

Unequal Longevities and Lifestyles Transmission*

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Abstract

Whereas studies on the optimal taxation under endogenous longevity assume a fixed heterogeneity of lifestyles, this paper analyses the optimal tax policy in an economy where unequal longevities are the unintended outcome of differences in lifestyles, and where lifestyles are transmitted across generations. For that purpose, we develop a three-period OLG model where the population, who ignores the negative impact of excessive work on longevity, is partitioned in two groups with different tastes for leisure, and follows an adaptation/imitation process *à la* Bisin and Verdier (2001). The optimal short-run and long-run Pigouvian taxes on wages are shown to differ, because the latter correct agents' myopia, but also internalize intergenerational externalities due to the socialization process. The internalization of composition effects raises the Pigouvian tax on the wage of one type of agents, but reduces it on the other type, in such a way as to induce the optimal long-run partition of the population.

Keywords: longevity, OLG model, lifestyle, socialization, intergenerational externalities, Pigouvian taxes.

JEL codes: I12, I18, Z13.

1 Introduction

Although longevity differentials can be partly explained by genetic and environmental factors, *how long* one lives depends also on *how* one lives. Actually, various dimensions of lifestyles are shown to affect longevity, so that unequal longevities reflect - among other things - the diversity of lifestyles.¹

The existence of longevity inequalities due to heterogeneous lifestyles raises the question of the necessity, for a government, to intervene in that context. In

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¹Longevity was shown to be affected, *ceteris paribus*, by smoking (Doll and Hill, 1950), by alcoholism (Poikolainen, 1982), by the quantity and quality of food (Bender *et al*, 1998; Stamler, 1973), and by physical activity (Paffenbarger *et al*, 1978).

particular, should a utilitarian government do anything in front of such inequalities in longevity due to lifestyle differentials?

That question was recently explored by Leroux *et al* (2008), who studied the optimal tax policy in an economy where agents, because of differences in productivity, in genetic backgrounds, and in their degree of myopia, choose different levels of longevity-enhancing (monetary) efforts. In that paper, whether health spending should be taxed or subsidized depends on two major determinants. On the one hand, the myopia of agents, which prevents them from seeing the positive impact of health spending on longevity, invites a corrective subsidy on health spending. On the other hand, given that investing in health can involve an externality, in the sense that agents do not necessarily internalize the impact of health spending on the resources available per person at the old age, a tax on health expenditures may be required to internalize that externality.²

Whereas that study highlights some determinants of the optimal subsidy on health spending, one of its limitations is to take the partition of the population into different groups as *fixed*. The postulate of a fixed heterogeneity of the population is strong, as one may expect that the composition of a population into different groups having distinct lifestyles is likely to vary over time. Moreover, the heterogeneity of the population may also be influenced by the government itself, so that the question of the optimal structure of heterogeneity can hardly be escaped. Hence, the endogeneity of the composition of the population raises additional questions regarding the optimal public intervention.

The goal of this paper is to study the optimal taxation policy in an economy where unequal longevity result from differences in lifestyles, and where the composition of the population into different lifestyles is endogenously determined in the economy. As in Leroux *et al* (2008), the optimal policy will still depend here not only on how ‘rational’ agents are in their decisions (i.e. on the extent to which one’s lifestyle maximizes one’s lifetime welfare), but, also, on how individual behaviours affect social welfare (i.e. on the extent to which one’s lifestyle affects others’ welfares). In order to do justice to those two determinants of the optimal policy, the specificity of this paper will be twofold.

First, this paper focuses on an economy where longevity differentials are the *unintended* outcome of differences in lifestyles, in the sense that agents suffer from a myopia preventing them from seeing the impact of their lifestyle on longevity. Under that myopia, the chosen lifestyle does not maximize individual lifetime welfare, so that a State correction is required. Second, this paper considers another unintended effect of lifestyles: their impact on the *taste composition* of future cohorts. Actually, the partition of a population into groups with distinct lifestyles is not exogenous here, but is affected by agents’ choices. Given that agents’ lifestyles shape the tastes composition of next cohorts, the resulting unintended welfare effects invite also a correction.

Note that the corrections required by agents’ myopia may differ from the ones aimed at internalizing intergenerational ‘composition effects’. Hence, it is not easy to see what a utilitarian government should do in this context.

²That externality was firstly emphasized by Becker and Philipson (1998).

This paper aims at casting a new light on that issue, by studying the optimal tax policy in an economy where the partition of the population in distinct (longevity-affecting) lifestyles is endogenous. That issue has not been studied so far, as dynamic studies on the optimal policy under endogenous longevity focussed on homogeneous populations (see Zhang *et al*, 2006), or involved an unexplained heterogeneity (see Pestieau *et al*, 2008).³ On the contrary, this paper studies optimal policy under unequal longevity due to an explicit cause, the adopted lifestyle, whose transmission across generations is modelled.

For that purpose, we set up a three-period OLG model, where the length of the retirement period (i.e. the third period) depends on the lifestyle adopted during the active life (i.e. the second period). For simplicity, agents differ on a *single* dimension of lifestyle: the length of working time, following Nylen *et al*'s (2001) study showing that overwork leads, *ceteris paribus*, to a shorter life.⁴ Thus, each cohort, who ignores the impact of work on longevity, is, at adulthood, divided into two types of agents, having a more or less strong taste for leisure, and choosing their working time accordingly.

To study the optimal policy under an endogenous composition of the population, it is assumed that the adherence to a lifestyle is acquired, during childhood, through an adaptation and imitation process *à la* Bisin and Verdier (2001), where parents' socialization efforts and the social environment interact. Note that the socialization process and the demography influence each others strongly in our economy: while the (aggregated) life expectancy depends on the composition of the population into different types, the socialization efforts chosen by parents are increasing in their (type-specific) time horizons, so that the long-run composition of the population is determined by the demography.

The major result of this paper is to highlight that the policies required for the decentralizations of the short-run social optimum - i.e. under a *fixed* partition of the population - and of the long-run social optimum - i.e. under an *endogenous* partition - differ, as the latter involves not only a correction for agents' myopia, but, also, a correction for unintended 'composition effects' of individual labour choices on future cohorts' tastes under the prevailing socialization mechanism. More precisely, the internalization of composition effects raises the Pigouvian tax on the wage of one type of agents, but reduces it on the other type, in such a way as to induce the optimal long-run partition of the population. Hence, this paper emphasizes a dilemma between, on the one hand, correcting the myopia of existing people and, on the other hand, the internalization of the intergenerational externalities due to the socialization process.

The rest of this paper is organized as follows. Section 2 presents the model, describes the temporary equilibrium, and examines the existence, uniqueness and stability of steady-state equilibria. Section 3 describes the short-run and long-run social optima, and studies their decentralization. Section 4 concludes.

³By 'unexplained heterogeneity', I mean that nothing explains, in the model, why some members of a cohort live longer than others.

⁴See also Van Echtelt (2005) on the negative impact of overwork on longevity. Given the tendency of extra work to favor heart-diseases (see Hayashi *et al*, 1996), diabetes (see Kawakami *et al*, 1999) and obesity (see Nakamura *et al*, 1998), that relation is quite robust.

2 The model

2.1 Environment

Let us consider a three-period OLG model. Each cohort is a continuum of agents, with a measure normalized to 1.

Period 1, which is of unitary length, is a period of childhood, during which one has no consumption, does not work, and is only subject to socialization.⁵ During period 2, which is also of unitary length, individuals work, enjoy some leisure and consume. Period 3 is a period of retirement, whose length depends on the lifestyle adopted during the active life (i.e. period 2).

During their childhood (period 1), all members of a cohort are the same, and have no cultural type. However, once adult (in periods 2 and 3), a cohort becomes divided in two kinds of agents, depending on their preferences. We shall, in this paper, assume that the two kinds of agents differ only regarding the intensity of their taste for leisure with respect to consumption. Thus, each adult agent within a cohort is either of type C , i.e. ‘consumption-lover’, or of type L , i.e. ‘leisure-lover’. Agents of type $i \in \{C, L\}$ work during their second period of life, have one child, and are retired in period 3.

Throughout the paper, the mass of agents born at time t who are of type C when being young adults (at time $t+1$) shall be denoted by q_{t+1} ($0 \leq q_{t+1} \leq 1$), while the mass of young adults of type L is equal to $1 - q_{t+1}$.

Longevity Agents of type $i \in \{C, L\}$ born at t live periods 1 and 2, of length one. The retirement period (period 3) has a length h_{t+2}^i ($0 \leq h_{t+2}^i \leq 1$) determined by the lifestyle adopted during the active life (period 2). While the length of the retirement period is a function of various factors (including, for instance, sin goods consumption), we shall, for simplicity, assume that h_{t+2}^i depends on a single aspect of lifestyle. Following studies by Nylen *et al* (2001) and Van Echtelt (2005) on the impact of work on longevity, it is assumed that the working time has a negative and increasing impact on longevity.⁶ This amounts to postulate that the length of the retirement period depends only on the leisure time enjoyed during the active life, l_{t+1}^i ($0 \leq l_{t+1}^i \leq 1$):

$$h_{t+2}^i = h(l_{t+1}^i) \quad (1)$$

where $h'(l_{t+1}^i) \geq 0$, $h''(l_{t+1}^i) \leq 0$.

For analytical convenience, we shall, in this paper, assume that:

$$h(l_{t+1}^i) = \frac{l_{t+1}^i}{1 + l_{t+1}^i} \quad (2)$$

That functional form, based on Chakraborty (2004), satisfies the above conditions, and allows us to solve the model analytically.

⁵Hence, the model behaves like a two-period OLG model.

⁶On the impact of work on longevity, see also Hayashi *et al* (1996), Kawakami *et al* (1999) and Nakamura *et al* (1998).

Production For simplicity, production is assumed to be linear in labour:

$$y_t = wL_t \quad (3)$$

where y_t is the output, w is a productivity parameter, and L_t is the total labour force, defined as $L_t = q_t(1 - l_t^C) + (1 - q_t)(1 - l_t^L)$, where $1 - l_t^i$ is the labour supplied by agents of type $i \in \{C, L\}$.⁷ Factors are paid at their marginal productivity: each unit of labour supplied is paid at a wage w .

Socialization The population follows an adaptation and imitation process *à la* Bisin and Verdier (2001). The transmission of the cultural trait $i \in \{C, L\}$ is modelled as a mechanism where socialization *inside* the family ('direct vertical' socialization) and socialization *outside* the family ('oblique' socialization) interact.

Families are composed of a single parent and a single child. Children are born at time t without any defined cultural trait, and are first exposed to their parent's lifestyle. Direct vertical socialization to the parent's trait $i \in \{C, L\}$ occurs with a probability ρ_{t+1}^i . If the direct vertical socialization to the parent's trait does not take place, which happens with a probability $1 - \rho_{t+1}^i$, the child will then pick up the trait of a person - i.e. a 'role model' - chosen randomly in the population of young adults, and, thus, will pick the trait C with a probability q_t , and the trait L with a probability $1 - q_t$.

Hence, if p_{t+1}^{CC} and p_{t+1}^{CL} (resp. p_{t+1}^{LL} and p_{t+1}^{LC}) denote the probabilities that a child born at t in a family with trait C (resp. L) is socialized to, respectively, trait C and trait L (resp. L and C), the transition probabilities are:

$$\begin{aligned} p_{t+1}^{CC} &= \rho_{t+1}^C + (1 - \rho_{t+1}^C) q_t & p_{t+1}^{CL} &= (1 - \rho_{t+1}^C) (1 - q_t) \\ p_{t+1}^{LL} &= \rho_{t+1}^L + (1 - \rho_{t+1}^L) (1 - q_t) & p_{t+1}^{LC} &= (1 - \rho_{t+1}^L) (q_t) \end{aligned} \quad (4)$$

By the Law of Large Numbers, p_{t+1}^{ij} is also equal to the proportion of children whose parents are of type i who have the cultural trait j . Hence, the proportion q_{t+1} of agents born at time t who become of type C follows the dynamic law:

$$q_{t+1} = [\rho_{t+1}^C + (1 - \rho_{t+1}^C) q_t] q_t + [(1 - \rho_{t+1}^L) q_t] (1 - q_t) \quad (5)$$

The first term of (5) is the probability to be socialized to trait C when having a family of type C , multiplied by the probability to belong to a family of type C . The second term is the probability to acquire trait C when being born in a family of type L , multiplied by the probability to belong to a family of type L .

Following Bisin and Verdier (2001), we assume that parents of type $i \in \{C, L\}$ can socialize their children born at time t vertically, by educating them through a (purely physical) socialization effort e_t^i ($0 \leq e_t^i \leq 1$). The socialization effort e_t^i is an input in the cultural production of their children as adults: $\rho_{t+1}^i = \rho(e_t^i)$. A welfare loss $C(e_t^i)$ is generated by a socialization effort e_t^i .⁸

⁷ The hourly productivities of the two types of agents are assumed to be equal.

⁸ We assume $C(0) = 0$, $C(e_t^i) > 0$ for $e_t^i > 0$, $C'(e_t^i) > 0$ and $C''(e_t^i) > 0$.

Parents have a welfare gain from coexisting with children with the *same* type as themselves. The welfare derived by a parent of type i born at $t - 1$ when he coexists with a child of type i , denoted by φ_{t+1}^{ii} , exceeds the welfare derived by a parent of type i when he coexists with a child of type $j \neq i$, denoted φ_{t+1}^{ij} . In this paper, welfare gains from having children of one's own type are independent from one's type and from time ($\varphi_{t+1}^{CC} = \varphi_{t+1}^{LL} = \hat{\varphi}$ and $\varphi_{t+1}^{CL} = \varphi_{t+1}^{LC} = \tilde{\varphi}$).

Hence, under additive lifetime welfare, the socialization effort e_t^i chosen by a parent of type i maximizes:

$$u_{t-1}^i = u(c_t^i, l_t^i) - C(e_t^i) + E(h_{t+1}^i) \left[u(E(d_{t+1}^i), 1) + p_{t+1}^{ii}(e_t^i)\hat{\varphi} + p_{t+1}^{ij}(e_t^i)\tilde{\varphi} \right] \quad (6)$$

where $u(c_t^i, l_t^i)$ is the utility of period 2, which involves a consumption c_t^i and a leisure l_t^i , and $E(h_{t+1}^i) \left[u(E(d_{t+1}^i), 1) + p_{t+1}^{ii}(e_t^i)\hat{\varphi} + p_{t+1}^{ij}(e_t^i)\tilde{\varphi} \right]$ is the expected utility of period 3, which involves an (expected) annuitized consumption $E(d_{t+1}^i)$, and no work. $E(h_{t+1}^i)$ is the expected length of period 3.⁹

Parents, when choosing e_t^i , weight the cost of socialization - $C(e_t^i)$ - against its expected gains - $E(h_{t+1}^i) \left[p_{t+1}^{ii}(e_t^i)\hat{\varphi} + p_{t+1}^{ij}(e_t^i)\tilde{\varphi} \right]$ -, which depend on the influence of their effort e_t^i on probabilities p_{t+1}^{ii} and p_{t+1}^{ij} , determined by the relation $\rho_{t+1}^i = \rho(e_t^i)$, and on the expected length of their retirement period, $E(h_{t+1}^i)$.

While there exist various ways to model the relation $\rho_{t+1}^i = \rho(e_t^i)$, we will assume that the probability of direct vertical socialization to trait i ρ_{t+1}^i equals parent's socialization effort e_t^i , in conformity with what Bisin and Verdier (2001) call the '*It's the family*' transmission technology:

$$\rho_{t+1}^i = e_t^i \quad (7)$$

That technology, which is most adequate for our purpose, exhibits what Bisin and Verdier call 'cultural substitution': a higher proportion of agents of type i in the population of reference makes parents of type i choose a *lower* effort.¹⁰

Agents' decisions In order to describe how agents of types C and L make their decisions, we need to make three additional assumptions.

First, temporal welfare $u(c_t^i, l_t^i)$ is, for analytical convenience, assumed to take a Cobb-Douglas form in consumption and leisure time:

$$u(c^i, l^i) = (c^i)^{\theta^i} (l^i)^{1-\theta^i} \quad (8)$$

where $0 < \theta^i < 1$. Given that θ^i reflects the importance of consumption with respect to leisure, we have here $\theta^C > \theta^L$.

⁹ For simplicity, there is no pure time preference here.

¹⁰ As shown by Bisin and Verdier (2001), technologies with no cultural substitutability can lead to stable equilibria without heterogeneity, which is inadequate for the purpose of this study.

Second, each agent - whatever his type - does not, when making his decisions, internalize their impact on longevity, but forms myopic anticipations, in the sense that he expects to have a life as long as the one of his ‘model’ adult:

$$E(h_{t+1}^i) = h_t^i \quad (9)$$

This assumption consists of type-specific myopic anticipations.

Third, the welfare cost from socialization efforts, $C(e_t^i)$, takes the form:

$$C(e_t^i) = \frac{\xi(e_t^i)^2}{2} \quad (10)$$

where ξ accounts for the disutility of socialization effort ($\xi > 0$).

Substituting $u(c^i, l^i)$, $E(h_{t+1}^i)$ and $C(e_t^i)$ in the function $u_{i,t-1}$ allows us to rewrite the expected lifetime welfare of an agent of type i born at $t - 1$ as:¹¹

$$u_{t-1}^i = (c^i)^{\theta^i} (l^i)^{1-\theta^i} - \frac{\xi(e_t^i)^2}{2} + h_t^i \left[(d_{t+1}^i)^{\theta^i} + p_{t+1}^{ii} \hat{\varphi} + p_{t+1}^{ij} \tilde{\varphi} \right] \quad (11)$$

Let us now focus on agents’ decisions, which are all taken at the beginning of adulthood (i.e. of period 2). At that time, each agent of type $i \in \{C, L\}$ chooses second and third-period consumptions, as well as socialization effort and leisure time, in such a way as to maximize his welfare subject to his budget constraint:

$$\begin{aligned} \max_{c_t^i, d_{t+1}^i, e_t^i, l_t^i} & (c_t^i)^{\theta^i} (l_t^i)^{1-\theta^i} - \frac{\xi(e_t^i)^2}{2} + h_t^i \left[(d_{t+1}^i)^{\theta^i} + p_{t+1}^{ii} \hat{\varphi} + p_{t+1}^{ij} \tilde{\varphi} \right] \\ \text{s.t.} \quad & c_t^i + h_t^i d_{t+1}^i \leq w(1 - l_t^i) \end{aligned}$$

Assuming an interior solution, the FOCs of that problem are:

$$\begin{aligned} \theta^C (c_t^C)^{\theta^C-1} (l_t^C)^{1-\theta^C} &= \lambda^C & \theta^L (c_t^L)^{\theta^L-1} (l_t^L)^{1-\theta^L} &= \lambda^L \\ \theta^C (d_{t+1}^C)^{\theta^C-1} &= \lambda^C & \theta^L (d_{t+1}^L)^{\theta^L-1} &= \lambda^L \\ e_t^C &= \frac{h_t^C (1-q_t)(\hat{\varphi}-\tilde{\varphi})}{\xi} & e_t^L &= \frac{h_t^L (q_t)(\hat{\varphi}-\tilde{\varphi})}{\xi} \\ (1-\theta^C)(c_t^C)^{\theta^C} (l_t^C)^{-\theta^C} &= \lambda^C w & (1-\theta^L)(c_t^L)^{\theta^L} (l_t^L)^{-\theta^L} &= \lambda^L w \end{aligned}$$

where λ^C and λ^L denote the Lagrange multipliers associated with the budget constraints of agents of types C and L .¹²

Grouping the first two FOCs shows that, for all agents, second-period consumption is lower than third-period consumption. That result comes from our utility function, under which the welfare return of consumption is higher the larger leisure time is, so that agents choose to consume more when retired.

Regarding socialization, one can notice that cultural substitution holds here: e_t^C is decreasing in q_t , and e_t^L is increasing in q_t . Note also that the expected longevity has a positive impact on the chosen socialization effort: the longer the

¹¹We abstract here from childhood.

¹²Given that e_t^C and e_t^L lie, for any value of q_t , between 0 and 1, it must be assumed here that $0 \leq h_t^C (\hat{\varphi} - \tilde{\varphi}) / \xi \leq 1$ and $0 \leq h_t^L (\hat{\varphi} - \tilde{\varphi}) / \xi \leq 1$.

third period is expected to be, the longer the expected period of coexistence with a child of one's type is. Hence, a larger expected longevity leads to a larger welfare gain from socialization, and, thus, to a higher socialization effort. If $h_t^C < h_t^L$, one can expect that parents of type L choose, *ceteris paribus*, a larger socialization effort than agents of type C . As we shall see, this *horizon effect* has a crucial impact on the long-run dynamics of the population structure.

As far as the choice of leisure is concerned, combining the first two FOCs with the fourth one and substituting the budget constraint yields:¹³

$$l_t^C = 1 - \theta^C(1 + h_t^C) \quad l_t^L = 1 - \theta^L(1 + h_t^L)$$

In the light of those expressions, it is not obvious to see whether agents with a lower taste for leisure will necessarily choose a lower leisure level, as there are two effects at work. If agents had the same expected longevity (i.e. $h_t^C = h_t^L$), it would follow, under $\theta^C > \theta^L$, that agents with a low taste for leisure would choose a lower leisure ($l_t^C < l_t^L$). However, the leisure decision depends also on the expected lifetime. Thus, under $h_t^C < h_t^L$, it might be the case that agents of type C choose, despite $\theta^C > \theta^L$, a higher leisure, because they expect to live a shorter life, which makes labour less useful. Note that, if $\theta^C/\theta^L > 2$, the first effect necessarily dominates the second one, so that, whatever the prevailing group-specific longevitys are, agents of type L choose necessarily a higher leisure, even though they expect to live longer than agents of type C . Our results are summarized in Proposition 1.¹⁴

Proposition 1 *Under the laissez-faire,*

- $c_t^i \leq d_{t+1}^i \forall i \in \{C, L\}$,
- if $h_t^C = h_t^L = \bar{h}_t$, then $l_t^C < l_t^L$, $d_{t+1}^C > d_{t+1}^L$ and $\frac{c_t^C}{l_t^C} > \frac{c_t^L}{l_t^L}$,
- if $h_t^C \neq h_t^L$, then $l_t^C \leq l_t^L$, $d_{t+1}^C \geq d_{t+1}^L$ and $\frac{c_t^C}{l_t^C} \geq \frac{c_t^L}{l_t^L}$ iff $\frac{\theta^C}{\theta^L} \geq \frac{1+h_t^L}{1+h_t^C}$,
- under $h_t^C < h_t^L$ and $\frac{\theta^C}{\theta^L} > 2$, we have $l_t^C < l_t^L$, $d_{t+1}^C > d_{t+1}^L$ and $\frac{c_t^C}{l_t^C} > \frac{c_t^L}{l_t^L}$,
- $e_t^C \geq e_t^L \iff \frac{1-q_t}{q_t} \geq \frac{h_t^L}{h_t^C}$.

Regarding the last part of Proposition 1, note that whether agents of type C make a larger socialization effort than agents of type L depends not only on the prevailing partition of the population (as in Bisin and Verdier, 2001), but, also, on group-specific longevitys. The longer the retirement period is expected to be, the more the adult will socialize his child *ceteris paribus*.

2.2 Temporary equilibrium

We are now in position to characterize the temporary equilibrium of the economy. As usual, the temporary equilibrium is such that all agents are 'reaching

¹³In order to guarantee a non-negative leisure level [i.e. $l_t^i \geq 0 \forall i \in \{C, L\}$], we shall, in the rest of this paper, assume that $\theta^i(1 + h_t^i) \leq 1 \forall h_t^i \in [0, 1]$, which is equivalent to $\theta^i \leq 1/2 \forall i \in \{C, L\}$.

¹⁴See the Appendix for the proof.

their best' given the constraints they face and given the expectations they have at the moment. The temporary equilibrium can be characterized as follows.¹⁵

Given the past partition q_{t-1} , given past group-specific longevities h_{t-1}^i , leisures l_{t-1}^i and savings s_{t-1}^i , and given group-specific expectations $E(h_{t+1}^i) = h_t^i$ for $i \in \{C, L\}$, the temporary equilibrium of time t is defined by:

- group-specific second-period leisures l_t^i , consumptions c_t^i , and savings $s_t^i = w(1 - l_t^i) - c_t^i$ satisfying the above FOCs;
- group-specific third-period consumptions d_t^i equal to s_{t-1}^i/h_t^i ;
- group-specific socialization efforts $e_t^i = h_t^i(1 - q_t^i)(\hat{\varphi} - \tilde{\varphi})/\xi$;¹⁶
- group-specific lengths of the retirement period h_t^i equal to $l_{t-1}^i/(1 + l_{t-1}^i)$;
- the partition of the population q_t equal to $q_t = F(q_{t-1}, h_{t-1}^C, h_{t-1}^L)$;
- the aggregate production $y_t = q_t(1 - l_t^C)w + (1 - q_t)(1 - l_t^L)w$.

Note that the temporary equilibrium at t can be written as¹⁷

$$\begin{aligned} l_t^i &= l(h_t^i) & e_t^i &= e(q_t, h_t^i) \\ c_t^i &= c(l_t^i) = c(l(h_t^i)) & q_t &= F(q_{t-1}, h_{t-1}^C, h_{t-1}^L) \\ s_t^i &= s(l_t^i, c(l_t^i)) = s(l_t^i) = s(l(h_t^i)) & h_t^i &= h(l_{t-1}^i) = h(l(h_{t-1}^i)) \\ d_t^i &= s_{t-1}^i/h_t^i & y_t &= y(q_t, l_t^C, l_t^L) = y(q_t, h_t^C, h_t^L) \end{aligned}$$

Given that $l(\cdot)$, $c(\cdot)$, $s(\cdot)$, $e(\cdot)$ and $y(\cdot)$ are (single-valued) functions, the above system defines a unique temporary equilibrium. This equilibrium can be characterized in terms of only three variables: q_t , h_t^C and h_t^L . Hence, the constancy of q_t , h_t^C and h_t^L over time leads to the constancy of all other variables.

2.3 Steady-state equilibria

Given that the constancy of q_t , h_t^C and h_t^L implies the stationarity of all other variables, we can now study the existence of a steady-state equilibrium in the three-dimensional space (q_t, h_t^C, h_t^L) . For that purpose, we shall first define the qq , hh^C and hh^L loci, along which respectively q_t , h_t^C and h_t^L are constant, and, then, examine whether those loci intersect in the (q_t, h_t^C, h_t^L) space.

Substituting for $\rho_{t+1}^C = e_t^C$ and $\rho_{t+1}^L = e_t^L$ in expression (5) yields:

$$q_{t+1} = q_t + \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} (h_t^C(1 - q_t)^2 q_t - h_t^L(q_t)^2(1 - q_t))$$

¹⁵Note that q_t depends on q_{t-1} and e_{t-1}^i for $i \in \{C, L\}$. Given that e_{t-1}^i is a function of q_{t-1} and h_{t-1}^i , the current partition q_t can be written as a function $F(q_{t-1}, h_{t-1}^C, h_{t-1}^L)$.

¹⁶ q_t^i denotes the proportion of agents of type i [i.e. $q_t^C \equiv q_t$; $q_t^L \equiv 1 - q_t$].

¹⁷Regarding consumption and savings, substituting the budget constraint in the FOCs yields $s_t^i = \frac{\theta^i h_t^i w}{1 - \theta^i}$, from which it follows that s_t^i and c_t^i can be written as functions of h_t^i only.

Fixing $q_{t+1} = q_t = q$ in that expression allows us to derive the qq locus, along which q is constant. Actually, there exist three values of q yielding the constancy of the composition of the population over time:

$$q = 0; \quad q = \frac{h^C}{h^L + h^C}; \quad q = 1$$

Thus, the qq locus is made of those three components.

At the steady-state, it must also be the case that agent-specific longevities are constant. h_{t+1}^C can be written as:

$$h_{t+1}^C = \frac{1 - \theta^C(1 + h_t^C)}{2 - \theta^C(1 + h_t^C)}$$

Hence, $h_{t+1}^C = h_t^C$ defines the hh^C locus as follows:

$$h^C = \frac{1 - \sqrt[2]{1 - \theta^C(1 - \theta^C)}}{\theta^C}$$

The steady-state value of h^C depends only on θ^C , and is decreasing in θ^C .

Similarly, from

$$h_{t+1}^L = \frac{1 - \theta^L(1 + h_t^L)}{2 - \theta^L(1 + h_t^L)}$$

one can derive the hh^L locus, along which h^L is constant over time:

$$h^L = \frac{1 - \sqrt[2]{1 - \theta^L(1 - \theta^L)}}{\theta^L}$$

That long-run value of h^L depends only on θ^L , and is decreasing in θ^L .

Regarding the existence of an intersection of the qq , hh^C and hh^L loci in the (q_t, h_t^C, h_t^L) space, it is clear that, if q equals 1 (resp. 0), the variable h^L (resp. h^C) is not defined, so that the system becomes 2-dimensional, and the steady-state consists merely of the intersection of the hh^C locus (resp. hh^L locus) with the qq locus corresponding to $q = 1$ (resp. $q = 0$). The qq and hh^C loci (resp. hh^L locus) being non parallel lines, these must intersect only once. Thus, the economy admits two steady-state equilibria with a homogeneous population, entirely made either of agents of type C (i.e. $q = 1$), or, alternatively, of agents of type L (i.e. $q = 0$).

The issue of the existence of a steady-state with a heterogeneous population can be reformulated as the question of the intersection of the qq , hh^C and hh^L loci strictly within the (q_t, h_t^C, h_t^L) space. Under the postulated functional forms, it can be shown that the three loci intersect only once at a value of $q \neq 0 \neq 1$, so that there exists a *unique* steady-state with a heterogeneous population. Moreover, it is shown in the Appendix that the intermediate steady-state is, under mild conditions, locally stable, in the sense that any economy that

does not lie initially at another steady-state equilibrium will converge towards the intermediate steady-state. Those results are summarized below.¹⁸

Proposition 2 *There exist three steady-state equilibria:*

- $q = 0$, $h^L = \frac{1 - \sqrt[2]{1 - \theta^L(1 - \theta^L)}}{\theta^L}$.
- $q = \frac{h^C}{h^L + h^C}$, $h^C = \frac{1 - \sqrt[2]{1 - \theta^C(1 - \theta^C)}}{\theta^C}$, $h^L = \frac{1 - \sqrt[2]{1 - \theta^L(1 - \theta^L)}}{\theta^L}$.
- $q = 1$, $h^C = \frac{1 - \sqrt[2]{1 - \theta^C(1 - \theta^C)}}{\theta^C}$.

The intermediate steady-state is, under mild conditions, locally stable.

Given that the intermediate steady-state (i.e. involving heterogeneity in tastes) is the equilibrium that is the most likely to emerge - except if the population is initially uniform - we shall, in the rest of this paper, pay a particular attention to that steady-state equilibrium, which is described by Proposition 3.

Proposition 3 *At the intermediate steady-state equilibrium, we have:*

- $h^C < h^L$,
- $q < 1/2$,
- $l^C < l^L$,
- $c^i < d^i \forall i \in \{C, L\}$, $d^C > d^L$ and $\frac{c^C}{l^C} > \frac{c^L}{l^L}$,
- $e^C = e^L$.

The proof of those results goes as follows. Given that $\theta^C > \theta^L$, the steady-state longevity of agents of type L is larger than the one of agents of type C , as equilibrium longevity h^i is decreasing in θ^i for $i \in \{C, L\}$. Another prediction of the model concerns the long-run composition of the population. Actually, given that $h^L > h^C$, the steady-state population will be composed by a larger proportion of ‘leisure-lovers’ than ‘consumption-lovers’: $q < 1/2$.¹⁹ Moreover, as agents of type L live longer at the steady-state, it must also be the case, under the postulated longevity production function, that these work less, so that $l^C < l^L$, $d^C > d^L$ and $c^C/l^C > c^L/l^L$. Finally, given that, at the steady-state, $q = \frac{h^C}{h^C + h^L}$, it is straightforward to see from the FOCs for optimal socialization efforts that socialization efforts are equalized at the steady-state.²⁰

In sum, this model states that the healthy lifestyle must, in the long-run, ‘dominate’ the unhealthy lifestyle. The intuition behind this result goes as follows: the defenders of the healthy lifestyle - parents of type L -, by facing, on average, a longer expected lifetime - and thus higher payoffs from having children like them -, tend, *ceteris paribus*, to make a stronger effort to transmit

¹⁸See the Appendix for the proof.

¹⁹This is so because $q = \frac{h^C}{h^C + h^L} < 1/2$ when $h^C < h^L$.

²⁰One can also deduce the equality of efforts by substituting for efforts in expression (5). This yields: $q_{t+1} = q_t + q_t(1 - q_t)(e_t^C - e_t^L)$, from which it is easy to see that, when $q_{t+1} = q_t \neq 0 \neq 1$, one must have $e_t^C = e_t^L$. Intuitively, the constancy of the composition of the population is achieved when socialization efforts are equal across groups. Alternatively, one can deduce the equality of efforts from the last line of Proposition 1: under $q = \frac{h^C}{h^C + h^L}$, we have $\frac{1-q}{q} = \frac{h^L}{h^C}$ implying $e^C = e^L$.

their trait, which will lead, in the long-run, to the larger propagation of their lifestyle. Moreover, given that the healthy lifestyle coincides with a high taste for leisure, this model can also predict, provided $q_0 > \frac{h^C}{h^C + h^L}$, the rise of the ‘leisure society’ (a fall of q_t over time).

3 Social optimum and optimal policy

Let us now characterize the social optimum in the economy under study. For that purpose, social welfare will be defined in the standard utilitarian way, as the sum of utilities of each group of agents, weighted by their size.²¹

Given that this paper aims at emphasizing the importance of ‘composition effects’ for the definition of the optimal public intervention, we will proceed here in two stages, and consider first the short-run social optimum, i.e. the optimum for a *given* partition q_t (with $0 < q_t < 1$), and, then, the long-run social optimum. That two-stage procedure will also allow us, when studying the decentralization of the social optimum, to compare the optimal policies under exogenous and endogenous composition of the population in terms of tastes.

3.1 The short-run first-best problem

The short-run first-best solution The short-run social optimum is the octuplet $\{c_t^C, c_t^L, d_{t+1}^C, d_{t+1}^L, e_t^C, e_t^L, l_t^C, l_t^L\}$ maximizing the following Lagrangian:

$$\begin{aligned} \mathcal{L} = & (q_t) \left[(c_t^C)^{\theta^C} (l_t^C)^{1-\theta^C} - \frac{\xi(e_t^C)^2}{2} + h_{t+1}^C \left[(d_{t+1}^C)^{\theta^C} + p_{t+1}^{CC} \hat{\varphi} + p_{t+1}^{CL} \tilde{\varphi} \right] \right] \\ & + (1 - q_t) \left[(c_t^L)^{\theta^L} (l_t^L)^{1-\theta^L} - \frac{\xi(e_t^L)^2}{2} + h_{t+1}^L \left[(d_{t+1}^L)^{\theta^L} + p_{t+1}^{LL} \hat{\varphi} + p_{t+1}^{LC} \tilde{\varphi} \right] \right] \\ & + \lambda \left[w (q_t(1 - l_t^C) + (1 - q_t)(1 - l_t^L)) - q_t c_t^C - (1 - q_t) c_t^L - q_t h_{t+1}^C d_{t+1}^C - (1 - q_t) h_{t+1}^L d_{t+1}^L \right] \end{aligned}$$

where λ is the Lagrange multiplier associated to the resource constraint. Note that the partition of the population q_t is here taken as given.

The FOCs for optimal consumptions and socialization efforts are:

$$\begin{aligned} \theta^C (c_t^C)^{\theta^C - 1} (l_t^C)^{1-\theta^C} &= \lambda = \theta^L (c_t^L)^{\theta^L - 1} (l_t^L)^{1-\theta^L} & e_t^C &= \frac{h_{t+1}^C (1 - q_t) (\hat{\varphi} - \tilde{\varphi})}{\xi} \\ \theta^C (d_{t+1}^C)^{\theta^C - 1} &= \lambda = \theta^L (d_{t+1}^L)^{\theta^L - 1} & e_t^L &= \frac{h_{t+1}^L (q_t) (\hat{\varphi} - \tilde{\varphi})}{\xi} \end{aligned}$$

The first two FOCs involve the standard equalization of marginal utilities of consumption for the different types of agents at the two periods. Those FOCs

²¹ As it is well-known, the aggregation of utilities of agents having different preferences makes sense only if individual utilities are interpersonally comparable. One way to achieve this is, as proposed by Mirrlees (1982, p. 78-80), by means of discussions between agents about utilities. Although agents of types C and L have different tastes, they may communicate about their life experiences, and reach an agreement as to how their welfare should be included in a social welfare function. We assume that such an agreement can be reached, so that the aggregation of utilities of agents of different types in a social welfare function makes sense.

imply that second-period consumption cannot exceed third-period consumption. These imply also that d_{t+1}^C exceeds d_{t+1}^L , and that $c_t^C/l_t^C > c_t^L/l_t^L$.

As indicated by the two FOCs concerning socialization efforts, the short-run social optimum requires socialization effort levels to be chosen on the basis of optimal - rather than expected - future (group-specific) longevities. Hence, the optimal socialization effort is larger than the one under laissez-faire if and only if the expected longevity is lower than the optimal longevity. Regarding how optimal socialization efforts differ across agents of different types, the FOCs suggest that this depends on whether $(1 - q_t)/q_t$ exceeds h_{t+1}^L/h_{t+1}^C or not.

The FOCs for optimal leisure times are:

$$\frac{(c_t^C)^{\theta^C} (1 - \theta^C) (l_t^C)^{-\theta^C} + h_{t+1}^{C'} \left[(d_{t+1}^C)^{\theta^C} + p_{t+1}^{CC} \hat{\varphi} + p_{t+1}^{CL} \hat{\varphi} \right]}{w + h_{t+1}^{C'} d_{t+1}^C} = \lambda$$

$$\frac{(c_t^L)^{\theta^L} (1 - \theta^L) (l_t^L)^{-\theta^L} + h_{t+1}^{L'} \left[(d_{t+1}^L)^{\theta^L} + p_{t+1}^{LL} \hat{\varphi} + p_{t+1}^{LC} \hat{\varphi} \right]}{w + h_{t+1}^{L'} d_{t+1}^L} = \lambda$$

Note that those FOCs differ from the FOCs for optimal leisure under laissez-faire. Clearly, both types of agents tend, under laissez-faire, to have a too low level of leisure with respect to what is socially optimal. The reason for this lies in agents' tendency to take the length of the third period of life as fixed, whereas this depends on their working time during the second period.²² This myopia implies that longevities under laissez-faire are lower than their first-best levels. It implies also that $c_t^{iLF}/d_{t+1}^{iLF} < c_t^{iFB}/d_{t+1}^{iFB}$ for all agents of type $i \in \{C, L\}$, meaning that all agents tend to oversave with respect to what is optimal. In the light of this, we can describe the short-run social optimum as follows.²³

Proposition 4 *The short-run social optimum is such that:*

- $c_t^{iFB} \leq d_{t+1}^{iFB} \quad \forall i \in \{C, L\}; d_{t+1}^{CFB} > d_{t+1}^{LFB}, \frac{c_t^{CFB}}{l_t^{CFB}} > \frac{c_t^{LFB}}{l_t^{LFB}},$
- $e_t^{iLF} \leq e_t^{iFB} \iff h_t^{iLF} \leq h_{t+1}^{iFB} \quad \forall i \in \{C, L\},$
- $e_t^{CFB} \leq e_t^{LFB} \iff \frac{1-q_t}{q_t} \leq \frac{h_{t+1}^{LFB}}{h_{t+1}^{CFB}},$
- $l_t^{iLF} < l_t^{iFB} \quad \forall i \in \{C, L\},$ so that $\frac{c_t^{iLF}}{d_{t+1}^{iLF}} < \frac{c_t^{iFB}}{d_{t+1}^{iFB}} \quad \forall i \in \{C, L\},$
- $h_{t+1}^{iLF} < h_{t+1}^{iFB} \quad \forall i \in \{C, L\}.$

Decentralization of the short-run social optimum This subsection studies the decentralization of the short-run social optimum by means of group-specific tax/transfers instruments. Note that such instruments suppose that the

²²The tendency of agents to choose a too low leisure - independently from the expected welfare gains from coexistence with children - is here due to the assumed functional form, which implies the 'fear of ruin' property $u'(d_{t+1}^i) d_{t+1}^i < u(d_{t+1}^i)$ [as $\theta^i d_{t+1}^{i\theta^i-1} d_{t+1}^i < d_{t+1}^{i\theta^i}$ under $\theta^i < 1$] (see Eeckhoudt and Pestieau, 2007).

²³See the Appendix for a proof of those statements.

government can observe the type of agents.²⁴ Moreover, this analysis assumes also that agents cannot change their type.²⁵

Comparing the FOCs under laissez-faire with the ones describing the short-run social optimum shows that the decentralization of the short-run optimum requires a lump sum transfer system $\{T^C, T^L\}$ equalizing marginal utilities of consumption across agents and periods. However, because of myopia, such transfers do not suffice to decentralize the optimum. Actually, the decentralization of the optimal leisures for agents C and L requires group-specific Pigouvian taxes on wages taking the impact of work on longevity into account:

$$\tau^C = h_{t+1}^{C'} \frac{(d_{t+1}^C)^{\theta^C} (1 - \theta^C) + p_{t+1}^{CC} \hat{\varphi} + p_{t+1}^{CL} \tilde{\varphi}}{\theta^C (d_{t+1}^C)^{\theta^C - 1} w} > 0 \quad (12)$$

$$\tau^L = h_{t+1}^{L'} \frac{(d_{t+1}^L)^{\theta^L} (1 - \theta^L) + p_{t+1}^{LL} \hat{\varphi} + p_{t+1}^{LC} \tilde{\varphi}}{\theta^L (d_{t+1}^L)^{\theta^L - 1} w} > 0 \quad (13)$$

Those Pigouvian taxes τ^C and τ^L are positive. Agents, whatever their type is, tend, by ignoring the negative impact of work on longevity, to work too much, so that a positive tax on wages is required to decentralize the social optimum. Naturally, those taxes depend on the impact of leisure on longevity.²⁶ Taxes τ^C and τ^L depend also on the expected welfare gains from coexistence with children (i.e. the last two terms of the numerators): the larger these are, the larger is the welfare loss due to myopia, inviting a stronger correction.

Note that, although q_t does not appear explicitly in the Pigouvian taxes formulae, it influences τ^C and τ^L through its impact on the expected welfare gains from socialization. As shown by substitutions, τ^C is actually increasing in q_t , while τ^L is decreasing in q_t .²⁷ Thus, the larger a group is, the higher the Pigouvian tax faced by that group is *ceteris paribus*. The intuition behind that result is that when a group is larger, the expected gains, for a member of that group, from coexisting with his child at the old age are also larger, making the myopia more damageable, which requires a larger correction. Hence, the level of the - exogenous - partition q_t influences Pigouvian taxes.

Whereas those Pigouvian taxes, coupled with lump sum transfers, are necessary to decentralize the social optimum, these are not sufficient. Actually, comparing the FOCs for optimal socialization efforts with the ones under laissez-faire shows that nothing guarantees the equality of optimal and laissez-faire socialization efforts. However, socialization efforts - which are here of purely physical nature (i.e. non-monetary and non-temporal) - can hardly be taxed or subsidized, as these are personal, non-monitored, variables. Hence, the unique

²⁴The observability of types is in conformity with the model: the fact that one derives welfare from coexisting with children of one's type requires the observability of types.

²⁵As described in Section 2, the trait $i \in \{C, L\}$ is taken *once-and-for-all* during the socialization process, so that there can be no 'adaptation' to public policy.

²⁶If $h_{t+1}^{C'}$ and $h_{t+1}^{L'}$ equal 0, the myopia is benign, so that τ^C and τ^L equal zero.

²⁷To see that τ^C is increasing in q_t , it suffices to substitute for p_{t+1}^{CC} and p_{t+1}^{CL} using expressions (4) and (7). This yields: $p_{t+1}^{CC} \hat{\varphi} + p_{t+1}^{CL} \tilde{\varphi} = \tilde{\varphi} + (\hat{\varphi} - \tilde{\varphi}) [q_t + e_t^C (1 - q_t)]$. Given that the socialization effort cannot exceed 1, this expression is increasing in q_t .

possibility to decentralize the optimal effort levels would be to proceed indirectly, through the influence of taxation on the time horizon governing the choice of effort. But here is the problem: agents' choices of socialization efforts are based on myopic anticipations, so that the time horizon governing the socialization decision is, at the temporary equilibrium, *fixed*, and thus cannot be affected by a Pigouvian tax, as this can only influence the *future* longevity, but not the past one (h_t^i is, at the temporary equilibrium, *taken as given* by agents, and thus independent from taxes, which can only affect h_{t+1}^i). Hence, transfers T^C and T^L and Pigouvian taxes τ^C and τ^L on wages do not suffice here to decentralize the short-run optimum. Those results are summarized below.²⁸

Proposition 5 *The decentralization of the short-run social optimum requires:*

- lump sum transfers T^C and T^L equalizing the marginal utilities of consumption for all agents at all periods,
- group-specific Pigouvian taxes τ^C and τ^L on wages, equal to:

$$\begin{aligned}\tau^C &= h_{t+1}^{C'} \frac{(d_{t+1}^C)^{\theta^C} (1 - \theta^C) + p_{t+1}^{CC} \hat{\varphi} + p_{t+1}^{CL} \tilde{\varphi}}{\theta^C (d_{t+1}^C)^{\theta^C - 1} w} > 0 \\ \tau^L &= h_{t+1}^{L'} \frac{(d_{t+1}^L)^{\theta^L} (1 - \theta^L) + p_{t+1}^{LL} \hat{\varphi} + p_{t+1}^{LC} \tilde{\varphi}}{\theta^L (d_{t+1}^L)^{\theta^L - 1} w} > 0\end{aligned}$$

However, due to the physical nature of e_t^C and e_t^L , the optimal socialization effort levels cannot be decentralized under the prevailing myopic anticipations.

Thus, the fact that the government cannot interfere with the socialization decision, which relies on past - and thus fixed - longevities, prevents the decentralization of the short-run optimum. Note that, in the *long-run* first-best problem, that difficulty will disappear.²⁹ However, as we shall now see, the decentralization of the long-run social optimum requires also to deal with the - so far neglected - effects of agents' decisions on future cohorts' compositions.

3.2 The long-run first-best problem

The long-run first-best solution The above analysis, which studied the decentralization of the social optimum, was carried out under a *fixed* proportion of the two types of agents in the population. However, that postulate is a simplification, as the partition of the population is actually a variable, and not a constant. This subsection studies the long-run social optimum, that is, the optimum under an *endogenous* composition of the population. In that analysis,

²⁸See the Appendix for the proof of that proposition, which holds under $0 < q_t < 1$. Note that, in a homogeneous society ($q_t = 0$ or $q_t = 1$), the laissez-faire and optimal socialization efforts are equal to 0. Hence, in that specific case, a single tax on wages suffices to decentralize the short-run optimum (as no lump sum transfers are required).

²⁹Indeed, in the long-run first-best problem, Pigouvian taxes influence longevity h_t^i at *all* times - including the previous period -, and thus allow the decentralization of the optimal socialization effort through Pigouvian taxes on wages.

the social planner will no longer take the partition of the population as given, unlike in the short-run first-best problem.

In order to characterize the long-run social optimum, we shall here consider that the social planner cannot choose the partition of the population q directly, but, rather, can only choose the individual variables influencing q under the prevailing socialization process. That modelling strategy is based on a simple analogy with the modelling of individual longevity: the socialization process is like a survival process for *types* of agents, so that it makes sense to assume, as with standard survival processes, that the socialization process is imposed as a constraint to the social planner, who can only choose the inputs entering that process, but cannot alter the process itself.

Note that, if the population is initially homogeneous - i.e. q_0 is 0 or 1 -, the partition of the population remains the same over time, and is unaffected by agents' lifestyles, so that a social planner controlling individual variables cannot change the composition of the population. Hence, in that special case, there is no difference between the short-run and the long-run social optima.³⁰ However, in the general case where some heterogeneity exists initially, the steady-state level of q depends on the longevity of each group, and, thus, on their leisure, which are under the control of the social planner. Hence, in that case, the short-run and long-run social optima are likely to differ, as we shall now see.

The first-best long-run problem can be stated as the search for the octuplet $\{c^C, c^L, d^C, d^L, e^C, e^L, l^C, l^L\}$ maximizing the Lagrangian:³¹

$$\begin{aligned} \mathcal{L} = & (q) \left[(c^C)^{\theta^C} (l^C)^{1-\theta^C} - \frac{\xi(e^C)^2}{2} + h^C \left[(d^C)^{\theta^C} + p^{CC} \hat{\varphi} + p^{CL} \tilde{\varphi} \right] \right] \\ & + (1-q) \left[(c^L)^{\theta^L} (l^L)^{1-\theta^L} - \frac{\xi(e^L)^2}{2} + h^L \left[(d^L)^{\theta^L} + p^{LL} \hat{\varphi} + p^{LC} \tilde{\varphi} \right] \right] \\ & + \lambda \left[w (q(1-l^C) + (1-q)(1-l^L)) - qc^C - (1-q)c^L - qh^C d^C - (1-q)h^L d^L \right] \end{aligned}$$

where q takes its steady-state value $\frac{h^C}{h^C+h^L}$.

The FOCs for optimal consumptions are:

$$\begin{aligned} \theta^C (c^C)^{\theta^C-1} (l^C)^{1-\theta^C} &= \lambda = \theta^L (c^L)^{\theta^L-1} (l^L)^{1-\theta^L} \\ \theta^C (d^C)^{\theta^C-1} &= \lambda = \theta^L (d^L)^{\theta^L-1} \end{aligned}$$

These are conditions of equalization of marginal utilities of consumption across periods and agents. As in the short-run social optimum, agents of type C have a larger third-period consumption than agents of type L .

The FOCs for optimal socialization efforts are:

$$e^C = \frac{h^C(1-q)(\hat{\varphi}-\tilde{\varphi})}{\xi} \quad e^L = \frac{h^L(q)(\hat{\varphi}-\tilde{\varphi})}{\xi}$$

³⁰ In each case, the decentralization involves only a single Pigouvian tax on wages (correcting the myopia), and no transfers, as the population is uniform.

³¹ For simplicity, time indices are omitted here.

where h^C , h^L and q are at their optimal levels. Note that, given that $q = \frac{h^C}{h^C + h^L}$ at the steady-state, optimal socialization efforts are equal for all agents. The FOCs for the optimal leisure time for agents of types C and L are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial l^C} &= q'_{h^C} h^{C'} [u^C - u^L] + q(c^C)^{\theta^C} (1 - \theta^C)(l^C)^{-\theta^C} \\ &\quad + qh'^C \left[(d^C)^{\theta^C} (1 - \theta^C) + p^{CC} \hat{\varphi} + p^{CL} \hat{\varphi} \right] - \lambda q w \\ &\quad + \lambda q'_{h^C} h^{C'} [(1 - l^C)w - (1 - l^L)w - c^C + c^L - h^C d^C + h^L d^L] \\ &= 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial l^L} &= q'_{h^L} h^{L'} [u^C - u^L] + (1 - q) (c^L)^{\theta^L} (1 - \theta^L)(l^L)^{-\theta^L} \\ &\quad + (1 - q)h'^L \left[(d^L)^{\theta^L} (1 - \theta^L) + p^{LL} \hat{\varphi} + p^{LC} \hat{\varphi} \right] - \lambda (1 - q) w \\ &\quad + \lambda q'_{h^L} h^{L'} [(1 - l^C)w - (1 - l^L)w - c^C + c^L - h^C d^C + h^L d^L] \\ &= 0 \end{aligned}$$

Unlike the FOCs characterizing the short-run social optimum, those FOCs capture the impact of agents' lifestyles (i.e. leisure times) on the long-run composition of the population, through the terms multiplied by $q'_{h^i} h^{i'}$ for $i \in \{C, L\}$. The intuition behind those *intergenerational composition effects* goes as follows. When an agent chooses his leisure time, this determines his longevity, which, under myopic anticipations on longevity, will also affect the socialization efforts of agents of the same type (taking him as a 'model') at the *next* generation (through its impact on agents' time horizons). Thus there exists a chain linking all generations' lifestyles. That chain is not taken into account by the social planner in the *short-run* first-best problem, but well in the *long-run* first-best problem. This explains the difference between the FOCs of the two problems.

Note that it is only in the special case where the partition of the population is fixed - $q'_{h^C} = q'_{h^L} = 0$ - that those FOCs collapse to the ones describing the short-run optimum. In comparison with the FOCs characterizing the short-run optimum, each of those FOC includes two additional terms.

The first term, $q'_{h^C} h^{C'} [u^C - u^L]$, captures the impact, in welfare terms, of the change in the partition of the population resulting from a variation in the agent's leisure time. If, for instance, the lifetime welfare of an agent of type C - denoted by u^C - exceeds the one of an agent of type L , the first term of the FOC for l^C is positive, implying, *ceteris paribus*, a higher l^C , in such a way as to raise the proportion of agents of type C at the steady-state. On the contrary, under $u^C < u^L$, the first term of the FOC for l^L is negative, implying a lower leisure l^L for agents of type L , in such a way as to reduce their proportion in the long-run. Hence, the first term has opposite signs for the two types of agents.

The other additional term (the last term of the FOC) consists of the impact of a marginal change in the partition of the population on the resources of the economy. If, for instance, agents of type C are net contributors, the factor in brackets is positive, inviting, *ceteris paribus*, a higher proportion of agents of

that type, and, thus, a higher leisure level for them. Here again, those additional terms are of opposite signs in the two FOCs, as the derivatives q'_{h^C} and q'_{h^L} multiplying the factors in brackets are of opposite signs in the two FOCs.

Comparing the FOCs for optimal leisure with the ones under laissez-faire shows that the leisure of one type of agent at the social optimum may be superior or inferior to its level under laissez-faire, depending on whether the sum of those two additional terms with the myopia term of the FOC is positive or negative. The following proposition summarizes the above findings.³²

Proposition 6 *The long-run social optimum is such that:*

- $c^{iFB} \leq d^{iFB} \quad \forall i \in \{C, L\}, d^{CFB} > d^{LFB}, \frac{c^{CFB}}{l^{CFB}} > \frac{c^{LFB}}{l^{LFB}},$
- $e^{CFB} = e^{LFB}$
- $l^{iLF} \leq l^{iFB} \iff \Omega^i \geq 0 \quad \forall i \in \{C, L\},$

$$\text{where } \Omega^i \equiv q^i h^i \left[(d^{iFB})^{\theta^i} (1 - \theta^i) + p^{iiFB} \hat{\varphi} + p^{ijFB} \check{\varphi} \right] + q'_{h^i} h^{i'} [u^{CFB} - u^{LFB}] \\ + \lambda q'_{h^i} h^{i'} [(1 - l^{CFB})w - (1 - l^{LFB})w - c^{CFB} + c^{LFB} - h^{CFB} d^{CFB} + h^{LFB} d^{LFB}].$$

Decentralization of the long-run social optimum Comparing the FOCs for optimal long-run consumptions with the ones at the laissez-faire shows that the decentralization of the long-run social optimum requires, as in the short-run case, lump sum transfers $\{T^C, T^L\}$ equalizing marginal utilities of consumption across agents and periods.

Regarding the decentralization of the optimal socialization efforts levels, the comparison of the FOCs of the laissez-faire with the ones characterizing the social optimum shows that, as long as leisure times - and, through h^C and h^L , the composition of the population q - take their optimal values, the optimal socialization effort levels will also prevail.

Contrasting the FOCs relative to the choice of leisure times reveals that the Pigouvian taxes designed for the decentralization of the short-run social optimum are no longer appropriate here, because of the endogeneity of the partition of the population. Agents choose their leisure time *without* considering its impact on the composition of future cohorts. Hence, besides the correction for myopia, the decentralization of the long-run optimum requires also a correction for those intergenerational externalities due to the socialization process.³³

Proposition 7 *The long-run social optimum can be decentralized by means of:*

- lump sum transfers T^C and T^L yielding an equalization of marginal utilities of consumption across agents and periods;

³²See the Appendix for the proof.

³³See the Appendix for the proof.

- Pigouvian taxes τ^C and τ^L on wages equal to:

$$\begin{aligned}\tau^C &= h^{C'} \frac{(d^C)^{\theta^C} (1 - \theta^C)^{[1]} + p^{CC} \hat{\varphi} + p^{CL} \tilde{\varphi}}{\lambda w} + \frac{q'_{h^C}}{q} h^{C'} \frac{u^C - u^L}{\lambda w} \\ &\quad + \frac{q'_{h^C}}{q} h^{C'} \frac{w(1 - l^C) - w(1 - l^L)^{[3]} - c^C + c^L - h^C d^C + h^L d^L}{w} \\ \tau^L &= h^{L'} \frac{(d^L)^{\theta^L} (1 - \theta^L)^{[1']} + p^{LL} \hat{\varphi} + p^{LC} \tilde{\varphi}}{\lambda w} + \frac{q'_{h^L}}{1 - q} h^{L'} \frac{u^C - u^L}{\lambda w} \\ &\quad + \frac{q'_{h^L}}{1 - q} h^{L'} \frac{w(1 - l^C) - w(1 - l^L)^{[3']} - c^C + c^L - h^C d^C + h^L d^L}{w}\end{aligned}$$

The comparison of those Pigouvian taxes τ^C and τ^L allowing the decentralization of the long-run optimum with the ones in the short-run first-best problem shows that the long-run optimal tax formula is equal to short-run optimal tax formula [i.e. term [1]] - internalizing the impact of myopia on one's own lifetime welfare - *plus* the correction for the unintended 'composition effects' of leisure decisions on the composition of future populations [i.e. terms [2] and [3]].

Note first that, if each agent's longevity had no impact on the composition of the population - i.e. $q'_{h^C} = q'_{h^L} = 0$ - Pigouvian taxes τ^C and τ^L would be equal to their short-run levels. However, given that, for an initial $q_0 \in]0, 1[$, the steady-state q equals $\frac{h^C}{h^C + h^L}$, the longevity of each group influences the long-run composition of the population. Hence, the social planner, when fixing the Pigouvian taxes, must take that intergenerational externality into account.

At this stage, it should be stressed that, if longevity has an impact on the composition of the population, the first terms [1] and [1'] of the Pigouvian taxes, although formally similar to the short-run Pigouvian taxes, take different values in the long-run problem, as these are now functions of the *optimal* q , and no longer of the (arbitrary) exogenous q_t . Note, however, that long-run [1] and [1'] still remain positive, as these aim at correcting the myopia.

Let us now interpret the two new components of the optimal tax rates.³⁴

The term [2] accounts for the marginal social welfare gains from changing, *ceteris paribus*, the composition of the population q . That term is a product of three factors: the first is the marginal effect, on q , of a rise of agents C 's longevity; the second is the marginal effect, on agents C 's longevity, of a higher leisure l^C ; the third factor, $u^C - u^L$, is the difference between lifetime welfares enjoyed by agents of types C and L .

The term [3] accounts for the marginal effect, on the available resources, of a rise of the proportion of agents of type C as a result of a higher l^C . The sign of that term is also ambiguous: it is not clear that a rise of q resulting from a higher l^C increases or reduces the amount of available resources in the economy.

³⁴For that purpose, we shall concentrate on terms [2] and [3] in τ^C .

Note that, contrary to the decentralization of the short-run optimum, it is not easy to see which *signs* Pigouvian taxes τ^C and τ^L have: this depends on the signs and sizes of their three components. If, for instance, the lifestyle of agents of type C brings a higher lifetime welfare than the one of agents of type L [i.e. $[2] > 0$], and if increasing q tends also to raise the resources in the economy [i.e. $[3] > 0$], the internalization of the myopia and of unintended composition effects go in the same directions, and require agents of type C to work less. Hence, a positive tax on the labour income of agents of type C is required to restore the social optimum by making them work less than under the laissez-faire. If, on the contrary, agents of type C have a lower lifetime welfare than agents of type L , and if $[1] + [3] < |[2]|$, it follows, despite the myopia of agents of type C , that the decentralization of the long-run social optimum requires to subsidize their labour, as these tend to work too little from a social perspective.

Finally, it should be stressed that Pigouvian taxes and lump sum transfers allow the decentralization of the long-run optimum, as socialization efforts are, under these, optimally chosen, unlike what prevailed in the short-run. Actually, agents, when choosing how much to socialize their child, still rely here on *expected* future longevities, but given the constancy of longevity over time, this does not prevent the optimal level of effort to be chosen. Thus the decentralization of the long-run social optimum can be carried out.

Comparison of short-run and long-run Pigouvian taxes A major corollary of our analysis is that the optimal *short-run* and *long-run* Pigouvian taxes differ. The reason for this is that, in the short-run problem, the maximization of the social welfare of the cohort can be performed by the social planner without being concerned with the taste composition of future cohorts: correcting current myopia is the only thing that governs $\{\tau^C, \tau^L\}$ from a short-run perspective. However, from a long-run perspective, agents' decisions are imperfect not only from the point of view of individual lifetime welfare maximization, but, also, from the perspective of the composition of future cohorts' tastes. This double imperfection of leisure decisions invites a double correction.

Regarding the extent to which the short-run Pigouvian taxes differ from the long-run ones, the following proposition states that the correction for composition effects raises the tax on the labour of one type of agents, and reduces the tax on the labour of the other type.³⁵

Proposition 8 *Let τ^{iSR} and τ^{iLR} denote the Pigouvian taxes on agent i 's wage required for the decentralization of the short-run and long-run social optima, for $i \in \{C, L\}$. We have: $\tau^{CLR} - \tau^{CSR} \geq 0 \iff \tau^{LLR} - \tau^{LSR} \leq 0$.*

Hence, the optimal long-run tax on labour is, for one group, higher than its optimal short-run level, whereas the opposite holds for the other group. In other words, shifting from the maximization of short-run social welfare to the maximization of long-run social welfare tends, for one type of agents, to reinforce the correction required by the myopia - and make those agents live

³⁵See the Appendix for the proof.

more than what maximizes their lifetime welfare -, whereas that shift weakens the correction required by the myopia for the other type of agents, and makes those other agents live less than what maximizes their lifetime welfare.

Although that result seems counter-intuitive, this comes from the fact that, from a utilitarian point of view, the tendency of agents to work too much with respect to what is the best for them is *not* the unique determinant of public intervention. A utilitarian government must also, in its intervention, take the composition effects of agents' decisions into account. When agents decide how much to work - and unconsciously choose their longevity -, they also affect the time horizon of the next cohort taking them as 'models' (either as a parent or as a 'role model'). The members of the next cohort will then choose, on the basis of their 'models', their socialization efforts, determining the composition of the next generation, who will proceed similarly, and so forth. Hence, internalizing those intergenerational externalities involves making agents work more or less - and live less or more - than what maximizes their *own* lifetime welfares.

4 Conclusions

This paper aimed at studying the optimal tax policy in an economy where unequal longevities are the unintended outcome of differences in lifestyles, and where lifestyles are transmitted across generations. For that purpose, we developed a three-period OLG model, where the length of the third period depends on second-period leisure, and where the population, who ignores the impact of work on longevity, is partitioned in two groups with different tastes for leisure, and follows a socialization process *à la* Bisin and Verdier (2001).

We showed that the long-run social optimum can be decentralized by means of (1) inter-group lump sum transfers; (2) group-specific Pigouvian taxes on wages, which internalize not only the unintended effects of leisure on individual lifetime welfare, but, also, its impact on the composition of future cohorts. This paper shows also that the shift from the maximization of short-run social welfare to the maximization of long-run social welfare changes the optimal policy: taking intergenerational composition effects into account raises the optimal tax on labour for one type of agents, but reduces it for the other type.

Thus, the endogeneity of the population's preferences is far from neutral for the optimal public intervention. Therefore, while it is tempting to discuss the optimal public policy in a static framework - for a *given* partition of the population in terms of tastes -, the policy recommendations from such a static study may, by neglecting intergenerational externalities, differ significantly from what would best serve the long-run interests of the economy.

Note that our conclusions should be treated with extreme caution. A first, obvious reason for this is that these rely on standard utilitarianism, and thus leave little room for - possibly strong - intuitions about compensation and responsibility. Moreover, the government, by its fiscal policy, affects here what future people will *be*, which is, from a liberal point of view, questionable, as such

an intervention may match the behaviour of a uniforming totalitarian State.³⁶ However, it should be stressed that the State interferences studied here, which shape the tastes of (future) *non existing* people, differ from influences on the tastes of *existing* people, so that one cannot accuse the social planner of totalitarianism.³⁷

At the end of the day, the major question raised by this paper concerns the arbitrages to be made between the interests of existing people *versus* the ones of future, non-existing people. Correcting a myopia may well improve the welfare of current people, but, by the composition effects it generates, may also reduce long-run welfare, so that an arbitrage is to be made between the welfare of current cohorts and the welfare of future cohorts. This kind of dilemma, raised, for instance, by Malthus (1798) in the context of poverty reduction, goes far beyond the scope of this paper.³⁸ But the complexity of solving intergenerational trade-offs should not make us do *as if* such arbitrages did not exist. Tensions between short-run and long-run social welfare maximization exist, and governments have no other reasonable choices than to take these into account.

5 References

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³⁶The tensions between utilitarianism and liberalism in the context of a heterogeneous society are best summarized in Mill’s (1859) work on the conditions required for State intervention.

³⁷On the latter, see Hahn (1982).

³⁸Malthus’s argument against the Poor Laws was that those laws, although reducing poverty in the short-run, would, by favouring population growth, lead to a rise in the proportion of poor people in the long-run.

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6 Appendix

6.1 Laissez-faire

6.1.1 Temporary equilibrium

The expression $\frac{c_t^i}{d_{t+1}^i} = l_t^i$ is obtained from rearranging the first two FOCs. From this, it follows that, under $0 \leq l_t^i \leq 1$, it must be the case that $c_t^i \leq d_{t+1}^i \forall i \in \{C, L\}$.

Actually, if the budget constraint is binding, we have $c_t^i + h_t^i d_{t+1}^i = w(1 - l_t^i)$, so that, given the above expression, we can deduce that: $c_t^i = \frac{l_t^i w(1 - l_t^i)}{l_t^i + h_t^i}$.

From the first and third FOCs, we have: $l_t^i = \frac{1 - \theta^i}{\theta^i} \frac{c_t^i}{w}$. Substituting for c_t^i yields: $l_t^i = 1 - \theta^i(1 + h_t^i)$. From that expression, one can see that, if $h_t^L = h_t^C = \bar{h}_t$, we have also, given $\theta^L < \theta^C$, $l_t^C < l_t^L$. Moreover, given that $d_{t+1}^i = \frac{c_t^i}{l_t^i} = \frac{w(1 - l_t^i)}{l_t^i + h_t^i}$, $l_t^C < l_t^L$ and $h_t^L = h_t^C = \bar{h}_t$ imply also $d_{t+1}^C > d_{t+1}^L$. Given that

$\frac{c_t^i}{d_{t+1}^i} = l_t^i$, this implies also $\frac{c_t^C}{l_{t+1}^C} > \frac{c_t^L}{l_{t+1}^L}$.

Under the general case where $h_t^L \neq h_t^C$, we know that agents of type C choose a lower leisure than agents of type L if and only if: $1 - \theta^C(1 + h_t^C) < 1 - \theta^L(1 + h_t^L)$. This is true if and only if: $\frac{\theta^C}{\theta^L} > \frac{1 + h_t^L}{1 + h_t^C}$.

Given that $h_t^L, h_t^C \in [0, 1]$ and $h_t^L > h_t^C$, that condition is satisfied for all longevities if and only if $\frac{\theta^C}{\theta^L} > 2$. Note that if $l_t^C < l_t^L$, we have $d_{t+1}^C > d_{t+1}^L$, as $\frac{w(1-l_t^C)}{l_t^C + h_t^C} > \frac{w(1-l_t^L)}{l_t^L + h_t^L}$ under $h_t^L > h_t^C$. Given that $\frac{c_t^i}{d_{t+1}^i} = l_t^i$, this implies also $\frac{c_t^C}{l_{t+1}^C} > \frac{c_t^L}{l_{t+1}^L}$.

The inequality condition on efforts levels follows from comparing the two efforts levels in the above FOC: $\frac{h_t^C(1-q_t)(\hat{\varphi}-\tilde{\varphi})}{\xi} \geq \frac{h_t^L(q_t)(\hat{\varphi}-\tilde{\varphi})}{\xi} \iff \frac{h_t^C}{h_t^L} \geq \frac{q_t}{1-q_t}$.

6.1.2 Steady-state equilibrium

Existence of steady-states The dynamics of the economy is given by:

$$\begin{aligned} q_{t+1} &= F(q_t, h_t^C, h_t^L) = q_t + \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} (h_t^C(1 - q_t)^2 q_t - h_t^L(q_t)^2(1 - q_t)) \\ h_{t+1}^C &= G(h_t^C) = \frac{1 - \theta^C(1 + h_t^C)}{2 - \theta^C(1 + h_t^C)} \\ h_{t+1}^L &= H(h_t^L) = \frac{1 - \theta^L(1 + h_t^L)}{2 - \theta^L(1 + h_t^L)} \end{aligned}$$

Imposing $h_{t+1}^C = h_t^C$ and $h_{t+1}^L = h_t^L$ yields:

$$h^C = \frac{1 - \sqrt[2]{1 - \theta^C(1 - \theta^C)}}{\theta^C}; h^L = \frac{1 - \sqrt[2]{1 - \theta^L(1 - \theta^L)}}{\theta^L}$$

Those two levels are independent from each others, and independent from q .

Substituting for these in the dynamic law for q and defining $\phi(q_t) \equiv q_{t+1}$ yields:

$$\phi(q_t) = q_t + \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \left(\frac{1 - \sqrt[2]{1 - \theta^C(1 - \theta^C)}}{\theta^C} (1 - q_t)^2 q_t - \frac{1 - \sqrt[2]{1 - \theta^L(1 - \theta^L)}}{\theta^L} (q_t)^2 (1 - q_t) \right)$$

The existence of a steady-state equilibrium amounts to find a fixed point for $\phi(q_t)$, that is, a value of q_t such that $\phi(q_t) = q_t$. It is obvious that $q_t = 0$ and $q_t = 1$ are such fixed points, as $\phi(0) = 0$ and $\phi(1) = 1$. Hence, the homogenous society is a steady-state equilibrium.

The existence of an intermediate steady-state equilibrium - i.e. an equilibrium involving some heterogeneity - amounts to find a value of $q_t \in]0, 1[$ such that $\phi(q_t) = q_t$.

Note that $\phi(q_t)$ satisfies the following conditions:

- (i) $\phi(0) = 0$
- (ii) $\phi(1) = 1$
- (iii) $0 \leq \phi(q_t) \leq 1$
- (iv) $\lim_{q \rightarrow 0} \phi'(q_t) = 1 + \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{1 - \sqrt{1 - \theta^C(1 - \theta^C)}}{\theta^C} > 1$
- (v) $\lim_{q \rightarrow 1} \phi'(q_t) = 1 + \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{1 - \sqrt{1 - \theta^L(1 - \theta^L)}}{\theta^L} > 1$

Thus, given that $\phi(q_t)$ crosses the 45° line at $q_t = 0$ and $q_t = 1$, and has a slope that exceeds 1 when q_t tends towards 0 and when q_t tends towards 1, it must be the case, given the continuity of $\phi(q_t)$, that $\phi(q_t)$ intersects the 45° line for $q_t^* \in]0, 1[$.

The uniqueness of that intermediate steady-state can be shown by *reductio ad absurdum*. Suppose that such another intermediate equilibrium $\tilde{q} \in]0, 1[$ exists, and differs from $\frac{h^C}{h^L + h^C}$. This amounts to assuming that equals $\tilde{q} = \frac{h^C}{h^L + h^C} + \varepsilon$, with $\varepsilon \neq 0$, $\varepsilon \neq \frac{h^L}{h^L + h^C}$ and $\varepsilon \neq \frac{-h^C}{h^L + h^C}$. We have, by definition of the steady-state, $\tilde{q} = \tilde{q} + \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} (h^C(1 - \tilde{q})^2 \tilde{q} - h^L(\tilde{q})^2(1 - \tilde{q}))$. Given that $\frac{\hat{\varphi} - \tilde{\varphi}}{\xi} > 0$, this simplifies to: $h^C(1 - \tilde{q})^2 \tilde{q} - h^L(\tilde{q})^2(1 - \tilde{q}) = 0$. As $\tilde{q} \in]0, 1[$, we can divide by $\tilde{q}$ and $(1 - \tilde{q})$, and we obtain: $\tilde{q} = \frac{h^C}{h^L + h^C}$, implying $\varepsilon = 0$, contrary to what was assumed. Thus the intermediate steady-state is unique.

Stability of the intermediate steady-state In order to study the local stability of the intermediate steady-state equilibrium, let us first notice that the present system is non-linear, so that the conventional analysis of the Jacobian matrix (composed of the first-order derivatives of dynamic equations with respect to state variables) can only inform us on the stability of equilibria *provided* these are hyperbolic. Actually, if a fixed-point is hyperbolic, the Hartman-Grobman Theorem states that the stability of the linearized system (or its non-stability) implies the local stability of the non-linear system (or its non-stability) (see Medio and Lines, 2001). However, if the fixed-point is not hyperbolic, then the analysis of the linearized system does not allow us to draw any conclusion on the local stability of the non-linear system.

As stated in Medio and Lines (2001), fixed-points are, in discrete-time systems, hyperbolic if none of the eigenvalues of the Jacobian matrix, evaluated at the equilibrium, is equal to 1 in modulo.

The Jacobian matrix is defined as follows.

$$J \equiv \begin{pmatrix} \frac{\partial F}{\partial q_t} & \frac{\partial F}{\partial h_t^C} & \frac{\partial F}{\partial h_t^L} \\ \frac{\partial G}{\partial q_t} & \frac{\partial G}{\partial h_t^C} & \frac{\partial G}{\partial h_t^L} \\ \frac{\partial H}{\partial q_t} & \frac{\partial H}{\partial h_t^C} & \frac{\partial H}{\partial h_t^L} \end{pmatrix}$$

We have, at the intermediate steady-state:

$$\begin{aligned} \frac{\partial F}{\partial q_t} &= 1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} < 1 \text{ and positive (as } \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} h^C \leq 1). \\ \frac{\partial F}{\partial h_t^C} &= \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \left(\frac{h^L}{h^C + h^L} \right)^2 \frac{h^C}{h^C + h^L} < 1 \text{ and positive (as } \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} h^C \leq 1). \\ \frac{\partial F}{\partial h_t^L} &= -\frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \left(\frac{h^C}{h^C + h^L} \right)^2 \frac{h^L}{h^C + h^L} > -1 \text{ and negative.} \end{aligned}$$

$$\begin{aligned}
\frac{\partial G}{\partial q_t} &= 0 \\
\frac{\partial G}{\partial h_t^C} &= \frac{-\theta^C}{[2-\theta^C(1+h_t^C)]^2} > -1 \text{ and negative.} \\
\frac{\partial G}{\partial h_t^L} &= \frac{\partial H}{\partial q_t} = \frac{\partial H}{\partial h_t^C} = 0 \\
\frac{\partial H}{\partial h_t^L} &= \frac{-\theta^L}{[2-\theta^L(1+h_t^L)]^2} > -1 \text{ and negative.}
\end{aligned}$$

Note first that the trace of the Jacobian matrix is:

$$1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} + \frac{-\theta^C}{[2 - \theta^C(1 + h_t^C)]^2} + \frac{-\theta^L}{[2 - \theta^L(1 + h_t^L)]^2}$$

which lies, under our assumptions on $\frac{\hat{\varphi} - \tilde{\varphi}}{\xi}$, θ^C and θ^L , between -1 and 1.

The determinant of the Jacobian matrix is:

$$\left[1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L}\right] \frac{-\theta^C}{[2 - \theta^C(1 + h_t^C)]^2} \frac{-\theta^L}{[2 - \theta^L(1 + h_t^L)]^2}$$

which lies between 0 and 1.

We would like the linearized system to be stable. As this is well-known, a necessary and sufficient condition for the stability of the linearized system is that all eigenvalues of the Jacobian matrix estimated at the equilibrium are magnitudes less than 1. That condition implies also that the equilibrium is hyperbolic, leading to the (local) stability of the equilibrium.

Following Brooks (2004)'s study of stability in first-order three-dimensional dynamic systems, we know that all eigenvalues of a 3x3 Jacobian matrix are lower than 1 in modulo provided the following three conditions are satisfied:

- (i) $|\det(J)| < 1$
- (ii) $1 > [\sum M_i(J)] - [tr(J)] [\det(J)] + [\det(J)]^2$
- (iii) $-\left[\sum M_i(J) + 1\right] < tr(J) + \det(J) < [\sum M_i(J) + 1]$

where $\det(J)$, $tr(J)$ and $\sum M_i(J)$ denote respectively the determinant, the trace and the sum of the principal minors of the Jacobian matrix.

Condition (i) is satisfied (see above).

Regarding condition (ii), note first that $\sum M_i(J)$ is here equal to:

$$\begin{aligned}
&\left[1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L}\right] \left[1 - \frac{\theta^C + \theta^L}{[2 - \theta^C(1 + h_t^C)]^2}\right] + \frac{-\theta^C}{[2 - \theta^C(1 + h_t^C)]^2} + \frac{-\theta^L}{[2 - \theta^L(1 + h_t^L)]^2} \\
&+ \left[2 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L}\right] \frac{-\theta^C}{[2 - \theta^C(1 + h_t^C)]^2} \frac{-\theta^L}{[2 - \theta^L(1 + h_t^L)]^2}
\end{aligned}$$

Hence, condition (ii) requires: $1 > [\sum M_i(J)] - [tr(J)] [\det(J)] + [\det(J)]^2$

$$1 > \left[1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L}\right] \left[1 - \frac{\theta^C + \theta^L}{[2 - \theta^C(1 + h_t^C)]^2}\right] + \frac{-\theta^C}{[2 - \theta^C(1 + h_t^C)]^2} + \frac{-\theta^L}{[2 - \theta^L(1 + h_t^L)]^2}$$

$$\begin{aligned}
& + \left[2 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \frac{-\theta^C}{[2 - \theta^C(1 + h^C)]^2} \frac{-\theta^L}{[2 - \theta^L(1 + h^L)]^2} \\
& - \left[1 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} - \frac{\theta^C}{[2 - \theta^C(1 + h^C)]^2} - \frac{\theta^L}{[2 - \theta^L(1 + h^L)]^2} \right] \left[\left[1 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \frac{\theta^C}{[2 - \theta^C(1 + h^C)]^2} \frac{\theta^L}{[2 - \theta^L(1 + h^L)]^2} \right] \\
& + \left[\left[1 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \frac{\theta^C}{[2 - \theta^C(1 + h^C)]^2} \frac{\theta^L}{[2 - \theta^L(1 + h^L)]^2} \right]^2
\end{aligned}$$

That condition holds for low values of θ^C and θ^L , in conformity with our assumptions.

Condition (iii) requires: $-\left[\sum M_i(J) + 1\right] < \text{tr}(J) + \det(J) < \left[\sum M_i(J) + 1\right]$

That condition simplifies to

$$\begin{aligned}
& - \left[1 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \left[1 - \frac{\theta^C + \theta^L}{[2 - \theta^C(1 + h^C)]^2} \right] - \left[2 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \frac{\theta^C}{[2 - \theta^C(1 + h^C)]^2} \frac{\theta^L}{[2 - \theta^L(1 + h^L)]^2} - \\
& < \left[1 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \left[1 + \frac{\theta^C}{[2 - \theta^C(1 + h^C)]^2} \frac{\theta^L}{[2 - \theta^L(1 + h^L)]^2} \right] \\
& < \left[1 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \left[1 - \frac{\theta^C + \theta^L}{[2 - \theta^C(1 + h^C)]^2} \right] + \left[2 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \frac{\theta^C}{[2 - \theta^C(1 + h^C)]^2} \frac{\theta^L}{[2 - \theta^L(1 + h^L)]^2} +
\end{aligned}$$

It is straightforward to see that, under our assumptions on $\frac{\hat{\varphi} - \bar{\varphi}}{\xi}$, θ^C and θ^L , those two inequalities are satisfied, so that condition (iii) holds.

Under conditions (i)-(iii), all eigenvalues of the Jacobian matrix are less than 1 in modulo, so that the equilibrium is stable. Moreover, those conditions guarantee also that no eigenvalue equals 1 in modulo, and, thus, the hyperbolicity of the equilibrium. Hence, from the Hartman-Grobman Theorem, it can be concluded that, under conditions (i)-(iii), the steady-state is locally stable.

6.2 Social optimum

6.2.1 Short-run first-best problem

The first-best short-run solution From the first two FOCs, we have $\frac{c_t^i}{d_{t+1}^i} = l_t^i \forall i \in \{C, L\}$. Note also that $\theta^C(d_{t+1}^C)^{\theta^C - 1} = \lambda = \theta^L(d_{t+1}^L)^{\theta^L - 1}$. Given that $\theta^C > \theta^L$, we have $(d_{t+1}^C)^{\theta^C - 1} < (d_{t+1}^L)^{\theta^L - 1}$. Thus, given that $\theta^C, \theta^L < 1$ and $\theta^C > \theta^L$, this implies $d_{t+1}^C > d_{t+1}^L$. Combining this with the above expression yields: $\frac{c_t^C}{l_t^C} > \frac{c_t^L}{l_t^L}$.

The FOC for optimal leisure can be written as

$$\frac{1 - \theta^i}{\theta^i} \frac{c_t^i}{l_t^i} + h^{i'} \frac{d_{t+1}^{i\theta^i} (1 - \theta^i) + p_{t+1}^{ii} \hat{\varphi} + p_{t+1}^{ij} \tilde{\varphi}}{\theta^i d_{t+1}^{i(\theta^i - 1)}} = w$$

Comparing this with the FOC under laissez-faire, $\frac{1 - \theta^i}{\theta^i} \frac{c_t^i}{l_t^i} = w$, implies that we must have $l_t^{iLF} < l_t^{iFB} \forall i \in \{C, L\}$.

The optimal efforts levels are given by: $e_t^{CFB} = \frac{h_{t+1}^C(1-q_t)\varphi}{\xi}$ and $e_t^{LFB} = \frac{h_{t+1}^L(q_t)\varphi}{\xi}$. Comparing these with the ones under laissez-faire: $e_t^{CLF} = \frac{h_t^C(1-q_t)\varphi}{\xi}$ and $e_t^{LLF} = \frac{h_t^L(q_t)\varphi}{\xi}$ implies that e_t^{iFB} exceeds e_t^{iLF} if and only if the socially optimal longevity h_{t+1}^i exceeds the prevailing longevity h_t^i , which may or may not be the case.

Decentralization of short-run optimum The decentralized FOCs are:

$$\begin{aligned} \theta^C (c_t^C)^{\theta^C-1} (l_t^C)^{1-\theta^C} &= \lambda^C & \theta^L (c_t^L)^{\theta^L-1} (l_t^L)^{1-\theta^L} &= \lambda^L \\ \theta^C (d_{t+1}^C)^{\theta^C-1} &= \lambda^C & \theta^L (d_{t+1}^L)^{\theta^L-1} &= \lambda^L \\ (1-\theta^C)(c_t^C)^{\theta^C} (l_t^C)^{-\theta^C} &= \lambda^C w & (1-\theta^L)(c_t^L)^{\theta^L} (l_t^L)^{-\theta^L} &= \lambda^L w \\ e_t^C &= \frac{h_{t+1}^C(1-q_t)(\hat{\varphi}-\tilde{\varphi})}{\xi} & e_t^L &= \frac{h_{t+1}^L(q_t)(\hat{\varphi}-\tilde{\varphi})}{\xi} \end{aligned}$$

Actually, if transfers T^C and T^L are such that, after redistribution:

$$\begin{aligned} \theta^C (c_t^C)^{\theta^C-1} (l_t^C)^{1-\theta^C} &= \theta^L (c_t^L)^{\theta^L-1} (l_t^L)^{1-\theta^L} = \lambda \\ \theta^C (d_{t+1}^C)^{\theta^C-1} &= \lambda = \theta^L (d_{t+1}^L)^{\theta^L-1} \end{aligned}$$

as required in the social optimum, substituting for λ in the FOCs for optimal leisure yields:

$$\frac{(1-\theta^C)c_t^C}{\theta^C l_t^C} = w \quad \frac{(1-\theta^L)c_t^L}{\theta^L l_t^L} = w$$

Comparing these with the FOCs for socially optimal leisures:

$$\begin{aligned} \frac{(1-\theta^C)c_t^C}{\theta^C l_t^C} + h_{t+1}^{C'} \frac{(d_{t+1}^C)^{\theta^C} (1-\theta^C) + p_{t+1}^{CC}\hat{\varphi} + p_{t+1}^{CL}\tilde{\varphi}}{\theta^C (d_{t+1}^C)^{\theta^C-1}} &= w \\ \frac{(1-\theta^L)c_t^L}{\theta^L l_t^L} + h_{t+1}^{L'} \frac{(d_{t+1}^L)^{\theta^L} (1-\theta^L) + p_{t+1}^{LL}\hat{\varphi} + p_{t+1}^{LC}\tilde{\varphi}}{\theta^L (d_{t+1}^L)^{\theta^L-1}} &= w \end{aligned}$$

suggests that taxes

$$\begin{aligned} \tau^C &= h_{t+1}^{C'} \frac{(d_{t+1}^C)^{\theta^C} (1-\theta^C) + p_{t+1}^{CC}\hat{\varphi} + p_{t+1}^{CL}\tilde{\varphi}}{\theta^C (d_{t+1}^C)^{\theta^C-1} w} \\ \tau^L &= h_{t+1}^{L'} \frac{(d_{t+1}^L)^{\theta^L} (1-\theta^L) + p_{t+1}^{LL}\hat{\varphi} + p_{t+1}^{LC}\tilde{\varphi}}{\theta^L (d_{t+1}^L)^{\theta^L-1} w} \end{aligned}$$

on wages allow the satisfaction of above FOCs.

Note that, provided the socialization efforts took their optimal values, those taxes, if combined with the lump sum transfers defined above, would allow the decentralization of the social optimum. However, efforts are not optimally chosen, and can hardly be decentralized (see above).

6.2.2 Long-run first-best problem

The first-best long-run solution From the first two FOCs, we have $\frac{c^i}{d^i} = l^i \forall i \in \{C, L\}$. Note also that $\theta^C(d^C)^{\theta^C-1} = \lambda = \theta^L(d^L)^{\theta^L-1}$. Given that $\theta^C > \theta^L$, we have $(d^C)^{\theta^C-1} < (d^L)^{\theta^L-1}$. Thus, given that $\theta^C, \theta^L < 1$ and $\theta^C > \theta^L$, this implies $d^C > d^L$. Combining this with the above expression yields: $\frac{c^C}{l^C} > \frac{c^L}{l^L}$.

The equality of socialization efforts levels follows from the FOCs and the fact that $q = \frac{h^C}{h^C + h^L}$ at the steady-state equilibrium.

Regarding leisure times, comparing the FOCs for optimal leisure times with the ones under laissez-faire shows that the laissez-faire level of l^i is inferior to its socially optimum level if and only if:

$$\begin{aligned} & q^i h'^i \left[(d^i)^{\theta^i} (1 - \theta^i) + p^{ii} \hat{\varphi} + p^{ij} \tilde{\varphi} \right] + q'_{hi} h^{i'} [u^C - u^L] \\ & + \lambda q'_{hi} h^{i'} [(1 - l^C)w - (1 - l^L)w - c^C + c^L - h^C d^C + h^L d^L] \\ & > 0 \end{aligned}$$

This inequality coincides with the one in Proposition 6.

Decentralization of the long-run social optimum The FOCs are:

$$\begin{aligned} \theta^C (c^C)^{\theta^C-1} (l^C)^{1-\theta^C} &= \lambda^C & \theta^L (c^L)^{\theta^L-1} (l^L)^{1-\theta^L} &= \lambda^L \\ \theta^C (d^C)^{\theta^C-1} &= \lambda^C & \theta^L (d^L)^{\theta^L-1} &= \lambda^L \\ (1 - \theta^C) (c^C)^{\theta^C} (l^C)^{-\theta^C} &= \lambda^C w & (1 - \theta^L) (c^L)^{\theta^L} (l^L)^{-\theta^L} &= \lambda^L w \\ e^C &= \frac{h^C (1-q)(\hat{\varphi}-\tilde{\varphi})}{\xi} & e^L &= \frac{h^L (q)(\hat{\varphi}-\tilde{\varphi})}{\xi} \end{aligned}$$

Actually, transfers T^C and T^L are such that, after redistribution:

$$\begin{aligned} \theta^C (c^C)^{\theta^C-1} (l^C)^{1-\theta^C} &= \theta^L (c^L)^{\theta^L-1} (l^L)^{1-\theta^L} = \lambda \\ \theta^C (d^C)^{\theta^C-1} &= \lambda = \theta^L (d^L)^{\theta^L-1} \end{aligned}$$

Substituting for λ in the FOCs for optimal leisure yields:

$$\frac{(1-\theta^C)c^C}{\theta^C l^C} = w \quad \frac{(1-\theta^L)c^L}{\theta^L l^L} = w$$

Comparing these with the FOCs for socially optimal leisures

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial l^C} &= q'_{h^C} h^{C'} [u^C - u^L] + q (c^C)^{\theta^C} (1 - \theta^C) (l^C)^{-\theta^C} \\ &+ q h'^C \left[(d^C)^{\theta^C} (1 - \theta^C) + p^{CC} \hat{\varphi} + p^{CL} \tilde{\varphi} \right] - \lambda q w \\ &+ \lambda q'_{h^C} h^{C'} [(1 - l^C)w - (1 - l^L)w - c^C + c^L - h^C d^C + h^L d^L] \\ &= 0 \end{aligned}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial l^L} &= q'_{h^L} h^{L'} [u^C - u^L] + (1-q) (c^L)^{\theta^L} (1-\theta^L) (l^L)^{-\theta^L} \\
&\quad + (1-q) h^{L'} \left[(d^L)^{\theta^L} (1-\theta^L) + p^{LL} \hat{\varphi} + p^{LC} \tilde{\varphi} \right] - \lambda (1-q) w \\
&\quad + \lambda q'_{h^L} h^{L'} [(1-l^C)w - (1-l^L)w - c^C + c^L - h^C d^C + h^L d^L] \\
&= 0
\end{aligned}$$

suggests that taxes

$$\begin{aligned}
\tau^C &= \frac{q'_{h^C} h^{C'} \frac{u^C - u^L}{\lambda w}}{q} + \frac{h^{C'} (d^C)^{\theta^C} (1-\theta^C) + p^{CC} \hat{\varphi} + p^{CL} \tilde{\varphi}}{\lambda w} \\
&\quad + \frac{q'_{h^C} h^{C'} \frac{w(1-l^C) - w(1-l^L) - c^C + c^L - h^C d^C + h^L d^L}{w}}{q} \\
\tau^L &= \frac{q'_{h^L} h^{L'} \frac{u^C - u^L}{\lambda w}}{1-q} + \frac{h^{L'} (d^L)^{\theta^L} (1-\theta^L) + p^{LL} \hat{\varphi} + p^{LC} \tilde{\varphi}}{\lambda w} \\
&\quad + \frac{q'_{h^L} h^{L'} \frac{w(1-l^C) - w(1-l^L) - c^C + c^L - h^C d^C + h^L d^L}{w}}{1-q}
\end{aligned}$$

on wages allow the satisfaction of above FOCs.

Those taxes, if combined with the lump sum transfers defined above, allow the decentralization of the social optimum, as efforts are, under that tax and transfer system, optimally chosen, unlike in the short-run.

Comparison of short-run and long-run Pigouvian taxes To prove Proposition 8, we shall use the following notations, taken from Section 3. $\tau^{CLR} = [1]^{LR} + [2] + [3]$ and $\tau^{CSR} = [1]^{SR}$. Moreover, $\tau^{LLR} = [1']^{LR} + [2'] + [3']$ and $\tau^{LSR} = [1']^{SR}$. Hence we have $\tau^{CLR} \geq \tau^{CSR} \iff [1]^{LR} + [2] + [3] \geq [1]^{SR}$ and $\tau^{LLR} \geq \tau^{LSR} \iff [1']^{LR} + [2'] + [3'] \geq [1']^{SR}$.

Let us first notice that terms $[1]$ and $[1']$ differ from their short-run counterparts in opposite ways: if the short-run q exceeds its optimal level, $[1]^{LR}$ is below $[1]^{SR}$, and the opposite holds for the term $[1']$, as these are both increasing in the size of the subgroup considered. Thus, we have $[1]^{LR} - [1]^{SR} \geq 0 \iff [1']^{LR} - [1']^{SR} \leq 0$.

Moreover, terms $[2] + [3]$ and $[2'] + [3']$ are of opposite signs. Actually, we have:

$$\frac{q'_{h^C}}{q} = \frac{h^L}{h^C(h^C + h^L)} > 0 \quad \frac{q'_{h^L}}{1-q} = \frac{-h^C}{h^L(h^C + h^L)} < 0$$

Given that the factors $\frac{u^C - u^L}{\lambda w}$ and $\frac{w(1-l^C) - w(1-l^L) - c^C + c^L - h^C d^C + h^L d^L}{w}$ are present in the formulae of τ^{CLR} and τ^L , it follows that, whatever those factors are, $[2] + [3]$ and $[2'] + [3']$ are of opposite signs: $[2] + [3] \geq 0 \iff [2'] + [3'] \leq 0$. Furthermore, we have $[2] + [3] \geq 0 \iff [1]^{LR} - [1]^{SR} \geq 0$ and $[2'] + [3'] \geq 0 \iff [1']^{LR} - [1']^{SR} \geq 0$.

It follows from all this that $[1]^{LR} - [1]^{SR} + [2] + [3] \geq 0 \iff [1']^{LR} - [1']^{SR} + [2'] + [3'] \leq 0$, that is, that $\tau^{CLR} - \tau^{CSR} \geq 0 \iff \tau^{LLR} - \tau^{LSR} \leq 0$.