Age of river sediment: impact of the vertical structure of the model

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Introduction

Timescales such as the age or the residence time are frequently used to help understand the results of complex hydrodynamic models. Such diagnoses are usually related to passive tracers, including the water. Over recent years, attempts have been made to evaluate the age of other types of constituents of the water such as sediment (e.g. Mercier and Delhez 2007, Gong and Shen 2010, Ralston and Geyer 2017). This was achieved by having recourse to the tools of CART (Constituent-oriented Age and Residence time Theory, www.climate.be/cart).

Though calculating the age of sediment particles undoubtedly is a promising diagnostic method, there are specific issues (i.e. issues that do not arise when dealing with timescales related to dissolved constituents) that need be addressed for this approach to be considered trustworthy. Two of these problems are related to the evaluation of age fluxes at the lower boundary of the water column (i.e. age fluxes related to sediment settling and erosion) and the modelling of vertical fluxes inside the deposited sediment layer.

The aforementioned studies implicitly assumed that the deposited sediment layer (at least the layer exchanging sediment particles with the water column over timescales that are at most of the same order of magnitude as those considered in the model) is vertically well mixed. It is far from clear that such a simplifying hypothesis is acceptable (e.g. Toorman, 1996, 1999). This is why, in the present working note, the sensitivity of the sediment age to the vertical structure of the bottom sediment layer is investigated. A highly simplified approach is resorted to, suggesting that the sediment age in the bottom layer may significantly depend on the representation of vertical sediment fluxes in the deposited layer.

Hereinafter, only steady-state (i.e. time-independent) configurations are considered. Accordingly, the net vertical sediment flux at the bottom of the water column is zero. This does not imply that there is no vertical sediment motion through this boundary: the upward sediment flux must be equal to the downward one, but neither of these fluxes is necessarily zero (e.g. Einstein and Krone 1962). As far as the sediment mass budget is concerned, only the net flux is to be taken into account. However, for age calculations, the upward flux and the downward one must be determined so as to estimate the related age fluxes, the sum of which is unlikely to be zero.

In the present working note, the general principles of mass and age content budgets are briefly recalled. Then, a steady-state, one-dimensional model of a river is constructed. The age of the water is evaluated as well as that of sediment particles in both the water column and the deposited layer. A vertically well-mixed deposited sediment layer is first considered. Secondly, a two-layer model is worked out, leading to markedly different results in the lowest part of the domain of interest. Surprisingly, the sediment age in both the water column and in the upper part of the deposited layer is unchanged.

Though the features of the sediment particles under consideration are not explicitly specified, the idealised models dealt with hereinafter are likely to be relevant to fine, cohesive sediment. This is why no bedload transport is taken into account.

Mass and age content budgets: general principles

Consider a fixed (i.e. time-independent) elemental control domain denoted $\delta\Omega$. Its volume is δV (m^3), with $\delta V \to 0$. The control domain contains a number of constituents (water is one of them), be they dissolved or in particulate form. The evolution of the age of one of them is to be evaluated. The density of this constituent is denoted $\rho(t)$ ($\mathrm{kg\,m^{-3}}$) (t denotes the time), so that the mass of this constituent that is present in the elemental control volume is $\rho(t)\delta V$. At time t, the mean age of the particles of this constituent is $a(t)$, so that the age content, as defined in Deleersnijder et al. (2001), is $\rho(t)a(t)\delta V$.

Prior to examining the age budget of the elemental control domain, it is necessary to address its mass budget. The outgoing and incoming mass fluxes ($\mathrm{kg\,s^{-1}}$) are denoted $\Phi^{\mathrm{out}}(t)$ and $\Phi^{\mathrm{in}}(t)$, respectively. During the time interval $[t, t+\delta t]$, with $\delta t \to 0$, the mass of the constituent under study that is present in $\delta\Omega$ obeys (Figure 1)

$$\rho(t+\delta t)\delta V \sim \rho(t)\delta V + \left[\Phi^{\mathrm{in}}(t) - \Phi^{\mathrm{out}}(t)\right]\delta t \quad , \quad \delta t, \delta V \to 0 \quad . \tag{1}$$

In the limit $\delta t \to 0$, asymptotic expression (1) transforms to

$$\frac{d\rho}{dt} = \frac{\Phi^{\mathrm{in}} - \Phi^{\mathrm{out}}}{\delta V} \quad . \tag{2}$$

In accordance with the age averaging principle[1] laid out in Deleersnijder et al. (2001), the age content is an extensive — or additive — variable, whereas the age is an intensive variable. Accordingly, a budget of the age content may be established in a manner bearing similarities with the mass budget. The incoming and outgoing particles may have various ages, depending on the nature and origin of the associated fluxes. This is why it is appropriate to distinguish the fluxes of particles according to their age:

$$\Phi^{\mathrm{in}}(t) = \sum_{i=1}^{I} \Phi_i^{\mathrm{in}}(t) \tag{3a}$$

and

$$\Phi^{\mathrm{out}}(t) = \sum_{j=1}^{J} \Phi_j^{\mathrm{out}}(t) \quad . \tag{3b}$$

[1] Consider, for instance, two particles that are identified by means of subscripts "A" and "B". Their mass and age are denoted m^X and a^X, respectively, with $X=A,B$. The mass of the system "A+B" obvioulsy is $m^{A+B} = m^A + m^B$. This is in agreement with basic physical principles that stipulate that mass is an additive quantity. No such principle exists for the age. Therefore, to obtain the mean age of the system "A+B", an arbitrary decision is to be made. The latter was formulated as the so-called age-averaging hypothesis (Deleersnijder et al., 2001), which is the only arbitrary element of CART. It stipulates that the mean age of a system made up of various particles is the mass-weighted average of the ages of the particles. Accordingly, the mean age of the system "A+B", a^{A+B}, satisfies the follow relation: $m^{A+B}a^{A+B} = m^A a^A + m^B a^B$. Clearly, the age is not an additive quantity, but the age content is — the age content of a particle being defined as the product of its mass and its age. Ages other than CART's, such as the carbon-14 age, implicitly or explicitly rely on an age-averaging hypothesis (Deleersnijder et al., 2001). However, it is believed that CART's age-averaging hypothesis is the simplest one could think of.

Then, $a_i^{\rm in}(t)$ is the age of the particles of the i-th incoming flux, whilst $a_j^{\rm out}(t)$ is the age associated with $\Phi_j^{\rm out}(t)$. Thus, the associated incoming and outgoing age fluxes are $\Phi_i^{\rm in} a_i^{\rm in}$ and $\Phi_j^{\rm out} a_j^{\rm out}$. Another process, namely the ageing, must be taken into account in the age content budget: the age of a particle tends to increases as the same rate as time progresses, leading to the age content increment $\rho(t)\delta t \delta V$.

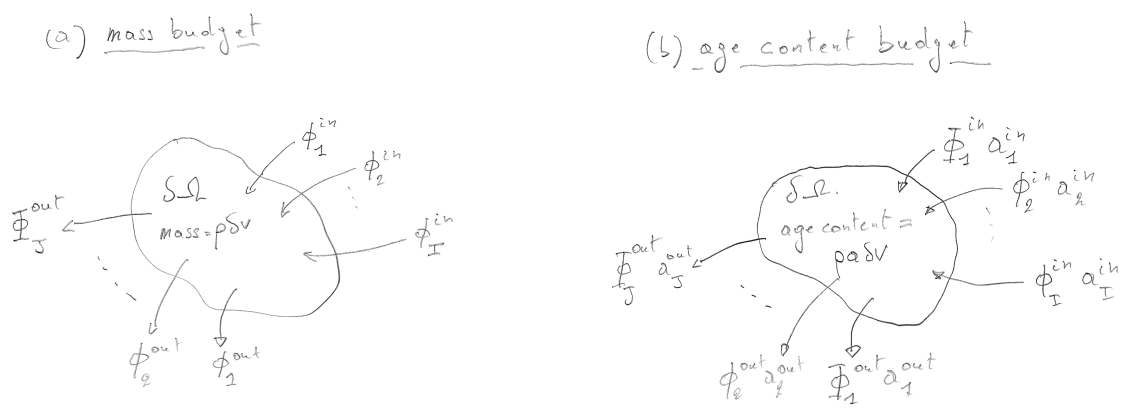

Figure 1. Illustration of the mass fluxes to be taken into account for establishing the mass budget of elemental control volume $\delta\Omega$ (a). The age fluxes contributing to the age content budget of the same elemental control volume are illustrated in panel (b).

Based on the above considerations, the age budget reads (Figure 1)

$$\overbrace{\rho(t+\delta t)a(t+\delta t)\delta V}^{\text{age content at time } t+\delta t} \sim \overbrace{\rho(t)a(t)\delta V}^{\text{age content at time } t} + \overbrace{\rho(t)\delta t \delta V}^{\text{ageing term}} + \underbrace{\left[\sum_{i=1}^{I} \Phi_i^{\rm in}(t) a_i^{\rm in}(t) - \sum_{j=1}^{J} \Phi_j^{\rm out}(t) a_j^{\rm out}(t)\right]\delta t}_{\text{contribution to the age content of the incoming and outgoing mass fluxes}} \quad , \quad \delta t \to 0 \tag{4}$$

In the limit $\delta t \to 0$, asymptotic expression (4) transforms to

$$\frac{d}{dt}(\rho a) = \rho + \frac{1}{\delta V}\sum_{i=1}^{I} \Phi_i^{\rm in} a_i^{\rm in} - \frac{1}{\delta V}\sum_{j=1}^{J} \Phi_j^{\rm out} a_j^{\rm out} \tag{5}$$

Combining (2), (3) and (5) readily yields

$$\frac{da}{dt} = 1 + \frac{1}{\rho \delta V}\sum_{i=1}^{i} \Phi_i^{\rm in}(a_i^{\rm in} - a) - \frac{1}{\rho \delta V}\sum_{j=1}^{J} \Phi_j^{\rm out}(a_j^{\rm out} - a) \ . \tag{6}$$

The first term in the right hand side of (6) obviously is related to the ageing process. The incoming fluxes tend to nudge the age of the control domain toward the age of the incoming particles. By contrast, the outgoing flux is such that $a(t)$ tends to increase (decrease) if the age of the outgoing particles is smaller (greater) than $a(t)$.

For the sake of illustration, assume that the control domain is sufficiently well mixed that the age of the outgoing particles is equal to the mean age of the domain ($a_j^{\mathrm{out}} = a$). Further assuming that there is no incoming flux and the age is prescribed to be zero at the initial instant ($t = 0$), the solution of (6) reads $a(t) = t$ whatever the value of the outgoing flux. This is because the age of the outgoing particles is equal to the mean age of the domain.

Now consider a slightly more complex case. The outgoing and incoming fluxes are time-independent and equal to eachother, implying that ρ is a constant. The age of the incoming particles is also constant, implying that the solution of age equation (6) is

$$a(t) \;=\; (\tau + a^{in})\left(1 - e^{-t/\tau}\right) \;, \tag{7}$$

with $\tau = \rho\delta V / \Phi^{\mathrm{in}} = \rho\delta V / \Phi^{\mathrm{out}}$. Clearly, age (7) tends to $\tau + a^{in}$ as time progresses. This is because the incoming particles have age a^{in} at the moment they enter the control domain and stay in it for a time that is, on average, equal to τ. All this is in agreement with elementary physical intuition.

Now that the basic principles of the establishment of the age content budget have been recalled, it is possible to turn to the riverine transport problem that is the central focus of this working note.

Riverine water age

The river under consideration is rectilinear with a rectangular cross section. Its constant depth and width are denoted H and W, respectively (Figure 2). Thus, the cross-sectional area is $S = WH$. Let x and z represent the along-river coordinate and the vertical one, with $z = 0$ at the water-air interface and $z = -H$ at the riverbed. The other horizontal coordinate is y, which is such that $-W/2 \le y \le W/2$. The water velocity, U, is constant and parallel to the x-axis, hence the volumetric flow rate ($\mathrm{m^3\,s^{-1}}$) is equal to $Q = SU = WHU$. The water density, ρ_w ($\mathrm{kg\,m^{-3}}$), is constant (Boussinesq approximation). Along-river diffusive processes are assumed to be negligible.

All of the variables are assumed to be at a steady-state.

The elemental control domain whose mass and age budgets are to be established is as follows:

$$\delta\Omega : \left[x - \frac{\delta x}{2} \,,\, x + \frac{\delta x}{2}\right] \times \left[y - \frac{W}{2} \,,\, y + \frac{W}{2}\right] \times \left[-H \,,\, 0\right] \,, \quad \delta x \to 0 \;. \tag{8}$$

The volume is $\delta V = S\delta x$. The incoming and outgoing mass fluxes are obviously equal to $\Phi^{\mathrm{in}} = \rho_w Q = \Phi^{\mathrm{out}}$, so that the mass of water present in the control domain remains equal to $\rho_w \delta V$. This is readily understood.

The water age, a_w, is assumed to be a function of only the longitudinal coordinate x. Since a steady state is established, the age content of the control domain must remain constant, implying that the ageing term and the transport term must balance each other:

$$0 \sim \underbrace{\rho_w\,\delta t\,\delta x W H}_{\text{ageing}} + \underbrace{\rho_w Q a_w(x - \delta x/2)\delta t}_{\text{incoming advection}} - \underbrace{\rho_w Q a_w(x + \delta x/2)\delta t}_{\text{outgoing advection}} \;, \quad \delta t, \delta x \to 0 \;. \tag{9}$$

In the limit $\delta x \to 0$, this asymptotic expression readily transforms to

$$0 = 1 - U\frac{da_w}{dx} \quad . \tag{10}$$

The water age is prescribed to zero at $x = 0$, yielding

$$a_w(x) = \frac{x}{U} \tag{11}$$

Thus, the water age is the time needed to travel distance x at speed U. This result is unsurprising.

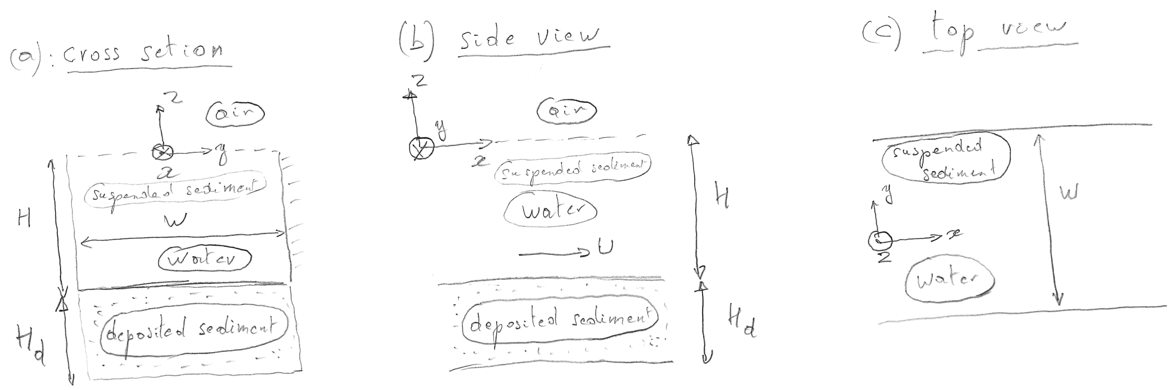

Figure 2. Geometry of the one-dimensional river model dealt with herein.

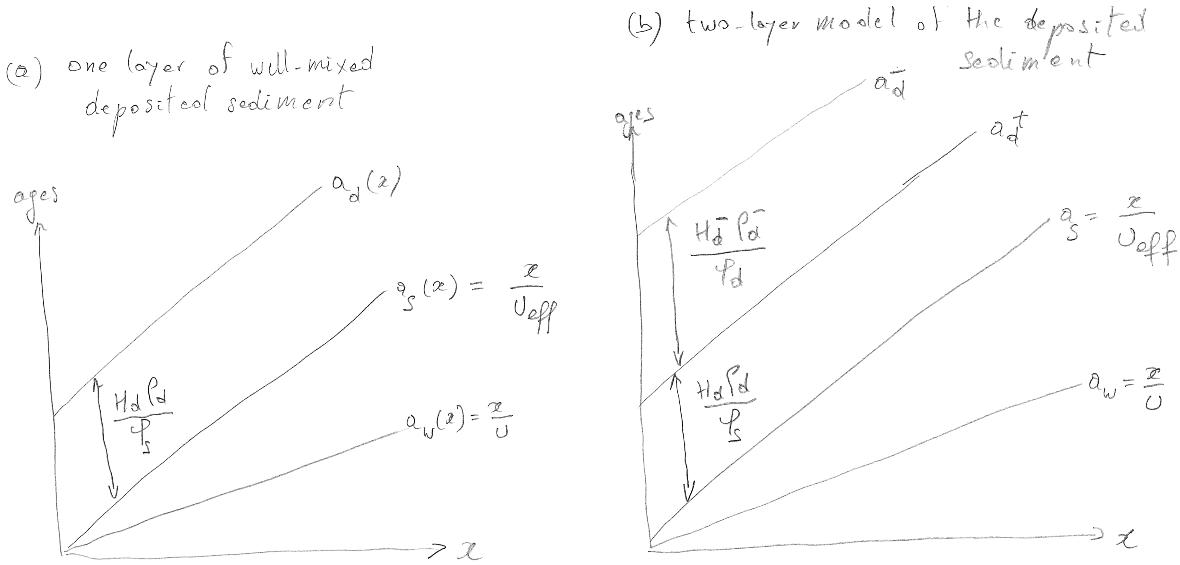

Figure 3. Illustration of the water and sediments ages for a well-mixed deposited sediment layer (a) and for a two-layer model of the deposited sediment (b).

Sediment age

Now consider sediment particles. There are sediment particles suspended in the water column and in the layer deposited on the riverbed, whose thickness is constant H_d (Figure 1). Both layers are assumed to be sufficiently well mixed that the associated sediment densities are independent of the vertical coordinate. The density of sediment particles that are in the water column and in the deposited layer is denoted $\rho_s(x)$ and $\rho_d(x)$, respectively. Since a steady state is assumed to prevail, the net flux of sediment at $z=-H$ is zero. However, sediment particles are continuously being deposited and resuspended. The erosion and settling sediment fluxes are equal to each other. The settling flux is denoted φ_s $(\mathrm{kg\,m^{-2}\,s^{-1}})$, with $\varphi_s > 0$ (Figure 4). Since all geometric and hydrodynamic variables do not depend on the along-river coordinate x, the aforementioned sediment densities must be independent of the along-river coordinate x. This can be verified easily by establishing the sediment budgets. For the control domain defined in (8), the sediment mass budget reads

$$0 \sim \overbrace{Q\rho_s(x-\delta x/2)\delta t}^{\text{incoming advection}} - \overbrace{Q\rho_s(x+\delta x/2)\delta t}^{\text{outgoing advection}} + \overbrace{\varphi_s\,\delta t\,\delta x W}^{\text{erosion}} - \overbrace{\varphi_s\,\delta t\,\delta x W}^{\text{settling}} \qquad (\delta t, \delta x \to 0) \tag{12}$$

In the limit $\delta x \to 0$, this asymptotic expression transforms to

$$0 = -U\frac{d\rho_s}{dx} \ . \tag{13}$$

Thus, as expected, the density of the suspended sediment is a constant. On the other hand, in the deposited sediment layer, there is no horizontal flux and the net vertical flux is zero. Therefore one may safely assume that the associated sediment density, ρ_d, is a constant.

The sediment ages are denoted $a_s(x)$ (water column) and $a_d(x)$ (deposited layer). Age $a_s(x)$ is prescribed to be zero at x=0, implying that the age is a measure of the time elapsed since leaving the point x=0. For the suspended sediment mass budget, the elemental control domain to be considered is that defined by (8). At a steady-state, its sediment age content budget reads

$$\begin{aligned} 0 \sim\ & \overbrace{\rho_s\,\delta t\,\delta x W H}^{\text{ageing}} + \overbrace{\rho_s Q a_s(x-\delta x/2)\delta t}^{\text{incoming advection}} - \overbrace{\rho_w Q a(x+\delta x/2)\delta t}^{\text{outgoing advection}} \\ & + \underbrace{\varphi_s a_d\,\delta t\,\delta x W}_{\text{erosion}} - \underbrace{\varphi_s a_s\,\delta t\,\delta x W}_{\text{settling}} \ , \quad \delta t, \delta x \to 0 \end{aligned} \tag{14}$$

In the limit $\delta x \to 0$, this asymptotic expression transforms to

$$0 = 1 - U\frac{da_s}{dx} + \frac{\varphi_s}{H\rho_s}(a_d - a_s) \ . \tag{15}$$

For the age budget in the deposited layer, the control domain is defined as follows

$$\delta\Omega : \left[x-\frac{\delta x}{2}\,,\ x+\frac{\delta x}{2}\right] \times \left[y-\frac{W}{2}\,,\ y+\frac{W}{2}\right] \times \left[-H-H_d\,,\ -H\right] \ , \quad \delta x \to 0 \ . \tag{16}$$

There is no horizontal sediment flux. The vertical flux at the bottom of the deposited layer is zero and so is the net flux at the riverbed-water interface. As a result, the mass budget is trivial: the density in the sediment layer is a constant, as expected. The associated age content budget reads

$$0 \sim \underbrace{\rho_d\,\delta t\,\delta x W H_d}_{\text{ageing}} - \underbrace{\varphi_s a_d\,\delta t\,\delta x W}_{\text{erosion}} + \underbrace{\varphi_s a_s\,\delta t\,\delta x W}_{\text{settling}} \ , \quad \delta t, \delta x \to 0 \ . \tag{17}$$

In the limit $\delta x \to 0$, this asymptotic expression transforms to

$$0 = 1 + \frac{\varphi_s}{H_d \rho_d}(a_s - a_d) \quad . \tag{18}$$

Combining (15) and (18) yields the age of the suspended sediment,

$$a_s(x) = \frac{H\rho_s + H_d\rho_d}{H\rho_s}\frac{x}{U} > \frac{x}{U} \tag{19}$$

as well as that of the deposited sediment (Figure 3)

$$a_d(x) = a_s(x) + \frac{H_d\rho_d}{\varphi_s} > a_s(x) \tag{20}$$

The suspended sediment age may be rewritten as $a_s(x) = x / U_{eff}$, where

$$U_{eff} = \frac{H\rho_s}{H\rho_s + H_d\rho_d} U < U \tag{21}$$

is the effective velocity of the sediment particles, which is smaller than the water velocity. This means that the downstream motion of the sediment proceeds at a speed that is smaller than that of the water, simply because sediment particles spend a fraction of the time in the deposited layer, in which there is no motion in the downstream direction. It is noteworthy that the effective velocity is independent of flux φ_s.

A more detailed physical interpretation of (19)-(21) should be obtained by having recourse to the concept of partial age (see Section 4 of Mouchet et al. 2017).

A two-layer model of the sediment deposited on the riverbed

Regarding the deposited sediment layer as vertically well mixed is most certainly an oversimplification. This is why a more elaborate model may need to be considered. The next level of complexity consists of two well-mixed layers, exchanging sediment particles. The net flux at the interface between the two layers is zero so as to be consistent with the steady state assumption that is the keystone of the present study. In other words, the upward flux φ_d (>0) is balanced by a flux in the opposite direction (Figure 4).

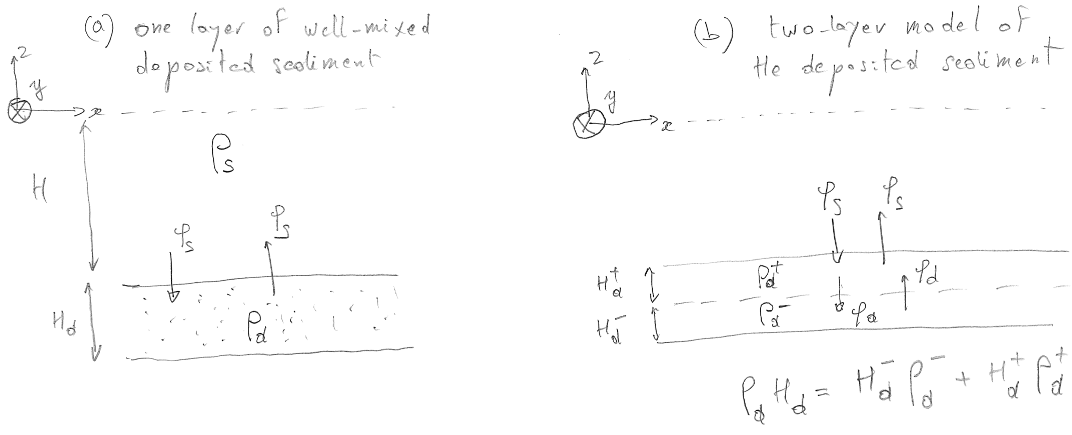

Figure 4. Vertical fluxes of sediment for the two models of the deposited sediment layer considered herein.

Let H_d^+ and H_d^- represent the height of the top layer of deposited sediment and the thickness of the lower one (Figure 1), respectively, with

$$H_d = H_d^- + H_d^+ \quad . \tag{22}$$

The corresponding densities are denoted ρ_d^+ and ρ_d^-. The vertical inventory of deposited sediment ($\mathrm{kg\,m^{-2}}$) must remain unchanged, implying

$$H_d\,\rho_d = H_d^-\,\rho_d^- + H_d^+\,\rho_d^+ \quad . \tag{23}$$

The age content budget of the upper layer of deposited sediment reads

$$\begin{aligned} 0 \sim\ & \overbrace{\rho_d^+\,\delta t\,\delta x W\,H_d^+}^{\text{ageing}} - \overbrace{\varphi_s\,a_d^+\,\delta t\,\delta x W}^{\text{erosion}} + \overbrace{\varphi_s\,a_s\,\delta t\,\delta x W}^{\text{settling}} \\ & \underbrace{-\ \varphi_d\,a_d^+\,\delta t\,\delta x W + \varphi_d\,a_d^-\,\delta t\,\delta x W}_{\text{exchange between the deposited layers}} \quad , \quad \delta t, \delta x \to 0 \end{aligned} \tag{24}$$

In the limit $\delta x \to 0$, this asymptotic expression transforms to

$$0 = 1 + \frac{\varphi_s}{H_d^+\rho_d^+}(a_s - a_d^+) - \frac{\varphi_d}{H_d^+\rho_d^+}(a_d^+ - a_d^-) \quad . \tag{25}$$

The age content budget of the lower layer of deposited sediment is as follows:

$$0 \sim \underbrace{\rho_d^-\,\delta t\,\delta x W\,H_d^-}_{\text{ageing}} \underbrace{+\ \varphi_d\,a_d^+\,\delta t\,\delta x W - \varphi_d\,a_d^-\,\delta t\,\delta x W}_{\text{exchange between the deposited layers}} \quad , \quad \delta t, \delta x \to 0 \tag{26}$$

In the limit $\delta x \to 0$, this asymptotic expression transforms to

$$0 = 1 + \frac{\varphi_d}{H_d^-\rho_d^-}(a_d^+ - a_d^-) \quad . \tag{27}$$

It is readily seen that seen the age of the suspended sediment is unchanged and that the age if the upper part of the deposited layer is equal to that obtained with the one-layer model, i.e.

$$a_d^+(x) = a_d(x) = a_s(x) + \frac{H_d\rho_d}{\varphi_s} \tag{28}$$

A convincing physical interpretation of these surprising results is yet to be developed. Presumably, the concept of partial age will be of use in this regard. On the other hand, the age in the lower layer of deposited sediment is (Figure 3)

$$a_d^-(x) = a_d^+(x) + \frac{H_d^-\rho_d^-}{\varphi_d} \tag{29}$$

Unsurprisingly, this age is a decreasing function of flux φ_d: the smaller this flux, the slower the sediment exchanges inside the deposited layer and, hence, the larger the age in the lower layer of deposited sediment.

Conclusion

The calculations above are based on several simplifying hypotheses (one-dimensional approach, steady-state assumption, no longitudinal diffusion). The age of the water simply is the ratio of the distance to the upstream boundary to the water velocity. Unsurprisingly, the age of suspended sediment particles is larger than that of the water. This is because sediment particles spend part of the time on the riverbed, where there are motionless: while they lie on the riverbed, their age keeps increasing at the same pace as time progresses. This led to the introduction of the effective sediment velocity, which is smaller than the water velocity. Increasing the complexity of the model of the deposited sediment layer (by switching from a one-layer approach to a two-layer one) does not modify the age of the suspended sediment particles. This has yet to be explained. However, the sediment age in the lower layer may increase significantly.

Initialising the sediment age, especially in the deposited layer, is far from trivial. This issue was barely touched upon in previous publications. It is possible that the developments carried out above will turn out to be instrumental in helping to solve this problem.

It must be stressed that all of the vertical sediment fluxes were assumed to be known in this study. In practice, however, they would have to be parameterised, which may lead to significant difficulties. This is because a number of widely-used parameterisations focus on net sediment fluxes and do not allow splitting in a straightforward manner the net fluxes in upward and downward fluxes. This might become a major stumbling block in the way towards the improvement of the estimation of the age of sediment particles.

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