Université Catholique De Louvain

Institut Des Sciences Actuarielles

Individual and Institutional Asset Liability Management.

Membres du jury:

Prof. Pierre Ars
Prof. Griselda Deelstra
Prof. Michel Denuit
Prof. Pierre Devolder *Promoteur*
Prof. Andrew Cairns

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Donatien Hainaut

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Preface.

This dissertation has been prepared in partial fulfillment of the requirements for the Ph.D. degree at the institute of actuarial sciences, of the Université Catholique de Louvain. The research was carried out in the period from November 2004 to june 2007 under the supervision of Pierre Devolder, professor and president of the institute of actuarial sciences and was financed by the "Communauté Française de Belgique" under the "Projet d' Actions de Recherches concertées".

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Contents

Chapter 1

Introduction.

One of the classical problems in financial economics is that of an economic unit who aims at maximizing his expected life-time utility from consumption and/or terminal wealth by an effective asset-liability management. This question is not only of interest to each single agent, but also on a macro level, since market equilibrium prices and allocations are determined by the optimal behaviour of all agents in the market.

We can distinguish two main categories of economic agents: the individual and institutional investors. In this dissertation, we study some of their assets allocation issues. Their concerns differ in the following way. Individuals invest in the financial markets mainly to finance future consumption of which they obtain some felicity or utility. Whereas institutionals, such as pension funds, aim to finance future liabilities and maximize utility drawn from some surplus, dividends or contributions rates.

The work consists of three parts. The first one addresses the individual assets allocation decision at the age of retirement. Two chapters develop this problem known as the annuity puzzle in the literature. The second part is devoted to the asset liability management of pension funds exposed to mortality risk. Three chapters analyse this issue for defined benefits and defined contributions pension plans. One chapter examines the modelling of longevity risk. The third part studies two particular assets allocation problems in complete markets: the wealth maximization under a value at risk constraint and the valuation of a fair profit sharing rate.

The first mathematical tool used in this dissertation is the stochastic control, which is a powerful theory allowing to study a wide range of individual consumption and assets allocation issues. This theory states that the optimal expected utility of an investor, usually known as the value function, is solution of a non linear second order differential equation, namely the Hamilton Jacobi Bellman equation (denoted HJB in the sequel). Under smooth assumptions, a verification theorem guarantees the existence and uniqueness of a potential solution. This approach was first successfully applied by Merton (1969, 1971) to assets allocation issues and gives rise to a prolific literature. Nevertheless, many interesting problems don't have any closed form solution or have a candidate solution non-differentiable at some points. To cope with this lack of smothness, Crandall & Lions (1984) have developed a weak formulation of solutions to differential equations, the viscosity solutions. Despite the interest of this theory, it rarely leads to analytical solutions and one has to rely upon numerical methods. Kushner (1977) has proposed a finite difference scheme for approximate solution of the HJB and has proved the convergence of this numerical solution toward the viscosity solution.

A second mathematical concept applied in this work is the martingale approach, developed in the eighties in papers such as Karatzas et al. (1987) and Cox & Huang (1989, 1991) in the setting of complete markets. This method is a powerful alternative to stochastic control and leads in many cases to closed form solutions. Furthermore, this theory has been extended to allow conic constraints on portfolio proportions or bounds on shortfall risk. Excepted in the work of Brennan & Xia (2002), very few papers use this approach to optimize assets allocation in an incomplete market. One of purposes of this dissertation is to show that this method has still sense in an actuarial context and leads to ALM strategies similar to those applied in practice by actuaries.

Another important concept addressed are Lévy processes which encompass Brownian motion, Poisson and Compound Poisson processes. Those processes were initially investigated by Paul Lévy and are mainly used to model heavy tailed distributions of assets (see e.g the work of Madan 1999). In this dissertation, Lévy processes are utilized in the modelling of mortality intensities.

A last tool used is the Fourier inversion in order to compute option prices. This technique is based upon the Parseval's theorem (for details see Dufresne et al. 2005) and allows to find formulas for option prices in term of the characteristic function of assets logprices. This technique is applied to value a fair profit sharing rate, for life insurance participating policies.

Contribution of the dissertation.

First part: chapters 2 and 3.

The first part of the thesis is devoted to the individual assets allocation decision at the age of retirement, between life annuities and financial investments such cash or stocks. We can identify two reasons motivating the increasing public awareness of longevity risk. Firstly, as the funding of pay as you go systems is problematic in the long run, nations are tempted (at least partly) to switch to privately funded pension systems. Secondly, employers are shifting from defined benefit plans to defined contribution plans. The pensioner becomes responsible for managing his own longevity risk over his remaining lifetime and is exposed to old-age poverty. Clearly, life annuities are the most adapted hedging tools that ensure the standard of living till death. Despites their advantages, the pensioners still ignore life annuities and prefer to invest their savings in financial markets, mainly because they fear to lose control over their funds. This issue is known as the annuity puzzle in the economic literature and gives rise to a whole branch of papers. The starting point of our research are the seminal papers of Yaari (1965) and Richards (1975). Yaari has proved that a pensioner without any bequest motive should prefer a full annuitization rather than a cash account. The intuition underlying this result is that the rate of return of an annuity is always higher than the risk free rate, because it includes a mortality risk credit. Richard extends the Merton's work (1971) to include deterministic incomes and instantaneous term life insurance. Others references are mentioned in introductions of chapter 2 and 3.

We consider an initial annuitization strategy that mixes withdrawal decisions and life-annuities, but does not allow for deferring annuitization into future periods. This assumption fits relatively well the reality of the Belgian market: individuals generally purchase a life annuity once for all at the age of retirement. In this setting, chapter 2 determines the optimal level of annuitization for a retiree who doesn't bear any bequest motive, and under the budget constraint guaranteeing that wealth remains non negative. The financial market considered only contains one riskless asset. The optimization tool adopted to address this issue is stochastic control. The novelty of our approach is to insert the budget constraint whithin the HJB equation by Lagrange multipliers. We are then able to find a semi-analytical solution and to characterize the optimal consumption pattern in function of the amount annuitized. As we observe that a part of the annuity is reinvested in cash in certain cases (section 2.3.3), we have determined the optimal allocation of wealth between cash and life annuities.

Chapter 3 addresses exactly the same issue as chapter 2 but allows for an utility of a bequest and for investment in a risky asset. Due to the presence of a stochastic return on assets, it is no more possible to infer a closed form solution of the HJB equation. To tackle this problem, we have relied on an adapted version of the numerical method developed by Kushner and Dupuis (2001). It comes down to replacing derivatives present in the HJB equation, by their finite difference approximations. The main technical difficulty consists of the choice of parameters of discretization to ensure the convergence of the algorithm. With respect to existing papers such as those of Purcal (2004) and Purcal and Pigott (2004) , we innovate by using a semi implicit approximation for derivatives. This approach speeds up the time of computation by decreasing the number of meshes of discretization.

The main results of chapters 2 and 3 can be summarized as follows. For an individual without any bequest motive, the optimal consumption pattern can be subdivided into two periods. During the first one, he will consume the entire annuity and a part of his wealth. Beyond a certain age, the consumption is equal to the annuity and his savings are depleted. If the return of riskless asset or of risky assets is sufficiently high, a fraction of the capital should be dedicated to their purchase. An interesting observation is that optimal assets allocation still includes a life annuity if the retiree wishes to pass on a bequest to his relatives. In this case, the optimal investment strategy is in agreement with the generally admitted principle that: the quantities of risky assets owned in portfolio should decrease with age.

Second part: chapters 4 to 7.

In the second part of the dissertation, we have adopted the point of view of an institutional investor who manages life insurance or pension liabilities and is exposed to mortality risk. The appropriate investment strategy for pension funds involves constructing a portfolio of financial assets performing sufficiently to meet pension liabilities. However the presence of mortality risk is at the origin of the incompleteness of the market and there does not exist an unified way to cover liabilities. In this context, hedging strategies are usually established by the means of quadratic criteria such mean-variance minimization or risk minimization (for a survey see Schweizer (2001)). Superhedging is another alternative (see El Karoui and Quenez (1995)). However the cost of those strategies is too high to be realistic. For example, it may be easily proved that the superhegding cost of a pure life endowment is the price of a zero coupon of same maturity. Eventually, one can also rely on stochastic control even if it rarely leads to a closed form solution (excepted when utilities are exponential, e.g. see Young & Zariphopoulou 2002). In this thesis, we have opted for the martingale optimization method already mentioned earlier in this introduction. It has been successfully applied to numerous issues in complete markets such the management and design of a pension fund: see e.g. the papers of Boulier et al. (2001), Deelstra et al. (2003, 2004). One of our purposes is to illustrate that the martingale approach makes still sense in an actuarial setting. Working in incomplete markets has however two consequences. Firstly, there exist more than one equivalent martingale measure. We need then to fix the deflator (or the market price of mortality risk) used by the pension fund manager to value liabilities, in order to apply the Cox & Huang martingale method. Secondly, the optimal wealth process found by this approach may only be partly hedged. Chapters 4 and 5 focus on wealth maximization issues that could be met in the management of defined contributions pension plans. Whereas chapter 6 studies a minimization problem typically related to defined benefits pension schemes. Chapter 7 develops a model of longevity risk, that could be inserted in the previous problems.

Chapter 4 addresses the management of pension fund liabilities. Its purpose being methodological, we have voluntarily worked in a simplified setting: the financial market contains only two assets (cash, stocks) and the mortality is modelled by a Poisson process. The liabilities are made of pure endowment policies which are the building blocks of the most of pension commitments. We have assumed here that the insurer's martingale measure is equal to the product of the financial risk neutral measure and of the historical actuarial measure. This particular measure is referred in the financial literature as the minimal measure and was initially introduced by Föllmer and Sonderman (1986). This minimal measure plays an important role in the risk minimization theory and was exhaustively studied by Schweizer (1991). The method developed in this chapter will be used in chapter 5 and 6 to address more complex issues met by pension funds.

In chapter 4, the fund manager aims at maximizing under a budget constraint, the expected utility of a terminal surplus, defined as the difference between a target terminal wealth and the sum of capitals paid out to alive affiliates. The budget constraint guarantees the actuarial equilibrium between the current richness and the market value of expected future benefits. The self financed investment strategy replicating at best the optimal wealth process is next obtained either by martingale decomposition of the wealth process (Kunita Watanabe decomposition) or either by dynamic programming. At first, we have considered that manager's preferences are reflected by an exponential (CARA) utility. The value function and the optimal investment policy are compared with the results of Young and Zariphopoulou (2002) obtained by direct resolution of the HJB equation. In the second instance, manager's utility is assumed to be of the power type (CRRA). At our knowledge, solutions for such kind of utility were not yet developed in incomplete markets. Our results reveal that, compared to exponential utilities, the choice of a power utility leads to an optimal assets allocation depending on the fund equity and is particularly well adapted to ALM purposes.

The Belgian regulator (CBFA) imposes an interest rate guarantee for defined contributions (DC) pension schemes. Chapter 5 addresses precisely the management of this kind of funds that are exposed to mortality and financial risks. As in chapter 4, the affiliates' mortality process is a Poisson process and is only hedgeable by diversification. The most new features of our work are the introduction of an actuarial change of measure in the deflator, the combined optimization of dividends and assets allocation in a market with stochastic interest rates. The return of the riskless asset is driven by a Vasicek model. The objectives pursued by the fund manager are on one side to maximize the expected utility from dividends and on another side to maximize a terminal surplus. This surplus is here the difference between a target asset and an accounting reserve.

Again, the optimization tool is the Cox & Huang's approach and requires to define a priori the fund manager's deflator. Contrary to chapter 4, the pricing measure is now different from the minimal one and modifies the mortality hazard rate by a constant multiplicative factor. This multiplicative factor may be seen either as a bad anticipation of mortality evolution or either as a mortality market price of risk depending on the insurer's preferences. At first, we have inferred the optimal dividends and investment policies without specifying the type of utility functions. The structure of the assets portfolio is obtained by the dynamic programming principle and maximization of the value function generator. This way of doing is quite easier than the martingale decomposition, given that the proportion of stocks and bonds are expressed in term of partial derivatives of the value function.

Two particular cases are next studied wherein the fund manager's utility belongs respectively to the power (CRRA) and exponential (CARA) families. In the power case, we arrive as in chapter 4 at the conclusion that dividends and assets allocation depends on the fund equity and are then well adapted for ALM purposes. For exponential utilities, the ALM policy is neither tied to assets nor liabilities. It is clear that this characteristic is not adapted to the concerns of a pension manager.

The theme of the next chapter is the management of defined benefit (DB) pension plans. In a defined benefit pension plan, the employer promises the employee a specific monetary benefit at a specific age, that is based on factors such as salary and length of service. The employer is responsible for setting aside the money to fund this pension scheme and the appropriate ALM strategy involves to optimize the assets allocation and the contribution rate, so as to match promised future pension liabilities. In this setting, the expected contribution paid in by the sponsor is named normal cost. Its calculation requires to forecast about how a pension fund 's liabilities are going to accrue over a particular time horizon. However, when the estimated value of liabilities changes as a result of salary developments, inflation or bad investment performances, a revised level of contributions is assessed at the periodical actuarial valuation. The fund manager aims then at optimizing the contribution adjustments in order to minimize the total unanticipated cost for the sponsor. The interested reader may refer to Blake (2003) for a survey of similarities and differences between DB and DC pension schemes.

Contrary to previous chapters, we face here a minimization of a loss function: one seeks to reduce as much as possible the terminal surplus (which is here interpreted as a final contribution) and the quadratic spread between normal cost and contributions. The novelty of our work with respect to the existing literature on DB funds, is precisely to address this minimization problem in a model including stochastic mortality (a Poisson process), stochastic stocks return, salaries and interest rates. As in chapter 5, the return of the riskless asset is driven by a Vasicek model. The optimization program is solved by martingale approach and the best investment strategy is obtained by dynamic programming.

The major drawback of quadratic penalty functions is to penalize in a similar way positive and negative contribution adjustments. Despite this shortcoming, this allows to obtain very intuitive analytical results. We observe indeed that optimal contribution rate and investment policy are both function of unfunded liabilities. This quantity, well known from actuaries is the difference between the market value of pension commitments and the market value of assets and future normal costs. Furthermore, the contribution rate is the sum of the normal cost and of an actuarial adjustment equal to unfunded liabilities multiplied by an amortization factor. This factor is dynamic and depends on market conditions and on the residual time horizon.

In chapter 7, we have focused on a key risk factor for life insurers: the longevity risk. Over the last few decades it has become clear that the timing of death is getting latter on average. A bad anticipation of this evolution threads pension funds and, to a lesser extent, pensioners self managing their savings. To cope with this uncertainty over the mortality trend, actuaries have replaced deterministic mortality rates by stochastic processes, directly inspired from the financial literature and in particular from credit risk theory. A whole branch of papers have studied the use of affine processes as mortality hazard rates (see e.g. the works of Biffis 2005, Biffis and Denuit 2006).

Over recent years, Lévy processes have been developed to model the fat tail behaviour of stocks (see e.g. the numerous papers of Carr and Madan), stochastic volatility, or the default intensity of credit derivatives (see e.g. Schoutens (2006)). Based upon those results, we have addressed the modelling of human mortality by the aid of doubly stochastic processes which have a mean reverting intensity, driven by a Lévy process. Due to their analytical tractability, the chosen Lévy processes belong to the family of so-called $\alpha-$stable subordinators. For Gamma and Inverse Gaussian Lévy processes, survival probabilities have closed form expressions. In other cases, a numerical integration is required.

We propose two methods in order to valuate actuarial liabilities. The first one relies on a change of measure toward a risk neutral world. The second approach is based upon indifference pricing and on utility maximization. We also emphasize that the Lévy mortality can be embedded in an ALM framework such as those developed in previous chapters. Finally, numerical examples illustrate the ability of our model to capture the recent trends observed in longevity such as the rectangularization and the expansion.

Third part: chapters 8 and 9.

The third part of the dissertation is still devoted to the asset liability management of an institutional investor, but in the setting of complete markets. Chapter 8 studies the optimization of terminal wealth under a risk management constraint and Chapter 9 proposes a method to determine a fair bonus for life insurance participating policies with a guaranteed return.

The nature of employer-sponsored pension plans in Europe has changed dramatically in the past 20 years. Specifically, more employees are now participating in defined contribution plans than in defined benefit plans. In a defined contribution plan, the employer (and often the employee as well) makes specific contributions to an employee's pension fund and in most of Anglo-Saxon countries, the amount of the benefit is not guaranteed and depends on how well the investments perform. In this case, the fund manager monitors the exposure to financial risk and aligns the assets allocation on the risk tolerance of the scheme sponsor. The exposure to market risk is usually assessed by the Value-at-Risk (VaR) which measures the maximum shortfall for a given confidence level. The chapter 8 revisits the theory developed by Basak & Shapiro (2001) who embedded this risk management criterion into a utility maximization problem. Our main contribution is to determine the investment strategy maximizing the expected power utility of terminal wealth, under a VaR limitation and in presence of stochastic interest rates. Three assets may be purchased by the fund manager: cash, a zero coupon bond and a stock. Due to its ability to match perfectly the real yield curve, we assume that the dynamics of interest rates obeys to the Hull & White model.

The optimization method used in chapter 8 is based upon the martingale approach. The optimal wealth process is here the sum of three options payoffs and the calculation of its expectation and hedging strategy needs a forward change of measure. Our work also reveals the similarity between the investment strategies with and without bound on shortfall risk. Finally, examples illustrate the influence of confidence levels, preference factors and bond maturities on the assets allocation.

Finally, chapter 9 proposes a model to valuate a fair profit sharing rate for participating policies with a minimum interest rate guaranteed. The bonus credited to policies depends on the guarantee and on the performance of a basket of two assets: a stock and a zero coupon bond. The dynamics of the instantaneous short rates is driven by a Hull and White model. The participation level is determined such that the return retained by the insurer is sufficient to hedge the interest rate guarantee.

Again, we ignore the mortality risk and work in a complete market. Contrary to previous chapters, we don't need to define the investor's utility. The assumption of no arbitrage entails indeed that the pricing of a given contingent claim is done under an unique equivalent risk neutral measure. And under this measure, all investors are indifferent to the risk. This chapter also stands apart from the previous ones by considering only static investment strategies. The insurance company does not reallocate his assets till distribution of the bonus. The asset structure is then an input of our model: we only optimize the participating rate such that the transaction is fair both for policyholders and stockholders.

The novelty of our model with respect to the existing literature (based mainly on papers of Briys and de Varenne (1997a, 1997b)) is to consider that the insurer's asset is a non-rebalanced basket of stocks and zero coupon bonds. In this setting, the distribution of the whole portfolio is the sum of two lognormals and has no analytical expression. However, one can rely on the FFT algorithm to price the options embedded in participating policies. More precisely, we have applied the the multi-factor FFT method of Dempster and Hong (2000), initially developed to price spread options, in a faster way than Monte Carlo methods. The interested reader may refer to the paper of Barbarin & Devolder (2005) for a similar issue when assets are continuously rebalanced (constant proportion strategies): in this particular case, the distribution of the portfolio remains lognormal.

The numerical results of chapter 9, allow us to highlight the link existing between static investment policies and the cost of interest guarantees. Furthermore, they also reveal that the valuation of a fair participating rate is directly embedded in the ALM decisions: the profit sharing rate is fully determined by the assets portfolio and the interest guarantee.

Papers.

The chapters of the dissertation give rise to the following list of research papers:

Chapter 2 : "The annuity puzzle revisited: a deterministic version with Lagrangian methods". Belgian Actuarial Bulletin. 2006, vol 6 (1).

Chapter 3: "Life annuitization: Why and how much?" ASTIN Bulletin. 2006, vol 36 (2).

Chapter 4: "A martingale approach applied to the management of life insurances." Accepted for publication in the ICFAI journal of risk and insurance.

Chapter 5: "Management of a pension fund under financial and mortality risks." Insurance: Mathematics and Economics. 2007, vol 41.

Chapter 6: "Optimal funding of a defined benefits pension plan". Submitted to the Journal of Economics dynamics and Control.

Chapter 7: "Mortality modelling with Lévy processes". Accepted for publication in Insurance: Mathematics and Economics.

Chapter 8: "Dynamic assets allocation under VaR constraint, with stochastic interest rates". Submitted to the Annals of operations research. Referred on GloriaMundi.org.

Chapter 9: "Optimal design of the profit sharing rate by Fast Fourier Transform". Submitted to the Annals of actuarial sciences.