

¹ Graphical Abstract

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³ **bility during overtopping event**

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5 Highlights

6 **Surface-Subsurface flow effect on earthen dikes geomechanical sta-**
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- 9 • A coupled surface-subsurface flow model has been developed.
- 10 • Evolution of the water content shows a big influence on the longitudinal
11 and lateral slope stability during an overtopping event.
- 12 • The model reproduces well the water content evolution and slope in-
13 stability of small scale experimental data.

Surface-Subsurface flow effect on earthen dikes geomechanical stability during overtopping event

Nathan Delpierre^a, Hadrien Rattez^a, Sandra Soares-Frazão^a

*^aInstitute of Mechanics Materials and Civil Engineering,
UCLouvain, Louvain-la-Neuve, Belgium*

Abstract

Droughts and extreme precipitation events, exacerbated by climate change, are causing growing threats to earthen dams. The increasing frequency of extreme events, which were not always considered when these earthen structures were built means that these factors may not have been fully accounted for in their design. The risk of overtopping flow is therefore increased and the initial soil's saturation level has a significant impact on the structure's strength that should not be overlooked during the assessment of the dike risk of failure when overtopped. In this paper, we present an innovative physically-based numerical framework to investigate how combined surface-subsurface flows affect slope stability.

In this paper, we present a novel numerical framework that consists in a physically-based approach which allows to study the effects of combined surface-subsurface flows on the slope stability evolution, using an effective stress formulation for the mechanical analysis. The surface flows are described using a one-dimensional shallow-water equations solver and are coupled in a conservative way with the subsurface flow, through a two-dimensional Richards equation solver. The shear strength reduction method is employed, in combination with an effective stress approach, to assess the longitudinal and lateral slope safety factor evolution taking into account the pore-pressure and saturation degree changes in time during overtopping.

Keywords: surface-subsurface flow, Dike overtopping, Unsaturated soil mechanics, Slope stability, Shear strength reduction method

1. Introduction

Geotechnical structures made of soil, such as dams and dikes, play a crucial role in managing floods, facilitating fluvial transport, and disposing of mining wastes. The consequences of their failure can be severe, resulting in human casualties, destruction of cities, and damage to infrastructure.

In September 2023, the city of Derna in Libya experienced the devastating aftermath of two consecutive dam failures. The Mansour dam, located 12 km upstream of Derna and designed to hold 22.5 million cubic meters of water, collapsed after an overtopping event likely due to intense rainfall from the storm Daniel. The massive volume of water cascaded towards the Derna dam, located near the city, triggering a second dam failure. This chain of events resulted in an estimated 20,000 human casualties and widespread devastation in the city. These two consecutive dam failures are representative of many issues related to dam and dikes exploitation in the current global warming context.

In particular, two phenomena can be pointed out when dealing with such structures in the current global warming context. First, the increasing frequency of long droughts and extreme precipitation events leads earthen structures to undergo high variations of internal water content. When long droughts occur, dessication cracks might appear due to high suction pressures^[1]. This phenomenon locally leads to mechanically weaker zones but also to fast infiltration paths for water, during subsequent rainfall events. On the other hand, long wet periods can lead to a decrease of shear strength in the soil when it becomes saturated or close to saturation, which can result in the dam's failure. Second, the return periods for extreme precipitation events, considered for the design of these structures, are currently showing a significant decrease^[2]. This suggests that many structures designed in the past for events with a (e.g) 100-year return period may now be undersized for the more intense events of the 21st century.

Failure of earthen dams are complex phenomena. In 34% of the cases, they are triggered by overtopping events^[3]. It combines fast erosive flows that generate a deep channel with geomechanical block failure on the breach sides. Tingsanchali et al.^[4] investigated the competition between the longitudinal slope failure and the erosive process using a numerical model and small scale experiments for sandy dikes. Their findings highlighted that the influence of longitudinal slope failure was important to consider in downstream slopes with a height-length ratio below 1:2.5, emphasizing that the erosive

77 process alone was insufficient to fully characterize the underlying physical
78 phenomena. As most river dikes have a downstream slope ratio of 1:2, the
79 longitudinal sliding should thus be considered to model accurately their fail-
80 ure.

81 The dynamics of breach opening play a major role in the way water
82 rushes into areas that should be protected by dams or dikes. This is why
83 numerical models have been developed with the aim of making better pre-
84 dictions of breach outflows. Nevertheless, most of these models are based
85 on the idea that the breaching is mostly dominated by the erosion process
86 of a non-cohesive material. Therefore, these models consist in solving the
87 shallow-water equations combined with the Exner equation, which is a mass
88 conservation equation^{[5], [6], [7]}. To perform a reliable simulation, specific sed-
89 iment transport laws are required to catch the general behaviour of the sys-
90 tem. The choice of the appropriate law is in general critical for the simulation
91 quality and require specific calibration, as underlined by Van Emelen et al.^[8].
92 Tingsanchali et al.^[4] used such a model combined with a slope stability model
93 based on limit equilibrium, but they did not solve for the water infiltration
94 and their results show a critical role of an empirical parameter that estimates
95 the pore water pressure inside the dam.

96 It should be noted that, especially for cohesive materials, the flow through
97 the breach is also influenced to a large extent by the breach widening resulting
98 from lateral block failures^[9], while the erosive process mainly participates
99 to the breach deepening. Some attempts to consider these lateral blocks
100 failures led to the developments of simplified numerical models, such as the
101 bank failure operator, developed by Swartenbroekx et al.^[10], which consists
102 in adjusting the slopes based only on a given friction angle. Recent works by
103 Zhong et al.^[11] or Xue et al.^[12] proposed to account for lateral block failures
104 by using the limit equilibrium method. While a sediment transport law is
105 used for the breach deepening, a planar block collapse is supposed for the
106 breach sides.

107 These simplified method do not account for soil saturation that also sig-
108 nificantly affects the mechanical strength of the dam, which evolves greatly
109 when the dam is exposed to intense rainfalls, droughts and overtopping. Such
110 evolutions are clearly identified as landslides triggering effects and cannot be
111 disregarded^[13]. In recent years, extensive research has been conducted to
112 investigate the mechanical behaviour of unsaturated soils, building upon the
113 foundational contributions of Bishop^[14]. The exploration of the role of suc-
114 tion and soil saturation on shear strength gave rise to a substantial research

115 effort, leading to the development of new constitutive models using two stress
116 state variables, such as the Barcelona Basic Model^[15], or models based on
117 the effective stress principle^[16]. Arguments against the validity of the effective
118 stress in unsaturated soils argue that it cannot explain the volumetric
119 collapse phenomenon upon wetting^[17], but as here we are interested only in
120 the shear failure, both approaches lead to similar results. Moreover, effective
121 stress approach provides a smooth transition from unsaturated to saturated
122 states, which is particularly useful for cases involving fluctuations of the water
123 table like for dikes and dams. Effective stress approaches also requires less
124 model parameters than total stress approaches and capture well the evolution
125 of the shear strength with suction^[16].

126 In this paper, we present an innovative numerical modelling framework
127 for the dynamic assessment of the safety factor of earthen embankments
128 during overtopping. By integrating a coupled surface-subsurface flow solver
129 from Delpierre et al.^[18] with an effective stress approach from Alonso et
130 al.^[16], our model provides a robust and physically based method. Using
131 only measurable quantities such as the soil-water retention curve and shear
132 strength, the model is able to provide valuable insight into the potential
133 failure of a given embankment during an overtopping event.

134 The present paper is divided in 3 different sections. In section 2, we recall
135 our methodology to solve in a coupled way the surface and subsurface flows.
136 Section 3 describes the constitutive model we use to describe the influence
137 of suction pressure on the shear strength of the soil. Finally, in section
138 4, we combine the outcomes of the coupled surface-subsurface flows solver
139 with the effective stress approach. This results in a comprehensive model,
140 able to predict the evolution of the dike's stability under various theoretical
141 situations.

142 We successfully compare our model with experimental data from the literature,
143 accurately predicting the failure onset of an experimental dike subjected to a
144 specific overtopping event. This alignment with experimental results provides
145 additional confidence in our methodology.

146 **2. Coupled surface-subsurface flows**

147 *2.1. Subsurface flow: the Richards equation*

148 Based on a combination of the Darcy law and mass conservation principle,
149 the Richards equation describes water transfers in porous unsaturated and
150 saturated media, such as soils. In a mixed form, meaning the simultaneous

151 presence of suction and water content terms, the Richards equation can be
 152 written as follows:

$$\frac{\partial \theta(h)}{\partial t} - \nabla[K(h)\nabla(h+z)] = 0 \quad (1)$$

153 where θ is the volumetric water content, h the pore pressure and K the
 154 hydraulic conductivity. The high non-linearity associated to the Richards
 155 equation mainly results from the constitutive law employed to describe the
 156 relationship between pore pressure and the other physical parameters, θ and
 157 K . In this work, we make use of the classical Mualem-van Genuchten con-
 158 stitutive law, describing the so-called soil-water retention curve (SWRC; see
 159 Fig.(1)). These relationships, describing the transitioning effects between
 160 saturated and unsaturated soils, are explicitly provided in Delpierre et al. ^[18].

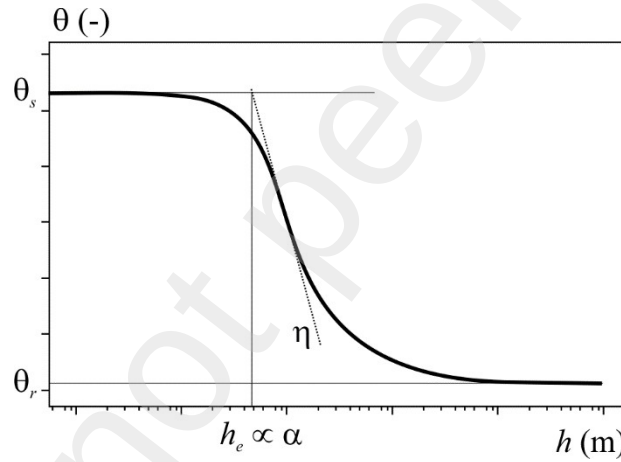


Figure 1: Example soil-water retention curve (following ^[18]).

161 Fig. 1 recalls the main parameters leading any SWRC shape. θ_s is the
 162 saturated volumetric water content, θ_r the residual water content. These
 163 values constitute the bounds for the water content of a given soil. These
 164 quantities are also used to express the equivalent volumetric water content
 165 as

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r}. \quad (2)$$

166 The parameter α is, together with the saturated hydraulic conductivity K_s ,
 167 the one which influence in the most critical way the saturation process. It is
 168 inversely proportionally linked to the air-entry pressure, usually defined as

the pressure required for the water to escape from a saturated soil. Typically, in sand-like soil, we can expect low air-entry pressure for this parameter and quite high air-entry pressure values for clay-like soil.

The numerical resolution scheme and its validation have been extensively described in Delpierre et al.^[18]. The solver is based on a finite volume formulation with an implicit temporal integration. The terms are obtained by applying the Gauss-Ostrogradsky's theorem to Eq. (1). As the process of groundwater flow can be considered as slow, the implicit formulation conveniently allows big time steps. At each time step, a system of equation is solved and the state variables (θ, h, K) are updated in each cell of the mesh.

2.2. Surface flow and system coupling

As presented in Delpierre et al.^[18], we use the shallow-water equations to evaluate the surface flow. Based on mass and momentum conservation principle, this system describes how waters level and velocities evolve in time. Considering a unit-width system, the equations can be written as follows:

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = q_{\text{in}} \quad (3)$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{q^2}{h} + g \frac{h^2}{2} \right) = gh (S_0 - S_f) \quad (4)$$

where h stands for water depth, q the unit discharge, S_0 the bed slope and S_f the friction slope with friction losses evaluated through the Manning's friction formulation. A finite volume scheme with an explicit time integration combined with fluxes evaluated using Roe's scheme is employed as presented in Soares-Frazao and Zech^[19].

Spatially, both physical domains are fully coupled. The one-dimensionnal shallow-water equations cells are aligned with the two-dimensionnal triangular cells depicting the geometry of the subsurface domain. With this feature, no error propagation through reinterpolation of physical quantities are expected. In a temporal perspective, the model is decoupled. We take advantage of the implicit nature of the Richards equation solver and of the relatively slow subsurface flow velocity to use large time steps. As regards the surface flow, the explicit nature of the solver implies the use of small time steps according to the CFL condition. The Richards equation solver time step is thus a multiple of the surface flow time step to allow for exchange of information between the two flow domains.

200 Strict mass conservation is achieved using a proper definition for the
 201 source term q_{in} , as described in Delpierre et al.^[18], which depicts exchanges
 202 between the surface and subsurface flow.

203 **3. Effective stress constitutive model for unsaturated soil**

204 *3.1. On the effective stress principle*

205 The effective stress principle is defined by Khalili et al.^[20] as the macro-
 206 scopic effect on the soil skeleton, which means that the effective stress should
 207 be seen as the stress responsible for strain. In the context of unsaturated
 208 soil, Bishop^[14] initially formulated effective stresses as follows:

$$\sigma'_{ij} = \sigma_{ij} + \chi s \delta_{ij} \quad (5)$$

209 where σ'_{ij} represents the effective stress tensor, σ_{ij} is the total stress tensor,
 210 $s = \gamma_w h$ denotes the suction pressure and χ stands for the effective stress
 211 parameter. Combining equation(5) with the Mohr Coulomb criterion, gives
 212 the following theoretical expression for the shear strength:

$$\tau = c' + \sigma \tan(\phi) + \chi s \tan(\phi) \quad (6)$$

213 where c' stands for the cohesion and ϕ is the friction angle. It is noteworthy
 214 that χ should be equal to one in a saturated soil and zero for a completely dry
 215 soil^[20]. While the χ parameter has often been considered equivalent to the
 216 saturation degree S_e , it was initially intended to be more general. Neverthe-
 217 less, this definition, while satisfactory for coarse granular soils, falls short in
 218 explaining the behavior of finer-grained soils like clay^[16]. The main concern
 219 with using the saturation degree S_e as representation of χ for clay soil arises
 220 from the unrealistic shear strength reached during a drying process, as the
 221 product $S_r s$ tends to extremely large values. As highlighted by Alonso et
 222 al.^[16], it is commonly observed that as the saturation degree decreases, the
 223 impact of pore pressure on shear strength diminishes. Following this obser-
 224 vation, Alonso et al.^[16] proposed a formulation that considers two distinct
 225 contributions for the saturation degree:

$$S_e = S_r^m + S_r^M \quad (7)$$

226 where S_r^M is the macroscopic degree of saturation, describing the pres-
 227 ence of water in the macropores, which can evolve in time. S_r^m stands for the

microscopic degree of saturation, describing the water content in the micropores. This quantity does not change in time and is independent from load and applied suction. This particular feature allows to differentiate between granular soil and fine soil with higher clay content. In a granular soil, we can expect that water can escape from the pores during a drying process leading to $S_r^m = 0$. On the contrary, as the clay content increases, the microscopic degree of saturation S_r^m should increase accordingly as water can be trapped in small pores.

Following all these developments, Alonso et al.^[16] defined an effective degree of saturation such that:

$$S_r^e = \chi = \left\langle \frac{S_e - S_r^m}{1 - S_r^m} \right\rangle \quad (8)$$

with $\langle x \rangle = 0.5(x + |x|)$ corresponding to the Macauley brackets. According to this formulation, χ becomes an expression describing the available water in the macropores, ranging from 0, when water only remains in micropores, to 1, for a fully saturated soil.

3.2. Numerical validation

To validate the implementation of the effective stress formulation, we conducted a comparison between direct shear test data sourced from the literature and finite element method numerical simulations performed using the Moose software^[21].

The comparison is based on the direct shear tests conducted by Vanapalli et al.^[22] on glacial till. The soil, characterized by a specific sand/silt/clay distribution of respectively 28%, 42%, and 30%, underwent a drying process over several days. The measured friction angle was $\phi = 25^\circ$, and an effective cohesion of zero was observed. Multiple tests were carried out with matric suction ranging from 0 to 500 kPa, maintaining a constant normal confining stress of 25 kPa. The experiments were performed using a modified direct shear test apparatus designed by Gan et al.^[23]. Alonso et al.^[16] proposed to fit the soil-water retention curve using a pore-air entry pressure related parameter $\alpha = 7.2e^{-05} cm^{-1}$, $\eta = 0.59$, $\mu = 0.67$ and a microscopic degree of saturation $S_r^m = 0.64$.

The direct shear test are conducted numerically using a unit cubic cell. The prescribed boundary conditions included a normal confining pressure of 25 kPa and a continuously increasing horizontal displacement applied to the

261 top, while the bottom of the cell remained fully fixed.
 262 As depicted in Fig.(2), the numerical results exhibit a strong correlation with
 263 the experimental shear strength data. The commonly assumed relationship,
 264 where χ is considered equal to the saturation degree, is also illustrated (la-
 265 belled as Bishop shear strength). It can be observed that this assumption
 266 proves to be unrealistic for the considered glacial till soil, as the shear strength
 267 does not constantly increase over time.

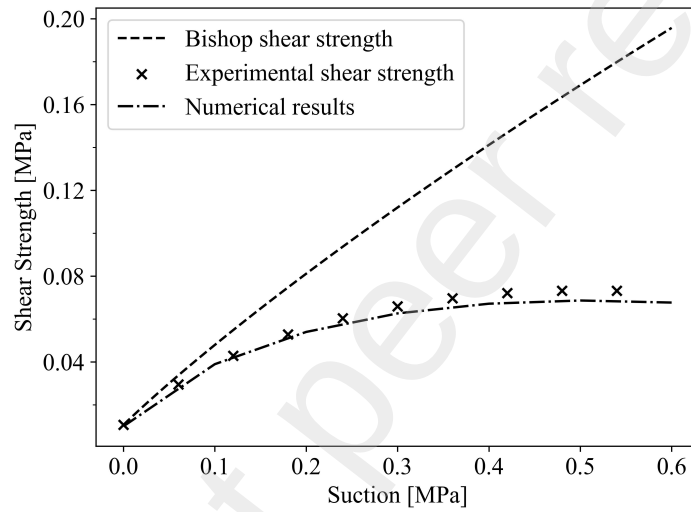


Figure 2: Comparison of numerical and experimental direct shear test results.

268 The numerical validation shows that integrating the concept of micro-
 269 scopic degree of saturation enhances the accuracy of shear strength evolution
 270 predictions. This aspect will be further explored in the subsequent section,
 271 which is dedicated to practical applications.

272 4. Model application

273 In this section, we combine the subsurface-surface flow model with the
 274 effective stress model to study realistic cases. The safety factor is assessed at
 275 various time intervals during a dam overtopping event, employing the shear-
 276 strength reduction method (SSRM). This method, first described by Matsui
 277 et al.^[24], consists in reducing in a linear way the geomechanical properties in

the Mohr-Coulomb criteria (see Eq.(6)), defined by a cohesion and a friction angle, until convergence is no longer achieved in the FEM calculation:

$$c'_{SF} = \frac{c'}{SF}, \quad \phi_{SF} = \frac{\phi}{SF} \quad (9)$$

where c'_{SF} is the reduced cohesion and ϕ_{SF} is the reduced friction angle. The resulting reduction factor corresponds to the so-called safety factor (SF). Contrariwise to Limit Equilibrium Methods (LEM), the failure surface does not need to be guessed. In addition, suction, which provides additional strength to the soil, is easily taken into account by remapping the solution of the groundwater flow to Gauss points constituting the FEM mesh.

Applications of this method are presented in the next sections. First a sensitivity analysis is conducted to investigate the influence of initial conditions and soil-water retention curve parameters. The longitudinal sliding failure mechanism is studied in subsection 4.1 whereas subsection 4.2 is dedicated to transversal side failure. Then, subsection 4.3 shows a successful validation of the model through a comparison with an experimental example retrieved from the literature.

4.1. Longitudinal sliding failure

To enhance comprehension across various cases, the same dam geometry, described in Fig.(3), is used for both longitudinal failure test cases. It consists in an asymmetric dam with a 1:3 upstream slope and a 1:2 downstream slope with a 3 m crest height. The initial conditions for the geomechanical calculation consist in a the pore pressure distribution within the dam, obtained by solving the subsurface flow. The influence of both fully saturated and unsaturated regions, as well as their temporal changes, in relation to the stability of the dam is highlighted in subsection 4.1.1.

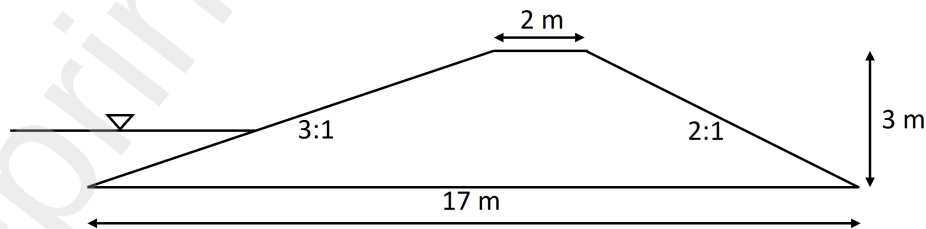


Figure 3: Asymmetric dike geometry.

302 The soil hydraulic properties employed in both section 4.1.1 and section
 303 4.1.2 are identical and are described in Table 1, with the exception of the pore
 304 air-entry pressure α , whose influence is specifically investigated in section
 305 4.1.2. A cohesion $c' = 5$ kPa and a friction angle $\phi = 25^\circ$ are considered for
 306 the geomechanical properties.

Parameter	Value	Unit	Parameter	Value	Units
K_s	0.001	cm s^{-1}	α	$7.2e^{-05}$	cm^{-1}
θ_r	0.05	-	$\hat{\eta}$	0.59	-
θ_s	0.6	-	μ	0.67	-

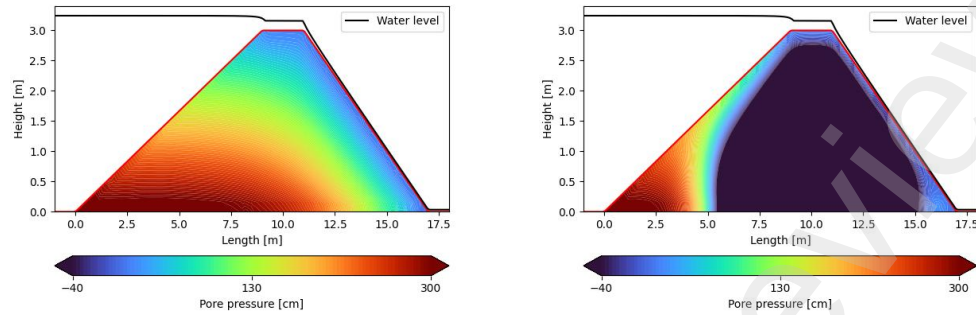
Table 1: Soil-water retention curve properties.

307 During the overtopping event, generated by a constant upstream flow of
 308 200 l.s^{-1} , we expect the failure mechanism to develop at the downstream
 309 part of the dam as no load is available to counterbalance the internal pore
 310 pressure.

311 4.1.1. Influence of the initial saturation degree

312 Throughout a classical exploitation, an earthen dam undergoes various
 313 meteorological conditions. Rain and droughts significantly affect the soil's
 314 water content and thus its shear strength. In this test case, we compare the
 315 evolution of the safety factor during an overtopping event for an initially dry
 316 dike, corresponding to a homogeneous initial pore pressure $h = -400\text{cm}$, and
 317 for an initially wet dike, with an initial pore pressure $h = -40\text{cm}$. Fig.(4)
 318 shows the pore pressure distribution after a 300 s flow for both initial con-
 319 ditions. The steady state is already reached for the wet dike, whereas the
 320 water struggles to infiltrate the dry dike.

321



(a) Pore pressure distribution for the wet dike. (b) Pore pressure distribution for the dry dike.

Figure 4: Comparison of the pore pressure distribution after 300 s.

As observed in Fig.(5), the safety factor is influenced to a large extent by the initial water content condition. The decrease in safety factor, associated with the evolution of the saturation degree in the dam during the overtopping, is particularly pronounced in the case of the initially wet dam. Notably, the time to reach the threshold safety factor of 1.5 is observed to be four times faster for the initially wet dam than for the initially dry one.

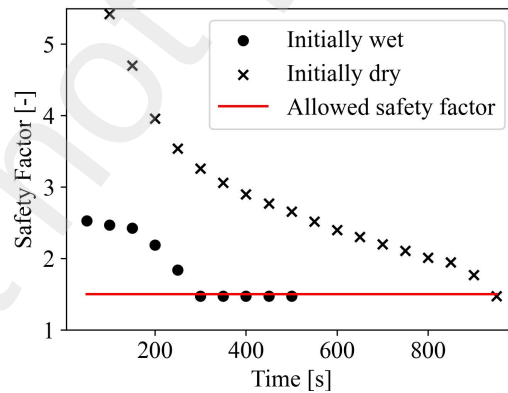


Figure 5: Safety Factor comparison for different initial conditions.

The initial presence of water influences greatly the infiltration rate as water tends to circulate in the already filled areas. These results allow to underline the need of maintaining proper saturation degree conditions inside a dam in operation.

332 4.1.2. Influence of pore air entry pressure

333 Among the parameters of the soil-water retention curve, the pore air-entry
 334 pressure seems to be the one which influences the most the shear strength.
 335 By strongly influencing the value of the α parameter, it also strongly influ-
 336 ences the saturated water content plateau (see Fig.(1)). If the α parameter
 337 is low, the soil tends to stay at high volumetric water content values even for
 338 high suction pressures. This particular behavior causes the soil to reach full
 339 saturation more rapidly, consequently accelerating the reduction in mechan-
 340 ical capacities. This, in turn, results in a faster decline of the safety factor,
 341 as illustrated in Fig.(6).

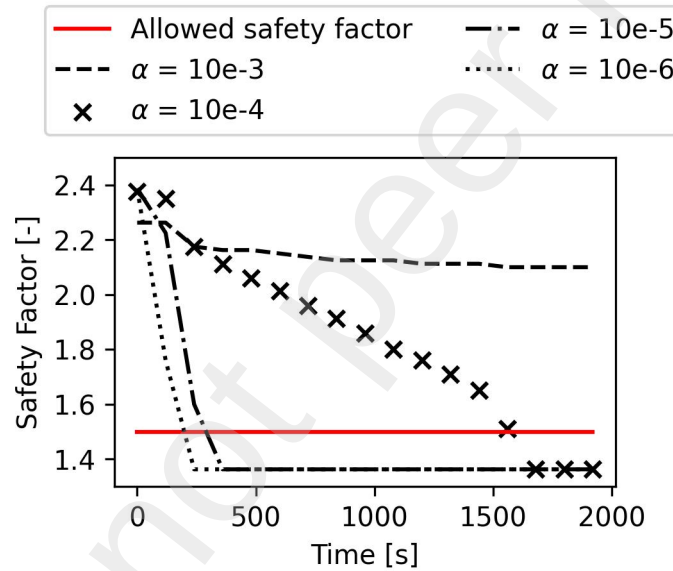


Figure 6: Safety Factor comparison for different pore-air entry pressure.

342 4.2. Breach widening

343 In this test case, the lateral breach widening is studied. The influence
 344 of the initial groundwater level, which is a result of the past hydrological
 345 conditions, on the potential breach side failure is emphasized. The geometry
 346 of the test case is presented in Fig.(7), where the water table is represented
 347 as well as a water depth representing the flow in the normal direction.

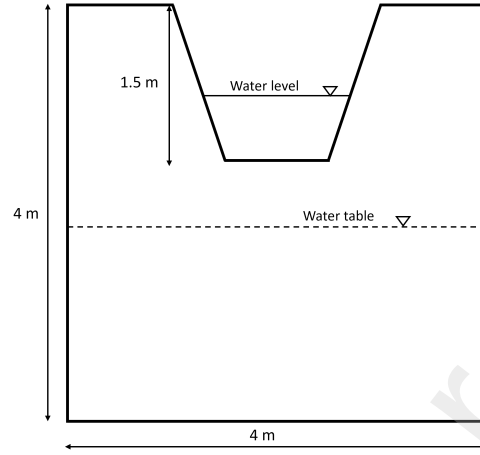


Figure 7: Geometry of the test case.

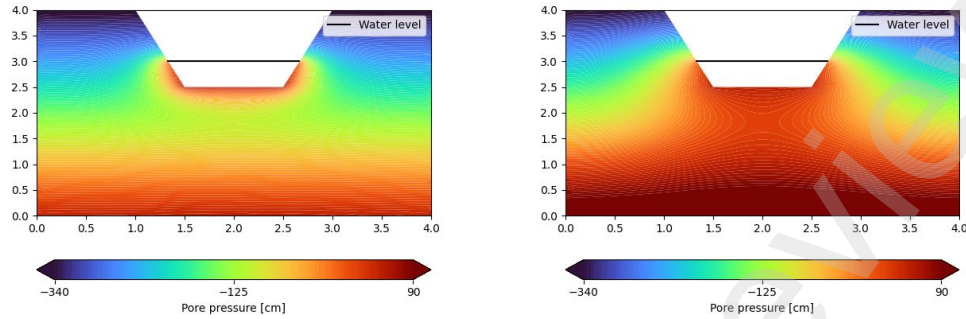
348 A 50 cm water depth is considered in the breach and the soil's hydraulic
 349 parameters are presented at Table 2.

Parameter	Value	Unit	Parameter	Value	Units
K_s	0.01	cm s^{-1}	α	10^{-3}	cm^{-1}
θ_r	0.05	-	$\hat{\eta}$	2	-
θ_s	0.6	-	μ	0.5	-

Table 2: Soil-water retention curve properties.

350 The same soil's geomechanical properties are used than in the previous
 351 section: a 5 kPa cohesion and a 25° friction angle with a microscopic degree
 352 of saturation $S_r^m = 0.8$.

353 The pore pressure evolves in time and a specific distribution is obtained
 354 depending on the initial groundwater level as observed in Fig.(8).



(a) Pore pressure distribution at $t = 10$ s.

(b) Pore pressure distribution at $t = 50$ s.

Figure 8: Pore pressure evolution for a 0.5 m high initial groundwater level.

355 The safety factor is evaluated for each scenario after a 50-second ground-
 356 water flow simulation, providing an objective basis for comparing the different
 357 cases. A rotational block failure mechanism is visible as the FEM calculation
 358 stops to converge. An illustration of a typical potential failure surface can
 359 be seen at Fig.(9). This shows that in the case of a cohesive material, led by
 360 suction forces, a rotational failure mechanism is likely observed rather than
 361 a slide failure.

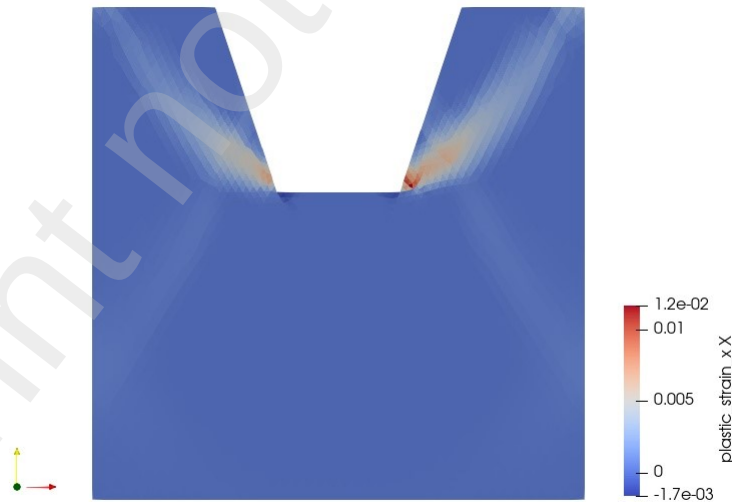


Figure 9: Rotational failure surface.

362 The safety factor associated to each situation is strongly influenced by
 363 the initial groundwater level, as shown at Fig.(10). This is mainly due to the
 364 fact that the initial groundwater level influences the rate of saturation in the
 365 dam. The higher the initial groundwater level, the lower the safety factor
 366 becomes. Again, this underlines the importance of maintaining an optimal
 367 degree of saturation in earthen structures. It can be concluded that keeping
 368 optimal saturation degrees is crucial in the way breaching will propagate
 369 laterally due to block failures.

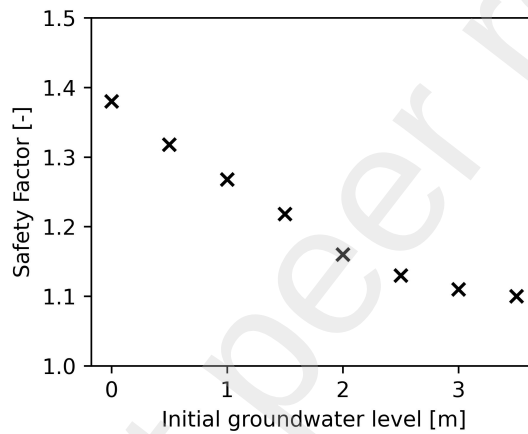


Figure 10: Safety Factor comparison for different initial groundwater level after 50s.

370 4.3. An experimental test case: dam overtopping

371 Awal et al.^[25] presented the results of various experimental overtopping
 372 embankment failures. One of these tests exhibited a geomechanical failure
 373 by longitudinal sliding caused by the combined action of surface flow forces
 374 and the variation in water content in the soil. The geometry of the test case
 375 is represented in Fig.(11).

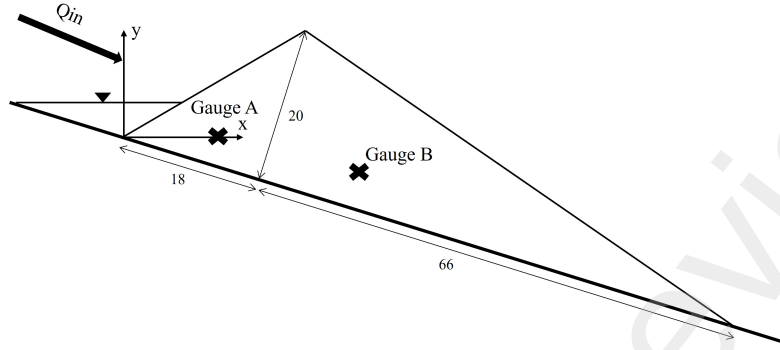


Figure 11: Geometry of the test case.

It consists of a sandy dike, built on a 5-meter long slope, with a steep upstream slope and a smoother downstream slope. The soil-water retention curve parameters were measured and are described in Table 3. Initially the saturation degree S_e is set to 20 %.

Parameter	Value	Unit	Parameter	Value	Units
K_s	0.03	cm s^{-1}	α	0.055	cm^{-1}
θ_r	0.045	-	$\hat{\eta}$	3.2	-
θ_s	0.287	-	μ	0.312	-

Table 3: Soil-water retention curve properties.

The geomechanical properties of the sand are the one of a cohesionless sand with a 34° friction angle.

A $30.5 \text{ cm}^3 \text{ s}^{-1}$ constant inflow discharge was imposed at the upstream end of the setup. The sandy dike is overtopped after approximatively 300 s and a sliding failure is observed after 447 s.

4.3.1. Comparison with the coupled surface-subsurface flows model

The outcomes generated by the numerical model were compared with the volumetric water content data recorded at two gauges throughout the experiment. Although the exact positions of these gauges were not explicitly specified in the original paper, they were estimated to be at the coordinates listed in Table 4.

Initial simulations with the parameters of Table 3 resulted in an overshoot in the saturation degree evolution, with the model response outpacing

Gauge	X	Y
A	0.13	0
B	0.31	-0.05

Table 4: Gauges position.

the measured data. Given the significant uncertainties associated with both gauge positions and the measurement of saturated hydraulic conductivity, the latter parameter was adjusted to $K_s = 0.015 \text{ cm.s}^{-1}$, which remains in the same order of magnitude. The final results are presented at Fig.(12), where the wetting process is compared and visually validated.

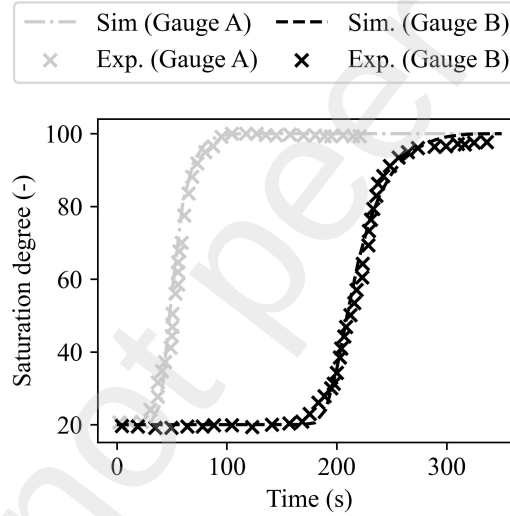


Figure 12: Comparison between experimental (crosses) and numerical results (dashed).

4.3.2. Safety factor evolution

The safety factor is assessed during the event using the shear strength reduction method detailed earlier. Considering the nature of the soil, with likely coarse grains, a microscopic degree of saturation $S_r^m = 0$ was selected. Also, a 60 Pa cohesion was considered for the material, preventing local non physical convergence of the SSRM process. The safety factor evolved greatly throughout the event as shown on Fig.(13).

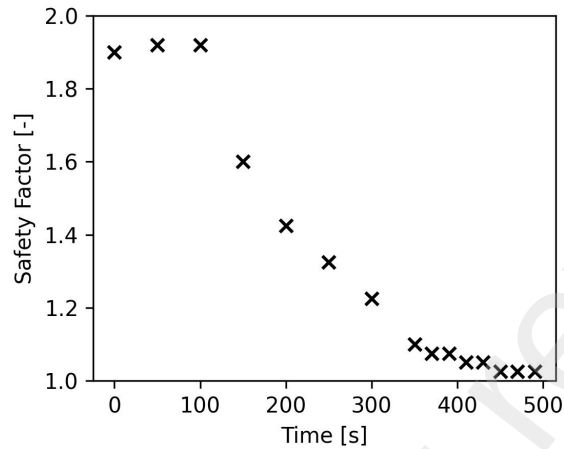


Figure 13: Safety factor evolution of the downstream slope during overtopping.

Initially, the safety factor is high, with a 1.9 value. Then, the safety factor evolves as a consequence of the changing water content, leading to a higher density but also to important cohesion losses. Finally, after 440 seconds the safety factor drops to 1.02 meaning that the dike is close to instability. The obtained failure surface is evaluated and represented on Fig.(14). Strong similarities appear between the calculated surface and the experimental one displayed^[25].

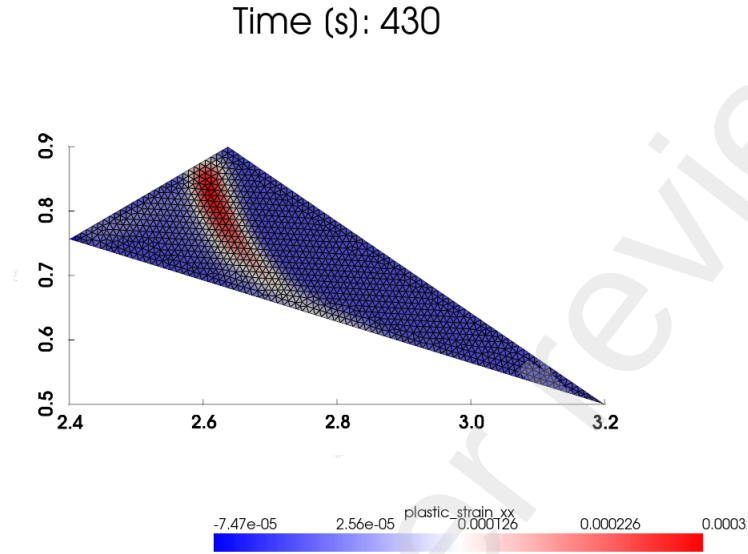


Figure 14: Failure surface: plastic strain in the x direction.

5. Conclusion

We presented an advanced model that predicts the evolution of stability in earthen embankments subjected to overtopping flows, relying solely on physical parameters. The model accounts for the impact of overtopping on the groundwater flow, which in turn influences the shear strength, the soil's density and thus the stability of the structure.

We first recalled the physical principles governing the surface-subsurface flows model, as presented in Delpierre et al.^[18], which provides information on the pore pressure evolution during a given overtopping event. Secondly, we presented the constitutive law, developed by Alonso et al.^[16], used to describe the relationship between pore pressure and shear strength evolution. We showed that, using this law, we were able to numerically reproduce direct shear test data, retrieved from the literature. This proves that the selection of an appropriate plastic law combined with an effective stress formulation is an adequate way of approaching the simulation of shear failure occurring in unsaturated soils.

The third part, which consists in combining the surface-subsurface flows with an appropriate effective stress formulation, is the main novelty of this work.

From the soil water retention curve and the saturated shear strength, we are able to predict the evolution of the stability of the embankment subjected to different events. We showed several theoretical applications where the model proved to perform physically. We then showed that the model is effectively able to predict the onset of failure by successfully comparing with a complex experimental failure. We were able to reproduce not only the groundwater flow, but also the failure mechanism and the onset time. With these developments we want to show that pore pressures variations effects on dams stability during overtopping events should not be overlooked.

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