On vector-based structural optimization and design of wind bracing of isostatic tetragonal grid systems for timber buildings

Abstract. This contribution addresses the optimization of bracing systems for innovative and efficient timber structures with the aid of a computational tool based on vector-based graphic statics.

Keywords: Timber construction, Structural design, Digital fabrication, Graphic-statics, VGS, Research-by-design.

1 Structural Design: Environment and Innovative Approaches

Abstract. The design of sustainable and efficient structures demands a multi-disciplinary approach that integrates environmental, geometric, and structural considerations. This article examines the environmental challenges of structural design, emphasizing the role of material optimization and the significance of timber construction as a low-carbon alternative. It explores the principles of graphic statics and the lower bound theorem of plasticity, demonstrating their applicability in analyzing and optimizing force flows within structural systems. Additionally, strut-and-tie models and the integration of elastic-plastic principles in timber structures are presented as tools to enhance structural efficiency. The chapter also highlights the benefits of isostatic design strategies and topology optimization for achieving material efficiency and structural robustness. Finally, the computational tool Vector-based Graphic Statics is introduced, offering interactive capabilities for visualizing and manipulating 3D structural solutions, advancing the conceptual design process towards innovation and sustainability.

1.1 Environment and Structural Design

1.1.1 Environmental Challenges

The design of structures is a multi-factorial process that must remain firmly rooted in the physical context of the project. Beyond ensuring mechanical resistance, it must consider multiple factors such as functionality, geometry, construction, cost and environmental impact, among others. In a world of limited and diminishing resources, it is also crucial to consider the structure's disassembly, recycling, and the reuse of its components. (1)

A structural solution can only be optimal depending on the importance assigned to each criterion. Therefore, the first challenge lies in defining the comparison parameters and their respective weighting.

1.1.2 Structural Efficiency

The current environmental crisis demands a significant reduction in CO2 emissions to mitigate the effects of global warming. In this context, structural engineers play a crucial role as the construction industry accounts for approximately 37% of greenhouse gas emissions (2). Load bearing systems contribute the most to embodied carbon emissions and waste generation in construction because of their high material mass and energy intensive production (3).

Given this climate emergency, the article assumes an optimization based on the amount of material used and, consequently, on greenhouse gas emissions.

1.1.3 Timber Construction

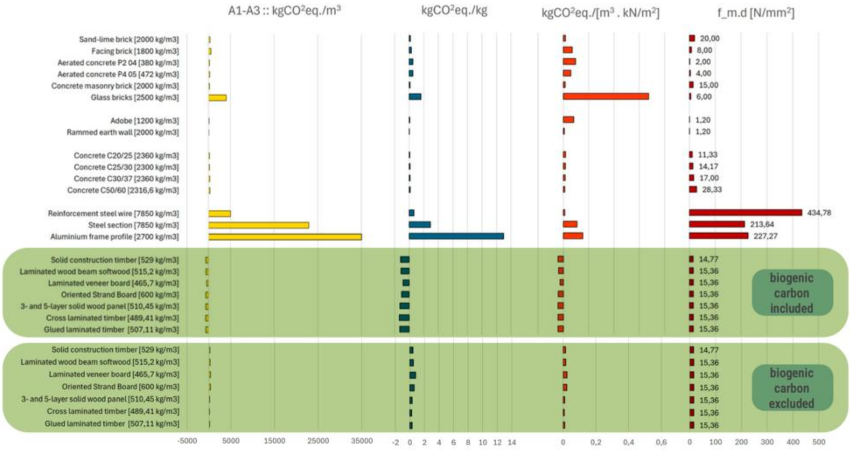

Fig. 1 Comparative diagram of the carbon assessment A1-A3 of construction materials including wood with (highlighted above) or without (below) biogenic carbon (Zastavni & Pollet 2025 -unpublished- dataset Sphera Solution GmbH 2023 and Thünen-Institut für Holzforschung); results are sensitive to hypothesis set.

Timber appears to be a very interesting material because it has the capacity to store CO2 and so is considered as having a positive carbon (negative index) footprint compared to other material.

With comparable performance, the CO2 footprint of wood in construction is significantly lower than that of steel or concrete, (4) even when calculated without biogenic carbon. Rough numbers show that the GWP impact, excluding the influence of biogenic carbon, is in favour of timber construction even when these values are balanced by their respective stress resisting capacities (see Fig.1).

Excluding biogenic carbon from the CO2 balance of timber prevent an overestimation of its ecological benefits. The carbon stored in wood during its growth is released at the end of its lifecycle (through decomposition or burning), meaning its long term-impact can be neutral or even negative if forest management is not sustainable. By omitting biogenic carbon from the CO2 balance, the assessment of emissions related to extraction, processing, and end-of-life disposal becomes more realistic, promoting a more rigorous approach to evaluating its environmental impact.

1.2 Graphic Statics and Plasticity Theory in Structural Design

1.2.1 Graphic Statics

Regarding the design and analysis of structures, graphic statics has proven its considerable potential for achieving efficient and elegant structures. This method relies on two interdependent diagrams, namely the form and force diagrams (5). The first represents the geometry of the structure together with its external force, the latter embeds a synthetic vector representation of the forces applied to each node of the structure, thus representing vectorially the equilibrium of forces acting on the structures.

The interdependency of the two diagrams and their visual convenience provides a visual and intuitive understanding of the relationship between a structural shape and its inner stresses.

1.2.2 Lower Bound Theorem of Plasticity

The theory of plasticity brings an answer to the inconsistencies of elastic theory, considering the ductility of the material and its consequences on the redistribution of stresses. (6)

The two theorems of plastic design were theorized in 1936 by Gvosdev and Feinberg, which Greenberg and Prager proved in 1949. The lower bound theorem particularly suited for design purposes establishes the following: for an ideal plastic material, any limiting load obtained from a distribution of internal forces is less than or equal to the actual limiting load. As a result, a graphic static design is one possible solution to the static theorem. (7)

1.2.3 Strut and Tie Modelling

Thanks to the lower-bound theorem of plasticity, any continuous structural system made of a plastic material can be modeled as a strut-and-tie network. The strut-and-tie approaches were developed for the analysis of shear walls and structural details in concrete structures based on plastic theorems.

Strut-and-tie models are composed only of rods in compression – struts – or traction – ties – linked together in reticular systems with pin-jointed nodes on which point forces are applied. They can be used as generic abstractions for many types of structures. (8, 9)

1.2.4 Plastic Wood

Timber, as a lightweight and sustainable material, presents unique challenges and opportunities in structural design.

Plastic principles are used for structural dimensioning of timber in most structural standards. Stress-strain relationship of timber demonstrates clearly plastic capacities in compression, both parallel and perpendicular to fibers. In usual timber structures, redistributions capacities are reached through plastic behavior of timber in compressed member and steel connections. Even in that case, the application of the plastic theoretical framework requires a special attention that plastic capacities for redistributing forces develop in compression before tensile strength is exceeded.

Therefore, the assumption of an elastic plastic behavior of wood related to its orthotropic characteristics is usual for timber dimensioning and can be considered for a careful application of plastic theorems for structural analysis of timber structures.

1.3 Optimizing Structural Geometry

1.3.1 Isostatic Structure as a Design Strategy

Timber structures represent specific challenges. The optimum use of material can be found in timber structures avoiding bending forces or complex behaviors involving torsion or shear damaging (10). Only direct transmission of forces with compression or tension members maximizes structural efficiency and reduces the cost of wood joins.

In modelling strategies with FEM, models offer by default embedding characteristic in joints that radically modifies the local behavior, even in trussed systems. Bending-resistant metal connections can no longer be avoided since bending behaviors are activated and hence distorting stress distributions. There are huge consequences on the required degree of complexity of joints. Above this, in the case of complex structures of entire buildings, it is extremely common for the designer to be unable to find how to re-establish the hinging of the nodes of his model while maintaining its global structural equilibrium.

Beyond being less sensitive to settings and dimensional variation, isostatic hinged frames offer several advantages. As far as the structural system is designed isostatic – and is consistent with this from the theoretical point of view – redistributions of forces will not occur under loads that do not change in nature, unlike hyperstatic structures, allowing for direct plastic design. They can be analyzed geometrically only – or with static equilibrium equations – without needing to consider mechanical properties and a preliminary dimensioning. Isostatic structures require careful design of support conditions, non-redundancy and capacities to withstand all load conditions and help to understand and control the structural behavior.

For this, graphic statics – with all its advantages: visual representation, self-checking for structural equilibrium, interactivity – can be used. Isostatic structures require careful design of support conditions, non-redundancy and capacities to withstand all load conditions and help to understand and control the structural behavior. Graphic static designs, as reticular equilibrium that have not reached the level of a complete structural system, can reveal unstable in the face of certain load cases (11). If their design and construction are simpler, in case of overload or local failure, isostatic structures do not benefit from redundancy as part of a robustness strategy. Among the 16 strategies of robustness proposed by Knoll & Vogel (12, 13), five approaches only are focusing on structural design in itself (14). Among them strength, stiffness strategies and post-buckling strength fits to isostatic designs, whereas second line of defense and multiple load paths & redundancy are specific to hyperstatic systems. In practice however depending on the structural and nodes' dynamics, bending behavior and alternative load paths can still act as backup mechanisms.

1.3.2 Topology Optimization

Topology refers to the spatial organization and arrangement of a structure's components, regardless of their dimensions or material properties. It focuses on the connections between elements (columns, beams, etc.).

In the specific case of a truss structure, topology corresponds to the layout of nodes and the bars connecting them.

In isostatic systems, each loading case is connected to unique load path that needs to be designed. Topology plays a crucial role in optimizing the bracing of a truss structure made of bars and nodes subjected to horizontal wind loads. It involves identifying the most efficient paths to transmit loads to the supports. By determining critical areas where forces are concentrated, topology guides the optimal placement of bracing bars to minimize deformations and enhance stiffness.

By optimizing the distribution of bracing bars, it is possible to reduce their number or size in less stressed areas.

1.3.3 VGS – a Computational Tool for 3d Vector-based Graphic Statics

Numerical analysis approaches are not the best suited for the early stage of design since their way of working relies more on an analytical process and structural models requiring several hypotheses to give a result, resulting in a lack of interactivity.

However, the latest theoretical and practical developments of 3d Graphic statics opened new possibilities in terms of complex 3d structural typologies. In this, two main methods were developed, namely the vector-based (15) and the polyhedral-based ((16). Both approaches have their own specificities and benefits.

Vector-based Graphic Statics (VGS) is a plugin in the digital environment of Grasshopper whose main purpose is to automatically generate 3d vector-base interdependent form and force diagrams. The theoretical foundations are based on vector-based graphic statics and its implementation results in a computational tool (17).

The advantage of VGS lies in its geometric nature, as it exclusively manages trusses, reticular and strut-and-tie models without requiring material or section definitions. VGS allows users to manipulate equilibrated models, potentially verify deformations, and evaluate the utilization of structural elements.

Furthermore VGS is naturally suited for analysing and designing both joints and spatial bar networks in static equilibrium, making it particularly well-suited for timber structures that exhibit these characteristics.

2 Structural Explorations

Abstract. This chapter investigates the optimization of bracing systems for innovative timber structures using a computational tool based on Vector-based Graphic Statics. A research-by-design approach is applied to evaluate bracing strategies for a five-story timber structure exposed to wind loads. The structure is modeled as a tetragonal grid analyzed through various configurations including core, corner and façade bracing systems. Using VGS within Rhino & Grasshopper, the workflow integrates structural performance analysis with carbon footprint assessments, enabling iterative design refinement. The study underscores the potential of computational tools to achieve structurally efficient and environmentally responsible timber designs.

2.1 Research-by-Design

2.1.1 Case Study

The case study involves defining structural bracing strategies for a five-story building to resist wind forces through numerical simulations. It is discretized into a tetragonal grid which represents structural system of post-and-beam truss of equal spans which bracing will be managed with diagonal members or struts as commonly seen in various timber construction systems.

Drawing upon the theories of graphic statics, the plastic lower bound theorem and the strut and tie model, the experiments aim provide insights into the comparative structural performance of the bracing of a timber-based pavilion design.

By employing the computational optional tab Vector-based graphic statics (VGS) and complementary panels in the Rhino & Grasshopper digital environment, a comparative exploration of the bracing system performance is undertaken. The bracing members are assumed to be rigid bars, allowing them to be considered under both tension and compression.

The approach of isostatic models requires a sharp definition of the flow of forces and a clear vision on the structural behavior that both represents a decisive advantage for the structural reliability.

2.1.2 Assumptions

The structure has a rectangular plan consisting of three by five square grids, each six meters on each side. The building rises over five floors, each three meters high. The tetragonal grid system is analyzed under a series of bracing strategies. The bracing elements are composed of timber cores or facade lattice elements such as glulam frames. Key assumptions involve the uniformity of material properties (Glulam GL 28) and the simplification of joints as pinned connections. For environmental assessment, a minimum dimension of 10*10 cm is given to all the bars of the strut-and-tie model.

We take into account the acting forces resulting from a horizontal wind oriented Southwest/Northeast: a positive pressure of 1kN/m2 on the South and West faces and a negative pressure of 1kN/m2 on the 2 other adjacent faces. Neither live nor dead loads are considered.

For the dimensioning of the bracing systems, the following material properties of the glulam GL28 are considered:

• Modulus of Elasticity	10 500 000 kN/m2;
• Admissible compressive stress	16900 kN/m2;
• Admissible tensile stress	12 400 kN/m2;
• Characteristic resistance to bending	28 000 kN/m2.

2.1.3 Workflow

Although primarily an analysis tool, VGS is combined here with a module for section sizing and carbon footprint calculation. This integration transforms the framework into a design and preliminary sizing tool allowing for iterative exploration of bracing strategies. The capabilities are critical to identify bracing configurations that optimize the structural performance of the tetragonal grid system.

The framework allows the extraction of the characteristics of each variant of the structural bracing system: the material volume and consequently, the amount of CO2 equivalent per square meter.

2.1.4 Pedagocial framework

The design and analysis exercice for bracing systems presented in this article is part of the “Architectural Structures 3” course (UCLouvain – LOCI faculty). The pedagogical objective of this exercise was to introduce students to the principles of structural design and bracing system analysis through iterative research on form diagram and force evaluations.

These explorations enabled students to develop structural proposals, the results of which are classified and analyzed in the following section. It is important to note that this article does not replicate the exporatory processes involving form diagrams or force diagrams carried out by the students.

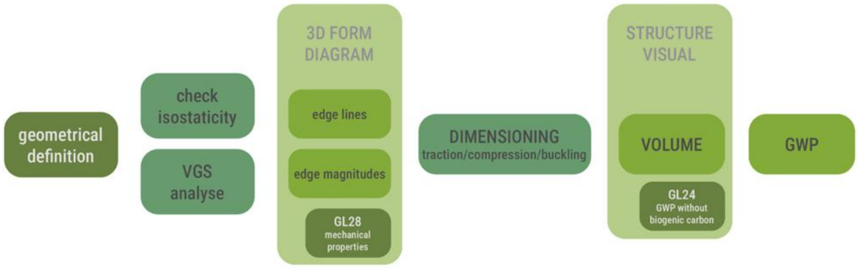

Fig. 2 Flow diagram of the algorithm used for the design of equilibrated and dimensioned structures in the Rhino & Grasshopper digital environment

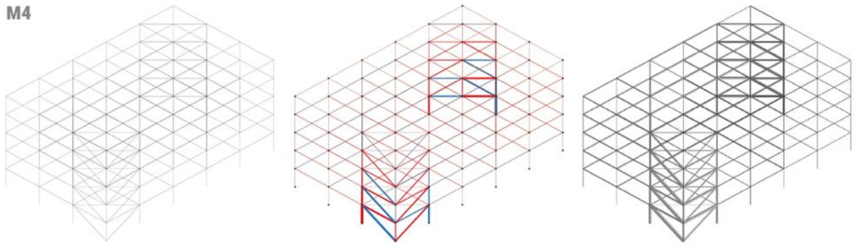

Fig. 3 Isometric views from the southwest of the model n°4: wireframe structure (left), the structure analyzed by VGS (center) and the structure dimensioned for compression, tension and buckling (right)

2.2 Comparative Explorations

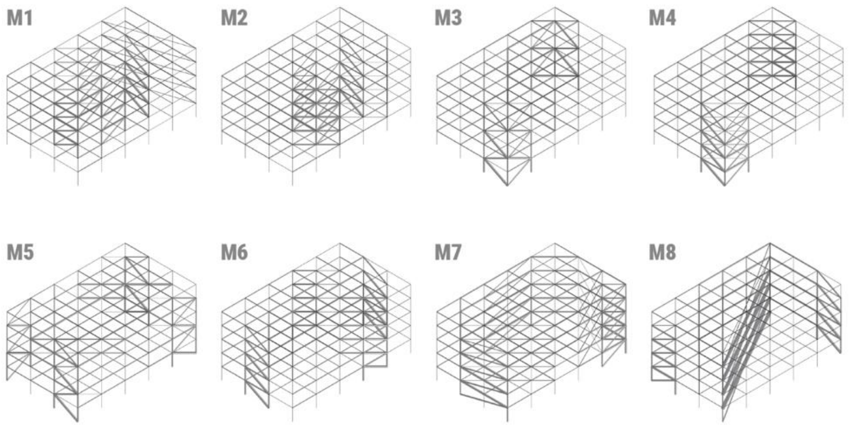

Fig. 4 Isometric views of the eight compared bracing systems

	BRACING SYSTEM	ANTHROPOGENIC GH. GAS KG CO2 EQ.	WOOD VOLUME M3
M1	OPEN CORE	1163	16,02
M2	CENTRAL CORE	1254	17,268
M3	CONTINUOUS CORNER	1311	18,059
M4	SEQUENTIAL CORNER	1330	18,314
M5	CONTINUOUS FACADE	1410	19,412
M6	SEQUENTIAL FACADE	1547	21,31
M7	SEQUENTIAL FACADE (DOUBLED)	1661	22,87
M8	HYBRID GLOBAL	3789	52,17

Table 1 Quantitative results from dimensioning. The volume of carbon emissions is calculated based on the carbon weight values of the wood, excluding biogenic carbon.

2.2.1 Corner Bracing System

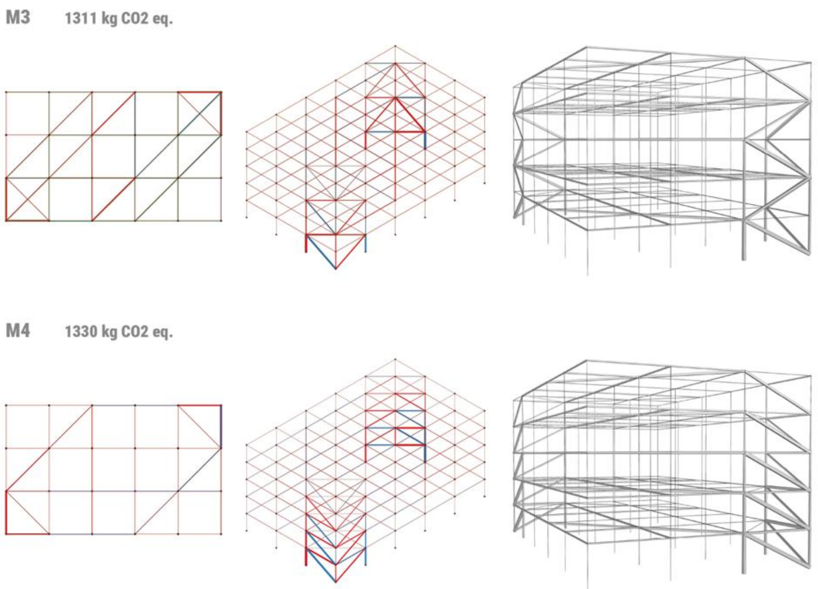

Fig. 5 Plan, isometric and two-point perspective views of the bracing models M3 (continuous corner) and M4 (sequential corner)

The comparison of these two models highlights the advantage of continuous bracing, which more directly balances the stresses. Logically, the sequential repetition of bracing at the corner generates greater horizontal forces, complicating the load path.

It is worth noting the higher number of horizontal bracings in the M3 model. The more uniform distribution of these bars also contributes to reducing the structural volume required.

Overall, both structures demonstrate remarkable efficiency due to the concentration of forces, which effectively stiffens the facades against the wind, allowing the loads to be transferred more quickly to the supports.

2.2.2 Core Bracing System

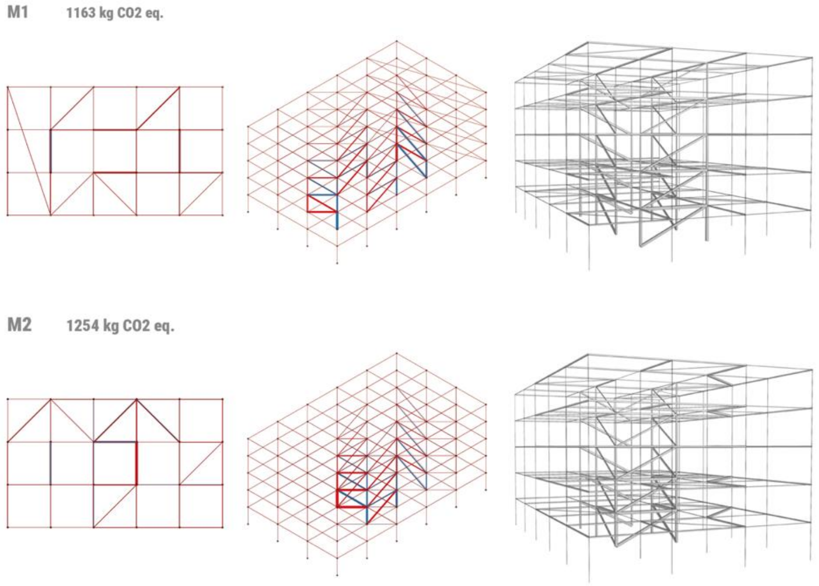

Fig. 6 Plan, isometric and two-point perspective views of the bracing models M1 (open core) and M2 (central core)

Considering supports evenly distributed beneath the 24 columns, the location of bracing at the center of the grid helps to equalize the stresses within the structure. Once again, this approach reduces the material required to stabilize the structure.

The increased number of loaded supports in the M1 model further optimizes the CO2 emissions generated by the structure by simplifying the load paths.

Similarly, the number of braced grids in the horizontal plane serves as an indicator of the energy efficiency of both structures.

2.2.3 Facade Bracing System

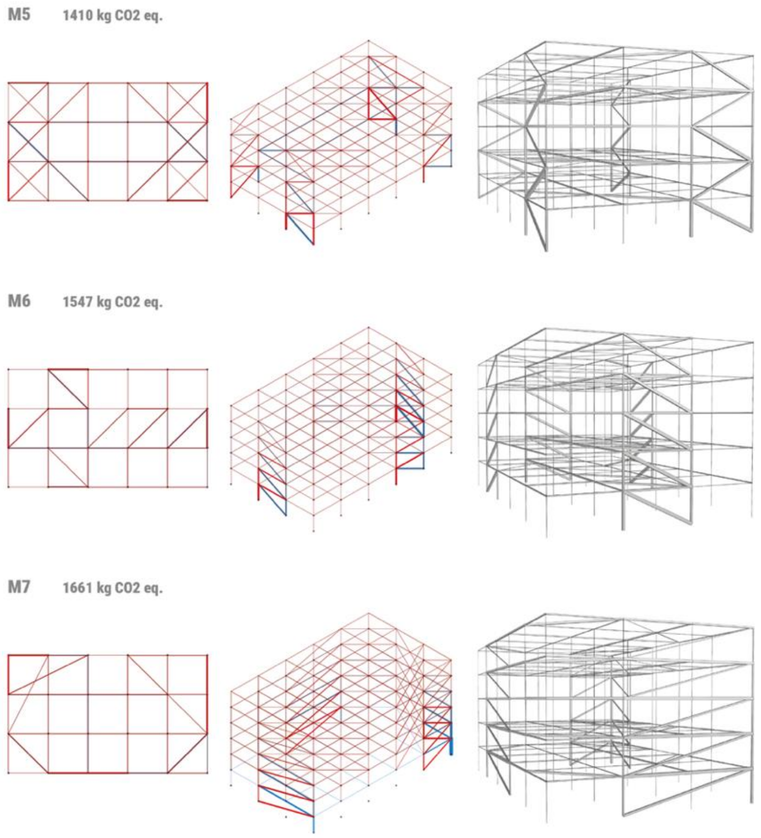

Fig. 7 Plan, isometric and two-point perspective views of the bracing models M5 (continuous façade), M6 (sequential façade) and M7 (sequential facade doubled)

This comparison of façade bracing models highlights the advantages of shorter bars over longer ones, which are sensitive to buckling. This is obvious in the M7 model which suffers from the resizing of its longer bars. On the one hand, the longer bars are resized due to their length and the forces they carry. On the other hand, the length of the bracing reduces the number of loaded supports, which undermines structural efficiency.

The M5 and M6 models differ in their bracing strategies: continuous bracing (M5) versus sequential bracing (M6). Once again, the continuity of bracing bars around the nodes ensures greater structural efficiency. This is further enhanced by the number of braced grids in the horizontal plane, helping to balance stresses throughout the structure.

3 Conclusion

Abstract. This chapter concludes by emphasizing the significant advantages of isostatic systems in timber construction, particularly their predictable load paths and compatibility with digital manufacturing techniques. Key insights into the behavior of isostatic tetragonal grid systems include the efficiency of shorter bars due to reduced buckling risk, the effectiveness of alternating in-plane floor bracing to uniform stress distribution, and the improved stability achieved by aligning bracing elements with corner supports. These findings highlight the potential to optimize wind bracing systems, minimizing material consumption and carbon emissions while enhancing structural efficiency. The integration of Vector-based Graphic Statics with section dimensioning and carbon footprint assessments demonstrates a streamlined and innovative approach for designing efficient, sustainable timber structures.

Isostatic systems present numerous advantages linked to their global behavior. Among them, the certainty of load paths and structural working opens a wide field of application in the direction of contact joints and digital manufacturing with timber. The above-mentioned exploration reveals several key insights into the behavior of isostatic tetragonal grid systems. First, shorter bars are consistently better than long bars which are more sensitive to buckling. Second, alternating in-plane floor bracing emerges as a highly effective strategy for achieving a uniform stress distribution within the structure. This configuration reduces localized stress concentrations, enhancing the overall resilience of the system. Third, the alignment of bracing elements toward support points, particularly those located at corners significantly improves structural stability. This finding underscores the importance of directing force flows to areas of maximum stiffness and support.

Optimizing wind bracing isostatic tetragonal grid systems for timber buildings presents a valuable opportunity to enhance structural efficiency while minimizing material consumption and carbon emissions. By integrating vector-based graphic statics (VGS) with principles from plasticity theory and topology optimization, this research demonstrates the potential for achieving optimized timber constructions.

This study highlights the crucial role of bracing strategies in governing the overall efficiency and stability of timber structures. Comparative analyses of various bracing configurations highlight the importance of continuous load paths. Configurations that ensure a direct and uniform load transfer demonstrate superior stiffness; reduced material use and lower embodied carbon.

The research-by-design approach supported by computational simulations proves to be an effective methodology for iterating design alternatives and refining bracing configurations.

By integrating VGS with section dimensioning and carbon footprint calculations, this framework extends beyond structural analysis, serving as a preliminary design tool for sustainable timber construction. Ultimately, this research contributes to the development of efficient structural systems, highlighting the importance of digital innovation for sustainability in contemporary architectural practice.

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