

## 5.2 Studied parameters – two-layers rotor

### 5.2.1 Airgap

The very first of all the parameters is the airgap  $e$ . Intuition suggests us, that it is an important parameter and in this chapter I will prove it and show, how changing the airgap can influence the value of the torque produced by the asynchronous machine.

For everybody who knows a little bit about electromagnetism is obvious, that the airgap filled with air, which has the magnetic relative permeability equal to one, is an important element in the magnetic circuit of the asynchronous machine. The bigger the airgap is, the higher the reluctance of the magnetic circuit will be, resulting in the smaller magnetic flux circulating in the circuit. The smaller magnetic flux will induce less current in the rotor of the asynchronous machine and as the

final result our machine will produce less torque. That is why we have the interest to minimize the size of the airgap.

In the classical asynchronous machines the dimension of the airgap could vary between  $0.2 \dots 0.3 \text{ mm}$  for the small ones up to few millimeters for the medium and big ones [14]. In my study I will choose  $0.5 \text{ mm}$  (Table 5.1) as the basic value but, taking into account the use of the aerostatic suspension of the rotor (4.1.1), we can consider descending with the airgap size up to even  $0.1 \text{ mm}$ .

The value of  $0.5 \text{ mm}$  was chosen as a reasonably small one, but still high enough to be implemented and calculated easily by the finite element method software. In fact, the relatively big difference between the size of the airgap equal to  $0.1 \text{ mm}$  and the dimensions of surrounding it the rotor and the stator, will result in the need of much denser meshing of the airgap comparing to the meshing of the rotor and stator. Such a dense meshing of the airgap will influence the meshing of the rotor and the stator at the boundaries with the airgap as well, to guarantee the continuity of the meshing. All of this will cause the increase of the total number of finite elements and as a result the increase of the total calculation time. It can and will be of course done to study the influence of the airgap size but apart of this chapter the chosen basic value of  $0.5 \text{ mm}$  will be used.

### Influence of airgap – simulations results

To learn about the influence of the airgap size  $e$  I have chosen several values of it, lower and higher comparing to the basic value of  $0.5 \text{ mm}$ , namely  $0.1$ ,  $0.3$ ,  $0.7$  and  $1 \text{ mm}$ . As I have mentioned at the beginning of this chapter I will continue to use the two-layers rotor model with the external layer of  $0.7 \text{ mm}$  of thickness and the stator ‘C - model’. To keep this value constant, changing the airgap will result in changing the internal layer thickness, what however should not influence the results, because the magnetic permeability of steel is much higher comparing to air.

On the figure 5.10 we can see the characteristics torque-speed of the actuator, computed for the chosen values of the airgap dimension  $e$ . As it was to expect, decreasing the  $e$  value increases the maximal generated torque value and improves the electromechanical conversion represented by the  $\tau$  parameter.

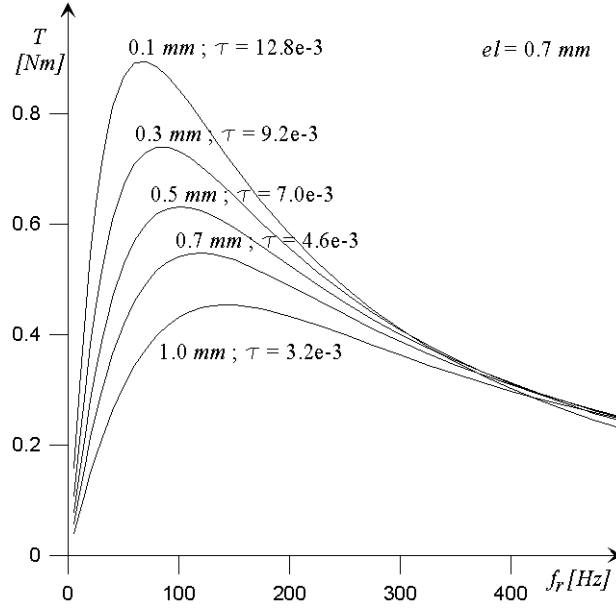
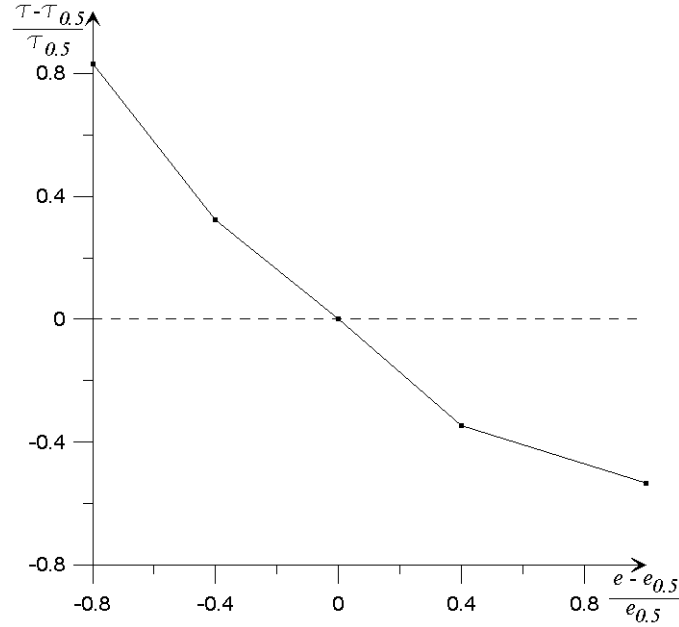


Figure 5.10 Characteristics torque-speed and  $\tau$  parameter for different airgap  $e$  values

Figure 5.11 shows the relationship between the airgap  $e$  and the electromechanical conversion  $\tau$ , expressed using the relative values of the both parameters, calculated with the reference to the airgap basic dimension of  $0.5 \text{ mm}$ . As we can observe, around the  $e$  value of  $0.5 \text{ mm}$  the relation is linear and inversely proportional to the airgap dimension. However, further increasing the  $e$  value over  $0.7 \text{ mm}$  deteriorates the electromechanical conversion with lower intensity and on the contrary further decreasing the  $e$  value below  $0.3 \text{ mm}$  improves the  $\tau$  parameter with higher intensity, comparing to the changes of  $e$  between  $0.3$  and  $0.7 \text{ mm}$ . In any case the utility of decreasing the airgap dimension  $e$  is proven.

Finally there is a need to mention that the possibility to build, thanks to the aerostatic suspension, the airgap as thin as  $0.1 \text{ mm}$  allows the actuator to achieve an electromechanical conversion  $\tau$  almost twice as high as the one computed in the simulations performed hereafter.

Figure 5.11 *Relative  $\tau$  versus relative  $e$* 

### 5.2.2 Rotor external layer thickness versus airgap

The influence of the rotor external layer thickness  $el$  on the actuator performance was already presented while speaking about the rotor basic structure first improvement – adding the external layer to the solid one-layer rotor (5.1.2). In this part of my dissertation I will however investigate the link between the  $el$  optimal value and the airgap dimension  $e$  value.

#### External layer vs airgap – simulations results

Previously I showed, that for the airgap  $e$  of 0.5 mm the optimal value of the  $el$  is 0.7 mm. Now I will present how does it change for the  $e$  value varying between 0.1 and 1 mm. For all these airgap dimensions I created the family of the torque-speed characteristics, corresponding to the different external layer thickness, which served me to find out the optimal value of  $el$  for each value of  $e$ .

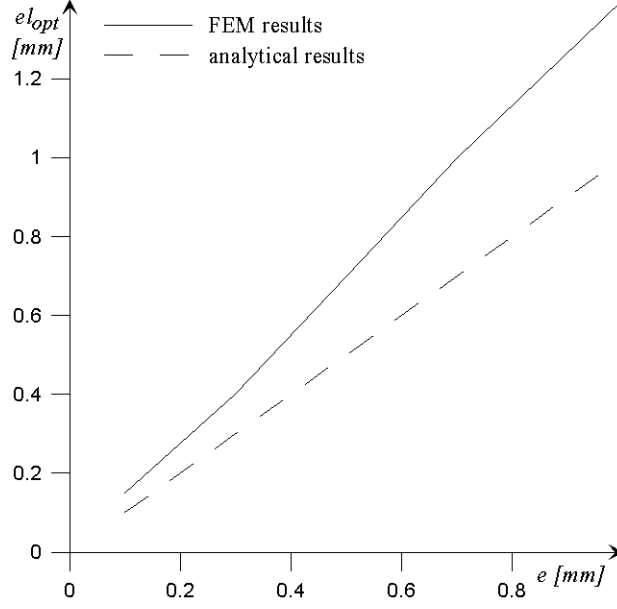


Figure 5.12 Rotor external layer optimal thickness  $el_{opt}$  versus airgap size  $e$  - FEM and analytical results

On the figure 5.12 we can observe, that the relation between the airgap size  $e$  and the rotor external layer optimal thickness  $el_{opt}$  is quite linear. The wider the airgap, the thicker the optimal rotor external layer.

According to the previous analytical calculations [5] discussed characteristic is not only linear but the rotor external layer optimal thickness should be equal to the corresponding airgap size. In fact, if the torque maximal value  $T_{max}$  is equal to (as presented in A.2):

$$T_{max} = 3p \frac{L_{\mu}^2 I_s^2}{L_{\mu} + l_r'} \quad (5.3)$$

and the corresponding rotor pulsation  $\omega_{r \max}$  is:

$$\omega_{r \max} = \frac{R_r'}{L_{\mu} + l_r'} \quad (5.4)$$

and finally the magnetizing inductance  $L_\mu$  and the rotor resistance (brought to the stator side)  $R_r'$  are proportional to, respectively:

$$L_\mu \propto \frac{1}{e + el} \quad (5.5)$$

$$R_r' \propto \frac{1}{el} \quad (5.6)$$

it is possible to show that the maximum of the electromagnetic conversion parameter  $\tau$  corresponds to the following ratio:

$$\frac{e}{el} = 1 \quad (5.7)$$

It is however not the case of my results. On the other hand, taking into account the hypothesis made in the analytical and the finite element modeling, it is possible to explain such a difference between the results and I will do it on the next few pages.

### External layer vs airgap – results explanation

I said already before, that increasing the airgap size  $e$  results in higher actuator magnetic circuit reluctance. At the same time the magnetic flux in the circuit is decreasing. In order to achieve the optimal performance we need to compensate the loss of the flux by the increase of the rotor external layer conductivity to maximize the currents induced in the rotor. To obtain such result, we need to increase the rotor external layer thickness  $el$ . That is, what can explain the linear relation between the  $e$  value and the  $el_{opt}$  value.

To explain the difference between the analytical and finite element results we need to have a closer look at both models. In the analytical model there were not any slots in the stator and the source currents were defined on the stator surface in the smooth airgap. In my finite element model there are slots in the stator, that is why the airgap width  $e$  in my model corresponds to the bigger width of the equivalent airgap in the analytical model  $e_{eq}$  (Figure 5.13). The relation between the two airgap dimensions is described by the parameter called Carter coefficient [2, 3], which depends on the slot pitch  $\lambda$  and slot width  $s$  as well as the airgap width  $e$ .

### Carter coefficient $C_c$

In general we can describe the Carter coefficient  $C_c$  as a parameter, which allows, in the approximate way, to take into account, in the simplified model without slots, the existence of the slots in the real machine (Figure 5.13). When we homogenize the structure (here: stator containing the slots) we cannot forget about the surface phenomena appearing on this structure limits. If the particular dimensions of this structure are small comparing to the whole structure size, the surface phenomena can be often neglected. It is however not the case of our stator, when calculating the magnetic field flux density  $B$  in the equivalent smooth airgap we must take into account the existence of the stator slots. If we define the real machine in the idealized form, as presented on the figure 5.13, we suppose that the slots are straight and of the infinite depth, that the structure is flat and of the infinite length in the  $Z$  dimension perpendicular to the drawing. We suppose as well that the magnetic materials have the infinite permeability ( $\mu = \infty$ ,  $H = 0$ ) and that there are not

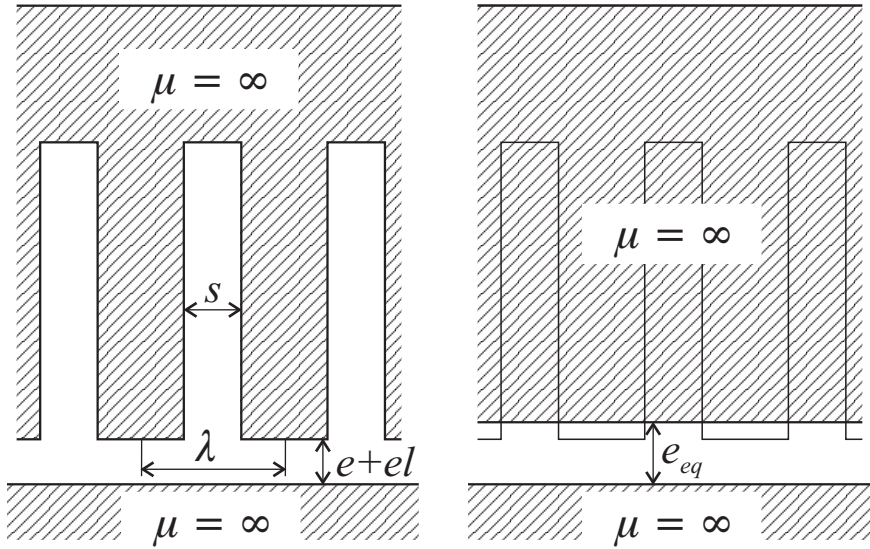


Figure 5.13 Structure with stator slots and its equivalent with 'smooth' airgap

any currents in the slots. As the result the magnetomotive force  $F_{mm}$  (integral of the magnetic field  $\mathbf{H}$  lines) is constant for each airgap crossing independently of the chosen path. Finally the airgap is supposed to be filled with the linear and uniform medium, which the most often is vacuum with its  $\mu_0$  permeability. We should notice here that in our case the external layer of thickness  $el$ , because of its permeability, is also treated as a part of the airgap (Figure 5.13).

The solution of such a simplified problem was given by Gibbs [12] and in general it says that in the model with the homogenized stator structure the additional vacuum layer should be inserted in the airgap comparing to the initial model with slots (Figure 5.13). It can be presented also differently telling that the initial airgap remains unchanged but on its surface an additional surface reluctance  $Rel_{surface}$  appears. Both of these interpretations are connected to each other by the following equation:

$$Rel_{surface} = \frac{(e_{eq} + el) - (e + el)}{\mu_0} = \frac{e_{eq} - e}{\mu_0} \quad (5.8)$$

Even though the  $Rel_{surface}$  value is calculated for the strong simplifying assumptions, once obtained it can be used to calculate the airgap magnetic field flux density  $B$  without any assumptions. In particular, even if the slots are not straight (most of the cases) we use as the  $s$  - width of the slot - value the dimension of the slot opening.

Although the Gibbs' solution is a simple function of two variables (ratios:  $s/(e + el)$  and  $\lambda/(e + el)$ , where  $\lambda$  is the slot pitch - distance between two consecutive slots), it is the implicit one. That is why very often the explicit expression is used, which corresponds to one of the extreme cases, when:

$$\begin{array}{ll} \text{either} & (e + el) \ll \lambda \quad : \text{ narrow airgap} \\ \text{or} & (e + el) \gg \lambda \quad : \text{ very wide airgap} \end{array}$$

In both cases the solution is explicit and depends only on one variable.

The first case of narrow airgap, which is our case, was studied by Carter and its solution is closest to the accurate one for the following conditions:

- $(e + el)/\lambda < 0.125$  or  $0.125 < (e + el)/\lambda < 0.5$
- $s/\lambda$  is smaller than 1

For the narrow airgap we can make an assumption that the magnetic field flux density  $B$  surrounding a slot will not be influenced by the presence of an other slot. In our simplified case there is only one slot (Figure 5.14) and in some distance from it the  $B$  value is constant and described by the following expression:

$$B = \frac{\mu_0 F_{mm}}{e + el} \quad (5.9)$$

Equation 5.9 gives the exact value of the  $B$  parameter on the surface of the airgap without slots. Vector of the magnetic field flux density  $\mathbf{B}$  is perpendicular to the airgap surface. Value found by Carter corresponds also to the  $B$  value in some distance from the slot, however nearby the slot this value decreases (Figure 5.14 -a-). Thus the existence of a slot results in the deficiency of the magnetic flux crossing the airgap. Carter explained it telling that the  $B$  value (Equation 5.9) is constant in the whole airgap except the section of the length  $\alpha \cdot s$  which corresponds to the fraction of the slot width  $s$  (Figure 5.14 -b-).

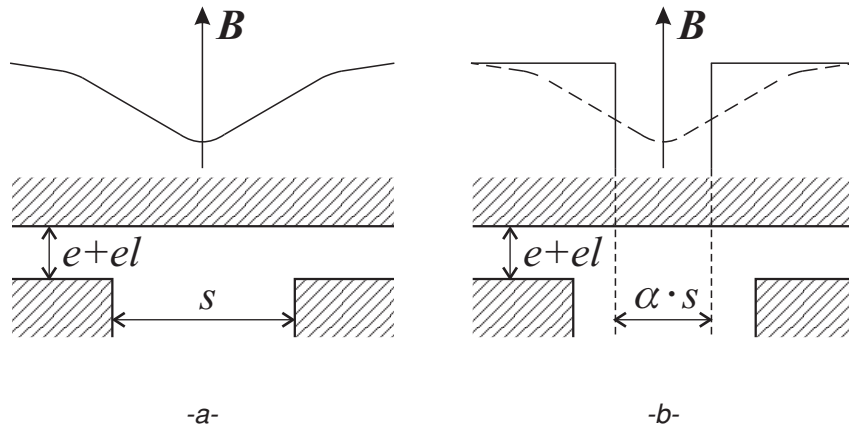


Figure 5.14 Flux density  $B$  value along the airgap

The analytic expression describing the  $\alpha$  parameter is, as follows:

$$\alpha = \frac{2}{\pi} \arctan \left[ \frac{s}{2(e+el)} \right] - \frac{2(e+el)}{\pi s} \ln \left[ 1 + \left( \frac{s}{2(e+el)} \right)^2 \right] \quad (5.10)$$

for the ‘arctan’ given in radians. The slot pitch  $\lambda$  which does not appear in the equation 5.10 is introduced in the description of the Carter coefficient  $C_c$ :

$$C_c = \frac{\lambda}{\lambda - \alpha s} \quad (5.11)$$

Finally the equivalent airgap  $e_{eq}$  can be expressed as follows:

$$e_{eq} + el = C_c (e + el) \quad (5.12)$$

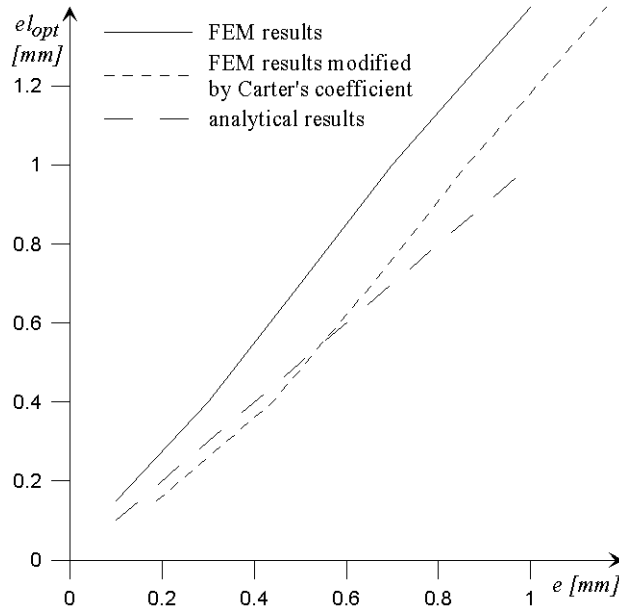


Figure 5.15 Rotor external layer optimal thickness  $el_{opt}$  versus airgap size  $e$  - FEM, modified FEM and analytical results

The second case of the very wide airgap does not concern our actuator that is why it will not be discussed closer in my dissertation.

### External layer vs airgap – results explanation (continuation)

We know now the relation between the ‘real’ and the smooth equivalent airgap (Equation 5.12). However, because there is not an easy mathematical way to find the  $e$  value knowing the  $e_{eq}$  value, what in our case corresponds to passing from the analytical to the real structure with slots, we will do the opposite and modify the FEM results to represent them as a function of the smooth equivalent airgap. That will allow the comparison between the FEM and analytical results (Figure 5.15). We can see that the analytical results are much closer to the modified FEM results what means that the use of Carter coefficient can explain in general the differences between the analytical and FEM results (not modified).

### 5.2.3 Rotor internal layer thickness

Having presented, while speaking about the rotor structure first improvement (5.1.2), the influence on the actuator performance of the rotor external layer thickness  $el$ , I will discuss now the rotor internal layer thickness  $il$  influence. From the electromagnetic point of view this parameter should change nothing for a wide range of its values, however decreasing it to much below some critical thickness  $il$  should result in an important deterioration of performance. It would be caused by the increase of the magnetic circuit reluctance connected directly with the non-sufficient thickness of the rotor internal layer.

To prove the above-mentioned theory I will decrease the  $il$  value, which corresponds at the moment to the radius of the rotor internal layer, in the way that in the middle of the rotor I will create an empty air-space (Figure 5.16). This air-space allows to reduce the rotor inertia and to improve the dynamic performance of the actuator.

Additional computations taking into account the material non-linear characteristic of magnetization (5.1.1) will be performed as a validation.

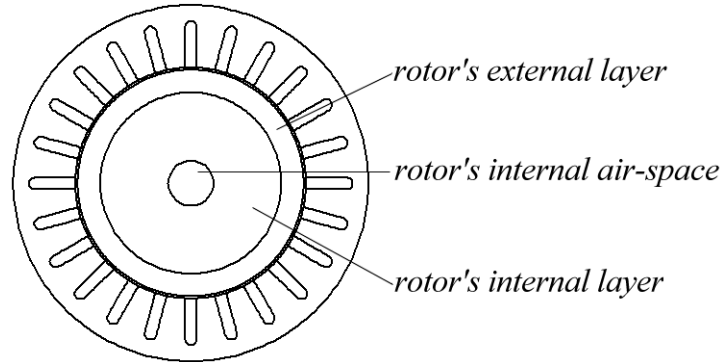


Figure 5.16 Structure with rotor internal air-space

#### Two-layers rotor internal layer – simulations' results and explanation

Figure 5.17 presents the results of computations for both the linear and non-linear material magnetization curves. Looking at the characteristics of the electromechanical conversion  $\tau$  as a function of rotor internal layer thickness, we can see clearly that until some critical value of  $il$  parameter is reached, it has no influence on the actuator performance. Reaching critical value, for the  $il$  smaller than  $1.8\text{ mm}$  (linear) and  $1.3\text{ mm}$  (non-linear), the  $\tau$  parameter decreases rapidly, as it was to expect. It happens, because the rotor internal layer becomes too thin and the magnetic circuit reluctance increases significantly (Figure 5.18). That is why the magnetic flux and, what follows, the induced currents and generated torque decrease.

The difference between the characteristics of the electromechanical conversion for the linear and non-linear material magnetization curves can be explain by the fact, that the linear magnetization curve represents in some way the 'mean' value of the material magnetic permeability in the whole range of magnetic fields (figure 5.2). In such a way it does not take into account the extreme values of permeability represented by the non-linear magnetization curve. Figure 5.17 shows us, that for values of the rotor internal layer thickness higher than the critical one, the real material permeability is higher than assumed by the linear magnetization curve, so that we obtain higher torque and  $\tau$  parameter. On the contrary, for very thin internal layers, when the magnetic saturation

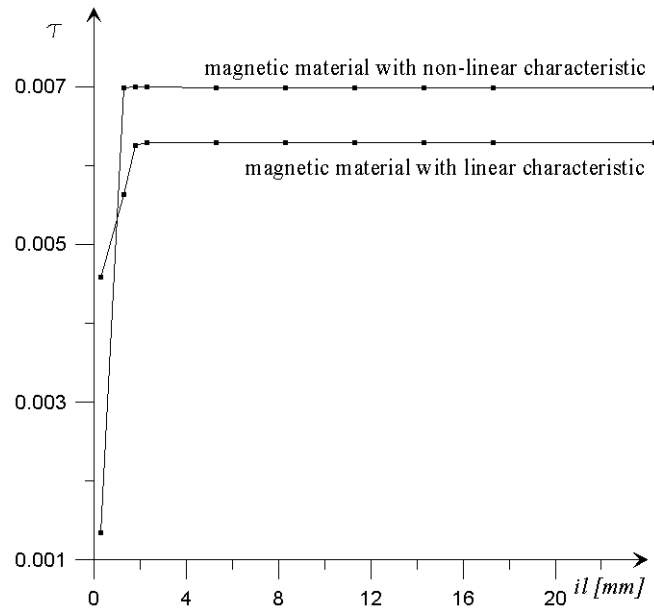


Figure 5.17 Conversion  $\tau$  as a function of the rotor internal layer thickness  $il$  for the linear and non-linear magnetization curve

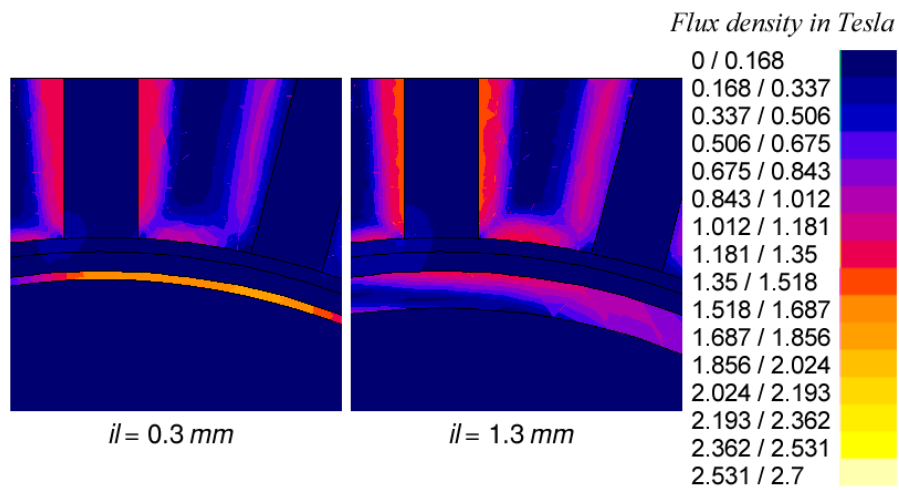


Figure 5.18 Flux density in rotor internal layer for different values of its thickness  $il$  (non-linear case)

starts to play an important role, the real permeability is smaller than the ‘linear’ one and the  $\tau$  value decreases much lower comparing to the use of the linear magnetization curve.

Results presented on figure 5.17 proves, that it is possible to change the thickness of the rotor internal layer in a wide range without influencing the value of the maximal generated torque and the electromechanical conversion  $\tau$  parameter.

#### 5.2.4 Supply currents

The next parameter to study is the supply current density  $I$ . It is a parameter, which can easily influence performance of the actuator because, in the induction machine, the maximal torque and, what follows, the electromechanical conversion  $\tau$  parameter are directly depending on its value. However, the infinite increase of the supply current value is in practice limited by the acceptable level of the actuator heating and, in other way, by the saturation of magnetic material, what makes further increase of the supply current useless.

As I said before in 4.3, the basic value of the  $I$  current was chosen to be  $10 \text{ A/mm}^2$  (Table 4.1). Now I will perform the simulations for two additional  $I$  values of 5 and  $20 \text{ A/mm}^2$ .

#### Supply currents – simulations results and explanation

On figure 5.19 we can see the characteristics torque-speed of the actuator calculated for three different values of the supply current density  $I$ . As it was to expect, the higher the supply current, the higher the torque and the electromechanical conversion  $\tau$ .

As I explain in A.2, the maximal value of the electromagnetic torque  $T_{max}$  generated by the actuator can be found using the following equation:

$$T_{max} = 3p \frac{L_{\mu}^2 I_s^2}{L_{\mu} + l_r'} \quad (5.13)$$

where :

$L_{\mu}$  – magnetizing inductance

$l_r'$  – rotor inductance (brought to the stator side)

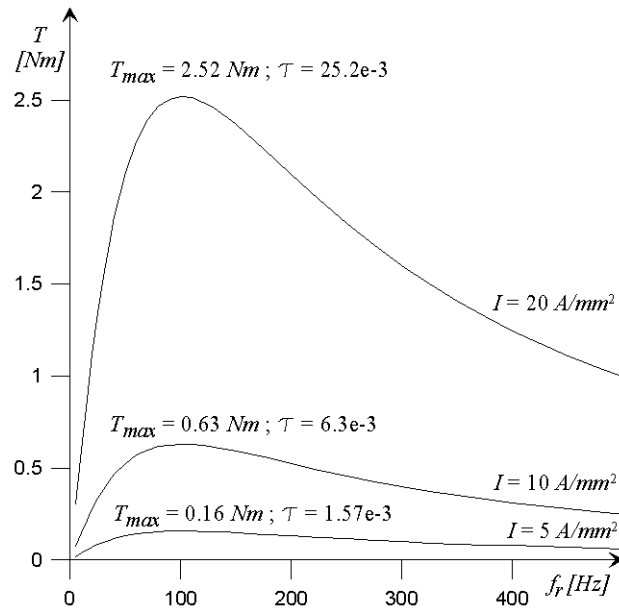


Figure 5.19 Characteristics torque-speed for different values of the supply current density  $I$

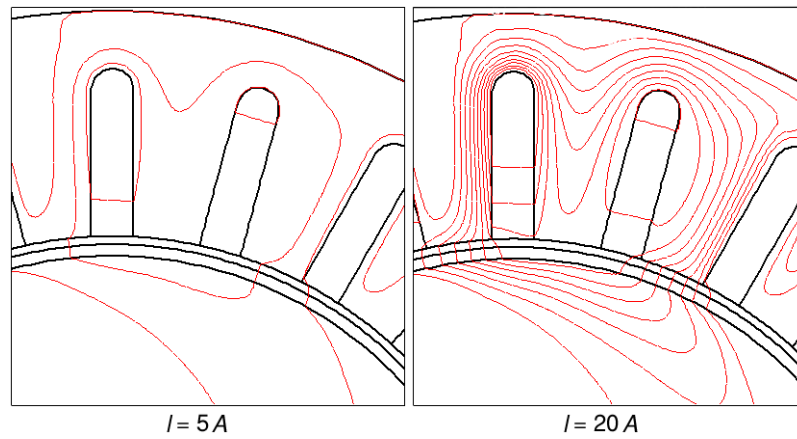


Figure 5.20 Equiflux lines for different values of the supply current density  $I$

Equation 5.13 shows, that the torque maximal value corresponds to the supply current squared value. It is clearly confirmed by the results of my simulations presented on figure 5.19.

Note here again the interest of the aerostatic suspension that improves the cooling of the actuator and hence increases the acceptable stator current level. Allowing to double the current and hence to quadruple the copper losses permits indeed at the same time to quadruple the maximal torque value. It will however simultaneously modify all the optimal parameters values obtained hereafter for the  $I$  value of  $10\text{ A/mm}^2$ . To determine the optimal current value it is necessary to perform a thermal study of the actuator, it is however not a subject of my thesis.



## CHAPTER 6

# TWO-LAYERS-WITH-TEETH ROTOR

This chapter will present the next step in the evolution of the rotor and the influence of some further parameters, which will be tested using this new rotor structure.

### 6.1 Rotor structure advanced evolution

In this section the second, and final at this level of my research, change of the rotor structure will be introduced, leading to improve performance of the spherical actuator.

#### 6.1.1 Two-layers-with-teeth rotor

The second improvement of the rotor structure are the so called ‘teeth’ made of the same material as the rotor internal layer and passing through the external layer (Figure 6.1).

The main idea of such a solution is to make the circulation of the magnetic flux through the external layer easier, it means to decrease the magnetic circuit reluctance and to increase the flux value, what should result in a higher torque value. Instead of increasing the torque it could also allow us to obtain the same performance but by the use of the lower supply currents.

For my first simulation I will use an equivalent 2D structure with 22 teeth, that will occupy 50 % of the rotor external surface, and the stator ‘C - model’. Such a number of teeth is the closest one to the value of “ $0.9 \cdot \text{stator slot number}$ ” (Equation 6.1), which is proposed in the

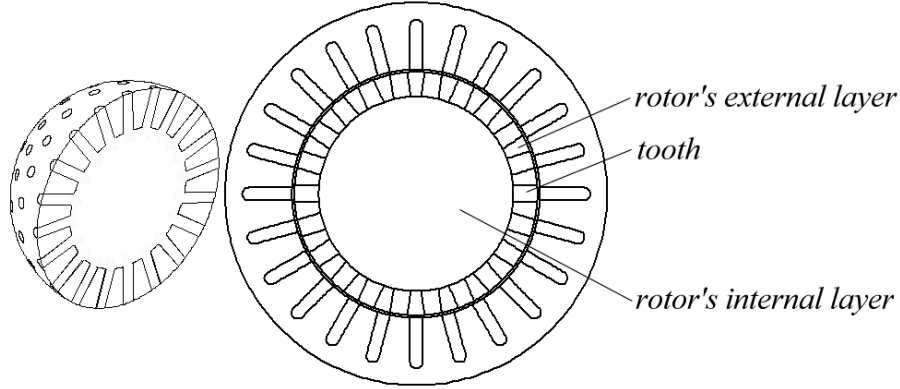


Figure 6.1 *Two-layers-with-teeth rotor and its 2D model*

literature [7]. The 50/50 value of the copper/steel ratio was chosen as the initial value and its possible influence on the actuator performance will be tested in another section. As the easiest geometrical form to manufacture the straight teeth were chosen. Teeth form influence will be however verified in one of the next sections.

#### Relation between the two-layers-with-teeth rotor and its 2D model

The parameter allowing us to transfer the results obtained using the 2D model of the rotor to the real 3D rotor structure (Figure 6.1) is obviously not the number of teeth but the distance between two consecutive teeth. Of course in a 2D model we can reproduce only a one distance at a time comparing to the 3D structure where the distance between two closest teeth can vary according to the rotor direction of rotation (Figure 6.2). For that reason the histograms representing the distribution of all possible angular distances separating the two consecutive teeth located under an inductor for all possible orientations of the rotor will be created later on in my thesis (Figure 6.31). Such histograms give us for example an information about the distances which appear more frequently than the others. For the same reason as well the 2D study were performed for the varying distance between the two consecutive teeth (Figure 6.30).

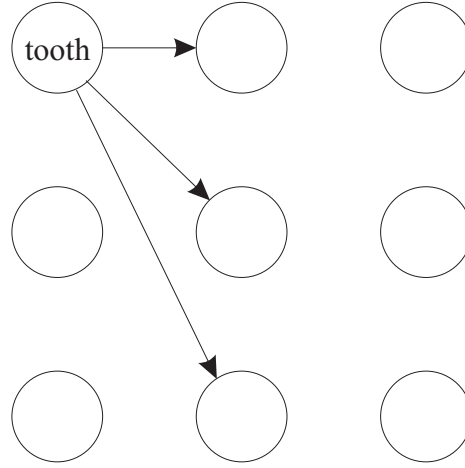


Figure 6.2 *Distance between the closest teeth depending on the rotor direction of rotation*

### 6.1.2 Comparison between two-layers and two-layers-with-teeth rotor

For some values of the rotor external layer thickness  $el$ , including the value of  $0.7\text{ mm}$ , which corresponds to the maximal value of the  $\tau$  parameter calculated for the two-layers rotor (5.1.3), I will now compute the characteristics torque-speed of the actuator with the two-layers-with-teeth rotor.

As we can see on the figure 6.3, the actuator with teeth generates not only higher torque but its electromechanical conversion is better as well. Figure 6.4 shows, that the two-layers-with-teeth rotor guarantees the  $\tau$  parameter values even ten times higher (for  $el = 10\text{ mm}$ ) than the two-layers rotor. There is a need to mention that such results were obtained for the first chosen copper/steel ratio value (50/50) and, as I have already said before, influence of this parameter will be verified in another section, what could lead us to further improvement of the actuator performance.

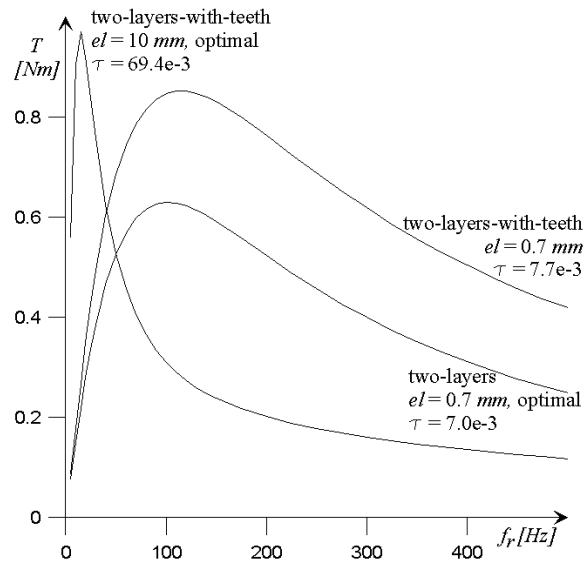


Figure 6.3 Characteristics torque-speed and  $\tau$  parameter for two-layers and two-layers-with-teeth rotor

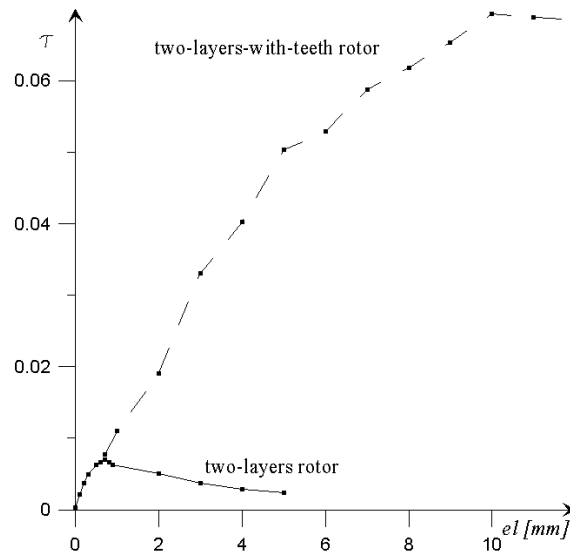


Figure 6.4 Electromechanical conversion  $\tau$  as a function of rotor external layer thickness  $el$

### Two-layers and two-layers-with-teeth rotors – results explanation

In order to better explain the obtained results and to present the physical phenomenon appearing in the two-layers-with-teeth rotor, I have performed some simulations to find out the mean value of the flux density radial component  $B_r$  and the total current  $I_{REL}$  induced in the rotor external layer, as a function of this layer thickness  $el$ . These computations were done in a similar way as for the two-layers rotor (5.1.3), however in the case of the rotor with teeth the  $I_{REL}$  value were found for a part of the external layer made of copper, the  $B_r$  value for the neighbouring tooth made of steel (Figure 6.5), where the magnetic flux is concentrated, and the calculations were done assuming the homogenized structure of the rotor. Figure 6.6 presents results of these computations. As in the case of the two-layers rotor, we can observe, that  $B_r$  value decreases with the increase of the external layer thickness  $el$ . It can be explained by the increase of the magnetic circuit reluctance. At the same time the total current value  $I_{REL}$  is growing, at the beginning rapidly, then much slower, to achieve finally its maximum value for the  $el$  of 9 mm. After that the  $I_{REL}$  current starts to decrease. The initial increase of the current value can be explained, as for the two-layers rotor, by the decrease of the rotor external layer resistivity. However, the simultaneous decrease of the  $B_r$  value is the reason, why the current achieves its maximum and begins to decrease as well.

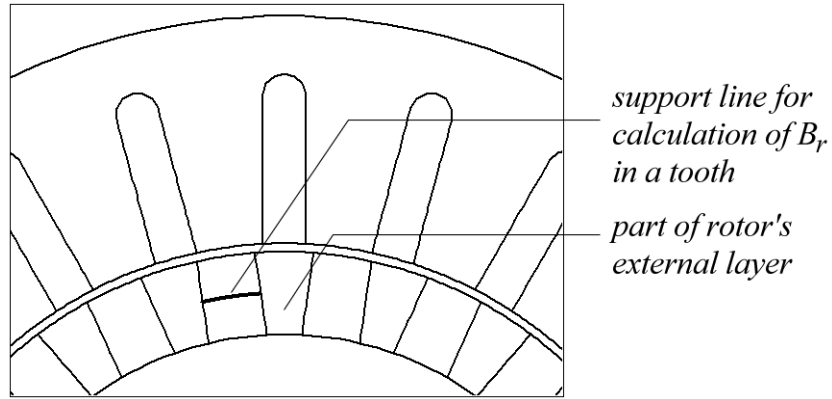


Figure 6.5 Part of the rotor external layer, neighbouring tooth and support line for calculation of  $B_r$

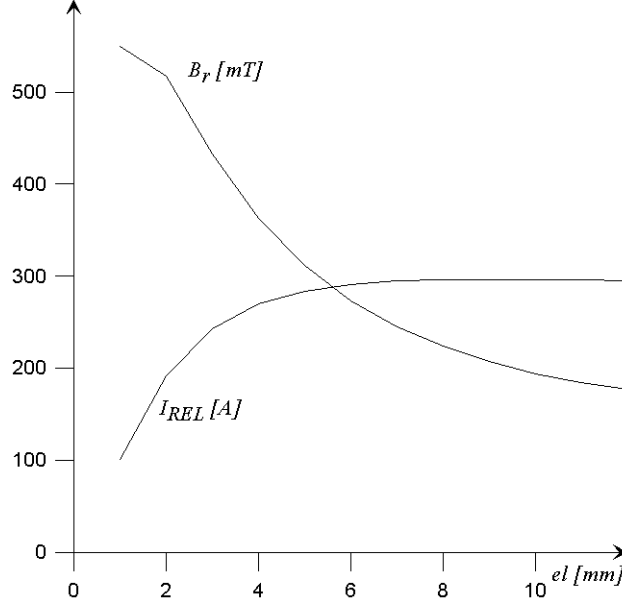


Figure 6.6 Current  $I_{REL}$  and flux density radial component  $B_r$  in part of the rotor external layer and neighbouring tooth, respectively

On the figure 6.7 we can see the product of multiplication of  $I_{REL}$ ,  $B_r$ ,  $l$  and  $r$ , which gives a partial value of the maximal torque, from the Laplace law (Equation 5.2), and the maximal torque  $T_{max}$  obtained directly from my simulations. The evolution of the both torque values as a function of the rotor external layer thickness  $el$  is highly similar, what confirms the correctness of the ' $I_{REL} B_r$ ' approach presented above.

On the figure 6.8 both the cases of the two-layers and two-layers-with-teeth rotor are presented. For the same rotor external layer thickness ( $0.7\text{ mm}$ ) and using the same scale of presentation we can see clearly that the use of the teeth increased the number of the equiflux lines passing through the external layer. It means that the circulating magnetic flux is higher and, what we can observe on the same figure, it is concentrated in the teeth area, where its passage is much easier. Such results confirm my expectations and show the interest of the deeper analysis of the two-layers-with-teeth rotor structure and that is, what I have decided to do in my further work.

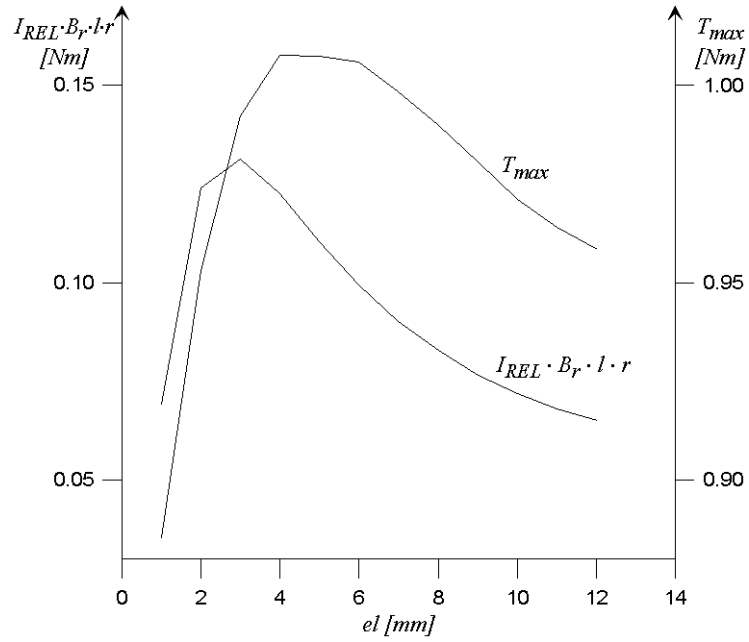


Figure 6.7 Maximal torque values  $T_{max}$  and  $I_{REL} \cdot B_r \cdot l \cdot r$  product as a function of rotor external layer thickness  $el$

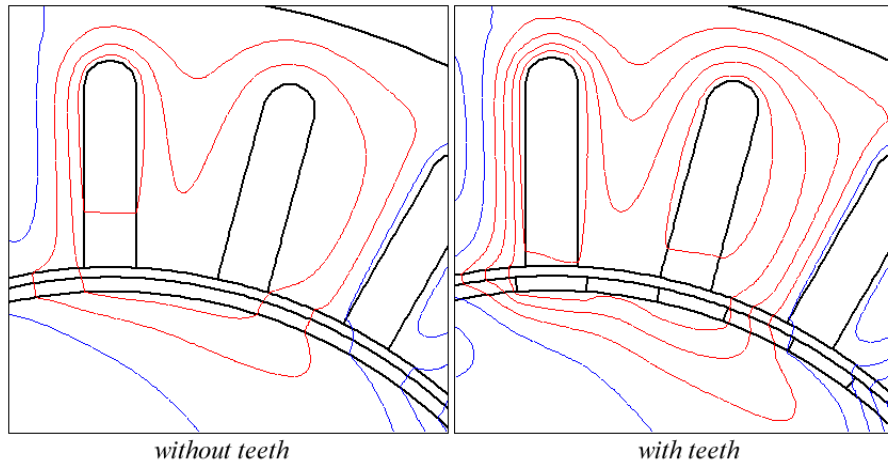


Figure 6.8 Equiflux lines for two-layers and two-layers-with-teeth rotor

## 6.2 Studied parameters – two-layers-with-teeth rotor

Having presented the superiority of the two-layers-with-teeth rotor solution comparing to the two-layers rotor, I will use now this improved rotor structure and the stator ‘C - model’ to examine the influence of some parameters, especially these ones connected directly with the existence of the rotor teeth, on the actuator performance. Two of these parameters, namely tooth size  $ts$  and number of teeth  $nt$ , seem to be the most interesting to study because of their dominating role in the process of the actuator performance improvement.

### 6.2.1 Rotor external layer thickness versus copper/steel ratio – size of teeth

The study presented in this subsection was performed, because I supposed, that the external layer thickness optimal value vary with the copper/steel ratio. When we change the size of the teeth, the reluctance of the magnetic circuit and the resistance of the external layer change as well. If the teeth become smaller for example, the rotor reluctance increases and the resistance decreases, respectively. The next parameter, rotor magnetic flux leakage, becomes in this case smaller. Finally, we cannot forget that the decreasing tooth size will influence the equivalent airgap  $e_{eq}$ , calculated using the Carter coefficient (page 56), which will increase (Figure 6.22, page 92) and cause the increase of the airgap reluctance. In such a multi-parameter case it is more than probable that the ‘balance’ between all these values will not be kept any more and that it will be necessary to change the external layer thickness, in order to find the new optimal combination of all these parameters and, what follows, optimize the electromechanical conversion  $\tau$ .

Previously (6.1.2) I have shown that, for the teeth occupying 50 % of the rotor external surface, the maximal value of the electromechanical conversion  $\tau$  parameter was obtained for the rotor external layer thickness  $el$  of 10 mm. Now I will follow this way of thinking and search for the optimal  $el$  values and corresponding  $\tau$  values for the different copper/steel ratios, namely for the teeth occupying between 10 and 70 % of the rotor surface. To obtain such a variation I will only change the size of the teeth, keeping the number of teeth constant at the value of 22.

Similar study will be done for the teeth numbers of 44 and 88, however, as my main interest concentrates here on the ‘size of teeth’ parameter, the number of teeth influence will be investigated closer in one of the following subsections.

Changing only the size of teeth, which have a straight form, results also in the maximal surface occupied by them no bigger than about 70 %, because for the concerned external layer thickness values, the teeth of big size risk to cross each other in the external layer before reaching the inner layer.

### Size of teeth – simulations results

Figure 6.9 presents the optimal values of the external layer thickness  $el_{opt}$  and the corresponding values of the electromechanical conversion parameter  $\tau_{opt}$  for the varying steel ratio  $sr$ , which represents the

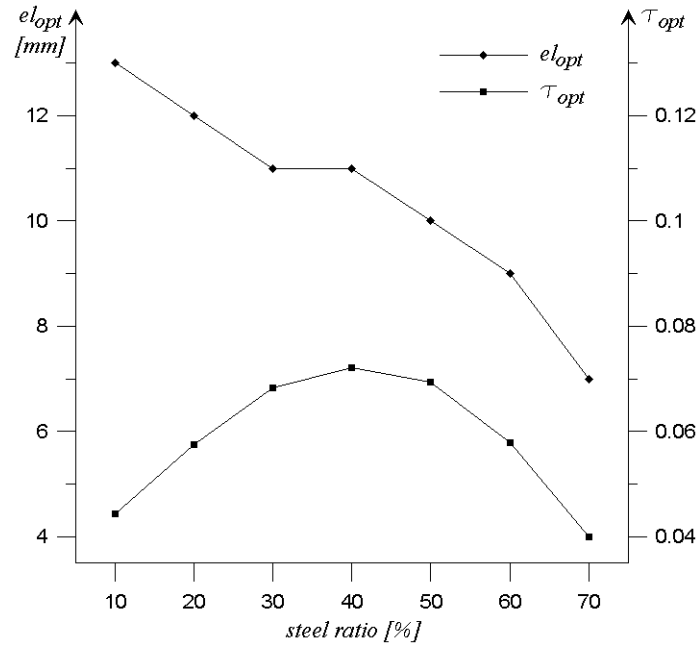


Figure 6.9 Rotor external layer optimal value  $el_{opt}$  and the corresponding conversion value  $\tau_{opt}$  as a function of the steel ratio

percentage of the rotor surface occupied by the teeth. As we can observe, the highest among the  $\tau_{opt}$  values corresponds to the steel ratio of 40 % and was obtained for the rotor external layer thickness of 11 mm.

Looking at the rotor external layer thickness optimal values  $el_{opt}$  we can see directly, that the smaller the teeth are (lower steel ratio), the thicker external layer should be, in order to achieve the maximal electromechanical conversion for the corresponding steel ratio.

### Size of teeth – results explanation

I will explain now, why the rotor  $el_{opt}$  values increase with the decrease of the steel ratio. As I said at the beginning of this subsection (6.2.1, page 74) changing the steel ratio  $sr$  influences the reluctances of a tooth  $Rel_{tooth}$  and the airgap  $Rel_{airgap}$ , the rotor external layer resistance  $R_{Cu}$  as well as the rotor magnetic flux leakage. If the teeth become smaller (decreasing steel ratio), first two of these phenomena, namely  $Rel_{tooth}$  and  $Rel_{airgap}$ , contribute to the reduction of the magnetic flux circulating in the rotor and hence of the value of the rotor current  $I_r$  and finally of the actuator torque. But on the other hand the external layer resistance decreases allowing the circulation of higher currents and reducing the rotor pulsation at maximal torque  $\omega_{r\ max}$  (compare: Equation 5.4), both contributing to the increase of the electromechanical conversion  $\tau$  (see definition of  $\tau$  parameter: Equations 3.8 and A.9).

All these phenomena appear at the same time (Table 6.1) and it is impossible to predict, which one will be dominant. The influence of the rotor flux leakage can be neglected in our case, because the  $\tau$  parameter does not depend on the rotor leakage inductance  $l_r'$  which concerns only the maximal value of the generated torque  $T_{max}$ .

The similar reasoning can be presented for the increasing rotor external layer thickness  $el$  (Table 6.2). In such a case the reluctance of a tooth increases, the airgap reluctance remains unchanged and finally the rotor external layer resistance decreases.

If we combine the two cases presented above, namely the decreasing steel ratio  $sr$  and the increasing rotor external layer thickness  $el$ , we obtain the description of my simulation results presented on the figure 6.9. Looking at the characteristics shown on this figure leads us to the conclusion that the initial reduction of the magnetic flux (due to the decrease of the steel ratio), which continues to decrease with the increasing  $el$

Table 6.1 *Reluctances and rotor resistance for the decreasing steel ratio  $sr$* 

| if steel ratio $sr \searrow$ :                      |                             |                            |                                        |               |                 |
|-----------------------------------------------------|-----------------------------|----------------------------|----------------------------------------|---------------|-----------------|
| $R_{Cu} \searrow$                                   | $\Rightarrow$               | $\Rightarrow$              | $\Rightarrow$                          | $\Rightarrow$ | $\tau \nearrow$ |
| $R_{Cu} \searrow$                                   | $\Rightarrow$               | $\Rightarrow$              | $\Rightarrow \omega_{r \max} \searrow$ | $\Rightarrow$ | $\tau \nearrow$ |
| $Rel_{tooth} \nearrow$                              | $\Rightarrow \phi \searrow$ | $\Rightarrow I_r \searrow$ | $\Rightarrow T_{\max} \searrow$        | $\Rightarrow$ | $\tau \searrow$ |
| $e_{eq} \nearrow \Rightarrow Rel_{airgap} \nearrow$ | $\Rightarrow \phi \searrow$ | $\Rightarrow I_r \searrow$ | $\Rightarrow T_{\max} \searrow$        | $\Rightarrow$ | $\tau \searrow$ |

where:

|                   |   |                                                       |
|-------------------|---|-------------------------------------------------------|
| $R_{Cu}$          | – | resistance of a $\frac{1}{22}$ part of external layer |
| $Rel_{tooth}$     | – | reluctance of a tooth                                 |
| $Rel_{airgap}$    | – | reluctance of the airgap                              |
| $e_{eq}$          | – | equivalent airgap                                     |
| $\phi$            | – | magnetic flux                                         |
| $I_r$             | – | rotor current                                         |
| $T_{\max}$        | – | maximal torque                                        |
| $\omega_{r \max}$ | – | rotor pulsation at maximal torque                     |
| $\tau$            | – | conversion parameter                                  |

thickness value, can be at the same time partially compensated by the increasing inductive currents, because of the decreasing rotor external layer resistance. In addition to the decreasing  $\omega_{r \max}$  value it leads to the increase of the electromechanical conversion parameter  $\tau$  (in our case for the steel ratio higher than 40 %). Such an increase continues until the new balance between the resistance and the reluctance will be found. In other words, it is possible that the smaller magnetic flux (here: smaller maximal torque) corresponds to better electromechanical conversion (Figures 6.10 and 6.11).

Table 6.2 *Reluctances and rotor resistance for the increasing rotor external layer thickness  $el$*

| if external layer $el \nearrow$ : |                                        |                                |                                        |                                    |                                |
|-----------------------------------|----------------------------------------|--------------------------------|----------------------------------------|------------------------------------|--------------------------------|
| $R_{Cu} \searrow$                 | $\Rightarrow$                          | $\Rightarrow$                  | $\Rightarrow$                          | $\Rightarrow$                      | $\tau \nearrow$                |
| $R_{Cu} \searrow$                 | $\Rightarrow$                          | $\Rightarrow$                  | $\Rightarrow \omega_{r \max} \searrow$ | $\Rightarrow$                      | $\tau \nearrow$                |
| $Rel_{tooth} \nearrow$            | $\Rightarrow \phi \searrow$            | $\Rightarrow I_r \searrow$     | $\Rightarrow T_{\max} \searrow$        | $\Rightarrow$                      | $\tau \searrow$                |
| $e_{eq} \rightarrow$              | $\Rightarrow Rel_{airgap} \rightarrow$ | $\Rightarrow \phi \rightarrow$ | $\Rightarrow I_r \rightarrow$          | $\Rightarrow T_{\max} \rightarrow$ | $\Rightarrow \tau \rightarrow$ |

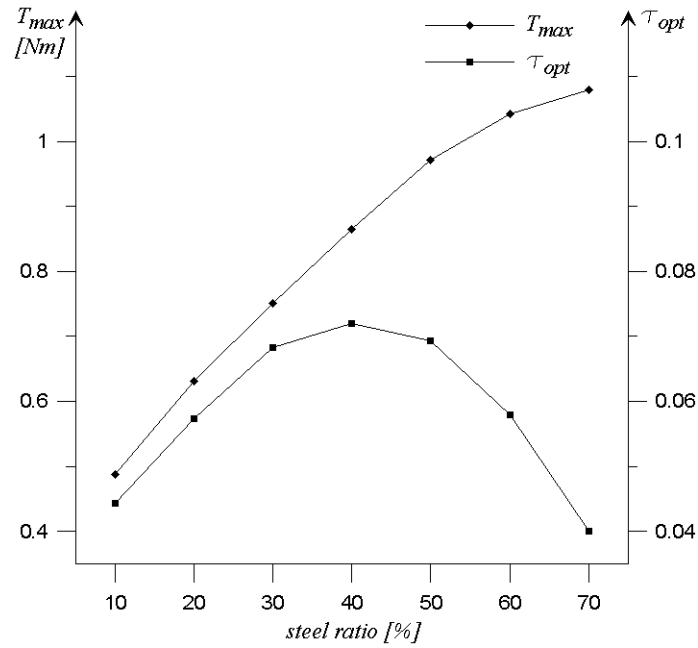


Figure 6.10 *Optimal conversion value  $\tau_{opt}$  and the corresponding maximal torque  $T_{\max}$  as a function of the steel ratio*

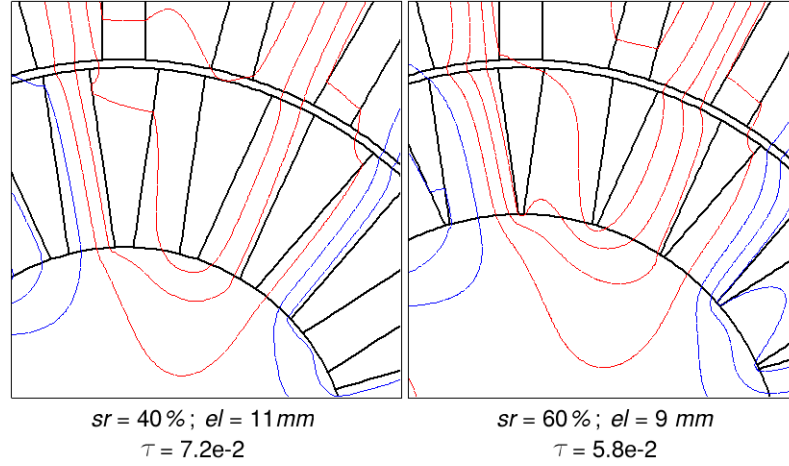


Figure 6.11 *Equiflux lines and conversion  $\tau$  for different values of steel ratio  $sr$*

Of course for smaller steel ratio values (in our case smaller than 40 %) the both effects of the decrease of the resistance will only partially compensate the increase of the reluctance and as the result the  $\tau_{opt}$  value will start to decrease, but the tendency of the  $el_{opt}$  value will be still increasing.

We can ask a question why not to do the opposite and decrease the external layer thickness  $el$  to simply increase the magnetic flux decreased by the steel ratio reduction. The answer is that with the decreasing  $el$  value the magnetic flux is increasing linearly and these changes are relatively small comparing to the initial decrease of the magnetic flux due to the increased airgap reluctance (because of the steel ratio reduction). At the same time the rotor external layer resistance is changing hyperbolically (Figure 6.12). If we reduce the  $el$  value we can increase the magnetic flux but the resistance increases ‘faster’ what limits the growth of the inductive currents and increases the rotor pulsation at maximal torque  $\omega_{r\ max}$ .

For the same reason of the hyperbolically changing rotor external layer resistance we cannot increase the  $el$  thickness without limits. It means that for the high enough values of the thickness  $el$  the resistance will decrease slower and slower. At the same time the reluctance

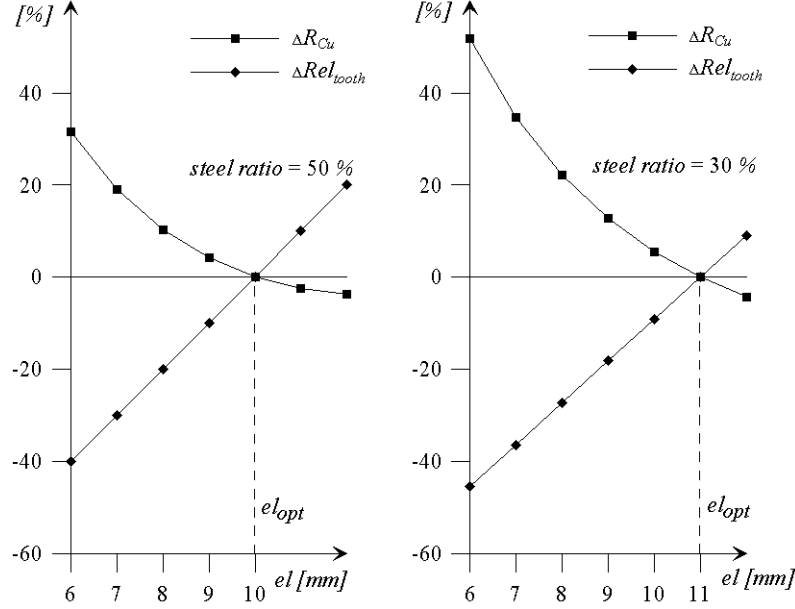


Figure 6.12 Percentage changes of  $R_{Cu}$  and  $Rel_{tooth}$  around the optimum point (10 and 11 mm for steel ratio of 50 and 30 %, respectively)

continues to increase linearly, what results directly in the further reduction of the magnetic flux. With a relatively small variation of the resistance the continuous decrease of the magnetic flux becomes dominant and is no more compensated by the easier circulation of the rotor current and decreasing  $\omega_{r\ max}$  value. That is why, after reaching the optimal combination of the rotor external layer resistance and the total reluctance the further increase of the rotor external layer thickness will result in the deterioration of the electromechanical conversion  $\tau$ .

To better explain and verify the correctness of the results obtained during the simulations I have performed also the analytical study, which confirms my conclusion that the rotor external layer thickness  $el$  increases with the decrease of the steel ratio  $sr$ . The details of this analytical study are presented in the appendix B.

### Size of teeth – ‘non-linear’ validation

The first simulation performed to validate the results presented previously in this subsection took into account the non-linear magnetization curve (5.1.1) of the magnetic material used in the rotor and the stator, and was done, as before, for the rotor with 22 teeth and the stator ‘C - model’. As we can observe on the figure 6.13 the results obtained using the non-linear characteristic of magnetization show the same tendencies as the results obtained for the simplified, linear characteristic. As for the rotor external layer optimal values  $el_{opt}$ , they are increasing with the decrease of the steel ratio and the  $el_{opt}$  value corresponding to the maximal value of the  $\tau_{opt}$  parameter is again 11 mm. Concerning the  $\tau_{opt}$  parameter we can affirm that the use of the non-linear magnetization curve gives in general better results (higher electromechanical conversion) comparing to the simplified linear model simulations, but with one very important condition - the saturation is not present. In such a case we can continue to use the simplified model.

The non-linear results, which are higher comparing to the linear ones, shows that it remains a possibility to increase the supply current, and hence the torque, in order to obtain an acceptable average saturation level. That could be done however only after a thermal study.

As for the reason, why the non-linear results are higher comparing to the linear ones when the saturation is not present (and are smaller when the saturation is present) it can be explained by the fact that the magnetic permeability described by the non-linear magnetization curve can vary, so that is why it can become higher (without saturation) or lower (with saturation) comparing to the fixed and in some sort ‘mean’ value chosen for linear model (figure 5.2). It is, what happens in our case. However when the saturation exists in the machine, the non-linear results show that the linear model is no more valid. It is a case of the steel ratio of 10 % (for 22 teeth) when the teeth have the smallest diameter in the tested range and the saturation just occurs. It is confirmed by the non-linear result which is slightly smaller than the linear one, what means that the linear model was just at the limit of use for this particular size of teeth. This subject will be discussed further in the subsection concerning the number of teeth parameter, where the results for 22, 44 and 88 teeth will be presented.

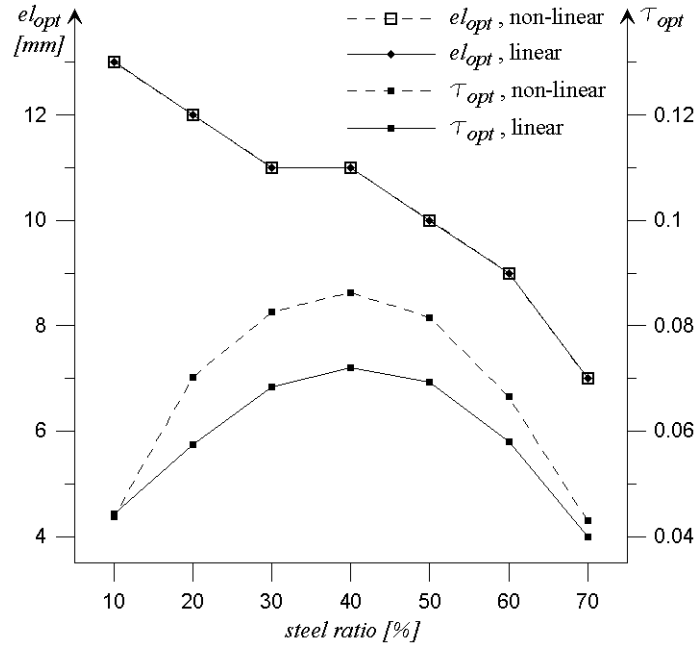


Figure 6.13 The  $el_{opt}$  and  $\tau_{opt}$  values as a function of the steel ratio for linear and non-linear magnetization curve

### Size of teeth – ‘material magnetic permeability’ validation

The next validation also treated the case of the magnetic permeability, but its objective was to check, if the use of the material with quite high permeability can change the tendencies in the obtained results, especially the increase of the rotor external layer optimal value  $el_{opt}$  connected with the decrease of the steel ratio. With such an objective the simulation for the rotor with 22 teeth and the material magnetic relative permeability ten times higher than in the previous case ( $\mu_r = 5000$  instead of 500, respectively) was performed. Figure 6.14 shows that even high permeability of the material does not influence the general rules describing the obtained results, which rules are the same in both cases.

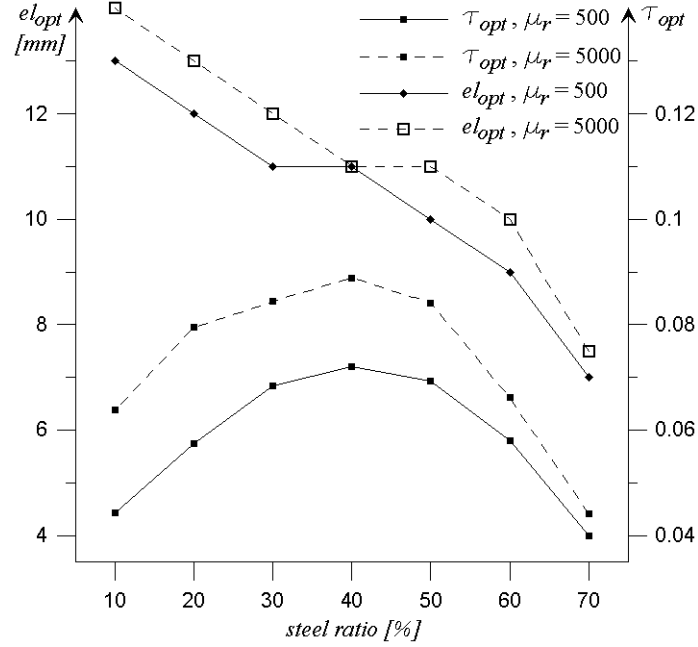


Figure 6.14 The  $e_{l_{opt}}$  and  $\tau_{opt}$  values as a function of the steel ratio for different values of the magnetic material relative permeability  $\mu_r$

### Size of teeth – ‘transient magnetic’ validation

The following simulation used the transient magnetic formulation (TM, 2.4.3) which, as I said before, takes into account not only the rotating movement of the rotor and, what follows, the existence of the slot effect, but as well the non-sinusoidal coupling between the stator and the rotor. The both of these phenomena will be presented closer in the following parts of my thesis. The transient magnetic formulation being the most complete one and combined with the mechanic equation allowed me the simulation of the actuator no-load start-up and, as the result, the creation of the torque-speed characteristic (Figure 6.15). We can observe the torque oscillations connected with the start-up transients and later on with the slot effect.

To compare with the TM results I present on the figure 6.16 the characteristics torque-speed obtained using the magnetodynamic formula-

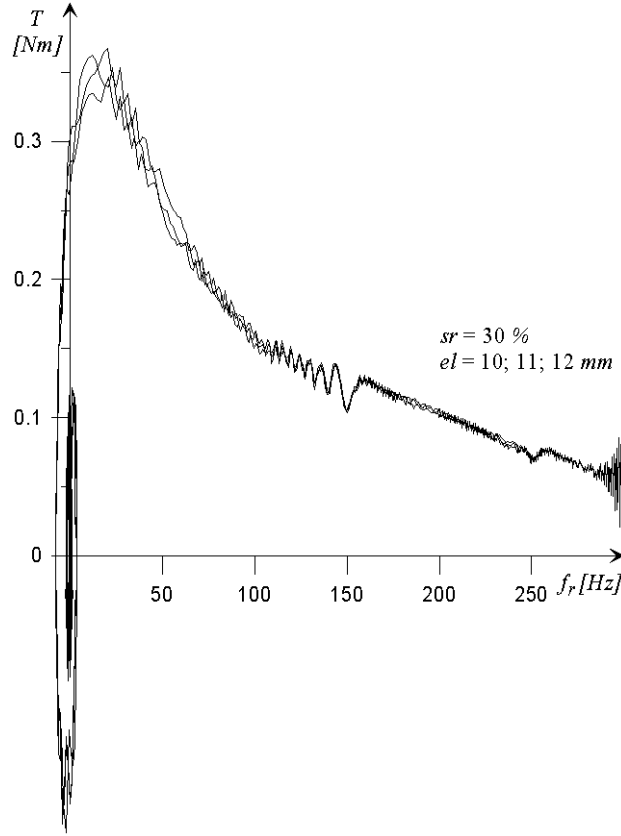


Figure 6.15 *Characteristics torque-speed obtained using transient magnetic formulation TM*

tion (MD, 2.4.4). We can see that the curves shown on the both figures represent in general the same tendencies, although the torque values for the TM formulation are 50 % smaller than for the MD formulation, which however is more general and less precise.

Finally on the figure 6.17 I compare the  $\tau_{opt}$  and  $el_{opt}$  results obtained, as before, for the rotor with 22 teeth using the transient magnetic (TM) and magnetodynamic (MD) formulations. Also in this case we can observe the similar tendencies. For both formulations the rotor external layer optimal thickness  $el_{opt}$  increases with the decrease of the steel ratio  $sr$  and the exact values are the same or close to each other.

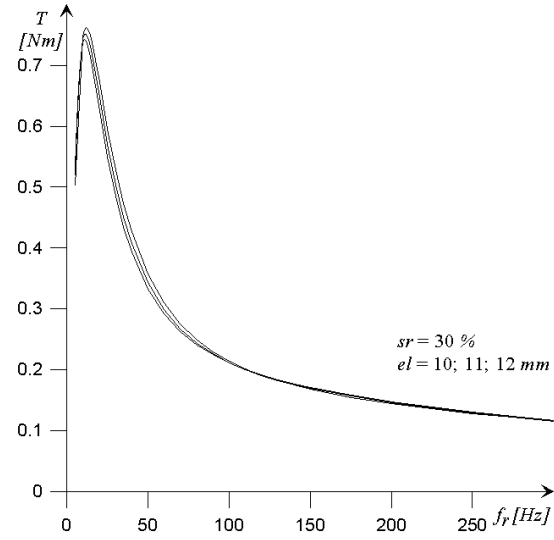


Figure 6.16 Characteristics torque-speed obtained using magnetodynamic formulation MD

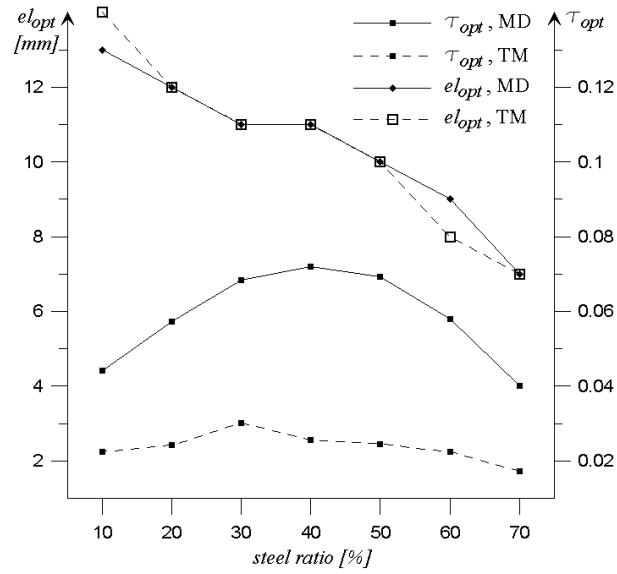


Figure 6.17 The  $el_{opt}$  and  $\tau_{opt}$  values obtained using the MD and TM formulations as a function of the steel ratio

In both cases as well the  $el_{opt}$  value corresponding to the  $\tau_{opt}$  parameter maximal value is once more 11 mm. As for the  $\tau_{opt}$  parameter, in both cases there is one clear maximal value situated in the same zone (steel ratio of  $30 \div 40$  %), although for the TM formulation the results are much less diversified with the changing steel ratio and are generally of the lower values comparing to the use of the MD formulation. However this validation confirms that in order to find the major rules describing the influence of some particular parameters on the actuator performance, the easier to apply magnetodynamic formulation can be used. It is not the case for the non-sinusoidal coupling phenomenon but it is of course excluded by definition and requires the use of the transient magnetic formulation.

#### Size of teeth – ‘3-dimensional magnetodynamic’ validation

Because the 3-dimensional computations demand much more time to produce a result comparing to 2-dimensional ones, what considers the time of model, especially 3D geometry, preparation and the computation itself, only a few cases were tested. One of them is the magnetodynamic validation of the results presented previously on the figure 6.9 (2D model).

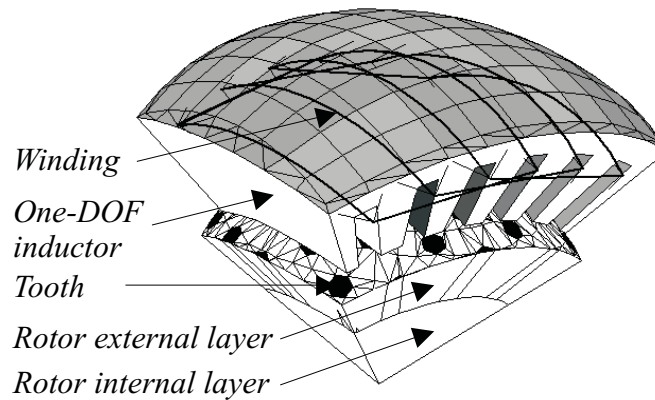


Figure 6.18 3-dimensional structure used in simulations

The 3D simulations were performed using the structure containing one one-degree of freedom inductor - ‘A - model’ - and a section of the rotor with teeth (Figure 6.18). Three values of the steel ratio were checked, namely 20, 40 and 60 %. The step of 20 % was chosen to limit the number and hence the time of calculations.

The obtained results are presented on the figure 6.19. For the chosen range and checked values of the steel ratio the maximal value of the electromechanical conversion parameter  $\tau_{opt}$  corresponds to the steel ratio of 40 %. At the same time the rotor external layer optimal thickness  $el_{opt}$  increases with the decreasing steel ratio. All of this as well as the exact  $el_{opt}$  values (especially 11 mm for steel ratio of 40 %) confirm the previously presented results obtained using the 2D simplified model.

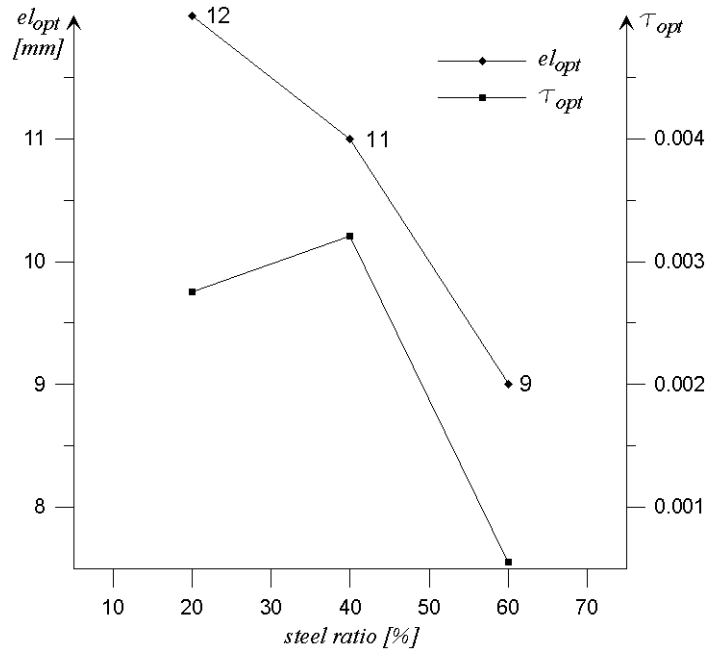


Figure 6.19 Rotor external layer optimal value  $el_{opt}$  and the corresponding conversion value  $\tau_{opt}$  as a function of the steel ratio (3D model)

### 6.2.2 Number of teeth

The number of teeth parameter is closely connected with the previously presented size of teeth and steel ratio parameters. If we change the number of teeth and keep the size of teeth constant, in such a case the steel ratio changes. But if we change the teeth number and keep the steel ratio constant, then it is the teeth size which changes. In this part of my dissertation I will use both of these approaches to test the influence of the number of teeth parameter. Although the teeth number, size and steel ratio parameters are presented separately in two different subsections we must always keep in mind that they cannot be treated totally independently.

#### Number of teeth and constant steel ratio – simulations results

The influence of the teeth number on the actuator performance calculated for a few chosen values of the steel ratio will be investigated here. I have performed simulations for the teeth number values of 22, 44 and 88, and for the steel ratio varying, like in the previous study, between 10 and 70 %. Because increasing the number of teeth and keeping the steel ratio constant results in the decrease of the teeth size, that is why additional computations using the non-linear magnetization curve were performed. In fact there is a risk that in the smaller teeth the saturation phenomenon could occur what would mean that the linear model is no more valid.

The results presented on the figure 6.20 show clearly that the higher teeth number guarantees the better electromechanical conversion  $\tau_{opt}$ . It is confirmed in general by the linear and non-linear simulations. The both type of simulations confirm also the fact that the electromechanical conversion depends on the steel ratio. As for the more precise interpretation, if we compare the linear and non-linear results, we can see that increasing the teeth number results in the saturation of the teeth, which becomes smaller. Of course it occurs first of all for the smaller values of the steel ratio - up to 10, 20 and almost 30 % for 22, 44 and 88 teeth, respectively. For the higher values of the teeth number the critical value of the steel ratio will continue to increase. Because the linear model is no more valid when the saturation appears, it can no more be used to find for example the steel ratio value corresponding to the maximal value of the  $\tau_{opt}$  parameter. As for 22 and 44 teeth the linear and non-linear

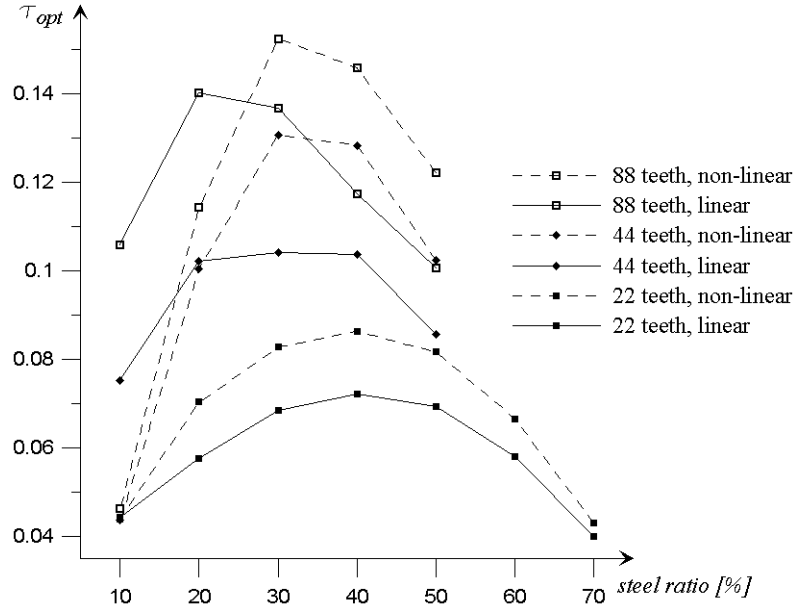


Figure 6.20 The  $\tau_{opt}$  value as a function of the steel ratio for different teeth numbers and linear and non-linear magnetization curve

models give the same results, steel ratio of 40 and 30 %, respectively. It is no more the case for 88 teeth, when the saturation is too important and the linear model gives the wrong result - the maximal  $\tau_{opt}$  value corresponds to the steel ratio of 20 %. In fact the non-linear model shows, that in the reality it corresponds to the 30 %. As for the exact values the presented comparison of the linear and non-linear results reveal that the  $\tau_{opt}$  parameter can be even higher as estimated using only the linear model, of course in the range of its validity.

On the basis of the presented results we can come to the conclusion that the maximal values of the electromechanical conversion parameter  $\tau_{opt}$  for each number of teeth correspond to the steel ratio values from the range between 30 and 40 %. Because the chosen step of changing the steel ratio value during the simulations was equal to 10 %, the closer analysis could reveal that the variation of the optimal steel ratio value for all teeth numbers is even smaller than 10 %.

The next results I want to present are the characteristics of the optimal rotor external layer thickness  $el_{opt}$  as a function of the steel ratio for the three different teeth numbers, namely 22, 44 and 88 teeth, calculated using the linear and non-linear magnetization curve (Figure 6.21).

The first conclusion we can make is that all these curves are almost identical. The highest differences appear for the smallest steel ratio of 10 %, especially for the linear and non-linear characteristics for 88 teeth (difference of 3 mm). It is however easy to explain because for such a small steel ratio value and 88 teeth the saturation phenomenon is already important (compare: Figure 6.20) and the linear results are no more valid.

These results confirm also the tendency shown already for 22 teeth, that the rotor external layer optimal thickness  $el_{opt}$  increases with the decrease of the steel ratio  $sr$  (Figure 6.9). They were also confirmed by the analytical study presented in the appendix B.

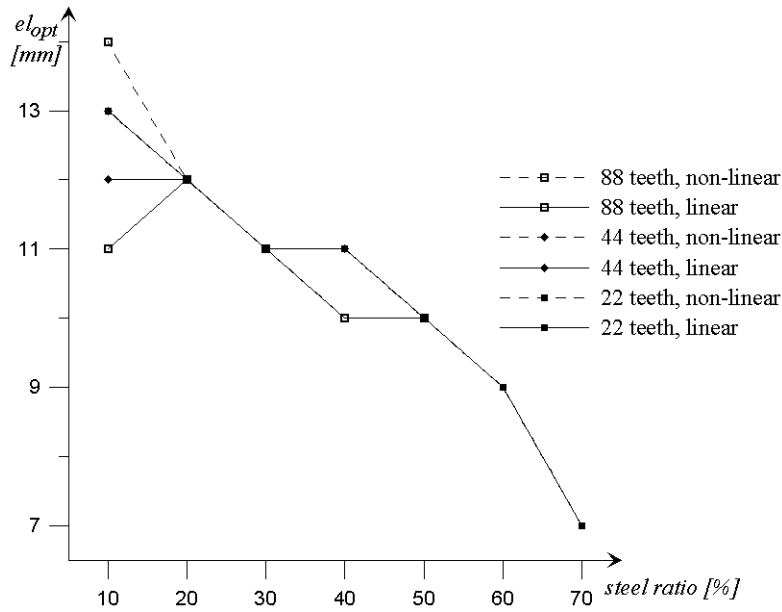


Figure 6.21 The  $el_{opt}$  value as a function of the steel ratio for different teeth numbers and linear and non-linear magnetization curve

If we now have a look simultaneously at the figures 6.20 and 6.21, and consider only the non-linear results which are reliable for the all three tested number of teeth values, we can notice that each time the maximal electromechanical conversion parameter  $\tau_{opt}$  value were obtained for the same thickness of the rotor external layer  $el_{opt}$ , namely 11 mm.

### Number of teeth and constant steel ratio – results explanation

I will try to explain now why the higher teeth number  $nt$  guarantees the better electromechanical conversion  $\tau$ . Such a result tells us that not only the quantity of steel forming the teeth is important, but also changing its layout can improve the performance of the actuator. In fact when the steel ratio  $sr$  and the rotor external layer thickness  $el$  remain constant, it does not matter how many teeth there are to keep the total volumes, occupied by the external layer and by the teeth, constant as well. In this case it is equal to keeping the total reluctance of the teeth and total resistance of the external layer constant. If these general parameters are the same for 22, 44 and 88 teeth, it can explain why the  $el_{opt}$  curves are almost the same for these three teeth numbers. According to the analytical study (Appendix B) for the teeth number higher than 10 the unique differences between the  $el_{opt}$  curves appear due to the airgap  $e$  variation and not the influence of the rotor reluctance and resistance.

However it is not the case of the reluctance describing the airgap which is influenced by the changing number of teeth. My theory is based on the supposition, that increasing the number of teeth decreases the reluctance of the equivalent airgap, which depends on the distance to cover by the magnetic flux passing through the airgap. In fact the magnetic flux tries to connect two closest teeth in the rotor and the stator to decrease the reluctance which stays on his way. In my opinion the higher teeth number results in the more uniform rotor structure, what makes the passage of the magnetic flux through the airgap much easier, because it does not need to cover a distance along the airgap looking for the closest teeth to connect. Because air is not a magnetic material, that is why reducing the distance to cover can highly increase the performance of the actuator, what was confirmed by the obtained results.

To support the presented theory I will again make use of the Carter coefficient. The airgap  $e$  of  $0.5 \text{ mm}$  with teeth and slots will be represented as an equivalent airgap  $e_{eq}$  (with only stator slots) for the rotors with 22, 44 and 88 teeth and different values of the steel ratio. Comparing the values of the equivalent airgap  $e_{eq}$  shown on the figure 6.22 we can see clearly that the ‘real’ airgap to cover by the passing magnetic flux becomes smaller and smaller with the increasing teeth number. The highest decrease of the airgap value can be observed for the smallest tested teeth ratio (10 %), however we cannot forget that in this case everything gained ‘in the airgap’, or even more, will be lost because of the saturation in the small teeth. For the higher values of the steel ratio the differences between the characteristics of the equivalent airgap  $e_{eq}$  corresponding to the 22, 44 and 88 teeth are decreasing, but they still show the interest of increasing the number of teeth in the rotor. The smaller the real airgap to cover, the smaller the reluctance describing the airgap and the higher the circulating magnetic flux.

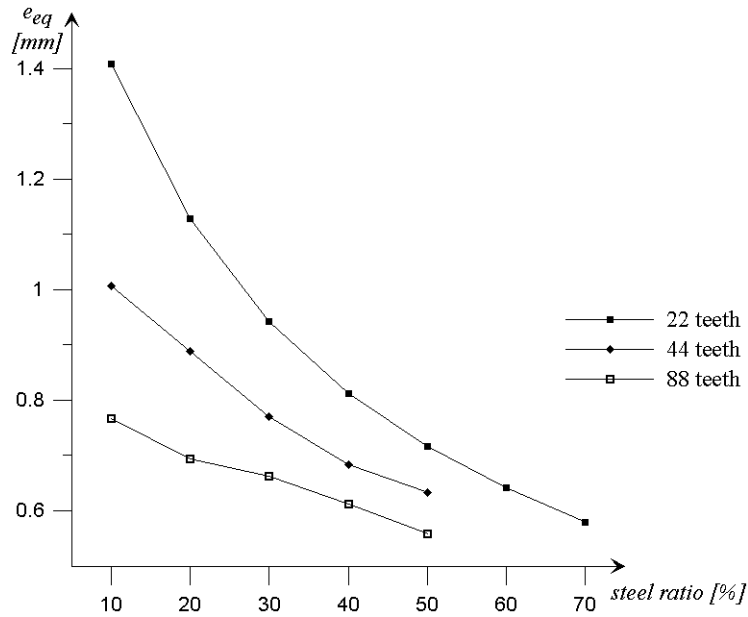


Figure 6.22 Equivalent airgap according to Carter for different teeth number

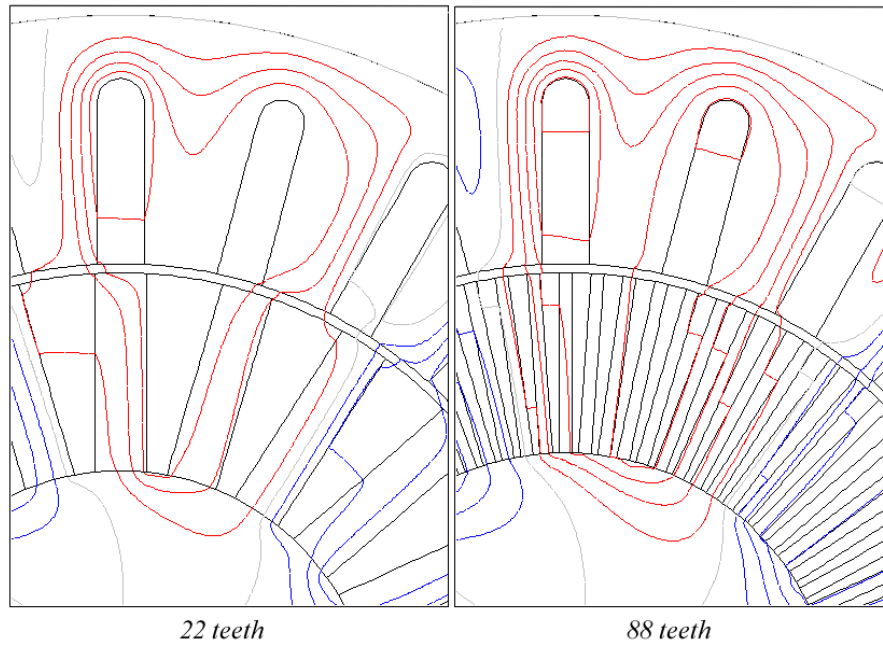


Figure 6.23 *Equiflux lines for rotors with 22 and 88 teeth*

On the figure 6.23 we can observe the cases of the rotor with 22 and 88 teeth, both with the steel ratio of 40 % and both calculated using the non-linear magnetization curve. The corresponding  $\tau$  parameter values are to be found on the figure 6.20 and the one corresponding to 88 teeth is twice as high as the one for 22 teeth. The equiflux lines are presented on the figure 6.23 in the same scale and we can see clearly that for 88 teeth there are more lines penetrating inside the rotor than for 22 teeth. It means of course that the magnetic flux circulating in the magnetic circuit is higher for 88 teeth. In my opinion it happens due to the decreasing reluctance of the airgap, which is the dominant factor when the number of teeth increases.

#### Number of teeth and constant teeth size

I will introduce now some results presented already during the International Conference on Electrical Machine (ICEM) in September 2004 [11].

That paper has been selected to be published in a special issue of the *COMPEL* journal [9]. Using the two-dimensional simplified model with teeth I kept the constant size of teeth and increased their number. The simulations were performed in the transient magnetic (TM) mode for the constant speed of rotation equal to 500 *rpm*.

Figure 6.24 shows the torque mean value  $T_{mean}$  calculated for the number of teeth varying between 10 and 120. The clearly visible tendency is that  $T_{mean}$  value increases with the increasing number of teeth. In our case (constant rotating speed) it means that the electromechanical conversion  $\tau$  improves as well. However, because the size of teeth is constant, increasing the teeth number increases the steel ratio. It could lead to the conclusion that the higher the steel ratio the better electromechanical conversion, what stays in the contrary to the results presented previously (optimal steel ratio between 30 and 40 %). We cannot forget however that the chosen constant rotating speed of 500 *rpm* do not need to correspond in each case with the optimal rotor current pulsation

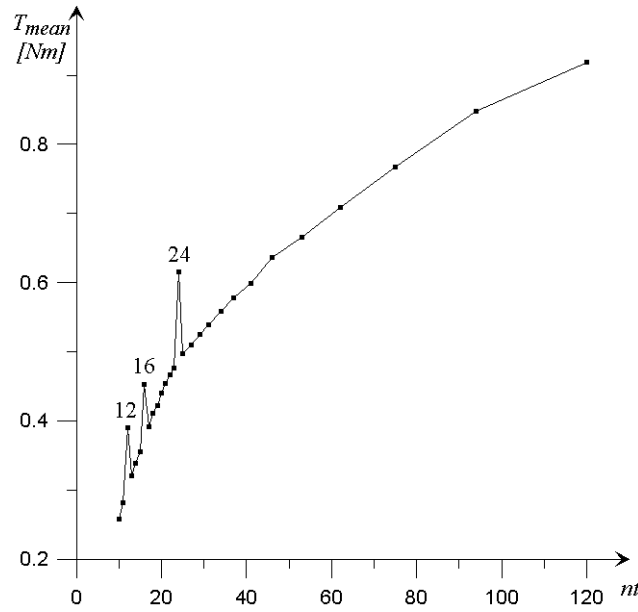


Figure 6.24 Torque mean value  $T_{mean}$  as a function of the number of teeth  $nt$

$\omega_{r \max}$  and that is why the general conclusion about the optimal steel ratio value must not be obtained from these results.

The local extrema of the  $T_{mean}$  characteristic for 12, 16 and 24 teeth appear because of the specific layout of teeth, however this phenomenon will be discussed later on in my thesis (6.2.3).

### Number and size of teeth in 3-dimensional simulations

Another results presented already at the ICEM conference [9, 11] were obtained using the 3-dimensional model (Figure 6.18). On the figure 6.25 we can see the characteristics torque-speed calculated using the magnetodynamic formulation for two different numbers of teeth and few teeth sizes. As I said before (6.2.1, page 86) the 3D computations demand much more time, that is why only a few cases were tested. Looking at the presented characteristics we can notice the influence of the teeth size  $ts$  and the number of teeth  $nt$ , what have been shown already before for the 2D model. Another interesting conclusion is that the teeth number

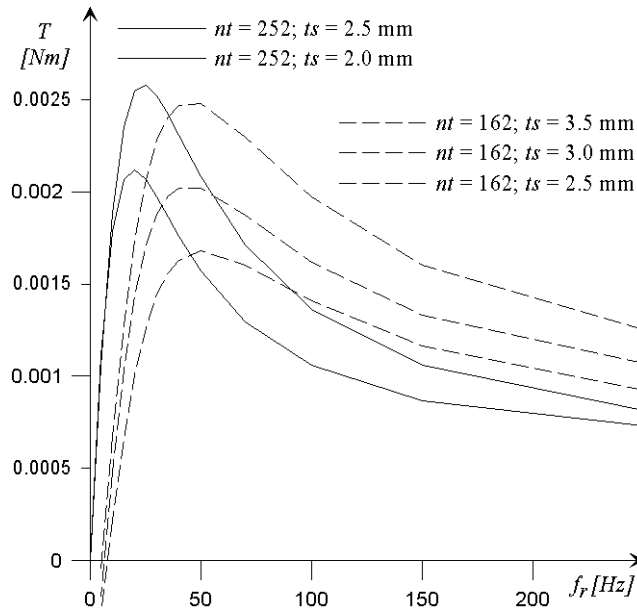


Figure 6.25 Characteristics torque-speed for different teeth numbers  $nt$  and sizes  $ts$

influences the shape of the torque-speed characteristic - the higher the  $nt$  the lower the  $\omega_r \text{ max}$ , what can lead directly to the better electromechanical conversion  $\tau$  (presented already for the 2D model). Finally we are coming to the most important remark - if we have a closer look at the curves we can see what is presented better on the figure 6.26. The characteristics torque-speed for the  $nt$  of 162 show that for the synchronous speed of rotation ( $f_r = 0$ ) the torque is not equal to 0. Because the simulations were performed for the rotor locked in one chosen position, it means that the torque is position sensitive, what results in the oscillations around the torque mean value during the rotor rotation. Such a phenomenon is called a slot effect and was already mentioned before. At the same time however for the  $nt$  of 252 the characteristics torque-speed show no sign of the slot effect and it is not caused by the coincidence in the choice of the rotor lock position. That is why we can come to the conclusion that the higher teeth number reduces the slot effect what means the reduction of the torque oscillations. This phenomenon will be however closer discussed in the next subsection of my dissertation.

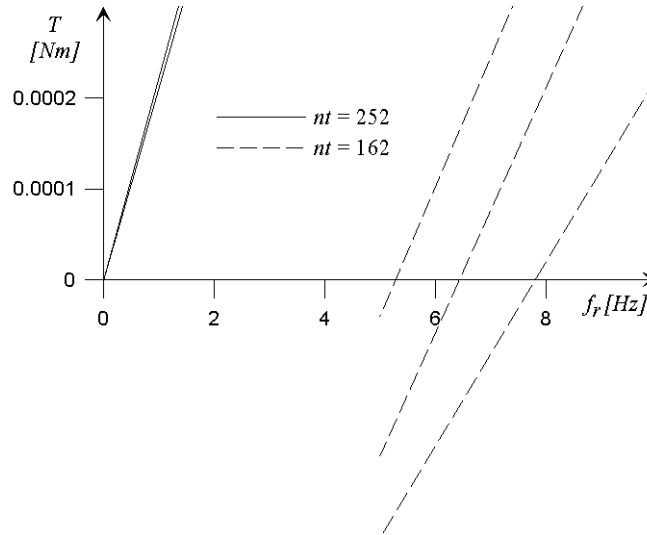


Figure 6.26 Slot effect for 162 teeth

### 6.2.3 Torque oscillations

In this part of my thesis I will discuss once again the results presented already during the ICEM conference [9, 11] which this time concerns the problem of the torque oscillations. Using the 2-dimensional model presented before (page 93) with constant teeth size and number of teeth varying between 10 and 120, I performed simulations in the transient magnetic mode, what gave as the result the amplitude of the actuator torque ripple  $\Delta T$  (Figure 6.27) and the torque mean value  $T_{mean}$  (presented already before, Figure 6.24).

On the figure 6.27 we can observe, that quite important torque oscillations appear for the number of teeth  $nt$  equal to 12, 16, 20 and 24. For the higher teeth number the ripple value decreases and becomes negligible. The specific layout of teeth for their number  $nt$  equal to 12, 16 and 24 results also in the local increase of the torque mean value  $T_{mean}$  (compare: Figure 6.24), what is however not worth of consideration because of accompanying it such a high oscillations.

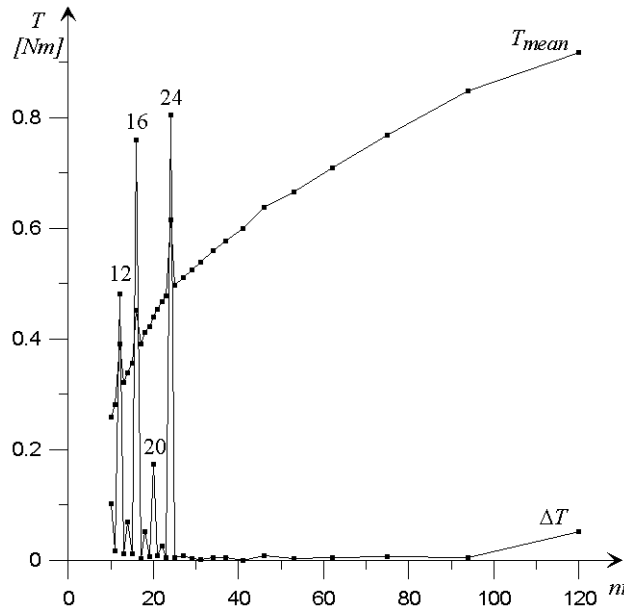


Figure 6.27 Torque mean value  $T_{mean}$  and ripple  $\Delta T$  as a function of the number of teeth  $nt$

The presented results are interesting regarding the empiric ratio found in the literature [7], which says that the optimal teeth number  $nt$  can be found using the following relation:

$$nt \approx 0,9 \cdot ns \quad (6.1)$$

where:

$ns$  — number of stator slots

In our case the  $ns$  value is equal to 24 what should give approximately 22 teeth in the rotor as the best solution. However we can see clearly that it is possible to increase the torque mean value without increasing the torque ripple, using just the higher teeth number.

Another aspect influencing the torque oscillations is the distribution of the teeth in the 3-dimensional rotor. Independently of the teeth number, the distribution must be as regular as possible in order to guarantee the torque isotropy. Ideally, the distance between the teeth must be constant in every direction. We must realize that such a configuration obviously does not exist and the distance between two closest teeth can vary according to the rotor direction of rotation (Figure 6.2).

Not ideal but the most uniform teeth distribution on a sphere is defined by the vertices of a regular icosahedron (Figure 6.28 -c-), i.e. formed of identical equilateral triangles, inscribed in this sphere.

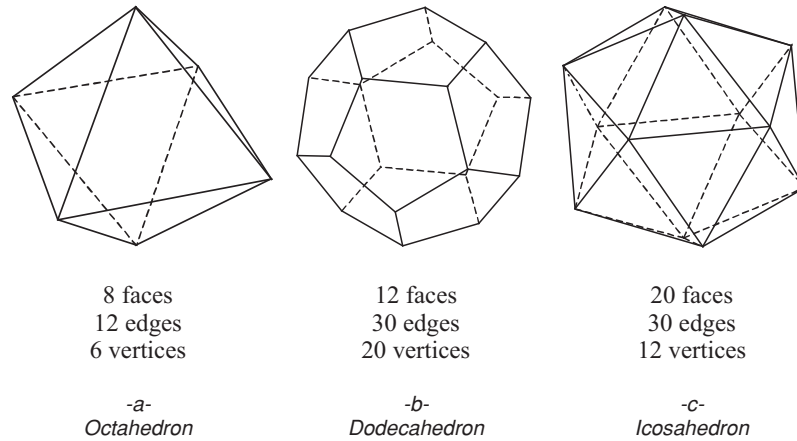


Figure 6.28 *Platonic solids and their principal characteristics*

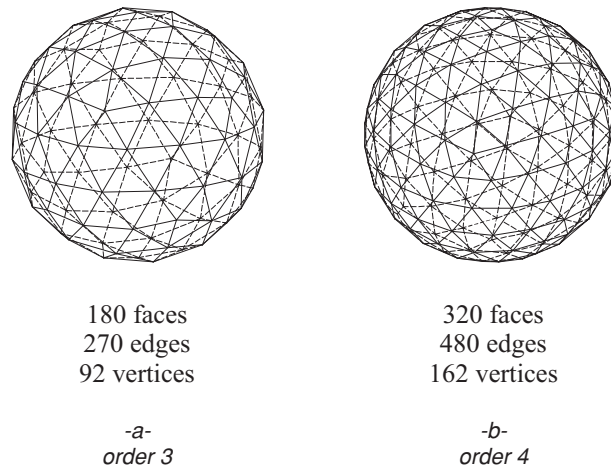


Figure 6.29 *Evolution of the faces, edges and vertex number of the icosahedron with the tessellation order*

In our study we considered several tessellations (Figure 6.29) of a regular icosahedron. In a very simplified way, the tessellation consists in multiplying the vertex and faces number by adding new vertices in the center of each face of the considered solid. The vertices created like this are connected to the face vertices from which they result. Table 6.3 presents the relation between the tessellation order and the number of vertices (equivalent to the teeth number).

In order to minimize the torque oscillations, we were trying to find the optimal teeth number and distribution. That is why I have redrawn the characteristic from the figure 6.27 as a function of the angular distance between the two following each other teeth (in a 2-dimensional

Table 6.3 *Icosahedron tessellation orders and the corresponding teeth number*

| Tessellation order | 1  | 2  | 3  | 4   | 5   | 6   | 7   |
|--------------------|----|----|----|-----|-----|-----|-----|
| Teeth number       | 12 | 42 | 92 | 162 | 252 | 362 | 492 |

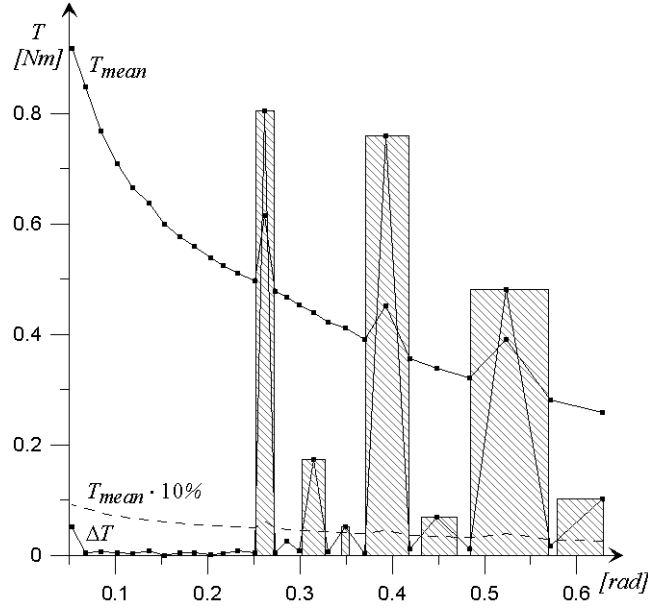


Figure 6.30 *Actuator torque mean value and ripple value as a function of the angular distance between the two following each other teeth*

model), as presented on the figure 6.30. On this figure we can see the angular intervals or zones, where the amplitude of the torque oscillations  $\Delta T$  exceeds 10 % of the torque mean value  $T_{mean}$ . These ‘forbidden’ zones are preferred to be avoided in order to guarantee the low or even possible to neglect level of the torque ripple value.

At the same time we also established the angular spectrum of various icosahedrons (Figure 6.31) - kind of histogram showing, for all the possible orientations of the rotor, the distribution of all the possible angular distances separating the two consecutive teeth located under an inductor (Figure 6.2). These histograms reveal a series of peaks indicating the existence of some angular distances more frequent than the others (Figure 6.31). By superimposing them on the angular ‘forbidden’ zones presented on the figure 6.30, it becomes possible to compare the various icosahedrons and to seek the tessellation orders for which peaks of distribution avoid the concerned zones. It is the case of the fifth order tessellation and the higher ones.

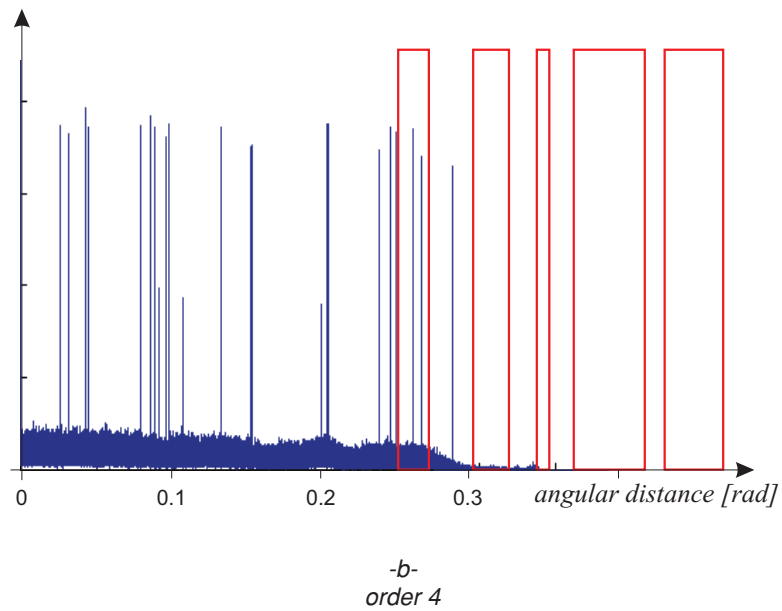
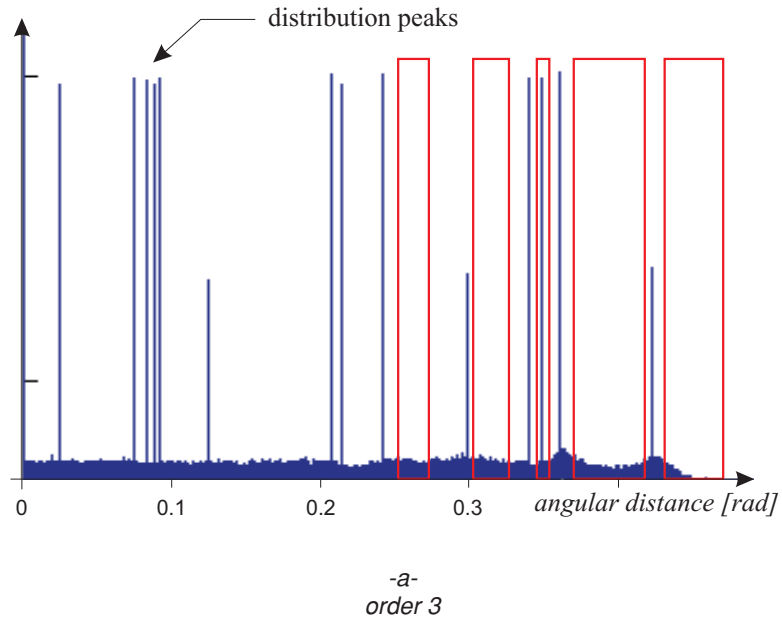


Figure 6.31 Superposition of icosahedron histograms and angular 'forbidden' zones for the tessellations of different order  
(continues on next page)

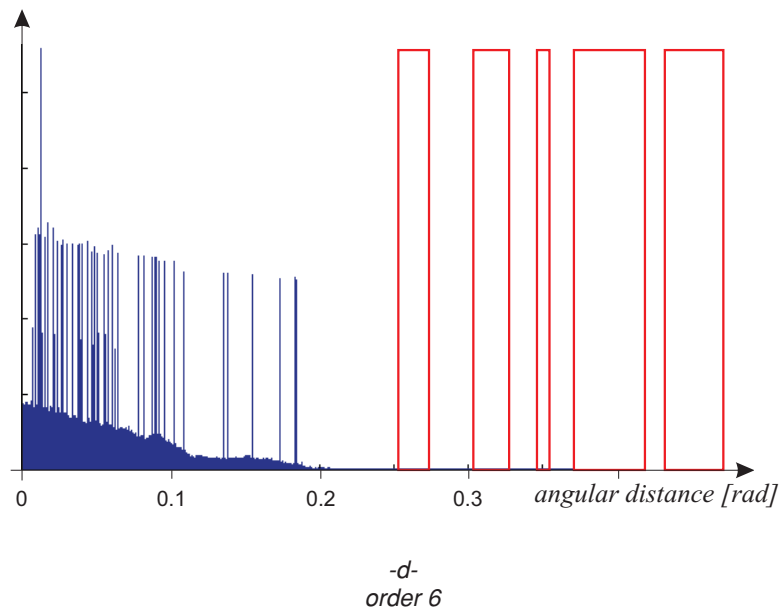
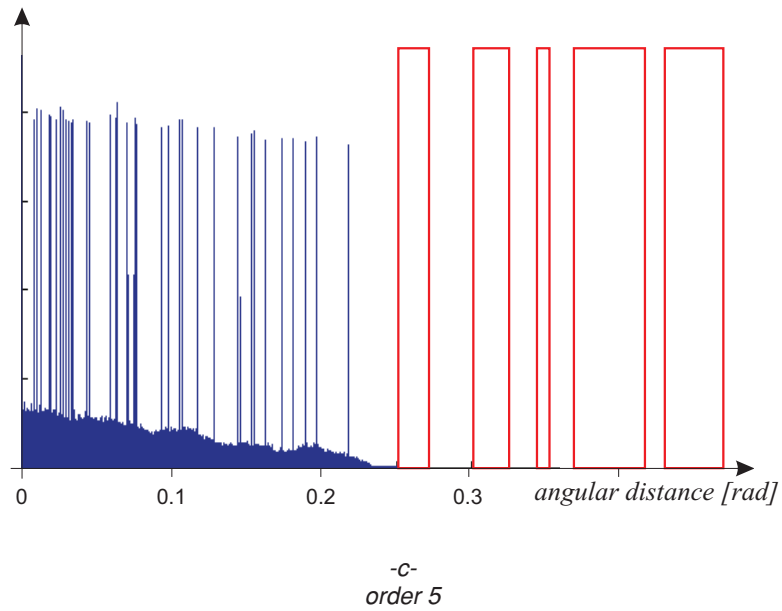


Figure 6.31 Superposition of icosahedron histograms and angular 'forbidden' zones for the tessellations of different order (continuation)

The use of the higher order tessellations (what corresponds to the higher teeth number) not only guarantees the torque isotropy but also results in the higher torque mean value (Figure 6.27). It correlates with the results presented in the previous subsection concerning the number of teeth parameter itself in the 2- and 3-dimensional simulations (6.2.2). However, taking into account the manufacturing complexity and the high frequency losses in the rotor conducting parts (which losses resulted in the restrictive rule of De Jong, Equation 6.1) depending directly on the teeth number, it is rather advisable to reduce this number. Having this in mind we have decided to keep the fifth order tessellation and not higher, especially as the lack of the slot effect was confirmed for it by the 3-D simulation (Figures 6.25 and 6.26), and to find another way to increase the torque mean value.

#### 6.2.4 Form of teeth

I will present now some results showing the importance of the rotor teeth geometrical form and its influence on the actuator performance.

As a starting point of my analysis I chose the rotor with 22 straight teeth, steel ratio  $sr$  of 50 % and the corresponding external layer optimal thickness  $el$  of 10 mm (Figure 6.32 -a-). Keeping the same steel ratio (defined for the surface of the rotor) and the same external layer thickness I changed the form of the teeth - deeper inside the rotor external layer they become thinner (Figure 6.32 -b-). We can see clearly that such a change improved the electromechanical conversion  $\tau$  and, what is more interesting, the obtained  $\tau$  value is even higher than the global optimal value for the straight teeth (Figure 6.32 -c-) corresponding to the steel ratio  $sr$  of 40 % and the external layer thickness  $el$  of 11 mm (compare: Figure 6.9). It shows us the importance of the rotor teeth form in the optimization of the actuator performance.

Such positive influence of the teeth geometrical modification, which I performed, can be explained by the fact, that the steel ratio remains unchanged what means the same value of the equivalent airgap  $e_{eq}$  and, what follows, the airgap reluctance  $Rel_{airgap}$ . At the same time, although the teeth become thinner what results in the higher teeth reluctance  $Rel_{tooth}$ , the rotor external layer resistance  $R_{Cu}$  decreases and, apparently, it creates better conditions for the electromechanical conversion  $\tau$ , which increases. There is a need to mention that such a improved

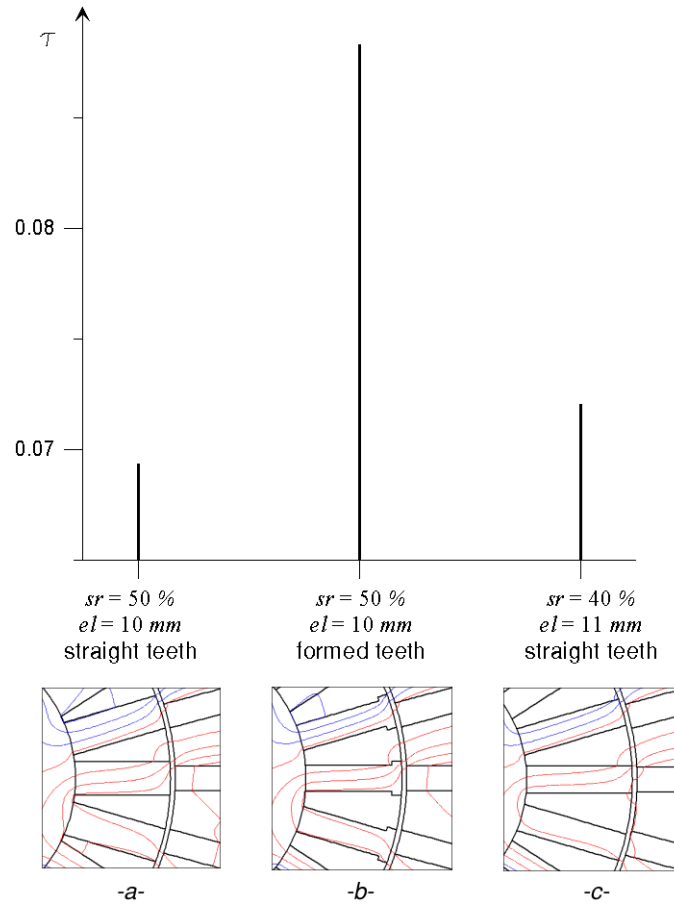


Figure 6.32 Teeth form influence on the electromechanical conversion

performance was obtained for the first tested case of the formed teeth and this form was not optimized at all. However I believe to have proven the great need of such an optimization.

### 6.2.5 Magnetic material conductivity and permeability

In this subsection I will present a closer analysis of the last parameters I have verified, namely the magnetic material parameters such as the conductivity  $\sigma$  (or resistivity  $\rho$ ) and the relative permeability  $\mu_r$ .

### Relative permeability

In case of the magnetic permeability it is quite obvious that the higher its value the better, because the circulation of higher magnetic flux will be allowed and as a result the higher torque will be generated. Looking at the figure 6.33 we can confirm our supposition, however we can see as well that the increase of the electromechanical conversion  $\tau$  is not linear and aims at some specific value. Such a characteristic can be explained by the use of the equations A.9 and B.1. Especially the second one of these equations is interesting because it shows that the increase of the magnetic material permeability reduces the total reluctance  $Rel_{total}$  only partially. It is caused by the airgap reluctance  $Rel_{airgap}$  which remains unchanged. All of this means that the magnetizing inductance  $L_\mu$  will increase only in some range not overcoming some specific value and, what follows according to the equation A.9, it applies also to the electromechanical conversion  $\tau$ .

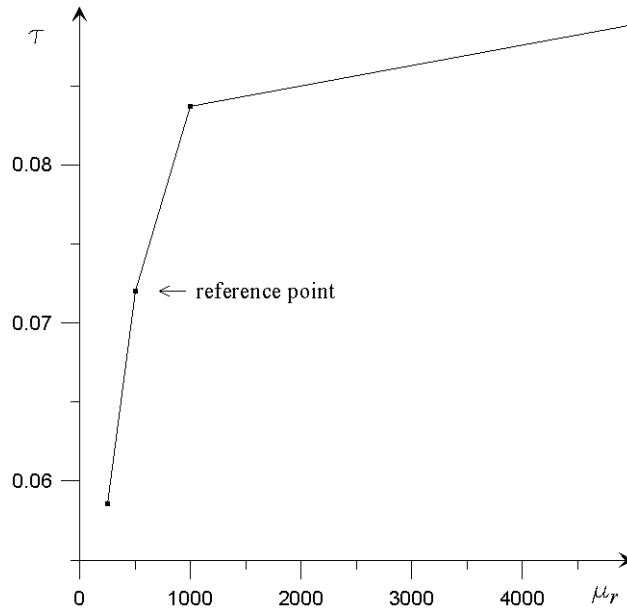


Figure 6.33 Conversion  $\tau$  as a function of the magnetic material relative permeability  $\mu_r$ . Reference point corresponds to the value used in the majority of simulations

Thus such results not only confirm the interest of increasing the magnetic material permeability but remind us as well the need of reducing the airgap size, if only it is applicable.

### Conductivity

As in the case of the rotor conducting parts (rotor external layer) increasing their conductivity is evident, it is not so clear for the rest of the rotor made of the magnetic material, despite the fact that inductive currents from everywhere in the rotor generate a torque. Figure 6.34 shows us even something opposite - we need to increase the magnetic material resistivity  $\rho$  (and not the conductivity) in order to increase the electromechanical conversion  $\tau$ . In this case the shown relation is linear but of course it should not mean that for the infinite resistivity we will obtain the infinite conversion. It will be for sure limited by the other factors.

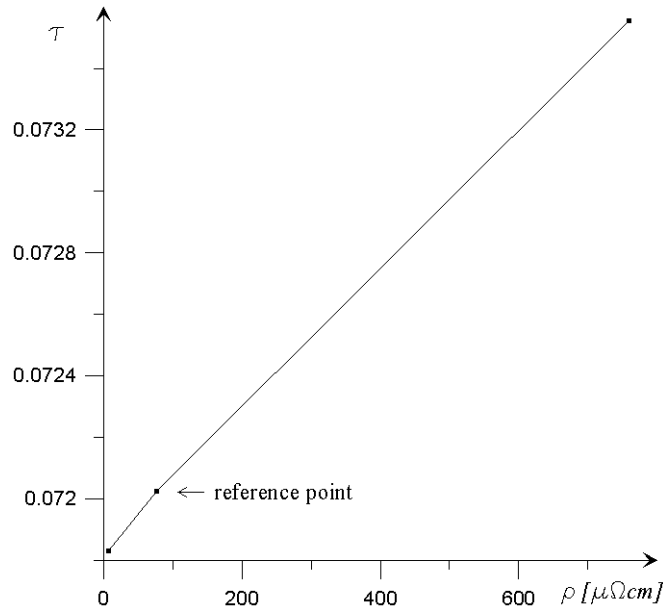


Figure 6.34 Conversion  $\tau$  as a function of the magnetic material resistivity  $\rho$ . Reference point corresponds to the value used in the majority of simulations

The existence of the conductivity in the magnetic material results in the decrease of the electromechanical conversion because it allows the development of the inductive currents in the magnetic parts of the rotor. Such currents disrupt the circulation of the magnetic flux (Figure 6.35). That negative phenomenon is called skin effect [5] and was already mentioned before (5.1.2, page 43).

In any case such results show the interest of reducing the magnetic material conductivity what, in relation to the previously presented results concerning the permeability, can be summarized and simplified in one sentence - the materials should be ‘specialized’ what means that the conducting material should have the high conductivity while the magnetic material should have the high permeability and no conductivity at all. It is related to the different roles which the both materials play in the actuator structure.

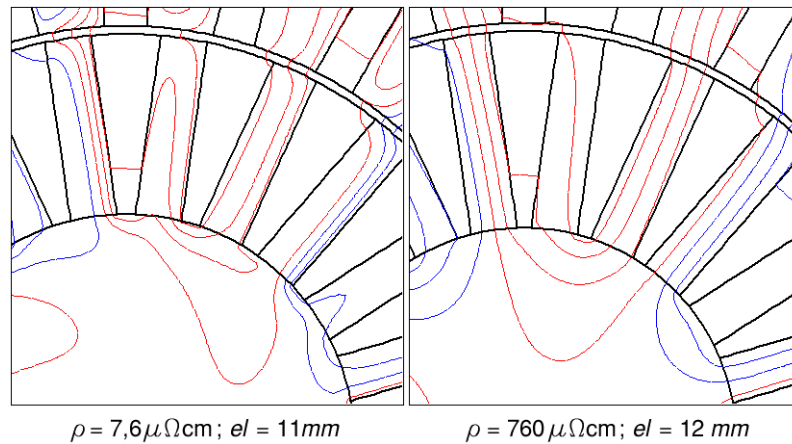


Figure 6.35 Equipotential lines for different values of the magnetic material resistivity  $\rho$

### 6.3 Non-sinusoidal coupling

The classic model of an asynchronous motor (Appendix A) takes into account only the first, main harmonic of the magnetic flux passing through the airgap. In certain cases such a model can be insufficient to describe the functioning of the motor. If we are taking into account the higher harmonics up to the seventh one and because of the symmetries, we can write the matrix of the mutual inductance between the stator and rotor windings  $\mathbf{M}_{sr}$ , as follows [8]:

$$\begin{aligned}
 \mathbf{M}_{sr} &= \mathbf{M}_{sr1} + \mathbf{M}_{sr3} + \mathbf{M}_{sr5} + \mathbf{M}_{sr7} \\
 &= M_1 \begin{bmatrix} \cos \theta_{em} & \cos(\theta_{em} + \frac{2\pi}{3}) & \cos(\theta_{em} - \frac{2\pi}{3}) \\ \cos(\theta_{em} - \frac{2\pi}{3}) & \cos \theta_{em} & \cos(\theta_{em} + \frac{2\pi}{3}) \\ \cos(\theta_{em} + \frac{2\pi}{3}) & \cos(\theta_{em} - \frac{2\pi}{3}) & \cos \theta_{em} \end{bmatrix} \\
 &\quad + M_3 \begin{bmatrix} \cos(3\theta_{em}) & \cos(3\theta_{em}) & \cos(3\theta_{em}) \\ \cos(3\theta_{em}) & \cos(3\theta_{em}) & \cos(3\theta_{em}) \\ \cos(3\theta_{em}) & \cos(3\theta_{em}) & \cos(3\theta_{em}) \end{bmatrix} \\
 &\quad + M_5 \begin{bmatrix} \cos(5\theta_{em}) & \cos(5\theta_{em} - \frac{2\pi}{3}) & \cos(5\theta_{em} + \frac{2\pi}{3}) \\ \cos(5\theta_{em} + \frac{2\pi}{3}) & \cos(5\theta_{em}) & \cos(5\theta_{em} - \frac{2\pi}{3}) \\ \cos(5\theta_{em} - \frac{2\pi}{3}) & \cos(5\theta_{em} + \frac{2\pi}{3}) & \cos(5\theta_{em}) \end{bmatrix} \\
 &\quad + M_7 \begin{bmatrix} \cos(7\theta_{em}) & \cos(7\theta_{em} + \frac{2\pi}{3}) & \cos(7\theta_{em} - \frac{2\pi}{3}) \\ \cos(7\theta_{em} - \frac{2\pi}{3}) & \cos(7\theta_{em}) & \cos(7\theta_{em} + \frac{2\pi}{3}) \\ \cos(7\theta_{em} + \frac{2\pi}{3}) & \cos(7\theta_{em} - \frac{2\pi}{3}) & \cos(7\theta_{em}) \end{bmatrix}
 \end{aligned} \tag{6.2}$$

The  $\mathbf{M}_{sr}$  matrix varies with the relative position  $\theta_{em}$  of the rotor equivalent windings with reference to the stator windings.

This problem was already discussed in the literature [8, 14] and here I will limit myself to presenting it only in context of the numerical simulations. I said already before that the only formulation taking into account the existence of the higher harmonics resulting in the non-sinusoidal coupling between the stator and the rotor is the transient magnetic TM formulation (2.4.3). Using this formulation I performed three simulations, one for the two-layers rotor and the others for the two-layers-with-teeth rotor, for 22 and 30 teeth. In each case I simulated the no-load start-up of the actuator for the stator supply current frequency  $f$  of 50 Hz. The friction was defined as equal to zero and only pure inertia was imposed. For such a conditions according to the classic theory the actuator

rotating speed  $n_r$  should achieve the value of the synchronous speed  $n_s$  defined as follows:

$$n_s = \frac{60 f}{p} \quad (6.3)$$

what in our case results in the  $n_s$  of 750 *rpm*.

Looking at the rotating speed characteristic of the two-layers rotor (Figure 6.36) we can see clearly that the synchronous speed was not achieved and  $n_r$  stabilized itself at the level of 709 *rpm*. It shows of course the existence of higher harmonics and non-sinusoidal coupling in the machine. However if we compare the speed characteristics of the two-layers-with-teeth rotors we can notice that after some initial start-up transients the rotating speed oscillated around the value of 748 *rpm* and 746 *rpm*, for 22 and 30 teeth respectively, which are close to the synchronous speed. The both characteristics for the rotors with teeth almost overlap each other. We can notice as well that the two-layers-

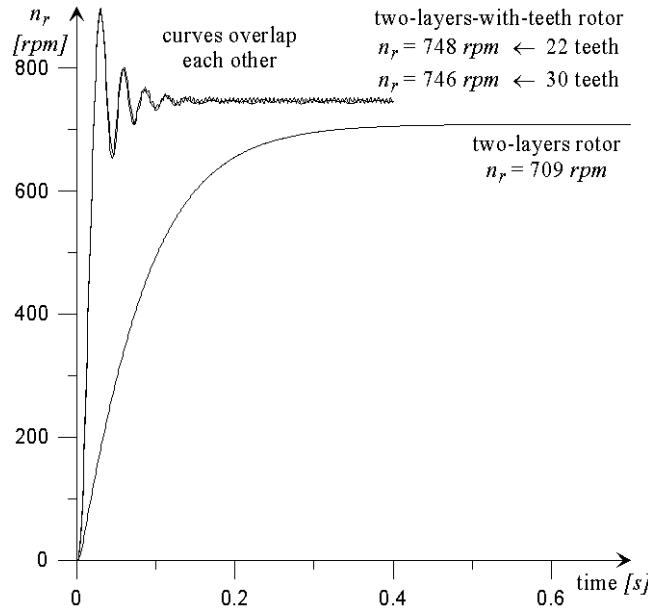


Figure 6.36 Rotating speed of two-layers and two-layers-with-teeth rotor during the no-load start-up

with-teeth rotor corresponds here to the classic rotor with slots, which is the basic rotor structure in the asynchronous machines. Such a rotor structure is commonly considered as a good one so it is not a surprise that the higher harmonics are not present for it. However for the two-layers rotor which is a bad rotor structure from the electromagnetic point of view the existence of higher harmonics is not a surprise as well.

Knowing this we can make an assumption that using the magneto-dynamic MD formulation, which does not take into account the higher harmonics, for the simulations with the two-layers-with-teeth rotor is acceptable and not charged with errors. Such an assumption validates the use of the MD formulation in my previous simulations.

## CHAPTER 7

# CONCLUSIONS

The general aim of this thesis was to show the superiority of the new two-layers-with-teeth spherical rotor developed for the two-degree of freedom actuator and to perform some first optimization of its structure. This aim was achieved by numerous simulations comparing first the one-layer, two-layers and two-layers-with-teeth rotor structures, and then verifying the influence of many different rotor parameters on the actuator performance.

### Finite element method discussion

The majority of the simulations were performed using a simplified 2D model with the stator ‘C - model’ structure (see: 4.1.4), magnetodynamic formulation and a linear magnetization curve for characterizing the magnetic materials. Such approximation have been done in order to reduce the computation time as well as the need of memory for the finite element method based software. Such a model does obviously not take into account the real 3D form of the structure, the movement of the rotor, the possibility of non-sinusoidal coupling between the rotor and the stator windings and finally the magnetic saturation phenomena. However, each of these aspects were verified separately and confirmed the correctness of the simplified model by giving same or similar results.

There is a need to mention that the rotor movement could anyway not be simulated at all using the 3D structure because of the software limitations. The few 3D software able to take into account the effects of the movement are indeed limited to plane or cylindrical interface between

the moving parts and are hence anyway not able to simulate the motion of a spherical rotor. In this conditions, it has been decided to do all the computation with magnetodynamic formulation. This formulation does not take into account also the non-sinusoidal coupling between the rotor and the stator (it requires the use of a transient formulation which is much more computational time consuming). However it has been shown that the better is performance of the actuator (performance evaluated only on the base of the first harmonic model), the more these non-sinusoidal effects can be neglected.

#### Classic results - what could have been expected

As I have said earlier, first the three rotor structures were compared using the introduced electromechanical conversion parameter  $\tau$  (see: chapter 3). Taking as the reference the  $\tau$  value for the one-layer rotor the maximal found conversion parameter values for the two-layers rotor is over 18 times higher. The maximal found conversion parameter value for the two-layers-with-teeth rotor is still 20 times higher (and hence 360 times higher than for the one-layer rotor). It confirms clearly the correct choice of the rotor structure evolution path finalized by the rotor with teeth.

The optimal two-layers rotor is obtained with an external-layer thickness about the same size than the airgap (or more precisely, the same size than the airgap corrected by the Carter coefficient in order to take into account the slot effects in the stator). The use of aerostatic suspension which allows

- to obtain very thin airgaps (up to even 0.1 *mm*) and hence very high electromechanical conversion ratio since it is more or less inversely proportional to the airgap thickness
- to put in the stator windings higher current density thanks to the side effect of cooling

leads to the use of about 0.1 *mm* copper sheet to make the external layer.

For the two-layers-with-teeth rotor, the obtained results show that it exists an optimal tooth area expressed by the steel ratio parameter value from the range between 30 and 40 %. The corresponding optimal external layer thickness (and hence tooth height) is about 11 *mm*.

The process for realizing such a layer will probably not be the same than for a two-layers rotor. Such results were obtained using the magnetodynamic formulation and the linear magnetization curve for a 2D equivalent rotor with 22 teeth, however they were confirmed by transient magnetic simulations, use of the non-linear magnetization curve, use of the magnetic material with much higher (10 times) permeability  $\mu_r$ , 3D simulations and finally for rotors with 44 and 88 teeth. It has equally been shown that the form of teeth has a great importance on the electromechanical conversion but this one has not been optimized. The use of reduced-opening slots is however strongly recommended with a shape that remains to be optimized in future works but by keeping in mind the fabrication limits.

In both cases, the inner layer has just to be permeable and thick enough to ensure the magnetic flux circulation. Above this minimal value, increasing its thickness will not improve the electromechanical conversion. Its conductivity should be as low as possible, leaving the conducting tasks to the conducting material.

All these results are not very surprising and confirm just the well-known results obtained for classic, cylindrical, one-degree-of-freedom induction motors: benefit of using a two-material rotor, if possible disposed in conductive slots and ferromagnetic teeth, existence of an optimal tooth shape (optimal height, diameter, advantage of use of wide-mouthed shape equivalent to reduce the slot opening...)

### Original results - what was not expected

Comparison of rotor structures with 22, 44 and 88 teeth, for the constant steel ratio, shows the correlation between the higher teeth number and better electromechanical conversion due to the thinner equivalent airgap  $e_{eq}$  expressed using the Carter coefficient.

The same study shows that a high enough teeth number guarantees the reduction of the torque oscillations what was confirmed as well by the 3D simulations.

All of that gives the double reason to increase the teeth number - improvement of the electromechanical conversion and reduction of the torque ripple. In case of classic, cylindrical, one-degree-of-freedom induction motors, the number of teeth of the rotor is usually limited to about 0.9 times the number of teeth of the stator in order to limit high

frequency losses. Making that, a local minimum for torque ripple is reached thanks to a spread between the teeth of the rotor very close but not exactly equal to the spread between the teeth of the stator. Such a local minimum is not however reachable for a two-degree-of-freedom rotor since the spread between successive teeth depends in that case on the direction of rotation. There is then a need to increase as many as possible the teeth number, even if it entails the use of a ferromagnetic material with a very small hysteresis cycle for making the teeth.

Knowing that the most uniform teeth distribution on the rotor surface corresponds to the vertices of the icosahedron and its tessellations, it was showed that the lower teeth number ensuring an acceptable level of torque oscillations is 252, what corresponds to the fifth order tessellation. Using a tessellation of an higher order would only increase the manufacturing complexity.

Such a result does not correspond to the classic empirical rules presented in the literature and would need then to be experimentally validated.

### Directions for future

The suggested directions of further investigation in the future are:

- to experimentally validate the two-layer-with-teeth rotor structure and especially the fact that a very large number of teeth (comparing to classic cylindrical motor) is necessary in order to avoid a too high level of torque ripple without leading to a weakening of the electromechanical conversion ratio due to a too high level of high frequency losses;
- to optimize the tooth shape, conceivably by starting from the shape already optimized in case of classic one-degree-of-freedom motor;
- to make the same work done on the rotor in order to optimize the stator; first experimental results show indeed very poor performance due probably to very high iron losses in the stator; a deeper research about the choice of the material to be used to build the stator structure could be from this point of view, fruitful;

- finally, in order to achieve to complete the optimization process, to develop 3D-FEM tools able to take into account the movement of the rotor even in the other degrees of freedom.

It is still the cost to be able to realize an as good as possible spherical induction motor, able to drive a two-degree-of-freedom movement without any complex, fragile and often source of vibration mechanical transmissions.



# EPILOGUE

Two-degree of freedom spherical actuator is a very complex machine and so was the concerning it project. Although it took me four years to accomplish my study and my dissertation is already the second one treating this subject there are still a lot of work to do as for example the whole problem of the stator which has not been optimized at all. Even this does not mean that my work has not been limited - on the contrary, regarding the mentioned complexity of the problem I have chosen to study only the most interesting and obvious parameters and cases which have been presented in this dissertation. Describing it all the data and results are stored in form of the .zip files on about twenty discs CD, but to describe the whole amount of work which I performed I should mention as well that it exists over one hundred other discs CD containing plenty of 'introductory' results which allowed me the final and structured analysis presented in my thesis.

Despite the amount of work and the difficulties I encountered I find the project very interesting and challenging. Now I am glad it is over and satisfied with all what I have done and learned. I wish as well a successful continuation to my successors.



## APPENDICES



## APPENDIX A

# CHARACTERISTIC OF CLASSIC ASYNCHRONOUS MOTORS

### A.1 Equivalent scheme

The electrical characteristics of an asynchronous motor can be deduced from its equivalent scheme. Such a scheme (Figure A.1), representing the one phase of the motor, can be created on the basis of the electrical equations describing the evolution of the motor currents and voltages [13].

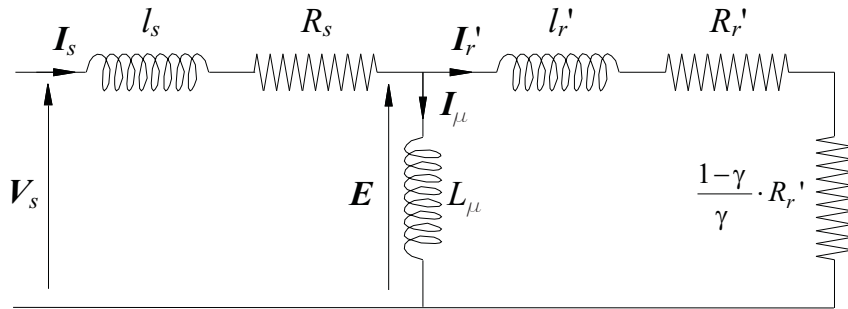


Figure A.1 *Equivalent scheme of an classic asynchronous motor*

Figure A.1 presents the equivalent scheme of an classic asynchronous motor, however in our case, by the current supply of the motor, the

stator parameters could be neglected. The parameters presented in the scheme are as follows:

$R_s$  – stator resistance

$l_s$  – leakage inductance of stator windings

$L_\mu$  – magnetizing inductance

$R_r'$  – rotor resistance brought to the stator side

$l_r'$  – leakage inductance of rotor windings brought to the stator side

$\gamma$  – slip

$V_s$  – stator voltage

$I_s$  – stator supply current

$I_r'$  – rotor current brought to the stator side

$I_\mu$  – magnetizing current

where:

$$L_\mu = k \frac{3}{2} M_{sr} \quad (\text{A.1})$$

$$R_r' = k^2 R_r \quad (\text{A.2})$$

$$l_r' = k^2 l_r \quad (\text{A.3})$$

$$\gamma = \frac{\omega_s/p - \omega_m}{\omega_s/p} = \frac{\omega_s - p \omega_m}{\omega_s} = \frac{\omega_r}{\omega_s} \quad (\text{A.4})$$

$$I_r' = -\frac{1}{k} I_r \quad (\text{A.5})$$

and:

$M_{sr}$  – mutual inductance between stator and rotor windings

$R_r$  – rotor resistance

$l_r$  – leakage inductance of rotor windings

$I_r$  – rotor current

$k$  – current ratio

$\omega_s$  – stator supply current pulsation

$\omega_m$  – rotor rotating speed in radians/s

$\omega_r$  – rotor current pulsation

$p$  – pole pairs number

## A.2 Electromagnetic torque

Knowing the different motor parameters we cannot only define the evolution of the currents and voltages, but as well the function describing the electromagnetic torque  $T_{em}$ , which in our case, by the current supply of the motor, presents itself as follows [5]:

$$T_{em} = \frac{6p}{\omega_s} \frac{R_r'}{\gamma} \frac{(\omega_s L_\mu)^2 I_s^2}{(R_r'/\gamma)^2 + (\omega_s (L_\mu + l_r'))^2} \quad (\text{A.6})$$

where:

$I_s$  – effective stator supply current

The maximal value of this torque  $T_{max}$  is equal to:

$$T_{max} = 3p \frac{L_\mu^2 I_s^2}{L_\mu + l_r'} \quad (\text{A.7})$$

and corresponds to the slip  $\gamma_{max}$ :

$$\gamma_{max} = \frac{R_r'}{\omega_s (L_\mu + l_r')} \quad (\text{A.8})$$

Looking at the equation A.7 we can see clearly that in the case of the current supply of the motor, the amplitude of the maximal torque does not depend on the supply current frequency. As for the magnetizing inductance  $L_\mu$  however, it influences the maximal torque amplitude  $T_{max}$  and the corresponding slip  $\gamma_{max}$ . What is more, we have the interest in maximizing the  $L_\mu$  value to increase the efficiency of the electromechanical conversion  $\tau$  in the spherical actuator (3), what is clearly visible in the following equation:

$$\tau = \frac{T_{max}}{\omega_{r\ max}} = \frac{T_{max}}{\gamma_{max} \omega_s} = 3p \frac{L_\mu^2 I_s^2}{R_r'} \quad (\text{A.9})$$

There is a need to mention that the  $\tau$  parameter does not depend on the rotor leakage inductance  $l_r'$ .



## APPENDIX B

### SIZE OF TEETH – ANALYTICAL RESULTS EXPLANATION

In the subsection 6.2.1 I have presented the results of my simulations concerning the size of teeth parameter (steel ratio parameter), where two of the main conclusions were that the rotor external layer optimal value  $el_{opt}$  increases with the decreasing size of teeth (steel ratio) and that the  $el_{opt}$  values does not depend visibly (in our case) on the teeth number. Now I will present the analytical study confirming the above-mentioned results.

In order to optimize the electromechanical conversion parameter  $\tau$  as a function of the rotor external layer thickness  $el$  and, what follows, find the optimal  $el_{opt}$  values, we need to know the expression describing the  $\tau$  parameter. It was already presented in the previous appendix as the equation A.9. The next step to do is to find the expressions describing the  $L_\mu$  and  $R_r'$  parameters.

The magnetizing inductance  $L_\mu$  and the rotor resistance  $R_r'$  can be expressed using the geometrical parameters (Figure B.1) of the actuator which, in particular, are the airgap thickness  $e$ , the rotor external layer thickness  $el$ , the tooth size  $ts$ , the rotor radius  $r$  and the actuator length  $l$ . We will use also the number of teeth parameter  $nt$  and the steel ratio parameter  $sr$ . In fact, the magnetizing inductance can be, in a first approximation, expressed as a function of the reluctance of the magnetic circuit linking stator and rotor (Figure B.2) as:

$$L_\mu = \frac{k_1}{Rel_{total}} = \frac{k_1}{Rel_{stator} + 2 Rel_{airgap} + 2 Rel_{tooth} + Rel_{rotor}} \quad (B.1)$$

where:

$k_1$  – constant depending on the stator and rotor turn number  
and on the winding distribution in the stator and rotor slots

In this expression, we can neglect the stator and rotor reluctances,  $Rel_{stator}$  and  $Rel_{rotor}$  respectively, compared to the dominant tooth and airgap reluctances,  $Rel_{tooth}$  and  $Rel_{airgap}$ .

Considering the following expression for the reluctance  $Rel$  of a magnetic circuit of relative permeability  $\mu_r$ , length  $l_{mc}$  and section  $S_{mc}$ :

$$Rel = \frac{l_{mc}}{\mu_0 \mu_r S_{mc}} \quad (B.2)$$

the magnetizing inductance can finally be written:

$$L_\mu = \frac{k_1}{\frac{2e}{\mu_0 l ts} + \frac{2el}{\mu_0 \mu_r l ts}} \quad (B.3)$$

In the same way, the rotor resistance  $R_r'$  can be expressed as follows:

$$R_r' = k_2 \frac{1}{\sigma} \frac{2l}{S_{ec}} \quad (B.4)$$

where:

$k_2$  – constant depending on the stator and rotor turn number  
and on the winding distribution in the stator and rotor slots

$\sigma$  – electric conductivity of the rotor external layer

$S_{ec}$  – section of electric circuit

and:

$$S_{ec} \approx \frac{\left( \pi r^2 - \pi (r - el)^2 \right) - nt ts el}{nt} \quad (B.5)$$

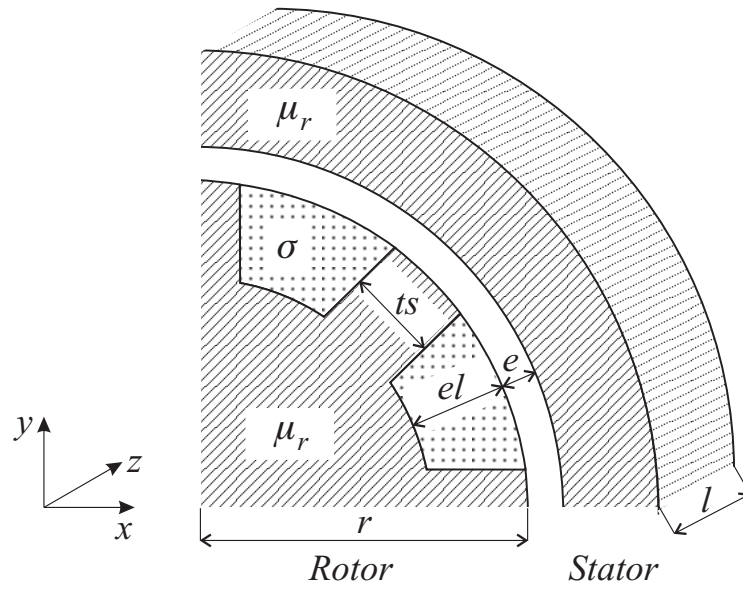


Figure B.1 Actuator simplified geometry

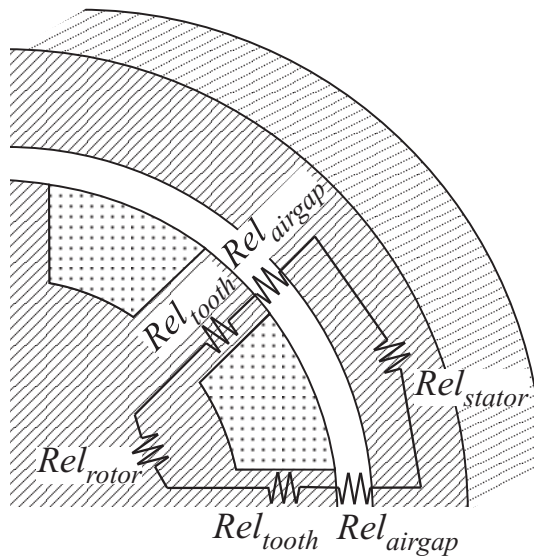


Figure B.2 Reluctance of the simplified magnetic circuit

Finally I will replace the tooth size parameter  $ts$  with the function of the steel ratio parameter  $sr$ :

$$ts = 2r \sin \left[ \frac{\pi sr}{nt} \right] \quad (\text{B.6})$$

On the basis of the equations A.9, B.3 and B.4 it then becomes possible to express the electromechanical conversion  $\tau$  parameter in terms of the geometrical parameters:

$$\tau \approx \frac{k_1^2}{k_2} \text{const} \frac{el \sin^2 \left[ \frac{\pi sr}{nt} \right] \left( \pi (2r - el) - 2r nt \sin \left[ \frac{\pi sr}{nt} \right] \right)}{nt (el + e\mu_r)^2} \quad (\text{B.7})$$

where:

$$\text{const} = \frac{3}{2} I_s^2 \sigma p \mu_0^2 \mu_r^2 r^2 l \quad (\text{B.8})$$

is the value constant in my simulations. It is also possible to find the value of the rotor external layer thickness  $el$  which optimizes  $\tau$  parameter. After some calculations we obtain the expression describing approximately the optimal thickness  $el_{opt}$ , which is:

$$el_{opt} \approx \frac{e\mu_r r \left( \pi - nt \sin \left[ \frac{\pi sr}{nt} \right] \right)}{e\mu_r \pi + r \left( \pi - nt \sin \left[ \frac{\pi sr}{nt} \right] \right)} \quad (\text{B.9})$$

We can see clearly that the rotor external layer optimal thickness  $el_{opt}$  depends on the airgap thickness  $e$  and the steel ratio parameter  $sr$  ( $\mu_r$  and  $r$  are constant). As for the number of teeth  $nt$  it does not play a role for its value higher than 10.

The characteristics presented on the figure B.3 confirm my simulation results and show that the rotor external layer optimal thickness  $el_{opt}$  increases with the decreasing steel ratio. Concerning the exact values of the  $el_{opt}$  parameter, the results of the simplified analytical study cannot be directly compared with my simulation results.

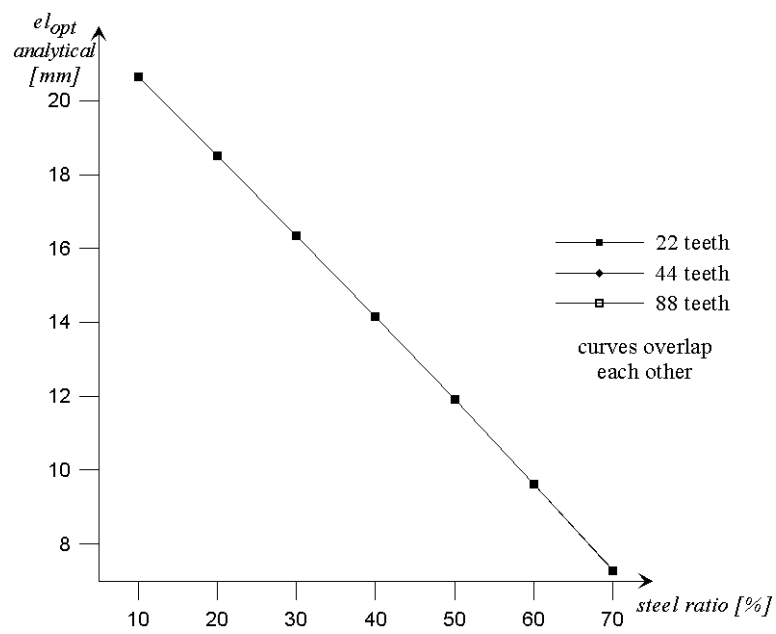


Figure B.3 Analytical results of the rotor external layer optimal value  $el_{opt}$  for different teeth numbers



## APPENDIX C

# SOME DEFINITIONS

### C.1 Maxwell equations

Maxwell equations are the set of four equations, that describe the behavior of both the electric and magnetic fields, as well as their interactions with matter. They express, respectively, how currents produce magnetic fields (Ampere law), how changing magnetic fields produce electric fields (Faraday law of induction), how electric charges produce electric fields (Gauss' law), and the experimental absence of magnetic charges (Gauss' law for magnetism).

$$\text{rot } \mathbf{H} = \mathbf{J}_{ext} + \sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \quad (\text{C.1})$$

$$\text{rot } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (\text{C.2})$$

$$\text{div } (\varepsilon \mathbf{E}) = \varrho \quad (\text{C.3})$$

$$\text{div } \mathbf{B} = 0 \quad (\text{C.4})$$

## C.2 Biot-Savart law

It describes the magnetic field set up by a steadily flowing line current. The field produced by a current element  $d\mathbf{l}_c$  is:

$$d\mathbf{B} = \frac{\mu_0 I_c}{4\pi} \frac{d\mathbf{l}_c \times \mathbf{r}_c}{r_c^3} \quad (\text{C.5})$$

## C.3 Nabla operator

In vector calculus, the nabla operator is a first order differential operator:

$$\nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \quad (\text{C.6})$$

## C.4 Gradient

In vector calculus, the gradient of a scalar field is a vector field which points in the direction of the greatest rate of change of the scalar field, and whose magnitude is the greatest rate of change. For example gradient of scalar function  $\varphi$  is:

$$\text{grad } \varphi = \nabla \varphi = \left( \frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial y}, \frac{\partial \varphi}{\partial z} \right) \quad (\text{C.7})$$

## C.5 Divergence

In vector calculus, the divergence is an operator that measures a vector field tendency to originate from or converge upon a given point. For example divergence of vector field  $\mathbf{B}$  is:

$$\text{div } \mathbf{B} = \nabla \cdot \mathbf{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \quad (\text{C.8})$$

## C.6 Rotation

In vector calculus, rotation is a vector operator that shows a vector field rate of rotation about a point. For example rotation of vector field  $\mathbf{H}$  is:

$$\text{rot } \mathbf{H} = \nabla \times \mathbf{H} = \begin{bmatrix} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \end{bmatrix} \quad (\text{C.9})$$

## C.7 Divergence of gradient - Laplace operator (Laplacian)

The Laplace operator is a second order differential operator, defined as the divergence of the gradient:

$$\Delta = \text{div}(\text{grad}) = \nabla \cdot \nabla = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad (\text{C.10})$$

## C.8 Divergence of rotation

Divergence of rotation of any vector field is always equal to zero:

$$\text{div}(\text{rot } \mathbf{H}) = \nabla \cdot (\nabla \times \mathbf{H}) \equiv 0 \quad (\text{C.11})$$

## C.9 Rotation of gradient

Rotation of gradient of any scalar function is always equal to zero:

$$\text{rot}(\text{grad } \varphi) = \nabla \times (\nabla \varphi) \equiv 0 \quad (\text{C.12})$$



## APPENDIX D

## PAPERS



## D.1 COMPEL [9]

### **Finite element modeling of a two-degree of freedom spherical actuator**

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#### **Introduction**

In the field of robotic and automation the need of actuation with multiple degrees of freedom is obvious. Generally multi-degree of freedom motions are achieved by combination of several one-degree of freedom actuators, what requires complex transmission systems. The idea of creation of a two-degree of freedom spherical actuator is based on the presumption that such construction should ensure higher level of precision, dynamic performance and lower friction comparing to a two-degree of freedom structure created using two independent one-degree of freedom actuators.

In the paper general concept of the two-degree of freedom spherical actuator with aerostatic suspension of the rotor is presented, as well as the first approach study leading to optimize the rotor's structure and parameters, realized using finite element method based software. Rotor with two layers and rotor with ferromagnetic so-called 'teeth' crossing external layer are discussed. It will be shown that in the last case higher torque and efficiency are obtained but a slot effect appears.

#### **Concept of spherical actuator**

The two-degree of freedom spherical actuator is working as an induction (asynchronous) machine and its concept is based on three mayor constructional solutions:

- **Stator**

Stator consists of five separated inductors. There are four one-degree of freedom classic side-inductors and one two-degree of freedom (with crossed windings) inductor below the rotor (Figures 1 and 2).

Such a solution, combined with adequate alimentation of both systems of windings, provides to the actuator the two degrees of freedom.

- **Suspension of rotor**

In order to minimize the air-gap and avoid friction, aerostatic suspension of the rotor was used. The compressed air is delivered through small diameter canals (Fig.1) created in the lower and side inductors. A positive side effect of such a solution is cooling of the actuator.

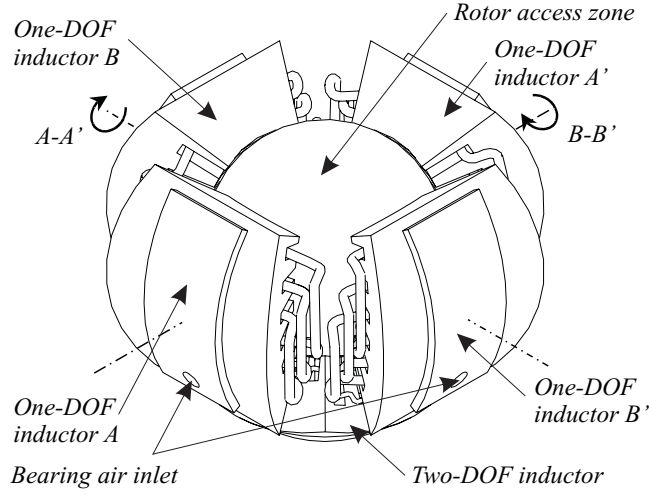


Figure 1. A two-degree of freedom spherical actuator

- Rotor

We analyzed three different rotor's structures: the massive rotor made of steel, the two-layers rotor with inner steel layer and external copper layer and the two-layers-with-teeth rotor.

The last structure was created by analogy to classic induction machines. However, as in the spherical rotor we do not have any direction of rotation, which is privileged, we created the teeth (crossing the external layer) rather than slots (Fig.2). The inner layer and the teeth are made of ferromagnetic material providing high permeability and average conductivity (steel) while the external layer is created from copper to allow the circulation of inductive currents.

### Finite element modeling

Numerical modeling of the two-degree of freedom spherical actuator was performed using finite element based software called Flux2D and Flux3D in cooperation with the creator of this software, Cedrat company from Grenoble, France. Choose of the right formulations describing simulated model of the actuator was one of the most important steps in our work.

In two-dimensional modeling the **AV** formulation is used. Starting with Maxwell equations and defining the magnetic vector potential **A** and the electric scalar potential **V** as follows:

- potential **A** such that:  $\mathbf{B} = \text{rot } \mathbf{A}$  (1)

- potential **V** such that:  $\mathbf{E} = - \text{grad } V - \frac{\partial \mathbf{A}}{\partial t}$  (2)

respectively, we obtain the following equation:

$$\text{rot} (1/\mu \cdot \text{rot} \mathbf{A}) = \mathbf{J}_{\text{ext}} - \sigma \cdot \partial \mathbf{A} / \partial t - \sigma \cdot \text{grad} V \quad (3)$$

where  $\mathbf{J}_{\text{ext}}$  is the source current and  $\sigma$  is material's conductivity.

Having the external conducting layer short-circuited the last element in (3) will disappear, what gives us:

$$\text{rot} (1/\mu \cdot \text{rot} \mathbf{A}) = \mathbf{J}_{\text{ext}} - \sigma \cdot \partial \mathbf{A} / \partial t \quad (4)$$

Depending on the hypothesis, we make, we can solve (4) in the two different modes. In the transient magnetic mode, we keep it as it is and we get the step-by-step resolution. In the magnetodynamic mode we make the assumption, that only the first harmonic component is present and we get it modified:

$$\text{rot} (1/\mu \cdot \text{rot} \mathbf{A}) = \mathbf{J}_{\text{ext}} - j \cdot \sigma \cdot \omega \cdot \mathbf{A} \quad (5)$$

In three-dimensional modeling we apply the  $\mathbf{T}\Phi$ - $\Phi$ - $\Phi_{\text{red}}$  formulation. The sub-formulation  $\mathbf{T}\Phi$  – electric vector potential  $\mathbf{T}$  and magnetic scalar potential  $\Phi$  – is used for the conducting regions not possessing the external magnetic field sources while the reduced magnetic scalar potential  $\Phi_{\text{red}}$  and the total magnetic scalar potential  $\Phi$  are used for the no-conducting regions, possessing or not the external magnetic field sources respectively. Based on the Maxwell equations we define as follows:

- potential  $\mathbf{T}$  such that:  $\sigma \mathbf{E} = \text{rot} \mathbf{T}$  (6)
- potential ( $\Phi_{\text{red}}$ ) such that:  $\mathbf{H} = \mathbf{H}_J - \text{grad} \Phi_{\text{red}}$  (7)

where  $\mathbf{H}_J$  is the magnetic field created by the no-meshed coils and ‘- grad  $\Phi_{\text{red}}$ ’ is the material's reaction to this field

- potential  $\Phi$  such that:  $\mathbf{H} = - \text{grad} \Phi$  (8)

Using the Lorentz' calibration (Bolkowski, 1993):

$$\text{div} \mathbf{T} = - \mu \cdot \sigma \cdot \partial \Phi / \partial t \quad (9)$$

and depending on the region properties we obtain finally three different equations to solve:

- for the conducting regions:  $\text{div} (\mu \cdot \text{grad} \Phi) - \mu \cdot \sigma \cdot \partial \Phi / \partial t = 0$  (10)

- for the no-conducting regions with source:  $\text{div} (\mu \cdot \text{grad } \Phi_{\text{red}}) = 0$  (11)
- for the no-conducting regions without source:  $\text{div} (\mu \cdot \text{grad } \Phi) = 0$  (12)

All the regions of our three-dimensional model and the corresponding formulations are presented on the Fig.2.

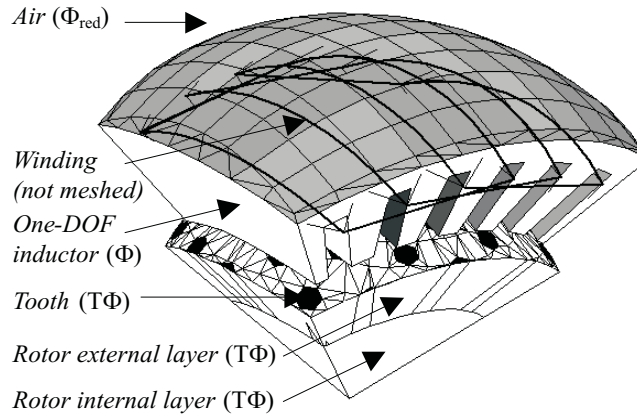


Figure 2. Formulations' layout in the model

### Electromechanical conversion's parameter

In order to evaluate the different results of our study we have decided to use the corresponding actuator's efficiency of the electromechanical conversion. The model we used in our simulations did not let us calculate the exact value of this efficiency, however we can estimate its value on a basis of the actuator's torque-speed characteristics.

Considering the characteristics torque-speed of a classic induction actuator and a load (Fig.3) we define the working point of the actuator-load system in the intersection point of the both given characteristics. At that point, the mechanic power transferred to the rotor is given by the equation:

$$P_{\text{mechanic}} = \omega_{\text{wp}} \cdot T_{\text{wp}}, \quad (13)$$

while the Joule's losses power in the rotor is equal to:

$$P_{\text{Joule rotor}} = (\omega_s / p - \omega_{\text{wp}}) \cdot T_{\text{wp}} \quad (14)$$

where  $\omega_s$  is the source's frequency,  $p$  is the number of the poles' pairs and  $\omega_s / p$  is the synchronous speed.

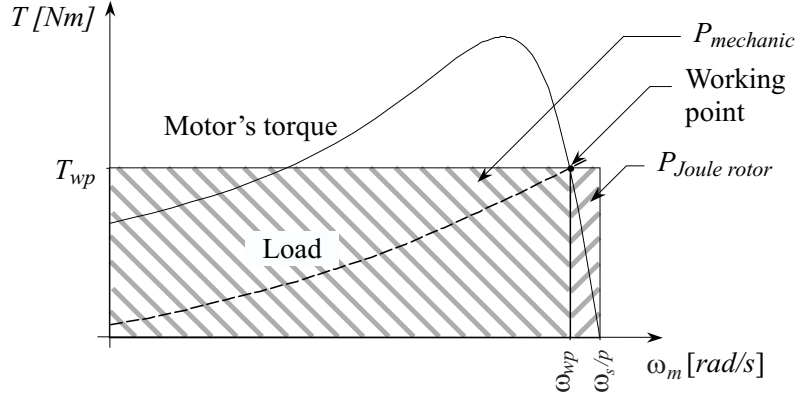


Figure 3. Working point, delivered mechanic power and Joule's losses in rotor, presented on a basis of the torque-speed characteristic

Knowing these two parameters we can calculate the efficiency of the conversion between the power transferred from the stator to the rotor and the outgoing mechanic power:

$$\eta_{s \rightarrow r} = \frac{P_{mechanic}}{P_{s \rightarrow r}} = \frac{P_{mechanic}}{P_{mechanic} + P_{Joule\_rotor}} \quad (15)$$

Taking into account (13) and (14) we can rewrite (15) and get:

$$\eta_{s \rightarrow r} = 1 - \gamma_{wp} \quad (16)$$

where  $\gamma$  is the slip between the rotor's speed and the synchronous speed, defined as follows:

$$\gamma = \frac{\omega_s - p \cdot \omega_m}{\omega_s} \quad (17)$$

Looking at (16) we realize, that the lower the slip, the higher electromechanical conversion's efficiency we have. However, the slip depends not only on the load but as well on the motor's characteristic and especially on its form around the synchronous speed (Fig.4). In this area the torque-speed characteristic is quasi-linear and generally can be replaced with a straight line. The more this line inclined, the lower the slip corresponding to the working point and the higher the  $\eta_{s \rightarrow r}$  conversion.

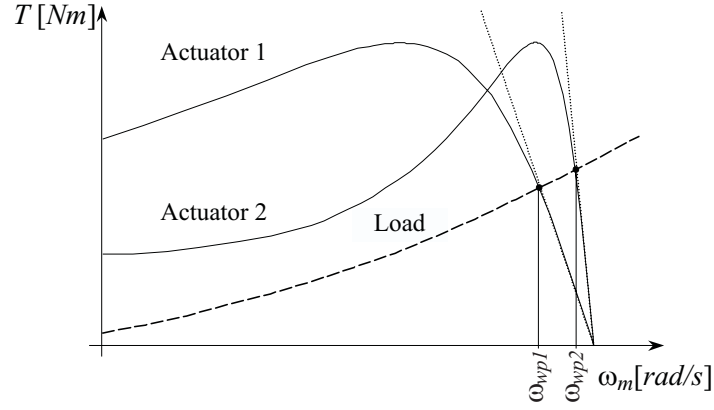


Figure 4. Working point dependence on the actuator's torque-speed characteristic

Finally, in a first approximation, it seems possible to compare the  $\eta_{s \rightarrow r}$  conversions of two different actuators on a basis of the torque-speed characteristic's slope around the synchronous speed. This angular coefficient can be approximated by the following rapport:

$$\frac{T_{\max}}{\omega_s/p - \omega_{\max}} \quad (18)$$

where the  $T_{\max}$  is the actuator's highest torque and the  $\omega_s/p - \omega_{\max}$  is the difference between the synchronous and rotor's speed (Fig.5).

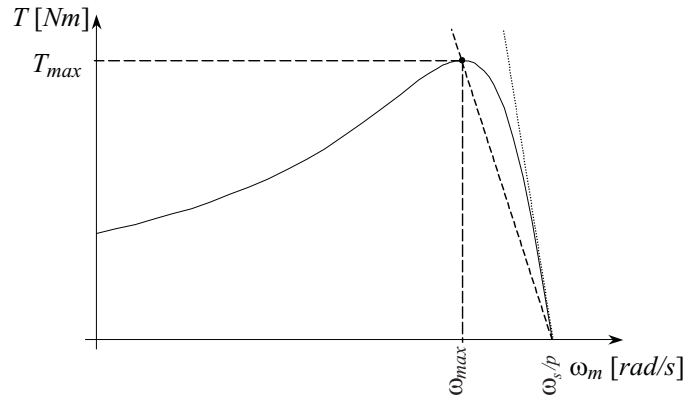


Figure 5. Approximation of the torque-speed characteristic's slope around the working point

Using the rotor's currents frequency  $\omega_r = \omega_s - p \cdot \omega_m$  that is proportional to the difference between the synchronous and rotor's speed we can replace (18) by the rapport:

$$\frac{T_{\max}}{\omega_{r\_max}} = \tau \quad (19)$$

where  $\omega_{r\_max}$  is the rotor's frequency at the maximal torque and  $\tau$  is the defined by us electromechanical conversion's parameter, which we will use in our study.

### Rotor's structure comparison

Analytical (Dehez, 1999) and finite element modelling proved, that massive rotor's performances are much worse comparing to the rotor with internal layer composed of iron and external layer created of copper.

In the next step, we have simplified our three-dimensional structure and performed our simulations using the two-dimensional models of the rotor without and with teeth (Fig.6).

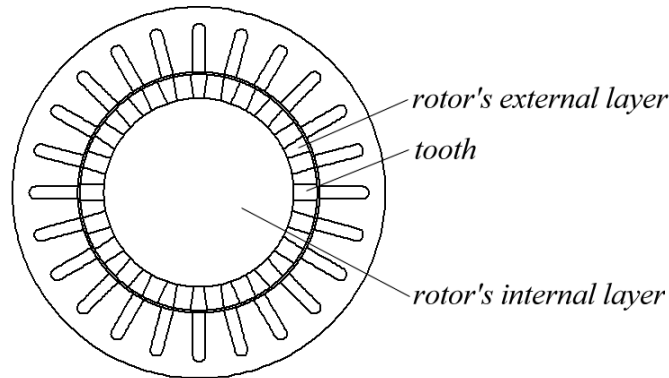


Figure 6. Two-dimensional structure with teeth

For both rotors, we were looking for the best electro-mechanical conversion, represented by the  $\tau$  parameter, as a function of the rotor's external layer thickness  $el$ .

As we can see on the Fig.7, for the actuator with two-layers-with-teeth rotor the  $\tau$  parameter can achieve values ten times higher, than for the actuator using simple two-layers rotor. The optimal values of the rotor's external layer thickness  $el$  are of course not the same in both cases.

Looking at the Fig.8 we can observe, that for the rotor with teeth not only the electromechanical conversion is better, but also the maximal torque produced is higher.

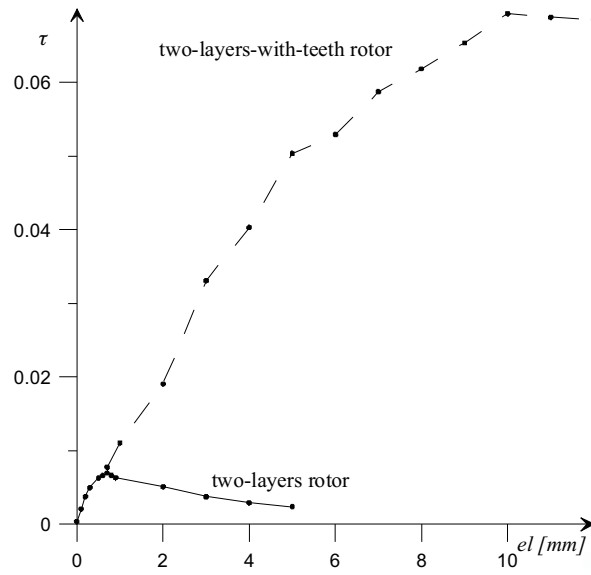


Figure 7. Electromechanical conversion  $\tau$  as a function of the rotor's external layer thickness  $el$

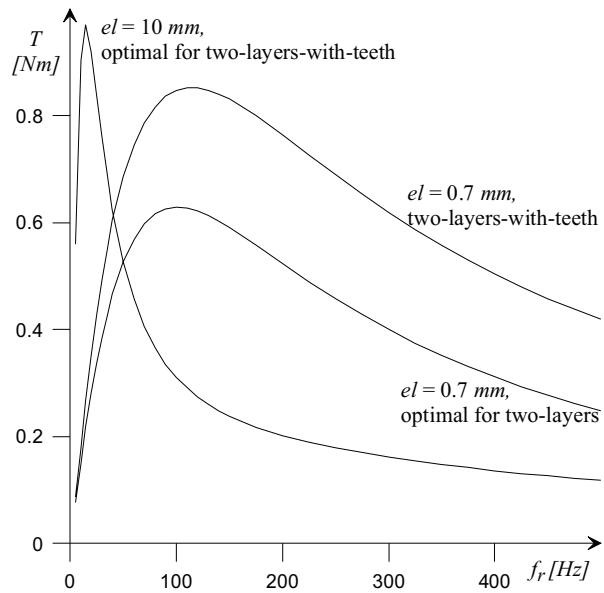


Figure 8. Comparison between the two-layers and two-layers-with-teeth rotor

### Torque's oscillations

Having proved, that the two-layers-with-teeth rotor ensures the higher actuator's torque than the two-layers rotor, we decided to find the optimal teeth' number.

For several values of this number, using the simplified two-dimensional model, we performed simulations in the transient magnetic mode, what gave us as the result the actuator's torque mean value  $T_{mean}$  and the amplitude of torque's oscillation value  $\Delta T$  (Fig.9).

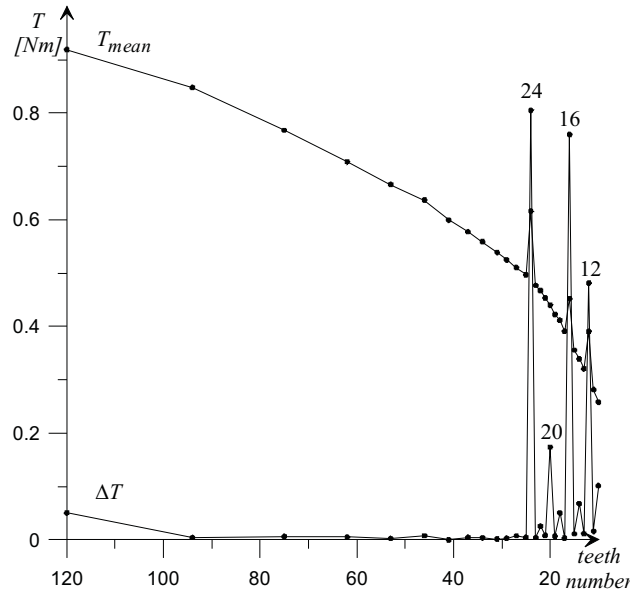


Figure 9. Actuator's torque mean value and oscillation value as a function of the teeth' number

On Fig.9 we can observe, that quite important torque's oscillations appear for the teeth' number equal to 12, 16, 20 and 24. For the higher teeth' number the oscillation value decreases and becomes negligible. As well the torque's mean value increases with the increase of the teeth' number. It stays in the contrary to the empiric ratio found in the literature (De Jong, 1976), which says that the optimal teeth' number can be found using the following relation:

$$N_r \approx 0.9 * N_s \quad (20)$$

where  $N_r$  and  $N_s$  are the rotor's teeth number and the stator's slots number, respectively. In our case  $N_s = 24$ .

Another aspect influencing the torque's value oscillations is the distribution of the teeth in the three-dimensional rotor. Independently of teeth' number, the

distribution must be as regular as possible in order to ensure torque's isotropy. Ideally, the distance between the teeth must be constant in every direction. We must realize that such a configuration does not exist obviously and the distance between two closest teeth can vary according to the rotor's direction of rotation (Fig.10).

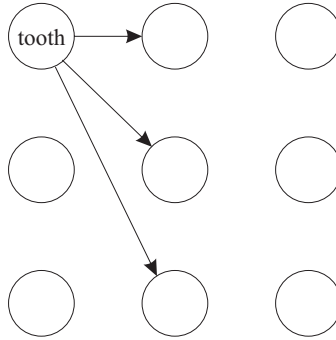


Figure 10. Distance between the closest teeth depending on the rotor's direction of rotation

Not ideal but the most uniform distribution on a sphere is defined by the vertices of a regular icosahedron (Fig.11-c-), i.e. formed of identical equilateral triangles, inscribed in this sphere.

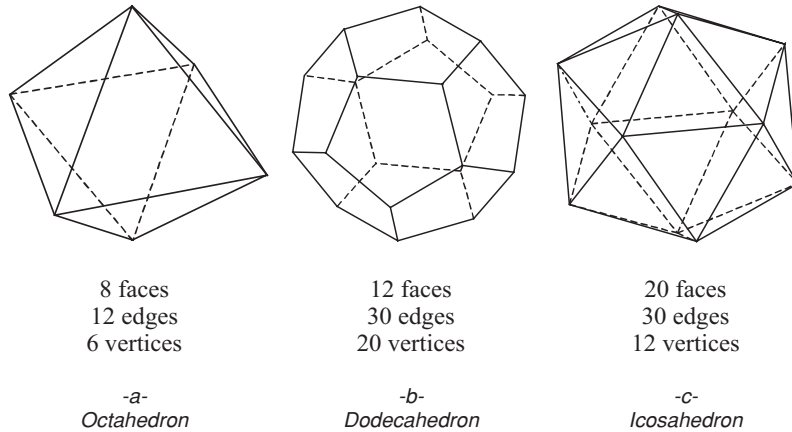


Figure 11. Platonic solids and their principal characteristics

In our study we considered several tessellations (Fig.12) of a regular icosahedron. In a very simplified way, the tessellation consists in multiplying the vertex and face number by adding new vertices in the center of each face of the considered solid. The vertices created like this are connected to the face vertices from which they result.

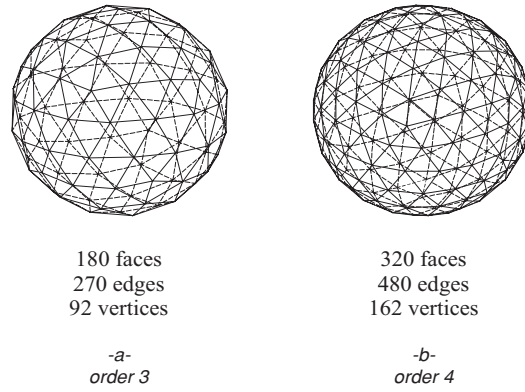


Figure 12. Evolution of the face, edge and vertex number of the icosahedron with the tessellation order

In order to minimize the torque's oscillations, we were trying to find the optimal teeth' number and distribution. That is why we have redrawn the characteristic from the Fig.9 as a function of the angular distance between the two following each other teeth (in a two-dimensional model), as presented on the Fig.13.

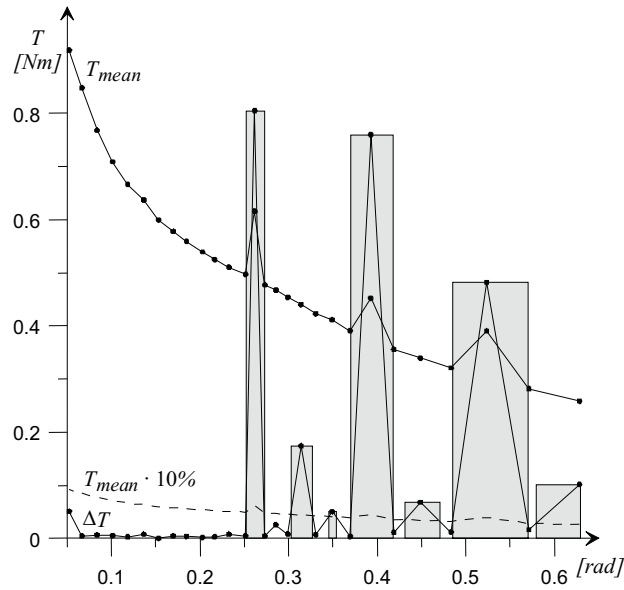


Figure 13. Actuator's torque mean value and oscillation value as a function of the angular distance between the two following each other teeth

On the Fig.13 we can see the angular intervals or zones, where the amplitude of the torque's oscillations exceeds 10 % of the torque's mean value. These 'forbidden' zones are preferred to be avoided.

At the same time we also established the angular spectrum of various icosahedrons (Fig.14): kind of histogram showing, for all the possible orientations of the rotor, the distribution of all the possible angular distances separating the two consecutive teeth located under an inductor (Fig.10).

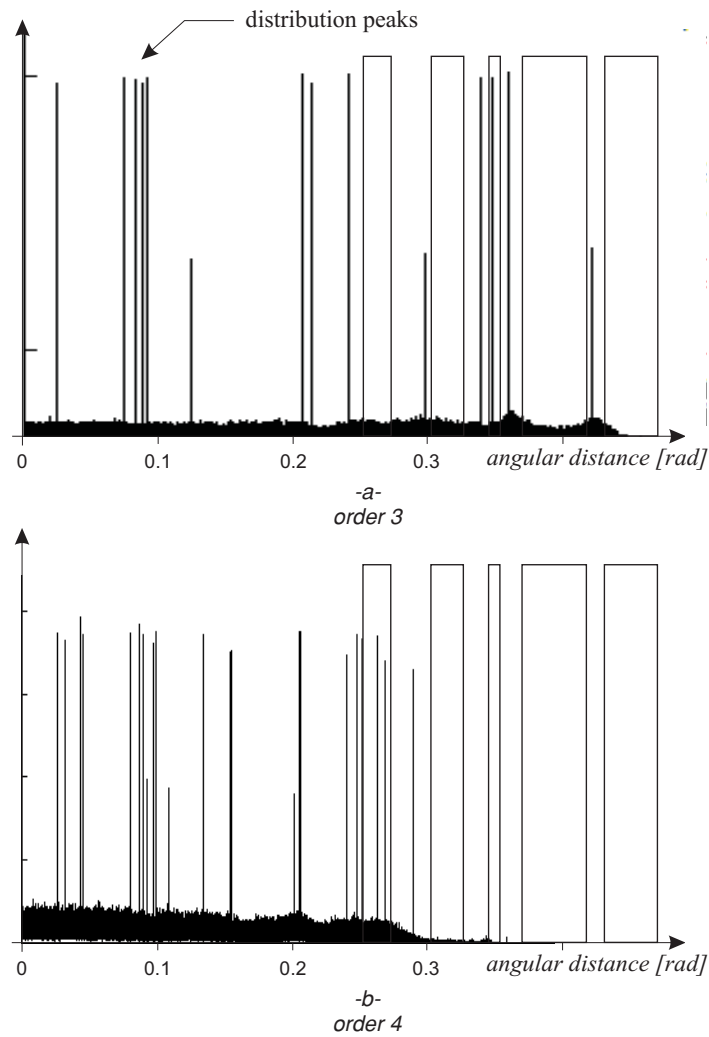


Figure 14. Superposition of icosahedron histograms and angular 'forbidden' zones for the tessellations of different order (continues on next page)

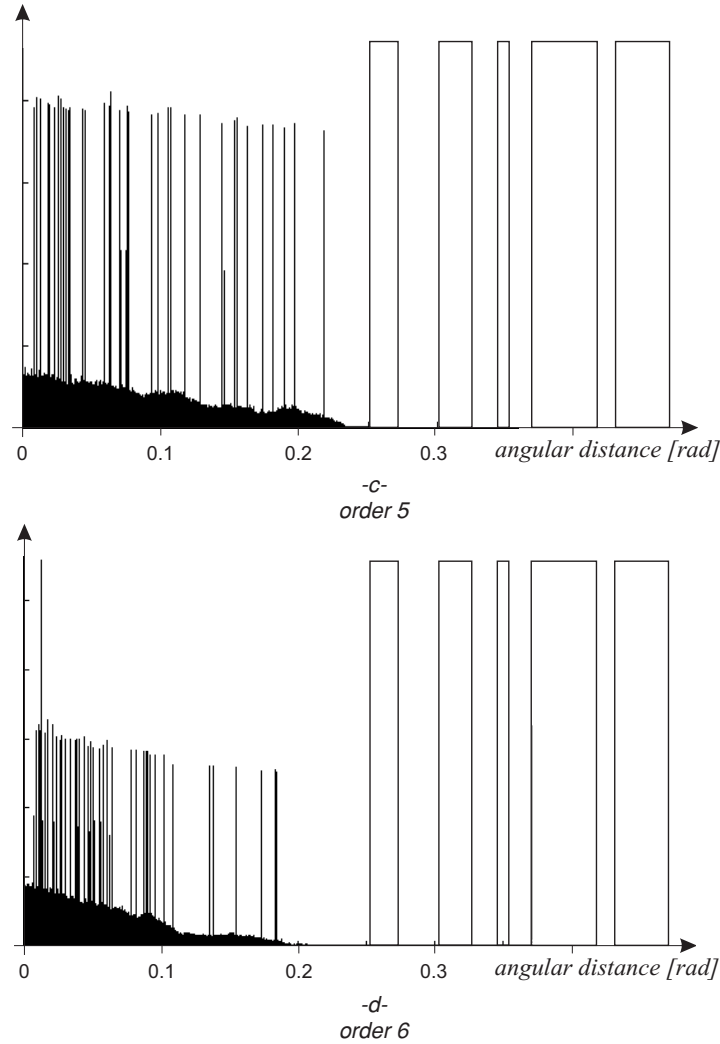


Figure 14. Superposition of icosahedron histograms and angular 'forbidden' zones for the tessellations of different order (continuation)

These histograms reveal a series of peaks indicating the existence of some angular distances more frequent than the others (Fig.14). By superimposing them on the angular 'forbidden' zones presented on the Fig.13, it becomes possible to compare the various icosahedrons and to seek the tessellation orders for which peaks of distribution avoid the concerned zones. It is the case of the fifth order tessellation and the higher ones.

The use of the higher order tessellations (what corresponds to the higher teeth' number) not only ensures the torque's isotropy but also results in the higher torque's mean value (Fig.9). However, taking into account the manufacturing complexity and the high frequency losses in the rotor's conducting parts depending directly on the teeth' number, it is rather advisable to reduce this number. Having this in mind we have decided to keep the fifth order tessellation and not higher, and to find another way to increase the torque's mean value.

### Other parameters

In our simulations up to now, we changed the teeth' number, but the tooth diameter stayed constant. It means that increasing the teeth' number increased the ratio between the rotor's surface covered by the teeth made of steel and the rest of this surface belonging to the external layer made of copper. We proposed the hypothesis, that the increase of this ratio for the constant teeth' number will have the same result as increasing the number of the constant diameter teeth.

For the reason of the simulations' complexity, which were conducted using the three-dimensional model, we decided to study the influence of the steel/copper ratio for the forth and fifth, and not sixth, order tessellations (Fig.15).

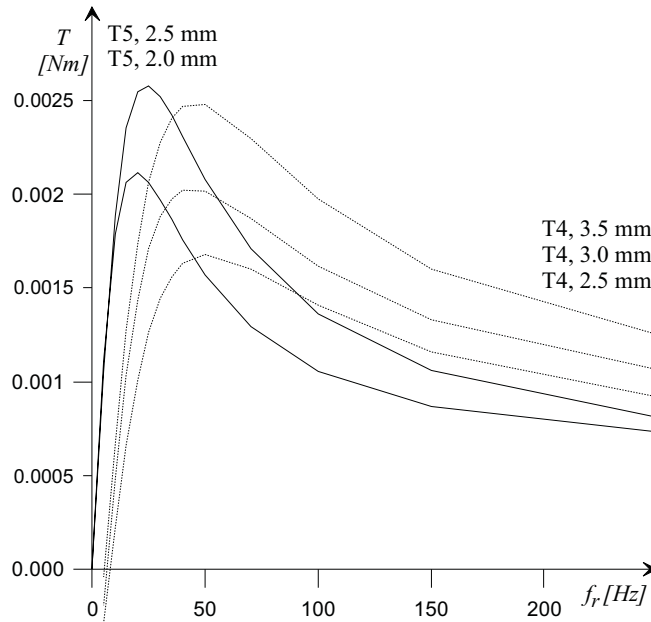


Figure 15. Characteristics torque-speed as a function of the tessellation order (T4, T5) and tooth's diameter (steel/copper ratio)

The results presented on the Fig.15 prove our theory that keeping the teeth' number constant and increasing the steel/copper ratio results in the increase of the torque's mean value. These results confirm also the previous conclusions, that for the constant tooth's diameter the higher teeth' number (higher tessellation order) ensures higher torque and lower torque's oscillations. The last conclusion is confirmed by the appearance of the slot effect for the forth order tessellation only.

### Conclusions

The new concept of the two-degree of freedom spherical induction actuator was presented. The need of ferromagnetic teeth in the rotor was shown. Some ways of optimization of the rotor's teeth number in order to increase the torque's mean value and decrease the torque's oscillations, were proposed.

After having performed this introductory study, there is a need of the experimental confirmation of our results.

For the future we consider to study some other parameters such as the influence of the magnetic material's saturation, the geometrical form of the tooth and the appearance of the higher harmonics in the signal connected to the no-sinusoidal coupling between the rotor and the stator.

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## D.2 ELECTRIMACS [10]

### Advanced Study of a Two-Degree-of-Freedom Asynchronous Spherical Actuator

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**Abstract** – A two-degree-of-freedom induction spherical actuator concept and first important choices concerning its rotor structure are first presented. Next, an advanced study on the rotor parameters with the view to optimize the actuator performances is performed. In particular, number of rotor teeth, their distribution and size are first considered in order to reduce torque oscillations and secondly to maximize the electromechanical conversion efficiency. Exploitation of the 2-D results in the 3-D study is also claimed and validated thanks to modeling of a 3-D geometry.

#### I. INTRODUCTION

At present, most electrical motors are able to actuate only one degree of freedom (DOF). However, many applications require complex motions with multiple DOFs, for example the actuation of a robot wrist [1], the omnidirectional motion of a mobile robot [2]. In these cases, many single DOF motors are generally used and the adaptation of these unidirectional motions achieved by the use of complex mechanisms such as parallel transmissions [1] or universal wheels [2].

The idea of a multi-DOF actuator, and especially a two-DOF spherical actuator, is based on the presumption that such a construction should ensure a higher level of precision, dynamic performance and lower friction compared to two-DOF structures using several independent one-DOF actuators.

A number of actuators able to generate multi-DOF have been developed over recent years. Wang *et al.* proposed a synchronous actuator with a permanent magnet spherical rotor and a stator consisting of three orthogonal windings [3]. Roth *et al.* developed a three-DOF spherical actuator operating on the principle of variable reluctance [4]. Lee *et al.* present in [5] a spherical stepper motor capable of three-DOF motion in a single joint. Davey *et al.*, as for them, proposed an asynchronous motor consisting of a homogeneous spherical rotor surrounded by three sets of windings given rotations around three perpendicular axes [6]. Finally, the ultrasonic spherical motor developed by Takemura *et al.* [7] can generate a three-DOF motion of a spherical rotor with a bar-shaped stator.

Among these many actuation principles, we chose induction essentially because the torque then depends on the inducing current frequency and rotor angular rate and not on the rotor angular position as in an synchronous or variable reluctant actuator. Now, it is obviously more complicated to determine with precision a phase displacement than an angular rate.

In a first step, this paper presents the general concept of the two-DOF spherical actuator we developed and the first important choices concerning its rotor structure.

Next, we study the influence of some rotor parameters (number of rotor teeth and their distribution) on actuator characteristics (torque oscillations and actuator conversion efficiency) using 2-D and 3-D finite element method based software.

#### II. CONCEPT

The proposed concept of two-DOF spherical actuator is based on three major constructional solutions:

##### A. Stator

The stator consists of five separated inductors. There are four one-DOF classical side-inductors (Fig. 1 -a-) and one two-DOF cross-wound inductor (Fig. 1 -b-) below the rotor (Fig. 2).

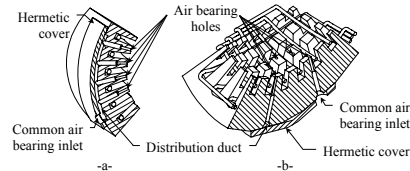


Fig. 1. Section views of one and two-DOF inductors

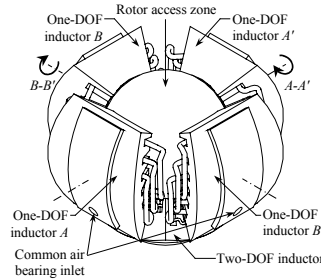


Fig. 2. Two-DOF spherical actuator

Such a solution, combined with adequate supply to both systems of windings, provides the actuator with two DOF motion around the  $A-A'$  and  $B-B'$  axes.

### B. Rotor suspension

In order to minimize the air-gap and avoid friction, an aerostatic suspension of the rotor was used. The compressed air is delivered through small diameter holes (Fig. 1) created in the lower and side inductors. A positive side effect of such a solution is cooling of the actuator.

### C. Rotor

We considered three different rotor structures: a massive rotor (Fig. 3 -a-), a two-layer rotor (Fig. 3 -b-) and a two-layer-with-teeth rotor (Fig. 3 -c-).

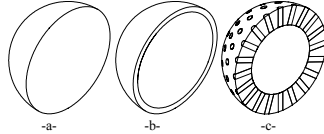


Fig. 3. Section views of the different rotor structures

The last structure was created by analogy with classical induction machines. However, since in the spherical rotor we do not have any privileged direction of rotation, we created teeth (crossing the external layer) rather than slots. The inner layer and the teeth are made of ferromagnetic material exhibiting high permeability and average conductivity (like silicon steel) while the external layer is made of copper to allow the circulation of inductive currents.

Analytical [8] and numerical [9] modeling proved, following the electromechanical conversion parameter defined below, that two-layer-with-teeth rotor is the most efficient comparing to one and two-layer rotor structures. This is why we focused our study on the two-layer-with-teeth structure.

## III. ELECTROMECHANICAL CONVERSION PARAMETER

In order to evaluate the various results of our study we decided to use the electromechanical conversion efficiency of the corresponding actuator. The model we used in our simulations did not allow us to calculate the exact value of this efficiency. However we can estimate its value on a basis of the actuator torque-speed characteristics.

Indeed, operating point is defined at the intersection of the actuator torque-speed and load characteristics (Fig. 4). The efficiency  $\eta_{s \rightarrow r}$  of the conversion between the power transferred from the stator to the rotor and the outgoing mechanical power can be expressed as a function of this operating point considering that:

$$\eta_{s \rightarrow r} = \frac{P_{mechanic}}{P_{s \rightarrow r}} = 1 - \gamma_{sp} \quad (1)$$

and,  $\gamma$ , the slip between the rotor speed and the synchronous speed, is defined as follows:

$$\gamma = \frac{\omega_s / p - \omega_m}{\omega_s / p}, \quad (2)$$

where  $\omega_s$  is the source frequency,  $p$  the number of pairs poles and  $\omega_s/p$  the synchronous speed.

Taking (1) and (2) into account we can write:

$$\eta_{s \rightarrow r} = \frac{p \omega_{mop}}{\omega_s}. \quad (3)$$

Looking at (3) we realize that the closer the rotor speed  $\omega_m$  is to synchronous speed  $\omega_s/p$ , the higher the electromechanical conversion efficiency. However, the speed depends not only on the load but also on the motor characteristic and especially on its shape around the synchronous speed (Fig. 4). In this zone the torque-speed characteristic is quasi-linear and generally can be replaced by a straight line. The more this line is inclined, the lower the slip corresponding to the operating point and the higher the  $\eta_{s \rightarrow r}$  conversion.

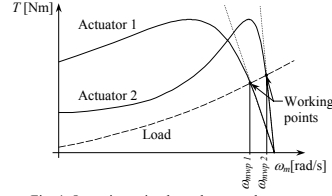


Fig. 4. Operating point dependence on the actuator torque-speed characteristic

Finally, in a first approximation, it seems possible to compare the  $\eta_{s \rightarrow r}$  conversions of two different actuators on the basis of the torque-speed characteristic slope around the synchronous speed. This angular coefficient can be approximated by the following ratio:

$$\frac{T_{max}}{\omega_s / p - \omega_{mmax}} \quad (4)$$

where  $T_{max}$  is the maximum actuator torque and  $\omega_{mmax}$  the corresponding rotor speed (Fig. 5).

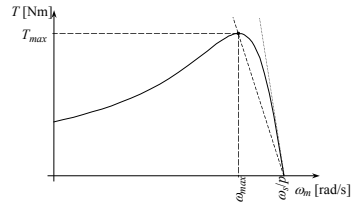


Fig. 5. Approximation of the torque-speed characteristic slope around the operating point

Using the rotor current frequency  $\omega_r = \omega_s - p \omega_m$  which is proportional to the difference between the synchronous and rotor speed we can replace (4) by the ratio:

$$\frac{T_{max}}{\omega_{rmax}} = \tau \quad (5)$$

where  $\omega_{rmax}$  is the rotor frequency at maximum torque and  $\tau$

the electromechanical conversion parameter we use in our study as an image of the actuator efficiency.

#### IV. ROTOR OPTIMIZATION

##### A. Number and distribution of teeth

In a previous work [9], we studied the number of teeth and their distribution around the spherical rotor in order to minimize torque oscillations.

In order to achieve this, we first performed simulations in transient magnetic mode on a simplified 2-D structure (Fig. 6 and Table I) for a constant rotational speed. This allowed us to study the evolution of the actuator's torque mean value and oscillation amplitude. We observed that quite important torque oscillations appear for certain intervals of angular distances between rotor teeth depending directly on the angular distances between stator teeth. It corresponds to the critical intervals shown on Fig. 7.

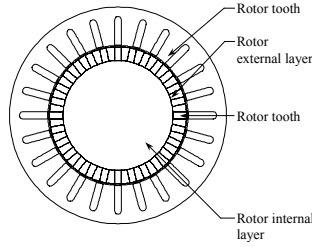


Fig. 6. 2-D structure for FEM modeling

Secondly, in 3-D, we consider particular tooth distributions: teeth corresponding to the vertices of regular polyhedrons and their tessellations. For each of these, we established an angular spectrum: a kind of histogram showing, for all possible orientations of the rotor, the distribution of angular distances separating two consecutive teeth located under an inductor (Fig. 7). These histograms reveal a series of peaks indicating the existence of some angular distances occurring more frequently than others.

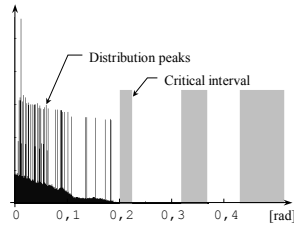


Fig. 7. Superposition of icosahedron histograms and angular 'forbidden' zones for the tessellations of different order

The choice of the tooth distribution and, a fortiori, of their number was finally made by superimposing these histograms and the critical intervals revealed by the 2-D study (Fig. 7).

We observed that a tooth distribution corresponding to a sixth order tessellation on an icosahedron allows us to avoid all critical intervals.

##### B. Steel ratio

After this first study, we began to analyze the influence, on the actuator performances, of the distribution of rotor surface between the teeth (steel) and external layer (copper) surfaces.

TABLE I

| MAIN MODELING PARAMETERS                           |                                                |
|----------------------------------------------------|------------------------------------------------|
| Rotor diameter                                     | 50 mm                                          |
| Airgap thickness                                   | 0,5 mm                                         |
| Material of external rotor layer                   | Copper                                         |
| Material of internal rotor layer, teeth and stator | Stainless steel type 430FR<br>Solenoid Quality |

Indeed, this distribution can evolve according to the number of teeth and their diameter and influence the actuator characteristics, namely, its maximal torque and the corresponding rotor frequency, and consequently, the electromechanical conversion parameter  $\tau$  used to compare performances.

To carry out this study, we started with a 2-D structure (Fig. 6) characterized by the values given in the Table I. For a given number of teeth, we then determined by FEM modeling, in magnetodynamic mode, the optimum of parameter  $\tau$  according to the thickness of the external layer  $h$  for various tooth diameters and, a fortiori, steel ratio. Fig. 8 shows, for two different numbers of teeth, the evolution of the optimal parameter  $\tau$  and thickness  $h$  according to steel ratio on the rotor surface.

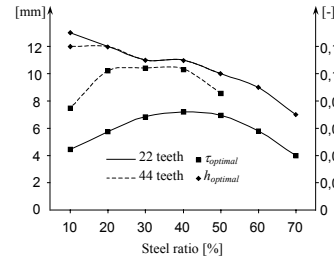


Fig. 8. Evolution of the optimal electromechanical conversion parameter and external rotor layer thickness with the steel ratio

##### - Optimal external layer thickness $h$

A first observation relates to the evolution of the optimal external layer thickness  $h$ . It tends to increase when the steel ratio decreases. It means that the less copper we have on the rotor, the less copper thickness we need.

This surprising evolution can be explained by considering the expression of the electromechanical parameter  $\tau$  through the equivalent electrical scheme of the asynchronous motor (Fig. 9).

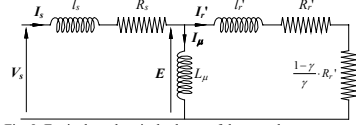


Fig. 9. Equivalent electrical scheme of the asynchronous motor

In the particular case of a current supply of the actuator, the electromagnetic torque  $T_{em}$  can be written:

$$T_{em} = \frac{3 \cdot p}{\omega_s} \cdot R_r' / \gamma \cdot \frac{(\omega_s L_\mu)^2 \|\bar{I}_s\|^2}{(R_r' / \gamma)^2 + (\omega_s (L_\mu + l_r'))^2} \quad (6)$$

where  $\bar{I}_s$  is the stator current,  $R_r'$  rotor resistance,  $L_\mu$  the magnetizing inductance and  $l_r'$  rotor leakage inductance.

From this expression, we can determine the maximum torque  $T_{max}$  developed as a function of the slip  $\gamma$  and, similarly, of the rotor frequency  $\omega_r$ . The electromechanical parameter  $\tau$ , given by the ratio between the maximum torque  $T_{max}$  and the rotor frequency corresponding to this torque  $\omega_{r_{max}}$ , is finally:

$$\tau = \frac{3 \cdot p}{2 \cdot \omega_s} \cdot \frac{L_\mu^2}{R_r'} \|\bar{I}_s\|^2 \quad (7)$$

and no longer depends on the rotor leakage inductance  $l_r'$ .

Magnetizing inductance  $L_\mu$  and rotor resistance  $R_r'$  can be expressed according to the geometrical parameters (Fig. 10) of the actuator and particularly the airgap thickness  $e$ , the external rotor layer thickness  $h$ , the distance between rotor teeth  $a$  and the tooth width  $b$ .

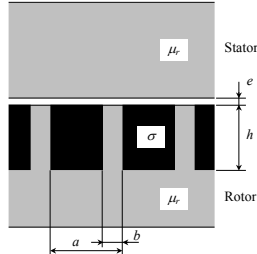


Fig. 10. 2-D simplified geometry

Indeed, the magnetizing inductance can be, in a first approximation, expressed as a function of the reluctance of the magnetic circuit linking stator and rotor (Fig. 11) as:

$$L_\mu = \frac{k_1}{\mathfrak{R}_{totale}} = \frac{k_1}{\mathfrak{R}_{stator} + 2 \cdot \mathfrak{R}_{airgap} + 2 \cdot \mathfrak{R}_{tooth} + \mathfrak{R}_{rotor}} \quad (8)$$

where  $k_1$  is a constant depending on the stator and rotor turn number and on the winding distribution in the stator and rotor slots. In this expression, we can neglect stator and rotor reluctances,  $\mathfrak{R}_{stator}$  and  $\mathfrak{R}_{rotor}$ , compared to tooth and airgap reluctances,  $\mathfrak{R}_{tooth}$  and  $\mathfrak{R}_{airgap}$ .

Considering the following expression for the reluctance  $\mathfrak{R}$  of a magnetic circuit of relative permeability  $\mu_r$ , length  $l$  and

section  $S$ :

$$\mathfrak{R} = \frac{l}{\mu_0 \cdot \mu_r \cdot S} \quad (9)$$

the magnetizing inductance can finally be written :

$$L_\mu = \frac{k_1}{\frac{2 \cdot e}{\mu_0 \cdot c \cdot b} + \frac{2 \cdot h}{\mu_0 \cdot \mu_r \cdot c \cdot b}} \quad (10)$$

where  $c$  is the thickness of the considered geometry (Fig. 10).

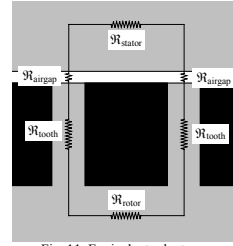


Fig. 11. Equivalent reluctance

In the same way, the rotor resistance  $R_r'$  can be expressed as follows:

$$R_r' = k_2 \cdot \frac{1}{\sigma} \cdot \frac{2 \cdot c}{h \cdot (a - b)} \quad (11)$$

where  $\sigma$  is the electrical conductivity of the external rotor layer and  $k_2$  a constant depending on the stator and rotor turn number and on the winding distribution in the stator and rotor slots.

On the basis of equations (7), (10) and (11) it then becomes possible to express the electromechanical parameter  $\tau$  in terms of the geometrical parameters. It is also possible to seek the value of the external rotor layer thickness  $h$  which optimizes this parameter. After some calculations, this optimal thickness is:

$$h_{optimal} = \mu_r \cdot e \quad (12)$$

This thickness thus depends only on the rotor relative magnetic permeability  $\mu_r$  and airgap thickness  $e$ . Without going any further, we cannot explain why, according to 2-D modeling results (Fig. 8), the optimal thickness  $h_{opt}$  decreases when the steel ratio increases.

To understand this evolution, it is finally necessary to take into account the influence of the tooth size  $b$  and spacing  $a$  on the field distribution in the airgap. Gibbs [10] and Carter [11] studied the influence of these parameters on the thickness of a homogenized airgap. By applying the Carter theory to our study case (Fig. 10), we observe (Fig. 12) that the equivalent airgap varies according to tooth size and thus to steel ratio.

To conclude, the evolution of the optimal external layer rotor thickness  $h_{opt}$  with steel ratio is mainly due to the influence of tooth size and spacing on the magnetic field distribution in the airgap.

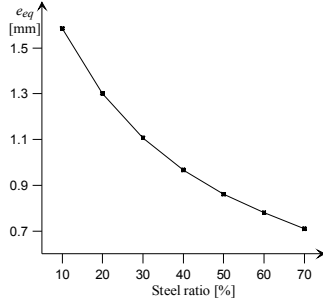


Fig. 12. Evolution of the equivalent airgap with steel ratio for the rotor with 22 teeth

#### - Electromechanical conversion parameter $\tau$

Another observation relates to the evolution of the optimal electromechanical conversion parameter  $\tau$  and its position with number of teeth. Fig. 8 shows that the optimal  $\tau$  values correspond for 22 and 44 teeth to steel ratios of, respectively, 40 and 30 %. Moreover, both optimal values were obtained for the same external layer thickness of 11 mm.

We realized with the increase of the number of teeth that the teeth become smaller and the saturation phenomenon is more likely to occur. This was the reason why the linear magnetization model (Fig. 13) could no longer be valid for smaller steel ratio values and the necessity of some additional simulations using the non-linear magnetization curve appeared.

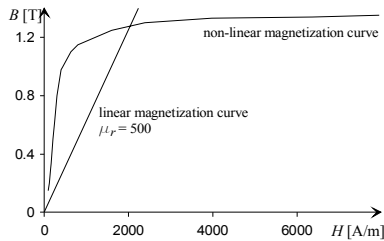


Fig. 13. Linear and non-linear magnetization curves

In our next step, we decided to compare the results of the electromechanical conversion parameter  $\tau$  obtained for 22, 44 and 88 teeth using the linear and non-linear magnetization curves (Fig. 14).

We clearly see that the results obtained using the non-linear magnetization curve are, for some values of number of teeth and of steel ratio, smaller than those obtained for the linear characteristic, which indicates the presence of saturation (the real material's magnetic permeability is smaller than that chosen for the linear characteristics). For 22 teeth, saturation appears up to steel ratios of 10 %, for 44 teeth it is already 20 % and for 88 teeth the critical steel ratio

value is almost 30 %. This is of course normal, because the higher is the number of teeth the smaller the tooth size.

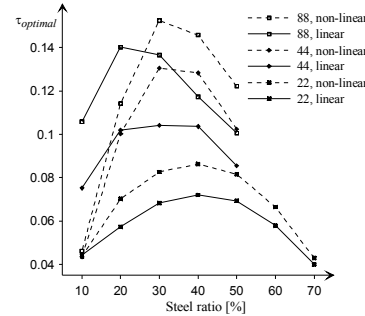


Fig. 14. Evolution of the electromechanical parameter with steel ratio for linear and non-linear magnetization curves

The results shown in Fig. 14 also confirm the previously presented observation (Fig. 8), that a higher number of teeth ensures a better electromechanical conversion and that its optimal values are to be found in the range of steel ratio values between 20 and 40 %, regardless of the changing number of teeth. We can also add that the calculations performed for 88 teeth confirmed the fact that the optimal external layer thickness value is 11 mm.

In conclusion, it appears that the optimal steel ratio and external layer thickness do not depend on the number of teeth and hence the angular distance between them. It is thus possible to implement the 2-D results in 3-D structures, where the number of teeth does not define only one constant angular distance between the neighboring teeth but a complete spectrum as illustrated on Fig. 7.

In order to confirm this, we performed some FEM simulations on 3-D structures (Fig. 15).

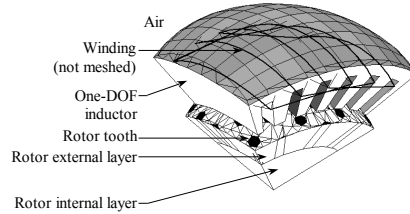


Fig. 15. 3-D structure for FEM modeling

Due to the complexity of this 3-D problem, our study is still not completed. Nonetheless, first results show (Fig. 16), for a rotor with 252 teeth and a steel ratio of 20 %, that the optimal electromechanical parameter  $\tau$  is achieved for a external rotor layer of about 12 mm. This value effectively corresponds to previsions obtained in 2-D (Fig. 8) for a steel ratio of 20 %. We expect that further 3-D simulations will continue to confirm the suitability of the 2-D results for the optimal external layer as well as for the optimal steel ratio.

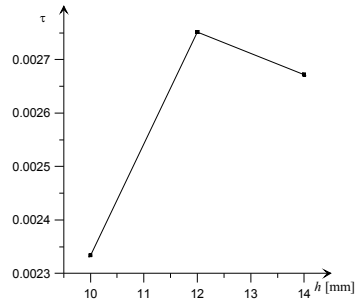


Fig. 16. Evolution of the electromechanical parameter with external rotor thickness  $h$  for 3-D structure

As regards the comparison of a exact values of the optimal parameter  $\tau$  obtained in 2-D and 3-D simulations, it is quite impossible, even taking into account that the 3-D structure represents only a part of the whole structure. It is indeed very difficult to determine the 2-D equivalent number of teeth of a 3-D structure where teeth spacing depends on the rotor position and its motion direction.

The reason for this is that one cannot take the number of teeth from a 2-D model and find the corresponding number of teeth in the 3-D model.

## V. CONCLUSIONS

We first presented a new concept of two-DOF spherical induction actuator and the justified choice for a two-layer rotor structure with teeth.

We also recalled the main results of a previous study concerning the choice of the number of teeth and their distribution over the spherical rotor surface, and their choice in order to minimize torque oscillations. We also show that the angular distance between rotor teeth is not unique but spread out over a broad spectrum.

Next, we discussed the results of an advanced study concerning the influence of tooth size and their number on the actuator electromechanical conversion efficiency. The conclusion is that the best level of actuator performances is obtained for almost the same steel ratio, around 30% for the parameters used in FEM modeling. Moreover, the optimal thickness, around 11 mm for the same parameters, of the external rotor layer being independent of the number of teeth. We thus estimate to be able to exploit the results of the study performed in 2-D for the dimensioning of the 3-D structure. Some modeling performed on a 3-D geometry approaching the real geometry of the actuator actually allowed us to consolidate this assumption.

Lastly we observe that the electromechanical conversion efficiency seems to be better when the number of teeth increases. Knowing that the torque ripple equally decrease with the increase of the teeth number [9], it confirms the interest, despite its fabrication cost, to build rotor with the highest possible number of teeth.

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